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Holographic cosmology from “dimensional reduction” of $\mathcal{N} = 4$ SYM vs. $\text{AdS}_5 \times S^5$

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ABSTRACT: We propose a way to obtain holographic cosmology models for 3+1 dimensional cosmologies vs. 3 dimensional field theories from a “dimensional reduction” procedure, obtained by integrating over the time direction, of (modified) standard holographic duals of 3+1 dimensional field theories. The example of a modified $\mathcal{N} = 4$ SYM vs. $\text{AdS}_5 \times S^5$ is presented, and in perturbation theory doesn’t match observations, though at strong coupling it might. But the proposed mechanism is more general, and it could in principle be applied to other top down holographic models.

KEYWORDS: AdS-CFT Correspondence, Cosmology of Theories beyond the SM

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1 Introduction

The idea of a holographic cosmology has been around for a long time. The first concrete proposal of how that would look like was put forward by Maldacena in [1], stating that the wave function of the Universe, as a function of spatial 3-metrics (and scalars), $\psi[h_{ij}, \phi]$ in some gravity dual background (in his specific case, proposed for some space that asymptotes to de Sitter), equals the partition function of some (3 dimensional) field theory, with sources (for the energy-momentum tensor T_{ij} and some scalar operator $\mathcal{O}$) h_{ij}, ϕ , i.e., $Z[h_{ij}, \phi] = \psi[h_{ij}, \phi]$. However, at the time, there was no concrete proposal for a gravity dual pair.

In [2], such a model was proposed, and a sort of phenomenological holographic cosmology approach was born. It was first noted that, for cosmological scale factors $a(t)$ that are both exponential (as in standard inflation, and corresponding to AdS space) or power law (as in power law inflation, and corresponding to nonconformal D-branes, for instance), a specific Wick rotation, the “domain wall/cosmology correspondence”, turns the cosmology into a standard holographic space like a domain wall, that should have a field theory dual in 3 Euclidean dimensions. A holographic computation then relates the cosmological power spectrum, coming from the $\langle \delta h_{ij}(\vec{x}) \delta h_{kl}(\vec{y}) \rangle$ correlators in the bulk, with $\langle T_{ij}(\vec{x}) T_{kl}(\vec{y}) \rangle$ correlators in the boundary field theory. One can assume a regime where the field theory is perturbative, and the latter correlators can be calculated from Feynman diagrams. Then by comparing the cosmological power spectrum with CMBR data, we can find the best fit in a phenomenological class of field theories, with a “generalized conformal structure”. In [3, 4] (see [5] for an early attempt to match to the CMBR, in WMAP data) it was shown that the phenomenological fit matches the CMBR as well as the (different) standard Λ CDM with inflation, though the perturbative field theory approximation breaks down for modes with $l < 30$. But this holographic cosmology paradigm is more general than the specific class of phenomenological models: it includes standard inflationary cosmology, where the

gravitational side is weakly coupled, as well as intermediate coupling field theory models, that can be treated non-perturbatively on the lattice.¹

Another approach to holographic cosmology was considered in [6–8], where one starts with a “top down” construction (a well-defined gravity dual pair, derived as the decoupling limit of some system of branes), specifically a modified version of the original $\mathcal{N} = 4$ SYM vs. string theory in $\text{AdS}_5 \times S^5$, where an FLRW cosmology with $a(t)$ replaces the Minkowski metric, and a nontrivial dilaton is introduced. On the field theory side, one has a time-dependent coupling now. The model has been used in [7, 8] to show how perturbations entering a Big Crunch exit after the Big Bang, one issue that has been very contentious in ekpyrotic and cyclic cosmologies. It was shown that the spectral index of perturbations exits unchanged, but there was no simple mechanism in [7, 8] of calculating the power spectrum of fluctuations for CMBR.

A natural question to ask then is: can one modify the top down construction of [6–8], to fit it into the holographic paradigm of [2], for which the common concrete realization so far is a phenomenological (bottom up) approach? In this paper, we want to give an answer in the affirmative. We will find that we can modify the general proposal of Maldacena for $Z[h_{ij}, \phi] = \psi[h_{ij}, \phi]$ to deal with this case of having both time and a radial coordinate, and then use an integration over the time coordinate, from close to zero until an arbitrary time t_0 (but not to the future of it, in this way obtaining a function of t_0), to argue that we have effectively a “dimensional reduction” over the time direction. The result is a specific theory with “generalized conformal structure”, but we will see that in perturbation theory it doesn’t fit the CMBR data. However, it could be that by considering a nonperturbative coupling, we have a match. It could also be that one has to apply the above procedure to some other top down holographic duality construction.

The paper is organized as follows. In section 2 we review the holographic cosmology paradigm of McFadden and Skenderis. In section 3 we consider the top-down model coming from the $\mathcal{N} = 4$ SYM model vs. $\text{AdS}_5 \times S^5$, and present our proposal for the extension of the Maldacena map, and the resulting “dimensional reduction” in the time direction. We also show that the dilaton transforms in the bulk, resulting in an operator VEV on the boundary, that depends on the cosmological solution. In section 4 we conclude.

2 Holographic cosmology paradigm

In this section we review the holographic cosmology paradigm of [2].² One considers a cosmological FLRW model, coupled with a scalar ϕ , and having fluctuations in both,

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)[\delta_{ij} + h_{ij}(t, \vec{x})]dx^i dx^j, \\ \phi(t, \vec{x}) &= \phi(t) + \delta\phi(t, \vec{x})a. \end{aligned} \tag{2.1}$$

After a Wick rotation, the “domain wall/cosmology correspondence”, putting $t = -iz$, but also $\bar{\kappa}^2 = -\kappa^2$, $\bar{q} = -iq$ (here κ is the Newton constant and q is momentum), which in

¹Lattice work on this is ongoing.

²See [9] for an early attempt to relate inflation with holography.

field theory corresponds to $\bar{q} = -iq$, $\bar{N} = -iN$, we obtain the domain wall gravity dual

$$\begin{aligned} ds^2 &= +dz^2 + a^2(z)[\delta_{ij} + h_{ij}(z, \vec{x})]dx^i dx^j, \\ \phi(z, \vec{x}) &= \phi(z) + \delta\phi(z, \vec{x})a. \end{aligned} \quad (2.2)$$

The generic “domain wall” above can correspond to (asymptotically) AdS space, for (asymptotically) exponential $a(z)$, in which case expect a field theory that is conformal in the UV. Or it can correspond to some holographic dual of the type of nonconformal branes, for power law $a(z)$, in which case one expects a “generalized conformal structure”: the theory has as only dimensional parameter the YM coupling g_{YM} , which appears as an overall factor in front of the action. Therefore it is of the type that we would obtain by dimensionally reducing a 4 dimensional conformal field theory.

Specifically, the phenomenological class of models considered for the fit to the CMBR is a super-renormalizable theory of $SU(N)$ gauge fields A_i^a , scalars ϕ^{aM} and fermions ψ^{aL} , where a is an adjoint $SU(N)$ index and M, L are flavour indices, with action

$$\begin{aligned} S_{\text{QFT}} &= \int d^3x \text{Tr} \left[\frac{1}{2} F_{ij} F^{ij} + \delta_{M_1 M_2} D_i \Phi^{M_1} D^i \Phi^{M_2} + 2\delta_{L_1 L_2} \bar{\psi}^{L_1} \gamma^i D_i \psi^{L_2} \right. \\ &\quad \left. + \sqrt{2} g_{YM} \mu_{ML_1 L_2} \Phi^M \bar{\psi}^{L_1} \psi^{L_2} + \frac{1}{6} g_{YM}^2 \lambda_{M_1 \dots M_4} \Phi^{M_1} \dots \Phi^{M_4} \right] \\ &= \frac{1}{g_{YM}^2} \int d^3x \text{Tr} \left[\frac{1}{2} F_{ij} F^{ij} + \delta_{M_1 M_2} D_i \Phi^{M_1} D^i \Phi^{M_2} + 2\delta_{L_1 L_2} \bar{\psi}^{L_1} \gamma^i D_i \psi^{L_2} \right. \\ &\quad \left. + \sqrt{2} \mu_{ML_1 L_2} \Phi^M \bar{\psi}^{L_1} \psi^{L_2} + \frac{1}{6} \lambda_{M_1 \dots M_4} \Phi^{M_1} \dots \Phi^{M_4} \right]. \end{aligned} \quad (2.3)$$

Here $\lambda_{M_1 \dots M_4}$ and $\mu_{ML_1 L_2}$ are dimensionless, and only g_{YM} is dimensional, and in the second line the fields have been rescaled by g_{YM} in order to obtain g_{YM} as an overall factor, and the dimensions of the fields to be the ones in 4 dimensions.

The generalized conformal structure means that the momentum dependence organizes into a dependence on the effective dimensionless coupling of the theory,

$$g_{\text{eff}}^2 = \frac{g_{YM}^2 N}{q}. \quad (2.4)$$

Correlators will thus depend on g_{eff}^2 , and in perturbation theory one obtains, as usual, a combination of powers of g_{eff}^2 and $\ln g_{\text{eff}}^2$.

The CMBR power spectrum is defined in terms of the standard scalar and tensor fluctuations in momentum space $\zeta(q)$ and $\gamma_{ij}(q)$ as

$$\begin{aligned} \Delta_S^2(q) &\equiv \frac{q^3}{2\pi^3} \langle \zeta(q) \zeta(-q) \rangle \\ \Delta_T^2(q) &\equiv \frac{q^3}{2\pi^3} \langle \gamma_{ij}(q) \gamma_{ij}(-q) \rangle. \end{aligned} \quad (2.5)$$

In principle, one could relate them to the two-point functions of the energy-momentum tensors via the Maldacena relation $Z[h_{ij}] = \psi[h_{ij}]$ as follows. From general theory, the

partition function is represented as the generating functional of correlators as

$$Z[h_{ij}] = \exp \left[\int \frac{1}{2} h^{ij} \langle T_{ij} T_{kl} \rangle h^{kl} + \dots \right], \quad (2.6)$$

which leads to the 2-point function of cosmological fluctuations h_{ij} as

$$\langle h_{ij} h_{kl} \rangle = \int \mathcal{D}h_{mn} |\psi[h_{pq}]|^2 h_{ij} h_{kl} \sim \frac{1}{\text{Im} \langle T_{ij} T_{kl} \rangle}, \quad (2.7)$$

where the last equality is qualitative, and involves a nontrivial calculation.

The more precise relation was found in [10], based on the formalism in [11, 12], and is reviewed in the appendix. Decomposing the energy-momentum tensor correlators as

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl}, \quad (2.8)$$

where

$$\Pi_{ijkl} = \pi_{i(k} \pi_{l)j} - \frac{1}{2} \pi_{ij} \pi_{kl}, \quad \pi_{ij} = \delta_{ij} - \frac{\bar{q}_i \bar{q}_j}{\bar{q}^2} \quad (2.9)$$

are the 4-index transverse traceless projection operator (Π_{ijkl}), and the 2-index transverse projection operator (π_{ij}), we obtain the power spectra

$$\begin{aligned} \Delta_S^2(q) &= -\frac{q^3}{16\pi^2 \text{Im} B(-iq)} \\ \Delta_T^2(q) &= -\frac{2q^3}{\pi^2 \text{Im} A(-iq)}, \end{aligned} \quad (2.10)$$

where we have already performed the analytical continuation to Lorentzian signature through $\bar{q} = -iq$ and $\bar{N} = -iN$.

3 Top-down model from dimensional reduction of $\mathcal{N}=4$ SYM vs. $\text{AdS}_5 \times S^5$

Another holographic approach was developed in [6–8], and we will present it in a way that can fit into the holographic cosmology paradigm from the previous section.

We consider a 4+1 dimensional geometry that is a solution of the 10 dimensional type IIB equations of motion, with a metric ansatz

$$ds^2 = \frac{R^2}{z^2} [dz^2 + (-dT^2 + a^2(T) d\vec{x}^2)] + R^2 d\Omega_5^2, \quad (3.1)$$

and with a nontrivial dilaton $\phi = \phi(T)$.

More generally, for the metric ansatz

$$ds^2 = \frac{R^2}{z^2} [dz^2 + g_{\mu\nu}(x) dx^\mu dx^\nu] + R^2 d\Omega_5^2, \quad (3.2)$$

the equations of motion are

$$R_{\mu\nu}[g_{\rho\sigma}] = \frac{1}{2} \partial_\mu \phi \partial_\nu \phi, \quad \partial_A \left(\sqrt{-G} G^{AB} \partial_B \phi \right) = 0, \quad (3.3)$$

where G_{AB} is the 5-dimensional metric. With a flat FLRW cosmological ansatz as in (3.1), one finds the unique solution

$$a(T) \propto T^{1/3}, \quad e^{\phi(T)} = \left(\frac{T}{R}\right)^{2/\sqrt{3}}, \quad (3.4)$$

which corresponds to a “stiff matter” cosmology, with equation of state $P = w\rho$, with $w = +1$. Indeed, in general for FLRW we have $a(T) \propto T^{\frac{2}{3(1+w)}}$.

Making a transformation to conformal time t (usually called η), we obtain

$$-dT^2 + a^2(T)d\vec{x}^2 = a^2(t)[-dt^2 + d\vec{x}^2] \Rightarrow a \sim T^{1/3} \sim t^{1/2}, \quad (3.5)$$

so in particular

$$e^{\phi(t)} = \left(\frac{t}{R}\right)^{\sqrt{3}}. \quad (3.6)$$

In fact, for a general homogeneous and isotropic cosmological ansatz for the metric, we have

$$R_{00}[g_{\rho\sigma}] = -3\frac{\ddot{a}}{a}, \quad R_{ij}[g_{\rho\sigma}] = \left(\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2}\right)\delta_{ij}, \quad (3.7)$$

with $k = -1, 0, 1$ for open, flat and closed universes. So, to have a 10 dimensional solution with homogeneous dilaton, we should have

$$\dot{\phi}^2 = -6\frac{\ddot{a}}{a}, \quad \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{k}{a^2} = 0, \quad (3.8)$$

which in conformal time reads

$$\left(\frac{d\phi}{dt}\right)^2 = 6\left[\frac{1}{2a^4}\left(\frac{d}{dt}(a^2)\right)^2 + 2k\right], \quad \frac{1}{2a^2}\frac{d^2}{dt^2}(a^2) + 2k = 0. \quad (3.9)$$

Solving these equations for $k = 0$ gives the results before. For $k = 1$ the solution is

$$a(t) \propto |\sin(2t)|^{1/2}, \quad e^{\phi(t)} \propto |\tan(t/R)|^{\sqrt{3}}, \quad (3.10)$$

and for $k = -1$ we have

$$a(t) \propto |\sinh(2t)|^{1/2}, \quad e^{\phi(t)} \propto |\tanh(t/R)|^{\sqrt{3}}. \quad (3.11)$$

We conclude that, for homogeneous dilaton, there is unique solution for each possible spatial “topology” (in the restricted sense associated with the sign of the curvature, of closed, open, or flat).

Note that the original G_{AB} metric was in Einstein frame, and $\phi(T)$ was the dilaton. If we make the conformal transformation by $a(T)$ we move away from the Einstein frame. Then $\phi(T) = \phi(t)$ is the dilaton, thus $e^{\phi(t)}$ is the string coupling, corresponding in the boundary field theory to the YM coupling $g_{YM}^2/(4\pi)$. In terms of the time t of Minkowski space, we have then a time-dependent SYM coupling,

$$g_{YM}(t) = g_{YM,0} \left(\frac{|t|}{R}\right)^{\sqrt{3}}. \quad (3.12)$$

The conformal transformation on the boundary is allowed, given that the boundary field theory is conformal. However, when doing that in holography, we will obtain a modification of the holographic map, that will be calculated in the next subsection.

As an aside, note that the solution

$$ds_4^2 = |\sinh(2t)| \left[-dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2^2 \right]$$

$$e^{\phi(t)} = g_s |\tanh(t/R)|^{\sqrt{3}} \quad (3.13)$$

is not just conformally flat, but actually asymptotically flat. For $t \rightarrow \pm\infty$, there is a coordinate transformation that takes away the conformal factor, giving $\text{AdS}_5 \times S^5$ and constant dilaton in these regimes, as analyzed in [6]. However, *close to the strong coupling gravity region* $t \sim 0$, we get still a conformal factor deviating from 1, and the solution is the same as before, $a^2(t) \propto t$ and $e^{\phi(t)} \propto t^{\sqrt{3}}$.

Until now, we have presented solutions with Lorentzian signature. However, the AdS/CFT correspondence is known to be better understood and defined in Euclidean signature space, and the Wick rotation to Lorentzian signature to be a difficult issue.³ Therefore, one does not simply Wick rotate the solutions we found to Euclidean signature, but rather considers a *mapping* from the above solutions to solutions of the Euclidean version of supergravity (thus string theory), and that is where we assume that our correspondence is defined.⁴ Specifically, the equations of motion (3.9) for $k = 0$ are invariant under the standard Wick rotation $t = -it_E$, which means both in Lorentzian and Euclidean signature we have the same solutions (3.4), and we can choose real prefactors in both cases, i.e. $(a(t) = a_0(t/t_0)^{1/2}, e^{\phi(t)} = e^{\phi_0}(t/R)^{\sqrt{3}})$ and $(a_E(t_E) = a_{0,E}(t_E/t_{0,E})^{1/2}, e^{\phi_E(t_E)} = e^{\phi_{0,E}}(t_E/R)^{\sqrt{3}})$, even though the two solutions are not Wick rotations of each other (in which case we would need to define branch cuts, obtain complex prefactors, etc.). From now on, we will assume the Euclidean signature solutions and dual field theory.

In order to embed the approach presented in this section into the paradigm from the last section, we need to consider how to extend it to the case when there is both a radial coordinate, and a time coordinate. For the general set-up of Maldacena, the wavefunction of the Universe $\psi[h_{ij}]$ is evolved in time with the Hamiltonian, which corresponds on the boundary to the RG flow of the correlators obtained from $Z[h_{ij}]$, as the energy scale is varied. In the framework of [2], the Wick rotation (“domain wall/cosmology correspondence”)

³In particular, in many cases the correspondence is better defined in global coordinates, but there is not a clear relation between the natural Wick rotation in global coordinates (mapped to “radial” Wick rotation in the boundary Minkowski space, $r = -ir_E$) and the standard Wick rotation in Poincaré coordinates (mapped to the usual $t = -it_E$). Also in the case of the pp wave correspondence [13], obtained as a Penrose limit of AdS/CFT, the issue of Wick rotation is extremely subtle, as found for instance in [14]. In all these cases, the implicit assumption is that the field theory dual to the Euclidean signature solution is still the Euclidean version of the field theory, and that is the starting point for defining the correspondence.

⁴This is similar to the “domain wall/cosmology correspondence” of Skenderis and Townsend [15], used in the definition of the phenomenological holographic cosmology model of McFadden and Skenderis [2] and reviewed in the previous section. There also, one does not have a Wick rotation per se, but rather a mapping of solutions from ones of a domain wall type to ones of a cosmology type (similar to a double Wick rotation), and only then Wick rotates the domain wall from Lorentzian to Euclidean signature.

means that time evolution is replaced by a radial “Hamiltonian” evolution, corresponding to the same, and in line with the usual AdS/CFT construction.

The Maldacena map is based on the fact that the wavefunction of the Universe can be thought of as a path integral, integrated over time (in the past), but with the boundary condition of spatial 3-metric h_{ij} at the corresponding time t . Then it is really just a type of analytical continuation of the usual AdS/CFT map between the partition function of the field theory, with sources h_{ij} , and the partition function of the gravity or string theory (written as a path integral), with a boundary condition of h_{ij} .

But now we have both a radial direction and a time direction, and we have to decide how to generalize the set-up of Maldacena to this situation, so that maybe in a second step, we can relate it to the paradigm of [2].⁵

There are now two possible Hamiltonians in the gravitational theory: both the radial one, who gives the evolution that, via the usual AdS/CFT correspondence, corresponds to the RG flow of the boundary field theory, and the true Hamiltonian, which gives the evolution of gravity along the time direction, and should similarly correspond to a Hamiltonian evolution in time in the boundary field theory.

It seems therefore reasonable to assume that the correct prescription to use is to have, on the gravity side, a partition function with boundary condition both at time t and at radial size r , which therefore is still a wavefunction of the Universe, corresponding in field theory to a partition function integrated over time until the corresponding time t , and both be as usual functions of spatial 3-metrics h_{ij} ,

$$\psi[h_{ij}]_{t,r} = Z[h_{ij}]_{t,q}. \quad (3.14)$$

Here q is the energy scale corresponding holographically to the radial direction r in the bulk. The time t is arbitrary, and the path integration is assumed to be for times between $-\infty$ and t , but not in its future. In this way, both sides of the equation are functions of this time t , which are evolved with the Hamiltonian. Of course, in the context of the “top-down” model, the bulk will have a time-dependent Hamiltonian, which can be interpreted in terms of particle production. The evolution with the Hamiltonian from t to t' should be equivalent with the path integration until a later time t' .

Next, we need to understand the effect of the integration over t on both sides of the equality, and how to take it into account. On the gravity side, the integration over time gives the wavefunction of the Universe, and there is nothing we need to do with it. Since the holographic map is the same, the calculation of the correlators of metric fluctuations in (2.7) is unchanged, and we should obtain the same relation (2.10).

On the field theory side, we should do the path integral over the time direction until the time t . Because of the fact that $g_{YM}(t)$ is a positive power law, and appears in the denominator in the action,

$$e^{-S} = e^{-\int dt \frac{1}{g_{YM}^2(t)} \int d^3x \mathcal{L}_{\text{SYM}}}, \quad (3.15)$$

⁵See [16–18] for early treatments of having both time and radial direction, though outside the holographic cosmology context we introduce here.

the largest contribution to the weight e^{-S} will be from small times. But then, if at small t the fields are *positive* power laws in time (which should be the case since fields must not be singular at $t = 0$, and must be Taylor expandable), which would correspond to “massive KK modes” in a “KK” expansion in t of the fields of SYM, these would give small contributions to the path integral. The leading contribution must be from the time-independent fields, i.e., the “KK dimensionally reduced” fields. We also should split the Lorentz indices according to this dimensional reduction, finally obtaining a 3 dimensional field theory action, with coupling factor integrated over time from a time $t_X \sim t_{\text{Pl}}$ of the order of the Planck scale up to the relevant t ,

$$\int dt \frac{1}{g_{YM}^2(t)} \sim \frac{1}{g_{YM,0}^2} \int_{t_{\text{Pl}}}^{t_X} \frac{dt}{(t/R)^{\sqrt{3}}} = \frac{R}{g_{YM,0}^2} (t/R)^{1-\sqrt{3}} \Big|_{t_{\text{Pl}}}^{t_X} \equiv \frac{RK}{g_{YM,0}^2} \equiv \frac{1}{g_{3d}^2}. \quad (3.16)$$

Here $K = (t_X/R)^{1-\sqrt{3}} - (t_{\text{Pl}}/R)^{1-\sqrt{3}}$ is very large.

But then the *effective* (dimensionless) 3 dimensional coupling is

$$g_{\text{eff}}^2 \equiv \frac{g_{3d}^2 N}{\bar{q}} = \frac{g_{YM,0}^2 N}{K(R\bar{q})}. \quad (3.17)$$

Since both $g_{YM,0}^2 N \gg 1$ (from the usual holographic condition on the validity of the supergravity approximation for $\text{AdS}_5 \times S^5$) and $K \gg 1$, we can have even $R\bar{q} \sim 1$, and still *we can choose* the effective coupling to be perturbative, $g_{\text{eff}}^2 < 1$, though that is not necessary.

In this case, we see that we obtain a specific 3 dimensional field theory with generalized conformal structure, one obtained from the dimensional reduction of $\mathcal{N} = 4$ SYM. However, in [3, 4] the best fit to the CMBR data of the *perturbative* phenomenological field theory was analyzed, and it was found that for no fermions (introducing fermions moves the fit away from the desired region), the number of adjoint scalars for a good match is of the order of 10^4 , which is much larger than the one obtained from dimensionally reducing $\mathcal{N} = 4$ SYM (which is 7: 6 originally, and one from the A_0 component of the gauge field). That means that this theory does not fit the CMBR data *perturbatively*.

It could be that one needs to choose a larger coupling (so as not to have $g_{\text{eff}}^2 < 1$) in order to find the fit, though to test that we would need access to lattice data. Or it could be that $\mathcal{N} = 4$ SYM is just a toy model, and we would need to apply the same methods to other top down gravity dual pairs, though we will leave that for further work.⁶ In particular, we saw that the $a(t)$ uniquely selected by the type IIB equations of motion corresponded to a “stiff matter” cosmology, with $w = 1$, which is different than, say, inflation.

3.1 Transformation of dilaton and operator VEV⁷

We could ask: where do we see the dependence on the cosmological model $a(t)$? There is not much dependence in the constant K , defining g_{3d}^2 , and there would be a small dependence

⁶Note that supersymmetry itself for the gravity dual pair is not ruled out by the CMBR data, since bosons and fermions give different contributions to the fit; only a small number of fields in 3 dimensions is, since as we said, we need of the order of 10^4 bosons.

⁷This section was done in collaboration with Kostas Skenderis.

if we took into account corrections due to non-constant field theory modes (considering “the full KK tower” of fields, instead of the dimensionally reduced ones only). Of course, the type IIB equations of motion only allow a specific $a(t)$, so it cannot be varied, but it still seems strange. Here we want to see that there is in fact one quantity that depends on it, though it should affect only correlators away from the perturbative regime.

We have already noted that a conformal rescaling on the boundary, to go from a conformally flat space to a flat space (by the $a^2(t)$ factor that takes us from a cosmological model to a simple flat space), corresponds in the bulk to a coordinate transformation.

Indeed, a conformal transformation *on the boundary* can be thought of as embedded in the set of general coordinate transformations *on the boundary* (conformal transformations are global $SO(4, 2)$ transformations in $d = 4$, embedded in the infinite dimensional “group” of general coordinate transformations). But by applying a conformal transformation, we just obtain a specific coordinate transformation, differing from what we have, which means that we can’t remove the conformal factor by a conformal transformation *on the boundary*.

But we can remove *any* conformal factor on the boundary by a coordinate transformation *in the bulk*, as shown in [19], eqs. 8, 9, 10.

Let us apply this procedure to our case. Writing $\rho = z^2$, the general coordinate transformation is expanded as

$$\begin{aligned}\rho &= \rho' e^{-2\sigma(x')} + \sum_{k \geq 2} a_{(k)}(x') \rho'^k \\ x^i &= x'^i + \sum_{k \geq 1} a_{(k)}^i(x') \rho'^k,\end{aligned}\tag{3.18}$$

which gives

$$g'_{(0)ij} = e^{2\sigma} g_{(0)ij}\tag{3.19}$$

and higher orders, which don’t interest us.

For us, we have

$$g_{(0)ij} = a^2(t) \delta_{ij}, \quad g'_{(0)ij} = \delta_{ij},\tag{3.20}$$

so $e^{2\sigma} = a^{-2}(t)$. That means that we only need to transform time, as

$$t = t' + \sum_{k \geq 1} a_{(k)}^0(t') \rho'^k,\tag{3.21}$$

but not space (since the metric is space independent). Then the formulas for the relevant coefficients are (note that we are not interested in the transformation on ρ , so we don’t care about $a_{(k)}$ ’s)

$$\begin{aligned}a_{(1)}^0 &= \frac{1}{2} \partial^t \sigma e^{-2\sigma} \\ a_{(2)}^0 &= -\frac{1}{4} e^{-4\sigma} \left(\partial_t \sigma g_{(2)}^{tt} + \frac{1}{2} \partial^t \sigma (\partial \sigma)^2 + \frac{1}{2} \Gamma_{tt}^t \partial^t \sigma \partial^t \sigma \right), \quad \text{where} \\ g_{(2)ij} &= \frac{1}{d-2} \left(R_{ij} - \frac{1}{2(d-1)} R g_{(0)ij} \right).\end{aligned}\tag{3.22}$$

Here indices are raised and lowered with $g_{(0)ij} = a^2(t) \delta_{ij}$.

We consider in particular the cosmological solution of the type IIB equations of motion, which has

$$a^2(t) = t, \quad e^{\phi(t)} = t^{\sqrt{3}} \Rightarrow \phi = \sqrt{3} \ln t, \quad (3.23)$$

and solves

$$R_{ij} = \frac{1}{2} \partial_i \phi \partial_j \phi, \quad (3.24)$$

giving for $g_{(2)ij}$ the value

$$g_{(2)ij} = \frac{1}{2} \left(\frac{\partial_i \phi \partial_j \phi}{2} - \frac{1}{6} (\partial \phi)^2 g_{(0)ij} \right), \quad (3.25)$$

or more precisely

$$g_{(2)tt} = \frac{1}{6} (\partial_t \phi)^2. \quad (3.26)$$

We also calculate the relevant Christoffel symbol,

$$\Gamma^t_{tt} = \frac{1}{2t}. \quad (3.27)$$

Then, after a bit of algebra, we find the coefficients

$$a_{(1)}^0 = \frac{1}{4t}, \quad a_{(2)}^0 = \frac{1}{16t^3}. \quad (3.28)$$

Substituting in the coordinate transformation of the time direction, we find

$$t = t' + \frac{1}{4t'} \rho' + \frac{1}{16t'^3} \rho'^2. \quad (3.29)$$

The scalar transformation law is $\phi'(t') = \phi(t)$, so we obtain

$$\phi'(t') = \phi(t) = \phi \left(t' + \frac{\rho'}{4t'} + \frac{\rho'^2}{16t'^3} \right) = \sqrt{3} \ln \left[t' + \frac{\rho'}{4t'} + \frac{\rho'^2}{16t'^3} \right]. \quad (3.30)$$

Expanding near the boundary at $\rho = 0$, we find

$$\phi'(t') = \sqrt{3} \left[\ln t' + \ln \left(1 + \frac{\rho'}{4t'^2} + \frac{\rho'^2}{16t'^4} \right) \right] \simeq \sqrt{3} \left[\ln t' + \frac{\rho'}{4t'^2} + \frac{\rho'^2}{32t'^4} \right]. \quad (3.31)$$

The leading term in the ρ' expansion of on-shell fields is the source on the boundary, and we see that it is unmodified in the case of $\phi(t)$. The second term in the expansion of $\phi(t)$ (with ρ') is related to the first, but the third (with ρ'^2) is related to an operator VEV on the boundary.

That means that we have, besides the source, also an operator VEV in the $\mathcal{N} = 4$ SYM with time dependent coupling. This coupling $g_{YM}^2(t)$ is unchanged, but we have obtained a nonzero VEV, of

$$\langle \text{Tr}[F_{\mu\nu}^2] \rangle \propto \frac{1}{32t'^4} \neq 0. \quad (3.32)$$

This operator VEV is truly dependent on the cosmological solution $a(t)$, as we have seen, and its presence should modify nonperturbatively the SYM correlators. But in the perturbation theory we have considered, there is no modification.⁸

4 Conclusions and discussion

In this paper we have extended the holographic cosmology map $Z[h_{ij}, \phi] = \psi[h_{ij}, \phi]$ of Maldacena, between the wavefunction of the Universe and the boundary partition function, to the case where there is *both* an Euclidean holographic direction, and a Minkowskian time direction, obtaining $\psi[h_{ij}, \phi]_{t,r} = Z[h_{ij}, \phi]_{t,q}$. Specifically, we applied this prescription to the case of a cosmological solution of the type IIB equations of motion with a time-dependent dilaton $\phi(t)$, where the conformal factor $a^2(t)$ relates it conformally to a flat space solution, corresponding to the usual $\text{AdS}_5 \times S^5$ vs. $\mathcal{N} = 4$ SYM in flat space. This is therefore a “top down” holographic cosmology, obtained by a modification of the original AdS/CFT case. We have then proposed that to integrate over the time direction as needed, we can, in the boundary partition function, “dimensionally reduce” the theory on the time direction, by considering only time-independent quantities, except for the overall coupling $g_{YM}(t)$. In so doing, we obtain the set-up of [2], just that from a top down, as opposed to bottom up, construction. While the resulting cosmology was not, perturbatively, consistent with the CMBR data, we could think of the possibility of either a non-perturbative match, where the SYM results would be obtained on the lattice, or of using the same construction for a different top down starting point. These possibilities are left for further work. We have also shown that the effect of the scale factor $a(t)$ (of the cosmology) on the correlators of SYM is to introduce a nonzero time-dependent VEV $\langle \text{Tr}[F_{\mu\nu}^2] \rangle$ non-perturbatively.

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⁸In a CFT, a state is created by a local operator, so a correlator in a different state is equivalent with adding two more operators in the vacuum correlator. However, the perturbation theory considered here is in 3 Euclidean dimensions, after the “dimensional reduction” of the time direction. From this theory’s point of view, we are in a nonperturbative state: at fixed time, the VEV calculated here is constant throughout the space, and thus is not the effect of a local operator.

A Holographic calculation of the scalar and tensor two-point functions

In this appendix, we review the holographic calculation in [10–12], relating $\langle \delta h_{ij} \delta h_{kl} \rangle$ correlators (experimentally derived from the CMBR) to $\langle T_{ij} T_{kl} \rangle$ correlators in the $\mathcal{N} = 4$ SYM field theory, using the radial Hamiltonian formalism.

Consider an asymptotically AdS metric in Fefferman-Graham coordinates,

$$ds^2 = \frac{1}{z^2} \left[dz^2 + \left(g_{(0)ij} + \dots + z^d g_{(d)ij} + \dots \right) \right]. \quad (\text{A.1})$$

The one-point function of the energy-momentum tensor in the presence of sources is then

$$\langle T_{ij}(x) \rangle = - \frac{1}{\sqrt{g_{(0)}(x)}} \frac{\delta W[g_{(0)}, \dots]}{\delta g_{(0)}^{ij}(x)}, \quad (\text{A.2})$$

where W , the generating functional of connected graphs, equals by the AdS/CFT prescription (minus) the on-shell action $S_{\text{on-shell}}$.

We use a radial Hamiltonian formulation for AdS gravity, with r ,

$$z = e^{-r}, \quad (\text{A.3})$$

acting as “time” in the “ADM parametrization”

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \hat{\gamma}_{ij} d\hat{x}^i d\hat{x}^j + 2N_i d\hat{x}^i dr + (N^2 + N_i N^i) dr^2. \quad (\text{A.4})$$

Then the asymptotically AdS metric is

$$ds^2 = dr^2 + g_{ij}(r, x) dx^i dx^j \quad (\text{A.5})$$

and $g_{(p)ij}$ means (as in (A.1)) the expansion of g_{ij} in $z^{2p-2} = e^{(2-2p)r}$.

Then, like in the usual ADM construction, we can always choose a gauge such that $N = 1$, $N_i = 0$, and the ADM parametrization becomes the same as the Fefferman-Graham expansion above, with

$$\hat{\gamma}_{ij} = g_{ij} = \frac{1}{z^2} (g_{(0)ij}(x) + \mathcal{O}(z^2)) \simeq e^{2r} g_{(0)ij}(x). \quad (\text{A.6})$$

The resulting on-shell action is

$$S_{\text{on-shell}} = - \frac{1}{8\pi G_N} \int_{r_0}^{r_\epsilon} dr \int d^d x \sqrt{\hat{\gamma}} N \left[\hat{R} + 8\pi G_N (\tilde{T}_{ij} - \mathcal{L}_m) \right], \quad (\text{A.7})$$

and one defines the canonically conjugate momentum to $\hat{\gamma}_{ij}$ as (at the position $r_\epsilon = 1/\epsilon$, close to the boundary at $z = 0$)

$$\pi^{ij}(r_\epsilon, x) = \frac{\delta S_{\text{on-shell}}}{\delta \hat{\gamma}_{ij}(r_\epsilon, x)}. \quad (\text{A.8})$$

We obtain

$$\begin{aligned} \partial_r &\simeq \int d^d x \, 2\hat{\gamma}_{ij} \frac{\delta}{\delta \hat{\gamma}_{ij}} + \int d^d x (\Delta_I - d) \Phi_I \frac{\delta}{\delta \Phi^I} \\ &= \delta_D (1 + \mathcal{O}(e^{-2r})), \end{aligned} \quad (\text{A.9})$$

where D is the dilatation operator.

Thus we can identify the radial expansion with the expansion in the eigenfunctions of the dilation operator. In particular, we could do that for the canonical momentum, which is found in the radial picture to equal

$$\pi_{ij} = \frac{\sqrt{g}}{16\pi G_N} (K_{ij} - K\hat{\gamma}_{ij}), \quad (\text{A.10})$$

where K_{ij} is the extrinsic curvature of the radial surface,

$$K_{ij} = \frac{1}{2}\partial_r g_{ij} \rightarrow \frac{1}{2}\delta_D g_{ij}, \quad (\text{A.11})$$

$K = K_{ij}\hat{\gamma}^{ij}$, and expand the canonical momentum in eigenvalues of δ_D ,

$$\delta_D \pi_{ij}^{(n)} = -n\pi_{ij}^{(n)}. \quad (\text{A.12})$$

This would not be important in the unrenormalized case, but in the renormalized case, it is.

Then, identifying $S_{\text{on-shell}}$ with $-W$ as before, we obtain a relation between the one-point function of the energy-momentum tensor and the canonical momentum conjugate to $\hat{\gamma}_{ij}$,

$$\langle T_{ij} \rangle = -\frac{2}{\sqrt{g}}\pi_{ij}, \quad (\text{A.13})$$

which is valid even in the renormalized case, provided we keep the piece of engineering dimension equal to the spatial one, d (3 in the physical case), so

$$\begin{aligned} \langle T_{ij} \rangle &= \left(-\frac{2}{\sqrt{g}}\pi_{ij} \right)_{(d)} = -\frac{1}{8\pi G_N} (K_{ij} - K\hat{\gamma}_{ij})_{(d)} = -\frac{1}{8\pi G_N} (K_{(d)ij} - K_{(d)}\hat{\gamma}_{ij}) \\ &= -\frac{1}{16\pi G_N} (\partial_r g_{(d)ij} - \hat{\gamma}^{kl}\partial_r g_{(d)kl}\hat{\gamma}_{ij}) \\ &\simeq -\frac{d}{16\pi G_N} g_{(d)ij}. \end{aligned} \quad (\text{A.14})$$

The 2-point function is found from the variation of the one-point function in the presence of sources,

$$\delta \langle T_{ij}(x) \rangle = - \int d^3y \sqrt{g_{(0)}} \left(\frac{1}{2} \langle T_{ij}(x) T_{kl}(y) \rangle \delta g_{(0)}^{kl}(y) + \mathcal{O}(\delta\phi_I) \right), \quad (\text{A.15})$$

so that

$$\langle T_{ij}(x) T_{kl}(y) \rangle = \frac{1}{\sqrt{g_{(0)}}} \frac{\delta}{\delta g_{kl}^{(0)}(y)} \langle T_{ij}(x) \rangle = \frac{1}{\sqrt{g_{(0)}}} \frac{\delta}{\delta g_{kl}^{(0)}(y)} \left(-\frac{2}{\sqrt{g}}\pi_{ij} \right)_{(d)}. \quad (\text{A.16})$$

The right-hand side, when we take out the trivial index structure, was in a sense the definition of the linear response functions, which to linear order satisfy

$$E = \frac{\delta\pi_q^\gamma}{\delta\gamma_q} + \text{nonlinear}, \quad \Omega = \frac{\delta\pi^\zeta}{\delta\zeta_q} + \text{nonlinear}, \quad (\text{A.17})$$

so after decomposing, in momentum space

$$\langle T_{ij}(q) T_{kl}(-q) \rangle = A(q)\pi_{ijkl} + B(q)\pi_{ij}\pi_{kl}, \quad (\text{A.18})$$

we find

$$A(q) = 4E_{(0)}(q), \quad B(q) = \frac{1}{4}\Omega_{(0)}(q). \quad (\text{A.19})$$

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