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Correlators in the Gaussian and chiral supereigenvalue models in the Neveu-Schwarz sector

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ABSTRACT: We analyze the Gaussian and chiral supereigenvalue models in the Neveu-Schwarz sector. We show that their partition functions can be expressed as the infinite sums of the homogeneous operators acting on the elementary functions. In spite of the fact that the usual W -representations of these matrix models can not be provided here, we can still derive the compact expressions of the correlators in these two supereigenvalue models. Furthermore, the non-Gaussian (chiral) cases are also discussed.

KEYWORDS: Conformal and W Symmetry, Matrix Models

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1 Introduction

The supereigenvalue models have attracted considerable attention. They can be regarded as supersymmetric generalizations of matrix models [1]–[17]. Many of the important features of matrix models, such as the Virasoro constraints, the loop equations, the genus expansions and the moment descriptions have their supersymmetric counterparts in the supereigenvalue models. Two types of supereigenvalue models in the Ramond sector, i.e., Ramond-NS and Ramond-R supereigenvalue models, were derived in ref. [14] by means of the conformal field theory formalism. They obey the super Virasoro constraints, where the bosonic and fermionic operators yield the Ramond version of the super-Virasoro algebra. Moreover, the corresponding Ramond super quantum curves were analyzed and the underlying Ramond super spectral curves were identified. It is well-known that the W -representations can be used to realize partition functions of various matrix models, such as the Gaussian Hermitian and complex matrix models [18]–[21], the Kontsevich matrix model [22] and the generalized Brezin-Gross-Witten model [23]. Namely, by acting on elementary functions with exponents of the given W -operators, we can give the corresponding partition functions of the matrix models. Recently it was proved that Ramond-NS supereigenvalue model can be obtained in terms of the W -representation [24]

$$\begin{aligned}
 Z &= \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \Delta_R(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N \left(\sum_{k=0}^{\infty} (t_k z_i^k + \xi_k z_i^{k-\frac{1}{2}} \theta_i) + z_i \right)} \\
 &= e^{-W} e^{-\frac{\sqrt{\beta}}{\hbar} N t_0},
 \end{aligned} \tag{1.1}$$

where N is even, z_i are bosonic variables and θ_i are Grassmann variables, t_k and ξ_k are bosonic and fermionic coupling constants, respectively, $\Delta_R(z, \theta)$ is the Vandermonde-like determinant, $\Delta_R(z, \theta) = \prod_{1 \leq i < j \leq N} \left(z_i - z_j - \frac{1}{2} (z_i + z_j) \frac{\theta_i \theta_j}{\sqrt{z_i z_j}} \right)$. Due to the W -representation, the compact expressions of correlators in the supereigenvalue model (1.1)

can be derived. The final result shows that the correlators are determined by the certain coefficients in the power of the operator W .

The supereigenvalue model in the Neveu-Schwarz sector describes the coupling between $(2, 4m)$ superconformal models and world-sheet supergravity [1]. The β -deformed case of this supereigenvalue model is given by [12]

$$\tilde{Z} = \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N V_{NS}(z_i, \theta_i)}, \quad (1.2)$$

where N is even, $\Delta_{NS}(z, \theta) = \prod_{1 \leq i < j \leq N} (z_i - z_j - \theta_i \theta_j)$, $V_{NS}(z, \theta) = \sum_{k=0}^{\infty} t_k z^k + \sum_{k=0}^{\infty} \xi_{k+\frac{1}{2}} z^k \theta$, and $\xi_{k+\frac{1}{2}}$ are fermionic coupling constants. There are the super Virasoro constraints for the partition function (1.2), where the bosonic and fermionic operators yield the Neveu-Schwarz version of the super-Virasoro algebra. When the bosonic variables z_i in (1.2) are integrated from 0 to $+\infty$, it gives the chiral supereigenvalue model, which can be seen as the supersymmetric generalization of the complex one-matrix model. The closed expressions for all planar multi-superloop correlation functions were derived for $\beta = 1$ chiral supereigenvalue model. It was found that the higher genus contributions to the free energy and the correlators can be calculated by an iterative scheme [10].

It is known that the compact expressions of the correlators in the supereigenvalue model (1.1) in the Ramond sector have been presented. However, for the cases of the supereigenvalue model in the Neveu-Schwarz sector, it still remains to be seen whether there are the similar results. In this paper, we analyze the (non-)Gaussian supereigenvalue model in the Neveu-Schwarz sector and give the correlators in these matrix models. Moreover, the cases of the chiral supereigenvalue model will be also investigated.

This paper is organized as follows. In section 2, we focus on the Gaussian supereigenvalue model in the Neveu-Schwarz sector and show that its partition function can be expressed as the infinite sums of the homogeneous operators acting on the elementary functions. Moreover, we derive the compact expressions of the correlators. In section 3, we consider the non-Gaussian supereigenvalue model in the Neveu-Schwarz sector. In section 4, we consider the chiral supereigenvalue model and present the compact expressions of the correlators. The non-Gaussian chiral case is analyzed in section 5. We end this paper with the conclusions in section 6.

2 Gaussian supereigenvalue model in the Neveu-Schwarz sector

The Gaussian Hermitian matrix model is one of the most studied and best understood matrix models. Its partition function can be generated by the W -representation [18]. The correlators in this matrix model have been well discussed [25]–[35] and the compact expressions of the correlators have been presented [33]–[35]. For the non-Gaussian Hermitian matrix model, the correlators can be evaluated by the recursive formulas derived from the Virasoro constraints and the additional constraints. However, it is hard to give the compact expressions of the correlators [36–40].

In this section, we focus on the Gaussian supereigenvalue model which is obtained by taking the shift $t_2 \rightarrow t_2 + \frac{1}{2}$ in the potential of the partition function (1.2)

$$Z_G = \frac{1}{\Lambda_G} \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N (V_{NS}(z_i, \theta_i) + \frac{1}{2} z_i^2)}, \quad (2.1)$$

where the normalization factor Λ_G is given by

$$\Lambda_G = \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{2\hbar} \sum_{i=1}^N z_i^2}. \quad (2.2)$$

From the invariance of the integral (2.1) under the changes of integration variables $(z_i \rightarrow z_i + \epsilon z_i^{n+1}, \quad \theta_i \rightarrow \theta_i + \frac{1}{2} \epsilon (n+1) \theta_i z_i^n)$ and $(z_i \rightarrow z_i + z_i^{n+1} \theta_i \delta, \quad \theta_i \rightarrow \theta_i + z_i^{n+1} \delta)$, where ϵ and δ are the infinitesimal bosonic and fermionic constants, respectively, we can give the super Virasoro constraints

$$\ell_n Z_G = 0, \quad g_{n+\frac{1}{2}} Z_G = 0, \quad n \geq -1, \quad (2.3)$$

where

$$\begin{aligned} \ell_n &= \frac{\partial}{\partial t_{n+2}} + \sum_{k=1}^{\infty} k t_k \frac{\partial}{\partial t_{n+k}} + \frac{\hbar^2}{2} \sum_{k=0}^n \frac{\partial}{\partial t_k} \frac{\partial}{\partial t_{n-k}} + \sum_{k=0}^{\infty} \left(k + \frac{n+1}{2} \right) \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+n+\frac{1}{2}}} \\ &\quad + \frac{\hbar^2}{2} \sum_{k=1}^n k \frac{\partial}{\partial \xi_{n-k+\frac{1}{2}}} \frac{\partial}{\partial \xi_{k-\frac{1}{2}}} - \frac{\hbar}{2\sqrt{\beta}} (1-\beta)(n+1) \frac{\partial}{\partial t_n}, \\ g_{n+\frac{1}{2}} &= \frac{\partial}{\partial \xi_{n+\frac{5}{2}}} + \sum_{k=1}^{\infty} k t_k \frac{\partial}{\partial \xi_{n+k+\frac{1}{2}}} + \sum_{k=0}^{\infty} \xi_{k+\frac{1}{2}} \frac{\partial}{\partial t_{k+n+1}} + \hbar^2 \sum_{k=0}^n \frac{\partial}{\partial \xi_{k+\frac{1}{2}}} \frac{\partial}{\partial t_{n-k}} \\ &\quad - \frac{\hbar}{\sqrt{\beta}} (1-\beta)(n+1) \frac{\partial}{\partial \xi_{n+\frac{1}{2}}}. \end{aligned} \quad (2.4)$$

The constraint operators obey the Neveu-Schwarz version of the super-Virasoro algebra

$$\begin{aligned} [\ell_m, \ell_n] &= (m-n) \ell_{m+n}, \\ [\ell_m, g_{n+\frac{1}{2}}] &= \left(\frac{m-1}{2} - n \right) g_{m+n+\frac{1}{2}}, \\ \{g_{m+\frac{1}{2}}, g_{n+\frac{1}{2}}\} &= 2\ell_{m+n+1}. \end{aligned} \quad (2.5)$$

Let us now take the change of variables given by

$$z_i \rightarrow z_i + \epsilon \sum_{n=1}^{\infty} n t_n z_i^{n-1}, \quad \theta_i \rightarrow \theta_i + \frac{1}{2} \epsilon \sum_{n=2}^{\infty} n(n-1) t_n z_i^{n-2} \theta_i. \quad (2.6)$$

By requiring that the partition function is invariant under the infinitesimal transformations (2.6), it leads to the constraint

$$(\tilde{D}_1 + \tilde{W}_1) Z_G = 0, \quad (2.7)$$

where

$$\begin{aligned}
 \tilde{D}_1 &= \sum_{k=1}^{\infty} k t_k \frac{\partial}{\partial t_k}, \\
 \tilde{W}_1 &= \sum_{n,k=1}^{\infty} n k t_n t_k \frac{\partial}{\partial t_{n+k-2}} + \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} n \left(k + \frac{n-1}{2} \right) t_n \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{n+k-\frac{3}{2}}} \\
 &\quad + \frac{\hbar^2}{2} \sum_{n=2}^{\infty} \sum_{k=0}^{n-2} n t_n \frac{\partial}{\partial t_k} \frac{\partial}{\partial t_{n-k-2}} + \frac{\hbar^2}{2} \sum_{n=3}^{\infty} \sum_{k=1}^{n-2} n k t_n \frac{\partial}{\partial \xi_{n-k-\frac{3}{2}}} \frac{\partial}{\partial \xi_{k-\frac{1}{2}}} \\
 &\quad - \frac{\hbar}{2\sqrt{\beta}} (1-\beta) \sum_{n=2}^{\infty} n(n-1) t_n \frac{\partial}{\partial t_{n-2}}.
 \end{aligned} \tag{2.8}$$

Similarly, by requiring the invariance of the partition function (2.1) under

$$z_i \rightarrow z_i + \epsilon \sum_{n=1}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} z_i^{n-1} \theta_i, \quad \theta_i \rightarrow \theta_i - \epsilon \sum_{n=1}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} z_i^{n-1}, \tag{2.9}$$

it gives another constraint

$$(\tilde{D}_2 + \tilde{W}_2) Z_G = 0, \tag{2.10}$$

where

$$\begin{aligned}
 \tilde{D}_2 &= \sum_{k=1}^{\infty} \left(k + \frac{1}{2} \right) \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+\frac{1}{2}}}, \\
 \tilde{W}_2 &= \sum_{n,k=1}^{\infty} n \left(k + \frac{1}{2} \right) t_n \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{n+k-\frac{3}{2}}} - \frac{\hbar}{\sqrt{\beta}} (1-\beta) \sum_{n=2}^{\infty} (n-1) \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{n-\frac{3}{2}}} \\
 &\quad + \hbar^2 \sum_{n=2}^{\infty} \sum_{k=0}^{n-2} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+\frac{1}{2}}} \frac{\partial}{\partial t_{n-k-2}} + \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \xi_{k+\frac{1}{2}} \frac{\partial}{\partial t_{n+k-1}}.
 \end{aligned} \tag{2.11}$$

Combining (2.7) and (2.10), we have

$$(\tilde{D} + \tilde{W}) Z_G = 0, \tag{2.12}$$

where $\tilde{D} = \tilde{D}_1 + \tilde{D}_2$, $\tilde{W} = \tilde{W}_1 + \tilde{W}_2$.

The partition function (2.1) can be formally expanded as

$$\begin{aligned}
 Z_G &= e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} + \sum_{s=1}^{\infty} Z_G^{(s)} \\
 &= e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} \left[1 - \frac{\sqrt{\beta}}{\hbar} \tilde{C}_{k_1} t_{k_1} + \frac{1}{2!} \left(\frac{\sqrt{\beta}}{\hbar} \right)^2 \tilde{C}_{k_1, k_2} t_{k_1} t_{k_2} - \frac{1}{2!} \left(\frac{\sqrt{\beta}}{\hbar} \right)^2 \tilde{C}^{s_1+\frac{1}{2}, s_2+\frac{1}{2}} \xi_{s_1+\frac{1}{2}} \xi_{s_2+\frac{1}{2}} \right. \\
 &\quad \left. - \frac{1}{3!} \left(\frac{\sqrt{\beta}}{\hbar} \right)^3 \tilde{C}_{k_1, k_2, k_3} t_{k_1} t_{k_2} t_{k_3} + \frac{1}{2!} \left(\frac{\sqrt{\beta}}{\hbar} \right)^3 \tilde{C}_{k_1}^{s_1+\frac{1}{2}, s_2+\frac{1}{2}} t_{k_1} \xi_{s_1+\frac{1}{2}} \xi_{s_2+\frac{1}{2}} + \dots \right], \tag{2.13}
 \end{aligned}$$

where

$$Z_G^{(s)} = e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} \left[\sum_{n=0}^{\infty} \sum_{\substack{m=0 \\ m \text{ is even}}}^{\infty} \sum_{\substack{k_1+\dots+k_n+s_1+\dots+s_m+\frac{m}{2}=s \\ k_1, \dots, k_n \geq 1 \\ s_1, \dots, s_m \geq 0}} \frac{(-1)^{\frac{m(m+1)}{2}} \left(-\frac{\sqrt{\beta}}{\hbar}\right)^{n+m}}{n!m!} \tilde{C}_{k_1, \dots, k_n}^{s_1+\frac{1}{2}, \dots, s_m+\frac{1}{2}} \cdot t_{k_1} \cdots t_{k_n} \xi_{s_1+\frac{1}{2}} \cdots \xi_{s_m+\frac{1}{2}} \right], \quad (2.14)$$

and the coefficients $\tilde{C}_{k_1, \dots, k_n}^{s_1+\frac{1}{2}, \dots, s_m+\frac{1}{2}}$ are the correlators defined by

$$\begin{aligned} \tilde{C}_{k_1, \dots, k_n}^{s_1+\frac{1}{2}, \dots, s_m+\frac{1}{2}} &= \frac{1}{\Lambda_G} \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \sum_{\substack{i_1, \dots, i_n=1 \\ j_1, \dots, j_m=1}}^N z_{i_1}^{k_1} \cdots z_{i_n}^{k_n} z_{j_1}^{s_1} \theta_{j_1} \cdots z_{j_m}^{s_m} \theta_{j_m} \\ &\cdot \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{2\hbar} \sum_{i=1}^N z_i^2}. \end{aligned} \quad (2.15)$$

Let V be the infinite dimensional vector space on $\mathbb{C}[[t_0]]$ generated by the basis

$$\{t_{k_1} \cdots t_{k_n} \xi_{s_1+\frac{1}{2}} \cdots \xi_{s_m+\frac{1}{2}} | k_1, \dots, k_n \geq 1, s_1, \dots, s_m \geq 0, n \in \mathbb{N}, m \in 2\mathbb{N}\}. \quad (2.16)$$

Thus the partition function Z_G can be seen as a vector in V , the operators $\tilde{D}$ and $\tilde{W}$ are differential operators on V . Under the natural gradation $\deg(t_k) = k$, $\deg\left(\frac{\partial}{\partial t_k}\right) = -k$, $\deg\left(\xi_{k+\frac{1}{2}}\right) = k + \frac{1}{2}$, and $\deg\left(\frac{\partial}{\partial \xi_{k+\frac{1}{2}}}\right) = -k - \frac{1}{2}$, $k = 0, 1, \dots, \infty$, we have $\deg(\tilde{D}) = 0$ and $\deg(\tilde{W}) = 2$. In addition, the kernel of $\tilde{D}$ denoted as $\text{Ker}(\tilde{D}) = \{v \in V | \tilde{D}v = 0\}$, is one dimensional. It is not difficult to see that $\text{Ker}(\tilde{D} + \tilde{W})$ is still one dimensional. Therefore, the partition function Z_G is uniquely determined by the constraint (2.12). As a consequence, the correlators (2.15) can be totally derived from (2.12).

Let us collect the coefficients of t_1^l and t_2^l in (2.12) and set to zero, respectively, we obtain

$$\tilde{C}_1 = 0, \quad \tilde{C}_{1,1} = \frac{\hbar}{\sqrt{\beta}} N, \quad \tilde{C}_2 = \frac{\hbar}{2\sqrt{\beta}} N \tilde{N}, \quad (2.17)$$

and the recursive relations

$$\begin{aligned} \tilde{C}_{\underbrace{1, \dots, 1}_l} &= \frac{\hbar}{\sqrt{\beta}} N(l-1) \tilde{C}_{\underbrace{1, \dots, 1}_{l-2}}, \\ \tilde{C}_{\underbrace{2, \dots, 2}_l} &= \frac{\hbar}{\sqrt{\beta}} \left[\frac{N \tilde{N}}{2} + 2(l-1) \right] \tilde{C}_{\underbrace{2, \dots, 2}_{l-1}}, \end{aligned} \quad (2.18)$$

where $\tilde{N} = \beta N + 1 - \beta$.

Then it is easy to obtain

$$\tilde{C}_{\underbrace{1, \dots, 1}_l} = \begin{cases} \left(\frac{\hbar}{\sqrt{\beta}} N\right)^{\frac{l}{2}} (l-1)!!, & \text{for } l \text{ even;} \\ 0, & \text{for } l \text{ odd,} \end{cases} \quad (2.19)$$

and

$$\tilde{C}_{\underbrace{2, \dots, 2}_l} = \left(\frac{\hbar}{\sqrt{\beta}} \right)^l \prod_{j=0}^{l-1} \left(\frac{N\tilde{N}}{2} + 2j \right). \quad (2.20)$$

Similarly, by collecting the coefficients of $t_1^l t_2$, $t_1^l \xi_{\frac{1}{2}} \xi_{\frac{3}{2}}$, $t_2^l t_1^2$ and $t_2^l \xi_{\frac{1}{2}} \xi_{\frac{3}{2}}$ in (2.12) and setting to zero, respectively, we may also obtain the corresponding recursive relations and derive the exact correlators

$$\begin{aligned} \tilde{C}_{2, \underbrace{1, \dots, 1}_l} &= \begin{cases} \left(\frac{\hbar}{\sqrt{\beta}} \right)^{\frac{l}{2}+1} (l-1)!! \left(l + \frac{N\tilde{N}}{2} \right) N^{\frac{l}{2}}, & \text{for } l \text{ even;} \\ 0, & \text{for } l \text{ odd,} \end{cases} \\ \tilde{C}_{\underbrace{1, \dots, 1}_l}^{\frac{1}{2}, \frac{3}{2}} &= \begin{cases} \left(\frac{\hbar}{\sqrt{\beta}} N \right)^{\frac{l}{2}+1} (l-1)!!, & \text{for } l \text{ even;} \\ 0, & \text{for } l \text{ odd,} \end{cases} \\ \tilde{C}_{1,1, \underbrace{2, \dots, 2}_l} &= \tilde{C}_{\underbrace{2, \dots, 2}_l}^{\frac{1}{2}, \frac{3}{2}} = \left(\frac{\hbar}{\sqrt{\beta}} \right)^{l+1} N \prod_{j=1}^l \left(\frac{N\tilde{N}}{2} + 2j \right). \end{aligned} \quad (2.21)$$

It should be mentioned that we can also calculate the correlators (2.19)–(2.21) from the super Virasoro constraints (2.3). However, due to the complicated form of the recursive formulas for correlators from (2.3), it is hard to give the general expressions of the correlators. In order to achieve more results, let us further analyze the constraint (2.12).

Since $\tilde{D}e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} = 0$, the constraint (2.12) can be rewritten as

$$(\tilde{D} + \tilde{W}) \sum_{s=1}^{\infty} Z_G^{(s)} = -\tilde{W}e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0}. \quad (2.22)$$

It is noted that the function $-\tilde{W}e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0}$ on the right hand side of (2.22) has degree 2. Hence the operators $\tilde{D}$ and $\tilde{D} + \tilde{W}$ are invertible on $-\tilde{W}e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0}$.

From (2.22), we have

$$\begin{aligned} \sum_{s=1}^{\infty} Z_G^{(s)} &= -(\tilde{D} + \tilde{W})^{-1} \tilde{W}e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} \\ &= \sum_{k=1}^{\infty} (-\tilde{D}^{-1} \tilde{W})^k e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0}. \end{aligned} \quad (2.23)$$

Thus the partition function (2.1) can be expressed as

$$Z_G = \sum_{k=0}^{\infty} (-\tilde{D}^{-1} \tilde{W})^k e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0}. \quad (2.24)$$

We denote an operator $\mathcal{O}$ on V the degree operator if $\mathcal{O}f = \deg(f)f$ for any homogeneous function $f \in V$. Note that $\tilde{W}$ is a homogeneous operator with degree 2 in (2.24).

It is not difficult to show that if $\tilde{D}$ is a degree operator, the partition function (2.24) can be generated by the W -representation. Unfortunately, it is clear that $\tilde{D}$ in (2.24) is not the degree operator. It causes the usual W -representation of the matrix model (2.24) to fail here. In other words, the partition function (2.1) can not be obtained by acting on elementary functions with exponents of the operator $\tilde{W}$. In spite of this negative result, by evaluating the action of the homogeneous operator $(-\tilde{D}^{-1}\tilde{W})^k$ on the function $e^{-\frac{\sqrt{\beta}}{h}Nt_0}$, we can still derive the compact expressions of the correlators from the representation (2.24). In what follows let us continue to discuss the correlators.

Since $\tilde{D}^{-1}\tilde{W}$ is an operator with degree 2, we can see from (2.24) that $Z_G^{(s)} = 0$, when s is odd. It leads to $\tilde{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}} = 0$, when $\sum_{\mu=1}^n k_\mu + \sum_{\nu=1}^m s_\nu + \frac{m}{2} = s$ is odd. In order to give the general expressions of the correlators for the s even case, we need to write out the action $(-\tilde{D}^{-1}\tilde{W})^k e^{-\frac{\sqrt{\beta}}{h}Nt_0}$ explicitly.

The operator $\tilde{D}^{-1}$ can be expressed as

$$\begin{aligned}\tilde{D}^{-1} &= \left(D - \frac{1}{2} \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}} \right)^{-1} \\ &= D^{-1} + D^{-1}(2D-1)^{-1} \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}},\end{aligned}\quad (2.25)$$

where D is the degree operator

$$D = \sum_{k=1}^{\infty} k t_k \frac{\partial}{\partial t_k} + \sum_{k=0}^{\infty} \left(k + \frac{1}{2} \right) \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+\frac{1}{2}}}, \quad (2.26)$$

and the relations $\left[D, \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}} \right] = 0$ and $\left(\xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}} \right)^2 = \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}}$ are used to give (2.25).

Thus the operators in (2.24) now take the following form:

$$\begin{aligned}(\tilde{D}^{-1}\tilde{W})^k &= (D^{-1}\tilde{W} + D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^k \\ &= \sum_{r=1}^{\lfloor \frac{k+1}{2} \rfloor} \sum_{\substack{l_1+\dots+l_{2r-1}=k \\ l_1, \dots, l_{2r-1} \geq 1}} (T_1 + T_2) + \sum_{r=1}^{\lfloor \frac{k}{2} \rfloor} \sum_{\substack{l_1+\dots+l_{2r}=k \\ l_1, \dots, l_{2r} \geq 1}} (T_3 + T_4),\end{aligned}\quad (2.27)$$

where $\lfloor k \rfloor = \text{Max}\{m \in \mathbb{Z} | m \leq k\}$ is the floor function, $\tilde{\mathbb{W}} = \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}} \tilde{W}$, and T_i , $i = 1, \dots, 4$ are

$$\begin{aligned}T_1 &= (D^{-1}\tilde{W})^{l_{2r-1}} (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_{2r-2}} \dots (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_2} (D^{-1}\tilde{W})^{l_1}, \\ T_2 &= (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_{2r-1}} (D^{-1}\tilde{W})^{l_{2r-2}} \dots (D^{-1}\tilde{W})^{l_2} (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_1}, \\ T_3 &= (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_{2r}} (D^{-1}\tilde{W})^{l_{2r-1}} \dots (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_2} (D^{-1}\tilde{W})^{l_1}, \\ T_4 &= (D^{-1}\tilde{W})^{l_{2r}} (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_{2r-1}} \dots (D^{-1}\tilde{W})^{l_2} (D^{-1}(2D-1)^{-1}\tilde{\mathbb{W}})^{l_1}.\end{aligned}\quad (2.28)$$

Using the fact that for the homogeneous function $f \in V$, $D^{-1}f = \deg(f)^{-1}f$, the action of the operator T_1 on the function $e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0}$ gives

$$\begin{aligned}
 T_1 e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} &= \frac{1}{k!2^k} \frac{1}{\prod_{i=l_1+1}^k (4i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [4(l_1 + \dots + l_{2i} + j) - 1] \\
 &\quad \cdot \tilde{W}^{l_{2r-1}} \tilde{W}^{l_{2r-2}} \dots \tilde{W}^{l_2} \tilde{W}^{l_1} e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} \\
 &= \frac{1}{k!2^k} \frac{1}{\prod_{i=l_1+1}^k (4i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [4(l_1 + \dots + l_{2i} + j) - 1] e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} \\
 &\quad \cdot \sum_{\alpha=1}^{2k} \sum_{\substack{k_1+\dots+k_c+\frac{d}{2}+ \\ s_1+\dots+s_d=2k \\ k_1,\dots,k_c \geq 1 \\ s_1,\dots,s_d \geq 0}} \left(-\frac{\sqrt{\beta}}{\hbar}N\right)^\alpha E_{\underbrace{(0,\dots,0)}_\alpha}^{(k_1,\dots,k_c|s_1+\frac{1}{2},\dots,s_d+\frac{1}{2})} t_{k_1} \dots t_{k_c} \xi_{s_1+\frac{1}{2}} \dots \xi_{s_d+\frac{1}{2}},
 \end{aligned} \tag{2.29}$$

where we have used the formal expression of $\tilde{W}^{l_{2r-1}} \tilde{W}^{l_{2r-2}} \dots \tilde{W}^{l_2} \tilde{W}^{l_1}$ with degree $2k$

$$\begin{aligned}
 \tilde{W}^{l_{2r-1}} \tilde{W}^{l_{2r-2}} \dots \tilde{W}^{l_2} \tilde{W}^{l_1} &= \sum_{a,c=0}^{2k} \sum_{b,d=0}^{2k+1} \sum_{i_1,\dots,i_a=0}^{\infty} \sum_{\substack{j_1,\dots,j_b=0 \\ k_1+\dots+k_c+ \\ s_1+\dots+s_d=\rho \\ k_1,\dots,k_c \geq 1 \\ s_1,\dots,s_d \geq 0}} E_{\left(i_1,\dots,i_a|j_1+\frac{1}{2},\dots,j_b+\frac{1}{2}\right)}^{(k_1,\dots,k_c|s_1+\frac{1}{2},\dots,s_d+\frac{1}{2})} t_{k_1} \dots t_{k_c} \\
 &\quad \cdot \xi_{s_1+\frac{1}{2}} \dots \xi_{s_d+\frac{1}{2}} \frac{\partial}{\partial t_{i_1}} \dots \frac{\partial}{\partial t_{i_a}} \frac{\partial}{\partial \xi_{j_1+\frac{1}{2}}} \dots \frac{\partial}{\partial \xi_{j_b+\frac{1}{2}}},
 \end{aligned} \tag{2.30}$$

$\rho = \sum_{\mu=1}^a i_\mu + \sum_{\nu=1}^b j_\nu + 2k + \frac{b}{2} - \frac{d}{2}$, the coefficients $E_{\left(i_1,\dots,i_a|j_1+\frac{1}{2},\dots,j_b+\frac{1}{2}\right)}^{(k_1,\dots,k_c|s_1+\frac{1}{2},\dots,s_d+\frac{1}{2})}$ are polynomials with respect to i_μ , j_ν , $k_{\bar{\mu}}$ and $s_{\bar{\nu}}$, $\bar{\mu} = 1, \dots, c$, $\bar{\nu} = 1, \dots, d$.

Similarly, we have

$$\begin{aligned}
 T_2 e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} &= \frac{1}{k!2^k} \frac{1}{\prod_{i=1}^k (4i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i}} [4(l_1 + \dots + l_{2i-1} + j) - 1] e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} \\
 &\quad \cdot \sum_{\alpha=1}^{2k} \sum_{\substack{k_1+\dots+k_c+\frac{d}{2}+ \\ s_1+\dots+s_d=2k \\ k_1,\dots,k_c \geq 1 \\ s_1,\dots,s_d \geq 0}} \left(-\frac{\sqrt{\beta}}{\hbar}N\right)^\alpha F_{\underbrace{(0,\dots,0)}_\alpha}^{(k_1,\dots,k_c|s_1+\frac{1}{2},\dots,s_d+\frac{1}{2})} t_{k_1} \dots t_{k_c} \xi_{s_1+\frac{1}{2}} \dots \xi_{s_d+\frac{1}{2}}, \\
 T_3 e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} &= \frac{1}{k!2^k} \frac{1}{\prod_{i=l_1+1}^k (4i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [4(l_1 + \dots + l_{2i} + j) - 1] e^{-\frac{\sqrt{\beta}}{\hbar}Nt_0} \\
 &\quad \cdot \sum_{\alpha=1}^{2k} \sum_{\substack{k_1+\dots+k_c+\frac{d}{2}+ \\ s_1+\dots+s_d=2k \\ k_1,\dots,k_c \geq 1 \\ s_1,\dots,s_d \geq 0}} \left(-\frac{\sqrt{\beta}}{\hbar}N\right)^\alpha G_{\underbrace{(0,\dots,0)}_\alpha}^{(k_1,\dots,k_c|s_1+\frac{1}{2},\dots,s_d+\frac{1}{2})} t_{k_1} \dots t_{k_c} \xi_{s_1+\frac{1}{2}} \dots \xi_{s_d+\frac{1}{2}},
 \end{aligned}$$

$$\begin{aligned}
 T_4 e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} &= \frac{1}{k! 2^k} \frac{1}{\prod_{i=1}^k (4i-1)} \prod_{i=1}^r \prod_{j=1}^{l_{2i}} [4(l_1 + \dots + l_{2i-1} + j) - 1] e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} \\
 &\quad \cdot \sum_{\alpha=1}^{2k} \sum_{\substack{k_1 + \dots + k_c + \frac{d}{2} + \\ s_1 + \dots + s_d = 2k \\ k_1, \dots, k_c \geq 1 \\ s_1, \dots, s_d \geq 0}} \left(-\frac{\sqrt{\beta}}{\hbar} N \right)^\alpha H_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2} \right)} t_{k_1} \dots t_{k_c} \xi_{s_1 + \frac{1}{2}} \dots \xi_{s_d + \frac{1}{2}},
 \end{aligned} \tag{2.31}$$

where $F_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2} \right)}$, $G_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2} \right)}$ and $H_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2} \right)}$ are the coefficients of the terms $t_{k_1} \dots t_{k_c} \xi_{s_1 + \frac{1}{2}} \dots \xi_{s_d + \frac{1}{2}} \frac{\partial^\alpha}{\partial t_0^\alpha}$ in the formal expansions of the corresponding operators $\tilde{W}^{l_{2r-1}} \tilde{W}^{l_{2r-2}} \dots \tilde{W}^{l_2} \tilde{W}^{l_1}$, $\tilde{W}^{l_{2r}} \tilde{W}^{l_{2r-1}} \dots \tilde{W}^{l_2} \tilde{W}^{l_1}$ and $\tilde{W}^{l_{2r}} \tilde{W}^{l_{2r-1}} \dots \tilde{W}^{l_2} \tilde{W}^{l_1}$, respectively.

Using the actions (2.29) and (2.31), we may give the coefficients of $t_{k_1} \dots t_{k_n} \xi_{s_1 + \frac{1}{2}} \dots \xi_{s_m + \frac{1}{2}}$ with $\sum_{\mu=1}^n k_\mu + \sum_{\nu=1}^m s_\nu + \frac{m}{2} = 2k$, $k_\mu \geq 1$, $s_\nu \geq 0$ in (2.24). On the other hand, it is easy to give the corresponding coefficients in (2.13). Then from the equivalence of these coefficients, we obtain the desired expressions for the correlators

$$\tilde{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}} = \frac{(-1)^{k + \frac{m(m+1)}{2}} n! \left(-\frac{\hbar}{\sqrt{\beta}} \right)^{n+m}}{k! 2^k \lambda_{(k_1, \dots, k_n)}} \sum_{\alpha=1}^{2k} \left(-\frac{\sqrt{\beta}}{\hbar} N \right)^\alpha \tilde{P}_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(k_1, \dots, k_n | s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2} \right)}, \tag{2.32}$$

where $k = \frac{1}{2} \left(\sum_{\mu=1}^n k_\mu + \sum_{\nu=1}^m s_\nu + \frac{m}{2} \right)$, $k_\mu \geq 1$, $s_\nu \geq 0$, $\lambda_{(k_1, \dots, k_n)}$ is the number of distinct permutations of $(k_1, \dots, k_n)$,

$$\begin{aligned}
 &\tilde{P}_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(k_1, \dots, k_n | s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2} \right)} \\
 &= \sum_{\sigma_1, \sigma_2} (-1)^{\tau(\sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} \left[\sum_{r=1}^{\lfloor \frac{k+1}{2} \rfloor} \sum_{\substack{l_1 + \dots + l_{2r-1} = k \\ l_1, \dots, l_{2r-1} \geq 1}} \left(\frac{1}{\prod_{i=l_1+1}^k (4i-1)} \right. \right. \\
 &\quad \cdot \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [4(l_1 + \dots + l_{2i} + j) - 1] E_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}) \right)} \\
 &\quad \left. \left. + \frac{1}{\prod_{i=1}^k (4i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i}} [4(l_1 + \dots + l_{2i-1} + j) - 1] F_{\underbrace{(0, \dots, 0)_\alpha}}^{\left(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}) \right)} \right) \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{r=1}^{\lfloor \frac{k}{2} \rfloor} \sum_{\substack{l_1+\dots+l_{2r}=k \\ l_1, \dots, l_{2r} \geq 1}} \left(\frac{1}{\prod_{i=l_1+1}^k (4i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [4(l_1 + \dots + l_{2i} + j) - 1] \right. \\
 & \cdot G_{\underbrace{(0, \dots, 0)}_{\alpha}}^{(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} + \frac{1}{\prod_{i=1}^k (4i-1)} \prod_{i=1}^r \prod_{j=1}^{l_{2i}} [4(l_1 + \dots + l_{2i-1} + j) - 1] \\
 & \cdot H_{\underbrace{(0, \dots, 0)}_{\alpha}}^{(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} \left. \right) \Bigg]. \quad (2.33)
 \end{aligned}$$

Here σ_1 denotes all the distinct permutations of $(k_1, \dots, k_n)$, σ_2 is all the distinct permutations of $(s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2})$ and its inverse number is denoted as $\tau(\sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))$.

Let us list the correlators for the case of $k = 2$ in (2.32) as follows:

$$\begin{aligned}
 \tilde{C}_4 &= \frac{\hbar^2}{4\beta} N[2\tilde{N}^2 + (1 - \beta)\tilde{N} + 4\beta], & \tilde{C}_{3,1} &= \frac{3\hbar^2}{2\beta} N\tilde{N}, \\
 \tilde{C}_{2,2} &= \frac{\hbar^2}{4\beta} N\tilde{N}(N\tilde{N} + 4), & \tilde{C}_{1,1,1,1} &= \frac{3\hbar^2}{\beta} N^2, \\
 \tilde{C}_{2,1,1} &= \frac{\hbar^2}{2\beta} N(N\tilde{N} + 4), & C_{1,1}^{\frac{1}{2}, \frac{3}{2}} &= \frac{\hbar^2}{\beta} N^2, \\
 \tilde{C}_2^{\frac{1}{2}, \frac{3}{2}} &= \frac{\hbar^2}{2\beta} N(N\tilde{N} + 4), & C_1^{\frac{1}{2}, \frac{5}{2}} &= \frac{2\hbar^2}{\beta} N, \\
 \tilde{C}_{\frac{1}{2}, \frac{7}{2}} &= \frac{\hbar^2}{2\beta} N[3\tilde{N} + 2(1 - \beta)], & \tilde{C}_{\frac{3}{2}, \frac{5}{2}} &= -\frac{\hbar^2}{2\beta} N\tilde{N}. \quad (2.34)
 \end{aligned}$$

3 Non-Gaussian supereigenvalue model in the Neveu-Schwarz sector

Let us consider the non-Gaussian supereigenvalue model in the Neveu-Schwarz sector

$$Z_{NG}(t, \xi; a, \varepsilon) = \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N (V_{NS}(z_i, \theta_i) + \tilde{V}_{NS}(z_i, \theta_i))}, \quad (3.1)$$

where N is even, $\tilde{V}_{NS}(z, \theta) = \frac{1}{2p+2} z^{2p+2} + \sum_{k=1}^{2p} \frac{1}{k} a_k z^k + \sum_{l=0}^{2p} \varepsilon_l z^l \theta$, $p \geq 1$, a_k and ε_l are nonzero bosonic and fermionic coupling constants, respectively.

By applying the changes of integration variables $(z_i \rightarrow z_i + \epsilon \sum_{n=2p+1}^{\infty} n t_n z_i^{n-2p-1}, \theta_i \rightarrow \theta_i + \frac{1}{2} \epsilon \sum_{n=2p+1}^{\infty} n(n-2p-1) t_n z_i^{n-2p-2} \theta_i)$ and $(z_i \rightarrow z_i + \epsilon \sum_{n=2p+1}^{\infty} (n + \frac{1}{2}) \xi_{n+\frac{1}{2}} z_i^{n-2p-1} \theta_i, \theta_i \rightarrow \theta_i - \epsilon \sum_{n=2p+1}^{\infty} (n + \frac{1}{2}) \xi_{n+\frac{1}{2}} z_i^{n-2p-1})$ for the partition function (3.1), we may derive the constraint from the invariance of the integral

$$(\hat{D} + \hat{W}) Z_{NG}(t, \xi; a, \varepsilon) = 0, \quad (3.2)$$

where

$$\hat{D} = \sum_{n=2p+1}^{\infty} n t_n \frac{\partial}{\partial t_n} + \sum_{n=2p+1}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{n+\frac{1}{2}}}, \quad (3.3)$$

$\hat{W} = \hat{W}_{2p+2} + \sum_{k=2}^{2p+1} \hat{W}_k + \sum_{l=1}^{2p+1} \hat{W}_{l+\frac{1}{2}}$, the operators $\hat{W}_{2p+2}$, $\hat{W}_k$ and $\hat{W}_{l+\frac{1}{2}}$ are, respectively, given by

$$\begin{aligned}
 \hat{W}_{2p+2} = & \sum_{n=2p+1}^{\infty} nt_n \left[\sum_{k=1}^{\infty} kt_k \frac{\partial}{\partial t_{n+k-2p-2}} + \frac{\hbar^2}{2} \sum_{k=0}^{n-2p-2} \frac{\partial}{\partial t_k} \frac{\partial}{\partial t_{n-k-2p-2}} \right. \\
 & + \sum_{k=0}^{\infty} \left(\frac{n-1}{2} + k - p \right) \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+n-2p-\frac{3}{2}}} + \frac{\hbar^2}{2} \sum_{k=1}^{n-2p-2} k \frac{\partial}{\partial \xi_{n-k-2p-\frac{3}{2}}} \frac{\partial}{\partial \xi_{k-\frac{1}{2}}} \\
 & \left. - \frac{\hbar}{2\sqrt{\beta}} (1-\beta)(n-2p-1) \frac{\partial}{\partial t_{n-2p-2}} \right] + \sum_{n=2p+1}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \left[\sum_{k=1}^{\infty} kt_k \frac{\partial}{\partial \xi_{n+k-2p-\frac{3}{2}}} \right. \\
 & + \hbar^2 \sum_{k=0}^{n-2p-2} \frac{\partial}{\partial \xi_{k+\frac{1}{2}}} \frac{\partial}{\partial t_{n-k-2p-2}} - \frac{\hbar}{\sqrt{\beta}} (1-\beta)(n-2p-1) \frac{\partial}{\partial \xi_{n-2p-\frac{3}{2}}} \\
 & \left. + \sum_{k=0}^{\infty} \xi_{k+\frac{1}{2}} \frac{\partial}{\partial t_{k+n-2p-1}} \right], \\
 \hat{W}_k = & a_{2p+2-k} \sum_{n=2p+1}^{\infty} nt_n \frac{\partial}{\partial t_{n-k}} + a_{2p+2-k} \sum_{n=2p+1}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{n+\frac{1}{2}-k}}, \\
 \hat{W}_{l+\frac{1}{2}} = & \varepsilon_{2p+1-l} \sum_{n=2p+1}^{\infty} n \left(p-l + \frac{n+1}{2} \right) t_n \frac{\partial}{\partial \xi_{n-\frac{1}{2}-l}} \\
 & + \varepsilon_{2p+1-l} \sum_{n=2p+1}^{\infty} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial t_{n-l}}. \tag{3.4}
 \end{aligned}$$

The partition function (3.1) is now viewed as a vector in the space $\tilde{V}$ generated by the basis

$$\{t_{k_1} \cdots t_{k_n} \xi_{s_1+\frac{1}{2}} \cdots \xi_{s_m+\frac{1}{2}} | k_1, \dots, k_n \geq 1, s_1, \dots, s_m \geq 0, n, m \in \mathbb{N}\}, \tag{3.5}$$

with coefficients on $\mathbb{C}[[t_0, a, \varepsilon]]$. Let us define the degrees of the coupling constants a_k and ε_l are 0. Since $m \in \mathbb{N}$ in the basis (3.5), the partition function (3.1) is graded from $0, \frac{1}{2}, 1, \frac{3}{2}, \dots, \infty$. Thus we have the expansion

$$\begin{aligned}
 Z_{NG}(t, \xi; a, \varepsilon) = & e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} Z_{NG}(a, \varepsilon) + \sum_{s \in \frac{1}{2}\mathbb{N}_+} Z_{NG}^{(s)}(t, \xi; a, \varepsilon) \\
 = & e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} \left[Z_{NG}(a, \varepsilon) - \frac{\sqrt{\beta}}{\hbar} t_{k_1} C_{k_1}(a, \varepsilon) - \frac{\sqrt{\beta}}{\hbar} \xi_{s_1+\frac{1}{2}} C^{s_1+\frac{1}{2}}(a, \varepsilon) \right. \\
 & + \frac{1}{2!} \left(\frac{\sqrt{\beta}}{\hbar} \right)^2 t_{k_1} t_{k_2} C_{k_1, k_2}(a, \varepsilon) + \left(\frac{\sqrt{\beta}}{\hbar} \right)^2 t_{k_1} \xi_{s_1+\frac{1}{2}} C_{k_1}^{s_1+\frac{1}{2}}(a, \varepsilon) \\
 & \left. - \frac{1}{2!} \left(\frac{\sqrt{\beta}}{\hbar} \right)^2 \xi_{s_1+\frac{1}{2}} \xi_{s_2+\frac{1}{2}} C^{s_1+\frac{1}{2}, s_2+\frac{1}{2}}(a, \varepsilon) + \dots \right], \tag{3.6}
 \end{aligned}$$

where

$$Z_{NG}(a, \varepsilon) = \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{h} \sum_{i=1}^N \tilde{V}_{NS}(z_i, \theta_i)}, \quad (3.7)$$

$$Z_{NG}^{(s)}(t, \xi; a, \varepsilon) = e^{-\frac{\sqrt{\beta}}{h} N t_0} \left[\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \sum_{\substack{k_1 + \dots + k_n + \\ s_1 + \dots + s_m + \frac{m}{2} = s \\ k_1, \dots, k_n \geq 1 \\ s_1, \dots, s_m \geq 0}} \frac{(-1)^{\frac{m(m-1)}{2}} \left(-\frac{\sqrt{\beta}}{h}\right)^{n+m}}{n! m!} t_{k_1} \dots t_{k_n} \right. \\ \left. \cdot \xi_{s_1 + \frac{1}{2}} \dots \xi_{s_m + \frac{1}{2}} C_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}(a, \varepsilon) \right], \quad (3.8)$$

and the correlators $C_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}(a, \varepsilon)$ are defined by

$$C_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}(a, \varepsilon) = \left(\prod_{i=1}^N \int_{-\infty}^{+\infty} dz_i \int d\theta_i \right) \sum_{\substack{i_1, \dots, i_n = 1 \\ j_1, \dots, j_m = 1}}^N z_{i_1}^{k_1} \dots z_{i_n}^{k_n} z_{j_1}^{s_1} \theta_{j_1} \dots z_{j_m}^{s_m} \theta_{j_m} \\ \cdot \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{h} \sum_{i=1}^N \tilde{V}_{NS}(z_i, \theta_i)}. \quad (3.9)$$

It should be noted that the correlators $C_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}(a, \varepsilon)$ with the fermionic coupling constants are written on the right side of $t_{k_1} \dots t_{k_n} \xi_{s_1 + \frac{1}{2}} \dots \xi_{s_m + \frac{1}{2}}$ in (3.6) for the convenience of the following discussions.

Let us further analyze the constraint (3.2). We observe that in (3.2) $\hat{D} = D - \sum_{k=1}^{2p} k t_k \frac{\partial}{\partial t_k} - \sum_{l=0}^{2p} (l + \frac{1}{2}) \xi_{l + \frac{1}{2}} \frac{\partial}{\partial \xi_{l + \frac{1}{2}}}$ has degree 0, $\hat{W}_{2p+2}$, $\hat{W}_k$ and $\hat{W}_{l + \frac{1}{2}}$ are operators with degrees $2p+2$, k and $l + \frac{1}{2}$, respectively. Since $\text{Ker}(\hat{D} + \hat{W})$ is no longer one dimensional on $\tilde{V}$, the partition function (3.1) can not be uniquely determined by the constraint (3.2). On the other hand, there are the additional constraints for the partition function (3.1)

$$\frac{\partial}{\partial t_k} Z_{NG}(t, \xi; a, \varepsilon) = k \frac{\partial}{\partial a_k} Z_{NG}(t, \xi; a, \varepsilon), \quad k = 1, \dots, 2p, \\ \frac{\partial}{\partial \xi_{l + \frac{1}{2}}} Z_{NG}(t, \xi; a, \varepsilon) = \frac{\partial}{\partial \varepsilon_l} Z_{NG}(t, \xi; a, \varepsilon), \quad l = 0, \dots, 2p. \quad (3.10)$$

Substituting (3.10) into (3.2), we obtain

$$(D + \hat{W}) Z_{NG}(t, \xi; a, \varepsilon) = 0, \quad (3.11)$$

where $\hat{W} = \hat{W} - \sum_{k=1}^{2p} k^2 t_k \frac{\partial}{\partial a_k} - \sum_{l=0}^{2p} (l + \frac{1}{2}) \xi_{l + \frac{1}{2}} \frac{\partial}{\partial \varepsilon_l}$.

Since the operators D and $D + \hat{W}$ are invertible on $-\hat{W} e^{-\frac{\sqrt{\beta}}{h} N t_0} Z_{NG}(a, \varepsilon)$, by the constraint (3.11), we have

$$\sum_{s \in \frac{1}{2} \mathbb{N}_+}^{\infty} Z_{NG}^{(s)}(t, \xi; a, \varepsilon) = -(D + \hat{W})^{-1} \hat{W} e^{-\frac{\sqrt{\beta}}{h} N t_0}. \quad (3.12)$$

Hence the partition function (3.1) can be expressed as

$$Z_{NG}(t, \xi; a, \varepsilon) = \sum_{k=0}^{\infty} (-D^{-1} \hat{\mathcal{W}})^k e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} Z_{NG}(a, \varepsilon). \quad (3.13)$$

Similar to the representation for the non-Gaussian Hermitian matrix model presented in ref. [40], we see that $\hat{\mathcal{W}}$ in (3.13) is not a homogeneous operator. Since $\hat{\mathcal{W}}$ contains the noncommutative operators with degrees ranging from $\frac{1}{2}$ to $2p+2$, it not only leads to the fact that the partition function (3.1) can not be obtained by acting on elementary functions with exponents of the operator $\hat{\mathcal{W}}$, but also makes the handling of the correlators quite difficult from (3.13). We can in principle derive the correlators step by step from (3.13).

For example, we give some correlators as follows:

$$\begin{aligned} C^{\frac{1}{2}}(a, \varepsilon) &= -\frac{\hbar}{\sqrt{\beta}} \frac{\partial}{\partial \varepsilon_0} Z_{NG}(a, \varepsilon), & C_1(a, \varepsilon) &= -\frac{\hbar}{\sqrt{\beta}} \frac{\partial}{\partial a_1} Z_{NG}(a, \varepsilon), \\ C_1^{\frac{1}{2}}(a, \varepsilon) &= \frac{\hbar^2}{\beta} \frac{\partial^2}{\partial a_1 \partial \varepsilon_0} Z_{NG}(a, \varepsilon), & C^{\frac{3}{2}}(a, \varepsilon) &= -\frac{\hbar}{\sqrt{\beta}} \frac{\partial}{\partial \varepsilon_1} Z_{NG}(a, \varepsilon), \\ C_2(a, \varepsilon) &= -\frac{2\hbar}{\sqrt{\beta}} \frac{\partial}{\partial a_2} Z_{NG}(a, \varepsilon), & C_{1,1}(a, \varepsilon) &= \frac{\hbar^2}{\beta} \frac{\partial^2}{\partial a_1^2} Z_{NG}(a, \varepsilon), \\ C^{\frac{1}{2}, \frac{3}{2}}(a, \varepsilon) &= \frac{2\hbar^2}{\beta} \frac{\partial^2}{\partial \varepsilon_0 \partial \varepsilon_1} Z_{NG}(a, \varepsilon), & C_2^{\frac{1}{2}}(a, \varepsilon) &= \frac{2\hbar^2}{\beta} \frac{\partial^2}{\partial a_2 \partial \varepsilon_0} Z_{NG}(a, \varepsilon), \\ C_{1,1}^{\frac{1}{2}}(a, \varepsilon) &= -\left(\frac{\hbar}{\sqrt{\beta}}\right)^3 \frac{\partial^3}{\partial a_1^2 \partial \varepsilon_0} Z_{NG}(a, \varepsilon), & C_1^{\frac{3}{2}}(a, \varepsilon) &= \frac{\hbar^2}{\beta} \frac{\partial^2}{\partial a_1 \partial \varepsilon_1} Z_{NG}(a, \varepsilon), \\ C^{\frac{5}{2}}(a, \varepsilon) &= -\frac{\hbar}{\sqrt{\beta}} \frac{\partial}{\partial \varepsilon_2} Z_{NG}(a, \varepsilon), & C_{2,1}(a, \varepsilon) &= \frac{4\hbar^2}{\beta} \frac{\partial^2}{\partial a_1 \partial a_2} Z_{NG}(a, \varepsilon), \\ C_{1,1,1}(a, \varepsilon) &= -\left(\frac{\hbar}{\sqrt{\beta}}\right)^3 \frac{\partial^3}{\partial a_1^3} Z_{NG}(a, \varepsilon), & C^{\frac{1}{2}, \frac{5}{2}}(a, \varepsilon) &= \frac{2\hbar^2}{\beta} \frac{\partial^2}{\partial \varepsilon_0 \partial \varepsilon_2} Z_{NG}(a, \varepsilon), \\ C_3(a, \varepsilon) &= \begin{cases} \left[-Na_1 + \frac{\hbar}{\sqrt{\beta}} \left(a_2 \frac{\partial}{\partial a_1} + \varepsilon_1 \frac{\partial}{\partial \varepsilon_0} + 2\varepsilon_2 \frac{\partial}{\partial \varepsilon_1} \right) \right] Z_{NG}(a, \varepsilon), & p=1; \\ -\frac{3\hbar}{\sqrt{\beta}} \frac{\partial}{\partial a_3} Z_{NG}(a, \varepsilon), & p \geq 2, \end{cases} \\ C_1^{\frac{1}{2}, \frac{3}{2}}(a, \varepsilon) &= -2 \left(\frac{\hbar}{\sqrt{\beta}}\right)^3 \frac{\partial^3}{\partial a_1 \partial \varepsilon_0 \partial \varepsilon_1} Z_{NG}(a, \varepsilon). \end{aligned} \quad (3.14)$$

4 Chiral supereigenvalue model in the Neveu-Schwarz sector

Let us consider the chiral supereigenvalue model in the Neveu-Schwarz sector

$$Z_C = \frac{1}{\Lambda_C} \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N (V_{NS}(z_i, \theta_i) + z_i)}, \quad (4.1)$$

where N is even, the normalization factor Λ_C is given by

$$\Lambda_C = \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N z_i}. \quad (4.2)$$

The correlators $\bar{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}$ in the chiral supereigenvalue model are defined by

$$\bar{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}} = \frac{1}{\Lambda_C} \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \sum_{\substack{i_1, \dots, i_n=1 \\ j_1, \dots, j_m=1}}^N z_{i_1}^{k_1} \dots z_{i_n}^{k_n} z_{j_1}^{s_1} \theta_{j_1} \dots z_{j_m}^{s_m} \theta_{j_m} \cdot \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N z_i}. \quad (4.3)$$

Note that (4.3) is not convergent when the bosonic variables z_i are integrated from $-\infty$ to $+\infty$. This is the reason why we consider the chiral case (4.1) instead of the supereigenvalue model (1.2).

By the invariance of the partition function (4.1) under two pairs of the changes of integration variables ($z_i \rightarrow z_i + \epsilon \sum_{n=1}^\infty n t_n z_i^n$, $\theta_i \rightarrow \theta_i + \frac{1}{2} \epsilon \sum_{n=1}^\infty n^2 t_n z_i^{n-1} \theta_i$) and ($z_i \rightarrow z_i + \epsilon \sum_{n=1}^\infty (n + \frac{1}{2}) \xi_{n+\frac{1}{2}} z_i^n \theta_i$, $\theta_i \rightarrow \theta_i - \epsilon \sum_{n=1}^\infty (n + \frac{1}{2}) \xi_{n+\frac{1}{2}} z_i^n$), we obtain

$$(\tilde{D} + \bar{W})Z_C = 0, \quad (4.4)$$

where

$$\begin{aligned} \bar{W} = & \sum_{n,k=1}^\infty n k t_n t_k \frac{\partial}{\partial t_{n+k-1}} + \sum_{n,k=1}^\infty n \left(2k + \frac{n+1}{2} \right) t_n \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{n+k-\frac{1}{2}}} \\ & + \frac{1}{2} \sum_{n=1}^\infty n^2 t_n \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{n-\frac{1}{2}}} + \frac{\hbar^2}{2} \sum_{n=1}^\infty \sum_{k=0}^{n-1} n t_n \frac{\partial}{\partial t_k} \frac{\partial}{\partial t_{n-k-1}} \\ & + \frac{\hbar^2}{2} \sum_{n=2}^\infty \sum_{k=1}^{n-1} n k t_n \frac{\partial}{\partial \xi_{n-k-\frac{1}{2}}} \frac{\partial}{\partial \xi_{k-\frac{1}{2}}} - \frac{\hbar}{2\sqrt{\beta}} (1 - \beta) \sum_{n=1}^\infty n^2 t_n \frac{\partial}{\partial t_{n-1}} \\ & + \hbar^2 \sum_{n=1}^\infty \sum_{k=0}^{n-1} \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+\frac{1}{2}}} \frac{\partial}{\partial t_{n-k-1}} + \sum_{n=1}^\infty \sum_{k=0}^\infty \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \xi_{k+\frac{1}{2}} \frac{\partial}{\partial t_{n+k}} \\ & - \frac{\hbar}{\sqrt{\beta}} (1 - \beta) \sum_{n=1}^\infty n \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{n-\frac{1}{2}}}. \end{aligned} \quad (4.5)$$

Some specific types of the correlators can be recursively derived from the constraint (4.4). Taking the coefficients of t_1^{l+1} , $t_1^l \xi_{\frac{1}{2}} \xi_{\frac{3}{2}}$, $t_1^l t_2$ in (4.4) and setting to zero, respectively, we obtain

$$\bar{C}_1 = \frac{\hbar}{2\sqrt{\beta}} N \tilde{N}, \quad (4.6)$$

and the recursive relations

$$\begin{aligned} \bar{C}_{\underbrace{1, \dots, 1}_{l+1}} &= \frac{\hbar}{2\sqrt{\beta}} (N \tilde{N} + 2l) \bar{C}_{\underbrace{1, \dots, 1}_l}, \\ \bar{C}_{\underbrace{1, \dots, 1}_l}^{\frac{1}{2}, \frac{3}{2}} &= \frac{\hbar}{\sqrt{\beta}(2l+3)} \left[l(2l+5 + N \tilde{N}) \bar{C}_{\underbrace{1, \dots, 1}_{l-1}}^{\frac{1}{2}, \frac{3}{2}} + 3 \bar{C}_{\underbrace{1, \dots, 1}_{l+1}} \right], \\ \bar{C}_{\underbrace{2, 1, \dots, 1}_l} &= \frac{\hbar}{\sqrt{\beta}(l+2)} \left[l \left(l + 3 + \frac{1}{2} N \tilde{N} \right) \bar{C}_{\underbrace{2, 1, \dots, 1}_{l-1}} + 2 \tilde{N} \bar{C}_{\underbrace{1, \dots, 1}_{l+1}} \right]. \end{aligned} \quad (4.7)$$

From (4.6) and (4.7), we can further derive the correlators

$$\begin{aligned}
 \bar{C}_{\underbrace{1, \dots, 1}_i} &= \left(\frac{\hbar}{\sqrt{\beta}} \right)^l \prod_{j=0}^{l-1} \left(\frac{N\tilde{N}}{2} + j \right), \\
 \bar{C}_{\underbrace{1, \dots, 1}_{\frac{1}{2}, \frac{3}{2}}} &= \left(\frac{\hbar}{\sqrt{\beta}} \right)^{l+2} \frac{N\tilde{N}}{2} \prod_{j=1}^l \left(\frac{N\tilde{N}}{2} + j + 1 \right), \\
 \bar{C}_{2, \underbrace{1, \dots, 1}_i} &= \left(\frac{\hbar}{\sqrt{\beta}} \right)^{l+2} \frac{N\tilde{N}^2}{2} \prod_{j=1}^l \left(\frac{N\tilde{N}}{2} + j + 1 \right).
 \end{aligned} \tag{4.8}$$

We observe that the operator $\tilde{D}$ in (4.4) is the same as in (2.12), and the operator $\bar{W}$ in (4.4) is a homogeneous operator with degree 1. Following the similar considerations in the Gaussian supereigenvalue model, we reach the final expressions for the partition function (4.1)

$$Z_C = \sum_{k=0}^{\infty} (-\tilde{D}^{-1}\bar{W})^k e^{-\frac{\sqrt{\beta}}{\hbar} N t_0}, \tag{4.9}$$

and the correlators

$$\bar{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}} = \frac{(-1)^{s + \frac{m(m+1)}{2}} n! \left(-\frac{\hbar}{\sqrt{\beta}} \right)^{n+m}}{s! \lambda_{(k_1, \dots, k_n)}} \sum_{\alpha=1}^{2s} \left(-\frac{\sqrt{\beta}}{\hbar} N \right)^{\alpha} \bar{P}_{\underbrace{(0, \dots, 0)_\alpha}}^{(k_1, \dots, k_n | s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2})}, \tag{4.10}$$

where $s = \sum_{\mu=1}^n k_\mu + \sum_{\nu=1}^m s_\nu + \frac{m}{2}$, $k_\mu \geq 1$, $s_\nu \geq 0$,

$$\begin{aligned}
 &\bar{P}_{\underbrace{(0, \dots, 0)_\alpha}}^{(k_1, \dots, k_n | s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2})} \\
 &= \sum_{\sigma_1, \sigma_2} (-1)^{\tau(\sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} \left[\sum_{r=1}^{\lfloor \frac{s+1}{2} \rfloor} \sum_{\substack{l_1 + \dots + l_{2r-1} = s \\ l_1, \dots, l_{2r-1} \geq 1}} \left(\frac{1}{\prod_{i=l_1+1}^s (2i-1)} \right. \right. \\
 &\quad \cdot \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [2(l_1 + \dots + l_{2i} + j) - 1] \bar{E}_{\underbrace{(0, \dots, 0)_\alpha}}^{(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} \\
 &\quad \left. \left. + \frac{1}{\prod_{i=1}^s (2i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i}} [2(l_1 + \dots + l_{2i-1} + j) - 1] \bar{E}_{\underbrace{(0, \dots, 0)_\alpha}}^{(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} \right) \right] \\
 &\quad + \sum_{r=1}^{\lfloor \frac{s}{2} \rfloor} \sum_{\substack{l_1 + \dots + l_{2r} = s \\ l_1, \dots, l_{2r} \geq 1}} \left(\frac{1}{\prod_{i=l_1+1}^s (2i-1)} \prod_{i=1}^{r-1} \prod_{j=1}^{l_{2i+1}} [2(l_1 + \dots + l_{2i} + j) - 1] \right. \\
 &\quad \left. \left. + \prod_{j=1}^{l_{2r}} [2(l_1 + \dots + l_{2r-1} + j) - 1] \bar{E}_{\underbrace{(0, \dots, 0)_\alpha}}^{(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2}))} \right) \right]
 \end{aligned}$$

$$\left. \begin{aligned} & \bar{G}_{\alpha}(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2})) + \frac{1}{\prod_{i=1}^s (2i-1)} \prod_{i=1}^r \prod_{j=1}^{l_{2i}} [2(l_1 + \dots + l_{2i-1} + j) - 1] \\ & \cdot \bar{H}_{\alpha}(\sigma_1(k_1), \dots, \sigma_1(k_n) | \sigma_2(s_1 + \frac{1}{2}), \dots, \sigma_2(s_m + \frac{1}{2})) \end{aligned} \right] \Bigg| \quad (4.11)$$

Here the four functions $\bar{E}_{\alpha}^{(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2})}$, $\bar{F}_{\alpha}^{(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2})}$, $\bar{G}_{\alpha}^{(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2})}$ and $\bar{H}_{\alpha}^{(k_1, \dots, k_c | s_1 + \frac{1}{2}, \dots, s_d + \frac{1}{2})}$ are, respectively, the coefficients of $t_{k_1} \dots t_{k_c} \xi_{s_1 + \frac{1}{2}} \dots \xi_{s_d + \frac{1}{2}} \frac{\partial^\alpha}{\partial t_0^\alpha}$ in the formal expansions of the corresponding operators $\bar{W}^{l_{2r-1}} \bar{\mathbb{W}}^{l_{2r-2}} \dots \bar{\mathbb{W}}^{l_2} \bar{W}^{l_1}$, $\bar{\mathbb{W}}^{l_{2r-1}} \bar{W}^{l_{2r-2}} \dots \bar{W}^{l_2} \bar{\mathbb{W}}^{l_1}$, $\bar{\mathbb{W}}^{l_{2r}} \bar{W}^{l_{2r-1}} \dots \bar{\mathbb{W}}^{l_2} \bar{W}^{l_1}$ and $\bar{W}^{l_{2r}} \bar{\mathbb{W}}^{l_{2r-1}} \dots \bar{W}^{l_2} \bar{\mathbb{W}}^{l_1}$, the operator $\bar{\mathbb{W}}$ is given by $\bar{\mathbb{W}} = \xi_{\frac{1}{2}} \frac{\partial}{\partial \xi_{\frac{1}{2}}} \bar{W}$.

Let us list the correlators (4.10) with $\sum_{\mu=1}^n k_\mu + \sum_{\nu=1}^m s_\nu + \frac{m}{2} \leq 3$ as follows:

$$\begin{aligned} \bar{C}_1 &= \frac{\hbar}{2\sqrt{\beta}} N \tilde{N}, & \bar{C}_2 &= \frac{\hbar^2}{2\beta} N \tilde{N}^2, \\ \bar{C}_{1,1} &= \frac{\hbar^2}{4\beta} N \tilde{N} (N \tilde{N} + 2), & \bar{C}_{\frac{1}{2}, \frac{3}{2}} &= \frac{\hbar^2}{2\beta} N \tilde{N}, \\ \bar{C}_3 &= \frac{1}{8} \left(\frac{\hbar}{\sqrt{\beta}} \right)^3 N \tilde{N} [4\beta + \tilde{N}(1 - \beta) + 5\tilde{N}^2], \\ \bar{C}_{2,1} &= \frac{1}{4} \left(\frac{\hbar}{\sqrt{\beta}} \right)^3 N \tilde{N}^2 (N \tilde{N} + 4), \\ \bar{C}_{1,1,1} &= \frac{1}{8} \left(\frac{\hbar}{\sqrt{\beta}} \right)^3 N \tilde{N} (N \tilde{N} + 2) (N \tilde{N} + 4), \\ \bar{C}_1^{\frac{1}{2}, \frac{3}{2}} &= \frac{1}{4} \left(\frac{\hbar}{\sqrt{\beta}} \right)^3 N \tilde{N} (N \tilde{N} + 4), \\ \bar{C}_{\frac{1}{2}, \frac{5}{2}} &= \frac{1}{2} \left(\frac{\hbar}{\sqrt{\beta}} \right)^3 N \tilde{N} (2\tilde{N} + 1 - \beta). \end{aligned} \quad (4.12)$$

5 Non-Gaussian chiral supereigenvalue model in the Neveu-Schwarz sector

The non-Gaussian chiral supereigenvalue model in the Neveu-Schwarz sector is

$$Z_{CNG}(t, \xi; a, \varepsilon) = \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N (V_{NS}(z_i, \theta_i) + \bar{V}_{NS}(z_i, \theta_i))}, \quad (5.1)$$

where N is even, $\bar{V}_{NS}(z, \theta) = \frac{1}{2p+1} z^{2p+1} + \sum_{k=1}^{2p} \frac{1}{k} a_k z^k + \sum_{l=0}^{2p} \varepsilon_l z^l \theta$, $p \geq 1$, a_k and ε_l are nonzero bosonic and fermionic coupling constants, respectively. The correlators

$\hat{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}(a, \varepsilon)$ are defined by

$$\begin{aligned} \hat{C}_{k_1, \dots, k_n}^{s_1 + \frac{1}{2}, \dots, s_m + \frac{1}{2}}(a, \varepsilon) = & \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \sum_{\substack{i_1, \dots, i_n=1 \\ j_1, \dots, j_m=1}}^N z_{i_1}^{k_1} \dots z_{i_n}^{k_n} z_{j_1}^{s_1} \theta_{j_1} \dots z_{j_m}^{s_m} \theta_{j_m} \\ & \cdot \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N \tilde{V}_{NS}(z_i, \theta_i)}. \end{aligned} \quad (5.2)$$

By applying the changes of integration variables ($z_i \rightarrow z_i + \epsilon \sum_{n=2p+1}^\infty n t_n z_i^{n-2p}$, $\theta_i \rightarrow \theta_i + \frac{1}{2} \epsilon \sum_{n=2p+1}^\infty n(n-2p) t_n z_i^{n-2p-1} \theta_i$) and ($z_i \rightarrow z_i + \epsilon \sum_{n=2p+1}^\infty (n + \frac{1}{2}) \xi_{n+\frac{1}{2}} z_i^{n-2p} \theta_i$, $\theta_i \rightarrow \theta_i - \epsilon \sum_{n=2p+1}^\infty (n + \frac{1}{2}) \xi_{n+\frac{1}{2}} z_i^{n-2p}$) for the partition function (5.1), we may derive the constraint from the invariance of the integral

$$(\hat{D} + \check{W}) Z_{CNG}(t, \xi; a, \varepsilon) = 0, \quad (5.3)$$

where $\check{W} = \check{W}_{2p+1} + \sum_{k=1}^{2p} \check{W}_k + \sum_{l=0}^{2p} \check{W}_{l+\frac{1}{2}}$, the operators $\check{W}_{2p+1}$, $\check{W}_k$ and $\check{W}_{l+\frac{1}{2}}$ are, respectively, given by

$$\begin{aligned} \check{W}_{2p+1} = & \sum_{n=2p+1}^\infty n t_n \left[\sum_{k=1}^\infty k t_k \frac{\partial}{\partial t_{n+k-2p-1}} + \frac{\hbar^2}{2} \sum_{k=0}^{n-2p-1} \frac{\partial}{\partial t_k} \frac{\partial}{\partial t_{n-k-2p-1}} \right. \\ & + \sum_{k=0}^\infty \left(\frac{n}{2} + k - p \right) \xi_{k+\frac{1}{2}} \frac{\partial}{\partial \xi_{k+n-2p-\frac{1}{2}}} + \frac{\hbar^2}{2} \sum_{k=1}^{n-2p-1} k \frac{\partial}{\partial \xi_{n-k-2p-\frac{1}{2}}} \frac{\partial}{\partial \xi_{k-\frac{1}{2}}} \\ & \left. - \frac{\hbar}{2\sqrt{\beta}} (1-\beta)(n-2p) \frac{\partial}{\partial t_{n-2p-1}} \right] + \sum_{n=2p+1}^\infty \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \left[\sum_{k=1}^\infty k t_k \frac{\partial}{\partial \xi_{n+k-2p-\frac{1}{2}}} \right. \\ & + \hbar^2 \sum_{k=0}^{n-2p-1} \frac{\partial}{\partial \xi_{k+\frac{1}{2}}} \frac{\partial}{\partial t_{n-k-2p-1}} - \frac{\hbar}{\sqrt{\beta}} (1-\beta)(n-2p) \frac{\partial}{\partial \xi_{n-2p-\frac{1}{2}}} \\ & \left. + \sum_{k=0}^\infty \xi_{k+\frac{1}{2}} \frac{\partial}{\partial t_{k+n-2p}} \right], \\ \check{W}_k = & a_{2p+1-k} \sum_{n=2p+1}^\infty n t_n \frac{\partial}{\partial t_{n-k}} + a_{2p+1-k} \sum_{n=2p+1}^\infty \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial \xi_{n+\frac{1}{2}-k}}, \\ \check{W}_{l+\frac{1}{2}} = & \varepsilon_{2p-l} \sum_{n=2p+1}^\infty n \left(p-l + \frac{n}{2} \right) t_n \frac{\partial}{\partial \xi_{n-\frac{1}{2}-l}} + \varepsilon_{2p-l} \sum_{n=2p+1}^\infty \left(n + \frac{1}{2} \right) \xi_{n+\frac{1}{2}} \frac{\partial}{\partial t_{n-l}}. \end{aligned} \quad (5.4)$$

In similarity with the case of non-Gaussian supereigenvalue model (3.1), the partition function (5.1) can not be uniquely determined by the constraint (5.3). We have to consider the additional constraints. It is noted that the partition function (5.1) also satisfies the relations (3.10). Substituting (3.10) into (5.3), we obtain

$$(D + \check{W}) Z_{CNG}(t, \xi; a, \varepsilon) = 0, \quad (5.5)$$

where $\check{W} = \check{W} - \sum_{k=1}^{2p} k^2 t_k \frac{\partial}{\partial a_k} - \sum_{l=0}^{2p} \left(l + \frac{1}{2} \right) \xi_{l+\frac{1}{2}} \frac{\partial}{\partial \varepsilon_l}$.

Furthermore, from (5.5) we can derive

$$Z_{CNG}(t, \xi; a, \varepsilon) = \sum_{k=0}^{\infty} (-D^{-1}\check{W})^k e^{-\frac{\sqrt{\beta}}{\hbar} N t_0} Z_{CNG}(a, \varepsilon), \quad (5.6)$$

where $Z_{CNG}(a, \varepsilon)$ is given by

$$Z_{CNG}(a, \varepsilon) = \left(\prod_{i=1}^N \int_0^{+\infty} dz_i \int d\theta_i \right) \Delta_{NS}(z, \theta)^\beta e^{-\frac{\sqrt{\beta}}{\hbar} \sum_{i=1}^N \bar{V}_{NS}(z_i, \theta_i)}. \quad (5.7)$$

Note that $\check{W}$ in (5.6) contains noncommutative operators with degrees ranging from $\frac{1}{2}$ to $2p+1$, it causes that the general expressions of the correlators (5.2) can not be obtained. In principle, we can calculate the correlators step by step from (5.6).

For example, we present the correlators (5.2) with $\sum_{\mu=1}^n k_\mu + \sum_{\nu=1}^m s_\nu + \frac{m}{2} \leq 3$. They are the same as (3.14) by replacing $Z_{NG}(a, \varepsilon)$ by $Z_{CNG}(a, \varepsilon)$, except for the case of $p = 1$ in (5.2)

$$\hat{C}_3(a, \varepsilon) = \frac{\hbar}{\sqrt{\beta}} \left[\frac{N\tilde{N}}{2} + \sum_{k=1}^2 k a_k \frac{\partial}{\partial a_k} + \sum_{l=0}^2 \left(l + \frac{1}{2} \right) \varepsilon_l \frac{\partial}{\partial \varepsilon_l} \right] Z_{CNG}(a, \varepsilon). \quad (5.8)$$

6 Conclusions

The W -representations of matrix models may realize the partition functions by acting on elementary functions with exponents of the given W operators. They play an important role in the calculations of the correlators. For the supereigenvalue model in the Ramond sector, one has found its W -representation and derived the compact expressions of the correlators. In this paper, we have analyzed the Gaussian and chiral supereigenvalue models in the Neveu-Schwarz sector. Their partition functions (2.1) and (4.1) can be expressed as the infinite sums of the homogeneous operators $(-\tilde{D}^{-1}\tilde{W})^k$ and $(-\tilde{D}^{-1}\bar{W})^k$ acting on the function $e^{-\frac{\sqrt{\beta}}{\hbar} N t_0}$, i.e., (2.24) and (4.9), respectively. We noted that $\tilde{D}$ is not the degree operator. It leads to that the usual W -representations of these matrix models can not be provided here. In spite of this negative result, the notable feature has emerged from the studies of the correlators. Since the actions of the homogeneous operators $(-\tilde{D}^{-1}\tilde{W})^k$ and $(-\tilde{D}^{-1}\bar{W})^k$ on the function $e^{-\frac{\sqrt{\beta}}{\hbar} N t_0}$ can be evaluated explicitly, we have derived the compact expressions of the correlators (2.32) and (4.10) from (2.24) and (4.9), respectively. Our analysis provides additional insight into these supereigenvalue models.

We have also considered the non-Gaussian (chiral) supereigenvalue models in the Neveu-Schwarz sector. Unlike the cases of previous Gaussian and chiral supereigenvalue models, it was noted that the operators $\hat{W}$ and $\check{W}$ in (3.13) and (5.6) are not the homogeneous operators, which are constituted by the noncommutative operators with degrees ranging from $\frac{1}{2}$ to $2p+2$ and $2p+1$, respectively. This makes it quite difficult to derive the compact expressions of the correlators from (3.13) and (5.6).

Finally, it should be mentioned that the partition function for (deformed) Gaussian matrix models have the explicit character expansions [19]–[21], [33, 34, 41]. The character

expansions imply that the expectation values of characters are again characters, which is considered as the superintegrability property of the models [42]. For the supereigenvalue models, whether there exists the character decomposition formulas deserves further study.

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