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On resolving the dark LMA solution at neutrino oscillation experiments

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ABSTRACT: In presence of non standard interactions (NSI), the solar neutrino data is consistent with two solutions, one close to the standard LMA solution with $\sin^2 \theta_{12} \simeq 0.31$ and another with $\sin^2 \theta_{12}^D \simeq 0.69 (= 1 - \sin^2 \theta_{12})$. The latter has been called the Dark LMA (DLMA) solution in the literature and essentially brings an octant degeneracy in the measurement of the mixing angle θ_{12} . This θ_{12} octant degeneracy is hard to resolve via oscillations because of the existence of the so-called “generalised mass hierarchy degeneracy” of the neutrino mass matrix in presence of NSI. One might think that if the mass hierarchy is independently determined in a non-oscillation experiment such as neutrino-less double beta decay, one might be able to break the θ_{12} octant degeneracy. In this paper we study this in detail in the context of long-baseline experiments ($P_{\mu\mu}$ channel) as well as reactor experiments (P_{ee} channel) and show that if we combine information from both long-baseline and reactor experiments we can find the correct octant and hence value of θ_{12} . We elaborate the reasons for it and study the prospects of determining the θ_{12} octant using T2HK, DUNE and JUNO experiments. Of course, one would need information on the neutrino mass hierarchy as well.

KEYWORDS: Neutrino Detectors and Telescopes (experiments), Oscillation

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1 Introduction

Within the standard three-generation neutrino oscillation paradigm, the Large Mixing Angle (LMA) solution provides an excellent explanation to the solar neutrino problem [1] and has been independently tested and confirmed by the KamLAND [2] reactor antineutrino experiment. The solar and KamLAND data jointly restrict the LMA solution to $\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$ and $\sin^2 \theta_{12} \simeq 0.31$ [3]. In particular, the high significance compatibility of the solar neutrino data with resonant MSW matter effect inside the sun rules out all values of $\sin^2 \theta_{12} > 0.5$ for $\Delta m_{21}^2 > 0$. However, this apparently robust conclusion runs into trouble [4, 5] as soon as one allows for the presence of (large) Non-Standard neutrino Interactions (NSIs) [6]. It has been shown that in presence of NSI we get two nearly degenerate solutions that fits the solar neutrino data — the standard LMA solution ($\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$, $\sin^2 \theta_{12} \simeq 0.3$) mentioned above, and another solution at ($\Delta m_{21}^{2D} \simeq 7.5 \times 10^{-5} \text{ eV}^2$, $\sin^2 \theta_{12}^D \simeq 0.7$). This latter solution has been called the Dark LMA (DLMA) solution [4] which has the same best-fit Δm_{21}^2 as in LMA solution but the best-fit mixing angle is $\sin^2 \theta_{12}^D \simeq 1 - \sin^2 \theta_{12}$. This is therefore like an octant degeneracy in θ_{12} . The conditions that need to be satisfied for the degeneracy to be exact, include NSIs corresponding to scattering off both up and down type quarks with NSI couplings proportional to their respective charges [7–10]. However even without this assumption, global analysis of solar neutrino and KamLAND data give the above mentioned approximately degenerate solutions at LMA and DLMA [11].

The existence of the DLMA solution not only has consequences for model building but has phenomenological implications as well. In particular, once both “octants” of θ_{12} become viable, neutrino mass hierarchy determination at oscillation experiments becomes a problem owing to the “generalised mass hierarchy degeneracy” [9], which can be understood as follows. For pure vacuum oscillations, the neutrino mass matrix and hence the neutrino oscillation probabilities remain invariant under the simultaneous transformations $\Delta m_{31}^2 \leftrightarrow -\Delta m_{32}^2$, $\sin^2 \theta_{12} \leftrightarrow \cos^2 \theta_{12}$ and $\delta_{cp} \leftrightarrow \pi - \delta_{cp}$. This poses a serious challenge for experiments such as JUNO [12, 13] that are attempting to measure the sign of Δm_{31}^2 via vacuum oscillations [8]. Presence of matter effects could in principle break this degeneracy, however, if one adds the possibility of having NSI in the neutrino sector, then in addition to the transformations mentioned above if we also transform the NSI parameters $\epsilon_{ee} \rightarrow \epsilon_{ee} - 2$, $\epsilon_{e\mu} \rightarrow \epsilon_{e\mu}^*$, $\epsilon_{e\tau} \rightarrow \epsilon_{e\tau}^*$ and all other $\epsilon_{\alpha\beta} \rightarrow -\epsilon_{\alpha\beta}^*$, then the mass hierarchy degeneracy is restored [9] and now even experiments such as DUNE [14] that aim to determine the sign of Δm_{31}^2 via observation of earth matter effects are unable to do so [9]. As long as this degeneracy remains unresolved, oscillation experiments will not be able to measure the sign of Δm_{31}^2 , unless the issue of the octant of θ_{12} is resolved. Likewise, the octant of θ_{12} can be determined at oscillation experiments only if the sign of Δm_{31}^2 is known.

Therefore, it is obvious that both cannot be determined simultaneously using oscillation experiments alone and one needs to look for inputs from non-oscillation experiments for the resolution of the LMA vs DLMA conundrum. This can be done in two ways. The first way is of course to independently constrain the NSI parameters themselves such that the values of the NSI parameters required for the DLMA solution are disfavored, and hence disfavoring the DLMA solution [10, 15]. Constraints on NSI parameters have been obtained from neutrino scattering experiments. However, these constraints depend on the mass of the mediator particle. For heavy mediators, the best constraints come from the CHARM [16] and NuTeV experiments [17], while the current data from COHERENT [18] severely constrains the NSI parameters to mediator masses down to 10 MeV. However, there still remains allowed a narrow window of NSI parameters for which the DLMA solution is allowed [19]. Indeed the global analysis [20] of neutrino oscillation data and COHERENT data allows the DLMA solution for certain (restricted) class of NSI models, with some cases even favoring the DLMA solution over the LMA solution (cf. figure 8 of [20]). Future data from CONUS and more data from COHERENT could narrow down this range severely [19], however, this remains to be seen, and it is possible that the DLMA solution could survive the constraints from neutrino scattering experiments for very low mediator masses and/or specific NSI models. Hence, neutrino scattering data may or may not be able to completely rule out the DLMA solution in a truly model independent way.

The other way of resolving the θ_{12} vs θ_{12}^D conundrum could be to determine the neutrino mass hierarchy in a non-oscillation experiment and then test octant of θ_{12} in an oscillation experiment. Here, we will study in detail the possibility of determining the octant of θ_{12} at long-baseline experiments such as T2HK [25] or DUNE [14] and show that owing to an intrinsic correlation between θ_{12} and $|\Delta m_{31}^2|$ in $P_{\mu\mu}$, it is impossible to say anything about θ_{12} at all from data from long-baseline experiments alone, even if the sign of Δm_{31}^2 was known. We will then show how we can break this correlation using data from a complementary oscillation experiment such as JUNO which measures P_{ee} . We show that

while Δm_{31}^2 and $\sin^2 \theta_{12}$ are correlated for P_{ee} , they are anti-correlated for $P_{\mu\mu}$. Therefore, adding information from both channels can break the θ_{12} octant degeneracy.

Some comments on our assumption that neutrino mass hierarchy will be determined in a non-oscillation experiment is in order. The main non-oscillation experiments which depend on the neutrino mass hierarchy are neutrino-less double beta decay experiments and cosmology. The implications of DLMA for neutrino-less double beta decay have been presented in [21]. It was pointed out in that work that predictions for neutrino-less double beta decay for inverted hierarchy (IH) remain invariant under change from LMA to DLMA. However, for normal hierarchy (NH), the predicted value of the effective mass $m_{\beta\beta}$ shifts to higher values into the “desert region”, increasing the prospects of a positive signal in this very important class of experiments. Since the next generation neutrino-less double beta decay experiments are expected to cover the entire predicted range for IH, we could know the neutrino mass hierarchy in the near future, if neutrinos are Majorana particles. While this is an optimistic view-point, one must keep in mind the caveats associated with it. Firstly, as pointed out above, non-observation of neutrino-less double beta decay does not necessarily imply that IH is ruled out - it may well be that the neutrinos are not Majorana particles. On the other hand, if we do observe a positive signal, this would clearly prove that neutrinos are Majorana. However, this would still not be fool-proof confirmation of IH since a variety of new physics scenarios (see for e.g. [22]) could lead to a neutrino-less double beta half-life that is comparable to that predicted by neutrino masses with IH. Separation between the predicted $m_{\beta\beta}$ for NH and IH also depends on the lightest neutrino mass, and the limits on this are obtained from cosmological data, which have their own set of caveats. Finally, $m_{\beta\beta}$ depends on the nuclear matrix element uncertainties and this needs to be taken into consideration when drawing conclusions from neutrino-less double beta decay experiments. Determining the neutrino mass hierarchy to some extent is also possible from future cosmological data (see e.g. [23]).

The rest of the paper is organised as follows. In section 2, we give a brief description of our numerical techniques and experimental details. In section 3 we discuss the neutrino oscillation probabilities to show how the long-baseline experiments and reactor experiments depend on the octant of θ_{12} . In section 4 we discuss the (anti)correlation between θ_{12} and $|\Delta m_{31}^2|$ and the role of combining JUNO with T2HK (or DUNE) in breaking this correlation, thereby solving the θ_{12} octant degeneracy. In section 5 we compare the sensitivity of T2HK vs DUNE in resolving the DLMA solution. Section 6 discusses the role of neutrino-less double beta decay experiments briefly. Finally, we end in section 7 with our conclusions.

2 Experimental and numerical details

We first describe here the specifications we use in our analysis for the DUNE and T2HK experiments. We next detail the simulation technique used in our analysis.

2.1 DUNE

Deep Underground Neutrino Experiment (DUNE) is a future long-baseline experiment program to be built in the U.S.A.. It will have an intense neutrino source at Fermilab

and a far detector 1300 km away at the Sanford Underground Research Facility in South Dakota. The neutrino beam will be generated by a 80–120 GeV proton beam at 1.2–2.4 MW. The far detector will be made of 4 identical 10 kt liquid Argon detectors which will give a fiducial mass of 34 kt.

In this work we followed the specifications given in [24]. We have considered a 34 kt. detector at baseline 1300 km. The neutrino flux is given by the 120 GeV, 1.2 MW proton beam. Liquid Argon detectors can measure both μ and e type events. So we have taken into consideration both the ν_e ($\bar{\nu}_e$) appearance as well as the ν_μ ($\bar{\nu}_\mu$) disappearance channels in the analysis. We consider 5 years running of the experiment in the neutrino mode and 5 years in the antineutrino mode. We have taken $20\%/\sqrt{E}$ energy resolution for the μ and $15\%/\sqrt{E}$ resolution for the e channels. We took the signal efficiency to be 85%. For the neutrino (antineutrino) mode, the signal normalization error is taken as 2%(5%), the background normalization error is taken as 10%(10%) and the energy calibration error is taken as 5% (5%). Although there is an updated version of the DUNE specifications available in the literature [14], we explicitly checked that the main conclusions of our analysis do not change significantly for the new specifications. So we have kept the older specifications in our analysis.

2.2 T2HK

T2HK [25] is the upgradation program of the currently running T2K experiment [26]. Here the neutrino beam will come from an upgraded J-PARC accelerator source at Tokai. The far detector will be an upgraded version of the currently running Super-Kamiokande (SK) detector, and will be called Hyper-Kamiokande (HK). HK will be made of two identical 187 kt water Cherenkov detectors which will be placed at a baseline of about 295 km from the source. The HK detector will be at a location 2.5° off-axis with respect to the J-PARC beam-line.

For this work we consider 1.3 MW beam power and the 2.5° off-axis flux. We take 295 km as the baseline and 374 kt as the fiducial mass. For the run-time we consider 2.5 years running in the neutrino mode and 7.5 years running in the antineutrino mode. We match our simulated number of events with tables 2 and 3 of [27]. We take $15\%/\sqrt{E}$ energy resolution for all channels. The signal normalization errors for ν_e ($\bar{\nu}_e$) and ν_μ ($\bar{\nu}_\mu$) channel are 3.2% (3.6%) and 3.9%(3.6%), respectively. We also take 10% and 5% background and energy calibration errors in all channels, respectively.

We stress that we have not done a full simulation of the JUNO experiment. Instead we just put the expected constraints obtained by the JUNO collaboration [12] as a prior in our analysis. While we understand that a full analysis would of course be more desirable, however, since the purpose of this paper is only to present the idea that one can use to find the octant of θ_{12} , this serves the purpose. For a more rigorous quantitative analysis, one could simulate the JUNO data. We leave that for a future work.

2.3 Simulation techniques

We have used GLoBES (Global Long-baseline Experiment Simulator) [28, 29] for our simulation of the long-baseline experiments. For the calculation of neutrino oscillation prob-

Parameter Name	True value	Marginalization
$\sin^2 \theta_{12}$	0.31 (0.69)	[0.0–1.0]
θ_{13}	8.48°	Nil
θ_{23}	42°	Nil
δ_{CP}	-90°	$[-\pi, \pi]$
Δm_{21}^2	$7.50 \times 10^{-5} \text{ eV}^2$	Nil
Δm_{31}^2	$2.52 \times 10^{-3} \text{ eV}^2$	$[2.3, 2.6] \times 10^{-3} \text{ eV}^2$
ϵ_{ee}	0	$[-3.0, 3.0]$
$\epsilon_{\alpha\beta}, \alpha\beta \neq ee$	0	Nil

Table 1. Values of parameters taken for generating mock data used in this work and the range over which they are marginalised in the fit.

abilities in presence of NSI, we used NSI probability engine of MonteCUBES [30]. We have taken the following values of the oscillation parameters for generating our mock data: $\sin^2 \theta_{12} = 0.31$, $\theta_{13} = 8.48^\circ$, $\theta_{23} = 42^\circ$, $\delta_{\text{CP}} = -90^\circ$, $\Delta m_{21}^2 = 7.5 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 = 2.52 \times 10^{-3} \text{ eV}^2$. This is consistent with the currently allowed values of the standard oscillation parameters from global fits [3]. We assume normal hierarchy in data for all the results presented in the paper, except the last figure, where inverted hierarchy is taken in the mock data. In the fit we marginalise over standard oscillation parameters Δm_{31}^2 and δ_{CP} . The range used in our marginalization is given in the table 1 and is consistent with the currently allowed ranges. Note that for θ_{12} we consider the entire range in the fit since studying the octant degeneracy of θ_{12} is the main concern of this work. We keep Δm_{21}^2 fixed in the fit since varying it has practically no impact on the final conclusions. We keep θ_{23} fixed because we explicitly checked that the effect of θ_{23} marginalisation on the final results is negligible. We also keep θ_{13} fixed in the fit since it is well determined by the Daya Bay data. While mass hierarchy is considered fixed in most of the plots since we assume normal hierarchy throughout this work, for figure 5 we marginalise over the mass hierarchy in order to show the impact of the generalised mass hierarchy degeneracy and the importance of measuring the mass hierarchy in a non-oscillation experiment.

The mock data is always generated for no NSIs, since the data always correspond to the LMA solution. Whenever we allow NSI to vary in the fit we marginalised over a large range of ϵ_{ee} . We keep all other NSI parameters at zero in both data and the fit. While this choice of marginalisation is not the most general one, it is sufficient for the purpose on this paper. The choices of all the parameters in the data and the marginalization ranges in the fit are summarized in table. 1.

3 LMA vs. DLMA and oscillation probabilities

All results in this work are obtained within the exact three-generation paradigm, taking earth matter effect into account. However, the resolution of the octant of θ_{12} can be understood simply from the expressions of oscillation probabilities in vacuum. Therefore for simplicity, we begin by discussing oscillation probabilities in vacuum. In vacuum, the

ν_μ survival probability using the standard parametrisation of the neutrino mixing matrix is given as

$$\begin{aligned}
 P_{\mu\mu} = & 1 - P_{21} \sin^2 \left(\frac{\Delta m_{21}^2 L}{4E} \right) \\
 & - 4s_{23}^2 c_{13}^2 \left(c_{12}^2 c_{23}^2 + s_{12}^2 s_{23}^2 s_{13}^2 - \frac{1}{2} \sin 2\theta_{12} \sin 2\theta_{23} s_{13}^2 \cos \delta \right) \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right) \\
 & - 4s_{23}^2 c_{13}^2 \left(s_{12}^2 c_{23}^2 + c_{12}^2 s_{23}^2 s_{13}^2 - \frac{1}{2} \sin 2\theta_{12} \sin 2\theta_{23} s_{13}^2 \cos \delta \right) \sin^2 \left(\frac{\Delta m_{32}^2 L}{4E} \right). \quad (3.1)
 \end{aligned}$$

Here, P_{21} is given by

$$\begin{aligned}
 P_{21} = & \sin^2 2\theta_{12} (c_{23}^4 + s_{23}^4 s_{13}^2) + \left(1 - \frac{1}{2} \sin^2 2\theta_{12} \right) \sin^2 2\theta_{23} s_{13}^2 \\
 & + \sin 4\theta_{12} \sin 2\theta_{23} (c_{23}^2 - s_{23}^2 s_{13}^2) s_{13} \cos \delta_{13} - \sin^2 2\theta_{12} \sin^2 2\theta_{23} s_{13}^2 \cos^2 \delta_{13}. \quad (3.2)
 \end{aligned}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$, s_{ij}^2 and c_{ij}^2 imply $\sin^2 \theta_{ij}$ and $\cos^2 \theta_{ij}$, respectively, while E and L are the neutrino energy and distance travelled. P_{21} associated with the Δm_{21}^2 -driven oscillatory term is symmetric with respect to the interchange $\sin^2 \theta_{12} \leftrightarrow \cos^2 \theta_{12}$ and hence not interesting to us. The expression in eq. (3.1) shows that $P_{\mu\mu}$ is invariant under simultaneous exchange of $\sin^2 \theta_{12} \leftrightarrow \cos^2 \theta_{12}$ and $|\Delta m_{31}^2| \leftrightarrow |\Delta m_{32}^2|$. Therefore, if the goal is to distinguish $\sin^2 \theta_{12}$ from $\cos^2 \theta_{12}$ ($= \sin^2 \theta_{12}^D$), the experiment should be able to distinguish the oscillatory frequency corresponding to $|\Delta m_{31}^2|$ from the frequency corresponding to $|\Delta m_{32}^2|$. The difference in $P_{\mu\mu}$ between $\sin^2 \theta_{12}$ and $\sin^2 \theta_{12}^D = \cos^2 \theta_{12}$ is given as,

$$\Delta P_{\mu\mu} = 4s_{23}^2 c_{13}^2 \cos 2\theta_{12} \left(c_{23}^2 - s_{23}^2 s_{13}^2 \right) \sin \left(\frac{\Delta m_{21}^2 L}{4E} \right) \sin \left(\frac{(2\Delta m_{31}^2 - \Delta m_{21}^2)L}{4E} \right). \quad (3.3)$$

Note that this difference goes to zero close to the oscillation maximum corresponding to Δm_{31}^2 , and is extremal when $\sin(2\Delta m_{31}^2 - \Delta m_{21}^2)L/4E \simeq 1$. The difference will also go to zero if $c_{23}^2 = s_{23}^2 s_{13}^2$, which is safely avoided for the current allowed values of θ_{13} and θ_{23} . The corresponding difference in the $P_{\mu e}$ channel between $\sin^2 \theta_{12}$ and $\sin^2 \theta_{12}^D$ is

$$\Delta P_{\mu e} = s_{23}^2 \sin^2 2\theta_{13} \cos 2\theta_{12} \sin \left(\frac{\Delta m_{21}^2 L}{4E} \right) \sin \left(\frac{(2\Delta m_{31}^2 - \Delta m_{21}^2)L}{4E} \right). \quad (3.4)$$

In figure 1 we show $\Delta P_{\mu\mu}$ and $\Delta P_{\mu e}$ as a function of energy for the T2HK baseline of $L = 295$ km for normal (NH) and inverted (IH) mass hierarchy. We define NH and IH corresponding to $\Delta m_{31}^2 > 0$ and $\Delta m_{31}^2 < 0$, respectively. For a given mass hierarchy, extrema in $\Delta P_{\mu\mu}$ and $\Delta P_{\mu e}$ appear close to $E \sim \Delta m_{31}^2 L / (2n + 1)\pi$, with $n = 2, 1, 0$. We see a maxima in $\Delta P_{\mu\mu}$ and $\Delta P_{\mu e}$ for $n = 2$, minima for $n = 1$ and back again a maxima for $n = 0$, as was expected from eqs. (3.3) and (3.4). This flipping of $\Delta P_{\mu\mu}$ with neutrino energy is also a signal for the octant of θ_{12} . The presence of Δm_{21}^2 has two-fold effect. Firstly, it dampens ΔP as we go to higher energies. This comes from the first oscillatory term $\sin(\Delta m_{21}^2 L / 4E)$ in eqs. (3.3) and (3.4). Since this term decreases as we increase

Figure 1. Difference in probability as a function of energy. The plot is shown for $L = 295$ km and without matter effect. The black lines show $\Delta P_{\mu\mu}$ while the green lines show $\Delta P_{\mu e}$. The solid lines are for NH while the dashed lines are for IH.

energy, the damping is seen to increase with energy in figure 1. Secondly, it determines the exact position of the ΔP extrema according to the second oscillatory term.

A comparison of eqs. (3.3) and (3.4) shows that $\Delta P_{\mu e}/\Delta P_{\mu\mu} = s_{13}^2/(\cos 2\theta_{23} + s_{23}^2 c_{13}^2)$, which for maximal θ_{23} gives $\Delta P_{\mu e}/\Delta P_{\mu\mu} = 2s_{13}^2/c_{13}^2$. Thus, for the measured value of θ_{13} , $\Delta P_{\mu e}$ is suppressed by two orders of magnitude compared to $\Delta P_{\mu\mu}$. This difference can be seen in figure 1 and hence we conclude that in long-baseline experiments, it is the $P_{\mu\mu}$ (disappearance) channel which will bring in the sensitivity to the octant of θ_{12} , while the role of the $P_{\mu e}$ (appearance) channel will be seen to be minimal.

Let us next look at the $\bar{\nu}_e$ survival probability that is relevant for the reactor experiments,

$$P_{ee} = 1 - \sin^2 2\theta_{12} c_{13}^4 \sin^2 \frac{\Delta m_{21}^2 L}{4E} - \sin^2 2\theta_{13} \left[c_{12}^2 \sin^2 \frac{\Delta m_{31}^2 L}{4E} + s_{12}^2 \sin^2 \frac{\Delta m_{32}^2 L}{4E} \right]. \quad (3.5)$$

The second term in this expression is the leading term in the probability driven by Δm_{21}^2 oscillations and depends on the solar mixing angle as $\sin^2 2\theta_{12}$. This is the term that gives an experiment like JUNO its sensitivity to $\sin^2 2\theta_{12}$. However, since this term depends on $\sin^2 2\theta_{12}$ and not on $\sin^2 \theta_{12}$ (or $\cos^2 \theta_{12}$), it does not bring any sensitivity to the octant of θ_{12} . The sensitivity to the octant of θ_{12} comes from the sub-dominant third term. This is the interference term between the solar and atmospheric frequencies. It comes from the full three-generation probability and indeed is the term that also brings the hierarchy sensitivity in JUNO. The similarity between this term and the third and fourth terms in eq. (3.1) is uncanny. Just as in the case of $P_{\mu\mu}$, this term depends on the octant of θ_{12} as long as one can distinguish between the oscillations driven by Δm_{31}^2 and Δm_{32}^2 . The difference in P_{ee} between $\sin^2 \theta_{12}$ and $\sin^2 \theta_{12}^D = \cos^2 \theta_{12}$ is given as

$$\Delta P_{ee} = \sin^2 2\theta_{13} \cos 2\theta_{12} \sin \left(\frac{\Delta m_{21}^2 L}{4E} \right) \sin \left(\frac{(2\Delta m_{31}^2 - \Delta m_{21}^2)L}{4E} \right). \quad (3.6)$$

Comparison of eqs. (3.3), (3.4) and (3.6) shows that the oscillatory part in the respective $\Delta P_{\alpha\beta}$ are the same. The only difference is in the part which contain the mixing angles. We see that

$$\frac{\Delta P_{ee}}{\Delta P_{\mu\mu}} = \frac{s_{13}^2}{s_{23}^2(c_{23}^2 - s_{23}^2 s_{13}^2)}, \quad (3.7)$$

which for maximal θ_{23} mixing reduces to

$$\frac{\Delta P_{ee}}{\Delta P_{\mu\mu}} = 4 \frac{s_{13}^2}{c_{13}^2}. \quad (3.8)$$

Hence, we see that the θ_{12} octant sensitivity is suppressed in the P_{ee} channel compared to the $P_{\mu\mu}$ channel by a factor of $4 \tan^2 \theta_{13}$. For the sake of completeness, we also note that

$$\frac{\Delta P_{ee}}{\Delta P_{\mu e}} = \frac{1}{s_{23}^2}, \quad (3.9)$$

which means that the octant sensitivity in the P_{ee} channel is about twice as much as that in the $P_{\mu e}$ channel. Hence, from the discussion with the probabilities we see that the θ_{12} octant sensitivity appears to be maximum for $P_{\mu\mu}$ and minimum for $P_{\mu e}$, with the P_{ee} channel being significantly less sensitive to θ_{12} octant compared to $P_{\mu\mu}$, but a factor of two more sensitive compared to the $P_{\mu e}$ channel. Nonetheless, as we will see in the next section, the contribution from the reactor antineutrino experiments is crucial for determining the octant of θ_{12} and without combining information from both $P_{\mu\mu}$ and P_{ee} channels it will be impossible to determine the octant of θ_{12} .

4 Mass squared difference and θ_{12}

We begin by showing the sensitivity of the long-baseline experiments and then add the information from reactor experiment JUNO to study the sensitivity of oscillation experiments to octant of θ_{12} . We show in figure 2 the χ^2 and hence the significance with which T2HK will be able to distinguish LMA from DLMA, if all oscillation parameters are kept fixed at their assumed true value in the fit. We have taken the experimental settings for T2HK as given in [25] and considered the mock data corresponding to running the experiment for 2.5+7.5 years in $\nu+\bar{\nu}$ mode, respectively. The mock data is generated at $\sin^2 \theta_{12} = 0.31$ and we want to rule out the higher octant of θ_{12} in general and the DLMA solution in particular. Shown are the χ^2 obtained from the analysis of disappearance data, appearance data, and both data sets combined. The appearance channel on its own is unable to resolve the degeneracy. However, it puts a weak lower and upper limit on $\sin^2 \theta_{12}$, as $P_{\mu e}$ depends on $\sin^2 2\theta_{12}$. We see that the disappearance data alone can rule out the DLMA solution at more than 5σ , *if all the other parameters were known*.

However, the other oscillation parameters are not completely known, and hence should be allowed to vary to account for correlations between them. Indeed there is a correlation between Δm_{31}^2 and θ_{12} , which completely spoils this sensitivity and which we discuss next. Instead of keeping the oscillation parameters fixed as in figure 2, we now allow them to vary

Figure 2. χ^2 as a function of $\sin^2 \theta_{12}$ for T2HK mock data generated at $\sin^2 \theta_{12} = 0.31$ corresponding to the LMA solution and fitted by varying $\sin^2 \theta_{12}$ while all other parameters are fixed. Results are shown for analysis of appearance channel data only, disappearance channel data only and for the combination of both appearance and disappearance channel data.

Figure 3. The 95% C.L. expected allowed areas in the $\Delta m_{31}^2 - \sin^2 \theta_{12}$ plane for T2HK. The blue dot-dashed contour is obtained using T2HK disappearance channel only, the green dashed curve using T2HK appearance channel only and the black dotted curve using both. The blue shaded region is the JUNO prior on Δm_{ee}^2 . The red solid contour is obtained when we apply the prior from JUNO on Δm_{ee}^2 and the dark red contour is obtained when we also add the $\sin^2 2\theta_{12}$ prior along with the prior on Δm_{ee}^2 .

as detailed in table 1. Figure 3 presents the 95% C.L. allowed area in the $\Delta m_{31}^2 - \sin^2 \theta_{12}$ plane expected at T2HK from disappearance channel (blue dot-dashed curve), appearance channel (green dashed curve) and by combining the two (black dotted curve). The contours are marginalised over θ_{13} and δ_{CP} . The assumed true value in this parameter space is shown by the black star. We find that once we allow Δm_{31}^2 to vary, the constraint on $\sin^2 \theta_{12}$ is seen to be completely lost. The appearance channel puts only a weak constraint on

$\sin^2 2\theta_{12}$, as was the case in figure 2. However, for the disappearance channel the effect is drastic and we find that nearly the entire $\sin^2 \theta_{12}$ range gets allowed. This happens because of the anti-correlation between Δm_{31}^2 and $\sin^2 \theta_{12}$ in the ν_μ survival probability which can be explained easily if we rewrite eq. (3.1) in terms of an effective mass squared difference $\Delta m_{\mu\mu}^2$ such that [31–33]

$$P_{\mu\mu} \simeq 1 - 4 \sin^2 \theta_{23} \cos^2 \theta_{13} (1 - \sin^2 \theta_{23} \cos^2 \theta_{13}) \sin^2 \left(\frac{\Delta m_{\mu\mu}^2 L}{4E} \right),$$

$$\Delta m_{\mu\mu}^2 = \Delta m_{31}^2 \Delta m_{21}^2 (\cos^2 \theta_{12} - \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}) \quad (4.1)$$

The long-baseline experiments measure the effective parameter $\Delta m_{\mu\mu}^2$ which depends on Δm_{31}^2 and $\sin^2 \theta_{12}$ in an anti-correlated way and hence $P_{\mu\mu}$ alone will never be able to measure $\sin^2 \theta_{12}$ in a long-baseline experiment. A given measured value of $\Delta m_{\mu\mu}^2$ can always be reproduced for lower Δm_{31}^2 and higher $\sin^2 \theta_{12}$, and/or vice-versa. Adding the $P_{\mu e}$ channel to the analysis along with the $P_{\mu\mu}$ channel (shown by black short-dashed line) improves the constraint on $\sin^2 \theta_{12}$ only marginally on the two edges of the scale, as was expected from the green-dashed lines in figures 2. However, this is unable to break the Δm_{31}^2 and $\sin^2 \theta_{12}$ anti-correlation coming from $\Delta m_{\mu\mu}^2$ and as a result the DLMA solution continues to get allowed. In order to remedy this, we need an independent measurement of Δm_{31}^2 which can break the Δm_{31}^2 and $\sin^2 \theta_{12}$ anti-correlation in the $P_{\mu\mu}$ channel.

Such an independent channel can be the P_{ee} channel, as discussed before. As in the case of ν_μ disappearance, we can write the $\bar{\nu}_e$ disappearance probability in terms of an effective mass squared difference Δm_{ee}^2 [12] as

$$P_{ee} = 1 - \sin^2 2\theta_{13} \left[1 - \sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}} \cos \left(2\Delta m_{ee}^2 \pm \phi \right) \right] - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21}, \quad (4.2)$$

where Δm_{ee}^2 and ϕ carry the θ_{12} octant dependence of P_{ee} and are given by

$$\Delta m_{ee}^2 = \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2, \quad (4.3)$$

$$\sin \phi = \frac{c_{12}^2 \sin(2s_{12}^2 \Delta_{21}) - s_{12}^2 \sin(2c_{12}^2 \Delta_{21})}{\sqrt{1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}}}. \quad (4.4)$$

We have used $\Delta_{21} = \Delta m_{21}^2 L / 4E$ in the above as shorthand and $\pm$ in eq. (4.2) corresponds to the neutrino mass hierarchy, $+$ for normal and $-$ for inverted. From eqs. (4.3) and (4.4) it is clear that ϕ is smaller than Δm_{ee}^2 . Note that the dependence of Δm_{ee}^2 on θ_{12} is complementary to what we had for $\Delta m_{\mu\mu}^2$. While the former depends on $\sin^2 \theta_{12}$, the latter on $\cos^2 \theta_{12}$. We use here the expected constraints from the JUNO [12] experiment which will observe $\bar{\nu}_e$ disappearance probability P_{ee} and will measure the effective mass squared difference Δm_{ee}^2 and $\sin^2 2\theta_{12}$ to a very high precision. We show in the blue band the 95% C.L. expected allowed area in the $\Delta m_{31}^2 - \sin^2 \theta_{12}$ plane from JUNO taken from [12]. Since Δm_{31}^2 and $\sin^2 \theta_{12}$ are correlated for Δm_{ee}^2 measured in JUNO, we get a slanting bar in this plane, corresponding to this constraint. This band intersects with the band/contour

Figure 4. χ^2 as a function of $\sin^2 \theta_{12}$ for mock data generated at $\sin^2 \theta_{12} = 0.31$. The parameters δ_{CP} , $|\Delta m_{31}^2|$ and ϵ_{ee} are allowed to vary freely in the fit. Shown are the sensitivity expected at T2HK as well as DUNE with and without prior from JUNO.

corresponding to T2HK around the best-fit point (star). This happens because Δm_{31}^2 and $\sin^2 \theta_{12}$ are correlated for Δm_{ee}^2 , while they are anti-correlated for $\Delta m_{\mu\mu}^2$, making the two bands slant in opposite directions. Including the JUNO constraint on Δm_{ee}^2 as a prior in our analysis of the T2HK data gives us the contour shown by the red solid curve. The degeneracy between Δm_{31}^2 and $\sin^2 \theta_{12}$ is broken, giving us a unique solution in $\sin^2 \theta_{12}$ and the DLMA solution can now be completely ruled out. If we also add the expected JUNO constraint on θ_{12} , we get the sharp narrow contour shown by the maroon solid line.

As mentioned above, in addition to Δm_{ee}^2 , P_{ee} also depends on the octant of θ_{12} via the ϕ terms. While this term is sub-dominant, it brings a difference in the P_{ee} channel between the two octants of θ_{12} . This ϕ dependence in P_{ee} can be seen only in an experiment like JUNO which has very good energy resolution. This could enable JUNO alone to measure the octant of θ_{12} if the neutrino mass hierarchy was already known, as was pointed out in [8].

5 Comparing T2HK with DUNE

In figure 4 we show the χ^2 as a function of $\sin^2 \theta_{12}$ for mock data generated at $\sin^2 \theta_{12} = 0.31$. We have shown here the expected χ^2 from combined appearance and disappearance data at T2HK as well as DUNE. The oscillation parameters δ_{CP} as well as $|\Delta m_{31}^2|$ are varied freely in the fit. We have also varied ϵ_{ee} in the fit. Results are shown without taking any prior from JUNO (dashed lines), when the JUNO prior on Δm_{ee}^2 is included (solid lines) and when the JUNO prior on $\sin^2 2\theta_{12}$ is also added along with the Δm_{ee}^2 prior (dotted

Figure 5. The impact of the neutrino mass hierarchy on the determination of the octant of θ_{12} .

lines). As in figure 3, we see that without the prior on Δm_{ee}^2 from JUNO, it is impossible for the long-baseline experiments to put any meaningful constraint on $\sin^2 \theta_{12}$. However, once the information on Δm_{ee}^2 from JUNO is included, the disappearance channel at these experiments can rule out the DLMA solution at more than 5σ significance at T2HK and more than 4σ at DUNE. We also see that when the $\sin^2 2\theta_{12}$ prior is added from JUNO, the precision of θ_{12} is strictly determined by the JUNO prior.

6 Role of neutrino mass hierarchy

So long we had fixed the neutrino mass hierarchy, assuming that it will be determined by a non-oscillation experiment. Neutrino-less double beta decay could fix the neutrino mass hierarchy [34, 35]. Information on neutrino mass hierarchy could also come from future cosmological data [23, 36]. We have already stressed the caveats associated with this assumption in the introduction. Here we lift this assumption and study the phenomenological implications of what happens if hierarchy is unknown.

In figure 5, we show the impact of mass hierarchy on ruling out the DLMA solution. We present four cases each for T2HK and DUNE. The solid lines are for the cases when we take hierarchy to be known and add both Δm_{ee}^2 and $\sin^2 2\theta_{12}$ priors from JUNO, the dotted lines are for the cases when both priors are added but the hierarchy is assumed to be unknown, the dashed lines are for only Δm_{ee}^2 prior and hierarchy unknown and the dashed-dotted lines are with Δm_{ee}^2 prior but hierarchy known. The data is generated using the parameters given in section 2 and mass hierarchy is assumed to be inverted in data. For the cases where the mass hierarchy is taken as known, we keep the mass hierarchy

fixed in our fit, while for cases when it is taken as unknown we marginalise our χ^2 over mass hierarchy. We take no NSI in the data while in the fit, we varied the oscillation parameters as well as NSI parameters in the ranges gives in table 1. We see that due to the generalised mass hierarchy degeneracy, for all the hierarchy unknown cases, the fit in the presence of NSI parameters gives two degenerate solutions corresponding to the LMA (true solution) and DLMA (fake solution). On the other hand, if we know the mass hierarchy through measurements at neutrino-less double beta decay experiments and/or cosmology, the corresponding χ^2 show that the DLMA solution can be disfavoured at a very high significance once prior on Δm_{ee}^2 is applied along with the long-baseline data. We note that the entire higher octant corresponding to $\sin^2 \theta_{12} > 0.5$ can be ruled out at greater than 3σ by data from T2HK. For DUNE the significance is slightly less, but still competitive. As we saw in figure 4, the role of the prior on $\sin^2 2\theta_{12}$ is to increase the precision on the solar mixing angle.

7 Conclusions

In presence of NSIs the solar neutrino data is known to give two (nearly) degenerate solutions with same Δm_{21}^2 but with $\sin^2 \theta_{12} \simeq 0.3$ and $\sin^2 \theta_{12}^D \simeq 0.7$, known as the LMA and DLMA solutions, respectively. The existence of this degeneracy has both model building as well as phenomenological implications. One being the so-called generalised mass hierarchy degeneracy wherein the neutrino mass matrix in vacuum remains invariant under the simultaneous exchange of $\Delta m_{31}^2 \leftrightarrow -\Delta m_{32}^2$, $\sin^2 \theta_{12} \leftrightarrow \cos^2 \theta_{12}$ and $\delta_{cp} \leftrightarrow \pi - \delta_{cp}$. This is a problem for experiments like JUNO which aim to measure the neutrino mass hierarchy by observing neutrino oscillations in vacuum. While presence of matter effect could resolve this degeneracy for standard neutrinos, in presence of NSI the transformations $\epsilon_{ee} \rightarrow \epsilon_{ee} - 2$, $\epsilon_{e\mu} \rightarrow \epsilon_{e\mu}^*$, $\epsilon_{e\tau} \rightarrow \epsilon_{e\tau}^*$ and all other $\epsilon_{\alpha\beta} \rightarrow -\epsilon_{\alpha\beta}^*$ are allowed and hence the generalised mass hierarchy degeneracy is restored, making simultaneous measurement of neutrino mass hierarchy and octant of θ_{12} impossible at neutrino oscillation experiments alone.

While this is well known in the literature, in this paper we showed that even if the neutrino mass hierarchy was measured in a non-oscillation such as neutrino-less double beta decay, long-baseline experiments alone would still not be able to determine the octant of θ_{12} . This happens because the dependence on octant of θ_{12} in these experiments come from the measurement of the effective atmospheric mass squared difference. For given measured value of $\Delta m_{\mu\mu}^2$, oscillation parameters Δm_{31}^2 and $\sin^2 \theta_{12}$ are anticorrelated to each other. This intrinsic anticorrelation makes it impossible to measure the octant of θ_{12} using the $P_{\mu\mu}$ channel alone. The P_{ee} channel has Δm_{31}^2 and $\sin^2 \theta_{12}$ correlated and hence, combining the reactor data with the long-baseline data cancels the mentioned (anti)correlation and gives us a unique solution for the octant of $\sin^2 \theta_{12}$. This can therefore resolve the LMA vs DLMA conundrum.

To summarise,

- Due to the generalised mass hierarchy degeneracy, one cannot measure θ_{12} and mass hierarchy simultaneously in any oscillation experiment. This is well known in the literature.

- In this paper we showed that *even* if the mass hierarchy was known, it would *still not be possible* to determine the octant of θ_{12} in a long-baseline experiment.
- We showed that the dependence of the probability on the octant of θ_{12} came via the effective atmospheric mass squared difference that is $\Delta m_{\mu\mu}^2$ for long baseline experiments. Since $\Delta m_{\mu\mu}^2 = \Delta m_{31}^2 - \Delta m_{21}^2(\cos^2 \theta_{12} - \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23})$, this gives a anticorrelation between Δm_{31}^2 and $\sin^2 \theta_{12}$, and hence, no matter how precisely the effective mass squared difference is measured, we get a whole band of allowed values of $\sin^2 \theta_{12}$.
- However, once JUNO and long-baseline experiment(s) are combined, the (anti)correlation between Δm_{31}^2 and $\sin^2 \theta_{12}$ gets cancelled breaking the θ_{12} octant degeneracy.

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