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Unstable 'black branes' from scaled membranes at large D

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Abstract: It has recently been demonstrated that the dynamics of black holes at large D can be recast as a set of non gravitational membrane equations. These membrane equations admit a simple static solution with shape $S^{D-p-2} \times R^{p,1}$. In this note we study the equations for small fluctuations about this solution in a limit in which amplitude and length scale of the fluctuations are simultaneously scaled to zero as D is taken to infinity. We demonstrate that the resultant nonlinear equations, which capture the Gregory-Laflamme instability and its end point, exactly agree with the effective dynamical 'black brane' equations of Emparan Suzuki and Tanabe. Our results thus identify the 'black brane' equations as a special limit of the membrane equations and so unify these approaches to large D black hole dynamics.

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Contents

1 Introduction

A few years ago Emparan, Suzuki, Tanabe and collaborators observed [1–7] (see also [8, 9]) that the classical equations that govern the dynamics of black holes in D dimensions simplify in the large D limit. Motivated by this observation, several papers written over the last year or so have demonstrated that black hole physics at large D can be reformulated in terms of dual non gravitational equations. In broad terms there have been two different approaches to this problem.

The first of these approaches is laid out in the 'membrane paradigm' papers of [10–12] (see also [13–15][1]). The authors of these papers have demonstrated that nonlinear black hole dynamics can be reformulated in terms of the equations of motion of a non gravitational membrane that lives in flat space. The variables of this problem are the shape of the membrane and a velocity field on this membrane.[2] Einstein's equations force the membrane variables to obey a set of equations of motion. There are as many equations of motion as variables, so the membrane description defines a good initial value problem. We emphasize that the membrane equations of [10–12] apply to arbitrarily nonlinear and completely dynamical black hole motions. There are, in particular, no restrictions on the initial shape of the membrane which can be chosen to be any sufficiently smooth codimension one submanifold of flat spacetime; the evolution of this shape (and the membrane velocity fields) in time is, of course, governed by the membrane equations of motion.

A second approach is that of the 'scaled black brane' papers of [16, 17] (see also [18, 19]). These papers study small fluctuations about the p dimensional 'black brane'; a spacetime given by the direct product of the Schwarzschild solution in $R^{D-p-1,1}$ and R^p. The authors of [16] consider fluctuations that preserve $SO(D-p-1)$ isometry but vary in the R^p

[1] These papers worked out the equations that govern the shape of the membrane, described later in this paragraph, for stationary configurations. Atleast in the absence of a cosmological constant, these equations may be shown to follow from the more general dynamical membrane equations of [10–12] upon inserting an appropriate stationary ansatz, and so are special cases of the general membrane equations.

[2] The variables of the membrane also include a charge field for charged black holes. In this note, however, we focus solely on solutions of the vacuum Einstein equations $R_{MN} = 0$. We leave the generalization of our study to charged black holes, and to dynamics in spaces with nonzero cosmological constants, to future work.

direction over length scales of order $\frac{1}{\sqrt{D}}$ and time scales of order unity. Focusing attention on wiggles of the event horizon of amplitude $\frac{1}{D}$ and on boost velocities of the horizon of order $\frac{1}{\sqrt{D}}$, the authors of [16] were able to derive a set of effective non gravitational nonlinear equations that completely reproduce black brane dynamics in the scaled large D limit described above. This scaling limit is of particular interest because it turns out to capture the Gregory-Laflamme instability of black branes at large D.[3,4]

In this brief note we derive the 'black brane' equations of [16] starting from the membrane equations of [10–12]. The starting point of our analysis is the simple exact solution to the membrane equations of motion that is dual to the p dimensional 'black brane' described in the previous paragraph. This solution is static, which means that the membrane velocity field is simply given by $u = -dt$. The shape of the membrane on this solution is $S^{D-p-2} \times R^{p,1}$. We then proceed to study the scaling limit of [16] directly within the membrane picture. In other words we study fluctuations of the membrane that preserve $\mathrm{SO}(D-p-1)$ isometry but vary in the R^p direction over length scales of order $\frac{1}{\sqrt{D}}$ and time scales of order unity. We then focus on wiggles of the shape of the membrane with amplitude of order $\frac{1}{D}$ and on membrane velocities of order $\frac{1}{\sqrt{D}}$. At leading order in the large D limit we obtain a simple set of scaled equations of membrane dynamics which (after the appropriate field redefinitions) turn out to agree exactly with the equations of [16]. We view our derivation of the (uncharged) black brane equations from the membrane equations as a unification of these two approaches to horizon dynamics at large D. Note it follows, in particular, that the dynamics of the Gregory-Laflamme instability is captured by scaling limit of membrane equations described above.

The limit of the previous paragraph is loosely reminiscent of the scaling limit that yields the nonrelativistic Navier-Stokes equations starting from the more general relativistic equations [26]. The membrane equations may also admit other interesting scaling limits. We leave the investigation of this point to future work.

2 A scaling limit of the membrane equations

In this note we study the equations of motion [10–12] of an uncharged large D membrane propagating in flat Minkowski spacetime. To leading order in $\frac{1}{D}$ these equations take the form

$$\left[\frac{\nabla^2 u_A}{\mathcal{K}} - \frac{\nabla_A \mathcal{K}}{\mathcal{K}} + u^B K_{BA} - u^B \nabla_B u_A\right] \mathcal{P}^A_C = 0 \tag{2.1}$$

with,

$$\nabla . u = 0 \,. \tag{2.2}$$

Here K_{AB} is the extrinsic curvature of the membrane, $\mathcal{K}$ is its trace and u is the local world volume velocity field of the membrane. All covariant derivatives in (2.1) and (2.2)

[3] [16] has subsequently been generalized to the the study of charged black branes in [17]. As mentioned above, however, in this note we focus attention on uncharged black holes and black branes.

[4] Additional recent studies of black hole physics at large D include [20–25].

are defined with respect to the induced metric on the membrane. Also

$$\mathcal{P}^{AB} = \hat{g}^{AB} + u^A u^B \tag{2.3}$$

where $\hat{g}^{AB}$ is the metric induced from the ambient flat space on the world volume of the membrane. In other words $\mathcal{P}^{AB}$ is the projector, on the membrane world volume, orthogonal to the velocity field u.

2.1 Linearized fluctuations

In our study we will find it useful to use coordinates in which the flat space D dimensional metric takes the form

$$ds^2 = -dt^2 + d\tilde{x}^a d\tilde{x}^a + dr^2 + r^2 d\Omega_n^2 \tag{2.4}$$

where

$$n = D - p - 2\,.$$

and $a = 1 \ldots p$ label the spatial directions on the black brane. A simple solution to the equations (2.1) and (2.2) is given by the membrane shape $r = 1$ and constant static velocity field $u = -dt$.[5]

The solution of the membrane equations described in the previous paragraph is dual to a 'black brane' — the solution of general relativity given by the direct product of R^p and the Schwarschild black hole in $D - p$ dimensions. It is well known that this solution of general relativity is unstable in an arbitrary number of dimensions. We will now use the membrane equations to exhibit this instability, by linearizing these equations about the simple solution. The Gregorry Laflamme instability of black branes is known to preserve the SO$(n+1)$ symmetry of the sphere but to break translational invariance along R^p, so we study fluctuations with the same property. In other words we set

$$\begin{aligned} r &= 1 + \tilde{\delta r}(t, \tilde{x}^a) \\ u &= -dt + \tilde{\delta u_a}(t, \tilde{x}^a) d\tilde{x}^a\,. \end{aligned} \tag{2.5}$$

Note that our velocity fluctuations lie entirely in the black brane directions and none of our fluctuations fields depend on the angular variables on S^n.

Following the method described in section 5 of [11], it is not difficult to linearize the membrane equations around the 'black brane' solution. The equation (2.2) reduces to

$$n\, \partial_t \tilde{\delta r} + \tilde{\partial}_a \tilde{\delta u^a} = 0 \tag{2.6}$$

(recall $n = D - p - 2$).[6] The equation with a free index in the (spatial) R^p direction turns out to take the form

$$\left(\tilde{\partial}_a \tilde{\delta r} - \partial_t \tilde{\partial}_a \tilde{\delta r} - \partial_t \tilde{\delta u_a}\right) + \left(\frac{-\partial_t^2 + \tilde{\partial}_b \tilde{\partial}^b}{n}\right)\left(\tilde{\delta u_a} + \tilde{\partial}_a \tilde{\delta r}\right) = 0 \tag{2.7}$$

[5]The choice $r = 1$ involves no loss of generality, as the scale invariance of the classical Einstein equations relate the solution with $r = 1$ to the solution with $r = r_0$ for any constant r_0.

[6]The factor of n, which plays a crucial role in the analysis below, has its origin in the fact that the induced metric on the world volume of the membrane is given, to leading order in fluctuations by

$$ds^2 = -dt^2 + d\tilde{x}^a d\tilde{x}_a + (1 + 2\tilde{\delta r}) d\Omega_n^2$$

so that $\sqrt{g} = 1 + n\tilde{\delta r}$ in these coordinates.

where $\tilde{\partial}_a$ is the derivative with respect to the coordinate $\tilde{x}^a$ defined in (2.4). When all spatial and time derivatives are of order unity or smaller, the term

$$\left(\frac{-\partial_t^2 + \tilde{\partial}_b \tilde{\partial}^b}{n}\right)\left(\delta \tilde{u}_a + \tilde{\partial}_a \tilde{\delta r}\right)$$

in (2.7) is subleading in the $\frac{1}{n}$ expansion and so can naively be dropped at leading order. However we will soon find ourselves interested in configurations with spatial derivatives of order $\sqrt{n}$ but time derivatives of order unity. For such configurations the term proportional to time derivatives in (2.7) is indeed subleading in $\frac{1}{n}$. On the other hand the term proportional to the spatial laplacian is comparable to the other terms in (2.7) and so must be retained. Over the parameter ranges of interest to this paper, therefore, we can replace (2.7) with the slightly simpler equation[7]

$$\left(\tilde{\partial}_a \tilde{\delta r} - \partial_t \tilde{\partial}_a \tilde{\delta r} - \partial_t \delta \tilde{u}_a\right) + \left(\frac{\tilde{\partial}_b \tilde{\partial}^b}{n}\right)\left(\delta \tilde{u}_a + \tilde{\partial}_a \tilde{\delta r}\right) = 0\,. \tag{2.8}$$

The equations (2.8) and (2.6) are easily analysed. Substituting the plane wave expansion

$$\begin{aligned} \tilde{\delta r}(t, \tilde{x}^a) &= \delta r^0 e^{-i\omega t} e^{i\tilde{k}_a \tilde{x}^a} \\ \delta \tilde{u}_a(t, \tilde{x}^a) &= \delta u_a^0 e^{-i\omega t} e^{i\tilde{k}_a \tilde{x}^a} \end{aligned} \tag{2.9}$$

into (2.8) and (2.6) turns these equations into eigenvalue equations for the fluctuation frequencies ω. Solving the resultant cubic equation in ω we find find that the most general solution to these equations is given by

$$\begin{aligned} &\tilde{\delta r}(t,\tilde{x}^a) = \delta r_1^0 e^{-i\omega_1 t} e^{i\tilde{k}_a \tilde{x}^a} + \delta r_2^0 e^{-i\omega_2 t} e^{i\tilde{k}_a \tilde{x}^a} \\ &\delta \tilde{u}_a(t,\tilde{x}^a) = \delta r_1^0 \tilde{k}_a \left(-i + \frac{\sqrt{n}}{\tilde{k}}\right) e^{-i\omega_1 t} e^{i\tilde{k}_a \tilde{x}^a} + \delta r_2^0 \tilde{k}_a \left(-i - \frac{\sqrt{n}}{\tilde{k}}\right) e^{-i\omega_2 t} e^{i\tilde{k}_a \tilde{x}^a} + v_a e^{-i\omega_3 t} e^{i\tilde{k}_a \tilde{x}^a} \\ &w_1 = i\left(\frac{\tilde{k}}{\sqrt{n}} - \frac{\tilde{k}^2}{n}\right), \quad w_2 = i\left(-\frac{\tilde{k}}{\sqrt{n}} - \frac{\tilde{k}^2}{n}\right), \quad w_3 = -i\frac{\tilde{k}^2}{n}, \quad \text{where } \tilde{k}^2 = \tilde{k}_a \tilde{k}^a. \end{aligned} \tag{2.10}$$

(2.10) is a solution to the linearized membrane equations for arbitrary constant values of δr_1^0 and δr_2^0 and for any constant vector v_a s.t. $\tilde{k}^a v_a = 0$.

Note that the mode proportional to δr_1^0 — i.e. the mode with frequency ω_1 — is unstable when $\tilde{k} < \sqrt{n}$. This IR instability (i.e. an instability that occurs at distance scales lareger than a minimum) is the membrane dual of the Gregory-Laflamme instability. When $\tilde{k}$ is of order unity time scale associated with this frequency is of order $\sqrt{n}$ and so is very large. The minimum time scale for an instability, however, occurs at $\tilde{k} = \frac{\sqrt{n}}{2}$. At this wavelength the time scale of the instability is order unity.[8]

[7]The membrane equations with free index in sphere direction is trivially satisfied, while the equation in the time direction is also a triviality (this is a consequence of the projector in (2.1)).

[8]The expression for the unstable mode w_1 was conjectured earlier from fluid/gravity methods in [27]. See also [28] and [1] for further evidence for the above proposal.

At the level of the linearized equations the Gregory-Laflamme unstable modes simply grow forever. Nonlinear effects, however, stabilize these modes. The discussion of the previous paragraph makes it clear that the length scale relevant to this physics is $\frac{1}{\sqrt{n}}$. We will now proceed to find the effective nonlinear theory within which the Gregory-Laflamme instability and its end point can be reliably studied.

2.2 Scaled nonlinear equations

In order to restrict attention to distance of order $\frac{1}{\sqrt{n}}$ in the spatial black brane directions we work with the scaled coordinate x^a defined by $\tilde{x}^a = \frac{x^a}{\sqrt{n}}$. Unstable modes with finite wavelength in this new coordinate have frequencies of order unity. The background flat space metric now takes the form

$$ds^2 = -dt^2 + dr^2 + \frac{1}{n} dx_a dx^a + r^2 d\Omega_n^2 \,. \tag{2.11}$$

As our fluctuations field all vary over distances of order unity and time scales of order unity in scaled coordinates, the velocity field u^a should thus also be of order unity. This implies that $u_a \sim \mathcal{O}(\frac{1}{n})$. Translating back to unscaled coordinates it follows that $\tilde{u}^a = \mathcal{O}(\frac{1}{\sqrt{n}})$. In order to ensure this scaling in our solution (2.10) we must choose $v_a \sim \mathcal{O}(\frac{1}{\sqrt{n}})$, $\delta r_1^0 \sim \delta r_2^0 \sim \mathcal{O}(\frac{1}{n})$. These choices, in turn, ensure that $\tilde{\delta r} \sim \mathcal{O}(\frac{1}{n})$ (see (2.10)). It is thus natural to make the further coordinate change

$$r = 1 + \frac{y}{n} \,. \tag{2.12}$$

The flat space metric is now given by

$$ds^2 = -dt^2 + \frac{dy^2}{n^2} + \frac{1}{n} dx_a dx^a + \left(1 + \frac{y}{n}\right)^2 d\Omega_n^2 \,. \tag{2.13}$$

With our scalings now in place we focus attention on membrane configurations of the form

$$\begin{aligned} y &= y(x^a, t) \\ u^a &= u^a(x^a, t) \end{aligned} \tag{2.14}$$

where the functions $y(x^a, t)$ and $u^a(x^a, t)$ are independent of n. We then evaluate the membrane equations (2.2) and (2.1) for such configurations propagating on the metric (2.13). Retaining only terms of leading order at large n we find that the equation (2.2) (which we call E^s below) and the a components of (2.1) (which we call E^v_a below) reduce to

$$\begin{aligned} &E^s \equiv u^b \partial_b y + \partial^b u_b + \partial_t y = 0 \\ &E^v_a \equiv \partial^b \partial_b u_a + \partial_a y - u^b \partial_b u_a + \partial^b y \partial_b u_a - u^b \partial_b \partial_a y + \partial^b y \partial_b \partial_a y + \partial^b \partial_b \partial_a y - \partial_t u_a - \partial_t \partial_a y = 0 \,. \end{aligned} \tag{2.15}$$

Note that the equations (2.15) are nonlinear. If we linearize these equations around the background $y = u^a = 0$ we obtain the linearized equations

$$\begin{aligned} \partial^b \delta u_b + \partial_t \delta r &= 0 \\ \partial^b \partial_b \delta u_a + \partial_a \delta r + \partial^b \partial_b \partial_a \delta r - \partial_t \delta u_a - \partial_t \partial_a \delta r &= 0 \,. \end{aligned} \tag{2.16}$$

The first and second of (2.16) are simply (2.6) and (2.8) expressed in scaled variables. It follows that (2.15) are nonlinear generalizations of the linearized fluctuation equations of the previous subsection. The (2.16) are exact at large n within the scaling limit described in this section.

The nonlinear equations (2.15) capture both the linear exponential growth as well as the nonlinear settling down of the Gregory-Laflamme instability. We do not need to perform the analysis of this fact, however, because it has already been done! We will now demonstrate that the equations (2.15) are equivalent to those that Emparan Suzuki and Tanabe [16] derived to study large D 'black branes' — and used to perform an extensive study of the Gregory-Laflamme instability.

In order to make contact with the work of [16] we make the following field redefinitions

$$\begin{aligned} y(t,x^a) &= \log m(t,x^a) \\ u_a(t,x^a) &= \frac{p_a(t,x^a) - \partial_a\left(m(t,x^a)\right)}{m(t,x^a)} \end{aligned} \tag{2.17}$$

and work with the following linear combinations of (2.15)

$$E_1 = m(t,x^a)E^s \qquad \text{and} \qquad E_a = p_a(t,x^a)E^s - m(t,x^a)E^v_a\,. \tag{2.18}$$

It is easily verified that E_1 and E_a take the form

$$\begin{aligned} E_1 &= \partial_t m - \partial_b\partial^b m + \partial_b p^b = 0 \\ E_a &= \partial_t p_a - \partial_b\partial^b p_a - \partial_a m + \partial_b\left(\frac{p_a p^b}{m}\right) = 0\,. \end{aligned} \tag{2.19}$$

The equations (2.19) are precisely the nonlinear black brane equations (11) and (12) of [16]. It follows that these black brane equations are simply a particular scaled limit of the general leading order (in an expansion in $\frac{1}{D}$) equations (2.2) and (2.1).

3 Discussion

In this note we have demonstrated by explicit computation that the uncharged 'black brane' equations of [16] may be obtained from a scaling limit of the general membrane equations (2.2) and (2.1). The reader may, at first, find herself puzzled at this agreement, given the scaling limit described in this note focuses on length scales of order $\frac{1}{\sqrt{D}}$ while that the membrane equations (2.2) and (2.1) were derived as the first term in a systematic expansion in $\frac{1}{D}$ under the assumption that the horizon and velocity fields all vary on length scale unity. We will now explain why this agreement was infact to be expected despite the apparent conflict of regimes of validity.

The equations (2.2) and (2.1) would fail to accurately capture dynamics at leading order in the large D limit if the explicit factors of D in the metric (2.13) ensured that a higher order term[9] were to contribute to the equations at same (or higher) order in $\frac{1}{D}$ as

[9] I.e. a term that appears at higher order in the expansion in $\frac{1}{D}$ in the membrane equations of [12].

the terms in (2.2) and (2.1). We will now explain that this never happens. Potentially dangerous terms are those that contain one or more factors of the inverse metric g^{ab} where the indices a and b are spatial black brane directions. These terms are potentially dangerous as g^{ab} (see (2.13)) is of order D. However these factors never actually lead to a mixing of orders because the extra indices a and b each need to contract with something. When these indices contract with u_a the extra factor of D is nullified by the fact that u_a is of order $\frac{1}{D}$. When these indices contract with a derivative, the derivative acts on some quantity built out of fluctuation fields. However all such quantities are of order $\frac{1}{D}$ (recall, for instance, that every fluctuation component of the extrinsic curvature is proportional to $\tilde{\delta r}$ which is of order $\frac{1}{D}$). The smallness of fluctuations in our scaling limit once again counteracts the potential enhancement of powers of D. It follows that leading order equations (2.2) and (2.1) is infact sufficient to capture the leading order large D dynamics of the scaling limit described in this note despite the fact that the scaling limit zooms in on distance scales of order $\frac{1}{\sqrt{D}}$.

It should not be difficult to generalize the discussion of this note to obtain the first corrections, in an expansion in $\frac{1}{D}$, to the black brane equations of (2.19). These corrections have been obtained from 'scaled black brane' approach in [18, 22, 23]. The starting point for such an analysis would be the first order corrected membrane equations derived in [12]. It would also be interesting to check whether the analysis of this note generalizes to a derivation of the charged 'black brane' equations of [17] starting with the charged membrane equations of [11].[10] We leave a study of these issues to future work.

We end this note by reiterating that we have demonstrated that the black brane equations of [16] can be derived as a special case of the more general membrane equations of [10–12], leading to a satisfying unification recent attempts to reformulate large D horizon dynamics in non gravitational terms.

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[10]The discussion of the last paragraph suggests that this is guaranteed to work only if the scaling limit of [17] turns on membrane charge fluctuations that scale like $\frac{1}{D}$.

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