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BSM master formula for ε'/ε in the WET basis at NLO in QCD

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ABSTRACT: As an important step towards a complete next-to-leading (NLO) QCD analysis of the ratio ε'/ε within the Standard Model Effective Field Theory (SMEFT), we present for the first time the NLO master formula for the BSM part of this ratio expressed in terms of the Wilson coefficients of all contributing operators evaluated at the electroweak scale. To this end we use the common Weak Effective Theory (WET) basis (the so-called JMS basis) for which tree-level and one-loop matching to the SMEFT are already known. The relevant hadronic matrix elements of BSM operators at the electroweak scale are taken from Dual QCD approach and the SM ones from lattice QCD. It includes the renormalization group evolution and quark-flavour threshold effects at NLO in QCD from hadronic scales, at which these matrix elements have been calculated, to the electroweak scale.

KEYWORDS: Beyond Standard Model, CP violation, Kaon Physics

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1 Introduction

The ratio ε'/ε measures the size of the direct CP violation in $K_L \rightarrow \pi\pi$ decays (ε') relative to the indirect one (ε). It is very sensitive to new sources of CP violation. As such it played a prominent role in particle physics already for 45 years [1]. However, due to large hadronic uncertainties in the evaluation of ε'/ε no consensus between theorists has been reached on its value within the Standard Model (SM). Even 20 years after its measurements from NA48 [2] and KTeV [3, 4] collaborations implying the world average

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}, \quad (1.1)$$

we do not know how much room is left for new physics (NP) contributions to this ratio.

Within the SM ε'/ε is governed by the difference between QCD penguin (QCDP) and electroweak penguin (EWP) contributions. Interestingly last year a consensus between the lattice QCD (LQCD) RBC-UKQCD collaboration [5] and the Dual QCD (DQCD) estimate [6] of the EWP contribution to ε'/ε in the SM has been reached with the result

$$(\varepsilon'/\varepsilon)_{\text{SM}}^{\text{EWP}} = -(7 \pm 1) \times 10^{-4}, \quad (\text{LQCD and DQCD}). \quad (1.2)$$

The estimate of this contribution from Chiral Perturbation Theory (ChPT), having the same sign, is by roughly a factor of two lower in magnitude [7], see [8] for latest developments, but in view of large uncertainties consistent with (1.2).

On the other hand there is no consensus on the size of the QCDP contribution, which originates on the one hand in different estimates of the hadronic matrix elements of QCDP operators and on the other hand in the estimate of isospin-breaking effects that suppress the QCDP contributions. While the RBC-UKQCD collaboration did not provide an estimate of the isospin-breaking effects until now, the suppression of QCDP by them provided by ChPT [9] amounts to roughly 15%, while the advocates of DQCD find the suppression as large as 30% [10].

Adding isospin-breaking effects from [10] to the RBC-UKQCD results for QCDP hadronic matrix elements [5] one finds [10, 11]

$$(\varepsilon'/\varepsilon)_{\text{SM}} = (13.9 \pm 5.2) \times 10^{-4}. \quad (1.3)$$

A similar result is found using ChPT [7], but as discussed in [1] it is a numerical coincidence in view of the smaller estimate of EWP contributions by these authors relative to LQCD, and a different estimate of isospin-breaking corrections. In any case, while this estimate is fully compatible with the experimental value in (1.1), the large uncertainty related primarily to QCDP contributions does not exclude the modifications of ε'/ε by NP as large as 10^{-3} . In fact as argued on the basis of the DQCD approach [6] in [1, 12] the values of ε'/ε in the SM in the ballpark of 5×10^{-4} are still possible.

This situation motivated various authors already for many years to perform analyses of ε'/ε in various extensions of the SM with the goal to identify which models could allow for significant contributions to ε'/ε taking existing constraints from other processes into account. They are listed in [13]. However, one could question the usefulness of these studies in view of very large uncertainties in the estimate of the SM value of ε'/ε and thereby of the room left for NP contributions. In particular one could question the usefulness of sophisticated NLO calculations of Wilson coefficients of various operators within extensions of the SM at present that are multiplied by very uncertain hadronic matrix elements.

Here comes a good news. We have seen that the EWP contribution could be determined within the SM with a respectable precision already now. Being dominated by the $\Delta I = 3/2$ transitions and contributing dominantly to the isospin amplitude A_2 , its contribution to ε'/ε turns out to be automatically enhanced by a factor of 22.4 relative to the QCDP contribution that dominates the A_0 amplitude. The only reason why in the SM the QCDP contribution is, despite of this strong suppression relative to A_2 , larger in magnitude than the EWP contribution is that at scales relevant to the $K_L \rightarrow \pi\pi$ decays the value of α_s is by roughly a factor of 50 larger than the one of α_{em} . Consequently the Wilson coefficients of QCDP operators are much larger than those of EWP operators and this implies a positive sign of ε'/ε as opposed to the result in (1.2).

While one cannot exclude that such a pattern of different contributions is present in some of the NP scenarios, in most of the BSM analyses to date the contributions of NP to the imaginary part of A_2 were by far dominant. It could be then that BSM contributions to ε'/ε , as far as hadronic uncertainties are concerned, can be more precisely calculated than the SM contribution. Therefore, as proposed in [14], it is useful to write ε'/ε as a sum of the SM and BSM contributions,

$$\frac{\varepsilon'}{\varepsilon} = \left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} + \left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{BSM}}, \quad (1.4)$$

and concentrate on the BSM one, until an improved estimate of the SM contribution will be available.

Now, the novelty of the BSM contributions to ε'/ε is the presence of new operators, absent or strongly suppressed in the SM. The hadronic matrix elements of all BSM four-quark operators have been calculated to date only within DQCD in [15] and it will still take some time before we know LQCD estimates for them. But already this estimate allowed to derive a master formula for ε'/ε in [16]. Knowing the Wilson coefficients in any BSM scenario at the electroweak scale, this formula provides the BSM contribution to ε'/ε . This then allowed to perform for the first time a SMEFT anatomy of ε'/ε [17], demonstrating the possible interplay of NP contributions to ε'/ε with BSM effects in other observables.

So far the master formula for ε'/ε within WET in [16] is based on LO RG evolution from the low-energy scale μ_{had} to the electroweak scale μ_{ew} , at which the Wilson coefficients (WCs) are obtained from a UV completion or SMEFT. However, beyond the LO, the RG evolution from μ_{had} to μ_{ew} is known only in the so-called *BMU basis* [18], which is less suitable for the matching to the SMEFT. Moreover, for this matching the most convenient WET basis is the one introduced in [19], the so-called *JMS basis*.¹ In particular, for this basis the tree-level matching of SMEFT on to WET [19] and the corresponding one-loop matching [21, 22] are known. Therefore it is evident that a master formula for ε'/ε written in terms of WCs in the JMS basis would be more useful for BSM analyses than the one that uses BMU basis at the electroweak scale. Moreover, for future combination of WCs with hadronic matrix elements from LQCD it is mandatory to calculate the RG evolution at the NLO level.

With this goal in mind we have recently calculated, including NLO QCD corrections, the RG evolution matrix $\hat{U}_{\text{BMU}}(\mu_{\text{had}}, \mu_{\text{ew}})$ [23] that allows to express the WCs $\vec{C}_{\text{BMU}}(\mu_{\text{had}})$ at the hadronic scale in the BMU basis in terms of the $\vec{C}_{\text{JMS}}(\mu_{\text{ew}})$ in the JMS basis at the electroweak scale as follows

$$\vec{C}_{\text{BMU}}(\mu_{\text{had}}) = \hat{U}_{\text{BMU}}(\mu_{\text{had}}, \mu_{\text{ew}}) \hat{M}_{\text{JMS}}(\mu_{\text{ew}}) \vec{C}_{\text{JMS}}(\mu_{\text{ew}}). \quad (1.5)$$

To this end the RG evolution from μ_{had} to μ_{ew} has been performed in the BMU basis, including quark-flavour threshold crossings at NLO in QCD, and subsequently the resulting $\vec{C}_{\text{BMU}}(\mu_{\text{ew}})$ in the BMU basis have been transformed to $\vec{C}_{\text{JMS}}(\mu_{\text{ew}})$ in the JMS basis at the one-loop level. This required a careful treatment of the evanescent operators, encoded in the matrix $\hat{M}_{\text{JMS}}(\mu_{\text{ew}})$.

Having the matrices $\hat{U}_{\text{BMU}}$ and $\hat{M}_{\text{JMS}}$ at our disposal, the main result of our work is a new master formula for $(\varepsilon'/\varepsilon)_{\text{BSM}}$, which is superior to the one in [11, 16, 17] in two ways

- It includes NLO QCD RG evolution below the electroweak scale. Further, also the LO QED and NLO QED $\times$ QCD evolution is known, and can be applied if needed [23].
- It is expressed in terms of the WCs evaluated at the electroweak scale in the JMS basis, thereby allowing with the help of the results in [21] to express it eventually in terms of the SMEFT WCs including one-loop corrections in the matching of the SMEFT on to WET. As such it can be readily used for all UV completions that reduce to SMEFT in their infra-red regime.

¹We use here the **WCxf** convention for the JMS basis, defined in [20].

Optimally for model building it would be useful to have a master formula for ε'/ε expressed in terms of WCs in the Warsaw SMEFT basis that are evaluated at the NP scale as we have done for $\Delta F = 2$ transitions in [24]. We will present such a formula in a future publication.

As the two items above have been accomplished in [23], the main goal of the present paper is the evaluation of all hadronic matrix elements for $I = 0$ and $I = 2$ in the BMU basis at μ_{had} using the results of [15] obtained in the DQCD approach. Combining them with (1.5), we will be able to present a new master formula for ε'/ε with the virtues listed above.

Our paper is organized as follows. In section 2 we present the general expression for the updated master formula for ε'/ε . In section 3 we present the numerical values of the relevant hadronic matrix elements and in section 4 the values of the coefficients $P_b(\mu_{\text{ew}})$ for the 40 WCs in the JMS basis will be presented. The summary of our results and an outlook are presented in section 5. The definitions of the JMS and BMU operator bases are given in appendix A and appendix B, respectively.

2 Master formula for ε'/ε

The general formula for ε'/ε reads [25]

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}|\varepsilon_K|} \left[\frac{\text{Im } A_0}{\text{Re } A_0} (1 - \hat{\Omega}_{\text{eff}}) - \frac{1}{a} \frac{\text{Im } A_2}{\text{Re } A_2} \right], \quad (2.1)$$

with ω_+ , a and $\hat{\Omega}_{\text{eff}}$ given as follows

$$\omega_+ = a \frac{\text{Re } A_2}{\text{Re } A_0} = (4.53 \pm 0.02) \times 10^{-2}, \quad a = 1.017, \quad \hat{\Omega}_{\text{eff}} = (29 \pm 7) \times 10^{-2}. \quad (2.2)$$

Here a and $\hat{\Omega}_{\text{eff}}$ summarize isospin-breaking corrections and include strong isospin violation ($m_u \neq m_d$), the correction to the isospin limit coming from $\Delta I = 5/2$ transitions and electromagnetic corrections [26–28]. The value for $\hat{\Omega}_{\text{eff}}$ quoted above is the most recent one from DQCD approach that includes explicitly the effect of $\eta - \eta'$ mixing [10]. Within ChPT η' does not appear as an explicit degree of freedom, but the effect of $\eta - \eta'$ mixing is included through the appropriate low energy constant L_7 . The present uncertain value of L_7 does not allow for an accurate estimate of this effect resulting in the value $\hat{\Omega}_{\text{eff}} = (17 \pm 9) \times 10^{-2}$ [9] that within the errors is consistent with the one from DQCD. It would be desirable to make a better estimate of this effect in the future.

In the SM $\text{Im } A_0$ receives dominantly contributions from QCDs, but also from EWP. On the other hand, $\text{Im } A_2$ receives contributions exclusively from EWPs. However, in BSM scenarios they are both affected by contributions of BSM operators and also the WCs of SM operators are modified. The amplitudes $\text{Re } A_{0,2}$ are extracted from the branching ratios of $K \rightarrow \pi\pi$ decays in the isospin limit. Their values are given by

$$\text{Re } A_0 = 27.04(1) \times 10^{-8} \text{ GeV}, \quad \text{Re } A_2 = 1.210(2) \times 10^{-8} \text{ GeV}. \quad (2.3)$$

The main reason for taking these amplitudes from the data is that we do not know presently the BSM contributions to them and proceeding in this manner one automatically includes both SM and BSM contributions.

For the discussion of the BSM contributions with the WCs normalized at the electroweak scale we define

$$\mathcal{H}_{\Delta S=1} \equiv - \sum_b \left[C_b(\mu_{\text{ew}}) Q_b + C'_b(\mu_{\text{ew}}) Q'_b \right], \quad (2.4)$$

with the operators Q_b and Q'_b given in the JMS basis as defined in appendix A and the ordering of operators in VLL, VLR and SRR sectors as given in [23]. The chirality-flipped operators Q'_b in the sectors VRR, VRL and SLL are obtained from Q_b by interchanging the chirality-projectors $P_L \leftrightarrow P_R$. The relation of $C'_a(\mu_{\text{had}})$ in the BMU basis to $C'_b(\mu_{\text{ew}})$ in the JMS basis is given in (1.5).

The updated master formula [16] for the BSM part in (1.4) reads

$$\left(\frac{\varepsilon'}{\varepsilon} \right)_{\text{BSM}} = \sum_b P_b(\mu_{\text{ew}}) \text{Im} [C_b(\mu_{\text{ew}}) - C'_b(\mu_{\text{ew}})] \times (1 \text{ TeV})^2, \quad (2.5)$$

where

$$P_b(\mu_{\text{ew}}) = \sum_a \sum_{I=0,2} p_{ba}^{(I)}(\mu_{\text{ew}}, \mu_{\text{had}}) \left[\frac{\langle Q_a(\mu_{\text{had}}) \rangle_I}{\text{GeV}^3} \right], \quad (2.6)$$

with the sum over b running over the Wilson coefficients C_b of all operators in the JMS basis and their chirality-flipped counterparts denoted by C'_b . The relative minus sign accounts for the fact that their $K \rightarrow \pi\pi$ matrix elements differ by a sign. Among the contributing operators are also operators present already in the SM, but their WCs in (2.5) are meant to include only BSM contributions. The list of all contributing operators in the JMS basis is given in appendix A and the one for the BMU basis in appendix B. We mention further that the choice of the JMS basis in [23] contains hermitian conjugated operators $[Q_{uddu}^{V1,LR}]^\dagger_{1211}$, $[Q_{uddu}^{V8,LR}]^\dagger_{1211}$, $[Q_{uddu}^{V1,LR}]^\dagger_{2212}$ and $[Q_{uddu}^{V8,LR}]^\dagger_{2212}$ in (2.4), such that actually their complex-conjugated Wilson coefficients enter ε'/ε , but using $\text{Im} C_b(\mu_{\text{ew}})^* = -\text{Im} C_b(\mu_{\text{ew}})$, in (2.5) the minus sign is absorbed into P_b .

It should be emphasized that the BMU operators

$$Q_i, \quad i = 19 - 22, \quad i = 25 - 28, \quad i = 37 - 40, \quad (2.7)$$

can not be generated in the tree-level matching from SMEFT operators, because they do not conserve hypercharge.

Comparing (2.5) with (2.1) and taking the minus sign in (2.4) into account we have²

$$p_{ba}^{(0)}(\mu_{\text{ew}}, \mu_{\text{had}}) = \frac{\omega_+}{\sqrt{2} |\varepsilon_K|} \left[\frac{10^{-6} \text{ GeV}}{\text{Re } A_0} \right] (1 - \hat{\Omega}_{\text{eff}}) [\hat{U}_{\text{BMU}}(\mu_{\text{had}}, \mu_{\text{ew}}) \hat{M}_{\text{JMS}}(\mu_{\text{ew}})]_{ab}, \quad (2.8)$$

$$p_{ba}^{(2)}(\mu_{\text{ew}}, \mu_{\text{had}}) = - \frac{\omega_+}{\sqrt{2} a |\varepsilon_K|} \left[\frac{10^{-6} \text{ GeV}}{\text{Re } A_2} \right] [\hat{U}_{\text{BMU}}(\mu_{\text{had}}, \mu_{\text{ew}}) \hat{M}_{\text{JMS}}(\mu_{\text{ew}})]_{ab}, \quad (2.9)$$

²Note that on the l.h.s. the indices are ba , while on the r.h.s. ab .

leading to

$$p_{ba}^{(0)}(\mu_{\text{ew}}, \mu_{\text{had}}) = -a(1 - \hat{\Omega}_{\text{eff}}) \frac{\text{Re } A_2}{\text{Re } A_0} p_{ba}^{(2)}(\mu_{\text{ew}}, \mu_{\text{had}}) \quad (2.10)$$

and therefore implying a strong suppression of $I = 0$ coefficients relative to the $I = 2$ ones by $\text{Re } A_2/\text{Re } A_0 \approx 1/22$. On the other hand, this suppression is partially lifted because in most cases $\langle Q_a \rangle_2 < \langle Q_a \rangle_0$.

So far only four-quark operators have been considered, however also the chromo- and electromagnetic dipole operators (Q_{8g} and $Q_{7\gamma}$ respectively) contribute to ε'/ε in principle. Whereas the electromagnetic dipole operator has been neglected, the Q_{8g} was included in the previous LO master formula [11, 16, 17]. Also in this case an improvement towards NLO QCD evolution would be desirable, but here the corresponding anomalous dimension matrices for the mixing of the four-quark operators into Q_{8g} are not yet available, except the NLO mixing of Q_{8g} and $Q_{7\gamma}$ [29]. In view of this, we include contributions that are proportional to $\langle Q_{8g} \rangle_I$ at LO only, based on the results of [17]. The involved operators belong to the three sectors SRR, s, c, b (called class B in [16]) and the sectors SRR, u (class C) and SRR, d (class D), which are part of the BMU basis. Eventually it remains to transform their Wilson coefficients at μ_{ew} to the JMS basis, where we use the LO transformations given in [23], see also appendix D. In the JMS and BMU bases the chromomagnetic dipole operator are defined as

$$[Q_{dG}]_{21} = [\bar{s}\sigma^{\mu\nu} P_R T^A d] G_{\mu\nu}^a, \quad (\text{JMS}) \quad (2.11)$$

$$Q'_{8g} = m_s [\bar{s}\sigma^{\mu\nu} P_R T^A d] G_{\mu\nu}^a, \quad (\text{BMU}) \quad (2.12)$$

respectively, which differ by a factor of the strange quark mass (in the $\overline{\text{MS}}$ scheme). The corresponding chirality-flipped operator in the JMS basis is $[Q_{dG}]_{12}^\dagger$. Class B operators contribute only via mixing into $Q_{8g}^{(\prime)}$ operators, hence in (2.6)

$$P_b = P_b^{\text{dipole}}. \quad (\text{Class B}) \quad (2.13)$$

The class C and D operators have non-vanishing $K \rightarrow \pi\pi$ matrix elements of their own, resulting in a contribution due to four-quark operators and due to mixing a contribution from the dipole operator

$$P_b = P_b^{\text{four-quark}} + P_b^{\text{dipole}}, \quad (\text{Class C and D}) \quad (2.14)$$

where the $P_b^{\text{four-quark}}$ are updated below at NLO in QCD. On the other hand the P_b^{dipole} can be found at LO for classes C and D from tables 8 and 9 [17], respectively. As can be seen there, in practice $P_b^{\text{dipole}} \ll P_b^{\text{four-quark}}$, such that within theory uncertainties one can neglect for class C and D operators the P_b^{dipole} due to mixing into Q_{8g} .

3 Hadronic matrix elements

The calculation of the coefficients $P_b(\mu_{\text{ew}})$ requires the knowledge of the hadronic matrix elements

$$\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I | Q_i | K \rangle(\mu) \quad I = 0, 2. \quad (3.1)$$

MEs	$[C_{ud}^{V1,LL}]_{1121}$	$[C_{ud}^{V8,LL}]_{1121}$	$[C_{ud}^{V1,LL}]_{2221}$	$[C_{ud}^{V8,LL}]_{2221}$	$[C_{dd}^{VLL}]_{2133}$	$[C_{dd}^{VLL}]_{2331}$	$[C_{dd}^{VLL}]_{2111}$	$[C_{dd}^{VLL}]_{2122}$
Q_1	-1494.8	554.0	1494.8	-554.0	—	—	—	—
Q_2	609.7	-849.0	-609.7	849.0	—	—	—	—
Q_3	21.6	-23.0	-476.8	161.6	-282.5	1092.4	-368.8	-368.8
Q_4	-24.4	44.9	179.0	-238.1	286.9	-1090.6	114.6	114.6
Q_9	—	—	-996.5	369.3	291.8	-1111.2	352.4	352.4
Q_{10}	—	—	406.5	-566.0	-291.8	1111.2	-57.3	-57.3
Q_{14}	—	—	—	—	—	—	-442.5	442.5
Q_5	-6.7	-4.1	-6.3	-5.1	-5.5	3.8	-18.0	-18.0
Q_6	-32.2	68.7	-31.1	66.9	-13.5	65.1	90.2	90.2

MEs	$[C_{du}^{V1,LR}]_{2111}$	$[C_{du}^{V8,LR}]_{2111}$	$[C_{du}^{V1,LR}]_{2122}$	$[C_{du}^{V8,LR}]_{2122}$	$[C_{dd}^{V1,LR}]_{2133}$	$[C_{dd}^{V8,LR}]_{2133}$	$[C_{dd}^{V1,LR}]_{2111}$	$[C_{dd}^{V8,LR}]_{2111}$
Q_3	-19.4	-27.1	-26.2	-39.2	-10.3	-19.2	-19.4	-27.1
Q_4	18.1	49.9	38.4	86.1	10.3	32.7	18.1	49.9
Q_5	-345.6	3.8	1.1	-17.3	3.4	-2.9	-345.6	3.8
Q_6	-363.9	-605.1	52.7	135.0	19.2	71.0	-363.9	-605.1
Q_7	-707.0	18.0	—	—	—	—	353.5	-9.0
Q_8	-792.5	-1407.8	—	—	—	—	396.2	703.9
Q_{15}	—	—	—	—	—	—	-594.4	-1055.8
Q_{16}	—	—	—	—	—	—	-530.2	13.5

MEs	$[C_{dd}^{V1,LR}]_{2122}$	$[C_{dd}^{V8,LR}]_{2122}$	$[C_{uddu}^{V1,LR}]_{1211}$	$[C_{uddu}^{V8,LR}]_{1211}$	$[C_{uddu}^{V1,LR}]_{2212}$	$[C_{uddu}^{V8,LR}]_{2212}$	$[C_{dd}^{V1,LR}]_{2331}$	$[C_{dd}^{V8,LR}]_{2331}$
Q_3	-19.4	-27.1	—	—	—	—	—	—
Q_4	18.1	49.9	—	—	—	—	—	—
Q_5	-345.6	3.8	—	—	—	—	—	—
Q_6	-363.9	-605.1	—	—	—	—	—	—
Q_7	353.5	-9.0	—	—	—	—	—	—
Q_8	396.2	703.9	—	—	—	—	—	—
Q_{15}	594.4	1055.8	—	—	—	—	—	—
Q_{16}	530.2	-13.5	—	—	—	—	—	—
Q_{19}	—	—	2120.9	-53.9	—	—	—	—
Q_{20}	—	—	2377.4	4223.4	—	—	—	—

MEs	$[C_{dd}^{S1,RR}]_{2111}$	$[C_{dd}^{S8,RR}]_{2111}$	$[C_{dd}^{S1,RR}]_{2122}$	$[C_{dd}^{S8,RR}]_{2122}$	$[C_{ud}^{S1,RR}]_{1121}$	$[C_{ud}^{S8,RR}]_{1121}$	$[C_{uddu}^{S1,RR}]_{1121}$	$[C_{uddu}^{S8,RR}]_{1121}$
Q_{25}	-2723.9	922.2	—	—	—	—	—	—
Q_{26}	-12.4	30.3	—	—	—	—	—	—
Q_{27}	—	—	-2723.9	922.2	—	—	—	—
Q_{28}	—	—	-12.4	30.3	—	—	—	—
Q_{29}	—	—	—	—	-695.0	-693.0	2176.2	55.0
Q_{30}	—	—	—	—	-5763.7	-80.6	3393.1	927.7
Q_{31}	—	—	—	—	-403.0	-89.8	733.3	-2.8
Q_{32}	—	—	—	—	111.0	-0.9	-138.3	37.9

Table 1. The values for $p_{ba}^{(2)}(\mu_{\text{ew}}, \mu_{\text{had}})$ (see (2.9) for definition) are shown. Here the first index a refers to the matrix elements in the BMU basis as shown in the left most column and b is for the Wilson coefficients in the JMS basis shown in the top row of the table. The corresponding $p_{ba}^{(0)}$ can be obtained through the relation (2.10).

The number of $K \rightarrow \pi\pi$ matrix elements of four-quark operators in the $N_f = 3$ quark-flavour theory has been discussed in full generality in [17]. Here we recall the most important points:

- The matrix elements of the chirality-flipped operators are related to the non-flipped ones due to parity conservation as $\langle Q'_i \rangle_I = -\langle Q_i \rangle_I$, thereby reducing the number of independent matrix elements by a factor of two.
- We assume that operators with quark-flavour content $(\bar{s}d)(\bar{s}s)$ are strongly suppressed and neglect them, see explanations in [17]. In practice this concerns only the BSM operators, because for matrix elements of SM operators the LQCD calculations include these contributions.
- For the operators with quark-flavour content $(\bar{s}d)(\bar{u}u)$ and $(\bar{s}d)(\bar{d}d)$ there are 15_0 and 8_2 matrix elements for $I = 0$ and $I = 2$, respectively, when using exact or nearly exact symmetries of parity and isospin. Out of these, 7_0 and 3_2 are known from the SM operators from LQCD calculations [5, 30]. The remaining 8_0 and 2_2 matrix elements of the BSM operators are available from the DQCD calculation [15].

As already stressed in the previous section it is useful to perform this calculation in the BMU operator basis because in this basis the RG evolution is available at NLO in QCD, contrary to the JMS basis. The SM operators are part of the BMU basis and their matrix elements from the LQCD calculations [5] for $I = 0$ and [30] for $I = 2$ can be used after RG evolution from the scales $\mu_0 = 4.006 \text{ GeV}$ and $\mu_2 = 3.0 \text{ GeV}$, respectively, to the scale $\mu = 1.3 \text{ GeV}$ that we will use here. The results and details of the RG evolution are given in [11].³

The matrix elements of the BSM operators from the DQCD calculation [15] are given in the BMU basis, with the exception of the SRR, d sector shown in appendix C, at the scale $\mu_{\text{had}} = 1.0 \text{ GeV}$. The results after the RG evolution to $\mu = 1.3 \text{ GeV}$ are provided in [11]. In the case of the SRR, d sector Fierz identities have to be used in order to relate BMU operators to the ones used in [15]. To this end we will use tree-level relations and thereby neglect possible effects of evanescent operators required for a consistent treatment at NLO in QCD, simply because the present accuracy of the matrix elements obtained in the DQCD approach is at best at the level of 10%, such that effects at NLO in QCD can be neglected.⁴ However, we kept in [23] the NLO corrections in the evaluation of the RG evolution contained in $p_{ba}^{(I)}(\mu_{\text{ew}}, \mu_{\text{had}})$. Indeed, anticipating more accurate future results from LQCD that should be performed directly in the BMU basis, such effects can be included consistently one day.

Working then for $N_f = 3$, but neglecting $(\bar{s}s)$ contributions, most BSM operators in the BMU basis coincide with the ones in the basis of [15] and hence have the same matrix

³As explained in [11], the perturbative RG flow contains isospin-breaking corrections from quark-charges and leads to deviations from the isospin limits of the matrix elements after evolution.

⁴In particular, while the scale dependence of the BSM matrix elements obtained in [15] matches well the one of Wilson coefficients, the inclusion of the renormalization scheme dependences beyond the LO would require a more advanced calculation that was performed in DQCD for SM operators in [31].

Op.	$\langle Q_i \rangle_0$	$\langle Q_i \rangle_2$	Op.	$\langle Q_i \rangle_0$	$\langle Q_i \rangle_2$
Q_1	-0.0163(43)(25)	0.0021(1)(1)	Q_{15}	-0.3708(92)(448)	-0.1138(32)(68)
Q_2	0.0217(32)(35)	0.0021(1)(1)	Q_{16}	-0.0562(92)(65)	-0.0165(11)(5)
Q_3	-0.0187(143)(30)		Q_{19}	0.0150(30)	-0.0030(6)
Q_4	0.0232(130)(37)		Q_{20}	0.1410(280)	-0.0500(100)
Q_9	-0.0152(43)(22)	0.0032(1)(2)	Q_{25}	0.0880(180)	-0.0310(60)
Q_{10}	0.0230(45)(35)	0.0032(1)(2)	Q_{26}	-0.2960(729)	0.1080(400)
Q_{14}	-0.0025(149)(39)	-0.0021(1)(1)	Q_{29}	0.0050(10)	0.0030(6)
Q_5	-0.0302(132)(47)		Q_{30}	0.0440(90)	0.0310(60)
Q_6	-0.1610(115)(253)		Q_{31}	-0.3710(740)	-0.2620(520)
Q_7	0.0540(40)(85)	0.0247(17)(8)	Q_{32}	-0.2140(430)	-0.1510(300)
Q_8	0.3952(75)(622)	0.1708(47)(103)			

Table 2. The matrix elements $\langle Q_i \rangle_{0,2}$ [GeV³] for $N_f = 3$ BMU basis at scale $\mu = 1.3$ GeV are shown. The statistical and systematic uncertainties are given for results from lattice calculations $Q_{1,\dots,16}$, whereas only parametric uncertainties for results from DQCD calculations $Q_{19,\dots,32}$.

elements.⁵ In consequence they can be directly obtained from the DQCD results in [15] at $\mu = 1.3$ GeV with appropriate RG evolution. These BSM operators in the BMU basis are

$$Q_i, \quad i = 14, 15, 16, 19, 20, 25, 26, 29, 30, 31, 32. \quad (3.2)$$

The operators $Q_{14,15,16}$ become linearly dependent on the SM operators when neglecting the $(\bar{s}s)$ contributions and we can obtain their matrix elements in this approximation as follows:

$$\langle Q_{14} \rangle_I \simeq \langle Q_3 - Q_1 \rangle_I, \quad \langle Q_{15} \rangle_I \simeq \frac{2}{3} \langle Q_6 - Q_8 \rangle_I, \quad \langle Q_{16} \rangle_I \simeq \frac{2}{3} \langle Q_5 - Q_7 \rangle_I, \quad (3.3)$$

from known LQCD results.⁶ This leaves us with only 8 BSM operators ($Q_{19,20}, Q_{25,26}, Q_{29-32}$), that can be mapped onto the SD basis in [15]. Among these, the only BSM operator in the BMU basis for which a Fierz transformation is required is Q_{26} , see (C.1). The complete basis transformation of the BMU operators to the SD basis of [15] is given in appendix C and allows to determine the $I = 0, 2$ matrix elements of these operators at the input scale $\mu_{\text{had}} = 1.0$ GeV.

We collect the values of hadronic matrix elements in table 2. We notice in particular that the matrix elements of the operators Q_i with $i = 6, 8, 15, 20, 26, 31, 32$ have the largest values. These are the VLR, scalar and tensor operators. In comparing the values of the matrix elements of the SM operators with those present in the literature one should take into account that our SM operators are defined with $P_{L,R}$ instead of traditional $V \mp A$, which brings in a factor of four: $\langle Q_i^{\text{our}} \rangle = \langle Q_i^{\text{trad}} \rangle / 4$. The chromomagnetic dipole operator has the nonvanishing matrix element $\langle Q'_{8g} \rangle_0 = +0.013(4) \text{ GeV}^3$ [32, 33].

⁵Equal up to $P_L \leftrightarrow P_R$, which implies the flip of sign in $K \rightarrow \pi\pi$ matrix elements, or differing by a sign due to different definitions of tensor operators.

⁶Based on the assumption that the $(\bar{s}s)$ contribution is strongly suppressed.

$C_b(\mu_{\text{ew}})$	$P_b(\mu_{\text{ew}})$		$C_b(\mu_{\text{ew}})$	$P_b(\mu_{\text{ew}})$	
	NLO	LO		NLO	LO
$[C_{ud}^{V1,LL}]_{1121}$	−3.47	−3.34	$[C_{ud}^{V8,LL}]_{1121}$	0.77	0.63
$[C_{ud}^{V1,LL}]_{2221}$	−0.21	−0.13	$[C_{ud}^{V8,LL}]_{2221}$	0.39	0.28
$[C_{dd}^{VLL}]_{2133}$	−0.12	−0.08	$[C_{dd}^{VLL}]_{2331}$	0.53	0.46
$[C_{dd}^{VLL}]_{2111}$	2.26	2.15	$[C_{dd}^{VLL}]_{2122}$	0.46	0.36
$[C_{du}^{V1,LR}]_{2111}$	−142.16	−108.08	$[C_{du}^{V8,LR}]_{2111}$	−222.69	−174.02
$[C_{du}^{V1,LR}]_{2122}$	0.27	0.10	$[C_{du}^{V8,LR}]_{2122}$	0.70	0.33
$[C_{dd}^{V1,LR}]_{2133}$	0.10	0.04	$[C_{dd}^{V8,LR}]_{2133}$	0.39	0.20
$[C_{dd}^{V1,LR}]_{2111}$	134.07	101.82	$[C_{dd}^{V8,LR}]_{2111}$	210.94	164.71
$[C_{dd}^{V1,LR}]_{2122}$	0.18	0.10	$[C_{dd}^{V8,LR}]_{2122}$	0.53	0.33
$[C_{uddu}^{V1,LR}]_{1211}$	139.10	103.83	$[C_{uddu}^{V8,LR}]_{1211}$	233.47	183.69
$[C_{dd}^{S1,RR}]_{2111}$	92.02	78.90	$[C_{dd}^{S8,RR}]_{2111}$	−28.05	−21.69
$[C_{ud}^{S1,RR}]_{1121}$	−86.98	−66.64	$[C_{ud}^{S8,RR}]_{1121}$	18.09	17.26
$[C_{uddu}^{S1,RR}]_{1121}$	−56.42	−67.69	$[C_{uddu}^{S8,RR}]_{1121}$	22.65	16.99
$[C_{dd}^{S1,RR}]_{2122}$		−0.05	$[C_{dd}^{S8,RR}]_{2122}$		0.01
$[C_{ud}^{S1,RR}]_{2221}$		0.26	$[C_{ud}^{S8,RR}]_{2221}$		0.07
$[C_{uddu}^{S1,RR}]_{2221}$		−0.85	$[C_{uddu}^{S8,RR}]_{2221}$		0.07
$[C_{dd}^{S1,RR}]_{2133}$		0.35	$[C_{dd}^{S8,RR}]_{2133}$		0.11
$[C_{dd}^{S1,RR}]_{2331}$		−1.85	$[C_{dd}^{S8,RR}]_{2331}$		0.21

Table 3. The numerical values for the quantities $P_b(\mu_{\text{ew}})$ that enter in the master formula (2.5) are shown for all JMS WCs contributing to ε'/ε for $\mu_{\text{ew}} = 160$ GeV. The upper part is due to operators with non-vanishing $K \rightarrow \pi\pi$ matrix elements at NLO in QCD with LO results for comparison. The lower part is due to operators that contribute via mixing into Q_{8g} at LO in QCD.

4 Results for $P_b(\mu_{\text{ew}})$ in the JMS basis

In table 3 we give the values of $P_b(\mu_{\text{ew}})$ entering in the master formula (2.5) at the electroweak scale. These values are self-explanatory and confirm what one would expect on the basis of the values of hadronic matrix elements and the size of the anomalous dimensions: the coefficients P_b that multiply the WCs of left-right and scalar operators are typically by one order of magnitude and in a few cases by two orders of magnitude larger than of vector left-left and vector right-right operators. However, the importance of a given contribution to ε'/ε will eventually depend on the size of WCs at the electroweak scale and this will depend on NP scenario considered. We recall that there are four out of the 40 four-quark operators that do not contribute to ε'/ε [16, 17], neither directly via a non-vanishing $K \rightarrow \pi\pi$ matrix element nor via RG mixing. They correspond to the operators $[Q_{uddu}^{V1,LR}]_{2212}$, $[Q_{uddu}^{V8,LR}]_{2212}$, $[Q_{dd}^{V1,LR}]_{2331}$ and $[Q_{dd}^{V8,LR}]_{2331}$ in the JMS basis.

We departed from the previous treatment of certain BSM matrix elements in the master formula [11, 16, 17], by using here directly DQCD results [15] without employing isospin relations for $I = 2$ matrix elements. Using instead the previous treatment, following entries change

$$\begin{aligned}
P_{[C_{uddu}^{V1,LR}]_{1211}} &= 131.68, & P_{[C_{uddu}^{V8,LR}]_{1211}} &= 263.26, \\
P_{[C_{ud}^{S1,RR}]_{1121}} &= -86.80, & P_{[C_{ud}^{S8,RR}]_{1121}} &= 19.54, \\
P_{[C_{uddu}^{S1,RR}]_{1121}} &= -58.11, & P_{[C_{uddu}^{S8,RR}]_{1121}} &= 24.21.
\end{aligned} \tag{4.1}$$

The changes are in the ballpark of 10%, which is in general smaller compared to the uncertainties of about 20% of the matrix elements in the DQCD approach.

It is however clear that this table will be modified with time when LQCD will improve the values of the SM hadronic matrix elements and calculate the corresponding matrix elements for BSM operators. Whether also in this case there will be consensus between LQCD and DQCD on the $I = 2$ matrix elements remains to be seen. Such contributions are expected to dominate NP contributions to ε'/ε because of the factor $\text{Re } A_2/\text{Re } A_0 \sim 1/22$. However, to obtain the full picture some consensus has to be reached on $I = 0$ matrix elements, in particular within the SM because this determines the room left for NP contributions.

In using the master formula (2.5) it is crucial to take into account the definition of Wilson coefficients as given in (2.4). That is they enter the Hamiltonian with the minus sign and have the dimension energy⁻². While it is convenient to use the units 1/TeV², it is not essential because at the end the final results for P_b are dimensionless.

The theoretical uncertainties of the coefficients P_b are determined firstly by the uncertainties of the hadronic matrix elements $\langle Q_i \rangle_{0,2}$, listed in table 2. They enter the P_b linearly and result in 10% – 50% relative uncertainty, depending on the P_b [16]. Secondly, scheme dependences due to higher order corrections in the RG evolution are reduced here due to the inclusion of the NLO QCD corrections in the RG evolution and the threshold corrections. As discussed in [23], such effects are considerably reduced now. The comparison of the LO results that do not include NLO QCD corrections in the RG evolution of the Wilson coefficients in table 3 shows sizeable effects. They are up to 60% for left-right vector operators, but also for left-left vector operators these corrections can become larger than 30%, whereas for the scalar right-right operators they are up to 25%. The NLO QCD effects of the RG evolution are sizeable, but unfortunately these improvements can not be fully appreciated in view of the remaining large hadronic uncertainties of the matrix elements.

5 Conclusions

The main result of our paper is the master formula (2.5) with the numerical values of the coefficients $P_b(\mu_{\text{ew}})$ given in table 3. The advantage of this formula over the one presented in [16] is the inclusion of next-to-leading order (NLO) QCD corrections in the RG evolution in the Weak Effective Theory (WET) from the hadronic scale μ_{had} to the electroweak scale

μ_{ew} and the use of the JMS basis [19]. NLO QCD evolution allows a correct matching of WCs to the matrix elements calculated by lattice QCD (LQCD) or other non-perturbative methods sensitive to renormalization scheme dependences. The use of the JMS basis on the other hand allows to generalize this formula to the Standard Model Effective Field Theory (SMEFT) one because in this basis the tree-level matching of SMEFT on to WET [19] and the one-loop matching [21, 22] are known.

Our master formula can already be used for approximate estimates of BSM contributions to ε'/ε using the JMS basis in the SMEFT. It constitutes an important step towards a complete NLO QCD analysis of the ratio ε'/ε within the SMEFT. However, in order to finish this very ambitious project the following steps have still to be performed:

- The $K \rightarrow \pi\pi$ matrix elements of all operators of the BMU basis, in particular of the BSM ones, should be calculated by LQCD and also the ones already found using DQCD [15] could be in principle improved.
- The contributions due to the chromomagnetic dipole operator and mixing of four-quark operators into the former, including threshold corrections, need to be extended to the NLO level.
- The tree-level and one-loop matchings of SMEFT on WET that are already known [19, 21, 22] should be included. We have not done it because consistently also the next step should be made simultaneously.
- The RG QCD evolution within the SMEFT up to the NP scale should be generalized to the NLO level, i.e. using two-loop anomalous dimensions.

Only then one will be able to cancel all unphysical renormalization scheme dependences that we discussed in [23] at length.

While the second step can be performed already now and the third one could be accomplished relatively soon, the first one is most difficult and could still take several years of intensive calculation by several LQCD collaborations. We are looking forward to the improvements of our formula and hope to report on the progress in a not too distant future.

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A JMS operator basis

The JMS basis [19] categorizes dimension-six four-quark operators with four different chirality structures $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$, $(\bar{L}L)(\bar{R}R)$ and $(\bar{L}R)(\bar{L}R)$. The non-leptonic operators

that contain at least one pair of down-type quarks are given as

$$\begin{aligned}
 & (\bar{L}L)(\bar{L}L) \\
 & [Q_{dd}^{VLL}]_{prst} = (\bar{d}_L^p \gamma_\mu d_L^r)(\bar{d}_L^s \gamma^\mu d_L^t), \\
 & [Q_{ud}^{V1,LL}]_{prst} = (\bar{u}_L^p \gamma_\mu u_L^r)(\bar{d}_L^s \gamma^\mu d_L^t), \quad [Q_{ud}^{V8,LL}]_{prst} = (\bar{u}_L^p \gamma_\mu T^A u_L^r)(\bar{d}_L^s \gamma^\mu T^A d_L^t), \quad (A.1)
 \end{aligned}$$

$$\begin{aligned}
 & (\bar{R}R)(\bar{R}R) \\
 & [Q_{dd}^{VRR}]_{prst} = (\bar{d}_R^p \gamma_\mu d_R^r)(\bar{d}_R^s \gamma^\mu d_R^t), \\
 & [Q_{ud}^{V1,RR}]_{prst} = (\bar{u}_R^p \gamma_\mu u_R^r)(\bar{d}_R^s \gamma^\mu d_R^t), \quad [Q_{ud}^{V8,RR}]_{prst} = (\bar{u}_R^p \gamma_\mu T^A u_R^r)(\bar{d}_R^s \gamma^\mu T^A d_R^t), \quad (A.2)
 \end{aligned}$$

$$\begin{aligned}
 & (\bar{L}L)(\bar{R}R) \\
 & [Q_{dd}^{V1,LR}]_{prst} = (\bar{d}_L^p \gamma_\mu d_L^r)(\bar{d}_R^s \gamma^\mu d_R^t), \quad [Q_{dd}^{V8,LR}]_{prst} = (\bar{d}_L^p \gamma_\mu T^A d_L^r)(\bar{d}_R^s \gamma^\mu T^A d_R^t), \\
 & [Q_{ud}^{V1,LR}]_{prst} = (\bar{u}_L^p \gamma_\mu u_L^r)(\bar{d}_R^s \gamma^\mu d_R^t), \quad [Q_{ud}^{V8,LR}]_{prst} = (\bar{u}_L^p \gamma_\mu T^A u_L^r)(\bar{d}_R^s \gamma^\mu T^A d_R^t), \\
 & [Q_{du}^{V1,LR}]_{prst} = (\bar{d}_L^p \gamma_\mu d_L^r)(\bar{u}_R^s \gamma^\mu u_R^t), \quad [Q_{du}^{V8,LR}]_{prst} = (\bar{d}_L^p \gamma_\mu T^A d_L^r)(\bar{u}_R^s \gamma^\mu T^A u_R^t), \\
 & [Q_{uddu}^{V1,LR}]_{prst} = (\bar{u}_L^p \gamma_\mu d_L^r)(\bar{d}_R^s \gamma^\mu u_R^t), \quad [Q_{uddu}^{V8,LR}]_{prst} = (\bar{u}_L^p \gamma_\mu T^A d_L^r)(\bar{d}_R^s \gamma^\mu T^A u_R^t), \quad (A.3)
 \end{aligned}$$

$$\begin{aligned}
 & (\bar{L}R)(\bar{L}R) \\
 & [Q_{dd}^{S1,RR}]_{prst} = (\bar{d}_L^p d_R^r)(\bar{d}_L^s d_R^t), \quad [Q_{dd}^{S8,RR}]_{prst} = (\bar{d}_L^p T^A d_R^r)(\bar{d}_L^s T^A d_R^t), \\
 & [Q_{ud}^{S1,RR}]_{prst} = (\bar{u}_L^p u_R^r)(\bar{d}_L^s d_R^t), \quad [Q_{ud}^{S8,RR}]_{prst} = (\bar{u}_L^p T^A u_R^r)(\bar{d}_L^s T^A d_R^t), \\
 & [Q_{uddu}^{S1,RR}]_{prst} = (\bar{u}_L^p d_R^r)(\bar{d}_L^s u_R^t), \quad [Q_{uddu}^{S8,RR}]_{prst} = (\bar{u}_L^p T^A d_R^r)(\bar{d}_L^s T^A u_R^t), \quad (A.4)
 \end{aligned}$$

Some of the listed operators are not equal to their hermitian conjugates with permuted generation indices. Although we do not list them explicitly here, they have to be treated as independent operators. Note that also $Q_{uddu}^{V1,LR}$ and $Q_{uddu}^{V8,LR}$ have Hermitian conjugates. The same holds for the operators $(\bar{L}R)(\bar{L}R)$. This choice of basis eliminates all operators with Dirac structures $\sigma^{\mu\nu}$. The colour structure is expressed in terms of the generators T^A of $SU(3)_c$. Above p, r, s, t denote quark flavour indices in the mass-eigenstate basis. These operators include both the “non-flipped” and the chirality-flipped ones.

B BMU operator basis for $K \rightarrow \pi\pi$

The BMU basis [18] contains several sectors of operators, depending on their quark-flavour content and Lorentz structures. They form separate blocks under RG evolution, thereby minimizing the operator mixing. General description of this basis valid also for the B -meson system is discussed in detail in the appendix A.2 in [23]. Here we specify this basis to the case relevant for $K \rightarrow \pi\pi$ decays. This amounts to setting $d_j = s$ and $d_i = d$ in the expressions listed in [23]. Moreover in some operators $d_k = b$.

The SM basis consists of two current-current-type operators

$$\begin{aligned}
 Q_1 &= Q_1^{VLL,u} = (\bar{s}^\alpha \gamma_\mu P_L u^\beta)(\bar{u}^\beta \gamma^\mu P_L d^\alpha), \\
 Q_2 &= Q_2^{VLL,u} = (\bar{s}^\alpha \gamma_\mu P_L u^\alpha)(\bar{u}^\beta \gamma^\mu P_L d^\beta), \quad (B.1)
 \end{aligned}$$

where the naming used in [18] is given as well. The $SU(3)_c$ colour indices are denoted by α and β . The choice of $u_k = u$ implies that the corresponding operators for $u_k = c$ are eliminated in favour of QCD- and QED-penguin operators, see below, which is most suited for Kaon physics.

There are eight penguin-type operators with left-handed $(\bar{s}\gamma_\mu P_L d)$ vector currents: the P_{QCD}

$$\begin{aligned} Q_3 &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_q (\bar{q}^\beta \gamma^\mu P_L q^\beta), & Q_4 &= (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_q (\bar{q}^\beta \gamma^\mu P_L q^\alpha), \\ Q_5 &= (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_q (\bar{q}^\beta \gamma^\mu P_R q^\beta), & Q_6 &= (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_q (\bar{q}^\beta \gamma^\mu P_R q^\alpha), \end{aligned} \quad (\text{B.2})$$

and P_{QED}

$$\begin{aligned} Q_7 &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_q Q_q (\bar{q}^\beta \gamma^\mu P_R q^\beta), & Q_8 &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_q Q_q (\bar{q}^\beta \gamma^\mu P_R q^\alpha), \\ Q_9 &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) \sum_q Q_q (\bar{q}^\beta \gamma^\mu P_L q^\beta), & Q_{10} &= \frac{3}{2} (\bar{s}^\alpha \gamma_\mu P_L d^\beta) \sum_q Q_q (\bar{q}^\beta \gamma^\mu P_L q^\alpha). \end{aligned} \quad (\text{B.3})$$

The sum over $q = u, d, s, c, b$ includes all five active quark flavours and Q_q their electromagnetic charges: $Q_u = +2/3$ and $Q_d = -1/3$. Their chirality-flipped counterparts are not present in the SM.

Then the SM basis contains ten operators only because the chirality-flipped operators are absent. No Wilson coefficients are generated for them due to the left-handed nature of electroweak interactions of the SM.

Beyond the SM, we have first operators with vectorial Lorentz structure [18]

$$\begin{aligned} Q_{11} &= Q_1^{\text{VLL},d+s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) [(\bar{d}^\beta \gamma^\mu P_L d^\beta) + (\bar{s}^\beta \gamma^\mu P_L s^\beta)], \\ Q_{12} &= Q_1^{\text{VLR},d+s} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta) [(\bar{d}^\beta \gamma^\mu P_R d^\alpha) + (\bar{s}^\beta \gamma^\mu P_R s^\alpha)], \\ Q_{13} &= Q_2^{\text{VLR},d+s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) [(\bar{d}^\beta \gamma^\mu P_R d^\beta) + (\bar{s}^\beta \gamma^\mu P_R s^\beta)], \end{aligned} \quad (\text{B.4})$$

$$\begin{aligned} Q_{14} &= Q_1^{\text{VLL},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) [(\bar{d}^\beta \gamma^\mu P_L d^\beta) - (\bar{s}^\beta \gamma^\mu P_L s^\beta)], \\ Q_{15} &= Q_1^{\text{VLR},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta) [(\bar{d}^\beta \gamma^\mu P_R d^\alpha) - (\bar{s}^\beta \gamma^\mu P_R s^\alpha)], \\ Q_{16} &= Q_2^{\text{VLR},d-s} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) [(\bar{d}^\beta \gamma^\mu P_R d^\beta) - (\bar{s}^\beta \gamma^\mu P_R s^\beta)]. \end{aligned} \quad (\text{B.5})$$

Only $d + s$ operators mix into QCD- and QED-penguin operators $Q_{3,\dots,10}$, whereas $d - s$ ones do not. Analogous operators for $d \rightarrow s u_k \bar{u}_k$ are

$$\begin{aligned} Q_{17} &= Q_1^{\text{VLR},u-c} = (\bar{s}^\alpha \gamma_\mu P_L d^\beta) [(\bar{u}^\beta \gamma^\mu P_R u^\alpha) - (\bar{c}^\beta \gamma^\mu P_R c^\alpha)], \\ Q_{18} &= Q_2^{\text{VLR},u-c} = (\bar{s}^\alpha \gamma_\mu P_L d^\alpha) [(\bar{u}^\beta \gamma^\mu P_R u^\beta) - (\bar{c}^\beta \gamma^\mu P_R c^\beta)], \end{aligned} \quad (\text{B.6})$$

which do not mix into QCD- and QED-penguin operators $Q_{3,\dots,10}$, but give threshold corrections when decoupling the c quark.

The operators of the SRL sector for $d \rightarrow s Q \bar{Q}$ with $Q = u, c, b$ are given by [18]

$$Q_1^{\text{SRL},Q} = (\bar{s}^\alpha P_R d^\beta) (\bar{Q}^\beta P_L Q^\alpha), \quad Q_2^{\text{SRL},Q} = (\bar{s}^\alpha P_R d^\alpha) (\bar{Q}^\beta P_L Q^\beta), \quad (\text{B.7})$$

which are numbered as

$$\begin{aligned} (Q_{19}, Q_{20}) &= (Q_1^{\text{SRL},u}, Q_2^{\text{SRL},u}), & (Q_{23}, Q_{24}) &= (Q_1^{\text{SRL},b}, Q_2^{\text{SRL},b}), \\ (Q_{21}, Q_{22}) &= (Q_1^{\text{SRL},c}, Q_2^{\text{SRL},c}), \end{aligned} \quad (\text{B.8})$$

The scalar operators of the *SRR sectors* for $d \rightarrow s d \bar{d}$ and $d \rightarrow s s \bar{s}$ have been chosen in [18] as colour-singlet operators, see $\Delta F = 2$ sector. Due to Fierz symmetries there are only two operators per sector SRR, d

$$Q_{25} = Q_1^{\text{SRR},d} = (\bar{s}^\alpha P_R d^\alpha) (\bar{d}^\beta P_R d^\beta), \quad Q_{26} = Q_2^{\text{SRR},d} = (\bar{s}^\alpha \sigma_{\mu\nu} P_R d^\alpha) (\bar{d}^\beta \sigma^{\mu\nu} P_R d^\beta), \quad (\text{B.9})$$

and SRR, s

$$Q_{27} = Q_1^{\text{SRR},s} = (\bar{s}^\alpha P_R d^\alpha) (\bar{s}^\beta P_R s^\beta), \quad Q_{28} = Q_2^{\text{SRR},s} = (\bar{s}^\alpha \sigma_{\mu\nu} P_R d^\alpha) (\bar{s}^\beta \sigma^{\mu\nu} P_R s^\beta). \quad (\text{B.10})$$

The case of $d \rightarrow s Q \bar{Q}$ with $Q = u, c, b$ comprises four operators per sector

$$\begin{aligned} Q_1^{\text{SRR},Q} &= (\bar{s}^\alpha P_R d^\beta) (\bar{Q}^\beta P_R Q^\alpha), & Q_3^{\text{SRR},Q} &= (\bar{s}^\alpha \sigma_{\mu\nu} P_R d^\beta) (\bar{Q}^\beta \sigma^{\mu\nu} P_R Q^\alpha), \\ Q_2^{\text{SRR},Q} &= (\bar{s}^\alpha P_R d^\alpha) (\bar{Q}^\beta P_R Q^\beta), & Q_4^{\text{SRR},Q} &= (\bar{s}^\alpha \sigma_{\mu\nu} P_R d^\alpha) (\bar{Q}^\beta \sigma^{\mu\nu} P_R Q^\beta). \end{aligned} \quad (\text{B.11})$$

They are numbered consecutively

$$\begin{aligned} (Q_{29}, Q_{30}, Q_{31}, Q_{32}) &= (Q_1^{\text{SRR},u}, Q_2^{\text{SRR},u}, Q_3^{\text{SRR},u}, Q_4^{\text{SRR},u}), \\ (Q_{33}, Q_{34}, Q_{35}, Q_{36}) &= (Q_1^{\text{SRR},c}, Q_2^{\text{SRR},c}, Q_3^{\text{SRR},c}, Q_4^{\text{SRR},c}), \\ (Q_{37}, Q_{38}, Q_{39}, Q_{40}) &= (Q_1^{\text{SRR},b}, Q_2^{\text{SRR},b}, Q_3^{\text{SRR},b}, Q_4^{\text{SRR},b}). \end{aligned} \quad (\text{B.12})$$

The definition of chirality-flipped operators is not explicitly shown here. The numbering of these operators is given as

$$Q_{40+i} = Q_i [P_L \leftrightarrow P_R], \quad (\text{B.13})$$

i.e. they are found by interchange of $P_L \leftrightarrow P_R$ from the “non-flipped” operators.

C Basis transformation: BMU to SD

In this appendix we report the basis change from the BMU basis used in the RG evolution to the SD basis defined in [15], in which the matrix elements for the BSM operators are given. Both bases almost coincide, except for a few exceptions. The relations for the BSM operators Q_i in the BMU basis and O_i in the SD basis, respectively, read:

$$\begin{aligned} Q_{14} &= O_1^{\text{VLL},d-s}, & Q_{15} &= O_1^{\text{VLR},d-s}, & Q_{16} &= O_2^{\text{VLR},d-s}, \\ Q_{19} &= (O_1^{\text{SLR},u})' = O_1^{\text{SRL},u}, & Q_{20} &= (O_2^{\text{SLR},u})' = O_2^{\text{SRL},u}, \end{aligned} \quad (\text{C.1})$$

$$\begin{aligned}
 Q_{29} &= (O_1^{\text{SLL},u})' = O_1^{\text{SRR},u}, & Q_{31} &= -(O_3^{\text{SLL},u})' = -O_3^{\text{SRR},u}, \\
 Q_{30} &= (O_2^{\text{SLL},u})' = O_2^{\text{SRR},u}, & Q_{32} &= -(O_4^{\text{SLL},u})' = -O_4^{\text{SRR},u}, \\
 Q_{25} &= (O_2^{\text{SLL},d})' = O_2^{\text{SRR},d}, & Q_{26} &= -8(O_1^{\text{SLL},d})' - 4(O_2^{\text{SLL},d})' = -8O_1^{\text{SRR},d} - 4O_2^{\text{SRR},d}, \\
 Q_{27} &= (O_2^{\text{SLL},s})' = O_2^{\text{SRR},s}, & Q_{28} &= -8(O_1^{\text{SLL},s})' - 4(O_2^{\text{SLL},s})' = -8O_1^{\text{SRR},s} - 4O_2^{\text{SRR},s}.
 \end{aligned}$$

D Basis transformation: BMU to JMS

Here we report the basis transformation from the basis used in [16, 17, 34], which coincides with the BMU basis, to the JMS basis. This basis change allows us to use the results obtained in [34] for the operators in class B [16], which consists of operators that only contribute to ε'/ε via RG mixing into the dipole operators $Q_{8g}^{(r)}$. The mixing of these operators was computed using `wilson` [35], which includes the one-loop QCD and QED [36, 37] running below the EW scale. In the notation of [16] we find the following relations:

$$O_{SRR}^s = Q_{27} = [Q_{dd}^{S1,RR}]_{2122}, \quad (\text{D.1})$$

$$O_{TRR}^s = Q_{28} = -\frac{20}{3}[Q_{dd}^{S1,RR}]_{2122} - 16[Q_{dd}^{S8,RR}]_{2122} \quad (\text{D.2})$$

$$O_{SRR}^c = Q_{34} = [Q_{ud}^{S1,RR}]_{2221}, \quad (\text{D.3})$$

$$O_{TRR}^c = Q_{36} = -\frac{8}{3}[Q_{uddu}^{S1,RR}]_{2122} - 4[Q_{ud}^{S1,RR}]_{2221} - 16[Q_{uddu}^{S8,RR}]_{2122}, \quad (\text{D.4})$$

$$\tilde{O}_{SRR}^c = Q_{33} = \frac{1}{3}[Q_{ud}^{S1,RR}]_{2221} + 2[Q_{ud}^{S8,RR}]_{2221}, \quad (\text{D.5})$$

$$\tilde{O}_{TRR}^c = Q_{35} = -8[Q_{uddu}^{S1,RR}]_{2122} - \frac{4}{3}[Q_{ud}^{S1,RR}]_{2221} - 8[Q_{ud}^{S8,RR}]_{2221}, \quad (\text{D.6})$$

$$O_{SRR}^b = Q_{38} = [Q_{dd}^{S1,RR}]_{2133}, \quad (\text{D.7})$$

$$O_{TRR}^b = Q_{40} = -\frac{8}{3}[Q_{dd}^{S1,RR}]_{2331} - 4[Q_{dd}^{S1,RR}]_{2133} - 16[Q_{dd}^{S8,RR}]_{2331}, \quad (\text{D.8})$$

$$\tilde{O}_{SRR}^b = Q_{37} = \frac{1}{3}[Q_{dd}^{S1,RR}]_{2133} + 2[Q_{dd}^{S8,RR}]_{2133}, \quad (\text{D.9})$$

$$\tilde{O}_{TRR}^b = Q_{39} = -8[Q_{dd}^{S1,RR}]_{2331} - \frac{4}{3}[Q_{dd}^{S1,RR}]_{2133} - 8[Q_{dd}^{S8,RR}]_{2133}. \quad (\text{D.10})$$

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References

- [1] A.J. Buras, *The ϵ'/ϵ -story: 1976–2021*, *Acta Phys. Polon. B* **52** (2021) 7 [[arXiv:2101.00020](https://arxiv.org/abs/2101.00020)] [[INSPIRE](#)].
- [2] NA48 collaboration, *A precision measurement of direct CP-violation in the decay of neutral kaons into two pions*, *Phys. Lett. B* **544** (2002) 97 [[hep-ex/0208009](https://arxiv.org/abs/hep-ex/0208009)] [[INSPIRE](#)].

- [3] KTeV collaboration, *Measurements of direct CP-violation, CPT symmetry, and other parameters in the neutral kaon system*, *Phys. Rev. D* **67** (2003) 012005 [Erratum *ibid.* **70** (2004) 079904] [[hep-ex/0208007](#)] [[INSPIRE](#)].
- [4] KTeV collaboration, *Precise measurements of direct CP-violation, CPT symmetry, and other parameters in the neutral kaon system*, *Phys. Rev. D* **83** (2011) 092001 [[arXiv:1011.0127](#)] [[INSPIRE](#)].
- [5] RBC, UKQCD collaboration, *Direct CP-violation and the $\Delta I = 1/2$ rule in $K \rightarrow \pi\pi$ decay from the standard model*, *Phys. Rev. D* **102** (2020) 054509 [[arXiv:2004.09440](#)] [[INSPIRE](#)].
- [6] A.J. Buras and J.-M. Gérard, *Upper bounds on ϵ'/ϵ parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from large N QCD and other news*, *JHEP* **12** (2015) 008 [[arXiv:1507.06326](#)] [[INSPIRE](#)].
- [7] V. Cirigliano, H. Gisbert, A. Pich and A. Rodríguez-Sánchez, *Theoretical status of ϵ'/ϵ* , *J. Phys. Conf. Ser.* **1526** (2020) 012011 [[arXiv:1912.04736](#)] [[INSPIRE](#)].
- [8] A. Pich and A. Rodríguez-Sánchez, *SU(3) analysis of four-quark operators: $K \rightarrow \pi\pi$ and vacuum matrix elements*, *JHEP* **06** (2021) 005 [[arXiv:2102.09308](#)] [[INSPIRE](#)].
- [9] V. Cirigliano, H. Gisbert, A. Pich and A. Rodríguez-Sánchez, *Isospin-violating contributions to ϵ'/ϵ* , *JHEP* **02** (2020) 032 [[arXiv:1911.01359](#)] [[INSPIRE](#)].
- [10] A.J. Buras and J.-M. Gérard, *Isospin-breaking in ϵ'/ϵ : impact of η_0 at the dawn of the 2020s*, *Eur. Phys. J. C* **80** (2020) 701 [[arXiv:2005.08976](#)] [[INSPIRE](#)].
- [11] J. Aebischer, C. Bobeth and A.J. Buras, *ϵ'/ϵ in the standard model at the dawn of the 2020s*, *Eur. Phys. J. C* **80** (2020) 705 [[arXiv:2005.05978](#)] [[INSPIRE](#)].
- [12] A.J. Buras, *ϵ'/ϵ -2018: a Christmas story*, [arXiv:1812.06102](#) [[INSPIRE](#)].
- [13] J. Aebischer, C. Bobeth and A.J. Buras, *On the importance of NNLO QCD and isospin-breaking corrections in ϵ'/ϵ* , *Eur. Phys. J. C* **80** (2020) 1 [[arXiv:1909.05610](#)] [[INSPIRE](#)].
- [14] A.J. Buras, *New physics patterns in ϵ'/ϵ and ϵ_K with implications for rare kaon decays and ΔM_K* , *JHEP* **04** (2016) 071 [[arXiv:1601.00005](#)] [[INSPIRE](#)].
- [15] J. Aebischer, A.J. Buras and J.-M. Gérard, *BSM hadronic matrix elements for ϵ'/ϵ and $K \rightarrow \pi\pi$ decays in the Dual QCD approach*, *JHEP* **02** (2019) 021 [[arXiv:1807.01709](#)] [[INSPIRE](#)].
- [16] J. Aebischer, C. Bobeth, A.J. Buras, J.-M. Gérard and D.M. Straub, *Master formula for ϵ'/ϵ beyond the standard model*, *Phys. Lett. B* **792** (2019) 465 [[arXiv:1807.02520](#)] [[INSPIRE](#)].
- [17] J. Aebischer, C. Bobeth, A.J. Buras and D.M. Straub, *Anatomy of ϵ'/ϵ beyond the standard model*, *Eur. Phys. J. C* **79** (2019) 219 [[arXiv:1808.00466](#)] [[INSPIRE](#)].
- [18] A.J. Buras, M. Misiak and J. Urban, *Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the standard model*, *Nucl. Phys. B* **586** (2000) 397 [[hep-ph/0005183](#)] [[INSPIRE](#)].
- [19] E.E. Jenkins, A.V. Manohar and P. Stoffer, *Low-energy effective field theory below the electroweak scale: operators and matching*, *JHEP* **03** (2018) 016 [[arXiv:1709.04486](#)] [[INSPIRE](#)].
- [20] J. Aebischer et al., *WCxf: an exchange format for Wilson coefficients beyond the standard model*, *Comput. Phys. Commun.* **232** (2018) 71 [[arXiv:1712.05298](#)] [[INSPIRE](#)].

- [21] W. Dekens and P. Stoffer, *Low-energy effective field theory below the electroweak scale: matching at one loop*, *JHEP* **10** (2019) 197 [[arXiv:1908.05295](#)] [[INSPIRE](#)].
- [22] J. Aebischer, A. Crivellin, M. Fael and C. Greub, *Matching of gauge invariant dimension-six operators for $b \rightarrow s$ and $b \rightarrow c$ transitions*, *JHEP* **05** (2016) 037 [[arXiv:1512.02830](#)] [[INSPIRE](#)].
- [23] J. Aebischer, C. Bobeth, A.J. Buras, J. Kumar and M. Misiak, *General non-leptonic $\Delta F = 1$ WET at the NLO in QCD*, *JHEP* **11** (2021) 227 [[arXiv:2107.10262](#)] [[INSPIRE](#)].
- [24] J. Aebischer, C. Bobeth, A.J. Buras and J. Kumar, *SMEFT ATLAS of $\Delta F = 2$ transitions*, *JHEP* **12** (2020) 187 [[arXiv:2009.07276](#)] [[INSPIRE](#)].
- [25] A.J. Buras, M. Gorbahn, S. Jäger and M. Jamin, *Improved anatomy of ε'/ε in the standard model*, *JHEP* **11** (2015) 202 [[arXiv:1507.06345](#)] [[INSPIRE](#)].
- [26] V. Cirigliano, A. Pich, G. Ecker and H. Neufeld, *Isospin violation in ε'* , *Phys. Rev. Lett.* **91** (2003) 162001 [[hep-ph/0307030](#)] [[INSPIRE](#)].
- [27] V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, *Isospin breaking in $K \rightarrow \pi\pi$ decays*, *Eur. Phys. J. C* **33** (2004) 369 [[hep-ph/0310351](#)] [[INSPIRE](#)].
- [28] J. Bijnens and F. Borg, *Isospin breaking in $K \rightarrow 3\pi$ decays III: Bremsstrahlung and fit to experiment*, *Eur. Phys. J. C* **40** (2005) 383 [[hep-ph/0501163](#)] [[INSPIRE](#)].
- [29] M. Misiak and M. Münz, *Two loop mixing of dimension five flavor changing operators*, *Phys. Lett. B* **344** (1995) 308 [[hep-ph/9409454](#)] [[INSPIRE](#)].
- [30] T. Blum et al., *$K \rightarrow \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit*, *Phys. Rev. D* **91** (2015) 074502 [[arXiv:1502.00263](#)] [[INSPIRE](#)].
- [31] A.J. Buras, J.-M. Gérard and W.A. Bardeen, *Large N approach to kaon decays and mixing 28 years later: $\Delta I = 1/2$ rule, $\hat{B}_K$ and ΔM_K* , *Eur. Phys. J. C* **74** (2014) 2871 [[arXiv:1401.1385](#)] [[INSPIRE](#)].
- [32] ETM collaboration, *$K \rightarrow \pi$ matrix elements of the chromomagnetic operator on the lattice*, *Phys. Rev. D* **97** (2018) 074501 [[arXiv:1712.09824](#)] [[INSPIRE](#)].
- [33] A.J. Buras and J.-M. Gérard, *$K \rightarrow \pi\pi$ and $K - \pi$ matrix elements of the chromomagnetic operators from dual QCD*, *JHEP* **07** (2018) 126 [[arXiv:1803.08052](#)] [[INSPIRE](#)].
- [34] J. Aebischer, A.J. Buras and J. Kumar, *Another SMEFT story: Z' facing new results on ε'/ε , ΔM_K and $K \rightarrow \pi\nu\bar{\nu}$* , *JHEP* **12** (2020) 097 [[arXiv:2006.01138](#)] [[INSPIRE](#)].
- [35] J. Aebischer, J. Kumar and D.M. Straub, *Wilson: a Python package for the running and matching of Wilson coefficients above and below the electroweak scale*, *Eur. Phys. J. C* **78** (2018) 1026 [[arXiv:1804.05033](#)] [[INSPIRE](#)].
- [36] J. Aebischer, M. Fael, C. Greub and J. Virto, *B physics beyond the standard model at one loop: complete renormalization group evolution below the electroweak scale*, *JHEP* **09** (2017) 158 [[arXiv:1704.06639](#)] [[INSPIRE](#)].
- [37] E.E. Jenkins, A.V. Manohar and P. Stoffer, *Low-energy effective field theory below the electroweak scale: anomalous dimensions*, *JHEP* **01** (2018) 084 [[arXiv:1711.05270](#)] [[INSPIRE](#)].