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SO(10) × S_4 grand unified theory of flavour and leptogenesis

Francisco J. de Anda,[a] **Stephen F. King**[b] **and Elena Perdomo**[b]

[a] *School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, U.K.*

[b] *Tepatitlán's Institute for Theoretical Studies, C.P. 47600, Jalisco, México*

E-mail: franciscojosedea@gmail.com, king@soton.ac.uk, e.perdomo-mendez@soton.ac.uk

Abstract: We propose a Grand Unified Theory of Flavour, based on SO(10) together with a non-Abelian discrete group S_4, under which the unified three quark and lepton 16-plets are unified into a single triplet $3'$. The model involves a further discrete group $\mathbb{Z}_4^R \times \mathbb{Z}_4^3$ which controls the Higgs and flavon symmetry breaking sectors. The CSD2 flavon vacuum alignment is discussed, along with the GUT breaking potential and the doublet-triplet splitting, and proton decay is shown to be under control. The Yukawa matrices are derived in detail, from renormalisable diagrams, and neutrino masses emerge from the type I seesaw mechanism. A full numerical fit is performed with 15 input parameters generating 19 presently constrained observables, taking into account supersymmetry threshold corrections. The model predicts a normal neutrino mass ordering with a CP oscillation phase of 260°, an atmospheric angle in the first octant and neutrinoless double beta decay with $m_{\beta\beta} = 11$ meV. We discuss N_2 leptogenesis, which fixes the second right-handed neutrino mass to be $M_2 \simeq 2 \times 10^{11}$ GeV, in the natural range predicted by the model.

Keywords: GUT, Discrete Symmetries, Neutrino Physics

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JHEP12(2017)075

Contents

JHEP12(2017)075

Introduction. The Standard Model (SM) [1], though highly successful, does not address the origin of neutrino mass and lepton mixing [2, 3]. One attractive possibility is the type I seesaw mechanism, which can account for the smallness of neutrino masses by introducing three right-handed neutrinos with very large Majorana masses [4–9]. Such right-handed neutrinos arise very naturally from SO(10) Grand Unified Theories (GUTs) [10] in which a single family of quarks and leptons, together with a right-handed neutrino, is unified into a single 16-plet. Supersymmetry (SUSY) is then naturally suggested for gauge coupling unification and to ameliorate the gauge hierarchy problem. However the origin of the three families, and their hierarchical masses are not explained by traditional SO(10) SUSY GUTs.

The almost tri-bimaximal lepton mixing observed over recent years [11], combined with a reactor angle of order 8.5° [12], suggests that some sort of non-Abelian family symmetry may be at work in the lepton sector [13–16]. The first models to consider a non-Abelian

SU(3) symmetry as an explanation of bi-large lepton mixing were put forward in [17–19]. Models based on SO(10) with a non-Abelian discrete symmetry were first proposed in [19–30], and further flavoured GUTs were considered in [31–56]. A more general study of flavour symmetries in SO(10) can be found in [57–59]. Here we shall be interested in a SUSY GUT theory of flavour in which all quarks and leptons are fitted into a single $(3, 16)$ representation of $S_4 \times \mathrm{SO}(10)$ [60–67]. While the former model predicted a zero reactor angle [60–66], the latter model [67] was based on CSD3[1] flavon vacuum alignment [69–73], leading to approximate tri-bimaximal mixing with the correct value of the reactor angle. However the latter model is so far incomplete since it did not include any explicit discussion of the flavon vacuum alignment, or GUT breaking potential, and also did not include any discussion of leptogenesis.

In the present paper we consider a more complete $S_4 \times \mathrm{SO}(10)$ SUSY GUT of flavour, which also involves a further discrete group $\mathbb{Z}_4^R \times \mathbb{Z}_4^3$ which controls the Higgs and flavon symmetry breaking sectors. In the model here, we prefer the simpler CSD2 [75, 76] vacuum alignment, which, in conjunction with small charged lepton corrections arising from the SO(10) structure of Yukawa matrices, is capable of yielding the desired reactor angle. It also allows successful leptogenesis, as discussed below. Here the flavon vacuum alignment potential is discussed, along with the GUT breaking potential and the doublet-triplet splitting, and proton decay are shown to be under control. The Yukawa matrices are derived in detail, from renormalisable diagrams, and neutrino masses emerge from the type I seesaw mechanism. A full numerical fit is performed with 15 input parameters describing 19 observables, taking into account supersymmetry threshold corrections. The model predicts a normal neutrino mass ordering with a CP oscillation phase of 260°, an atmospheric angle in the first octant and neutrinoless double beta decay with $m_{\beta\beta} = 11$ meV. We also discuss N_2 leptogenesis [77–83], which fixes the second right-handed neutrino mass $M_2 \simeq 2 \times 10^{11}$ GeV, in the natural range predicted by the model.[2]

The layout of the remainder of the paper is as follows. In section 1 we describe the symmetries of the model and the superfields related to the low energy fields. In section 2 we list the complete set of fields together with the effective Yukawa terms they generate. Section 3 shows how the flavon VEVs are aligned in the CSD2 direction. In section 4 we show the symmetry breaking superpotential that produces a hierarchy between the flavon VEVs and drives them, together with the GUT breaking fields. Section 5 shows how doublet-triplet splitting is achieved. In section 6, proton decay is discussed. In section 7 we give the complete Yukawa superpotential and the fermion mass matrices structure arising from it. In section 8 we give a numerical fit of model parameters to data, as well as the parametrization of SUSY threshold corrections and the parameter counting of the model. In section 9 we show how the model can achieve successful N_2 leptogenesis. Section 10 lists our conclusions.

[1] CSD refers to "constrained sequential dominance" first introduced in [68]. In this paper CSD is simply used as a label which refers to a particular flavon vacuum alignment as discussed later. Such vacuum alignments motivates the choice of S_4 as the family symmetry, as discussed by Luhn et al. [69–73].

[2] Interestingly we find that N_2 leptogenesis is not consistent with the earlier model based on CSD3 vacuum alignment [67], which is a significant motivation for considering the new model based on CSD2.

JHEP12(2017)075

Field	Representation					
	S_4	SO(10)	$\mathbb{Z}_4^R$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$
ψ	$3'$	16	1	0	0	0
H_{10}^u	1	10	0	0	0	0
H_{10}^d	1	10	0	0	2	0
$H_{\overline{16}}$	1	$\overline{16}$	0	0	0	0
H_{16}	1	16	0	0	1	0
$H_{45}^{X,Y}$	1	45	0	0	1	0
$H_{45}^{W,Z}$	1	45	0	2	0	0
H_{45}^{B-L}	1	45	2	0	2	0
ζ	1	1	0	0	2	0

(a) Matter, and Higgs superfields.

Field	Representation					
	S_4	SO(10)	$\mathbb{Z}_4^R$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$
ϕ_1	$3'$	1	0	2	2	0
ϕ_2	$3'$	1	0	2	0	0
ϕ_3	$3'$	1	0	0	2	0
$\phi_{S,U}$	$3'$	1	0	0	0	1
ϕ_T	3	1	0	1	0	1
ξ	1	1	0	3	0	2
ϕ_t	3	1	0	0	1	3

(b) Flavon superfields.

Table 1. Field content of the model that relates directly to the low energy fields.

1 Overview of the model

The symmetry of the model is $\mathrm{SO}(10) \times S_4 \times \mathbb{Z}_4^R \times \mathbb{Z}_4^3$. The model has a gauge symmetry SO(10) which is the GUT symmetry. The symmetry S_4 is the flavour symmetry which gives the specific CSD2 structure to the fermion mass matrices. The $\mathbb{Z}_4^R$ is an R symmetry while the other three $\mathbb{Z}_4$'s are shaping symmetries. Furthermore, we assume that the GUT theory is invariant under trivial CP symmetry, which is spontaneously broken by the complex VEVs of the flavons.

In the table 1 we present the fields that contain the Higgs, flavons and matter fields, which are relevant to Yukawa sector. The field ψ contains the full SM fermion content. The fields $H_{10}^{u,d}$ contain the MSSM Higgs doublets $h_{u,d}$ respectively. The $H_{\overline{16}}$ breaks $\mathrm{SO}(10) \to \mathrm{SU}(5)$ and gives masses to the right handed neutrinos (RHN). The H_{45}'s break $\mathrm{SU}(5) \to SM$ and introduce the necessary Clebsch-Gordan (CG) relations to generate correct charged lepton and down quark masses. The flavon fields ϕ_i, with $i = 1, 2, 3$ break S_4 completely with the CSD2 vacuum alignment [75, 76],

$$\langle\phi_1\rangle = v_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \langle\phi_2\rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle\phi_3\rangle = v_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \tag{1.1}$$

with $|v_1| \ll |v_2| \ll |v_3|$. This CSD2 flavon alignment is fixed by a superpotential as discussed in section 3.

With these fields, a very specific mass structure for the SM fermion fields is generated. For the up-type quark and the neutrino sectors, the Yukawa terms look like

$$H_{10}^u(\psi\phi_1)(\psi\phi_1) + H_{10}^u(\psi\phi_2)(\psi\phi_2) + H_{10}^u(\psi\phi_3)(\psi\phi_3), \tag{1.2}$$

where the brackets denote S_4 singlet contractions. Each of these terms generates a rank-1 matrix. The hierarchy between the flavon VEVs, shown in section 4, gives a natural

JHEP12(2017)075

explanation of the hierarchical Yukawa couplings $y_u \sim v_1^2/M_{\rm GUT}^2$, $y_c \sim v_2^2/M_{\rm GUT}^2$, $y_t \sim v_3^2/M_{\rm GUT}^2$. The RHN Majorana masses are similar to eq. (1.2) replacing H_{10}^u by $H_{\overline{16}}H_{\overline{16}}$. The fact that the RHN masses have the same structure as the Dirac neutrino masses generate exactly the same structure for the left handed neutrino Majorana masses, as shown in section 7.3, after the seesaw mechanism.

For the down-type quark and the charged lepton sectors, the Yukawa terms look like

$$H_{10}^d(\psi\phi_1)(\psi\phi_2) + H_{10}^d(\psi\phi_2)(\psi\phi_2) + H_{10}^d(\psi\phi_3)(\psi\phi_3) + H_{10}^d(\psi\psi)_{3'}(\phi_3), \tag{1.3}$$

where the brackets denote S_4 singlet contractions apart from the $3'$ contraction which is necessary to combine with $\phi_3 \sim 3'$ into a singlet. They have a different structure compared to the up sector, due to a mixing term between the flavons ϕ_1 and ϕ_2, which explains why there is a milder hierarchy in the down and charged lepton sectors compared to the up one. It also introduces a texture zero in the (1,1) element of the down Yukawa matrix, reproducing the GST relation [84], i.e. the Cabibbo angle is predicted to be $\theta_{12}^q \simeq \sqrt{y_d/y_s}$. With this setup the full SM fermion masses are generated in a very specific and predictive way, this being the main aim of the paper.

After GUT symmetry breaking, all the messenger fields and adjoints obtain a GUT scale mass. Furthermore, the triplets inside the $H_{10}^{u,d}$ also get a GUT scale mass through the Dimopoulos-Wiclzeck mechanism [85–87], as shown in the section 5. This way we make sure that at low energies, only the MSSM remains.

2 Effective Yukawa structure

We now present the effective Yukawa terms in more detail than in the previous section. With the fields in the table 1 we may write the superpotential relevant to the Yukawa terms, including terms $O(1/M_P)$, as

$$\begin{aligned} W_Y \sim & \frac{H_{10}^u(\psi\phi_1)(\psi\phi_1)}{\langle H_{45}^{W,Z}\rangle^2} + \frac{H_{10}^u(\psi\phi_2)(\psi\phi_2)}{\langle H_{45}^{W,Z}\rangle^2} + \frac{H_{10}^u(\psi\phi_3)(\psi\phi_3)}{\langle H_{45}^{W,Z}\rangle^2} \\ & + \frac{H_{10}^d(\psi\phi_1)(\psi\phi_2)}{\langle H_{45}^{W,Z}\rangle^2} + \frac{H_{10}^d(\psi\phi_2)(\psi\phi_2)}{\langle H_{45}^{X,Y}\rangle^2} + \frac{H_{10}^d(\psi\phi_3)(\psi\phi_3)}{\langle H_{45}^{X,Y}\rangle^2} \\ & + \frac{H_{\overline{16}}H_{\overline{16}}(\psi\phi_1)(\psi\phi_1)}{M_P\langle H_{45}^{W,Z}\rangle^2} + \frac{H_{\overline{16}}H_{\overline{16}}(\psi\phi_2)(\psi\phi_2)}{M_P\langle H_{45}^{W,Z}\rangle^2} + \frac{H_{\overline{16}}H_{\overline{16}}(\psi\phi_3)(\psi\phi_3)}{M_P\langle H_{45}^{W,Z}\rangle^2} \\ & + \frac{H_{10}^d(\psi\psi)_{3'}(\phi_3)}{M_P} \end{aligned} \tag{2.1}$$

where $(\ )_{3'}$ means a $3'$ contraction, while $(\ )$ without any subscript means the singlet contraction of S_4. There are plenty of terms supressed by M_P^2 and they are expected to make small mass contributions of $O(M_{\rm GUT}^2/M_P^2) < 10^{-6}$, and therefore negligible.[3]

[3]The most important correction, of $O(10^{-6})$, is made to the up-quark Yukawa coupling. From table 5, we see that it is of comparable magnitude. We performed the fit ignoring these corrections. If they were included, they would shift the fit parameters. The largest contribution to the electron Yukawa coupling is of $O(10^{-8})$ and therefore negligible.

JHEP12(2017)075

Field	Representation					
	S_4	SO(10)	$\mathbb{Z}_4^R$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$
$\bar{\chi}_1$	1	$\overline{16}$	1	2	2	0
χ_1	1	16	1	0	2	0
$\bar{\chi}_2$	1	$\overline{16}$	1	2	0	0
χ_2	1	16	1	0	0	0
$\bar{\chi}_3$	1	$\overline{16}$	1	0	2	0
χ_3	1	16	1	2	2	0
χ_3^d	1	16	1	0	1	0
χ_2^d	1	16	1	2	3	0
$\bar{\chi}_u$	1	$\overline{16}$	2	0	0	0
χ_u	1	16	0	0	2	0
$\bar{\chi}_d$	1	$\overline{16}$	0	0	1	0
χ_d	1	16	2	0	1	0
ζ_1	1	45	2	0	3	0
ζ_2	1	45	0	0	3	0

(a) Messenger superfields.

Field	Representation					
	S_4	SO(10)	$\mathbb{Z}_4^R$	$\mathbb{Z}_4$	$\mathbb{Z}_4$	$\mathbb{Z}_4$
$X_{3'}$	$3'$	1	2	0	0	2
X_2	2	1	2	2	0	2
$\tilde{X}_2$	2	1	2	0	1	1
X_1	1	1	2	0	2	2
$\tilde{X}_1$	1	1	2	3	3	0
$X_{1'}$	$1'$	1	2	3	2	2
$Z_{3'}$	$3'$	1	2	3	0	2
$\tilde{Z}_{3'}$	$3'$	1	2	2	2	0
$\tilde{Z}$	1	1	2	3	2	3
Z	1	1	2	0	0	0

(b) Alignment superfields.

Table 2. Fields that appear only at high energies. Together with the ones in table 1 they list the complete field content of the model.

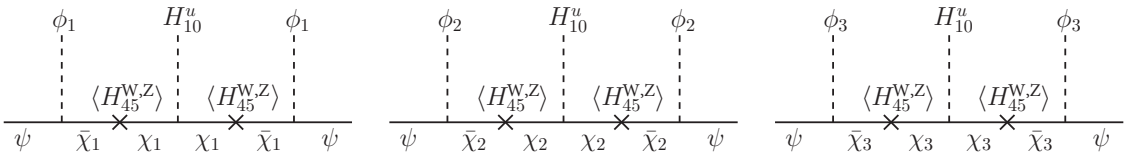

Figure 1. Diagrams coupling ψ to H_{10}^u. When flavons acquire VEVs, these give the up-type quark and Dirac neutrino Yukawa matrices.

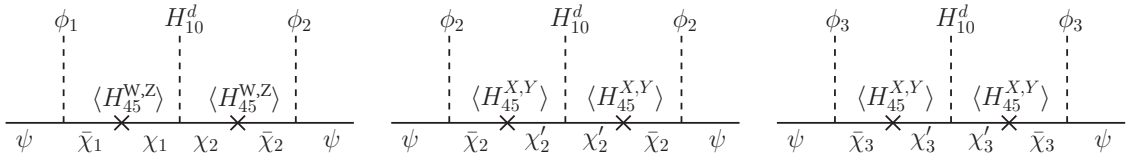

Figure 2. Diagrams coupling ψ to H_{10}^d. These generate the down-type quark and charged lepton Yukawa matrices.

We have ignored all the $O(1)$ dimensionless couplings for simplicity. The diagrams that generate these terms are shown in figures 1–3, where they include the messengers χ, listed in table 2. In the section 7 we present them in full detail together with the specific messenger structure.

The full field content of the model is listed in tables 1–2. Even though the list of fields seems large, it is substantially smaller than previous flavoured GUT models that attempt to be complete [29, 30].

JHEP12(2017)075

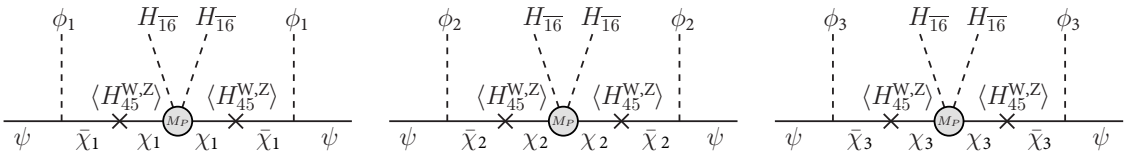

Figure 3. Diagrams coupling ψ to $H_{\overline{16}}$. These give the RH neutrino mass matrix.

3 Vacuum alignment

The flavon superpotential fixes the symmetry breaking flavon VEVs in eq. (1.1). To derive this alignment we use a set of driving fields, listed in table 2, coupled to the flavon fields in table 1. We follow a sequence of steps using supersymmetric F-terms equations to align all the flavons. The letter subscript in the flavons refers to the symmetry preserving generator. The alignments depend on the S_4 representation of the alignment field, denoted by its index. The superpotential is given by

$$\begin{aligned} W_\phi \sim\ & X_{3'}(\phi_{S,U})^2 + X_2(\phi_T)^2 + X_1(\phi_t)^2 + \tilde{X}_1\phi_T\phi_t + X_{1'}\phi_T\phi_3 + \tilde{X}_2\phi_t\phi_3 \\ & + Z_{3'}(\phi_{S,U}\phi_T + \xi\phi_2) + \tilde{Z}_{3'}\xi\left(\frac{\phi_2\phi_3}{M_P} - \phi_1\right), \end{aligned} \tag{3.1}$$

where we have ignored dimensionless $O(1)$ parameters since they are not relevant. Solving the F-term equations from the alignment fields fixes the flavon VEV alignment, while the F-term equations from flavons forbid the alignment fields from getting a VEV.

The three S_4 generators, working in the T diagonal basis, are

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad \text{for } \mathbf{3} \text{ or } \mathbf{3'}, \tag{3.2}$$

and

$$U = \mp\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad SU = US = \mp\frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & -1 \\ 2 & -1 & 2 \end{pmatrix}, \quad \text{for } \mathbf{3}, \mathbf{3'} \text{ respectively.} \tag{3.3}$$

The first 3 terms in the superpotential in eq. (3.1) fix the alignments

$$X_{3'}(\phi_{S,U})^2 \longrightarrow \begin{pmatrix} 1 \\ \omega^n \\ \omega^{2n} \end{pmatrix}, \tag{3.4}$$

$$X_2(\phi_T)^2 \longrightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2\omega^n \\ -2\omega^{2n} \end{pmatrix}, \tag{3.5}$$

$$X_1(\phi_t)^2 \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2x \\ -1/x \end{pmatrix}, \tag{3.6}$$

JHEP12(2017)075

up to an integer ($n \in \mathbb{Z}$) or continuos ($x \in \mathbb{R}$) parameter, with $\omega = e^{2\pi i/3}$. We may notice that the three solutions for $\langle \phi_{S,U} \rangle$ are related one to another by a T transformation. We may choose it to be $(1,1,1)^T$ without loss of generality.

The $\langle \phi_T \rangle$ has four different solutions. The last three solutions are related by a T transformation. From these three, the one without any ω is related to the first solution by an S transformation. Since they are all related, we may choose $(1,0,0)^T$ without loss of generality.

The $\langle \phi_t \rangle$ has three different solutions. The third solution is not related to the first two by any symmetry transformations. The fourth term in the superpotential fixes the solution to be either $(0,0,1)^T$ or $(0,1,0)^T$, which are related by an U transformation and we choose the former without loss of generality.

The fifth and sixth terms fix ϕ_3 to be orthogonal to ϕ_t and ϕ_T so that it is fixed to be $(0,1,0)^T$.

The first term from the second line in eq. (3.1) involves

$$(\langle \phi_{S,U} \rangle \cdot \langle \phi_T \rangle)_{3'} \quad \propto \quad \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \tag{3.7}$$

and together with the fifth one fixes $\langle \phi_2 \rangle$ into this direction. The third term in the second line involves

$$(\langle \phi_2 \rangle \cdot \langle \phi_3 \rangle)_{3'} \quad \propto \quad \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \tag{3.8}$$

and together with the seventh term we fix $\langle \phi_1 \rangle$ into this direction. Furthermore the ξ field that does not add anything to the alignment but it plays a role in the VEV driving as explained below.

The F-term equations from the X, Z fix the alignments to be

$$\begin{aligned} \langle \phi_{S,U} \rangle &= v_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \langle \phi_T \rangle = v_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \phi_t \rangle = v_t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\ \langle \phi_1 \rangle &= v_1 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad \langle \phi_2 \rangle = v_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \langle \phi_3 \rangle = v_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \end{aligned} \tag{3.9}$$

where the last three flavons couple to the matter superfield ψ and determine the fermion mass matrix structure. The flavon VEVs v_i are, in general, complex, and spontaneously break the assumed CP symmetry of the high energy theory.

4 Symmetry breaking

The model gives a natural understanding of the SM fermion masses through the hierarchy between the flavon VEVs $|v_1| \ll |v_2| \ll |v_3|$. Here, we show the symmetry breaking

JHEP12(2017)075

superpotential that produces such hierarchy between the VEVs,

$$\begin{aligned}\mathcal{W}_{\rm DV} \sim\ & \tilde{Z}_3\xi\left(\phi_1 - \frac{\phi_2\phi_3}{M_P}\right) + \tilde{Z}\frac{\phi_T}{M_P}\left(\phi_1\phi_2 - \frac{\phi_3\sum_i\phi_i^2}{M_P} + O(1/M_P^2)\right)\\ & + Z\left(M_{\rm GUT}^2 + \sum_i \phi_i^2 + (H_{45}^{W,Z})^2 + (H_{45}^{B-L})^2 + \zeta^2 + Z^2 + O(1/M_P)\right)\\ & + H_{45}^{B-L}\left((H_{45}^{X,Y})^2 + \frac{\zeta}{M_P}\left((H_{45}^{W,Z})^2 + (H_{45}^{B-L})^2\right) + H_{45}^{X,Y}\frac{H_{\overline{16}}H_{16}}{M_P} + DT + O(1/M_P^2)\right),\end{aligned} \tag{4.1}$$

where we have ignored dimensionless couplings for simplicity.

The first term of eq. (4.1) also appears in the alignment potential in eq. (3.1) and fixes

$$|\tilde{\kappa}_1 v_1| = \left|\frac{v_2 v_3}{M_P}\right|, \tag{4.2}$$

where $\tilde{\kappa}_1$ denotes an effective dimensionless coupling coming from the ones in the superpotential. Note that we have written this equation as only fixing the modulus. This happens due to the appearance of the field ξ; we assume there are two copies of that field, which get a VEV with an arbitrary phase. This phase, together with the dimensionless couplings for each term, does not allow to relate the phases of the v_i.

The second term of eq. (4.1) fixes the VEVs

$$\tilde{\kappa}_2\ v_1 v_2 = \frac{v_3}{M_P}\sum_i v_i^2, \tag{4.3}$$

where $\tilde{\kappa}_2$ denotes an effective dimensionless coupling coming from the ones in the superpotential. This equation, together with the previous one, require a hierarchy in the v_i's. Specifically it requires $v_{2,3} \gg v_1$.

The field $\tilde{Z}$ does not obtain a VEV to comply with the F-term equations from the flavons.

The second line of eq. (4.1) drives the linear combination

$$M_{\rm GUT}^2 \sim \sum_i v_i^2 + \langle H_{45}^{W,Z}\rangle^2 + \langle H_{45}^{B-L}\rangle^2 + \langle\zeta\rangle^2 + \langle Z\rangle^2, \tag{4.4}$$

where we assume that the sum of v_i and the all adjoints get a GUT scale VEV. The field Z does not get a VEV due to the F-term equations coming from the adjoints. This equation does not fix the phases of the VEVs. We assume that the $\langle H_{45}^{W,Z}\rangle$ are real while the phase of the sum of flavon VEVs is unconstrained (only related to the one of $\langle\zeta\rangle$ which does not appear at low energies). We assume that the flavons obtain a VEV that break the CP symmetry with an arbitrary phase.

The third line of eq. (4.1) drives

$$\frac{\langle\zeta\rangle}{M_P}\left(\langle H_{45}^{W,Z}\rangle^2 + \langle H_{45}^{B-L}\rangle^2\right) \sim \langle H_{45}^{X,Y}\rangle^2 + \langle H_{45}^{X,Y}\rangle\frac{\langle H_{\overline{16}}H_{16}\rangle}{M_P}, \tag{4.5}$$

JHEP12(2017)075

where we assume that the $\langle H_{45}^{X,Y} \rangle$ is real. The DT represent all the terms involved in the D-T splitting (shown in section 5) that do not contribute to the F-term equation, but they are there nonetheless. The F-term equations coming from the $H_{45}^{X,Y}$ force the $\chi_{u,d}$ to also get a VEV and does not change any low energy phenomenology.

The F-term equations previously discussed can give a VEV to the adjoint fields but do not fix their direction. The adjoint fields can get a VEV in any SM preserving direction. In general they can be written as a linear combination of the $\mathrm{U}(1)_{X,Y}$ directions. We do not assume any specific direction for the VEVs $\langle H_{45}^{W,X,Y,Z}, \zeta \rangle$. We assume that $\langle H_{45}^{B-L} \rangle$ lies in the $\mathrm{U}(1)_{B-L}$ direction.[4] We assume that the $\langle H_{\overline{16},16} \rangle$ lie in the right handed neutrino direction.

Using the first three equations (4.2)–4.4, we may find that the flavon VEVs

$$v_1 = \frac{\tilde{\kappa}_3^2 M_{\mathrm{GUT}}^2}{\sqrt{\tilde{\kappa}_1 \tilde{\kappa}_2} M_P} \, v_2, \quad v_2 = \frac{\sqrt{\tilde{\kappa}_1} \tilde{\kappa}_3 M_{\mathrm{GUT}}}{\sqrt{\tilde{\kappa}_2}}, \quad v_3 = \tilde{\kappa}_3 M_{\mathrm{GUT}}, \tag{4.6}$$

in this way, if we assume that $\tilde{\kappa}_1 \simeq 0.1$, $\tilde{\kappa}_2 \simeq 10$, $\tilde{\kappa}_3 \simeq 1$, we have

$$v_3 \simeq M_{\mathrm{GUT}}, \quad v_2 \simeq 0.1 \; M_{\mathrm{GUT}}, \quad v_1 \simeq 0.001 \; M_{\mathrm{GUT}}, \tag{4.7}$$

which generates the hierarchy between the fermion families. We note that the hierachy between v_1 and v_2 is given by the structure of the F-term equations. The hierarchy between v_2 and v_3 is assumed and realized by a much milder hierarchy between the couplings in the superpotential.

Using eq. (4.3) and knowing that $v_3 \gg v_{1,2}$, we approximately get

$$\tilde{\kappa}_2 \; v_1 v_2 \simeq \frac{v_3^3}{M_P}, \tag{4.8}$$

which also fixes the VEV phases to be

$$\arg v_1 + \arg v_2 \simeq 3 \arg v_3, \tag{4.9}$$

that in terms of the physical phases is

$$\eta \simeq 4\eta' - 2\gamma, \tag{4.10}$$

this way there are only 2 free physical phases.

5 Doublet-Triplet splitting

The Higgs fields $H_{10}^{u,d}$ and $H_{16,\overline{16}}$ contain SU(2) doublets and SU(3) triplets. We need the triplets to be heavy since they mediate proton decay, while two of the doublets remain light so they can be associated to the MSSM Higgs doublets. This is known as the doublet-triplet splitting problem and can be solved using the Dimopolous-Wilczek mechanism [85–87]. In our case this mechanism is in place since we assume that $\langle H_{45}^{B-L} \rangle$ lies in the $\mathrm{U}(1)_{B-L}$

[4] It can be written as the linear combination $B - L = (-X + 4Y)/5$.

direction. Furthermore, there are extra pairs of doublets, and they are required to be heavy to preserve gauge coupling unification. Using the fields in tables 1–2, we may write the superpotential involving the Higgs fields (ignoring dimensionless parameters)

$$\begin{aligned}\mathcal{W}_H = H_{45}^{B-L}\Big(&H_{10}^u H_{10}^d + \zeta_2\zeta_2 + H_{\overline{16}}\chi_u + H_{16}\overline{\chi}_d\Big)\\ &+ H_{\overline{16}}H_{10}^u\overline{\chi}_u + H_{16}H_{10}^d\chi_d + H_{16}H_{\overline{16}}\zeta_1 + \zeta\left(\zeta_1\zeta_2 + \overline{\chi}_u\chi_u + \overline{\chi}_d\chi_d\right)\\ &+ H_{45}^{B-L}\left(\frac{H_{\overline{16}}H_{\overline{16}}H_{10}^d}{M_P} + \frac{H_{16}H_{16}H_{10}^u}{M_P} + H_{10}^u H_{10}^d\frac{(H_{45}^{X,Y,W,Z})^4}{M_P^4}\right).\end{aligned} \tag{5.1}$$

After integrating out the messengers ζ_i, χ_j, the superpotential becomes

$$\begin{aligned}\mathcal{W}_H = H_{45}^{B-L}\Bigg(&\kappa_1 H_{10}^u H_{10}^d + \kappa_2\frac{(H_{16}H_{\overline{16}})^2}{\langle\zeta\rangle^2} + \kappa_7 H_{10}^u H_{10}^d\frac{(H_{45}^{X,Y,W,Z})^4}{M_P^4}\\ &+ \kappa_3\frac{H_{\overline{16}}H_{\overline{16}}H_{10}^u}{\langle\zeta\rangle} + \kappa_4\frac{H_{\overline{16}}H_{\overline{16}}H_{10}^d}{M_P} + \kappa_5\frac{H_{16}H_{16}H_{10}^u}{M_P} + \kappa_6\frac{H_{16}H_{16}H_{10}^d}{\langle\zeta\rangle}\Bigg).\end{aligned} \tag{5.2}$$

We remember that the magnitude of the VEVs is

$$\langle H_{16}\rangle \simeq \langle H_{\overline{16}}\rangle \simeq \langle H_{45}\rangle = M_{\rm GUT}, \tag{5.3}$$

and we define

$$z = M_{\rm GUT}/\langle\zeta\rangle, \quad y = M_{\rm GUT}/M_P. \tag{5.4}$$

Denoting the up (down)-type doublet inside each H_{10} as $\mathbf{2}_{u(d)}(H_{10}^{u,(d)})$, and similarly for the triplets, the mass matrix for the triplets becomes

$$M_T \sim \begin{array}{c} \\ \mathbf{3}_d(H_{10}^d)\\ \mathbf{3}_d(H_{10}^u)\\ \mathbf{3}_d(H_{16})\end{array}\begin{array}{c}\begin{array}{ccc}\mathbf{3}_u(H_{10}^u) & \mathbf{3}_u(H_{10}^d) & \mathbf{3}_u(H_{\overline{16}})\end{array}\\ \begin{pmatrix}\kappa_1 & 0 & \kappa_4 y\\ 0 & -\kappa_1 & \kappa_3 z\\ \kappa_5 y & \kappa_6 z & \kappa_2 z^2\end{pmatrix}\end{array} M_{\rm GUT}, \tag{5.5}$$

that has as approximate eigenvalues

$$m_T \sim \quad \kappa_1 M_{\rm GUT}, \quad \kappa_1 M_{\rm GUT}, \quad \kappa_2 z^2 M_{\rm GUT}, \tag{5.6}$$

so that it requires $\kappa_1 \sim \kappa_2 z^2 \sim 1$, to get the triplets at the GUT scale. The doublets mass matrix is

$$M_D \sim \begin{array}{c} \\ \mathbf{2}_d(H_{10}^d)\\ \mathbf{2}_d(H_{10}^u)\\ \mathbf{2}_d(H_{16})\end{array}\begin{array}{c}\begin{array}{ccc}\mathbf{2}_u(H_{10}^u) & \mathbf{2}_u(H_{10}^d) & \mathbf{2}_u(H_{\overline{16}})\end{array}\\ \begin{pmatrix}-\kappa_7 y^4 & 0 & \kappa_4 y\\ 0 & \kappa_7 y^4 & \kappa_3 z\\ \kappa_5 y & \kappa_6 z & \kappa_2 z^2\end{pmatrix}\end{array} M_{\rm GUT}, \tag{5.7}$$

JHEP12(2017)075

that has as eigenvalues

$$m_D \sim \quad -y^4 M_{\rm GUT}, \quad \kappa_6\kappa_3 z^2 M_{\rm GUT}, \quad \kappa_2 z^2 M_{\rm GUT}, \tag{5.8}$$

so that we must have $\kappa_6\kappa_3 z^2 \sim \kappa_2 z^2 \sim 1$, to get two doublet pairs at the GUT scale. Furthermore, there is a μ term generated by

$$\mu \sim y^4 M_{\rm GUT} \sim 1\ TeV, \tag{5.9}$$

which happens at the correct order.

The light MSSM doublets are

$$h_u \simeq \mathbf{2}_u(H^u_{10}) + \frac{\kappa_4 y}{\kappa_3 z}\mathbf{2}_u(H^d_{10}), \qquad h_d \simeq \mathbf{2}_d(H^d_{10}) + \frac{\kappa_5 y}{\kappa_6 z}\mathbf{2}_u(H^d_{10}), \tag{5.10}$$

so that the second term is suppressed to be $< 10^{-3}$ and we may safely assume that $h_{u(d)}$ lies only inside $H^{u(d)}_{10}$.

6 Proton decay

One of the characteristic features of GUTs is the prediction of proton decay. It has not been observed and the proton lifetime is constrained to be $\tau_p > 10^{34}$ years [1].

Proton decay can be mediated by the extra gauge bosons of the GUT and by the triplets accompanying the Higgs doublets. In SUSY SO(10) GUTs, the main source for proton decay comes from the triplet Higgsinos. The decay width is dependent on SUSY breaking and the specific coupling texture of the triplets and determining it exactly lies beyond the scope of this paper. In general the constraints are barely met when the triplets have a mass at the GUT scale [88–90], and in section 5 we have shown this is our case.

The existence of additional fields in the model may allow proton decay to arise from effective terms of the type

$$gQQQL\frac{\langle X\rangle}{M_P^2}. \tag{6.1}$$

Such terms must obey the constraint $g\langle X\rangle < 3\times 10^9\,\text{GeV}$ [88–90]. In our model, the largest contribution of this type comes from the term

$$\psi\psi\psi\psi\frac{\langle H^{B-L}_{45}(H^{X,Y}_{45})^2\rangle}{M_P^4} \quad \Rightarrow \quad \langle X\rangle = \frac{(M_{\rm GUT})^3}{M_P^2} \sim 10^{10}\ \text{GeV}. \tag{6.2}$$

The constraints are met when $g < 0.3$. With an $O(1)$ g parameter, the contributions coming from these terms are the same order as the ones coming from the Higgs triplets. In this model, proton decay complies with experimental constraints but lies fairly close to detection.

JHEP12(2017)075

7 Detailed Yukawa structure

Now that we have given VEVs to the fields in a specific direction, we may write the fully detailed Yukawa structure.

With the fields in the table 1, together with the messenger fields in table 2 we may write the superpotential relevant to the Yukawa terms, up to $O(1/M_P)$,

$$W_Y = \sum_{a=1,2,3} \left(\lambda_a^\phi \left(\psi\phi_a\right) \bar{\chi}_a + (\lambda_a^W H_{45}^W + \lambda_a^Z H_{45}^Z)\chi_a\bar{\chi}_a + \lambda_a^u \chi_a\chi_a H_{10}^u + \lambda_a^N \chi_a\chi_a \frac{H_{\overline{16}}H_{\overline{16}}}{M_P} \right) + \sum_{b=2,3} \left(\bar{\chi}_b\chi_b^d(\lambda_a^X H_{45}^X + \lambda_a^Y H_{45}^Y) + \lambda_b^d \chi_b^d\chi_b^d H_{10}^d \right) + \lambda_{12}^d \chi_1\chi_2 H_{10}^d + \lambda_t^d \frac{(\psi\psi)_{3'}\,\phi_3 H_{10}^d}{M_P}, \tag{7.1}$$

where $(\,)\,,\ (\,)_{3'}$ means an S_4 singlet or $3'$ contraction respectively. The λ's are dimensionless and real coupling constants (due to CP conservation) and are all expected to be $O(1)$.

After integrating the messengers χ, we obtain the superpotential

$$\begin{aligned} W_Y = \sum_{a=1,2,3} \Bigg(& \frac{(\lambda_a^\phi)^2 \left(\psi \langle\phi_a\rangle\right)\left(\psi \langle\phi_a\rangle\right)}{(\lambda_a^W \langle H_{45}^W\rangle + \lambda_a^Z \langle H_{45}^Z\rangle)^2} \lambda_\chi^u H_{10}^u \\ & + \frac{(\lambda_a^\phi)^2 \left(\psi \langle\phi_a\rangle\right)\left(\psi \langle\phi_a\rangle\right)}{(\lambda_a^W \langle H_{45}^W\rangle + \lambda_a^Z \langle H_{45}^Z\rangle)^2} \frac{\lambda_a^N}{M_P} \langle H_{\overline{16}}\rangle \langle H_{\overline{16}}\rangle \Bigg) \\ & + \Bigg(\sum_{b=2,3} \lambda_a^u \frac{(\lambda_b^\phi)^2 \left(\psi \langle\phi_b\rangle\right)\left(\psi \langle\phi_b\rangle\right)}{(\lambda_b^X \langle H_{45}^X\rangle + \lambda_b^Y \langle H_{45}^Y\rangle)^2} \\ & + \lambda_{12}^d \frac{\lambda_1^\phi \lambda_2^\phi \left(\psi \langle\phi_1\rangle\right)\left(\psi \langle\phi_2\rangle\right)}{(\lambda_1^W \langle H_{45}^W\rangle + \lambda_1^Z \langle H_{45}^Z\rangle)(\lambda_2^W \langle H_{45}^W\rangle + \lambda_2^Z \langle H_{45}^Z\rangle)} + \lambda_t^d \frac{(\psi\psi)_{3'} \langle\phi_3\rangle}{M_P} \Bigg) H_{10}^d. \end{aligned} \tag{7.2}$$

This superpotential generates all the SM fermion masses.

7.1 Renormalisability of the third family

In eq. (7.2), all the terms suppressed by $\langle H_{45}^{X,Y,W,Z}\rangle$ involve integrating out the messengers by assuming $M_{\rm GUT} \gg v_i$. This naive integration is not possible for the third flavon since it has a much larger VEV $v_3 \sim M_{\rm GUT}$. Let us single out the terms in W_Y involving these fields. Ignoring $\mathcal{O}(1)$ couplings, and after the fields get their VEV, the relevant terms are

$$W_Y^{(3)} \sim v_3\psi_3\bar{\chi}_3 + \langle H_{45}^{W,Z}\rangle\, \chi_3\bar{\chi}_3. \tag{7.3}$$

Naively, one would interpret ψ_3 as the set of third-family particles, but the first term in eq. (7.3) generates mixing with $\bar{\chi}_3$. To obtain the physical (massless) states, which we label t, we rotate into a physical basis $(\psi_3, \chi_3) \to (t, \chi)$

$$\psi_3 = \frac{\langle H_{45}^{W,Z}\rangle\, t + v_3\,\chi}{r}, \quad \chi_3 = \frac{-v_3\, t + \langle H_{45}^{W,Z}\rangle\,\chi}{r}; \quad r = \sqrt{v_3^2 + \langle H_{45}^{W,Z}\rangle^2}. \tag{7.4}$$

JHEP12(2017)075

Physically, it may be interpreted as follows: inside the original superpotential W_Y lie the terms

$$\mathcal{W}_Y \supset \chi_3\chi_3 H_{10}^{u,d} \supset \frac{v_3^2}{v_3^2 + \langle H_{45}^{W,Z}\rangle^2}\, t\, t\, H_{10}^{u,d}, \tag{7.5}$$

which generate renormalisable mass terms for the third family at the electroweak scale.

7.2 Mass matrix structure

The superpotential in eq. (7.2) generates all the SM fermion mass matrices. The structure of the mass matrices is fixed by the flavon VEV structure shown in eq. (1.1). We may redefine the dimensionless couplings to obtain the mass structure of the SM fermions at low energies

$$\begin{aligned}
y^u_{a=1,2} &= \lambda^u_a \frac{(\lambda^\phi_a)^2|v_a|^2}{[\lambda^W_a \langle H^W_{45}\rangle + \lambda^Z_a \langle H^Z_{45}\rangle]_Q [\lambda^W_a \langle H^W_{45}\rangle + \lambda^Z_a \langle H^Z_{45}\rangle]_{u^c}}, \\
y^u_3 &= \frac{(\lambda^\phi_3)^2|v_3|^2}{(\lambda^\phi_3)^2 v_3^2 + [\lambda^W_3 \langle H^W_{45}\rangle + \lambda^Z_3 \langle H^Z_{45}\rangle]_Q [\lambda^W_3 \langle H^W_{45}\rangle + \lambda^Z_3 \langle H^Z_{45}\rangle]_{u^c}}, \\
y^\nu_{a=1,2} &= \lambda^u_a \frac{(\lambda^\phi_a)^2|v_a|^2}{[\lambda^W_a \langle H^W_{45}\rangle + \lambda^Z_a \langle H^Z_{45}\rangle]_L [\lambda^W_a \langle H^W_{45}\rangle + \lambda^Z_a \langle H^Z_{45}\rangle]_{\nu^c}}, \\
y^\nu_3 &= \frac{(\lambda^\phi_3)^2|v_3|^2}{(\lambda^\phi_3)^2 v_3^2 + (\lambda^\chi_3)^2 [\lambda^W_3 \langle H^W_{45}\rangle + \lambda^Z_3 \langle H^Z_{45}\rangle]_L [\lambda^W_3 \langle H^W_{45}\rangle + \lambda^Z_3 \langle H^Z_{45}\rangle]_{\nu^c}}, \\
y^e_2 &= \lambda^d_2 \frac{(\lambda^\phi_2)^2|v_2|^2}{[\lambda^X_2 \langle H^X_{45}\rangle + \lambda^Y_2 \langle H^Y_{45}\rangle]_L [\lambda^X_2 \langle H^X_{45}\rangle + \lambda^Y_2 \langle H^Y_{45}\rangle]_{e^c}}, \\
y^e_3 &= \lambda^d_3 \frac{(\lambda^\phi_3)^2|v_3|^2}{(\lambda^\phi_3)^2 v_3^2 + [\lambda^X_3 \langle H^X_{45}\rangle + \lambda^Y_3 \langle H^Y_{45}\rangle]_L [\lambda^X_3 \langle H^X_{45}\rangle + \lambda^Y_3 \langle H^Y_{45}\rangle]_{e^c}}, \\
y^d_2 &= \lambda^d_2 \frac{(\lambda^\phi_2)^2|v_2|^2}{[\lambda^X_2 \langle H^X_{45}\rangle + \lambda^Y_2 \langle H^Y_{45}\rangle]_Q [\lambda^X_2 \langle H^X_{45}\rangle + \lambda^Y_2 \langle H^Y_{45}\rangle]_{d^c}}, \\
y^d_3 &= \lambda^d_3 \frac{(\lambda^\phi_3)^2|v_3|^2}{(\lambda^\phi_3)^2 v_3^2 + [\lambda^X_3 \langle H^X_{45}\rangle + \lambda^Y_3 \langle H^Y_{45}\rangle]_Q [\lambda^X_3 \langle H^X_{45}\rangle + \lambda^Y_3 \langle H^Y_{45}\rangle]_{d^c}}, \\
y^e_{12} &= \lambda^d_{12} \frac{\lambda^\phi_1 \lambda^\phi_2 |v_1 v_2|}{[\lambda^W_1 \langle H^W_{45}\rangle + \lambda^Z_1 \langle H^Z_{45}\rangle]_{L+e^c} [\lambda^W_2 \langle H^W_{45}\rangle + \lambda^Z_2 \langle H^Z_{45}\rangle]_{L+e^c}}, \\
y^d_{12} &= \lambda^d_{12} \frac{\lambda^\phi_1 \lambda^\phi_2 |v_1 v_2|}{[\lambda^W_1 \langle H^W_{45}\rangle + \lambda^Z_1 \langle H^Z_{45}\rangle]_{Q+d^c} [\lambda^W_2 \langle H^W_{45}\rangle + \lambda^Z_2 \langle H^Z_{45}\rangle]_{Q+d^c}}, \\
M^{\mathrm{R}}_{a=1,2} &= \frac{\lambda^N_a v_{\overline{16}}^2}{M_P} \frac{(\lambda^\phi_a)^2|v_a|^2}{[\lambda^W_a \langle H^W_{45}\rangle + \lambda^Z_a \langle H^Z_{45}\rangle]^2_{\nu^c}}, \\
M^{\mathrm{R}}_3 &= \frac{\lambda^N_3 v_{\overline{16}}^2}{M_P} \frac{(\lambda^\phi_3)^2|v_3|^2}{(\lambda^\phi_3)^2 v_3^2 + [\lambda^W_3 \langle H^W_{45}\rangle + \lambda^Z_3 \langle H^Z_{45}\rangle]^2_{\nu^c}}, \\
y^P &= \lambda^d_t \frac{Y_P v_3}{M_P},
\end{aligned} \tag{7.6}$$

where $\langle H_{45}^{X,Y,W,Z}\rangle_f$ denotes the adjoint VEV with the corresponding CG coefficients for each SM fermion f. This allows for each y, M parameter in eq. (7.6) to be independent.

JHEP12(2017)075

The VEVs $\langle H^{X,Y}\rangle$ obtain a VEV in an arbitrary SO(10) breaking direction. They need to be different from one another.

For a better understanding we can show an explicit example. Let us assume that $\langle H_{45}^{X,Y}\rangle$ is aligned in the $\mathrm{U}(1)_{X,Y}$ direction respectively with an M_{GUT} magnitude. In this case the effective Yukawa couplings $y_2^{e,d}$ would be

$$y_2^e = \lambda_2^d \frac{(\lambda_2^\phi)^2|v_2|^2}{[3\lambda_2^X - \lambda_2^Y/2][-\lambda_2^X + \lambda_2^Y]M_{\mathrm{GUT}}^2}, \quad y_2^d = \lambda_2^d \frac{(\lambda_2^\phi)^2|v_2|^2}{[-\lambda_2^X + \lambda_2^Y/6][3\lambda_2^X + \lambda_2^Y/3]M_{\mathrm{GUT}}^2}, \tag{7.7}$$

where the coefficients multiplying each $\lambda^{X,Y}$ are the $\mathrm{U}(1)_{X,Y}$ charges of the corresponding SM field. Since the $\lambda_2^{X,Y}$ appear with different coefficients in $y_2^{e,d}$, we can use them to obtain a arbitrarily different effective Yukawa coupling for charged leptons and down type quarks.

Assuming the adjoints have all real VEVs, the physical phases are

$$\begin{aligned} \eta &= 2\arg v_1 - 2\arg v_2 \\ \eta' &= 2\arg v_3 - 2\arg v_2 \\ \gamma &= \arg v_3 - 2\arg v_2, \end{aligned} \tag{7.8}$$

while all the $y's$ and $M's$ are real.

With these definitions we may write the fermion mass matrices

$$\begin{aligned}
M^e/v_d = &\quad y_{12}^e e^{i\eta/2}\begin{pmatrix}0&1&1\\1&4&2\\1&2&0\end{pmatrix} + \quad y_2^e\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + \quad y_3^e e^{i\eta'}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} \quad + y^P e^{i\gamma}\begin{pmatrix}0&0&1\\0&2&0\\1&0&0\end{pmatrix}, \\
M^d/v_d = &\quad y_{12}^d e^{i\eta/2}\begin{pmatrix}0&1&1\\1&4&2\\1&2&0\end{pmatrix} + \quad y_2^d\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + \quad y_3^d e^{i\eta'}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} \quad + y^P e^{i\gamma}\begin{pmatrix}0&0&1\\0&2&0\\1&0&0\end{pmatrix}, \\
M^u/v_u = &\quad y_1^u e^{i\eta}\begin{pmatrix}1&2&0\\2&4&0\\0&0&0\end{pmatrix} + \quad y_2^u\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + \quad y_3^u e^{i\eta'}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}, \\
M_D^\nu/v_u = &\quad y_1^\nu e^{i\eta}\begin{pmatrix}1&2&0\\2&4&0\\0&0&0\end{pmatrix} + \quad y_2^\nu\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + \quad y_3^\nu e^{i\eta'}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}, \\
M^R = &\quad M_1^R e^{i\eta}\begin{pmatrix}1&2&0\\2&4&0\\0&0&0\end{pmatrix} + \quad M_2^R\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} + \quad M_3^R e^{i\eta'}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}.
\end{aligned} \tag{7.9}$$

We note the remarkable universal structure of the matrices in the up and neutrino sectors, which differ from the down and charged lepton sectors.

The y and M parameters are all free and independent while there is a constraint in the phases

$$\eta \simeq 4\eta' - 2\gamma, \tag{7.10}$$

JHEP12(2017)075

as shown in the section 4. We have in total 18 free parameters that fix the whole spectrum of fermion masses and mixing angles, as discussed in section 8.2.

As shown in the section 4, the flavons and adjoint fields get a VEV

$$\begin{aligned} M_{\rm GUT} &\sim v_3 \sim 10\ v_2 \sim 1000\ v_1, \\ M_{\rm GUT} &\sim v_{45}^{X,Y,W,Z} \sim\ M_\rho \sim v_{\overline{16}}. \end{aligned} \tag{7.11}$$

Assuming all the parameters in the superpotential we have ignored are $O(1)$, and $\tan\beta \sim 20$, the mass matrix parameters are expected to be

$$\begin{aligned} &y_1^u \sim y_1^\nu \sim v_1^2/v_{45}^2 \sim 10^{-6}, && y_2^u \sim y_2^\nu \sim v_2^2/v_{45}^2 \sim 10^{-2}, \\ &y_3^u \sim y_3^\nu \sim v_3^2/v_{45}^2 \sim 1, && y_{12}^d \sim y_{12}^e \sim \cos\beta\ v_1 v_2/v_{45}^2 \sim 10^{-5}, \\ &y_2^d \sim y_2^e \sim \cos\beta\ v_2^2/v_{45}^2 \sim 10^{-3}, && y_3^d \sim y_3^e \sim \cos\beta\ v_3^2/v_{45}^2 \sim 0.1, \\ &y^P \sim \cos\beta\ v_3/M_P \sim 10^{-4}, && M_1^R \sim v_{\overline{16}}^2 v_1^2/v_{45}^2 M_P \sim 10^7\ GeV, \\ &M_2^R \sim v_{\overline{16}}^2 v_2^2/v_{45}^2 M_P \sim 10^{11}\ GeV, && M_3^R \sim v_{\overline{16}}^2 v_3^2/v_{45}^2 M_P \sim 10^{13}\ GeV. \end{aligned} \tag{7.12}$$

These values denote only an approximate order of magnitude for each parameter and are expected to be different due to the appearance of dimensionless couplings. This applies specially to the last 4 parameters that come from unknown Planck suppressed physics and may deviate significantly from our naive expectation.

7.3 Seesaw mechanism

Since we have very heavy RHN Majorana masses, the left handed neutrinos get a very small Majorana mass through type I seesaw

$$m_L^\nu = M_D^\nu (M^R)^{-1} (M_D^\nu)^T. \tag{7.13}$$

As we see in eq. (7.9), the Dirac neutrino masses M_D^ν and RHN Majorana masses M^R have the same matrix structure. These are rank one matrices so that we may write them as

$$\begin{aligned} M_D^\nu/v_u &= y_1^\nu e^{i\eta}\ \varphi_1\varphi_1^T + y_2^\nu\ \varphi_2\varphi_2^T + y_3^\nu e^{i\eta'}\ \varphi_3\varphi_3^T, \\ M^R &= M_1^R e^{i\eta}\ \varphi_1\varphi_1^T + M_2^R\ \varphi_2\varphi_2^T + M_3^R e^{i\eta'}\ \varphi_3\varphi_3^T, \end{aligned} \tag{7.14}$$

with

$$\varphi_1^T = (1,2,0), \quad \varphi_2^T = (0,1,1), \quad \varphi_3^T = (0,0,1). \tag{7.15}$$

We may always find vectors $\tilde\varphi_a$ such that

$$\tilde\varphi_i^T \varphi_j = \delta_{ij}, \tag{7.16}$$

this way we may write the inverse matrix as

$$(M^R)^{-1} = \frac{e^{-i\eta}}{M_1^R}\tilde\varphi_1\tilde\varphi_1^T + \frac{1}{M_2^R}\tilde\varphi_2\tilde\varphi_2^T + \frac{e^{-i\eta'}}{M_3^R}\tilde\varphi_3\tilde\varphi_3^T. \tag{7.17}$$

JHEP12(2017)075

Plugging this into the seesaw mechanism we obtain the light effective left-handed Majorana neutrino mass matrix m_L^ν,

$$m_L^\nu = \mu_1 e^{i\eta}\ \varphi_1\varphi_1^T + \mu_2\ \varphi_2\varphi_2^T + \mu_3 e^{i\eta'}\ \varphi_3\varphi_3^T, \quad \text{with} \quad \mu_a = \frac{(y_a^\nu v_u)^2}{M_a^R} \tag{7.18}$$

so that we may conclude that the small left handed neutrino mass matrix has the same universal structure

$$m_L^\nu = \mu_1 e^{i\eta} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \mu_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \mu_3 e^{i\eta'} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{7.19}$$

after the seesaw mechanism.

8 Numerical fit

To test our model we perform a numerical fit using a χ^2 test function. We have a set of input parameters $x = \{y_i^u, y_i^d, y_i^e, y^P, \mu_i, \eta', \gamma\}$, from which we obtain a set of observables $P_n(x)$. We minimize the function defined as

$$\chi^2 = \sum_n \left(\frac{P_n(x) - P_n^{\text{obs}}}{\sigma_n} \right)^2, \tag{8.1}$$

where the 19 observables are given by $P_n^{\text{obs}} \in \{\theta_{ij}^q, \delta^q, y_{u,c,t}, y_{d,s,b}, \theta_{ij}^\ell, \delta^l, y_{e,\mu,\tau}, \Delta m_{ij}^2\}$ with statistical errors σ_n. This test assumes data is normally (Gaussian) distributed, which is true for most of the observables except for θ_{23}^ℓ. The atmospheric mixing angle octant, i.e. $\theta_{23}^\ell < 45°$ or $\theta_{23}^\ell > 45°$, has not been determined yet. Current data favours $\theta_{23}^\ell = 41.6$ [93] and we assume such scenario.

We need to run up all the measured Yukawa couplings and mixing angles up to the GUT scale in order to compare it with the predictions of our model.[5] In doing so, we need to match the SM to the MSSM at the SUSY scale, M_{SUSY}, which involves adding the supersymmetric radiative threshold corrections. This has been done in [94]. At the GUT scale, the values depend to a good approximation only on $\bar{\eta}_b$ and $\tan\beta$. A good fit is found for large $\bar{\eta}_b$, which can be explained if $\tan\beta \gtrsim 10$, as shown in the section 8.1. We also need $\tan\beta < 30$ to keep Yukawa couplings perturbative. The best fit is found for $\bar{\eta}_b = -0.9$ and $\tan\beta = 20$. The SUSY scale does not affect the fit and we choose $M_{\text{SUSY}} = 1\,\text{TeV}$. The fit has been performed using the Mixing Parameter Tools (MPT) package [95].

The best fit found has a $\chi^2 = 11.9$. Table 3 shows the best fit to the charged leptons and neutrinos observables. Neutrino data is taken from the Nufit global fit [93]. Only the neutrino mass-squared differences are known but our model also predicts the neutrino masses themselves as well as the Majorana phases. The model predicts normal ordered neutrino masses and we also give the effective Majorana mass $m_{\beta\beta}$. All the lepton sector is fitted to within 1σ except for the leptonic CP phase. δ^ℓ is not yet well measured, although a negative CP phase is preferred [96, 97].

[5] Note that we are performing the numerical fit in terms of the effective neutrino mass parameters μ_i defined in eq. (7.19). We are ignoring any renormalisation group running corrections in the neutrino sector.

JHEP12(2017)075

Observable	Data		Model
	Central value	1σ range	Best fit
θ_{12}^{ℓ} /°	33.57	$32.81 \to 34.33$	33.53
θ_{13}^{ℓ} /°	8.460	$8.310 \to 8.610$	8.452
θ_{23}^{ℓ} /°	41.75	$40.40 \to 43.10$	41.88
δ^{ℓ} /°	261.0	$206.0 \to 316.0$	200.3
y_e /10^{-5}	6.023	$5.987 \to 6.059$	6.023
y_μ /10^{-2}	1.272	$1.264 \to 1.280$	1.272
y_τ	0.222	$0.219 \to 0.225$	0.222
$\Delta m_{21}^2/(10^{-5}\,\mathrm{eV}^2)$	7.510	$7.330 \to 7.690$	7.507
$\Delta m_{31}^2/(10^{-3}\,\mathrm{eV}^2)$	2.524	$2.484 \to 2.564$	2.524
m_1 /meV			10.94
m_2 /meV			13.95
m_3 /meV			51.42
$\sum m_i$ /meV		< 230	76.31
α_{21} /°			134.3
α_{31} /°			6.415
$m_{\beta\beta}$ /meV		< 61–165	11.10

Table 3. Model predictions in the lepton sector for $\tan\beta = 20$, $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$ and $\bar{\eta}_b = -0.9$. The observables are at the GUT scale. The lepton contribution to the total χ^2 is 1.2. The neutrino masses m_i as well as the Majorana phases are pure predictions of our model. The bound on $\sum m_i$ is taken from [91]. The bound on $m_{\beta\beta}$ is taken from [92].

Observable	Data		Model
	Central value	1σ range	Best fit
θ_{12}^{q} /°	13.03	$12.99 \to 13.07$	13.02
θ_{13}^{q} /°	0.016	$0.016 \to 0.017$	0.016
θ_{23}^{q} /°	0.189	$0.186 \to 0.192$	0.186
δ^{q} /°	69.22	$66.12 \to 72.31$	70.66
y_u /10^{-6}	3.060	$2.111 \to 4.009$	3.253
y_c /10^{-3}	1.497	$1.444 \to 1.549$	1.567
y_t	0.666	$0.637 \to 0.694$	0.611
y_d /10^{-4}	1.473	$1.311 \to 1.635$	1.614
y_s /10^{-3}	2.918	$2.760 \to 3.075$	3.098
y_b	2.363	$2.268 \to 2.457$	2.238

Table 4. Model predictions in the quark sector for $\tan\beta = 20$, $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$ and $\bar{\eta}_b = -0.9$. The observables are at the GUT scale. The quark contribution to the total χ^2 is 10.7.

In table 4, we have all the quark Yukawa couplings and mixing parameters for the minimum χ^2. The biggest contribution to the χ^2 is coming from this sector, as shown in figure 4. This figure shows the corresponding pulls for lepton (light orange) and quark (blue) observables. As we can see, all parameters lie inside the 2σ region and the biggest pulls are in the quark Yukawa couplings.

JHEP12(2017)075

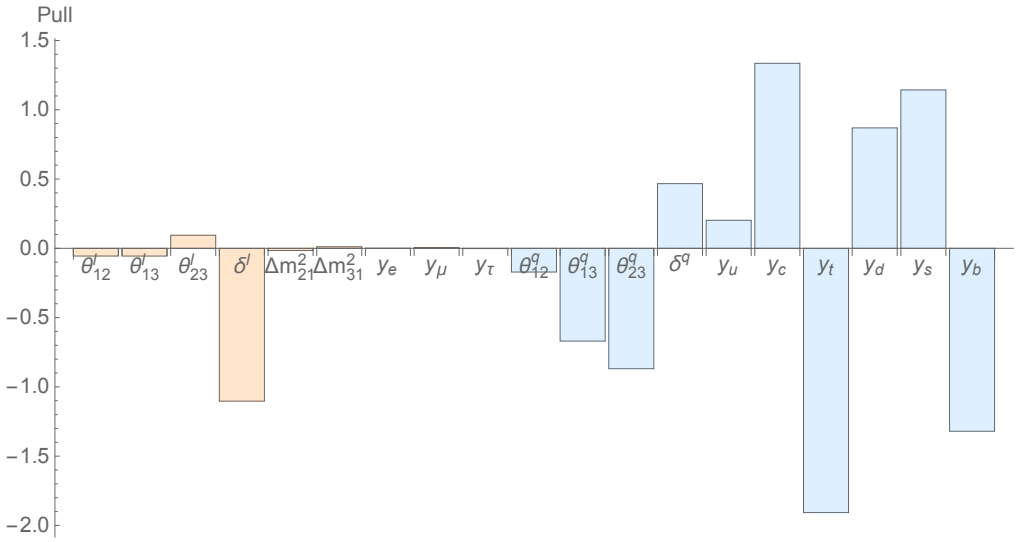

Figure 4. Pulls for the best fit of model to data, as shown in tables 3–4, for lepton (light orange) and quark (blue) parameters.

Parameter	Value
$y_1^u\,/10^{-6}$	3.232
$y_2^u\,/10^{-3}$	1.580
y_3^u	−0.610
$y_{12}^d\,/10^{-4}$	−7.068
$y_2^d\,/10^{-4}$	−8.737
y_3^d	−2.238

Parameter	Value
$y_{12}^e\,/10^{-4}$	8.616
$y_2^e\,/10^{-2}$	1.013
y_3^e	0.229
μ_1 /meV	6.845
μ_2 /meV	27.18
μ_3 /meV	42.17

Parameter	Value
$y^P\,/10^{-4}$	2.475
γ	1.968π
η'	0.790π

Table 5. Best fit input parameter values. The model has 13 real parameters: y_i^u, y_i^d, y_i^e, μ_i and y^P and two additional free phases: η' and γ. The total χ^2 is 11.9.

Table 5 shows the input parameter values.[6] There are 13 real parameters plus two additional phases, a total of 15 input parameters to fit 19 data points. Naively, we can measure the goodness of the fit computing the reduced χ^2, i.e. the χ^2 per degree of freedom $\chi^2_\nu = \chi^2/\nu$. The number of degrees of freedom is given by $\nu = n - n_i$, where $n = 19$ is the number of measured observables, while $n_i = 15$ is the number of input parameters. A good fit is expected to have $\chi^2_\nu \sim 1$. We have 4 degrees of freedom and the best fit has a reduced $\chi^2_\nu \simeq 3$. We view this as a good fit and it also remarks the predictivity of the model, not only fitting to all available quark and lepton data but also fixing the neutrino masses and Majorana phases.

[6] Assuming the Dirac neutrino Yukawa parameters y_i^ν in eq. (7.12), we can compute the RHN masses, using the Seesaw formula in eq. (7.18) and taking the μ_i values from the fit, such that $M_1^R \sim 10^4\,\text{GeV}$, $M_2^R \sim 10^{11}\,\text{GeV}$ and $M_3^R \sim 10^{15}\,\text{GeV}$. Only M_2 has the expected natural value given in eq. (7.12). We remark that RHN Majorana masses come from unknown Planck suppressed physics, which is presumably responsible for the mismatch.

JHEP12(2017)075

8.1 SUSY threshold corrections

The running of the MSSM Yukawa couplings to the GUT scale, $M_{\rm GUT}$, was performed in [94]. Here, the threshold corrections at the SUSY scale, $M_{\rm SUSY}$, are parametrized by

$$\begin{aligned} y_{u,c,t}^{\rm MSSM} &\simeq y_{u,c,t}^{\rm SM} \csc\bar{\beta}, \\ y_{d,s}^{\rm MSSM} &\simeq (1+\bar{\eta}_q)^{-1}\, y_{d,s}^{\rm SM} \sec\bar{\beta}, \\ y_{b}^{\rm MSSM} &\simeq (1+\bar{\eta}_b)^{-1}\, y_{b}^{\rm SM} \sec\bar{\beta}, \\ y_{e,\mu}^{\rm MSSM} &\simeq (1+\bar{\eta}_\ell)^{-1}\, y_{e,\mu}^{\rm SM} \sec\bar{\beta}, \\ y_{\tau}^{\rm MSSM} &\simeq y_{\tau}^{\rm MSSM} \sec\bar{\beta}. \end{aligned} \tag{8.2}$$

The CKM parameters become

$$\theta_{i3}^{q,\rm MSSM} \simeq \frac{1+\bar{\eta}_b}{1+\bar{\eta}_q}\,\theta_{i3}^{q,\rm SM}, \qquad \theta_{12}^{q,\rm MSSM} \simeq \theta_{12}^{q,\rm SM}, \qquad \delta^{q,\rm MSSM} \simeq \delta^{q,\rm SM}. \tag{8.3}$$

When running between $M_{\rm SUSY}$ and $M_{\rm GUT}$, the most relevant parameters are $\bar{\eta}_b$ and $\tan\bar{\beta}$. Due to their small contribtutions, we assume $\bar{\eta}_q = \bar{\eta}_\ell = 0$ and $\bar{\beta} = \beta$. These assumptions do not affect the quality of the fit. Similarly, we fix $M_{\rm SUSY} = 1\,\text{TeV}$. The effect on the fit, of having it up to $\mathcal{O}(10)\,\text{TeV}$, is minor.

Specifically, the parameter $\bar{\eta}_b$ is required to be somewhat large $\bar{\eta}_b = -0.9$ to obtain a good quality fit. Ignoring it would yield a fit of $\chi^2 \sim 400$. The leading contributions to this parameter come from loops either sbottoms and gluinos or stops and higgsinos that add up to [98]

$$\bar{\eta}_b \simeq \frac{\tan\beta}{16\pi^2}\left(\frac{8}{3}g_3^2\frac{m_{\tilde{g}}\mu}{2m_0^2} + \lambda_t^2\frac{\mu A_t}{m_0^2}\right), \tag{8.4}$$

where m_0 represents the squark masses, g_3 the strong coupling, $m_{\tilde{g}}$ the gluino mass and A_t the SUSY softly breaking trilinear coupling involving the stops. We see that a large contribution can be achieved when

$$m_{\tilde{g}}, \mu, A_t > m_0, \qquad \tan\beta \gtrsim 10. \tag{8.5}$$

Since SUSY breaking lies beyond the scope of our paper, it is sufficient for us to show that there is a parameter space in the softly broken SUSY that generates the necessary corrections.

8.2 Parameter counting

In this section we explain and clarify the number of parameters in our model. Clearly at the high energy scale there are many parameters associated with the undetermined $O(1)$ Yukawa couplings of the 43 superfields of the model. For example the renormalisable Yukawa superpotential in eq. (7.1) contains 23 parameters alone. Then we must add to this all the $O(1)$ Yukawa couplings associated with vacuum alignment, GUT symmetry breaking and doublet-triplet splitting, many of which we have not defined explicitly. Despite this, we are claiming that our model is predictive at low energies. How can this be? The short answer is that most of these parameters are irrelevant for physics below the GUT scale, as discussed in detail below.

JHEP12(2017)075

The effective fermion mass matrices generated below the GUT scale are given in eq. (7.9) as function of 18 free effective parameters (remembering the constraint on η) that will fix all the fermion masses and mixing angles, including RHN Majorana masses and Majorana phases. This compares favourably to the 31 parameters of a general high energy model, comprising 21 parameters in the lepton sector of a general 3 right-handed neutrino seesaw model [99], plus the 6 quark masses and 4 CKM parameters. However, below the seesaw scale of right-handed neutrino masses, the effective parameter counting is different again and requires further discussion below.

In order to perform the fit and compare our model with available data, we apply the seesaw mechanism to write the light effective left-handed Majorana neutrino mass matrix as a function of the new parameters μ_i in Eq 7.18. Therefore, we have 15 effective parameters at low energy (shown in table 5) that fit the 19 so far measured or constrained observables in figure 4.[7] After the fit is performed, the model predicts all the three light neutrino masses with a normal ordering, a CP oscillation phase of 260° and both the Majorana phases, corresponding to a total of 22 low energy observables which will be eventually observable (10 from the quark sector discussed above and 12 from the lepton sector, including the two Majorana phases). Therefore we see that, below the seesaw scales, the model contains 15 effective parameters which generate 22 observables, making the model eminently testable, as these observables become better determined.

9 N_2 leptogenesis

The source of the Baryon Asymmetry of the Universe (BAU)

$$\eta_B^{\rm CMB} = (6.1 \pm 0.1) \times 10^{-10}, \tag{9.1}$$

remains unexplained in the SM. One of the most convincing sources for it is Leptogenesis, where the asymmetry is generated through CP breaking decays of heavy RHNs into leptons, then converted into baryons through sphalerons [100].

The simplest mechanism to generate the correct BAU, happens when the lightest RHNs has CP breaking decays and has a mass of about $\sim 10^{10} GeV$. In our model, according to eq. (7.12), it is the second RHN the one that is expected to be at that scale. When leptogenesis is generated mainly by the decays of the second RHN it is called N_2 leptogenesis. This has already been calculated in [83] and we will apply such calculations to our specific model.

9.1 General N_2 leptogenesis

Leptogenesis calculations are done in the so called Flavor Basis, where the charged lepton and RHN mass matrices are diagonal and we work with the Dirac neutrino mass matrix

$$\begin{aligned} m_D &= V_{eL} M_D^{\nu} U_N^T, \\ V_{eL} M^{e\dagger} M^e V_{eL}^{\dagger} &= diag(y_e^2, y_\mu^2, y_\tau^2) v_d^2, \qquad U_N M^R U_N^T = diag(M_1, M_2, M_3). \end{aligned} \tag{9.2}$$

[7] We need to run up to the GUT scale these observables and, therefore, we need to include SUSY threshold corrections. The fit is therefore also dependent on $\bar{\eta}_b$ and $\tan\beta$. As shown earlier, we find a good fit for $\bar{\eta}_b = -0.9$ and $\tan\beta = 20$.

JHEP12(2017)075

The total and flavoured decay parameters, K_i and $K_{i\alpha}$ respectively, can be written as

$$K_{i\alpha} = \frac{|m_{D\alpha i}|^2}{m_\star^{\rm MSSM}\, M_i} \quad \text{and} \quad K_i = \sum_\alpha K_{i\alpha} = \frac{(m_D^\dagger\, m_D)_{ii}}{m_\star^{\rm MSSM}\, M_i}\,, \tag{9.3}$$

where the equilibrium neutrino mass is given by

$$m_\star^{\rm MSSM} \simeq 0.78 \times 10^{-3}\,{\rm eV}\,\sin^2\beta. \tag{9.4}$$

The wash-out at the production is described by the efficiency factor $\kappa(K_{2\alpha})$ that for an initial thermal N_2 abundance can be calculated as

$$\kappa(K_{2\alpha}) = \frac{2}{z_B(K_{2\alpha})\, K_{2\alpha}} \left(1 - e^{-\frac{K_{2\alpha}\, z_B(K_{2\alpha})}{2}}\right)\,, \quad z_B(K_{2\alpha}) \simeq 2 + 4\, K_{2\alpha}^{0.13}\, e^{-\frac{2.5}{K_{2\alpha}}}\,. \tag{9.5}$$

In the hierarchical RH neutrino mass limit, as our model is, the CP asymmetries can be approximated to

$$\varepsilon_2 = \sum_\alpha \varepsilon_{2\alpha}, \quad \varepsilon_{2\alpha} \simeq \frac{3}{8\,\pi}\frac{M_2}{v^2}\,\frac{{\rm Im}\Big[\big(m_D^\dagger\big)_{i\alpha}\,\big(m_D\big)_{\alpha 3}\big(m_D^\dagger m_D\big)_{i3}\Big]}{M_2\, M_3\, \widetilde{m}_2}, \tag{9.6}$$

where $\widetilde{m}_2 \equiv (m_D^\dagger\, m_D)_{22}/M_2$.

In the regime where $5 \times 10^{11}\,{\rm GeV}\,(1+\tan^2\beta) \gg M_2 \gg 5 \times 10^{8}\,{\rm GeV}\,(1+\tan^2\beta)$, the final $B-L$ asymmetry can be calculated using

$$\begin{aligned} N_{B-L}^{\rm f} \simeq & \left[\frac{K_{2e}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2e} - \frac{K_{2e}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\,\kappa(K_{2\tau_2^\perp}/2)\right]\, e^{-\frac{3\pi}{8}\,K_{1e}} \\ & + \left[\frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\,\kappa(K_{2\tau_2^\perp}) + \left(\varepsilon_{2\mu} - \frac{K_{2\mu}}{K_{2\tau_2^\perp}}\,\varepsilon_{2\tau_2^\perp}\right)\,\kappa(K_{2\tau_2^\perp}/2)\right]\, e^{-\frac{3\pi}{8}\,K_{1\mu}} \\ & + \varepsilon_{2\tau}\,\kappa(K_{2\tau})\, e^{-\frac{3\pi}{8}\,K_{1\tau}}\,, \end{aligned} \tag{9.7}$$

where we indicated with $\tau_2^\perp$ the electron plus muon component of the quantum flavour states produced by the N_2-decays defining $K_{2\tau_2^\perp} \equiv K_{2e} + K_{2\mu}$, $\varepsilon_{2\tau_2^\perp} \equiv \varepsilon_{2e} + \varepsilon_{2\mu}$. The final asymmetry, in terms of the baryon to photon number ratio is

$$\eta_B \simeq 2\; a_{\rm sph}\frac{N_{B-L}}{N_\gamma^{\rm rec}}, \tag{9.8}$$

where $\alpha_{\rm sph} = 8/23$ is the fraction of $B-L$ asymmetry converted into baryon asymmetry by sphalerons. The photon asymmetry at recombination is $(N_\gamma^{\rm rec})^{\rm MSSM} \simeq 78$. The factor of 2 accounts for the asymmetry generated by the RH neutrinos and sneutrinos.

9.2 Leptogenesis in our model

Using the matrices in eq. (7.9) and the fit in table 5, we may calculate the BAU generated through N_2 Leptogenesis in our model. The first thing to note is that the parameters are

JHEP12(2017)075

quite hierarchical so that the rotation angles of the diagonalizing matrices can be neglected since they only give 1% contributions

$$V_{eL} \simeq \mathbb{1}, \qquad U_N \simeq diag(e^{-i\eta/2}, 0, e^{-i\eta'/2}), \tag{9.9}$$

and the neutrino mass matrix in the Flavor Basis becomes

$$m_{Dij} \simeq \begin{pmatrix} y_1^\nu e^{i\eta/2} & 2y_1^\nu e^{i\eta} & 0 \\ 2\, y_1^\nu e^{i\eta/2} & y_2^\nu & y_2^\nu e^{-i\eta'/2} \\ 0 & y_2^\nu & y_3^\nu e^{i\eta'/2} \end{pmatrix} v_u. \tag{9.10}$$

Also, due to the hierarchical nature of the couplings we may safely assume that the RHN mass parameter are equal to their mass eigenvalues $M_a^R \simeq M_a$.

One of the features of the matrix structure is that $K_{1\tau}$ vanishes, due to the approximate zero in the (3,1) entry of the Dirac mass matrix,[8] so that the last term in eq. (9.7) is greatly enhanced since it overcomes the exponential suppression. With these approximations, the baryon asymmetry becomes

$$\begin{aligned} \eta_B &\simeq \frac{2\alpha_{\rm sph}}{N_\gamma^{\rm rec}}\, \kappa(K_{2\tau})\, \varepsilon_{2\tau}, \\ K_{2\tau} &= \frac{(y_2^\nu)^2 v_u^2}{m_\star^{\rm MSSM} M_2}, \qquad \epsilon_{2\tau} = \sin\eta' \frac{3}{8\pi} \frac{M_2}{M_3} \frac{(y_3^\nu)^2}{2} \sin^2\beta. \end{aligned} \tag{9.11}$$

We note that η' is identified with the leptogenesis phase. With use of eq. (7.18), we may write the neutrino Yukawa couplings as $y_a^\nu = \sqrt{\mu_a M_a^R}/v_u$ so that

$$\eta_B \simeq \sin\eta' \frac{3}{8\pi} \frac{\alpha_{\rm sph}}{N_\gamma^{\rm rec}}\, \kappa\left(\frac{\mu_2}{m_\star^{\rm MSSM}}\right) \frac{\mu_3 M_2}{v^2}, \tag{9.12}$$

where we note that the only free parameter is M_2. Using the parameters from the fit, in table 5, the correct BAU is generated when[9]

$$M_2 \simeq 1.9 \times 10^{11}\ GeV\,. \tag{9.13}$$

From eq. (7.12) we see that this is the natural value for the second RHN mass, so that the model naturally explains the origin of the BAU through N_2 leptogenesis without any need for tuning.

10 Conclusion

We have constructed a SUSY GUT of flavour based on the symmetry $S_4 \times {\rm SO}(10) \times \mathbb{Z}_4^3 \times \mathbb{Z}_4^R$ that is relatively simple, predictive and fairly complete. The Higgs sector of the model involves two SO(10) 10-plets, a 16-plet and its conjugate representation, and three 45-plets. These low dimensional Higgs representations are all that is required to break the

[8] The zero is a consequence of the CSD2 vaccum alignment; it would not be zero for CSD3 vacuum alignment.

[9] M_2 has been computed numerically, including the rotation angles of the diagonalizing matrices in eq. (9.10).

JHEP12(2017)075

GUT symmetry, yield the Clebsch relations responsible for the difference of the charged fermion masses, and account for heavy Majorana right-handed neutrino masses. In order to account for the hierarchical mixing structure of the Yukawa matrices, we also need a particular set of S_4 triplet flavons with hierarchical VEVs and particular CSD2 vacuum alignments, where both features are fully discussed here. To complete the model we also require a rather rich spectrum of high energy messenger and alignment superfields, which, like most of the Higgs fields, do not appear in the low energy effective theory.

We highlight and summarise the main successes and features of the model as follows:

- The model is succesfully built with an SO(10) gauge symmetry where all of the fields belong to the small "named" representations: fundamental, spinorial and adjoints; this could be helpful for a possible future string embedding.

- It contains a superpotential that spontaneously breaks the original symmetry: $S_4{\times}\mathrm{SO}(10){\times}\mathbb{Z}_4^3{\times}\mathbb{Z}_4^R \to \mathrm{SU}(3)_C{\times}\mathrm{SU}(2)_L{\times}\mathrm{U}(1)_Y{\times}\mathbb{Z}_2^R$. The model also spontaneously breaks CP.

- The S_4 breaking superpotential that yields the CSD2 vacuum alignment is fairly simple.

- All the GUT scale parameters are natural and $\sim O(1)$, explaining the hierarchy of the low energy parameters, where the family mass hierarchy is due the derived hierarchy of flavon VEVs $|v_1| \ll |v_2| \ll |v_3|$, rather than by Froggatt-Nielsen.

- The model contains a working doublet-triplet mechanism, that yields exactly two light Higgs doublets from two SO(10) Higgs multiplets, respectively and without mixing, apart from the μ term which is generated at the correct scale. It also has well behaved proton decay.

- The model naturally generates sufficient BAU through N_2 Leptogenesis, which fixes the second right-handed neutrino mass $M_2 \simeq 2 \times 10^{11}\,\mathrm{GeV}$, in the natural range predicted by the model.

- At low energies, the model contains 15 free parameters that generate 19 presently constrained observables so that it is quite predictive. The model achieves an excellent fit of the SM fermion masses and mixing angles, with $\chi^2 = 11.9$.

We find it remarkable that all of the above can be achieved consistently within a single model. It contains 43 supermultiplet fields, which is the minimal number for any such complete model in the literature so far.

Despite the above successes of the model, it also has a few drawbacks. It does not explain SUSY breaking, and it relies on specific threshold corrections. Even though it has an almost complete UV completion, it still relies on $O(1/M_P)$ terms for the right-handed neutrino masses. Indeed M_1 and M_3 apparently do not have such natural values as M_2, and we are forced to explain this away by appealing to the unknown physics at the Planck scale. The symmetry breaking superpotential gives VEVs to most of the GUT breaking fields but

JHEP12(2017)075

it does not drive all of them. Also we do not address the strong CP problem, inflation or Dark Matter (DM) (which may in principle be due to the lightest SUSY particle, stabilised by the R-parity). Indeed we have not considered the SUSY spectrum at all. Such issues are beyond the stated aims of the present paper, which is to propose a complete grand unified theory of flavour and leptogenesis, consistent with the latest data on quark and lepton masses and mixing parameters, in which the three families of quarks and leptons are unified into a single $(3', 16)$ representation of $S_4 \times \mathrm{SO}(10)$.

Importantly, the model can be tested due to its robust predictions of a normal neutrino mass ordering, a CP oscillation phase of 260°, an atmospheric angle of 42° in the first octant and a neutrinoless double beta decay parameter $m_{\beta\beta} = 11$ meV, with the sum of neutrino masses being 76 meV. These predictions, together with the other lepton mixing angles given earlier, will enable the model to be tested by the forthcoming neutrino experiments.

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References

[1] Particle Data Group collaboration, C. Patrignani et al., *Review of particle physics*, *Chin. Phys.* **C 40** (2016) 100001 [INSPIRE].

[2] Super-Kamiokande collaboration, Y. Fukuda et al., *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003] [INSPIRE].

[3] SNO collaboration, Q.R. Ahmad et al., *Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory*, *Phys. Rev. Lett.* **89** (2002) 011301 [nucl-ex/0204008] [INSPIRE].

[4] P. Minkowski, $\mu \to e\gamma$ *at a rate of one out of* 10^9 *muon decays?*, *Phys. Lett.* **67B** (1977) 421 [INSPIRE].

[5] T. Yanagida, *Horizontal symmetry and masses of neutrinos*, in the proceedings of the *Workshop on unified theory and baryon number in the universe*, O. Sawada and A. Sugamoto eds., KEK, Tsukuba, Japan (1979).

[6] M. Gell-Mann, P. Ramond, R. Slansky, *Complex spinors and unified theories*, in *Supergravity*, D.Z. Freedman and P.van Nieuwenhuizen eds., North Holland, Amsterdam, The Netherlands (1979).

[7] P. Ramond, *The family group in grand unified theories*, hep-ph/9809459 [INSPIRE] [C79-02-25].

JHEP12(2017)075

[8] R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity violation*, *Phys. Rev. Lett.* **44** (1980) 912 [INSPIRE].

[9] J. Schechter and J.W.F. Valle, *Neutrino masses in* SU(2) × U(1) *theories*, *Phys. Rev.* **D 22** (1980) 2227 [INSPIRE].

[10] H. Fritzsch and P. Minkowski, *Unified interactions of leptons and hadrons*, *Annals Phys.* **93** (1975) 193 [INSPIRE].

[11] T. Ohlsson, *Neutrino oscillations: celebrating the Nobel prize in physics 2015*, *Nucl. Phys.* **B 908** (2016) 1.

[12] S.F. King, *Tri-bimaximal-Cabibbo Mixing*, *Phys. Lett.* **B 718** (2012) 136 [arXiv:1205.0506] [INSPIRE].

[13] S.F. King, *Unified models of neutrinos, flavour and CP-violation*, *Prog. Part. Nucl. Phys.* **94** (2017) 217 [arXiv:1701.04413] [INSPIRE].

[14] S.F. King, *Models of neutrino mass, mixing and CP-violation*, *J. Phys.* **G 42** (2015) 123001 [arXiv:1510.02091] [INSPIRE].

[15] S.F. King and C. Luhn, *Neutrino mass and mixing with discrete symmetry*, *Rept. Prog. Phys.* **76** (2013) 056201 [arXiv:1301.1340] [INSPIRE].

[16] S.F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, *Neutrino mass and mixing: from theory to experiment*, *New J. Phys.* **16** (2014) 045018 [arXiv:1402.4271] [INSPIRE].

[17] S.F. King and G.G. Ross, *Fermion masses and mixing angles from* SU(3) *family symmetry*, *Phys. Lett.* **B 520** (2001) 243 [hep-ph/0108112] [INSPIRE].

[18] S.F. King and G.G. Ross, *Fermion masses and mixing angles from* SU(3) *family symmetry and unification*, *Phys. Lett.* **B 574** (2003) 239 [hep-ph/0307190] [INSPIRE].

[19] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-Abelian discrete family symmetry*, *Phys. Lett.* **B 648** (2007) 201 [hep-ph/0607045] [INSPIRE].

[20] S.F. King and M. Malinsky, A_4 *family symmetry and quark-lepton unification*, *Phys. Lett.* **B 645** (2007) 351 [hep-ph/0610250] [INSPIRE].

[21] S. Morisi, M. Picariello and E. Torrente-Lujan, *Model for fermion masses and lepton mixing in* SO(10) × A_4, *Phys. Rev.* **D 75** (2007) 075015 [hep-ph/0702034] [INSPIRE].

[22] F. Bazzocchi, M. Frigerio and S. Morisi, *Fermion masses and mixing in models with* SO(10) × A_4 *symmetry*, *Phys. Rev.* **D 78** (2008) 116018 [arXiv:0809.3573] [INSPIRE].

[23] C. Hagedorn, M.A. Schmidt and A.Yu. Smirnov, *Lepton mixing and cancellation of the Dirac mass hierarchy in* SO(10) *GUTs with flavor symmetries* $T(7)$ *and* $\Sigma(81)$, *Phys. Rev.* **D 79** (2009) 036002 [arXiv:0811.2955] [INSPIRE].

[24] F. Bazzocchi and I. de Medeiros Varzielas, *Tri-bi-maximal mixing in viable family symmetry unified model with extended seesaw*, *Phys. Rev.* **D 79** (2009) 093001 [arXiv:0902.3250] [INSPIRE].

[25] S.F. King and C. Luhn, *A new family symmetry for* SO(10) *GUTs*, *Nucl. Phys.* **B 820** (2009) 269 [arXiv:0905.1686] [INSPIRE].

[26] S.F. King and C. Luhn, *A Supersymmetric grand unified theory of flavour with* PSL(2)(7) × SO(10), *Nucl. Phys.* **B 832** (2010) 414 [arXiv:0912.1344] [INSPIRE].

JHEP12(2017)075

[27] I. de Medeiros Varzielas and G.G. Ross, *Discrete family symmetry, Higgs mediators and* θ_{13}, *JHEP* **12** (2012) 041 [arXiv:1203.6636] [INSPIRE].

[28] A. Anandakrishnan, S. Raby and A. Wingerter, *Yukawa unification predictions for the LHC*, *Phys. Rev.* **D 87** (2013) 055005 [arXiv:1212.0542] [INSPIRE].

[29] F. Björkeroth, F.J. de Anda, I. de Medeiros Varzielas and S.F. King, *Towards a complete* $\Delta(27) \times \mathrm{SO}(10)$ *SUSY GUT*, *Phys. Rev.* **D 94** (2016) 016006 [arXiv:1512.00850] [INSPIRE].

[30] F. Björkeroth, F.J. de Anda, I. de Medeiros Varzielas and S.F. King, *Towards a complete* $A_4\times$ SU(5) *SUSY GUT*, *JHEP* **06** (2015) 141 [arXiv:1503.03306] [INSPIRE].

[31] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Tri-bimaximal neutrino mixing from discrete subgroups of* SU(3) *and* SO(3) *family symmetry*, *Phys. Lett.* **B 644** (2007) 153 [hep-ph/0512313] [INSPIRE].

[32] S.F. King and M. Malinsky, *Towards a complete theory of fermion masses and mixings with* SO(3) *family symmetry and* $5-D$ SO(10) *unification*, *JHEP* **11** (2006) 071 [hep-ph/0608021] [INSPIRE].

[33] G. Altarelli, F. Feruglio and C. Hagedorn, *A SUSY* SU(5) *grand unified model of tri-bimaximal mixing from* A_4, *JHEP* **03** (2008) 052 [arXiv:0802.0090] [INSPIRE].

[34] P. Ciafaloni, M. Picariello, E. Torrente-Lujan and A. Urbano, *Neutrino masses and tribimaximal mixing in minimal renormalizable SUSY* SU(5) *grand unified model with* A_4 *flavor symmetry*, *Phys. Rev.* **D 79** (2009) 116010 [arXiv:0901.2236] [INSPIRE].

[35] T.J. Burrows and S.F. King, A_4 *family symmetry from* SU(5) *SUSY GUTs in* 6*d*, *Nucl. Phys.* **B 835** (2010) 174 [arXiv:0909.1433] [INSPIRE].

[36] I.K. Cooper, S.F. King and C. Luhn, *SUSY* SU(5) *with singlet plus adjoint matter and* A_4 *family symmetry*, *Phys. Lett.* **B 690** (2010) 396 [arXiv:1004.3243] [INSPIRE].

[37] I.K. Cooper, S.F. King and C. Luhn, *A4x*SU(5) *SUSY GUT of flavour with trimaximal neutrino mixing*, *JHEP* **06** (2012) 130 [arXiv:1203.1324] [INSPIRE].

[38] S. Antusch, S.F. King and M. Spinrath, *Measurable neutrino mass scale in* $A_4 \times$ SU(5), *Phys. Rev.* **D 83** (2011) 013005 [arXiv:1005.0708] [INSPIRE].

[39] S. Antusch, S.F. King, C. Luhn and M. Spinrath, *Right unitarity triangles and tri-bimaximal mixing from discrete symmetries and unification*, *Nucl. Phys.* **B 850** (2011) 477 [arXiv:1103.5930] [INSPIRE].

[40] D. Meloni, *Bimaximal mixing and large* θ_{13} *in a SUSY* SU(5) *model based on* S^4, *JHEP* **10** (2011) 010 [arXiv:1107.0221] [INSPIRE].

[41] T.J. Burrows and S.F. King, $A_4 \times$ SU(5) *SUSY GUT of flavour in* 8*D*, *Nucl. Phys.* **B 842** (2011) 107 [arXiv:1007.2310] [INSPIRE].

[42] C. Hagedorn, S.F. King and C. Luhn, *SUSY* $S_4 \times$ SU(5) *revisited*, *Phys. Lett.* **B 717** (2012) 207 [arXiv:1205.3114] [INSPIRE].

[43] J. Gehrlein, J.P. Oppermann, D. Schäfer and M. Spinrath, *An* SU(5) $\times$ A_5 *golden ratio flavour model*, *Nucl. Phys.* **B 890** (2014) 539 [arXiv:1410.2057] [INSPIRE].

[44] B.D. Callen and R.R. Volkas, *Large lepton mixing angles from a* $4+1$*-dimensional* SU(5) $\times$ A_4 *domain-wall braneworld model*, *Phys. Rev.* **D 86** (2012) 056007 [arXiv:1205.3617] [INSPIRE].

JHEP12(2017)075

[45] A. Meroni, S.T. Petcov and M. Spinrath, *A SUSY* SU(5) × T' *unified model of flavour with large* θ_{13}, *Phys. Rev.* **D 86** (2012) 113003 [arXiv:1205.5241] [INSPIRE].

[46] M.-C. Chen, J. Huang, K.T. Mahanthappa and A.M. Wijangco, *Large* $\theta_1 3$ *in a SUSY* SU(5) × T' *model*, *JHEP* **10** (2013) 112 [arXiv:1307.7711] [INSPIRE].

[47] S.F. King, C. Luhn and A.J. Stuart, *A grand* $\Delta(96) \times$ SU(5) *flavour model*, *Nucl. Phys.* **B 867** (2013) 203 [arXiv:1207.5741] [INSPIRE].

[48] S. Antusch, C. Gross, V. Maurer and C. Sluka, *A flavour GUT model with* $\theta_{13}^{PMNS} \simeq \theta_C/\sqrt{2}$, *Nucl. Phys.* **B 877** (2013) 772 [arXiv:1305.6612] [INSPIRE].

[49] S. Antusch, C. Gross, V. Maurer and C. Sluka, *Inverse neutrino mass hierarchy in a flavour GUT model*, *Nucl. Phys.* **B 879** (2014) 19 [arXiv:1306.3984] [INSPIRE].

[50] S. Antusch, I. de Medeiros Varzielas, V. Maurer, C. Sluka and M. Spinrath, *Towards predictive flavour models in SUSY* SU(5) *GUTs with doublet-triplet splitting*, *JHEP* **09** (2014) 141 [arXiv:1405.6962] [INSPIRE].

[51] S.F. King, *A model of quark and lepton mixing*, *JHEP* **01** (2014) 119 [arXiv:1311.3295] [INSPIRE].

[52] S.F. King, *A to Z of Flavour with Pati-Salam*, *JHEP* **08** (2014) 130 [arXiv:1406.7005] [INSPIRE].

[53] R. de Adelhart Toorop, F. Bazzocchi and L. Merlo, *The interplay between GUT and flavour symmetries in a Pati-Salam* $\times S^4$ *model*, *JHEP* **08** (2010) 001 [arXiv:1003.4502] [INSPIRE].

[54] T. Feldmann, F. Hartmann, W. Kilian and C. Luhn, *Combining Pati-Salam and flavour symmetries*, *JHEP* **10** (2015) 160 [arXiv:1506.00782] [INSPIRE].

[55] I. de Medeiros Varzielas, *Non-Abelian family symmetries in Pati-Salam unification*, *JHEP* **01** (2012) 097 [arXiv:1111.3952] [INSPIRE].

[56] I. de Medeiros Varzielas, G.G. Ross and J. Talbert, *A unified model of quarks and leptons with a universal texture zero*, arXiv:1710.01741 [INSPIRE].

[57] I.P. Ivanov and L. Lavoura, SO(10) *models with flavour symmetries: classification and examples*, *J. Phys.* **G 43** (2016) 105005 [arXiv:1511.02720] [INSPIRE].

[58] P.M. Ferreira, W. Grimus, D. Jurčiukonis and L. Lavoura, *Flavour symmetries in a renormalizable* SO(10) *model*, *Nucl. Phys.* **B 906** (2016) 289 [arXiv:1510.02641] [INSPIRE].

[59] D. Meloni, *GUT and flavor models for neutrino masses and mixing*, *Front. in Phys.* **5** (2017) 43 [arXiv:1709.02662] [INSPIRE].

[60] D.-G. Lee and R.N. Mohapatra, *An* SO(10) × S_4 *scenario for naturally degenerate neutrinos*, *Phys. Lett.* **B 329** (1994) 463 [hep-ph/9403201] [INSPIRE].

[61] C. Hagedorn, M. Lindner and R.N. Mohapatra, S_4 *flavor symmetry and fermion masses: towards a grand unified theory of flavor*, *JHEP* **06** (2006) 042 [hep-ph/0602244] [INSPIRE].

[62] Y. Cai and H.-B. Yu, *A* SO(10) *GUT model with* S^4 *flavor symmetry*, *Phys. Rev.* **D 74** (2006) 115005 [hep-ph/0608022] [INSPIRE].

[63] B. Dutta, Y. Mimura and R.N. Mohapatra, *An* SO(10) *grand unified theory of flavor*, *JHEP* **05** (2010) 034 [arXiv:0911.2242] [INSPIRE].

[64] P.S. Bhupal Dev, R.N. Mohapatra and M. Severson, *Neutrino mixings in* SO(10) *with Type II seesaw and* θ_{13}, *Phys. Rev.* **D 84** (2011) 053005 [arXiv:1107.2378] [INSPIRE].

JHEP12(2017)075

[65] K.M. Patel, *An* SO(10) × S^4 *model of quark-lepton complementarity*, *Phys. Lett.* **B 695** (2011) 225 [arXiv:1008.5061] [INSPIRE].

[66] P.S. Bhupal Dev, B. Dutta, R.N. Mohapatra and M. Severson, θ_{13} *and proton decay in a minimal* SO(10) × S_4 *model of flavor*, *Phys. Rev.* **D 86** (2012) 035002 [arXiv:1202.4012] [INSPIRE].

[67] F. Björkeroth, F.J. de Anda, S.F. King and E. Perdomo, *A natural* S_4 × SO(10) *model of flavour*, *JHEP* **10** (2017) 148 [arXiv:1705.01555] [INSPIRE].

[68] S.F. King, *Predicting neutrino parameters from* SO(3) *family symmetry and quark-lepton unification*, *JHEP* **08** (2005) 105 [hep-ph/0506297] [INSPIRE].

[69] S.F. King, *Minimal predictive see-saw model with normal neutrino mass hierarchy*, *JHEP* **07** (2013) 137 [arXiv:1304.6264] [INSPIRE].

[70] F. Björkeroth and S.F. King, *Testing constrained sequential dominance models of neutrinos*, *J. Phys.* **G 42** (2015) 125002 [arXiv:1412.6996] [INSPIRE].

[71] S.F. King, *Littlest seesaw*, *JHEP* **02** (2016) 085 [arXiv:1512.07531] [INSPIRE].

[72] S.F. King and C. Luhn, *Littlest Seesaw model from* S_4 × U(1), *JHEP* **09** (2016) 023 [arXiv:1607.05276] [INSPIRE].

[73] T. Geib and S.F. King, *A comprehensive renormalisation group analysis of the littlest seesaw model*, arXiv:1709.07425 [INSPIRE].

[74] H.M. Lee et al., *A unique* $\mathbb{Z}_4^R$ *symmetry for the MSSM*, *Phys. Lett.* **B 694** (2011) 491 [arXiv:1009.0905] [INSPIRE].

[75] S. Antusch, S.F. King, C. Luhn and M. Spinrath, *Trimaximal mixing with predicted* θ_{13} *from a new type of constrained sequential dominance*, *Nucl. Phys.* **B 856** (2012) 328 [arXiv:1108.4278] [INSPIRE].

[76] S. Antusch, S.F. King and M. Spinrath, *Spontaneous CP-violation in* A_4 × SU(5) *with constrained sequential dominance 2*, *Phys. Rev.* **D 87** (2013) 096018 [arXiv:1301.6764] [INSPIRE].

[77] P. Di Bari, *Seesaw geometry and leptogenesis*, *Nucl. Phys.* **B 727** (2005) 318 [hep-ph/0502082] [INSPIRE].

[78] E. Bertuzzo, P. Di Bari and L. Marzola, *The problem of the initial conditions in flavoured leptogenesis and the tauon* N_2*-dominated scenario*, *Nucl. Phys.* **B 849** (2011) 521 [arXiv:1007.1641] [INSPIRE].

[79] S. Antusch, P. Di Bari, D.A. Jones and S.F. King, *A fuller flavour treatment of* N_2*-dominated leptogenesis*, *Nucl. Phys.* **B 856** (2012) 180 [arXiv:1003.5132] [INSPIRE].

[80] P. Di Bari, L. Marzola and M. Re Fiorentin, *Decrypting* SO(10)*-inspired leptogenesis*, *Nucl. Phys.* **B 893** (2015) 122 [arXiv:1411.5478] [INSPIRE].

[81] P. Di Bari and S.F. King, *Successful* N_2 *leptogenesis with flavour coupling effects in realistic unified models*, *JCAP* **10** (2015) 008 [arXiv:1507.06431] [INSPIRE].

[82] P. Di Bari and M. Re Fiorentin, *A full analytic solution of* SO(10)*-inspired leptogenesis*, *JHEP* **10** (2017) 029 [arXiv:1705.01935] [INSPIRE].

[83] P. Di Bari and M. Re Fiorentin, *Supersymmetric* SO(10)*-inspired leptogenesis and a new* N_2*-dominated scenario*, *JCAP* **03** (2016) 039 [arXiv:1512.06739] [INSPIRE].

JHEP12(2017)075

[84] R. Gatto, G. Sartori and M. Tonin, *Weak selfmasses, cabibbo angle, and broken* SU(2) × SU(2), *Phys. Lett.* **B 28** (1968) 128.

[85] F. Wilczek and S. Dimopoulos, *Incomplete multiplets in supersymmetric unified models*, NSF-ITP-82-07 (1981).

[86] K.S. Babu and S.M. Barr, *Natural suppression of Higgsino mediated proton decay in supersymmetric* SO(10), *Phys. Rev.* **D 48** (1993) 5354 [hep-ph/9306242] [INSPIRE].

[87] S.M. Barr and S. Raby, *Minimal* SO(10) *unification*, *Phys. Rev. Lett.* **79** (1997) 4748 [hep-ph/9705366] [INSPIRE].

[88] H. Murayama and D.B. Kaplan, *Family symmetries and proton decay*, *Phys. Lett.* **B 336** (1994) 221 [hep-ph/9406423] [INSPIRE].

[89] P. Nath and P. Fileviez Perez, *Proton stability in grand unified theories, in strings and in branes*, *Phys. Rept.* **441** (2007) 191 [hep-ph/0601023] [INSPIRE].

[90] A. Bueno et al., *Nucleon decay searches with large liquid argon TPC detectors at shallow depths: Atmospheric neutrinos and cosmogenic backgrounds*, *JHEP* **04** (2007) 041 [hep-ph/0701101] [INSPIRE].

[91] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13 [arXiv:1502.01589] [INSPIRE].

[92] KAMLAND-ZEN collaboration, A. Gando et al., *Search for Majorana neutrinos near the inverted mass hierarchy region with KamLAND-Zen*, *Phys. Rev. Lett.* **117** (2016) 082503 [arXiv:1605.02889] [INSPIRE].

[93] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwetz, *Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity*, *JHEP* **01** (2017) 087 [arXiv:1611.01514] [INSPIRE].

[94] S. Antusch and V. Maurer, *Running quark and lepton parameters at various scales*, *JHEP* **11** (2013) 115 [arXiv:1306.6879] [INSPIRE].

[95] S. Antusch, J. Kersten, M. Lindner, M. Ratz and M.A. Schmidt, *Running neutrino mass parameters in see-saw scenarios*, *JHEP* **03** (2005) 024 [hep-ph/0501272] [INSPIRE].

[96] T2K collaboration, K. Abe et al., *Combined analysis of neutrino and antineutrino oscillations at T2K*, *Phys. Rev. Lett.* **118** (2017) 151801 [arXiv:1701.00432] [INSPIRE].

[97] NOvA collaboration, P. Adamson et al., *Measurement of the neutrino mixing angle* θ_{23} *in NOvA*, *Phys. Rev. Lett.* **118** (2017) 151802 [arXiv:1701.05891] [INSPIRE].

[98] L.J. Hall, R. Rattazzi and U. Sarid, *The top quark mass in supersymmetric* SO(10) *unification*, *Phys. Rev.* **D 50** (1994) 7048 [hep-ph/9306309] [INSPIRE].

[99] S. Davidson, *Parametrizations of the seesaw, or, can the seesaw be tested?*, hep-ph/0409339 [INSPIRE].

[100] P. Di Bari, *An introduction to leptogenesis and neutrino properties*, *Contemp. Phys.* **53** (2012) 315 [arXiv:1206.3168] [INSPIRE].

JHEP12(2017)075