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SMEFT atlas of $\Delta F = 2$ transitions

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ABSTRACT: We present a model-independent anatomy of the $\Delta F = 2$ transitions $K^0 - \bar{K}^0$, $B_{s,d} - \bar{B}_{s,d}$ and $D^0 - \bar{D}^0$ in the context of the Standard Model Effective Field Theory (SMEFT). We present two master formulae for the mixing amplitude $[M_{12}]_{\text{BSM}}$. One in terms of the Wilson coefficients (WCs) of the Low-Energy Effective Theory (LEFT) operators evaluated at the electroweak scale μ_{ew} and one in terms of the WCs of the SMEFT operators evaluated at the BSM scale Λ . The coefficients P_a^{ij} entering these formulae contain all the information below the scales μ_{ew} and Λ , respectively. Renormalization group effects from the top-quark Yukawa coupling play the most important role. The collection of the individual contributions of the SMEFT operators to $[M_{12}]_{\text{BSM}}$ can be considered as the SMEFT atlas of $\Delta F = 2$ transitions and constitutes a travel guide to such transitions far beyond the scales explored by the LHC. We emphasize that this atlas depends on whether the down-basis or the up-basis for SMEFT operators is considered. We illustrate this technology with tree-level exchanges of heavy gauge bosons (Z' , G') and corresponding heavy scalars.

KEYWORDS: QCD Phenomenology

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1 Introduction

$K^0 - \bar{K}^0$, $B_{s,d} - \bar{B}_{s,d}$ and $D^0 - \bar{D}^0$ mixings have been already for many years the stars among the flavour-changing neutral-current processes (FCNC) [1, 2]. This is in particular the case of the parameter ε_K , of the $B_{s,d} - \bar{B}_{s,d}$ mass differences $\Delta M_{B_{s,d}}$ and of mixing-induced CP asymmetries in the latter systems. The $K_L - K_S$ mass difference ΔM_K remained due to large theoretical uncertainties until recently under the shadow of these observables although it played a very important role in the past in estimating successfully the charm-quark mass prior to its discovery [3]. However, recently progress in evaluating ΔM_K within the Standard Model (SM) has been made by the RBC-UKQCD collaboration [4–6] so that ΔM_K begins to play again an important role in phenomenology, not only to bound effects beyond the SM (BSM) [7–12], but also to help identifying what this new physics (NP) could be.

These days, the absence of the discovery of new particles at the Large Hadron Collider (LHC) points towards a mass gap between the electroweak (EW) scale μ_{ew} and the next threshold scale Λ of new heavy degrees of freedom. For such a scenario, NP effects can be discussed for all processes between scales sufficiently below Λ down to the EW scale conveniently within the Standard Model Effective Field Theory (SMEFT), the renormalizable SM augmented with higher-dimensional operators invariant under the full SM group $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$. The flavour-changing processes of light quarks and leptons further below the EW scale are described by the usual respective Low-Energy Effective Field Theories (LEFT) that are invariant under $\text{SU}(3)_c \times \text{U}(1)_{\text{em}}$. The LEFTs are characterized by disjoint sets of flavour-changing operators for each process and are conveniently separated from each other. They have been studied systematically in the SM and generalizations in terms of the renormalization group (RG) equations also at higher orders in QCD and in some cases in QED. Currently this program is extended within the framework of SMEFT and recently the complete matching of SMEFT to LEFTs at μ_{ew} at tree- and one-loop level

has been summarized in [13, 14]. The matching conditions of SMEFT to LEFT together with the RG equations within SMEFT and LEFTs are the essential ingredients to study phenomenological correlations of low- and high-energy¹ observables.

Several model-independent SMEFT analyses of FCNC processes can be found in the literature [11, 15–25]. In particular in [11] a systematic analysis of the RG effects above the EW scale has been performed and for the first time the full set of constraints on all relevant dimension-six operators resulting from $\Delta F = 2$ transitions has been presented in numerous tables.

In the present paper we want to explore two different avenues involving $\Delta F = 2$ transitions. First we will provide in the spirit of the SMEFT anatomy of the ratio ε'/ε [26, 27] a master formula for $\Delta F = 2$ processes in terms of the standard LEFT operator basis used already by many authors for two decades following the expressions presented in [28] with Wilson coefficients (WCs) of the LEFT given at the EW scale μ_{ew} . The second avenue leads us to a master formula for $\Delta F = 2$ processes given directly in terms of the SMEFT operator basis with SMEFT WCs, evaluated at the NP scale Λ . To our knowledge the SMEFT formula in question is presented here for the first time. It allows to include automatically SMEFT effects above the EW scale, in particular the ones from the RG running of the top-Yukawa coupling. Moreover, in contrast to recent SMEFT analyses found in the literature it includes in addition to the leading order (LO) QCD RG corrections also the next-to-leading order (NLO) ones below the electroweak scale. This is necessary to have a proper matching of LEFT WCs to the hadronic matrix elements from lattice QCD (LQCD).

The collection of the individual contributions of the SMEFT operators to this master formula can be considered as the SMEFT atlas for $\Delta F = 2$ transitions. As such it will allow model builders to obtain efficiently predictions for $\Delta F = 2$ processes in a plethora of NP models that are consistent with the rules of the SMEFT. As we will see such an analysis is also useful for an analytic insight into model-independent analyses, which complements the very extensive numerical analysis in [11].² To this end our atlas exhibits in addition to usually assumed most important contributions of a given operator to $[M_{12}^{ij}]_{\text{BSM}}$ with $ij = ds, db, sb, cu$ also subleading ones, which in some NP scenarios could turn out to be the most important ones. Seeing these contributions to various meson systems side-by-side illustrates possible correlations generated by RG evolution between $\Delta F = 2$ and $\Delta F = 1$ transitions among the various meson systems that have to be taken into account. This is in particular the case between B_d^0 and B_s^0 as well as K^0 and D^0 meson systems. Our atlas casts in this manner some doubts on the validity of many analyses present in the literature that consider only one or two operators at the time and restrict the analyses to a single meson system. Such analyses can only be considered as a first look and have to be supplemented eventually by a more complete SMEFT analysis that optimally includes a concrete UV completion. This is undermined by the fact that within the SMEFT the results depend on the chosen basis for SMEFT operators that signals the need for the UV completions that include some aspects of a theory for Yukawa couplings.

¹“High-energy” refers here to scales sufficiently below Λ .

²While the RG analysis in [11] is based on the first-leading-log approximation, we perform full resummation of leading logarithms which allows to include automatically the mixing between operators that is absent if only the first leading logarithm is kept. See [29] for a detailed analysis of such effects.

Our paper is organized as follows. In section 2 we recall the LEFT for the $\Delta F = 2$ processes in any BSM scenario. We pay particular attention to scheme transformations among the various operator bases in the literature to enable a consistent use of RG equations, hadronic matrix elements as well as UV matching conditions to SMEFT at the EW scale. In particular we discuss the treatment of the evanescent operators, which must be consistent with the available two-loop anomalous dimensions of the involved operators.

In section 2.4 we present an update of the master formula for $\Delta F = 2$ processes ($K^0 - \bar{K}^0$, $B_{s,d} - \bar{B}_{s,d}$ and $D^0 - \bar{D}^0$) in LEFT valid in any BSM scenario first presented in [28]. It depends on the *model-independent* matrix elements evaluated at $\mu_{\text{ew}} = 160 \text{ GeV}$ and on Wilson coefficients of these operators evaluated at the same scale. All the model dependence is collected in the values of these coefficients. As a byproduct we present in section 2.3 a review of various estimates of hadronic matrix elements found in the literature.

In section 3 we perform a general SMEFT anatomy of $\Delta F = 2$ processes, which eventually leads us to the most important formula for $\Delta F = 2$ processes in our paper, the one given entirely in terms of SMEFT WCs in the Warsaw basis [30] at the NP scale Λ . In this context we stress the importance of the Yukawa RG effects in the evolution from Λ down to μ_{ew} . We also emphasize the differences between results obtained in *down*- and *up*-Warsaw bases. In this section we discuss one-loop matching of SMEFT onto LEFT in the analytic form and collect the relevant RG equations accompanied by RG flow charts. This section culminates in the SMEFT atlas, mentioned previously, built out of numerous formulae for the individual contributions of the relevant operators. In the main text we present these formulae for $\Lambda = 5 \text{ TeV}$ while in the appendix A the corresponding expressions are given for $\Lambda = 100 \text{ TeV}$. The full set of contributions from all relevant SMEFT operators can be found in the supplementary material of this article. We also present the effective sensitivity scales Λ_i of the Wilson coefficients of the dominant operators.

In section 4 we illustrate this technology on a number of simplified models that allow for $\Delta F = 2$ processes at tree-level via heavy spin-zero or spin-one boson exchange. This includes models with colourless heavy gauge bosons (Z') and scalars and models with coloured heavy gauge bosons (G'_a) and scalars. Also the cases of vector-like quarks and leptoquarks are briefly considered.

In section 5 we summarize the main results of our paper and present a brief outlook for the coming years. In appendices we present the SMEFT atlas for $\Lambda = 100 \text{ TeV}$ and its version for NP scenarios of section 4 at $\Lambda = 5 \text{ TeV}$ in the up-basis. We list the relations between various operator bases and we report the one-loop matching of the SMEFT onto the LEFT in an analytic form both in the down basis and the up basis. Finally, we elaborate on the issue of evanescent operators.

2 LEFT anatomy of $\Delta F = 2$ processes

2.1 Preliminaries

The $\Delta F = 2$ LEFTs for $K^0 - \bar{K}^0$, $B_{s,d} - \bar{B}_{s,d}$ and $D^0 - \bar{D}^0$ mixing arise in the SM from the decoupling of the heavy electroweak gauge bosons, the Higgs field and the top quark at μ_{ew} and similarly by the decoupling of heavy degrees of freedom in any UV completion

where additional light degrees of freedom below μ_{ew} are absent, as is the case for SMEFT. We decompose the corresponding effective Hamiltonian [31, 32] into the SM and BSM contribution as follows

$$\mathcal{H}_{\Delta F=2}^{ij} = [\mathcal{H}_{\Delta F=2}^{ij}]_{\text{SM}} + \sum_a C_a^{ij}(\mu) Q_a^{ij} + \text{h.c.}, \quad (2.1)$$

with $ij = ds$ for $K^0 - \bar{K}^0$ mixing and $ij = sb, db$ for $B_{s,d} - \bar{B}_{s,d}$ mixing, respectively. In the SM there is only a single $\Delta F = 2$ operator in each meson system

$$[\mathcal{H}_{\Delta F=2}^{ij}]_{\text{SM}} = [C_{\text{VLL}}^{ij}(\mu)]_{\text{SM}} Q_{\text{VLL}}^{ij} + \text{h.c.}, \quad (2.2)$$

with the expression for Q_{VLL}^{ij} given in (2.8). The SM Wilson coefficients at the scale μ_{ew}

$$C_{\text{VLL}}^{ij}(\mu_{\text{ew}})|_{\text{SM}} = \mathcal{N} (\lambda_t^{ij})^2 S_0(x_t), \quad S_0(x) = \frac{x(4 - 11x + x^2)}{4(x-1)^2} + \frac{3x^3 \ln x}{2(x-1)^3}, \quad (2.3)$$

contain the normalisation factor $\mathcal{N}$ and the CKM combinations given respectively by

$$\mathcal{N} = \frac{G_F^2 m_W^2}{4\pi^2}, \quad \lambda_t^{ij} = V_{ti}^* V_{tj}, \quad (2.4)$$

where we show the case of down-type mixing. The effect of the SM one-loop box diagrams with top-quark exchange for down-type mixing is contained in³ the function $S_0(x_t)$, which depends on the ratio of the top-quark and W -boson masses $x_t \equiv m_t^2/m_W^2$. The NLO QCD matching correction is known from [33] and NLO EW matching corrections from [34].

The case of $\Delta C = 2$ is found by exchanging the down-type quarks with up-type quarks in the operators and using $ij = cu$. The pure short-distance box diagrams with W -boson exchange and light down-type quarks yield a vanishing contribution due to the unitarity of the CKM matrix, i.e. GIM cancellation. This can be also inferred from $S_0(x) \rightarrow 0$ in the limit $x \rightarrow 0$ and in consequence $[\mathcal{H}_{\Delta F=2}^{cu}]_{\text{SM}} = 0$ in (2.1).

The most general LEFT operator basis consists of eight operators Q_a^{ij} , with different basis conventions in the literature. We anticipate that in SMEFT only four out of the eight operators are generated. We will specify them below. The Wilson coefficients $C_a^{ij}(\mu)$ depend on the renormalization scale μ . They are obtained at the matching scale μ_{ew} and can be evolved via the RG equations to a typical low-energy scale μ_{had} of the order of a few GeV, i.e. the order of the relevant external scales.

The $\Delta F = 2$ operators contribute to the off-diagonal element of the mass matrix of neutral meson ($M^0 = K^0, D^0, B_{s,d}$) mixing as follows

$$M_{12}^{ij} \equiv [M_{12}^{ij}]_{\text{SM}} + [M_{12}^{ij}]_{\text{BSM}} = \frac{\langle M^0 | \mathcal{H}_{\Delta F=2}^{ij} | \bar{M}^0 \rangle}{2M_{M^0}} + \mathcal{O}(\text{dim-8}), \quad (2.5)$$

where

$$[M_{12}^{ij}]_{\text{BSM}} = \frac{1}{2M_{M^0}} \sum_a C_a^{ij}(\mu) \langle Q_a^{ij} \rangle(\mu) + \mathcal{O}(\text{dim-8}), \quad (2.6)$$

³Note that here $C_{\text{VLL}}^{ij}(\mu_{\text{ew}})|_{\text{SM}}$ carries a mass dimension contrary to most literature on the SM, but in line with our choice for the BSM part of the Wilson coefficients that carry dimension $1/\text{TeV}^2$.

the central quantity in our paper, is given in terms of Wilson coefficients and hadronic matrix elements of the operators,

$$\langle Q_a^{ij} \rangle = \langle Q_a^{ij} \rangle(\mu) \equiv \langle M^0 | Q_a^{ij} | \overline{M}^0 \rangle(\mu). \quad (2.7)$$

The μ dependence and more generally renormalization-scheme dependences of Wilson coefficients and hadronic matrix elements cancel in (2.6), such that observables that depend on the M_{12}^{ij} are independent of the renormalization scheme. We will list the most important observables in section 2.4.

The hadronic matrix elements $\langle Q_a^{ij} \rangle(\mu)$ are calculated with nonperturbative methods like LQCD or QCD sum rules at the typical low energy scales $\mu = \mu_{\text{had}}$ of a few GeV set by the masses of the neutral mesons M_{M^0} . For $K^0 - \bar{K}^0$ mixing also the dual QCD (DQCD) approach is useful [35].

The leading contribution in the expansion of the ratio $\mu_{\text{had}}/\mu_{\text{ew}} \ll 1$ to the off-diagonal element M_{12}^{ij} is due to the dimension-six contributions of the $\Delta F = 2$ effective Hamiltonian at the low-energy scales as given in (2.5). In fact, this is a very good approximation for $B_{s,d} - \bar{B}_{s,d}$ mixing in the SM [36].⁴ However, in the case of $K^0 - \bar{K}^0$ mixing the strong CKM hierarchies suppress in the SM this contribution such that dimension-eight contributions are CKM-enhanced and compete numerically with it. In the SM the numerically most important contributions at dimension eight are from double-insertions of $\Delta F = 1$ dimension-six operators. Here the contributions of $\Delta F = 1$ operators with charm quarks can be still decoupled perturbatively and actually absorbed as additional $\Delta F = 2$ contributions in (2.5). However, there is a remaining dispersive contribution to M_{12}^{ij} from $\Delta F = 1$ operators with light quarks ($q = u, d, s$), which can be evaluated only with nonperturbative methods and prevents up to now the full prediction of ΔM_K in the SM and also beyond. These dimension-eight contributions, those that can still be decoupled at the charm scale and also the genuine long-distance ones, are indicated in (2.5) and (2.6). Although our objective is to provide a general anatomy of $\Delta F = 2$, our main focus will be the anatomy of the leading dimension-six term of (2.5) in SMEFT.

The Wilson coefficients $C_a^{ij}(\mu)$ in (2.6) are calculated perturbatively and are renormalised in the $\overline{\text{MS}}$ scheme because of the particularly simple RG evolution from μ_{ew} to any other scale μ . The necessary anomalous dimension matrices (ADM) are known at NLO in QCD [32] for the so-called BMU operator basis, see (2.8). It is important to bear in mind that these higher-order calculations in dimensional regularisation with $D \neq 4$ dimensions require a generalization of the four-dimensional Levi-Civita tensor, and specifically γ_5 , which can be conveniently done with the help of so-called evanescent operators [39–41] and naively anti-commuting γ_5 (NDR). The presence of evanescent operators besides the physical ones requires for example also a finite renormalization of those parts of matrix elements of evanescent operators that are proportional to physical operators. Therefore the Wilson coefficients and ADMs are dependent on the choice of evanescent operators as well. The hadronic matrix elements $\langle Q_a^{ij} \rangle$ are calculated by LQCD collaborations employing nonperturbative renormalization schemes. These results are then converted to the

⁴Constraints from $B_{s,d} - \bar{B}_{s,d}$ mixing on NP contributions in current-current operators have been studied in [37, 38].

$\overline{\text{MS}}$ -NDR including the very same evanescent operators to guarantee a cancellation of the renormalization scheme dependences in (2.5) to the NLO in QCD. The typical scale μ_{had} after conversion is of order of a few GeV, depending on the meson type. The NLO QCD corrections are particularly important at these low scales because the QCD coupling α_s is large. Of course, $\langle Q_a^{ij} \rangle(\mu)$ obey the very same RG equations as the Wilson coefficients except that the ADMs used in the latter are replaced by the transposed ones.⁵ Therefore they can be evolved to any (perturbative) scale μ , in particular μ_{ew} or even Λ .

Unfortunately there is no unique choice of the operator basis for the calculation of hadronic matrix elements in the literature, requiring basis changes at NLO in QCD, that we will explain in more detail now.

2.2 $\Delta F = 2$ operator bases

Several $\Delta F = 2$ operator bases have been chosen for various reasons in the past. We begin with the so-called BMU basis [32] for which the complete ADMs at NLO in QCD have been calculated in [32]. The BMU basis consists of $(5 + 3) = 8$ physical operators

$$\begin{aligned} Q_{\text{VLL}}^{ij} &= [\bar{d}_i \gamma_\mu P_L d_j][\bar{d}_i \gamma^\mu P_L d_j], \\ Q_{\text{LR},1}^{ij} &= [\bar{d}_i \gamma_\mu P_L d_j][\bar{d}_i \gamma^\mu P_R d_j], & Q_{\text{LR},2}^{ij} &= [\bar{d}_i P_L d_j][\bar{d}_i P_R d_j], \\ Q_{\text{SLL},1}^{ij} &= [\bar{d}_i P_L d_j][\bar{d}_i P_L d_j], & Q_{\text{SLL},2}^{ij} &= -[\bar{d}_i \sigma_{\mu\nu} P_L d_j][\bar{d}_i \sigma^{\mu\nu} P_L d_j], \end{aligned} \quad (2.8)$$

which are built exclusively out of colour-singlet currents $[\bar{d}_i^\alpha \dots d_j^\alpha][\bar{d}_i^\beta \dots d_j^\beta]$, where α, β denote colour indices. This feature is very useful for calculations in DQCD [35, 43], because the matrix elements in the large- N_c limit can be obtained directly without using Fierz identities. The chirality-flipped sectors VRR and SRR are obtained from interchanging $P_L \leftrightarrow P_R$ in VLL and SLL. Note that the minus sign in $Q_{\text{SLL},2}^{ij}$ arises from different definitions of $\tilde{\sigma}_{\mu\nu} \equiv [\gamma_\mu, \gamma_\nu]/2$ in [32] w.r.t. $\sigma_{\mu\nu} = i\tilde{\sigma}_{\mu\nu}$ used here. The ADMs of the five distinct sectors (VLL, SLL, LR, VRR, SRR) have been calculated at NLO in QCD [32], and numerical solutions for $ij = ds, bd, bs$ are given in [28].

The so-called SUSY basis [8, 44] instead uses the operators

$$\begin{aligned} Q_1^{ij} &= Q_{\text{VLL}}^{ij}, \\ Q_4^{ij} &= Q_{\text{LR},2}^{ij}, & Q_5^{ij} &= [\bar{d}_i^\alpha P_L d_j^\beta][\bar{d}_i^\beta P_R d_j^\alpha] = -\frac{1}{2}Q_{\text{LR},1}^{ij}, \\ Q_2^{ij} &= Q_{\text{SLL},1}^{ij}, & Q_3^{ij} &= [\bar{d}_i^\alpha P_L d_j^\beta][\bar{d}_i^\beta P_L d_j^\alpha] = -\frac{1}{2}Q_{\text{SLL},1}^{ij} + \frac{1}{8}Q_{\text{SLL},2}^{ij}, \end{aligned} \quad (2.9)$$

and $Q_{1',2',3'}^{ij}$ obtained from $Q_{1,2,3}^{ij}$ via $P_L \rightarrow P_R$. The relations for $Q_{3,5}^{ij}$ are the usual Fierz relations valid in $D = 4$ only. Beyond the LO evanescent operators must be added to the r.h.s. of these relations. However, as demonstrated in [39], and subsequently discussed in [40, 41], a particular definition of these operators can be made so that these operators affect only two-loop anomalous dimensions, but have no impact on one-loop matching and allow to use $D = 4$ Fierz relations in transforming one operator basis to another one. In

⁵Usually in LEFT the RG evolution of WCs is governed by transposed ADMs of the operators [42], but in the SMEFT literature these transposed ADMs are just called ADMs.

the present paper we use exclusively this definition (BMU) of the evanescent operators⁶ that is consistent with the two-loop anomalous dimensions calculated in [32] that are used in the master formulae of [28]. Therefore we did not show them explicitly in the formulae above. However, this is an important issue for the future NLO SMEFT analyses and we elaborate on it in appendix E. The ADMs of the $Q_{1,1'}^{ij}$ at NLO in QCD are identical to the results of the ones of the VLL and VRR sectors in the BMU basis.

Yet another basis is relevant for our study, which has been introduced to facilitate the classification of the complete LEFT operator basis [13] for the purpose of matching with SMEFT. We will refer to it as JMS basis. The full one-loop matching of SMEFT to LEFT in the JMS basis was recently given in [14]. The relevant $\Delta F = 2$ operators are

$$\begin{aligned}
 [Q_{dd}^{VLL}]_{ijij} &= Q_{VLL}^{ij}, \\
 [Q_{dd}^{VRR}]_{ijij} &= Q_{VRR}^{ij}, \\
 [Q_{dd}^{V1,LR}]_{ijij} &= Q_{LR,1}^{ij}, \\
 [Q_{dd}^{V8,LR}]_{ijij} &= [\bar{d}_i \gamma_\mu P_L T^A d_j] [\bar{d}_i \gamma^\mu P_R T^A d_j] = -\frac{1}{6} Q_{LR,1}^{ij} - Q_{LR,2}^{ij}, \\
 [Q_{dd}^{S1,RR}]_{ijij} &= Q_{SRR,1}^{ij}, \\
 [Q_{dd}^{S8,RR}]_{ijij} &= [\bar{d}_i P_R T^A d_j] [\bar{d}_i P_R T^A d_j] = -\frac{5}{12} Q_{SRR,1}^{ij} + \frac{1}{16} Q_{SRR,2}^{ij},
 \end{aligned} \tag{2.10}$$

where T^A are $SU(3)_c$ colour generators of the fundamental representation. Note that $([Q_{dd}^{S1,RR}]_{ijij})^\dagger = Q_{SLL,1}^{ij}$ etc. To make use of the one-loop matching results in the JMS basis and connect them with the hadronic matrix elements from LQCD collaborations it is hence necessary to transform the Wilson coefficients in the JMS basis to the BMU (or SUSY) basis in order to cancel the scheme dependence of hadronic matrix elements at NLO in QCD. We collect these relations in appendix C.

Eventually we note that the four operators $Q_{1,2}^{SLL}$ and $Q_{1,2}^{SRR}$ violate hypercharge and, although allowed by $SU(3)_c \times U(1)_{em}$, cannot be generated at and above the EW scale in the context of SMEFT and moreover cannot be generated through RG evolution.

2.3 Hadronic matrix elements

The matrix elements $\langle Q_a^{ij} \rangle$ in (2.6) are provided by LQCD collaborations that present results either for the BMU or the SUSY basis. The matrix elements of the LR and SLL/SRR sectors are chirally enhanced compared to the VLL/VRR sector, as the latter vanish in the chiral limit. The corresponding chiral enhancement factor

$$r_\chi^{ij}(\mu) \equiv \frac{(f_{M^0} M_{M^0})^{-2}}{\langle M^0 | \bar{d}_i \gamma_5 d_j | 0 \rangle \langle 0 | \bar{d}_i \gamma_5 d_j | M^0 \rangle} \stackrel{\text{(VIA)}}{\approx} \left(\frac{M_{M^0}}{m_i(\mu) + m_j(\mu)} \right)^2 \tag{2.11}$$

is related to the meson decay constant f_{M^0} and the overlap of the scalar densities with the meson states. It is renormalization-scheme dependent and involves in the vacuum insertion

⁶There is a second choice of evanescent operators known as BBGLN [45] in the context of SM calculations of life times and decay width differences for B_q mesons, which does not preserve Fierz relations beyond LO QCD. The transformation from the BMU to the BBGLN basis at NLO QCD is given in [46].

approximation (VIA) and DQCD approach the $\overline{\text{MS}}$ quark masses. It becomes especially large for $K^0 - \bar{K}^0$ mixing.

Usually the LQCD collaborations prefer not to calculate directly $\langle Q_a^{ij} \rangle$, but rather ratios, as for example [47]

$$R_a^{ij}(\mu) \equiv \frac{\langle Q_a^{ij} \rangle(\mu)}{\langle Q_{\text{VLL}}^{ij} \rangle(\mu)}, \quad a \neq \text{VLL}, \quad (2.12)$$

which exhibit a cancellation of LQCD-specific systematic uncertainties. The $R_a^{ij}(\mu)$ advantageously include the nonperturbative evaluation of r_χ^{ij} .

However, for historical reasons the $\langle Q_a^{ij} \rangle(\mu)$ are expressed often also in terms of bag factors $B_a^{ij}(\mu)$ that are unity in the VIA, i.e. they quantify the deviation from VIA. This allows also for getting insight in their LQCD values for $K^0 - \bar{K}^0$ mixing with the help of DQCD [35]. The bag parameters are also subject to cancellation of systematic uncertainties in LQCD calculations [48, 49]. There are different conventions for the various neutral meson systems, but all are for the SUSY basis

$$\begin{aligned} \langle Q_1^{ij} \rangle(\mu) &= \frac{2}{3}(F_{M^0} M_{M^0})^2 B_1^{ij}(\mu), \\ \langle Q_a^{ij} \rangle(\mu) &= N_a^{ij} \left(r_\chi^{ij} + d_a^{ij} \right) (F_{M^0} M_{M^0})^2 B_a^{ij}(\mu), \quad a \in (2, 3, 4, 5) \end{aligned} \quad (2.13)$$

with $N_a^{ij} = (-5/12, 1/12, 1/2, 1/6)$. The B_1^{ds} is the well-known B_K parameter of $K^0 - \bar{K}^0$ mixing. Further, the constants $d_a^{ij} = 0$ for $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ [50] mixing, whereas $d_a^{ib} = (0, 0, 1/6, 3/2)$ for $B_{s,d} - \bar{B}_{s,d}$ mixing [49, 51].

It is obvious that the bag factors themselves are not sufficient to calculate the $\langle Q_a^{ij} \rangle$, but require the knowledge of r_χ^{ij} . For example, FNAL/MILC calculates $\langle Q_a^{ij} \rangle$ for $B_{s,d} - \bar{B}_{s,d}$ [51] and $D^0 - \bar{D}^0$ [52] mixing directly and converts them to bag factors using the VIA form of r_χ^{ib} in (2.11) with fixed numerical values of the quark masses and decay constant. In this way r_χ^{ib} can be regarded as a fixed numerical convenience factor that in principle would not introduce additional uncertainties related to quark masses and decay constant, despite being scheme dependent. Consequently, in all applications then strictly the numerical values of quark masses and decay constants used by FNAL/MILC must be used as well for the conversion of $B_a^{ij} \rightarrow \langle Q_a^{ij} \rangle$. On the other hand the Flavour Lattice Averaging Group (FLAG) [44] provides currently $K^0 - \bar{K}^0$ bag factors without r_χ^{ds} , and hence in phenomenological predictions the applicant is forced to introduce unknown systematic uncertainties when using the VIA approximation of r_χ^{ds} together with the parametric uncertainties from the quark masses and the decay constants. The latter two quantities are usually obtained from different dedicated LQCD calculations that in principle involve different systematic uncertainties as well. Therefore it is important that the FLAG provides also the r_χ^{ds} or alternatively directly averages of the matrix elements $\langle Q_a^{ij} \rangle$ or the ratios R_a^{ij} . The summary of the current status is as follows.

- In the case of $K^0 - \bar{K}^0$ mixing, FLAG provides a summary [44] of the available results for the bag factors B_a^{ds} ($a = 1, 2, 3, 4, 5$) of the SUSY basis.⁷ They are from ETM

⁷Note that we use the convention $ij = ds$ as opposed to FLAG's $ij = sd$, which complies with the convention in WCxf [53].

for $N_f = 2$ [54] and for $N_f = 2 + 1 + 1$ from [50], as well as the $N_f = 2 + 1$ average from the SWME [48] and RBC-UKQCD [55, 56] LQCD collaborations. We will use here the FLAG average for $N_f = 2 + 1$, for further details see also [44, 57]. We note that SWME provides results for the BMU basis, which have been converted to the SUSY basis by FLAG. In principle RBC-UKQCD provides also the ratios R_a^{ds} , which do not require the knowledge of r_χ^{ds} . We will use the averages of the $N_f = 2 + 1$ bag factors from FLAG.

- For the case of $B_{s,d} - \bar{B}_{s,d}$ mixing, FLAG provides only averages for the bag factor B_1^{ib} of the SM operator. However, the full set of matrix elements has been calculated with LQCD methods for $N_f = 2$ by ETM [58], $N_f = 2 + 1$ by FNAL/MILC [51] and for $N_f = 2 + 1 + 1$ by HPQCD [49]. Further, the bag factors were also calculated with sum rules [59–62] and the average of LQCD (except [58]) and sum rule results can be found in [63]. We use the averages of the bag factors from HPQCD and FNAL/MILC, as given in [49].
- For the case of $D^0 - \bar{D}^0$ mixing, ETM calculated bag factors for $N_f = 2 + 1 + 1$ [50] and more recently FNAL/MILC the matrix elements for $N_f = 2 + 1$ [52] for the full set of $\Delta C = 2$ operators (BBGLN and BMU). Both results are consistent with largest tensions for bag factors $B_{4,5}^{cu}$. The sum rule determination [60] suffers from larger uncertainties and is consistent with the LQCD determinations. We prefer to use the more recent direct determinations of matrix elements from FNAL/MILC [52] over the bag factors of ETM [50].

The numerical values of the $\Delta F = 2$ nonperturbative input for $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B_{s,d} - \bar{B}_{s,d}$ mixing is collected in table 2. There we provide the matrix elements in the $\overline{\text{MS}}\text{-NDR}$ scheme at a low-energy scale μ_{had} for the SUSY basis, which can be converted to the BMU basis with the help of (2.9). The number of flavours, N_f , in table 2 gives the starting number of active flavours used in the running of α_s for the RG evolution.

In the absence of any information on the chiral enhancement factor from FLAG, we use for the conversion of the $K^0 - \bar{K}^0$ bag factors to matrix elements the $N_f = 2 + 1$ $\overline{\text{MS}}$ quark masses from FLAG [44] together with the Kaon decay constant and masses [64] listed in table 1. The chiral enhancement factor is $r_\chi^{ds} = 33.44(78)$ at $\mu_{\text{had}} = 3.0 \text{ GeV}$ for $N_f = 3$. Note that B_1^{ds} is given by FLAG at $\mu = 2.0 \text{ GeV}$, whereas all the other bag factors $B_{2,3,4,5}^{ds}$ at $\mu_{\text{had}} = 3.0 \text{ GeV}$, thus we have evolved $\langle Q_1^{ds} \rangle$ to that scale as well.

The conversion of the averaged bag factors for $B_{s,d} - \bar{B}_{s,d}$ mixing given in [49] to matrix elements is done with the very same values of the $\overline{\text{MS}}$ quark masses as given in table 1 of [49]

$$\overline{m}_b(\overline{m}_b) = 4.162(48) \text{ GeV}, \quad \frac{\overline{m}_b}{\overline{m}_s} = 52.55(55), \quad \frac{\overline{m}_s}{\overline{m}_d} = 27.18(10), \quad (2.14)$$

the B_q -meson decay constants from [71] and B_q -meson masses in table 1. Thus, we use the very same values as HPQCD to calculate the chiral enhancement factors $r_\chi^{db} = 1.607(37)$ and $r_\chi^{sb} = 1.601(37)$ at $\mu_{\text{had}} = 4.16 \text{ GeV}$.

Parameter	Value	Ref.	Parameter	Value	Ref.
$\alpha_s^{(5)}(m_Z)$	0.1181(11)	[64]	m_Z	91.1876(21) GeV	[64]
M_{K^0}	497.611(13) MeV	[64]	f_K/f_π	1.194(5)	[44]
ΔM_K	$3.484(6) \cdot 10^{-15}$ GeV	[64]	f_π	130.41(20) MeV	[64]
M_{D^0}	1864.83(5) MeV	[64]	τ_{D^0}	$4.101(15) \cdot 10^{-13}$ s	[64]
ΔM_D	$6.3(^{+1.8}_{-1.9}) \cdot 10^{-15}$ GeV		x	$0.39(^{+11}_{-12}) \cdot 10^{-2}$	[64]
M_{B_s}	5366.88(17) MeV	[64]	M_{B_d}	5279.64(13) MeV	[64]
ΔM_{B_s}	$1.1683(13) \cdot 10^{-11}$ GeV	[64]	ΔM_{B_d}	$3.334(13) \cdot 10^{-13}$ GeV	[64]
f_{B_s}	230.3(1.3) MeV	[44]	f_{B_d}	190.0(1.3) MeV	[44]
$\overline{m}_s(2 \text{ GeV})$	92.0(1.1) MeV	[44]	$\overline{m}_d(2 \text{ GeV})$	4.67(9) MeV	[44]

Table 1. Numerical input values for parameters entering the conversion of bag factors to matrix elements and the LEFT master formula (2.15). The values of the strange- and down-quark masses in the $\overline{\text{MS}}$ scheme are the $N_f = 2 + 1$ averages of lattice determinations from the FLAG group from [65–70]. The B_q -meson decay constants f_{B_q} are averages from the FLAG group for $N_f = 2 + 1 + 1$ from [71–74]. They are almost identical to the single determination of FNAL/MILC $f_{B_s} = 230.7(1.3)$ MeV and $f_{B_d} = 190.5(1.3)$ MeV [71]. We determine $\Delta M_D = x/\tau_{D^0}$ with the value of x from a global fit when allowing CP violation in all decays [64], for comparison it would be $x = 0.50(^{+13}_{-14}) \cdot 10^{-2}$ assuming no CP violation.

ij	μ_{had} [GeV]	N_f	$\langle Q_1^{ij} \rangle$ [GeV ⁴]	$\langle Q_2^{ij} \rangle$ [GeV ⁴]	$\langle Q_3^{ij} \rangle$ [GeV ⁴]	$\langle Q_4^{ij} \rangle$ [GeV ⁴]	$\langle Q_5^{ij} \rangle$ [GeV ⁴]
sd	3.0	3	0.002156(34)	−0.0420(16)	0.0128(6)	0.0930(30)	0.0241(14)
cu	3.0	4	0.0806(56)	−0.1442(72)	0.0452(31)	0.2745(140)	0.1035(74)
db	4.16	5	0.56(2)	−0.53(3)	0.106(8)	0.96(5)	0.51(2)
sb	4.16	5	0.86(3)	−0.85(5)	0.174(11)	1.40(6)	0.74(3)

Table 2. The values of the matrix elements in the SUSY basis in the $\overline{\text{MS}}$ -NDR scheme at the low-energy scale μ_{had} for number of flavours N_f .

The RG evolution of the matrix elements depends strongly on the running coupling α_s , for which we use the initial value $\alpha_s^{(5)}(m_Z)$ with $N_f = 5$ given in table 1 and three-loop equations for the RG evolution. The quark-threshold crossings to $N_f = 4$ is set to $\mu_4 = 4.2$ GeV and for $N_f = 3$ to $\mu_3 = 1.3$ GeV, whereas when going to $N_f = 6$ we use $\mu_5 = \mu_{\text{ew}} = 160$ GeV.

Inspecting table 2 we observe the following pattern:

- The matrix elements of LR operators are in the $K^0 - \bar{K}^0$ system much larger than the matrix element of the SM VLL operator due to the chiral enhancement. In particular $\langle Q_{\text{LR},2}^{ds} \rangle = \langle Q_4^{ds} \rangle$ is very large with significant impact on phenomenology as known already for decades [31, 32, 75, 76].

BMU	P_{VLL}	$P_{SLL,1}$	$P_{SLL,2}$	$P_{LR,1}$	$P_{LR,2}$	units
K^0	0.102(2)	-4.32(16)	-7.93(37)	-8.55(28)	14.14(82)	10^7 TeV^2
D^0	0.56(4)	-2.20(11)	-4.04(28)	-4.23(22)	6.18(44)	10^7 TeV^2
B_d	2.67(10)	-4.99(28)	-9.05(68)	-10.29(54)	12.75(50)	10^5 TeV^2
B_s	1.15(4)	-2.24(13)	-4.08(26)	-4.20(18)	5.22(21)	10^4 TeV^2
SUSY	P_1	P_2	P_3	P_4	P_5	units
K^0	0.102(2)	-4.32(16)	1.09(5)	14.14(82)	4.28(14)	10^7 TeV^2
D^0	0.56(4)	-2.20(11)	0.56(4)	6.18(44)	2.12(11)	10^7 TeV^2
B_d	2.67(10)	-4.99(28)	1.12(8)	12.75(50)	5.15(27)	10^5 TeV^2
B_s	1.15(4)	-2.24(13)	0.51(3)	5.22(21)	2.10(9)	10^4 TeV^2
JMS	P_{dd}^{VLL}	$P_{dd}^{S1,RR}$	$P_{dd}^{S8,RR}$	$P_{dd}^{V1,LR}$	$P_{dd}^{V8,LR}$	units
K^0	0.102(2)	-4.32(16)	1.31(5)	-8.55(28)	-12.72(74)	10^7 TeV^2
D^0	0.56(4)	-2.20(11)	0.66(3)	-4.23(22)	-5.48(39)	10^7 TeV^2
B_d	2.67(10)	-4.99(28)	1.51(9)	-10.29(54)	-11.04(44)	10^5 TeV^2
B_s	1.15(4)	-2.24(13)	0.68(4)	-4.20(18)	-4.52(18)	10^4 TeV^2

Table 3. The values of the coefficients $P_a^{ij}(\mu_{\text{ew}})$ entering the LEFT master formula (2.15) at $\mu_{\text{ew}} = 160 \text{ GeV}$ in the BMU, SUSY and JMS bases, using as input the $\overline{\text{MS}}$ -NDR matrix elements at the low-energy scale μ_{had} from table 2. The shown uncertainties are due to the either matrix elements or the bag factors and their corresponding chiral enhancement factors.

- While this pattern is also seen to some extent in the charm system it is practically absent at these scales in the $B_{s,d} - \bar{B}_{s,d}$ systems where chiral enhancement is absent.
- However, as we will see in the master formulas for M_{12}^{ij} in LEFT, and in particular in SMEFT, the hierarchy in question is further enhanced in the $K^0 - \bar{K}^0$ system through RG effects. Even in the $B_{s,d} - \bar{B}_{s,d}$ and charm systems the matrix elements of the LR operators and consequently the coefficients in the master formulae are significantly larger at these high scales than the SM one. This feature is known from [77, 78].

2.4 LEFT master formula

Hadronic matrix elements from LQCD, DQCD and sum rules are usually obtained at low energy scales μ_{had} of a few GeV. However, for a transparent study of NP contributions that are generated at higher scales it is useful to evaluate these matrix elements at the EW scale μ_{ew} , the largest scale of validity of LEFT. This can be done by means of RG methods as explained in great detail in [28]. In this section we adapt these results to derive the first master formula for $\Delta F = 2$ processes given in terms of LEFT Wilson coefficients evaluated at the EW scale. The one involving SMEFT Wilson coefficients will be presented in section 3.3.

We find it convenient to present the numerical results in the form of the master formula

$$2[M_{12}^{ij}]_{\text{BSM}} = (\Delta M_{ij})_{\text{exp}} \sum_a P_a^{ij}(\mu_{\text{ew}}) C_a^{ij}(\mu_{\text{ew}}). \quad (2.15)$$

The normalization to the experimental value of $(\Delta M_{ij})_{\text{exp}}$ allows easily to infer the size of the BSM Wilson coefficients $C_a^{ij}(\mu_{\text{ew}})$ that would generate a certain fraction of this measured value, in view of the known numerical values of the coefficients $P_a^{ij}(\mu_{\text{ew}})$, which are collected in table 3. The $P_a^{ij}(\mu_{\text{ew}}) = \langle Q_a^{ij} \rangle(\mu_{\text{ew}}) / (M_{M^0}(\Delta M_{ij})_{\text{exp}})$ are given in terms of the matrix elements at the EW scale as follows from (2.6). They are related to the $[P_a^{ij}(\mu_{\text{ew}})]^{\text{BJU}}$ from [28] as

$$P_a^{ij}(\mu_{\text{ew}}) = \frac{2}{3} \frac{M_{M^0} f_{M^0}^2}{(\Delta M_{ij})_{\text{exp}}} [P_a^{ij}(\mu_{\text{ew}})]^{\text{BJU}}. \quad (2.16)$$

The expressions of $[P_a^{ij}(\mu_{\text{ew}})]^{\text{BJU}}$ and $P_a^{ij}(\mu_{\text{ew}})$ summarize the RG evolution from the low-energy scale μ_{had} and the matrix elements $\langle Q_a^{ij} \rangle(\mu_{\text{had}})$ from table 2, such that the μ_{had} dependence cancels [28].

For the numerical evaluation of the RG evolution of the matrix elements at NLO in QCD, we use for $B_{s,d} - \bar{B}_{s,d}$ mixing the initial scale and $N_f = 5$ as given in table 2. In the case of $D^0 - \bar{D}^0$ mixing the RG evolution starts at $\mu = 3.0 \text{ GeV}$ with $N_f = 4$ and is switched to $N_f = 5$ at $\mu_4 = 4.2 \text{ GeV}$. In the case of $K^0 - \bar{K}^0$ mixing the evolution is done first with $N_f = 3$ from $\mu = 3.0 \text{ GeV}$ down to $\mu_3 = 1.3 \text{ GeV}$ and only then we switch to $N_f = 4$ to evolve up in scale to μ_{ew} , with the intermediate threshold crossing to $N_f = 5$ at μ_4 . The NLO QCD corrections always lead to an increase of the LO results for the P_a^{ij} of about $\{1.5 - 1.9, 4.7 - 6.0, 2.0 - 2.7, 6.0 - 10.4, 9.7 - 11.4\}\%$ for $a = \{\text{VLL}, \text{SLL1}, \text{SLL2}, \text{LR1}, \text{LR2}\}$ in the BMU basis, with smallest numbers for $B_{s,d} - \bar{B}_{s,d}$ mixing and largest for $K^0 - \bar{K}^0$ mixing. The size of the NLO corrections is up to roughly a factor of two larger than the current hadronic uncertainties due to the matrix elements. The effect is also shown in figure 1 for $K^0 - \bar{K}^0$ and $B_s - \bar{B}_s$ mixing.

The formulae for observables in terms of these matrix elements can be found in many papers, in particular in [1, 2, 38, 79–81]. Here we recall just the general dependence on M_{12}^{ij}

$$ij = ds : \quad \Delta M_K = 2 \text{Re}(M_{12}^{ds}), \quad \varepsilon_K \propto \text{Im}(M_{12}^{ds}), \quad (2.17)$$

$$ij = ib : \quad \Delta M_{B_i} = 2|M_{12}^{ib}|, \quad \phi_i = \text{Arg}(M_{12}^{ib}) + \dots, \quad (2.18)$$

where ΔM_K denotes the mass difference of the K_L and K_S , ε_K is the parameter characterizing indirect CP violation in the Kaon sector, ΔM_{B_i} denotes the mass difference of the B -meson mass eigenstates and ϕ_i are the CP violating phases in the B sector. We remind the reader that $\Delta M_{K,D}$ receive also substantial long-distance corrections. Further, we have assumed that SM QCD penguin pollution and new physics in $b \rightarrow s c \bar{c}$ processes are negligible in the relations of ϕ_i , indicated by the dots.

The magnitude of the dimensionfull $P_a^{ij} \sim \mu^2$ must be cancelled by the $C_a^{ij} \sim (\mu')^{-2}$, such that their ratio $\mu/\mu' < 1$ does not lead to excessive BSM contributions to $[M_{12}^{ij}]_{\text{BSM}}$ that would be ruled out by observations. In the case of BSM contributions at tree-level the

Figure 1. The QCD RG evolution at NLO [solid] versus LO [dashed] for some of the coefficients $P_a^{ij}(\mu)$ for $K^0 - \bar{K}^0$ [left] and $B_s - \bar{B}_s$ mixing [right]. The coloured band around the NLO results shows the hadronic uncertainties from the matrix elements. The vertical dashed line indicates $\mu_{\text{ew}} = 160 \text{ GeV}$ and for $K^0 - \bar{K}^0$ mixing also the $N_f = 4, 5$ flavour thresholds. For scales larger than μ_{ew} the full SMEFT RG evolution should be used in principle, including Yukawa and the full SM gauge sector contributions.

μ' can be interpreted as the mass scale Λ of the new physics. It is instructive to calculate the size of C_{VLL}^{ij} in the SM using (2.2). The normalization factor and the universal one-loop function entering down-type meson mixing are of order $\mathcal{N}S_0(x_t) \approx 0.0518 \text{ TeV}^{-2}$. The CKM combinations entering $B_{s,d} - \bar{B}_{s,d}$ and $K^0 - \bar{K}^0$ mixing found from a SM CKM fit are $|(V_{tb}V_{td}^*)^2| \approx 8.0 \cdot 10^{-5}$, $|(V_{tb}V_{ts}^*)^2| \approx 1.7 \cdot 10^{-3}$ and $(V_{td}^*V_{ts})^2 \approx (9.5 + 9.9i) \cdot 10^{-8}$.⁸ Then the corresponding inverse, now given in the BSM normalization, is found to be

$$(C_{\text{VLL}}^{ij})_{\text{SM}}^{-1} = \frac{1}{\mathcal{N}S_0(x_t)|(\lambda_t^{ij})^2|} \approx \begin{cases} 2.0 \cdot 10^8 \text{ TeV}^2 & ij = ds \\ 2.4 \cdot 10^5 \text{ TeV}^2 & ij = db \\ 1.1 \cdot 10^4 \text{ TeV}^2 & ij = sb \end{cases} \quad (2.19)$$

It shows that for $B_{s,d} - \bar{B}_{s,d}$ mixing the short-distance SM contribution yields to a good accuracy the experimental value as can be seen by comparison with $P_{\text{VLL}}^{db} = 2.67 \cdot 10^5 \text{ TeV}^2$ and $P_{\text{VLL}}^{sb} = 1.15 \cdot 10^4 \text{ TeV}^2$ from table 3. On the other hand for $K^0 - \bar{K}^0$ mixing the “experimental” $P_{\text{VLL}}^{ds} = 0.10 \cdot 10^7 \text{ TeV}^2$ in table 3 is not made up by the short-distance SM contribution of the top-quark in (2.19) alone, which is only about 0.5% of the measured value of $\Delta M_K = 2 \text{ Re}(M_{12}^{ds})$. Similarly $\epsilon_K \propto \text{Im}(M_{12}^{ds})$ is not nearly reproduced by the top-quark contribution alone. Here there are also charm-top and charm-charm contributions, as reanalysed recently in the SM at NNLO in QCD [82].

We have completed the calculation of the dynamics below the EW scale by obtaining the values of the coefficients $P_a^{ij}(\mu_{\text{ew}})$ accompanying the LEFT Wilson coefficients

⁸The CKM input is found from the Wolfenstein parameters $\lambda = 0.22453(44)$, $A = 0.836(15)$, $\bar{\rho} = 0.122(^{+18}_{-17})$ and $\bar{\eta} = 0.355(^{+12}_{-11})$ [64], respectively.

$C_a^{ij}(\mu_{\text{ew}})$. In the next section we will present a detailed SMEFT anatomy of the corresponding coefficients $P_a^{ij}(\Lambda)$ that collect the information about the SMEFT dynamics up to the new physics scale Λ .

3 SMEFT anatomy of $\Delta F = 2$ processes

3.1 Preliminaries

The main assumption inherent to the SMEFT framework is that NP interactions have been integrated out at some high scale $\Lambda \gg \mu_{\text{ew}}$ above the electroweak scale. The field content of the SMEFT Lagrangian

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \sum_a \mathcal{C}_a(\mu) \mathcal{O}_a \quad (3.1)$$

are the SM fields and the interactions are locally invariant under the SM gauge group $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$. Here $\mathcal{L}_{\text{SM}}^{(4)}$ is the renormalizable part known from the SM, whereas the $\mathcal{O}_a$ are higher-dimensional ($\text{dim} = 5, 6$) operators that parametrize the effects of new physics. We use here the non-redundant set of operators as classified in [30], also known as the “Warsaw” basis. The dimension-four part $\mathcal{L}_{\text{SM}}^{(4)}$ contains all the couplings known from the SM, however, their numerical values can be significantly altered in the presence of higher-dimensional SMEFT operators, i.e. nonvanishing Wilson coefficients $\mathcal{C}_a(\mu)$.

In SMEFT it is convenient to work above μ_{ew} in the unbroken $\text{SU}(2)_L \times \text{U}(1)_Y$ phase, however electroweak symmetry breaking (EWSB) is taking place at μ_{ew} and it is more convenient to transform gauge bosons and fermions from the weak to their mass eigenstates, see details on SMEFT-specific modifications due to the presence of higher-dimensional operators in [83]. The weak eigenstates of down- and up-quarks have in general both flavour-off-diagonal⁹ mass matrices, but the flavour-symmetry of the kinetic term in SMEFT allows for some freedom in the choice of the weak eigenstates of quarks. This freedom can be used in particular to diagonalize in flavour space the mass term of either the down-type quarks or the up-type quarks already at the NP scale Λ , to which we refer as *down-basis* and *up-basis*, respectively. In principle, there are many valid basis choices, which are related through unitary rotations in flavour space. Since we are interested in FCNC observables in the down- and up-sector we will however focus in our discussion on the two aforementioned bases. For example the transition to the down-basis significantly simplifies phenomenology of the corresponding down-type quark transitions but is a rather inconvenient choice for up-type quark transitions and the same holds vice versa for the up-basis and up-type quark transitions. Usually the rotation from a general weak basis to the down- or up-basis is done at the electroweak scale, but our choice at the new physics scale seems more appropriate in the context of matching a UV completion onto SMEFT. Note that in SMEFT the mass term consists of dimension-four Yukawa couplings and a dimension-six contribution, and hence in SMEFT their sum is chosen diagonal. This fixes then also the definition of all SMEFT Wilson coefficients as explained in more detail in [84]. Throughout we denote Wilson coefficients in the down-basis by $\mathcal{C}_a$ and those of the up-basis are denoted with a hat as $\hat{\mathcal{C}}_a$.

⁹We use here “flavour” as synonymous to “generation” of fermions.

The RG evolution of the dimension-four couplings in $\mathcal{L}_{\text{SM}}^{(4)}$ and of the Wilson coefficients $\mathcal{C}_a(\mu)$ from the scale Λ to μ_{ew} is governed by the ADMs in SMEFT. Given some initial coefficients $\mathcal{C}_a(\Lambda)$, they can be evolved down to μ_{ew} , thereby resumming leading logarithmic (LL) effects to all orders in the quartic Higgs, gauge and Yukawa couplings into $\mathcal{C}_b(\mu_{\text{ew}})$. The formal solution of the coupled ordinary first order differential equations is given as

$$\mathcal{C}_b(\mu_{\text{ew}}) = \sum_a U_{ba}(\mu_{\text{ew}}, \Lambda) \mathcal{C}_a(\Lambda), \quad (3.2)$$

which can be obtained in the most general case numerically. The following comments should be made.

- The matrix $U_{ba}(\mu_{\text{ew}}, \Lambda)$ is the RG evolution matrix. It is presently known from the one-loop ADMs of the Warsaw basis [83, 85, 86]. At NLO it requires the calculation of the two-loop ADMs, of which some results, in particular for QCD, are scattered over the literature and recent discussions can be found in [87, 88]. At NLO $U_{ba}(\mu_{\text{ew}}, \Lambda)$ depends on the renormalization scheme used for the evaluation of two-loop ADMs.
- In order to cancel the renormalization scheme and scale dependences in $\mathcal{C}_a(\Lambda)$ around the Λ scale the matching between a given NP model (UV completion) and SMEFT has to include tree- and one-loop corrections. The $\mathcal{C}_a(\Lambda)$ carry then dependence on the fundamental parameters of the NP model.
- The RG evolution of the dimension-four couplings has to be performed in the presence of nonvanishing $\mathcal{C}_a(\Lambda)$.

In particular the RG evolution will reintroduce flavour off-diagonal entries in the mass terms at μ_{ew} , which can be undone in principle with an additional *back-rotation* [25] to the down- or up-basis at μ_{ew} . We will return soon to this issue.

Subsequently, the SMEFT is matched on the LEFT when decoupling the heavy W and Z bosons, the Higgs boson and the top-quark. In this matching the LEFT Wilson coefficients $\mathcal{C}_d(\mu_{\text{ew}})$ are determined in terms of the SMEFT Wilson coefficients $\mathcal{C}_b(\mu_{\text{ew}})$ at the electroweak scale. The $\mathcal{L}_{\text{SM}}^{(4)}$ part is responsible for the known SM expressions of the LEFT Wilson coefficients, for which often higher order QCD and partially also EW radiative corrections are known. The effects of NP parameterized by the higher-dimensional operators in the matching is known nowadays at tree-level [13] and at one-loop level [14].¹⁰ As mentioned before, this is done in the broken phase in terms of mass eigenstates of the Higgs and gauge bosons and also fermions. In the literature the matching is done in terms of SMEFT Wilson coefficients in either down- or up-basis at μ_{ew} , in particular the one-loop results [14] are given for the up-basis. The transformation of the SMEFT Wilson coefficients between down- and up-basis is governed entirely by the quark-mixing matrix (CKM matrix) at μ_{ew} [84]. As can be seen, the back-rotation at μ_{ew} to either the down- or the up-basis is required to make use of these matching results. The phenomenological impact of the back-rotation on B -physics observables, including $B_{s,d} - \bar{B}_{s,d}$ mixing, has

¹⁰Numerous partial results have been known in the literature before.

been discussed in [25]. The matching equation for the BSM contribution to a LEFT Wilson coefficient is¹¹

$$C_d(\mu_{\text{ew}}) = \sum_{b \in B} M_{db}^{(0)}(\mu_{\text{ew}}) \mathcal{C}_b(\mu_{\text{ew}}) + \sum_{c \in C} M_{dc}^{(1)}(\mu_{\text{ew}}) \mathcal{C}_c(\mu_{\text{ew}}) + \dots, \quad (3.3)$$

where $M^{(0)}(\mu_{\text{ew}})$ and $M^{(1)}(\mu_{\text{ew}})$ denote the tree-level and one-loop threshold corrections between SMEFT and LEFT, see [13] and [14] respectively. The following comments are in order:

- The combination of (3.2) with (3.3) expresses the LEFT coefficients in terms of the SMEFT coefficients at the NP scale Λ . Note that the second term in (3.3) is a NLO correction, which requires in principle to include in the RG solution (3.2) the two-loop ADMs, which are not fully available yet.
- The one-loop threshold corrections $M_{dc}^{(1)}(\mu_{\text{ew}})$ depend on logarithms $\ln(m_i/\mu_{\text{ew}})$ with $i = W, Z, t, h$ that cancel μ_{ew} dependences present in the tree-level term $M_{db}^{(0)}(\mu_{\text{ew}}) U_{ba}(\mu_{\text{ew}}, \Lambda) \mathcal{C}_a(\Lambda)$, and replacing them by large logarithms $\ln(m_i/\Lambda)$, which are resummed in $U_{ba}(m_i, \Lambda)$.
- In particular all scheme dependences related to the top-Yukawa coupling cancel up to neglected higher order effects. This is the case because the top quark is integrated out at μ_{ew} and therefore the cancellation has to happen at this scale.
- In the case of $\Delta F = 2$ Wilson coefficients $C_d(\mu_{\text{ew}})$, its dependence on the QCD renormalization scheme and μ_{ew} , given by $M_{dc}^{(1)}(\mu_{\text{ew}})$, cancels the one present in $P_d^{ij}(\mu_{\text{ew}})$ of the LEFT master formula (2.15).
- It is well understood that the set of operators C contains all operators that mix at one-loop into set B to guarantee the aforementioned renormalization scheme independence. However, set C can contain in principle additional operators that do not mix into set B .

Based on these preliminaries, we explain in the following the extension of the LEFT master formula (2.15) of $[M_{12}^{ij}]_{\text{BSM}}$ to the one of SMEFT

$$2[M_{12}^{ij}]_{\text{BSM}} = (\Delta M_{ij})_{\text{exp}} \sum_a P_a^{ij}(\Lambda) [\mathcal{C}_a]_{ij}(\Lambda) = (\Delta M_{ij})_{\text{exp}} \sum_a P_a^{ij}(\Lambda) \frac{[c_a]_{ij}(\Lambda)}{\Lambda^2} \quad (3.4)$$

in terms of all relevant SMEFT Wilson coefficients at the NP scale Λ of the operators collected in table 4. Here the $P_a^{ij}(\Lambda)$ generalize the $P_a^{ij}(\mu_{\text{ew}})$ of (2.15) by including effects in (3.2) and (3.3). Now the sum over a has to be taken over SMEFT Wilson coefficients at the scale Λ . For later convenience we introduce also dimensionless Wilson coefficients $[c_a]_{ij}(\Lambda)$. To begin with we will use approximate solutions of the RG evolution to explain the most important contributions for $\Delta F = 2$ processes in SMEFT. The actual calculation

¹¹We have reserved the index “ a ” for SMEFT coefficients at Λ so that we use in this section the index “ d ” for LEFT coefficients.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{u}_s \gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{d}_s \gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{\ell}_p \gamma_\mu \tau^I \ell_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$\psi^2 X \phi$		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$\mathcal{O}_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\phi} W_{\mu\nu}^I$	$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$(\bar{L}R)(\bar{R}L) \text{ and } (\bar{L}R)(\bar{L}R)$		$\psi^2 \phi^2 D$			
$\mathcal{O}_{ledq}$	$(\bar{\ell}_p^j e_r)(\bar{d}_s^k q_t^j)$	$\mathcal{O}_{\phi q}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{q}_p \gamma^\mu q_r)$		
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \epsilon_{jk} (\bar{q}_s^k d_t)$	$\mathcal{O}_{\phi q}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^I \phi)(\bar{q}_p \tau^I \gamma^\mu q_r)$		
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \epsilon_{jk} (\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{\phi u}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{u}_p \gamma^\mu u_r)$		
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{\ell}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t)$	$\mathcal{O}_{\phi d}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{d}_p \gamma^\mu d_r)$		
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{\ell}_p^j \sigma_{\mu\nu} e_r) \epsilon_{jk} (\bar{q}_s^k \sigma^{\mu\nu} u_t)$	$\mathcal{O}_{\phi ud}$	$(\phi^\dagger i D_\mu \phi)(\bar{u}_p \gamma^\mu d_r)$		

Table 4. The SMEFT operators entering the master formulae.

of the $P_a^{ij}(\Lambda)$ is based on all known results and has to be done numerically, as explained in more detail in section 3.3. As an example we will present the “SMEFT atlas” of $[M_{12}^{ij}]_{\text{BSM}}$ for the specific scale $\Lambda = 5 \text{ TeV}$ in section 3.4. The case of $\Lambda = 100 \text{ TeV}$ is presented in appendix A.

3.2 $\Delta F = 2$ processes in SMEFT

The most important Wilson coefficients of SMEFT operators that enter (3.3) for $\Delta F = 2$ processes are

$$B = \left\{ \mathcal{C}_{qq}^{(1)}, \quad \mathcal{C}_{qq}^{(3)}, \quad \mathcal{C}_{qa}^{(1)}, \quad \mathcal{C}_{qa}^{(8)}, \quad \mathcal{C}_{aa} \right\}, \quad (3.5)$$

in the down ($a = d$) and up ($a = u$) sector, respectively [84].

At tree-level for $B_{s,d} - \bar{B}_{s,d}$ and $K^0 - \bar{K}^0$ mixing one finds the following matching conditions at μ_{ew} in the **down-basis**

$$\begin{aligned} [C_{dd}^{V,LL}]_{ijij} &= -[\mathcal{C}_{qq}^{(1)}]_{ijij} - [\mathcal{C}_{qq}^{(3)}]_{ijij}, & [C_{dd}^{V1,LR}]_{ijij} &= -[\mathcal{C}_{qd}^{(1)}]_{ijij}, \\ [C_{dd}^{V,RR}]_{ijij} &= -[\mathcal{C}_{dd}]_{ijij}, & [C_{dd}^{V8,RR}]_{ijij} &= -[\mathcal{C}_{qd}^{(8)}]_{ijij}, \end{aligned} \quad (3.6)$$

and for $D^0 - \bar{D}^0$ mixing in the **up-basis**

$$\begin{aligned} [C_{uu}^{V,LL}]_{ijij} &= -[\hat{\mathcal{C}}_{qq}^{(1)}]_{ijij} - [\hat{\mathcal{C}}_{qq}^{(3)}]_{ijij}, & [C_{uu}^{V1,LR}]_{ijij} &= -[\hat{\mathcal{C}}_{qu}^{(1)}]_{ijij}, \\ [C_{uu}^{V,RR}]_{ijij} &= -[\hat{\mathcal{C}}_{uu}]_{ijij}, & [C_{uu}^{V8,RR}]_{ijij} &= -[\hat{\mathcal{C}}_{qu}^{(8)}]_{ijij}. \end{aligned} \quad (3.7)$$

Here we have chosen the JMS basis in the LEFT.¹²

¹²Note that we use the Hamiltonian for LEFT to define Wilson coefficients contrary to [13, 14], who use the Lagrangian, in consequence minus signs are present in the matching conditions.

The transformations of the SMEFT Wilson coefficients from the down to the up basis is governed by elements of the CKM matrix [84] as follows

$$[\hat{\mathcal{C}}_{qq}^{(1,3)}]_{ijij} = \sum_{prst} V_{ip} V_{jr}^* V_{is} V_{jt}^* [\mathcal{C}_{qq}^{(1,3)}]_{prst}, \quad (3.8)$$

$$[\hat{\mathcal{C}}_{qa}^{(1,8)}]_{ijij} = \sum_{pr} V_{ip} V_{jr}^* [\mathcal{C}_{qa}^{(1,8)}]_{pri}, \quad (a = u, d). \quad (3.9)$$

The SMEFT four-quark operators in (3.6) and (3.7) form the operator set B of the first term in (3.3) at μ_{ew} . All these operators undergo self-mixing,¹³ under gauge-interactions and also Yukawa interactions. Consequently, they remain the coefficients with largest $P_a^{ij}(\Lambda)$ also at Λ . Whether large Wilson coefficients are generated for these operators is then a matter of the flavour structures in the considered UV completion and whether tree-level or loop mediation occurs.

The RG evolution (3.2) however will also introduce via mixing all Wilson coefficients that mix into set B , which we call set $B' \subset C$ in the following. Their numerical impact is loop-suppressed and depends on the size of the ADMs γ_{ba} , but a large logarithm $\ln(\mu_{\text{ew}}/\Lambda)$ appears as in the self-mixing of set B . This can be illustrated with the approximate solution of (3.2)

$$\mathcal{C}_b(\mu_{\text{ew}}) \approx \sum_a [\delta_{ba} + \gamma_{ba} L] \mathcal{C}_a(\Lambda), \quad L \equiv \frac{1}{(4\pi)^2} \ln \left(\frac{\mu_{\text{ew}}}{\Lambda} \right), \quad (3.10)$$

when retaining only the first leading logarithm L .

The most sizeable ADMs from Yukawa-mixing is due to the top-Yukawa coupling $y_t \approx 1$. The contributions $\gamma_{ba} \propto y_t$ for down-type mixing in (3.10) (in the **down-basis**) are

$$[\mathcal{C}_{qq}^{(1)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qq}^{(1)}]_{ijij} + y_t^2 \left[\lambda_t^{ik} [\mathcal{C}_{qq}^{(1)}]_{kjj} + \lambda_t^{kj} [\mathcal{C}_{qq}^{(1)}]_{ikij} - \lambda_t^{ij} \left([\mathcal{C}_{qu}^{(1)}]_{ij33} + \frac{1}{12} [\mathcal{C}_{qu}^{(8)}]_{ij33} - [\mathcal{C}_{\phi q}^{(1)}]_{ij} \right) \right] L, \quad (3.11)$$

$$[\mathcal{C}_{qq}^{(3)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qq}^{(3)}]_{ijij} + y_t^2 \left[\lambda_t^{ik} [\mathcal{C}_{qq}^{(3)}]_{kjj} + \lambda_t^{kj} [\mathcal{C}_{qq}^{(3)}]_{ikij} - \lambda_t^{ij} \left(\frac{1}{4} [\mathcal{C}_{qu}^{(8)}]_{ij33} + [\mathcal{C}_{\phi q}^{(3)}]_{ij} \right) \right] L, \quad (3.12)$$

$$[\mathcal{C}_{qd}^{(1)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qd}^{(1)}]_{ijij} + y_t^2 \left[\frac{\lambda_t^{ik}}{2} [\mathcal{C}_{qd}^{(1)}]_{kjj} + \frac{\lambda_t^{kj}}{2} [\mathcal{C}_{qd}^{(1)}]_{ikij} - \lambda_t^{ij} ([\mathcal{C}_{ud}^{(1)}]_{33ij} - [\mathcal{C}_{\phi d}]_{ij}) \right] L, \quad (3.13)$$

$$[\mathcal{C}_{qd}^{(8)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qd}^{(8)}]_{ijij} + y_t^2 \left[\frac{\lambda_t^{ik}}{2} [\mathcal{C}_{qd}^{(8)}]_{kjj} + \frac{\lambda_t^{kj}}{2} [\mathcal{C}_{qd}^{(8)}]_{ikij} - \lambda_t^{ij} [\mathcal{C}_{ud}^{(8)}]_{33ij} \right] L, \quad (3.14)$$

where a summation over k is implied. We have suppressed the argument of the NP scale Λ in the Wilson coefficients on the r.h.s. to simplify the notation. Note that the up-type

¹³A strict use of the term “self-mixing” implies that the flavour structure of the $\Delta F = 2$ Wilson coefficient is conserved to be $[\mathcal{C}_b]_{ijij}$.

Yukawa matrix in the down-basis is given by¹⁴

$$Y^U = \frac{\sqrt{2}}{v} V_{\text{CKM}}^\dagger m_U^{\text{diag}}. \quad (3.15)$$

The flavour-mixing of the Yukawa couplings is seen by the presence of CKM elements λ_t^{ij} . Although at first sight numerically suppressed, other than top-Yukawa mixings can be phenomenologically important, depending on the UV completion and also on the SM suppression factors for the observable under consideration.

Concerning the gauge sector, indeed the most sizeable ADMs are those due to the strong coupling $4\pi\alpha_s = g_s^2 \approx 1.4$ and less sizeable due to $\text{SU}(2)_L \times \text{U}(1)_Y$, however this mixing is flavour-diagonal. The most important evolution due to gauge couplings are

$$[\mathcal{C}_{qq}^{(1)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qq}^{(1)}]_{ijij} + \left[\left(\frac{g'^2}{3} + g_s^2 \right) [\mathcal{C}_{qq}^{(1)}]_{ijij} + 9(g^2 + g_s^2) [\mathcal{C}_{qq}^{(3)}]_{ijij} \right] L, \quad (3.16)$$

$$[\mathcal{C}_{qq}^{(3)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qq}^{(3)}]_{ijij} + \left[\left(\frac{g'^2}{3} - 6g^2 - 5g_s^2 \right) [\mathcal{C}_{qq}^{(3)}]_{ijij} + 3(g^2 + g_s^2) [\mathcal{C}_{qq}^{(1)}]_{ijij} \right] L, \quad (3.17)$$

$$[\mathcal{C}_{qd}^{(1)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qd}^{(1)}]_{ijij} + \frac{2}{3} \left[g'^2 [\mathcal{C}_{qd}^{(1)}]_{ijij} - 4g_s^2 [\mathcal{C}_{qd}^{(8)}]_{ijij} \right] L, \quad (3.18)$$

$$[\mathcal{C}_{qd}^{(8)}]_{ijij}(\mu_{\text{ew}}) = [\mathcal{C}_{qd}^{(8)}]_{ijij} + \left[\left(\frac{2}{3}g'^2 - 14g_s^2 \right) [\mathcal{C}_{qd}^{(8)}]_{ijij} - 12g_s^2 [\mathcal{C}_{qd}^{(1)}]_{ijij} \right] L, \quad (3.19)$$

$$[\mathcal{C}_{dd}]_{ijij}(\mu_{\text{ew}}) = \left(1 + 4 \left[\frac{g'^2}{3} + g_s^2 \right] L \right) [\mathcal{C}_{dd}]_{ijij}. \quad (3.20)$$

In **the up-basis**, where the up-type Yukawa matrix $Y^U = \sqrt{2} m_U^{\text{diag}}/v$ is diagonal, all flavour-changing mixing terms in (3.11)–(3.14) disappear, whereas mixing due to gauge couplings shown in (3.16)–(3.20) remains unaltered.

From the above equations one sees that the operator set B' for down-type mixing contains at least the following operators

$$\left\{ \mathcal{C}_{qu}^{(1)}, \mathcal{C}_{qu}^{(8)}, \mathcal{C}_{ud}^{(1)}, \mathcal{C}_{ud}^{(8)}, \mathcal{C}_{\phi q}^{(1)}, \mathcal{C}_{\phi q}^{(3)}, \mathcal{C}_{\phi d} \right\}. \quad (3.21)$$

They are additional four-quark operators and modified Z - and W -couplings of quarks parameterized by the $\psi^2\phi^2 D$ operators $\mathcal{O}_{\phi q}^{(1,3)}$ and $\mathcal{O}_{\phi d}$. The RG running of SMEFT Wilson coefficients from the NP scale down to the EW scale contributing to $K^0 - \bar{K}^0$ and $B_{s,d} - \bar{B}_{s,d}$ mixing in the down-basis is displayed in figure 2 and for $D^0 - \bar{D}^0$ mixing in the down-basis in figure 3. The following clarifying comments are in order:

- At the scale Λ operators are listed that contribute to $\Delta F = 2$ operators at μ_{ew} either directly or through operator mixing in the RG evolution. In case the former operators are absent at Λ but are generated at μ_{ew} solely via RG evolution they are placed on a lighter background than the original operators.

¹⁴For illustration we neglect here the dimension-six terms to the mass matrix, but take them into account in our numerics.

Figure 2. Mixing of operators relevant for $\Delta F = 2$ observables for the $K^0 - \bar{K}^0$ and $B_{s,d} - \bar{B}_{s,d}$ mixing in the Warsaw down-basis. The red, green and black lines indicate the mixing due to strong, electroweak and top-Yukawa couplings, respectively. The self-mixing is shown by a dashed black line. Here $k = 1, 2, 3$. For the up-basis the mixing due to top-Yukawa, i.e. all black lines disappear.

- The distinction between strong, weak and Yukawa interactions is made with the help of colours as described in the figure caption.
- We split the operator mixing into two parts (a) referred as *flavour independent* in which the flavour structure of the Wilson coefficients at Λ remains intact at μ_{ew} and (b) the *flavour dependent* part in which the original flavour structure is modified due to RG running. Specifically, the operator mixing due to gauge couplings is flavour independent whereas the Yukawa couplings give rise to both, flavour-dependent and flavour-independent mixing. These are shown in the upper and lower panels of figure 2 for $K^0 - \bar{K}^0$ and $B_{s,d} - \bar{B}_{s,d}$ mixing. For $D^0 - \bar{D}^0$ the operator mixing is shown in figure 3, which is flavour independent.

In the numerical evaluation we include all Yukawa mixings, which lead to additional operators in set B' than we listed in (3.21).

So far we have discussed the effects of operator mixing in the RG evolution. But as already mentioned above, the set C in the second term of (3.3) can contain further operators. Indeed the only one is the up-type dipole operator $\mathcal{O}_{uW}$, such that

$$C = B' + \{\mathcal{C}_{uW}\}. \quad (3.22)$$

Its Wilson coefficient at μ_{ew} contributes via $M_{dc}^{(1)}(\mu_{\text{ew}})$ in (3.3) and as such it is one-loop suppressed. Despite being formally a NLO correction, the absence of mixing into set B

Figure 3. Mixing of operators relevant for $\Delta F = 2$ observables for $D^0 - \bar{D}^0$ -mixing in the Warsaw down-basis. The red, green and black lines indicate the mixing due to strong, electroweak and top-Yukawa couplings, respectively. Here $k = 1, 2, 3$. The self-mixing is shown by a dashed black line. For the up-basis the mixing due to top-Yukawa, i.e. all black lines disappear.

guarantees scheme-independence and allows actually to include this correction without full knowledge of two-loop ADMs.

The complete set of one-loop SMEFT threshold corrections $M_{dc}^{(1)}(\mu_{ew})$ has been calculated in [14], but is provided there only in electronic form. We have extracted those relevant for $\Delta F = 2$ processes and collected them in appendix D. With the help of these results in (D.1)–(D.3), one can verify the cancellation of the μ_{ew} dependence with the corresponding dependences in (3.11)–(3.13). This can be seen explicitly by making the replacement $y_t^2 = 2\pi x_t \alpha / s_W^2$.

3.3 Derivation of the SMEFT master formula

In the previous section we provided an approximate analytic insight into the one-loop RG evolution relevant for $\Delta F = 2$ processes, with some details on the cancellation of scale dependences. It also showed that Yukawa couplings are responsible for a complex flavour-mixing. As also mentioned before, the complete solution of the RG has to be performed numerically in order to determine the $P_a^{ij}(\Lambda)$ in (3.4). Here we provide the details of their determination before we go to an explicit example with $\Lambda = 5$ TeV in the next section.

We first calculate $[M_{12}^{ij}]_{\text{BSM}}$ and determine then the coefficients $P_a^{ij}(\Lambda)$ via (3.4) with the following steps:

- From the high scale Λ down to μ_{ew} , the full one-loop RG equations [83, 85, 86] are taken into account, using the package `wilson` [89]. As a result of operator mixing several secondary WCs are generated at μ_{ew} through operator mixing as depicted in the RG charts of figure 2, and 3.
- At μ_{ew} the full one-loop matching onto the LEFT Wilson coefficients (2.8) is considered. Here we have used the results from [14], which are implemented in `wilson`. The back-rotation [25] is taken into account automatically at μ_{ew} when using `wilson`.
- In the next step the complete LO [90, 91] and NLO QCD [32] running of the LEFT Wilson coefficients down to lower scales μ_{had} is taken into account following section 2.4.
- Finally, at the low scale μ_{had} the LEFT Wilson coefficients are combined with the hadronic matrix elements given in table 2 to calculate $[M_{12}^{ij}]_{\text{BSM}}$.

Note that, apart from the set B' defined in (3.21), the following SMEFT Wilson coefficients can contribute to $\Delta F = 2$ observables at one-loop

$$\text{four-quark :} \quad [\mathcal{C}_{quqd}^{(1)}], [\mathcal{C}_{quqd}^{(8)}]; \quad (3.23)$$

$$\text{semileptonic :} \quad [\mathcal{C}_{lq}^{(1)}], [\mathcal{C}_{lq}^{(3)}], [\mathcal{C}_{ld}], [\mathcal{C}_{qe}], [\mathcal{C}_{ledq}], [\mathcal{C}_{lequ}^{(1)}], [\mathcal{C}_{lequ}^{(3)}]. \quad (3.24)$$

In the process of obtaining the P_a^{ij} factors at the electroweak scale, the dimension-four parameters are needed at the NP scale Λ for the RG evolution of the Wilson coefficients. They are obtained through an iterative process explained in [24], where we assume no NP contributions to the CKM parameters stemming from four-quark operators. In particular as input serve tree-level determinations of $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ from semi-leptonic processes and the determination of the CKM angle γ from hadronic tree-level decays [89].

3.4 SMEFT atlas at $\Lambda = 5 \text{ TeV}$

As a numerical example we present here the SMEFT atlas at the scale $\Lambda = 5 \text{ TeV}$. It consists of the master formulae for the contributions of individual operators to the sum in (3.4) given in terms of dimensionless Wilson coefficients c_a introduced in (3.4). The numerical coefficients in these formulae are just the central values of the coefficients $P_a^{ij}(\Lambda)$ divided by Λ^2 with $\Lambda = 5 \text{ TeV}$.

The numerical values in the master formulae of this section have been evaluated at $\Lambda = 5 \text{ TeV}$. Corresponding results for $\Lambda = 100 \text{ TeV}$ are given in appendix A. While some visible changes are present in the left-right operators they are fully subdominant relative to the change of Λ , which amounts to the suppression of BSM contributions by a factor of 400 relative to the $\Lambda = 5 \text{ TeV}$ case considered here.

Before describing this $\Lambda = 5 \text{ TeV}$ SMEFT atlas in more detail let us make the following observations on the general pattern of these coefficients

- In contrast to the analogous coefficients entering (2.15) that were real valued, the ones in (3.4) are complex quantities. The phases in these formulae originate from

the complex CKM factors λ_t^{ij} in the RG equations (3.11)–(3.14) and from one-loop matching as seen in the formulae in appendix D. They are often represented by the phase $(-\beta)$ of the CKM element V_{td} . Its presence is signaled by the values of the phases in the ballpark of $\pm 22^\circ$ or $\pm 44^\circ$ often shifted by the small phase of V_{ts} . But in non-leading contributions also other phases are present. They result from the interplay of the complex values of CKM elements. We do not include in the numerical coefficients phases smaller than three degrees to simplify the formulae.

- The by far largest coefficients are the ones with indices (1212), (1313) and (2323) for K^0 , B_d and B_s systems, respectively, in particular for the coefficients $[\mathcal{C}_{qd}^{(1,8)}]_{1212}$, but also $[\mathcal{C}_{qd}^{(1,8)}]_{1313}$ and $[\mathcal{C}_{qd}^{(1,8)}]_{2323}$. Also for charm (1212) dominates. Yet, in particular in the up-basis among the contributions with smaller numerical coefficients multiplying c_a there are several with repeated indices signaling flavour conserving contributions. These are often weaker constrained by $\Delta F = 1$ observables than the flavour violating ones so that larger size of the corresponding c_a relative to the ones with larger $P_a^{ij}(\Lambda)$ factors could enhance their importance. We will return to this phenomenon in section 4 in the context of simplified models.
- Yet, when comparing different meson systems there is a large hierarchy in the values of the largest coefficients, which results not only from chiral enhancement of hadronic matrix elements of left-right operators in the K^0 meson system, but also from a large hierarchy in the normalization factors $(\Delta M_{ij})_{\text{exp}}$. Consequently, while for $K^0 - \bar{K}^0$ mixing the largest P_a^{12} are $\mathcal{O}(10^8)$, for $B_d - \bar{B}_d$ mixing the largest P_a^{13} are $\mathcal{O}(10^6)$ and for $B_s - \bar{B}_s$ mixing the largest P_a^{23} is in the ballpark of $\mathcal{O}(10^5)$. This clearly confirms the known fact that $K^0 - \bar{K}^0$ mixing can probe much larger energy scales than $B_d - \bar{B}_d$ or $B_s - \bar{B}_s$ mixing. The largest P_a^{12} for $D^0 - \bar{D}^0$ mixing is $\mathcal{O}(10^8)$ and can probe very short distance scales. We will quantify this in section 3.5.
- While at first sight the $[\mathcal{C}_{\phi X}]_{ij}$ coefficients with $X = q, d, u$ would appear irrelevant when compared with $\mathcal{C}_{qd}^{(1,8)}$, in models in which the latter are not generated at the high scale, they can play a dominant role. This is in particular the case of vector-like quark models [92].
- The $P_a^{ij}(\Lambda)$ for semileptonic operators are much smaller than those of nonleptonic ones, implying that the corresponding WCs are much weaker constrained, if at all, by $\Delta F = 2$ transitions.

After these general statements on the SMEFT atlas let us look at it a bit closer. Below we retain only those contributions to the sum in (3.4), which amount to at least 5% of $(\Delta M_{ij})_{\text{exp}}$ when setting $c_a = 10$. The 5% cut is in the ballpark of the uncertainties of the hadronic matrix elements in table 2 that enter the prediction of $[M_{12}^{ij}]_{\text{BSM}}$ linearly. The choice of a maximal value $c_a = 10$ is close to the generic value of 4π , which one would still consider a magnitude that allows perturbative expansions in couplings. We show only results for the three down-type meson systems K^0 , B_d and B_s , whereas the results for D^0

can be found in the supplementary material of this article, together with non-vanishing $P_a^{ij}(\Lambda)$ that yield numerically subleading contributions below 5% to the down-type meson systems.

3.4.1 $\mathcal{C}_{qq}^{(1)}$ and $\mathcal{C}_{qq}^{(3)}$

Before presenting the master formulae for the contributions of $[\mathcal{C}_{qq}^{(1,3)}]_{ijkl}$ to various meson systems in the down and up bases let us already make general statements on them on the basis of the size of numerical coefficients multiplying WCs.

- In the down basis constraints from $D^0 - \bar{D}^0$ mixing dominate for many entries, but $K^0 - \bar{K}^0$ mixing provides a very important constraint for $[\mathcal{C}_{qq}^{(1,3)}]_{1212}$. In view of potential poorly known long distance effects in the former it appears that presently it is plausible to put the constraint on $[\mathcal{C}_{qq}^{(1,3)}]_{1212}$ mainly from $K^0 - \bar{K}^0$ mixing.
- In the up basis constraints from $K^0 - \bar{K}^0$ mixing dominate for many entries but $D^0 - \bar{D}^0$ mixing could still play the role for $[\mathcal{C}_{qq}^{(1,3)}]_{1212}$ if long-distance contributions were under better control.
- The entries 1313 and 2323 are at first sight in both down and up bases dominantly constrained by $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing, respectively.
- However, as already mentioned above, it should also be noticed that beyond the leading $\Delta F = 2$ entries 1212, 1313, 2323, the $\Delta F = 1$ entries in particular in the up-basis could play a role dependently on the size of WCs in a given model. Indeed, in the up-basis strong correlations between $\Delta F = 2$ and $\Delta F = 1$ should be expected. This will be the case if in a given NP scenario also flavour-conserving couplings can contribute through RG effects to $\Delta F = 2$ transitions.
- There is a tendency of correlated constraints from $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixing on one hand and $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings on the other hand. This is for instance the case of 1212 for $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings in both bases and of 1323 for $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings in the up basis. Even if the coefficient for 1323 in $B_d - \bar{B}_d$ mixing is much larger than in $B_s - \bar{B}_s$ mixing, their relative importance to the leading terms in these mixings is similar.
- The results for $[\mathcal{C}_{qq}^{(3)}]_{ijkl}$ are practically equal to the ones for $[\mathcal{C}_{qq}^{(1)}]_{ijkl}$ for the dominant terms and the differences are only in subleading terms. Therefore we present only the terms which have different numerical coefficients, although it should be kept in mind that the WCs entering here are $[\mathcal{C}_{qq}^{(3)}]_{ijkl}$ and not $[\mathcal{C}_{qq}^{(1)}]_{ijkl}$. They could differ in specific NP scenarios.

This pattern is summarized by the following rather accurate formulae for the sum in (3.4) in **the down-basis**

$$\begin{aligned}\Sigma_{qq1}^{B_s} = & -3.9 \cdot 10^2 [c_{qq}^{(1)}]_{2323} - 5.4 \cdot 10^{-1} [c_{qq}^{(1)}]_{2333} \\ & + 3.1 \cdot 10^{-1} [c_{qq}^{(1)}]_{2223} - 6.8 \cdot 10^{-2} e^{i22^\circ} [c_{qq}^{(1)}]_{1232},\end{aligned}\quad (3.25)$$

$$\begin{aligned}\Sigma_{qq1}^{B_d} = & -9.1 \cdot 10^3 [c_{qq}^{(1)}]_{1313} + 7.2 [c_{qq}^{(1)}]_{1213} + 2.7 e^{i22^\circ} [c_{qq}^{(1)}]_{1333} - 1.6 e^{i22^\circ} [c_{qq}^{(1)}]_{1113} \\ & - 1.2 \cdot 10^{-1} e^{i23^\circ} [c_{qq}^{(1)}]_{1323} + 2.3 \cdot 10^{-2} e^{i21^\circ} [c_{qq}^{(1)}]_{1332} + 5.8 \cdot 10^{-3} e^{i22^\circ} [c_{qq}^{(1)}]_{1223} \\ & - 5.7 \cdot 10^{-3} [c_{qq}^{(1)}]_{1212} - 5.1 \cdot 10^{-3} e^{i44^\circ} [c_{qq}^{(1)}]_{1331},\end{aligned}\quad (3.26)$$

$$\begin{aligned}\Sigma_{qq1}^K = & -3.6 \cdot 10^4 [c_{qq}^{(1)}]_{1212} - 6.0 \cdot 10^1 [c_{qq}^{(1)}]_{1213} + 1.3 \cdot 10^1 e^{i22^\circ} [c_{qq}^{(1)}]_{1232} \\ & - 5.7 \cdot 10^{-1} e^{i23^\circ} [c_{qq}^{(1)}]_{1222} + 2.5 \cdot 10^{-1} e^{i23^\circ} [c_{qq}^{(1)}]_{1112} + 1.2 \cdot 10^{-1} e^{i23^\circ} [c_{qq}^{(1)}]_{1233} \\ & - 1.0 \cdot 10^{-1} [c_{qq}^{(1)}]_{1313} + 2.6 \cdot 10^{-2} e^{i23^\circ} [c_{qq}^{(1)}]_{1332} - 5.2 \cdot 10^{-3} e^{i24^\circ} [c_{qq}^{(1)}]_{1223}.\end{aligned}\quad (3.27)$$

Before continuing let us explain shortly on the above example the meaning of these results. Taking the case of $B_s - \bar{B}_s$ mixing, the numerical value of $\Sigma_{qq1}^{B_s}$ corresponds to its contribution to the ratio $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{B_s})_{\text{exp}}$ in (3.4). This choice of normalization allows to read off the fraction of the NP contribution to the prediction of ΔM_{B_s} compared to its central experimental value. For illustration, the value $\Sigma_{qq1}^{B_s} = 0.1$ is a deviation from the SM prediction of the size of 10% of the measured experimental value. Since the SM prediction of $B_s - \bar{B}_s$ mixing and the experimental measurement are in close agreement, this corresponds also to a 10% deviation due to NP from the SM prediction of ΔM_{B_s} . The numerically leading $\Delta F = 2$ contribution $\Sigma_{qq1}^{B_s} \approx -3.9 \cdot 10^2 [c_{qq}^{(1)}]_{2323}$ yields then a constraint on $|[c_{qq}^{(1)}]_{2323}| \lesssim 2.6 \cdot 10^{-4}$ for the NP contribution to ΔM_{B_s} not to exceed 10% deviation from the SM prediction. The same arguments apply also to $B_d - \bar{B}_d$ mixing, because the SM prediction is in close agreement with the experimental measurement. The SM predictions of $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixing receive in part unknown nonperturbative contributions and here the fraction refers to the experimentally measured $(\Delta M_{K,D})_{\text{exp}}$.

If the numerically leading terms are absent in a UV completion, then the subleading $\Delta F = 1$ terms like $\Sigma_{qq1}^{B_s} \approx -5.4 \cdot 10^{-1} [c_{qq}^{(1)}]_{2333}$ would be subject to a much weaker constraint $|[c_{qq}^{(1)}]_{2333}| \lesssim 1.9 \cdot 10^{-1}$. Complicated interference between the numerically leading $\Delta F = 2$ term and the subleading $\Delta F = 1$ terms will arise in UV completions that admit all contributions. Thus there can be strong correlations to $\Delta F = 1$ processes, depending on the suppression factors in the SM predictions of the $\Delta F = 1$ observables. Then it remains to be seen whether $\Delta F = 1$ observables will impose stronger constraints.

The result for **the up-basis** reads

$$\begin{aligned}\hat{\Sigma}_{qq1}^{B_s} = & -3.7 \cdot 10^2 [\hat{c}_{qq}^{(1)}]_{2323} - 8.5 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1323} - 2.0 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1313} - 1.5 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{2223} \\ & + 1.5 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{2333} - 3.5 [\hat{c}_{qq}^{(1)}]_{1223} - 3.5 [\hat{c}_{qq}^{(1)}]_{1322} + 3.5 [\hat{c}_{qq}^{(1)}]_{1333} - 1.3 e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1232} \\ & - 8.2 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1213} - 6.4 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{2222} + 6.3 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{2233} + 6.3 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{2332} \\ & - 6.3 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{3333} - 3.0 \cdot 10^{-1} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1231} - 3.0 \cdot 10^{-1} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1123} \\ & - 1.7 \cdot 10^{-1} e^{-i18^\circ} [\hat{c}_{qq}^{(1)}]_{1222} + 1.7 \cdot 10^{-1} e^{-i19^\circ} [\hat{c}_{qq}^{(1)}]_{1233} + 1.7 \cdot 10^{-1} e^{-i19^\circ} [\hat{c}_{qq}^{(1)}]_{1332}\end{aligned}$$

$$\begin{aligned}
 & -7.0 \cdot 10^{-2} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1113} - 3.0 \cdot 10^{-2} e^{-i5^\circ} [\hat{c}_{qq}^{(1)}]_{1212} - 1.3 \cdot 10^{-2} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1221} \\
 & - 1.3 \cdot 10^{-2} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1122} + 1.2 \cdot 10^{-2} e^{-i74^\circ} [\hat{c}_{qq}^{(1)}]_{1133} + 1.2 \cdot 10^{-2} e^{-i74^\circ} [\hat{c}_{qq}^{(1)}]_{1331}, \quad (3.28) \\
 \hat{\Sigma}_{qq1}^{B_d} = & -8.6 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1313} + 2.0 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1323} - 4.6 \cdot 10^2 [\hat{c}_{qq}^{(1)}]_{2323} - 3.6 \cdot 10^2 [\hat{c}_{qq}^{(1)}]_{1213} \\
 & + 8.2 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1223} + 8.2 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1322} - 7.8 \cdot 10^1 e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{1333} \\
 & - 3.1 \cdot 10^1 e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1113} - 1.9 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{2223} + 1.8 \cdot 10^1 e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{2333} - 1.5 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1212} \\
 & + 7.0 e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1231} + 7.0 e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1123} + 3.4 [\hat{c}_{qq}^{(1)}]_{1222} - 3.2 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1233} \\
 & - 3.2 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1332} - 1.6 e^{-i68^\circ} [\hat{c}_{qq}^{(1)}]_{1232} - 1.3 e^{-i72^\circ} [\hat{c}_{qq}^{(1)}]_{1112} - 7.8 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{2222} \\
 & + 7.4 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{2233} + 7.3 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{2332} - 7.0 \cdot 10^{-1} e^{i44^\circ} [\hat{c}_{qq}^{(1)}]_{3333} \\
 & + 2.9 \cdot 10^{-1} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1221} + 2.9 \cdot 10^{-1} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1122} - 2.8 \cdot 10^{-1} e^{-i51^\circ} [\hat{c}_{qq}^{(1)}]_{1133} \\
 & - 2.7 \cdot 10^{-1} e^{-i51^\circ} [\hat{c}_{qq}^{(1)}]_{1331} + 1.1 \cdot 10^{-1} e^{i34^\circ} [\hat{c}_{qq}^{(1)}]_{1111}, \quad (3.29) \\
 \hat{\Sigma}_{qq1}^K = & -3.2 \cdot 10^4 [\hat{c}_{qq}^{(1)}]_{1212} - 7.0 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1112} + 7.0 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1222} - 1.7 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1111} \\
 & + 1.7 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1221} + 1.7 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1122} - 1.7 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{2222} + 1.3 \cdot 10^3 [\hat{c}_{qq}^{(1)}]_{1213} \\
 & + 2.9 \cdot 10^2 [\hat{c}_{qq}^{(1)}]_{1113} - 2.9 \cdot 10^2 [\hat{c}_{qq}^{(1)}]_{1322} - 2.9 \cdot 10^2 [\hat{c}_{qq}^{(1)}]_{1223} - 2.7 \cdot 10^2 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1232} \\
 & - 1.3 \cdot 10^2 e^{i11^\circ} [\hat{c}_{qq}^{(1)}]_{1123} - 1.3 \cdot 10^2 e^{i11^\circ} [\hat{c}_{qq}^{(1)}]_{1231} + 1.3 \cdot 10^2 e^{i11^\circ} [\hat{c}_{qq}^{(1)}]_{2223} \\
 & - 5.3 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1313} + 1.2 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1323} + 1.1 \cdot 10^1 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1233} + 1.1 \cdot 10^1 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1332} \\
 & - 5.0 e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{2323} + 2.7 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1133} - 2.7 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{2233} + 2.7 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1331} \\
 & - 2.7 e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{2332} - 4.6 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1333} + 2.1 \cdot 10^{-1} e^{i34^\circ} [\hat{c}_{qq}^{(1)}]_{2333}. \quad (3.30)
 \end{aligned}$$

For $\mathcal{C}_{qq}^{(3)}$ one finds:

$$\Sigma_{qq3}^{B_s} = \Sigma_{qq1}^{B_s} |^{1 \rightarrow 3} - 8.3 \cdot 10^{-3} [c_{qq}^{(3)}]_{2233} - 6.4 \cdot 10^{-3} [c_{qq}^{(3)}]_{1123}, \quad (3.31)$$

$$\begin{aligned}
 \Sigma_{qq3}^{B_d} = \Sigma_{qq1}^{B_d} |^{1 \rightarrow 3} & + 4.3 \cdot 10^{-2} e^{i21^\circ} [c_{qq}^{(3)}]_{1233} - 4.1 \cdot 10^{-2} e^{i21^\circ} [c_{qq}^{(3)}]_{1332} + 3.3 \cdot 10^{-2} e^{i22^\circ} [c_{qq}^{(3)}]_{1322} \\
 & - 9.2 \cdot 10^{-3} e^{i44^\circ} [c_{qq}^{(3)}]_{1133} - 5.8 \cdot 10^{-3} e^{i22^\circ} [c_{qq}^{(3)}]_{1223} + 5.1 \cdot 10^{-3} e^{i44^\circ} [c_{qq}^{(3)}]_{1331}, \quad (3.32)
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{qq3}^K = \Sigma_{qq1}^K |^{1 \rightarrow 3} & - 0.5 \cdot 10^{-1} e^{i23^\circ} [c_{qq}^{(3)}]_{1332} + 8.7 \cdot 10^{-2} e^{i23^\circ} [c_{qq}^{(3)}]_{1233} \\
 & - 8.7 \cdot 10^{-3} e^{i24^\circ} [c_{qq}^{(3)}]_{1322} + 5.2 \cdot 10^{-3} e^{i24^\circ} [c_{qq}^{(3)}]_{1223}, \quad (3.33)
 \end{aligned}$$

and in the up-basis

$$\hat{\Sigma}_{qq3}^{B_s} = \hat{\Sigma}_{qq1}^{B_s} |^{1 \rightarrow 3}, \quad \hat{\Sigma}_{qq3}^{B_d} = \hat{\Sigma}_{qq1}^{B_d} |^{1 \rightarrow 3}, \quad \hat{\Sigma}_{qq3}^K = \hat{\Sigma}_{qq1}^K |^{1 \rightarrow 3}. \quad (3.34)$$

3.4.2 $\mathcal{C}_{qd}^{(1)}$ and $\mathcal{C}_{qd}^{(8)}$

We next present the results for $[\mathcal{C}_{qd}^{(1)}]_{ijkl}$ and $[\mathcal{C}_{qd}^{(8)}]_{ijkl}$ in the down and up bases. Evidently $\Delta F = 2$ processes provide very strong constraints for these coefficients. We observe

- The $K^0 - \bar{K}^0$ mixing is most constraining except for 1313 and 2323 elements for which constraints from $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing dominate. Yet, inspection of the formulae below shows that contributions from other WCs with different indices could also play a role in some UV completions.

- In certain cases there is a large difference between down and up bases.
- The results for $[C_{qd}^{(1)}]_{ijkl}$ and $[C_{qd}^{(8)}]_{ijkl}$ are similar.
- Constraints from $D^0 - \bar{D}^0$ mixing are irrelevant for both bases.

This pattern can be summarized by the following rather accurate formulae for the sum in (3.4) in **the down-basis**

$$\begin{aligned} \Sigma_{qd1}^{B_s} = & 2.3 \cdot 10^3 [c_{qd}^{(1)}]_{2323} + 3.3 [c_{qd}^{(1)}]_{3323} - 1.8 [c_{qd}^{(1)}]_{2223} + 4.0 \cdot 10^{-1} e^{i22^\circ} [c_{qd}^{(1)}]_{1232} \\ & - 1.1 \cdot 10^{-1} [c_{qd}^{(1)}]_{2322} + 1.1 \cdot 10^{-1} [c_{qd}^{(1)}]_{2333} + 2.5 \cdot 10^{-2} [c_{qd}^{(1)}]_{2332} \\ & - 2.3 \cdot 10^{-2} e^{-i23^\circ} [c_{qd}^{(1)}]_{1323} - 5.5 \cdot 10^{-3} e^{i21^\circ} [c_{qd}^{(1)}]_{1332}, \end{aligned} \quad (3.35)$$

$$\begin{aligned} \Sigma_{qd1}^{B_d} = & 5.7 \cdot 10^4 [c_{qd}^{(1)}]_{1313} - 4.5 \cdot 10^1 [c_{qd}^{(1)}]_{1213} - 1.8 \cdot 10^1 e^{i22^\circ} [c_{qd}^{(1)}]_{3313} \\ & + 9.8 e^{i22^\circ} [c_{qd}^{(1)}]_{1113} - 2.8 [c_{qd}^{(1)}]_{1312} + 7.6 \cdot 10^{-1} e^{i23^\circ} [c_{qd}^{(1)}]_{2313} \\ & - 1.4 \cdot 10^{-1} e^{i21^\circ} [c_{qd}^{(1)}]_{2331} + 6.5 \cdot 10^{-2} e^{i23^\circ} [c_{qd}^{(1)}]_{1323} - 3.1 \cdot 10^{-2} e^{i21^\circ} [c_{qd}^{(1)}]_{1333} \\ & + 3.1 \cdot 10^{-2} e^{i21^\circ} [c_{qd}^{(1)}]_{1311} + 3.0 \cdot 10^{-2} e^{i44^\circ} [c_{qd}^{(1)}]_{1331} + 7.2 \cdot 10^{-3} e^{i22^\circ} [c_{qd}^{(1)}]_{2213}, \end{aligned} \quad (3.36)$$

$$\begin{aligned} \Sigma_{qd1}^K = & 5.3 \cdot 10^6 [c_{qd}^{(1)}]_{1212} + 9.0 \cdot 10^3 [c_{qd}^{(1)}]_{1312} - 2.0 \cdot 10^3 e^{i22^\circ} [c_{qd}^{(1)}]_{2321} + 2.6 \cdot 10^2 [c_{qd}^{(1)}]_{1213} \\ & + 8.1 \cdot 10^1 e^{i24^\circ} [c_{qd}^{(1)}]_{2212} - 3.4 \cdot 10^1 e^{i26^\circ} [c_{qd}^{(1)}]_{1112} - 1.7 \cdot 10^1 e^{i23^\circ} [c_{qd}^{(1)}]_{3312} \\ & - 2.8 e^{i22^\circ} [c_{qd}^{(1)}]_{1232} - 2.5 e^{i66^\circ} [c_{qd}^{(1)}]_{1211} + 2.5 e^{i66^\circ} [c_{qd}^{(1)}]_{1222} + 7.2 \cdot 10^{-1} e^{i24^\circ} [c_{qd}^{(1)}]_{2312} \\ & + 4.4 \cdot 10^{-1} [c_{qd}^{(1)}]_{1313} - 1.1 \cdot 10^{-1} e^{i45^\circ} [c_{qd}^{(1)}]_{1321} - 9.6 \cdot 10^{-2} e^{i23^\circ} [c_{qd}^{(1)}]_{2331} \\ & + 1.0 \cdot 10^{-2} e^{i24^\circ} [c_{qd}^{(1)}]_{1322} - 1.0 \cdot 10^{-2} e^{i24^\circ} [c_{qd}^{(1)}]_{1311}, \end{aligned} \quad (3.37)$$

and

$$\begin{aligned} \Sigma_{qd8}^{B_s} = & 2.7 \cdot 10^3 [c_{qd}^{(8)}]_{2323} + 3.8 [c_{qd}^{(8)}]_{3323} - 2.1 [c_{qd}^{(8)}]_{2223} + 4.7 \cdot 10^{-1} e^{i22^\circ} [c_{qd}^{(8)}]_{1232} \\ & - 1.3 \cdot 10^{-1} [c_{qd}^{(8)}]_{2322} + 1.3 \cdot 10^{-1} [c_{qd}^{(8)}]_{2333} + 2.7 \cdot 10^{-2} [c_{qd}^{(8)}]_{2332} \\ & - 2.6 \cdot 10^{-2} e^{-i23^\circ} [c_{qd}^{(8)}]_{1323} - 5.8 \cdot 10^{-3} e^{i21^\circ} [c_{qd}^{(8)}]_{1332}, \end{aligned} \quad (3.38)$$

$$\begin{aligned} \Sigma_{qd8}^{B_d} = & 6.6 \cdot 10^4 [c_{qd}^{(8)}]_{1313} - 5.2 \cdot 10^1 [c_{qd}^{(8)}]_{1213} - 2.1 \cdot 10^1 e^{i22^\circ} [c_{qd}^{(8)}]_{3313} \\ & + 1.1 \cdot 10^1 e^{i22^\circ} [c_{qd}^{(8)}]_{1113} - 3.2 [c_{qd}^{(8)}]_{1312} + 8.9 \cdot 10^{-1} e^{i23^\circ} [c_{qd}^{(8)}]_{2313} \\ & - 1.4 \cdot 10^{-1} e^{i21^\circ} [c_{qd}^{(8)}]_{2331} + 7.5 \cdot 10^{-2} e^{i23^\circ} [c_{qd}^{(8)}]_{1323} + 3.6 \cdot 10^{-2} e^{i21^\circ} [c_{qd}^{(8)}]_{1311} \\ & - 3.5 \cdot 10^{-2} e^{i21^\circ} [c_{qd}^{(8)}]_{1333} + 3.1 \cdot 10^{-2} e^{i44^\circ} [c_{qd}^{(8)}]_{1331} + 1.8 \cdot 10^{-2} e^{i22^\circ} [c_{qd}^{(8)}]_{2213}, \end{aligned} \quad (3.39)$$

$$\begin{aligned} \Sigma_{qd8}^K = & 7.5 \cdot 10^6 [c_{qd}^{(8)}]_{1212} + 1.3 \cdot 10^4 [c_{qd}^{(8)}]_{1312} - 2.8 \cdot 10^3 e^{i22^\circ} [c_{qd}^{(8)}]_{2321} + 3.7 \cdot 10^2 [c_{qd}^{(8)}]_{1213} \\ & + 1.1 \cdot 10^2 e^{i24^\circ} [c_{qd}^{(8)}]_{2212} - 4.7 \cdot 10^1 e^{i27^\circ} [c_{qd}^{(8)}]_{1112} - 2.3 \cdot 10^1 e^{i23^\circ} [c_{qd}^{(8)}]_{3312} \\ & - 4.0 e^{i22^\circ} [c_{qd}^{(8)}]_{1232} - 3.4 e^{-i79^\circ} [c_{qd}^{(8)}]_{1222} + 3.4 e^{-i79^\circ} [c_{qd}^{(8)}]_{1211} \\ & + 9.7 \cdot 10^{-1} e^{i24^\circ} [c_{qd}^{(8)}]_{2312} + 6.3 \cdot 10^{-1} [c_{qd}^{(8)}]_{1313} - 1.5 \cdot 10^{-1} e^{i45^\circ} [c_{qd}^{(8)}]_{1321} \\ & - 1.4 \cdot 10^{-1} e^{i23^\circ} [c_{qd}^{(8)}]_{2331} + 1.5 \cdot 10^{-2} e^{i24^\circ} [c_{qd}^{(8)}]_{1322} - 1.4 \cdot 10^{-2} e^{i24^\circ} [c_{qd}^{(8)}]_{1311} \\ & - 6.7 \cdot 10^{-3} e^{i23^\circ} [c_{qd}^{(8)}]_{1332} + 6.3 \cdot 10^{-3} e^{i46^\circ} [c_{qd}^{(8)}]_{1221} + 6.2 \cdot 10^{-3} e^{i23^\circ} [c_{qd}^{(8)}]_{2213}, \end{aligned} \quad (3.40)$$

For the up-basis we have

$$\begin{aligned}\hat{\Sigma}_{qd1}^{B_s} = & 2.3 \cdot 10^3 [\hat{c}_{qd}^{(1)}]_{2323} + 5.2 \cdot 10^2 [\hat{c}_{qd}^{(1)}]_{1323} + 9.3 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{2223} - 9.3 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{3323} \\ & + 2.2 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{1223} + 8.0 e^{-i73^\circ} [\hat{c}_{qd}^{(1)}]_{1232} - 3.8 [\hat{c}_{qd}^{(1)}]_{2332} + 1.8 e^{-i73^\circ} [\hat{c}_{qd}^{(1)}]_{1123} \\ & - 3.3 \cdot 10^{-1} e^{-i74^\circ} [\hat{c}_{qd}^{(1)}]_{1332} - 1.1 \cdot 10^{-1} [\hat{c}_{qd}^{(1)}]_{2322} + 1.1 \cdot 10^{-1} [\hat{c}_{qd}^{(1)}]_{2333} \\ & - 2.5 \cdot 10^{-2} [\hat{c}_{qd}^{(1)}]_{1322} + 2.5 \cdot 10^{-2} [\hat{c}_{qd}^{(1)}]_{1333} - 5.4 \cdot 10^{-3} e^{-i11^\circ} [\hat{c}_{qd}^{(1)}]_{2313},\end{aligned}\quad (3.41)$$

$$\begin{aligned}\hat{\Sigma}_{qd1}^{B_d} = & 5.5 \cdot 10^4 [\hat{c}_{qd}^{(1)}]_{1313} - 1.3 \cdot 10^4 [\hat{c}_{qd}^{(1)}]_{2313} + 2.3 \cdot 10^3 [\hat{c}_{qd}^{(1)}]_{1213} - 5.3 \cdot 10^2 [\hat{c}_{qd}^{(1)}]_{2213} \\ & + 5.0 \cdot 10^2 e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{3313} + 2.0 \cdot 10^2 e^{-i73^\circ} [\hat{c}_{qd}^{(1)}]_{1113} - 4.5 \cdot 10^1 e^{-i73^\circ} [\hat{c}_{qd}^{(1)}]_{1231} \\ & + 2.0 \cdot 10^1 e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{2331} - 2.7 [\hat{c}_{qd}^{(1)}]_{1312} + 1.8 e^{-i51^\circ} [\hat{c}_{qd}^{(1)}]_{1331} + 6.2 \cdot 10^{-1} [\hat{c}_{qd}^{(1)}]_{2312} \\ & - 1.1 \cdot 10^{-1} [\hat{c}_{qd}^{(1)}]_{1212} + 6.3 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{1323} - 3.0 \cdot 10^{-2} e^{i21^\circ} [\hat{c}_{qd}^{(1)}]_{1333} \\ & + 3.0 \cdot 10^{-2} e^{i21^\circ} [\hat{c}_{qd}^{(1)}]_{1311} + 2.6 \cdot 10^{-2} [\hat{c}_{qd}^{(1)}]_{2212} - 2.4 \cdot 10^{-2} e^{i21^\circ} [\hat{c}_{qd}^{(1)}]_{3312} \\ & - 1.4 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{2323} - 9.6 \cdot 10^{-3} e^{-i74^\circ} [\hat{c}_{qd}^{(1)}]_{1112} + 6.6 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{2333} \\ & - 6.6 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{2311},\end{aligned}\quad (3.42)$$

$$\begin{aligned}\hat{\Sigma}_{qd1}^K = & 5.0 \cdot 10^6 [\hat{c}_{qd}^{(1)}]_{1212} + 1.2 \cdot 10^6 [\hat{c}_{qd}^{(1)}]_{1112} - 1.2 \cdot 10^6 [\hat{c}_{qd}^{(1)}]_{2212} - 2.7 \cdot 10^5 [\hat{c}_{qd}^{(1)}]_{1221} \\ & - 2.0 \cdot 10^5 [\hat{c}_{qd}^{(1)}]_{1312} + 4.7 \cdot 10^4 [\hat{c}_{qd}^{(1)}]_{2312} + 4.5 \cdot 10^4 e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{2321} + 1.0 \cdot 10^4 e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{1321} \\ & - 1.8 \cdot 10^3 e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{3312} + 2.5 \cdot 10^2 [\hat{c}_{qd}^{(1)}]_{1213} + 5.6 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{1113} - 5.6 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{2213} \\ & - 1.3 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{1231} - 1.0 \cdot 10^1 [\hat{c}_{qd}^{(1)}]_{1313} - 2.6 e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{1232} - 2.5 e^{i57^\circ} [\hat{c}_{qd}^{(1)}]_{1211} \\ & + 2.5 e^{i57^\circ} [\hat{c}_{qd}^{(1)}]_{1222} + 2.3 [\hat{c}_{qd}^{(1)}]_{2313} + 2.2 e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{2331} - 1.5 e^{i20^\circ} [\hat{c}_{qd}^{(1)}]_{2222} \\ & + 1.5 e^{i20^\circ} [\hat{c}_{qd}^{(1)}]_{2211} + 1.1 e^{i27^\circ} [\hat{c}_{qd}^{(1)}]_{1122} - 1.1 e^{i27^\circ} [\hat{c}_{qd}^{(1)}]_{1111} - 6.1 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{1123} \\ & + 6.0 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{2223} + 5.0 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{1331} - 2.3 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{1322} \\ & + 2.3 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{1311} + 1.4 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(1)}]_{1223} + 1.1 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{1332} \\ & + 1.0 \cdot 10^{-1} e^{i34^\circ} [\hat{c}_{qd}^{(1)}]_{2322} - 1.0 \cdot 10^{-1} e^{i34^\circ} [\hat{c}_{qd}^{(1)}]_{2311} - 9.0 \cdot 10^{-2} e^{i24^\circ} [\hat{c}_{qd}^{(1)}]_{3313} \\ & - 2.5 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qd}^{(1)}]_{2332} - 2.3 \cdot 10^{-2} e^{i44^\circ} [\hat{c}_{qd}^{(1)}]_{2323} - 5.4 \cdot 10^{-3} e^{i44^\circ} [\hat{c}_{qd}^{(1)}]_{1323},\end{aligned}\quad (3.43)$$

and

$$\begin{aligned}\hat{\Sigma}_{qd8}^{B_s} = & 2.6 \cdot 10^3 [\hat{c}_{qd}^{(8)}]_{2323} + 6.0 \cdot 10^2 [\hat{c}_{qd}^{(8)}]_{1323} + 1.1 \cdot 10^2 [\hat{c}_{qd}^{(8)}]_{2223} - 1.1 \cdot 10^2 [\hat{c}_{qd}^{(8)}]_{3323} \\ & + 2.5 \cdot 10^1 [\hat{c}_{qd}^{(8)}]_{1223} + 9.3 e^{-i73^\circ} [\hat{c}_{qd}^{(8)}]_{1232} - 4.4 [\hat{c}_{qd}^{(8)}]_{2332} + 2.1 e^{-i73^\circ} [\hat{c}_{qd}^{(8)}]_{1123} \\ & - 3.8 \cdot 10^{-1} e^{-i74^\circ} [\hat{c}_{qd}^{(8)}]_{1332} - 1.3 \cdot 10^{-1} [\hat{c}_{qd}^{(8)}]_{2322} + 1.3 \cdot 10^{-1} [\hat{c}_{qd}^{(8)}]_{2333} - 2.9 \cdot 10^{-2} [\hat{c}_{qd}^{(8)}]_{1322} \\ & + 2.9 \cdot 10^{-2} [\hat{c}_{qd}^{(8)}]_{1333} - 7.4 \cdot 10^{-3} e^{-i10^\circ} [\hat{c}_{qd}^{(8)}]_{2313} - 6.5 \cdot 10^{-3} [\hat{c}_{qd}^{(8)}]_{2331} \\ & + 6.0 \cdot 10^{-3} e^{-i3^\circ} [\hat{c}_{qd}^{(8)}]_{3313} - 5.3 \cdot 10^{-3} [\hat{c}_{qd}^{(8)}]_{2222} + 5.3 \cdot 10^{-3} [\hat{c}_{qd}^{(8)}]_{2233} \\ & + 5.3 \cdot 10^{-3} [\hat{c}_{qd}^{(8)}]_{3322} - 5.2 \cdot 10^{-3} [\hat{c}_{qd}^{(8)}]_{3333} + 5.0 \cdot 10^{-3} [\hat{c}_{qd}^{(8)}]_{2213},\end{aligned}\quad (3.44)$$

$$\begin{aligned}\hat{\Sigma}_{qd8}^{B_d} = & 6.4 \cdot 10^4 [\hat{c}_{qd}^{(8)}]_{1313} - 1.5 \cdot 10^4 [\hat{c}_{qd}^{(8)}]_{2313} + 2.6 \cdot 10^3 [\hat{c}_{qd}^{(8)}]_{1213} - 6.1 \cdot 10^2 [\hat{c}_{qd}^{(8)}]_{2213} \\ & + 5.7 \cdot 10^2 e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{3313} + 2.3 \cdot 10^2 e^{-i73^\circ} [\hat{c}_{qd}^{(8)}]_{1113} - 5.2 \cdot 10^1 e^{-i73^\circ} [\hat{c}_{qd}^{(8)}]_{1231}\end{aligned}$$

$$\begin{aligned}
 & + 2.4 \cdot 10^1 e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{2331} - 3.1 [\hat{c}_{qd}^{(8)}]_{1312} + 2.0 e^{-i51^\circ} [\hat{c}_{qd}^{(8)}]_{1331} + 7.2 \cdot 10^{-1} [\hat{c}_{qd}^{(8)}]_{2312} \\
 & - 1.3 \cdot 10^{-1} [\hat{c}_{qd}^{(8)}]_{1212} + 7.3 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{1323} + 3.6 \cdot 10^{-2} e^{i21^\circ} [\hat{c}_{qd}^{(8)}]_{1311} \\
 & - 3.5 \cdot 10^{-2} e^{i21^\circ} [\hat{c}_{qd}^{(8)}]_{1333} + 3.0 \cdot 10^{-2} [\hat{c}_{qd}^{(8)}]_{2212} - 2.8 \cdot 10^{-2} e^{i21^\circ} [\hat{c}_{qd}^{(8)}]_{3312} \\
 & - 1.6 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{2323} - 1.1 \cdot 10^{-2} e^{-i74^\circ} [\hat{c}_{qd}^{(8)}]_{1112} - 7.8 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{2311} \\
 & + 7.4 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{2333}, \tag{3.45} \\
 \hat{\Sigma}_{qd8}^K = & 7.2 \cdot 10^6 [\hat{c}_{qd}^{(8)}]_{1212} + 1.6 \cdot 10^6 [\hat{c}_{qd}^{(8)}]_{1112} - 1.6 \cdot 10^6 [\hat{c}_{qd}^{(8)}]_{2212} - 3.8 \cdot 10^5 [\hat{c}_{qd}^{(8)}]_{1221} \\
 & - 2.9 \cdot 10^5 [\hat{c}_{qd}^{(8)}]_{1312} + 6.7 \cdot 10^4 [\hat{c}_{qd}^{(8)}]_{2312} + 6.4 \cdot 10^4 e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{2321} + 1.5 \cdot 10^4 e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{1321} \\
 & - 2.6 \cdot 10^3 e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{3312} + 3.5 \cdot 10^2 [\hat{c}_{qd}^{(8)}]_{1213} + 8.1 \cdot 10^1 [\hat{c}_{qd}^{(8)}]_{1113} - 8.0 \cdot 10^1 [\hat{c}_{qd}^{(8)}]_{2213} \\
 & - 1.9 \cdot 10^1 [\hat{c}_{qd}^{(8)}]_{1231} - 1.4 \cdot 10^1 [\hat{c}_{qd}^{(8)}]_{1313} - 3.8 e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{1232} + 3.3 [\hat{c}_{qd}^{(8)}]_{2313} \\
 & + 3.1 e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{2331} - 3.0 e^{i87^\circ} [\hat{c}_{qd}^{(8)}]_{1211} + 3.0 e^{i87^\circ} [\hat{c}_{qd}^{(8)}]_{1222} - 2.3 e^{i19^\circ} [\hat{c}_{qd}^{(8)}]_{2222} \\
 & + 2.3 e^{i19^\circ} [\hat{c}_{qd}^{(8)}]_{2211} + 1.5 e^{i29^\circ} [\hat{c}_{qd}^{(8)}]_{1122} - 1.5 e^{i29^\circ} [\hat{c}_{qd}^{(8)}]_{1111} - 8.7 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{1123} \\
 & + 8.6 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{2223} + 7.2 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{1331} - 3.3 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{1322} \\
 & + 3.3 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{1311} + 2.0 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qd}^{(8)}]_{1223} + 1.5 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{1332} \\
 & + 1.4 \cdot 10^{-1} e^{i34^\circ} [\hat{c}_{qd}^{(8)}]_{2322} - 1.4 \cdot 10^{-1} e^{i34^\circ} [\hat{c}_{qd}^{(8)}]_{2311} - 1.3 \cdot 10^{-1} e^{i24^\circ} [\hat{c}_{qd}^{(8)}]_{3313} \\
 & - 3.5 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qd}^{(8)}]_{2332} - 3.4 \cdot 10^{-2} e^{i44^\circ} [\hat{c}_{qd}^{(8)}]_{2323} - 7.7 \cdot 10^{-3} e^{i44^\circ} [\hat{c}_{qd}^{(8)}]_{1323}. \tag{3.46}
 \end{aligned}$$

3.4.3 $\mathcal{C}_{qu}^{(1)}$ and $\mathcal{C}_{qu}^{(8)}$

We next present the results for $[\mathcal{C}_{qu}^{(1)}]_{ijkl}$ and $[\mathcal{C}_{qu}^{(8)}]_{ijkl}$ in the down and up bases. Here $D^0 - \bar{D}^0$ mixing dominates the scene, in particular for 1212, but for certain elements like 1313 and 2323 the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings are more important. Interestingly, the entries 1233, 1333 and 2333, can be relevant for K^0 , B_d and B_s , respectively. The index “3” indicates the top Yukawa at work. Yet, the constraints are much weaker than in previous cases.

This pattern can be summarized by the following rather accurate formulae for the sum in (3.4) in the down-basis

$$\Sigma_{qu1}^{B_s} = 2.6 \cdot 10^{-1} [c_{qu}^{(1)}]_{2333} - 3.2 \cdot 10^{-2} [c_{qu}^{(1)}]_{2323}, \tag{3.47}$$

$$\Sigma_{qu1}^{B_d} = -1.3 e^{i22^\circ} [c_{qu}^{(1)}]_{1333} + 1.7 \cdot 10^{-1} [c_{qu}^{(1)}]_{1323}, \tag{3.48}$$

$$\Sigma_{qu1}^K = 2.2 \cdot 10^{-1} e^{i23^\circ} [c_{qu}^{(1)}]_{1233} - 2.8 \cdot 10^{-2} [c_{qu}^{(1)}]_{1223} - 2.6 \cdot 10^{-2} e^{i22^\circ} [c_{qu}^{(1)}]_{1232}, \tag{3.49}$$

and

$$\Sigma_{qu8}^{B_s} = 1.1 \cdot 10^{-1} [c_{qu}^{(8)}]_{2333} - 1.4 \cdot 10^{-2} [c_{qu}^{(8)}]_{2323}, \tag{3.50}$$

$$\Sigma_{qu8}^{B_d} = -5.2 \cdot 10^{-1} e^{i22^\circ} [c_{qu}^{(8)}]_{1333} + 7.5 \cdot 10^{-2} [c_{qu}^{(8)}]_{1323}, \tag{3.51}$$

$$\Sigma_{qu8}^K = 9.2 \cdot 10^{-2} e^{i23^\circ} [c_{qu}^{(8)}]_{1233} - 1.2 \cdot 10^{-2} [c_{qu}^{(8)}]_{1223} - 1.1 \cdot 10^{-2} e^{i22^\circ} [c_{qu}^{(8)}]_{1232}. \tag{3.52}$$

For the up-basis we have

$$\begin{aligned}\hat{\Sigma}_{qu1}^{B_s} = & 2.6 \cdot 10^{-1} [\hat{c}_{qu}^{(1)}]_{2333} + 5.9 \cdot 10^{-2} [\hat{c}_{qu}^{(1)}]_{1333} - 3.1 \cdot 10^{-2} [\hat{c}_{qu}^{(1)}]_{2323} \\ & + 1.1 \cdot 10^{-2} [\hat{c}_{qu}^{(1)}]_{2233} - 1.0 \cdot 10^{-2} [\hat{c}_{qu}^{(1)}]_{3333} - 7.2 \cdot 10^{-3} [\hat{c}_{qu}^{(1)}]_{1323},\end{aligned}\quad (3.53)$$

$$\begin{aligned}\hat{\Sigma}_{qu1}^{B_d} = & -1.3 e^{i22^\circ} [\hat{c}_{qu}^{(1)}]_{1333} + 3.0 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qu}^{(1)}]_{2333} + 1.7 \cdot 10^{-1} [\hat{c}_{qu}^{(1)}]_{1323} \\ & - 5.5 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(1)}]_{1233} - 3.8 \cdot 10^{-2} [\hat{c}_{qu}^{(1)}]_{2323} + 1.3 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{qu}^{(1)}]_{2233} \\ & - 1.1 \cdot 10^{-2} e^{i44^\circ} [\hat{c}_{qu}^{(1)}]_{3333} + 7.1 \cdot 10^{-3} [\hat{c}_{qu}^{(1)}]_{1223},\end{aligned}\quad (3.54)$$

$$\begin{aligned}\hat{\Sigma}_{qu1}^K = & 2.0 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qu}^{(1)}]_{1233} + 4.8 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(1)}]_{1133} - 4.8 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(1)}]_{2233} \\ & - 2.5 \cdot 10^{-2} [\hat{c}_{qu}^{(1)}]_{1223} - 2.4 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(1)}]_{1232} - 1.2 \cdot 10^{-2} e^{i11^\circ} [\hat{c}_{qu}^{(1)}]_{1123} \\ & + 1.2 \cdot 10^{-2} e^{i11^\circ} [\hat{c}_{qu}^{(1)}]_{2223} - 7.7 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qu}^{(1)}]_{1333},\end{aligned}\quad (3.55)$$

and

$$\hat{\Sigma}_{qu8}^{B_s} = 1.0 \cdot 10^{-1} [\hat{c}_{qu}^{(8)}]_{2333} + 2.4 \cdot 10^{-2} [\hat{c}_{qu}^{(8)}]_{1333} - 1.4 \cdot 10^{-2} [\hat{c}_{qu}^{(8)}]_{2323}, \quad (3.56)$$

$$\begin{aligned}\hat{\Sigma}_{qu8}^{B_d} = & -5.1 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qu}^{(8)}]_{1333} + 1.2 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qu}^{(8)}]_{2333} + 7.3 \cdot 10^{-2} [\hat{c}_{qu}^{(8)}]_{1323} \\ & - 2.3 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(8)}]_{1233} - 1.7 \cdot 10^{-2} [\hat{c}_{qu}^{(8)}]_{2323} + 5.3 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{qu}^{(8)}]_{2233},\end{aligned}\quad (3.57)$$

$$\begin{aligned}\hat{\Sigma}_{qu8}^K = & 8.2 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(8)}]_{1233} + 2.0 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(8)}]_{1133} - 2.0 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(8)}]_{2233} \\ & - 1.1 \cdot 10^{-2} [\hat{c}_{qu}^{(8)}]_{1223} - 1.0 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qu}^{(8)}]_{1232} - 5.0 \cdot 10^{-3} e^{i11^\circ} [\hat{c}_{qu}^{(8)}]_{1123} \\ & + 5.0 \cdot 10^{-3} e^{i11^\circ} [\hat{c}_{qu}^{(8)}]_{2223}.\end{aligned}\quad (3.58)$$

3.4.4 $\mathcal{C}_{dd}$, $\mathcal{C}_{uu}$, $\mathcal{C}_{ud}^{(1)}$ and $\mathcal{C}_{ud}^{(8)}$

We collect the results for these four operators in one section together, because the corresponding $P_a^{ij}(\Lambda)$ are the same in the down and up bases.

In the case of $[\mathcal{C}_{dd}]_{ijkl}$ large coefficients are only found in 1212, 1313 and 2323 entries from $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings, respectively. The corresponding coefficients in $D^0 - \bar{D}^0$ mixing are very small.

On the other hand in the case of $[\mathcal{C}_{uu}]_{ijkl}$ only constraints from $D^0 - \bar{D}^0$ mixing are relevant. Indeed, as seen in (3.62) the contributions to down-quark mixings are eliminated by our constraints.

In the case of $[\mathcal{C}_{ud}^{(1)}]_{ijkl}$ and $[\mathcal{C}_{ud}^{(8)}]_{ijkl}$ the numerical coefficients are very small, in particular for charm, implying that $\Delta F = 2$ transitions do not play any role in constraining these Wilson coefficients. Yet, the entries 3312, 3313 and 3323 for $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings, respectively, could play some role in specific models.

This pattern can be summarized by the following rather accurate formulae for the sum in (3.4) for **both down and up basis**

$$\Sigma_{dd}^{B_s} = -4.1 \cdot 10^2 [c_{dd}]_{2323} - 2.2 \cdot 10^{-2} [c_{dd}]_{2333} + 1.9 \cdot 10^{-2} [c_{dd}]_{2223}, \quad (3.59)$$

$$\begin{aligned} \Sigma_{dd}^{B_d} = & -9.6 \cdot 10^3 [c_{dd}]_{1313} + 4.7 \cdot 10^{-1} [c_{dd}]_{1213} + 1.4 \cdot 10^{-2} e^{i22^\circ} [c_{dd}]_{1333} \\ & - 1.1 \cdot 10^{-2} e^{i23^\circ} [c_{dd}]_{1323} + 1.1 \cdot 10^{-2} e^{i22^\circ} [c_{dd}]_{1223}, \end{aligned} \quad (3.60)$$

$$\begin{aligned} \Sigma_{dd}^K = & -3.7 \cdot 10^4 [c_{dd}]_{1212} - 1.8 [c_{dd}]_{1213} - 8.2 \cdot 10^{-2} e^{i23^\circ} [c_{dd}]_{1222} \\ & - 4.8 \cdot 10^{-2} e^{i22^\circ} [c_{dd}]_{1332} + 1.9 \cdot 10^{-2} e^{i22^\circ} [c_{dd}]_{1232} + 5.5 \cdot 10^{-3} e^{i22^\circ} [c_{dd}]_{1233}, \end{aligned} \quad (3.61)$$

$$\Sigma_{uu}^{B_s} = 0, \quad \Sigma_{uu}^{B_d} = 0, \quad \Sigma_{uu}^K = 0, \quad (3.62)$$

$$\Sigma_{ud1}^{B_s} = -1.0 [c_{ud}^{(1)}]_{3323} + 1.1 \cdot 10^{-1} [c_{ud}^{(1)}]_{2323}, \quad (3.63)$$

$$\Sigma_{ud1}^{B_d} = 5.6 e^{i22^\circ} [c_{ud}^{(1)}]_{3313} - 6.2 \cdot 10^{-1} [c_{ud}^{(1)}]_{2313} + 5.4 \cdot 10^{-3} [c_{ud}^{(1)}]_{1313}, \quad (3.64)$$

$$\begin{aligned} \Sigma_{ud1}^K = & -2.0 \cdot 10^1 e^{i23^\circ} [c_{ud}^{(1)}]_{3312} + 2.1 [c_{ud}^{(1)}]_{2312} + 2.0 e^{i22^\circ} [c_{ud}^{(1)}]_{2321} \\ & - 1.9 \cdot 10^{-1} [c_{ud}^{(1)}]_{2212} - 1.8 \cdot 10^{-2} [c_{ud}^{(1)}]_{1312} - 8.1 \cdot 10^{-3} e^{i23^\circ} [c_{ud}^{(1)}]_{1112}, \end{aligned} \quad (3.65)$$

and

$$\Sigma_{ud8}^{B_s} = -1.5 [c_{ud}^{(8)}]_{3323} + 1.6 \cdot 10^{-1} [c_{ud}^{(8)}]_{2323}, \quad (3.66)$$

$$\begin{aligned} \Sigma_{ud8}^{B_d} = & 7.9 e^{i22^\circ} [c_{ud}^{(8)}]_{3313} - 8.7 \cdot 10^{-1} [c_{ud}^{(8)}]_{2313} + 7.6 \cdot 10^{-3} [c_{ud}^{(8)}]_{1313} \\ & + 5.5 \cdot 10^{-3} e^{i22^\circ} [c_{ud}^{(8)}]_{1113} + 5.4 \cdot 10^{-3} e^{i22^\circ} [c_{ud}^{(8)}]_{2213}, \end{aligned} \quad (3.67)$$

$$\begin{aligned} \Sigma_{ud8}^K = & -3.8 \cdot 10^1 e^{i23^\circ} [c_{ud}^{(8)}]_{3312} + 4.2 [c_{ud}^{(8)}]_{2312} + 4.0 e^{i22^\circ} [c_{ud}^{(8)}]_{2321} \\ & - 3.9 \cdot 10^{-1} [c_{ud}^{(8)}]_{2212} - 3.6 \cdot 10^{-2} [c_{ud}^{(8)}]_{1312} - 2.5 \cdot 10^{-2} e^{i22^\circ} [c_{ud}^{(8)}]_{1112}. \end{aligned} \quad (3.68)$$

3.4.5 $\mathcal{C}_{\phi q}^{(1,3)}$, $\mathcal{C}_{\phi d}$ and $\mathcal{C}_{\phi u}$

Here of particular interest are the entries 12, 13, 23 in $[\mathcal{C}_{\phi d}]$, which in certain scenarios like VLQ models imply rather strong constraints [16, 92].

Explicitly we have in **the down basis** for $\mathcal{C}_{\phi q}^{(1,3)}$

$$\Sigma_{\phi q1}^{B_s} = -2.4 \cdot 10^{-1} [c_{\phi q}^{(1)}]_{23}, \quad \Sigma_{\phi q1}^{B_d} = 1.2 e^{i22^\circ} [c_{\phi q}^{(1)}]_{13}, \quad \Sigma_{\phi q1}^K = -2.1 \cdot 10^{-1} e^{i23^\circ} [c_{\phi q}^{(1)}]_{12}, \quad (3.69)$$

and

$$\Sigma_{\phi q3}^{B_s} = 5.4 \cdot 10^{-1} [c_{\phi q}^{(3)}]_{23} + 6.3 \cdot 10^{-3} [c_{\phi q}^{(3)}]_{33} + 5.1 \cdot 10^{-3} [c_{\phi q}^{(3)}]_{22}, \quad (3.70)$$

$$\begin{aligned} \Sigma_{\phi q3}^{B_d} = & -2.7 e^{i22^\circ} [c_{\phi q}^{(3)}]_{13} - 2.6 \cdot 10^{-2} e^{i21^\circ} [c_{\phi q}^{(3)}]_{12} \\ & + 7.0 \cdot 10^{-3} e^{i44^\circ} [c_{\phi q}^{(3)}]_{33} + 5.7 \cdot 10^{-3} e^{i44^\circ} [c_{\phi q}^{(3)}]_{11}, \end{aligned} \quad (3.71)$$

$$\Sigma_{\phi q3}^K = 5.4 \cdot 10^{-1} e^{i23^\circ} [c_{\phi q}^{(3)}]_{12} + 5.3 \cdot 10^{-3} e^{i24^\circ} [c_{\phi q}^{(3)}]_{13}, \quad (3.72)$$

and $\mathcal{C}_{\phi d}$

$$\Sigma_{\phi d}^{B_s} = 1.2 [c_{\phi d}]_{23}, \quad \Sigma_{\phi d}^{B_d} = -6.6 e^{i22^\circ} [c_{\phi d}]_{13}, \quad \Sigma_{\phi d}^K = 2.5 \cdot 10^1 e^{i23^\circ} [c_{\phi d}]_{12}. \quad (3.73)$$

In the case $\mathcal{C}_{\phi u}$ the corresponding contributions of $\Sigma_{\phi u}$ are too small to allow for a 5% effect in $[M_{12}^{ij}]_{\text{BSM}}$ and hence are eliminated by our conditions.

For **the up basis** we have

$$\hat{\Sigma}_{\phi q 1}^{B_s} = -2.3 \cdot 10^{-1} [\hat{c}_{\phi q}^{(1)}]_{23} - 5.4 \cdot 10^{-2} [\hat{c}_{\phi q}^{(1)}]_{13} - 1.0 \cdot 10^{-2} [\hat{c}_{\phi q}^{(1)}]_{22} + 9.4 \cdot 10^{-3} [\hat{c}_{\phi q}^{(1)}]_{33}, \quad (3.74)$$

$$\begin{aligned} \hat{\Sigma}_{\phi q 1}^{B_d} &= 1.2 e^{i22^\circ} [\hat{c}_{\phi q}^{(1)}]_{13} - 2.7 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{\phi q}^{(1)}]_{23} + 5.2 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{\phi q}^{(1)}]_{12} \\ &\quad - 1.2 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{\phi q}^{(1)}]_{22} + 1.0 \cdot 10^{-2} e^{i44^\circ} [\hat{c}_{\phi q}^{(1)}]_{33}, \end{aligned} \quad (3.75)$$

$$\begin{aligned} \hat{\Sigma}_{\phi q 1}^K &= -1.9 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{\phi q}^{(1)}]_{12} - 4.6 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{\phi q}^{(1)}]_{11} + 4.6 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{\phi q}^{(1)}]_{22} \\ &\quad + 7.3 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{\phi q}^{(1)}]_{13}, \end{aligned} \quad (3.76)$$

and

$$\begin{aligned} \hat{\Sigma}_{\phi q 3}^{B_s} &= 5.2 \cdot 10^{-1} [\hat{c}_{\phi q}^{(3)}]_{23} + 1.2 \cdot 10^{-1} [\hat{c}_{\phi q}^{(3)}]_{13} + 2.7 \cdot 10^{-2} [\hat{c}_{\phi q}^{(3)}]_{22} \\ &\quad - 1.6 \cdot 10^{-2} [\hat{c}_{\phi q}^{(3)}]_{33} + 7.3 \cdot 10^{-3} e^{-i19^\circ} [\hat{c}_{\phi q}^{(3)}]_{12}, \end{aligned} \quad (3.77)$$

$$\begin{aligned} \hat{\Sigma}_{\phi q 3}^{B_d} &= -2.6 e^{i22^\circ} [\hat{c}_{\phi q}^{(3)}]_{13} + 6.1 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{\phi q}^{(3)}]_{23} - 1.4 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{\phi q}^{(3)}]_{12} \\ &\quad + 3.2 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{\phi q}^{(3)}]_{22} - 1.8 \cdot 10^{-2} e^{i44^\circ} [\hat{c}_{\phi q}^{(3)}]_{33} - 1.2 \cdot 10^{-2} e^{-i51^\circ} [\hat{c}_{\phi q}^{(3)}]_{11}, \end{aligned} \quad (3.78)$$

$$\begin{aligned} \hat{\Sigma}_{\phi q 3}^K &= 4.9 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{\phi q}^{(3)}]_{12} + 1.2 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{\phi q}^{(3)}]_{11} - 1.2 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{\phi q}^{(3)}]_{22} \\ &\quad - 1.6 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{\phi q}^{(3)}]_{13} + 7.4 \cdot 10^{-3} e^{i34^\circ} [\hat{c}_{\phi q}^{(3)}]_{23}, \end{aligned} \quad (3.79)$$

whereas results for $\hat{\mathcal{C}}_{\phi d}$ and $\hat{\mathcal{C}}_{\phi u}$ are the same as in the down basis.

3.4.6 $\mathcal{C}_{quqd}^{(1,8)}$

The contributions from these coefficients are very small. However, what is interesting is the dominance of $\Delta F = 1$ transitions in all meson systems, demonstrating top Yukawa at work.

Explicitly we have in **the down basis**

$$\Sigma_{quqd 1}^{B_s} = 1.0 \cdot 10^{-1} [c_{quqd}^{(1)}]_{3332} - 5.0 \cdot 10^{-2} [c_{quqd}^{(1)}]_{2323} - 9.2 \cdot 10^{-3} [c_{quqd}^{(1)}]_{3232}, \quad (3.80)$$

$$\Sigma_{quqd 1}^{B_d} = -5.4 \cdot 10^{-1} e^{i22^\circ} [c_{quqd}^{(1)}]_{3331} - 6.0 \cdot 10^{-2} [c_{quqd}^{(1)}]_{1313} + 5.2 \cdot 10^{-2} [c_{quqd}^{(1)}]_{3231}, \quad (3.81)$$

$$\begin{aligned} \Sigma_{quqd 1}^K &= -2.8 [c_{quqd}^{(1)}]_{3321} - 2.8 [c_{quqd}^{(1)}]_{3312} - 1.2 e^{i22^\circ} [c_{quqd}^{(1)}]_{2321} \\ &\quad + 2.7 \cdot 10^{-1} [c_{quqd}^{(1)}]_{1312} + 1.1 \cdot 10^{-1} [c_{quqd}^{(1)}]_{2221} - 4.3 \cdot 10^{-2} [c_{quqd}^{(1)}]_{2331} \\ &\quad - 4.1 \cdot 10^{-2} [c_{quqd}^{(1)}]_{1332} - 2.4 \cdot 10^{-2} [c_{quqd}^{(1)}]_{1212}, \end{aligned} \quad (3.82)$$

and

$$\Sigma_{quqd 8}^{B_s} = 1.8 \cdot 10^{-2} [c_{quqd}^{(8)}]_{3332} - 8.8 \cdot 10^{-3} [c_{quqd}^{(8)}]_{2323}, \quad (3.83)$$

$$\Sigma_{quqd 8}^{B_d} = -9.6 \cdot 10^{-2} e^{i22^\circ} [c_{quqd}^{(8)}]_{3331} - 1.1 \cdot 10^{-2} [c_{quqd}^{(8)}]_{1313} + 9.0 \cdot 10^{-3} [c_{quqd}^{(8)}]_{3231}, \quad (3.84)$$

$$\begin{aligned} \Sigma_{quqd 8}^K &= -1.9 \cdot 10^{-1} e^{i22^\circ} [c_{quqd}^{(8)}]_{2321} + 4.2 \cdot 10^{-2} [c_{quqd}^{(8)}]_{1312} + 1.8 \cdot 10^{-2} [c_{quqd}^{(8)}]_{2221} \\ &\quad - 6.2 \cdot 10^{-3} [c_{quqd}^{(8)}]_{2331} - 5.7 \cdot 10^{-3} [c_{quqd}^{(8)}]_{1332}. \end{aligned} \quad (3.85)$$

In the up basis we find

$$\begin{aligned}\hat{\Sigma}_{quqd1}^{B_s} &= 1.0 \cdot 10^{-1} [\hat{c}_{quqd}^{(1)}]_{3332} - 4.7 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2323} - 1.1 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2313} \\ &\quad - 1.1 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{1323} - 9.2 \cdot 10^{-3} [\hat{c}_{quqd}^{(1)}]_{3232},\end{aligned}\quad (3.86)$$

$$\begin{aligned}\hat{\Sigma}_{quqd1}^{B_d} &= -5.5 \cdot 10^{-1} e^{i20^\circ} [\hat{c}_{quqd}^{(1)}]_{3331} - 5.7 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{1313} + 5.2 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{3231} \\ &\quad - 2.2 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{quqd}^{(1)}]_{2331} - 1.7 \cdot 10^{-2} e^{i28^\circ} [\hat{c}_{quqd}^{(1)}]_{3321} + 1.3 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2313} \\ &\quad + 1.3 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{1323},\end{aligned}\quad (3.87)$$

$$\begin{aligned}\hat{\Sigma}_{quqd1}^K &= 3.4 \cdot 10^1 [\hat{c}_{quqd}^{(1)}]_{3321} + 1.5 \cdot 10^1 [\hat{c}_{quqd}^{(1)}]_{3311} + 2.2 [\hat{c}_{quqd}^{(1)}]_{3322} - 1.7 [\hat{c}_{quqd}^{(1)}]_{3312} \\ &\quad + 1.2 [\hat{c}_{quqd}^{(1)}]_{2331} - 1.1 e^{i22^\circ} [\hat{c}_{quqd}^{(1)}]_{2321} + 2.7 \cdot 10^{-1} [\hat{c}_{quqd}^{(1)}]_{1331} \\ &\quad - 2.6 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{quqd}^{(1)}]_{1321} - 2.6 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{quqd}^{(1)}]_{2311} + 2.5 \cdot 10^{-1} [\hat{c}_{quqd}^{(1)}]_{1312} \\ &\quad + 1.0 \cdot 10^{-1} [\hat{c}_{quqd}^{(1)}]_{2221} - 6.0 \cdot 10^{-2} e^{i20^\circ} [\hat{c}_{quqd}^{(1)}]_{1311} - 5.7 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{1322} \\ &\quad - 5.5 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2312} + 2.4 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2211} + 2.4 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{1221} \\ &\quad - 2.2 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{1212} + 2.0 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2332} + 2.0 \cdot 10^{-2} e^{i3^\circ} [\hat{c}_{quqd}^{(1)}]_{1332} \\ &\quad + 1.4 \cdot 10^{-2} [\hat{c}_{quqd}^{(1)}]_{2322} - 6.0 \cdot 10^{-3} e^{i21^\circ} [\hat{c}_{quqd}^{(1)}]_{3331} + 5.6 \cdot 10^{-3} [\hat{c}_{quqd}^{(1)}]_{1211} \\ &\quad + 5.1 \cdot 10^{-3} [\hat{c}_{quqd}^{(1)}]_{2212} + 5.1 \cdot 10^{-3} [\hat{c}_{quqd}^{(1)}]_{1222},\end{aligned}\quad (3.88)$$

and

$$\hat{\Sigma}_{quqd8}^{B_s} = 1.8 \cdot 10^{-2} [\hat{c}_{quqd}^{(8)}]_{3332} - 8.4 \cdot 10^{-3} [\hat{c}_{quqd}^{(8)}]_{2323}, \quad (3.89)$$

$$\hat{\Sigma}_{quqd8}^{B_d} = -9.5 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{quqd}^{(8)}]_{3331} - 1.0 \cdot 10^{-2} [\hat{c}_{quqd}^{(8)}]_{1313} + 9.0 \cdot 10^{-3} [\hat{c}_{quqd}^{(8)}]_{3231}, \quad (3.90)$$

$$\begin{aligned}\hat{\Sigma}_{quqd8}^K &= 3.6 \cdot 10^{-1} [\hat{c}_{quqd}^{(8)}]_{2331} - 1.8 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{quqd}^{(8)}]_{2321} - 8.2 \cdot 10^{-2} [\hat{c}_{quqd}^{(8)}]_{1331} \\ &\quad - 4.1 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{quqd}^{(8)}]_{2311} - 4.1 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{quqd}^{(8)}]_{1321} + 4.0 \cdot 10^{-2} [\hat{c}_{quqd}^{(8)}]_{1312} \\ &\quad + 1.7 \cdot 10^{-2} [\hat{c}_{quqd}^{(8)}]_{2221} + 1.3 \cdot 10^{-2} [\hat{c}_{quqd}^{(8)}]_{1332} - 9.6 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{quqd}^{(8)}]_{1311} \\ &\quad - 9.3 \cdot 10^{-3} [\hat{c}_{quqd}^{(8)}]_{2312} - 9.2 \cdot 10^{-3} [\hat{c}_{quqd}^{(8)}]_{1322} + 7.3 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{quqd}^{(8)}]_{3321} \\ &\quad + 5.1 \cdot 10^{-3} [\hat{c}_{quqd}^{(8)}]_{2332}.\end{aligned}\quad (3.91)$$

3.4.7 $\mathcal{C}_{lequ}^{(1,3)}$, $\mathcal{C}_{lq}^{(1,3)}$, $\mathcal{C}_{ld}$ and $\mathcal{C}_{qe}$

The contributions of the semileptonic operators are even smaller than the previous ones. Most of them give too small contributions and we show here only those that meet our criteria.¹⁵ The correlations between rare Kaon decays, $K^0 - \bar{K}^0$ mixing and the $\Delta F = 1$ process ε'/ε have been discussed in the framework of leptoquark models in [93]. Explicitly under our exclusion principle stated above we find in **the down basis**

$$\Sigma_{lq3}^{B_s} = 0, \quad \Sigma_{lq3}^{B_d} = 1.1 \cdot 10^{-2} e^{i22^\circ} \left([c_{lq}^{(3)}]_{3313} + [c_{lq}^{(3)}]_{1113} + [c_{lq}^{(3)}]_{2213} \right), \quad \Sigma_{lq3}^K = 0, \quad (3.92)$$

¹⁵For the interested reader we refer to the tables in the supplementary material, which provide all contributions.

and in **the up-basis**

$$\hat{\Sigma}_{lq3}^{B_s} = 0, \quad \hat{\Sigma}_{lq3}^{B_d} = 1.1 \cdot 10^{-2} e^{i22^\circ} \left([\hat{c}_{lq}^{(3)}]_{1113} + [\hat{c}_{lq}^{(3)}]_{2213} + [\hat{c}_{lq}^{(3)}]_{3313} \right), \quad \hat{\Sigma}_{lq3}^K = 0. \quad (3.93)$$

While the numerical coefficients in the sums are the same in up and down bases, the WCs could be different in these two bases.

3.4.8 $\mathcal{C}_{uW}$

For the operator $\mathcal{O}_{uW}$ one finds in **the down-basis**:

$$\Sigma_{uW}^{B_s} = -6.1 \cdot 10^{-2} [c_{uW}]_{23}, \quad \Sigma_{uW}^{B_d} = 3.1 \cdot 10^{-1} e^{i22^\circ} [c_{uW}]_{13}, \quad \Sigma_{uW}^K = 0, \quad (3.94)$$

and the same for **the up-basis**.

3.5 Probing large values of Λ

In this section we present maximal values of the scale Λ at which the NP contribution from a single Wilson coefficient could provide a shift in the mass difference ΔM_{ij} of at least 10% of $(\Delta M_{ij})_{\text{exp}}$ when the dimensionless SMEFT coefficient, introduced in (3.4), becomes $c_a = 10$. This choice corresponds to an approximate upper bound of $c_a \sim 4\pi$ for which a UV completion would be still considered to be in the perturbative regime assuming a tree-level exchange of a heavy mediator. For lower values of c_a these maximal values of Λ will be smaller. On the other hand, keeping $c_a = 10$ and increasing Λ will imply NP effects below 10% to ΔM_{ij} . In view of the small hadronic uncertainties in most of the $\Delta F = 2$ matrix elements in table 2, NP effects below 10% should be still of interest when constraining BSM scenarios. The results are presented in figures 4–13 for both down and up bases. For an operator with a given flavour structure, strikingly different values of Λ are found in the two bases. These results are self-explanatory.

4 Simplified models

In this section we discuss the tree-level models, which match onto one or several operators relevant for the considered $\Delta F = 2$ processes.¹⁶ Their complete matching onto SMEFT can be found in [96]. Table 5 shows all models with tree-level exchange of scalars that match onto relevant four-quark operators. Table 6 lists the fermion and vector models that match onto modified Z - and W couplings or dipole operators at tree-level. Finally, table 7 lists all tree-level mediated vector models, which generate four-quark operators relevant for $\Delta F = 2$ transitions. As examples we choose the models with vector bosons that are singlets under $\text{SU}(2)_L \times \text{U}(1)_Y$ and with scalar bosons that are doublets under $\text{SU}(2)_L$, i.e. for vector exchanges a colourless heavy Z' and a heavy coloured gluon G' and similarly for scalar exchanges a colourless φ and a coloured Φ .

¹⁶The matching for simplified models with loop effects involving generic scalars and fermions were calculated in [94, 95].

Figure 4. The maximal NP scale Λ for $[c_{qq}^{(1)}]_{ijkl} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Spin	Rep.	$\mathcal{O}_{qq}^{(1)}$	$\mathcal{O}_{qq}^{(3)}$	$\mathcal{O}_{qd}^{(1)}$	$\mathcal{O}_{qd}^{(8)}$	$\mathcal{O}_{qu}^{(1)}$	$\mathcal{O}_{qu}^{(8)}$	$\mathcal{O}_{dd}$	$\mathcal{O}_{uu}$	$\mathcal{O}_{ud}^{(1)}$	$\mathcal{O}_{ud}^{(8)}$
0	$(1, 2)_{\frac{1}{2}}$			×	×	×	×				
	$(3, 1)_{-\frac{1}{3}}$	×	×							×	×
	$(3, 1)_{\frac{2}{3}}$							×			
	$(3, 1)_{-\frac{4}{3}}$								×		
	$(3, 3)_{-\frac{1}{3}}$	×	×								
	$(6, 1)_{\frac{1}{3}}$	×	×								
	$(6, 1)_{-\frac{2}{3}}$							×			
	$(6, 1)_{\frac{4}{3}}$								×		
	$(6, 3)_{\frac{1}{3}}$	×	×								
	$(8, 2)_{\frac{1}{2}}$			×	×	×	×				

Table 5. Four-quark (ψ^4) operators generated from additional scalar fields.

Figure 5. The maximal NP scale Λ for $[c_{qd}^{(1)}]_{ijkl} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Spin	Rep.	$\mathcal{O}_{\phi q}^{(1)}$	$\mathcal{O}_{\phi q}^{(3)}$	$\mathcal{O}_{\phi d}$	$\mathcal{O}_{uW}$
$\frac{1}{2}$	$(3, 1)_{\frac{2}{3}}$	$\times$	$\times$		
	$(3, 1)_{-\frac{1}{3}}$	$\times$	$\times$		
	$(3, 3)_{-\frac{1}{3}}$	$\times$	$\times$		
	$(3, 3)_{\frac{2}{3}}$	$\times$	$\times$		$\times$
	$(3, 2)_{\frac{1}{6}}$			$\times$	$\times$
	$(3, 2)_{-\frac{5}{6}}$			$\times$	
1	$(1, 1)_0$	$\times$		$\times$	
	$(1, 3)_0$		$\times$		

Table 6. $\psi^2\phi^2D$ and $\psi^2X\phi$ operators generated from additional fermion or vector fields.

Figure 6. The maximal NP scale Λ for $[c_{dd}]_{ijkl} = 10$ (upper) and $[c_{ud}^{(1)}]_{ijkl} = 10$ (lower) that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Spin	Rep.	$\mathcal{O}_{qq}^{(1)}$	$\mathcal{O}_{qq}^{(3)}$	$\mathcal{O}_{qd}^{(1)}$	$\mathcal{O}_{qd}^{(8)}$	$\mathcal{O}_{qu}^{(1)}$	$\mathcal{O}_{qu}^{(8)}$	$\mathcal{O}_{dd}$	$\mathcal{O}_{uu}$	$\mathcal{O}_{ud}^{(1)}$	$\mathcal{O}_{ud}^{(8)}$
1	$(1,1)_0$	×		×		×		×	×	×	
	$(1,1)_1$									×	×
	$(1,3)_0$		×								
	$(8,1)_0$	×	×		×		×	×	×		×
	$(8,1)_1$									×	×
	$(8,3)_0$	×	×								
	$(3,2)_{\frac{1}{6}}$			×	×						
	$(3,2)_{-\frac{5}{6}}$					×	×				
	$(\bar{6},2)_{\frac{1}{6}}$			×	×						
	$(\bar{6},2)_{-\frac{5}{6}}$					×	×				

Table 7. Four-quark (ψ^4) operators generated from additional vector fields.

Figure 7. The maximal NP scale Λ for $[c_{qu}^{(1)}]_{ijkl} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Figure 8. The maximal NP scale Λ for $[c_{\phi q}^{(1)}]_{ij} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Figure 9. The maximal NP scale Λ for $[c_{\phi q}^{(3)}]_{ij} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Figure 10. The maximal NP scale Λ for $[c_{\phi d}]_{ij} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

4.1 Z' model

The interaction Lagrangian of a $Z' = (1, 1)_0$ field coupling to the quarks and the SM Higgs doublet ϕ reads:

$$\mathcal{L}_{Z'} = - \left[z_q^{ij} (\bar{q}^i \gamma^\mu q^j) + z_u^{ij} (\bar{u}^i \gamma^\mu u^j) + z_d^{ij} (\bar{d}^i \gamma^\mu d^j) \right] Z'_\mu + z_\phi (\phi^\dagger i \mathcal{D}^\mu \phi) Z'_\mu + \text{h.c.} \quad (4.1)$$

Matching this model onto the relevant SMEFT Wilson coefficients leads to the following matching conditions at Λ :

$$\begin{aligned} [C_{qq}^{(1)}]_{ijkl} &= -\frac{z_q^{ij} z_q^{kl}}{2M_{Z'}^2}, & [C_{qd}^{(1)}]_{ijkl} &= -\frac{z_q^{ij} z_d^{kl}}{M_{Z'}^2}, & [C_{qu}^{(1)}]_{ijkl} &= -\frac{z_q^{ij} z_u^{kl}}{M_{Z'}^2}, \\ [C_{dd}^{(1)}]_{ijkl} &= -\frac{z_d^{ij} z_d^{kl}}{2M_{Z'}^2}, & [C_{uu}^{(1)}]_{ijkl} &= -\frac{z_u^{ij} z_u^{kl}}{2M_{Z'}^2}, & [C_{ud}^{(1)}]_{ijkl} &= -\frac{z_u^{ij} z_d^{kl}}{M_{Z'}^2}, \\ [C_{\phi q}^{(1)}]_{ij} &= -\frac{\text{Re}(z_\phi) z_q^{ij}}{M_{Z'}^2}, & [C_{\phi d}]_{ij} &= -\frac{\text{Re}(z_\phi) z_d^{ij}}{M_{Z'}^2}. \end{aligned} \quad (4.2)$$

Figure 11. The maximal NP scale Λ for $[\mathcal{C}_{quqd}^{(1)}]_{ijkl} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

These equations imply the following tree-level relations between different coefficients valid at the NP matching scale that are independent of $M_{Z'}$ and are universal in all meson systems considered ($a = u, d$)

$$[\mathcal{C}_{qa}^{(1)}]_{ijij}^2 = 4[\mathcal{C}_{qq}^{(1)}]_{ijij}[\mathcal{C}_{aa}]_{ijij}, \quad [\mathcal{C}_{ud}^{(1)}]_{ijij}^2 = 4[\mathcal{C}_{dd}]_{ijij}[\mathcal{C}_{uu}]_{ijij}. \quad (4.3)$$

Albeit they are generally modified through RG effects and matching at one-loop level.

We note that the coefficients $[\mathcal{C}_{qa}^{(8)}]_{ijij}$ are absent in this list, which in the BMU basis implies $C_{\text{LR},2}^{ij} = 0$, while $C_{\text{LR},1}^{ij}$ are non-vanishing. But including QCD corrections we find in the NDR scheme using the results in [97] at Λ

$$[\mathcal{C}_{qa}^{(1)}]_{ijij} = -\frac{z_q^{ij} z_a^{ij}}{M_{Z'}^2} \left(1 - \frac{\alpha_s}{4\pi} \left[2 \ln \frac{M_{Z'}^2}{\Lambda^2} + \frac{1}{3} \right] \right), \quad (a = u, d) \quad (4.4)$$

$$[\mathcal{C}_{qa}^{(8)}]_{ijij} = \frac{\alpha_s}{4\pi} \frac{z_q^{ij} z_a^{ij}}{M_{Z'}^2} \left[6 \ln \frac{M_{Z'}^2}{\Lambda^2} + 1 \right], \quad (4.5)$$

and this implies non-vanishing $C_{\text{LR},2}^{ij}$. The presence of the logarithms cancels the dependence on the choice of the matching scale Λ present in the NLO RG evolution and the constant terms remove the corresponding renormalization scheme dependence.

Figure 12. The maximal NP scale Λ for $[c_{quqd}^{(8)}]_{ijkl} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

Using the master formulae of the previous section we find following master formulae for the Z' model in **the down-basis**

$$\begin{aligned} \frac{M_{Z', \Sigma_{Z'}^{B_s}}^2}{(5\text{TeV})^2} = & -2.3 \cdot 10^3 z_q^{23} z_d^{23} + 2.1 \cdot 10^2 z_d^{23} z_d^{23} + 2.0 \cdot 10^2 z_q^{23} z_q^{23} - 3.3 z_q^{33} z_d^{23} + 1.8 z_q^{22} z_d^{23} \\ & - 1.2 \text{Re}(z_\phi) z_d^{23} + 1.0 z_u^{33} z_d^{23} - 4.0 \cdot 10^{-1} e^{i22^\circ} z_q^{12} z_d^{32} + 2.7 \cdot 10^{-1} z_q^{23} z_q^{33} \\ & - 2.6 \cdot 10^{-1} z_q^{23} z_u^{33} + 2.4 \cdot 10^{-1} \text{Re}(z_\phi) z_q^{23} - 1.5 \cdot 10^{-1} z_q^{22} z_q^{23} + 1.1 \cdot 10^{-1} z_q^{23} z_d^{22} \\ & - 1.1 \cdot 10^{-1} z_q^{23} z_d^{33} - 1.1 \cdot 10^{-1} z_u^{23} z_d^{23} + 3.4 \cdot 10^{-2} e^{i22^\circ} z_q^{12} z_q^{32} + 3.2 \cdot 10^{-2} z_q^{23} z_u^{23} \\ & - 2.5 \cdot 10^{-2} z_q^{23} z_d^{32} + 2.3 \cdot 10^{-2} e^{-i23^\circ} z_q^{13} z_d^{23} + 1.1 \cdot 10^{-2} z_d^{23} z_d^{33} - 9.3 \cdot 10^{-3} z_d^{22} z_d^{23} \\ & + 5.5 \cdot 10^{-3} e^{i21^\circ} z_q^{13} z_d^{32}, \end{aligned} \quad (4.6)$$

$$\begin{aligned} \frac{M_{Z', \Sigma_{Z'}^{B_d}}^2}{(5\text{TeV})^2} = & -5.7 \cdot 10^4 z_q^{13} z_d^{13} + 4.8 \cdot 10^3 z_d^{13} z_d^{13} + 4.5 \cdot 10^3 z_q^{13} z_q^{13} + 4.5 \cdot 10^1 z_q^{12} z_d^{13} \\ & + 1.8 \cdot 10^1 e^{i22^\circ} z_q^{33} z_d^{13} - 9.8 e^{i22^\circ} z_q^{11} z_d^{13} + 6.6 e^{i22^\circ} \text{Re}(z_\phi) z_d^{13} - 5.6 e^{i22^\circ} z_u^{33} z_d^{13} \\ & - 3.6 z_q^{12} z_q^{13} + 2.8 z_q^{13} z_d^{12} - 1.4 e^{i22^\circ} z_q^{13} z_q^{33} + 1.3 e^{i22^\circ} z_q^{13} z_u^{33} - 1.2 e^{i22^\circ} \text{Re}(z_\phi) z_q^{13} \\ & + 7.8 \cdot 10^{-1} e^{i22^\circ} z_q^{11} z_q^{13} - 7.6 \cdot 10^{-1} e^{i23^\circ} z_q^{23} z_d^{13} + 6.2 \cdot 10^{-1} z_u^{23} z_d^{13} - 2.4 \cdot 10^{-1} z_d^{12} z_d^{13} \\ & - 1.7 \cdot 10^{-1} z_q^{13} z_u^{23} + 1.4 \cdot 10^{-1} e^{i21^\circ} z_q^{23} z_d^{31} - 6.5 \cdot 10^{-2} e^{i23^\circ} z_q^{13} z_d^{23} + 5.9 \cdot 10^{-2} e^{i23^\circ} z_q^{13} z_q^{23} \end{aligned}$$

Figure 13. The maximal NP scale Λ for $[\mathcal{C}_{\ell q}^{(3)}]_{ijkl} = 10$ that corresponds to a 10% effect in $2[M_{12}^{ij}]_{\text{BSM}}/(\Delta M_{ij})_{\text{exp}}$ for B_s (blue), B_d (red) and K^0 (green), respectively.

$$\begin{aligned}
 &+3.1 \cdot 10^{-2} e^{i21^\circ} z_q^{13} z_d^{33} - 3.1 \cdot 10^{-2} e^{i21^\circ} z_q^{13} z_d^{11} - 3.0 \cdot 10^{-2} e^{i44^\circ} z_q^{13} z_d^{31} \\
 &- 1.2 \cdot 10^{-2} e^{i21^\circ} z_q^{13} z_q^{32} - 7.2 \cdot 10^{-3} e^{i22^\circ} z_q^{22} z_d^{13} - 7.2 \cdot 10^{-3} e^{i22^\circ} z_d^{13} z_d^{33} \\
 &+ 5.5 \cdot 10^{-3} e^{i23^\circ} z_d^{13} z_d^{23} - 5.4 \cdot 10^{-3} z_u^{13} z_d^{13} - 5.3 \cdot 10^{-3} e^{i22^\circ} z_d^{12} z_d^{23}, \quad (4.7)
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_{Z'}^2 \Sigma_{Z'}^K}{(5 \text{ TeV})^2} = & -5.3 \cdot 10^6 z_q^{12} z_d^{12} + 1.8 \cdot 10^4 z_d^{12} z_d^{12} + 1.8 \cdot 10^4 z_q^{12} z_q^{12} - 9.0 \cdot 10^3 z_q^{13} z_d^{12} \\
 & + 2.0 \cdot 10^3 e^{i22^\circ} z_q^{23} z_d^{21} - 2.6 \cdot 10^2 z_q^{12} z_d^{13} - 8.1 \cdot 10^1 e^{i24^\circ} z_q^{22} z_d^{12} + 3.4 \cdot 10^1 e^{i26^\circ} z_q^{11} z_d^{12} \\
 & + 3.0 \cdot 10^1 z_q^{12} z_q^{13} - 2.5 \cdot 10^1 e^{i23^\circ} \text{Re}(z_\phi) z_d^{12} + 2.0 \cdot 10^1 e^{i23^\circ} z_u^{33} z_d^{12} + 1.7 \cdot 10^1 e^{i23^\circ} z_q^{33} z_d^{12} \\
 & - 6.6 e^{i22^\circ} z_q^{12} z_q^{32} + 2.8 e^{i22^\circ} z_q^{12} z_d^{32} + 2.5 e^{i66^\circ} z_q^{12} z_d^{11} - 2.5 e^{i66^\circ} z_q^{12} z_d^{22} - 2.1 z_u^{23} z_d^{12} \\
 & - 2.0 e^{i22^\circ} z_u^{23} z_d^{21} + 9.0 \cdot 10^{-1} z_d^{12} z_d^{13} - 7.2 \cdot 10^{-1} e^{i24^\circ} z_q^{23} z_d^{12} - 4.4 \cdot 10^{-1} z_q^{13} z_d^{13} \\
 & + 2.8 \cdot 10^{-1} e^{i23^\circ} z_q^{12} z_q^{22} - 2.2 \cdot 10^{-1} e^{i23^\circ} z_q^{12} z_u^{33} + 2.1 \cdot 10^{-1} e^{i23^\circ} \text{Re}(z_\phi) z_q^{12} \\
 & + 1.9 \cdot 10^{-1} z_u^{22} z_d^{12} - 1.3 \cdot 10^{-1} e^{i23^\circ} z_q^{11} z_q^{12} + 1.1 \cdot 10^{-1} e^{i45^\circ} z_q^{13} z_d^{21} + 9.6 \cdot 10^{-2} e^{i23^\circ} z_q^{23} z_d^{31} \\
 & - 6.1 \cdot 10^{-2} e^{i23^\circ} z_q^{12} z_q^{33} + 5.1 \cdot 10^{-2} z_q^{13} z_q^{13} + 4.1 \cdot 10^{-2} e^{i23^\circ} z_d^{12} z_d^{22} + 2.8 \cdot 10^{-2} z_q^{12} z_u^{23} \\
 & + 2.6 \cdot 10^{-2} e^{i22^\circ} z_q^{12} z_u^{32} + 2.4 \cdot 10^{-2} e^{i22^\circ} z_d^{13} z_d^{32} + 1.8 \cdot 10^{-2} z_u^{13} z_d^{12} - 1.3 \cdot 10^{-2} e^{i23^\circ} z_q^{13} z_q^{32}
 \end{aligned}$$

$$\begin{aligned}
 & -1.0 \cdot 10^{-2} e^{i24^\circ} z_q^{13} z_d^{22} + 1.0 \cdot 10^{-2} e^{i24^\circ} z_q^{13} z_d^{11} - 9.5 \cdot 10^{-3} e^{i22^\circ} z_d^{12} z_d^{32} \\
 & + 8.1 \cdot 10^{-3} e^{i23^\circ} z_u^{11} z_d^{12}.
 \end{aligned} \tag{4.8}$$

The corresponding expressions in **the up-basis** are given in appendix B. Although the value of $M_{Z'}$ has been kept arbitrary, it should be of the order of Λ to avoid the appearance of large logarithms $\ln \Lambda/M_{Z'}$. The same applies to the other simplified models.

4.2 G' model

The interaction Lagrangian of a $G' = (8, 1)_0$ field coupling to the quarks reads:

$$\mathcal{L}_{G'} = - \left[g_q^{ij} (\bar{q}^i \gamma^\mu T^A q^j) + g_u^{ij} (\bar{u}^i \gamma^\mu T^A u^j) + g_d^{ij} (\bar{d}^i \gamma^\mu T^A d^j) \right] G_\mu^A. \tag{4.9}$$

Matching this model onto the relevant SMEFT Wilson coefficients leads to the following tree-level matching conditions at Λ

$$\begin{aligned}
 [\mathcal{C}_{qq}^{(1)}]_{ijkl} &= \frac{g_q^{ij} g_q^{kl}}{12 M_{G'}^2} - \frac{g_q^{il} g_q^{kj}}{8 M_{G'}^2}, & [\mathcal{C}_{qq}^{(3)}]_{ijkl} &= -\frac{g_q^{il} g_q^{kj}}{8 M_{G'}^2}, \\
 [\mathcal{C}_{qd}^{(8)}]_{ijkl} &= -\frac{g_q^{ij} g_d^{kl}}{M_{G'}^2}, & [\mathcal{C}_{qu}^{(8)}]_{ijkl} &= -\frac{g_q^{ij} g_u^{kl}}{M_{G'}^2}, \\
 [\mathcal{C}_{dd}]_{ijkl} &= \frac{g_d^{ij} g_d^{kl}}{12 M_{G'}^2} - \frac{g_d^{il} g_d^{kj}}{4 M_{G'}^2}, & [\mathcal{C}_{uu}]_{ijkl} &= \frac{g_u^{ij} g_u^{kl}}{12 M_{G'}^2} - \frac{g_u^{il} g_u^{kj}}{4 M_{G'}^2}, & [\mathcal{C}_{ud}^{(8)}]_{ijkl} &= -\frac{g_u^{ij} g_d^{kl}}{M_{G'}^2}.
 \end{aligned} \tag{4.10}$$

These equations imply the following tree-level relations between different coefficients that are independent of $M_{G'}$ and are universal in all meson systems considered

$$\begin{aligned}
 [\mathcal{C}_{ud}^{(8)}]_{ijij}^2 &= 36 [\mathcal{C}_{dd}]_{ijij} [\mathcal{C}_{uu}]_{ijij}, & [\mathcal{C}_{qd}^{(8)}]_{ijij}^2 &= 144 [\mathcal{C}_{qq}^{(1)}]_{ijij} [\mathcal{C}_{dd}]_{ijij}, \\
 [\mathcal{C}_{qu}^{(8)}]_{ijij}^2 &= 144 [\mathcal{C}_{qq}^{(1)}]_{ijij} [\mathcal{C}_{uu}]_{ijij}, & [\mathcal{C}_{qq}^{(3)}]_{ijij} &= 3 [\mathcal{C}_{qq}^{(1)}]_{ijij}.
 \end{aligned} \tag{4.11}$$

In this model we find in **the down-basis**:

$$\begin{aligned}
 \frac{M_{G'}^2 \Sigma_{G'}^{B_s}}{(5 \text{ TeV})^2} &= -2.7 \cdot 10^3 g_q^{23} g_d^{23} + 6.9 \cdot 10^1 g_d^{23} g_d^{23} + 6.5 \cdot 10^1 g_q^{23} g_q^{23} - 3.8 g_q^{33} g_d^{23} + 2.1 g_q^{22} g_d^{23} \\
 &+ 1.5 g_u^{33} g_d^{23} - 4.7 \cdot 10^{-1} e^{i22^\circ} g_q^{12} g_d^{32} - 1.6 \cdot 10^{-1} g_u^{23} g_d^{23} + 1.3 \cdot 10^{-1} g_q^{23} g_d^{22} \\
 &- 1.3 \cdot 10^{-1} g_q^{23} g_d^{33} - 1.1 \cdot 10^{-1} g_q^{23} g_u^{33} + 8.7 \cdot 10^{-2} g_q^{23} g_q^{33} - 5.2 \cdot 10^{-2} g_q^{23} g_q^{22} \\
 &- 2.7 \cdot 10^{-2} g_q^{23} g_d^{32} + 2.6 \cdot 10^{-2} e^{-i23^\circ} g_q^{13} g_d^{23} + 1.4 \cdot 10^{-2} g_q^{23} g_u^{23} \\
 &+ 1.1 \cdot 10^{-2} e^{i22^\circ} g_q^{12} g_q^{32} + 5.8 \cdot 10^{-3} e^{i21^\circ} g_q^{13} g_d^{32},
 \end{aligned} \tag{4.12}$$

$$\begin{aligned}
 \frac{M_{G'}^2 \Sigma_{G'}^{B_d}}{(5 \text{ TeV})^2} &= -6.6 \cdot 10^4 g_q^{13} g_d^{13} + 1.6 \cdot 10^3 g_d^{13} g_d^{13} + 1.5 \cdot 10^3 g_q^{13} g_q^{13} + 5.2 \cdot 10^1 g_q^{12} g_d^{13} \\
 &+ 2.1 \cdot 10^1 e^{i22^\circ} g_q^{33} g_d^{13} - 1.1 \cdot 10^1 e^{i22^\circ} g_q^{11} g_d^{13} - 7.9 e^{i22^\circ} g_u^{33} g_d^{13} + 3.2 g_q^{13} g_d^{12} \\
 &- 1.2 g_q^{13} g_q^{12} - 8.9 \cdot 10^{-1} e^{i23^\circ} g_q^{23} g_d^{13} + 8.7 \cdot 10^{-1} g_u^{23} g_d^{13} + 5.2 \cdot 10^{-1} e^{i22^\circ} g_q^{13} g_u^{33} \\
 &- 4.7 \cdot 10^{-1} e^{i21^\circ} g_q^{13} g_q^{33} + 2.7 \cdot 10^{-1} e^{i22^\circ} g_q^{13} g_q^{11} + 1.4 \cdot 10^{-1} e^{i21^\circ} g_q^{23} g_d^{31} \\
 &- 7.8 \cdot 10^{-2} g_d^{13} g_d^{12} - 7.5 \cdot 10^{-2} e^{i23^\circ} g_q^{13} g_d^{23} - 7.5 \cdot 10^{-2} g_q^{13} g_u^{23} \\
 &- 3.6 \cdot 10^{-2} e^{i21^\circ} g_q^{13} g_d^{11} + 3.5 \cdot 10^{-2} e^{i21^\circ} g_q^{13} g_d^{33} - 3.1 \cdot 10^{-2} e^{i44^\circ} g_q^{13} g_d^{31}
 \end{aligned}$$

$$\begin{aligned}
 & + 1.9 \cdot 10^{-2} e^{i23^\circ} g_q^{13} g_q^{23} - 1.8 \cdot 10^{-2} e^{i22^\circ} g_q^{22} g_d^{13} - 7.6 \cdot 10^{-3} g_u^{13} g_d^{13} \\
 & - 5.5 \cdot 10^{-3} e^{i22^\circ} g_u^{11} g_d^{13} - 5.4 \cdot 10^{-3} e^{i22^\circ} g_u^{22} g_d^{13}, \quad (4.13)
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_{G'}^2 \Sigma_{G'}^K}{(5 \text{ TeV})^2} = & -7.5 \cdot 10^6 g_q^{12} g_d^{12} - 1.3 \cdot 10^4 g_q^{13} g_d^{12} + 6.1 \cdot 10^3 g_d^{12} g_d^{12} + 5.9 \cdot 10^3 g_q^{12} g_q^{12} \\
 & + 2.8 \cdot 10^3 e^{i22^\circ} g_q^{23} g_d^{21} - 3.7 \cdot 10^2 g_q^{12} g_d^{13} - 1.1 \cdot 10^2 e^{i24^\circ} g_q^{22} g_d^{12} + 4.7 \cdot 10^1 e^{i27^\circ} g_q^{11} g_d^{12} \\
 & + 3.8 \cdot 10^1 e^{i23^\circ} g_u^{33} g_d^{12} + 2.3 \cdot 10^1 e^{i23^\circ} g_q^{33} g_d^{12} + 1.0 \cdot 10^1 g_q^{13} g_q^{12} - 4.2 g_u^{23} g_d^{12} \\
 & - 4.0 e^{i22^\circ} g_u^{23} g_d^{21} + 4.0 e^{i22^\circ} g_q^{12} g_d^{32} + 3.4 e^{-i79^\circ} g_q^{12} g_d^{22} - 3.4 e^{-i79^\circ} g_q^{12} g_d^{11} \\
 & - 2.2 e^{i22^\circ} g_q^{12} g_q^{32} - 9.7 \cdot 10^{-1} e^{i24^\circ} g_q^{23} g_d^{12} - 6.3 \cdot 10^{-1} g_q^{13} g_d^{13} \\
 & + 3.9 \cdot 10^{-1} g_u^{22} g_d^{12} + 3 \cdot 10^{-1} g_d^{12} g_d^{13} + 1.5 \cdot 10^{-1} e^{i45^\circ} g_q^{13} g_d^{21} + 1.4 \cdot 10^{-1} e^{i23^\circ} g_q^{23} g_d^{31} \\
 & + 9.5 \cdot 10^{-2} e^{i23^\circ} g_q^{12} g_q^{22} - 9.2 \cdot 10^{-2} e^{i23^\circ} g_q^{12} g_u^{33} - 4.2 \cdot 10^{-2} e^{i23^\circ} g_q^{12} g_q^{11} \\
 & - 3.9 \cdot 10^{-2} e^{i23^\circ} g_q^{13} g_q^{32} + 3.6 \cdot 10^{-2} g_u^{13} g_d^{12} + 2.5 \cdot 10^{-2} e^{i22^\circ} g_u^{11} g_d^{12} \\
 & + 1.7 \cdot 10^{-2} g_q^{13} g_q^{13} - 1.5 \cdot 10^{-2} e^{i24^\circ} g_q^{13} g_d^{22} + 1.4 \cdot 10^{-2} e^{i24^\circ} g_q^{13} g_d^{11} \\
 & + 1.4 \cdot 10^{-2} e^{i23^\circ} g_d^{12} g_d^{22} + 1.2 \cdot 10^{-2} g_q^{12} g_u^{23} + 1.2 \cdot 10^{-2} e^{i22^\circ} g_d^{12} g_d^{33} \\
 & + 1.1 \cdot 10^{-2} e^{i22^\circ} g_q^{12} g_u^{32} + 9.7 \cdot 10^{-3} e^{i23^\circ} g_q^{12} g_q^{33} + 6.7 \cdot 10^{-3} e^{i23^\circ} g_q^{13} g_d^{32} \\
 & - 6.3 \cdot 10^{-3} e^{i46^\circ} g_q^{12} g_d^{21} - 6.2 \cdot 10^{-3} e^{i23^\circ} g_q^{22} g_d^{13} - 5.4 \cdot 10^{-3} e^{i22^\circ} g_d^{13} g_d^{32}. \quad (4.14)
 \end{aligned}$$

The corresponding expressions in **the up-basis** are given in appendix B.

4.3 Colourless scalar model

The interaction Lagrangian of a $\varphi = (1, 2)_{1/2}$ scalar field coupling to the quarks reads:

$$\mathcal{L}_\varphi = -Y_d^{ij} (\bar{q}_i d_j) \varphi - Y_u^{ij} (\bar{q}_i u_j) \tilde{\varphi} + \text{h.c.} \quad \tilde{\varphi} \equiv i\sigma_2 \varphi^*. \quad (4.15)$$

Integrating out the heavy scalar leads to the following tree-level matching conditions for the four-quark SMEFT operators [96] at Λ

$$[\mathcal{C}_{qu}^{(8)}]_{ijkl} = 6[\mathcal{C}_{qu}^{(1)}]_{ijkl} = -\frac{Y_u^{jk} Y_u^{il*}}{M_\varphi^2}, \quad [\mathcal{C}_{qd}^{(8)}]_{ijkl} = 6[\mathcal{C}_{qd}^{(1)}]_{ijkl} = -\frac{Y_d^{li} Y_d^{kj*}}{M_\varphi^2}. \quad (4.16)$$

In the BMU basis these results imply $C_{\text{LR},1}^{ij} = 0$ and only $C_{\text{LR},2}^{ij}$ are non-vanishing. However, it is evident from the charts in the previous section that QCD RG evolution will generate non-vanishing $C_{\text{LR},1}^{ij}$ at different scales. This is already seen when QCD corrections to the matching in (4.16) are extracted from [97]. In the NDR scheme we find

$$[\mathcal{C}_{qa}^{(1)}]_{ijij} = -\frac{1}{6} \frac{Y_a^{ji} Y_a^{ij*}}{M_\varphi^2} \left(1 - \frac{5}{2} \frac{\alpha_s}{4\pi}\right), \quad [\mathcal{C}_{qa}^{(8)}]_{ijij} = -\frac{Y_a^{ji} Y_a^{ij*}}{M_\varphi^2} \left(1 - \frac{\alpha_s}{4\pi}\right), \quad (4.17)$$

so that the relation in (4.16) is violated and non-vanishing $C_{\text{LR},1}^{ij}$ are generated. These QCD corrections cancel the renormalization scheme dependence present in two-loop anomalous dimensions of the operators in question that enter the RG evolution at the NLO level. One should note that no logarithms involving the NP scale are present in these corrections. The reason for this is explained in [97].

We find for the master formulae in the **down-basis**:

$$\begin{aligned} \frac{M_\varphi^2 \Sigma_\varphi^{B_s}}{(5 \text{ TeV})^2} = & -3.1 \cdot 10^3 Y_d^{32} Y_d^{23*} - 4.4 Y_d^{33} Y_d^{23*} + 2.4 Y_d^{32} Y_d^{22*} \\ & - 5.3 \cdot 10^{-1} e^{i22^\circ} Y_d^{21} Y_d^{32*} - 1.5 \cdot 10^{-1} Y_u^{33} Y_u^{23*} + 1.5 \cdot 10^{-1} Y_d^{22} Y_d^{23*} \\ & - 1.5 \cdot 10^{-1} Y_d^{32} Y_d^{33*} - 3.1 \cdot 10^{-2} Y_d^{22} Y_d^{33*} + 3.0 \cdot 10^{-2} e^{-i23^\circ} Y_d^{31} Y_d^{23*} \\ & + 1.9 \cdot 10^{-2} Y_u^{32} Y_u^{23*} + 6.7 \cdot 10^{-3} e^{i21^\circ} Y_d^{21} Y_d^{33*}, \end{aligned} \quad (4.18)$$

$$\begin{aligned} \frac{M_\varphi^2 \Sigma_\varphi^{B_d}}{(5 \text{ TeV})^2} = & -7.5 \cdot 10^4 Y_d^{31} Y_d^{13*} + 5.9 \cdot 10^1 Y_d^{31} Y_d^{12*} + 2.4 \cdot 10^1 e^{i22^\circ} Y_d^{33} Y_d^{13*} \\ & - 1.3 \cdot 10^1 e^{i22^\circ} Y_d^{31} Y_d^{11*} + 3.7 Y_d^{21} Y_d^{13*} - 1.0 e^{i23^\circ} Y_d^{32} Y_d^{13*} \\ & + 7.4 \cdot 10^{-1} e^{i22^\circ} Y_u^{33} Y_u^{13*} + 1.7 \cdot 10^{-1} e^{i21^\circ} Y_d^{12} Y_d^{33*} - 1.0 \cdot 10^{-1} Y_u^{32} Y_u^{13*} \\ & - 8.5 \cdot 10^{-2} e^{i23^\circ} Y_d^{31} Y_d^{23*} - 4.2 \cdot 10^{-2} e^{i21^\circ} Y_d^{11} Y_d^{13*} + 4.1 \cdot 10^{-2} e^{i21^\circ} Y_d^{31} Y_d^{33*} \\ & - 3.6 \cdot 10^{-2} e^{i44^\circ} Y_d^{11} Y_d^{33*} - 1.9 \cdot 10^{-2} e^{i22^\circ} Y_d^{32} Y_d^{12*}, \end{aligned} \quad (4.19)$$

$$\begin{aligned} \frac{M_\varphi^2 \Sigma_\varphi^K}{(5 \text{ TeV})^2} = & -8.4 \cdot 10^6 Y_d^{21} Y_d^{12*} - 1.4 \cdot 10^4 Y_d^{21} Y_d^{13*} + 3.1 \cdot 10^3 e^{i22^\circ} Y_d^{12} Y_d^{23*} \\ & - 4.1 \cdot 10^2 Y_d^{31} Y_d^{12*} - 1.3 \cdot 10^2 e^{i24^\circ} Y_d^{22} Y_d^{12*} + 5.2 \cdot 10^1 e^{i27^\circ} Y_d^{21} Y_d^{11*} \\ & + 2.6 \cdot 10^1 e^{i23^\circ} Y_d^{23} Y_d^{13*} + 4.4 e^{i22^\circ} Y_d^{21} Y_d^{32*} + 3.7 e^{-i82^\circ} Y_d^{21} Y_d^{22*} \\ & - 3.7 e^{-i82^\circ} Y_d^{11} Y_d^{12*} - 1.1 e^{i24^\circ} Y_d^{22} Y_d^{13*} - 7.0 \cdot 10^{-1} Y_d^{31} Y_d^{13*} \\ & + 1.7 \cdot 10^{-1} e^{i45^\circ} Y_d^{11} Y_d^{23*} + 1.5 \cdot 10^{-1} e^{i23^\circ} Y_d^{12} Y_d^{33*} - 1.3 \cdot 10^{-1} e^{i23^\circ} Y_u^{23} Y_u^{13*} \\ & + 1.7 \cdot 10^{-2} Y_u^{22} Y_u^{13*} - 1.6 \cdot 10^{-2} e^{i24^\circ} Y_d^{21} Y_d^{23*} + 1.6 \cdot 10^{-2} e^{i24^\circ} Y_d^{11} Y_d^{13*} \\ & + 1.6 \cdot 10^{-2} e^{i22^\circ} Y_u^{23} Y_u^{12*} + 7.5 \cdot 10^{-3} e^{i23^\circ} Y_d^{21} Y_d^{33*} - 7.1 \cdot 10^{-3} e^{i46^\circ} Y_d^{11} Y_d^{22*} \\ & - 6.9 \cdot 10^{-3} e^{i23^\circ} Y_d^{32} Y_d^{12*}. \end{aligned} \quad (4.20)$$

The corresponding expressions in the **up-basis** are given in appendix B.

4.4 Coloured scalar model

The interaction Lagrangian of a $\Phi = (8, 2)_{1/2}$ scalar field coupling to the quarks reads:

$$\mathcal{L}_\Phi = -X_d^{ij} (\bar{q}_i T^A d_j) \Phi^A - X_u^{ij} (\bar{q}_i T^A u_j) \tilde{\Phi}^A + \text{h.c.}, \quad \tilde{\Phi}^A \equiv i\sigma_2 (\Phi^A)^*. \quad (4.21)$$

Integrating out the heavy scalar leads to the following tree-level matching conditions for the four-quark SMEFT operators [96] at Λ

$$\frac{4}{3} [\mathcal{C}_{qu}^{(8)}]_{ijkl} = -[\mathcal{C}_{qu}^{(1)}]_{ijkl} = \frac{2}{9} \frac{X_u^{jk} X_u^{il*}}{M_\Phi^2}, \quad \frac{4}{3} [\mathcal{C}_{qd}^{(8)}]_{ijkl} = -[\mathcal{C}_{qd}^{(1)}]_{ijkl} = \frac{2}{9} \frac{X_d^{li} X_d^{kj*}}{M_\Phi^2}. \quad (4.22)$$

Note that this time the relation between the two coefficients differs from (4.16) and both $C_{\text{LR},1}^{ij}$ and $C_{\text{LR},2}^{ij}$ are non-vanishing already at tree-level. For the master formula we find in

the down-basis:

$$\begin{aligned} \frac{M_\Phi^2 \Sigma_\Phi^{B_s}}{(5 \text{ TeV})^2} = & -6.8 \cdot 10^1 X_d^{32} X_d^{23*} - 8.6 \cdot 10^{-2} X_d^{33} X_d^{23*} + 5.3 \cdot 10^{-2} X_d^{32} X_d^{22*} \\ & - 4.1 \cdot 10^{-2} X_u^{33} X_u^{23*} - 1.2 \cdot 10^{-2} e^{i22^\circ} X_d^{21} X_d^{32*}, \end{aligned} \quad (4.23)$$

$$\begin{aligned} \frac{M_\Phi^2 \Sigma_\Phi^{B_d}}{(5 \text{ TeV})^2} = & -1.7 \cdot 10^3 X_d^{31} X_d^{13*} + 1.3 X_d^{31} X_d^{12*} + 4.7 \cdot 10^{-1} e^{i22^\circ} X_d^{33} X_d^{13*} \\ & - 2.9 \cdot 10^{-1} e^{i22^\circ} X_d^{31} X_d^{11*} + 2.0 \cdot 10^{-1} e^{i22^\circ} X_u^{33} X_u^{13*} + 8.2 \cdot 10^{-2} X_d^{21} X_d^{13*} \\ & - 2.6 \cdot 10^{-2} X_u^{32} X_u^{13*} - 2.0 \cdot 10^{-2} e^{i23^\circ} X_d^{32} X_d^{13*} + 6.2 \cdot 10^{-3} e^{i21^\circ} X_d^{12} X_d^{33*}, \end{aligned} \quad (4.24)$$

$$\begin{aligned} \frac{M_\Phi^2 \Sigma_\Phi^{B_K}}{(5 \text{ TeV})^2} = & 8.3 \cdot 10^4 X_d^{21} X_d^{12*} + 1.4 \cdot 10^2 X_d^{21} X_d^{13*} - 3.1 \cdot 10^1 e^{i22^\circ} X_d^{12} X_d^{23*} \\ & + 4.1 X_d^{31} X_d^{12*} + 9.5 \cdot 10^{-1} e^{i33^\circ} X_d^{22} X_d^{12*} - 3.4 \cdot 10^{-1} e^{-i6^\circ} X_d^{21} X_d^{22*} \\ & + 3.4 \cdot 10^{-1} e^{-i6^\circ} X_d^{11} X_d^{12*} - 2.7 \cdot 10^{-1} e^{i61^\circ} X_d^{21} X_d^{11*} - 4.4 \cdot 10^{-2} e^{i22^\circ} X_d^{21} X_d^{32*} \\ & - 4.0 \cdot 10^{-2} e^{i24^\circ} X_d^{23} X_d^{13*} - 3.3 \cdot 10^{-2} e^{i23^\circ} X_u^{23} X_u^{13*} + 6.9 \cdot 10^{-3} X_d^{31} X_d^{13*}. \end{aligned} \quad (4.25)$$

The corresponding expressions in **the up-basis** are given in appendix B.

4.5 A closer look at NP scenarios

Let us next get a better insight into different NP scenarios by collecting in table 8 the values of flavour violating couplings for $M_{Z',G',\varphi,\Phi} = 5 \text{ TeV}$ that give rise to 20% NP corrections to $2[M_{12}^{ij}]_{\text{BSM}}$ that is 0.2 for the sum entering (3.4). To this end we keep only the contribution with the largest $P_a^{ij}(\Lambda)$ in our results. These are the ones which come directly from $\Delta F = 2$ operators or from left-right operators, which for the Z' and G' models involve the products $z_d^{ij} z_q^{ij}$ with $ij = 12, 13, 23$ for K^0 , B_d and B_s , respectively. In order to get an idea of the size of the couplings we first take them to be real and assume the relations

$$z_d^{ij} = z_q^{ij}, \quad g_d^{ij} = g_q^{ij}, \quad X_d^{ij} = X_d^{ji}, \quad Y_d^{ij} = Y_d^{ji}, \quad (4.26)$$

that we will relax soon.

As the values of flavour-violating couplings turn out to be small the question arises whether non-leading terms with smaller $P_a^{ij}(\Lambda)$, or equivalently smaller numerical coefficients multiplying the products of couplings, could play a role, in particular those in which flavour-conserving couplings are present. While a detailed analysis would require a simultaneous study of $\Delta F = 1$ transitions it is of interest to see whether other terms generated by the RG evolution play eventually any role in the estimate of NP contributions to $\Delta F = 2$ observables. As we will see soon this indeed can be the case provided flavour-conserving couplings are sufficiently large but still in a perturbative regime. We will assume such couplings to be at most ~ 3 .

Evidently the outcome of such an analysis depends on the scenarios for couplings considered and it is common in the case of Z' and G' scenarios to investigate the following scenarios for couplings:

- **Left-handed Scenario (LHS)** in which only coefficients involving the couplings z_q^{ij} or g_q^{ij} are kept non-zero.

Model	Couplings		
Z'	z_q^{12}	z_q^{13}	z_q^{23}
	$2.0 \cdot 10^{-4}$	$1.9 \cdot 10^{-3}$	$9.3 \cdot 10^{-3}$
G'	g_q^{12}	g_q^{13}	g_q^{23}
	$1.6 \cdot 10^{-4}$	$1.7 \cdot 10^{-3}$	$8.6 \cdot 10^{-3}$
φ	Y_d^{12}	Y_d^{13}	Y_d^{23}
	$1.5 \cdot 10^{-4}$	$1.6 \cdot 10^{-3}$	$8.0 \cdot 10^{-3}$
Φ	X_d^{12}	X_d^{13}	X_d^{23}
	$1.6 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	$5.4 \cdot 10^{-2}$

Table 8. The values of flavour-violating couplings for $M_{Z', G', \varphi, \Phi} = 5 \text{ TeV}$ that give rise to 20% NP corrections to $2[M_{12}^{ij}]_{\text{BSM}}$.

- **Right-handed Scenario (RHS)** in which only coefficients involving the couplings z_d^{ij} and z_u^{ij} or g_d^{ij} and g_u^{ij} are kept non-zero.
- **Left-Right-handed Scenario (LRS)** in which all coefficients are kept nonzero.

Inspecting the master formulae for the sums in Z' , G' , φ and Φ scenarios listed above, we make the following observations:

- The pattern of various contributions depends on whether the model is formulated in the down basis or the up basis. While this could be at first sight surprising it can be explained as the explicit breakdown of the $U(3)^5$ flavour symmetry in the NP scenarios considered. We will return to this point in section 4.7.
- The pattern also depends on the scenario of couplings as already mentioned above.
- In the down basis the $\Delta F = 2$ contributions seem to dominate by far in all the down-type meson systems (K^0, B_d, B_s) in all four NP scenarios considered. This is in particular the case in LHS and RHS scenarios.
- In the up basis the terms involving flavour-diagonal couplings, that are representing $\Delta F = 1$ operators, can compete with direct $\Delta F = 2$ contributions. This is in particular the case for the LRS scenario with a hierarchy between left- and right-handed couplings, which, if necessary, can through cancellations between different important terms allow to suppress NP contributions to $\Delta F = 2$ processes in the presence of significant NP contributions to $\Delta F = 1$ transitions [78].

In particular the contributions (23)(33) and (23)(22) can compete with (23)(23) in the B_s system and (12)(22) and (12)(11) with (12)(12) in the K^0 system, provided the flavour-conserving couplings are much larger than flavour-violating ones, but still being in the perturbative regime. Such competition seems to be less likely in the B_d system.

- We also find that the $\Delta F = 1$ contributions are most relevant in the G' scenario followed by the Z' scenario. They can also be relevant in scalar scenarios but at most at the level of 30% of the numerically leading contributions.
- For the up basis we also find that contributions involving only flavour-violating couplings can compete with each other. In particular (23)(13) can compete with (23)(23) in the B_s system, while (13)(23) with (13)(13) in the B_d system. This also shows that the same couplings enter B_s and B_d systems implying correlations between these two systems. Such correlations are missed if RG effects are not considered. Similar correlations are found between K^0 and D^0 systems.

These findings demonstrate that just keeping the direct $\Delta F = 2$ contributions in the phenomenological analyses of $\Delta F = 2$ observables can miss important dynamics of SMEFT.

4.6 Comments on VLQ and LQ models

We have seen that in the considered scenarios the role of the operators

$$\mathcal{O}_{\phi q}^{(1)}, \quad \mathcal{O}_{\phi q}^{(3)}, \quad \mathcal{O}_{\phi d}, \quad (4.27)$$

was minor as they have been put under the shadow of the four-fermion operators, in particular $\mathcal{O}_{qd}^{(1)}$ and $\mathcal{O}_{qd}^{(8)}$, which are generated in the matching on SMEFT in these scenarios already at tree-level.

The situation changes in VLQ models based on the SM group in which operators in (4.27) play the dominant role by generating the operators $\mathcal{O}_{qd}^{(1)}$ and $\mathcal{O}_{qd}^{(8)}$ through RG running dominated by Yukawa couplings. In turn these operators give again important contributions to $\Delta F = 2$ transitions. The analysis of this scenario with correlations between $\Delta F = 2$ and $\Delta F = 1$ transitions has been already analyzed in [92] and we refer to this paper for details. However, let us point out that in these VLQ models the Wilson coefficients $\mathcal{C}_{qd}^{(1)}$ and $\mathcal{C}_{qd}^{(8)}$ receive at the electroweak scale two contributions: firstly the one due to tree-level VLQ exchange that generates $\mathcal{O}_{\phi d}$ at Λ and enters via Yukawa mixing at μ_{ew} with a logarithmic enhancement. And secondly due to a direct contribution from one-loop matching at Λ . Both contributions are loop-suppressed, but have a different dependence on the VLQ Yukawa couplings Y_{VLQ} : the former scales with $\propto (Y_{\text{VLQ}})^2 (V_{\text{CKM}})^2$, whereas the latter scales with $\propto (Y_{\text{VLQ}})^4$. The absolute numerical importance of both contributions depends strongly on the allowed size of Y_{VLQ} compared to V_{CKM} . The physics behind these effects are FCNCs mediated by the SM Z -boson that are generated through the mixing of VLQs and the SM quarks in the process of electroweak symmetry breaking. A more general discussion of this phenomenon in the context of the SMEFT can be found in [16, 22].

Still different is the case of LQ models in which four-fermion operators are generated at the electroweak scale through RG running from semileptonic operators

$$\mathcal{O}_{lq}^{(1)}, \quad \mathcal{O}_{lq}^{(3)}, \quad \mathcal{O}_{ld}, \quad \mathcal{O}_{qe}, \quad \mathcal{O}_{ledq}, \quad \mathcal{O}_{lequ}^{(1)}, \quad \mathcal{O}_{lequ}^{(3)}. \quad (4.28)$$

Only electroweak interactions are involved here and the contributions to $\Delta F = 2$ processes in these models are small. Indeed we have seen that the contributions of operators involving leptons in our master formulae are strongly suppressed. We refer to [93] for details.

4.7 The issue of the basis choice

Having the set of linearly independent SMEFT operators, we had to specify the weak-eigenstate basis in which we plan to perform calculations including the RG evolution above the electroweak scale. Performing the calculations in either the down basis or the up basis, we found different results which could be surprising because we are used to basis-independent results within the SM. In order to understand better what is going on let us repeat what is well known within the SM.

The gauge interactions in the SM are invariant under a $[U(3)]^5$ flavour symmetry

$$q_L \rightarrow V_L^q q_L, \quad u_R \rightarrow V_R^u u_R, \quad d_R \rightarrow V_R^d d_R, \quad (4.29)$$

$$\ell_L \rightarrow V_L^e \ell_L, \quad e_R \rightarrow V_R^e e_R, \quad (4.30)$$

where $V_L^q, V_R^u, V_R^d, V_L^e$ and V_R^e are unitary 3×3 matrices. This is the consequence of the fact that there is the universality of the gauge couplings for all fermion families of left- and right-handed fermions. In the SM the Yukawa sector breaks this universality and consequently $[U(3)]^5$ symmetry explicitly simply because the Yukawa couplings to fermions are not subject to further symmetry constraints, and in this way allows to account for the known mass spectrum of quarks and leptons. The preferred basis for calculations is the mass-eigenstate basis in which the Yukawa and consequently mass matrices are diagonalized as explicitly given by

$$(V_L^d)^\dagger Y^D V_R^d = \hat{Y}^D, \quad (V_L^u)^\dagger Y^U V_R^u = \hat{Y}^U, \quad (V_L^e)^\dagger Y^E V_R^e = \hat{Y}^E, \quad (4.31)$$

with $\hat{Y}^i$ being diagonal. Here V_L^u and V_L^d rotate the $SU(2)_L$ components of q_L individually contrary to V_L^q in (4.29).

Now because of the universality of gauge couplings and the unitarity of rotation matrices, FCNCs are absent and flavour changes appear only in the charged currents parametrized by CKM and PMNS matrices. It should be stressed that it is irrelevant whether we rotate the down-quarks from flavour to mass eigenstates and assume flavour and mass eigenstates in the up-quark system to be equal, or vice versa. The interactions in the mass-eigenstate basis remain unchanged. The same applies to the lepton sector.

Let us next assume that NP contributions, e.g. with non-universal but generation-diagonal gauge couplings, break the $[U(3)]^5$ flavour symmetry explicitly. In order to see the consequences of this breakdown let us consider a Z' model and choose the up basis, i.e. $V_L^u = \mathbb{1}$ and $V_R^u = \mathbb{1}$. This means that the Yukawa matrix or equivalently the mass matrix for up-quarks is diagonal and the same applies to the interactions of up-quarks with Z' . There is no flavour violation in the up-quark sector mediated by the Z' up to contributions from matching and back-rotation in SMEFT. But with $V_L^u = \mathbb{1}$ we have $V_L^d = V_{\text{CKM}}$. Therefore, performing the usual rotations in the down sector from flavour- to mass-eigenstate basis we find FCNC transitions in the down-quark sector with

$$\Delta_L^{ij}(Z') = g_{Z'} [V_{\text{CKM}}^\dagger \hat{Z}_L^d V_{\text{CKM}}]_{ij}, \quad \Delta_R^{ij}(Z') = g_{Z'} [(V_R^d)^\dagger \hat{Z}_R^d V_R^d]_{ij}, \quad (4.32)$$

with $(i, j = d, s, b)$ and $\hat{Z}_{L,R}^d$ being diagonal matrices collecting $U(1)'$ charges of left- and right-handed down-quarks.

However, $V_L^u = \mathbb{1}$ and $V_R^u = \mathbb{1}$ is an assumption which specifies our model. It assumes that in the basis in which Yukawa matrices for up-quarks are diagonal also the interactions of the Z' with the up-quarks are flavour diagonal. In other words $\hat{Y}_u$ and Z' interactions for the up-quarks are aligned with each other. But we could as well choose $V_L^d = \mathbb{1}$ and $V_R^d = \mathbb{1}$ which would result in FCNCs mediated by the Z' in the up-quark sector and no FCNCs in the down-quark sector again up to contributions from matching and back-rotation.

These simple examples show that in the absence of a $[U(3)]^5$ flavour symmetry in the gauge sector we have more freedom and the physics depends on how the Yukawa matrices and matrices describing interactions are oriented in flavour space. This also explains why the bounds on various coefficients found in [11] for the down-basis and up-basis differ from each other and also implied different SMEFT master formulae in our paper.

These findings underline the importance of the construction of UV completions in which also a flavour theory is specified so that the orientation between Yukawa matrices and the matrices describing the interactions are known. Interesting model constructions in this direction can be found in [98, 99].

5 Summary and outlook

In the present paper we have worked out the model-independent anatomy of the $\Delta F = 2$ transitions $K^0 - \bar{K}^0$, $B_{s,d} - \bar{B}_{s,d}$ and $D^0 - \bar{D}^0$ in the context of SMEFT and LEFT. On the technical side the two most important novel results are two master formulae for the new physics contribution of the mixing amplitude $[M_{12}^{ij}]_{\text{BSM}}$ with $ij = ds, db, sb, cu$.

The first eq. (2.15) is given in terms of the Wilson coefficients (WCs) of the LEFT operators evaluated at the electroweak scale μ_{ew} . For each meson system there are eight WCs and corresponding coefficients $P_a^{ij}(\mu_{\text{ew}})$ that collect all the information below the scales μ_{ew} . This means the existing results for hadronic matrix elements from LQCD combined with the presently known QCD renormalization group evolution at the NLO level up to μ_{ew} . The numerical values of $P_a^{ij}(\mu_{\text{ew}})$ in different operator bases are collected in table 3. Calculating the WCs of the eight operators in question at μ_{ew} in any BSM scenario this formula gives directly $[M_{12}^{ij}]_{\text{BSM}}$ and consequently allows to calculate all observables related to neutral-meson mixing. It is a modern version of the formula presented already in [28]. The advantage of this formula with respect to the second master formula for SMEFT is the paucity of the terms entering it and further it is not subject to constraints from $SU(2)_L \times U(1)_Y$ gauge invariance present in the SMEFT master formula. But the drawback is that it requires from the practitioners the calculation of the RG evolution in the context of the SMEFT from the NP scale Λ down to the electroweak scale μ_{ew} . Because of subtle and often important RG effects related to the top-Yukawa coupling taking place on the route from Λ down to μ_{ew} , it is not evident from the structure of a given extension of the SM that it is in agreement with the data at low energy or not.

The second master formula, given in (3.4), although containing many more terms than the LEFT one, is more powerful because it allows right away to see whether a given BSM scenario has a chance to be consistent with the data or not. It gives $[M_{12}^{ij}]_{\text{BSM}}$ directly in terms of the WCs of the SMEFT operators evaluated at the BSM scale Λ . The coefficients

$P_a^{ij}(\Lambda)$ entering this formula generalize the information below the scale μ_{ew} present already in $P_a^{ij}(\mu_{\text{ew}})$ to include all RG effects between μ_{ew} and Λ . Therefore performing the matching of a given NP model to the SMEFT, using in particular results of [96] at the scale Λ , and using the master formulae in section 3.4 allows to connect NP at a very high scale with the observables measured at low energy scales. While the numerical coefficients in these formulae are given for the example of $\Lambda = 5 \text{ TeV}$, the dominant change in the results for observables comes from the quadratic dependence on the masses of gauge bosons and scalars and can be calculated right away. The dependence of $P_a^{ij}(\Lambda)$ on Λ is logarithmic and much weaker. One can verify this by inspecting the corresponding master formulae for 100 TeV in appendix A.

We stress that although the solution to the relevant RG equations are collected in the first leading logarithmic approximation in section 3.2, the numerical values of $P_a^{ij}(\Lambda)$ that enter the master formulae are obtained by summing leading logarithms to all orders in the coupling numerically and including the one-loop matching from SMEFT onto LEFT [14] at μ_{ew} , which is collected in appendix D.

Presenting our master formulae in the down and the up basis, we have reemphasized their differences and the need for UV completions incorporating a flavour theory in order to be able to understand the full dynamics of the SMEFT. Whereas in the down basis the $\Delta F = 2$ contributions have the numerically largest coefficients compared to the $\Delta F = 1$ ones that enter mainly via Yukawa mixing, in the up basis this hierarchy is absent. This implies very strong correlations between $\Delta F = 2$ and $\Delta F = 1$ processes in the up basis, hence affecting many phenomenological analysis of collider processes, in particular top-quark phenomenology.

We have also illustrated this technology by applying the SMEFT formula to a number of simplified models containing colourless heavy gauge bosons (Z') and scalars and models with coloured heavy gauge bosons (G'_a) and scalars. Also the cases of vector-like quarks and leptoquarks have been briefly discussed.

Our analysis demonstrates that RG effects of the running from Λ down to μ_{ew} , in particular those related to QCD interactions and top-Yukawa couplings constitute an essential ingredient of any analysis of BSM scenarios. However, it should be kept in mind that in some NP scenarios the role of the model-dependent one-loop matching at the NP scale Λ could also be important if it generates the Wilson coefficients $\mathcal{C}_{qd}^{(1)}$ and $\mathcal{C}_{qd}^{(8)}$ which are multiplied by very large $P_a^{ij}(\Lambda)$ coefficients.

The corresponding analysis of $\Delta F = 1$ transitions is expected to be much more involved and subject to much larger hadronic uncertainties, but already our $\Delta F = 2$ atlas casts some doubts on the validity of many analyses present in the literature that consider only one or two operators at the time and restrict the analyses to a single meson system.

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A SMEFT atlas at $\Lambda = 100$ TeV

Here we report the master formulae for the contributions of different operators, this time generated at $\Lambda = 100$ TeV. The dominant effect relative to the $\Lambda = 5$ TeV case is suppression of these contributions by a factor of 400 originating in $1/\Lambda^2$. RG effects, although visible, amount to shifts of at most 50% and this only for left-right operators.

A.1 $\mathcal{C}_{qq}^{(1)}$ and $\mathcal{C}_{qq}^{(3)}$

In the down-basis

$$\Sigma_{qq1}^{B_s} = -8.9 \cdot 10^{-1} [c_{qq}^{(1)}]_{2323}, \quad (\text{A.1})$$

$$\begin{aligned} \Sigma_{qq1}^{B_d} = & -2.1 \cdot 10^1 [c_{qq}^{(1)}]_{1313} + 2.6 \cdot 10^{-2} [c_{qq}^{(1)}]_{1213} + 1.0 \cdot 10^{-2} e^{i22^\circ} [c_{qq}^{(1)}]_{1333} \\ & - 5.7 \cdot 10^{-3} e^{i22^\circ} [c_{qq}^{(1)}]_{1113}, \end{aligned} \quad (\text{A.2})$$

$$\Sigma_{qq1}^K = -8.1 \cdot 10^1 [c_{qq}^{(1)}]_{1212} - 2.2 \cdot 10^{-1} [c_{qq}^{(1)}]_{1213} + 4.7 \cdot 10^{-2} e^{i22^\circ} [c_{qq}^{(1)}]_{1232}, \quad (\text{A.3})$$

and the up-basis

$$\begin{aligned} \hat{\Sigma}_{qq1}^{B_s} = & -8.4 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{2323} - 1.9 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1323} - 4.4 \cdot 10^{-2} [\hat{c}_{qq}^{(1)}]_{1313} - 3.4 \cdot 10^{-2} [\hat{c}_{qq}^{(1)}]_{2223} \\ & + 3.4 \cdot 10^{-2} [\hat{c}_{qq}^{(1)}]_{2333} - 7.9 \cdot 10^{-3} [\hat{c}_{qq}^{(1)}]_{1223} - 7.9 \cdot 10^{-3} [\hat{c}_{qq}^{(1)}]_{1322} + 7.8 \cdot 10^{-3} [\hat{c}_{qq}^{(1)}]_{1333}, \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \hat{\Sigma}_{qq1}^{B_d} = & -2.0 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1313} + 4.5 [\hat{c}_{qq}^{(1)}]_{1323} - 1.0 [\hat{c}_{qq}^{(1)}]_{2323} - 8.0 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1213} \\ & + 1.8 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1223} + 1.8 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1322} - 1.7 \cdot 10^{-1} e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{1333} \\ & - 6.8 \cdot 10^{-2} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1113} - 4.2 \cdot 10^{-2} [\hat{c}_{qq}^{(1)}]_{2223} + 3.9 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{2333} \\ & - 3.3 \cdot 10^{-2} [\hat{c}_{qq}^{(1)}]_{1212} + 1.6 \cdot 10^{-2} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1231} + 1.6 \cdot 10^{-2} e^{-i73^\circ} [\hat{c}_{qq}^{(1)}]_{1123} \\ & + 7.5 \cdot 10^{-3} [\hat{c}_{qq}^{(1)}]_{1222} - 7.0 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1233} - 6.9 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1332}, \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned} \hat{\Sigma}_{qq1}^K = & -7.3 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1212} - 1.6 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1112} + 1.6 \cdot 10^1 [\hat{c}_{qq}^{(1)}]_{1222} - 3.9 [\hat{c}_{qq}^{(1)}]_{1111} \\ & + 3.9 [\hat{c}_{qq}^{(1)}]_{1122} + 3.9 [\hat{c}_{qq}^{(1)}]_{1221} - 3.9 [\hat{c}_{qq}^{(1)}]_{2222} + 2.9 [\hat{c}_{qq}^{(1)}]_{1213} + 6.4 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1113} \\ & - 6.4 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1223} - 6.4 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1322} - 6.0 \cdot 10^{-1} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1232} \\ & - 3.0 \cdot 10^{-1} e^{i11^\circ} [\hat{c}_{qq}^{(1)}]_{1123} - 3.0 \cdot 10^{-1} e^{i11^\circ} [\hat{c}_{qq}^{(1)}]_{1231} + 3.0 \cdot 10^{-1} e^{i11^\circ} [\hat{c}_{qq}^{(1)}]_{2223} \\ & - 1.2 \cdot 10^{-1} [\hat{c}_{qq}^{(1)}]_{1313} + 2.6 \cdot 10^{-2} [\hat{c}_{qq}^{(1)}]_{1323} + 2.4 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1233} \\ & + 2.4 \cdot 10^{-2} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1332} - 1.1 \cdot 10^{-2} e^{i22^\circ} [\hat{c}_{qq}^{(1)}]_{2323} + 5.9 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1133} \\ & - 5.9 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{2233} + 5.8 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{1331} - 5.8 \cdot 10^{-3} e^{i23^\circ} [\hat{c}_{qq}^{(1)}]_{2332}, \end{aligned} \quad (\text{A.6})$$

and to a very good approximation the same expressions for $\mathcal{C}_{qq}^{(3)}$ and $\hat{\mathcal{C}}_{qq}^{(3)}$.

A.2 $\mathcal{C}_{qd}^{(1)}$ and $\mathcal{C}_{qd}^{(8)}$

In the down basis

$$\Sigma_{qd1}^{B_s} = 7.3[c_{qd}^{(1)}]_{2323} + 1.7 \cdot 10^{-2}[c_{qd}^{(1)}]_{3323} - 9.3 \cdot 10^{-3}[c_{qd}^{(1)}]_{2223}, \quad (\text{A.7})$$

$$\begin{aligned} \Sigma_{qd1}^{B_d} = & 1.8 \cdot 10^2[c_{qd}^{(1)}]_{1313} - 2.3 \cdot 10^{-1}[c_{qd}^{(1)}]_{1213} - 9.0 \cdot 10^{-2}e^{i22^\circ}[c_{qd}^{(1)}]_{3313} \\ & + 5.0 \cdot 10^{-2}e^{i22^\circ}[c_{qd}^{(1)}]_{1113} - 1.4 \cdot 10^{-2}[c_{qd}^{(1)}]_{1312}, \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} \Sigma_{qd1}^K = & 1.7 \cdot 10^4[c_{qd}^{(1)}]_{1212} + 4.6 \cdot 10^1[c_{qd}^{(1)}]_{1312} - 1.0 \cdot 10^1e^{i22^\circ}[c_{qd}^{(1)}]_{2321} + 1.4[c_{qd}^{(1)}]_{1213} \\ & + 4.4 \cdot 10^{-1}e^{i23^\circ}[c_{qd}^{(1)}]_{2212} - 2.0 \cdot 10^{-1}e^{i23^\circ}[c_{qd}^{(1)}]_{1112} - 7.8 \cdot 10^{-2}e^{i23^\circ}[c_{qd}^{(1)}]_{3312} \\ & + 3.2 \cdot 10^{-2}e^{i23^\circ}[c_{qd}^{(1)}]_{1222} - 3.2 \cdot 10^{-2}e^{i23^\circ}[c_{qd}^{(1)}]_{1211} - 1.5 \cdot 10^{-2}e^{i22^\circ}[c_{qd}^{(1)}]_{1232}, \end{aligned} \quad (\text{A.9})$$

and

$$\Sigma_{qd8}^{B_s} = 8.7[c_{qd}^{(8)}]_{2323} + 2.1 \cdot 10^{-2}[c_{qd}^{(8)}]_{3323} - 1.1 \cdot 10^{-2}[c_{qd}^{(8)}]_{2223}, \quad (\text{A.10})$$

$$\begin{aligned} \Sigma_{qd8}^{B_d} = & 2.1 \cdot 10^2[c_{qd}^{(8)}]_{1313} - 2.7 \cdot 10^{-1}[c_{qd}^{(8)}]_{1213} - 1.1 \cdot 10^{-1}e^{i22^\circ}[c_{qd}^{(8)}]_{3313} \\ & + 6.0 \cdot 10^{-2}e^{i22^\circ}[c_{qd}^{(8)}]_{1113} - 1.7 \cdot 10^{-2}[c_{qd}^{(8)}]_{1312}, \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} \Sigma_{qd8}^K = & 2.4 \cdot 10^4[c_{qd}^{(8)}]_{1212} + 6.5 \cdot 10^1[c_{qd}^{(8)}]_{1312} - 1.4 \cdot 10^1e^{i22^\circ}[c_{qd}^{(8)}]_{2321} + 1.9[c_{qd}^{(8)}]_{1213} \\ & + 6.2 \cdot 10^{-1}e^{i23^\circ}[c_{qd}^{(8)}]_{2212} - 2.8 \cdot 10^{-1}e^{i23^\circ}[c_{qd}^{(8)}]_{1112} - 1.0 \cdot 10^{-1}e^{i23^\circ}[c_{qd}^{(8)}]_{3312} \\ & - 4.4 \cdot 10^{-2}e^{i23^\circ}[c_{qd}^{(8)}]_{1211} + 4.4 \cdot 10^{-2}e^{i23^\circ}[c_{qd}^{(8)}]_{1222} - 2.0 \cdot 10^{-2}e^{i22^\circ}[c_{qd}^{(8)}]_{1232} \\ & + 5.1 \cdot 10^{-3}[c_{qd}^{(8)}]_{1313}. \end{aligned} \quad (\text{A.12})$$

For the up basis we have

$$\begin{aligned} \hat{\Sigma}_{qd1}^{B_s} = & 7.1[\hat{c}_{qd}^{(1)}]_{2323} + 1.6[\hat{c}_{qd}^{(1)}]_{1323} + 2.9 \cdot 10^{-1}[\hat{c}_{qd}^{(1)}]_{2223} - 2.8 \cdot 10^{-1}[\hat{c}_{qd}^{(1)}]_{3323} + 6.7 \cdot 10^{-2}[\hat{c}_{qd}^{(1)}]_{1223} \\ & + 2.5 \cdot 10^{-2}e^{-i73^\circ}[\hat{c}_{qd}^{(1)}]_{1232} - 1.2 \cdot 10^{-2}[\hat{c}_{qd}^{(1)}]_{2332} + 5.7 \cdot 10^{-3}e^{-i73^\circ}[\hat{c}_{qd}^{(1)}]_{1123}, \end{aligned} \quad (\text{A.13})$$

$$\begin{aligned} \hat{\Sigma}_{qd1}^{B_d} = & 1.7 \cdot 10^2[\hat{c}_{qd}^{(1)}]_{1313} - 4.0 \cdot 10^1[\hat{c}_{qd}^{(1)}]_{2313} + 7.1[\hat{c}_{qd}^{(1)}]_{1213} - 1.6[\hat{c}_{qd}^{(1)}]_{2213} + 1.5e^{i22^\circ}[\hat{c}_{qd}^{(1)}]_{3313} \\ & + 6.1 \cdot 10^{-1}e^{-i73^\circ}[\hat{c}_{qd}^{(1)}]_{1113} - 1.4 \cdot 10^{-1}e^{-i73^\circ}[\hat{c}_{qd}^{(1)}]_{1231} + 6.2 \cdot 10^{-2}e^{i22^\circ}[\hat{c}_{qd}^{(1)}]_{2331} \\ & - 1.4 \cdot 10^{-2}[\hat{c}_{qd}^{(1)}]_{1312} + 5.3 \cdot 10^{-3}e^{-i51^\circ}[\hat{c}_{qd}^{(1)}]_{1331}, \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned} \hat{\Sigma}_{qd1}^K = & 1.7 \cdot 10^4[\hat{c}_{qd}^{(1)}]_{1212} + 3.8 \cdot 10^3[\hat{c}_{qd}^{(1)}]_{1112} - 3.8 \cdot 10^3[\hat{c}_{qd}^{(1)}]_{2212} - 8.8 \cdot 10^2[\hat{c}_{qd}^{(1)}]_{1221} \\ & - 6.6 \cdot 10^2[\hat{c}_{qd}^{(1)}]_{1312} + 1.5 \cdot 10^2[\hat{c}_{qd}^{(1)}]_{2312} + 1.4 \cdot 10^2e^{i22^\circ}[\hat{c}_{qd}^{(1)}]_{2321} + 3.3 \cdot 10^1e^{i22^\circ}[\hat{c}_{qd}^{(1)}]_{1321} \\ & - 5.8e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{3312} + 1.3[\hat{c}_{qd}^{(1)}]_{1213} + 3.0 \cdot 10^{-1}[\hat{c}_{qd}^{(1)}]_{1113} - 3.0 \cdot 10^{-1}[\hat{c}_{qd}^{(1)}]_{2213} \\ & - 6.8 \cdot 10^{-2}[\hat{c}_{qd}^{(1)}]_{1231} - 5.1 \cdot 10^{-2}[\hat{c}_{qd}^{(1)}]_{1313} + 2.8 \cdot 10^{-2}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{1222} \\ & - 2.8 \cdot 10^{-2}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{1211} - 1.4 \cdot 10^{-2}e^{i22^\circ}[\hat{c}_{qd}^{(1)}]_{1232} + 1.2 \cdot 10^{-2}[\hat{c}_{qd}^{(1)}]_{2313} \\ & + 1.1 \cdot 10^{-2}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{2331} + 6.9 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{1122} - 6.9 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{1111} \\ & - 6.9 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{2222} + 6.9 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(1)}]_{2211}, \end{aligned} \quad (\text{A.15})$$

and

$$\hat{\Sigma}_{qd8}^{B_s} = 8.5[\hat{c}_{qd}^{(8)}]_{2323} + 2.0[\hat{c}_{qd}^{(8)}]_{1323} + 3.5 \cdot 10^{-1}[\hat{c}_{qd}^{(8)}]_{2223} - 3.4 \cdot 10^{-1}[\hat{c}_{qd}^{(8)}]_{3323} + 8.0 \cdot 10^{-2}[\hat{c}_{qd}^{(8)}]_{1223} \\ + 3.0 \cdot 10^{-2}e^{-i73^\circ}[\hat{c}_{qd}^{(8)}]_{1232} - 1.4 \cdot 10^{-2}[\hat{c}_{qd}^{(8)}]_{2332} + 6.8 \cdot 10^{-3}e^{-i73^\circ}[\hat{c}_{qd}^{(8)}]_{1123}, \quad (\text{A.16})$$

$$\hat{\Sigma}_{qd8}^{B_d} = 2.1 \cdot 10^2[\hat{c}_{qd}^{(8)}]_{1313} - 4.8 \cdot 10^1[\hat{c}_{qd}^{(8)}]_{2313} + 8.5[\hat{c}_{qd}^{(8)}]_{1213} - 2.0[\hat{c}_{qd}^{(8)}]_{2213} + 1.8e^{i22^\circ}[\hat{c}_{qd}^{(8)}]_{3313} \\ + 7.3 \cdot 10^{-1}e^{-i73^\circ}[\hat{c}_{qd}^{(8)}]_{1113} - 1.7 \cdot 10^{-1}e^{-i73^\circ}[\hat{c}_{qd}^{(8)}]_{1231} + 7.4 \cdot 10^{-2}e^{i22^\circ}[\hat{c}_{qd}^{(8)}]_{2331} \\ - 1.6 \cdot 10^{-2}[\hat{c}_{qd}^{(8)}]_{1312} + 6.3 \cdot 10^{-3}e^{-i51^\circ}[\hat{c}_{qd}^{(8)}]_{1331}, \quad (\text{A.17})$$

$$\hat{\Sigma}_{qd8}^K = 2.3 \cdot 10^4[\hat{c}_{qd}^{(8)}]_{1212} + 5.3 \cdot 10^3[\hat{c}_{qd}^{(8)}]_{1112} - 5.3 \cdot 10^3[\hat{c}_{qd}^{(8)}]_{2212} - 1.2 \cdot 10^3[\hat{c}_{qd}^{(8)}]_{1221} \\ - 9.2 \cdot 10^2[\hat{c}_{qd}^{(8)}]_{1312} + 2.1 \cdot 10^2[\hat{c}_{qd}^{(8)}]_{2312} + 2.0 \cdot 10^2e^{i22^\circ}[\hat{c}_{qd}^{(8)}]_{2321} + 4.7 \cdot 10^1e^{i22^\circ}[\hat{c}_{qd}^{(8)}]_{1321} \\ - 8.1e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{3312} + 1.8[\hat{c}_{qd}^{(8)}]_{1213} + 4.2 \cdot 10^{-1}[\hat{c}_{qd}^{(8)}]_{1113} - 4.2 \cdot 10^{-1}[\hat{c}_{qd}^{(8)}]_{2213} \\ - 9.6 \cdot 10^{-2}[\hat{c}_{qd}^{(8)}]_{1231} - 7.2 \cdot 10^{-2}[\hat{c}_{qd}^{(8)}]_{1313} - 4.0 \cdot 10^{-2}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{1211} \\ + 4.0 \cdot 10^{-2}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{1222} - 1.9 \cdot 10^{-2}e^{i22^\circ}[\hat{c}_{qd}^{(8)}]_{1232} + 1.7 \cdot 10^{-2}[\hat{c}_{qd}^{(8)}]_{2313} \\ + 1.6 \cdot 10^{-2}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{2331} - 9.7 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{1111} + 9.7 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{1122} \\ + 9.7 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{2211} - 9.7 \cdot 10^{-3}e^{i23^\circ}[\hat{c}_{qd}^{(8)}]_{2222}. \quad (\text{A.18})$$

A.3 $\mathcal{C}_{qu}^{(1)}$ and $\mathcal{C}_{qu}^{(8)}$

The only non-vanishing expression following our criteria is in **the down-basis**

$$\Sigma_{qu1}^{B_d} = -5.4 \cdot 10^{-3}e^{i22^\circ}[\hat{c}_{qu}^{(1)}]_{1333}, \quad (\text{A.19})$$

and in **the up basis**

$$\hat{\Sigma}_{qu1}^{B_d} = -5.2 \cdot 10^{-3}e^{i22^\circ}[\hat{c}_{qu}^{(1)}]_{1333}. \quad (\text{A.20})$$

A.4 $\mathcal{C}_{dd}$, $\mathcal{C}_{uu}$, $\mathcal{C}_{ud}^{(1)}$ and $\mathcal{C}_{ud}^{(8)}$

In both bases

$$\Sigma_{dd}^{B_s} = -9.7 \cdot 10^{-1}[c_{dd}]_{2323}, \quad (\text{A.21})$$

$$\Sigma_{dd}^{B_d} = -2.2 \cdot 10^1[c_{dd}]_{1313}, \quad (\text{A.22})$$

$$\Sigma_{dd}^K = -8.6 \cdot 10^1[c_{dd}]_{1212} - 6.7 \cdot 10^{-3}[c_{dd}]_{1213}, \quad (\text{A.23})$$

$$\Sigma_{uu}^{B_s} = 0, \quad \Sigma_{uu}^{B_d} = 0, \quad \Sigma_{uu}^K = 0, \quad (\text{A.24})$$

$$\Sigma_{ud1}^{B_s} = 0, \quad (\text{A.25})$$

$$\Sigma_{ud1}^{B_d} = 2.3 \cdot 10^{-2}e^{i22^\circ}[c_{ud}^{(1)}]_{3313}, \quad (\text{A.26})$$

$$\Sigma_{ud1}^K = -7.8 \cdot 10^{-2}e^{i23^\circ}[c_{ud}^{(1)}]_{3312} + 7.9 \cdot 10^{-3}[c_{ud}^{(1)}]_{2312} + 7.6 \cdot 10^{-3}e^{i22^\circ}[c_{ud}^{(1)}]_{2321}, \quad (\text{A.27})$$

$$\Sigma_{ud8}^{B_s} = -7.4 \cdot 10^{-3}[c_{ud}^{(8)}]_{3323}, \quad (\text{A.28})$$

$$\Sigma_{ud8}^{B_d} = 4.0 \cdot 10^{-2}e^{i22^\circ}[c_{ud}^{(8)}]_{3313}, \quad (\text{A.29})$$

$$\Sigma_{ud8}^K = -1.9 \cdot 10^{-1}e^{i23^\circ}[c_{ud}^{(8)}]_{3312} + 1.9 \cdot 10^{-2}[c_{ud}^{(8)}]_{2312} + 1.8 \cdot 10^{-2}e^{i22^\circ}[c_{ud}^{(8)}]_{2321}. \quad (\text{A.30})$$

A.5 $\mathcal{C}_{\phi q}^{(1,3)}$, $\mathcal{C}_{\phi d}$ and $\mathcal{C}_{\phi u}$

The only non-vanishing expressions for these WCs in **the down and up basis** are

$$\Sigma_{\phi q 3}^{B_d} = -8.5 \cdot 10^{-3} e^{i22^\circ} [c_{\phi q}^{(3)}]_{13}, \quad (\text{A.31})$$

and

$$\Sigma_{\phi d}^{B_s} = 5.7 \cdot 10^{-3} [c_{\phi d}]_{23}, \quad (\text{A.32})$$

$$\Sigma_{\phi d}^{B_d} = -3.1 \cdot 10^{-2} e^{i22^\circ} [c_{\phi d}]_{13}, \quad (\text{A.33})$$

$$\Sigma_{\phi d}^K = 1.2 \cdot 10^{-1} e^{i23^\circ} [c_{\phi d}]_{12}. \quad (\text{A.34})$$

A.6 $\mathcal{C}_{quqd}^{(1,8)}$

For these operators the only non-zero contributions are in **the down basis**

$$\Sigma_{quqd1}^K = -6.2 \cdot 10^{-3} e^{i22^\circ} [c_{quqd}^{(1)}]_{2321}, \quad (\text{A.35})$$

and in **the up basis**

$$\hat{\Sigma}_{quqd1}^K = -5.9 \cdot 10^{-3} e^{i22^\circ} [\hat{c}_{quqd}^{(1)}]_{2321}. \quad (\text{A.36})$$

A.7 $\mathcal{C}_{lequ}^{(1,3)}$, $\mathcal{C}_{lq}^{(1,3)}$, $\mathcal{C}_{ld}$, $\mathcal{C}_{qe}$ and $\mathcal{C}_{uW}$

The contributions of all the semileptonic Wilson coefficients as well as $\mathcal{C}_{uW}$ are negligible.

B NP master formulae in the up-basis

In this appendix we present the NP master formulae for the four scenarios under consideration at 5 TeV in the up-basis.

B.1 Z'

$$\begin{aligned} \frac{M_{Z'}^2 \hat{\Sigma}_{Z'}^{B_s}}{(5 \text{ TeV})^2} = & -2.3 \cdot 10^3 \hat{z}_q^{23} \hat{z}_d^{23} - 5.2 \cdot 10^2 \hat{z}_q^{13} \hat{z}_d^{23} + 2.1 \cdot 10^2 \hat{z}_d^{23} \hat{z}_d^{23} + 1.9 \cdot 10^2 \hat{z}_q^{23} \hat{z}_q^{23} \\ & - 9.3 \cdot 10^1 \hat{z}_q^{22} \hat{z}_d^{23} + 9.3 \cdot 10^1 \hat{z}_q^{33} \hat{z}_d^{23} + 4.3 \cdot 10^1 \hat{z}_q^{13} \hat{z}_q^{23} - 2.2 \cdot 10^1 \hat{z}_q^{12} \hat{z}_d^{23} \\ & + 9.8 \hat{z}_q^{13} \hat{z}_q^{13} - 8.0 e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_d^{32} + 7.7 \hat{z}_q^{22} \hat{z}_q^{23} - 7.6 \hat{z}_q^{23} \hat{z}_q^{33} + 3.8 \hat{z}_q^{23} \hat{z}_d^{32} \\ & - 1.8 e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_d^{23} + 1.8 \hat{z}_q^{12} \hat{z}_q^{23} + 1.8 \hat{z}_q^{13} \hat{z}_q^{22} - 1.8 \hat{z}_q^{13} \hat{z}_q^{33} \\ & - 1.2 \text{Re}(\hat{z}_\phi) \hat{z}_d^{23} + 1.0 \hat{z}_u^{33} \hat{z}_d^{23} + 6.6 \cdot 10^{-1} e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_q^{32} + 4.1 \cdot 10^{-1} \hat{z}_q^{12} \hat{z}_q^{13} \\ & + 3.3 \cdot 10^{-1} e^{-i74^\circ} \hat{z}_q^{13} \hat{z}_d^{32} + 3.2 \cdot 10^{-1} \hat{z}_q^{22} \hat{z}_q^{22} - 3.2 \cdot 10^{-1} \hat{z}_q^{22} \hat{z}_q^{33} - 3.1 \cdot 10^{-1} \hat{z}_q^{23} \hat{z}_q^{32} \\ & + 3.1 \cdot 10^{-1} \hat{z}_q^{33} \hat{z}_q^{33} - 2.6 \cdot 10^{-1} \hat{z}_q^{23} \hat{z}_u^{33} + 2.3 \cdot 10^{-1} \text{Re}(\hat{z}_\phi) \hat{z}_q^{23} \\ & + 1.5 \cdot 10^{-1} e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_q^{31} + 1.5 \cdot 10^{-1} e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_q^{23} - 1.1 \cdot 10^{-1} \hat{z}_u^{23} \hat{z}_d^{23} \\ & + 1.1 \cdot 10^{-1} \hat{z}_q^{23} \hat{z}_d^{22} - 1.1 \cdot 10^{-1} \hat{z}_q^{23} \hat{z}_d^{33} + 8.5 \cdot 10^{-2} e^{-i18^\circ} \hat{z}_q^{12} \hat{z}_q^{22} \\ & - 8.5 \cdot 10^{-2} e^{-i19^\circ} \hat{z}_q^{12} \hat{z}_q^{33} - 8.4 \cdot 10^{-2} e^{-i19^\circ} \hat{z}_q^{13} \hat{z}_q^{32} - 5.9 \cdot 10^{-2} \hat{z}_q^{13} \hat{z}_u^{33} \\ & + 5.4 \cdot 10^{-2} \text{Re}(\hat{z}_\phi) \hat{z}_q^{13} + 3.5 \cdot 10^{-2} e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_q^{13} + 3.1 \cdot 10^{-2} \hat{z}_q^{23} \hat{z}_u^{23} \end{aligned}$$

$$\begin{aligned}
 & + 2.5 \cdot 10^{-2} \hat{z}_q^{13} \hat{z}_d^{22} - 2.5 \cdot 10^{-2} \hat{z}_q^{13} \hat{z}_d^{33} + 1.5 \cdot 10^{-2} e^{-i5^\circ} \hat{z}_q^{12} \hat{z}_q^{12} \\
 & + 1.1 \cdot 10^{-2} \hat{z}_d^{23} \hat{z}_d^{33} - 1.1 \cdot 10^{-2} \hat{z}_q^{22} \hat{z}_u^{33} + 1.0 \cdot 10^{-2} \text{Re}(\hat{z}_\phi) \hat{z}_q^{22} \\
 & + 1.0 \cdot 10^{-2} \hat{z}_q^{33} \hat{z}_u^{33} - 9.4 \cdot 10^{-3} \text{Re}(\hat{z}_\phi) \hat{z}_q^{33} - 9.3 \cdot 10^{-3} \hat{z}_d^{22} \hat{z}_d^{23} \\
 & + 7.2 \cdot 10^{-3} \hat{z}_q^{13} \hat{z}_u^{23} + 6.3 \cdot 10^{-3} e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_q^{21} + 6.3 \cdot 10^{-3} e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_q^{22} \\
 & - 6.2 \cdot 10^{-3} e^{-i74^\circ} \hat{z}_q^{11} \hat{z}_q^{33} - 6.2 \cdot 10^{-3} e^{-i74^\circ} \hat{z}_q^{13} \hat{z}_q^{31} + 5.4 \cdot 10^{-3} e^{-i11^\circ} \hat{z}_q^{23} \hat{z}_d^{13}, \quad (\text{B.1})
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_{Z'}^2 \hat{\Sigma}_{Z'}^{B_d}}{(5 \text{ TeV})^2} = & -5.5 \cdot 10^4 \hat{z}_q^{13} \hat{z}_d^{13} + 1.3 \cdot 10^4 \hat{z}_q^{23} \hat{z}_d^{13} + 4.8 \cdot 10^3 \hat{z}_d^{13} \hat{z}_d^{13} + 4.3 \cdot 10^3 \hat{z}_q^{13} \hat{z}_q^{13} \\
 & - 2.3 \cdot 10^3 \hat{z}_q^{12} \hat{z}_d^{13} - 9.9 \cdot 10^2 \hat{z}_q^{13} \hat{z}_q^{23} + 5.3 \cdot 10^2 \hat{z}_q^{22} \hat{z}_d^{13} - 5.0 \cdot 10^2 e^{i22^\circ} \hat{z}_q^{33} \hat{z}_d^{13} \\
 & + 2.3 \cdot 10^2 \hat{z}_q^{23} \hat{z}_q^{23} - 2.0 \cdot 10^2 e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_d^{13} + 1.8 \cdot 10^2 \hat{z}_q^{12} \hat{z}_q^{13} + 4.5 \cdot 10^1 e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_d^{31} \\
 & - 4.1 \cdot 10^1 \hat{z}_q^{12} \hat{z}_q^{23} - 4.1 \cdot 10^1 \hat{z}_q^{13} \hat{z}_q^{22} + 3.9 \cdot 10^1 e^{i22^\circ} \hat{z}_q^{13} \hat{z}_q^{33} - 2.0 \cdot 10^1 e^{i22^\circ} \hat{z}_q^{23} \hat{z}_d^{31} \\
 & + 1.5 \cdot 10^1 e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_q^{13} + 9.4 \hat{z}_q^{22} \hat{z}_q^{23} - 8.9 e^{i22^\circ} \hat{z}_q^{23} \hat{z}_q^{33} + 7.4 \hat{z}_q^{12} \hat{z}_q^{12} \\
 & + 6.6 e^{i22^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_d^{13} - 5.6 e^{i22^\circ} \hat{z}_u^{33} \hat{z}_d^{13} - 3.5 e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_q^{31} - 3.5 e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_q^{23} \\
 & + 2.7 \hat{z}_q^{13} \hat{z}_d^{12} - 1.8 e^{-i51^\circ} \hat{z}_q^{13} \hat{z}_d^{31} - 1.7 \hat{z}_q^{12} \hat{z}_q^{22} + 1.6 e^{i23^\circ} \hat{z}_q^{12} \hat{z}_q^{33} + 1.6 e^{i23^\circ} \hat{z}_q^{13} \hat{z}_q^{32} \\
 & + 1.3 e^{i22^\circ} \hat{z}_q^{13} \hat{z}_u^{33} - 1.2 e^{i22^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{13} + 8.1 \cdot 10^{-1} e^{-i68^\circ} \hat{z}_q^{12} \hat{z}_q^{32} \\
 & + 6.3 \cdot 10^{-1} e^{-i72^\circ} \hat{z}_q^{11} \hat{z}_q^{12} + 6.2 \cdot 10^{-1} \hat{z}_u^{23} \hat{z}_d^{13} - 6.2 \cdot 10^{-1} \hat{z}_q^{23} \hat{z}_d^{12} \\
 & + 3.9 \cdot 10^{-1} \hat{z}_q^{22} \hat{z}_q^{22} - 3.7 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{22} \hat{z}_q^{33} - 3.7 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{23} \hat{z}_q^{32} \\
 & + 3.5 \cdot 10^{-1} e^{i44^\circ} \hat{z}_q^{33} \hat{z}_q^{33} - 3.0 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{23} \hat{z}_u^{33} + 2.7 \cdot 10^{-1} e^{i22^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{23} \\
 & - 2.4 \cdot 10^{-1} \hat{z}_d^{12} \hat{z}_d^{13} - 1.7 \cdot 10^{-1} \hat{z}_q^{13} \hat{z}_u^{23} - 1.5 \cdot 10^{-1} e^{-i73^\circ} \hat{z}_q^{12} \hat{z}_q^{21} \\
 & - 1.5 \cdot 10^{-1} e^{-i73^\circ} \hat{z}_q^{11} \hat{z}_q^{22} + 1.4 \cdot 10^{-1} e^{-i51^\circ} \hat{z}_q^{11} \hat{z}_q^{33} + 1.4 \cdot 10^{-1} e^{-i51^\circ} \hat{z}_q^{13} \hat{z}_q^{31} \\
 & + 1.1 \cdot 10^{-1} \hat{z}_q^{12} \hat{z}_d^{12} - 6.3 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{13} \hat{z}_d^{23} + 5.5 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{12} \hat{z}_u^{33} \\
 & - 5.4 \cdot 10^{-2} e^{i34^\circ} \hat{z}_q^{11} \hat{z}_q^{11} - 5.2 \cdot 10^{-2} e^{i23^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{12} + 3.8 \cdot 10^{-2} \hat{z}_q^{23} \hat{z}_u^{23} \\
 & + 3.0 \cdot 10^{-2} e^{i21^\circ} \hat{z}_q^{13} \hat{z}_d^{33} - 3.0 \cdot 10^{-2} e^{i21^\circ} \hat{z}_q^{13} \hat{z}_d^{11} - 2.6 \cdot 10^{-2} \hat{z}_q^{22} \hat{z}_d^{12} \\
 & + 2.4 \cdot 10^{-2} e^{i21^\circ} \hat{z}_q^{33} \hat{z}_d^{12} + 1.4 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{23} \hat{z}_d^{23} - 1.3 \cdot 10^{-2} e^{i22^\circ} \hat{z}_q^{22} \hat{z}_u^{33} \\
 & + 1.2 \cdot 10^{-2} e^{i22^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{22} + 1.1 \cdot 10^{-2} e^{i44^\circ} \hat{z}_q^{33} \hat{z}_u^{33} - 1.0 \cdot 10^{-2} e^{i44^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{33} \\
 & + 9.6 \cdot 10^{-3} e^{-i74^\circ} \hat{z}_q^{11} \hat{z}_d^{12} - 7.2 \cdot 10^{-3} e^{i22^\circ} \hat{z}_d^{13} \hat{z}_d^{33} - 7.1 \cdot 10^{-3} \hat{z}_q^{12} \hat{z}_u^{23} \\
 & - 6.6 \cdot 10^{-3} e^{i22^\circ} \hat{z}_q^{23} \hat{z}_d^{33} + 6.6 \cdot 10^{-3} e^{i22^\circ} \hat{z}_q^{23} \hat{z}_d^{11} + 5.5 \cdot 10^{-3} e^{i23^\circ} \hat{z}_d^{13} \hat{z}_d^{23} \\
 & - 5.4 \cdot 10^{-3} \hat{z}_u^{13} \hat{z}_d^{13} - 5.3 \cdot 10^{-3} e^{i22^\circ} \hat{z}_d^{12} \hat{z}_d^{23}, \quad (\text{B.2})
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_{Z'}^2 \hat{\Sigma}_{Z'}^K}{(5 \text{ TeV})^2} = & -5.0 \cdot 10^6 \hat{z}_q^{12} \hat{z}_d^{12} - 1.2 \cdot 10^6 \hat{z}_q^{11} \hat{z}_d^{12} + 1.2 \cdot 10^6 \hat{z}_q^{22} \hat{z}_d^{12} + 2.7 \cdot 10^5 \hat{z}_q^{12} \hat{z}_d^{21} \\
 & + 2.0 \cdot 10^5 \hat{z}_q^{13} \hat{z}_d^{12} - 4.7 \cdot 10^4 \hat{z}_q^{23} \hat{z}_d^{12} - 4.5 \cdot 10^4 e^{i22^\circ} \hat{z}_q^{23} \hat{z}_d^{21} + 1.8 \cdot 10^4 \hat{z}_d^{12} \hat{z}_d^{12} \\
 & + 1.6 \cdot 10^4 \hat{z}_q^{12} \hat{z}_q^{12} - 1.0 \cdot 10^4 e^{i22^\circ} \hat{z}_q^{13} \hat{z}_d^{21} + 3.5 \cdot 10^3 \hat{z}_q^{11} \hat{z}_q^{12} - 3.5 \cdot 10^3 \hat{z}_q^{12} \hat{z}_q^{22} \\
 & + 1.8 \cdot 10^3 e^{i23^\circ} \hat{z}_q^{33} \hat{z}_d^{12} + 8.5 \cdot 10^2 \hat{z}_q^{11} \hat{z}_q^{11} - 8.5 \cdot 10^2 \hat{z}_q^{12} \hat{z}_q^{21} - 8.5 \cdot 10^2 \hat{z}_q^{12} \hat{z}_q^{22} \\
 & + 8.5 \cdot 10^2 \hat{z}_q^{22} \hat{z}_q^{22} - 6.5 \cdot 10^2 \hat{z}_q^{12} \hat{z}_q^{13} - 2.5 \cdot 10^2 \hat{z}_q^{12} \hat{z}_d^{13} - 1.4 \cdot 10^2 \hat{z}_q^{11} \hat{z}_q^{13} \\
 & + 1.4 \cdot 10^2 \hat{z}_q^{13} \hat{z}_q^{22} + 1.4 \cdot 10^2 \hat{z}_q^{12} \hat{z}_q^{23} + 1.4 \cdot 10^2 e^{i23^\circ} \hat{z}_q^{12} \hat{z}_q^{32} + 6.6 \cdot 10^1 e^{i11^\circ} \hat{z}_q^{11} \hat{z}_q^{23} \\
 & + 6.6 \cdot 10^1 e^{i11^\circ} \hat{z}_q^{12} \hat{z}_q^{31} - 6.6 \cdot 10^1 e^{i11^\circ} \hat{z}_q^{22} \hat{z}_q^{23} - 5.6 \cdot 10^1 \hat{z}_q^{11} \hat{z}_d^{13} + 5.6 \cdot 10^1 \hat{z}_q^{22} \hat{z}_d^{13} \\
 & + 2.7 \cdot 10^1 \hat{z}_q^{13} \hat{z}_q^{13} - 2.5 \cdot 10^1 e^{i23^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_d^{12} + 2.0 \cdot 10^1 e^{i23^\circ} \hat{z}_u^{33} \hat{z}_d^{12} + 1.3 \cdot 10^1 \hat{z}_q^{12} \hat{z}_d^{31}
 \end{aligned}$$

$$\begin{aligned}
 & + 1.0 \cdot 10^1 \hat{z}_q^{13} \hat{z}_d^{13} - 5.9 \hat{z}_q^{13} \hat{z}_q^{23} - 5.6 e^{i23^\circ} \hat{z}_q^{12} \hat{z}_q^{33} - 5.5 e^{i23^\circ} \hat{z}_q^{13} \hat{z}_q^{32} + 2.6 e^{i22^\circ} \hat{z}_q^{12} \hat{z}_d^{32} \\
 & + 2.5 e^{i22^\circ} \hat{z}_q^{23} \hat{z}_q^{23} + 2.5 e^{i57^\circ} \hat{z}_q^{12} \hat{z}_d^{11} - 2.5 e^{i57^\circ} \hat{z}_q^{12} \hat{z}_d^{22} - 2.3 \hat{z}_q^{23} \hat{z}_d^{13} \\
 & - 2.2 e^{i23^\circ} \hat{z}_q^{23} \hat{z}_d^{31} - 2.1 \hat{z}_u^{23} \hat{z}_d^{12} - 2.0 e^{i22^\circ} \hat{z}_u^{23} \hat{z}_d^{21} + 1.5 e^{i20^\circ} \hat{z}_q^{22} \hat{z}_d^{22} \\
 & - 1.5 e^{i20^\circ} \hat{z}_q^{22} \hat{z}_d^{11} - 1.4 e^{i23^\circ} \hat{z}_q^{11} \hat{z}_q^{33} + 1.3 e^{i23^\circ} \hat{z}_q^{22} \hat{z}_q^{33} - 1.3 e^{i23^\circ} \hat{z}_q^{13} \hat{z}_q^{31} \\
 & + 1.3 e^{i23^\circ} \hat{z}_q^{23} \hat{z}_q^{32} - 1.1 e^{i27^\circ} \hat{z}_q^{11} \hat{z}_d^{22} + 1.1 e^{i27^\circ} \hat{z}_q^{11} \hat{z}_d^{11} + 9.0 \cdot 10^{-1} \hat{z}_d^{12} \hat{z}_d^{13} \\
 & + 6.1 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{11} \hat{z}_d^{23} - 6.0 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{22} \hat{z}_d^{23} - 5.0 \cdot 10^{-1} e^{i23^\circ} \hat{z}_q^{13} \hat{z}_d^{31} \\
 & + 2.3 \cdot 10^{-1} e^{i23^\circ} \hat{z}_q^{13} \hat{z}_q^{33} + 2.3 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{13} \hat{z}_d^{22} - 2.3 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{13} \hat{z}_d^{11} \\
 & - 2.0 \cdot 10^{-1} e^{i23^\circ} \hat{z}_q^{12} \hat{z}_u^{33} + 1.9 \cdot 10^{-1} \hat{z}_u^{22} \hat{z}_d^{12} + 1.9 \cdot 10^{-1} e^{i23^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{12} \\
 & - 1.4 \cdot 10^{-1} e^{i22^\circ} \hat{z}_q^{12} \hat{z}_d^{23} - 1.1 \cdot 10^{-1} e^{i23^\circ} \hat{z}_q^{13} \hat{z}_d^{32} - 1.1 \cdot 10^{-1} e^{i34^\circ} \hat{z}_q^{23} \hat{z}_q^{33} \\
 & - 1.0 \cdot 10^{-1} e^{i34^\circ} \hat{z}_q^{23} \hat{z}_d^{22} + 1.0 \cdot 10^{-1} e^{i34^\circ} \hat{z}_q^{23} \hat{z}_d^{11} + 9.0 \cdot 10^{-2} e^{i24^\circ} \hat{z}_q^{33} \hat{z}_d^{13} \\
 & - 4.8 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{11} \hat{z}_u^{33} + 4.8 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{22} \hat{z}_u^{33} + 4.6 \cdot 10^{-2} e^{i23^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{11} \\
 & - 4.6 \cdot 10^{-2} e^{i23^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{22} + 4.1 \cdot 10^{-2} e^{i23^\circ} \hat{z}_d^{12} \hat{z}_d^{22} + 2.5 \cdot 10^{-2} \hat{z}_q^{12} \hat{z}_u^{23} \\
 & + 2.5 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{23} \hat{z}_d^{32} + 2.4 \cdot 10^{-2} e^{i22^\circ} \hat{z}_d^{13} \hat{z}_d^{32} + 2.4 \cdot 10^{-2} e^{i23^\circ} \hat{z}_q^{12} \hat{z}_u^{32} \\
 & + 2.3 \cdot 10^{-2} e^{i44^\circ} \hat{z}_q^{23} \hat{z}_d^{23} + 1.8 \cdot 10^{-2} \hat{z}_u^{13} \hat{z}_d^{12} + 1.2 \cdot 10^{-2} e^{i11^\circ} \hat{z}_q^{11} \hat{z}_u^{23} \\
 & - 1.2 \cdot 10^{-2} e^{i11^\circ} \hat{z}_q^{22} \hat{z}_u^{23} - 9.5 \cdot 10^{-3} e^{i22^\circ} \hat{z}_d^{12} \hat{z}_d^{32} + 8.1 \cdot 10^{-3} e^{i23^\circ} \hat{z}_u^{11} \hat{z}_d^{12} \\
 & + 7.7 \cdot 10^{-3} e^{i23^\circ} \hat{z}_q^{13} \hat{z}_u^{33} - 7.3 \cdot 10^{-3} e^{i23^\circ} \text{Re}(\hat{z}_\phi) \hat{z}_q^{13} + 5.4 \cdot 10^{-3} e^{i44^\circ} \hat{z}_q^{13} \hat{z}_d^{23}. \quad (\text{B.3})
 \end{aligned}$$

B.2 G'

$$\begin{aligned}
 \frac{M_{G'}^2 \hat{\Sigma}_{G'}^{B_s}}{(5 \text{ TeV})^2} = & -2.6 \cdot 10^3 \hat{g}_q^{23} \hat{g}_d^{23} - 6.0 \cdot 10^2 \hat{g}_q^{13} \hat{g}_d^{23} - 1.1 \cdot 10^2 \hat{g}_q^{22} \hat{g}_d^{23} + 1.1 \cdot 10^2 \hat{g}_q^{33} \hat{g}_d^{23} \\
 & + 6.9 \cdot 10^1 \hat{g}_d^{23} \hat{g}_q^{23} + 6.2 \cdot 10^1 \hat{g}_q^{23} \hat{g}_q^{23} - 2.5 \cdot 10^1 \hat{g}_q^{12} \hat{g}_d^{23} + 1.4 \cdot 10^1 \hat{g}_q^{13} \hat{g}_q^{23} \\
 & - 9.3 e^{-i73^\circ} \hat{g}_q^{12} \hat{g}_d^{32} + 4.4 \hat{g}_q^{23} \hat{g}_d^{32} + 3.3 \hat{g}_q^{13} \hat{g}_q^{13} - 2.5 \hat{g}_q^{23} \hat{g}_q^{33} + 2.5 \hat{g}_q^{23} \hat{g}_q^{22} \\
 & - 2.1 e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_d^{23} + 1.5 \hat{g}_u^{33} \hat{g}_d^{23} + 5.8 \cdot 10^{-1} \hat{g}_q^{12} \hat{g}_q^{23} + 5.8 \cdot 10^{-1} \hat{g}_q^{13} \hat{g}_q^{22} \\
 & - 5.9 \cdot 10^{-1} \hat{g}_q^{13} \hat{g}_q^{33} + 3.8 \cdot 10^{-1} e^{-i74^\circ} \hat{g}_q^{13} \hat{g}_d^{32} + 2.2 \cdot 10^{-1} e^{-i73^\circ} \hat{g}_q^{12} \hat{g}_q^{32} \\
 & - 1.6 \cdot 10^{-1} \hat{g}_u^{23} \hat{g}_d^{23} + 1.4 \cdot 10^{-1} \hat{g}_q^{12} \hat{g}_q^{13} + 1.3 \cdot 10^{-1} \hat{g}_q^{23} \hat{g}_d^{22} - 1.3 \cdot 10^{-1} \hat{g}_q^{23} \hat{g}_d^{33} \\
 & + 1.1 \cdot 10^{-1} \hat{g}_q^{22} \hat{g}_q^{22} - 1.1 \cdot 10^{-1} \hat{g}_q^{22} \hat{g}_q^{33} - 1.1 \cdot 10^{-1} \hat{g}_q^{23} \hat{g}_q^{32} + 1.0 \cdot 10^{-1} \hat{g}_q^{33} \hat{g}_q^{33} \\
 & - 1.0 \cdot 10^{-1} \hat{g}_q^{23} \hat{g}_u^{33} + 7.5 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_q^{32} + 7.5 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{13} \hat{g}_q^{21} + 2.9 \cdot 10^{-2} \hat{g}_q^{13} \hat{g}_d^{22} \\
 & - 2.9 \cdot 10^{-2} \hat{g}_q^{13} \hat{g}_d^{33} - 2.8 \cdot 10^{-2} e^{-i19^\circ} \hat{g}_q^{12} \hat{g}_q^{33} - 2.8 \cdot 10^{-2} e^{-i19^\circ} \hat{g}_q^{13} \hat{g}_q^{32} \\
 & + 2.8 \cdot 10^{-2} e^{-i18^\circ} \hat{g}_q^{12} \hat{g}_q^{22} - 2.5 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{12} \hat{g}_q^{31} - 2.5 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_q^{23} \\
 & - 2.4 \cdot 10^{-2} \hat{g}_q^{13} \hat{g}_u^{33} + 1.4 \cdot 10^{-2} \hat{g}_q^{23} \hat{g}_u^{23} + 1.2 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_d^{13} + 7.4 \cdot 10^{-3} e^{-i10^\circ} \hat{g}_q^{23} \hat{g}_d^{13} \\
 & + 6.5 \cdot 10^{-3} \hat{g}_q^{23} \hat{g}_d^{31} - 6.0 \cdot 10^{-3} e^{-i3^\circ} \hat{g}_q^{33} \hat{g}_d^{13} + 5.3 \cdot 10^{-3} \hat{g}_q^{22} \hat{g}_d^{22} - 5.3 \cdot 10^{-3} \hat{g}_q^{22} \hat{g}_d^{33} \\
 & - 5.3 \cdot 10^{-3} \hat{g}_q^{33} \hat{g}_d^{22} + 5.2 \cdot 10^{-3} \hat{g}_q^{33} \hat{g}_d^{33} - 5.0 \cdot 10^{-3} \hat{g}_q^{22} \hat{g}_d^{13} + 5.0 \cdot 10^{-3} e^{-i5^\circ} \hat{g}_q^{12} \hat{g}_q^{12}, \quad (\text{B.4})
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_{G'}^2 \hat{\Sigma}_{G'}^{B_d}}{(5 \text{ TeV})^2} = & -6.4 \cdot 10^4 \hat{g}_q^{13} \hat{g}_d^{13} + 1.5 \cdot 10^4 \hat{g}_q^{23} \hat{g}_d^{13} - 2.6 \cdot 10^3 \hat{g}_q^{12} \hat{g}_d^{13} + 1.6 \cdot 10^3 \hat{g}_d^{13} \hat{g}_d^{13} \\
 & + 1.4 \cdot 10^3 \hat{g}_q^{13} \hat{g}_q^{13} + 6.1 \cdot 10^2 \hat{g}_q^{22} \hat{g}_d^{13} - 5.7 \cdot 10^2 e^{i22^\circ} \hat{g}_q^{33} \hat{g}_d^{13} - 3.3 \cdot 10^2 \hat{g}_q^{13} \hat{g}_q^{23} \\
 & - 2.3 \cdot 10^2 e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_d^{13} + 7.6 \cdot 10^1 \hat{g}_q^{23} \hat{g}_q^{23} + 6.0 \cdot 10^1 \hat{g}_q^{12} \hat{g}_q^{13} + 5.2 \cdot 10^1 e^{-i73^\circ} \hat{g}_q^{12} \hat{g}_d^{31}
 \end{aligned}$$

$$\begin{aligned}
 & -2.4 \cdot 10^1 e^{i22^\circ} \hat{g}_q^{23} \hat{g}_d^{31} - 1.4 \cdot 10^1 \hat{g}_q^{12} \hat{g}_q^{23} - 1.4 \cdot 10^1 \hat{g}_q^{13} \hat{g}_q^{22} + 1.3 \cdot 10^1 e^{i22^\circ} \hat{g}_q^{13} \hat{g}_q^{33} \\
 & - 7.9 e^{i22^\circ} \hat{g}_u^{33} \hat{g}_d^{13} + 5.2 e^{-i73^\circ} \hat{g}_q^{13} \hat{g}_q^{11} + 3.2 \hat{g}_q^{22} \hat{g}_q^{23} + 3.1 \hat{g}_q^{13} \hat{g}_d^{12} - 3.0 e^{i22^\circ} \hat{g}_q^{23} \hat{g}_q^{33} \\
 & + 2.5 \hat{g}_q^{12} \hat{g}_q^{12} - 2.0 e^{-i51^\circ} \hat{g}_q^{13} \hat{g}_d^{31} - 1.8 e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_q^{32} - 1.8 e^{-i73^\circ} \hat{g}_q^{13} \hat{g}_q^{21} \\
 & + 8.7 \cdot 10^{-1} \hat{g}_u^{23} \hat{g}_d^{13} - 7.2 \cdot 10^{-1} \hat{g}_q^{23} \hat{g}_d^{12} + 5.8 \cdot 10^{-1} e^{-i73^\circ} \hat{g}_q^{12} \hat{g}_q^{31} + 5.8 \cdot 10^{-1} e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_q^{23} \\
 & - 5.6 \cdot 10^{-1} \hat{g}_q^{12} \hat{g}_q^{22} + 5.3 \cdot 10^{-1} e^{i23^\circ} \hat{g}_q^{13} \hat{g}_q^{32} + 5.3 \cdot 10^{-1} e^{i23^\circ} \hat{g}_q^{12} \hat{g}_q^{33} \\
 & + 5.1 \cdot 10^{-1} e^{i22^\circ} \hat{g}_q^{13} \hat{g}_u^{33} + 2.7 \cdot 10^{-1} e^{-i68^\circ} \hat{g}_q^{12} \hat{g}_q^{32} + 2.2 \cdot 10^{-1} e^{-i72^\circ} \hat{g}_q^{12} \hat{g}_q^{11} \\
 & + 1.3 \cdot 10^{-1} \hat{g}_q^{22} \hat{g}_q^{22} + 1.3 \cdot 10^{-1} \hat{g}_q^{12} \hat{g}_d^{12} - 1.2 \cdot 10^{-1} e^{i22^\circ} \hat{g}_q^{23} \hat{g}_q^{32} \\
 & - 1.2 \cdot 10^{-1} e^{i22^\circ} \hat{g}_q^{22} \hat{g}_q^{33} - 1.2 \cdot 10^{-1} e^{i22^\circ} \hat{g}_q^{23} \hat{g}_u^{33} + 1.2 \cdot 10^{-1} e^{i44^\circ} \hat{g}_q^{33} \hat{g}_q^{33} \\
 & - 7.8 \cdot 10^{-2} \hat{g}_d^{12} \hat{g}_d^{13} - 7.3 \cdot 10^{-2} \hat{g}_q^{13} \hat{g}_q^{23} - 7.3 \cdot 10^{-2} e^{i23^\circ} \hat{g}_d^{23} \hat{g}_q^{13} - 4.8 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{11} \hat{g}_q^{22} \\
 & - 4.8 \cdot 10^{-2} e^{-i73^\circ} \hat{g}_q^{12} \hat{g}_q^{21} + 4.7 \cdot 10^{-2} e^{-i51^\circ} \hat{g}_q^{13} \hat{g}_q^{31} + 4.4 \cdot 10^{-2} e^{-i51^\circ} \hat{g}_q^{11} \hat{g}_q^{33} \\
 & - 3.6 \cdot 10^{-2} e^{i21^\circ} \hat{g}_d^{11} \hat{g}_q^{13} + 3.5 \cdot 10^{-2} e^{i21^\circ} \hat{g}_d^{33} \hat{g}_q^{13} - 3.0 \cdot 10^{-2} \hat{g}_d^{12} \hat{g}_q^{22} \\
 & + 2.8 \cdot 10^{-2} e^{i21^\circ} \hat{g}_d^{12} \hat{g}_q^{33} + 2.3 \cdot 10^{-2} e^{i23^\circ} \hat{g}_q^{12} \hat{g}_u^{33} - 1.8 \cdot 10^{-2} e^{i34^\circ} \hat{g}_q^{11} \hat{g}_q^{11} \\
 & + 1.7 \cdot 10^{-2} \hat{g}_q^{23} \hat{g}_u^{23} + 1.6 \cdot 10^{-2} e^{i23^\circ} \hat{g}_d^{23} \hat{g}_q^{23} + 1.1 \cdot 10^{-2} e^{-i74^\circ} \hat{g}_d^{12} \hat{g}_q^{11} \\
 & + 7.8 \cdot 10^{-3} e^{i22^\circ} \hat{g}_q^{23} \hat{g}_d^{11} - 7.6 \cdot 10^{-3} \hat{g}_u^{13} \hat{g}_d^{13} - 7.4 \cdot 10^{-3} e^{i22^\circ} \hat{g}_q^{23} \hat{g}_d^{33} \\
 & - 5.5 \cdot 10^{-3} e^{i22^\circ} \hat{g}_u^{11} \hat{g}_d^{13} - 5.4 \cdot 10^{-3} e^{i22^\circ} \hat{g}_u^{22} \hat{g}_d^{13} - 5.3 \cdot 10^{-3} e^{i22^\circ} \hat{g}_d^{22} \hat{g}_u^{33}, \tag{B.5}
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_{G'}^2 \hat{\Sigma}_{G'}^K}{(5 \text{ TeV})^2} = & -7.2 \cdot 10^6 \hat{g}_q^{12} \hat{g}_d^{12} - 1.6 \cdot 10^6 \hat{g}_q^{11} \hat{g}_d^{12} + 1.6 \cdot 10^6 \hat{g}_q^{22} \hat{g}_d^{12} + 3.8 \cdot 10^5 \hat{g}_q^{12} \hat{g}_d^{21} \\
 & + 2.9 \cdot 10^5 \hat{g}_q^{13} \hat{g}_d^{12} - 6.7 \cdot 10^4 \hat{g}_q^{23} \hat{g}_d^{12} - 6.4 \cdot 10^4 e^{i22^\circ} \hat{g}_q^{23} \hat{g}_d^{21} - 1.5 \cdot 10^4 e^{i22^\circ} \hat{g}_q^{13} \hat{g}_d^{21} \\
 & + 6.1 \cdot 10^3 \hat{g}_d^{12} \hat{g}_d^{12} + 5.3 \cdot 10^3 \hat{g}_q^{12} \hat{g}_q^{12} + 2.6 \cdot 10^3 e^{i23^\circ} \hat{g}_q^{33} \hat{g}_d^{12} - 1.2 \cdot 10^3 \hat{g}_q^{12} \hat{g}_q^{22} \\
 & + 1.2 \cdot 10^3 \hat{g}_q^{11} \hat{g}_q^{12} - 3.5 \cdot 10^2 \hat{g}_d^{13} \hat{g}_q^{12} - 2.8 \cdot 10^2 \hat{g}_q^{11} \hat{g}_q^{22} - 2.8 \cdot 10^2 \hat{g}_q^{12} \hat{g}_q^{21} \\
 & + 2.8 \cdot 10^2 \hat{g}_q^{22} \hat{g}_q^{22} + 2.8 \cdot 10^2 \hat{g}_q^{11} \hat{g}_q^{11} - 2.2 \cdot 10^2 \hat{g}_q^{12} \hat{g}_q^{13} - 8.1 \cdot 10^1 \hat{g}_d^{13} \hat{g}_q^{11} \\
 & + 8.0 \cdot 10^1 \hat{g}_d^{13} \hat{g}_q^{22} + 4.8 \cdot 10^1 \hat{g}_q^{12} \hat{g}_q^{23} + 4.8 \cdot 10^1 \hat{g}_q^{13} \hat{g}_q^{22} - 4.8 \cdot 10^1 \hat{g}_q^{11} \hat{g}_q^{13} \\
 & + 4.5 \cdot 10^1 e^{i23^\circ} \hat{g}_q^{12} \hat{g}_q^{32} + 3.8 \cdot 10^1 e^{i23^\circ} \hat{g}_d^{12} \hat{g}_u^{33} + 3.3 \cdot 10^1 e^{i11^\circ} \hat{g}_q^{11} \hat{g}_q^{32} \\
 & + 3.3 \cdot 10^1 e^{i11^\circ} \hat{g}_q^{13} \hat{g}_q^{21} - 2.2 \cdot 10^1 e^{i11^\circ} \hat{g}_q^{22} \hat{g}_q^{23} + 1.9 \cdot 10^1 \hat{g}_d^{31} \hat{g}_q^{12} + 1.4 \cdot 10^1 \hat{g}_d^{13} \hat{g}_q^{13} \\
 & - 1.1 \cdot 10^1 e^{i11^\circ} \hat{g}_q^{12} \hat{g}_q^{31} - 1.1 \cdot 10^1 e^{i11^\circ} \hat{g}_q^{11} \hat{g}_q^{23} + 8.8 \hat{g}_q^{13} \hat{g}_q^{13} - 4.2 \hat{g}_d^{12} \hat{g}_u^{23} \\
 & - 4.2 e^{i22^\circ} \hat{g}_d^{21} \hat{g}_u^{23} + 3.8 e^{i22^\circ} \hat{g}_d^{32} \hat{g}_q^{12} - 3.3 \hat{g}_d^{13} \hat{g}_q^{23} - 3.1 e^{i23^\circ} \hat{g}_d^{31} \hat{g}_q^{23} - 3.2 e^{i87^\circ} \hat{g}_d^{22} \hat{g}_q^{12} \\
 & + 3.2 e^{i87^\circ} \hat{g}_d^{11} \hat{g}_q^{12} + 2.3 e^{i19^\circ} \hat{g}_d^{22} \hat{g}_q^{22} - 2.3 e^{i19^\circ} \hat{g}_d^{11} \hat{g}_q^{22} - 2.0 \hat{g}_q^{13} \hat{g}_q^{23} - 1.8 e^{i23^\circ} \hat{g}_q^{12} \hat{g}_q^{33} \\
 & - 1.8 e^{i23^\circ} \hat{g}_q^{13} \hat{g}_q^{32} - 1.5 e^{i29^\circ} \hat{g}_d^{22} \hat{g}_q^{11} + 1.5 e^{i29^\circ} \hat{g}_d^{11} \hat{g}_q^{11} + 8.7 \cdot 10^{-1} e^{i22^\circ} \hat{g}_d^{23} \hat{g}_q^{11} \\
 & - 8.6 \cdot 10^{-1} e^{i22^\circ} \hat{g}_d^{23} \hat{g}_q^{22} + 8.3 \cdot 10^{-1} e^{i22^\circ} \hat{g}_q^{23} \hat{g}_q^{23} - 7.2 \cdot 10^{-1} e^{i23^\circ} \hat{g}_d^{31} \hat{g}_q^{13} \\
 & + 4.5 \cdot 10^{-1} e^{i23^\circ} \hat{g}_q^{22} \hat{g}_q^{33} - 4.5 \cdot 10^{-1} e^{i23^\circ} \hat{g}_q^{11} \hat{g}_q^{33} + 4.5 \cdot 10^{-1} e^{i23^\circ} \hat{g}_q^{23} \hat{g}_q^{32} \\
 & - 4.5 \cdot 10^{-1} e^{i23^\circ} \hat{g}_q^{13} \hat{g}_q^{31} + 3.9 \cdot 10^{-1} \hat{g}_d^{12} \hat{g}_u^{22} + 3.3 \cdot 10^{-1} e^{i22^\circ} \hat{g}_d^{22} \hat{g}_q^{13} \\
 & - 3.3 \cdot 10^{-1} e^{i22^\circ} \hat{g}_d^{11} \hat{g}_q^{13} + 3.0 \cdot 10^{-1} \hat{g}_d^{12} \hat{g}_d^{13} - 2.0 \cdot 10^{-1} e^{i22^\circ} \hat{g}_d^{23} \hat{g}_q^{12} \\
 & - 1.5 \cdot 10^{-1} e^{i23^\circ} \hat{g}_d^{32} \hat{g}_q^{13} - 1.4 \cdot 10^{-1} e^{i34^\circ} \hat{g}_d^{22} \hat{g}_q^{23} + 1.4 \cdot 10^{-1} e^{i34^\circ} \hat{g}_d^{11} \hat{g}_q^{23} \\
 & + 1.3 \cdot 10^{-1} e^{i24^\circ} \hat{g}_d^{13} \hat{g}_q^{33} - 8.2 \cdot 10^{-2} e^{i23^\circ} \hat{g}_q^{12} \hat{g}_u^{33} + 7.7 \cdot 10^{-2} e^{i23^\circ} \hat{g}_q^{13} \hat{g}_q^{33} \\
 & + 3.6 \cdot 10^{-2} \hat{g}_d^{12} \hat{g}_u^{13} - 3.5 \cdot 10^{-2} e^{i34^\circ} \hat{g}_q^{23} \hat{g}_q^{33} + 3.5 \cdot 10^{-2} e^{i23^\circ} \hat{g}_d^{32} \hat{g}_u^{23} \\
 & + 3.4 \cdot 10^{-2} e^{i44^\circ} \hat{g}_d^{23} \hat{g}_q^{23} + 2.5 \cdot 10^{-2} e^{i22^\circ} \hat{g}_d^{12} \hat{g}_u^{11} + 2.0 \cdot 10^{-2} e^{i23^\circ} \hat{g}_q^{22} \hat{g}_u^{33}
 \end{aligned}$$

$$\begin{aligned}
 & -2.0 \cdot 10^{-2} e^{i23^\circ} \hat{g}_q^{11} \hat{g}_u^{33} + 1.4 \cdot 10^{-2} e^{i23^\circ} \hat{g}_d^{12} \hat{g}_d^{22} + 1.2 \cdot 10^{-2} e^{i22^\circ} \hat{g}_d^{12} \hat{g}_d^{33} \\
 & + 1.1 \cdot 10^{-2} \hat{g}_q^{12} \hat{g}_u^{23} + 1.0 \cdot 10^{-2} e^{i23^\circ} \hat{g}_q^{12} \hat{g}_u^{32} + 7.7 \cdot 10^{-3} e^{i44^\circ} \hat{g}_d^{23} \hat{g}_q^{13} \\
 & - 5.4 \cdot 10^{-3} e^{i22^\circ} \hat{g}_d^{13} \hat{g}_d^{32} - 5.0 \cdot 10^{-3} e^{i11^\circ} \hat{g}_q^{22} \hat{g}_u^{23} + 5.0 \cdot 10^{-3} e^{i11^\circ} \hat{g}_q^{11} \hat{g}_u^{23}.
 \end{aligned} \tag{B.6}$$

B.3 φ

$$\begin{aligned}
 \frac{M_\varphi^2 \hat{\Sigma}_\varphi^{B_s}}{(5 \text{ TeV})^2} = & -3.0 \cdot 10^3 \hat{Y}_d^{32} \hat{Y}_d^{23*} - 6.9 \cdot 10^2 \hat{Y}_d^{31} \hat{Y}_d^{23*} - 1.2 \cdot 10^2 \hat{Y}_d^{32} \hat{Y}_d^{22*} + 1.2 \cdot 10^2 \hat{Y}_d^{33} \hat{Y}_d^{23*} \\
 & - 2.9 \cdot 10^1 \hat{Y}_d^{31} \hat{Y}_d^{22*} - 1.1 \cdot 10^1 e^{-i73^\circ} \hat{Y}_d^{21} \hat{Y}_d^{32*} + 5.0 \hat{Y}_d^{22} \hat{Y}_d^{33*} - 2.4 e^{-i73^\circ} \hat{Y}_d^{31} \hat{Y}_d^{21*} \\
 & + 4.3 \cdot 10^{-1} e^{-i74^\circ} \hat{Y}_d^{21} \hat{Y}_d^{33*} - 1.5 \cdot 10^{-1} \hat{Y}_u^{33} \hat{Y}_u^{23*} + 1.5 \cdot 10^{-1} \hat{Y}_d^{22} \hat{Y}_d^{23*} \\
 & - 1.4 \cdot 10^{-1} \hat{Y}_d^{32} \hat{Y}_d^{33*} + 3.4 \cdot 10^{-2} \hat{Y}_d^{21} \hat{Y}_d^{23*} - 3.4 \cdot 10^{-2} \hat{Y}_d^{31} \hat{Y}_d^{33*} \\
 & - 3.4 \cdot 10^{-2} \hat{Y}_u^{33} \hat{Y}_u^{13*} + 1.9 \cdot 10^{-2} \hat{Y}_u^{32} \hat{Y}_u^{23*} + 8.3 \cdot 10^{-3} e^{-i10^\circ} \hat{Y}_d^{32} \hat{Y}_d^{13*} \\
 & + 7.2 \cdot 10^{-3} \hat{Y}_d^{12} \hat{Y}_d^{33*} - 6.7 \cdot 10^{-3} e^{-i3^\circ} \hat{Y}_d^{33} \hat{Y}_d^{13*} - 6.4 \cdot 10^{-3} \hat{Y}_u^{23} \hat{Y}_u^{23*} \\
 & + 6.1 \cdot 10^{-3} \hat{Y}_d^{22} \hat{Y}_d^{22*} - 6.1 \cdot 10^{-3} \hat{Y}_d^{32} \hat{Y}_d^{32*} - 6.0 \cdot 10^{-3} \hat{Y}_d^{23} \hat{Y}_d^{23*} \\
 & + 6.0 \cdot 10^{-3} \hat{Y}_d^{33} \hat{Y}_d^{33*} + 5.7 \cdot 10^{-3} \hat{Y}_u^{33} \hat{Y}_u^{33*} - 5.6 \cdot 10^{-3} \hat{Y}_d^{32} \hat{Y}_d^{12*},
 \end{aligned} \tag{B.7}$$

$$\begin{aligned}
 \frac{M_\varphi^2 \hat{\Sigma}_\varphi^{B_d}}{(5 \text{ TeV})^2} = & -7.3 \cdot 10^4 \hat{Y}_d^{31} \hat{Y}_d^{13*} + 1.7 \cdot 10^4 \hat{Y}_d^{32} \hat{Y}_d^{13*} - 3.0 \cdot 10^3 \hat{Y}_d^{31} \hat{Y}_d^{12*} + 7.0 \cdot 10^2 \hat{Y}_d^{32} \hat{Y}_d^{12*} \\
 & - 6.6 \cdot 10^2 e^{i22^\circ} \hat{Y}_d^{33} \hat{Y}_d^{13*} - 2.6 \cdot 10^2 e^{-i73^\circ} \hat{Y}_d^{31} \hat{Y}_d^{11*} + 6.0 \cdot 10^1 e^{-i73^\circ} \hat{Y}_d^{11} \hat{Y}_d^{32*} \\
 & - 2.7 \cdot 10^1 e^{i22^\circ} \hat{Y}_d^{12} \hat{Y}_d^{33*} + 3.6 \hat{Y}_d^{21} \hat{Y}_d^{13*} - 2.3 e^{-i51^\circ} \hat{Y}_d^{11} \hat{Y}_d^{33*} - 8.2 \cdot 10^{-1} \hat{Y}_d^{22} \hat{Y}_d^{13*} \\
 & + 7.2 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_u^{33} \hat{Y}_u^{13*} - 1.7 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_u^{33} \hat{Y}_u^{23*} + 1.5 \cdot 10^{-1} \hat{Y}_d^{21} \hat{Y}_d^{12*} \\
 & - 1.0 \cdot 10^{-1} \hat{Y}_u^{32} \hat{Y}_u^{13*} - 8.3 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_d^{31} \hat{Y}_d^{23*} - 4.0 \cdot 10^{-2} e^{i21^\circ} \hat{Y}_d^{11} \hat{Y}_d^{13*} \\
 & + 4.0 \cdot 10^{-2} e^{i21^\circ} \hat{Y}_d^{31} \hat{Y}_d^{33*} - 3.4 \cdot 10^{-2} \hat{Y}_d^{22} \hat{Y}_d^{12*} + 3.2 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_u^{23} \hat{Y}_u^{13*} \\
 & + 3.2 \cdot 10^{-2} e^{i21^\circ} \hat{Y}_d^{23} \hat{Y}_d^{13*} + 2.3 \cdot 10^{-2} \hat{Y}_u^{32} \hat{Y}_u^{23*} + 1.9 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_d^{32} \hat{Y}_d^{23*} \\
 & + 1.3 \cdot 10^{-2} e^{-i74^\circ} \hat{Y}_d^{21} \hat{Y}_d^{11*} + 9.0 \cdot 10^{-3} e^{i22^\circ} \hat{Y}_d^{12} \hat{Y}_d^{13*} - 8.5 \cdot 10^{-3} e^{i22^\circ} \hat{Y}_d^{32} \hat{Y}_d^{33*} \\
 & - 7.4 \cdot 10^{-3} e^{i22^\circ} \hat{Y}_u^{23} \hat{Y}_u^{23*} + 6.3 \cdot 10^{-3} e^{i44^\circ} \hat{Y}_u^{33} \hat{Y}_u^{33*},
 \end{aligned} \tag{B.8}$$

$$\begin{aligned}
 \frac{M_\varphi^2 \hat{\Sigma}_\varphi^K}{(5 \text{ TeV})^2} = & -8.0 \cdot 10^6 \hat{Y}_d^{21} \hat{Y}_d^{12*} - 1.8 \cdot 10^6 \hat{Y}_d^{21} \hat{Y}_d^{11*} + 1.8 \cdot 10^6 \hat{Y}_d^{22} \hat{Y}_d^{12*} + 4.2 \cdot 10^5 \hat{Y}_d^{11} \hat{Y}_d^{22*} \\
 & + 3.3 \cdot 10^5 \hat{Y}_d^{21} \hat{Y}_d^{13*} - 7.5 \cdot 10^4 \hat{Y}_d^{22} \hat{Y}_d^{13*} - 7.1 \cdot 10^4 e^{i22^\circ} \hat{Y}_d^{12} \hat{Y}_d^{23*} \\
 & - 1.6 \cdot 10^4 e^{i22^\circ} \hat{Y}_d^{11} \hat{Y}_d^{23*} + 2.9 \cdot 10^3 e^{i23^\circ} \hat{Y}_d^{23} \hat{Y}_d^{13*} - 3.9 \cdot 10^2 \hat{Y}_d^{31} \hat{Y}_d^{12*} \\
 & - 9.0 \cdot 10^1 \hat{Y}_d^{31} \hat{Y}_d^{11*} + 9.0 \cdot 10^1 \hat{Y}_d^{32} \hat{Y}_d^{12*} + 2.1 \cdot 10^1 \hat{Y}_d^{11} \hat{Y}_d^{32*} + 1.6 \cdot 10^1 \hat{Y}_d^{31} \hat{Y}_d^{13*} \\
 & + 4.2 e^{i22^\circ} \hat{Y}_d^{21} \hat{Y}_d^{32*} - 3.7 \hat{Y}_d^{32} \hat{Y}_d^{13*} - 3.5 e^{i23^\circ} \hat{Y}_d^{12} \hat{Y}_d^{33*} + 3.3 e^{i83^\circ} \hat{Y}_d^{11} \hat{Y}_d^{12*} \\
 & - 3.3 e^{i83^\circ} \hat{Y}_d^{21} \hat{Y}_d^{22*} + 2.5 e^{i19^\circ} \hat{Y}_d^{22} \hat{Y}_d^{22*} - 2.5 e^{i19^\circ} \hat{Y}_d^{12} \hat{Y}_d^{12*} - 1.7 e^{i29^\circ} \hat{Y}_d^{21} \hat{Y}_d^{21*} \\
 & + 1.7 e^{i29^\circ} \hat{Y}_d^{11} \hat{Y}_d^{11*} + 9.7 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_d^{31} \hat{Y}_d^{21*} - 9.6 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_d^{32} \hat{Y}_d^{22*} \\
 & - 8.0 \cdot 10^{-1} e^{i23^\circ} \hat{Y}_d^{11} \hat{Y}_d^{33*} + 3.6 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_d^{21} \hat{Y}_d^{23*} - 3.6 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_d^{11} \hat{Y}_d^{13*} \\
 & - 2.2 \cdot 10^{-1} e^{i22^\circ} \hat{Y}_d^{31} \hat{Y}_d^{22*} - 1.7 \cdot 10^{-1} e^{i23^\circ} \hat{Y}_d^{21} \hat{Y}_d^{33*} - 1.6 \cdot 10^{-1} e^{i34^\circ} \hat{Y}_d^{22} \hat{Y}_d^{23*} \\
 & + 1.6 \cdot 10^{-1} e^{i34^\circ} \hat{Y}_d^{12} \hat{Y}_d^{13*} + 1.4 \cdot 10^{-1} e^{i24^\circ} \hat{Y}_d^{33} \hat{Y}_d^{13*} - 1.2 \cdot 10^{-1} e^{i23^\circ} \hat{Y}_u^{23} \hat{Y}_u^{13*} \\
 & + 3.9 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_d^{22} \hat{Y}_d^{33*} + 3.7 \cdot 10^{-2} e^{i44^\circ} \hat{Y}_d^{32} \hat{Y}_d^{23*} - 2.8 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_u^{13} \hat{Y}_u^{13*}
 \end{aligned}$$

$$\begin{aligned}
 &+ 2.8 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_u^{23} \hat{Y}_u^{23*} + 1.5 \cdot 10^{-2} \hat{Y}_u^{22} \hat{Y}_u^{13*} + 1.4 \cdot 10^{-2} e^{i23^\circ} \hat{Y}_u^{23} \hat{Y}_u^{12*} \\
 &+ 8.6 \cdot 10^{-3} e^{i44^\circ} \hat{Y}_d^{31} \hat{Y}_d^{23*} + 7.0 \cdot 10^{-3} e^{i11^\circ} \hat{Y}_u^{12} \hat{Y}_u^{13*} - 7.0 \cdot 10^{-3} e^{i11^\circ} \hat{Y}_u^{22} \hat{Y}_u^{23*}. \quad (\text{B.9})
 \end{aligned}$$

B.4 Φ^A

$$\begin{aligned}
 \frac{M_\Phi^2 \hat{\Sigma}_\Phi^{B_s}}{(5 \text{ TeV})^2} = & -6.6 \cdot 10^1 \hat{X}_d^{32} \hat{X}_d^{23*} - 1.5 \cdot 10^1 \hat{X}_d^{31} \hat{X}_d^{23*} - 2.7 \hat{X}_d^{32} \hat{X}_d^{22*} + 2.7 \hat{X}_d^{33} \hat{X}_d^{23*} \\
 & - 6.3 \cdot 10^{-1} \hat{X}_d^{31} \hat{X}_d^{22*} - 2.3 \cdot 10^{-1} e^{-i73^\circ} \hat{X}_d^{21} \hat{X}_d^{32*} + 1.1 \cdot 10^{-1} \hat{X}_d^{22} \hat{X}_d^{33*} \\
 & - 5.4 \cdot 10^{-2} e^{-i72^\circ} \hat{X}_d^{31} \hat{X}_d^{21*} - 4.0 \cdot 10^{-2} \hat{X}_u^{33} \hat{X}_u^{23*} + 9.5 \cdot 10^{-3} e^{-i74^\circ} \hat{X}_d^{21} \hat{X}_d^{33*} \\
 & - 9.1 \cdot 10^{-3} \hat{X}_u^{33} \hat{X}_u^{13*}, \quad (\text{B.10})
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_\Phi^2 \hat{\Sigma}_\Phi^{B_d}}{(5 \text{ TeV})^2} = & -1.6 \cdot 10^3 \hat{X}_d^{31} \hat{X}_d^{13*} + 3.8 \cdot 10^2 \hat{X}_d^{32} \hat{X}_d^{13*} - 6.8 \cdot 10^1 \hat{X}_d^{31} \hat{X}_d^{12*} + 1.6 \cdot 10^1 \hat{X}_d^{32} \hat{X}_d^{12*} \\
 & - 1.5 \cdot 10^1 e^{i22^\circ} \hat{X}_d^{33} \hat{X}_d^{13*} - 5.8 e^{-i73^\circ} \hat{X}_d^{31} \hat{X}_d^{11*} + 1.3 e^{-i73^\circ} \hat{X}_d^{11} \hat{X}_d^{32*} \\
 & - 6.0 \cdot 10^{-1} e^{i22^\circ} \hat{X}_d^{12} \hat{X}_d^{33*} + 2.0 \cdot 10^{-1} e^{i22^\circ} \hat{X}_u^{33} \hat{X}_u^{13*} + 8.0 \cdot 10^{-2} \hat{X}_d^{21} \hat{X}_d^{13*} \\
 & - 5.2 \cdot 10^{-2} e^{-i51^\circ} \hat{X}_d^{11} \hat{X}_d^{33*} - 4.7 \cdot 10^{-2} e^{i22^\circ} \hat{X}_u^{33} \hat{X}_u^{23*} - 2.5 \cdot 10^{-2} \hat{X}_u^{32} \hat{X}_u^{13*} \\
 & - 1.8 \cdot 10^{-2} \hat{X}_d^{22} \hat{X}_d^{13*} + 8.4 \cdot 10^{-3} e^{i23^\circ} \hat{X}_u^{23} \hat{X}_u^{13*} + 5.8 \cdot 10^{-3} \hat{X}_u^{32} \hat{X}_u^{23*}, \quad (\text{B.11})
 \end{aligned}$$

$$\begin{aligned}
 \frac{M_\Phi^2 \hat{\Sigma}_\Phi^K}{(5 \text{ TeV})^2} = & 7.9 \cdot 10^4 \hat{X}_d^{21} \hat{X}_d^{12*} + 1.8 \cdot 10^4 \hat{X}_d^{21} \hat{X}_d^{11*} - 1.8 \cdot 10^4 \hat{X}_d^{22} \hat{X}_d^{12*} \\
 & - 4.2 \cdot 10^3 \hat{X}_d^{11} \hat{X}_d^{22*} - 3.2 \cdot 10^3 \hat{X}_d^{21} \hat{X}_d^{13*} + 7.4 \cdot 10^2 \hat{X}_d^{22} \hat{X}_d^{13*} \\
 & + 7.1 \cdot 10^2 e^{i22^\circ} \hat{X}_d^{12} \hat{X}_d^{23*} + 1.6 \cdot 10^2 e^{i22^\circ} \hat{X}_d^{11} \hat{X}_d^{23*} - 2.9 \cdot 10^1 e^{i23^\circ} \hat{X}_d^{23} \hat{X}_d^{13*} \\
 & + 3.9 \hat{X}_d^{31} \hat{X}_d^{12*} + 8.9 \cdot 10^{-1} \hat{X}_d^{31} \hat{X}_d^{11*} - 8.9 \cdot 10^{-1} \hat{X}_d^{32} \hat{X}_d^{12*} \\
 & - 2.7 \cdot 10^{-1} e^{-i7^\circ} \hat{X}_d^{21} \hat{X}_d^{22*} + 2.7 \cdot 10^{-1} e^{-i7^\circ} \hat{X}_d^{11} \hat{X}_d^{12*} - 2.1 \cdot 10^{-1} \hat{X}_d^{11} \hat{X}_d^{32*} \\
 & - 1.6 \cdot 10^{-1} \hat{X}_d^{31} \hat{X}_d^{13*} - 4.2 \cdot 10^{-2} e^{i22^\circ} \hat{X}_d^{21} \hat{X}_d^{32*} - 4.0 \cdot 10^{-2} e^{i11^\circ} \hat{X}_d^{22} \hat{X}_d^{22*} \\
 & + 4.0 \cdot 10^{-2} e^{i11^\circ} \hat{X}_d^{12} \hat{X}_d^{12*} + 3.6 \cdot 10^{-2} \hat{X}_d^{32} \hat{X}_d^{13*} + 3.5 \cdot 10^{-2} e^{i23^\circ} \hat{X}_d^{12} \hat{X}_d^{33*} \\
 & - 3.0 \cdot 10^{-2} e^{i23^\circ} \hat{X}_u^{23} \hat{X}_u^{13*} - 9.6 \cdot 10^{-3} e^{i22^\circ} \hat{X}_d^{31} \hat{X}_d^{21*} + 9.6 \cdot 10^{-3} e^{i22^\circ} \hat{X}_d^{32} \hat{X}_d^{22*} \\
 & - 8.1 \cdot 10^{-3} e^{-i80^\circ} \hat{X}_d^{21} \hat{X}_d^{21*} + 8.1 \cdot 10^{-3} e^{-i80^\circ} \hat{X}_d^{11} \hat{X}_d^{11*} + 8.0 \cdot 10^{-3} e^{i23^\circ} \hat{X}_d^{11} \hat{X}_d^{33*} \\
 & - 7.3 \cdot 10^{-3} e^{i23^\circ} \hat{X}_u^{13} \hat{X}_u^{13*} + 7.3 \cdot 10^{-3} e^{i23^\circ} \hat{X}_u^{23} \hat{X}_u^{23*}. \quad (\text{B.12})
 \end{aligned}$$

C LEFT bases changes

The transformation of the Wilson coefficients between the various LEFT bases read:

SUSY $\rightarrow$ BMU:

$$C_{\text{VLL}} = C_1, \quad C_{\text{LR},1} = -\frac{1}{2} C_5, \quad C_{\text{LR},2} = C_4, \quad (\text{C.1})$$

JMS $\rightarrow$ BMU:

$$C_{\text{VLL}} = C_{dd}^{V,LL}, \quad C_{\text{LR},1} = C_{dd}^{V1,LR} - \frac{1}{6} C_{dd}^{V8,LR}, \quad C_{\text{LR},2} = -C_{dd}^{V8,LR}, \quad (\text{C.2})$$

BMU $\rightarrow$ JMS:

$$C_{dd}^{V,LL} = C_{\text{VLL}}, \quad C_{dd}^{V1,LR} = C_{\text{LR},1} - \frac{1}{6} C_{\text{LR},2}, \quad C_{dd}^{V8,LR} = -C_{\text{LR},2}, \quad (\text{C.3})$$

SUSY $\rightarrow$ JMS:

$$C_{dd}^{V,LL} = C_1, \quad C_{dd}^{V1,LR} = \frac{1}{6}C_4 + \frac{1}{2}C_5, \quad C_{dd}^{V8,LR} = C_4, \quad (\text{C.4})$$

JMS $\rightarrow$ SUSY:

$$C_1 = C_{dd}^{V,LL}, \quad C_4 = C_{dd}^{V1,LR}, \quad C_5 = 2C_{dd}^{V1,LR} - \frac{1}{3}C_{dd}^{V8,LR}, \quad (\text{C.5})$$

where we have omitted flavour indices $[C_a]_{ijij}$ on all Wilson coefficients.

Here we report the basis transformation from the JMS basis to the SUSY operator basis introduced in [13]. The basis change can be written in a compact form as:

$$\begin{aligned} Q_1^{ij} &= [Q_{aa}^{V,LL}]_{ijij}, & Q_5^{ij} &= -\frac{1}{2}[Q_{aa}^{V1,LR}]_{ijij}, \\ Q_2^{ij} &= [Q_{aa}^{S1,RR\dagger}]_{jiji}, & Q_{1'}^{ij} &= [Q_{aa}^{V,RR}]_{ijij}, \\ Q_3^{ij} &= \frac{1}{3}[Q_{aa}^{S1,RR\dagger}]_{jiji} + 2[Q_{aa}^{S8,RR\dagger}]_{jiji}, & Q_{2'}^{ij} &= [Q_{aa}^{S1,RR}]_{ijij}, \\ Q_4^{ij} &= -\frac{1}{6}[Q_{aa}^{V1,LR}]_{ijij} - [Q_{aa}^{V8,LR}]_{ijij}, & Q_{3'}^{ij} &= \frac{1}{3}[Q_{aa}^{S1,RR}]_{ijij} + 2[Q_{aa}^{S8,RR}]_{ijij}, \end{aligned} \quad (\text{C.6})$$

where we have for the down- ($aa = dd$) and up-sector ($aa = uu$), respectively.

D SMEFT one-loop matching

We report the matching of the SMEFT Wilson coefficient onto LEFT (JMS basis) derived in ref. [14]. The SMEFT Wilson coefficients are given in the down-basis as opposed to [14]. Furthermore, the matching is performed in a redundant basis containing operators which are related to each other. For our purpose we adopt the non-redundant basis defined in [53] and find for $K^0 - \bar{K}^0$ and $B_{s,d} - \bar{B}_{s,d}$ down-type meson mixing in **the down-basis**

$$\begin{aligned} [C_{dd}^{V,LL}]_{ijij} &= -\frac{\alpha}{\pi s_W^2} \left([\mathcal{C}_{qq}^{(1)}]_{ijij} + [\mathcal{C}_{qq}^{(3)}]_{ijij} \right) I_3(m_W, m_Z, \mu_{\text{ew}}) \\ &\quad - \frac{2\alpha\lambda_t^{im}\lambda_t^{nj}}{\pi s_W^2} \left([\mathcal{C}_{qq}^{(1)}]_{ijmn} + [\mathcal{C}_{qq}^{(1)}]_{mni j} - [\mathcal{C}_{qq}^{(3)}]_{ijmn} - [\mathcal{C}_{qq}^{(3)}]_{mni j} \right. \\ &\quad \left. + 2[\mathcal{C}_{qq}^{(3)}]_{inmj} + 2[\mathcal{C}_{qq}^{(3)}]_{mj in} \right) J(x_t) \\ &\quad - \frac{\alpha}{\pi s_W^2} \left[\lambda_t^{im} \left([\mathcal{C}_{qq}^{(1)}]_{mji j} + [\mathcal{C}_{qq}^{(1)}]_{ijmj} + [\mathcal{C}_{qq}^{(3)}]_{mji j} + [\mathcal{C}_{qq}^{(3)}]_{ijmj} \right) \right. \\ &\quad \left. + \lambda_t^{mj} \left([\mathcal{C}_{qq}^{(1)}]_{imij} + [\mathcal{C}_{qq}^{(1)}]_{ijim} + [\mathcal{C}_{qq}^{(3)}]_{imij} + [\mathcal{C}_{qq}^{(3)}]_{ijim} \right) \right] K(x_t, \mu_{\text{ew}}) \\ &\quad + \frac{\alpha\lambda_t^{ij}}{\pi s_W^2} [\mathcal{C}_{\phi q}^{(1)}]_{ij} I_1(x_t, \mu_{\text{ew}}) - \frac{\alpha\lambda_t^{ij}}{\pi s_W^2} [\mathcal{C}_{\phi q}^{(3)}]_{ij} I_2(x_t, \mu_{\text{ew}}) \\ &\quad + \frac{\alpha\lambda_t^{ij}}{4\pi s_W^2} \left(\lambda_t^{im} [\mathcal{C}_{\phi q}^{(3)}]_{mj} + \lambda_t^{mj} [\mathcal{C}_{\phi q}^{(3)}]_{im} \right) S_0(x_t) \end{aligned}$$

$$\begin{aligned}
 & -\frac{\alpha\lambda_t^{ij}}{\pi s_W^2} \left([\mathcal{C}_{qu}^{(1)}]_{ij33} + \frac{N_c-1}{2N_c} [\mathcal{C}_{qu}^{(8)}]_{ij33} \right) I_1(x_t, \mu_{ew}) \\
 & + \frac{\alpha\lambda_t^{ij}}{\pi s_W^2} \left([\mathcal{C}_{uW}]_{i3} V_{3j} + [\mathcal{C}_{uW}]_{j3}^* V_{3i}^* \right) I_4(x_t, \mu_{ew}), \tag{D.1}
 \end{aligned}$$

$$\begin{aligned}
 [C_{dd}^{V1,LR}]_{ijij} &= -\frac{\alpha}{\pi s_W^2} [\mathcal{C}_{qd}^{(1)}]_{ijij} I_5(m_W, m_Z, \mu_{ew}) + \frac{\alpha\lambda_t^{ij}}{\pi s_W^2} [\mathcal{C}_{\phi d}]_{ij} I_1(x_t, \mu_{ew}) \\
 & -\frac{\alpha\lambda_t^{ij}}{\pi s_W^2} [\mathcal{C}_{ud}^{(1)}]_{33ij} I_1(x_t, \mu_{ew}) - \frac{\alpha}{\pi s_W^2} \left(\lambda_t^{im} [\mathcal{C}_{qd}^{(1)}]_{mjj} + \lambda_t^{mj} [\mathcal{C}_{qd}^{(1)}]_{imij} \right) K(x_t, \mu_{ew}) \\
 & -\frac{2\alpha\lambda_t^{ij}\lambda_t^{mn}}{\pi s_W^2} [\mathcal{C}_{qd}^{(1)}]_{nmij} J(x_t), \tag{D.2}
 \end{aligned}$$

$$\begin{aligned}
 [C_{dd}^{V8,LR}]_{ijij} &= -\frac{\alpha}{\pi s_W^2} [\mathcal{C}_{qd}^{(8)}]_{ijij} I_5(m_W, m_Z, \mu_{ew}) - \frac{2\alpha\lambda_t^{ij}\lambda_t^{mn}}{\pi s_W^2} [\mathcal{C}_{qd}^{(8)}]_{nmij} J(x_t) \\
 & -\frac{\alpha\lambda_t^{ij}}{\pi s_W^2} [\mathcal{C}_{ud}^{(8)}]_{33ij} I_1(x_t, \mu_{ew}) - \frac{\alpha}{\pi s_W^2} \left(\lambda_t^{im} [\mathcal{C}_{qd}^{(8)}]_{mjj} + \lambda_t^{mj} [\mathcal{C}_{qd}^{(8)}]_{imij} \right) K(x_t, \mu_{ew}), \tag{D.3}
 \end{aligned}$$

$$[C_{dd}^{V,RR}]_{ijij} = -\frac{\alpha}{\pi s_W^2} [\mathcal{C}_{dd}]_{ijij} I_6(m_W, m_Z, \mu_{ew}), \tag{D.4}$$

where we use $x_t = m_t^2/m_W^2$ and the loop-functions:

$$K(x, \mu_{ew}) = -\frac{3x(1+x)}{64(x-1)} + \frac{x(4-2x+x^2)}{32(x-1)^2} \ln x - \frac{x}{16} \ln \frac{\mu_{ew}}{m_W}, \tag{D.5}$$

$$I_1(x, \mu_{ew}) = -\frac{x(x-7)}{32(x-1)} - \frac{x(x^2-2x+4)}{16(x-1)^2} \ln x + \frac{x}{8} \ln \frac{\mu_{ew}}{m_W}, \tag{D.6}$$

$$I_2(x, \mu_{ew}) = \frac{7x^2-25x}{32(x-1)} - \frac{x(x^2-14x+4)}{16(x-1)^2} \ln x + \frac{x}{8} \ln \frac{\mu_{ew}}{m_W}, \tag{D.7}$$

$$I_4(x) = \frac{3x^{3/2}(x+1)}{8\sqrt{2}(x-1)^2} - \frac{3\sqrt{x}x^2}{4\sqrt{2}(x-1)^3} \ln x, \tag{D.8}$$

$$J(x) = \frac{x}{16}, \tag{D.9}$$

$$S_0(x) = \frac{x(x^2-11x+4)}{4(x-1)^2} + \frac{3x^3}{2(x-1)^3} \ln x, \tag{D.10}$$

$$I_3(m_W, m_Z, \mu_{ew}) = -\frac{11(2m_W^2+m_Z^2)^2}{144m_W^2m_Z^2} - \frac{(2m_W^2+m_Z^2)^2}{12m_W^2m_Z^2} \ln \frac{\mu_{ew}}{m_Z}, \tag{D.11}$$

$$I_5(m_W, m_Z, \mu_{ew}) = \frac{-2m_W^4+m_W^2m_Z^2+m_Z^4}{72m_W^2m_Z^2} + \frac{-m_Z^4-m_W^2m_Z^2+2m_W^4}{6m_W^2m_Z^2} \ln \frac{\mu_{ew}}{m_Z}, \tag{D.12}$$

$$I_6(m_W, m_Z, \mu_{ew}) = -\frac{11(m_W^2-m_Z^2)^2}{36m_W^2m_Z^2} - \frac{(m_W^2-m_Z^2)^2}{3m_W^2m_Z^2} \ln \frac{\mu_{ew}}{m_Z}. \tag{D.13}$$

The remaining Wilson coefficients $C_{dd}^{S1,RR}$ and $C_{dd}^{S8,RR}$ do not get a matching contribution neither at tree-level nor at one-loop.

For the up-sector one finds at one-loop for $D^0 - \bar{D}^0$ up-type meson mixing in **the up-basis**:

$$[C_{uu}^{V,LL}]_{ijij} = -\frac{\alpha}{\pi s_W^2} \left([\hat{\mathcal{C}}_{qq}^{(1)}]_{ijij} + [\hat{\mathcal{C}}_{qq}^{(3)}]_{ijij} \right) I_7(m_W, m_Z, \mu_{\text{ew}}), \quad (\text{D.14})$$

$$[C_{uu}^{V1,LR}]_{ijij} = -\frac{\alpha}{\pi s_W^2} [\hat{\mathcal{C}}_{qu}^{(1)}]_{ijij} I_8(m_W, m_Z, \mu_{\text{ew}}), \quad (\text{D.15})$$

$$[C_{uu}^{V8,LR}]_{ijij} = -\frac{\alpha}{\pi s_W^2} [\hat{\mathcal{C}}_{qu}^{(8)}]_{ijij} I_8(m_W, m_Z, \mu_{\text{ew}}), \quad (\text{D.16})$$

$$[C_{uu}^{V,RR}]_{ijij} = -\frac{4\alpha}{\pi s_W^2} [\hat{\mathcal{C}}_{uu}]_{ijij} I_6(m_W, m_Z, \mu_{\text{ew}}), \quad (\text{D.17})$$

with the loop functions

$$I_7(m_W, m_Z, \mu_{\text{ew}}) = -\frac{11(m_Z^2 - 4m_W^2)^2}{144m_W^2 m_Z^2} - \frac{(m_Z^2 - 4m_W^2)^2}{12m_W^2 m_Z^2} \ln \frac{\mu_{\text{ew}}}{m_Z}, \quad (\text{D.18})$$

$$I_8(m_W, m_Z, \mu_{\text{ew}}) = \frac{(4m_W^4 - 5m_W^2 m_Z^2 + m_Z^4)}{36m_W^2 m_Z^2} \left(-1 + 12 \ln \frac{\mu_{\text{ew}}}{m_Z} \right). \quad (\text{D.19})$$

E The issue of evanescent operators

The one-loop matching conditions discussed in our paper constitute a part of an NLO calculation. It is well known that in the process of NLO calculations in the NDR- $\overline{\text{MS}}$ scheme, where ultraviolet divergences are regulated dimensionally, the so-called evanescent operators that vanish in $D = 4$ dimensions have to be considered [32, 39]. They arise in particular when complicated Dirac structures are projected onto the chosen basis of physical operators and can also arise when different operator bases are related by performing usual Fierz transformations that generally are not valid in $D \neq 4$ dimensions.

The treatment of these operators in the process of one-loop matching must be consistent with the one used in the calculation of two-loop anomalous dimensions. In performing the NLO QCD evolutions we have used the $P_a^{ij}(\mu_{\text{ew}})$ from [28], see (2.16), which are based on the two-loop anomalous dimensions of operators calculated in [32]. Therefore it is mandatory for us to treat evanescent operators appearing in the one-loop matching in the same manner as done in [32]. Now, the latter paper used the treatment of evanescent operators as proposed in the context of the formulation of the NDR- $\overline{\text{MS}}$ scheme introduced in [39]. The virtue of this treatment is that the evanescent operators defined in this scheme influence only two-loop anomalous dimensions. Indeed

- By definition they do not contribute to the matching and to the finite corrections to the matrix elements of renormalized physical operators. They are simply subtracted away in the process of renormalization. This issue is summarized in section 6.9.4 of [100], where further references can be found. Important papers in this context are also [40, 41].
- Similarly using the projections of products of gamma matrices consistent with [39] also the usual Fierz transformations are not affected by the so-called Fierz-vanishing evanescent operators as long as the infrared divergences are not regulated dimensionally [32]. Moreover, this issue is absent in the operators considered by us.

Even if these issues have been discussed in the literature it is instructive to demonstrate in our case that indeed the usual Fierz identities can be used to relate different bases by simply calculating the matrix elements of the involved operators and checking that the $D = 4$ relations between different bases do not receive any $\mathcal{O}(\alpha_s)$ corrections.

In the case of the relation between the operators $Q_{\text{LR},1}$ and Q_5 such a test can be performed by calculating the matrix elements of these operators inserting them in the current-current topologies. Master formulae for these one-loop matrix elements can be found in (6.64)–(6.69) of [100]. There, these formulae have been used for the calculation of one-loop anomalous dimensions. Strictly speaking, when finite parts of one-loop diagrams are considered these formulae should include a universal factor $(1 + 2\epsilon)$ that results from the calculation of the integrals. We will keep this factor but it is irrelevant for our test.

According to the procedure outlined above, in doing the reduction of products of γ_μ matrices one can omit the usual evanescent operators that are relevant for two-loop anomalous dimensions but are chosen not to contribute to one-loop matchings. They are simply subtracted in the process of renormalization [39]. Consequently the reductions to be used are as follows. In the case of the $Q_{\text{LR},1}$ operator we have

$$\gamma_\mu \gamma_\rho \gamma_\tau P_L \gamma^\rho \gamma^\mu \otimes \gamma^\tau P_R = 4(1 - 2\epsilon) \gamma_\tau P_L \otimes \gamma^\tau P_R, \quad (\text{E.1})$$

$$\gamma_\tau P_L \gamma_\rho \gamma_\mu \otimes \gamma^\tau P_R \gamma^\rho \gamma^\mu = 4(1 + \epsilon) \gamma_\tau P_L \otimes \gamma^\tau P_R, \quad (\text{E.2})$$

$$\gamma_\tau P_L \gamma_\rho \gamma_\mu \otimes \gamma^\mu \gamma^\rho \gamma^\tau P_R = 16(1 - \epsilon) \gamma_\tau P_L \otimes \gamma^\tau P_R. \quad (\text{E.3})$$

In the case Q_5 we have

$$\gamma_\mu \gamma_\rho P_L \gamma^\rho \gamma^\mu \otimes P_R = 16(1 - \epsilon) P_L \otimes P_R, \quad (\text{E.4})$$

$$P_L \gamma_\rho \gamma_\mu \otimes P_R \gamma^\rho \gamma^\mu = 4(1 + \epsilon) P_L \otimes P_R, \quad (\text{E.5})$$

$$P_L \gamma_\rho \gamma_\mu \otimes \gamma^\mu \gamma^\rho P_R = 4(1 - 2\epsilon) P_L \otimes P_R. \quad (\text{E.6})$$

We find then the following $\mathcal{O}(\alpha_s)$ contributions to the matrix elements in question:

$$\langle Q_{\text{LR},1} \rangle = \frac{1}{\epsilon} \frac{\alpha_s}{16\pi} (1 + 2\epsilon) \left[\frac{(20 - 44\epsilon)}{3} \langle Q_{\text{LR},1} \rangle_0 + (12 - 20\epsilon) \langle \tilde{Q}_{\text{LR},1} \rangle_0 \right], \quad (\text{E.7})$$

$$\langle Q_5 \rangle = \frac{1}{\epsilon} \frac{\alpha_s}{16\pi} (1 + 2\epsilon) \left[\frac{(20 - 44\epsilon)}{3} \langle Q_5 \rangle_0 + (12 - 20\epsilon) \langle Q_4 \rangle_0 \right], \quad (\text{E.8})$$

where

$$\tilde{Q}_{\text{LR},1}^{ij} = (\bar{d}_i^\alpha \gamma_\mu P_L d_j^\beta) (\bar{d}_i^\beta \gamma^\mu P_R d_j^\alpha), \quad (\text{E.9})$$

and "0" indicates that these are tree-level matrix elements. We can use for the latter $D = 4$ Fierz relations

$$Q_{\text{LR},1} = -2Q_5, \quad \tilde{Q}_{\text{LR},1} = -2Q_4. \quad (\text{E.10})$$

Using these results we indeed find that the $D = 4$ Fierz relation between $Q_{\text{LR},1}$ and Q_5

$$Q_{\text{LR},1} = -2Q_5 + \mathcal{O}(\alpha_s^2) \quad (\text{E.11})$$

is satisfied.

For the transformation from the BMU to the JMS basis we still need one-loop matrix elements of $\tilde{Q}_{\text{LR},1}$ and $Q_{\text{LR},2}$. The reason is that

$$[Q_{dd}^{V8,LR}]_{ijij} = [\bar{d}_i \gamma_\mu P_L T^A d_j][\bar{d}_i \gamma^\mu P_R T^A d_j] = \frac{1}{2} \left(\tilde{Q}_{\text{LR},1}^{ij} - \frac{1}{3} Q_{\text{LR},1}^{ij} \right) \quad (\text{E.12})$$

and $\tilde{Q}_{\text{LR},1}$ do not belong to the BMU basis. We find

$$\langle \tilde{Q}_{\text{LR},1} \rangle = \frac{1}{\epsilon} \frac{\alpha_s}{16\pi} (1 + 2\epsilon) \left[\frac{4}{3} (32 - 29\epsilon) \langle \tilde{Q}_{\text{LR},1} \rangle_0 - 12\epsilon \langle Q_{\text{LR},1} \rangle_0 \right], \quad (\text{E.13})$$

$$\langle Q_{\text{LR},2} \rangle = \frac{1}{\epsilon} \frac{\alpha_s}{16\pi} (1 + 2\epsilon) \left[\frac{4}{3} (32 - 29\epsilon) \langle Q_{\text{LR},2} \rangle_0 - 12\epsilon \langle Q_5 \rangle_0 \right]. \quad (\text{E.14})$$

Using these results and the Fierz identities for tree-level matrix elements in (E.10) we indeed find

$$\tilde{Q}_{\text{LR},1} = -2 Q_{\text{LR},2} + \mathcal{O}(\alpha_s^2), \quad (\text{E.15})$$

which implies the relation between the JMS and the BMU basis given in the text.

Our results for the matrix elements of $Q_{\text{LR},1}$ and $Q_{\text{LR},2} = Q_4$ confirm the ones obtained in [97] but to test the Fierz relations we had to calculate the ones of Q_5 and $\tilde{Q}_{\text{LR},1}$ as well.

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References

- [1] A.J. Buras and J. Girrbach, *Towards the identification of new physics through quark flavour violating processes*, *Rept. Prog. Phys.* **77** (2014) 086201 [[arXiv:1306.3775](https://arxiv.org/abs/1306.3775)] [[INSPIRE](#)].
- [2] A.J. Buras, *Gauge theory of weak decays*, Cambridge University Press, Cambridge, U.K. (2020).
- [3] M.K. Gaillard and B.W. Lee, *Rare decay modes of the K-mesons in gauge theories*, *Phys. Rev. D* **10** (1974) 897 [[INSPIRE](#)].
- [4] Z. Bai, N.H. Christ, T. Izubuchi, C.T. Sachrajda, A. Soni and J. Yu, *K_L - K_S mass difference from lattice QCD*, *Phys. Rev. Lett.* **113** (2014) 112003 [[arXiv:1406.0916](https://arxiv.org/abs/1406.0916)] [[INSPIRE](#)].
- [5] N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, *Effects of finite volume on the K_L - K_S mass difference*, *Phys. Rev. D* **91** (2015) 114510 [[arXiv:1504.01170](https://arxiv.org/abs/1504.01170)] [[INSPIRE](#)].
- [6] Z. Bai, N.H. Christ and C.T. Sachrajda, *The K_L - K_S mass difference*, *EPJ Web Conf.* **175** (2018) 13017 [[INSPIRE](#)].
- [7] J.M. Gérard, W. Grimus, A. Raychaudhuri and G. Zoupanos, *Super Kobayashi-Maskawa CP-violation*, *Phys. Lett. B* **140** (1984) 349 [[INSPIRE](#)].
- [8] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of FCNC and CP constraints in general SUSY extensions of the Standard Model*, *Nucl. Phys. B* **477** (1996) 321 [[hep-ph/9604387](https://arxiv.org/abs/hep-ph/9604387)] [[INSPIRE](#)].
- [9] UTFIT collaboration, *Model-independent constraints on $\Delta F = 2$ operators and the scale of new physics*, *JHEP* **03** (2008) 049 [[arXiv:0707.0636](https://arxiv.org/abs/0707.0636)] [[INSPIRE](#)].

- [10] G. Isidori, Y. Nir and G. Perez, *Flavor physics constraints for physics beyond the Standard Model*, *Ann. Rev. Nucl. Part. Sci.* **60** (2010) 355 [[arXiv:1002.0900](#)] [[INSPIRE](#)].
- [11] L. Silvestrini and M. Valli, *Model-independent bounds on the Standard Model effective theory from flavour physics*, *Phys. Lett. B* **799** (2019) 135062 [[arXiv:1812.10913](#)] [[INSPIRE](#)].
- [12] L. Calibbi, A. Crivellin, F. Kirk, C.A. Manzari and L. Vernazza, *Z' models with less-minimal flavour violation*, *Phys. Rev. D* **101** (2020) 095003 [[arXiv:1910.00014](#)] [[INSPIRE](#)].
- [13] E.E. Jenkins, A.V. Manohar and P. Stoffer, *Low-energy effective field theory below the electroweak scale: operators and matching*, *JHEP* **03** (2018) 016 [[arXiv:1709.04486](#)] [[INSPIRE](#)].
- [14] W. Dekens and P. Stoffer, *Low-energy effective field theory below the electroweak scale: matching at one loop*, *JHEP* **10** (2019) 197 [[arXiv:1908.05295](#)] [[INSPIRE](#)].
- [15] M. Endo, T. Kitahara, S. Mishima and K. Yamamoto, *Revisiting kaon physics in general Z scenario*, *Phys. Lett. B* **771** (2017) 37 [[arXiv:1612.08839](#)] [[INSPIRE](#)].
- [16] C. Bobeth, A.J. Buras, A. Celis and M. Jung, *Yukawa enhancement of Z -mediated new physics in $\Delta S = 2$ and $\Delta B = 2$ processes*, *JHEP* **07** (2017) 124 [[arXiv:1703.04753](#)] [[INSPIRE](#)].
- [17] F. Feruglio, P. Paradisi and A. Pattori, *On the importance of electroweak corrections for B anomalies*, *JHEP* **09** (2017) 061 [[arXiv:1705.00929](#)] [[INSPIRE](#)].
- [18] M. González-Alonso, J. Martin Camalich and K. Mimouni, *Renormalization-group evolution of new physics contributions to (semi)leptonic meson decays*, *Phys. Lett. B* **772** (2017) 777 [[arXiv:1706.00410](#)] [[INSPIRE](#)].
- [19] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B -physics anomalies: a guide to combined explanations*, *JHEP* **11** (2017) 044 [[arXiv:1706.07808](#)] [[INSPIRE](#)].
- [20] J. Kumar, D. London and R. Watanabe, *Combined explanations of the $b \rightarrow s\mu^+\mu^-$ and $b \rightarrow c\tau^-\bar{\nu}$ anomalies: a general model analysis*, *Phys. Rev. D* **99** (2019) 015007 [[arXiv:1806.07403](#)] [[INSPIRE](#)].
- [21] J. Aebischer, J. Kumar, P. Stangl and D.M. Straub, *A global likelihood for precision constraints and flavour anomalies*, *Eur. Phys. J. C* **79** (2019) 509 [[arXiv:1810.07698](#)] [[INSPIRE](#)].
- [22] M. Endo, T. Kitahara and D. Ueda, *SMEFT top-quark effects on $\Delta F = 2$ observables*, *JHEP* **07** (2019) 182 [[arXiv:1811.04961](#)] [[INSPIRE](#)].
- [23] J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl and D.M. Straub, *B -decay discrepancies after Moriond 2019*, *Eur. Phys. J. C* **80** (2020) 252 [[arXiv:1903.10434](#)] [[INSPIRE](#)].
- [24] J. Aebischer, A.J. Buras and J. Kumar, *Another SMEFT story: Z' facing new results on ε'/ε , ΔM_K and $K \rightarrow \pi\nu\bar{\nu}$* , [arXiv:2006.01138](#) [[INSPIRE](#)].
- [25] J. Aebischer and J. Kumar, *Flavour violating effects of Yukawa running in SMEFT*, *JHEP* **09** (2020) 187 [[arXiv:2005.12283](#)] [[INSPIRE](#)].
- [26] J. Aebischer, C. Bobeth, A.J. Buras and D.M. Straub, *Anatomy of ε'/ε beyond the Standard Model*, *Eur. Phys. J. C* **79** (2019) 219 [[arXiv:1808.00466](#)] [[INSPIRE](#)].

- [27] J. Aebischer, C. Bobeth, A.J. Buras, J.-M. Gérard and D.M. Straub, *Master formula for ε'/ε beyond the Standard Model*, *Phys. Lett. B* **792** (2019) 465 [[arXiv:1807.02520](#)] [[INSPIRE](#)].
- [28] A.J. Buras, S. Jäger and J. Urban, *Master formulae for $\Delta F = 2$ NLO QCD factors in the Standard Model and beyond*, *Nucl. Phys. B* **605** (2001) 600 [[hep-ph/0102316](#)] [[INSPIRE](#)].
- [29] A.J. Buras and M. Jung, *Analytic inclusion of the scale dependence of the anomalous dimension matrix in Standard Model effective theory*, *JHEP* **06** (2018) 067 [[arXiv:1804.05852](#)] [[INSPIRE](#)].
- [30] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-six terms in the Standard Model Lagrangian*, *JHEP* **10** (2010) 085 [[arXiv:1008.4884](#)] [[INSPIRE](#)].
- [31] M. Ciuchini, E. Franco, V. Lubicz, G. Martinelli, I. Scimemi and L. Silvestrini, *Next-to-leading order QCD corrections to $\Delta F = 2$ effective Hamiltonians*, *Nucl. Phys. B* **523** (1998) 501 [[hep-ph/9711402](#)] [[INSPIRE](#)].
- [32] A.J. Buras, M. Misiak and J. Urban, *Two loop QCD anomalous dimensions of flavor changing four quark operators within and beyond the Standard Model*, *Nucl. Phys. B* **586** (2000) 397 [[hep-ph/0005183](#)] [[INSPIRE](#)].
- [33] A.J. Buras, M. Jamin and P.H. Weisz, *Leading and next-to-leading QCD corrections to ε parameter and B^0 - $\bar{B}^0$ mixing in the presence of a heavy top quark*, *Nucl. Phys. B* **347** (1990) 491 [[INSPIRE](#)].
- [34] P. Gambino, A. Kwiatkowski and N. Pott, *Electroweak effects in the B^0 - $\bar{B}^0$ mixing*, *Nucl. Phys. B* **544** (1999) 532 [[hep-ph/9810400](#)] [[INSPIRE](#)].
- [35] A.J. Buras and J.-M. Gérard, *Dual QCD insight into BSM hadronic matrix elements for K^0 - $\bar{K}^0$ mixing from lattice QCD*, *Acta Phys. Polon. B* **50** (2019) 121 [[arXiv:1804.02401](#)] [[INSPIRE](#)].
- [36] H. Boos, T. Mannel and J. Reuter, *The gold plated mode revisited: $\sin(2\beta)$ and $B^0 \rightarrow J/\Psi K_S$ in the Standard Model*, *Phys. Rev. D* **70** (2004) 036006 [[hep-ph/0403085](#)] [[INSPIRE](#)].
- [37] C. Bobeth, U. Haisch, A. Lenz, B. Pecjak and G. Tetlalmatzi-Xolocotzi, *On new physics in $\Delta\Gamma_d$* , *JHEP* **06** (2014) 040 [[arXiv:1404.2531](#)] [[INSPIRE](#)].
- [38] A. Lenz and G. Tetlalmatzi-Xolocotzi, *Model-independent bounds on new physics effects in non-leptonic tree-level decays of B-mesons*, *JHEP* **07** (2020) 177 [[arXiv:1912.07621](#)] [[INSPIRE](#)].
- [39] A.J. Buras and P.H. Weisz, *QCD nonleading corrections to weak decays in dimensional regularization and 't Hooft-Veltman schemes*, *Nucl. Phys. B* **333** (1990) 66 [[INSPIRE](#)].
- [40] M.J. Dugan and B. Grinstein, *On the vanishing of evanescent operators*, *Phys. Lett. B* **256** (1991) 239 [[INSPIRE](#)].
- [41] S. Herrlich and U. Nierste, *Evanescent operators, scheme dependences and double insertions*, *Nucl. Phys. B* **455** (1995) 39 [[hep-ph/9412375](#)] [[INSPIRE](#)].
- [42] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)] [[INSPIRE](#)].

- [43] J. Aebischer, A.J. Buras and J.-M. Gérard, *BSM hadronic matrix elements for ε'/ε and $K \rightarrow \pi\pi$ decays in the dual QCD approach*, *JHEP* **02** (2019) 021 [[arXiv:1807.01709](#)] [[INSPIRE](#)].
- [44] FLAVOUR LATTICE AVERAGING GROUP collaboration, *FLAG review 2019: Flavour Lattice Averaging Group (FLAG)*, *Eur. Phys. J. C* **80** (2020) 113 [[arXiv:1902.08191](#)] [[INSPIRE](#)].
- [45] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, *Next-to-leading order QCD corrections to the lifetime difference of B_s mesons*, *Phys. Lett. B* **459** (1999) 631 [[hep-ph/9808385](#)] [[INSPIRE](#)].
- [46] M. Gorbahn, S. Jager, U. Nierste and S. Trine, *The supersymmetric Higgs sector and B - $\bar{B}$ mixing for large $\tan \beta$* , *Phys. Rev. D* **84** (2011) 034030 [[arXiv:0901.2065](#)] [[INSPIRE](#)].
- [47] R. Babich, N. Garron, C. Höbbling, J. Howard, L. Lellouch and C. Rebbi, *K^0 - $\bar{K}^0$ mixing beyond the standard model and CP-violating electroweak penguins in quenched QCD with exact chiral symmetry*, *Phys. Rev. D* **74** (2006) 073009 [[hep-lat/0605016](#)] [[INSPIRE](#)].
- [48] SWME collaboration, *Kaon BSM B -parameters using improved staggered fermions from $N_f = 2 + 1$ unquenched QCD*, *Phys. Rev. D* **93** (2016) 014511 [[arXiv:1509.00592](#)] [[INSPIRE](#)].
- [49] R.J. Dowdall et al., *Neutral B -meson mixing from full lattice QCD at the physical point*, *Phys. Rev. D* **100** (2019) 094508 [[arXiv:1907.01025](#)] [[INSPIRE](#)].
- [50] ETM collaboration, *$\Delta S = 2$ and $\Delta C = 2$ bag parameters in the Standard Model and beyond from $N_f = 2 + 1 + 1$ twisted-mass lattice QCD*, *Phys. Rev. D* **92** (2015) 034516 [[arXiv:1505.06639](#)] [[INSPIRE](#)].
- [51] FERMILAB LATTICE and MILC collaborations, *$B_{(s)}^0$ -mixing matrix elements from lattice QCD for the Standard Model and beyond*, *Phys. Rev. D* **93** (2016) 113016 [[arXiv:1602.03560](#)] [[INSPIRE](#)].
- [52] A. Bazavov et al., *Short-distance matrix elements for D^0 -meson mixing for $N_f = 2 + 1$ lattice QCD*, *Phys. Rev. D* **97** (2018) 034513 [[arXiv:1706.04622](#)] [[INSPIRE](#)].
- [53] J. Aebischer et al., *WCxf: an exchange format for Wilson coefficients beyond the Standard Model*, *Comput. Phys. Commun.* **232** (2018) 71 [[arXiv:1712.05298](#)] [[INSPIRE](#)].
- [54] ETM collaboration, *Kaon mixing beyond the SM from $N_f = 2$ tmQCD and model independent constraints from the UTA*, *JHEP* **03** (2013) 089 [Erratum *ibid.* **07** (2013) 143] [[arXiv:1207.1287](#)] [[INSPIRE](#)].
- [55] RBC/UKQCD collaboration, *Neutral kaon mixing beyond the Standard Model with $n_f = 2 + 1$ chiral fermions part 1: bare matrix elements and physical results*, *JHEP* **11** (2016) 001 [[arXiv:1609.03334](#)] [[INSPIRE](#)].
- [56] RBC and UKQCD collaborations, *Neutral kaon mixing beyond the Standard Model with $n_f = 2 + 1$ chiral fermions. Part 2: non perturbative renormalisation of the $\Delta F = 2$ four-quark operators*, *JHEP* **10** (2017) 054 [[arXiv:1708.03552](#)] [[INSPIRE](#)].
- [57] P. Boyle, N. Garron, J. Kettle, A. Khamseh and J.T. Tsang, *BSM kaon mixing at the physical point*, *EPJ Web Conf.* **175** (2018) 13010 [[arXiv:1710.09176](#)] [[INSPIRE](#)].
- [58] ETM collaboration, *B -physics from $N_f = 2$ tmQCD: the Standard Model and beyond*, *JHEP* **03** (2014) 016 [[arXiv:1308.1851](#)] [[INSPIRE](#)].

- [59] A.G. Grozin, R. Klein, T. Mannel and A.A. Pivovarov, B^0 - $\bar{B}^0$ mixing at next-to-leading order, *Phys. Rev. D* **94** (2016) 034024 [[arXiv:1606.06054](#)] [[INSPIRE](#)].
- [60] M. Kirk, A. Lenz and T. Rauh, Dimension-six matrix elements for meson mixing and lifetimes from sum rules, *JHEP* **12** (2017) 068 [Erratum *ibid.* **06** (2020) 162] [[arXiv:1711.02100](#)] [[INSPIRE](#)].
- [61] A.G. Grozin, T. Mannel and A.A. Pivovarov, B^0 - $\bar{B}^0$ mixing: matching to HQET at NNLO, *Phys. Rev. D* **98** (2018) 054020 [[arXiv:1806.00253](#)] [[INSPIRE](#)].
- [62] D. King, A. Lenz and T. Rauh, B_s mixing observables and $|V_{td}/V_{ts}|$ from sum rules, *JHEP* **05** (2019) 034 [[arXiv:1904.00940](#)] [[INSPIRE](#)].
- [63] L. Di Luzio, M. Kirk, A. Lenz and T. Rauh, ΔM_s theory precision confronts flavour anomalies, *JHEP* **12** (2019) 009 [[arXiv:1909.11087](#)] [[INSPIRE](#)].
- [64] PARTICLE DATA GROUP collaboration, Review of Particle Physics, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [65] RBC and UKQCD collaborations, Domain wall QCD with physical quark masses, *Phys. Rev. D* **93** (2016) 074505 [[arXiv:1411.7017](#)] [[INSPIRE](#)].
- [66] S. Dürer et al., Lattice QCD at the physical point: light quark masses, *Phys. Lett. B* **701** (2011) 265 [[arXiv:1011.2403](#)] [[INSPIRE](#)].
- [67] S. Dürer et al., Lattice QCD at the physical point: simulation and analysis details, *JHEP* **08** (2011) 148 [[arXiv:1011.2711](#)] [[INSPIRE](#)].
- [68] C. McNeile, C.T.H. Davies, E. Follana, K. Hornbostel and G.P. Lepage, High-precision c and b masses, and QCD coupling from current-current correlators in lattice and continuum QCD, *Phys. Rev. D* **82** (2010) 034512 [[arXiv:1004.4285](#)] [[INSPIRE](#)].
- [69] MILC collaboration, MILC results for light pseudoscalars, *PoS(CD09)007* (2009) [[arXiv:0910.2966](#)] [[INSPIRE](#)].
- [70] Z. Fodor et al., Up and down quark masses and corrections to Dashen's theorem from lattice QCD and quenched QED, *Phys. Rev. Lett.* **117** (2016) 082001 [[arXiv:1604.07112](#)] [[INSPIRE](#)].
- [71] A. Bazavov et al., B - and D -meson leptonic decay constants from four-flavor lattice QCD, *Phys. Rev. D* **98** (2018) 074512 [[arXiv:1712.09262](#)] [[INSPIRE](#)].
- [72] ETM collaboration, Mass of the b quark and B -meson decay constants from $N_f = 2 + 1 + 1$ twisted-mass lattice QCD, *Phys. Rev. D* **93** (2016) 114505 [[arXiv:1603.04306](#)] [[INSPIRE](#)].
- [73] HPQCD collaboration, B -meson decay constants from improved lattice nonrelativistic QCD with physical u , d , s , and c quarks, *Phys. Rev. Lett.* **110** (2013) 222003 [[arXiv:1302.2644](#)] [[INSPIRE](#)].
- [74] C. Hughes, C.T.H. Davies and C.J. Monahan, New methods for B meson decay constants and form factors from lattice NRQCD, *Phys. Rev. D* **97** (2018) 054509 [[arXiv:1711.09981](#)] [[INSPIRE](#)].
- [75] G. Beall, M. Bander and A. Soni, Constraint on the mass scale of a left-right symmetric electroweak theory from the K_L - K_S mass difference, *Phys. Rev. Lett.* **48** (1982) 848 [[INSPIRE](#)].

- [76] J.A. Bagger, K.T. Matchev and R.-J. Zhang, *QCD corrections to flavor changing neutral currents in the supersymmetric Standard Model*, *Phys. Lett. B* **412** (1997) 77 [[hep-ph/9707225](#)] [[INSPIRE](#)].
- [77] M. Blanke, A.J. Buras, B. Duling, S. Gori and A. Weiler, *$\Delta F = 2$ observables and fine-tuning in a warped extra dimension with custodial protection*, *JHEP* **03** (2009) 001 [[arXiv:0809.1073](#)] [[INSPIRE](#)].
- [78] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, *Can we reach the Zeptouniverse with rare K and $B_{s,d}$ decays?*, *JHEP* **11** (2014) 121 [[arXiv:1408.0728](#)] [[INSPIRE](#)].
- [79] A.J. Buras, *New physics patterns in ε'/ε and ε_K with implications for rare kaon decays and ΔM_K* , *JHEP* **04** (2016) 071 [[arXiv:1601.00005](#)] [[INSPIRE](#)].
- [80] M. Artuso, G. Borissov and A. Lenz, *CP violation in the B_s^0 system*, *Rev. Mod. Phys.* **88** (2016) 045002 [*Addendum ibid.* **91** (2019) 049901] [[arXiv:1511.09466](#)] [[INSPIRE](#)].
- [81] S. Esen and A. Lenz, *CKM 2018 summary of working group 4: mixing and mixing-related CP-violation in the B system ΔM , $\Delta\Gamma$, ϕ_s , ϕ_1/β , ϕ_2/α , ϕ_3/γ* , in 10th International Workshop on the CKM Unitarity Triangle, (2019) [[arXiv:1901.05000](#)] [[INSPIRE](#)].
- [82] J. Brod, M. Gorbahn and E. Stamou, *Standard-Model prediction of ϵ_K with manifest quark-mixing unitarity*, *Phys. Rev. Lett.* **125** (2020) 171803 [[arXiv:1911.06822](#)] [[INSPIRE](#)].
- [83] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization group evolution of the Standard Model dimension six operators III: gauge coupling dependence and phenomenology*, *JHEP* **04** (2014) 159 [[arXiv:1312.2014](#)] [[INSPIRE](#)].
- [84] J. Aebischer, A. Crivellin, M. Fael and C. Greub, *Matching of gauge invariant dimension-six operators for $b \rightarrow s$ and $b \rightarrow c$ transitions*, *JHEP* **05** (2016) 037 [[arXiv:1512.02830](#)] [[INSPIRE](#)].
- [85] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization group evolution of the Standard Model dimension six operators I: formalism and λ dependence*, *JHEP* **10** (2013) 087 [[arXiv:1308.2627](#)] [[INSPIRE](#)].
- [86] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization group evolution of the Standard Model dimension six operators II: Yukawa dependence*, *JHEP* **01** (2014) 035 [[arXiv:1310.4838](#)] [[INSPIRE](#)].
- [87] Z. Bern, J. Parra-Martinez and E. Sawyer, *Structure of two-loop SMEFT anomalous dimensions via on-shell methods*, *JHEP* **10** (2020) 211 [[arXiv:2005.12917](#)] [[INSPIRE](#)].
- [88] J. de Vries, G. Falcioni, F. Herzog and B. Ruijl, *Two- and three-loop anomalous dimensions of Weinberg's dimension-six CP-odd gluonic operator*, *Phys. Rev. D* **102** (2020) 016010 [[arXiv:1907.04923](#)] [[INSPIRE](#)].
- [89] J. Aebischer, J. Kumar and D.M. Straub, *Wilson: a Python package for the running and matching of Wilson coefficients above and below the electroweak scale*, *Eur. Phys. J. C* **78** (2018) 1026 [[arXiv:1804.05033](#)] [[INSPIRE](#)].
- [90] J. Aebischer, M. Fael, C. Greub and J. Virto, *B physics beyond the Standard Model at one loop: complete renormalization group evolution below the electroweak scale*, *JHEP* **09** (2017) 158 [[arXiv:1704.06639](#)] [[INSPIRE](#)].
- [91] E.E. Jenkins, A.V. Manohar and P. Stoffer, *Low-energy effective field theory below the electroweak scale: anomalous dimensions*, *JHEP* **01** (2018) 084 [[arXiv:1711.05270](#)] [[INSPIRE](#)].

- [92] C. Bobeth, A.J. Buras, A. Celis and M. Jung, *Patterns of flavour violation in models with vector-like quarks*, *JHEP* **04** (2017) 079 [[arXiv:1609.04783](#)] [[INSPIRE](#)].
- [93] C. Bobeth and A.J. Buras, *Leptoquarks meet ε'/ε and rare kaon processes*, *JHEP* **02** (2018) 101 [[arXiv:1712.01295](#)] [[INSPIRE](#)].
- [94] P. Arnan, L. Hofer, F. Mescia and A. Crivellin, *Loop effects of heavy new scalars and fermions in $b \rightarrow s\mu^+\mu^-$* , *JHEP* **04** (2017) 043 [[arXiv:1608.07832](#)] [[INSPIRE](#)].
- [95] P. Arnan, A. Crivellin, M. Fedele and F. Mescia, *Generic loop effects of new scalars and fermions in $b \rightarrow s\ell^+\ell^-$ and a vector-like 4th generation*, *JHEP* **06** (2019) 118 [[arXiv:1904.05890](#)] [[INSPIRE](#)].
- [96] J. de Blas, J.C. Criado, M. Pérez-Victoria and J. Santiago, *Effective description of general extensions of the Standard Model: the complete tree-level dictionary*, *JHEP* **03** (2018) 109 [[arXiv:1711.10391](#)] [[INSPIRE](#)].
- [97] A.J. Buras and J. Girrbach, *Complete NLO QCD corrections for tree level $\Delta F = 2$ FCNC processes*, *JHEP* **03** (2012) 052 [[arXiv:1201.1302](#)] [[INSPIRE](#)].
- [98] M. Bordone, C. Cornella, J. Fuentes-Martin and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, *Phys. Lett. B* **779** (2018) 317 [[arXiv:1712.01368](#)] [[INSPIRE](#)].
- [99] V. Gherardi, D. Marzocca, M. Nardecchia and A. Romanino, *Rank-one flavor violation and B-meson anomalies*, *JHEP* **10** (2019) 112 [[arXiv:1903.10954](#)] [[INSPIRE](#)].
- [100] A.J. Buras, *Weak Hamiltonian, CP-violation and rare decays*, in *Les Houches summer school in theoretical physics, session 68: probing the Standard Model of particle interactions*, (1998), pg. 281 [[hep-ph/9806471](#)] [[INSPIRE](#)].