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Amplitudes involving massive states using pure spinor formalism

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Abstract: Same amplitudes evaluated independently using RNS and pure spinor formalism are expected to agree. While for massless states, this fact has been firmly established, for massive states such an explicit check has been lacking so far. We compute all massless-massless-massive 3-point functions in open supertrings in pure spinor formalism for the first massive states and compare them with the corresponding RNS results. We fix the normalization of the vertex operators of the massive states by comparing same set of 3-point functions for a fixed ordering in the two formalisms. Once fixed, the subsequent 3-point functions for each inequivalent ordering match exactly. This extends the explicit demonstration of equivalence of pure spinor and RNS formalism from massless states to first massive states.

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1 Introduction

Since the turn of last century, the pure spinor formalism has emerged as a powerful alternative to the RNS and Green Schwarz formalisms for superstring theory [1–3]. The main advantage of the pure spinor formalism over RNS is that it maintains manifest spacetime super-Poincaré covariance at all stages. There are several general arguments demonstrating how the cohomology of the pure spinor formalism is equivalent to the cohomology of the RNS and Green Schwarz formalisms [4–6]. For massless states several explicit calculations of scattering amplitude in pure spinor formalism have allowed a direct comparison with the corresponding amplitude calculation in RNS formalism [7–10]. This direct comparison fixed the relative normalization between the vertex operators in PS and RNS formalism for the massless states.

In this paper we extend such explicit direct comparison for the first massive states ($m^2 = \frac{1}{\alpha'}$) of open superstring. We explicitly compute all possible tree level 3 point

functions for a fixed ordering of vertex operators in PS formalism involving 2 massless and 1 massive state and compare them directly with their RNS counterparts. The massless spectrum contains a bosonic gluon field a_m (with 8 degrees of freedom) and a fermionic gluino field χ^α(with 8 degrees of freedom). The first massive states include a bosonic 3 form field b_{mnp} (with 84 degrees of freedom), a bosonic symmetric rank 2 tensor field g_{mn} (with 44 degrees of freedom) and a fermionic spin $\frac{3}{2}$ field $\psi_{m\alpha}$ (with 128 = 84+44 degrees of freedom).

The computation of the above mentioned 3 point functions in PS formalism requires the massless unintegrated vertex and its covariant θ expansion [1, 11–13] along with the massive unintegrated vertex and its covariant θ expansion [14, 15]. While the tree level amplitude prescription in PS is straightforward, one typically encounters a large number of terms in θ expansion of massive vertex and the calculation becomes cumbersome to compute solely by hand. For this reason, the computation was done using Cadabra [16, 17] and benchmarked by computing 3-point massless-massless-massless amplitudes which are already known in literature (see, e.g. [18]). For use of Cadabra in pure spinor calculations, also see [19]. The final amplitudes were computed directly using this code and compared with their corresponding RNS results. After fixing the normalization of the three massive vertex operators by comparing three correlators (for three massive states), we find that the rest of the correlators agree perfectly. Moreover, while doing the calculations, we have not fixed the positions of the vertex operators to some specific values but evaluated the correlation functions for general positions x_1, x_2, x_3 and have checked that their dependence drops out in the final result which is expected from the $\mathrm{SL}(2,\mathbb{R})$ invariance.

The rest of the paper is organized as follows. In section 2, we review the tree level amplitude prescription in PS. In section 3, we summarize the θ expansion procedure for massive states given in [15] and give the result for the θ expansion to the relevant order we need in this work. Section 4 gives the detailed derivation and results of 3-point amplitudes in PS formalism. In section 5, we compare our results obtained in PS formalism directly with the amplitudes computed using RNS formalism. We conclude in section 6 with some brief discussion. In appendix A, we give our conventions for the PS formalism. In appendix B, we give our conventions for the RNS calculations and summarize the results for 3-point functions in the RNS formalism. Finally, in appendix C, we give the theta expansion of the massless vertex operator in our conventions.

2 Tree amplitude prescription in pure spinor formalism

Both minimal as well as the non minimal pure spinor formalisms give the same amplitude prescription at tree level. In this section, we shall review this prescription [1, 2]. In particular, we shall focus on 3-point functions on the disk with a specific ordering of vertex operators on the boundary. To evaluate any 3-point correlator, the knowledge of the unintegrated form of the vertex operator for external states is sufficient. All the amplitudes of interest in this paper are of the form

$$\mathcal{A}_3 = \langle V_1 V_2 V_3 \rangle \tag{2.1}$$

where $\langle V_1 V_2 V_3 \rangle$ denotes 3-point function on the disk with a fixed ordering.

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In this paper, V_1 and V_2 will be taken to be unintegrated vertex operators creating massless states (gluon or gluino) and V_3 will be the unintegrated vertex operator creating a massive state (b_{mnp}, g_{mn} or $\psi_{m\alpha}$). However, we must emphasize, none of the strategy that we shall outline below is particular to this specific kind of 3-point functions. The tree-level amplitude prescription will be equally valid for any 3-point functions of open strings states (massive or massless).

Our choice of normalization of pure spinor measure is the standard one in the literature

$$\left\langle (\lambda\gamma^s\theta)(\lambda\gamma^t\theta)(\lambda\gamma^u\theta)(\theta\gamma_{stu}\theta)\right\rangle_{PSS} = 1 \tag{2.2}$$

The process of 3-point amplitude computation can be succinctly summarized in a series of steps that is given below.

- *Step 1:* assume a particular order of the vertex operator inserted on the disk and make use of the OPEs between various conformal weight 1 objects to reduce the three point function to a correlation function involving only the three superfields and three pure spinor ghost λ^α coming from the three vertices. The correlation function which contains $\partial\theta^\alpha$ terms do not contribute since they always fail to have the correct number of θ zero modes to give non-zero answer as required by 2.2. Now perform the θ expansion for each of the vertex, viz. V_1, V_2 and V_3, explicitly. Although one can do the full θ expansion, but that is rendered redundant due to 2.2 which states that only terms involving exactly five θs can give a non-zero contribution. The relevant order of θ expansion for each of the vertex for a given amplitude can therefore be deduced from this consideration.

- *Step 2:* from the product $V_1V_2V_3$, retain only the terms which have precisely five θs as all other terms will give zero contribution trivially due to 2.2.[1]

- *Step 3:* expand the physical fields appearing in various vertices in a basis of plane waves with polarizations as the coefficients

  $$H^{\beta_1\ldots\beta_k}_{p_1\ldots p_n}(X) = h^{\beta_1\ldots\beta_k}_{p_1\ldots p_n}\, e^{ik\cdot X}$$

  Here $h^{\beta_1\ldots\beta_k}_{p_1\ldots p_n}$ are the constant tensor-spinor of appropriate index structure denoting the polarizations for the physical field $H^{\beta_1\ldots\beta_k}_{p_1\ldots p_n}(X)$.

  The correlation function $\langle V_1V_2V_3\rangle$ at this stage factorizes completely into two separate correlation functions, one involving X^m fields and the other involving pure spinor fields. These can be evaluated independently and their result can be multiplied to obtain the final answer for a given ordering.

- *Step 4:* evaluate the 3-point function on disk by the usual methodology of bosonic open strings. Typically these correlation functions can be put into the schematic form $\langle :\, e^{ik_1\cdot X(x_1)} :: e^{ik_2\cdot X(x_2)} :: \mathcal{F}(\partial X^m(x_3))\, e^{ik_3\cdot X(x_3)} :\rangle_{Disk}$ where $\mathcal{F}(\partial X^m(x)) = \prod_i \partial X^{m_i}(x)$.

[1] Notice that 2.2 also suggests we should retain only terms with exactly three number of λs. But this is automatically ensured by the fact that by construction all unintegrated vertex in PS formalism have ghost number 1 and therefore they each carry exactly a single factor of λ. Consequentially the product $V_1V_2V_3$ always goes as λ^3.

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- *Step 5:* the correlators living on pure spinor superspace can be evaluated following the list of identities first derived in [8, 20]. We list in appendix A.3 the subset of those identities that were needed in evaluating the 3-point functions in our case.

- *Step 6:* multiply the results obtained from step 4 and step 5 to obtain the full contribution for a given order in theta expansion of the 3 vertices.

Once the answer has been obtained for a given order, the final answer for all other inequivalent orders can be readily obtained by suitable permutation of momenta and polarization labels. Since each ordering carries different Chan-Paton factors, the full amplitude cannot be obtained by simply adding the contribution coming from different ordering before multiplying them by Chan-Paton factors. Therefore, to compare our result with the result obtained in RNS formalism, we shall directly compare each inequivalent ordering separately.

While the algorithm described above is quite straightforward in principle, the growing number of terms in θ expansion of the vertex operators implies that the cleanest way to perform these amplitude calculations beyond a point is to employ the help of an available computer algebra system. As mentioned in the introduction, we used Cadabra [16, 17] which is an open source computer algebra system developed to aid in field-theoretic computations. For our present purpose, we found Cadabra to be unparalleled in implementing the algorithm described above.

3 First massive vertex operator and its θ expansion

At the 1st mass level of the open string, there are 128 fermionic and 128 bosonic degrees of freedom. The fermionic degrees of freedom are contained in a spin-3/2 field $\psi_{m\alpha}$ satisfying

$$\partial^m \psi_{m\alpha} = 0 \qquad ; \qquad \gamma^{m\alpha\beta}\psi_{m\beta} = 0. \tag{3.1}$$

The bosonic degrees of freedom are contained in a traceless symmetric tensor g_{mn} and a 3-form field b_{mnp} satisfying

$$\partial^m b_{mnp} = 0 \qquad ; \qquad \eta^{mn} g_{mn} = 0 \qquad ; \qquad \partial^m g_{mn} = 0. \tag{3.2}$$

The constraints (3.1) and (3.2) ensure that the number of independent components in the fields $\psi_{m\beta}, b_{mnp}$ and g_{mn} are $128, 84$ and 44 respectively. These fields form a massive spin-2 supermultiplet in 10 dimensions. In pure spinor formalism, we use the language of superspace to describe the system in a manifestly supersymmetric invariant manner. This is done by introducing three basic superfields $\Psi_{m\alpha}, B_{mnp}$ and G_{mn} whose theta independent components are $\psi_{m\alpha}, b_{mnp}$ and g_{mn} respectively. These basic superfields satisfy the superspace equations[2] [15]

$$D_\alpha G_{sm} = 16\, \partial^p (\gamma_{p(s}\Psi_{m)})_\alpha \tag{3.3}$$

[2]We use the same conventions as in the reference [15]. Thus, e.g., our convention for the anti-symmetrization and symmetrizations are

$$A_{[mn]} = \frac{1}{2}(A_{mn} - A_{nm}) \qquad , \qquad A_{(mn)} = \frac{1}{2}(A_{mn} + A_{nm})\,.$$

$$D_\alpha B_{mnp} = 12(\gamma_{[mn}\Psi_{p]})_\alpha - 24\alpha' \partial^t \partial_{[m}(\gamma_{|t|n}\Psi_{p]})_\alpha \tag{3.4}$$

$$D_\alpha \Psi_{s\beta} = \frac{1}{16} G_{sm}\gamma^m_{\alpha\beta} + \frac{1}{24}\partial_m B_{nps}(\gamma^{mnp})_{\alpha\beta} - \frac{1}{144}\partial^m B^{npq}(\gamma_{smnpq})_{\alpha\beta} \tag{3.5}$$

and the constraints

$$(\gamma^m)^{\alpha\beta}\Psi_{m\beta} = 0 \quad ; \quad \partial^m \Psi_{m\beta} = 0 \quad ; \quad \partial^m B_{mnp} = 0 \quad ; \quad \partial^m G_{mn} = 0 \quad ; \quad \eta^{mn} G_{mn} = 0 \tag{3.6}$$

where, $D_\alpha = \partial_\alpha + \gamma^m_{\alpha\beta}\theta^\beta \partial_m$.

The equations (3.3), (3.4) and (3.5) give recursion relations to determine the complete theta expansion of the superfields B_{mnp}, G_{mn} and $\Psi_{m\alpha}$ [15]. Before giving the theta expansion results, we note the unintegrated vertex operator for the 1st massive states of the open string which can be expressed in terms of the basic superfields as [14]

$$V = \partial\theta^\beta \lambda^\alpha B_{\alpha\beta} + d_\beta \lambda^\alpha C^\beta_\alpha + \Pi^m \lambda^\alpha H_{m\alpha} + N^{mn}\lambda^\alpha F_{\alpha mn} \tag{3.7}$$

where [14, 15],

$$B_{\alpha\beta} = \gamma^{mnp}_{\alpha\beta} B_{mnp} \quad , \qquad C^\beta_\alpha = \frac{1}{2}(\gamma^{mnpq})^\beta_\alpha \partial_{[m} B_{npq]}$$
$$H_{m\alpha} = -72\Psi_{m\alpha} \ , \qquad F_{\alpha mn} = -9\Big(7\partial_{[m}\Psi_{n]\alpha} + \partial^q (\gamma_{q[m})_\alpha{}^\beta \Psi_{n]\beta}\Big). \tag{3.8}$$

For calculating any amplitude involving the massive states in pure spinor formalism, we shall need the theta expansion result of the above vertex operator. Clearly, the theta expansion of the basic superfields B_{mnp} and $\Psi_{m\alpha}$ automatically implies the theta expansion of the full vertex operator. For our purposes, we shall need the theta expansion results of $\Psi_{m\alpha}$ upto order θ^3 and that of $B_{\alpha\beta}$ upto order θ^4. We used the mathematica package GAMMA to do this computation [21]. Using equations (3.3), (3.4) and (3.5), we obtain (for details, please see [15])

$$\begin{aligned}
\Psi_{s\beta} = {} & \psi_{s\beta} + \frac{1}{16}(\gamma^m\theta)_\beta\ g_{sm} - \frac{i}{24}(\gamma^{mnp}\theta)_\beta k_m b_{nps} - \frac{i}{144}(\gamma_s{}^{npqr}\theta)_\beta k_n b_{pqr} \\
& - \frac{i}{2}k^p(\gamma^m\theta)_\beta(\psi_{(m}\gamma_{s)p}\theta) - \frac{i}{4}k_m(\gamma^{mnp}\theta)_\beta(\psi_{[s}\gamma_{np]}\theta) - \frac{i}{24}(\gamma_s{}^{mnpq}\theta)_\beta k_m(\psi_q\gamma_{np}\theta) \\
& - \frac{i}{6}\alpha' k_m k^r k_s(\gamma^{mnp}\theta)_\beta(\psi_p\gamma_{rn}\theta) + \frac{i}{288}\alpha'(\gamma^{mnp}\theta)_\beta k_m k^r k_s(\theta\gamma^q{}_{nr}\theta)\ g_{pq} \\
& - \frac{i}{192}(\gamma^{mnp}\theta)_\beta k_m(\theta\gamma^q{}_{[np}\theta)g_{s]q} - \frac{i}{1152}(\gamma_{smnpq}\theta)_\beta k^m(\theta\gamma_{npt}\theta)\ g^{qt} \\
& - \frac{i}{96}k^p(\gamma^m\theta)_\beta(\theta\gamma_{pq(s}\theta)\ g_{m)q} - \frac{1}{1728}(\gamma^{mnp}\theta)_\beta k_m(\theta\gamma^{tuvw}{}_{nps}\theta)k_t b_{uvw} \\
& - \frac{1}{864\alpha'}(\gamma_s\theta)_\beta(\theta\gamma^{npq}\theta)b_{npq} - \frac{1}{10368}(\gamma_s{}^{mnpq}\theta)_\beta k_m(\theta\gamma_{tuvwnpq}\theta)k^t b^{uvw} \\
& - \frac{1}{864}(\gamma^m\theta)_\beta(\theta\gamma^{npq}\theta)b_{npq}k_m k_s - \frac{1}{576}(\gamma_{smnpq}\theta)_\beta k^m(\theta\gamma^{tun}\theta)b_u{}^{pq}k_t \\
& - \frac{1}{96\alpha'}(\gamma^m\theta)_\beta(\theta\gamma^{qr}{}_{(s}\theta)b_{m)rq} + \frac{1}{96}(\gamma^m\theta)_\beta(\theta\gamma^{nqr}\theta)k_n k_{(s}b_{m)qr} \\
& + \frac{1}{96}(\gamma^{mnp}\theta)_\beta k_m(\theta\gamma^r{}_{q[n}\theta)b_{ps]r}k^q + O(\theta^4)
\end{aligned} \tag{3.9}$$

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Similarly, the θ expansion of the superfield $B_{\alpha\beta}$ is given by

$$
\begin{aligned}
B_{\alpha\beta} = \gamma^{mnp}_{\alpha\beta}\Big[& b_{mnp} + 12(\psi_p\gamma_{mn}\theta) + 24\alpha' k^r k_m(\psi_p\gamma_{rn}\theta) + \frac{3}{8}(\theta\gamma_{mn}{}^q\theta)\ g_{pq} - \frac{3i}{4}(\theta\gamma^{tu}{}_m\theta)k_t b_{unp} \\
& + \frac{3}{4}\alpha' k^r k_m(\theta\gamma_{rn}{}^q\theta)\ g_{pq} - \frac{i}{24}(\theta\gamma_{tuvwmnp}\theta)k^t b^{uvw} - \frac{1}{6}ik_s\,(\psi_v\gamma_{tu}\theta)\,(\theta\gamma_{stuvmnp}\theta) \\
& - 4i\alpha k_s k_t k_m\,(\theta\gamma_{tun}\theta)\,(\psi_p\gamma_{su}\theta) + ik_s\,(\theta\gamma_{tmn}\theta)\,(\psi_p\gamma_{st}\theta) + ik_s\,(\theta\gamma_{tmn}\theta)\,(\psi_t\gamma_{sp}\theta) \\
& + 2ik_s\,(\theta\gamma_{stm}\theta)\,(\psi_n\gamma_{tp}\theta) - ik_s\,(\theta\gamma_{stm}\theta)\,(\psi_t\gamma_{np}\theta) + \frac{1}{64\alpha'}(\theta\gamma_{smn}\theta)(\theta\gamma_{tup}\theta)b_{stu} \\
& - \frac{1}{288\alpha'}(\theta\gamma_{stu}\theta)(\theta\gamma_{mnp}\theta)b_{stu} + \frac{1}{64\alpha'}(\theta\gamma_{stu}\theta)(\theta\gamma_{unp}\theta)b_{stm} \\
& + \frac{1}{32}(\theta\gamma_{sux}\theta)(\theta\gamma_{txp}\theta)b_{smn}k_t k_u - \frac{1}{16}(\theta\gamma_{sun}\theta)(\theta\gamma_{txp}\theta)b_{stm}k_u k_x \\
& + \frac{1}{64}(\theta\gamma_{stx}\theta)(\theta\gamma_{unp}\theta)b_{stm}k_u k_x + \frac{1}{192}(\theta\gamma_{xzm}\theta)(\theta\gamma_{stuyznp}\theta)b_{stu}k_x k_y \\
& + \frac{1}{192}(\theta\gamma_{uyz}\theta)(\theta\gamma_{stxzmnp}\theta)b_{stu}k_x k_y + \frac{1}{3456}(\theta\gamma_{stuwxyz}\theta)(\theta\gamma_{vxyzmnp}\theta)b_{stu}k_v k_w \\
& + \frac{1}{32}(\theta\gamma_{svn}\theta)(\theta\gamma_{tup}\theta)b_{stu}k_v k_m + \frac{1}{64}(\theta\gamma_{tuv}\theta)(\theta\gamma_{snp}\theta)b_{stu}k_v k_m \\
& - \frac{1}{96}(\theta\gamma_{stu}\theta)(\theta\gamma_{vnp}\theta)b_{stu}k_v k_m - \frac{1}{32}(\theta\gamma_{stv}\theta)(\theta\gamma_{uvp}\theta)b_{stm}k_u k_n \\
& + \frac{1}{384}i\,(\theta\gamma_{tvw}\theta)\,(\theta\gamma_{suvwmnp}\theta)\,k_u g_{st} + \frac{1}{32}i\,(\theta\gamma_{sun}\theta)\,(\theta\gamma_{tup}\theta)\,k_t g_{sm} \\
& + \frac{1}{64}i\,(\theta\gamma_{stu}\theta)\,(\theta\gamma_{unp}\theta)\,k_t g_{sm} + \frac{1}{64}i\,(\theta\gamma_{smn}\theta)\,(\theta\gamma_{tup}\theta)\,k_u g_{st} \\
& + \frac{1}{64}i\,(\theta\gamma_{sum}\theta)\,(\theta\gamma_{tnp}\theta)\,k_u g_{st} - \frac{i\alpha'}{16}\,(\theta\gamma_{suv}\theta)\,(\theta\gamma_{tvp}\theta)\,k_t k_u k_n g_{sm} \ + \ O(\theta^5)\Big]
\end{aligned}
\tag{3.10}
$$

where,

$$
b_{mnp} = e_{mnp}e^{ik\cdot X} \quad , \qquad e_{mn} = e_{mn}e^{ik\cdot X} \quad , \qquad \psi_{m\alpha} = e_{m\alpha}e^{ik\cdot X} \tag{3.11}
$$

Using these theta expansion results in (3.7) we get the theta expansion of the unintegrated vertex operator.

4 Three point functions using pure spinor formalism

In this section, we calculate the 3-point functions involving two massless and one massive state using the pure spinor formalism. In subsection 4.1, we simplify the pure spinor correlators and set up the problem at the superfield level and in subsection 4.2, we evaluate the resulting correlators.

4.1 Simplifying 3-point correlator

Since we are only interested in the 3-point functions on the disk, the location of all the vertex operators can be fixed and hence we only need to consider the unintegrated vertex

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operators. Thus, the correlator we need to evaluate is given by

$$\mathcal{A}_3 = \langle\ V_A^{(1)}(x_1)V_A^{(2)}(x_2)V_{b,g,\psi}^{(3)}(x_3)\ \rangle \tag{4.1}$$

where, $V_{b,g,\psi}$ denotes the massive vertex operator (3.7) and V_A denotes the unintegrated vertex operator of the massless states given by[3]

$$V_A = \lambda^\alpha A_\alpha \tag{4.2}$$

The unintegrated vertex operators in (4.1) are fixed at some arbitrary locations x_i. The $SL(2,\mathbb{R})$ invariance on the disc guarantees that the 3-point function is independent of the choices of x_i. Using the expressions of the unintegrated vertex operators, the desired 3-point function is given by

$$\mathcal{A}_3 = \Big\langle \lambda^\alpha A_\alpha^{(1)}(x_1)\lambda^\beta A_\beta^{(2)}(x_2)\Big(\partial\theta^\rho\lambda^\sigma B_{\sigma\rho} + d_\rho\lambda^\sigma C_\sigma^\rho + \Pi^m\lambda^\sigma H_{m\sigma} + N^{mn}\lambda^\sigma F_{\sigma mn}\Big)(x_3)\Big\rangle \tag{4.3}$$

We have suppressed the superscript 3 from the massive superfields since there is only one massive state and hence there is no chance of any confusion. We now manipulate each term of (4.3) one by one.

First term. The first term is given by

$$\begin{aligned} T_1 &= \langle\lambda^\alpha A_\alpha^{(1)}(x_1)\lambda^\beta A_\beta^{(2)}(x_2)\partial\theta^\rho\lambda^\sigma B_{\sigma\rho}(x_3)\rangle \\ &= (\gamma^{mnp})_{\sigma\rho}\Big\langle\lambda^\alpha\tilde{A}_\alpha^{(1)}\lambda^\beta\tilde{A}_\beta^{(2)}\partial\theta^\rho\lambda^\sigma\tilde{B}_{mnp}\Big\rangle\Big\langle e^{ik_1\cdot X(x_1)}e^{ik_2\cdot X(x_2)}e^{ik_3\cdot X(x_3)}\Big\rangle \\ &= (\gamma^{mnp})_{\sigma\rho}\Big\langle\lambda^\alpha\tilde{A}_\alpha^{(1)}\lambda^\beta\tilde{A}_\beta^{(2)}\partial\theta^\rho\lambda^\sigma\tilde{B}_{mnp}\Big\rangle\frac{x_{13}x_{23}}{x_{12}}. \end{aligned} \tag{4.4}$$

where, the superfields with tilde denote the same superfields with $e^{ik\cdot X}$ factor stripped off. Thus, e.g., in the theta expansion of $\tilde{B}_{mnp}$, we just write the polarization tensor $e_{m\alpha}, e_{mnp}$ and e_{mn} instead of $\psi_{m\alpha}, b_{mnp}$ and g_{mn} respectively. We have also used the momentum conservation to write

$$\Big\langle e^{ik_1\cdot X(x_1)}e^{ik_2\cdot X(x_2)}e^{ik_3\cdot X(x_3)}\Big\rangle = |x_{12}|^{2\alpha' k_1\cdot k_2}|x_{23}|^{2\alpha' k_2\cdot k_3}|x_{13}|^{2\alpha' k_1\cdot k_3} = \frac{x_{13}x_{23}}{x_{12}}. \tag{4.5}$$

The T_1 does not contribute to any of the amplitude since it does not provide the 5 zero modes of θ^α which is required for the non vanishing of pure spinor correlators.

Second term. The 2nd term is given by

$$T_2' = \langle\lambda^\alpha A_\alpha^{(1)}(x_1)\lambda^\beta A_\beta^{(2)}(x_2)d_\rho\lambda^\sigma C_\sigma^\rho(x_3)\rangle$$

To simplify this term, we use the OPE of d_α with superfields to obtain

$$\begin{aligned} T_2' &= \langle\lambda^\alpha A_\alpha^{(1)}(x_1)\lambda^\beta A_\beta^{(2)}(x_2)d_\rho\lambda^\sigma C_\sigma^\rho(x_3)\rangle \\ &= \oint_{x_3}\frac{dw}{w-x_3}\,\langle\lambda^\alpha(x_1)A_\alpha^{(1)}(x_1)\lambda^\beta(x_2)A_\beta^{(2)}(x_2)d_\rho(w)\lambda^\sigma(x_3)C_\sigma^\rho(x_3)\rangle \end{aligned}$$

[3]The theta expansion of the massless superfields in our conventions is given in appendix C.

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$$= -\oint_{x_1}\frac{dw}{w-x_3}\left\langle\lambda^\alpha(x_1)\left[\frac{\alpha'}{2}\frac{D_\rho A^{(1)}_\alpha(x_1)}{w-x_1}\right]\lambda^\beta(x_2)A^{(2)}_\beta(x_2)\lambda^\sigma(x_3)C^\rho_{\ \sigma}(x_3)\right\rangle$$
$$+\oint_{x_2}\frac{dw}{w-x_3}\left\langle\lambda^\alpha(x_1)A^{(1)}_\alpha(x_1)\lambda^\beta(x_2)\left[\frac{\alpha'}{2}\frac{D_\rho A^{(2)}_\beta(x_2)}{w-x_2}\right]\lambda^\sigma(z_3)C^\rho_{\ \sigma}(x_3)\right\rangle \quad (4.6)$$

In going to the last line, we have unwrapped the contour to enclose the points x_1 and x_2. This gives a sign. A further sign comes while moving d_α across A_α. The signs in front of the individual terms in the last line are net effect of these sign factors. We now use equations (3.4), (3.8) and (C.1), the identity $QV = \frac{\alpha'}{2}\lambda^\alpha D_\alpha V$ for an arbitrary superfield V and unwrap the contour of Q. After using the on-shell condition and the pure spinor fierz identity

$$\lambda^\alpha\lambda^\beta = \frac{1}{5!\times 32}\gamma^{\alpha\beta}_{mnpqr}(\lambda\gamma^{mnpqr}\lambda) \quad (4.7)$$

we obtain (after some simplification using the gamma matrix identities)

$$T_2' = \tilde{T}_2 + T_2 \quad (4.8)$$

where,

$$\tilde{T}_2 \equiv -\frac{3i\alpha'}{64}\frac{x_{23}}{x_{12}}\left[(\gamma_{mstuv})^{\alpha\sigma}(k_3)_m + 4(\gamma_{tuv})^{\alpha\sigma}(k_3)_s\right]\left\langle\tilde{A}^{(1)}_\alpha\lambda^\beta\tilde{A}^{(2)}_\beta(\lambda\gamma^{stuvw}\lambda)\tilde{\Psi}_{w\sigma}\right\rangle$$
$$-\frac{3i\alpha'}{64}\frac{x_{13}}{x_{12}}\left[(\gamma_{mstuv})^{\alpha\sigma}(k_3)_m + 4(\gamma_{tuv})^{\alpha\sigma}(k_3)_s\right]\left\langle\lambda^\beta\tilde{A}^{(1)}_\beta\tilde{A}^{(2)}_\alpha(\lambda\gamma^{stuvw}\lambda)\tilde{\Psi}_{w\sigma}\right\rangle \quad (4.9)$$

and

$$T_2 \equiv -\frac{i\alpha'}{2}\frac{x_{23}}{x_{12}}(\gamma_m\gamma^{stuv})_{\alpha\sigma}(k_3)_s\left\langle\lambda^\alpha\tilde{A}^{(1)}_m\lambda^\beta\tilde{A}^{(2)}_\beta\lambda^\sigma\tilde{B}_{tuv}\right\rangle$$
$$+\frac{i\alpha'}{2}\frac{x_{13}}{x_{12}}(\gamma_m\gamma^{stuv})_{\alpha\sigma}(k_3)_s\left\langle\lambda^\beta\tilde{A}^{(1)}_\beta\lambda^\alpha\tilde{A}^{(2)}_m\lambda^\sigma\tilde{B}_{tuv}\right\rangle \quad (4.10)$$

Third term. The 3rd term is given by

$$\begin{aligned}T_3 &= \langle\lambda^\alpha A^{(1)}_\alpha(x_1)\lambda^\beta A^{(2)}_\beta(x_2)\Pi^m\lambda^\sigma H_{m\sigma}(x_3)\rangle\\ &= \oint_{x_3}\frac{dw}{w-x_3}\langle\lambda^\alpha A^{(1)}_\alpha(x_1)\lambda^\beta A^{(2)}_\beta(x_2)\Pi^m(w)\lambda^\sigma(x_3)H_{m\sigma}(x_3)\rangle\\ &= -\oint_{x_1}\frac{dw}{w-x_3}\left\langle\lambda^\alpha(x_1)\left[-i\alpha'(k_1)^m\frac{A^{(1)}_\alpha(x_1)}{w-x_1}\right]\lambda^\beta(x_2)A^{(2)}_\beta(x_2)\lambda^\sigma(x_3)H_{m\sigma}(x_3)\right\rangle\\ &\quad -\oint_{x_2}\frac{dw}{w-x_3}\left\langle\lambda^\alpha(x_1)A^{(1)}_\alpha(x_1)\lambda^\beta(z_2)\left[-i\alpha'(k_2)^m\frac{A^{(2)}_\beta(x_2)}{w-x_2}\right]\lambda^\sigma(x_3)H_{m\sigma}(x_3)\right\rangle\\ &= \frac{72i\alpha'}{5!\times 32}(\gamma_{stuvw})^{\alpha\sigma}(k_1)^m\left\langle\tilde{A}^{(1)}_\alpha\lambda^\beta\tilde{A}^{(2)}_\beta(\lambda\gamma^{stuvw}\lambda)\tilde{\Psi}_{m\sigma}\right\rangle\end{aligned} \quad (4.11)$$

Again, in going to the 3rd equality, we have unwrapped the contour and used the OPE between Π^m and superfields. In going to the last line, we have performed the contour integration and used the momentum conservation and the identity (4.7).

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Fourth term. Finally, the 4th term is given by

$$\begin{aligned}
T_4 &= \langle \lambda^\alpha A^{(1)}_\alpha(x_1)\lambda^\beta A^{(2)}_\beta(x_2) N^{mn}\lambda^\sigma F_{\sigma mn}(x_3)\rangle \\
&= \oint_{x_3}\frac{dw}{w-z_3}\langle \lambda^\alpha A^{(1)}_\alpha(x_1)\lambda^\beta A^{(2)}_\beta(x_2) N^{mn}(w)\lambda^\sigma(x_3) F_{\sigma mn}(x_3)\rangle \\
&= -\oint_{x_1}\frac{dw}{w-x_3}\left\langle \Big[\frac{\alpha'}{4}\frac{(\gamma^{mn})^\alpha{}_\rho\lambda^\rho(x_1)}{w-x_1}\Big] A^{(1)}_\alpha(x_1)\lambda^\beta(x_2)A^{(2)}_\beta(x_2)\lambda^\sigma(x_3)F_{\sigma mn}(x_3)\right\rangle \\
&\quad -\oint_{x_2}\frac{dw}{w-x_3}\left\langle \lambda^\alpha(x_1)A^{(1)}_\alpha(x_1)\Big[\frac{\alpha'}{4}\frac{(\gamma^{mn})^\beta{}_\rho\lambda^\rho(x_2)}{w-x_2}\Big] A^{(2)}_\beta(x_2)\lambda^\sigma(x_3)F_{\sigma mn}(x_3)\right\rangle \\
&= \frac{3i\alpha'}{64}\frac{x_{23}}{x_{12}}\Big[(\gamma_{mstuv})^{\alpha\sigma}(k_3)_m + 4(\gamma_{tuv})^{\alpha\sigma}(k_3)_s\Big]\Big\langle \tilde{A}^{(1)}_\alpha\lambda^\beta\tilde{A}^{(2)}_\beta(\lambda\gamma^{stuvw}\lambda)\tilde{\Psi}_{w\sigma}\Big\rangle \\
&\quad +\frac{3i\alpha'}{64}\frac{x_{13}}{x_{12}}\Big[(\gamma_{mstuv})^{\alpha\sigma}(k_3)_m + 4(\gamma_{tuv})^{\alpha\sigma}(k_3)_s\Big]\Big\langle \lambda^\beta\tilde{A}^{(1)}_\beta\tilde{A}^{(2)}_\alpha(\lambda\gamma^{stuvw}\lambda)\tilde{\Psi}_{w\sigma}\Big\rangle
\end{aligned} \tag{4.12}$$

In going to the 3rd equality, we have unwrapped the contour and used the OPE of N^{mn} with λ^α whereas in going to the 4th equality, we have used the expression of $F_{\alpha mn}$ in terms of $\Psi_{m\alpha}$ as given in (3.8). We note that T_4 is exactly minus of the $\tilde{T}_2$ as given in (4.9).

Combining all the terms, we find that the total 3-point function is given by

$$\mathcal{A}_3 \;=\; \langle V_1 V_2 V_3\rangle \;=\; T_2 + T_3 \tag{4.13}$$

where T_2 and T_3 are given in equations (4.10) and (4.11) respectively. Below, we shall give the results for the correlators $\langle V_1V_2V_3\rangle$ for different choices of the 2 massless and one massive external states.

4.2 Evaluation of correlators

The two terms T_2 and T_3 given in (4.13) are at the superfield level. To compute some specific 3-point function, we need to keep only the fields of interest to be non zero in the superfields. After specializing to some specific amplitude, $\mathcal{A}_3$ can be evaluated using the theta expansion results given in section 3 and appendix C and the pure spinor correlators listed in appendix A.3. We use the symbolic computer programme Cadabra to do this calculation [16, 17]. An important point to note is that T_2 as given in equation (4.10) depends upon the world-sheet coordinates x_i but T_3 does not. However, the dependence of T_2 on the world sheet coordinates automatically drops out on evaluating it using the theta expansion of the superfields as expected from the SL$(2,\mathbb{R})$ invariance. We now give the results for different 3-point correlators.

2 gluon and 1 b_{mnp} field. For this case, we have

$$T_2 = \frac{29i}{840}e^{mnp}e^1_m e^2_n (k_1)_p \quad , \qquad T_3 = \frac{13i}{840}e^{mnp}e^1_m e^2_n (k^1)_p$$

This gives

$$\langle aab\rangle \;=\; T_2 + T_3 \;=\; \frac{i}{20}e^{mnp}e^1_m e^2_n (k_1)_p \tag{4.14}$$

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2 gluon and 1 g_{mn} field. For the $\langle aag\rangle$ amplitude, we have

$$T_2 = -\frac{1}{80}(e^1\cdot g\cdot e^2) + \frac{\alpha'}{160}(e^2\cdot k^1)(e^1\cdot g\cdot k^1) - \frac{\alpha'}{160}(e^1\cdot k^2)(e^2\cdot g\cdot k^1)$$
$$T_3 = -\frac{3\alpha'}{160}(e^1\cdot k^2)(e^2\cdot g\cdot k^1) - \frac{\alpha'}{40}(e^1\cdot e^2)(k^1\cdot g\cdot k^1) + \frac{3\alpha'}{160}(e^2\cdot k^1)(e^1\cdot g\cdot k^1)$$

This gives,

$$\begin{aligned}\langle aag\rangle &= T_2 + T_3\\ &= -\frac{1}{80}\Big[2\alpha'(e^1\cdot k^2)(e^2\cdot g\cdot k^1) + 2\alpha'(e^2\cdot k^1)(e^1\cdot g\cdot k^2)\\ &\qquad -2\alpha'(e^1\cdot e^2)(k^1\cdot g\cdot k^2) + (e^1\cdot g\cdot e^2)\Big]\end{aligned} \tag{4.15}$$

2 gluino and 1 b_{mnp} field. For the $\langle\chi\chi b\rangle$ amplitude, we have

$$T_2 = \frac{23}{11340}(\xi^1\gamma^{mnp}\xi^2)e_{mnp} \quad , \qquad T_3 = \frac{1}{18144}(\xi^1\gamma^{mnp}\xi^2)e_{mnp}$$

This gives

$$\langle\chi\chi b\rangle \;=\; T_2 + T_3 = \frac{1}{480}(\xi^1\gamma^{mnp}\xi^2)e_{mnp} \tag{4.16}$$

2 gluino and 1 g_{mn} field. For the $\langle\chi\chi g\rangle$ amplitude, we have

$$T_2 = 0 \quad , \qquad T_3 = \frac{i\alpha'}{80}(\xi^1\gamma^m\xi^2)g_{mn}k_1^n$$

This gives,

$$\langle\chi\chi g\rangle \;=\; T_2 + T_3 = \frac{i\alpha'}{80}(\xi^1\gamma^m\xi^2)e_{mn}k_1^n \tag{4.17}$$

1 gluon, 1 gluino and 1 $\psi_{m\alpha}$ field. In this case, since all the external states are different, the two orderings $\langle a\chi\psi\rangle$ and $\langle\chi a\psi\rangle$ are different. We give the result for both cases. For the $\langle a\chi\psi\rangle$ correlator, we have

$$T_2 = \frac{4}{21}e_1^m(\xi^2\psi_m) - \frac{31}{210}\alpha'(\xi^2\psi_n)(e^1\cdot k^2)k_2^n - \frac{9}{140}\alpha'(\xi^2\gamma_{mn}\psi_p)e_1^m k_1^n k_2^p$$
$$T_3 = \frac{1}{105}e_1^m(\xi^2\psi_m) - \frac{53}{210}\alpha'(\xi^2\psi_n)(e^1\cdot k^2)k_2^n - \frac{19}{140}\alpha'(\xi^2\gamma_{mn}\psi_p)e_1^m k_1^n k_2^p$$

This gives

$$\begin{aligned}\langle a\chi\psi\rangle &= T_2 + T_3\\ &= \frac{1}{5}\Big[e_1^m(\xi^2\psi_m) - 2\alpha'(\xi^2\psi_n)(e^1\cdot k^2)k_2^n - \alpha'(\xi^2\gamma_{mn}\psi_p)e_1^m k_1^n k_2^p\Big]\end{aligned} \tag{4.18}$$

On the other hand, for the $\langle\chi a\psi\rangle$ correlator, we have

$$T_2 = \frac{4}{21}e_2^m(\xi^1\psi_m) - \frac{31}{210}\alpha'(\xi^1\psi_n)(e^2\cdot k^1)k_1^n - \frac{9}{140}\alpha'(\xi^1\gamma_{mn}\psi_p)e_2^m k_2^n k_1^p$$
$$T_3 = \frac{1}{105}e_2^m(\xi^1\psi_m) - \frac{53}{210}\alpha'(\xi^1\psi_n)(e^2\cdot k^1)k_1^n - \frac{19}{140}\alpha'(\xi^1\gamma_{mn}\psi_p)e_2^m k_2^n k_1^p$$

This gives

$$\begin{aligned}\langle\chi a\psi\rangle &= T_2 + T_3\\ &= \frac{1}{5}\Big[e_2^m(\xi^1\psi_m) - 2\alpha'(\xi^1\psi_n)(e^2\cdot k^1)k_1^n - \alpha'(\xi^1\gamma_{mn}\psi_p)e_2^m k_2^n k_1^p\Big]\end{aligned} \tag{4.19}$$

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3 massless fields. For comparing the pure spinor results with the corresponding RNS results, we also need the massless amplitudes. For $\langle aaa\rangle$ correlator, the pure spinor calculation gives the following result in our conventions

$$\langle aaa\rangle = \frac{i}{180}\left[(e^1\cdot e^2)(e^3\cdot k^1)+(e^1\cdot e^3)(e^2\cdot k^3)+(e^2\cdot e^3)(e^1\cdot k^2)\right] \tag{4.20}$$

On the other hand, for the $\langle a\chi\chi\rangle$ correlator, we get

$$\langle a\chi\chi\rangle = \frac{1}{360}(\chi^2\gamma^m\chi^3)e^1_m\ . \tag{4.21}$$

From the explicit expression of all massless-massless-massive amplitudes obtained in this section, we observe that all such 3-point functions are symmetric under a change of cyclic order. Therefore the inclusion of Chan-Paton factors (denoted by t^a, t^b t^c) will make the full amplitude proportional to $Tr(t^a,\{t^b,t^c\})$. Compare this with the 3-point functions involving all massless states, which are antisymmetric under a change of cyclic order and therefore after taking into account the Chan-Paton factors, the final answer becomes proportional to $Tr(t^a,[t^b,t^c])$. Another point to note is that all the amplitudes considered in this section are invariant under the gauge transformations $e^i\to e^i+k^i$. We now turn to comparing the RNS and PS results.

5 Comparing pure spinor and the RNS results

By comparing the pure spinor results given above with the corresponding RNS results given in appendix (B.2), we see that the tensor structures of the 3-point functions match perfectly. Moreover the relative coefficients of the various terms in the correlators $\langle aag\rangle$ and $\langle a\chi\psi\rangle$ which have more than one terms, also match exactly. This is a non trivial test. We shall now show that the overall numerical factors (i.e. normalizations) of the different 3-point functions in pure spinor and RNS are also in perfect agreement with each other.

We have denoted the overall normalization of the vertex operators in the RNS calculations relative to those in PS calculations by g_a, g_χ etc. (see appendix B.2). For example, if $\mathcal{N}_{\rm RNS}$ and $\mathcal{N}_{\rm PS}$ denote the normalizations of the gluon vertex operator in the RNS and PS respectively, then for the $\langle V_aV_aV_a\rangle$ correlator, we have (denoting $V\equiv\mathcal{N}\tilde{V}$)

$$(\mathcal{N}_{\rm PS})^3\langle\tilde{V}_a\tilde{V}_a\tilde{V}_a\rangle_{\rm PS} = (\mathcal{N}_{\rm RNS})^3\langle\tilde{V}_a\tilde{V}_a\tilde{V}_a\rangle_{\rm RNS} \tag{5.1}$$

From this, it is clear that only the relative normalization between RNS and PS vertex operators have any physical significance. To exploit this fact, we define

$$\mathcal{N}_{\rm RNS} = g_a\,\mathcal{N}_{\rm PS} \tag{5.2}$$

In our calculations, we have set the overall normalization of the PS vertex operators to be 1 and kept the relative normalization factor g_a, g_χ etc. to be in the RNS vertex operators.

With the above convention, the overall RNS and the pure spinor numerical factors for each correlator is given in the table 1. By comparing the RNS and pure spinor numerical factors for $\langle aaa\rangle$ and $\langle a\chi\chi\rangle$, we find

$$(g_a)^3 = \frac{-i}{180\sqrt{2\alpha'}}\quad,\qquad (g_\chi)^2 = \frac{\sqrt{2}}{360\ g_a}. \tag{5.3}$$

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Correlator	RNS	PS
$\langle aaa\rangle$	$-g_a^3\sqrt{2\alpha'}$	$\frac{i}{180}$
$\langle a\chi\chi\rangle$	$\frac{1}{\sqrt{2}}g_a g_\chi^2$	$\frac{1}{360}$
$\langle aab\rangle$	$6g_a^2 g_b\sqrt{2\alpha'}$	$\frac{i}{20}$
$\langle\chi\chi b\rangle$	$-\frac{1}{2\sqrt{2}}g_\chi^2 g_b$	$\frac{1}{480}$
$\langle aag\rangle$	$-g_a^2 g_g$	$-\frac{1}{80}$
$\langle\chi\chi g\rangle$	$\sqrt{\alpha'}g_\chi^2 g_g$	$\frac{i\alpha'}{80}$
$\langle a\chi\psi\rangle$	$-\frac{16\alpha'}{\sqrt{2}}g_a g_\chi g_\psi$	$\frac{1}{5}$
$\langle\chi a\psi\rangle$	$-\frac{16\alpha'}{\sqrt{2}}g_a g_\chi g_\psi$	$\frac{1}{5}$

Table 1. Numerical factors of RNS and pure spinor correlators.

In terms of g_a and g_χ, the numerical factors for $\langle aab\rangle$, $\langle aag\rangle$ and $\langle a\chi\psi\rangle$ give

$$g_b = \frac{i}{120\sqrt{2\alpha'}\ g_a^2} \quad , \qquad g_g = \frac{1}{80\ g_a^2} \quad , \qquad g_\psi = -\frac{\sqrt{2}}{80\alpha' g_a g_\chi} \tag{5.4}$$

The above values of g_b, g_g and g_ψ agree perfectly with the value obtained using $\langle\chi\chi b\rangle$, $\langle\chi\chi g\rangle$ and $\langle\chi a\psi\rangle$ correlators. This is a non trivial consistency check.

6 Discussions

In this paper we have explicitly shown the equivalence of all massless-massless-massive 3-point functions for each given ordering of vertex operators on the boundary for open superstrings computed in PS and RNS formalisms. For the massive states, the normalization factors, which are fixed, using say $\langle aab\rangle$, $\langle aag\rangle$ and $\langle a\chi\psi\rangle$ correlators, match perfectly with the $\langle\chi\chi b\rangle$, $\langle\chi\chi g\rangle$ and $\langle\chi a\psi\rangle$ correlators which establishes the consistency of the relative normalization determined in this paper and establishes firmly the equivalence between RNS and PS formalism for first massive states.

Once the vertex for open superstring is known, its extension to closed strings can be obtained in a straightforward manner by taking appropriate tensor products of left and right moving sectors [22].

To compare loop amplitudes one needs to compute the amplitude in PS formalism using integrated vertex constructed for massive states in [23]. With the relative normalization now fixed, they should match exactly with the corresponding RNS result. Also as argued in [23], the strategy behind constructing massive vertex (integrated or unintegrated) is not sensitive to mass level in question and is expected to be identical for all higher massive states. Therefore, one can reasonably expect that all 3-point functions involving higher massive states can also be directly compared with RNS results and yield a consistent set of normalizations for each mass level.

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A Some pure spinor results

In this appendix, we note some pure spinor results which are used in this work.

A.1 Pure spinor conventions

In this section, we give our conventions for the pure spinor calculations. For the tree amplitude calculations, both the minimal as well as the non minimal formalisms give the same prescription. For convenience, we shall only consider the minimal formalism. The world sheet action of the minimal formalism is given by

$$S = \frac{1}{\pi\alpha'}\int d^2z \left(\frac{1}{2}\partial X^m \bar{\partial} X_m + p_\alpha \bar{\partial}\theta^\alpha - w_\alpha \bar{\partial}\lambda^\alpha\right) \tag{A.1}$$

where, $m = 0, 1, , \cdots, 9$ and $\alpha = 1, \cdots, 16$.

The p_α and w_α are the conformal weight one conjugate momenta of the conformal weight zero fields θ^α and λ^α respectively. The fields with upper (lower) spinor indices transform as right (left) handed Weyl spinor. The λ^α satisfy the pure spinor constraint

$$\lambda^\alpha \gamma^m_{\alpha\beta}\lambda^\beta = 0 \tag{A.2}$$

The BRST operator is defined to be

$$Q = \oint dz\ (\lambda^\alpha d_\alpha) \tag{A.3}$$

Due to pure spinor constraint, the world-sheet fields can only appear in the following gauge invariant combinations[4]

$$N_{mn} = \frac{1}{2}w_\alpha(\gamma_{mn})^\alpha_{\ \beta}\lambda^\beta \quad , \quad J = w_\alpha\lambda^\alpha \quad , \quad T = w_\alpha\partial\lambda^\alpha$$

[4]The coincident operators are normal ordered via

$$:A(z)B(z):\equiv \frac{1}{2\pi i}\oint_z \frac{dw}{w-z}A(w)B(z) \tag{A.4}$$

where, A and B are any two operators and the contour surrounds the point z. However, we shall suppress the normal ordering symbol : : throughout this draft.

Any other gauge invariant combination can be expressed in terms of these objects. The ghost current is given by $J = \lambda^\alpha w_\alpha$ which implies that the λ^α carries the ghost number $+1$ and the rest of the fields carry zero ghost number.

Before proceeding ahead, we note down some important OPEs of the theory which arise frequently in calculations

$$\begin{aligned}
d_\alpha(z)d_\beta(w) &= -\frac{\alpha'\gamma^m_{\alpha\beta}}{2(z-w)}\Pi_m(w) + \cdots, \qquad d_\alpha(z)\Pi^m(w) = \frac{\alpha'\gamma^m_{\alpha\beta}}{2(z-w)}\partial\theta^\beta(w) + \cdots \\
d_\alpha(z)V(w) &= \frac{\alpha'}{2(z-w)}D_\alpha V(w) + \cdots, \qquad \Pi^m(z)V(w) = -\frac{\alpha'}{(z-w)}\partial^m V(w) + \cdots \\
\Pi^m(z)\Pi^n(w) &= -\frac{\alpha'\eta^{mn}}{2(z-w)^2} + \cdots, \qquad N^{mn}(z)\lambda^\alpha(w) = \frac{\alpha'(\gamma^{mn})^\alpha{}_\beta}{4(z-w)}\,\lambda^\beta(w) + \cdots \\
J(z)J(w) &= -\frac{(\alpha')^2}{(z-w)^2} + \cdots, \qquad J(z)\lambda^\alpha(w) = \frac{\alpha'}{2(z-w)}\lambda^\alpha(w) + \cdots \\
N^{mn}(z)N^{pq}(w) &= -\frac{3(\alpha')^2}{2(z-w)^2}\eta^{m[q}\eta^{p]n} - \frac{\alpha'}{(z-w)}\Big(\eta^{p[n}N^{m]q} - \eta^{q[n}N^{m]p}\Big) + \cdots
\end{aligned} \tag{A.5}$$

In the above OPEs, ∂_m is the derivative with respect to the spacetime coordinate X^m, ∂ is the derivative with respect to the world-sheet coordinate, V denotes an arbitrary superfield. The d_α and Π^m denote the supersymmetric combinations

$$\begin{aligned}
d_\alpha &= p_\alpha - \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\beta\partial X_m - \frac{1}{8}\gamma^m_{\alpha\beta}\gamma_{m\sigma\delta}\theta^\beta\theta^\sigma\partial\theta^\delta \\
\Pi^m &= \partial X^m + \frac{1}{2}\gamma^m_{\alpha\beta}\theta^\alpha\partial\theta^\beta
\end{aligned} \tag{A.6}$$

The D_α is the supercovariant derivative given by

$$D_\alpha \equiv \partial_\alpha + \gamma^m_{\alpha\beta}\theta^\beta\partial_m \qquad \Longrightarrow \qquad \{D_\alpha, D_\beta\} = 2(\gamma^m)_{\alpha\beta}\partial_m \tag{A.7}$$

For calculating the scattering amplitudes, we need to know the number of zero modes of the world-sheet fields. Due to the pure spinor constraints, it follows that all the world-sheet ghost sector fields, namely λ^α and w_α have 11 independent components. Now, it is a result from the theory of Riemann surfaces that the number of zero modes of the conformal weight one and conformal weight zero objects on a genus g Riemann surface is g and 1 respectively. Thus, the number of zero modes of λ^α on a genus g Riemann surface is 11 whereas the number of zero modes of w_α on a genus g Riemann surface is $11g$. On the other hand, the matter sector fields θ^α and p_α have 16 and $16g$ zero modes respectively on a genus g Riemann surface. As in RNS formalism, the integration over the zero modes give divergences and we need some way to absorb these zero modes. For details, see the original literatures.

Before moving further, we describe an important identity on the world sheet which is needed while solving the BRST equations of motion. The open string vertex operators live on the boundary, i.e., along the real axis on the complex plane. This implies that in the BRST equation $QU = \partial_{\mathbb{R}}V$, the derivative in the right hand side is along the real axis (and hence the subscript $\mathbb{R}$ in the right hand side). However, for doing calculations, it is

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convenient to convert it into the holomorphic derivative using the chain rule. If x denotes the coordinate along the real axis, then for an arbitrary function f on the real axis, we have

$$\frac{\partial f}{\partial x} = \left(\frac{\partial z}{\partial x}\frac{\partial f}{\partial z} + \frac{\partial \bar{z}}{\partial x}\frac{\partial f}{\partial \bar{z}}\right)\Big|_{\bar{z}=z} = \left(\frac{\partial f}{\partial z} + \frac{\partial f}{\partial \bar{z}}\right)\Big|_{\bar{z}=z} \tag{A.8}$$

Now, using the chain rule, we have

$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial X^m}\frac{\partial X^m}{\partial z} + \frac{\partial f}{\partial \theta^\alpha}\frac{\partial \theta^\alpha}{\partial z} \quad , \qquad \frac{\partial f}{\partial \bar{z}} = \frac{\partial f}{\partial X^m}\frac{\partial X^m}{\partial \bar{z}} + \frac{\partial f}{\partial \theta^\alpha}\frac{\partial \theta^\alpha}{\partial \bar{z}} \tag{A.9}$$

Using the equation of motion for p_α, namely, $\bar{\partial}\theta^\alpha = 0$ and the above two equations, we obtain

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial f}{\partial X^m}\frac{\partial X^m}{\partial z} + \frac{\partial f}{\partial X^m}\frac{\partial X^m}{\partial \bar{z}}\Big|_{\bar{z}=z} + \frac{\partial f}{\partial \theta^\alpha}\frac{\partial \theta^\alpha}{\partial z} \\ &= 2\frac{\partial f}{\partial X^m}\frac{\partial X^m}{\partial z} + \frac{\partial f}{\partial \theta^\alpha}\frac{\partial \theta^\alpha}{\partial z}\end{aligned} \tag{A.10}$$

Finally, using (A.6) and (A.7), we can express this as

$$\frac{\partial f}{\partial x} = 2\Pi^m \partial_m f + \partial\theta^\alpha D_\alpha f \tag{A.11}$$

In the above derivation, we have used the world sheet fields defined using the doubling trick. For a derivation without using doubling trick and more details, please see the section 2 of [23].

A.2 Hodge duality of gamma matrices in d=10

In 10 dimensions, the 16×16 gamma matrices satisfy the following Hodge dualities

$$(\gamma^{m_1\ldots m_{2n}})^\alpha{}_\beta = \frac{1}{(10-2n)!}(-1)^{(n+1)}\epsilon^{m_1\ldots m_{2n}p_1\ldots p_{10-2n}}(\gamma_{p_1\ldots p_{10-2n}})^\alpha{}_\beta \tag{A.12}$$

$$(\gamma^{m_1\ldots m_{2n}})_\alpha{}^\beta = -\frac{1}{(10-2n)!}(-1)^{(n+1)}\epsilon^{m_1\ldots m_{2n}p_1\ldots p_{10-2n}}(\gamma_{p_1\ldots p_{10-2n}})_\alpha{}^\beta \tag{A.13}$$

$$(\gamma^{m_1\ldots m_{2n+1}})^{\alpha\beta} = \frac{1}{(9-2n)!}(-1)^{n}\epsilon^{m_1\ldots m_{2n+1}p_1\ldots p_{9-2n}}(\gamma_{p_1\ldots p_{9-2n}})^{\alpha\beta} \tag{A.14}$$

$$(\gamma^{m_1\ldots m_{2n+1}})_{\alpha\beta} = -\frac{1}{(9-2n)!}(-1)^{n}\epsilon^{m_1\ldots m_{2n+1}p_1\ldots p_{9-2n}}(\gamma_{p_1\ldots p_{9-2n}})_{\alpha\beta} \tag{A.15}$$

where, $\epsilon^{m_1\cdots m_9}$ is the 10 dimensional epsilon tensor defined as

$$\epsilon_{0\,1\,\cdots\,9} = 1 \qquad \Longrightarrow \qquad \epsilon^{0\,1\,\cdots\,9} = -1 \tag{A.16}$$

Due to the above dualities, not all the antisymmetrized product of gamma matrices are independent. We shall take γ^{m_1}, $\gamma^{m_1m_2}$, $\gamma^{m_1m_2m_3}$, $\gamma^{m_1m_2m_3m_4}$ and $\gamma^{m_1m_2m_3m_4m_5}$ along with the identity matrix $\mathbb{I}_{16\times16}$ as the linearly independent basis elements for vector spaces of 16×16 complex matrices.

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A.3 Pure spinor superspace identities

While computing the scattering amplitudes in pure spinor formalism, the last step requires evaluation of integration over the zero modes of λ^α and θ^α. Due to the pure spinor constraints and the symmetry properties of θ^α and λ^α, there are only a finite number of these basic pure spinor correlators which are non zero. Below, we list those pure spinor correlators which are used in this note [8]

$$\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{stu}\theta)\rangle = \frac{1}{120}\delta^{mnp}_{stu} \tag{A.17}$$

$$\langle(\lambda\gamma^{pqr}\theta)(\lambda\gamma_m\theta)(\lambda\gamma_n\theta)(\theta\gamma_{stu}\theta)\rangle = \frac{1}{70}\delta^{[p}_{[m}\eta_{n][s}\delta^q_t\delta^{r]}_{u]} \tag{A.18}$$

$$\langle(\lambda\gamma^{mnpqr}\theta)(\lambda\gamma_s\theta)(\lambda\gamma_t\theta)(\theta\gamma_{uvw}\theta)\rangle = -\frac{1}{42}\delta^{mnpqr}_{stuvw} - \frac{1}{5040}\epsilon^{mnpqr}{}_{stuvw} \tag{A.19}$$

$$\begin{aligned}\langle(\lambda\gamma_q\theta)(\lambda\gamma^{mnp}\theta)(\lambda\gamma^{rst}\theta)(\theta\gamma_{uvw}\theta)\rangle = &-\frac{1}{280}\Big[\eta_{q[u}\eta^{z[r}\delta^s_v\eta^{t][m}\delta^n_{w]}\delta^{p]}_z - \eta_{q[u}\eta^{z[m}\delta^n_v\eta^{p][r}\delta^s_{w]}\delta^{t]}_z\Big]\\ &+\frac{1}{140}\Big[\delta^{[m}_q\delta^n_{[u}\eta^{p][r}\delta^s_v\delta^{t]}_{w]} - \delta^{[r}_q\delta^s_{[u}\eta^{t][m}\delta^n_v\delta^{p]}_{w]}\Big]\\ &-\frac{1}{8400}\epsilon^{qmnprstuvw}\end{aligned} \tag{A.20}$$

$$\begin{aligned}\langle(\lambda\gamma^{mnpqr}\theta)(\lambda\gamma_{stu}\theta)(\lambda\gamma^v\theta)(\theta\gamma_{wxy}\theta)\rangle = &\frac{1}{120}\epsilon^{mnpqr}{}_{ghijk}\Big(\frac{1}{35}\eta^{v[g}\delta^h_{[s}\delta^i_t\eta_{u][w}\delta^j_x\delta^{k]}_{y]}\\ &-\frac{2}{35}\delta^{[g}_{[s}\delta^h_t\delta^i_{u]}\delta^j_{[w}\delta^{k]}_x\delta^{v]}_{y]}\Big)\\ &+\frac{1}{35}\eta^{v[m}\delta^n_{[s}\delta^p_t\eta_{u][w}\delta^q_x\delta^{r]}_{y]} - \frac{2}{35}\delta^{[m}_{[s}\delta^n_t\delta^{p]}_{u]}\delta^q_{[w}\delta^{r]}_x\delta^{v]}_{y]}\end{aligned} \tag{A.21}$$

B RNS results

In this section, we summarize the results of the 3-point functions involving two massless and one massive states computed using the RNS formalism. In subsection B.1, we state our conventions and in subsection B.2, we give the results of 3-point computations. A useful reference for this section is [24] (also see [25, 26]).

B.1 Conventions for RNS calculations

Due to the picture number anomaly on a genus g Riemann surface, a non vanishing RNS correlator must have the picture number $2g-2$. This is ensured by working with vertex operators in appropriate picture number and the insertions of appropriate number of picture changing operators (PCOs). For the 3-point functions on Riemann sphere, we can avoid the insertions of PCOs by working with vertex operators of the appropriate picture number so that the total picture number adds up to -2 (which is the picture number anomaly on Riemann sphere).

We start by writing down the vertex operators for the massless states in various picture numbers. The gluon vertex operator in the -1 and 0 picture numbers is given by

$$\begin{aligned}V^{(-1)}_a(x) &= g_a\ e_m\psi^m(x)e^{-\phi(x)}e^{ik\cdot X(x)}\\ V^{(0)}_a(x) &= \frac{g_a}{\sqrt{2\alpha'}}e_m\Big(i\partial X^m(x) + 2\alpha' k_n\psi^n(x)\psi^m(x)\Big)e^{ik\cdot X(x)}\end{aligned} \tag{B.1}$$

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The gluino vertex operator in the $-1/2$ picture number is given by

$$V_\chi^{(-1/2)}(x) \;=\; g_\chi\; \xi^\alpha S_\alpha(x) e^{-\phi(x)/2} e^{ik\cdot X(x)} \tag{B.2}$$

The polarization vectors of the gluon and gluino satisfy the transversality conditions

$$e_m k^m \;=\; 0 \quad , \qquad \xi^\alpha \gamma^m_{\alpha\dot\beta} k_m \;=\; 0 \tag{B.3}$$

Now, we turn to the vertex operators for the first massive states [27, 28]. The vertex operators for the anti-symmetric 3-form field b_{mnp} in the picture numbers -1 and 0 are given by

$$\begin{aligned} V_b^{(-1)}(x) &= g_b\; e_{mnp}\psi^m(x)\psi^n(x)\psi^p(x) e^{-\phi(x)} e^{ik\cdot X(x)} \\ V_b^{(0)}(x) &= g_b\sqrt{2\alpha'}\; e_{mnp}\Big(\frac{3i}{2\alpha'}\partial X^m\psi^n\psi^p + k_q\psi^q\psi^m\psi^n\psi^p\Big) e^{ik\cdot X(x)} \end{aligned} \tag{B.4}$$

The vertex operators for the symmetric traceless massive graviton field g_{mn} are given by

$$\begin{aligned} V_g^{(-1)}(x) &= \frac{g_g}{\sqrt{2\alpha'}}\; e_{mn} i\partial X^m(x)\psi^n(x) e^{-\phi(x)} e^{ik\cdot X(x)} \\ V_g^{(0)}(x) &= g_g\; e_{mn}\Big(\frac{1}{2\alpha'} i\partial X^m i\partial X^n + \partial\psi^m\psi^n + i\partial X^m k_p\psi^p\psi^n\Big) e^{ik\cdot X} \end{aligned} \tag{B.5}$$

Finally, the vertex operators for the massive gravitino field $\psi_{m\alpha}$ in the $-1/2$ picture number is given by

$$V_\psi^{(-1/2)}(z) \;=\; \frac{g_\psi}{\sqrt{\alpha'}}\Big(\xi^\alpha_m i\partial X^m - \frac{\alpha'}{4}\xi^\beta_m\psi^m\gamma^n_{\beta\dot\beta}\gamma_p^{\dot\beta\alpha}k_n k^p\Big) S_\alpha e^{-\phi/2} e^{ik\cdot X}(z) \tag{B.6}$$

The polarization vectors of the 1st massive states satisfy the conditions

$$k^m e_{mnp} = k^m e_{mn} = k^m \xi^\alpha_m = 0 \tag{B.7}$$

For comparison with PS result, it is useful to parametrize the massive tensor spinor ξ^α_m as

$$\xi^\alpha_m = -8\alpha' \bar\rho_{m\dot\beta} k^p \bar\gamma_p^{\dot\beta\alpha} \qquad , \qquad k^m \bar\rho_{m\dot\beta} = \bar\rho_{m\dot\beta}\gamma_m^{\dot\beta\alpha} = 0 \tag{B.8}$$

We now turn to the OPEs and correlation functions of the world-sheet matter and ghost sector fields. The important correlators of the open string X^m fields are given by

$$\Big\langle \prod_{j=1}^n e^{ik_j\cdot X(y_j)} \Big\rangle = \prod_{i<j}^n |y_i - y_j|^{2\alpha' k_i\cdot k_j} \tag{B.9}$$

$$\begin{aligned} \Big\langle \prod_{\ell=1}^q i\partial X^{m_\ell}(w_\ell) \prod_{j=1}^n e^{ik_j\cdot X(y_j)} \Big\rangle &= \sum_{i=2}^p \frac{2\alpha'\eta^{m_1 m_i}}{(w_1 - w_i)^2} \Big\langle \prod_{\substack{\ell=2\\ \ell\neq i}}^p i\partial X^{m_\ell}(w_\ell) \prod_{j=1}^n e^{ik_j\cdot X(y_j)} \Big\rangle \\ &\quad + \sum_{\ell=1}^n \frac{2\alpha' k_\ell^{m_1}}{(w_1 - y_\ell)} \Big\langle \prod_{\ell=2}^p i\partial X^{m_\ell}(w_\ell) \prod_{j=1}^n e^{ik_j\cdot X(y_j)} \Big\rangle \end{aligned} \tag{B.10}$$

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This can be evaluated recursively using (B.9). We shall encounter the situation where some w_ℓ may coincide with some y_j. E.g., we shall need the following correlator

$$\begin{aligned}\Gamma^m &\equiv \left\langle e^{ik_1\cdot X(z_1)}e^{ik_2\cdot X(z_2)}\partial X^m(z_3)e^{ik_3\cdot X(z_3)}\right\rangle \\ &= \oint\frac{dw}{w-z_3}\left\langle e^{ik_1\cdot X(z_1)}e^{ik_2\cdot X(z_2)}e^{ik_3\cdot X(z_3)}\partial X^m(w)\right\rangle \\ &= \left(\frac{2i\alpha' k_1^m}{z_{13}}+\frac{2i\alpha' k_2^m}{z_{23}}\right)|z_{12}|^{2\alpha' k_1\cdot k_2}|z_{23}|^{2\alpha' k_2\cdot k_3}|z_{13}|^{2\alpha' k_1\cdot k_3}\end{aligned} \tag{B.11}$$

In going to the 2nd line, we have used the definition of the normal ordering (A.4) and in going to the 3rd line, we have used (B.10) for $q=1$ and $n=3$.

Similarly, we also need

$$\begin{aligned}&\left\langle e^{ik_1\cdot X(z_1)}e^{ik_2\cdot X(z_2)}i\partial X^p(z_3)i\partial X^q(z_3)e^{ik_3\cdot X(z_3)}\right\rangle \\ &\quad= 4(\alpha')^2\left[\frac{k_1^p}{z_{13}}\left(\frac{k_1^q}{z_{13}}+\frac{k_2^q}{z_{23}}\right)+\frac{k_2^p}{z_{23}}\left(\frac{k_1^q}{z_{13}}+\frac{k_2^q}{z_{23}}\right)\right]|z_{12}|^{2\alpha' k_1\cdot k_2}|z_{23}|^{2\alpha' k_2\cdot k_3}|z_{13}|^{2\alpha' k_1\cdot k_3}\end{aligned}$$

Next, we consider the world-sheet correlators involving the ψ^m fields

$$\left\langle\prod_{i=1}^n\psi^{m_i}(y_i)\right\rangle = \sum_{j=2}^n(-1)^j\frac{\eta^{m_1m_j}}{(y_1-y_j)}\left\langle\prod_{\substack{\ell=2\\ \ell\neq j}}^n\psi^{m_\ell}(y_\ell)\right\rangle \tag{B.12}$$

This expression can also be evaluated recursively. In the final step, the two point function can be evaluated by using the OPE

$$\psi^m(z)\psi^n(w) = \frac{\eta^{mn}}{z-w}+\cdots \tag{B.13}$$

Next, we consider the ghost sector. The basic correlator involving the reparametrization ghost $c(x)$ is given by

$$\langle c(x_1)c(x_2)c(x_3)\rangle = x_{12}x_{23}x_{13} \tag{B.14}$$

The basic correlator involving the bosonized ghost field $\phi(z)$ is given by

$$\left\langle\prod_{k=1}^n e^{q_k\phi(z_k)}\right\rangle = \prod_{k<\ell}^n\frac{1}{(z_k-z_\ell)^{q_kq_\ell}} \quad ; \quad \sum_{k=1}^n q_k = -2 \tag{B.15}$$

Finally, we consider the spin fields. The basic OPEs involving the spin fields are given by (a consistent set of these OPEs can be found in [29])

$$\psi^m(z)S_\alpha(w)e^{-\phi(w)/2} = \frac{(\bar\gamma^m)^{\dot\alpha\beta}S_\beta e^{-\phi(w)/2}}{\sqrt{2}(z-w)^{1/2}}+\cdots \tag{B.16}$$

$$\psi^m(z)S^{\dot\alpha}(w)e^{-\phi(w)/2} = \frac{(\gamma^m)_{\alpha\dot\beta}S^{\dot\beta}e^{-\phi(w)/2}}{\sqrt{2}(z-w)^{1/2}}+\cdots \tag{B.17}$$

$$S_\alpha(z)e^{-\phi(z)/2}S^{\dot\beta}(w)e^{-3\phi(w)/2} = \frac{C_\alpha{}^{\dot\beta}e^{-2\phi(w)}}{(z-w)^2}+\cdots \tag{B.18}$$

$$S_\alpha(z)e^{-\phi(z)/2}S_\beta(w)e^{-\phi(w)/2} = \frac{(\gamma_m C)_{\alpha\beta}\psi^m(w)e^{-\phi(w)}}{\sqrt{2}(z-w)} + \cdots \tag{B.19}$$

$$\psi_m(z)e^{-\phi(z)}S_\alpha(w)e^{-\phi(w)/2} = \frac{(\bar{\gamma}_m)_{\alpha\dot{\beta}}S^{\dot{\beta}}(w)e^{-3\phi(w)/2}}{\sqrt{2}(z-w)} + \cdots \tag{B.20}$$

$$j^{mn}(z)S_\alpha(w) = -\frac{(\gamma^{mn})_\alpha^{\ \beta}S_\beta(w)}{2(z-w)} \quad , \qquad j^{mn} \equiv :\psi^m\psi^n: \tag{B.21}$$

Using the above OPEs, we can work out the following useful correlators which are needed in our calculations

$$\langle \psi^m(x_1)e^{-\phi(x_1)}S_\alpha(x_2)e^{-\phi(x_2)/2}S_\beta(x_3)e^{-\phi(x_3)/2}\rangle = \frac{(\gamma^m C)_{\alpha\beta}}{\sqrt{2}z_{12}z_{13}z_{23}} \tag{B.22}$$

$$\xi_q^\beta\Big\langle \psi^m(x_1)e^{-\phi(x_1)}S_\alpha(x_2)e^{-\phi(x_2)/2}\psi^q(x_3)\psi^p(x_3)\gamma_p^{\dot{\beta}\sigma}S_\sigma(x_3)e^{-\phi(x_3)/2}\Big\rangle = \frac{8\ \xi_m^\beta C_\alpha^{\ \dot{\beta}}}{\sqrt{2}z_{13}^2z_{23}^2} \tag{B.23}$$

B.2 3-point functions

We are now ready to give the results of 3-point functions of two massless and one massive field (for the computation of 3-point functions of states in leading Regge trajectory in RNS, see [30]). These are straightforward to evaluate using the vertex operators and the correlators given in the previous subsection. Hence, we just state the final results below. For our calculations, we shall take the bosonic massless and massive fields to be either in the -1 or 0 picture numbers. On the other hand, the fermionic massless or massive fields will always be taken in the $-1/2$ picture. The picture numbers will be shown by a superscript on the vertex operators inside correlators. Thus, the 3-point amplitudes are given by

$$\mathcal{A}_3 = \langle\ c(x_1)V^{(1)}_{a,\chi}(x_1)c(x_2)V^{(2)}_{a,\chi}(x_2)c(x_3)V^{(3)}_{b,g,\psi}(x_3)\ \rangle \tag{B.24}$$

We start by considering the 3-point functions involving all massless fields. The two possible correlators in this case are $\langle aaa\rangle$ and $\langle a\chi\chi\rangle$. Using the world-sheet correlators given above, these 3-point functions can be evaluated to be

$$\begin{aligned}\langle aaa\rangle &\equiv \Big\langle c(x_1)V_a^{(-1)}(x_1)c(x_2)V_a^{(-1)}(x_2)c(x_3)V_a^{(0)}(x_3)\Big\rangle \\ &= -g_a^3\sqrt{2\alpha'}\Big[(e^1\cdot e^2)(e^3\cdot k^1)+(e^1\cdot e^3)(e^2\cdot k^3)+(e^2\cdot e^3)(e^1\cdot k^2)\Big]\end{aligned} \tag{B.25}$$

As an amplitude this vanishes on summing $(k_1 \leftrightarrow k_2)$ term if the gauge group is abelian.

Similarly, the $a\chi\chi$ correlator can be worked out to be

$$\begin{aligned}\langle a\chi\chi\rangle &\equiv \Big\langle c(x_1)V_a^{(-1)}(x_1)c(x_2)V_\chi^{(-1/2)}(x_2)c(x_3)V_\chi^{(-1/2)}(x_3)\Big\rangle \\ &= \frac{1}{\sqrt{2}}g_a g_\chi^2 e_m^1\Big(\xi^2\gamma^m C\xi^3\Big)\end{aligned} \tag{B.26}$$

Again, for the abelian gauge group, the corresponding amplitude vanishes.

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Next, we consider the two massless and one massive field. We start with two gluons and one b_{mnp} field

$$\begin{aligned}\langle aab\rangle &\equiv \Big\langle c(x_1)V_a^{(-1)}(x_1)c(x_2)V_a^{(-1)}(x_2)c(x_3)V_b^{(0)}(x_3)\Big\rangle \\ &= 6g_a^2 g_b\sqrt{2\alpha'}e_m^1 e_n^2 e_{mnp}k_1^p \end{aligned} \tag{B.27}$$

Next, we consider the 3-point function of two gluons and one massive g_{mn} field which is given by

$$\begin{aligned}\langle aag\rangle &\equiv \Big\langle c(x_1)V_a^{(-1)}(x_1)c(x_2)V_a^{(-1)}(x_2)c(x_3)V_g^{(0)}(x_3)\Big\rangle \\ &= -g_a^2 g_g\Big[2\alpha'\eta^{mn}e_m^1 e_n^2 e_{pq}k_1^p k_1^q + e_p^1 e_q^2 e_{pq} + 2\alpha' e_m^1 e_q^2 e_{pq}k_1^p k_2^m - 2\alpha' e_q^1 e_m^2 e_{pq}k_1^m k_1^p\Big]\end{aligned} \tag{B.28}$$

Next, we include gluino and consider the $\langle\chi\chi b\rangle$ and $\langle\chi\chi g\rangle$ correlators which can be worked out to be

$$\begin{aligned}\langle\chi\chi b\rangle &\equiv \Big\langle c(x_1)V_\chi^{(-1/2)}(x_1)c(x_2)V_\chi^{(-1/2)}(x_2)c(x_3)V_b^{(-1)}(x_3)\Big\rangle \\ &= -\frac{1}{2\sqrt{2}}g_\chi^2 g_b(\xi^1\gamma^{mnp}C\xi^2)e_{mnp}\end{aligned} \tag{B.29}$$

and,

$$\begin{aligned}\langle\chi\chi g\rangle &\equiv \langle c(x_1)V_\chi^{(-1/2)}(x_1)c(x_2)V_\chi^{(-1/2)}(x_2)c(x_3)V_g^{(-1)}(x_3)\rangle \\ &= \sqrt{\alpha'}g_\chi^2 g_g e_{mn}(\xi^1\gamma^n C\xi^2)k_1^m\end{aligned} \tag{B.30}$$

Finally, we consider the massive fermion. The two different 3-point functions with one massive fermion and two massless fields are $\langle a\chi\psi\rangle$ and $\langle\chi a\psi\rangle$ which are given by

$$\begin{aligned}\langle a\chi\psi\rangle &\equiv \Big\langle c(x_1)V_a^{(-1)}(x_1)c(x_2)V_\chi^{(-1/2)}(x_2)c(x_3)V_\psi^{(-1/2)}(x_3)\Big\rangle \\ &= -\frac{16\alpha'}{\sqrt{2}}g_a g_\chi g_\psi\Big[-\alpha' e_m^{(1)}\xi_{(2)}^\alpha k_2^q k_1^p\bar\rho_{q\dot\beta}(\bar\gamma_{mp})^{\dot\beta}{}_{\dot\sigma}C^{\dot\sigma}{}_\alpha \\ &\qquad -2\alpha' k_2^q k_2^m e_m^{(1)}\xi_{(2)}^\alpha\bar\rho_{q\dot\beta}C^{\dot\beta}{}_\alpha + e_m^{(1)}\xi_{(2)}^\alpha\bar\rho_{m\dot\sigma}C^{\dot\sigma}{}_\alpha\Big]\end{aligned} \tag{B.31}$$

and,

$$\begin{aligned}\langle\chi a\psi\rangle &\equiv \Big\langle c(x_1)V_\chi^{(-1/2)}(x_1)c(x_2)V_a^{(-1)}(x_2)c(x_3)V_\psi^{(-1/2)}(x_3)\Big\rangle \\ &= -\frac{16\alpha'}{\sqrt{2}}g_a g_\chi g_\psi\Big[-\alpha' e_m^{(2)}\xi_{(1)}^\alpha k_1^q k_2^p\bar\rho_{q\dot\beta}(\bar\gamma_{mp})^{\dot\beta}{}_{\dot\sigma}C^{\dot\sigma}{}_\alpha \\ &\qquad -2\alpha' k_1^q k_1^m e_m^{(2)}\xi_{(1)}^\alpha\bar\rho_{q\dot\beta}C^{\dot\beta}{}_\alpha + e_m^{(2)}\xi_{(1)}^\alpha\bar\rho_{m\dot\sigma}C^{\dot\sigma}{}_\alpha\Big]\end{aligned} \tag{B.32}$$

To compare the results involving the fermionic fields with the PS results, we need to first convert the RNS gamma matrix conventions into the PS gamma matrix conventions. This mainly involves setting the charge conjugation matrix to be the Kronecker delta $\delta^\alpha{}_\beta$ which implies (for details, see e.g., [24, 25])

$$(\gamma^m C)_{\alpha\beta}\to\gamma^m_{\alpha\beta}\quad,\quad(\bar\gamma_m C)^{\dot\alpha\dot\beta}\to\gamma_m^{\alpha\beta}\quad,\quad(\gamma^{mnp}C)_{\alpha\beta}\to(\gamma^{mnp})_{\alpha\beta} \tag{B.33}$$

and so on.

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C Theta expansion of massless vertex operator

Theta expansion of the massless vertex operator is known and has been extensively used in the literature (see, e.g., [18]). However, some of our conventions (e.g., equation (A.7)) are different from the literature in which the theta expansion of massless vertex operator is used. Below, we derive the results in our convention briefly indicating the steps. We start by recalling the $\mathcal{N}=1$ SYM equations in 10 dimensions [31].

$\mathcal{N}=1$ SYM equations in 10 dimensions. In 10 dimensions, the open string massless states are described by the 10 dimensional $\mathcal{N}=1$ super Yang Mills equations. The field strengths describing the theory are given by

$$F_{\alpha\beta} = \{\nabla_\alpha, \nabla_\beta\} - 2\gamma^m_{\alpha\beta}\nabla_m \quad , \quad F_{\alpha m} = [\nabla_\alpha, \nabla_m] = -F_{m\alpha} \quad , \quad F_{mn} = [\nabla_m, \nabla_n]$$

where $\nabla_m \equiv \partial_m + A_m$, $\nabla_\alpha \equiv D_\alpha + A_\alpha$ and D_α is defined in (A.7).

The 10 dimensional Yang-Mills equations of motion follow from

$$F_{\alpha\beta} = 0 \quad \Longrightarrow \quad D_\alpha A_\beta + D_\beta A_\alpha = 2\gamma^m_{\alpha\beta} A_m \tag{C.1}$$

Using this along with the Bianchi identities, we obtain the following equations at linearized level

$$\begin{aligned}
&F_{m\alpha} = (\gamma_m)_{\alpha\beta} W^\beta, && D_\alpha W^\beta = -\frac{1}{2}(\gamma^{mn})_\alpha{}^\beta F_{mn} \implies D_\alpha W^\alpha = 0 \\
&D_\alpha F_{mn} = \partial_m(\gamma_n W)_\alpha - \partial_n(\gamma_m W)_\alpha, && D_\alpha A^m = -\gamma^m_{\alpha\beta} W^\beta + \partial^m A_\alpha \\
&\partial_m F^{mn} = 0, && \gamma^m_{\alpha\beta}\partial_m W^\beta = 0
\end{aligned} \tag{C.2}$$

Further, the superfields A_m, W^α and F_{mn} can be expressed as

$$\begin{aligned}
A_m &= \frac{1}{16}\gamma^{\alpha\beta}_m D_\alpha A_\beta \quad , \quad F_{mn} = \partial_m A_n - \partial_n A_m \\
W^\alpha &= -\frac{1}{10}(\gamma^m)^{\alpha\beta}(D_\beta A_m - \partial_m A_\beta)
\end{aligned} \tag{C.3}$$

$\mathcal{N}=1$ SYM equations from pure spinor formalism. The pure spinor formalism gives the $\mathcal{N}=1$ SYM equations through its BRST equations of motion. To obtain the correct normalization of the superfields, we derive these equations using the BRST equation of motion $QU = \partial_{\mathbb{R}} V$ and match with the equations given above. At the massless level, the unintegrated vertex operator is constructed from the ghost number 1 and conformal weight 0 objects. The most general object with this property has the form $V = \lambda^\alpha A_\alpha$. Similarly, the most general integrated vertex operator has the form

$$U = \partial\theta^\alpha \tilde{A}_\alpha + \Pi^m \tilde{A}_m + d_\alpha \tilde{W}^\alpha + N^{mn}\tilde{F}_{mn} \tag{C.4}$$

Using the OPEs given in section 2, we obtain

$$\begin{aligned}
Q(\partial\theta^\alpha \tilde{A}_\alpha) &= -\frac{\alpha'}{2}\partial\theta^\alpha \lambda^\beta D_\beta \tilde{A}_\alpha + \frac{\alpha'}{2}\partial\lambda^\alpha \tilde{A}_\alpha \\
Q(\Pi^m \tilde{A}_m) &= \frac{\alpha'}{2}\Pi^m \lambda^\alpha D_\alpha \tilde{A}_m + \frac{\alpha'}{2}\partial\theta^\alpha \lambda^\beta \tilde{A}_m \gamma^m_{\alpha\beta}
\end{aligned}$$

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$$Q(d_\alpha \tilde{W}^\alpha) = -\frac{\alpha'}{2} d_\alpha \lambda^\beta D_\beta \tilde{W}^\alpha - \frac{\alpha'}{2} \Pi^m \lambda^\alpha \tilde{W}^\beta \gamma^m_{\alpha\beta} + \frac{(\alpha')^2}{2} \partial\lambda^\alpha \partial_m \tilde{W}^\beta \gamma^m_{\alpha\beta}$$
$$Q(N^{mn} \tilde{F}_{mn}) = \frac{\alpha'}{2} N^{mn} \lambda^\beta D_\beta \tilde{F}_{mn} - \frac{\alpha'}{4} d_\alpha \lambda^\beta \tilde{F}_{mn} (\gamma^{mn})^\alpha{}_\beta - \frac{(\alpha')^2}{8} \partial\lambda^\alpha D_\beta \tilde{F}_{mn} (\gamma^{mn})^\beta{}_\alpha$$

We also have,

$$\begin{aligned} \partial_{\mathbb{R}}(\lambda^\alpha A_\alpha) &= \partial\lambda^\alpha A_\alpha + \lambda^\alpha \partial_{\mathbb{R}} A_\alpha \\ &= \partial\lambda^\alpha A_\alpha + \partial\theta^\beta \lambda^\alpha D_\beta A_\alpha + 2\Pi^m \lambda^\alpha \partial_m A_\alpha \end{aligned} \tag{C.5}$$

where we have used the pure spinor open string identity (A.11) in going to the second line.

The BRST equation of motion now gives,

$$\begin{aligned} 0 &= QU - \partial_{\mathbb{R}} V \\ &= \partial\theta^\alpha \lambda^\beta \left(-\frac{\alpha'}{2} D_\beta \tilde{A}_\alpha - D_\alpha A_\beta + \frac{\alpha'}{2} \tilde{A}_m \gamma^m_{\alpha\beta} \right) + \Pi^m \lambda^\alpha \left(\frac{\alpha'}{2} D_\alpha \tilde{A}_m - \frac{\alpha'}{2} \tilde{W}^\beta \gamma^m_{\alpha\beta} - 2\partial_m A_\alpha \right) \\ &\quad + d_\alpha \lambda^\beta \left(-\frac{\alpha'}{2} D_\beta \tilde{W}^\alpha - \frac{\alpha'}{4} \tilde{F}_{mn} (\gamma^{mn})^\alpha{}_\beta \right) + \frac{\alpha'}{2} N^{mn} \lambda^\beta D_\beta \tilde{F}_{mn} \\ &\quad + \partial\lambda^\alpha \left(\frac{\alpha'}{2} \tilde{A}_\alpha + \frac{(\alpha')^2}{2} \partial_m \tilde{W}^\beta \gamma^m_{\alpha\beta} - \frac{(\alpha')^2}{8} D_\beta \tilde{F}_{mn} (\gamma^{mn})^\beta{}_\alpha - A_\alpha \right) \end{aligned} \tag{C.6}$$

To match these equations with the 10 dimensional SYM equations, we rescale the fields as

$$\tilde{A}_\alpha = \frac{2}{\alpha'} A_\alpha \quad , \quad \tilde{A}_m = \frac{4}{\alpha'} A_m \quad , \quad \tilde{W}^\alpha = -\frac{4}{\alpha'} W^\alpha \quad , \quad \tilde{F}_{mn} = \frac{4}{\alpha'} F_{mn} \tag{C.7}$$

After this rescaling, the BRST equation gives

$$D_\beta A_\alpha + D_\alpha A_\beta = 2A_m \gamma^m_{\alpha\beta}, \qquad D_\alpha A_m = -W^\beta \gamma^m_{\alpha\beta} + \partial_m A_\alpha, \qquad D_\beta W^\alpha = -\frac{1}{2} F_{mn} (\gamma^{mn})_\beta{}^\alpha$$
$$-\partial_m W^\beta \gamma^m_{\alpha\beta} = \frac{1}{4} D_\beta F_{mn} (\gamma^{mn})^\beta{}_\alpha, \qquad N^{mn} \lambda^\beta D_\beta F_{mn} = 0$$

The first 4 equations are precisely satisfied by the 10 dimensional $\mathcal{N} = 1$ SYM equations given in appendix C whereas the last equation is satisfied by the pure spinor constraint. The correctly normalized vertex operators of the massless states in our conventions thus become

$$\begin{aligned} V &= \lambda^\alpha A_\alpha \\ U &= \frac{2}{\alpha'} \partial\theta^\alpha A_\alpha + \frac{4}{\alpha'} \Pi^m A_m - \frac{4}{\alpha'} d_\alpha W^\alpha + \frac{4}{\alpha'} N^{mn} F_{mn} \\ &= \frac{4}{\alpha'} \left(\frac{1}{2} \partial\theta^\alpha A_\alpha + \Pi^m A_m - d_\alpha W^\alpha + N^{mn} F_{mn} \right) \end{aligned} \tag{C.8}$$

Theta expansion of massless superfields. Finally, we turn to the theta expansion. We shall follow the steps outlined in [13]. We shall need the following equations for doing the theta expansion

$$\begin{aligned} D_\alpha A_\beta + D_\beta A_\alpha &= 2A_m \gamma^m_{\alpha\beta} \quad , & D_\alpha A_m &= -W^\beta \gamma^m_{\alpha\beta} + \partial_m A_\alpha \\ D_\beta W^\alpha &= -\frac{1}{2} F_{mn} (\gamma^{mn})_\beta{}^\alpha \quad , & F_{mn} &= \partial_m A_n - \partial_n A_m \end{aligned} \tag{C.9}$$

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Before proceeding to do the theta expansion, we need to fix a gauge. We shall choose the gauge $\theta^\alpha A_\alpha = 0$. In this gauge choice, we have

$$0 = D_\beta(\theta^\alpha A_\alpha) = A_\beta - \theta^\alpha D_\beta A_\alpha \quad \Longrightarrow \quad A_\beta = \theta^\alpha D_\beta A_\alpha \tag{C.10}$$

Now, multiplying by θ^β in the 1st equation and by θ^α in the 2nd and 3rd equations of (C.9) and using the above identity along with the gauge choice $\theta^\alpha A_\alpha = 0$, we obtain

$$(1+D)A_\alpha = 2A_m(\gamma^m\theta)_\alpha \quad , \quad DA_m = -(\theta\gamma_m W) \quad , \quad DW^\alpha = \frac{1}{2}F_{mn}(\gamma^{mn}\theta)^\alpha \tag{C.11}$$

where we have defined $D \equiv \theta^\alpha D_\alpha = \theta^\alpha \partial_\alpha$.

We can use the above 3 equations along with the 4th equation of (C.9) to do the theta expansion. If we denote the ℓ^{th} order component of the superfield M by $M^{(\ell)}$, then we have

$$M^{(\ell)} = m_{\alpha_1\cdots\alpha_\ell}\theta^{\alpha_1}\cdots\theta^{\alpha_\ell} \quad \Longrightarrow \quad DM^{(\ell)} = \ell\, M^{(\ell)} \tag{C.12}$$

Using this, the 3 equations of (C.11) and the 4th equation of (C.9) give the following recursive relations

$$\begin{aligned}
(1+\ell)A^{(\ell)}_\alpha &= 2A^{(\ell-1)}_m(\gamma^m\theta)_\alpha && \Longrightarrow \quad A^{(\ell)}_\alpha = \frac{2}{1+\ell}A^{(\ell-1)}_m(\gamma^m\theta)_\alpha \\
\ell A^{(\ell)}_m &= -(\theta\gamma_m W^{(\ell-1)}) && \Longrightarrow \quad A^{(\ell)}_m = -\frac{1}{\ell}(\theta\gamma_m W^{(\ell-1)}) \\
\ell W^\alpha_{(\ell)} &= \frac{1}{2}F^{(\ell-1)}_{mn}(\gamma^{mn}\theta)^\alpha && \Longrightarrow \quad W^\alpha_{(\ell)} = \frac{1}{2\ell}(\gamma^{mn}\theta)^\alpha\ F^{(\ell-1)}_{mn} \\
F^{(\ell)}_{mn} &= \partial_m A^{(\ell)}_n - \partial_n A^{(\ell)}_m
\end{aligned} \tag{C.13}$$

Denoting the theta independent components of the superfields A_m and W^α to be

$$A^{(0)}_m \equiv a_m \quad , \qquad W^\alpha_{(0)} \equiv \chi^\alpha \tag{C.14}$$

the above recursive relations give

$$\begin{aligned}
A_\alpha &= a_m(\gamma^m\theta)_\alpha - \frac{2}{3}(\gamma^m\theta)_\alpha(\theta\gamma_m\chi) - \frac{1}{8}(\gamma_m\theta)_\alpha(\theta\gamma^{mpq}\theta)f_{pq} \\
&\quad -\frac{i}{15}(\gamma_m\theta)_\alpha(\theta\gamma_p\chi)(\theta\gamma^{mpq}\theta)k_q + \cdots \\
A_m &= a_m - (\theta\gamma_m\chi) - \frac{1}{4}(\theta\gamma_{mnp}\theta)f^{np} \ - \ \frac{1}{6}(\theta\gamma_m\gamma^{pq}\theta)(\theta\gamma_{[m}\partial_{n]}\chi) \\
&\quad +\frac{1}{48}(\theta\gamma_m\gamma^{rn}\theta)(\theta\gamma_{npq}\theta)\partial_r f^{pq} + \cdots
\end{aligned} \tag{C.15}$$

where $f_{mn} = \partial_m a_n - \partial_n a_m$ and the plane wave expansion of the gluon and gluino are given by

$$a_m = e_m e^{ik\cdot X} \quad , \qquad \chi^\alpha = \xi^\alpha e^{ik\cdot X} \tag{C.16}$$

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