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$1/N$ correction in holographic Wilson loop from quantum gravity

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ABSTRACT: We study $1/N$ corrections to a Wilson loop in holographic duality. Extending the AdS/CFT correspondence beyond the large N limit is an important but a subtle issue, as it needs quantum gravity corrections in the gravity side. To find a physical property of the quantum corrected geometry of near-horizon black 0-branes previously obtained by Hyakutake, we evaluate a Euclidean string worldsheet hanging down in the geometry, which corresponds to a rectangular Wilson loop in the $SU(N)$ quantum mechanics with 16 supercharges at a finite temperature with finite N . We find that the potential energy defined by the Wilson loop increases due to the $1/N$ correction, therefore the quantum gravity correction weakens the gravitational attraction.

KEYWORDS: AdS-CFT Correspondence, Wilson, 't Hooft and Polyakov loops, Models of Quantum Gravity

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1 Introduction

Whether the AdS/CFT correspondence [1–3] works beyond the large N limit is one of the central problems in holography and quantum gravity. This is simply because the question leads to the consistency of having the holographic principle as a definition of quantum gravity. In spite of the importance, the difficulty in calculating the quantum gravity corrections to the AdS/CFT observables prevented us from the explicit check of the validity of the AdS/CFT beyond the large N limit.

A nontrivial check was made in refs. [4, 5] at which the energy of the near horizon geometry of black 0-branes was shown to coincide with that of the dual $SU(N)$ supersymmetric quantum mechanics at finite temperature, to the first nontrivial correction order in the $1/N^2$ expansion. The analysis on the gravity side was based on the quantum-corrected near horizon geometry obtained by Hyakutake [6], therefore this nontrivial result shows that the geometry is a good test ground for the AdS/CFT correspondence beyond the large N limit.¹

The finite N AdS/CFT offers interesting issues, among which here we mention the phase transition. The Hawking-Page transition [9] in AdS/CFT is known [10] to correspond to confinement/deconfinement transition which is a first order transition at the large N

¹To obtain nontrivial $1/N$ terms in the gravity side, one needs to break at least a part of supersymmetries, as $AdS_5 \times S^5$ is expected to have no correction due to the largest number of supersymmetries. See for example ref. [7] for the check of the spectra at the $1/N^2$ level for an orbifolded S^5 . See also ref. [8] for recent discussions on the $1/N$ corrections to the AdS_5 blackhole entropy.

limit. On the other hand, theories such as QCD at $N = 3$ has a cross-over phase transition, thus the finite N drastically changes the thermal behavior. Even if one includes the $1/N^2$ correction to the gravity metric, intuitively the Hawking-Page transition is expected to persist, and the phase transition is still at the first order, which causes an issue about how the finite N effect should be seen in the gravity side. Interestingly, a gravity geometry inversely obtained to reproduce QCD lattice data [11] using a deep learning technique [12] has a weak repulsive wall just outside the black hole horizon, suggesting the coexistence of confinement and deconfinement phases in gravity.²

To find any local structure of the quantum-corrected black hole geometry, we need some probe. In this paper we evaluate a holographic Wilson loop of the $SU(N)$ quantum mechanics with 16 supercharges at a finite temperature. The gravity dual is a Euclidean string worldsheet ending on the contour of the Wilson loop [13]. Generally the rectangular Wilson loop is related to a potential energy of a quark and an antiquark pair, and here we consider the W -bosons, by separating two of the D0-branes from the remaining N D0-branes which compose the black hole.³ The quantum gravity corrections due to the geometry provides the $1/N^2$ corrections to the potential energy. We will find that the resultant contribution to the potential energy is positive, meaning that the gravitational attraction is weakened due to the quantum gravity corrections to the geometry. This appears to be consistent with the general argument above to make the phase transition weaker.

In addition to the correction due to the bulk quantum effect which we treat in this paper, as for the Wilson loop there exists another correction of the same order $\mathcal{O}(1/N^2)$, that is a higher-genus correction of the Euclidean string worldsheet for the probe. This effect was studied in [14] for the case of a circular Wilson loop in the 4-dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory.⁴ Applying the argument of [14], we find that the higher-genus correction has a temperature dependence different from that for the quantum gravity correction of the geometry, so these two $\mathcal{O}(1/N^2)$ corrections are distinguishable. In this paper, we focus on the quantum gravity correction to the geometry.

The organization of this paper is as follows. First in section 2, we review the quantum-corrected gravity geometry of the black 0-branes in type IIA string theory given by Hyakutake [6], and see that the parameters, N and the temperature, needs to satisfy certain inequalities so that the quantum corrections are dominant against α' corrections. Furthermore, we study the reality condition of the spacetime metric, and the spacetime region where the analysis is valid. In section 3, we evaluate the Wilson loop and its $1/N$ correction due to the quantum correction of the spacetime geometry in the gravity side. The

²The repulsive wall structure was also found in a probe D0-brane potential in the quantum corrected geometry [6].

³Since we treat D0-branes, there is only one direction in their worldvolume: time. To form a rectangular Wilson loop, the other direction is necessary, and here we take it to be a direction in S^8 which is spanned by the scalar field of the quantum mechanics. Therefore the two separated D0-branes are located in different positions. This kind of situation was treated originally in ref. [13].

⁴See also refs. [15–17] for the matching in the AdS/CFT at the leading order in $1/N$. For a different expansion, about circular Wilson loops with a totally symmetric/antisymmetric representation of $SU(N)$ whose dual is not a string worldsheet but a D3-brane/D5-brane, see refs. [18, 19].

obtained potential energy shows that the attractive force of the gravity is weakened by the $1/N$ correction. Section 4 is for our summary and discussions. Appendix A is a review of the details of the quantum corrected geometry [6] which we use in this paper. Appendix B provides the discussion of the temperature dependence of the quantum gravity correction of the geometry and the higher-genus contribution.

2 Spacetime conditions

In this section, we briefly review the result of [6] for introducing the target space of our interest, and analyze necessary conditions for our worldsheet analyses in the later sections to be valid.

2.1 Review: quantum correction of the near horizon black 0-brane geometry

Here we introduce the quantum-corrected near horizon geometry of the black N 0-branes obtained in ref. [6]. Let us describe first the geometry without the quantum correction, that is a near-horizon solution of the type IIA supergravity [20]:

$$ds^2 = l_s^2 \left(-H^{-1/2} F dt^2 + H^{1/2} F^{-1} dU^2 + H^{1/2} U^2 d\Omega_8^2 \right), \quad e^\phi = l_s^{-3} H^{3/4}, \quad (2.1)$$

$$H \equiv \frac{(2\pi)^4 15\pi\lambda}{U^7}, \quad F \equiv 1 - \frac{U_0^7}{U^7}. \quad (2.2)$$

Here ϕ is the dilaton field, while the Ramond-Ramond 1-form is omitted. λ is the 't Hooft coupling of the dual gauge theory, the $SU(N)$ quantum mechanics with 16 supercharges. The radial coordinate is $U \equiv r/l_s^2$ in which we rescaled the original supergravity coordinate r by the string length l_s , thus having the dimension of energy. U_0 is a parameter determining the event horizon of the geometry and resultantly the temperature,

$$\tilde{U}_0 \equiv \frac{U_0}{\lambda^{1/3}} = \left(\frac{7}{16\pi^3 \sqrt{15\pi}} \right)^{-2/5} \tilde{T}^{2/5}. \quad (2.3)$$

The dimensionless coordinate $\tilde{U}_0$ and the dimensionless temperature $\tilde{T} \equiv T/\lambda^{1/3}$ were introduced for convenience.

According to ref. [6], in order to obtain the quantum-corrected geometry, one makes use of the fact that the black 0-brane solution (2.1) is a dimensionally reduced M-wave solution [21] in 11-dimensional supergravity. At the 11 dimensions, some higher derivative corrections to the Einstein-Hilbert action are known, thus derive the 1-loop quantum corrections to the type IIA supergravity in string theory. The geometry solving the corrected 11-dimensional supergravity provides the quantum corrected black 0-brane solution at the dimensional reduction. The obtained geometry is [6]

$$ds^2 = l_s^2 \left(-H_1^{-1} H_2^{1/2} F_1 dt^2 + H_2^{1/2} F_1^{-1} U_0^2 dx^2 + H_2^{1/2} U_0^2 x^2 d\Omega_8^2 \right), \quad (2.4)$$

$$e^\phi = l_s^{-3} H_2^{3/4}. \quad (2.5)$$

Here $x \equiv U/U_0$ is a dimensionless radial coordinate, and the functions H_i, F_1 are defined in appendix A. In the limit $N \rightarrow \infty$, those functions reduce to the original uncorrected geometry (2.1), consistently. Note that the location of the horizon of the geometry, originally at $x = 1$, is shifted to some $x \neq 1$ in this coordinate system.

2.2 Constraints on the temperature and N

The higher derivative corrections of the gravity consist of quantum corrections and stringy corrections. In ref. [6], a condition for finding that the quantum corrections are dominant against the stringy corrections was calculated,

$$\tilde{T} < s^{5/9}, \quad \sqrt{\frac{c_2}{c_1 s}} \tilde{T}^{-3/2} < N < \sqrt{c_1 s} \tilde{T}^{-21/10}. \quad (2.6)$$

These inequalities are derived by the comparison of the various terms in the entropy of the black hole.⁵ The parameter $s (\ll 1)$ introduced here is a small constant to quantify the hierarchy among the magnitudes of the correction terms. In this paper we use $s = 0.1$ below, together with $c_1 = 0.779$ [6] and $c_2 = 0.00459$ [4]. We note that the heat capacity of the geometries (2.4) and (2.1) is positive when (2.6) is satisfied, so these spacetimes are thermodynamically stable [6].

In addition to the inequalities (2.6) found in ref. [6], we find a new inequality to be satisfied for a consistent geometry. The location of the horizon $x = x_h$ is determined by a solution of the equation $F_1(x_h) = 0$, while the other metric factors $H_i(x)$ could vanish in the region $x > x_h$. This leads to an inconsistency, because $H_i(x) = 0$ means that the perturbative correction is comparable to the original uncorrected metric. Furthermore, since the metric (2.4) includes a square root of $H_i(x)$, the metric becomes imaginary. Therefore we have to require

$$H_i(x) > 0 \quad \text{for} \quad x > x_h \quad (2.7)$$

for our geometry and the corrections to be consistent. This inequality is rephrased in terms of N and $\tilde{T}$. A straightforward numerical evaluation shows that a condition

$$H_2(x_h) > 0 \quad (2.8)$$

ensures the constraints for the metric functions (2.7) (see appendix A). So we choose this inequality (2.8) as a necessary condition for $(\tilde{T}, N)$ to be satisfied.

In figure 1, we plot all the inequalities including (2.6). We have to choose a set of values $(\tilde{T}, N)$ from the allowed region shown in figure 1.

2.3 Valid spacetime region

Up to now we have looked at some necessary conditions for the quantum gravity corrections to be a dominant correction and for the spacetime to make sense. In this subsection, we obtain conditions about valid spacetime regions, by assuming that string perturbation makes sense.

As is found in the supergravity metric (2.1), black p -brane supergravity solutions except for $p = 3$ are not scale invariant, and resultantly there appears spacetime regions where

⁵In ref. [6] the comparison was claimed to be made for the internal energy E , assuming a relation $E = TS$.

Figure 1. Allowed regions in $(\tilde{T}, N)$ plot. Blue region shows that for $H_2(x_h) > 0$. Red region shows (2.6) with $s = 0.1$. We also show the region (Yellow) where the heat capacity is negative and thus the spacetime is thermodynamically unstable.

quantum gravity corrections or stringy corrections grow to overwhelm the consistency of string perturbation theory. Perturbation consistency requires

$$l_s^2/\rho^2 \ll 1, \quad (2.9)$$

$$g_s e^\phi \ll 1. \quad (2.10)$$

The first inequality (2.9) means that the typical gravitational curvature scale ρ is much larger than the string scale so that stringy corrections are perturbative, and the second inequality (2.10) means that the string coupling is small enough such that string perturbation theory makes sense.

In our case, as ρ we may take the curvature scale of S^8 , so using the solution (2.4) we rephrase (2.9) and (2.10) as

$$\left[H_2(x)^{1/2} U_0^2 x^2 \right]^{-1} \ll 1, \quad (2.11)$$

$$\frac{(2\pi)^2 \lambda}{N} H_2(x)^{3/4} \ll 1. \quad (2.12)$$

These provides a region in the spacetime at which the string perturbation makes sense. Because the left hand side of (2.11) has a positive power in x and that of (2.12) has a negative one, these inequalities provide the upper bound and the lower bound of x , respectively.

To evaluate the spacetime region concretely for our later purpose, we assume a threshold value $r (< 1)$ for the left hand sides of the inequalities and define the upper bound x_u

Figure 2. Left: plot of the left hand sides of (2.11) and (2.12), for $N = 456$ and $\tilde{T} = 0.02$. A larger N results in a wider region in x . Right: valid spacetime region (White) and our probe string configuration. The shaded regions are excluded for use as the string perturbation breaks down.

and the lower bound x_l for the radial coordinate x ,⁶

$$H_2(x_u)^{1/2} U_0^2 x_u^2 = \frac{1}{r}, \quad (2.13)$$

$$\frac{(2\pi)^2 \lambda}{N} H_2(x_l)^{3/4} = r. \quad (2.14)$$

Solutions of these equations for a given small r , we obtain a spacetime region

$$x_u > x > x_l \quad (2.15)$$

at which the string perturbation expansion makes sense.⁷ Figure 2 Left explains the inequalities for our choice of $N = 456$ and $\tilde{T} = 0.02$ with $r = 0.1$.

In summary, the quantum-corrected geometry (2.4) can be used for $(\tilde{T}, N)$ satisfying (2.6) and (2.8), in the region (2.15). In the next section, we calculate the holographic Wilson loop by putting a probe string in the valid region of the spacetime given by a valid $(\tilde{T}, N)$.

3 $1/N$ corrections to Wilson loop

In this section, we calculate a holographic Wilson loop of the supersymmetric quantum mechanics at a finite temperature, using a string worldsheet in the quantum-corrected geometry (2.4).

⁶These two bounds can be parameterized by different values, but here for simplicity we parameterize them with the single parameter r .

⁷For the uncorrected geometry (2.1), $H_2(x)$ in these conditions is simply replaced by $H(x)$.

3.1 Set-up for the Wilson loop

We follow the original proposal [13, 22] for the holographic calculation of the Wilson loop. We have to make sure that the string worldsheet in the curved geometry exists only in the region where the calculation is valid, (2.15). For this purpose, we put two probe D-branes at $x = x_u$, and consider a fundamental string hanging down from there. See figure 2 Right. These D-branes can be D0-branes. In that case, the Wilson loop we consider is in a background of a nontrivial scalar expectation value:

$$\Phi^I = \begin{pmatrix} 0 & & 0 & 0 \\ & \cdots & 0 & 0 \\ & & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_1^I & 0 \\ 0 & 0 & 0 & 0 & \phi_2^I \end{pmatrix}. \quad (3.1)$$

The $(N+2) \times (N+2)$ matrix Φ^I is separated into $SU(N)$ sector and two $U(1)$ sectors, and the vacuum expectation values ϕ_1^I and ϕ_2^I ($I = 1, 2, \dots, 9$) provide the location of the two probe D0-branes.⁸ We assume $(\phi_1^I)^2 = (\phi_2^I)^2$, and the angle between the two vectors ϕ_1 and ϕ_2 is denoted as ϕ .

The Wilson loop operator relevant to this static string worldsheet is [13]

$$W(C) = \frac{1}{N} \text{tr} P \exp \left[i \oint_C ds \left(\frac{dt}{ds} A_0(t) + \theta^I(s) \Phi^I(t) \sqrt{|dt/ds|^2} \right) \right] \quad (3.2)$$

where the contour C has a rectangular shape in x^0 and θ spacetime, of the size $\tau \times \phi$. Here, θ is a coordinate for the equator of the spherical target space S^8 spanned by the angular part of the nine scalar fields Φ^I . So, when $dt/ds \neq 0$, $\theta^I(s)$ takes either $\phi_1^I/|\phi_1|$ or $\phi_2^I/|\phi_2|$. This generalized Wilson loop corresponding to a fundamental string hanging down from the two separate probe D-branes was first considered in ref. [13]. We take a long rectangle limit $\tau \rightarrow \infty$ so that the Wilson loop has the meaning of the potential energy between two “W-bosons.”

The location of the two D0-branes is given by the vacuum expectation values ϕ_1^I and ϕ_2^I . To place these at $x = x_u$ in the holographic spacetime, we need a physical relation between the scalar field and the spacetime coordinate. The definition of $x = x_u$ is given in (2.13). This equation means that the curvature of the spacetime geometry is not large compared to stringy scale. More precisely, the amplitude of the worldsheet instanton wrapping an S^2 of the S^8 is

$$S_{\text{inst}} = \frac{1}{2\pi l_s^2} \times \text{Vol}(S^2) = \frac{2\rho^2}{l_s^2} \quad (3.3)$$

⁸There should be a finite-temperature potential between the probe D0-branes and the N D0-branes, therefore the probe D0-branes are not stable. This problem would be absent for the case of other kinds of D-branes with different dimensionality. For example, one can additionally place probe D4-branes which form a bound state with the probe D0-branes, so that the location of the D0-branes in the target space can be fixed, like a flavor D-brane [23].

Figure 3. The configuration of the fundamental string in the space spanned by the coordinates (x, θ) . The string hangs down from the end points at $x = x_u$. It reaches $x = x_m$ at the bottom. The bottom needs to satisfy $x_m > x_l$ so that the string lies only in the valid spacetime region.

so the instanton correction is $\exp[-S_{\text{inst}}]$. This instanton correction should have a counterpart in the quantum mechanics, which in principle determines the vacuum expectation value $|\phi_1|$ and $|\phi_2|$, even with the $1/N$ corrections, by the relation $2/r = S_{\text{inst}}$.⁹

3.2 Evaluation of holographic Wilson loop and $1/N$ corrections

To calculate the string worldsheet area S_{NG} , we note that there exists a turning point $x = x_m$ which determines the whole configuration of the string, see figure 3. The configuration of the string is given by a function $\theta(x)$ which can be obtained by integrating the Nambu-Goto equation of motion in the background geometry. Denoting t_E as the Euclidean time, we find the worldsheet area of a string hanging down from the two D0-branes as

$$\begin{aligned}
 S_{\text{NG}} &= 2 \times \frac{1}{2\pi l_s^2} \int_{x_m}^{x_u} dx \int_0^\tau dt_E \sqrt{l_s^4 \frac{\sqrt{H_2(x)}}{H_1(x)} F_1(x) \left(\frac{\sqrt{H_2(x)}}{F_1(x)} U_0^2 + \sqrt{H_2(x)} U_0^2 x^2 \theta'^2 \right)} \\
 &= \frac{\tau U_0}{\pi} \int_{x_m}^{x_u} dx \sqrt{\frac{H_2(x)}{H_1(x)}} (1 + F_1(x) x^2 \theta'^2).
 \end{aligned} \tag{3.4}$$

⁹Note that the ordinary relation to identify $2\pi\alpha'|\phi|$ as the bulk radial coordinate may be corrected by $1/N$ and the temperature, so in our setup to see the quantum gravity correction in the Wilson loop we do not use the ordinary relation. To find a relation between the vacuum expectation value and the bulk location, a physical observable is necessary, and here we propose that it can be the string worldsheet amplitudes. Basically, the relation between our x_u and $|\phi_1|$ can be determined by comparing the potential energy of the D0-brane in the geometry with the free energy of the quantum mechanics. The former was calculated in [6], and the latter calculation, similar to the analysis given for example in [24], can determine the relation.

Here, we have defined $\theta' \equiv d\theta/dx$. The integrand does not depend on θ explicitly, so there exists a Noether invariant

$$p_\theta(x) \equiv \sqrt{\frac{H_2(x)}{H_1(x)}} \frac{x^2 F_1(x)}{\sqrt{\theta'(x)^{-2} + x^2 F_1(x)}} = \text{const.} \quad (3.5)$$

Since at the turning point $x = x_m$ we have $dx/d\theta = 0$, this constant can be easily evaluated at $x = x_m$. Using that, we can refine (3.4) so that the integrand is written only as an explicit function of x . The potential energy $V \equiv \lim_{\tau \rightarrow \infty} S_E/\tau$ which is related to the Wilson loop as $\langle W(C) \rangle = e^{-\tau V}$ is obtained as

$$V(x_m) = \frac{U_0}{\pi} \int_{x_m}^{x_u} dx \sqrt{\frac{H_2(x)}{H_1(x)} \left(1 + \left[\left(\frac{H_2(x)}{H_1(x)} x^2 F_1(x) \right) / \left(\frac{H_2(x_m)}{H_1(x_m)} x_m^2 F_1(x_m) \right) - 1 \right]^{-1} \right)}. \quad (3.6)$$

For this to be interpreted as an observable in the quantum mechanics, we need to calculate the relation between x_m and ϕ . It is obtained by an integration of θ' obtained by (3.5) along the string. We find

$$\phi = 2 \int_{x_m}^{x_u} dx \sqrt{x^2 F_1 \left[\left(\frac{H_2(x)}{H_1(x)} x^2 F_1(x) \right) / \left(\frac{H_2(x_m)}{H_1(x_m)} x_m^2 F_1(x_m) \right) - 1 \right]}. \quad (3.7)$$

Both the potential V , (3.6), and the angle ϕ , (3.7), are expressed by x_m , so eliminating x_m from these two equations, we obtain the potential $V(\phi)$.

The purpose of our study is to compare the potential energy V_0 for the original large N limit with the metric (2.1) and the potential energy V for the metric with the $1/N$ correction, (2.4). To see it, here we write the expression for the large N limit:

$$V_0 = \frac{U_0}{\pi} \int_{x_m}^{x_u} dx \sqrt{\left(1 + [(x^2 F(x)) / (x_m^2 F(x_m))] - 1 \right)^{-1}}, \quad (3.8)$$

$$\phi = 2 \int_{x_m}^{x_u} dx \sqrt{x^2 F(x) ((x^2 F(x)) / (x_m^2 F(x_m))) - 1}. \quad (3.9)$$

These are obtained just by replacing $H_1(x), H_2(x)$ by $H(x)$ and $F_1(x)$ by $F(x)$. From these expressions we can calculate the potential energy $V_0(\phi)$ for the large N limit.

Let us calculate the change of the potential energy by the $1/N$ correction,

$$\Delta V_{N, \tilde{T}}(\phi) \equiv V(\phi) - V_0(\phi). \quad (3.10)$$

Our procedures for numerically evaluating this correction are as follows. First, based on the argument given in section 2.2, we choose an appropriate set $(N, \tilde{T})$ to determine the spacetime geometry. Next, we calculate x_u and x_l by solving (2.13) and (2.14) to find the valid region of the spacetime, as shown in figure 2. In the valid region we vary x_m , and numerically calculate (3.6), (3.7), (3.8) and (3.9) for each x_m comprehensively, to finally evaluate the quantity $\Delta V_{N, \tilde{T}}(\phi)$.

Figure 4. Left: the uncorrected potential V_0 as a function of ϕ . The string worldsheet shrinks as $\phi \rightarrow 0$, so we have $V_0(\phi = 0) = 0$. Right: the $1/N$ energy correction (3.10) for $(N, \tilde{T}) = (456, 0.02)$.

Figure 5. Plot of $\Delta V_{N, \tilde{T}}/\epsilon$ for $N = 1050, 2735, 4000$ with fixed $\tilde{T} = 0.01$. They share the same N^2 dependence, as a check of our numerical calculations.

As an example, we choose $(N, \tilde{T}) = (456, 0.02)$, and show our result $\Delta V_{N, \tilde{T}}$ in figure 4. For any value of ϕ the energy correction ΔV is found to be positive, thus the gravitational attraction is weakened by the quantum gravity correction. Therefore, we conclude that *the $1/N$ correction provides a repulsive force*. For some other values of $(N, \tilde{T})$ we evaluate the potential energy corrections and found again that they are positive, so, as far as we search, the result is robust.

In a sense it is reasonable to expect that the quantum gravity correction may work as a screening of the gravitational force, thus weakening the total gravitational effect. An interesting note is that in ref. [6] the quantum gravity correction to the D0-brane potential due to the corrected spacetime (2.4) was found to be positive. This means that both the D0-brane and the fundamental string feel a repulsive force by the quantum gravity correction of the spacetime.

Let us finally check that the calculated correction ΔV behaves correctly as $1/N^2$. The quantum gravity correction of the metric (2.4) is governed by the small parameter $\epsilon \equiv \frac{\pi^6}{2^7 3^2} \frac{1}{N^2}$. So, the combination $\Delta V_{N, \tilde{T}}/\epsilon$ should not depend on N . In figure 5, we plot $\Delta V_{N, \tilde{T}}/\epsilon$ for $N = 1050, 2735, 4000$ with fixed $\tilde{T} = 0.01$. As expected, the plots with different N are found to be on top of each other.

4 Summary

In this paper we calculated the $1/N$ corrections to the rectangular Wilson loop in the $SU(N)$ quantum mechanics with 16 supercharges at finite temperature, by using the gauge/gravity correspondence. The $1/N$ correction corresponds to the quantum gravity correction in the gravity side, so we introduced the quantum corrected near horizon geometry of the black 0-brane solution (2.4) given in ref. [6]. We put a probe fundamental string in the geometry, then the traditional holographic dictionary provided in refs. [13, 22] gave us the expectation value of the $1/N$ corrected Wilson loop. For the configuration of our probe string, see figure 2 and figure 3.

For the holographic computations we needed to study the consistency conditions. First, we had to choose appropriate temperature and N as shown in figure 1 so that the quantum gravity corrections are dominant against other corrections (see (2.6)) and also that the spacetime metric is real, (2.7). Second, in some regions of the spacetime the background metric cannot be trusted as the correction overwhelms the classical geometry, so we need to work in a restricted spacetime region (2.15). The conditions for the region are given by (2.9) and (2.10), and we adopted the threshold value $r = 0.1$ to set the boundaries of the region. This determines the end points of the probe string, which are on two probe D0-branes whose location corresponds to the vacuum expectation values of the scalar field determined by the magnitude of the worldsheet instantons.

Our final result for the $1/N$ correction ΔV to the potential energy (which is the negative logarithm of the Wilson loop) is shown in figure 4 Right. It turned out that the correction is positive for any angle ϕ between the two end points of the probe string. This means that the $1/N$ correction weakens the potential. In the gravity side, the quantum gravity correction works as a repulsive force, like a screening effect of the attractive gravitational force.

Finally, we make comments on future directions. The important check of holography concerning our results can be made by explicit calculations in the gauge theory side. The D0-brane system admits numerical calculations of the observables [25–28], so we hope to see how the quantum gravity effect may be seen in gauge theories. Due to the validity region of the spacetime, we had to put two probe D0-branes in the valid region so that the Euclidean string worldsheet lies safely for the calculation. The precise location of the probe D0-branes needs to be specified by the string instanton amplitudes, which would be a challenge for the precise numerical matching check of the AdS/CFT correspondence for our case.

The understanding of the quantum gravity corrections is quite essential in holography. A unified understanding of the quantum gravity corrections is necessary for a further investigation of the quantum gravity in the AdS/CFT correspondence, and our finding of the gravitational screening may serve as a hint for it.

A Quantum gravity corrections to the near horizon geometry

Here we provide an explicit expression of the quantum-corrected geometry (2.4) obtained in ref. [6]. The functions H_i, F_1 used to define the geometry are given as

$$H_i \equiv \frac{(2\pi)^4 15\pi \lambda^{-4/3}}{\tilde{U}_0^7} \left(\frac{1}{x^7} + \frac{\epsilon}{\tilde{U}_0^6} h_i \right), \quad (\text{A.1})$$

$$F_1 \equiv 1 - \frac{1}{x^7} + \frac{\epsilon}{\tilde{U}_0^6} f_1, \quad (\text{A.2})$$

with the dimensionless $\tilde{U}_0 \equiv U_0/\lambda^{1/3}$. The corrections are $\mathcal{O}(\epsilon)$ where $\epsilon \equiv \frac{\pi^6}{2^7 3^2} \frac{1}{N^2}$. The functions appearing in these expressions are defined as follows.

$$h_1(x) \equiv \frac{1302501760}{9x^{34}} - \frac{57462469}{x^{27}} + \frac{12051648}{13x^{20}} - \frac{482400}{12x^{13}} - \frac{3747840}{x^7} + \frac{4099200}{x^6} - \frac{1639680(x-1)}{x^7-1} + 117120 \left(18 - \frac{23}{x^7} \right) I(x), \quad (\text{A.3})$$

$$h_2(x) \equiv \frac{19160960}{x^{34}} - \frac{58528288}{x^{27}} + \frac{2213568}{13x^{20}} - \frac{1229760}{13x^{13}} - \frac{2108160}{x^7} + \frac{2459520}{x^6} + 1054080 \left(2 - \frac{1}{x^7} \right) I(x), \quad (\text{A.4})$$

$$h_3(x) \equiv \frac{361110400}{9x^{34}} - \frac{5840032}{x^{27}} - \frac{24021312}{13x^{20}} - \frac{58072000}{13x^{13}} - \frac{210860}{x^7} + \frac{2459520}{x^6} + 117120 \left(1 - \frac{41}{x^7} \right) I(x), \quad (\text{A.5})$$

$$f_1(x) \equiv -\frac{1208170880}{9x^{34}} + \frac{161405664}{x^{27}} + \frac{5738880}{13x^{20}} + \frac{956480}{x^{13}} + \frac{819840}{x^7} I(x). \quad (\text{A.6})$$

The common function $I(x)$ is defined by

$$I(x) \equiv \log \frac{x^7(x-1)}{x^7-1} - \sum_{n=1,3,5} \cos \frac{n\pi}{7} \log \left(x^2 + 2x \cos \frac{n\pi}{7} + 1 \right) - 2 \sum_{n=1,3,5} \sin \frac{n\pi}{7} \left\{ \tan^{-1} \left(\frac{x + \cos \frac{n\pi}{7}}{\sin \frac{n\pi}{7}} \right) - \frac{\pi}{2} \right\}. \quad (\text{A.7})$$

The functions H_i and F_1 approach H and F in the limit $N \rightarrow \infty$ or at large x . Near $x \sim 1$, the correction terms are effective. See figure 6 for the behavior of the functions with the quantum corrections.

B Worldsheet higher-genus correction

As mentioned in the introduction, there exists a correction of the order $\mathcal{O}(1/N^2)$ coming from the higher-genus contribution to the string worldsheet in the gravity picture. Since the quantum gravity correction to the bulk geometry which we used in this paper gives the same $\mathcal{O}(1/N^2)$, the higher-genus correction can compete it. In this appendix we argue that the higher-genus contribution has a different temperature dependence, thus a careful

Figure 6. The behavior of the functions in the metric components. $H(x)$ is the classical geometry without the quantum correction, and $H_1(x)$, $H_2(x)$ and $H_3(x)$ are with the corrections. Left: the case with $N = 456$ and $\tilde{T} = 0.02$. Right: the case with $N = 20000$ and $\tilde{T} = 0.002$.

analysis of both can distinguish the two corrections. Note that in [4] the temperature dependence was important to identify the quantum gravity corrections in the free energy for the check of the AdS/CFT correspondence.

In order to estimate the temperature dependence of the higher-genus correction, we resort to the established argument for exactly calculable cases. The best example we can find for the higher-genus contribution to the worldsheet in the bulk is about the circular Wilson loop in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory [14]. In the following we simply assume that a similar counting of the power of α' and g_s can make sense for our geometry, as the exact argument just relies on a generic counting of the number of moduli of the Riemann surfaces.

In the $\mathcal{N} = 4$ super Yang Mills theory the Wilson loop expectation value can be expanded for large N and for large λ as

$$\langle W \rangle = \sum_{p=0}^{\infty} \frac{1}{N^{2p}} e^{-S_p(\lambda, N)} f_p(\lambda). \quad (\text{B.1})$$

Taking $S_0 = S_1$ as in [14], we find

$$-\log \langle W \rangle = S_0(\lambda, N) - \log f_0(\lambda) - \frac{1}{N^2} \frac{f_1(\lambda)}{f_0(\lambda)} + \mathcal{O}(1/N^4). \quad (\text{B.2})$$

When the background geometry is corrected by the quantum gravity, the first term S_0 has a correction

$$S_0(\lambda, N) = S_0(\lambda, N = \infty) + \frac{1}{N^2} S_0^{(2)}(\lambda) + \mathcal{O}(1/N^4). \quad (\text{B.3})$$

In this paper, we evaluated the second term of this (B.3). In this appendix, we are interested in the comparison between this term and the third term in (B.2) which is the higher-genus correction: both are at the same order in N , $\mathcal{O}(1/N^2)$, for fixed λ .

According to [14], the higher order term can be written as

$$\frac{1}{N^{2p}} f_p(\lambda) \sim (g_s)^{2p} \hbar^p \quad (\text{B.4})$$

where $\hbar$ is the effective Planck constant on the worldsheet theory. In the exactly solvable case with the AdS_5 geometry we have $g_s = \lambda/N$, and $\hbar \sim (l_s/R_{\text{AdS}})^2 \sim 1/\sqrt{\lambda}$ for the worldsheet quantum theory. Therefore

$$\frac{1}{N^{2p}} f_p(\lambda) \sim \frac{1}{N^{2p}} \lambda^{3p/2}. \quad (\text{B.5})$$

Adopting the same logic for our background, we can estimate

$$\hbar \sim (l_s/R)^2 \sim \tilde{T}^{3/5} \quad (\text{B.6})$$

where we have used the S^8 curvature scale $R \sim l_s(H_2)^{1/4}U$, and

$$g_s^{\text{eff}} = g_s e^{-\phi} \sim \frac{\lambda}{N} (H_2)^{3/4} \sim \frac{1}{N} \tilde{T}^{-21/10}. \quad (\text{B.7})$$

These quantities depend on x , so we have evaluated them near the horizon. Substituting these to (B.4), we obtain the temperature dependence of the third term of (B.2) for our case as

$$\frac{1}{N^2} \frac{f_1(\lambda)}{f_0(\lambda)} \sim \frac{1}{N^2} \tilde{T}^{-18/5}. \quad (\text{B.8})$$

On the other hand, our quantum gravity correction, the second term in (B.3), was calculated in this paper as

$$\frac{\epsilon}{(\tilde{U}_0)^6} h_i(x) \sim \frac{1}{N^2} \tilde{T}^{-12/5}, \quad (\text{B.9})$$

as seen from the quantum gravity correction to the metric, (A.1). Let us compare our quantum gravity correction (B.9) and the higher-genus correction (B.8). The important point is that they have a different temperature dependence. So, although they are at the same order in $1/N$, they can be distinguished once the temperature dependence is measured.

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