

Partially acoustic dark matter, interacting dark radiation, and large scale structure

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ABSTRACT: The standard paradigm of collisionless cold dark matter is in tension with measurements on large scales. In particular, the best fit values of the Hubble rate H_0 and the matter density perturbation σ_8 inferred from the cosmic microwave background seem inconsistent with the results from direct measurements. We show that both problems can be solved in a framework in which dark matter consists of two distinct components, a dominant component and a subdominant component. The primary component is cold and collisionless. The secondary component is also cold, but interacts strongly with dark radiation, which itself forms a tightly coupled fluid. The growth of density perturbations in the subdominant component is inhibited by dark acoustic oscillations due to its coupling to the dark radiation, solving the σ_8 problem, while the presence of tightly coupled dark radiation ameliorates the H_0 problem. The subdominant component of dark matter and dark radiation continue to remain in thermal equilibrium until late times, inhibiting the formation of a dark disk. We present an example of a simple model that naturally realizes this scenario in which both constituents of dark matter are thermal WIMPs. Our scenario can be tested by future stage-IV experiments designed to probe the CMB and large scale structure.

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1 Introduction

For nearly two decades, the Λ CDM paradigm in which dark matter (DM) is composed of cold, collisionless particles has provided an excellent fit to cosmological data. Although on galactic scales or smaller there have been long-standing issues such as the core-vs-cusp [1, 2] and too-big-to-fail [3] problems that are difficult to explain within this framework, it has been very successful on larger scales. However, in recent years, as the data has become more precise, the Λ CDM framework has also come into tension with measurements on large scales. In particular, the value of today's Hubble rate H_0 obtained from a fit to the cosmic microwave background (CMB) and baryon acoustic oscillation (BAO) data [4] is smaller than the results from local measurements [5–9], with a $\sim 3\sigma$ discrepancy. Similarly, the inferred value of σ_8 (the amplitude of matter density fluctuations at the scale of $8h^{-1}$ Mpc) is larger by $3\text{--}4\sigma$ [10–12] than the values from direct measurements such as weak lensing survey [13], CMB lensing [14], and Sunyaev-Zeldovich cluster counts [15–17].

These large-scale anomalies are particularly intriguing because the theoretical understanding of dynamics at large scales is rather simple and robust, essentially requiring only the application of linear perturbation theory to density fluctuations. This is in contrast to the studies of small scale structure, which not only require an understanding of the nonlinear evolution of a many-body system but also crucially depend on the detailed dynamics of baryons, which is quite challenging to simulate. Therefore, while it is possible that these large scale discrepancies are due to systematic errors in the associated experiments [18, 19], it is important to consider the possibility that they are in fact robust problems that require a fundamental shift away from the Λ CDM framework.

Several proposals have been put forward to explain one or the other of these anomalies by going beyond the Λ CDM paradigm. For example, DM that decays at late times, well after the CMB epoch, can reduce the size of σ_8 [20, 21]. Alternatively, neutrinos with masses near the top of the allowed range or sterile neutrinos with an eV mass can fit the CMB and BAO data with a smaller σ_8 [22], but have the effect of making the H_0

problem worse [4]. If there is dark radiation (DR) that behaves like a tightly coupled fluid (as opposed to free streaming like neutrinos), this can ameliorate the tension in H_0 measurements [23]. If DM further scatters with such DR, it may be possible to solve both the H_0 and σ_8 problems [24–27]. These proposals, however, require a rather tiny DM-DR interaction constrained in a very narrow range to solve both problems.

In this article we propose a new simple framework, “Partially Acoustic Dark Matter” (PAcDM), that can robustly solve both problems. We assume that DM consists of two components, χ_1 and χ_2 , and that there is also DR that behaves as a tightly coupled relativistic fluid.¹ The primary component χ_1 is cold and collisionless, and dominates the DM mass density. The subdominant component, χ_2 , is also cold, but is tightly coupled to the DR. The interactions within the DR, and between the DR and χ_2 , are both strong enough that the tight coupling treatment is valid not only during radiation domination before the CMB epoch but also well into the era of structure formation. Then, since our DR is a tightly coupled relativistic fluid as considered in ref. [23], we can solve the H_0 problem by choosing the amount of DR appropriately. We will then demonstrate that the persistent χ_2 -DR interaction inhibits the growth of density perturbations in χ_2 , which in turn reduces the growth of density fluctuations in the dominant DM component, χ_1 , provided that the modes in question enter the horizon before matter-radiation equality. The modes at the $8h^{-1}$ Mpc scale do indeed come inside the horizon before equality, so we can also solve the σ_8 problem just by choosing an appropriate amount of χ_2 to match the observed discrepancy in σ_8 .

This class of theories fits very naturally into a “hidden WIMP” scenario [35–38], in which the relic abundance of both DM components χ_1 and χ_2 is set by annihilation into a hidden sector, rather than into the SM. Massless states in this hidden sector could then constitute interacting DR [39], ameliorating the H_0 problem, while scattering of χ_2 off DR solves the σ_8 problem. In the next section, we construct an explicit model along these lines. However, we stress that the qualitative features of our scenario — an enhancement in H_0 and a reduction in σ_8 — are robust and only require the coupling constants in the DR- χ_2 sector to be sufficiently large that DR is a tightly coupled fluid and χ_2 remains in equilibrium with DR. The mechanism is therefore quite general, and is not restricted to a specific DM framework.

The PAcDM paradigm shares some features of the “double-disk dark matter” scenario explored in [40, 41], but there are several crucial differences. In particular, since our “dark electrons” are massless, they never form bound states and, consequently, the χ_2 -DR system never undergoes recombination. Therefore, after matter-radiation equality, it continues to undergo *dark acoustic oscillations* [42] without being disrupted by “dark recombination.” This has the effect of holding back the growth of density perturbations in χ_2 until the energy density in DR falls too much for the oscillations to be maintained. Furthermore, by assumption, the χ_2 -DR system remains tightly coupled throughout the evolution of the universe, even into the era of structure formation. At later times it continues to remain a *thermal system*, and hence does not virialize. Therefore, we expect that it does not collapse into a disk but instead forms a spherical halo around the galactic center.

¹Earlier work on multi-component DM may be found in, for example, [28–34].

The rest of this paper is organized as follows. In section 2, we describe a concrete, complete model that realizes our solution to the H_0 and σ_8 problems within the hidden WIMP framework. In section 3, we develop an analytical understanding of how the σ_8 problem is solved in the PAcDM scenario by studying the linear evolution equations for the density and metric perturbations. This is done without assuming any particular particle physics model of the DM and DR. In section 4, we present the results of our more detailed numerical simulations of the matter power spectrum to determine the precise fraction of χ_2 to match the observed discrepancy in σ_8 . We then show that the corrections to the CMB spectrum and the CMB lensing measurement are small and within the present uncertainties. We conclude in section 5.

2 A benchmark model

In this section, we construct a benchmark model based on the hidden WIMP paradigm that realizes these ideas. The hidden WIMP framework, in which DM annihilates into hidden sector states rather than into SM particles, has become increasingly attractive as the constraints from direct, indirect, and collider searches continue to get stronger. This class of theories retains the attractive feature of the WIMP paradigm that the DM mass and annihilation cross section are of order the weak scale, a scale that is motivated by a variety of solutions to the hierarchy problem. In the model we consider, both χ_1 and χ_2 are hidden WIMPs, and their abundances are set by annihilation within the hidden sector.

In a hidden WIMP model the couplings between DM and the SM states may be very small. Nevertheless, as emphasized in ref. [39], if the temperature of the hidden sector is comparable to that of the SM, this class of theories is potentially accessible to experiment. The nature of the signals depends on the masses of the lightest states in the hidden sector. If all the particles in the hidden sector have masses above an eV, these states must decay or annihilate into SM particles before the CMB epoch. This is so as to avoid the over-closure bounds if their masses are above a keV, or the limits on a warm subcomponent of DM if their masses lie between an eV and a keV. This then implies the existence of couplings between the hidden sector and the SM that can potentially be tested in experiments, as in the exciting DM [35, 43], secluded DM [36], and boosted DM [44, 45] scenarios. If instead the lightest states in the hidden sector have masses below an eV, they are likely to constitute a significant component of the energy density of the universe at the time of matter-radiation equality, potentially leading to observable signals in the CMB. The simplest possibility is that these states, if present, are massless and constitute DR at present times. Provided the hidden sector was in thermal equilibrium with the SM at temperatures at or above the weak scale, there is a lower bound on the contribution of DR to the energy density during the CMB epoch, $\Delta N_{\text{eff}} \gtrsim 0.027$ [39], as expressed in units of the energy density in a single SM neutrino species. This is potentially large enough to be detected in CMB Stage-IV experiments [46–49]. We see that the existence of observable DR is a natural feature in a large class of hidden WIMP models.

DR can take two distinct forms; it may free stream like neutrinos, or scatter with a short mean free path like the particles in a tightly coupled fluid. These two cases can be

experimentally distinguished, because the details of the CMB spectrum depend not just on the total energy density in radiation, but on the fraction that is free streaming [50–52]. This ratio impacts both the amplitudes of the modes, and the locations of the peaks in the CMB spectrum. The effects of scattering DR and free streaming DR on the CMB are therefore different. In the case when DR scatters, a fit to the Planck CMB data admits values of ΔN_{eff} large enough to address the H_0 problem [23]. As we shall see, scattering of χ_2 off the DR can address the σ_8 problem. In this way the hidden WIMP framework is well suited to address both the large scale puzzles.

A concrete, renormalizable and radiatively stable field theory model of a hidden WIMP that annihilates into interacting DR was presented and analyzed in ref. [39]. In this theory, the DR consists of one or more flavors of massless Dirac fermions $\hat{\psi}$ that carry charge under a new U(1) gauge symmetry. The massless gauge boson associated with this symmetry is denoted by \hat{A} . Provided that the U(1) gauge coupling is not too small, \hat{A} and $\hat{\psi}$ behave as tightly coupled DR at late times, and can solve the H_0 problem.

In the model of ref. [39], the DM was composed of a single component, a complex scalar χ , whose relic abundance was governed by the annihilation through a massive vector boson \hat{Z} into the dark fermions $\hat{\psi}$. It is straightforward to extend this model by introducing a second component of DM, so that we now have two complex scalars, χ_1 and χ_2 . The relic abundances of both χ_1 and χ_2 are now set by annihilation to the $\hat{\psi}$ through the massive vector boson \hat{Z} . A standard thermal freeze-out calculation then tells us that, in such a scenario, the ratio of their relic abundances is given by

$$\frac{\Omega_2}{\Omega_1} \sim \left(\frac{m_2 q_1}{m_1 q_2} \right)^2, \tag{2.1}$$

where $m_{1,2}$ and $q_{1,2}$ are the mass and \hat{Z} charge of $\chi_{1,2}$, respectively. We will see that a value of this ratio of order a few % is required to match the observed $\sim 10\%$ discrepancy in σ_8 . As is clear from the above expression, DM masses and \hat{Z} charges that differ by just a factor of a few can easily result in such a value. The SM and dark sectors can be arranged to be in thermal equilibrium at early times through small Higgs portal couplings $|\chi_{1,2}|^2 |H|^2$, ensuring that their temperatures are not very different at later times. This model shows that the thermal WIMP framework can provide a simple realization of our solution to the H_0 and σ_8 problems, without the need to introduce new scales or small parameters.

We now estimate the range of parameters for which this model gives rise to the desired dynamics. First, to solve the σ_8 problem, we want χ_2 to be in thermal equilibrium with DR until late times, including the era of structure formation. This requires the momentum transfer rate between the two systems to be much larger than the Hubble rate H until late times. To be quantitative, we define the momentum transfer rate Γ from DR to χ_2 per degree of freedom of χ_2 as

$$\Gamma \equiv \frac{1}{\langle p_2^2 \rangle} \frac{d\langle \delta p_2^2 \rangle}{dt}, \tag{2.2}$$

where $\delta \vec{p}_2$ is the change of momentum of χ_2 due to a single scattering event with a degree of freedom in DR, and the $\langle \rangle$ indicates thermal average with DR being treated as the bath. Here t is simply the Minkowski time, because Γ is a microscopic quantity independent of the

cosmological expansion, and can be determined in the flat spacetime limit. An elementary calculation shows $\Gamma \propto \hat{\alpha}^2 \ln(\hat{\alpha}^{-1}) \hat{T}^2/m_{\chi_2}$ [24], where $\hat{\alpha}$ is the dark fine structure constant and \hat{T} is the temperature of the dark sector. The proportionality factor is $\mathcal{O}(1)$ and not important, provided that all the dark charges involved in the scattering are $\mathcal{O}(1)$. On the other hand, the Hubble rate H scales as T^2 during the radiation dominated era, but as $T^{3/2}$ during the matter dominated era. Since we want $\Gamma \gg H$ at all times, it follows that the strongest bound comes from the latest times. Demanding $\Gamma \gg H_0$ then leads to

$$\hat{\alpha}^2 \ln(\hat{\alpha}^{-1}) \frac{10 \text{ GeV}}{m_{\chi_2}} \gg 10^{-16} \left(\frac{T_0}{\hat{T}} \right)^2, \tag{2.3}$$

where T_0 is the photon temperature today. It follows that we can easily maintain the thermal equilibrium between χ_2 and DR at all times even with fairly small values of the dark gauge coupling. We also need to maintain the equilibrium within DR itself, but this clearly leads to a much weaker condition than the above because the relevant momentum transfer rate simply scales as $\hat{\alpha}^2 \hat{T}$, without the additional \hat{T}/m_{χ_2} suppression. Finally, note that eq. (2.1) is only valid when the relic abundances of $\chi_{1,2}$ are set by the \hat{Z} gauge coupling, and not by $\hat{\alpha}$. This leads to the condition $\hat{\alpha} \ll \hat{\alpha}_{\hat{Z}} (m_{\chi_{1,2}}/m_{\hat{Z}})^2$. (If $\hat{\alpha}$ is larger than this, the annihilation through \hat{A} dominates and eq. (2.1) would have to be modified accordingly.) With a typical size of $\hat{\alpha}_{\hat{Z}}$ for WIMPs, we see that this leaves a wide range of parameters for which the σ_8 problems is solved.

Second, in order to solve the H_0 problem, we require $\Delta N_{\text{eff}}^{\text{scatt}} \gtrsim 0.4$ [7, 9], where $\Delta N_{\text{eff}}^{\text{scatt}}$ is the energy density in the scattering component of DR, measured in units of the energy density in a single SM neutrino species. The size of $\Delta N_{\text{eff}}^{\text{scatt}}$ in the concrete model being discussed in this section can be read off from ref. [39]. For example, if the SM and dark sectors kinetically decouple from each other between the QCD phase transition temperature and the c -quark mass (which can easily be arranged to be the case), we get $\Delta N_{\text{eff}}^{\text{scatt}} \simeq 0.25$ (or 0.42) for one (or two) flavor(s) of $\hat{\psi}$ and $\hat{T} \simeq 0.39 T_0$. These values of $\Delta N_{\text{eff}}^{\text{scatt}}$ are allowed by the current CMB constraints ref. [23, 25, 53], allowing a straightforward solution of the H_0 problem.

3 General evolution of DM density perturbations

In this section, we demonstrate how the σ_8 problem is robustly solved in our framework by performing a simple analysis of the perturbed Einstein equations. More detailed, numerical studies will be presented in the next section. Our analysis here is very general, and essentially depends on only the following assumptions:

- There is a significant contribution to the energy density of the universe from DR around the time of matter-radiation equality. This DR is tightly self-coupled not only during the CMB epoch but also at late times when large scale structure forms.
- DM is cold and comes in two types, a dominant component χ_1 that is collisionless, and a subdominant component χ_2 that interacts strongly with the DR, again until late times.

To study how the growth of the perturbations of χ_1 is reduced by the scattering of χ_2 with DR, we employ the general formalism of Ma and Bertschinger [54] for scalar perturbations. (The study of tensor perturbations is beyond the scope of the present work.) We work in the conformal Newtonian gauge,

$$ds^2 = a^2(\tau) [-(1 + 2\psi)d\tau^2 + (1 - 2\phi)\delta_{ij}dx^i dx^j], \quad (3.1)$$

where the fields ψ and ϕ describe scalar perturbations on the metric. They are determined by four scalar quantities associated with the perturbed energy-momentum tensor δT_μ^ν , namely, $\delta \equiv \delta\rho/\bar{\rho} = -\delta T_0^0/\bar{\rho}$, $\delta P = \delta T_i^i/3$, $\theta \equiv -\partial_i \delta T_0^i/(\bar{\rho} + \bar{P})$, and $\sigma \equiv -\hat{\partial}_i \hat{\partial}^j (\delta T_j^i - \delta P \delta_j^i)/(\bar{\rho} + \bar{P})$, where $\bar{\rho}$ and \bar{P} are the unperturbed total energy density and pressure, and $\hat{\partial}^i \equiv \partial_i/\sqrt{\partial_j \partial_j}$. For each particle species s , we define $\delta_s \equiv \delta\rho_s/\bar{\rho}_s$, $\theta_s \equiv -\partial_i \delta T_{s0}^i/(\bar{\rho}_s + \bar{P}_s)$, etc. We can also re-express θ_s as the divergence of comoving 3-velocity, $\theta_s = \partial_i v_s^i$, where $v^i \equiv dx^i/d\tau$. σ_s is the shear stress of the fluid component s and vanishes if s is a perfect fluid. For each s we assume an equation of state of the form $P_s = w_s \rho_s$ with constant w_s , so the pressures and energy densities are not independent quantities. The total δ and θ are given in terms of the individual δ_s and θ_s as $\delta = \sum_s \bar{\rho}_s \delta_s/\bar{\rho}$ and $\theta = \sum_s (\bar{\rho}_s + \bar{P}_s) \theta_s/(\bar{\rho} + \bar{P})$. σ is related to the individual σ_s by a sum rule analogous to that for θ . If the total σ is zero, the two metric perturbations become equal, $\psi = \phi$. An important parameter in our framework is the ratio r defined as

$$r \equiv \frac{\bar{\rho}_2}{\bar{\rho}_{\text{DM}}} \quad (3.2)$$

with $\bar{\rho}_{\text{DM}} \equiv \bar{\rho}_1 + \bar{\rho}_2$.

To linear order in the perturbations, the evolution of the dominant, collisionless component of DM, χ_1 , is described by

$$\dot{\delta}_1 = -\theta_1 + 3\dot{\phi}, \quad \dot{\theta}_1 = -\frac{\dot{a}}{a}\theta_1 + k^2\psi, \quad (3.3)$$

where the dots indicate derivatives with respect to τ , and we have switched to Fourier space via the transformation $e^{i\vec{k}\cdot\vec{x}}$. These equations are just the expressions of local conservation of energy and momentum of χ_1 in an expanding FRW background. Being collisionless and cold, χ_1 evolves exactly as standard cold DM (CDM) for *given* ϕ and ψ . But we will see that in the presence of scattering between χ_2 and DR, ϕ and ψ are modified away from their values in a standard, single-component CDM model. χ_2 evolves as

$$\dot{\delta}_2 = -\theta_2 + 3\dot{\phi}, \quad \dot{\theta}_2 = -\frac{\dot{a}}{a}\theta_2 + k^2\psi + a\Gamma(\theta_{\text{DR}} - \theta_2), \quad (3.4)$$

where Γ is the rate of momentum transfer as defined in (2.2), while DR evolves as

$$\begin{aligned} \dot{\delta}_{\text{DR}} &= -\frac{4}{3}\theta_{\text{DR}} + 4\dot{\phi}, \\ \dot{\theta}_{\text{DR}} &= k^2 \left(\frac{\delta_{\text{DR}}}{4} + \psi \right) + \frac{3}{4} \frac{\bar{\rho}_2}{\bar{\rho}_{\text{DR}}} a\Gamma(\theta_2 - \theta_{\text{DR}}). \end{aligned} \quad (3.5)$$

Equations (3.4) and (3.5) also just express local energy-momentum conservation, but now conservation applies only to the total χ_2 -DR system. Accordingly, it may be observed

that the Γ terms cancel for the total $\theta \propto \bar{\rho}_2\theta_2 + \frac{4}{3}\bar{\rho}_{\text{DR}}\theta_{\text{DR}}$, where we have made use of the sum rule for θ described earlier. It is implicitly assumed in (3.4) and (3.5) that all the constituents of DR (in our case, dark massless electrons and dark photons) are always sufficiently tightly coupled to each other that DR can be treated as a single fluid without a shear stress.

Now, one of our key assumptions is that the χ_2 -DR interaction rate, Γ , is also always large enough that the tight coupling approximation can be applied to (3.4) and (3.5) not only during the radiation dominated era but also throughout the matter dominated era. Then, expanding θ_2 , ψ , etc., in powers of $1/\Gamma$ in (3.4) and (3.5), we get $\theta_2 = \theta_{\text{DR}}$ to leading order. Then, cancelling the Γ terms by taking an appropriate linear combination of the $\dot{\theta}_2$ and $\dot{\theta}_{\text{DR}}$ equations and then setting $\theta_2 = \theta_{\text{DR}} \equiv \tilde{\theta}$, we obtain

$$\dot{\tilde{\theta}} = -\frac{\dot{a}}{a} \frac{f}{1+f} \tilde{\theta} + \frac{k^2}{4(1+f)} \delta_{\text{DR}} + k^2 \psi, \tag{3.6}$$

where $f \equiv 3\bar{\rho}_2/4\bar{\rho}_{\text{DR}}$. As it must, this reduces to the equation for $\dot{\theta}_2$ or $\dot{\theta}_{\text{DR}}$ *without* the scattering term in the $f \rightarrow \infty$ (no DR) or $f \rightarrow 0$ (no χ_2) limits, respectively. Since f grows linearly in a , it will eventually be $\gg 1$ and the linearized evolution equations for χ_2 will become identical to those for χ_1 . It follows that, in order to significantly modify the evolution of χ_2 , it is necessary to have $f \lesssim 1$ when the mode enters the horizon. Recalling the parametrization (3.2) and that $\Delta N_{\text{eff}} = \bar{\rho}_{\text{DR}}/\bar{\rho}_\nu$, the value of f at matter-radiation equality is given by

$$f_{\text{eq}} = \frac{3\bar{\rho}_2}{4\bar{\rho}_{\text{DR}}}\Big|_{\text{eq}} = \frac{3}{4} \frac{r}{\Delta N_{\text{eff}}} \frac{\bar{\rho}_{\text{DM}}}{\bar{\rho}_\nu}\Big|_{\text{eq}} \tag{3.7}$$

$$\simeq \frac{3}{4} \frac{r}{\Delta N_{\text{eff}}} \frac{3}{1 + \Omega_{\text{B}}/\Omega_{\text{DM}}} \left(1 + \frac{\bar{\rho}_\gamma}{3\bar{\rho}_\nu}\Big|_{\text{eq}}\right),$$

where we have approximated $\bar{\rho}_{\text{DM}}|_{\text{eq}}$ as $\bar{\rho}_\gamma + 3\bar{\rho}_\nu|_{\text{eq}}$ since we know that the amount of extra radiation, ΔN_{eff} , is constrained to be small. Using $\Omega_{\text{B}}/\Omega_{\text{DM}} = 0.19$, the above expression gives

$$f_{\text{eq}} \simeq 0.2 \cdot \frac{r}{0.02} \cdot \frac{0.5}{\Delta N_{\text{eff}}}, \tag{3.8}$$

where $r = 0.02$ and $\Delta N_{\text{eff}} = 0.5$ are close to the benchmark values we will use later in our numerical study. To see whether f_{eq} of 0.2 is small enough to qualify as “ $f \ll 1$ ”, note that it is comparable to the value of f for the standard baryon acoustic oscillation (BAO) case,

$$f_{\text{eq}}^{\text{BAO}} = \frac{3\bar{\rho}_{\text{B}}}{4\bar{\rho}_\gamma}\Big|_{\text{eq}} = \frac{3}{4} \frac{1}{1 + \Omega_{\text{DM}}/\Omega_{\text{B}}} \left(1 + \frac{3\bar{\rho}_\nu}{\bar{\rho}_\gamma}\Big|_{\text{eq}}\right) \simeq 0.2. \tag{3.9}$$

It follows that the initial behavior of the χ_2 -DR system after matter-radiation equality is just like that of baryon-photon fluid, namely the density perturbations in χ_2 undergo acoustic oscillations. In the meantime, the density perturbations in χ_1 grow monotonically. Eventually, around recombination or shortly after, f becomes $\gtrsim 1$ and the perturbations in χ_2 begin to grow monotonically as well. This is where the analogy with the baryon-photon fluid breaks down, because the χ_2 -DR system never undergoes recombination. Once ρ_{DR}

becomes subdominant to ρ_2 , DR particles do not have enough energy to maintain dark acoustic oscillations. After this the equations for χ_2 take the same form as those for collisionless CDM, and therefore its inhomogeneities in the linear regime evolve just like those of χ_1 . However, because χ_2 's growth was initially held back for a while, we continue to have $\delta_2 \ll \delta_1$. At very late times, the behavior of χ_1 and χ_2 are expected to differ on small scales, since χ_1 virializes while χ_2 remains thermal.

Needless to say, the above conclusion applies only to the modes that enter the horizon before matter-radiation equality, and to a lesser extent also to the modes that come in between the equality and recombination. The modes that come in much later enter the horizon when $f \gg 1$, so they evolve just like χ_1 from the beginning, leading to $\delta_2 = \delta_1$. This is a robust prediction of our scenario: *the modes with $k \ll k_{\text{eq}} \sim 0.01 \text{ Mpc}^{-1}$ should see no modifications in the matter power spectrum compared to standard collisionless CDM.*

We are now ready to analyze how the suppression of δ_2 affects the evolution of χ_1 for the modes with $k \gg k_{\text{eq}}$. At matter-radiation equality, these modes are already well within the horizon. Then, the relevant component of the perturbed Einstein equation,

$$k^2\phi + 3\frac{\dot{a}}{a}\left(\dot{\phi} + \frac{\dot{a}}{a}\psi\right) = -4\pi G a^2 \delta\rho, \quad (3.10)$$

just reduces to the Newtonian, Poisson equation, $k^2\phi \simeq -4\pi G a^2 \delta\rho$. (The a^2 on the right-hand side is there because the k on the left-hand side is the *comoving* wave number related to the physical wave number k_{phys} as $k_{\text{phys}} = k/a$, with $a = 1$ today by convention.) Here, recalling the sum rule $\delta\rho = \sum_s \bar{\rho}_s \delta_s$, we have

$$\begin{aligned} \delta\rho &= \bar{\rho}_1 \delta_1 + \bar{\rho}_2 \delta_2 \\ &= \bar{\rho}_{\text{DM}} [(1-r)\delta_1 + r\delta_2], \end{aligned} \quad (3.11)$$

where for simplicity we have only kept χ_1 and χ_2 , while ignoring the other components such as baryons. (However, these are included in our numerical studies to be discussed later.) If χ_2 were also collisionless CDM like χ_1 , we would have $\delta_1 = \delta_2 \equiv \delta_{\text{DM}}$. Then the above relation would just lead to $\delta\rho = \bar{\rho}_{\text{DM}} \delta_{\text{DM}}$, as if we only had a single component of CDM. However, as discussed above, the χ_2 -DR interactions lead to $\delta_1 \gg \delta_2$ for the modes with $k \gg k_{\text{eq}}$, so we instead have

$$\delta\rho \simeq (1-r)\bar{\rho}_{\text{DM}}\delta_1. \quad (3.12)$$

This result is completely intuitive: if χ_2 is not fluctuating, the fluctuations in the spacetime metric should only depend on the fluctuations in χ_1 , but χ_1 is only $(1-r)$ of the total mass density. Then, the Poisson equation becomes

$$\begin{aligned} k^2\phi &\simeq -4\pi a^2 G \bar{\rho}_{\text{DM}} \cdot (1-r)\delta_1 \\ &= -\frac{6}{\tau^2} \cdot (1-r)\delta_1, \end{aligned} \quad (3.13)$$

where we have used the Friedman equation and the fact that $a \propto \tau^2$ during matter domination. Defining a dimensionless variable $\eta \equiv k\tau$, the above equation becomes

$$\phi \simeq -\frac{6(1-r)}{\eta^2} \delta_1. \quad (3.14)$$

On the other hand, cancelling θ_1 between the two equations in (3.3), we get

$$\delta_1'' + \frac{2}{\eta} \delta_1' = -\psi + \frac{6}{\eta} \phi' + 3\phi'' . \quad (3.15)$$

where the primes denote derivatives with respect to η . Since the modes in question are well within the horizon, we have $\eta \gg 1$ by definition and the ϕ' and ϕ'' terms can be neglected compared to the ψ term. For simplicity, let us ignore the shear stresses in the photons and neutrinos, since our primary focus is on the difference between the $r = 0$ and $r \neq 0$ cases. Then we have $\psi = \phi$, and so combining the above equation with eq. (3.14) leads to

$$\delta_1'' + \frac{2}{\eta} \delta_1' \simeq -\phi \simeq \frac{6(1-r)}{\eta^2} \delta_1 . \quad (3.16)$$

This tells us that δ_1 is given by a linear combination of $\eta^{-1/2 \pm \sqrt{25/4 - 6r}}$. The solution with the negative power of η is important for matching the “initial” condition at $a = a_{\text{eq}}$, but then it quickly decays away. Then the late-time behavior of the modes of χ_1 with $k \gg k_{\text{eq}}$ is approximately given by

$$\delta_1 \propto \left(\frac{\eta}{\eta_{\text{eq}}} \right)^{-1/2 + \sqrt{25/4 - 6r}} \propto \left(\frac{a}{a_{\text{eq}}} \right)^{1 - 0.6r + \mathcal{O}(r^2)} . \quad (3.17)$$

Taking the ratio of our case ($r > 0$) to standard, single-component, collisionless CDM ($r = 0$), we obtain

$$\frac{\delta_1(r)}{\delta_1(0)} = \left(\frac{a}{a_{\text{eq}}} \right)^{-0.6r + \mathcal{O}(r^2)} , \quad (3.18)$$

where we have neglected the mild r dependence in the proportionality factor. This is justified because inflation gives us the same, universal initial conditions for the perturbations, irrespective of r (see eqs. (4.1) and (4.2)), and the subsequent evolution of χ_1 during radiation domination is controlled by radiation and hence is only mildly affected by the presence of χ_2 . We see from eq. (3.18) that δ_1 grows slower than in the CDM case.²

It follows that, in this approximation, the ratio of our power spectrum to the standard one is given by

$$\frac{P(r)}{P(0)} = \frac{[\delta\rho(r)]^2}{[\delta\rho(0)]^2} = \frac{(1-r)^2 [\delta_1(r)]^2}{[\delta_1(0)]^2} \simeq (1-2r) \left(\frac{a}{a_{\text{eq}}} \right)^{-1.2r} , \quad (3.19)$$

where we have used the relation (3.12) in the second step and dropped $\mathcal{O}(r^2)$ terms at the end. As discussed already, the above suppression applies only to the modes with $k \gg k_{\text{eq}}$, and the modes with $k \ll k_{\text{eq}}$ see no suppression. The upper and lower panels of figure 1 in section 4 show the results of a numerical analysis that confirms this analytical understanding.

²Although the underlying dynamics is different, it is possible to get a similar effect by increasing the neutrino masses [55].

4 Numerical results and corrections to the CMB spectrum

In this section, we obtain numerical results for the matter power spectrum in Λ CDM framework. We also show that the effects of χ_2 -DR scattering on the CMB spectrum are small and within current experimental uncertainties.

The evolution of the perturbations in $\chi_{1,2}$ and DR is governed by eqs. (3.3)–(3.5) and (3.10), where the dark U(1) coupling is chosen to satisfy the condition (2.3). The equations for baryons and photons are obtained simply by taking eqs. (3.4) and (3.5) and relabelling $\chi_2 \rightarrow B$ (baryon) and DR $\rightarrow \gamma$, again in the tight coupling limit. The tight coupling approximation means our analysis fails to properly capture the physics of recombination and photon diffusion. We also ignore the effect of neutrinos. These limitations are not an immediate concern for us because we are only interested in how χ_2 -DR scattering affects the CMB spectrum *compared to* the cases without such scattering, with or without DR, and the effects we ignore are common to all cases. Finally, photon polarizations are not distinguished in our treatment, but this is a small effect as we focus only on the matter and CMB temperature power spectra.

The tight coupling approximation and the absence of shear stress from neutrino free streaming imply that $\phi = \psi$. Assuming inflation, the initial conditions for the perturbations are then simply given by the superhorizon solutions of all the evolution equations above, together with the assumption of adiabatic perturbations that entropy per matter particle is unchanged by perturbations. For the modes that enter the horizon during radiation domination, the initial conditions are given by

$$\delta_{\gamma,DR} = \frac{4}{3}\delta_{1,2,B} = -2\psi, \quad \theta_{1,2,DR,B,\gamma} = \frac{k^2\tau}{2}\psi, \quad (4.1)$$

while for those that come in during matter domination we have

$$\frac{3}{4}\delta_{\gamma,DR} = \delta_{1,2,B} = -2\psi, \quad \theta_{1,2,DR,B,\gamma} = \frac{k^2\tau}{3}\psi. \quad (4.2)$$

We neglect the tilt in the primordial spectrum (i.e, $n_s = 1$) and use a k -independent value 10^{-4} for the initial perturbation -2ψ above, but the precise choice of this number is immaterial as we will only compare the ratio of the spectrum with $r \neq 0$ to that with $r = 0$.

We are now ready to evolve the perturbations numerically as governed by the equations and initial conditions described above, We choose the values $h = 0.68$, $\Omega_\gamma h^2 = 2.47 \times 10^{-5}$, $\Omega_\Lambda h^2 = 0.69$, $\Omega_b h^2 = 2.2 \times 10^{-2}$ and $\Omega_\nu = 0.69\Omega_\gamma$ [4]. Ω_Λ only has small effects on our results so its precise value is not important for our purpose here. We choose $\Delta N_{\text{eff}}^{\text{scatt}} = 0.4$ and a slightly larger value of $\Omega_{\text{DM}} h^2 = 0.13$ in order to keep the redshift at matter-radiation equality unchanged. This allows us to compare our matter power spectrum to that of a conventional single component DM model *without any* DR. We find the choice of $r = 2.0\%$ leads to a 10% suppression in the matter power spectrum around the scale $k \sim 0.2h \text{ Mpc}^{-1}$ compared to Λ CDM, thereby solving the σ_8 problem. It should be noted that this corresponds to a suppression of about 20% compared to the $r = 0$ case with

the same amount of DR, as shown in figure 1, and confirms the features we identified analytically in section 3.³

The lower panel of figure 1 shows the results with the same ratios r of DM components, but with a reduced amount of DR, $\Delta N_{\text{eff}}^{\text{scatt}} = 0.05$. Notice that the suppression in σ_8 is nevertheless almost identical to the case with $\Delta N_{\text{eff}}^{\text{scatt}} = 0.4$. The essential reason why the reduction in σ_8 is so robust is that, even with $\Delta N_{\text{eff}}^{\text{scatt}}$ as small as 0.05, there remains enough DR for χ_2 to scatter with at the time when the σ_8 modes enter the horizon, which is well before matter-radiation equality. Therefore, our mechanism would still constitute a solution to the σ_8 problem even if future measurements were to settle on a smaller $\Delta N_{\text{eff}}^{\text{scatt}}$.

Let us now discuss the impact of χ_2 -DR interactions on the CMB spectrum. We compare the temperature power spectra of the $r = 2.0\%$ and $r = 0$ cases with the same amount of DR. Since our equations do not include the physics of recombination or photon diffusion, we halt the evolution just before recombination at $a = 10^{-3}$ (which is when the electron number density begins to fall exponentially) and then evaluate the following quantity:

$$\left(\frac{\delta T}{T}\right)_* \equiv \frac{1}{4}\delta_\gamma + \psi. \quad (4.3)$$

This has the same form as the standard expression for $\delta T/T$ for the CMB, except for the absence of small Doppler effect corrections [57] and the fact that it is evaluated right before recombination as opposed to the time of photon decoupling. Nevertheless, we map $(\delta T/T)_*$ to the C_ℓ coefficients as if $(\delta T/T)_*$ were $\delta T/T$. In other words, we treat the system as if the photons instantaneously decouple right before recombination, and obtain a “snapshot” of the CMB spectrum at that time. This is sufficient for the purpose of showing that the effects of χ_2 -DR interactions have very small impact on the CMB spectrum, because the effects of such interactions on the photons at and after recombination are small. Moreover, there is very little time between recombination and photon decoupling. Hence, if the CMB spectrum with $r = 2.0\%$ is very similar to that with $r = 0$ right before recombination, we expect that they should continue to remain similar afterwards. In figure 2, we compare the primordial spectra of the $r \neq 0$ (red solid) and $r = 0$ (black solid) cases. The χ_2 -DR scattering damps the CMB spectrum, but the deviations are always less than 2%, which is within the uncertainties of current measurements. The largest deviations are for modes in the range $700 \lesssim \ell \lesssim 1600$. The reason that the modes with $\ell \lesssim 700$ see very little deviation is because they enter the horizon late and by that time the DR has become too cold to affect the physics, as discussed in section 3. On the other hand, the modes with $\ell \gtrsim 1600$ enter the horizon at a time when the energy density is completely dominated by radiation and so the metric perturbations damp quickly, resulting in small corrections to δ_γ .

The other aspect of the last point above is that the metric perturbations are appreciably modified only after matter-radiation equality. This means that the observations of CMB lensing effects can potentially constrain our scenario. The key quantity for calculating the impact of lensing on the temperature power spectrum is the lensing power spectrum $C_\ell^{\phi\phi}$ as

³The detailed CMB and Large Scale Structure constraints on Partially-Interacting DM were studied in [42] using the Planck 2013 data [56]. The bound allows $\lesssim 5\%$ of the DM density to be tightly coupled to DR. Since we require an r of order 2%, our result is consistent with this limit.

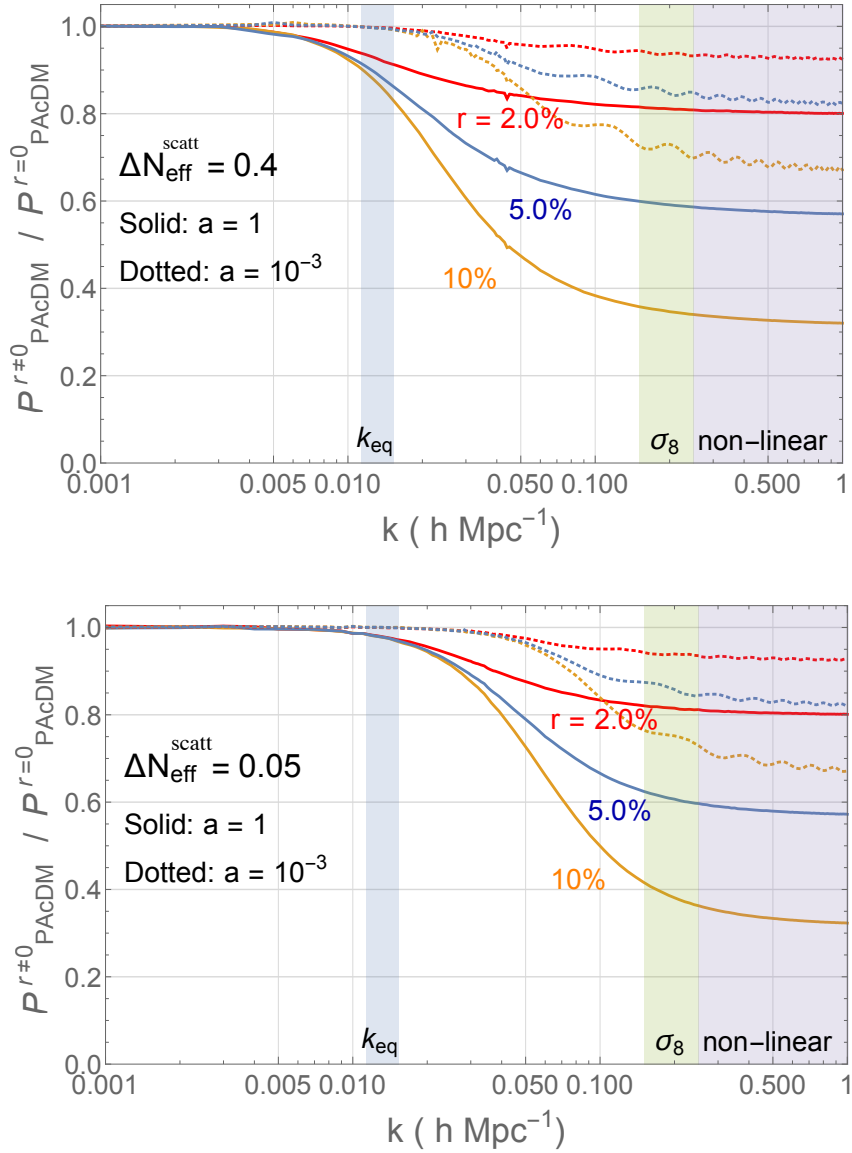


Figure 1. Upper: ratio of the DM power spectrum of the $r \neq 0$ case to the $r = 0$ case, both with $\Delta N_{\text{eff}}^{\text{scatt}} = 0.4$. The curves are obtained by numerically solving the linear evolution equations (3.3)–(3.5) and the perturbed Einstein equation (3.10), all in the tight coupling limit and assuming no anisotropic stress (hence $\sigma = 0$ and $\phi = \psi$). Results for different values of r are labelled in different colors, while earlier ($a = 10^{-3}$) and later ($a = 1$) times are indicated by dotted and solid lines, respectively. For the smaller scale structures $k \gtrsim 0.2h \text{ Mpc}^{-1}$, nonlinear gravitational effects become important so our linear approximation is no longer reliable. Lower: same plot but with a reduced amount of DR, $\Delta N_{\text{eff}}^{\text{scatt}} = 0.05$.

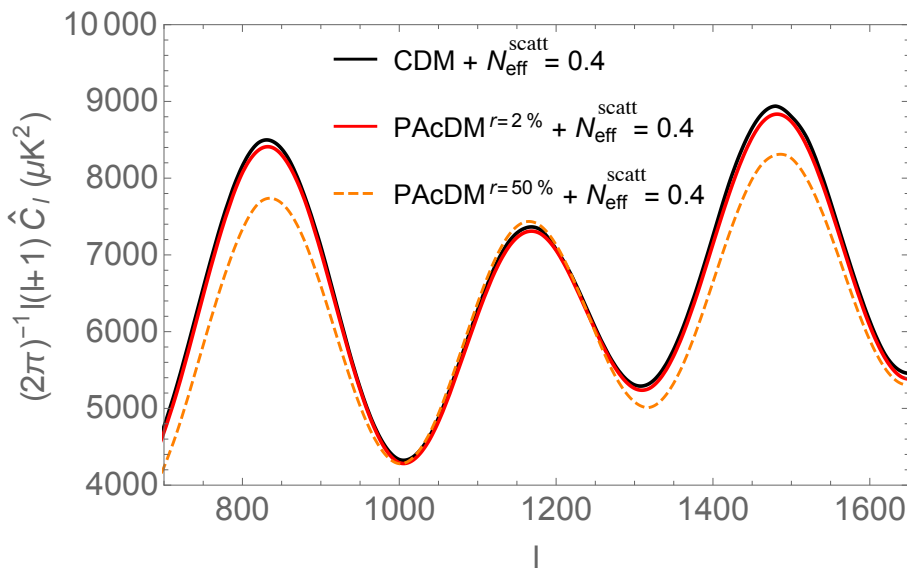


Figure 2. A comparison of the CMB spectrum between PAcDM and CDM models, assuming $\Delta N_{\text{eff}}^{\text{scatt}} = 0.4$ in both cases. The black (red) curve is for the Λ CDM (PAcDM) model, derived from the $\{\delta_\gamma, \theta_\gamma\}$ result in the linear evolution equations. The PAcDM model assumes the DM ratio $r = 2.0\%$. For comparison, we also show a PAcDM model with $r = 50\%$, which exhibits a clear enhancement of the expansion peaks and suppression of the compression peaks due to the pressure of tightly coupled DR.

a function of multipole ℓ . Using the Limber approximation [58], we estimate the spectrum using [59]

$$\ell^4 C_\ell^{\phi\phi} \propto \int_0^{\chi_*} d\chi (1 - \chi/\chi_*)^2 k \phi^2(k, a) g^2(\chi), \quad (4.4)$$

where χ is the comoving distance from the observer and hence $k = \ell/\chi$. In this expression the scale factor a is to be evaluated at χ , i.e., $a = a(\chi)$. The parameter $\chi_* \simeq 10^4$ Mpc corresponds to the value of χ of the last scattering surface. The lensing effect is captured by $(1 - \chi/\chi_*)^2$. The function $g(\chi)$ effectively describes the decays of metric perturbations due to the vacuum energy at late times. Its value is therefore equal to 1 by definition during matter domination ($\chi \gtrsim 0.5\chi_*$) and starts to decrease once χ drops below $\simeq 0.5\chi_*$. The transfer function is already folded into the above expression, so $\phi(k)$ is simply the primordial metric fluctuation determined by the linear perturbation equations discussed in section 3.

In the Planck data [14], the smallest percentage error in $C_\ell^{\phi\phi}$ is $\simeq 5\%$ at $\ell \simeq 150$. The $C_\ell^{\phi\phi}$ at this multipole gets its main contributions from $\chi \simeq 0.4\chi_*$ in eq. (4.4), which corresponds to the modes with $k \simeq \mathcal{O}(0.01)$ Mpc $^{-1}$. The deviation in ϕ^2 between the $r = 2.0\%$ and $r = 0$ cases is $\simeq 4\%$ for this value of k (see figure 1), and the integral (4.4) gives a 2.5% deviation between the two cases. This is within the current uncertainties.

5 Conclusions

We have presented a new framework in which DM is composed of two distinct components that can provide a solution to both the H_0 and σ_8 problems. While the dominant component of DM is cold and collisionless, the subdominant component is also cold but interacts strongly with DR, which itself constitutes a tightly coupled fluid. Our framework is very general and can be adopted in a wide variety of DM models. In particular, it can easily be accommodated in the hidden WIMP framework, with both constituents of DM arising as thermal relics. Our scenario predicts distinctive modifications to the matter and CMB power spectra, allowing it to be tested by future experiments.

By solving a set of linear evolution equations, we have shown that the observed 10% discrepancy in σ_8 requires the mass density in the subdominant, interacting DM species to constitute $\simeq 2.0\%$ of the total DM density, while the amount of DR can be separately chosen to fix the H_0 problem. This apparently small ratio of the two DM components could easily arise in, for example, the WIMP framework without introducing hierarchically small parameters into the Lagrangian. The required tight couplings between the interacting DM and DR and within DR itself can be obtained in a wide range of perturbative coupling constants, as we showed in a concrete model. We found that, with an interacting DM component of about 2% to solve the σ_8 problem and the appropriate amount of DR to address the H_0 problem, the deviations in the CMB spectrum and the CMB lensing measurements are well within the current uncertainties.

It is interesting to compare and contrast our proposal with the scenario put forward in [24–26]. In the PAcDM framework, only a subcomponent of DM experiences acoustic oscillations, while the primary component of DM is responsible for building up structure. As seen in eq. (3.19), this suppresses the rate of growth of power during the era of matter domination, with the result that most of the corrections to the DM density perturbations, and hence corrections to the gravitational potential, arise well after matter-radiation equality. However, in the proposal of [24–26], the entirety of DM undergoes oscillations prior to matter-radiation equality that continue through to the CMB epoch. In this case, the corrections to the DM density perturbations are already significant at the time of matter-radiation equality, and the resulting corrections to the CMB are expected to be significantly larger. Hence future precision studies of the CMB may be able to distinguish these two classes of models.

It is important to note that our mechanism to reduce σ_8 is not especially sensitive to the precise value of $\Delta N_{\text{eff}}^{\text{scatt}}$. Hence, if future measurements were to settle on a smaller $\Delta N_{\text{eff}}^{\text{scatt}}$, our mechanism would still constitute a solution to the σ_8 problem. Note that lowering $\Delta N_{\text{eff}}^{\text{scatt}}$ would also imply even smaller corrections to the CMB, since most of the modes that are observed in the CMB would now enter the horizon at a time when the contribution of DR to the energy density is small.

While the primary focus of this article is on large scale structure, our framework may also have potentially observable effects on smaller scales. It follows from eq. (2.3) that at each locale in the universe the χ_2 particles continue to experience a sufficient number of collisions with surrounding DR particles to maintain thermal equilibrium, at least locally,

at all times. This remains true during the era of structure formation. The same condition also ensures that the DR itself remains a tightly coupled relativistic fluid at all times. Being strictly massless, the dark charged particles in DR never undergo recombination. Being tightly coupled, DR is non-dissipative and hence behaves as a perfect thermal fluid. Because of these properties, which are qualitatively different from those of the baryon-photon system, we expect that the χ_2 -DR system does not collapse into a disk but instead forms a smooth, spherical halo around the galactic center. This is qualitatively distinct from the dynamics of the recently proposed “double-disk DM” or partially dissipative DM [40]. The existence of this halo would impact galactic dynamics, with the exact nature of its effects depending on the details of its density profile. We defer a careful study of these effects on the small scale structure of DM halos for future work.

The current discordance between direct and indirect measurements of H_0 and σ_8 may be the first hint for new cosmology beyond the Λ CDM paradigm. These discrepancies can be naturally addressed within the non-minimal dark sector structure we have proposed. In the coming years, the experimental precision in the indirect measurement of N_{eff} and σ_8 from the CMB [46–49] (e.g. CMB stage-IV), and in direct measurements of the Hubble constant [60, 61], and σ_8 [49, 62] (LSST, DESI) are all expected to improve significantly. If the current discrepancies in the H_0 and σ_8 measurements are indeed due to new physics, these future experiments have great potential for distinguishing between different candidate theories such as the framework presented in this paper.

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