Horndeski genesis: consistency of classical theory

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ABSTRACT: Genesis within the Horndeski theory is one of possible scenarios for the start of the Universe. In this model, the absence of instabilities is obtained at the expense of the property that coefficients, serving as effective Planck masses, vanish in the asymptotics $t \rightarrow -\infty$, which signals the danger of strong coupling and inconsistency of the classical treatment. We investigate this problem in a specific model and extend the analysis of cubic action for perturbations ([arXiv:2003.01202](https://arxiv.org/abs/2003.01202)) to arbitrary order. Our study is based on power counting and dimensional analysis of the higher order terms. We derive the latter, find characteristic strong coupling energy scales and obtain the conditions for the validity of the classical description. Curiously, we find that the strongest condition is the same as that obtained in already examined cubic case.

KEYWORDS: Classical Theories of Gravity, Cosmology of Theories beyond the SM

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1 Introduction

Genesis [1–6] is an interesting non-singular alternative to, or completion of inflationary cosmology. In this scenario, the Universe starts its expansion from static, Minkowski space-time at zero energy density. At the initial stage, energy density builds up and the Hubble parameter grows. This requires the violation of the null energy condition (NEC), see ref. [7] for a review of models with NEC-violation. Models with unusual matter which violates the NEC or null convergence condition [8] often suffer from pathological behavior because of various kinds of instabilities. It was noticed, however, that in Horndeski theory, the NEC can be violated in a stable way. Horndeski theory [9–17] is a scalar-tensor modification of gravity, with the Lagrangian containing second derivatives of the scalar field and yet with the second-order equations of motion. Stable NEC-violation is insufficient for constructing a complete cosmological model, though: it was shown in refs. [18, 19] that the absence of instabilities imposes strong constraints on Horndeski genesis. Nevertheless, there is an example of the Lagrangian [19] which yields stable genesis at the level of classical field theory and linear perturbations. A potential drawback of the model of ref. [19] is that

“effective Planck masses” vanish in the asymptotic past, which may lead to the strong coupling problem and make the classical treatment irrelevant.¹

In refs. [29, 30], the strong coupling problem in the model of ref. [19] has been addressed at the level of cubic action for perturbations. By making use of the dimensional analysis, it has been shown that there exists a region in the parameter space where the classical field theory treatment is legitimate despite the fact that “effective Planck masses” vanish as $t \rightarrow -\infty$. In ref. [31] the strong coupling problem was examined in another model of genesis, involving vector galileon.

The purpose of this paper is to extend the analysis of refs. [29, 30] to all orders of perturbation theory and figure out whether the same conclusion holds: the classical field theory is adequate for describing the Horndeski genesis of ref. [19] in a fairly large region of the parameter space.

Let us remind how the strong coupling problem arises in the genesis model of ref. [19]. Let h_{ij} and ζ denote tensor and scalar metric perturbations about spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background solution in the unitary gauge $\delta\phi = 0$, where ϕ is Horndeski scalar field. The unconstrained quadratic action for these perturbations has the general form

$$\mathcal{S}_{(2)} = \int dt d^3x N_0 a^3 \left[\mathcal{G}_S \frac{\dot{\zeta}^2}{N_0^2} - \frac{\mathcal{F}_S}{a^2} \zeta_{,i} \zeta_{,i} + \mathcal{G}_T \frac{\dot{h}_{ij}^2}{8N_0^2} - \frac{\mathcal{F}_T}{8a^2} h_{ij,k} h_{ij,k} \right], \quad (1.1)$$

where $\mathcal{F}_S$, $\mathcal{G}_S$, $\mathcal{F}_T$, and $\mathcal{G}_T$ are functions of cosmic time t , $a(t)$ is the scale factor, and N_0 is the background lapse function. To avoid ghost and gradient instabilities one requires that the coefficients satisfy

$$\mathcal{F}_S, \mathcal{G}_S, \mathcal{F}_T, \mathcal{G}_T > 0.$$

In the case of genesis, the background is nonsingular: $a(t) \rightarrow 1$ as $t \rightarrow -\infty$, while $N_0 = 1$. Thus, if the functions $\mathcal{F}_S$, $\mathcal{G}_S$, $\mathcal{F}_T$, and $\mathcal{G}_T$ are bounded from below by a strictly positive number, then the integral

$$\int_{-\infty}^t a(t) [\mathcal{F}_T(t) + \mathcal{F}_S(t)] dt$$

is divergent at the lower limit of integration. The no-go theorem of refs. [18, 19] states that in these circumstances, there is a gradient or ghost instability at some stage of the cosmological evolution.

The model of ref. [19] makes use of the observation that this no-go theorem no longer holds if $\mathcal{F}_T \rightarrow 0$, $\mathcal{F}_S \rightarrow 0$, $\mathcal{G}_T \rightarrow 0$ and $\mathcal{G}_S \rightarrow 0$ as $t \rightarrow -\infty$. Their asymptotics are [19]

$$\mathcal{G}_T \propto (-t)^{-2\mu}, \quad \mathcal{F}_T \propto (-t)^{-2\mu}, \quad \mathcal{G}_S \propto (-t)^{-2\mu+\delta}, \quad \mathcal{F}_S \propto (-t)^{-2\mu+\delta}, \quad (1.2)$$

where μ and δ are model parameters with $2\mu > 1 + \delta$ and $\delta > 0$. At the same time, this behavior implies that one may encounter strong coupling regime in the asymptotic past, since the coefficients of quadratic action for metric perturbations, which serve as effective Planck masses, tend to zero as $t \rightarrow -\infty$.

¹It has been shown [20–24] that another way to get around the constraints of refs. [18, 19] is to make use of beyond Horndeski [25, 26] or DHOST theories [27, 28].

However, it has been pointed out in ref. [29] that the fact that $\mathcal{F}_T$, $\mathcal{F}_S$, $\mathcal{G}_T$, and $\mathcal{G}_S$ tend to zero as $t \rightarrow -\infty$ does not necessarily mean that the classical field theory is not applicable for describing the evolution of the background in this asymptotics. To see what is going on, one has to estimate the actual strong coupling energy scale E_{strong} by studying cubic and higher-order interactions. The classical analysis is legitimate for Horndeski genesis if the energy scale E_{class} characteristic of the classical background evolution is lower than E_{strong} ,

$$E_{\text{class}} \ll E_{\text{strong}}.$$

Here the classical energy scale is the inverse time scale of the background evolution; for power-law behavior of the background one has

$$E_{\text{class}} \sim |t|^{-1}.$$

As we mentioned above, for the model of ref. [19] this program has been carried out in refs. [29, 30] at the level of cubic terms in the action for perturbations, and here we consider all orders. We make use of naive dimensional analysis based on power counting and find the strongest constraints on the parameters of the Lagrangian at each order in nonlinearity. When doing so, we disregard at all steps any possible cancellations and, in particular, do not take care of numerical coefficients. The cancellations, if any, can only enlarge the region of the parameter space where the strong coupling problem does not occur. We find that the strongest constraint is the same as that coming from the cubic action of the scalar perturbation studied in detail in ref. [30], i.e., higher order nonlinearities do not add anything new insofar as the strong coupling issue is concerned.

This paper is organized as follows. In section 2 we describe the model and derive the general form of conditions for the absence of strong coupling. In section 3 we introduce our technique based on naive power counting and dimensional analysis of higher order action. Using this technique we find the strongest constraint on model parameters that ensures that the strong coupling energy scale is parametrically above the classical energy scale in the asymptotics $t \rightarrow -\infty$. We discuss our results in section 4. In appendix A we present the expansion of the action in all metric perturbations; in appendix B we express non-dynamical variables through ζ and h_{ij} by solving the constraint equations, still within the perturbation theory and our power counting technique. Finally, appendix C is dedicated to the derivation of the unconstrained action.

2 Generalities

2.1 The model

We study the genesis model of ref. [19] which belongs to a simple subclass of Horndeski theories. The covariant form of the action for this subclass is

$$\mathcal{S} = \int d^3x dt \sqrt{-g} \mathcal{L}, \quad (2.1)$$

where

$$\begin{aligned} \mathcal{L} &= G_2(\phi, X) - G_3(\phi, X) \square \phi + G_4(\phi) R, \\ X &= -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \end{aligned} \quad (2.2)$$

R is the Ricci scalar, $\square\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$ and $G_{2,3,4}$ are some functions of their variables. We use mostly plus metric signature $(-, +, +, +)$ and work in natural units, i.e. $c = \hbar = G = 1$.

Instead of the covariant form, it is convenient for our purposes to use the Arnowitt-Deser-Misner (ADM) decomposition² of the Lagrangian (2.2):

$$\mathcal{L} = A_2(t, N) + A_3(t, N)K + A_4(t, N)(K^2 - K_j^i K_i^j) + B_4(t, N) {}^{(3)}R, \quad (2.3)$$

where $\phi = \text{const}$ hypersurfaces are taken to be constant time hypersurfaces. When it comes to perturbations, the latter property means that we choose the unitary gauge,

$$\delta\phi = 0.$$

The general form of metric is

$$ds^2 = -N^2 dt^2 + \gamma_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right), \quad (2.4)$$

where γ_{ij} is the spatial metric.

The extrinsic curvature and the spatial Ricci tensor are

$$\begin{aligned} K_{ij} &\equiv \frac{1}{2N} (\dot{\gamma}_{ij} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i), \\ {}^{(3)}R_{ij} &\equiv \partial_k {}^{(3)}\Gamma_{ij}^k - \partial_i {}^{(3)}\Gamma_{kj}^k + {}^{(3)}\Gamma_{lk}^k {}^{(3)}\Gamma_{ij}^l - {}^{(3)}\Gamma_{li}^k {}^{(3)}\Gamma_{jk}^l, \end{aligned}$$

while $K = \gamma^{ij}K_{ij}$, ${}^{(3)}R = \gamma^{ij} {}^{(3)}R_{ij}$. Finally, $\sqrt{-g} \equiv N\sqrt{\gamma}$ in action (2.1), where $\gamma \equiv \det({}^{(3)}\gamma_{ij})$.

We study concrete Horndeski theory of ref. [19], in which the Lagrangian functions are specified as follows:

$$A_2 = f^{-2\mu-2-\delta} a_2(N), \quad (2.5a)$$

$$A_3 = f^{-2\mu-1-\delta} a_3(N), \quad (2.5b)$$

$$B_4 = -A_4 = f^{-2\mu}, \quad (2.5c)$$

where μ and δ are constant parameters,³ the same as in (1.2), and $f(t)$ is some function of time such that

$$f \propto -t, \quad t \rightarrow -\infty.$$

It was shown in ref. [19] that one gets around the no-go theorem and builds genesis cosmology by choosing

$$2\mu > 1 + \delta, \quad \delta > 0. \quad (2.6)$$

We use this choice in what follows. The functions a_2 and a_3 entering (2.5) are given by

$$a_2(N) = -\frac{1}{N^2} + \frac{1}{3N^4}, \quad (2.7a)$$

$$a_3(N) = \frac{1}{4N^3}. \quad (2.7b)$$

²The way to convert one formalism to another can be found in refs. [26, 32, 33].

³Note that in refs. [19, 30], μ parameter was denoted by α . We save the notation α for a metric variable.

The asymptotics of the background genesis solution [19] is

$$a \propto 1 + \frac{1}{\delta(-t)^\delta}, \quad N_0 \rightarrow 1, \quad \text{as } t \rightarrow -\infty,$$

where $a(t)$ is the scale factor and N_0 is the background value of lapse function N . The Hubble parameter is $H = \dot{a}/(N_0 a)$ and equals

$$H \propto \frac{1}{(-t)^{1+\delta}}.$$

Wherever possible, we use the asymptotic values $a = N_0 = 1$.

In this paper we concentrate on the analysis of perturbations. The ADM decomposition of the metric (2.4), perturbations included, reads

$$\begin{aligned} N &= N_0(1 + \alpha), \\ N_i &= \partial_i \beta + N_i^T, \quad \text{where } \partial_i N^{Ti} = 0, \\ \gamma_{ij} &= a^2 \left(e^{2\zeta} (e^h)_{ij} + \partial_i \partial_j Y + \partial_i W_j^T + \partial_j W_i^T \right). \end{aligned}$$

We fix residual gauge freedom by setting $Y = 0$ and $W_i^T = 0$, so the spatial part of metric reads

$$\gamma_{ij} = a^2 e^{2\zeta} (e^h)_{ij}$$

with

$$(e^h)_{ij} = \delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} + \frac{1}{6} h_{ik} h_{kl} h_{lj} + \dots, \quad h_{ii} = 0, \quad \partial_i h_{ij} = 0.$$

Variables α , β and N^T enter the action without temporal derivatives; the dynamical degrees of freedom are ζ and transverse traceless h_{ij} , i.e., scalar and tensor perturbations.

2.2 Sketch of the analysis

The purpose of the further discussion in this section is to present the general scheme for deriving the strong coupling energy scales coming from interaction terms in the action. We adopt the most straightforward approach and perform our analysis by making use of the *unconstrained* action written in terms of variables ζ and h (we often omit indices in h_{ij}).

For power-counting purposes, we disregard all numerical coefficients, make use of (2.3), (2.5) and schematically write the asymptotic (large $-t$) expression for the integrand in the action as follows:

$$\sqrt{g} \mathcal{L} \propto (-t)^{-2\mu} \left[(-t)^{-2-\delta} a_2(N) + (-t)^{-1-\delta} a_3(N) K + (K^2 - K_j^i K_i^j) + {}^{(3)}R \right] N \sqrt{\gamma}. \quad (2.8)$$

By varying this action with respect to non-dynamical variables α , β and N_i^T one obtains constraint equations, then solves these equations for α , β and N_i^T , plugs them back into the action and obtains the unconstrained action for ζ and h . Importantly, the parameter μ enters the overall prefactor in (2.8) only, so the constraint equations and hence expressions for α , β and N_i^T are independent of μ , while the unconstrained action has the prefactor $(-t)^{-2\mu}$.

We then expand the unconstrained action in ζ and h_{ij} . Quadratic part is given by (1.1). In accordance to the above discussion, a higher order term of p -th order in scalar ζ and q -th order in tensor h_{ij} in the integrand of the unconstrained action ($p + q \geq 3$) has the following schematic form:

$$(\sqrt{-g}\mathcal{L})_{(pq)} \propto (-t)^{-2\mu} \sum_l (-t)^{d_l} \cdot (\partial_t)^{a_l} \cdot (\partial_i)^{b_l} \cdot \zeta^p \cdot h^q, \quad (2.9)$$

where subscript (pq) refers to orders in ζ and h (no multiplication of p and q), l labels different types of terms, a_l and b_l are the numbers of temporal and spatial derivatives (each acting on either ζ or h), respectively, and d_l are linear in δ . In our dimensional analysis we discard the numerical coefficients in (2.9). An example of the term (2.9) is the cubic action in the scalar sector ($p = 3, q = 0$), written in refs. [30, 34, 35]; in that case, the sum in (2.9) has 17 terms with different numbers of derivatives and/or different time-dependent coefficients. As the dimensionality of temporal and spatial derivatives is the same, for our dimensional analysis we rewrite (2.9) as

$$(\sqrt{-g}\mathcal{L})_{(pq)} \propto (-t)^{-2\mu} \sum_l (-t)^{d_l} \cdot (\partial)^{c_l} \cdot \zeta^p \cdot h^q, \quad (2.10)$$

where we introduce general derivative operator ∂ and count the number of these operators with $c_l \equiv a_l + b_l$. Clearly, the number of terms in the sum in (2.10) is smaller than in (2.9).

The next step is the canonical normalization of perturbations ζ and h_{ij} . The explicit form of the unconstrained quadratic action (1.1) and asymptotic behavior of coefficients (1.2) immediately give the canonically normalized fields

$$(h_{ij})_{(c)} \propto \sqrt{\mathcal{G}_T} h_{ij} \propto (-t)^{-\mu} h_{ij},$$

and

$$\zeta_{(c)} \propto \sqrt{\mathcal{G}_S} \zeta \propto (-t)^{-\mu + \frac{\delta}{2}} \zeta.$$

The fact that the coefficients here tend to zero as $t \rightarrow -\infty$ (due to the restrictions (2.6) imposed on the Lagrangian parameters) is crucial, as it signalizes possible strong coupling regime at early times. In terms of the canonically normalized fields, we have

$$\begin{aligned} (\sqrt{-g}\mathcal{L})_{(pq)} &\propto (-t)^{-2\mu} \sum_l (-t)^{d_l} \cdot (\partial)^{c_l} \cdot \mathcal{G}_S^{-p/2} \cdot \zeta_{(c)}^p \cdot \mathcal{G}_T^{-q/2} \cdot h_{(c)}^q \\ &\equiv \sum_l \Lambda_l \cdot (\partial)^{c_l} \cdot \zeta_{(c)}^p \cdot h_{(c)}^q, \end{aligned} \quad (2.11)$$

where

$$\Lambda_l \equiv (-t)^{-2\mu + d_l} \mathcal{G}_S^{-p/2} \mathcal{G}_T^{-q/2} = (-t)^{-2\mu + d_l + p(\mu - \delta/2) + q\mu}.$$

Now, we make use of dimensional analysis and find strong coupling energy scale E_l associated with each of the terms in the sum in (2.11). The dimension of canonically normalized fields is $[\zeta_{(c)}] = [(h_{ij})_{(c)}] = 1$, while $[\sqrt{-g}\mathcal{L}] = 4$. Thus, the dimension of Λ_l is

$$[\Lambda_l] = [\mathcal{L}] - [\zeta_{(c)}^p] - [h_{(c)}^q] - [\partial^{c_l}] = 4 - p - q - c_l.$$

Note that only terms with $4 - c_l - p - q < 0$ potentially lead to strong coupling. The strong coupling energy scale E_l is

$$E_l \propto \Lambda_l^{-\frac{1}{c_l+p+q-4}} \propto (-t)^{-\frac{-2\mu+d_l+p(\mu-\delta/2)+q\mu}{c_l+p+q-4}}.$$

The requirement of the legitimacy of the classical treatment is $E_{\text{class}} \ll E_l$ for any l . The classical energy scale is inferred from $\dot{H}/H \propto (-t)^{-1}$ (the scale $H \propto (-t)^{-1-\delta}$ is lower), so we have $E_{\text{class}} \propto (-t)^{-1}$. Thus, by requiring $E_{\text{class}} \ll E_l$, we obtain that a given monomial of (pq) -order yields the following condition imposed on the Lagrangian parameters μ and δ (which must obey (2.6)):

$$-2\mu + d_l + p(\mu - \delta/2) + q\mu < c_l + p + q - 4, \quad (2.12)$$

with $p + q \geq 3$, $4 - c_l - p - q < 0$. We rewrite (2.12) as

$$\mu < 1 + \frac{p\delta}{2(p+q-2)} - \frac{(d_l - c_l) + 2}{p+q-2}, \quad (2.13)$$

and see that the most dangerous of (pq) -terms are those with the largest difference $(d_l - c_l)$. So, at given (pq) -order, we have to find the term in $(\sqrt{-g}\mathcal{L})_{(pq)}$ with the largest $d_l - c_l$, and then obtain the smallest right hand side of (2.13) among all p and q (with $p + q \geq 3$). This will give the smallest region of healthy Lagrangian parameters.

We implement this procedure in section 3; details of calculations are given in appendices A, B and C.

3 Implementation of the procedure

3.1 Simplifications

Explicitly evaluating the perturbative expansion of the original action, solving constraints and obtaining the unconstrained action to arbitrary order appears notoriously difficult. However, we make a number of simplifications. We will find in the end that the strongest constraint on the model parameters is the same as that coming from the cubic terms which have been already analyzed in detail [30]. Therefore, all these simplifications do not modify our final result. Our simplifications are as follows.

1. As mentioned above, we discard all numerical factors thus neglecting any possible cancellations. Also, we do not keep track of the tensor structure of various terms in the cubic and higher order action.
2. In accordance with the above discussion, for given p and q we keep only those monomials entering (2.10) in the unconstrained action, which have the largest value of $(d_l - c_l)$. We write the value of $(d_l - c_l)$ as a superscript in front of the expression involving the fields, i.e., we employ the notation

$$(-t)^d (\partial)^c \zeta^p h^q = {}^{(d-c)} \zeta^p h^q \quad (3.1)$$

and do not distinguish monomials with different d and c but the same $(d - c)$.

3. We do the same in the cubic and higher order terms in the original action involving all variables α , β , N^T , ζ and h . To see why this is correct, we write the constraint equations:

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta\alpha} + \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\alpha} = 0, \quad (3.2a)$$

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta\beta} + \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\beta} = 0, \quad (3.2b)$$

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta N^T} + \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta N^T} = 0, \quad (3.2c)$$

where $(\sqrt{-g}\mathcal{L})^{(2)}$ is the quadratic part of the original integrand in action, which is known explicitly (and whose structure will be given below), and $(\sqrt{-g}\mathcal{L})^{H.O.}$ contains cubic and higher order terms. A general term in the latter is (to simplify formulas here, we do not write the overall factor $t^{-2\mu}$ in $(\sqrt{-g}\mathcal{L})$, see (2.8), (2.10))

$$(-t)^D (\partial)^C \alpha^{m_\alpha} \beta^{m_\beta} (N^T)^{m_{N^T}} \zeta^{m_\zeta} h^{m_h} \quad (3.3)$$

with positive integer $m_\alpha, \dots, m_h$. Let us compare effects of terms with the same set of parameters $(m_\alpha, \dots, m_h)$ and different C and D . Upon solving the constraint equations, one finds $\alpha(\zeta, h)$, $\beta(\zeta, h)$, $N^T(\zeta, h)$, again as series in ζ , h and their derivatives, with coefficients depending on t . By substituting them back into (3.3) one finds the term

$$(-t)^D (\partial)^C \alpha^{m_\alpha}(\zeta, h) \beta^{m_\beta}(\zeta, h) (N^T(\zeta, h))^{m_{N^T}} \zeta^{m_\zeta} h^{m_h}.$$

which is a linear combination of expressions (3.1). The largest value of $(d - c)$ for given (p, q) is obtained for the largest value of $(D - C)$. Also, order by order in perturbation theory, the largest contributions to α , β and N^T (in the sense of the largest $(d - c)$ for given (p, q) in the unconstrained action) come from the terms with the largest $(D - C)$. So, for given $(m_\alpha, \dots, m_h)$ we keep only the terms in the original $(\sqrt{-g}\mathcal{L})^{H.O.}$ with the largest $(D - C)$ and, in analogy with (3.1), use the notation

$$(-t)^D (\partial)^C \alpha^{m_\alpha} \beta^{m_\beta} (N^T)^{m_{N^T}} \zeta^{m_\zeta} h^{m_h} \stackrel{(D-C)}{=} \alpha^{m_\alpha} \beta^{m_\beta} (N^T)^{m_{N^T}} \zeta^{m_\zeta} h^{m_h} \quad (3.4)$$

and do not distinguish terms with the same $(D - C)$ but different D and C .

4. Yet another simplification is that we replace some terms in the original $(\sqrt{-g}\mathcal{L})^{H.O.}$, which have the form (3.3), with new ones with larger $(D - C)$. This can only strengthen the constraint on the model parameters (but in fact it does not). Concretely, we make the following replacements in $(\sqrt{-g}\mathcal{L})^{H.O.}$ (we use the notation (3.4)):

$$-2^{-\delta}(1 + \alpha + \dots) \rightarrow -2 (1 + \alpha + \dots), \quad (3.5a)$$

$$-3^{-\delta}\beta(1 + \alpha + \dots) \rightarrow -3 \beta(1 + \alpha + \dots), \quad (3.5b)$$

$$-2^{-\delta}(1 + \alpha + \dots)\zeta N^T \rightarrow -2 (1 + \alpha + \dots)\zeta N^T. \quad (3.5c)$$

We note in passing that similar replacements in quadratic part $(\sqrt{-g}\mathcal{L})^{(2)}$ (more precisely, in the part bilinear in non-dynamical variables) would be impossible, since they would have an effect of erroneously weakening the constraint on model parameters. Finally, we can add arbitrary extra terms to $(\sqrt{-g}\mathcal{L})^{H.O.}$; again, this can only strengthen the constraint on the model parameters (but in fact it does not). We use this observation to replace

$$(1 + \alpha) \rightarrow (1 + \alpha + \alpha^2 + \dots) \quad (3.6)$$

when expanding the term $\sqrt{-g}B_4^{(3)}R$ in (2.3) (and only at that point). These replacements simplify the calculations considerably.

3.2 Dominant terms in the action

With the above notations and simplifications, the dominant terms in the original action have fairly transparent form. We recall that there is the overall factor $t^{-2\mu}$ in $(\sqrt{-g}\mathcal{L})$, see (2.8), (2.10), and that it does not contribute to superscripts $(D - C)$ and $(d - c)$, which are independent of μ . Therefore, it is convenient to write the expressions for $t^{2\mu}\sqrt{-g}\mathcal{L}$ instead of $\sqrt{-g}\mathcal{L}$. The calculation is described in appendix A and gives

$$\begin{aligned} t^{2\mu}\sqrt{-g}\mathcal{L} \supset t^{2\mu}(\sqrt{-g}\mathcal{L})^{(2)} + \left\{ (1 + \alpha + \alpha^2 + \dots) \left[-^2(1 + \zeta + \zeta^2 + \dots) \right. \right. \\ +^{-2}(1 + \zeta + \zeta^2 + \dots)(h^2 + h^3 + \dots) \\ +^{-3}\beta(1 + \zeta + \zeta^2 + \dots)(1 + h + h^2 + \dots) \\ +^{-2}N^T(\zeta + \zeta^2 + \dots) +^{-2}N^T(1 + \zeta + \zeta^2 + \dots)(h + h^2 + \dots) \\ +^{-4}\beta^2(1 + \zeta + \zeta^2 + \dots)(1 + h + h^2 + \dots) +^{-2}(N^T)^2(1 + \zeta + \zeta^2 + \dots)(1 + h + h^2 + \dots) \\ \left. \left. +^{-3}\beta N^T(1 + \zeta + \zeta^2 + \dots)(1 + h + h^2 + \dots) \right] \right\}^{H.O.}, \end{aligned} \quad (3.7)$$

where notation $\{\dots\}^{H.O.}$ means that linear and quadratic parts are omitted. We write $\supset$ sign instead of equality or proportionality signs, since we make replacements (3.5) and (3.6), proceed with the naive analysis and do not care about numerical coefficients and possible cancellations.

The exact second order integrand $(\sqrt{-g}\mathcal{L})^{(2)}$ was evaluated in ref. [36], except for the term involving N^T . The latter term is straightforwardly calculated and has the form $(\partial_i N_j^T)^2$ due to transversality of N_i^T . We again omit numerical coefficients and write

$$\begin{aligned} t^{2\mu}(\sqrt{-g}\mathcal{L})^{(2)} \propto \dot{\zeta}^2 + \zeta_{,i}\dot{\zeta}_{,i} + (-t)^{-\delta-2}\alpha^2 \\ + (-t)^{-\delta-1}\alpha\beta_{,ii} + \dot{\zeta}\beta_{,ii} + (-t)^{-\delta-1}\alpha\dot{\zeta} + \alpha\zeta_{,ii} \\ + \dot{h}_{ij}^2 + h_{ij,k}h_{ij,k} \\ + (\partial_i N_j^T)^2. \end{aligned}$$

Let us compare terms $(-t)^{-\delta-1}\alpha\dot{\zeta}$ and $\alpha\zeta_{,ii}$. In our notations, these are written as $^{-\delta-2}\alpha\dot{\zeta}$ and $^{-2}\alpha\zeta$, respectively. Since $\delta > 0$, the latter term dominates over the former one both in

the constraint equation (3.2a) and in the unconstrained action. So, we neglect the former term. After that, the second order part of the integrand of the action is written as follows:

$$t^{2\mu}(\sqrt{-g}\mathcal{L})^{(2)} \supset -^2\zeta^2 + -^{2-\delta}\alpha^2 + -^{3-\delta}\alpha\beta + -^3\zeta\beta + -^2\zeta\alpha + -^2h^2 + -^2(N^T)^2. \quad (3.8)$$

Note that superscripts of bilinears of non-dynamical variables here (terms with α^2 , $\alpha\beta$ and $(N^T)^2$) have two-fold role. On the one hand, they determine the structure of the linear terms in the constraint equations (3.2) and hence enter the perturbative solution to these equations with flipped signs. As an example, to the linear order, the constraint equation (3.2b) has the form $-^{3-\delta}\alpha + -^3\zeta = 0$ and gives $\alpha \propto -^{\delta}\zeta$. On the other hand, these superscripts appear with their original signs in the expressions for the parts of the unconstrained action obtained by plugging the solutions to constraints $\alpha(\zeta, h)$, $\beta(\zeta, h)$ and $N^T(\zeta, h)$ back into (3.8).

3.3 Solutions for α , β and N_i^T

We now solve the constraint equations (3.2) and find α , β and N_i^T , still within our power-counting approach. To the linear order, we use the action (3.8), and find that the solutions are (see appendix B for details)

$$\alpha_{(pq)} = -^{\delta}\zeta, \quad (3.9a)$$

$$\beta_{(pq)} = -^{\delta+1}\zeta, \quad (3.9b)$$

$$N_{(pq)}^T = 0, \quad p + q = 1. \quad (3.9c)$$

Note that linear order solutions involve ζ only, which should be the case since h_{ij} is traceless and transverse tensor and thus there is no way to construct a linear scalar structure out of it. Also, the solution for N_i^T does not have a linear term.

The solutions to quadratic order are obtained by plugging (3.9) in the cubic part of the action (3.7) and using the result in (3.2). We do this calculation in appendix B. Keeping the dominant terms only (with the largest $(d-c)$ in each term) we obtain

$$\alpha_{(pq)} = -^{\delta}h^2 + ^{3\delta}\zeta^2 + ^{2\delta}\zeta h, \quad (3.10a)$$

$$\beta_{(pq)} = -^{\delta+1}h^2 + ^{3\delta+1}\zeta^2 + ^{2\delta+1}\zeta h, \quad (3.10b)$$

$$N_{(pq)}^T = h^2 + ^{2\delta}\zeta^2 + ^{\delta}\zeta h, \quad p + q = 2. \quad (3.10c)$$

We obtain higher order terms in appendix B by induction and get

$$\alpha_{(pq)} = ^{(2p+q-1)\delta}\zeta^p h^q, \quad (3.11a)$$

$$\beta_{(pq)} = ^{(2p+q-1)\delta+1}\zeta^p h^q, \quad (3.11b)$$

$$(N^T)_{(pq)} = ^{(2p+q-2)\delta}\zeta^p h^q, \quad p + q \geq 2. \quad (3.11c)$$

The complete solutions are linear combinations of (3.9) and (3.11) with all p and q such that $p + q \geq 2$.

3.4 Unconstrained action

Now, we substitute the solutions (3.11) into the second and higher order parts of the action integrand (3.7). We keep terms with maximum value of $(d-c)$ for each combination with fixed $\zeta^p h^q$ in the unconstrained action of cubic and higher order. The details of the calculation are given in appendix C, and here we quote the results. The contribution coming from the second, explicit higher order term in (3.7) is

$$t^{2\mu}(\sqrt{-g}\mathcal{L})^{H.O.} \supset \sum_{p+q \geq 3} (2p+q-3)\delta-2 \zeta^p h^q. \quad (3.12)$$

The quadratic action (3.8) gives the following contribution:

$$t^{2\mu}(\sqrt{-g}\mathcal{L})^{(2)} \supset \sum_{2p+q \geq 4, p+q \geq 3} (2p+q-3)\delta-2 \zeta^p h^q, \quad (3.13)$$

where the condition $p+q \geq 3$ reflects the fact that we are interested in cubic and higher order terms. We see that the structure of the leading terms in the unconstrained action is not particularly complicated.

3.5 Constraint on μ and δ

Now, we recall that a term $(d-c)\zeta^p h^q$ ($p+q \geq 3$) yields a constraint (2.13) on the model parameters, which we reproduce here:

$$\mu < 1 + \frac{p\delta}{2(p+q-2)} - \frac{(d-c)+2}{p+q-2}, \quad p+q \geq 3.$$

Each term in the expression (3.12) has $d-c = (2p+q-3)\delta-2$ and therefore gives the constraint

$$\mu + \delta + \frac{\delta(p-2)}{2(p+q-2)} < 1, \quad p+q \geq 3. \quad (3.14)$$

It is straightforward to see that the strongest of these constraints comes from the terms with $q=0$, $p \geq 3$ (which includes cubic order in the scalar sector), and reads

$$\mu + \frac{3}{2}\delta < 1. \quad (3.15)$$

This constraint coincides with the result of ref. [30] obtained at cubic order.

The cubic and higher order terms in $(\sqrt{-g}\mathcal{L})^{(2)}$, given by (3.13), also have $(d-c) = (2p+q-3)\delta-2$. The constraints on parameters have the same form as (3.14), but now the range of p and q is $2p+q \geq 4$, $p+q \geq 3$. The strongest of these constraints again comes from terms with $q=0$, $p \geq 3$ and has the form (3.15). Thus, the model with parameters obeying (3.15) (together with $2\mu > 1 + \delta$ and $\delta > 0$, see (2.6)) is free of strong coupling problem as long as the validity of classical description of genesis is concerned.

4 Conclusion

To summarize, the model we studied in this paper admits a consistent classical field theory description of the early genesis stage, provided its parameters are chosen in the range

$$\begin{aligned} 2\mu &> 1 + \delta > 1, \\ \mu + \frac{3}{2}\delta &< 1. \end{aligned}$$

This genesis epoch is peculiar, as it begins, as $t \rightarrow -\infty$, at zero “effective Planck masses”, which appears necessary in Horndeski theories (unlike in their generalizations) for avoiding instabilities during the entire evolution. Yet the quantum strong coupling energy scale E_{strong} stays well above the energy scale of classical evolution $E_{\text{class}} \sim t^{-1}$, to the extent that

$$\frac{E_{\text{strong}}(t)}{E_{\text{class}}(t)} \rightarrow \infty \quad \text{as } t \rightarrow -\infty.$$

This is because the interaction terms in the action for perturbations vanish rapidly enough in early-time asymptotics.

Clearly, the model studied in this paper is just an example of a consistent theory of so peculiar beginning of the Universe. Its advantage is that it is simple enough to allow for a reasonably straightforward analysis of the strong coupling issue, as we demonstrated in this paper. The power counting techniques we introduced may possibly be extended to more complicated models.

It would be interesting to examine various ways to incorporate such a genesis model into a full cosmological scenario, i.e. invent and study a healthy transition to the next stage like inflation or straight to the conventional hot epoch. First steps in this direction have been made already [37].

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A Expansion of $\sqrt{-g}\mathcal{L}$ in $\alpha, \beta, \zeta, N_i^T$ and h_{ij}

In this appendix we expand $\sqrt{-g}\mathcal{L}$ in metric perturbations. We discard all numerical factors and keep only the dominant terms, as described in sections 2.2 and 3.1. For this reason we use the symbol $\supset$ instead of equality sign.

The quadratic action is known explicitly, so we concentrate on cubic and higher order terms. We begin with the expression for the three-dimensional Christoffel symbol ${}^{(3)}\Gamma_{ij}^k = \frac{1}{2}\gamma^{ka}(\gamma_{ai,j} + \gamma_{aj,i} - \gamma_{ij,a})$. We substitute $\gamma_{ij} = a^2 e^{2\zeta}(e^h)_{ij}$ here and evaluate the derivative of tensor exponent:

$$(e^h)_{ij,l} = h_{ij,l} + \frac{1}{2}h_{ik,l}h_{kj} + \frac{1}{2}h_{ik}h_{kj,l} + \dots$$

We obtain

$$\begin{aligned}
 {}^{(3)}\Gamma_{ij}^k \supset & (e^{-h})^{ka} e^{-2\zeta} [((e^h)_{ai} e^{2\zeta} \partial_j \zeta + (e^h)_{aj} e^{2\zeta} \partial_i \zeta - (e^h)_{ij} e^{2\zeta} \partial_a \zeta) \\
 & + (e^{2\zeta} \partial_j h_{ai} + e^{2\zeta} \partial_i h_{aj} - e^{2\zeta} \partial_a h_{ij}) \\
 & + (e^{2\zeta} h_{ab} \partial_j h_{bi} + e^{2\zeta} h_{bi} \partial_j h_{ab} + e^{2\zeta} h_{ab} \partial_i h_{bj} + e^{2\zeta} h_{bj} \partial_i h_{ab} - e^{2\zeta} h_{ib} \partial_a h_{bj} - e^{2\zeta} h_{bj} \partial_a h_{ib}) \\
 & + \dots],
 \end{aligned}$$

where dots stand for higher order terms in h_{ij} . Since $(e^h)_{ik}(e^{-h})^{kj} = \delta_i^j$, we write Christoffel symbols schematically as

$$\begin{aligned}
 {}^{(3)}\Gamma_{ij}^k \supset & (\delta_{ki} \partial_j \zeta + \delta_{kj} \partial_i \zeta - \delta_{ij} \partial_k \zeta) \\
 & + (\partial_i h_{kj} + \partial_j h_{ik} - \partial_k h_{ij}) + (\partial h^2 + \partial h^3 + \dots)_{kij}.
 \end{aligned}$$

where $(\partial h^2 + \partial h^3 + \dots)_{kij}$ includes terms $h_{ab} \partial_j h_{bi}$, $h_{bi} \partial_j h_{ab}$, etc. We keep the tensor structure of the linear terms here and in appropriate places below, since we will encounter cancellations associated with it. Hereafter all spatial indices in final expressions are lower ones, so that there are no hidden metric factors like $e^{2\zeta}$ or $(e^h)_{ij}$.

The Lagrangian (2.3) involves extrinsic curvature which we write as

$$K_{ij} = \frac{E_{ij}}{N},$$

where

$$E_{ij} = \frac{1}{2} (\dot{\gamma}_{ij} - {}^{(3)}\nabla_i N_j - {}^{(3)}\nabla_j N_i),$$

with ${}^{(3)}\nabla_i N_j \equiv \partial_i N_j - {}^{(3)}\Gamma_{ij}^k N_k$, and $N_i = \partial_i \beta + N_i^T$. The term $\dot{\gamma}_{ij}$ reads

$$\dot{\gamma}_{ij} = \frac{\partial}{\partial t} (a^2 e^{2\zeta} (e^h)_{ij}) \supset H e^{2\zeta} (e^h)_{ij} + \dot{\zeta} e^{2\zeta} (e^h)_{ij} + e^{2\zeta} (\dot{h}_{ij} + h_{ik} \dot{h}_{kj} + h_{kj} \dot{h}_{ik} + \dots),$$

where dots again denote higher order terms in h_{ij} . We do not expand $e^{2\zeta}$ in all terms here and $(e^h)_{ij}$ in the first two terms, since the next step is the contraction $E_j^i = \gamma^{ik} E_{kj}$ where some of $e^{2\zeta}$ and $(e^h)_{ij}$ cancel out. In notations of section 3.1 we have

$$\dot{\gamma}_{ij} \supset (H + {}^{-1}\zeta) (e^h)_{ij} e^{2\zeta} + {}^{-1}(h + h^2 + h^3 + \dots)_{ij} e^{2\zeta}.$$

We make similar steps for the terms ${}^{(3)}\nabla_i N_j + {}^{(3)}\nabla_j N_i$ and obtain

$$\begin{aligned}
 E_j^i \supset & (H + {}^{-1}\zeta) \delta_j^i + {}^{-1}(h + h^2 + \dots)_{ij} \\
 & + (1 + \zeta + \zeta^2 + \dots) \left((1 + h + h^2 + \dots)_{ik} \partial_k \partial_j \beta + (1 + h + h^2 + \dots)_{lj} {}^{(3)}\Gamma_{il}^k \partial_k \beta \right. \\
 & \left. + (1 + h + h^2 + \dots)_{ik} (\partial_k N_j^T + \partial_j N_k^T) + (1 + h + h^2 + \dots)_{lj} {}^{(3)}\Gamma_{il}^k N_k^T \right).
 \end{aligned}$$

When evaluating the trace $E \equiv E_i^i$ and contraction $E_i^j E_j^i$, we will encounter cancellations due to the properties $h_{ii} = \partial_i h_{ij} = 0$ and $\partial_i N^{Ti} = 0$.

We are ready to expand various terms in $(-t)^{2\mu} \sqrt{-g} \mathcal{L}$ in metric perturbations (the reason for including the factor $(-t)^{2\mu}$ is explained in section 3.2). The factor $N e^{3\zeta}$ in the left hand sides below comes from $\sqrt{-g}$. We obtain the following.

- $\sqrt{-g}A_2$. This term is straightforwardly calculated by expanding in α the function $a_2(N)$ given by (2.7a):

$$(-t)^{2\mu} N e^{3\zeta} A_2 \supset^{-2-\delta} (1 + \alpha + \alpha^2 + \dots)(1 + \zeta + \zeta^2 + \dots) .$$

- $\sqrt{-g}A_3K$. Making use of expansion of function $a_3(N)$ given by (2.7b), we write

$$\begin{aligned} E \equiv K \cdot N \supset H +^{-1} \zeta +^{-1} (h^2 + \dots) \\ +^{-2} \beta(1 + \zeta + \dots)(1 + h + \dots) \\ +^{-1} N^T(1 + \zeta + \dots)(h + \dots) +^{-1} N^T(\zeta + \dots), \end{aligned}$$

and find

$$\begin{aligned} (-t)^{2\mu} N e^{3\zeta} A_3 K \supset^{-1-\delta} (1 + \alpha + \dots) \Big[^{-1-\delta} 1 +^{-1} (\zeta + \dots) +^{-1} (1 + \zeta + \dots)(h^2 + \dots) \\ +^{-2} \beta(1 + \zeta + \dots)(1 + h + \dots) +^{-1} N^T(1 + \zeta + \dots)(h + \dots) +^{-1} N^T(\zeta + \dots) \Big], \end{aligned}$$

where the term $^{-1-\delta} 1$ comes from the Hubble parameter $H \propto (-t)^{-1-\delta}$.

- $\sqrt{-g}A_4(K^2 - K_j^i K_i^j)$. A straightforward calculation gives

$$\begin{aligned} E^2 \supset^{-2-2\delta} 1 +^{-2} \zeta^2 +^{-2-\delta} (h^2 + h^3) +^{-2} (h^4 + \dots) +^{-2-\delta} \zeta +^{-2} \zeta(h^2 + \dots) \\ +^{-4} \beta^2(1 + \zeta + \dots)(1 + h + \dots) \\ +^{-2} (N^T)^2(h^2 + \dots) +^{-2} (N^T)^2(\zeta + \dots)(h + \dots) +^{-2} (N^T)^2(\zeta^2 + \dots) \\ +^{-3} \beta(\zeta + \dots)(1 + h + \dots) +^{-3-\delta} \beta +^{-3} \beta(1 + \zeta + \dots)(h^2 + \dots) \\ +^{-2-\delta} N^T \zeta +^{-2} N^T(\zeta^2 + \dots) \\ +^{-2-\delta} N^T(h + h^2) +^{-2} N^T(h^3 + \dots) +^{-2} N^T(\zeta + \dots)(h + \dots) \\ +^{-3} \beta N^T(h + \dots) +^{-3} \beta N^T(\zeta + \dots) +^{-3} \beta N^T(\zeta + \dots)(h + \dots), \\ E_j^i E_i^j \supset^{-2-2\delta} 1 +^{-2} \zeta^2 +^{-2} (h^2 + \dots) +^{-2-\delta} \zeta +^{-2} \zeta(h^2 + \dots) \\ +^{-4} \beta^2(1 + \zeta + \dots)(1 + h + \dots) +^{-2} (N^T)^2(1 + \zeta + \dots)(1 + h + \dots) \\ +^{-3} \beta(1 + \zeta + \dots)(h + \dots) +^{-3-\delta} \beta +^{-3} \beta(\zeta + \dots) \\ +^{-2-\delta} N^T \zeta +^{-2} N^T(\zeta^2 + \dots) +^{-2} N^T(1 + \zeta + \dots)(h + \dots) \\ +^{-3} \beta N^T(1 + \zeta + \dots)(1 + h + \dots) . \end{aligned}$$

There is some difference between E^2 and $E_j^i E_i^j$. In particular, E^2 contains $^{-2-\delta}(h^2 + h^3) +^{-2}(h^4 + \dots)$, while $E_j^i E_i^j$ includes another structure $^{-2}(h^2 + \dots)$. This happens due to the fact that h_{ij} is traceless and transverse. Together, the two expressions read

$$\begin{aligned} E^2 - E_j^i E_i^j \supset^{-2-2\delta} 1 +^{-2} \zeta^2 +^{-2}(h^2 + \dots) +^{-2-\delta} \zeta +^{-2} \zeta(h^2 + \dots) \\ +^{-4} \beta^2(1 + \zeta + \dots)(1 + h + \dots) +^{-2} (N^T)^2(1 + \zeta + \dots)(1 + h + \dots) \\ +^{-3} \beta(1 + \zeta + \dots)(h + \dots) +^{-3-\delta} \beta +^{-3} \beta(\zeta + \dots) \\ +^{-2-\delta} N^T \zeta +^{-2} N^T(\zeta^2 + \dots) +^{-2} N^T(1 + \zeta + \dots)(h + \dots) \\ +^{-3} \beta N^T(1 + \zeta + \dots)(1 + h + \dots) , \end{aligned}$$

and we obtain

$$\begin{aligned}
 (-t)^{2\mu} N e^{3\zeta} A_4 (K^2 - K_j^i K_i^j) \supset (1 + \alpha + \dots) \Big[&^{-2-2\delta} 1 \\
 &+^{-2-\delta} \zeta +^{-2} (\zeta^2 + \dots) +^{-2} (1 + \zeta + \dots)(h^2 + \dots) \\
 &+^{-4} \beta^2 (1 + \zeta + \dots)(1 + h + \dots) \\
 &+^{-2} (N^T)^2 (1 + \zeta + \dots)(1 + h + \dots) \\
 &+^{-3} \beta (1 + \zeta + \dots)(h + \dots) +^{-3-\delta} \beta +^{-3} \beta (\zeta + \dots) \\
 &+^{-2-\delta} N^T \zeta +^{-2} N^T (\zeta^2 + \dots) +^{-2} N^T (1 + \zeta + \dots)(h + \dots) \\
 &+^{-3} \beta N^T (1 + \zeta + \dots)(1 + h + \dots) \Big] .
 \end{aligned}$$

- $\sqrt{-g} B_4 {}^{(3)}R$. We again make use of the fact that h_{ij} is traceless and transverse and find

$$(-t)^{2\mu} N e^{3\zeta} B_4 {}^{(3)}R \supset (1 + \alpha) \Big[^{-2} (1 + \zeta + \dots)(h^2 + \dots) +^{-2} (\zeta + \dots) \Big] .$$

Note that unlike other terms, this term contains the factor $(1 + \alpha)$ instead of the full series $(1 + \alpha + \alpha^2 + \dots)$. The reason is that both $B_4 = B_4(t)$ and ${}^{(3)}R$ are independent of N .

Collecting all terms together, we find

$$\begin{aligned}
 (-t)^{2\mu} (\sqrt{-g} \mathcal{L})^{H.O.} \supset \Big[&^{-2} (1 + \alpha) (\zeta + \dots) +^{-2-\delta} (\alpha^2 + \dots) \zeta \\
 &+ (1 + \alpha + \dots) \left(^{-2-\delta} 1 +^{-3-\delta} \beta +^{-2-\delta} \zeta N^T +^{-2} (\zeta^2 + \dots) +^{-2} (1 + \zeta + \dots)(h^2 + \dots) \right. \\
 &+^{-3} \beta (1 + \zeta + \dots)(h + \dots) +^{-3} \beta (\zeta + \dots) +^{-2} N^T (\zeta^2 + \dots) +^{-2} N^T (1 + \zeta + \dots)(h + \dots) \\
 &+^{-4} \beta^2 (1 + \zeta + \dots)(1 + h + \dots) +^{-2} (N^T)^2 (1 + \zeta + \dots)(1 + h + \dots) \\
 &\left. +^{-3} \beta N^T (1 + \zeta + \dots)(1 + h + \dots) \right) \Big]^{H.O.} .
 \end{aligned}$$

This expression is simplified by making use of (3.5), i.e., removing δ from all superscripts, and using (3.6) in the first term in square brackets. The result is given by (3.7).

B Solution to constraint equations

In this appendix we solve the constraint equations (3.2) and find non-dynamical variables α , β and N_i^T in the form of series in ζ and h_{ij} .

We begin with eq. (3.2b). The calculation of the variations of (3.7) and (3.8) with respect to β is straightforward and gives (hereafter we do not write the overall factor

$(-t)^{2\mu})$

$$\begin{aligned} \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\beta} = & -^3(\alpha^2 + \dots) + ^3(h^2 + \dots) + ^3(\zeta^2 + \dots) \\ & + ^3(\alpha + \dots)(h + \dots) + ^3(\alpha + \dots)(\zeta + \dots) + ^3(\zeta + \dots)(h + \dots) \\ & + ^3(\alpha + \dots)(h + \dots)(\zeta + \dots) \\ & + (^3N^T + ^4\beta)[(\alpha + \dots) + (\zeta + \dots) + (h + \dots) \\ & + (h + \dots)(\alpha + \dots) + (h + \dots)(\zeta + \dots) + (\alpha + \dots)(\zeta + \dots) \\ & + (\alpha + \dots)(\zeta + \dots)(h + \dots)], \end{aligned} \quad (\text{B.1})$$

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta\beta} = -^{3-\delta}\alpha + ^3\zeta, \quad (\text{B.2})$$

To the linear order, the relevant equation is (B.2), and we immediately get

$$\alpha_{(pq)} = ^\delta\zeta, \quad p + q = 1. \quad (\text{B.3})$$

Next, we vary the action with respect to α and write

$$\begin{aligned} \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\alpha} = & -^2(\alpha^2 + \dots) + ^2(\zeta^2 + \dots) + ^2(\alpha + \dots)(\zeta + \dots) \\ & + ^2(1 + \alpha + \dots)(1 + \zeta + \dots)(h^2 + \dots) \\ & + ^3\beta(h + \dots) + ^3\beta(\zeta + \dots) + ^3\beta(\alpha + \dots) \\ & + ^3\beta(\alpha + \dots)(\zeta + \dots) + ^3\beta(h + \dots)(\zeta + \dots) + ^3\beta(\alpha + \dots)(h + \dots) \\ & + ^3\beta(\alpha + \dots)(\zeta + \dots)(h + \dots) \\ & + ^2N^T(1 + \alpha + \dots)(\zeta + \dots) + ^2N^T(1 + \alpha + \dots)(1 + \zeta + \dots)(h + \dots) \\ & + ^4\beta^2(1 + \alpha + \dots)(1 + \zeta + \dots)(1 + h + \dots) \\ & + ^2(N^T)^2(1 + \alpha + \dots)(1 + \zeta + \dots)(1 + h + \dots) \\ & + ^3\beta N^T(1 + \alpha + \dots)(1 + \zeta + \dots)(1 + h + \dots), \end{aligned} \quad (\text{B.4})$$

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta\alpha} = -^{2-\delta}\alpha + ^{3-\delta}\beta + ^2\zeta. \quad (\text{B.5})$$

Again considering linear order, and using (B.5) and (B.3), we find

$$\beta_{(pq)} = ^{\delta+1}\zeta, \quad p + q = 1. \quad (\text{B.6})$$

Finally, we turn to the variation of the action with respect to N^T :

$$\begin{aligned} \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta N^T} = & -^2(\zeta^2 + \dots) + ^2(\alpha + \dots)(\zeta + \dots) \\ & + (^2h + ^3\beta + ^2N^T)[(h + \dots) + (\zeta + \dots) + (\alpha + \dots) \\ & + (\alpha + \dots)(h + \dots) + (\zeta + \dots)(h + \dots) + (\alpha + \dots)(\zeta + \dots) \\ & + (\alpha + \dots)(\zeta + \dots)(h + \dots)], \end{aligned} \quad (\text{B.7})$$

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta N^T} = -^2N^T, \quad (\text{B.8})$$

so that to the linear order we have

$$N_{(pq)}^T = 0, \quad p + q = 1. \quad (\text{B.9})$$

The linear order solution is summarized in (3.9).

Obtaining perturbative solution is in principle straightforward: to find the solution to order $p + q = n$, one writes the unknown n -th order α , β and N^T in the linear parts of the constraint equations (B.2), (B.5), (B.8), plugs the known lower order expressions for α , β and N^T in non-linear parts (B.1), (B.4), (B.7), evaluates these parts to n -th order and solves the resulting equations for n -th order variables. To quadratic order, we use (B.3), (B.6), (B.9) in quadratic parts of the constraint equations. As an example, eq. (3.2b) reads at quadratic order

$$\begin{aligned} & \frac{\delta(\sqrt{-g}\mathcal{L})^{(2)}}{\delta\beta} \Big|_{(pq)} + \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\beta} \Big|_{(pq)} = \\ & = -3^{-\delta} \alpha_{(pq)} + {}^{-3+2\delta} \zeta^2 + {}^{-3} h^2 + {}^{-3} \zeta^2 + {}^{-3+\delta} \zeta h + {}^{-3+\delta} \zeta^2 + {}^{-3} \zeta h = 0, \quad p + q = 2. \end{aligned}$$

We keep the dominant terms (with the largest values of superscripts) and obtain the second order result

$$\alpha_{(pq)} = {}^{3\delta} \zeta^2 + {}^{\delta} h^2 + {}^{2\delta} \zeta h, \quad p + q = 2.$$

Similar procedure is used to find, with known second-order α , the expression for the second-order β from eq. (3.2a),

$$\beta_{(pq)} = {}^{3\delta+1} \zeta^2 + {}^{\delta+1} h^2 + {}^{2\delta+1} \zeta h, \quad p + q = 2,$$

and, finally, second-order N^T from eq. (3.2c),

$$(N^T)_{(pq)} = {}^{2\delta} \zeta^2 + h^2 + {}^{\delta} \zeta h.$$

Due to algebraic cancellations and the transversality and tracelessness of N_i^T and h_{ij} , some terms in the expressions above may possibly vanish. Nevertheless, we keep all terms, since we proceed with naive analysis and do not take into account these subtleties. The second order solution is summarized in (3.10).

Let us show by induction that the k -th order terms in the solutions to constraint equations are

$$\alpha_{(pq)} = {}^{(2p+q-1)\delta} \zeta^p h^q, \quad p + q = k, \quad (\text{B.10})$$

$$\beta_{(pq)} = {}^1 \alpha_{(pq)}, \quad p + q = k, \quad (\text{B.11})$$

$$N_{(pq)}^T = {}^{-\delta} \alpha_{(pq)} = {}^{-\delta-1} \beta_{(pq)}, \quad p + q = k, \quad k > 1. \quad (\text{B.12})$$

This is the case for $k = 2$. Let us assume that this is the case for $k \leq n - 1$ and show that the same formulas hold for $k = n$.

The general idea is that the n -th order of the non-linear parts of the constraint equations involves only $\alpha_{(pq)}$, $\beta_{(pq)}$ and $N_{(pq)}^T$ at orders $p + q \leq n - 1$, which are known by assumption of induction. Thus, the proof reduces to the evaluation of this non-linear parts (B.1), (B.4), (B.7).

One formula we use in what follows is

$$\begin{aligned}
 (\alpha^m)_{(pq)} &= \left[\left(\delta \zeta + \binom{3\delta}{2} \zeta^2 + \binom{2\delta}{1} \zeta h + \delta h^2 \right) + \dots \right. \\
 &\quad \left. + \left(\binom{(2l-1)\delta}{l} \zeta^l + \binom{(2(l-1)+1-1)\delta}{l-1} \zeta^{l-1} h + \dots + \binom{(l-1)\delta}{1} h^l \right) + \dots \right]_{(pq)}^m \\
 &= \binom{(2p+q-m)\delta}{m} \zeta^p h^q, \quad m > 1,
 \end{aligned} \tag{B.13}$$

which is valid for p and q obeying $2p + q \geq 2m$ (otherwise the left hand side vanishes) and, by assumption of induction, $p + q \leq n$. We also derive another useful formula, where $m > r + s$:

$$\begin{aligned}
 (\alpha^{m-r-s} \zeta^r h^s)_{(pq)} &= \sum_{p_1, q_1} (\alpha^{m-r-s})_{(p_1 q_1)} (\zeta^r h^s)_{(p-p_1, q-q_1)} \\
 &= \sum_{p_1, q_1} \left(\delta_{p-p_1}^r \delta_{q-q_1}^s \zeta^{p-p_1} h^{q-q_1} \right) \left(\binom{(2p_1+q_1-m+r+s)\delta}{p_1} \zeta^{p_1} h^{q_1} \right) \Big|_{2(m-k-r) \leq 2p_1+q_1; r \leq p; s \leq q} \\
 &= \binom{2(p-r)+(q-s)-m+s+r}{p} \zeta^p h^q \Big|_{2(m-s-r) \leq 2p-2r+q-s; r \leq p; s \leq q} \\
 &= \binom{(2p+q-m-r)\delta}{2p+q \geq 2m-s; r \leq p; s \leq q} \zeta^p h^q,
 \end{aligned} \tag{B.14}$$

Here $r \leq p$, $s \leq q$, and $2p + q \geq 2m - s$, otherwise the left hand side vanishes. We also have $p + q \leq n$ by assumption of induction.

Using formulas (B.13) and (B.14), we can compare various terms with one and the same structure $\zeta^p h^q$ in constraints (B.1), (B.4), (B.7) and keep only ones with the largest value of $(d - c)$. To this end, we examine each term in (B.1) one by one.

- We begin with the first term in eq. (B.1):

$$-^3(\alpha^2 + \alpha^3 + \dots)_{(pq)} = \underbrace{-^3(\alpha^2 + \alpha^3 + \dots)_{(pq)}}_{\text{from } (\alpha^2)_{(pq)}} + \underbrace{-^3(\alpha^2 + \alpha^3 + \dots)_{(pq)}}_{\text{from } (\alpha^3)_{(pq)}} + \dots \tag{B.15}$$

From (B.13) we observe that terms with minimum power of α lead to the contributions with the largest value of $(d - c)$ for every (p, q) . So, the dominant term is $(\alpha^2)_{(pq)}$, and we write

$$-^3(\alpha^2 + \alpha^3 + \dots)_{(pq)} = -^3(\alpha^2)_{(pq)}, \quad p + q \leq n,$$

where

$$(\alpha^2)_{(pq)} = \binom{(2p+q-2)\delta}{2} \zeta^p h^q, \quad 2p + q \geq 4.$$

Note that the term (B.15) does not have contributions of order h^2 and h^3 , because α is at least quadratic in h .

- The next two terms $-^3(h^2 + \dots)$ and $-^3(\zeta^2 + \dots)$ in (B.1) give contributions with smaller value of $(d - c)$ as compared to (B.15), except for the cases $q = 2$, $p = 0$ and $q = 3$, $p = 0$. We are not interested in quadratic terms, since we have already studied the quadratic order. So, from these two terms, we (temporarily) keep only $-^3h^3$.

- The next term is $^{-3}(\alpha + \dots)(h + \dots)$. Using the formula (B.14), we observe that the term with the minimum m provides the contribution with the largest $(d - c)$ for every (p, q) . Therefore,

$$^{-3}((\alpha + \dots)(h + \dots))_{(pq)} = ^{-3} \left(\sum_{m=2} \sum_{s=1}^{s=m-1} \alpha^{m-s} h^s \right)_{(pq)} = ^{-3} (\alpha h)_{(pq)}, \quad q \geq 1.$$

In accordance with (B.10), we have

$$(\alpha h)_{(pq)} = {}^{(2p+q-2)\delta} \zeta^p h^q, \quad 2p + q \geq 3, \quad q \geq 1,$$

so that for general (p, q) this term is contained in α^2 , coming from eq. (B.15), except that there are also the terms of order ζh and h^3 . The latter is actually the dominant cubic term of order $^{-3+\delta} h^3$.

- By the same logic as above, we write for the next term $^{-3}((\alpha + \dots)(\zeta + \dots))_{(pq)} = ^{-3}(\alpha \zeta)_{(pq)}$. However, using (B.14) we find that

$$^{-3}(\alpha \zeta)_{(pq)} = ^{-3} (\alpha^{2-1} \zeta^1)_{(pq)} = ^{-3+(2p+q-3)\delta} \zeta^p h^q,$$

and hence this term gives smaller contribution than (B.15).

- The next term $^{-3}(\zeta + \dots)(h + \dots)$ is obviously subdominant as compared to (B.15).
- Again applying the same logic, we write $^{-3}(\alpha + \dots)(\zeta + \dots)(h + \dots) = ^{-3} \alpha \zeta h$. The contributions due to this term are again subdominant.
- Finally, there is the set of terms in (B.1) which has the form $(^{-3} N^T + ^{-4} \beta)[\dots]$, where $[\dots]$ denotes $[(\alpha + \dots) + (\zeta + \dots) + (h + \dots) + (h + \dots)(\alpha + \dots) + (h + \dots)(\zeta + \dots) + (\alpha + \dots)(\zeta + \dots) + (\alpha + \dots)(\zeta + \dots)(h + \dots)]$. We use the assumption of induction (B.11) and (B.12) to write

$$\left((^{-3} N^T + ^{-4} \beta)[\dots] \right)_{(pq)} = \left((^{-3-\delta} \alpha + ^{-4+1} \alpha)[\dots] \right)_{(pq)} = \left((^{-3} \alpha)[\dots] \right)_{(pq)}, \quad p + q \leq n.$$

Then this set of terms involves precisely the same structures as some of the terms studied above, so this contribution gives nothing new.

To summarize, the non-linear term of the constraint equation (B.1) has the (p, q) part dominated by

$$\frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\beta} \Big|_{(pq)} \supset ^{-3+(2p+q-2)\delta} \zeta^p h^q.$$

We recall the form of the linear part, eq. (B.2), and write the equation for $\alpha_{(pq)}$ with $p + q = n$,

$$^{-3-\delta} \alpha_{(pq)} = ^{-3+(2p+q-2)\delta} \zeta^p h^q.$$

This gives

$$\alpha_{(pq)} = {}^{(2p+q-1)\delta} \zeta^p h^q, \quad p + q = n \geq 3,$$

as promised.

The analysis of non-linear parts of other constraint equations, eqs. (B.4) and (B.7), is essentially the same as above. We obtain

$$\begin{aligned} \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta\alpha}\Big|_{(pq)} &\supset^{-2+(2p+q-2)\delta} \zeta^p h^q, & p+q \leq n \\ \frac{\delta(\sqrt{-g}\mathcal{L})^{H.O.}}{\delta N^T}\Big|_{(pq)} &\supset^{-2+(2p+q-2)\delta} \zeta^p h^q, & p+q \leq n. \end{aligned}$$

With linear terms in the constraint equations given by (B.5) and (B.8), this yields (B.11) and (B.12). This completes the proof.

C Unconstrained action

Thus, the linear parts of the non-dynamical variables α , β , N^T are given by (B.3), (B.6), (B.9), while higher order parts are written in (B.10), (B.11) and (B.12). We plug these expressions in the terms (3.7) and (3.8) of the original action, obtain the unconstrained action in this way, and extract the dominant terms. We do this for the higher order action (3.7) explicitly, while the procedure for the quadratic action is similar (and simpler). Recall that we are interested in cubic and higher order terms there.

We firstly express β and N_i^T in terms of α using (B.11) and (B.12) and write:

$$\begin{aligned} (-t)^{2\mu}(\sqrt{-g}\mathcal{L})_{(pq)}^{H.O.} &\supset \left\{ (1+\alpha+\dots) \left[\underbrace{-^2(1+\zeta+\dots)}_{I.} + \underbrace{-^2(1+\zeta+\dots)(h^2+\dots)}_{II.} \right. \right. \\ &\quad \left. \left. + \underbrace{-^2\alpha(1+\zeta+\dots)(1+h+\dots)}_{III.} + \underbrace{-^2\alpha^2(1+\zeta+\dots)(1+h+\dots)}_{IV.} \right] \right\}_{(pq)}^{H.O.}, \end{aligned}$$

where superscript $H.O.$ still means that we keep only cubic and higher order terms in original variables α , β , N^T , ζ and h . As an example, there are no terms of order α and α^2 .

Let us consider each term separately, using (B.13) and (B.14) to extract the dominant contributions. Simple power counting similar to that employed in appendix B gives

$$\begin{aligned} I. \quad &\{(1+\alpha+\dots)(1+\zeta+\dots)\}_{(pq)}^{H.O.} \\ &\supset (\zeta^3+\dots)_{(pq)} + (\alpha^3+\dots)_{(pq)} + [(\alpha^2+\dots)(\zeta+\dots)]_{(pq)} + [(\alpha+\dots)(\zeta^2+\dots)]_{(pq)} \\ &\supset (\alpha^3)_{(pq)}, \end{aligned}$$

Similarly, we find for other three terms:

$$\begin{aligned} II. \quad &\{(1+\alpha+\dots)(1+\zeta+\dots)(h^2+\dots)\}_{(pq)}^{H.O.} \supset (\alpha h^2)_{(pq)} + (h^3)_{(pq)}. \\ III. \quad &\{(\alpha+\dots)(1+\zeta+\dots)(1+h+\dots)\}_{(pq)}^{H.O.} \supset (\alpha^3)_{(pq)} + (\alpha h^2)_{(pq)} + (\alpha^2 h)_{(pq)}, \\ IV. \quad &\{(\alpha^2+\dots)(1+\zeta+\dots)(1+h+\dots)\}_{(pq)}^{H.O.} \supset (\alpha^3)_{(pq)} + (\alpha^2 h)_{(pq)}. \end{aligned}$$

By combining these, we obtain

$$(-t)^{2\mu}(\sqrt{-g}\mathcal{L})_{(pq)}^{H.O.} \supset (-^2\alpha^3 + ^{-2}\alpha h^2 + ^{-2}\alpha^2 h + ^{-2}h^3)_{(pq)}, \quad p+q \geq 3. \quad (C.1)$$

Each term in this expression is non-zero in a certain range of p and q , see eqs. (B.13) and (B.14) and remarks below those formulas. Namely, we have ($p + q \geq 3$ everywhere)

$$\begin{aligned} (\alpha^3)_{(pq)} &= (2p+q-3)^\delta \zeta^p h^q, & \text{with } 2p + q \geq 6, \\ (\alpha h^2)_{(pq)} &= (2p+q-3)^\delta \zeta^p h^q, & \text{with } 2p + q \geq 4, \quad q \geq 2, \\ (\alpha^2 h)_{(pq)} &= (2p+q-3)^\delta \zeta^p h^q, & \text{with } 2p + q \geq 5, \quad q \geq 1. \end{aligned}$$

Still, the linear combination of these terms together with explicit h^3 in (C.1) exhausts all possibilities with $p + q \geq 3$, and we obtain finally

$$(-t)^{2\mu} (\sqrt{-g} \mathcal{L})_{(pq)}^{H.O.} \supset -2^{-(2p+q-3)\delta} \zeta^p h^q \Big|_{p+q \geq 3},$$

which is our formula (3.12). Similar analysis of quadratic part of the action gives (3.13).

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