Scalarization of compact stars in the scalar-Gauss-Bonnet gravity

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ABSTRACT: We study scalarization of horizonless neutral compact reflecting stars. In our model, the scalar hair can be induced by the coupling of static scalar fields to the Gauss-Bonnet invariant. We analytically obtain lower bounds on the coupling parameter. Below the bound, the static scalar hair cannot form. And above the bound, we numerically get the discrete coupling parameter that can support scalar hairs outside stars. We also disclose effects of model parameters on the discrete coupling parameter.

KEYWORDS: Classical Theories of Gravity, Models of Quantum Gravity

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1 Introduction

The recent direct detection of gravitational waves provided the strong evidence that extremely compact black holes exist in nature [1–3]. Astrophysical black holes were usually believed to be endowed with an absorbing horizon. Interestingly, however, some candidate quantum-gravity models suggested that quantum effects may prevent the formation of classical horizons [4]–[8]. An intriguing idea is replacing the horizon by a reflecting wall outside (but extremely close to) the gravitational radius [9]–[15]. From astrophysical and theoretical aspects, it is very interesting to search for similarities and differences between such horizonless reflecting compact stars and classical absorbing black holes.

For classical black holes, one famous property is the no hair theorem, see references [16]–[24] and reviews [25, 26]. The original no hair theorem states that static scalar fields cannot exist outside asymptotically flat black holes. Nevertheless, the scalar hair can form when violating some of their assumptions, such as considering stationary scalar fields or enclosing the black hole in a box [27]–[36]. Another well studied model is considering static scalar fields non-minimally coupled to the Gauss-Bonnet invariant, which allows the existence of thermodynamically stable hairy black holes [37]–[43]. This scalar-Gauss-Bonnet gravity attracted lots of attentions and more complete models were constructed [44]–[52].

For horizonless compact reflecting stars [9]–[15], intriguingly, such no scalar hair behavior also appears. Hod firstly showed that massive static scalar fields cannot exist outside asymptotically flat neutral horizonless compact stars with a scalar reflecting surface [53]. For massless static scalar fields outside such asymptotically flat neutral horizonless reflecting compact stars, the no hair theorem was firstly analysed in [54] and another proof was provided based on Cauchy-Kowalevski theorem [55]. In the asymptotically dS gravity, static scalar hairs still cannot form outside the neutral horizonless reflecting compact stars [56]. With field-curvature couplings, there is also no static hair theorem for the neutral horizonless reflecting stars [54, 57]. It seems that no static hair behavior is a very

general property in the background of neutral horizonless compact reflecting stars, for extended discussion in charged backgrounds see [58]–[67]. On the other side, as mentioned in the front paragraph, black holes can be hairy in the scalar-Gauss-Bonnet gravity. Along this line, it is interesting to examine whether the scalar-Gauss-Bonnet coupling can induce scalar hairs outside neutral horizonless compact reflecting stars.

This work is organized as follows. We firstly construct a system with a scalar field coupled to the Gauss-Bonnet invariant in the background of a neutral horizonless compact reflecting star. Then we analytically obtain a lower bound on the coupling parameter, below which there is no hair theorem. And above the bound, we numerically get discrete coupling parameters that can support horizonless neutral hairy compact stars. At last, we give the main conclusion.

2 Lower bounds on the scalar-Gauss-Bonnet coupling parameter

In this paper, we consider the coupling of massive scalar fields to the Gauss-Bonnet invariant in the asymptotically flat gravity. The general Lagrange density reads [37]–[43]

$$\mathcal{L} = R - |\nabla_\mu \psi|^2 - m^2 \psi^2 + f(\psi) \mathcal{R}_{\text{GB}}^2, \quad (2.1)$$

where R is the Ricci scalar curvature, $\psi(r)$ is the scalar field with mass m , $\mathcal{R}_{\text{GB}}^2$ is the Gauss-Bonnet invariant in the form $\mathcal{R}_{\text{GB}}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$ and $f(\psi)$ represents the coupling function. Neglecting backreaction of scalar fields, there is $\mathcal{R}_{\text{GB}}^2 = \frac{48M^2}{r^6}$. In the linear limit, without loss of generality, one can put the coupling function in a simple quadratic form $f(\psi) = \eta \psi^2$ with η as the coupling parameter [39, 40].

In the Schwarzschild coordinates, the spherically symmetric spacetime is described by [40]

$$ds^2 = -g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (2.2)$$

The metric outside the star is a Schwarzschild type solution $g(r) = 1 - \frac{2M}{r}$, where M is the star mass. We define the radial coordinate $r = r_s$ as the star surface radius. The horizonless condition of the compact star requires the relation $r_s > 2M$.

We take the static scalar field only depending on the radial coordinate in the simple form $\psi = \psi(r)$. After varying the action, the scalar field equation of motion is

$$\psi'' + \left(\frac{2}{r} + \frac{g'}{g}\right) \psi' + \left(\frac{\eta \mathcal{R}_{\text{GB}}^2}{g} - \frac{m^2}{g}\right) \psi = 0 \quad (2.3)$$

with $g = 1 - \frac{2M}{r}$ and $\mathcal{R}_{\text{GB}}^2 = \frac{48M^2}{r^6}$.

We assume that the scalar field vanishes at the star surface. The bound-state (spatially localized) massive scalar fields are characterized by asymptotically decaying behaviors $\psi(r \rightarrow \infty) \sim \frac{1}{r} e^{-mr}$. So the scalar field boundary conditions are

$$\psi(r_s) = 0, \quad \psi(\infty) = 0. \quad (2.4)$$

With a new function $\tilde{\psi} = \sqrt{r}\psi$, one can rewrite the differential equation (2.3) into

$$r^2\tilde{\psi}'' + \left(r + \frac{r^2g'}{g}\right)\tilde{\psi}' + \left(-\frac{1}{4} - \frac{rg'}{2g} + \frac{\eta\mathcal{R}_{\text{GB}}^2r^2}{g} - \frac{m^2r^2}{g}\right)\tilde{\psi} = 0. \quad (2.5)$$

According to boundary conditions (2.4), the new function $\tilde{\psi}$ satisfies

$$\tilde{\psi}(r_s) = 0, \quad \tilde{\psi}(\infty) = 0. \quad (2.6)$$

The function $\tilde{\psi}$ must possess one extremum point $r = r_{\text{peak}}$ between the star surface r_s and the infinity boundary. If r_{peak} is a positive maximum extremum point, there are relations

$$\tilde{\psi}(r_{\text{peak}}) > 0, \tilde{\psi}'(r_{\text{peak}}) = 0, \tilde{\psi}''(r_{\text{peak}}) \leq 0, \quad (2.7)$$

otherwise it will be negative minimum extremum point satisfying

$$\tilde{\psi}(r_{\text{peak}}) < 0, \tilde{\psi}'(r_{\text{peak}}) = 0, \tilde{\psi}''(r_{\text{peak}}) \geq 0. \quad (2.8)$$

In summary, at the extremum point $r = r_{\text{peak}}$, the scalar field is characterized by

$$\{\tilde{\psi} \neq 0, \tilde{\psi}' = 0 \text{ and } \tilde{\psi}\tilde{\psi}'' \leq 0\} \text{ for } r = r_{\text{peak}}. \quad (2.9)$$

Multiplying both sides of (2.5) with $\tilde{\psi}$, we arrive at the equation

$$r^2\tilde{\psi}\tilde{\psi}'' + \left(r + \frac{r^2g'}{g}\right)\tilde{\psi}\tilde{\psi}' + \left(-\frac{1}{4} - \frac{rg'}{2g} + \frac{\eta\mathcal{R}_{\text{GB}}^2r^2}{g} - \frac{m^2r^2}{g}\right)\tilde{\psi}^2 = 0. \quad (2.10)$$

Relations (2.9) and (2.10) yield the inequality

$$-\frac{1}{4} - \frac{rg'}{2g} + \frac{\eta\mathcal{R}_{\text{GB}}^2r^2}{g} - \frac{m^2r^2}{g} \geq 0 \quad \text{for } r = r_{\text{peak}}. \quad (2.11)$$

It can be transformed into

$$(m^2 - \eta\mathcal{R}_{\text{GB}}^2)r^2g \leq -\frac{rgg'}{2} - \frac{1}{4}g^2 \quad \text{for } r = r_{\text{peak}}. \quad (2.12)$$

Since $r \geq r_s > 2M$, we have

$$g = 1 - \frac{2M}{r} = \frac{1}{r}(r - 2M) > 0, \quad (2.13)$$

$$rg' = r \left(1 - \frac{2M}{r}\right)' = \frac{2M}{r} > 0. \quad (2.14)$$

From (2.12)–(2.14), one deduces that

$$m^2 - \eta\mathcal{R}_{\text{GB}}^2 < 0 \quad \text{for } r = r_{\text{peak}}. \quad (2.15)$$

According to (2.15), there are relations

$$\eta > \frac{m^2}{\mathcal{R}_{\text{GB}}^2} = \frac{m^2r_{\text{peak}}^6}{48M^2} \geq \frac{m^2r_s^6}{48M^2}. \quad (2.16)$$

Figure 1. (Color online) We plot the scalar field $\psi(r)$ as a function of the coordinate r with $mr_s = 3.5$ and $mM = 1.5$. The left panel corresponds to $m^2\eta = 132$ and the right panel is with $m^2\eta = 134$.

We describe the system with dimensionless quantity mr_s , mM and $m^2\eta$ in accordance with the symmetry of the equation (2.3) in the form

$$r \rightarrow kr, \quad m \rightarrow m/k, \quad M \rightarrow kM, \quad \eta \rightarrow k^2\eta. \quad (2.17)$$

From (2.16), we obtain a lower bound on the dimensionless coupling parameter $m^2\eta$ as

$$m^2\eta > \frac{m^4 r_s^6}{48M^2}. \quad (2.18)$$

Below this bound, the static scalar field cannot exist outside the horizonless neutral compact reflecting stars. Above this bound, in the following section, we will numerically obtain the scalar hairy configurations supported by horizonless neutral compact reflecting stars.

3 Static scalar hairy configurations supported by horizonless neutral compact reflecting stars

As the scalar field equation (2.3) is of the second order, we need values of $\psi(r_s)$, $\psi'(r_s)$, mr_s , mM and $m^2\eta$ to integrate the equation from the star surface to the infinity. Scalar reflecting surface conditions yield $\psi(r_s) = 0$. With the symmetry $\psi \rightarrow k\psi$, one can take $\psi'(r_s) = 1$. Fixing values of mr_s and mM , we search for the proper $m^2\eta$ that can support a scalar field asymptotically decaying at the infinity.

We show the numerical results with $mr_s = 3.5$, $mM = 1.5$ and various $m^2\eta$ in figure 1. In the left panel, for $m^2\eta = 132$, the scalar field increases to be more positive at the infinity. When we choose $m^2\eta = 134$ in the right panel, the scalar field decreases to be more negative in the larger r region. So $m^2\eta = 132$ and $m^2\eta = 134$ are not related to the bound-state scalar field.

The mathematical solutions of equation (2.3) have the general asymptotic behavior $\psi \approx A \cdot \frac{1}{r} e^{-mr} + B \cdot \frac{1}{r} e^{mr}$ with $r \rightarrow \infty$. We find $B > 0$ in cases of $m^2\eta < 133.002493$ and $B < 0$ in cases of $m^2\eta > 133.002493$, also in accordance with results in figure 1. It implies there is a critical value of $m^2\eta$ corresponding to $B = 0$, which leads to physical

Figure 2. (Color online) We show behaviors of the scalar field function $\psi(r)$ with respect to the coordinate r for $mr_s = 3.5$, $mM = 1.5$ and $m^2\eta = 133.002493$.

mM	1.50	1.55	1.60	1.65	1.70
$m^2\eta$	133.002493	122.364486	112.393858	102.719683	92.479122

Table 1. The coupling parameter $m^2\eta$ with $mr_s = 3.5$ and various mM .

mr_s	3.1	3.3	3.5	3.7	3.9
$m^2\eta$	65.351225	95.872104	133.002493	179.588365	237.794024

Table 2. The coupling parameter $m^2\eta$ with $mM = 1.5$ and various mr_s .

solutions with asymptotically decaying behaviors $\psi \propto \frac{1}{r}e^{-mr}$ at the infinity. With more detailed calculations, we obtain a discrete value $m^2\eta \approx 133.002493$, which corresponds to the scalar field satisfying $\psi(\infty) = 0$. We plot the scalar field with $mr_s = 3.5$, $mM = 1.5$ and $m^2\eta \approx 133.002493$ in figure 2. As shown in the picture, the scalar field approaches zero in the far region. In the case of black holes, as explicitly shown in refs. [38, 39], in the linearized regime, the non-minimally coupled scalar hair is also characterized by a discrete resonant set of the non-trivial coupling parameter. It should be noted that the continuous spectrum found numerically in [38, 39] belongs to non-linear (self-gravitating) field configurations.

In the following, we disclose effects of model parameters mr and mM on the dimensionless scalar-Gauss-Bonnet coupling parameter $m^2\eta$, which can support the existence of bound-state scalar field hairs. In table 1, we show $m^2\eta$ with respect to different mM in the case of $mr_s = 3.5$ and in table 2, we show values of $m^2\eta$ by choosing various mr_s with $mM = 1.5$. We find that $m^2\eta$ decreases as a function of mM and $m^2\eta$ becomes larger if we choose a larger value of mr_s . In particular, according to the data in table 1, $m^2\eta$ is almost a linear function with respect to mM .

4 Conclusions

We studied the formation of scalar field hairs in the background of asymptotically flat horizonless neutral compact stars. We considered a static scalar field coupled to the Gauss-Bonnet invariant. At the star surface, we took the scalar reflecting condition. With analytical methods, we obtained lower bounds on the scalar-Gauss-Bonnet coupling parameter as $m^2\eta > \frac{m^4 r_s^6}{48M^2}$, where η is the coupling parameter, m is the scalar field mass, M is the star mass and r_s represents star radii. Below the lower bound, static scalar fields cannot exist or there is no hair theorem. And above the bound, we numerically obtained discrete coupling parameters $m^2\eta$, which can support static scalar hairs outside horizonless neutral compact reflecting stars. Moreover, we disclosed effects of model parameters on the discrete coupling parameter.

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