RECEIVED: September 5, 2018

REVISED: November 28, 2018

ACCEPTED: December 11, 2018

PUBLISHED: December 17, 2018

Pati-Salam explanations of the B -meson anomalies

Julian Heeck^{a,b} and Daniele Teresi^{a,c}

^aService de Physique Théorique, Université Libre de Bruxelles,
Boulevard du Triomphe, CP225, 1050 Brussels, Belgium

^bDepartment of Physics and Astronomy, University of California,
Irvine, CA 92697-4575, U.S.A.

^cDipartimento di Fisica “E. Fermi”, Università di Pisa,
Largo Bruno Pontecorvo 3, I-56127 Pisa, Italy

E-mail: julian.heeck@uci.edu, daniele.teresi@df.unipi.it

ABSTRACT: We provide a combined explanation of the increasingly tantalizing B -meson anomalies, both in $R_{K^{(*)}}$ and $R_{D^{(*)}}$, in the Pati-Salam model with minimal matter content. This well-known model, based on the gauge group $SU(4)_{LC} \times SU(2)_L \times SU(2)_R$, naturally contains a variety of scalar leptoquarks with related and restricted couplings. In particular we show that the seesaw-motivated scalar leptoquark within the representation $(\overline{10}, 3, 1)$ and its right-handed parity partner $(\overline{10}, 1, 3)$ can solve both anomalies while making testable predictions for related observables such as $B \rightarrow K\nu\nu$ and $B \rightarrow K\mu\tau$. The solution of the $R_{K^{(*)}}$ anomaly alone can be related to a type-II seesaw neutrino mass structure. Explaining also $R_{D^{(*)}}$ requires the existence of a light right-handed neutrino, which constrains the UV structure of the model.

KEYWORDS: Beyond Standard Model, Gauge Symmetry, Heavy Quark Physics, Neutrino Physics

ARXIV EPRINT: [1808.07492](https://arxiv.org/abs/1808.07492)

Contents

1	Introduction	1
2	Pati-Salam model	3
3	Neutrino masses and $R_{K^{(*)}}$	7
3.1	Type II dominance	8
3.2	Type I dominance	9
4	Model-building challenges for combined explanations	11
5	Combined explanation of the B-meson anomalies in Pati-Salam	12
5.1	Fit of flavour observables	13
5.2	Collider phenomenology	14
5.3	Constraints on the UV structure of the model	16
6	Conclusion	18
A	Details of the fit for $R_{K^{(*)}}$ and type-II neutrino masses	19
B	Details of the fit for $R_{K^{(*)}}$ and $R_{D^{(*)}}$	20
C	Renormalization group running of Yukawas	22
C.1	Light right-handed neutrinos	23
C.2	Heavy right-handed neutrinos	24
C.3	Impact on the fit for the combined explanations	25
D	Left-right gauge coupling unification	26

1 Introduction

In the last few years various hints for new physics manifesting itself in lepton flavour universality violation (LFUV) in semileptonic B -meson decays have been reported. Several experimental results by different collaborations point at an increasingly coherent picture.

A first set of anomalies involves the charged-current transition $b \rightarrow c\tau\nu$, with LFUV between the τ and the other charged leptons in the decay $B \rightarrow D^{(*)}\ell\nu$, as parametrized by the double ratio $R_{D^{(*)}}$ with respect to the Standard Model (SM) prediction. This observable has been measured by the BaBar [1, 2], Belle [3–5], and LHCb [6–8] collaborations and is found to be consistently above the SM predictions. Global fits [9] give a combined statistical evidence at the $\sim 4\sigma$ level. A second set of anomalies appears in the neutral-current transition $b \rightarrow s\mu\mu$, with hints for LFUV in the decays $B \rightarrow K^{(*)}\ell\ell$ [10, 11],

parametrized by the double ratio $R_{K^{(*)}}$ between the μ and e channels, relative to the SM. This is further corroborated by a deviation measured in the angular distribution of the process $B \rightarrow K^* \mu^+ \mu^-$ [12, 13], for which the theoretical SM prediction is however still debated. Again, global fits give a deviation from the SM at the $\sim 4\sigma$ level or even higher (see e.g. refs. [14–19]).

Finding combined explanations in terms of physics beyond the SM is far from trivial, mainly because of the large new-physics effects needed to explain $R_{D^{(*)}}$ that have to compete with the tree-level SM contribution. The effective-field-theory (EFT) scale involved is rather low at $\Lambda \simeq 3.4 \text{ TeV}$ [20] — in that the effective operator $2(\bar{c}\gamma_\alpha P_L b \bar{\tau}\gamma^\alpha P_L \nu)/(3.4 \text{ TeV})^2$ significantly improves the fit [21] — and therefore a number of related observables pose stringent constraints; one of the most important observables here is $B \rightarrow K \nu \nu$ because the involved quark-level transition $b \rightarrow s \nu \nu$ is often related to the same new physics that contributes to the anomalous $b \rightarrow c \tau \nu$ by $\text{SU}(2)_L$ invariance. Explaining $R_{K^{(*)}}$ by itself is instead much easier since the EFT scale involved is rather high, $\Lambda \simeq 31 \text{ TeV}$ [20] — in that the single effective operator $(\bar{s}\gamma_\alpha P_L b \bar{\mu}\gamma^\alpha P_L \mu)/(31 \text{ TeV})^2$ significantly improves the global fit [14]. At the level of simplified models, combined explanations for $R_{K^{(*)}}$ and $R_{D^{(*)}}$ involving only left-handed new physics [22] make use of a single vector leptoquark (LQ) U_μ with SM $(\text{SU}(3)_C, \text{SU}(2)_L)_{\text{U}(1)_Y}$ quantum numbers $(\mathbf{3}, \mathbf{1})_{2/3}$, since this particle does not contribute to $B \rightarrow K \nu \nu$. With more than one new multiplet one can for example consider the combination of the scalar LQs $S_1 = (\mathbf{3}, \mathbf{1})_{1/3}$ and $S_3 = (\mathbf{3}, \mathbf{3})_{1/3}$, both with left-handed couplings, which can interfere destructively in the dangerous observables [22, 23]. Alternatively, simplified models explaining the charged-current anomaly by *right-handed* new physics by introducing a light right-handed neutrino that plays the role of the active ones in $R_{D^{(*)}}$ typically do not run into problems with $B \rightarrow K \nu \nu$ [24–28] because $b \rightarrow s \nu \nu$ is no longer the $\text{SU}(2)_L$ conjugate of the anomalous transition.

Recently, more and more effort has been put attempting to go beyond simplified models and consider instead full models for the combined explanation of both sets of anomalies. In this respect, lepton-colour unification as given by the Pati-Salam (PS) symmetry $\text{SU}(4)_{LC} \times \text{SU}(2)_L \times \text{SU}(2)_R$ has received a lot of attention because this is arguably the most natural framework to embed LQs in a complete model.¹ The obvious choice is to try to explain the anomalies with the vector LQ U_μ [31], since it is contained in the set of PS gauge bosons. However, since the anomalies point at new physics in the TeV range, such a light PS gauge boson would conflict with experiments involving first-generation fermions, for instance $K_L \rightarrow e^\pm \mu^\mp$, which exclude PS scales lower than $\sim 1000 \text{ TeV}$ [32–35]. A number of modifications to the PS model have been suggested to circumvent this problem, in particular by suppressing the coupling of the gauge boson U_μ to the first generation in different ways: i) introducing many vector-like fermions that mix with the SM ones and generate flavour-dependent gauge couplings [36, 37], ii) placing the PS symmetry in a Randall-Sundrum background to suppress the dangerous observables by a conveniently chosen localization in the extra dimension [38], iii) introducing three copies of the PS symmetry, one for each generation [39], iv) giving up the gauge character of the PS symmetry

¹In addition, we also mention very recent solutions based on lepton-quark unification different from PS [29, 30].

by considering it as a global symmetry of a strongly-interacting theory [40], which can also relate the B -meson anomalies to a possible solution of the gauge hierarchy problem.

While these are all perfectly viable possibilities, each with its own advantages, they often involve ad-hoc modifications of the Pati-Salam framework. This is mainly due to the usage of the vector LQ to explain the anomalies. However, the PS model also naturally contains *scalar* LQs, which have flavour-*dependent* couplings; the first-generation LQ couplings can therefore be relatively suppressed without any model-building effort. The PS model is typically (but not necessarily) supplemented by a left-right exchange symmetry, often in the form of parity. In this case, as we are going to show in this work, a *single* representation of the parity-symmetric PS group,

$$(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1}) \oplus (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}), \quad (1.1)$$

typically already present in the PS model with minimal matter content, is sufficient to explain both sets of anomalies, basically without any tension with other observables. This representation contains two scalar LQs that can be used to explain the anomalies, one with left-handed and one with right-handed couplings. However, it is far from trivial that a successful explanation is possible, because the PS symmetry restricts the structure of the couplings (quarks and leptons have the same couplings), parity makes the left-handed and right-handed ones equal, and their different chirality forbids them from interfering. The required suppression of $B \rightarrow K \nu \nu$ can therefore not be achieved by the destructive interference between several LQs, as discussed above, but instead requires one of the right-handed neutrinos to be light with a particular mixing pattern.

The plan of the paper is the following: in section 2 we introduce the PS model with minimal matter content, and discuss the restrictions on the couplings posed by its symmetries. In section 3 we show that explaining the neutral-current anomalies alone can be related to a type-II seesaw mass structure for the active neutrinos, leading to testable predictions. We then move to the combined explanation, by first discussing the model-building challenges involved in section 4 and then presenting the combined explanation, together with its phenomenological predictions, in section 5. We finally conclude in section 6 and report the details of the fits for the explanations of the anomalies in appendix A and B. We discuss the renormalization group running of the LQ and gauge couplings in appendix C and appendix D, respectively, to illustrate the validity of our assumed structures.

2 Pati-Salam model

The Pati-Salam model is based on the gauge group $SU(4)_{LC} \times SU(2)_L \times SU(2)_R$ [41], where colour and lepton number are unified in a single $SU(4)_{LC}$ factor. The PS group is eventually broken to the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, for which we define the hypercharge $Y = Q - T_L^3$. The SM fermions (plus right-handed neutrinos) are minimally unified in the following PS representations,

$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1}) \rightarrow (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \equiv Q_L \oplus L_L, \quad (2.1)$$

$$\Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2}) \rightarrow (\mathbf{3}, \mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{1})_{-1} \oplus (\mathbf{1}, \mathbf{1})_0 \equiv u_R \oplus d_R \oplus \ell_R \oplus N_R, \quad (2.2)$$

where we also give the branching rules with regards to the SM and identify the components with their usual SM notation, i.e. left-handed quark doublet Q_L etc, suppressing generation indices. In the presence of only this minimal matter content, fermion masses can be generated by introducing complex scalars in the representations in $(\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1}) \otimes (\mathbf{4}, \mathbf{1}, \mathbf{2}) = (\mathbf{1}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{15}, \mathbf{2}, \mathbf{2})$ [42]:

$$\phi_1 \sim (\mathbf{1}, \mathbf{2}, \mathbf{2}) \rightarrow (\mathbf{1}, \mathbf{2})_{\frac{1}{2}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}, \quad (2.3)$$

$$\begin{aligned} \phi_{15} \sim (\mathbf{15}, \mathbf{2}, \mathbf{2}) \rightarrow & (\mathbf{1}, \mathbf{2})_{\frac{1}{2}} \oplus (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} \oplus (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}} \\ & \oplus (\mathbf{3}, \mathbf{2})_{\frac{7}{6}} \oplus (\bar{\mathbf{3}}, \mathbf{2})_{-\frac{7}{6}} \oplus (\mathbf{8}, \mathbf{2})_{\frac{1}{2}} \oplus (\mathbf{8}, \mathbf{2})_{-\frac{1}{2}}, \end{aligned} \quad (2.4)$$

which have Yukawa couplings

$$\mathcal{L} \supset \bar{\Psi}_L (k_{1,A} \phi_1 + k_{1,B} \phi_1^c + k_{15,A} \phi_{15} + k_{15,B} \phi_{15}^c) \Psi_R + \text{h.c.}, \quad (2.5)$$

where the superscript c denotes an appropriate form of charge conjugation and we suppress all indices. ϕ_1 and ϕ_{15} contain two electroweak doublets each and can thus be used to both break $\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_Q$ and generate fermion masses. ϕ_1 by itself would preserve the $\text{SU}(4)_{LC}$ multiplet and thus lead to the mass-unification relations [42, 43]

$$m_d = m_\ell, \quad m_u = m_N^{\text{Dirac}} \quad (2.6)$$

for the Dirac mass matrices of down quarks with charged leptons, and up quarks with Dirac neutrinos. The latter relation will be broken by the seesaw mechanism, as discussed below, leaving $m_d = m_\ell$ as a PS prediction. Taking into account that these relations hold at the potentially high PS scale and that renormalization-group running strongly affects quarks, this is a potentially viable relation [43]. Keeping also the ϕ_{15} vacuum expectation values (VEVs) allows one to break the relation of eq. (2.6) at tree level and generate arbitrary masses for all fermions [42]. Since the bottom-tau unification seems to be a good approximation at least, we will assume that the ϕ_{15} VEVs only lead to small perturbations. As a result, the diagonalization of the mass matrices will be approximately the same for quarks and leptons, so diagonalization will proceed through

$$u_{L,R} \rightarrow V_{L,R}^\dagger u_{L,R}, \quad \nu_L \rightarrow V_L^\dagger \nu_L, \quad N_R \rightarrow V_R^\dagger N_R, \quad (2.7)$$

where we chose to pick the down-quark and charged-lepton basis without loss of generality. V_L is the usual unitary Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and V_R the right-handed analogue, which is physical due to the additional interactions beyond the SM. Imposing furthermore parity $\mathcal{P}$ on our theory, i.e. $f_L \leftrightarrow f_R$, leads to a relation between the two mixing matrices, approximately setting them equal if we ignore phases: $V_R \simeq V_L$ [44, 45]. We will mostly employ the above relations in the following, but it should be kept in mind that these are very strong assumptions that could easily be loosened, leading to more freedom in the couplings. Using generalized charge conjugation $\mathcal{C}$ instead of parity will lead to qualitatively similar results and will not be discussed further.

We still need additional scalars to break PS to the SM and ideally also for neutrino mass generation. A common choice to accomplish both tasks is given by the scalar representations [42]

$$\Delta_L \sim (\overline{10}, 3, 1) \rightarrow (6, 3)_{-\frac{1}{3}} \oplus (\overline{3}, 3)_{\frac{1}{3}} \oplus (1, 3)_1 \equiv \Sigma_3 \oplus S_3 \oplus \delta_3, \quad (2.8)$$

$$\begin{aligned} \Delta_R \sim (\overline{10}, 1, 3) \rightarrow (6, 1)_{-\frac{1}{3}} \oplus (6, 1)_{\frac{2}{3}} \oplus (6, 1)_{-\frac{4}{3}} \oplus (\overline{3}, 1)_{\frac{1}{3}} \oplus (\overline{3}, 1)_{-\frac{2}{3}} \oplus (\overline{3}, 1)_{\frac{4}{3}} \\ \oplus (1, 1)_0 \oplus (1, 1)_1 \oplus (1, 1)_2 \equiv \Sigma_1 \oplus \tilde{\Sigma}_1 \oplus \bar{\Sigma}_1 \oplus S_1 \oplus \tilde{S}_1 \oplus \bar{S}_1 \oplus \delta_1 \oplus \tilde{\delta}_1 \oplus \bar{\delta}_1, \end{aligned} \quad (2.9)$$

where $\langle \delta_1 \rangle$ has the right quantum numbers for PS breaking and to generate Majorana masses for N_R . This VEV thus breaks the PS group down to the SM in one step and provides a mass $\propto \langle \delta_1 \rangle$ to the non-SM gauge bosons X_μ (the vector LQ), $W_{\mu,R}^+$ and Z'_μ . As discussed below, the *left-handed* triplet VEV $\langle \delta_3 \rangle$ can generate type-II seesaw [46–48] masses for ν_L and is restricted to be below GeV from electroweak precision data [49]. Note that the bars and the tildes are part of the LQ names and do not denote charge conjugation. Also notice that, as will be detailed below, these representations do not lead to dangerous proton decay [50].

We have the Yukawa Lagrangian in the flavour basis, suppressing all indices,

$$\mathcal{L} = \bar{\Psi}_L^c y^L \Delta_L \Psi_L + \bar{\Psi}_R^c y^R \Delta_R \Psi_R + \text{h.c.} \quad (2.10)$$

$y^{L,R}$ are *symmetric* coupling matrices in flavour space, which is a true PS prediction. For instance, PS symmetry restricts the coupling of S_3 to a b quark and a μ to be the same as to an s quark and a τ ; this has a large impact on the explanations of the anomalies and the related phenomenological predictions, as will be clear in the next sections. Expanded, the couplings read

$$\begin{aligned} \bar{\Psi}_L^c y^L \Delta_L \Psi_L \equiv 6^{-1/4} (\bar{Q}_L^c y^L Q_L \Sigma_3)_{(1,1)_0} + (\bar{Q}_L^c y^L L_L S_3)_{(1,1)_0} + \frac{1}{\sqrt{2}} (\bar{L}_L^c y^L L_L \delta_3)_{(1,1)_0} \\ \supset \bar{\nu}_L^c V_L^* y^L V_L^\dagger u_L S_3^{-2/3} - \frac{1}{\sqrt{2}} \left(\bar{d}_L^c y^L V_L^\dagger \nu_L + \bar{\ell}_L^c y^L V_L^\dagger u_L \right) S_3^{1/3} + \bar{\ell}_L^c y^L d_L S_3^{4/3}, \end{aligned} \quad (2.11)$$

using some Fierz identities and the fact that y^L is symmetric. In the last line we give the couplings of the S_3 components S_3^Q in the quark mass basis, where the superscript Q denotes the electric charge. For the right-handed fermions in their mass basis one finds

$$\begin{aligned} \bar{\Psi}_R^c y^R \Delta_R \Psi_R \equiv -2^{1/4} 3^{-1/4} \bar{d}_R^c y^R V_R^\dagger u_R \Sigma_1 + 6^{-1/4} \bar{d}_R^c y^R d_R \tilde{\Sigma}_1 + 6^{-1/4} \bar{u}_R^c V_R^* y^R V_R^\dagger u_R \bar{\Sigma}_1 \\ + \bar{u}_R^c V_R^* y^R V_R^\dagger N_R \bar{S}_1 - \frac{1}{\sqrt{2}} \left(\bar{d}_R^c y^R V_R^\dagger N_R + \bar{\ell}_R^c y^R V_R^\dagger u_R \right) S_1 + \bar{d}_R^c y^R \ell_R \tilde{S}_1 \\ + \frac{1}{\sqrt{2}} \bar{\ell}_R^c y^R \ell_R \bar{\delta}_1 - \bar{\ell}_R^c y^R V_R^\dagger N_R \tilde{\delta}_1 + \frac{1}{\sqrt{2}} \bar{N}_R^c V_R^* y^R V_R^\dagger N_R \delta_1. \end{aligned} \quad (2.12)$$

We will be particularly interested in the couplings of the LQ S_1 , given in the second line. Parity $\mathcal{P}$ would exchange $\Psi_L \leftrightarrow \Psi_R$ and $\Delta_L \leftrightarrow \Delta_R$ and thus impose $y^L = y^R$ and $V_L = V_R$. These relations hold above the PS scale but are broken at lower energies, as can be seen by

studying the renormalization group running of the couplings. As we show in appendix C for some benchmark scenarios, the PS relations $y^R = y^L = (y^L)^\top$ at the TeV scale can still be satisfied to the $\mathcal{O}(10\%)$ level, especially if the PS scale is not too high. For simplicity we will therefore work with the relation $y = y^\top \equiv y^L = y^R$ in the following. Again, it should be kept in mind that this is a very constraining assumption that does not need to hold true, but we enforce it to maximize the predictivity of the framework. Note finally that within our minimal PS model the parity and PS breaking scales are identified so we must have the gauge-coupling relation $g_R = g_L$ at the PS scale. Given the low-energy particle content this fixes the PS scale to be typically around $\sim 10^{13}$ GeV, as discussed in appendix D, although this can be changed by several orders of magnitude by the presence of states in the desert up to the PS scale.

One of the most interesting features of PS is the very existence of LQs, which we have already used in the naming scheme above. We can identify the following scalar LQs within Δ_X in their common notation [51, 52]: $S_3 = (\bar{\mathbf{3}}, \mathbf{3})_{\frac{1}{3}}$, $S_1 = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$, $\tilde{S}_1 = (\bar{\mathbf{3}}, \mathbf{1})_{\frac{4}{3}}$, and $\bar{S}_1 = (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$.² The PS origin of the S_j LQs gives rise to many restrictions compared to naive SM extensions by S_j . For one, parity predicts right-handed partners to left-handed LQs. In particular, S_3 has a parity partner in the right-handed triplet $(S_1, \tilde{S}_1, \bar{S}_1)$. Below, we will use S_3 and S_1 to explain the B -meson anomalies. While this may look at first sight analogous to other existing approaches in the literature where scalar LQs with the same quantum numbers are used [22, 23, 54] (see also refs. [55–59] for other LQ approaches to the anomalies), our explanation to the B -meson anomalies is very different; our S_1 is forced to have purely right-handed couplings and therefore cannot be used for a destructive interference with S_3 in $B \rightarrow K\nu\nu$ in order to suppress this channel. We will see that it is still possible to explain both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ by invoking a *light right-handed* neutrino, similar to refs. [24–28].

Notice that the S_j here are pure LQs without any di-quark couplings, as enforced by the PS symmetry. This has the immediate consequence that the above Yukawa couplings will not lead to proton decay, a welcome feature that is usually enforced by imposing rather ad-hoc baryon [52, 60] or lepton-flavour symmetries [61].³ Furthermore, the S_j LQs here are purely chiral, every LQ only couples to one fermion chirality. This is again a welcome feature as it automatically leads to a chirality suppression in many rare decays and softens many constraints.

In section 3 we will assume that the PS breaking to the SM is given entirely by $\langle \delta_1 \rangle$, which also induces right-handed neutrino masses. If this is the only PS-breaking VEV, it needs to be above ~ 1000 TeV to satisfy constraints on $K_L \rightarrow e^\pm \mu^\mp$ induced by the heavy

² ϕ_{15} also contains (non-chiral) scalar LQs, namely R_2 and $\tilde{R}_2$. The combinations $\tilde{R}_2 + S_3$ [53] and $R_2 + S_3$ [29] have been discussed in relation to the B anomalies and it has been shown that $R_2 + S_3$ provides a good explanation. We will not discuss the LQs R_2 and $\tilde{R}_2$ in the following and simply assume them to be heavy.

³Our minimal PS model only has proton decay in the presence of ϕ_{15} through some of the quartic interactions in the scalar potential [42, 62]. This induces heavily suppressed dimension-10 proton-decay operators [61, 63] that also depend on the quartic coupling and the Yukawas of ϕ_{15} to the first generation, easy viable even for a low PS scale.

coloured PS gauge boson [32–35]. However, the key point of this work is that some of the PS *scalars* can be lighter than this scale.⁴ Quartic couplings such as $\text{Tr}\{\bar{\Delta}_X \Delta_X \bar{\Delta}_R \Delta_R\}$ will induce a mass splitting within the Δ_X multiplet proportional to $\langle \delta_1 \rangle$. This makes it possible to have e.g. S_1 at $\sim \text{TeV}$ but $\tilde{S}_1$ and $\bar{S}_1$ at a higher scale, so that we do not have to take their phenomenology into account in the analysis below. We will assume for the most part that S_3 is around 10 TeV and S_1 around 1 TeV in order to address $R_{K^{(*)}}$ and $R_{D^{(*)}}$, respectively, while all other scalars are heavy enough to be ignored.⁵

3 Neutrino masses and $R_{K^{(*)}}$

Assuming that $\langle \delta_1 \rangle$ is the only PS-breaking VEV, our PS setup brings both type I and II seesaw contributions to the light Majorana neutrino masses, similar to standard left-right models [64]:

$$M_\nu = M_\nu^I + M_\nu^{II} \simeq -\frac{1}{\sqrt{2}\langle \delta_1 \rangle} m_u (V_R y^{R*} V_R^\dagger)^{-1} m_u - \sqrt{2}\langle \delta_3 \rangle V_L^* y^L V_L^\dagger, \quad (3.1)$$

using eq. (2.6) and the fact that the Dirac matrices are real and diagonal. Diagonalizing $M_\nu = U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger$ with the usual unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix U [49],

$$U \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}), \quad (3.2)$$

allows us to predict the flavour structure of the coupling matrices $y^L = y^R$ using the measured neutrino masses and mixing angles (taken from ref. [65]).

The neutral-current B -meson anomalies can all be explained by a new-physics Wilson-coefficient contribution $\Delta C_9 = -\Delta C_{10} \simeq -0.61 \pm 0.12$ [14] to the effective-Hamiltonian operator

$$\mathcal{O}_{9\mu} - \mathcal{O}_{10\mu} = -\frac{\alpha}{\pi v^2} V_{L,tb} V_{L,ts}^* (\bar{s} \gamma_\mu P_L b) (\bar{\mu} \gamma^\mu P_L \mu), \quad (3.3)$$

where α is the fine-structure constant and $v \simeq 246 \text{ GeV}$ the electroweak VEV, both used purely for normalization. It is well-known that this can be generated by the LQ $S_3 \in (\bar{10}, \mathbf{3}, 1)$ as

$$\Delta C_9 = -\Delta C_{10} = \frac{\pi v^2}{\alpha} \frac{1}{V_{L,tb} V_{L,ts}^*} \frac{1}{M_{S_3}^2} y_{23}^L y_{22}^{L*}, \quad (3.4)$$

⁴These scalars then suffer from the gauge hierarchy problem much like the Brout-Englert-Higgs boson in the SM supplemented by physical high-scale thresholds. We will not attempt to solve this problem here.

⁵The presence of LQs lighter than the PS scale will affect the gauge-coupling running and thus potential unification. Guaranteeing successful unification, e.g. into $\text{SO}(10)$, requires the specification of the full particle spectrum. Since only particles up to few TeV are of phenomenological interest for the explanation of the anomalies, we will not specify the full spectrum and ignore gauge-coupling unification. However, partial unification of left and right gauge couplings, as required in our framework, is generically achieved as shown in appendix D.

with $\text{Re}(\Delta C_9) \stackrel{!}{\simeq} -0.61 \pm 0.12$. In principle, a successful contribution can be easily realized with $M_{S_3} \simeq 30 \text{ TeV}$ for $y_{23}^L, y_{22}^L = \mathcal{O}(1)$. However, the PS symmetry and the neutrino oscillation data pose stringent constraints on the structure of the couplings. Therefore, in this section we will aim at explaining $R_{K^{(*)}}$ in relation to the neutrino mass structure.

A combined explanation of also the charged-current anomaly $R_{D^{(*)}}$ is more involved and cannot be related directly to the neutrino mass structure. Hence, this will be postponed to the next sections (sections 4 and 5).

3.1 Type II dominance

The flavour structure of the couplings can be determined by eq. (3.1), under the assumptions discussed above, especially if one of the two terms dominate. Let us consider first the case in which the type-II seesaw contribution M_ν^{II} dominates, for instance because $\langle \delta_1 \rangle$ is around the Grand Unified Theory (GUT) scale. Then, eq. (3.1) gives

$$y^L = -\frac{1}{\sqrt{2}\langle \delta_3 \rangle} V_L^\dagger U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger V_L, \quad (3.5)$$

where U (V_L) is the standard PMNS (CKM) mixing matrix. Since we know the right-hand side with the exception of the phases within U and the overall neutrino mass scale m_{ν_1} , we can predict the flavour structure of y^L rather well.⁶ Assuming normal mass ordering with vanishing lightest neutrino mass $m_{\nu_1} \simeq 0$ as well as vanishing PMNS phases, the flavour structure is simply

$$y^L \propto \begin{pmatrix} 0.05 & 0.06 & -0.10 \\ 0.06 & 1 & 0.74 \\ -0.10 & 0.74 & 0.97 \end{pmatrix}, \quad (3.6)$$

setting all other neutrino [65] and CKM parameters to their best-fit values. For S_3 in the $\mathcal{O}(30 \text{ TeV})$ range we can easily fit $R_{K^{(*)}}$; however, the small but non-zero couplings to first generation fermions (both quarks and leptons due to the PS symmetry) give important constraints that need to be satisfied, in particular lepton flavour violation (LFV). The most stringent observable turns out to be $\mu \rightarrow e$ conversion in nuclei, because of the first and second-generation couplings to both leptons and the quarks in the nucleons. The relevant branching ratio in Au_{79}^{197} is given approximately by [52]

$$\begin{aligned} \text{BR}(\mu \rightarrow e) &\simeq 1 \times 10^{-6} \left(\frac{30 \text{ TeV}}{M_{S_3}} \right)^4 |2 y_{11}^L y_{12}^{L*} + V_{L,us} y_{11}^L y_{22}^{L*}|^2 \\ &\simeq 6 \times 10^{-8} \left(\frac{30 \text{ TeV}}{M_{S_3}} \right)^4 |y_{11}^L y_{22}^{L*}|^2, \end{aligned} \quad (3.7)$$

which has to be smaller than 7×10^{-13} at 90% C.L. [66]. The full expressions for this and the other relevant observables are given in appendix A. We see that the coupling y_{11}^L needs to be suppressed by a factor ~ 14 with respect to the estimate of eq. (3.6). This can

⁶Note that eq. (3.5) does not require left-right symmetry and can thus be obtained even in generalized PS models.

Parameter	Best fit	Observable	Best fit	Pull/bound
M_{S_3}	11.4 TeV	$\Delta C_9 = -\Delta C_{10}$	-0.62	-0.1σ
m_{ν_1}	2.1 meV	$KR^{e/\mu}$	2.8×10^{-5}	-1.1σ
δ	-0.2π	$\pi R^{e/\mu}$	-2.1×10^{-7}	$+1.1\sigma$
α	0.55π	$K_{\pi\nu\nu}$	1.67	-0.4σ
β	0.95π	$\text{BR}(\mu \rightarrow e)$	6.9×10^{-13}	7×10^{-13}
χ^2	2.45	$\text{BR}(\mu \rightarrow e\gamma)$	1.1×10^{-13}	4.2×10^{-13}
		$\Gamma(K_L \rightarrow e^\pm \mu^\mp)$	$3.2 \times 10^{-29} \text{ GeV}$	$6.0 \times 10^{-29} \text{ GeV}$

Table 1. Results of the fit for the neutral-current anomalies and type-II neutrino masses, fixing the type-II seesaw VEV $\langle \delta_3 \rangle = 50 \text{ meV}$.

be achieved by a moderate tuning of the phases in the PMNS matrix together with m_{ν_1} , in complete analogy to texture zeros in the neutrino mass matrix [67]. In this way, one can relate the explanation of $R_{K^{(*)}}$ with information on the unknown parameters in the neutrino sector, most importantly the lightest neutrino mass m_{ν_1} and the Dirac CP phase δ .

We perform a global fit including the most constraining observables discussed above together with other relevant ones involving first-generation fermions, detailed in appendix A. The results of the fit are given in table 1. The value of M_{S_3} needed to explain $R_{K^{(*)}}$ depends of course on the absolute scale of the Yukawa matrix, which in turn depends on the unknown VEV $\langle \delta_3 \rangle$. Therefore, we cannot be particularly predictive on the scale of M_{S_3} apart from the usual model-independent considerations based on perturbativity/unitarity [20]. Instead, a successful fit constrains the unknown neutrino-mass parameters, as discussed above. We show the two-dimensional correlation between m_{ν_1} and δ in figure 1. We note that the 1σ region for δ marginally overlaps with the 1σ range $\delta/\pi \simeq -0.7^{+0.24}_{-0.17}$ suggested by oscillation experiments [14].

Considering the small number of free parameters it is remarkable that the Ansatz of eq. (3.5) can indeed be used to resolve $R_{K^{(*)}}$. Ongoing and upcoming experimental efforts to significantly improve the limits on muonic LFV in μ to e conversion (Mu2e [68], COMET [69]), $\mu \rightarrow 3e$ (Mu3e [70]), and $\mu \rightarrow e\gamma$ (MEG-II [71]) will conclusively test this scenario, seeing as it is impossible to suppress all these observables much further.

3.2 Type I dominance

Similarly to the above discussion, we can assume type-I dominance, i.e. $M_\nu^{II} \ll M_\nu^I$, for example because of a tiny $\langle \delta_3 \rangle$. In this case the coupling structure takes the form

$$y^R = -\frac{1}{\sqrt{2}\langle \delta_1 \rangle} V_R^\dagger m_u U^* \text{diag} \left(\frac{1}{m_{\nu_1}}, \frac{1}{m_{\nu_2}}, \frac{1}{m_{\nu_3}} \right) U^\dagger m_u V_R. \quad (3.8)$$

Assuming once again normal ordering, vanishing PMNS phases and tiny $m_{\nu_1} \ll m_{\nu_2}$, as well as $V_R \simeq V_L$, we find

$$y \equiv y^L = y^R \propto \begin{pmatrix} 0.04 & -0.2 & 4 \\ -0.2 & 1 & -20 \\ 4 & -20 & 380 \end{pmatrix}. \quad (3.9)$$

Figure 1. Results of the fit for the neutral-current anomalies and type-II neutrino masses, fixing $\langle \delta_3 \rangle = 50$ meV. We show the $\Delta\chi^2 < 2.3$ (1σ), 6.2 (2σ), 11.8 (3σ) regions in green, yellow, and blue, respectively, marginalising over all other parameters. The star denotes the best-fit point.

The relative sign between y_{22}^R and y_{23}^R implies a wrong-sign contribution to the neutral-current anomaly (cf. (3.4)). Taking into account non-zero m_{ν_1} and PMNS phases δ, α, β does not change this, making it impossible to explain $R_{K^{(*)}}$ using our very restricted model. Taking into account that the general structure of the right-handed CKM-like matrix V_R is given by [44]

$$V_R \simeq \text{diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3}) V_L \text{diag}(e^{i\omega_4}, e^{i\omega_5}, 1), \quad (3.10)$$

where the 5 extra phases ω_i are arbitrary or multiple of π according to the discrete symmetry of the model, these phases ω_i can help in obtaining a correct contribution to ΔC_9 , as shown in figure 2.

However, even if one allows for these extra phases and makes use of the various phases to tune y_{11} small in order to suppress $\mu \rightarrow e$ conversion, the coupling structure in (3.9) does not seem to be successful for the following reason: the 22 and 23 entries, which are the ones that generate ΔC_9 , are very suppressed with respect to y_{33} . Fitting the anomaly with $M_{S_3} \gtrsim 1$ TeV then requires $y_{33} \gtrsim 3$; even neglecting possible concerns with perturbativity, this large coupling, together with y_{23} , would generate a too large contribution to other observables, such as $B \rightarrow K \nu \nu$ for instance (see also the discussion in the next section). Making use of the various phases does not seem to improve the situation significantly. Therefore, we do not follow this type-I dominance option any further, and instead move to combined explanations of both neutral-current and charged-current anomalies.

We have seen that type-II dominance is compatible with the coupling structure required for $R_{K^{(*)}}$, whereas the type-I contribution is not, at least in the rather stringent PS model with parity considered here. The general case of $M_\nu^{II} \sim M_\nu^I$ introduces more freedom [64] and will be considered elsewhere in its full extent. We stress again that it is a rather unique feature of PS that allows us to relate LQ couplings to the neutrino mass structure.

Figure 2. Scan of the S_3 contribution to $\Delta C_9 = -\Delta C_{10}$ obtained varying m_{ν_1} and the PMNS phases δ, α, β , as well as the extra phases ω_i in V_R , allowing them to be multiple of π . The 1σ , 2σ and 3σ regions for the explanation of the anomalies are enclosed in green, yellow, and blue lines, respectively.

4 Model-building challenges for combined explanations

Explaining both sets of anomalies is known to be significantly more challenging than the ones in $b \rightarrow s\mu\mu$ alone, essentially because of the low EFT scale $\Lambda \simeq 3.4$ TeV implied by the charged-current anomalies, as compared to $\Lambda \simeq 31$ TeV for the neutral-current ones.

A first possibility would be to try to explain both sets of anomalies by using the vector LQ $U_\mu = (\mathbf{3}, \mathbf{1})_{2/3}$, present in the adjoint representation of the PS group. One would like to identify this LQ as a PS gauge boson [31]. However, this does not work [37], at least in the minimal case, because U_μ , being a gauge boson, would couple with equal strength also to light quarks, and is thus required to be heavier than ~ 1000 TeV from the non-observation of new physics in $K_L \rightarrow e^\pm \mu^\mp$, for instance. To circumvent this problem, it has been proposed to generate flavour-dependent couplings to U_μ by introducing many vector-like fermions, charged under PS, that mix with the SM quarks [36] possibly extending the gauge group breaking the lepton-colour unification in $SU(4)_{LC}$ [37]. A more radical alternative is to consider flavour copies of the PS group (PS^3) [39] leading to generation-dependent *gauge* couplings.

Here we attempt a more minimal route by explaining both the anomalies not by the gauge PS leptoquarks, but by the *scalar* ones. Since the coupling of scalar LQ to SM fermions is of Yukawa type, these are naturally flavour dependent without any model-building effort. By limiting oneself to chiral LQs (as suggested by the explanation of R_K vs R_{K^*} assuming new physics in muons [18]) the only possibilities are the representations in $(\mathbf{4}, \mathbf{2}, \mathbf{1}) \otimes (\mathbf{4}, \mathbf{2}, \mathbf{1})$ plus their chiral partners:

$$(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1}), \quad (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{1}), \quad (\mathbf{6}, \mathbf{3}, \mathbf{1}), \quad (\mathbf{6}, \mathbf{1}, \mathbf{1}) \quad (4.1)$$

and their chiral partners. The $\mathbf{6}$ is a real representation of $SU(4)_{LC}$ and as a consequence the $(\bar{\mathbf{3}}, \mathbf{1})_{1/3}$ within has both leptoquark (from its embedding in $(\mathbf{6}, \mathbf{1}, \mathbf{1})$) and di-quark couplings (from $(\bar{\mathbf{6}}, \mathbf{1}, \mathbf{1}) \equiv (\mathbf{6}, \mathbf{1}, \mathbf{1})$), leading to unacceptable fast proton decay. The same holds for the $(\mathbf{6}, \mathbf{3}, \mathbf{1})$.

As we saw in the previous section, the S_3 in $(\bar{\mathbf{10}}, \mathbf{3}, \mathbf{1})$, which has left-handed couplings, is sufficient to explain the neutral-current anomalies at the scale $\Lambda \simeq 31 \text{ TeV}$. It also contributes to $R_{D^{(*)}}$; however, in order to explain the charged-current anomalies the scale for its contribution to $b \rightarrow c\tau\nu$ transitions needs to be $\Lambda \simeq 3.4 \text{ TeV}$ [20]. The $SU(2)_L$ conjugate process $b \rightarrow s\nu\nu$ would then contribute to $B \rightarrow K\nu\nu$, and such a low scale is already excluded by the Belle experiment. Therefore, one could be tempted to also use the S_1 in $(\bar{\mathbf{10}}, \mathbf{1}, \mathbf{1})$ with left-handed couplings, analogously to what is done in refs. [22, 23], to give a negatively interfering contribution to $B \rightarrow K\nu\nu$, bringing this observable in agreement with the existing null results. This does not work in the Pati-Salam model because of the strict constraints posed by the gauge symmetry to the Yukawa couplings. In particular, the couplings of the $(\bar{\mathbf{10}}, \mathbf{1}, \mathbf{1})$ are *antisymmetric* in flavour space. Therefore, in order to explain the charged current anomaly while at the same time suppressing $B \rightarrow K\nu\nu$ one needs to use large off-diagonal couplings between second and third generation fermions (and much smaller y_{33}). Thus, they would dominantly contribute to $b \rightarrow c\tau\nu_\mu$, which is non-interfering with the SM, thus lowering the scale needed to explain $R_{D^{(*)}}$ to $\Lambda \simeq 0.8 \text{ TeV}$. Hence, the contribution of S_3 and S_1 must be at similar scales $\approx \text{TeV}$, in order for the cancellation in $B \rightarrow K\nu\nu$ to occur, thus leading to an unacceptably large contribution of S_3 to the $SU(2)_L$ conjugate transition $b \rightarrow s\mu\tau$, and therefore to $B \rightarrow K\mu\tau$.

The only possibility left is to use $S_3 \in (\bar{\mathbf{10}}, \mathbf{3}, \mathbf{1})$ and its chiral partner $S_1 \in (\bar{\mathbf{10}}, \mathbf{1}, \mathbf{3})$, which has right-handed non-interfering couplings and therefore cannot be used to follow the strategy above. Nevertheless, in the next section we will show that the anomalies can be explained successfully by assuming that one of the right-handed neutrinos is light enough.

5 Combined explanation of the B -meson anomalies in Pati-Salam

As argued in the previous section, we consider the LQ $S_3 \in (\bar{\mathbf{10}}, \mathbf{3}, \mathbf{1})$ with left-handed couplings, and its chiral partner $S_1 \in (\bar{\mathbf{10}}, \mathbf{1}, \mathbf{3})$ with right-handed couplings. Their couplings are given in (2.11) and (2.12), and we take them to be real for simplicity. We assume parity, so that $y^L = y^R \equiv y$ and $V_L \simeq V_R$, neglecting the extra phases in V_R . Dropping the discrete left-right symmetry and considering independent couplings y_L and y_R would obviously make it easier to accommodate the B -meson anomalies, but turns out to not be necessary as a good fit can be obtained even with the strong extra constraint $y^L = y^R$; loop-induced corrections to this relation are discussed in appendix C but do not qualitatively change our conclusions. We furthermore assume that the couplings of the first generation are negligibly small, $y_{11} \simeq y_{12} \simeq y_{13} \simeq 0$, in order to satisfy the stringent limits involving first-generation fermions discussed in section 3.⁷

⁷In principle y_{13} can be relatively large, as in section 3, but we set it to zero anyway, to reduce the number of free parameters in the fit, and observables to take into account, since it would basically play no role in the explanation of the anomalies.

The LQ S_3 contributes to $\Delta C_9 = -\Delta C_{10}$ as given by eq. (3.4) and can easily explain the neutral-current anomalies, while S_1 does not contribute. To explain also the charged-current anomaly, we need to use S_1 with its right-handed couplings and the transition $b_R \rightarrow c_R \tau_R N_R$. This contributes to $R_{D^{(*)}}$ if (at least) one of the right-handed neutrino mass eigenstates is lighter than $\sim 100 \text{ MeV}$ [25–28]. We will denote this light neutrino mass eigenstate by $\hat{N}$. Then, assuming that this eigenstate has maximal overlap with the second generation $N_{R,2}$, there is a CKM enhanced contribution to $R_{D^{(*)}}$:

$$R_{D^{(*)}} \equiv \frac{\Gamma(B \rightarrow D\tau\nu, D\tau\hat{N})/\Gamma(B \rightarrow D\tau\nu_\tau)_{\text{SM}}}{\Gamma(B \rightarrow D\hat{\ell}\nu, D\hat{\ell}\hat{N})/\Gamma(B \rightarrow D\hat{\ell}\nu_\ell)_{\text{SM}}} \simeq 1 + \left(\frac{v^2}{4M_{S_1}^2} \frac{V_{R,cs}}{V_{L,cb}} y_{23}^R y_{23}^R \right)^2 \stackrel{!}{\simeq} 1.237 \pm 0.053, \quad (5.1)$$

with $\hat{\ell} = e, \mu$. The full expressions for this and the other relevant observables are given in appendix B. The relevant transition in flavour space is $b \rightarrow c\tau N_2$ and its $\text{SU}(2)_R$ conjugate is $b \rightarrow sN_3 N_2$. The latter would naively give a too large contribution to $B \rightarrow K\nu\nu$, analogously to the discussion in the previous section; however, if we postulate that the mixing between the light eigenstate $\hat{N}$ and N_3 is small, a sufficient suppression is obtained. This is the key point of the combined explanation of the B -meson anomalies in the PS model with parity. Then, setting this mixing to zero we have

$$B_{K\nu\nu} \equiv \frac{\Gamma(B \rightarrow K\nu\nu, K\hat{N}\hat{N})}{\Gamma(B \rightarrow K\nu_\ell\nu_\ell)_{\text{SM}}} \simeq 1 + \left(\frac{2\pi}{-6.4 \alpha V_{L,ts} V_{L,tb}} \frac{v^2}{4M_{S_1}^2} y_{22}^R y_{23}^R \right)^2 \stackrel{!}{\lesssim} 3.3 \text{ (95\% C.L.)} \quad (5.2)$$

which can be easily suppressed by y_{22} . A small $y_{22} \ll y_{23}$ is also functional to explain the difference in scales of the new-physics effects in $R_{K^{(*)}}$ and $R_{D^{(*)}}$, since the former depends on the flavour combination $y_{22}y_{23}$, as compared to $y_{23}y_{23}$.

If we allow for a small mixing $(V_R^\dagger U_R)_{3\hat{N}}$, the expression for $R_{D^{(*)}}$ in (5.1) would be modified by $y_{23}^R \rightarrow (V_R^\dagger U_R)_{2\hat{N}} y_{23}^R + (V_R^\dagger U_R)_{3\hat{N}} y_{33}^R$ with little effect. Instead, in (5.2) we would have $y_{22}^R \rightarrow (V_R^\dagger U_R)_{2\hat{N}} y_{22}^R + (V_R^\dagger U_R)_{3\hat{N}} y_{23}^R$, which can potentially increase significantly the new-physics contribution. A simple estimate obtained combining (5.1) and (5.2) then gives $(V_R^\dagger U_R)_{3\hat{N}} \lesssim 0.04$, implying a non-generic flavour structure in the right-handed neutrino sector. We will discuss the implications of this in section 5.3. Notice, however, that this upper limit for the mixing is similar to the CKM one between 2nd and 3rd generations, and therefore this should not be considered a priori as a fine tuning more than the structure of the CKM matrix itself. For simplicity, in the fit we will set $(V_R^\dagger U_R)_{3\hat{N}} = 0$, $(V_R^\dagger U_R)_{2\hat{N}} = 1$ and keep in mind that a non-zero value of the former would shrink the allowed parameter space.

5.1 Fit of flavour observables

We perform a combined fit as detailed in appendix B. In addition to the anomalies and $B_{K\nu\nu}$, we fit the LFUV in the first two generations as (non) observed in $R_{D^{(*)}}^{\mu/e}$ and the LFV process $B \rightarrow K\mu\tau$, which are generated at tree level. Moreover, we include several

Parameter	Best fit	Observable	Best fit	Pull/bound
M_{S_3}	6.5 TeV	$R_{D^{(*)}}$	1.24	+0.0 σ
y_{22}	0.034	$\Delta C_9 = -\Delta C_{10}$	-0.61	+0.0 σ
y_{23}	1.17	$R_{D^{(*)}}^{\mu/e}$	1.00	-0.0 σ
y_{33}	-0.01	$\delta g_{\tau\tau}^R$	0.5×10^{-4}	-1.2 σ
χ^2	2.4	$\delta g_{\mu\mu}^R$	-9.1×10^{-4}	-0.9 σ
		$B_{K\nu\nu}$	3.25	3.28
		$\text{BR}(B \rightarrow K\mu\tau)$	0.9×10^{-5}	4.8×10^{-5}
		$\text{BR}(\tau \rightarrow \mu\gamma)$	$2. \times 10^{-11}$	4.4×10^{-8}
		$\text{BR}(\tau \rightarrow 3\mu)$	$3. \times 10^{-10}$	2.1×10^{-8}

Table 2. Results of the fit for the combined explanation, fixing $M_{S_1} = 1$ TeV.

loop observables: the modifications to the Z couplings $\delta g_{\tau\tau}^R$, $\delta g_{\mu\mu}^R$, and the LFV processes $\tau \rightarrow \mu\gamma$, $\tau \rightarrow 3\mu$. One should keep in mind that loop observables can be affected by the other particles in the PS multiplets considered, for instance the other LQs and/or the di-leptons, in case these are light enough. Notice that $\Delta F = 2$ processes such as $B_s \leftrightarrow \bar{B}_s$ are small in our setup; this is one of the main virtues of explanations based on scalar LQs.

The results of the fit are given in table 2 for the fixed value of $M_{S_1} = 1$ TeV. Clearly, the fit depends only weakly on the absolute mass scale (up to constraints imposed by perturbativity/unitarity), since the tree-level observables are the most constraining and these depend only on ratios of couplings and $M_{S_{1,3}}$. As visible in table 2 the fit is very good and only has some minor pulls from the Z couplings.

We also show correlations between different parameters and observables in figure 3. Note that the charged-current anomaly is fitted basically with zero tension. This is a quite remarkable result since it is often difficult to fit the large observed value even in the much-less restricted simplified models [22]. Note also that one does not need to switch on y_{33} , although we did it for the sake of generality. Its inclusion does not improve the quality of the fit significantly. Finally, we stress that one of the phenomenological consequences of the model is the prediction of a large rate for the LFV process $B \rightarrow K\mu\tau$, since the relevant transition $b_L \rightarrow s_L \mu_L \tau_L$ is the parity plus $\text{SU}(2)_L$ partner of the $R_{D^{(*)}}$ one $b_R \rightarrow c_R N_{R,2} \tau_R$ (the former is mediated by S_3 , the latter by S_1). In particular,

$$\text{BR}(B \rightarrow K\mu\tau) \approx 8.6 \times 10^{-3} \left(\frac{\text{TeV}}{M_{S_3}} \right)^4 (y_{23}^L)^4 \stackrel{!}{\lesssim} 4.8 \times 10^{-5} \text{ (90\%C.L.)} \quad (5.3)$$

is required to be orders of magnitude higher than in other constructions. This, together with the large rate predicted for $B \rightarrow K\nu\nu$ is a smoking-gun phenomenological signature of our framework. As shown in figure 3 (bottom-right panel), Belle II [72] will test this model completely already with 5 ab^{-1} of data (with the assumption of parity).

5.2 Collider phenomenology

The LQ S_1 needs to be rather light and can be searched for at the LHC. S_3 can also be searched for at colliders in the parameter region where it is light enough, but we do not

Figure 3. Results of the fit for the combined explanation, with $M_{S_1} = 1 \text{ TeV}$. We show the $\Delta\chi^2 < 2.3$ (1σ), 6.2 (2σ), 11.8 (3σ) regions in green, yellow, and blue, respectively, after marginalising over all other parameters. The star denotes the best-fit point. In the bottom-left panel we show the observed anomalous values for these observables. In the bottom-right one we also show prospects for Belle II for certain luminosities [72, 73].

study this possibility here and rather focus on S_1 . The most important processes are the ones that involve the SM gauge couplings and/or the LQ Yukawa coupling y_{23} , which is required to be large to explain the B -meson anomalies. Hence, the most relevant searches are the ones for $pp \rightarrow \tau\nu$, $\nu\nu j$ and $t\mu t\mu$. The first two processes are given by a t -channel exchange of S_1 in the transitions $c\bar{b} \rightarrow \tau\hat{N}$, $b\bar{b} \rightarrow \hat{N}\hat{N}$, respectively (the latter accompanied by a jet to trigger on it), both proportional to $(y_{23})^2$ for the CKM-unsuppressed channels. The limits for the $\tau\nu$ final state are taken from the ATLAS analysis [74]. For the $\nu\nu j$ channel we adapt the results of [28], which recasts the CMS monojet search [75]. The most constraining channel is $pp \rightarrow S_1 S_1 \rightarrow t\mu t\mu$, where the LQs are pair-produced by QCD interactions. The CMS collaboration has recently performed a dedicated search for LQs decaying into third-generation quarks [76]. However, they provide results only for LQs decaying into either $t\mu$ or $b\nu$, i.e. with $\text{BR}(LQ \rightarrow t\mu) + \text{BR}(LQ \rightarrow b\nu) = 1$. In our model S_1 decays into $t\mu, c\tau, bN$ approximately with the same probability. Therefore, we recast the CMS search in the following way: we extract the results of [76] along the line $\text{BR}(LQ \rightarrow t\mu) = 1$ and rescale the observed limit on the cross-section by the inverse of our

Figure 4. Collider phenomenology for S_1 . The 1σ , 2σ , 3σ regions where both sets of anomalies are explained are denoted in green, yellow and blue, respectively. Existing LHC bounds are denoted by continuous lines and shaded regions, future prospects by dotted and dot-dashed lines.

$\text{BR}(S_1 \rightarrow t\mu)^2 \simeq 1/9$. We invite the experimental collaborations to analyse more general situations where the total branching ratio is not fixed to 1, and ideally to include the $c\tau$ final state in the analysis. The relation $\text{BR}(S_1 \rightarrow t\mu) \simeq \text{BR}(S_1 \rightarrow c\tau) \simeq \text{BR}(S_1 \rightarrow b \text{ invisible})$ is a smoking-gun signature of the Pati-Salam model. We show existing bounds and future limits in figure 4: the current bound on M_{S_1} is about 1 TeV. The LHC run III will cover the 1σ region, the high-luminosity LHC the entire parameter space that explains the B -meson anomalies.

5.3 Constraints on the UV structure of the model

The combined explanation of the anomalies poses constraints on the UV structure of the model, in particular regarding the PS breaking and right-handed neutrino mass generation. The successful explanation requires a right-handed neutrino $\hat{N}$ lighter than ≈ 100 MeV, and therefore much lighter than the PS-breaking scale $\gtrsim 1000$ TeV. In the discussion above, where we assumed parity to maximize the predictivity of the model, we also require a non-generic mixing pattern $(V_R^\dagger U_R)_{2\hat{N}} = \mathcal{O}(1)$, $(V_R^\dagger U_R)_{3\hat{N}} \lesssim 0.04$, at the level of the CKM one in the 2nd-3rd generation sector.

First, it is important to stress that this latter requirement is present only if parity is assumed. If the left- and right-handed CKM matrices were largely misaligned, with $V_{R,c b} = \mathcal{O}(1)$, one could explain the charged-current anomaly with an unsuppressed contribution:

$$R_{D^{(*)}} \supset \left(\frac{v^2}{4M_{S_1}^2} \frac{V_{R,c b}}{V_{L,c b}} y_{33}^R y_{23}^R \right)^2, \quad (5.4)$$

which in turns implies a contribution to $B_{K\nu\nu}$ as in (5.2), but with Yukawa couplings $y_{33}^R y_{23}^R$ and, most importantly, suppressed by the mixing *squared* $[(V_R^\dagger U_R)_{3\hat{N}}]^2$. Therefore, in this case the mixing needs to be smaller than ≈ 0.2 , which is not particularly constraining.

The lightness of $\hat{N}$, with $\mathcal{O}(1)$ couplings to S_1 , requires that $\langle\delta_1\rangle < \mathcal{O}(100)\text{MeV}$, if non-zero at all, and therefore this cannot be the only PS-breaking VEV. To show this, let us assume that $\langle\delta_1\rangle$ is the only source of masses for N_R , so that eq. (2.12) gives

$$U_R^* \text{diag}(M_1, M_2, M_3) U_R^\dagger = M_R = -\sqrt{2}\langle\delta_1\rangle V_R y^{R\dagger} V_R^\dagger \quad (5.5)$$

for the right-handed neutrino mass matrix. The couplings of S_1 to the mass eigenstates N_{mass} are then

$$\frac{1}{2\langle\delta_1\rangle} \bar{d}_R^c [V_R^\dagger U_R \text{diag}(M_1, M_2, M_3)] N_{\text{mass}} S_1. \quad (5.6)$$

Therefore, in order for the light eigenstate $\hat{N}$ to have $\mathcal{O}(1)$ couplings to S_1 its mass cannot be much smaller than $\langle\delta_1\rangle$. It is not possible to explain both B -meson anomalies without assuming additional sources of right-handed neutrino masses, in addition to the VEV $\langle\delta_1\rangle$, at some high scale.

Fortunately it is not difficult to extend the PS model in order to accommodate light right-handed neutrinos. Most importantly, whatever the mechanism is, its precise form is not expected to affect the explanation of the anomalies given above in this section, as long as it gives a light right-handed neutrino with suppressed mixing to the 3rd generation. For the sake of illustration, we now mention a couple of possibilities, although these should be considered only as toy examples. A detailed study of possible UV sectors goes beyond the scope of this paper.

A first possibility is to simply have a second scalar $(\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3})$ that gets a high-scale VEV, for instance at the 10^{12}GeV scale, but leaves one of the right-handed neutrinos very light, say 100MeV . The light LQs relevant for the B -meson anomalies then have Yukawa couplings that are unrelated to the neutrino mass structure and thus allow for $\mathcal{O}(1)$ couplings of this light N_R , leading precisely to the scenario fitted above. The large Dirac neutrino masses given by eq. (2.6) would then give a too large active neutrino mass of order $m_u^2/(100\text{MeV})$. This type-I contribution could be cancelled by a type-II contribution, which is clearly not well motivated. It is thus necessary to break the relation of eq. (2.6) and suppress the Yukawa coupling of the light right-handed neutrino to the active neutrinos, for example by invoking ϕ_{15} . One can then imagine that a Dirac mass $\mathcal{O}(\text{keV})$ would be suppressed sufficiently by $M_{\hat{N}} \sim 100\text{MeV}$ to generate sub-eV neutrino masses, giving a mixing angle $\sin\theta = \mathcal{O}(10^{-5})$ between $\hat{N}$ and at least one left-handed neutrino, so that $\hat{N}$ decays into 3ν in the early universe, with rate [77]

$$\Gamma(\hat{N} \rightarrow 3\nu) = \frac{G_F^2 \sin^2\theta}{96\pi^3} M_{\hat{N}}^5 \simeq (1.5 \times 10^4 \text{sec})^{-1} \left(\frac{\sin\theta}{10^{-5}}\right)^2 \left(\frac{M_{\hat{N}}}{100\text{MeV}}\right)^5, \quad (5.7)$$

well after Big-Bang nucleosynthesis,⁸ but well before the decoupling of the cosmic microwave background (thus not contributing to dark matter then). The light $\hat{N}$ can clearly

⁸Assuming also that it decouples non-relativistically from the thermal plasma, for instance being kept in equilibrium by the process $\hat{N}\hat{N} \leftrightarrow \nu_3\nu_3$, mediated by an $\mathcal{O}(10\text{TeV})$ admixture of the dileptons δ_3 and δ_1 .

lead to additional signatures, but generally unrelated to the B anomalies and thus severely model dependent [25–28].

A second, more radical, possibility is to get rid of the seesaw mechanism and change the neutrino mass generation. Following ref. [43] one could break PS by a scalar $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ and introduce singlet fermions, thus realizing an inverse-seesaw scenario that in principle allows for large couplings of the light $\hat{N}$, and to lower the PS scale down to ~ 1000 TeV, although in this latter case one needs to introduce also a mechanism to unify left and right gauge couplings at this scale (rather than at $\sim 10^{12}$ TeV) in the parity-symmetric scenario. Notice that the issue of neutrino-mass generation (or better, suppression) is often ignored in solutions of the B -meson anomalies via the PS-inspired models discussed in section 4. However, a first attempt in this direction has recently been given in ref. [78], for the model presented in ref. [37], along the lines discussed in this paragraph.

A full discussion of neutrino masses within PS needs to take the charged-fermion mass generation into account and must include effects of renormalization group running, which in turn depends on all potentially light particles such as the LQs. This is non-trivial even without attempting to resolve the B -meson anomalies and will therefore not be investigated here. At least qualitatively it seems that PS could indeed provide a consistent theory behind fermion masses and LFUV.

6 Conclusion

Recent hints for lepton-flavour non-universality in various B -meson decays point towards new physics around the TeV scale. Combined explanations for $R_{K^{(*)}}$ and $R_{D^{(*)}}$ can be found most easily in leptoquark models, although these are usually ad-hoc extensions without any relations to other processes or structures. Interestingly, they could originate from models that lead to (partial) unification, such as SU(5) or Pati-Salam, where the occurrence of LQs is an absolute necessity. Here we have shown that the Pati-Salam model naturally contains the relevant scalar LQs to address the B -meson anomalies and has additional welcome features: i) proton decay via LQs is avoided, ii) the LQ couplings are automatically chiral, iii) the couplings are restricted and related, making the model highly predictive.

The anomaly in $R_{K^{(*)}}$ can be easily addressed by assuming an S_3 LQ around 10 TeV. Within the Pati-Salam model the coupling structure is restricted to be *symmetric* in flavour space and can furthermore be related to the type-II seesaw neutrino mass structure, leading to interesting predictions for the neutrino mass parameters. This leads to testable rates for muonic LFV.

Explaining also the anomaly in $R_{D^{(*)}}$ requires one of the right-handed neutrinos to be sub-GeV, as well as an S_1 LQ around TeV. Pati-Salam once again heavily restricts the coupling structure, making it non-trivial to explain both $R_{K^{(*)}}$ and $R_{D^{(*)}}$ simultaneously. We have shown that one can obtain a near-perfect explanation for the anomalies despite the restricted couplings, leading again to highly testable predictions for processes such as $B \rightarrow K \nu \nu$ and $B \rightarrow K \mu \tau$, as well as direct collider searches. The connection to neutrino masses has to be sacrificed; it would be interesting to address it in an extension of the basic Pati-Salam model analyzed here, together with an UV sector of the model that gives rise to the required pattern of masses and mixing at the TeV scale and below.

Acknowledgments

We thank Thomas Hambye for discussions and collaboration on the early stages of this project. DT would like to thank Gian Giudice for stimulating discussions that motivated this work, Dario Buttazzo for clarifications on ref. [22], and Luca di Luzio for sharing his results on gauge-coupling unification. JH is a postdoctoral researcher of the F.R.S.-FNRS and furthermore supported, in part, by the National Science Foundation under Grant No. PHY-1620638, and by a Feodor Lynen Research Fellowship of the Alexander von Humboldt Foundation. DT is supported by an ULB postdoctoral fellowship, an ULB-ARC grant, the IAP P7/37, and the ERC grant NEO-NAT.

A Details of the fit for $R_{K^{(*)}}$ and type-II neutrino masses

In this appendix we give details about the fit performed in section 3 and present the explicit form of the observables used.

For the purposes of the discussion in section 3, we are interested in the contribution of the LQ S_3 . Its contribution to the neutral-current anomalies is given by (3.4). The relevant phenomenological constraints are given by observables involving first-generation fermions. In particular, the most stringent constraint is given by the LFV $\mu \rightarrow e$ conversion in Au_{79}^{197} nuclei [52, 79]:

$$\text{BR}(\mu \rightarrow e) = \frac{1}{13.07 \times 10^6 \text{ sec}^{-1}} \frac{4 M_\mu^5}{M_{S_3}^4} \left| (2V_p + V_n) V_{L,uq}^* y_{1q}^L V_{L,uq'} y_{2q'}^{L*} + (V_p + 2V_n) y_{11}^L y_{12}^{L*} \right|^2, \quad (\text{A.1})$$

with $q, q' = 1, 2, 3$ being summed over, $V_p = 0.0974$, and $V_n = 0.146$. We also consider the LFV process $\mu \rightarrow e\gamma$,

$$\Gamma(\mu \rightarrow e\gamma) = \frac{\alpha M_\mu^5}{16384\pi^4 M_{S_3}^4} \left| -4y_{1q}^{L*} y_{2q}^L + V_{L,qq'} y_{1q'}^{L*} V_{L,qq''}^* y_{2q''}^L \right|^2, \quad (\text{A.2})$$

as well as the LFV decay of the neutral kaon K_L [80],

$$\Gamma(K_L \rightarrow e^\pm \mu^\mp) \simeq 6.45 \times 10^{-19} \text{ GeV} \left(\frac{\text{TeV}}{M_{S_3}} \right)^4 |y_{12}^{L*} y_{12}^L + y_{11}^{L*} y_{22}^L|^2. \quad (\text{A.3})$$

Lepton-flavour *conserving* decays of K_L to light leptons receive an SM contribution that is difficult to evaluate, and therefore we do not consider them. Other potentially large effects are expected in the R -ratios measuring LFUV in kaon and pion decays:

$$R^{e/\mu}_{K} \equiv \frac{\Gamma(K \rightarrow e\nu)/\Gamma(K \rightarrow e\nu_e)_{\text{SM}}}{\Gamma(K \rightarrow \mu\nu)/\Gamma(K \rightarrow \mu\nu_\mu)_{\text{SM}}} - 1 \simeq -2C_3 \text{Re} \left(\frac{V_{L,uq}}{V_{L,us}} y_{1q}^{L*} y_{12}^L - \frac{V_{L,uq}}{V_{L,us}} y_{2q}^{L*} y_{22}^L \right), \quad (\text{A.4})$$

$$R^{e/\mu}_{\pi} \equiv \frac{\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow e\nu_e)_{\text{SM}}}{\Gamma(\pi \rightarrow \mu\nu)/\Gamma(\pi \rightarrow \mu\nu_\mu)_{\text{SM}}} - 1 \simeq -2C_3 \text{Re} \left(\frac{V_{L,uq}}{V_{L,ud}} y_{1q}^{L*} y_{11}^L - \frac{V_{L,uq}}{V_{L,ud}} y_{2q}^{L*} y_{12}^L \right), \quad (\text{A.5})$$

Observable	Value	Source
$\Delta C_9 = -\Delta C_{10}$	-0.61 ± 0.12	[14]
$K R^{e/\mu}$	$(4.4 \pm 4.0) \times 10^{-3}$	[52, 81]
$\pi R^{e/\mu}$	$(-2.0 \pm 1.9) \times 10^{-3}$	[52, 81]
$K_{\pi\nu\nu}$	2.2 ± 1.4	[52, 82]
$\text{BR}(\mu \rightarrow e)$	$< 7 \times 10^{-13}$	[66]
$\text{BR}(\mu \rightarrow e\gamma)$	$< 4.2 \times 10^{-13}$	[83]
$\Gamma(K_L \rightarrow e^\pm \mu^\mp)$	$< 6.0 \times 10^{-29} \text{ GeV}$	[84]

Table 3. Observables used in the fit for $R_{K^{(*)}}$ and neutrino masses in section 3.

having introduced $C_3 \equiv v^2/(4M_{S_3}^2)$. Finally, we also consider the contribution to the kaon decay into pions and neutrinos, $K \rightarrow \pi\nu\nu$:

$$K_{\pi\nu\nu} \equiv \frac{\Gamma(K^+ \rightarrow \pi^+ \nu\nu)}{\Gamma(K^+ \rightarrow \pi^+ \nu_\ell \nu_\ell)_{\text{SM}}} \simeq \frac{1}{3} \left(|1 + a_K C_3 y_{12}^L y_{11}^{L*}|^2 + |1 + a_K C_3 y_{22}^L y_{12}^{L*}|^2 + |1 + a_K C_3 y_{23}^L y_{13}^{L*}|^2 + |a_K C_3 y_{22}^L y_{11}^{L*}|^2 \right), \quad (\text{A.6})$$

with $a_K \equiv \frac{2\pi}{-6.4 \alpha V_{L,td}^* V_{L,ts}}$ [52].

To incorporate the information given by experimental *upper bounds* in the fit, we smoothly penalize points in the parameter space that violate a bound by introducing an artificial likelihood

$$-2 \log \mathcal{L}(x = \text{observable/bound}) = \begin{cases} 0, & x < \lambda, \\ 4 \left(\frac{x - \lambda}{1 - \lambda} \right)^2, & x \geq \lambda, \end{cases} \quad (\text{A.7})$$

with the choice $\lambda = 0.95$, since the real likelihood is not provided by the experiments. In this way, points at the bound are penalized by $-2 \log \mathcal{L} = 4$, corresponding to 2σ in the χ^2 fit, whereas points well within the experimental bound do not receive any penalization.

For the observables, we use the values given in table 3. In principle, eq. (A.7) has been chosen to emulate 95% C.L. exclusion limits ($\sim 2\sigma$), whereas the experimental bounds in table 3 are at 90% C.L.; we ignore this difference because the experimental bounds at these different C.L. are presumably very close to each other for these observables.

B Details of the fit for $R_{K^{(*)}}$ and $R_{D^{(*)}}$

In this appendix we give details about the fit performed in section 5 and give the explicit form of the observables used. We are interested in the contribution of the LQs S_3 and S_1 with left- and right-handed couplings, respectively. The couplings are taken to be real, and the ones to the first generation are considered to be negligible.

Only S_3 contributes to the neutral-current anomaly, as given by eq. (3.4). Instead, both of them contribute to the charged-current anomaly:

$$R_{D^{(*)}} \equiv \frac{\Gamma(B \rightarrow D\tau\nu, D\tau\hat{N})/\Gamma(B \rightarrow D\tau\nu_\tau)_{\text{SM}}}{\Gamma(B \rightarrow D\ell\nu, D\ell\hat{N})/\Gamma(B \rightarrow D\ell\nu_\ell)_{\text{SM}}} = \frac{R_{D^{(*)}}^\tau}{\frac{1}{2}(1 + R_{D^{(*)}}^\mu)} \quad (\text{B.1})$$

with

$$R_{D^{(*)}}^\tau = \left(1 - C_3 y_{33}^L \left(\frac{V_{L,cs}}{V_{L,cb}} y_{23}^L + y_{33}^L\right)\right)^2 + \left(C_3 y_{23}^L \left(\frac{V_{L,cs}}{V_{L,cb}} y_{23}^L + y_{33}^L\right)\right)^2 + \left(C_1 y_{23}^R \left(\frac{V_{R,cs}}{V_{L,cb}} y_{23}^R + \frac{V_{R,cb}}{V_{L,cb}} y_{33}^R\right)\right)^2, \quad (\text{B.2})$$

$$R_{D^{(*)}}^\mu = \left(1 - C_3 y_{23}^L \left(\frac{V_{L,cs}}{V_{L,cb}} y_{22}^L + y_{23}^L\right)\right)^2 + \left(C_3 y_{33}^L \left(\frac{V_{L,cs}}{V_{L,cb}} y_{22}^L + y_{23}^L\right)\right)^2 + \left(C_1 y_{23}^R \left(\frac{V_{R,cs}}{V_{L,cb}} y_{22}^R + \frac{V_{R,cb}}{V_{L,cb}} y_{23}^R\right)\right)^2, \quad (\text{B.3})$$

and $C_{1,3} \equiv v^2/(4M_{S_{1,3}}^2)$. The contribution of S_1 dominates, since it is significantly lighter than S_3 . We also consider the LFUV in the first two generations:

$$R_{D^{(*)}}^{\mu/e} \equiv \frac{\Gamma(B \rightarrow D\mu\nu, D\mu\hat{N})/\Gamma(B \rightarrow D\mu\nu_\mu)_{\text{SM}}}{\Gamma(B \rightarrow De\nu, De\hat{N})/\Gamma(B \rightarrow De\nu_e)_{\text{SM}}} = R_{D^{(*)}}^\mu. \quad (\text{B.4})$$

As discussed in the main body, a particularly constraining observable is given by the decay of B into a kaon and neutrinos

$$B_{K\nu\nu} \equiv \frac{\Gamma(B \rightarrow K\nu\nu, K\hat{N}\hat{N})}{\Gamma(B \rightarrow K\nu_\ell\nu_\ell)_{\text{SM}}} = \frac{1}{3} \left[1 + (1 + a C_3 y_{33}^L y_{23}^L)^2 + (1 + a C_3 y_{23}^L y_{22}^L)^2 + (a C_3 y_{33}^L y_{22}^L)^2 + (a C_3 y_{23}^L y_{23}^L)^2 + (a C_1 y_{22}^R y_{23}^R)^2 \right], \quad (\text{B.5})$$

with $a \equiv \frac{2\pi}{-6.4 \alpha V_{L,ts} V_{L,tb}}$. Since the coupling y_{23} needs to be relatively large to explain the anomalies, we have an enhanced contribution to the LFV decay [53]:

$$\text{BR}(B \rightarrow K\mu\tau) \simeq 8.6 \times 10^{-3} \left(\frac{\text{TeV}}{M_{S_3}}\right)^4 [(y_{23}^L y_{23}^L)^2 + (y_{22}^L y_{33}^L)^2]. \quad (\text{B.6})$$

We also consider loop modifications of the Z -boson couplings to right-handed leptons, normalized as $\frac{g}{c_W}(s_W^2 + \delta g_{\ell\ell'}^R)Z_\alpha \bar{\ell}\gamma^\alpha P_R \ell'$:

$$\delta g_{\tau\tau}^R \simeq \frac{1}{288\pi^2 M_{S_1}^2} \left[27M_t^2 (V_{R,ts} y_{23}^R + V_{R,tb} y_{33}^R)^2 \left(1 - 2 \log\left(\frac{M_{S_1}}{M_t}\right)\right) + M_Z^2 s_W^2 \left((V_{R,cs} y_{23}^R + V_{R,cb} y_{33}^R)^2 + (V_{R,ts} y_{23}^R + V_{R,tb} y_{33}^R)^2 \right) \left(5 + 24 \log\left(\frac{M_{S_1}}{M_Z}\right)\right) \right], \quad (\text{B.7})$$

$$\delta g_{\mu\mu}^R \simeq \frac{1}{288\pi^2 M_{S_1}^2} \left[27M_t^2 (V_{R,ts} y_{22}^R + V_{R,tb} y_{23}^R)^2 \left(1 - 2 \log\left(\frac{M_{S_1}}{M_t}\right)\right) + M_Z^2 s_W^2 \left((V_{R,cs} y_{22}^R + V_{R,cb} y_{23}^R)^2 + (V_{R,ts} y_{22}^R + V_{R,tb} y_{23}^R)^2 \right) \left(5 + 24 \log\left(\frac{M_{S_1}}{M_Z}\right)\right) \right], \quad (\text{B.8})$$

$$\delta g_{\mu\tau}^R \simeq \frac{1}{576\pi^2 M_{S_1}^2} \left[3(V_{R,ts} y_{22}^R + V_{R,tb} y_{23}^R)(V_{R,ts} y_{23}^R + V_{R,tb} y_{33}^R) \times \left[18M_t^2 - M_Z^2(3c_W^2 + 13s_W^2) - 4(9M_t^2 - 4M_Z^2 s_W^2) \log\left(\frac{M_{S_1}}{M_t}\right) \right] + 2M_Z^2 s_W^2 (V_{R,cs} y_{22}^R + V_{R,cb} y_{23}^R)(V_{R,cs} y_{23}^R + V_{R,cb} y_{33}^R) \left(5 + 24 \log\left(\frac{M_{S_1}}{M_Z}\right) + 12i\pi\right) \right], \quad (\text{B.9})$$

Observable	Value	Source
$\Delta C_9 = -\Delta C_{10}$	-0.61 ± 0.12	[14]
$R_{D^{(*)}}$	1.237 ± 0.053	[22]
$R_{D^{(*)}}^{\mu/e}$	1.00 ± 0.02	[49]
$\delta g_{\tau\tau}^R$	$(8 \pm 6) \times 10^{-4}$	[86]
$\delta g_{\mu\mu}^R$	$(3 \pm 13) \times 10^{-4}$	[86]
$B_{K\nu\nu}$	< 3.28	[87]
$\text{BR}(B \rightarrow K\mu\tau)$	$< 4.8 \times 10^{-5}$	[88]
$\text{BR}(\tau \rightarrow \mu\gamma)$	$< 4.4 \times 10^{-8}$	[89]
$\text{BR}(\tau \rightarrow \mu\mu\mu)$	$< 2.1 \times 10^{-8}$	[90]

Table 4. Observables used in the combined fit in section 5. $B_{K\nu\nu}$ was converted to 95% C.L., see the discussion in the text, all other upper bound are 90% C.L.

having neglected the non-interfering imaginary part for the lepton-flavour conserving couplings. Similar corrections but to the left-handed couplings $\delta g_{\ell\ell'}^L$, arise from S_3 but are correspondingly suppressed. The LFV coupling $\delta g_{\mu\tau}^R$ gives rise to the LFV decay of the τ lepton,

$$\frac{\Gamma(\tau \rightarrow \mu\mu\mu)}{\Gamma(\tau \rightarrow \mu\nu\nu)} \simeq [8s_W^4 + 4(s_W^2 - 1/2)^2] |\delta g_{\tau\mu}^R|^2, \quad (\text{B.10})$$

neglecting additional box diagrams [85]. Finally, we also consider the radiative LFV decay $\tau \rightarrow \mu\gamma$,

$$\begin{aligned} \Gamma(\tau \rightarrow \mu\gamma) \simeq & \frac{\alpha (M_\tau^2 - M_\mu^2)^3}{16384 \pi^4 M_{S_1}^4 M_\tau} [(V_{R,ts} y_{22}^R + V_{R,tb} y_{23}^R) (V_{R,ts} y_{23}^R + V_{R,tb} y_{33}^R) \\ & + (V_{R,cs} y_{22}^R + V_{R,cb} y_{23}^R) (V_{R,cs} y_{23}^R + V_{R,cb} y_{33}^R)]^2, \end{aligned} \quad (\text{B.11})$$

again neglecting the contribution of S_3 .

For the observables, we use the values given in table 4. We incorporate the information given by experimental upper bounds in the fit by means of the artificial likelihood (A.7). Since this is meant to emulate 95% C.L. exclusion limits ($\sim 2\sigma$), we chose to convert naively the 90% C.L. bound for $B_{K\nu\nu}$ given by Belle [87], as $2.7 \times (2\sigma/1.645\sigma) \simeq 3.28$, in order to be conservative for this particularly constraining observable. We ignore this difference for the other observables.

C Renormalization group running of Yukawas

The most interesting LQ-coupling predictions of our Pati-Salam model are the symmetric structure $y = y^\top$ as well as the left-right exchange symmetry $y^L = y^R$ (although this can be loosened even within PS). These relations hold above the PS-breaking scale μ_{PS} by construction, but will typically be broken at lower scales due to loop effects. In this appendix we will study this in two benchmark scenarios. For this purpose we make use of the one-loop renormalization group equations (RGEs) for the LQ Yukawa couplings y , $dy/d\log\mu = \beta_y/(4\pi)^2$, which have been known for a long time [91, 92] and have been

obtained here with SARAH [93, 94]. We will make a number of simplifying assumptions in order to focus on the main points: we will assume at low scales just the SM plus the S_x LQs and in some cases the right-handed neutrinos. All other particles, namely the non-SM gauge bosons and scalars, are assumed to sit closer to the PS scale and are thus neglected in the running. For the sake of simplicity, we assume that the SM Higgs boson H contributes predominantly to the SM fermion masses, thus approximating the Yukawa couplings by $y_f = m_f/\langle H \rangle$. In this approximation, only the top-quark Yukawa is large and relevant for the RGEs, so we will neglect all the other SM Yukawa couplings. Note that the LQ Yukawa matrices are written so that they have the quark index on the left side and the lepton index on the right, as is clear from the matrix multiplications below.

C.1 Light right-handed neutrinos

Let us start our discussion by assuming the right-handed neutrinos to be light enough to contribute to the RGEs down to the TeV scale. This is motivated by the fact that at least one of the right-handed neutrinos needs to be light for the combined explanation of the anomalies, as described in section 5. The beta functions for all the LQ couplings are then

$$\beta_{y_{S_3}^L} = - \left(\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4g_3^2 \right) y_{S_3}^L + \text{tr} \left(y_{S_3}^L y_{S_3}^{L\dagger} \right) y_{S_3}^L + 3y_{S_3}^L y_{S_3}^{L\dagger} y_{S_3}^L + \frac{1}{2}y_u^\dagger y_u^* y_{S_3}^L, \quad (\text{C.1})$$

$$\begin{aligned} \beta_{y_{S_1}^R} = & - (2g_1^2 + 4g_3^2) y_{S_1}^R + \text{tr} \left(y_{S_1}^R y_{S_1}^{R\dagger} \right) y_{S_1}^R + 2y_{S_1}^R y_{S_1}^{R\dagger} y_{S_1}^R + \frac{3}{4}y_{S_1}^R y_{S_{1,e}}^{R\dagger} y_{S_{1,e}}^R \\ & + \frac{1}{4}y_{S_{1,N}}^R y_{S_{1,N}}^{R\dagger} y_{S_1}^R, \end{aligned} \quad (\text{C.2})$$

$$\begin{aligned} \beta_{y_{S_{1,e}}^R} = & - \left(\frac{13}{5}g_1^2 + 4g_3^2 \right) y_{S_{1,e}}^R + \frac{1}{2}\text{tr} \left(y_{S_{1,e}}^R y_{S_{1,e}}^{R\dagger} \right) y_{S_{1,e}}^R + \frac{1}{2}\text{tr} \left(y_{S_{1,N}}^R y_{S_{1,N}}^{R\dagger} \right) y_{S_{1,e}}^R \\ & + y_{S_{1,e}}^R y_{S_{1,e}}^{R\dagger} y_{S_{1,e}}^R + \frac{3}{2}y_{S_{1,e}}^R y_{S_1}^{R\dagger} y_{S_1}^R + \frac{1}{2}y_{S_1}^R y_{S_1}^{R\dagger} y_{S_{1,e}}^R + y_u y_u^\dagger y_{S_{1,e}}^R, \end{aligned} \quad (\text{C.3})$$

$$\begin{aligned} \beta_{y_{S_{1,N}}^R} = & - \left(\frac{1}{5}g_1^2 + 4g_3^2 \right) y_{S_{1,N}}^R + \frac{1}{2}\text{tr} \left(y_{S_{1,N}}^R y_{S_{1,N}}^{R\dagger} \right) y_{S_{1,N}}^R + \frac{1}{2}\text{tr} \left(y_{S_{1,e}}^R y_{S_{1,e}}^{R\dagger} \right) y_{S_{1,N}}^R \\ & + y_{S_{1,N}}^R y_{S_{1,N}}^{R\dagger} y_{S_{1,N}}^R + \frac{3}{2}y_{S_{1,N}}^R y_{S_1}^{R\dagger} y_{S_1}^R + \frac{1}{2}y_{S_1}^R y_{S_1}^{R\dagger} y_{S_{1,N}}^R, \end{aligned} \quad (\text{C.4})$$

$$\begin{aligned} \beta_{y_{S_1}^R} = & - \left(\frac{4}{5}g_1^2 + 4g_3^2 \right) y_{S_1}^R + \text{tr} \left(y_{S_1}^R y_{S_1}^{R\dagger} \right) y_{S_1}^R + 2y_{S_1}^R y_{S_1}^{R\dagger} y_{S_1}^R + \frac{3}{4}y_{S_1}^R y_{S_{1,N}}^{R\dagger} y_{S_{1,N}}^R \\ & + \frac{1}{4}y_{S_{1,e}}^R y_{S_{1,e}}^{R\dagger} y_{S_1}^R + y_u y_u^\dagger y_{S_1}^R, \end{aligned} \quad (\text{C.5})$$

where we introduced two different couplings for S_1 , which are identical above the PS scale but split by the RG flow below it. All these couplings should be equal at the scale μ_{PS} , which serves as a boundary condition for the solution of these differential equations. We will be interested in the coupling structures at scales $\mu \sim \text{TeV}$, where the anomalies are generated. At this low scale, one of the coupling matrices can be fixed, e.g. by our fit, but the other ones are determined by RGE running.

All couplings receive the same contribution from the strong-coupling constant g_3 , but will clearly run apart due to the different contributions from the electroweak gauge couplings $g_{1,2}$ and the top-quark Yukawa $y_u = \text{diag}(0, 0, y_{\text{top}})$. (Note that g_1 has the standard

Figure 5. The relative difference of S_3 and S_1 LQ couplings at the TeV scale, $(y^R - y^L)/y^L|_{\mu=\text{TeV}}$, as a function of the PS scale μ_{PS} . The left (right) figure assumes light (heavy) right-handed neutrinos.

GUT normalization.) In the limit $g_{1,2} = 0 = y_u$, $y_{\dots}^R = y^R$ all the y^R beta functions become identical and equal to the one for y^L . This is to be expected because $(S_1, \tilde{S}_1, \tilde{\tilde{S}}_1)$ forms the parity partner of S_3 . The PS predictions $y_{\dots}^R = y^L = (y^L)^\top$ are then mainly broken by $y_{\text{top}} \sim 1$ with contributions of order $y \frac{y_{\text{top}}^2}{(4\pi)^2} \log(\mu_{\text{PS}}/\text{TeV})$. This corresponds to percent-level modifications of our formula even if the PS scale is close to the GUT scale.

This conclusion makes use of the fact that light right-handed neutrinos as well as light S_3 and $(S_1, \tilde{S}_1, \tilde{\tilde{S}}_1)$ LQs leave the left-right exchange symmetry approximately intact. Let us consider an explicit breaking of this symmetry by making $\tilde{\tilde{S}}_1$ and $\tilde{\tilde{S}}_1$ heavy, leaving only S_3 , S_1 , and the light N s in addition to the SM. This case, corresponding to $y_{\tilde{\tilde{S}}_1}^R = 0 = y_{\tilde{\tilde{S}}_1}^L$ in the above RGEs, is shown in figure 5 (left), assuming $y^L(\mu = \text{TeV})$ to be of the form relevant for our fit, i.e. taking the best-fit values from table 2. We see that the absence of $\tilde{\tilde{S}}_1$ and $\tilde{\tilde{S}}_1$ enhances the left-right asymmetry $y^R - y^L$, albeit only at the level of $\mathcal{O}(10\%)$ for not too large PS scales. Such corrections are not relevant for the fit performed in the main text and do not change our conclusions. This is especially true for the 23 entry, which is the largest of the Yukawa couplings and is left-right symmetric at the few percent level. Finally, in figure 6 we show the relative breaking of the most authentic PS prediction $y = y^\top$, i.e. colour-lepton unification, due to the running; again, for the 23 vs 32 entries, relevant for the explanation of the anomalies, the asymmetry is at most at the few percent level, as anticipated above.

C.2 Heavy right-handed neutrinos

Let us now consider the case in which all right-handed neutrinos are heavy. While, as pointed out above, at least one of the right-handed neutrinos needs to be light for the combined explanation of the anomalies, the case considered here provides a benchmark scenario to assess the dependence of our conclusions on the details of the spectrum not relevant for the explanation of the anomalies. The heaviness of the right-handed neutrinos leads to a larger violation of the PS predictions $y_{\dots}^R = y^L$. Assuming only the SM plus the

Figure 6. The relative difference of 23 and 32 LQ couplings at the TeV scale, $(y_{23}^R - y_{32}^R)/y_{23}^L|_{\mu=\text{TeV}}$, as a function of the PS scale μ_{PS} , assuming light right-handed neutrinos.

S_x LQs we find the RGEs

$$\beta_{y_{S_3}^L} = - \left(\frac{1}{2}g_1^2 + \frac{9}{2}g_2^2 + 4g_3^2 \right) y_{S_3}^L + \text{tr} \left(y_{S_3}^L y_{S_3}^{L\dagger} \right) y_{S_3}^L + 3y_{S_3}^L y_{S_3}^{L\dagger} y_{S_3}^L + \frac{1}{2}y_u^\dagger y_u^* y_{S_3}^L, \quad (\text{C.6})$$

$$\beta_{y_{\tilde{S}_1}^R} = - (2g_1^2 + 4g_3^2) y_{\tilde{S}_1}^R + \text{tr} \left(y_{\tilde{S}_1}^R y_{\tilde{S}_1}^{R\dagger} \right) y_{\tilde{S}_1}^R + 2y_{\tilde{S}_1}^R y_{\tilde{S}_1}^{R\dagger} y_{\tilde{S}_1}^R + \frac{3}{4}y_{\tilde{S}_1}^R y_{S_1}^{R\dagger} y_{S_1}^R, \quad (\text{C.7})$$

$$\beta_{y_{S_1}^R} = - \left(\frac{13}{5}g_1^2 + 4g_3^2 \right) y_{S_1}^R + \frac{1}{2}\text{tr} \left(y_{S_1}^R y_{S_1}^{R\dagger} \right) y_{S_1}^R + y_{S_1}^R y_{S_1}^{R\dagger} y_{S_1}^R + \frac{3}{2}y_{S_1}^R y_{\tilde{S}_1}^{R\dagger} y_{\tilde{S}_1}^R + y_u y_u^\dagger y_{S_1}^R, \quad (\text{C.8})$$

with $\tilde{S}_1$ being irrelevant since it only has couplings to the heavy N . We will further assume $\tilde{S}_1$ to be heavy, since we do not need it for the B -meson anomalies, leaving only the S_1 and S_3 Yukawa couplings to be compared. These two couplings will again run apart, more prominently than before due to all the additional explicit left-right breaking due to split multiplets (see figure 5 (right)). The large 23 entry is still roughly the same for S_1 and S_3 , the small $\mathcal{O}(20\%)$ corrections not being important in our fit. The 22 entry becomes potentially more asymmetric, but still at most at the $\mathcal{O}(60\%)$ level. Therefore, even in the most pessimistic case, our results in figure 3 will be at most changed by $\mathcal{O}(1)$ factors, leaving our qualitative conclusions unchanged.

Finally, we conclude by stressing the fact that the above exercise should be considered as a simple estimate of the accuracy of the approximations used in the main body. A quantitatively accurate treatment of the running requires the specification of the full spectrum up to the PS scale. However, only particles lighter than a few TeV contribute to the anomalies and therefore we do not need to specify the full spectrum for the purposes of this work. This will be left for future work, but below we will provide a brief discussion on the impact of $y_L \neq y_R$ on the B -anomaly fit.

C.3 Impact on the fit for the combined explanations

As clear from the discussion above, the relations $y_{\dots}^R = y^L = (y^L)^\dagger$ are deformed at the TeV scale from RG effects. Nevertheless, in our assumption of a left-right symmetry at the PS

scale, the two sets of couplings should not be considered independent for the combined fit as in section 5.

Knowing the full spectrum up to the PS scale one could then consider the couplings $y^R = y^L = (y^L)^\intercal$ at the PS scale as input parameters and, for each point in the parameter space, perform the RG running down to the TeV scale and use these value for the phenomenological observables entering the fitting procedure. However, since we do not presume to have knowledge of the full spectrum (only the LQs at the TeV scale have direct impact on the explanation of the anomalies), such a procedure would hardly seem sensible considering the combinatorial number of possible light states and thresholds. Moreover, from a more practical point of view, performing the RG running for each point in the parameter space during the likelihood maximization procedure would increase significantly the computing power required for the numerical fit.

Therefore, in order to assess the impact of the RG-running effects, we perform a fit with the relations $y_{\dots}^R = y^L = (y^L)^\intercal$ broken by a fixed typical amount. In detail, as shown in figure 5 and 6 assuming light right-handed neutrinos, the main breaking is in the equality $y^R = y^L$ for the relevant 22 and 23 entries. We thus choose the deformation suggested by the running of the best-fit values from table 2 (see figure 5 (left))

$$y_{22}^R = 1.5 \times y_{22}^L, \quad y_{23}^R = 1.1 \times y_{23}^L, \quad (\text{C.9})$$

with equal couplings of S_1 to N and e and keeping the rather accurate PS relation $y = y^\intercal$. The results of the fit are shown in figure 7. As can be seen by comparing with figure 3, the impact of RG effects is rather mild, thus justifying the usage of the unbroken relations $y_{\dots}^R = y^L = (y^L)^\intercal$ for qualitative and semi-quantitative conclusions, as in the main body of the paper.

D Left-right gauge coupling unification

The PS group breaking $\text{SU}(4)_{LC} \times \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ by itself does not impose any restrictions on the three gauge couplings. However, since we mostly focus on the PS model *with* left-right unification (parity $L \leftrightarrow R$) in order to constrain the left and right Yukawa couplings for a *combined* explanation of both sets of anomalies in section 5, one may wonder if such a scenario is viable from the UV point of view. In particular, this requires the gauge couplings of the $\text{SU}(2)_L$ and $\text{SU}(2)_R$ factors to be equal at a scale that defines the PS-breaking scale μ_{PS} , $g_R(\mu_{\text{PS}}) = g_L(\mu_{\text{PS}})$, which translates into a matching condition on the SM gauge-coupling fine-structure constants $\alpha_i \equiv g_i^2/(4\pi)$,

$$\frac{3}{5} \alpha_L^{-1}(\mu_{\text{PS}}) = \alpha_1^{-1}(\mu_{\text{PS}}) - \frac{2}{5} \alpha_C^{-1}(\mu_{\text{PS}}), \quad (\text{D.1})$$

with GUT normalization for the hypercharge generator $Q_1 = \sqrt{3/5} Y$. This is the lowest-order matching condition appropriate for the one-loop RGEs under consideration here, neglecting threshold corrections, see e.g. refs. [95–97] for improved formulae. Using only the SM particle content this fixes the PS scale $\mu_{\text{PS}} \sim 5 \times 10^{13} \text{ GeV}$.

Figure 7. Same as figure 3, but with the couplings deformed as in eq. (C.9).

Since two lines almost always meet at one point, eq. (D.1) can be generically satisfied even in presence of additional light particles beyond the SM, unless: (i) many new thresholds intervene that destroy the asymptotic freedom of the colour gauge coupling and make it grow faster than the hypercharge coupling; or (ii) the couplings would unify at non-perturbative values or beyond the Planck scale. Given that the running assuming only light SM degrees of freedom is rather far from these exceptional situations, we expect left-right unification to be generically possible. Since we do not presume to have knowledge of the full spectrum up to the PS scale, we simply show a proof of concept in figure 8, in which only the LQs required for the explanation of the anomalies lie below the PS scale. This shows that successful $g_L = g_R$ unification is achieved, at a scale slightly higher than in the SM. However, one should keep in mind that this should only be considered as an exercise because the presence of additional states in the *desert*, especially if coming in generation copies, could change the unification scale by a few orders of magnitude.

Note that it is in general possible to decouple the spontaneous parity and PS breaking scales [98] and thus have $g_R \neq g_L$ at μ_{PS} but still $y_L = y_R$ for the Yukawas, at least at tree level. This makes it possible to significantly lower the PS breaking scale, potentially down to the experimentally excluded value $\mu_{PS} \sim 1000 \text{ TeV}$ [43]. While we do not pursue this possibility here, it should be kept in mind as an interesting alternative.

Figure 8. Left-right gauge coupling unification in the SM and assuming only light additional S_1 and S_3 states. The dashed lines denote the scale at which the unification condition (D.1), $g_L = g_R$, is satisfied.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] BABAR collaboration, J.P. Lees et al., *Evidence for an excess of $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ decays*, *Phys. Rev. Lett.* **109** (2012) 101802 [[arXiv:1205.5442](https://arxiv.org/abs/1205.5442)] [[INSPIRE](#)].
- [2] BABAR collaboration, J.P. Lees et al., *Measurement of an Excess of $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ Decays and Implications for Charged Higgs Bosons*, *Phys. Rev. D* **88** (2013) 072012 [[arXiv:1303.0571](https://arxiv.org/abs/1303.0571)] [[INSPIRE](#)].
- [3] BELLE collaboration, M. Huschle et al., *Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$ relative to $\bar{B} \rightarrow D^{(*)}\ell^-\bar{\nu}_\ell$ decays with hadronic tagging at Belle*, *Phys. Rev. D* **92** (2015) 072014 [[arXiv:1507.03233](https://arxiv.org/abs/1507.03233)] [[INSPIRE](#)].
- [4] BELLE collaboration, Y. Sato et al., *Measurement of the branching ratio of $\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau$ relative to $\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}_\ell$ decays with a semileptonic tagging method*, *Phys. Rev. D* **94** (2016) 072007 [[arXiv:1607.07923](https://arxiv.org/abs/1607.07923)] [[INSPIRE](#)].
- [5] BELLE collaboration, S. Hirose et al., *Measurement of the τ lepton polarization and $R(D^*)$ in the decay $\bar{B} \rightarrow D^*\tau^-\bar{\nu}_\tau$* , *Phys. Rev. Lett.* **118** (2017) 211801 [[arXiv:1612.00529](https://arxiv.org/abs/1612.00529)] [[INSPIRE](#)].
- [6] LHCb collaboration, *Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\mu^-\bar{\nu}_\mu)$* , *Phys. Rev. Lett.* **115** (2015) 111803 [Erratum *ibid.* **115** (2015) 159901] [[arXiv:1506.08614](https://arxiv.org/abs/1506.08614)] [[INSPIRE](#)].
- [7] LHCb collaboration, *Measurement of the ratio of the $B^0 \rightarrow D^{*-}\tau^+\nu_\tau$ and $B^0 \rightarrow D^{*-}\mu^+\nu_\mu$ branching fractions using three-prong τ -lepton decays*, *Phys. Rev. Lett.* **120** (2018) 171802 [[arXiv:1708.08856](https://arxiv.org/abs/1708.08856)] [[INSPIRE](#)].
- [8] LHCb collaboration, *Test of Lepton Flavor Universality by the measurement of the $B^0 \rightarrow D^{*-}\tau^+\nu_\tau$ branching fraction using three-prong τ decays*, *Phys. Rev. D* **97** (2018) 072013 [[arXiv:1711.02505](https://arxiv.org/abs/1711.02505)] [[INSPIRE](#)].

- [9] HFLAV collaboration, Y. Amhis et al., *Averages of b -hadron, c -hadron and τ -lepton properties as of summer 2016*, *Eur. Phys. J. C* **77** (2017) 895 [[arXiv:1612.07233](#)] [[INSPIRE](#)].
- [10] LHCb collaboration, *Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **113** (2014) 151601 [[arXiv:1406.6482](#)] [[INSPIRE](#)].
- [11] LHCb collaboration, *Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays*, *JHEP* **08** (2017) 055 [[arXiv:1705.05802](#)] [[INSPIRE](#)].
- [12] LHCb collaboration, *Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay using 3fb^{-1} of integrated luminosity*, *JHEP* **02** (2016) 104 [[arXiv:1512.04442](#)] [[INSPIRE](#)].
- [13] LHCb collaboration, *Measurement of Form-Factor-Independent Observables in the Decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$* , *Phys. Rev. Lett.* **111** (2013) 191801 [[arXiv:1308.1707](#)] [[INSPIRE](#)].
- [14] B. Capdevila, A. Crivellin, S. Descotes-Genon, J. Matias and J. Virto, *Patterns of New Physics in $b \rightarrow s \ell^+ \ell^-$ transitions in the light of recent data*, *JHEP* **01** (2018) 093 [[arXiv:1704.05340](#)] [[INSPIRE](#)].
- [15] W. Altmannshofer, P. Stangl and D.M. Straub, *Interpreting Hints for Lepton Flavor Universality Violation*, *Phys. Rev. D* **96** (2017) 055008 [[arXiv:1704.05435](#)] [[INSPIRE](#)].
- [16] L.-S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X.-L. Ren and R.-X. Shi, *Towards the discovery of new physics with lepton-universality ratios of $b \rightarrow s \ell \ell$ decays*, *Phys. Rev. D* **96** (2017) 093006 [[arXiv:1704.05446](#)] [[INSPIRE](#)].
- [17] M. Ciuchini et al., *On Flavourful Easter eggs for New Physics hunger and Lepton Flavour Universality violation*, *Eur. Phys. J. C* **77** (2017) 688 [[arXiv:1704.05447](#)] [[INSPIRE](#)].
- [18] G. D'Amico et al., *Flavour anomalies after the R_{K^*} measurement*, *JHEP* **09** (2017) 010 [[arXiv:1704.05438](#)] [[INSPIRE](#)].
- [19] G. Hiller and I. Nisandzic, *R_K and R_{K^*} beyond the standard model*, *Phys. Rev. D* **96** (2017) 035003 [[arXiv:1704.05444](#)] [[INSPIRE](#)].
- [20] L. Di Luzio and M. Nardecchia, *What is the scale of new physics behind the B -flavour anomalies?*, *Eur. Phys. J. C* **77** (2017) 536 [[arXiv:1706.01868](#)] [[INSPIRE](#)].
- [21] R. Alonso, B. Grinstein and J. Martin Camalich, *Lepton universality violation and lepton flavor conservation in B -meson decays*, *JHEP* **10** (2015) 184 [[arXiv:1505.05164](#)] [[INSPIRE](#)].
- [22] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B -physics anomalies: a guide to combined explanations*, *JHEP* **11** (2017) 044 [[arXiv:1706.07808](#)] [[INSPIRE](#)].
- [23] A. Crivellin, D. Müller and T. Ota, *Simultaneous explanation of $R(D^{(*)})$ and $b \rightarrow s \mu^+ \mu^-$: the last scalar leptoquarks standing*, *JHEP* **09** (2017) 040 [[arXiv:1703.09226](#)] [[INSPIRE](#)].
- [24] D. Bečirević, S. Fajfer, N. Košnik and O. Sumensari, *Leptoquark model to explain the B -physics anomalies, R_K and R_D* , *Phys. Rev. D* **94** (2016) 115021 [[arXiv:1608.08501](#)] [[INSPIRE](#)].
- [25] P. Asadi, M.R. Buckley and D. Shih, *It's all right(-handed neutrinos): a new W' model for the $R_{D^{(*)}}$ anomaly*, *JHEP* **09** (2018) 010 [[arXiv:1804.04135](#)] [[INSPIRE](#)].
- [26] A. Greljo, D.J. Robinson, B. Shakya and J. Zupan, *$R(D^{(*)})$ from W' and right-handed neutrinos*, *JHEP* **09** (2018) 169 [[arXiv:1804.04642](#)] [[INSPIRE](#)].
- [27] D.J. Robinson, B. Shakya and J. Zupan, *Right-handed Neutrinos and $R(D^{(*)})$* , [[arXiv:1807.04753](#)] [[INSPIRE](#)].

- [28] A. Azatov, D. Barducci, D. Ghosh, D. Marzocca and L. Ubaldi, *Combined explanations of B-physics anomalies: the sterile neutrino solution*, *JHEP* **10** (2018) 092 [[arXiv:1807.10745](#)] [[INSPIRE](#)].
- [29] D. Bečirević, I. Doršner, S. Fajfer, N. Košnik, D.A. Faroughy and O. Sumensari, *Scalar leptoquarks from grand unified theories to accommodate the B-physics anomalies*, *Phys. Rev. D* **98** (2018) 055003 [[arXiv:1806.05689](#)] [[INSPIRE](#)].
- [30] T. Faber, M. Hudec, M. Malinský, P. Meinzinger, W. Porod and F. Staub, *A unified leptoquark model confronted with lepton non-universality in B-meson decays*, *Phys. Lett. B* **787** (2018) 159 [[arXiv:1808.05511](#)] [[INSPIRE](#)].
- [31] N. Assad, B. Fornal and B. Grinstein, *Baryon Number and Lepton Universality Violation in Leptoquark and Diquark Models*, *Phys. Lett. B* **777** (2018) 324 [[arXiv:1708.06350](#)] [[INSPIRE](#)].
- [32] G. Valencia and S. Willenbrock, *Quark-lepton unification and rare meson decays*, *Phys. Rev. D* **50** (1994) 6843 [[hep-ph/9409201](#)] [[INSPIRE](#)].
- [33] A.D. Smirnov, *Mass limits for scalar and gauge leptoquarks from $K_L^0 \rightarrow e^\pm \mu^\mp$, $B_0 \rightarrow e^\pm \tau^\mp$ decays*, *Mod. Phys. Lett. A* **22** (2007) 2353 [[arXiv:0705.0308](#)] [[INSPIRE](#)].
- [34] A.V. Kuznetsov, N.V. Mikheev and A.V. Serghienko, *The third type of fermion mixing in the lepton and quark interactions with leptoquarks*, *Int. J. Mod. Phys. A* **27** (2012) 1250062 [[arXiv:1203.0196](#)] [[INSPIRE](#)].
- [35] A.D. Smirnov, *Vector leptoquark mass limits and branching ratios of $K_L^0, B^0, B_s \rightarrow l_i^+ l_j^-$ decays with account of fermion mixing in leptoquark currents*, *Mod. Phys. Lett. A* **33** (2018) 1850019 [[arXiv:1801.02895](#)] [[INSPIRE](#)].
- [36] L. Calibbi, A. Crivellin and T. Li, *Model of vector leptoquarks in view of the B-physics anomalies*, *Phys. Rev. D* **98** (2018) 115002 [[arXiv:1709.00692](#)] [[INSPIRE](#)].
- [37] L. Di Luzio, A. Greljo and M. Nardecchia, *Gauge leptoquark as the origin of B-physics anomalies*, *Phys. Rev. D* **96** (2017) 115011 [[arXiv:1708.08450](#)] [[INSPIRE](#)].
- [38] M. Blanke and A. Crivellin, *B Meson Anomalies in a Pati-Salam Model within the Randall-Sundrum Background*, *Phys. Rev. Lett.* **121** (2018) 011801 [[arXiv:1801.07256](#)] [[INSPIRE](#)].
- [39] M. Bordone, C. Cornella, J. Fuentes-Martin and G. Isidori, *A three-site gauge model for flavor hierarchies and flavor anomalies*, *Phys. Lett. B* **779** (2018) 317 [[arXiv:1712.01368](#)] [[INSPIRE](#)].
- [40] R. Barbieri and A. Tesi, *B-decay anomalies in Pati-Salam SU(4)*, *Eur. Phys. J. C* **78** (2018) 193 [[arXiv:1712.06844](#)] [[INSPIRE](#)].
- [41] J.C. Pati and A. Salam, *Lepton Number as the Fourth Color*, *Phys. Rev. D* **10** (1974) 275 [*Erratum ibid.* **D 11** (1975) 703] [[INSPIRE](#)].
- [42] J.C. Pati, A. Salam and U. Sarkar, $\Delta B = -\Delta L$, $neutron \rightarrow e^- \pi^+, e^- K^+, \mu^- \pi^+$ and $\mu^- K^+$ DECAY modes in $SU(2)_L \times SU(2)_R \times SU(4)_C$ or $SO(10)$, *Phys. Lett.* **133B** (1983) 330 [[INSPIRE](#)].
- [43] R.R. Volkas, *Prospects for mass unification at low-energy scales*, *Phys. Rev. D* **53** (1996) 2681 [[hep-ph/9507215](#)] [[INSPIRE](#)].
- [44] A. Maiezza, M. Nemevšek, F. Nesti and G. Senjanović, *Left-Right Symmetry at LHC*, *Phys. Rev. D* **82** (2010) 055022 [[arXiv:1005.5160](#)] [[INSPIRE](#)].

- [45] G. Senjanović and V. Tello, *Right Handed Quark Mixing in Left-Right Symmetric Theory*, *Phys. Rev. Lett.* **114** (2015) 071801 [[arXiv:1408.3835](#)] [[INSPIRE](#)].
- [46] M. Magg and C. Wetterich, *Neutrino Mass Problem and Gauge Hierarchy*, *Phys. Lett.* **94B** (1980) 61 [[INSPIRE](#)].
- [47] J. Schechter and J.W.F. Valle, *Neutrino Masses in $SU(2) \times U(1)$ Theories*, *Phys. Rev. D* **22** (1980) 2227 [[INSPIRE](#)].
- [48] T.P. Cheng and L.-F. Li, *Neutrino Masses, Mixings and Oscillations in $SU(2) \times U(1)$ Models of Electroweak Interactions*, *Phys. Rev. D* **22** (1980) 2860 [[INSPIRE](#)].
- [49] PARTICLE DATA GROUP collaboration, M. Tanabashi et al., *Review of Particle Physics*, *Phys. Rev. D* **98** (2018) 030001 [[INSPIRE](#)].
- [50] R.N. Mohapatra and R.E. Marshak, *Local $B - L$ Symmetry of Electroweak Interactions, Majorana Neutrinos and Neutron Oscillations*, *Phys. Rev. Lett.* **44** (1980) 1316 [Erratum *ibid.* **44** (1980) 1643] [[INSPIRE](#)].
- [51] W. Buchmüller, R. Ruckl and D. Wyler, *Leptoquarks in lepton-quark collisions*, *Phys. Lett. B* **191** (1987) 442 [Erratum *ibid.* **B 448** (1999) 320] [[INSPIRE](#)].
- [52] I. Doršner, S. Fajfer, A. Greljo, J.F. Kamenik and N. Košnik, *Physics of leptoquarks in precision experiments and at particle colliders*, *Phys. Rept.* **641** (2016) 1 [[arXiv:1603.04993](#)] [[INSPIRE](#)].
- [53] I. Doršner, S. Fajfer, D.A. Faroughy and N. Košnik, *The role of the S_3 GUT leptoquark in flavor universality and collider searches*, [arXiv:1706.07779](#) [[INSPIRE](#)].
- [54] D. Marzocca, *Addressing the B -physics anomalies in a fundamental Composite Higgs Model*, *JHEP* **07** (2018) 121 [[arXiv:1803.10972](#)] [[INSPIRE](#)].
- [55] M. Bauer and M. Neubert, *Minimal Leptoquark Explanation for the $R_{D^{(*)}}$, R_K and $(g - 2)_g$ Anomalies*, *Phys. Rev. Lett.* **116** (2016) 141802 [[arXiv:1511.01900](#)] [[INSPIRE](#)].
- [56] D. Bečirević, N. Košnik, O. Sumensari and R. Zukanovich Funchal, *Palatable Leptoquark Scenarios for Lepton Flavor Violation in Exclusive $b \rightarrow s \ell_1 \ell_2$ modes*, *JHEP* **11** (2016) 035 [[arXiv:1608.07583](#)] [[INSPIRE](#)].
- [57] Y. Cai, J. Gargalionis, M.A. Schmidt and R.R. Volkas, *Reconsidering the One Leptoquark solution: flavor anomalies and neutrino mass*, *JHEP* **10** (2017) 047 [[arXiv:1704.05849](#)] [[INSPIRE](#)].
- [58] M. Freytsis, Z. Ligeti and J.T. Ruderman, *Flavor models for $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$* , *Phys. Rev. D* **92** (2015) 054018 [[arXiv:1506.08896](#)] [[INSPIRE](#)].
- [59] U. Aydemir, D. Minic, C. Sun and T. Takeuchi, *B -decay anomalies and scalar leptoquarks in unified Pati-Salam models from noncommutative geometry*, *JHEP* **09** (2018) 117 [[arXiv:1804.05844](#)] [[INSPIRE](#)].
- [60] J.F. Nieves, *Baryon and Lepton Number Nonconserving Processes and Intermediate Mass Scales*, *Nucl. Phys. B* **189** (1981) 182 [[INSPIRE](#)].
- [61] T. Hambye and J. Heeck, *Proton decay into charged leptons*, *Phys. Rev. Lett.* **120** (2018) 171801 [[arXiv:1712.04871](#)] [[INSPIRE](#)].
- [62] J.C. Pati, *Nucleon Decays Into Lepton + Lepton + Anti-lepton + Mesons Within $SU(4)$ of Color*, *Phys. Rev. D* **29** (1984) 1549 [[INSPIRE](#)].
- [63] P.J. O'Donnell and U. Sarkar, *Three lepton decay mode of the proton*, *Phys. Lett. B* **316** (1993) 121 [[hep-ph/9307254](#)] [[INSPIRE](#)].

- [64] E.K. Akhmedov and M. Frigerio, *Duality in Left-Right Symmetric Seesaw Mechanism*, *Phys. Rev. Lett.* **96** (2006) 061802 [[hep-ph/0509299](#)] [[INSPIRE](#)].
- [65] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwetz, *Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity*, *JHEP* **01** (2017) 087 [[arXiv:1611.01514](#)] [[INSPIRE](#)].
- [66] SINDRUM II collaboration, W.H. Bertl et al., *A Search for muon to electron conversion in muonic gold*, *Eur. Phys. J. C* **47** (2006) 337 [[INSPIRE](#)].
- [67] T. Araki, J. Heeck and J. Kubo, *Vanishing Minors in the Neutrino Mass Matrix from Abelian Gauge Symmetries*, *JHEP* **07** (2012) 083 [[arXiv:1203.4951](#)] [[INSPIRE](#)].
- [68] MU2E collaboration, L. Bartoszek et al., *Mu2e Technical Design Report*, [arXiv:1501.05241](#) [[INSPIRE](#)].
- [69] COMET collaboration, B.E. Krikler, *An Overview of the COMET Experiment and its Recent Progress*, in *Proceedings, 17th International Workshop on Neutrino Factories and Future Neutrino Facilities (NuFact15): Rio de Janeiro, Brazil, August 10-15, 2015*, 2015, [arXiv:1512.08564](#) [[INSPIRE](#)].
- [70] A. Blondel et al., *Research Proposal for an Experiment to Search for the Decay $\mu \rightarrow eee$* , [arXiv:1301.6113](#) [[INSPIRE](#)].
- [71] MEG II collaboration, A.M. Baldini et al., *The design of the MEG II experiment*, *Eur. Phys. J. C* **78** (2018) 380 [[arXiv:1801.04688](#)] [[INSPIRE](#)].
- [72] S. Cunliffe, *Lepton non-universality and lepton flavour violation at Belle II*, talk at *6th Workshop on Rare Semileptonic B Decays*, Munich Germany (2018).
- [73] BELLE II collaboration, E. Kou et al., *The Belle II Physics Book*, [arXiv:1808.10567](#) [[INSPIRE](#)].
- [74] ATLAS collaboration, *Search for High-Mass Resonances Decaying to $\tau\nu$ in pp Collisions at $\sqrt{s} = 13$ TeV with the ATLAS Detector*, *Phys. Rev. Lett.* **120** (2018) 161802 [[arXiv:1801.06992](#)] [[INSPIRE](#)].
- [75] CMS collaboration, *Search for new physics in final states with an energetic jet or a hadronically decaying W or Z boson and transverse momentum imbalance at $\sqrt{s} = 13$ TeV*, *Phys. Rev. D* **97** (2018) 092005 [[arXiv:1712.02345](#)] [[INSPIRE](#)].
- [76] CMS collaboration, *Search for leptoquarks coupling to third generation quarks*, *CMS-PAS-B2G-16-027* (2017).
- [77] M. Drewes et al., *A White Paper on keV Sterile Neutrino Dark Matter*, *JCAP* **01** (2017) 025 [[arXiv:1602.04816](#)] [[INSPIRE](#)].
- [78] A. Greljo and B.A. Stefanek, *Third family quark-lepton unification at the TeV scale*, *Phys. Lett. B* **782** (2018) 131 [[arXiv:1802.04274](#)] [[INSPIRE](#)].
- [79] R. Kitano, M. Koike and Y. Okada, *Detailed calculation of lepton flavor violating muon electron conversion rate for various nuclei*, *Phys. Rev. D* **66** (2002) 096002 [Erratum *ibid.* **D 76** (2007) 059902] [[hep-ph/0203110](#)] [[INSPIRE](#)].
- [80] S. Davidson, D.C. Bailey and B.A. Campbell, *Model independent constraints on leptoquarks from rare processes*, *Z. Phys. C* **61** (1994) 613 [[hep-ph/9309310](#)] [[INSPIRE](#)].
- [81] V. Cirigliano and I. Rosell, *Two-loop effective theory analysis of $\pi(K) \rightarrow e\bar{\nu}_e[\gamma]$ branching ratios*, *Phys. Rev. Lett.* **99** (2007) 231801 [[arXiv:0707.3439](#)] [[INSPIRE](#)].

- [82] J. Brod, M. Gorbahn and E. Stamou, *Two-Loop Electroweak Corrections for the $K \rightarrow \pi \nu \bar{\nu}$ Decays*, *Phys. Rev. D* **83** (2011) 034030 [[arXiv:1009.0947](#)] [[INSPIRE](#)].
- [83] MEG collaboration, A.M. Baldini et al., *Search for the lepton flavour violating decay $\mu^+ \rightarrow e^+ \gamma$ with the full dataset of the MEG experiment*, *Eur. Phys. J. C* **76** (2016) 434 [[arXiv:1605.05081](#)] [[INSPIRE](#)].
- [84] BNL collaboration, D. Ambrose et al., *New limit on muon and electron lepton number violation from $K_L^0 \rightarrow \mu^\pm e^\mp$ decay*, *Phys. Rev. Lett.* **81** (1998) 5734 [[hep-ex/9811038](#)] [[INSPIRE](#)].
- [85] E. Gabrielli, *Model independent constraints on leptoquarks from rare muon and tau lepton processes*, *Phys. Rev. D* **62** (2000) 055009 [[hep-ph/9911539](#)] [[INSPIRE](#)].
- [86] ALEPH, DELPHI, L3, OPAL, SLD, LEP ELECTROWEAK WORKING GROUP, SLD ELECTROWEAK GROUP and SLD HEAVY FLAVOUR GROUP collaborations, S. Schael et al., *Precision electroweak measurements on the Z resonance*, *Phys. Rept.* **427** (2006) 257 [[hep-ex/0509008](#)] [[INSPIRE](#)].
- [87] BELLE collaboration, J. Grygier et al., *Search for $B \rightarrow h \nu \bar{\nu}$ decays with semileptonic tagging at Belle*, *Phys. Rev. D* **96** (2017) 091101 [[arXiv:1702.03224](#)] [[INSPIRE](#)].
- [88] BABAR collaboration, J.P. Lees et al., *A search for the decay modes $B^{+-} \rightarrow h^{+-} \tau^{+-} l$* , *Phys. Rev. D* **86** (2012) 012004 [[arXiv:1204.2852](#)] [[INSPIRE](#)].
- [89] BABAR collaboration, B. Aubert et al., *Searches for Lepton Flavor Violation in the Decays $\tau \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$* , *Phys. Rev. Lett.* **104** (2010) 021802 [[arXiv:0908.2381](#)] [[INSPIRE](#)].
- [90] K. Hayasaka et al., *Search for Lepton Flavor Violating Tau Decays into Three Leptons with 719 Million Produced $\tau^+ \tau^-$ Pairs*, *Phys. Lett. B* **687** (2010) 139 [[arXiv:1001.3221](#)] [[INSPIRE](#)].
- [91] T.P. Cheng, E. Eichten and L.-F. Li, *Higgs Phenomena in Asymptotically Free Gauge Theories*, *Phys. Rev. D* **9** (1974) 2259 [[INSPIRE](#)].
- [92] M.E. Machacek and M.T. Vaughn, *Two Loop Renormalization Group Equations in a General Quantum Field Theory. 2. Yukawa Couplings*, *Nucl. Phys. B* **236** (1984) 221 [[INSPIRE](#)].
- [93] F. Staub, *SARAH 4: A tool for (not only SUSY) model builders*, *Comput. Phys. Commun.* **185** (2014) 1773 [[arXiv:1309.7223](#)] [[INSPIRE](#)].
- [94] F. Staub, *Exploring new models in all detail with SARAH*, *Adv. High Energy Phys.* **2015** (2015) 840780 [[arXiv:1503.04200](#)] [[INSPIRE](#)].
- [95] S. Weinberg, *Effective Gauge Theories*, *Phys. Lett.* **91B** (1980) 51 [[INSPIRE](#)].
- [96] L.J. Hall, *Grand Unification of Effective Gauge Theories*, *Nucl. Phys. B* **178** (1981) 75 [[INSPIRE](#)].
- [97] S. Bertolini, L. Di Luzio and M. Malinsky, *Intermediate mass scales in the non-supersymmetric SO(10) grand unification: A Reappraisal*, *Phys. Rev. D* **80** (2009) 015013 [[arXiv:0903.4049](#)] [[INSPIRE](#)].
- [98] D. Chang, R.N. Mohapatra and M.K. Parida, *Decoupling Parity and SU(2)-R Breaking Scales: A New Approach to Left-Right Symmetric Models*, *Phys. Rev. Lett.* **52** (1984) 1072 [[INSPIRE](#)].