A flavorful factoring of the strong CP problem

Prateek Agrawal^a and Kiel Howe^b

^a*Department of Physics, Harvard University,
17 Oxford Street, Cambridge, MA 02138, U.S.A.*

^b*Fermi National Accelerator Laboratory,
Wilson Street, Batavia, IL 60510, U.S.A.*

E-mail: prateek@fnal.gov, khowe@fnal.gov

ABSTRACT: Motivated by the intimate connection between the strong CP problem and the flavor structure of the Standard Model, we present a flavor model that revives and extends the classic $m_u = 0$ solution to the strong CP problem. QCD is embedded into a $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ gauge group, with each generation of quarks charged under the respective $SU(3)$. The non-zero value of the up-quark Yukawa coupling (along with the strange quark and bottom-quark Yukawas) is generated by contributions from small instantons at a new scale $M \gg \Lambda_{\text{QCD}}$. The Higgsing of $SU(3)^3 \rightarrow SU(3)_c$ allows dimension-5 operators that generate the Standard Model flavor structure and can be completed in a simple renormalizable theory. The smallness of the third generation mixing angles can naturally emerge in this picture, and is connected to the smallness of threshold corrections to $\bar{\theta}$. Remarkably, $\bar{\theta}$ is essentially fixed by the measured quark masses and mixings, and is estimated to be close to the current experimental bound and well within reach of the next generation of neutron and proton EDM experiments.

KEYWORDS: Anomalies in Field and String Theories, Beyond Standard Model, CP violation, Solitons Monopoles and Instantons

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1 Introduction

The standard model contains two physical CP violating parameters: (1) the perturbative CKM phase, which originates in the misalignment of the eigenvectors of the Yukawa matrices y_u and y_d [1],

$$\delta_{CKM} = \arg \det \left[y_u y_u^\dagger, y_d y_d^\dagger \right], \quad (1.1)$$

and (2) the strong CP phase

$$\bar{\theta} = -\arg \det \left[e^{-i\theta} y_u y_d \right], \quad (1.2)$$

which originates from the combination of the QCD θ angle and the determinant of the Yukawas. Although these two phases appear to be intimately related through their connection to the Yukawa matrices, δ_{CKM} is observed to be $\mathcal{O}(1)$, while current limits give $\bar{\theta} \lesssim 10^{-10}$ [2–4]. This is the strong CP problem: how can such a small value of $\bar{\theta}$ be explained when the quark sector appears to feel $\mathcal{O}(1)$ CP violation? In view of the strong connection of the flavor sector with the strong CP problem, it is natural to explore its solutions in the context of models which also generate the flavor structure in the standard model [5–9]. We present such a mechanism in this work.

One appealing class of solutions to this problem are those that contain a new anomalous $U(1)_{PQ}$ symmetry. The most economical possibility is the “massless up quark solution”, where setting $m_u = 0$ at a scale above the QCD scale leads to a $U(1)_{PQ}$ symmetry. This is

not a priori inconsistent with current algebra since non-perturbative effects can generate an effective up-quark mass [10–13] (see [14] for a review). In the simplest extensions of the standard model, non-perturbative QCD effects are only relevant at the scale $\Lambda_{\text{QCD}} \sim \text{GeV}$, and the mechanism can therefore remove any contributions to $\bar{\theta}$ generated above the scale Λ_{QCD} . Unfortunately, the massless up-quark solution is now strongly disfavored by lattice results, which find a non-zero $\overline{\text{MS}}$ value [15, 16],

$$m_u = 2.3^{+0.7}_{-0.5}|_{\mu=2 \text{ GeV}} . \quad (1.3)$$

The significance with which this rules out $m_u = 0$ solutions is more difficult to quantify. Refs. [14, 17] have recently pointed out some ambiguities and suggested further direct lattice tests that can support this conclusion.

In this work we consider an extension of the massless up quark solution into models where large non-perturbative effects are generated by embedding QCD as the diagonal subgroup of a $\text{SU}(3)^N$ gauge group. This mechanism for “factoring” the Strong CP problem was first presented in ref. [18], where all of the quarks are charged under a single $\text{SU}(3)$ factor, and the PQ symmetry is realized by a heavy axion in each sector. In this work, we instead give a flavorful embedding of the quarks in a $\text{SU}(3) \times \text{SU}(3) \times \text{SU}(3)$ gauge group, with each quark generation charged under a separate factor. Each factor contains an independent PQ symmetry implemented by a perturbatively massless quark instead of a heavy axion, and the observed non-vanishing Yukawa couplings are generated entirely by non-perturbative effects at a high scale M . These non-perturbative effects can be sizable because although the SM QCD coupling is weak at high scales $M \gg \Lambda_{\text{QCD}}$, each individual $\text{SU}(3)$ factor can easily be near strong coupling.¹ Higher dimension operators generate the quark mixing matrix upon the breaking to the diagonal group. Below the scale M the theory matches to the standard model with no additional matter. Since in the standard model $\bar{\theta}$ is very well sequestered from δ_{CKM} [19–22], solving the strong CP problem at the scale M solves it at low energy as long as no new sources of flavor or CP violation are introduced [23]. While $\bar{\theta}$ is suppressed in this model at tree-level, a non-vanishing radiative contribution is generated with a size directly connected to the observed quark masses and CKM angles. Remarkably, the model predicts $\bar{\theta} \sim 10^{-10}$, just below the sensitivity of current EDM experiments and within reach of proposed next generation neutron EDM [24, 25] and proton storage ring experiments [26].

Other models that can explain $\bar{\theta} = 0$ at tree level in the UV [27–29] typically require large discrete symmetries and extensions of the flavor structure, and may not preserve the radiative sequestering of $\bar{\theta}$ present in the SM. For example, in Nelson-Barr models [30–33], the radiative contributions $\Delta\bar{\theta}$ generally exclude the most appealing models unless some allowed couplings have unexplained suppressions or the symmetry structure of the SM is substantially extended [23, 34]. Theories which use parity (P) [35–37] to forbid a tree-level θ -angle often require new light colored particles [38]. There are also other mechanisms that introduce new non-perturbative PQ violating effects at higher energies $M \gg \Lambda_{\text{QCD}}$ to solve

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¹We will generalize this model to include additional $\text{SU}(3)$ factors with no charged matter, making each factor more strongly coupled, so that the non-perturbative effects can be made larger still.

the strong CP problem. Refs. [39–43] consider models where the $\bar{\theta}$ of the SM is related by a Z_2 symmetry to a mirror copy of the standard model with $\bar{\theta}' = \bar{\theta}$. Spontaneous Z_2 breaking [44] allows the states of the mirror sector to be decoupled, and non-perturbative mirror $SU(3)'$ effects to become strong at a scale $\Lambda'_{\text{QCD}} \gg \Lambda_{\text{QCD}}$ and simultaneously relax $\bar{\theta}'$ and θ either with a heavy-axion [39–43, 45] or a heavy perturbatively massless quark [41]. These theories are significantly constrained by the cosmology of the mirror sector and new colored TeV-scale particles. Another possibility is that the SM QCD itself becomes embedded in a strongly coupled gauge group at high energies — refs. [46–51] considered the possibility that extra matter causes QCD to run back to strong coupling at a scale M where it is embedded in a larger gauge group, e.g. $SU(3 + N)$. In general to obtain sizable effects these models also require the addition of new dynamics breaking the chiral symmetries, and contain new CP violating phases which cause a misalignment between the non-perturbative violations of the PQ symmetry at $\Lambda_{\text{QCD}'}$ and Λ_{QCD} , spoiling the solution to the strong CP problem [48].

2 Massless quark solution in QCD: the baby version

We start from a simpler version of the standard model with only a single generation of quarks — the $SU(2)$ doublet $q = (u, d)$ and two singlets u^c, d^c — charged as in the standard model. We include an $SU(2)$ doublet Higgs H and assume a UV cut-off Λ_{UV} . We make use of an anomalous $U(1)_{PQ}$ symmetry under which only u^c transforms,

$$u^c \rightarrow e^{i\alpha} u^c \quad (2.1)$$

which forbids an up Yukawa coupling at the perturbative level (more precisely, we assume that the dominant source of PQ breaking is from non-perturbative effects within the effective theory far below the scale Λ_{UV}). The relevant terms in the Lagrangian are

$$\mathcal{L}_{\Lambda_{UV}} \supset \frac{\alpha_s}{8\pi} \theta \tilde{G}_{\mu\nu}^a G^{a,\mu\nu} + y_d q H^\dagger d^c \quad (2.2)$$

The $U(1)_{PQ}$ symmetry and a chiral rotation of d^c can be used to remove the topological phase θ and the phase of the non-vanishing Yukawa coupling y_d . Therefore there is no physical CP violating parameter, $\bar{\theta} = 0$. This is effectively the massless up quark solution to the strong CP problem.

Non-perturbative $SU(3)$ effects violate the anomalous $U(1)_{PQ}$, so non-perturbative effects suppressed as $\sim e^{-2\pi/\alpha_s}$ will generate a non-vanishing effective y_u coupling at energies below Λ_{UV} . In the weak coupling limit, the dilute instanton gas approximation captures the leading non-perturbative effects, and the instantons can be integrated out to generate an effective Lagrangian for the fermions [11, 52, 53]. For $SU(3)$ with two flavors of quarks, both four-fermion and bilinear terms are generated from single-instanton effects,

$$\mathcal{L}_{\text{inst}} = \int_{\rho=\Lambda_{UV}^{-1}}^{\rho=\Lambda_{IR}^{-1}} \frac{d\rho}{\rho} e^{i\theta} D[\alpha_s(1/\rho)] \left(-c_0 y_d^* q H u^c + c_0 (2\pi^2) \rho^2 (u^\alpha d_\alpha^c d^\beta u_\beta^c - d^\alpha d_\alpha^c u^\alpha u_\alpha^c) \right) \Big|_{\mu=\rho^{-1}} \quad (2.3)$$

where α, β are QCD indices and the dimensionless instanton density is

$$D[\alpha] = D_0 \left(\frac{2\pi}{\alpha} \right)^6 e^{-\frac{2\pi}{\alpha}} \quad (2.4)$$

which features the non-perturbative exponential suppression factor at weak coupling. The analytic constants are $D_0 \approx 0.02$ and $c_0 \approx 1.79$ [11]. The couplings in the integrand are evaluated at the scale ρ^{-1} (higher order corrections can be found in ref. [14]). Higher dimension operators are suppressed by further powers of $D[\alpha]$, and $D[\alpha] \sim 1$ signals the breakdown of the dilute instanton gas approximation.

The effect of instantons on the Yukawa couplings can be conveniently described as a non-perturbative contribution to the running of the Yukawa couplings [11],

$$\frac{d}{d \ln \mu} \begin{pmatrix} y_u & 0 \\ 0 & y_d \end{pmatrix} = -c_0 D[\alpha(\mu)] e^{i\theta} \begin{pmatrix} y_d^* & 0 \\ 0 & y_u^* \end{pmatrix} \quad (2.5)$$

Recall that the perturbative contributions to the running of Yukawas are multiplicative, and are negligible here. Now that non-perturbative effects are included, $y_u \neq 0$ is generated and the PQ-symmetry appears to be violated perturbatively in the low energy effective Lagrangian. However, the physical CP angle $\bar{\theta}$ remains vanishing: the non-perturbatively generated y_u has just the right phase to allow the θ angle and the phase in y_d to be simultaneously rotated away, as is clear from eq. (2.3).

Two-flavor QCD is asymptotically free and the instanton density grows in the IR. If SU(3) is Higgsed at the scale M , the instanton contribution to y_u is cut-off and dominated by instantons of size comparable to the Higgsing, $\rho^{-1} \sim M$ (we will discuss the nature of the Higgsing sector in the following section). Using the one-loop running of the gauge coupling $d\alpha^{-1} = \frac{b}{4\pi} d \ln \mu$, with $b = 29/3$ for 2-flavor QCD, the linear solution to the running eq. (2.5) gives

$$\frac{|y_u|}{|y_d|} = -\frac{2c_0 D_0}{b} \int_{2\pi/\alpha(\Lambda_{UV})}^{2\pi/\alpha(M)} \left(\frac{2\pi}{\alpha} \right)^6 e^{-\frac{2\pi}{\alpha}} d(2\pi/\alpha) \approx \frac{2c_0 D_0}{b} \Gamma(7, 2\pi\alpha(M)) \quad (2.6)$$

where we have assumed $\alpha^{-1}(\Lambda_{UV}) \ll 1$ for the last equality, and $\Gamma(n, x)$ is the upper incomplete Γ -function. Figure 1 shows the ratio $|y_u|/|y_d|$ after integrating out effects above M as a function of the QCD coupling at the scale of Higgsing, $\alpha(M)$. As $\frac{|y_u|}{|y_d|}$ approaches ~ 1 , multiple-instanton effects captured by higher order solutions to eq. (2.5) become important, and the ratio asymptotes to $|y_u|/|y_d| = 1$. For $\alpha(M) \sim 0.4 - 0.8$ an $\mathcal{O}(1)$ ratio can be generated as required by the observed light quark masses. In this regime the dilute instanton gas approximation is only a qualitative picture of the non-perturbative QCD effects, but strongly suggests that they are $\mathcal{O}(1)$ and that a viable ratio $|y_u|/|y_d|$ can be realized before the theory enters the chiral-symmetry breaking phase which would be expected to occur at $\alpha(M) \gtrsim 0.7 - 1$ [54, 55]. As the theory flows to weak coupling at scales above M , the PQ violating effects are rapidly suppressed. For example for $\alpha \approx 0.1$, as in the SM near the weak scale, the non-perturbative contribution to y_u is $|y_u|/|y_d| \lesssim 10^{-16}$.

This simple 2-flavor example shows that instanton effects can generate large non-perturbative contributions to a perturbatively vanishing Yukawa coupling. In fact such

Figure 1. The ratio of Yukawa couplings generated by instanton effects as a function of $\alpha = g^2/4\pi$ evaluated at the scale M in the 2-flavor model assuming a vanishing perturbative value for one of the couplings. The solid curve cuts uses the full solution to eq. (2.5), cutting off the running at $\mu = M$. The dashed curves cut off the instanton effects at $\mu = M/2, 2M$, and are shown as a rough estimate of the theoretical uncertainty. The dotted line is the single-instanton approximation eq. (2.6). The horizontal dashed lines are, from top to bottom, the experimental values of y_u/y_d , y_s/y_c , and y_b/y_t .

effects are known to be important near the scale of QCD confinement, Λ_{QCD} , in the standard model, as reviewed in [14]. However, as mentioned above, lattice results strongly disfavor a massless up quark solution to the strong CP problem in the SM.

The suppression of this effect in the SM is partly due to the fact that the strange quark is also relevant at Λ_{QCD} , and instanton contributions to m_u are further suppressed by m_s . In fact, 2+1 flavor lattice QCD results fully include all instanton configurations and can be interpreted as a calculation of the 2nd order term in the Chiral Lagrangian giving an effective up-quark mass proportional to $m_d^* m_s^* / \Lambda_{\text{QCD}}$ — these results suggest that the size of the desired non-perturbative effect is only $\sim 10\text{--}40\%$ of the experimentally required value [14].

So, although qualitatively non-perturbative effects in the SM near the scale Λ_{QCD} are nearly the right size to allow $m_u = 0$ solution to the strong CP problem, quantitatively the possibility is strongly disfavored by precision lattice results. In the following section we will describe an extension to the SM in which non-perturbative effects can become important again at a high energy scale $M \gg \Lambda_{\text{QCD}}$, and these additional contributions allow a solution to the strong CP problem reminiscent of the massless up quark solution.

3 Massless quark solution in QCD: the real thing

Going beyond the illustrative two-flavor example, there are two challenges to generating a large non-perturbative contribution to the Yukawa couplings at a new scale $M \gg \Lambda_{\text{QCD}}$. The first is that QCD must be embedded in a strongly coupled theory at the scale M so that non-perturbative effects are important, but must match to the weak coupling of QCD in the standard model at high energies, e.g. $\alpha_s(1000 \text{ TeV}) \approx 0.05$. The second challenge is that at high energies in QCD, all three generations of quarks are relevant, leading to

Figure 2. An instanton contribution to the up quark mass in the SM at high energies. All six quark flavors appear, and the non-vanishing contributions are proportional to the products of all the Yukawa couplings. For $M \gg v$, diagrams with the Higgs looped off are more important than Higgs vev insertions.

Figure 3. The model is a 3-site $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ theory, with one generation charged under each $SU(3)$ factor. The link field Σ vevs break the gauge group down to the diagonal $SU(3)$ of the standard model. One quark in each generation obtains its mass from non-perturbative effects, making the $\bar{\theta}$ angle in each individual gauge factor unphysical.

further Yukawa suppressions of high energy contributions from instantons at small sizes $\rho^{-1} \gg v$. For example, as illustrated in figure 2, the high energy contributions to y_u in the 3-generation SM are further suppressed as

$$|y_u|/|y_d| \sim \frac{|y_c y_s y_t y_b|}{(16\pi^2)^2} \quad (3.1)$$

because the explicit breaking of each non-anomalous $U(1)_{PQ}$ by the Yukawa couplings must be felt to generate $y_u \neq 0$.

Both these challenges can be solved by embedding the standard model $SU(3)_c$ into a $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ product gauge group above the scale M , as depicted in figure 3. Each generation of quarks is charged under a separate $SU(3)$ factor. The theory will be Higgsed at the scale M to the diagonal gauge group by bifundamental scalar fields, as discussed in more detail in the following section. The unbroken diagonal $SU(3)_c$ group's coupling is

$$\frac{1}{\alpha_s} = \frac{1}{\alpha_{s_1}} + \frac{1}{\alpha_{s_2}} + \frac{1}{\alpha_{s_3}}, \quad (3.2)$$

allowing to match to the weakly coupled SM QCD even when each individual factor is more strongly coupled.

Since there are now three separate $SU(3)$ factors, there are now three separate θ problems! Fortunately, all the θ angles can be made unphysical if there is an independent

Figure 4. Schematically, high energy instantons act separately at scales above M to generate one of the Yukawa couplings in each individual $SU(3)$ factor.

anomalous $U(1)_{PQ}$ symmetry in each sector. The minimal realization of this PQ symmetry involves a perturbatively massless quark in *each* sector. Since the non-perturbatively generated Yukawa coupling is always smaller than the unprotected Yukawa, a natural choice is to choose PQ symmetries that enforce $y_u = 0, y_s = 0, y_b = 0$. Above the scale M of Higgsing, each site behaves as the two-flavor model of section 2. Schematically, the generation of the Yukawa couplings is depicted in figure 4. From figure 1, we can read off the size of the gauge couplings at the scale M that are necessary for the instantons in each factor to generate the observed Yukawa ratios:

$$\begin{aligned} y_u/y_d &\approx 2/5 \rightarrow \alpha_{s1}(M) \approx 0.45 - 0.85 \\ y_s/y_c &\approx 1/12 \rightarrow \alpha_{s2}(M) \approx 0.36 - 0.6 \\ y_b/y_t &\approx 1/35 \rightarrow \alpha_{s3}(M) \approx 0.33 - 0.55 \end{aligned} \quad (3.3)$$

Eq. (3.2) then gives the coupling of the unbroken diagonal group at the matching scale $\alpha_s(M) = 0.12 - 0.22$. Flavor constraints will require us to match to the SM at a scale $M \gtrsim 1000 \text{ TeV}$ where $\alpha_s(1000 \text{ TeV}) = 0.05$, so it appears unlikely that this minimum $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ model is viable unless our dilute instanton calculation significantly underestimates the size of non-perturbative effects.

One way to overcome this obstacle is to enlarge the product gauge group to $SU(3)_1 \times SU(3)_2 \times SU(3)_3 \times SU(3)_X^N$, where the extra gauge factors do not contain chiral matter and therefore can remain more weakly coupled. Removing the θ angle in these extra factors will involve introducing a PQ symmetry at each new site, as shown for example in figure 5. For example, the θ parameter in the extra sites can be removed with very heavy axion degrees of freedom as discussed in ref. [18]. Another simple viable possibility is to add $N = 3$ or 4 sites, each with a colored vectorlike particle Ψ_X, Ψ_X^c and $M_\Psi = 0$ perturbatively to realize a PQ symmetry. Instantons in each factor generate a mass $M_\Psi \sim D(\alpha(M))M$. For $N = 4$ and a scale $M = 1000 \text{ TeV}$, each extra site needs a coupling $\alpha_{sX}(M) \approx 0.26 - 0.35$, giving masses $M_\Psi \sim 1 \text{ TeV} - 100 \text{ TeV}$, while for $N > 4$ we find $M_\Psi \sim M$. The possible presence of these light vector-like colored fermions with masses $M_\Psi \ll M$ generated by non-perturbative effects could be an interesting signature of this theory to study in further work, but for the remainder of this work we assume these states decouple and focus on the details of the $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ theory.

Another alternative possibility to avoid enlarging the gauge group with extra $SU(3)$ factors is to consider a model with PQ symmetries ensuring $y_d = 0$ instead of $y_s = 0$, so

Figure 5. The 3-site $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ theory of figure 3 extended to contain an extra site with a more weakly coupled $SU(3)_4$ factor. There is no chiral matter at this site, and the θ_4 angle is removed by an anomalous $U(1)_{PQ}$ symmetry of a single vector-like quark species Ψ, Ψ^c . While $M_\Psi = 0$ perturbatively, non-perturbative effects violating the PQ symmetry generate $M_\Psi \neq 0$.

	$SU(3)_i$	$SU(3)_j$	$(B - L)_i$
q_i	$\square$	1	$\frac{1}{3}$
u_i^c	$\bar{\square}$	1	$-\frac{1}{3}$
d_i^c	$\bar{\square}$	1	$-\frac{1}{3}$
Σ_{ij}	$\bar{\square}$	$\square$	$\mp \frac{1}{3}$

Table 1. Particle content and charges under the family $SU(3)$ factors and family $B - L$ symmetries for a simple 3-generation model ($i, j = 1, 2, 3$).

that smaller non-perturbative effects are required to generate the quark mass ratios. This possibility is appealing but is in tension with constraints on $\bar{\theta}$, as described in appendix B.

3.1 The scalar sector

The full description of the relevant particle content of the $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ model is given in table 1. There are several possibilities for the scalar fields breaking the gauge group to the diagonal, here we take a simple choice motivated by CKM mixings as described in the following section.

We assume that the scalar link fields Σ_{12} , Σ_{23} , and Σ_{31} get vevs $f_{12} \sim f_{23} \sim f_{31}$ to break the gauge group down to the diagonal, with M corresponding to the scale of Higgsing $M \sim gf$ (only two link fields are necessary to break the gauge group, but the simplest renormalizable flavor models will involve three link fields). The renormalizable potential allowed by the symmetries leads to spontaneous breaking of the gauge group without introducing any new CP phases or uneaten light Goldstone boson degrees of freedom. A standard renormalizable Higgs-like potential drives a vev for each field,

$$V = V_0(\Sigma_{12}) + V_0(\Sigma_{23}) + V_0(\Sigma_{31}) + (\gamma \text{Tr}(\Sigma_{12}\Sigma_{23}\Sigma_{31}) + h.c.), \quad (3.4)$$

$$V_0(\Sigma_{ij}) = -m_{\Sigma_{ij}}^2 \text{Tr}(\Sigma_{ij}\Sigma_{ij}^\dagger) + \frac{\lambda_{ij}}{2} [\text{Tr}(\Sigma_{ij}\Sigma_{ij}^\dagger)]^2 + \frac{\kappa_{ij}}{2} \text{Tr}(\Sigma_{ij}\Sigma_{ij}^\dagger \Sigma_{ij}\Sigma_{ij}^\dagger) \quad (3.5)$$

(no sums on i, j)

The couplings λ_{ij} , δ_{ij} , and λ_{ij} are independent real parameters for each field Σ . The phase of γ can be removed by a field redefinition, and causes the vacuum to align with vevs

$\langle \Sigma_{12} \rangle$, $\langle \Sigma_{23} \rangle$, $\langle \Sigma_{31} \rangle$ that can all consistently be chosen to be real. Taking γ to be a small perturbation for simplicity, we find [56–59]

$$\langle \Sigma_{ij} \rangle = \frac{m_{\Sigma_{ij}}}{\sqrt{\kappa_{ij} + 3\lambda_{ij}}} \mathbb{I}_3 \equiv \frac{f_{\Sigma_{ij}}}{2} \mathbb{I}_3. \quad (3.6)$$

For simplicity assume all of the scales are comparable, $f_{12} \sim f_{23} \sim f_{31} \sim f$, giving a common scale $M \sim gf$ cutting off the instanton integrals in each $SU(3)$ factor.

Renormalizable cross-couplings of the form $\lambda' |\Sigma_{12}|^2 |\Sigma_{23}|^2$, $\lambda'' |\Sigma_{12} \Sigma_{23}|^2$, etc. are also allowed by the symmetries, but do not introduce any new CP phases and are qualitatively unimportant as long they do not destabilize the vevs. The symmetry breaking pattern at the level of the scalar potential so far is $SU(3)^3 \times U(1)_{B-L}^3 \times U(1)_\Sigma \rightarrow SU(3) \times U(1)_{B-L}$. The 16 colored Goldstone bosons are eaten by the Higgs mechanism for the broken $SU(3)$ factors. The 8 remaining colored pseudo-Goldstone bosons radiatively obtain masses at the scale $\sim g^2 f^2$. There remain three singlet Goldstones to lift. The renormalizable term, $\gamma \text{Tr}(\Sigma_{12} \Sigma_{23} \Sigma_{31}) + h.c.$ explicitly breaks the $U(1)_\Sigma$ symmetry, lifting one Goldstone. Gauging two of the $U(1)_{B-L}$ factors can lift the remaining Goldstones without introducing any new phases. Alternatively, explicitly breaking the $U(1)_{B-L}$ factors with terms of the form $\kappa' \Sigma_{12i}^a \Sigma_{12j}^b \Sigma_{12k}^c \epsilon_{abc} \epsilon^{ijk}$ would introduce new CP phases to the theory, but in a controlled way for $\kappa' \ll f$.

3.2 CKM and no $\bar{\theta}$ at tree level

The model we have introduced so far generates the diagonal Yukawa couplings and breaks the product gauge group down to the standard model $SU(3)_c$, all while maintaining an accidental CP symmetry at the renormalizable level. After integrating out the non-perturbative effects near the scale M , the theory matches to the standard model with non-vanishing diagonal Yukawa couplings for all of the quarks and phases that preserve $\bar{\theta} = 0$.

$$D^u = \begin{pmatrix} r_1 e^{i\theta} Y_{11}^{d*} & 0 \\ 0 & Y_{22}^u & 0 \\ 0 & 0 & Y_{33}^u \end{pmatrix} \quad (3.7)$$

$$D^d = \begin{pmatrix} Y_{11}^d & 0 & 0 \\ 0 & r_2 e^{i\theta_2} Y_{22}^{u*} & 0 \\ 0 & 0 & r_3 e^{i\theta_3} Y_{33}^{u*} \end{pmatrix} \quad (3.8)$$

The PQ symmetries and large non-perturbative effects are crucial to the accidental CP symmetry, since they allow the breaking of the quark chiral symmetries without introducing extra CP violating parameters.

The next challenge is to introduce the CKM mixing without spoiling this protection. When we introduce additional off-diagonal Yukawa couplings, the accidental CP symmetry can no longer survive, since the observed CKM phase must be generated. However, $\bar{\theta}_{SM}$ will still vanish at tree level and remain highly suppressed even at loop level due to the residual approximate flavor symmetries.

	PQ_1	PQ_2	PQ_3
q_1	0	0	-1
u_1^c	1	0	0
d_1^c	0	0	1
q_2	0	0	-1
u_2^c	0	0	1
d_2^c	0	1	0
q_3	0	0	0
u_3^c	0	0	0
d_3^c	0	0	1

Table 2. Particle charges under family PQ symmetries, for a simple 3-generation model with CKM mixings ($i, j = 1, 2, 3$).

Introducing quark-mixing between generations requires higher dimensional operators involving the link fields, e.g.

$$\mathcal{L}_{d=5} = \lambda_{ij}^u \left(q_i \frac{\Sigma_{ij}}{\Lambda_f} u_j^c \right) H \quad (3.9)$$

generates the effective Yukawa matrices when the Σ fields acquire vacuum expectation values. We can write the off-diagonal Yukawa couplings below the scale of Higgsing,

$$O^{u,d} = \frac{\lambda_{ij}^{u,d} f_{ij}}{\Lambda_f}. \quad (3.10)$$

Since the off-diagonal entries in the Yukawa matrices can be small, the flavor scale $\Lambda_f \gg M$ is possible, with a separation as large as $\Lambda_f \lesssim 10^4 M$ consistent with unitarity and the size of the observed off-diagonal Yukawa elements. However, a natural assumption that the couplings λ of the UV completion are comparable to the non-vanishing diagonal Yukawa couplings would require e.g. $\Lambda_f \sim f$ to generate the $\mathcal{O}(1)$ Cabibbo angle.

For general off-diagonal couplings, it is no longer true that the tree-level $\bar{\theta}$ vanishes after matching to the SM,

$$\bar{\theta} = \arg \det \left[e^{-i(\theta_1 + \theta_2 + \theta_3)} (D^u + O^u)(D^d + O^d) \right] \quad (3.11)$$

$$= \arg \left(\det \left[e^{-i(\theta_1 + \theta_2 + \theta_3)} D^u D^d \right] \det \left[(1 + (D^u)^{-1} O^u) \right] \det \left[(1 + (D^d)^{-1} O^d) \right] \right). \quad (3.12)$$

The first determinant factor is real, as shown above. For the other two factors, it is simple to see that we must require that the off-diagonal matrices $O^{u,d}$ can be put in a strictly triangular form (up to $SU(3)$ rotations). We would like the quarks to transform under (possibly anomalous) $U(1)$ symmetries that perturbatively protect this form, and in fact there are only two possible textures satisfying these constraints and giving viable CKM

mixings. The texture we will focus on is:

$$Y^u = \begin{pmatrix} 0 & Y_{12}^u & 0 \\ 0 & Y_{22}^u & 0 \\ 0 & 0 & Y_{33}^u \end{pmatrix} \quad (3.13)$$

$$Y^d = \begin{pmatrix} Y_{11}^d & 0 & Y_{13}^d \\ Y_{21}^d & 0 & Y_{23}^d \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.14)$$

and the assignment of PQ charges in table 2 protects this form of the Yukawa matrix. The other possible texture, described briefly in appendix B, gives a less natural realization of the CKM structure.

The three anomalous $U(1)_{PQ}$ symmetries allow us to rotate away the θ angle in each $SU(3)$ factor, and field redefinitions leave only two remaining physical CP phases in the Yukawa matrix, which we choose by convention to put in the Y_{23}^d and Y_{21}^d elements. Including non-perturbative instanton effects and for the moment ignoring all other radiative effects, below the scale M the theory matches to the SM with Yukawa matrices

$$y^u = \begin{pmatrix} r_1 e^{i\theta_1} Y_{11}^{d*} & Y_{12}^u & 0 \\ 0 & Y_{22}^u & 0 \\ 0 & 0 & Y_{33}^u \end{pmatrix} \quad (3.15)$$

$$y^d = \begin{pmatrix} Y_{11}^d & 0 & Y_{13}^d \\ Y_{21}^d & r_2 e^{i\theta_2} Y_{22}^{u*} & Y_{23}^d \\ 0 & 0 & r_3 e^{i\theta_3} Y_{33}^{u*} \end{pmatrix} \quad (3.16)$$

We can check explicitly that $\bar{\theta}_{SM} = 0$ at tree level,

$$\bar{\theta} = -\arg \det \left[e^{-i(\theta_1 + \theta_2 + \theta_3)} \begin{pmatrix} r_1 e^{i\theta_1} Y_{11}^{d*} & Y_{12}^u & 0 \\ 0 & Y_{22}^u & 0 \\ 0 & 0 & Y_{33}^u \end{pmatrix} \begin{pmatrix} Y_{11}^d & 0 & Y_{13}^d \\ Y_{21}^d & r_2 e^{i\theta_2} Y_{22}^{u*} & Y_{23}^d \\ 0 & 0 & r_3 e^{i\theta_3} Y_{33}^{u*} \end{pmatrix} \right] = 0. \quad (3.17)$$

The real coefficients $r_{1,2,3}$ parameterize the size of the instanton suppression of PQ breaking in each $SU(3)$ factor. The couplings Y can now be determined from the CKM matrix and the observed SM fermion masses. The only undetermined parameter is Y_{21}^d/Y_{11}^d , but we will be motivated shortly to focus on the limit $Y_{21}^d \ll Y_{11}^d$. Then to leading order in the small Yukawa ratios $y_{u,d}/y_{c,s,t,b}$, y_s/y_b , and small CKM mixings $|V_{31}| = 0.0089$, $|V_{32}| = 0.041$ [16] we obtain

$$\begin{aligned} Y_{33}^u &= y_t, & Y_{22}^u &= y_c \cos \theta_c, & Y_{12}^u &= y_c \sin \theta_c \\ Y_{11}^d &= y_d, & Y_{13}^d &= y_b |V_{31}| e^{i\delta_0}, & Y_{23}^d &= y_b |V_{32}| \\ r_1 &= \frac{y_u}{\cos \theta_c y_d}, & r_2 &= \frac{y_s}{\cos \theta_c y_c}, & r_3 &= \frac{y_b}{y_t}, \end{aligned} \quad (3.18)$$

where we have made a field definition choice to put the CKM phase entirely into Y_{13}^d and θ_c is the Cabibbo angle. An alternative solution with the same texture but flipping the role of the strange and down quarks is discussed in appendix B.

Now that the CKM elements are introduced, the gauge basis in the $SU(3) \times SU(3) \times SU(3)$ theory is no longer aligned with the flavor basis, and four-fermion operators generated by gauge interactions at the scale M will introduce non-MFV contributions to CP-preserving flavor observables. The dominant constraint is due to the $\Delta C = 2$ operator generated by exchange of the heavy broken $SU(3)$ gauge bosons, given in the quark mass basis as

$$\mathcal{O}_{\Delta C=2} \sim \frac{4\pi\alpha_{s1,2} \sin^2 \theta_C}{M^2} (\bar{c}\gamma^\mu u)^2. \quad (3.19)$$

Constraints on the D^0 splitting generated by this operator give $M \gtrsim 1000 \text{ TeV}$ [60]. The leading $\Delta B = 2$ and $\Delta S = 2$ operators are suppressed respectively by $|V_{13,23}|^2$ and $|V_{12,23}y_d/y_s|^2$ and give less stringent constraints.

3.3 $\Delta\bar{\Theta}$ from thresholds

With the two physical CP violating phases in the Y_{23}^d and Y_{12}^d elements, it is clear that at leading order in the Yukawa couplings, neither contributes to the low energy theta angle. However higher order perturbative corrections to the non-perturbative effects at M can give a non-vanishing threshold correction to $\bar{\theta}_{SM}$.

At energies below M , the additional breaking of the SM flavor symmetry generated by the gauging of $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ decouples and the theory is just the standard model, where the flavor symmetries suppress the running of $\bar{\theta}$ to negligible effects starting at 7-loops [19, 22]. At energies far above M , the non-perturbative PQ violating effects are exponentially suppressed by the weak coupling of the gauge groups, and the PQ symmetry protects the form of the Yukawa matrices with $\bar{\theta} = 0$ manifest, eq. (3.14). Therefore the dominant effect on $\bar{\theta}$ is a threshold effect at energies near M , where the non-perturbative violation of the PQ symmetries are still large and the extra breaking of the SM flavor symmetries through the gauging of $SU(3)_1 \times SU(3)_2 \times SU(3)_3$ has not decoupled.

The leading effects occur at third order in the Yukawa couplings, schematically generated from diagrams of the form of figure 6. Roughly, these diagrams describe how the Yukawa elements closing the instanton diagrams depend on the scale of the instanton — there is a mismatch of the phase between instantons at different scales because of the perturbative running of the Yukawas. Taking the Σ fields as background fields, the 1-loop running of the effective Yukawa couplings eq. (3.10) takes the same form as in the SM [61], with the non-vanishing CP phases entering through the terms 3rd order in the Yukawa couplings:

$$\beta_P(Y_{u,d}) = \frac{1}{16\pi^2} \frac{3}{2} (Y_{u,d} Y_{u,d}^\dagger - Y_{d,u} Y_{d,u}^\dagger) Y_{u,d} \quad (3.20)$$

Since the phases entering in the instantons no longer align exactly with the low energy perturbative values of the Yukawa couplings, there is no longer an exact cancellation in phase between the non-perturbatively generated eigenvalues and the perturbative eigenvalues of $Y_{u,d}$.

Figure 6. One of the leading diagrams generating a non-vanishing threshold correction to $\Delta\bar{\theta}$. The off-diagonal Yukawa couplings appear in order to introduce a CP phase, and the instanton violates the anomalous PQ symmetry protecting the UV form of the Yukawa couplings as in eq. (3.14).

To obtain a parametric estimate of these effects, we iteratively solve the RGE including the perturbative running eq. (3.20) and non-perturbative running eq. (2.5) of the Yukawas, as described in detail in appendix A. We ignore the effects of perturbative gauge interactions and the propagation of the Σ fields — all effects that modify $\bar{\theta}$ must involve both an instanton and a Yukawa loop, so these higher order effects can give at most $\mathcal{O}(1)$ corrections to our estimate if these states are strongly coupled. Finite effects not captured by the RGE are also expected to be of comparable size. There are two leading contributions to $\bar{\theta}$. The size of the first is fixed by the experimentally determined elements of the Yukawa matrix,

$$\Delta\bar{\theta} = \frac{3}{16\pi^2} \frac{1}{b} \tan\theta_C V_{31} V_{32} y_b^2 \sin(\delta_0) [f_I(\alpha_{s_1}) + f_I(\alpha_{s_2})] \quad (3.21)$$

$$\approx 10^{-10} \times \left(\frac{f_I(\alpha_{s_1}) + f_I(\alpha_{s_2})}{2} \right) \quad (3.22)$$

where

$$f_I(\alpha) = \frac{\Gamma(8, 2\pi\alpha^{-1})}{\Gamma(7, 2\pi\alpha^{-1})} - 2\pi\alpha^{-1} \approx 0.8 + 1.6\alpha \quad (3.23)$$

with this linear approximation holding well in the coupling region of interest $\alpha \sim 0.2 - 1$. The small size of $\Delta\bar{\theta} \sim 10^{-10}$ is due to the loop suppression and the smallness of the off-diagonal Yukawa elements. The form of $\Delta\bar{\theta}$ is consistent with the observation that Y_{13} and Y_{23} must appear as a product, since the physical phase can be rotated from one term to the other. The suppression by a factor of $1/b = 3/29$ arises because there is only a small range of energies where instanton effects are important, controlled by how rapidly the gauge coupling runs.

There is another contribution proportional to the undetermined Yukawa element Y_{21} ,

$$\begin{aligned}\Delta\bar{\theta}' &= -\frac{3}{16\pi^2} \frac{1}{b} \left(\cos\theta_C \sin\theta_C y_c^2 \text{Im}[Y_{21}^d] [f_I(\alpha_{s_1}) + f_I(\alpha_{s_2})] \right) \\ &\approx -(4 \times 10^{-8}) \times \frac{\text{Im}[Y_{21}^d]}{y_d} \left(\frac{f_I(\alpha_{s_1}) + f_I(\alpha_{s_2})}{2} \right)\end{aligned}\quad (3.24)$$

If Y_{21}^d takes on a value $\sim y_d$ with $\mathcal{O}(1)$ phase, this extra contribution is inconsistent with experimental limits. However, spurion arguments show that $|Y_{21}^d| \ll y_d$ can be naturally obtained. Since Y_{21}^d breaks a different set of flavor symmetries, its natural size can be as small as

$$|Y_{21}^d| \gtrsim Y_{11}^d \times \text{Max}(Y_{13}^{d*} Y_{23}^d, Y_{12}^{u*} Y_{22}^u) \approx y_d \times \text{Max}(y_b^2 V_{31} V_{32}, y_c^2 \sin\theta_C \cos\theta_C) \approx y_d \times 10^{-5} \quad (3.25)$$

making $\Delta\bar{\theta}'$ subdominant.

We have checked these estimates numerically at the one-loop level.

3.4 UV sensitivity

It is useful to discuss the degree to which this mechanism is insensitive to ultraviolet physics at some scale Λ_{UV} where CP may be violated in a sector strongly coupled to the standard model. For CP violation to be communicated from this sector to $\bar{\theta}$, the anomalous breaking of the PQ symmetry must be active. There are two possible contributions: small instantons of scale Λ_{UV}^{-1} interacting directly with the new UV physics, and the unsuppressed instantons at the scale M^{-1} interacting with the physics at Λ_{UV} through higher dimensional operators.

The contributions of small instantons of size Λ_{UV}^{-1} is suppressed by the exponentially small instanton density $D(\Lambda_{UV}^{-1})$ as long as the individual SU(3) factors have run back to weak coupling. For example, suppose the physics at Λ_{UV} introduces an $\mathcal{O}(1)$ phase α in the non-vanishing Yukawas, e.g. $y_d(\Lambda_{UV}) \approx e^{i\alpha} y_d(M)$. Then instantons at the scale generate a contribution to y_u with a phase that will appear in $\bar{\theta}$,

$$\frac{\Delta y_{uUV}}{y_u} \sim e^{i\alpha} \frac{D(\Lambda_{UV}^{-1})}{D(M^{-1})} \quad (3.26)$$

In a sector with two-flavors, this contribution is consistent with $\Delta\bar{\theta} \lesssim 10^{-10}$ if $\Lambda_{UV} \gtrsim 100M$.

The physics at the scale Λ_{UV} can also generate higher dimensional operators consistent with the PQ symmetries and other approximate chiral symmetries that carry CP phases and can interact with the unsuppressed instantons at the scale M (such operators also interact with instantons at the scale Λ_{QCD} and generate a shift in $\bar{\theta}$ even in the standard PQ axion or massless up quark solution [62], but here these effects are subdominant by a factor Λ_{QCD}^2/M^2). The most dangerous operators are momentum dependent contributions to the phase of the perturbatively allowed diagonal Yukawas,

$$\mathcal{L}_{d=6} \supset Y_{11}^{d'} H^\dagger Q_1 \frac{D_\mu^2}{\Lambda_{UV}^2} d_1^c + Y_{22}^{u'} H Q_2 \frac{D_\mu^2}{\Lambda_{UV}^2} u_2^c + Y_{33}^{u'} H Q_3 \frac{D_\mu^2}{\Lambda_{UV}^2} u_3^c \quad (3.27)$$

	$SU(3)_{1(B-L)_1, PQ_1}$	$SU(3)_{2(B-L)_2, PQ_2}$	$SU(3)_{3(B-L)_3, PQ_3}$
U_1^c	$\bar{\square}_{(-\frac{1}{3}, 0)}$	-	$-(0, 1)$
$\bar{U}_1^c$	$\bar{\square}_{(-\frac{1}{3}, 0)}$	-	$-(0, -1)$
Q_3	-	-	$\square_{(\frac{1}{3}, -1)}$
$\bar{Q}_3$	-	-	$\square_{(\frac{1}{3}, 1)}$

Table 3. Vector-like quark content allowing a simple UV completion of the off-diagonal higher dimension CKM mixing operators.

Combined with instanton insertions at M , these give

$$\Delta\bar{\theta} \sim \text{Im} \left[\frac{Y'}{Y} \right] \frac{M^2}{\Lambda_{UV}^2}. \quad (3.28)$$

When $Y' \sim Y$ and the phases are uncorrelated, this requires $\Lambda_{UV} \gtrsim 10^5 M$ to avoid generating $\Delta\bar{\theta}$.

Another dangerous $d = 6$ operator that can generate contributions to $\bar{\theta}$ even in the absence of PQ breaking are mixed topological terms, for example

$$\mathcal{L}_{d=6} \supset \hat{\theta}_{12} G_a^{(1)b} \tilde{G}_j^{(2)i} \frac{\Sigma_{12b}^j \Sigma_{12i}^{\dagger a}}{\Lambda_{UV}^2} \quad (3.29)$$

gives a contribution $\Delta\bar{\theta} \approx \hat{\theta}_{12} \frac{M^2}{\Lambda_{UV}^2}$, again requiring $\Lambda_{UV} \gtrsim 10^5 M$ unless $\hat{\theta}_{12}$ is suppressed.

4 A flavor UV completion

The $d = 5$ operators in eq. (3.9) generating the off-diagonal Yukawa elements require a UV completion at the scale Λ_f . Unitarity of the $d = 5$ operator in eq. (3.9) generating the off-diagonal Yukawas requires $\Lambda_f \lesssim 10^{-4} M$. Taking the effective action to $d = 6$ introduces operators consistent with the PQ symmetries that could allow the CP violation generating δ_{CKM} to enter directly into $\Delta\bar{\theta}$, as discussed in section 3.4.

In this section we give an example of a simple UV completion in which the higher dimension operators do not make large contributions to $\Delta\bar{\theta}$ and which can also explain the origin of the spurion argument giving $|Y_{21}^d| \ll y_d$. The model is extended to involve a set of vector-like fermions $Q_3, \bar{Q}_3, U_1^c, \bar{U}_1^c$, with charges under the gauge and PQ symmetries as given in table 3. Renormalizable mixings between heavy states and the SM-like fields generates the higher dimensional operators eq. (3.9) after integrating out the vector-like states.

The renormalizable terms in the Lagrangian consistent with the gauge and PQ symmetries are

$$\begin{aligned} \mathcal{L}_{UV} = & M_U U_1^c \bar{U}_1^c + M_Q Q_3 \bar{Q}_3 \\ & + z_{11}^u H q_1 U_1^c + z_{33}^d H^\dagger Q_3 d_3^c \\ & + x_{12}^U \Sigma_{12} u_2^c \bar{U}_1^c + x_{13}^Q \Sigma_{13} q_1 \bar{Q}_3 + x_{23}^Q \Sigma_{23} q_2 \bar{Q}_3 \\ & + \hat{Y}_{11}^d H^\dagger q_1 d_1^c + \hat{Y}_{22}^u H q_2 u_2^c + \hat{Y}_{33}^u H q_3 u_3^c \end{aligned} \quad (4.1)$$

The field redefinition freedom leaves one physics CP violating phase in this Lagrangian, which can be rotated between the parameters $M_U, x_{12}^U, x_{13}^Q, x_{23}^Q, z_{11}^u, \hat{Y}_{22}^u$. Taking $M_{Q,U} \gg M$ and integrating out these states at tree level, we obtain the effective theory of section 3.2, with couplings to leading order in $\langle \Sigma \rangle / M$

$$Y_{11}^d = \hat{Y}_{11}^d, \quad Y_{22}^u = \hat{Y}_{22}^u, \quad Y_{33}^u = \hat{Y}_{33}^u \quad (4.2)$$

$$Y_{12}^u = \frac{x_{12}^U \langle \Sigma_{12} \rangle}{M_U} z_{11}^u \quad (4.3)$$

$$Y_{13}^d = \frac{x_{13}^Q \langle \Sigma_{13} \rangle}{M_Q} z_{33}^d, \quad Y_{23}^d = \frac{x_{23}^Q \langle \Sigma_{23} \rangle}{M_Q} z_{33}^d, \quad (4.4)$$

$$Y_{21}^d = \hat{Y}_{11}^d \frac{x_{13}^Q \langle \Sigma_{13} \rangle (x_{23}^Q \langle \Sigma_{23} \rangle)^\dagger}{|M_Q|^2} \quad (4.5)$$

By convention we can rotate the physical phase entirely into Y_{23}^d and Y_{21}^d in the low energy Yukawa matrix, and to leading order this corresponds to rotating the phase entirely into x_{23}^Q in the full theory.

The contribution to $\Delta \bar{\theta}$ due to Y_{21} (eq. (3.24)) is suppressed by the mixing of q_1 and q_2 with the vectorlike Q_3 ,

$$\left| \frac{x_{13}^Q \langle \Sigma_{13} \rangle (x_{23}^Q \langle \Sigma_{23} \rangle)^\dagger}{|M_Q|^2} \right| = \left| \frac{Y_{13}^d Y_{23}^{d\dagger}}{z_{33}^{d\,2}} \right| \approx 3 \times 10^{-7} \times \left(\frac{1}{|z_{33}^d|^2} \right), \quad (4.6)$$

giving

$$\Delta \bar{\theta}' \approx 10^{-15} \times \left(\frac{1}{|z_{33}^d|^2} \right) \quad (4.7)$$

which requires the coupling $|z_{33}^d| \gtrsim 10^{-2}$ for this to be subdominant.

A more dangerous contribution in this model comes from the $d = 6$ operators of eq. (3.27) (the contributions from mixed topological terms eq. (3.29) are subdominant). There are unsuppressed terms generated a tree level giving $Y_{22}^u \sim Y_{22}^u$ and $Y_{11}^d \sim Y_{11}^d$, with scale $\Lambda_{UV} = M_Q$. However, these operators do not contribute to $\Delta \bar{\theta}$ because the phases of Y and this contribution to Y' are aligned. The leading contribution with a misaligned phase is

$$\Delta Y_{22}^{u'} = Y_{12}^u \frac{x_{13}^Q \langle \Sigma_{13} \rangle (x_{23}^Q \langle \Sigma_{23} \rangle)^\dagger}{|M_Q|^2} = Y_{12}^u \frac{Y_{13}^d Y_{23}^{d\dagger}}{|z_{33}^d|^2} \quad (4.8)$$

This term is also suppressed by the mixing of q_1 and q_2 with the vectorlike Q_3 . For $\langle \Sigma_{13} \rangle \sim \langle \Sigma_{23} \rangle \sim M$, the contribution to the theta angle is

$$\begin{aligned} \Delta \bar{\theta}_{D=6} &\sim \frac{\text{Im}[\Delta Y_{22}^{u'}]}{Y_{22}^u} \frac{M^2}{|M_Q|^2} \sim \frac{Y_{12}^u}{Y_{22}^u} |Y_{13}^d|^2 |Y_{23}^d|^2 \frac{g^2}{x_{13}^Q x_{23}^Q} \sin \delta_0 \\ &\approx 10^{-13} \times \left(\frac{\alpha_{s1,2,3}}{0.5} \right) \left(\frac{1}{|x_{13}^Q x_{23}^Q z_{33}^{d\,2}|} \right) \end{aligned} \quad (4.9)$$

As long as the marginal couplings generating the q_1 and q_2 mixings are not too weakly coupled, $x_{13}^Q, x_{23}^Q, z_{33} \gtrsim 0.2$, this contribution is subdominant. This corresponds to a rough

lower limit on the scale $M_Q \gtrsim 100M$. Note that a hierarchy $M_Q \gg M_U \sim M$ can naturally explain the small third-generation quark mixings and $O(1)$ Cabibbo angle.

This flavor model has a similar structure to minimal Nelson-Barr models [30–32], which obtain CKM mixings through vector-like quarks [33], forbidding a tree-level $\bar{\theta}$. However, in contrast to the present case where the $U(1)$ symmetries are sufficient to protect the structure of the theory, in Nelson-Barr models discrete symmetries and additional UV structure are required [23, 34]. In both cases, radiative contributions to $\bar{\theta}$ limit the allowed parameter space, but in Nelson-Barr models these limits appear to generically require unexplained suppressions of allowed couplings [23].

5 Conclusions

The solutions to the strong CP problem and the origins of the flavor structure of the standard model may be intricately tied to each other. In this work, we constructed a model where embedding QCD in a $SU(3)^3$ gauge group with flavorful anomalous PQ symmetries can naturally explain the non-observation of a neutron EDM, the smallness of the third generation CKM mixing angles, and the relative suppression of the down-like quark masses in the second and third generation. The theta angle in each $SU(3)$ factor can be set to zero using an anomalous PQ symmetry. This symmetry is realized by forbidding a bare mass for the lighter quark in each generation (i.e. u, s, b). Their masses are generated through instantons, dominantly at the scale of $SU(3)^3$ breaking, M , which can be far above the weak scale. The instanton-generated mass terms have phases that are naturally aligned with the theta angle, and hence do not reintroduce a non-zero $\bar{\theta}$.

There is a non-zero $\bar{\theta}$ generated at the threshold M through loop corrections that involve both the instanton vertex as well as the perturbative CKM phase. In fact, in our model the smallness of the CKM mixing angles is intimately tied to the smallness of $\bar{\theta}$ in this model, and the observed CKM elements give a prediction $\bar{\theta} \sim 10^{-10}$ that can be probed at the next generation of neutron EDM [24, 25] and proton storage ring experiments [26]. The solution to the strong CP problem is in the spirit of the massless up quark solution, and there are no axion-like states in the theory. An interesting future direction would be to study models which generate the full standard model flavor structure while also implementing our mechanism to solve the strong CP problem.

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A $\bar{\theta}$ threshold

The RGE can be written as an integral equation and solved iteratively to obtain the threshold corrections to $\bar{\theta}$,

$$Y_{ij}^{u,d}(t) = Y_{ij}^{u,d}(t_0) + \int_{t_0}^t dt' \left[\beta_P[Y_{ij}^{u,d}](Y^u(t'), Y^d(t')) + \beta_I[Y_{ij}^{u,d}](Y^u(t), Y^d(t), t') \right] \quad (\text{A.1})$$

where the perturbative beta function depends only on t' through the running of the couplings

$$\beta_P[Y_{ij}^{u,d}](Y^u(t'), Y^d(t')) = \frac{1}{16\pi^2} \frac{3}{2} \left(Y^{u,d}(t') Y^{u,d}(t')^\dagger - Y^{d,u}(t') Y^{d,u}(t')^\dagger \right)_i^k Y^{u,d}(t')_{kj} \quad (\text{A.2})$$

and the instanton contribution to the running depends explicitly on the scale through the instanton density

$$\begin{aligned} \beta_I[Y_{i=j}^{u,d}](Y^u(t'), Y^d(t'), t') &= -c_0 D[t'] Y_{i=j}^{d,u}(t')^* \\ \beta_I[Y_{i \neq j}^{u,d}] &= 0 \end{aligned} \quad (\text{A.3})$$

The leading terms in the series are given by

$$Y^{u,d}(t) = Y^{u,d}(t_0) + Y^{u,d(I)}(t) + Y^{u,d(P)}(t) + Y^{u,d(IP)}(t) + Y^{u,d(PI)}(t) + \dots \quad (\text{A.4})$$

where

$$\begin{aligned} Y^{u,d(P)}(t) &= \int_{t_0}^t dt' \beta_P[Y_{ij}^{u,d}](Y^u(t_0), Y^d(t_0)) \\ &= \frac{2}{b} (2\pi\alpha^{-1}(t) - 2\pi\alpha^{-1}(t_0)) \beta_P[Y_{ij}^{u,d}](Y^u(t_0), Y^d(t_0)) \\ Y^{u,d(I)}(t) &= \int_{t_0}^t dt' \beta_I[Y_{ij}^{u,d}](Y^u(t_0), Y^d(t_0), t') \\ &= -\frac{2c_0 D_0}{b} \Gamma(7, 2\pi\alpha^{-1}(t), 2\pi\alpha^{-1}(t_0)) \end{aligned} \quad (\text{A.5})$$

are the linear terms in the series. Note that although the dependence of the Yukawa couplings on scale has been discarded under the integral, the explicit t dependence of the instanton contribution is maintained. The leading cross terms between the instanton and perturbative running enter at second order,

$$\begin{aligned} Y_{ij}^{u,d(PI)}(t) &= -Y_{ij}^{u,d(P)}(t) + \int_{t_0}^t dt' \beta_P[Y_{ij}^{u,d}](Y^u(t_0) + Y^{u(I)}(t'), Y^d(t_0) + Y^{d(I)}(t')) \\ Y_{i=j}^{u,d(IP)}(t) &= -Y_{i=j}^{u,d(I)}(t) + \int_{t_0}^t dt' \beta_I[Y_{i=j}^{u,d}](Y^u(t_0) + Y^{u(P)}(t'), Y^d(t_0) + Y^{d(P)}(t'), t') \\ Y_{i \neq j}^{u,d(IP)}(t) &= 0 \end{aligned} \quad (\text{A.6})$$

where only terms linear in Y^I, Y^P are to be kept. $Y^{u,d (IP)}(t)$ takes a simple form and is illustrative to examine. In the limit $\alpha(t_0) \ll 1$,

$$Y_{i=j}^{u,d (IP)}(t) = Y_{i=j}^{u,d (I)}(t) \frac{Y_{i=j}^{d,u (P)}(t)^*}{Y_{i=j}^{d,u}(t_0)^*} \left(1 - \frac{f_I(\alpha(t))}{2\pi\alpha^{-1}(t_0) - 2\pi\alpha^{-1}(t)} \right) \quad (\text{A.7})$$

where $f_I(\alpha(t))$ is given in eq. (3.23). The term constant in the parentheses has a simple interpretation; we can write

$$Y_{i=j}^{u,d (I)}(t) + Y_{i=j}^{u,d (IP)}(t) = Y_{i=j}^{u,d (I)}(t) \frac{Y_{i=j}^{d,u}(t_0)^* + Y_{i=j}^{d,u (P)}(t)^*}{Y_{i=j}^{d,u}(t_0)^*} + \dots \quad (\text{A.8})$$

Clearly the physical effect of this term is to shift the Yukawa coupling entering the instanton to its value at the IR scale t where instanton effects are large, not t_0 where the theory is weakly coupled. This sets $\Delta\bar{\theta} = 0$. The second term in the parentheses is a threshold effect — since

$$Y^{d,u (P)} \propto (2\pi\alpha^{-1}(t_0) - 2\pi\alpha^{-1}(t)), \quad (\text{A.9})$$

it is finite while $2\pi\alpha^{-1}(t_0) \rightarrow \infty$ as $t_0 \rightarrow \infty$. This captures the fact that the instantons are active over a finite range of scale near the IR scale t , and are sensitive to changes in the phases of the Yukawas near t . This spoils the exact cancellation that set $\Delta\bar{\theta} = 0$ at leading order. The other term $Y^{u,d (PI)}$ describes similar threshold effects as the instanton-generated Yukawas themselves are rotated by the perturbative running. Evaluating $\Delta\bar{\theta}$ with these leading terms in the series eq. (A.4) gives the results eqs. (3.21) and (3.24).

B Alternative Yukawa structures

An alternative solution to the observed quark masses and CKM matrix is possible with the Yukawa texture of eq. (3.14) by switching the role of y_d and y_s . In this case the non-perturbative effects generate y_d from y_c , y_u from y_s , and y_b from y_t . This is an attractive possibility because it requires smaller non-perturbative effects, and therefore can more easily be accommodated without adding additional weakly coupled sites to the $\text{SU}(3) \times \text{SU}(3) \times \text{SU}(3)$ model. However, the size of the radiative contribution to $\Delta\bar{\theta}$ is increased by a factor of $(\cot \theta_c)^2 \approx 20$ in this model, which is excluded by current limits unless there is a $\sim 10\%$ tuned cancellation with another contribution to $\bar{\theta}$.

While we focused on the Yukawa texture eq. (3.14), there is one other possibility for a viable Yukawa texture that can be protected by $\text{U}(1)_{PQ}$ symmetries and has a vanishing tree-level contribution to $\bar{\theta}$,

$$Y^u = \begin{pmatrix} 0 & 0 & Y_{13}^u \\ 0 & Y_{22}^u & 0 \\ 0 & 0 & Y_{33}^u \end{pmatrix} \quad (\text{B.1})$$

$$Y^d = \begin{pmatrix} Y_{11}^d & Y_{12}^d & 0 \\ 0 & 0 & 0 \\ Y_{31}^d & Y_{32}^d & 0 \end{pmatrix}, \quad (\text{B.2})$$

The CKM structure emerges less naturally for this texture because of the right-handed dominant mixing structure in the down Yukawa matrix. Fitting the V_{31} and V_{32} CKM elements requires a cancellation between terms of order Y_{31}^u/y_t and $Y_{12}^d Y_{32}^d/Y_b^2$. Nonetheless this texture remains an interesting possibility, and viable models can be realized and also generically predict $\bar{\theta} \sim 10^{-10}$ from the radiative corrections.

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