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Non-Abelian rotating black holes in 4- and 5-dimensional gauged supergravity

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Abstract: We present new supersymmetric black-hole solutions of the 4- and 5-dimensional gauged supergravity theories that one obtains by dimensional reduction on T^5 and T^6 of Heterotic supergravity with a triplet of Yang-Mills fields. The new ingredient of our solutions is the presence of dyonic non-Abelian fields which allows us to obtain a generalization of the BMPV black hole with two independent angular momenta and the first example of a supersymmetric, rotating, asymptotically-flat black hole with a regular horizon in 4 dimensions.

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1 Introduction

The study of the classical solutions of General Relativity and its generalizations has been one of the major sources of information about the properties of those theories. Supergravity theories are a particularly interesting class of generalizations of General Relativity because many of them describe low-energy effective field theories of different superstring theories and a great deal of work has been devoted to them and their classical solutions (specially to the supersymmetric ones and specially to those describing black holes).[1]

One of the most important features of these theories is the presence of vector and scalar fields that give rise to many interesting phenomena and properties of the black-holes solutions. The electric and magnetic charges associated to those vectors play a crucial role in the stringy interpretation of the black holes that carry them and determine completely their entropy formula in the static, extremal cases. However, in most of the literature, only models with Abelian vector fields have been considered, even though non-Abelian vector fields play a more relevant role in our current understanding of Nature and most string

[1] For a review of superstring theories and their classical solutions from this point of view see ref. [1].

models (specially the more realistic ones) include them in their spectra. In the case of the Heterotic Superstring, non-Abelian vector fields occur at first order in α' and can play an important role in the suppression or even cancellation of α' corrections to black-hole geometries [2–4]. Thus, it is clearly important to study the interplay between gravity and Yang-Mills fields in this context and to understand how the results obtained in the Abelian case are modified by the presence of the later.

During the last decade our group has been trying to fill this gap in our understanding exploring supersymmetric solutions (specially black-hole or black-ring solutions) with non-Abelian fields in gauged supergravity theories [5–21]. They have also played an important role in the construction of black-hole microstate geometries [22, 23][2] and, furthermore, they have turned out to be the intermediate step necessary to construct α'-corrected stringy supersymmetric black-hole solutions [2–4, 25, 26]. This is due to the fact that, in the context of the Heterotic Superstring effective action, the non-Abelian gauge fields occur at first order in α' and it is known that the curvature squared of the torsionful spin connection occurs at the same order just as another gauged field [27], a feature of the theory that makes the Green-Schwarz anomaly-cancellation mechanism possible [28]. The same mechanism can be used to cancel some of the α' corrections as well.

Although the multicenter solutions constructed in ref. [18] have angular momentum, the only rotating, supersymmetric, single-center solutions constructed so far with non-Abelian fields are the black rings and black holes of ref. [21] which are the simplest generalizations of the Abelian ones. In particular, the rotating black-hole solution presented in that reference can be understood as a BMPV black hole (only one independent angular momentum) with additional non-Abelian hair.

The aim of this work is to extend the catalogue of known rotating, single-center, black-hole solutions in 4 and 5 dimensions by exploring the effect of adding dyonic non-Abelian fields defined on hyperKähler spaces. The supergravity theories that we are going to consider in this paper are the 4- and 5-dimensional versions of the SU(2)-gauged ST[2, 6] model (an extension of the STU model), whose main interest lies in the fact that it can be obtained by toroidal compactification and truncation of the $\mathcal{N} = 1, d = 10$ supergravity coupled to non-Abelian vector fields [25], which is often referred to as Heterotic supergravity.

The dyonic non-Abelian fields that we will consider are generalizations of those presented in refs. [22] and [29]. The latter were used in ref. [30] to construct a globally smooth solution of Heterotic supergravity. The solutions that we are going to consider here generalize that one by considering non-trivial hyperKähler base spaces on which the dyonic instanton is defined and by turning on additional fields that give rise to regular event horizons, which in 4 dimensions violates the no-go theorem for regular, supersymmetric, rotating black holes proven in ref. [31]. In 5 dimensions, they allow us to find supersymmetric black holes with two independent angular momenta.

This paper is organized as follows. In section 2 we describe the class of gauged $\mathcal{N} = 1, d = 5$ theories (8 supercharges) we are going to work with and a general solution-

[2]For a review on black-hole and black-ring microstate geometries see, for instance, ref. [24] and references therein.

generating technique for supersymmetric solutions of this kind of theories. In section 3 we apply this technique to the particular model we are interested in, the ST[2, 6] model, and discuss which kind of dyonic non-Abelian fields, in particular, can be added to it. We also describe the dimensional reduction of this model (and of the corresponding solutions) to 4 dimensions, where the theory becomes a model of gauged $\mathcal{N}=2, d=4$ supergravity. In section 4 we focus on the study of single-center black holes both in 5 and 4 dimensions. Section 5 contains a discussion of our results.

2 The general set up

2.1 $\mathcal{N}=1, d=5$ Super-Einstein-Yang-Mills theories

$\mathcal{N}=1, d=5$ super-Einstein-Yang-Mills (SEYM) theories are the simplest theories of 5-dimensional supergravity containing non-Abelian gauge fields. They can be described as the simplest and minimal supersymmetrization of 5-dimensional Einstein-Yang-Mills theories or as the simplest coupling of 5-dimensional super-Yang-Mills theory to supergravity.

For our purposes, it is convenient to describe these theories as the result of gauging a subgroup of the isometry group of the scalar manifold of a $\mathcal{N}=1, d=5$ supergravity coupled to vector multiplets.[3] Therefore, these theories describe

1. The supergravity multiplet containing the graviton $e^a{}_\mu$, the gravitino ψ^i_μ and the graviphoton $A^0{}_\mu$

2. n_v vector multiplets labeled by $x = 1, \cdots, n_v$ (each containing a real vector field $A^x{}_\mu$, a real scalar ϕ^x and a gaugino $\lambda^{i\,x}$).

The above field content does not determine completely the theory, since the matter fields can be coupled to supergravity in different ways, even before gauging. In order to describe the different possibilities, it is convenient to combine the indices of the matter vector fields and of the graviphoton into a single index $I, J, \ldots = 0, 1, \cdots, n_v$ so all the vector fields are denoted by a single object $A^I{}_\mu$. Then, all the couplings between the fields of a given ungauged theory (between scalars, $\mathfrak{g}_{xy}(\phi)$, between scalars and vectors $a_{IJ}(\phi)$ and the Chern-Simons couplings between vectors) are completely determined by a constant, completely symmetric tensor C_{IJK}.[4]

Generically, an ungauged theory of $\mathcal{N}=1, d=5$ supergravity coupled to vector multiplets will be invariant under certain group of symmetries acting only on the vector and scalar fields.[5] The action of these symmetries on the scalars has to preserve $g_{xy}(\phi)$, the metric of the scalar manifold and, therefore, it will act on them as the isometries generated by the Killing vectors, that we will label by $k_I{}^x(\phi)$, and which can vanish for some values of I. At the same time, because of the non-trivial couplings between scalar and vector fields, the vectors will be rotated by some given matrices.

[3]Our conventions are those of refs. [7, 32], which are based on ref. [33].

[4]Our description of these theories will be minimal, giving only the information required to obtain and explain the results presented in this paper. The reader interested in further details on these theories, such as how to derive the couplings between the fields from C_{IJK}, may consult the references quoted above.

[5]Here we are ignoring R-symmetry.

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In many cases, it is possible to gauge a (necessarily non-Abelian) subgroup of this symmetry group using as gauge fields a subset of the vector fields of the theory. We will denote the structure constants of the gauge group by $f_{IJ}{}^{K}$ using the convention that they and the associated Killing vectors, will just vanish for the values of the indices that do not correspond to the gauge fields. In the gauging procedure, the partial derivatives of the scalars are promoted to gauge-covariant derivatives $\mathfrak{D}_\mu\phi^x = \partial_\mu\phi^x + gA^I{}_\mu k_I{}^x$ and the Abelian vector field strengths are promoted to their non-Abelian counterparts $F^I{}_{\mu\nu} = 2\partial_{[\mu}A^I{}_{\nu]} + gf_{JK}{}^I A^J{}_\mu A^K{}_\nu$. Here g is the gauge coupling constant. Gauge symmetry also demands the addition of further terms to the Chern-Simons terms, but, as different from what happens in the gauging of many other supergravity theories, supersymmetry does not depmand the addition of a scalar potential and no effective cosmological constant is present in the theory and its solutions.

The bosonic action of these gauged supergravities, that we call $\mathcal{N}=1, d=5$ Super-Einstein-Yang-Mills (SEYM) theories, is given by

$$
\begin{aligned}
S = \frac{1}{16\pi G_N^{(5)}} \int d^5x\sqrt{|g|}\, \Big\{ & R + \tfrac{1}{2}\mathfrak{g}_{xy}\mathfrak{D}_\mu\phi^x\mathfrak{D}^\mu\phi^y - \tfrac{1}{4}a_{IJ}F^{I\,\mu\nu}F^J{}_{\mu\nu} \\
& + \frac{1}{12\sqrt{3}}C_{IJK}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{|g|}}\Big[F^I{}_{\mu\nu}F^J{}_{\rho\sigma}A^K{}_\alpha - \frac{1}{2}gf_{LM}{}^I F^J{}_{\mu\nu}A^K{}_\rho A^L{}_\sigma A^M{}_\alpha \\
& + \frac{1}{10}g^2 f_{LM}{}^I f_{NP}{}^J A^K{}_\mu A^L{}_\nu A^M{}_\rho A^N{}_\sigma A^P{}_\alpha\Big]\Big\}\,,
\end{aligned}
\tag{2.1}
$$

where $G_N^{(5)}$ is the 5-dimensional Newton constant and g is the Yang-Mills coupling constant.[6]

For the sake of completeness and also for their use in defining the charges of the solutions, we quote the equations of motion that follow from this action:

$$
\begin{aligned}
\mathcal{E}_{\mu\nu} &\equiv \frac{1}{2\sqrt{g}}e_{a(\mu}\frac{\delta S}{\delta e_a{}^{\nu)}} \\
&= G_{\mu\nu} - \frac{1}{2}a_{IJ}\left(F^I{}_\mu{}^\rho F^J{}_{\nu\rho} - \frac{1}{4}g_{\mu\nu}F^{I\,\rho\sigma}F^J{}_{\rho\sigma}\right) \\
&\quad + \frac{1}{2}\mathfrak{g}_{xy}\left(\mathfrak{D}_\mu\phi^x\mathfrak{D}_\nu\phi^y - \frac{1}{2}g_{\mu\nu}\mathfrak{D}_\rho\phi^x\mathfrak{D}^\rho\phi^y\right)
\end{aligned}
\tag{2.2}
$$

$$
\begin{aligned}
\mathcal{E}_I{}^\mu &\equiv \frac{1}{\sqrt{g}}\frac{\delta S}{\delta A^I{}_\mu} \\
&= \mathfrak{D}_\nu\left(a_{IJ}F^{J\,\nu\mu}\right) + \frac{1}{4\sqrt{3}}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}C_{IJK}F^J{}_{\nu\rho}F^k{}_{\sigma\alpha} + gk_{I\,x}\mathfrak{D}^\mu\phi^x
\end{aligned}
\tag{2.3}
$$

$$
\begin{aligned}
\mathcal{E}^x &\equiv -\frac{\mathfrak{g}^{xy}}{\sqrt{g}}\frac{\delta S}{\delta\phi^y} \\
&= \mathfrak{D}_\mu\mathfrak{D}^\mu\phi^x + \frac{1}{4}\mathfrak{g}^{xy}\partial_y a_{IJ}F^{I\,\rho\sigma}F^J{}_{\rho\sigma}.
\end{aligned}
\tag{2.4}
$$

[6]The symbol $|g|$ denotes, however, the determinant of the 5-dimensional metric $g_{\mu\nu} = e^a{}_\mu e^b{}_\nu\eta_{ab}$.

2.2 A solution-generating technique for $\mathcal{N}=1, d=5$ SEYM theories

Classical solutions of $\mathcal{N}=1, d=5$ SEYM theories with a symmetric scalar manifold can be constructed using the following building blocks:[7]

1. A 4-dimensional hyperKähler (HK) manifold with metric $d\sigma^2 = h_{\underline{mn}}\, dx^m dx^n$,[8] following fields defined on it:
2. n_v+1 vector fields $\hat{A}^I = \hat{A}^I{}_{\underline{m}}\, dx^m$.
3. n_v+1 functions Z_I, $I=0,\ldots,n_v$ defined
4. A 1-form $\omega = \omega_{\underline{m}}\, dx^m$.

In terms of these building blocks and the tensor C_{IJK} that defines the model, the 5-dimensional physical fields (metric $g_{\mu\nu}$, vector fields A^I and scalar fields ϕ^x) are given by

$$ds^2 = f^2\,(dt+\omega)^2 - f^{-1} d\sigma^2\,, \tag{2.5}$$

$$A^I = -27\sqrt{3}\, C^{IJK} Z_J Z_K f^3\,(dt+\omega) + \hat{A}^I\,, \tag{2.6}$$

$$\phi^x = \frac{Z_x}{Z_0}\,, \tag{2.7}$$

where the metric function f is given by

$$f^{-3} = 27 C^{IJK} Z_I Z_J Z_K\,. \tag{2.8}$$

The building blocks of the solution $(\hat{A}^I, Z_I, \omega)$ must satisfy the following differential equations on the HK manifold:

$$\hat{F}^I = \star_\sigma \hat{F}^I\,, \tag{2.9}$$

$$\hat{\mathfrak{D}} \star_\sigma \hat{\mathfrak{D}} Z_I = -\frac{1}{3} C_{IJK}\, \hat{F}^I \wedge \hat{F}^J\,, \tag{2.10}$$

$$d\omega + \star_\sigma d\omega = \sqrt{3} Z_I \hat{F}^I\,, \tag{2.11}$$

where $\star_\sigma$ is the restriction of the Hodge star to the 4-dimensional HK metric $d\sigma^2$, $\hat{\mathfrak{D}}$ is the gauge-covariant derivative with respect to the hatted gauge connection $\hat{A}^I$

$$\hat{\mathfrak{D}} Z_I = dZ_I + g f_{IJ}{}^K \hat{A}^J \wedge Z_K\,, \tag{2.12}$$

and $\hat{F}^I$ is the field strength of that connection

$$\hat{F}^I = d\hat{A}^I + \frac{g}{2} f_{JK}{}^I \hat{A}^J \wedge \hat{A}^K\,. \tag{2.13}$$

The solutions consrtucted in this way are time-independent and, in general ($\omega \neq 0$) stationary. They are also ("timelike") supersymmetric and preserve 1/2 of the 8 supersymmetries of these theories.

[7]This recipe stems from the characterization of timelike supersymmetric solutions of the most general $\mathcal{N}=1, d=5$ supergravity theory including vector supermultiplets and hypermultiplets and generic gaugings made in ref. [7], based in the results of ref. [32]. The inclusion of tensor supermultiplets was considered in ref. [10]. We have adapted those results to the case at hands. Furthermore, we have restricted this recipe to models with a symmetric scalar manifold, for simplicity (the model we are going to study belongs to this class). In these models, but not in general, the tensor C^{IJK} that one obtains by raising the indices with the inverse of $a_{IJ}(\phi)$ ($a^{IJ}(\phi)$) is constant and identical to C_{IJK}.

[8]In our conventions $m,n=1,\ldots,4$ are tangent space indices whereas $\underline{m},\underline{n}=1,\ldots,4$ are curved indices.

3 Dyonic solutions of the SU(2)-gauged ST[2, 6] model

In this section we are going to apply the solution-generating technique described in the previous section to the particular model we are interested in: the SU(2)-gauged ST[2, 6] model, which can be obtained by compactification of Heterotic Supergravity on T^4 followed by a truncation of all the fields related to the compact space. The ungauged model has $n_v = 5$ vector multiplets and is characterized by a C_{IJK} tensor whose only non-vanishing components are $C_{0xy} = \frac{1}{6}\eta_{xy}$, with $x = 1, \ldots, 5$. The last three vector fields ($x = 3, 4, 5$) will be used as SU(2) gauged fields in the gauged theory.

Then, it is convenient to split the index labelling the vector fields into a pair of indices $I = (i, A + 2)$, where $i = 0, 1, 2$ labels the Abelian vector fields and $A = 1, 2, 3$, the SU(2) gauged fields. Furthermore, we define the following combinations of Abelian vector fields

$$A^{\pm} \equiv A^1 \pm A^2 \,, \tag{3.1}$$

and we are going to use the scalar fields ϕ, k and ℓ^A. The scalar ϕ is the string dilaton field; k is the Kaluza-Klein scalar that measures the size of the circle of the compactification from 6 to 5 dimensions and the ℓ^A are just a convenient SU(2) triplet of scalar fields. The relation between these fields and those of the standard parametrization in eq. (2.7) is

$$e^{-2\phi} = \frac{1}{2}(\phi^1 - \phi^2)\,, \tag{3.2}$$

$$k^4 = 2\left[\frac{(\phi^1)^2 - (\phi^2)^2 - \phi^A\phi^A}{\phi^1 - \phi^2}\right]^2 , \tag{3.3}$$

$$\ell^A = \phi^A/(\phi^1 - \phi^2)\,. \tag{3.4}$$

Then, for this model and using these variables, the generic action eq. (2.1) takes the specific form

$$\begin{aligned} S = \int d^5x\,\sqrt{|g|}\,\Big\{ & R + \partial_\mu\phi\partial^\mu\phi + \frac{4}{3}\partial_\mu \log k\,\partial^\mu \log k + 2e^{-\phi}k^{-2}\mathfrak{D}_\mu\ell^A\mathfrak{D}^\mu\ell^A \\ & -\frac{1}{12}e^{2\phi}k^{-4/3}F^0\cdot F^0 - \frac{1}{48}k^{8/3}\left(1+2e^{-\phi}k^{-2}\ell^B\ell^B\right)^2 F^+\cdot F^+ - \frac{1}{12}e^{-2\phi}k^{-4/3}F^-\cdot F^- \\ & -\frac{1}{6}e^{-2\phi}k^{-4/3}\ell^B\ell^B F^+\cdot F^- - \frac{1}{12}\left(e^{-\phi}k^{2/3}\delta_{AB} + 4e^{-2\phi}k^{-4/3}\ell^A\ell^B\right)F^A\cdot F^B \\ & -\frac{1}{6}e^{-\phi}k^{2/3}\left(1+2e^{-\phi}k^{-2}\ell^B\ell^B\right)\ell^A F^+\cdot F^A - \frac{1}{3}e^{-2\phi}k^{-4/3}\ell^A F^-\cdot F^A \\ & + \frac{1}{24\sqrt{3}}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{|g|}}A^0{}_\mu\left(F^+{}_{\nu\rho}F^-{}_{\sigma\alpha} - F^A{}_{\nu\rho}F^A{}_{\sigma\alpha}\right)\Big\}\,. \end{aligned} \tag{3.5}$$

Following the redefinition of the vector fields, in order to describe the construction of the solutions, we will use the functions

$$Z_\pm \equiv Z_1 \pm Z_2\,, \quad \text{and} \quad \tilde{Z}_+ = Z_+ - Z_A Z_A/Z_-\,. \tag{3.6}$$

3.1 The metric

According to the prescription given in the previous section, the metric of the solutions of this model that we can construct with it will have the general form eq. (2.5), but now with the metric function f taking the form

$$f^{-3} = \frac{27}{2} Z_0 \tilde{Z}_+ Z_- \,. \tag{3.7}$$

3.2 The scalar fields

In terms of the functions that we have defined, the scalars are given by

$$e^{2\phi} = 2\frac{Z_0}{Z_-}\,, \qquad k = \left(\frac{2\tilde{Z}_+^2}{Z_0 Z_-}\right)^{1/4}\,, \qquad \ell^A = \frac{Z_A}{Z_-}\,. \tag{3.8}$$

3.3 The vector fields

The vector fields are generically given by eq. (2.6), but here we will restrict ourselves to solutions with $\hat{A}^0 = \hat{A}^\pm = 0$ for simplicity. We will keep the $\hat{A}^A \neq 0$, though. Then, the vector fields of our solutions will have the form

$$A^0 = -\frac{1}{\sqrt{3}}\frac{1}{Z_0}(dt+\omega)\,, \tag{3.9}$$

$$A^\pm = -\frac{2}{\sqrt{3}}\frac{Z_+}{Z_\pm \tilde{Z}_+}(dt+\omega)\,, \tag{3.10}$$

$$A^A = \frac{2}{\sqrt{3}}\frac{Z_A}{\tilde{Z}_+ Z_-}(dt+\omega) + \hat{A}^A\,, \tag{3.11}$$

and the building blocks for which we will have to solve eqs. (2.9), (2.10) and (2.11) are the functions $Z_0, Z_\pm, Z_A$ and the 1-forms $\omega, \hat{A}^A$.

Let us first consider eq. (2.9). This equation can be solved in an arbitrary HK metric by SU(2) gauge fields $\hat{A}^A$ defined on it via a *generalized 't Hooft ansatz* [3]:

$$g\hat{A}^A = \bar{\eta}^A{}_{mn}\partial_n \log P\, v^m\,, \tag{3.12}$$

where P is a harmonic function in the HK space, the $\bar{\eta}^A{}_{mn}$ are the three antiselfdual complex structures that are covariantly conserved in the HK space and which can be taken to be the constant 't Hooft symbols.[9] Finally, the v^m are the vierbein of the HK space: $d\sigma^2 = v^m v^m$.

[9]The 't Hooft symbols satisfy the following identities

$$\epsilon^{ABC}\bar{\eta}^B{}_{mp}\bar{\eta}^C{}_{nq} = -\delta_{mn}\bar{\eta}^A{}_{pq} - \delta_{pq}\bar{\eta}^A{}_{mn} + \delta_{mq}\bar{\eta}^A{}_{pn} + \delta_{pn}\bar{\eta}^A{}_{mq}\,, \tag{3.13}$$

$$\bar{\eta}^A{}_{mn}\bar{\eta}^A{}_{pq} = 2\delta_{m[p}\delta_{q]n} - \epsilon_{mnpq}\,, \tag{3.14}$$

$$\bar{\eta}^A{}_{mp}\bar{\eta}^B{}_{pn} = -\delta^{AB}\delta_{mn} + \epsilon^{ABC}\bar{\eta}^C{}_{mn}\,. \tag{3.15}$$

The selfdual gauge field strength is given by[10]

$$g\hat{F}^A = \left[\bar{\eta}^A{}_{np}\nabla_m\partial_p \log P + \bar{\eta}^A{}_{mp}\,\partial_p \log P\,\partial_n \log P - \frac{1}{2}\bar{\eta}^A{}_{mn}\,(\partial \log P)^2\right] v^m \wedge v^n\,. \tag{3.19}$$

Next, let us focus on eqs. (2.10), which in this case take the form

$$d\star_\sigma dZ_0 = \frac{1}{18}\hat{F}^A \wedge \hat{F}^A\,, \tag{3.20}$$

$$d\star_\sigma dZ_{1,2} = 0\,, \tag{3.21}$$

$$\hat{\mathfrak{D}}\star_\sigma \hat{\mathfrak{D}}Z_A = 0\,. \tag{3.22}$$

For the configurations considered, it was shown in ref. [3] that the instanton number density $\hat{F}^A \wedge \hat{F}^A$ enjoys the so-called "Laplacian property", i.e.

$$\hat{F}^A \wedge \hat{F}^A = -d\star_\sigma d\left[\frac{(\partial \log P)^2}{g^2}\right]\sqrt{h}\,d^4x\,, \tag{3.23}$$

where h is the determinant of the HK metric. Hence, we find that eqs. (3.20) and (3.21) are solved by

$$Z_0 = Z_0^{(0)} - \frac{(\partial \log P)^2}{18g^2}\,, \tag{3.24}$$

$$Z_{1,2} = Z_{1,2}^{(0)}\,. \tag{3.25}$$

where $Z_{0,1,2}^{(0)}$ are harmonic functions on the HK space.

As for eq. (3.22), two solutions of it are known to us:

3.3.1 Solution D1

This solution was found in ref. [22] for HK metrics admitting a triholomorphic isometry. These metrics are known as Gibbons-Hawking (GH) metrics [34, 35] and can be put in the form

$$d\sigma^2 = H^{-1}(d\eta + \chi)^2 + H\,dx^i dx^i\,, \qquad d\chi = \star_{(3)}\,dH\,, \tag{3.26}$$

where $\star_{(3)}$ is the Hodge star on $\mathbb{E}^3$ and H is a (η-independent) harmonic function on $\mathbb{E}^3$.[11]

This solution also makes use of SU(2) instantons obtained through the 't Hooft anstaz eq. (3.12) with a function P which is also independent of η and, therefore, harmonic on $\mathbb{E}^3$ as well. The consistency of the solution demands that the functions Z_A are also independent of η.

[10] The gauge field strength for the SU(2) group is given by

$$F^A = dA^A + \frac{1}{2}\epsilon^{ABC}\,A^B \wedge A^C\,, \tag{3.16}$$

and the covariant derivatives of the scalar functions are given by

$$\mathfrak{D}Z_A = dZ_A - \epsilon^{ABC}A^B Z_C\,, \tag{3.17}$$

$$\mathfrak{D}\varphi^A = d\varphi^A + \epsilon^{ABC}A^B\varphi^C\,. \tag{3.18}$$

[11] This is the integrability condition of the equation $d\chi = \star_{(3)}\,dH$.

Let us see in detail how this solution is obtained.

If P is independent of η and the HK metric is the above GH metric, the vector fields defined by 't Hooft ansatz eq. (3.12) can be written in the simple form

$$g\hat{A}^A = H^{-1}\varphi^A (d\eta + \chi) + \breve{A}^A \,, \tag{3.27}$$

where φ^A and $\breve{A}^A$ are fields (scalar and vector, resp.) defined on $\mathbb{E}^3$ and determined by the choice of P by

$$\varphi^A = \delta^{Ai}\,\partial_{\underline{i}} \log P \,, \tag{3.28}$$

$$\breve{A}^A = \epsilon^{Aij}\,\partial_{\underline{i}} \log P dx^j \,. \tag{3.29}$$

The selfduality of the field strength of $\hat{A}^A$ reduces in this case to the Bogmol'nyi equation relating the field strength of $\breve{A}^A$ and the covariant derivative of the φ^A with respect to that connection on $\mathbb{E}^3$:

$$\star_{(3)}\, \breve{F}^A = -\breve{\mathfrak{D}}\varphi^A \,. \tag{3.30}$$

Substituting eq. (3.27) into eq. (3.22), we find the following equation for Z_A

$$\partial_{\underline{i}}\partial_{\underline{i}} Z_A + 2\delta^i_A\,\partial_{\underline{i}} Z_B\,\varphi^B - 2\varphi^A\delta^{Bj}\partial_{\underline{j}} Z_B - 2Z_A\,\varphi^B\varphi^B = 0\,. \tag{3.31}$$

Following ref. [22], we make the following ansatz for Z_A:

$$Z_A = \delta^i_A \frac{\partial_{\underline{i}} Q}{gP} \,, \tag{3.32}$$

where Q is some function on $\mathbb{E}^3$. Plugging this ansatz into eq. (3.31), we arrive at the following equation for Q:

$$P\partial_{\underline{i}}\left(\frac{\partial_{\underline{j}}\partial_{\underline{j}} Q}{gP^2}\right) = 0\,, \qquad \Rightarrow \qquad \partial_{\underline{j}}\partial_{\underline{j}} Q = kP^2 \,, \tag{3.33}$$

for some constant k. If, as we will assume later, $P = 1 + \lambda^{-2}/r$ (a spherically-symmetric harmonic function in $\mathbb{E}^3$), then

$$Q = Q^{(0)} + k\left[\frac{1}{6}r^2 + \lambda^{-2} r + \lambda^{-4}\log r\right] \,, \tag{3.34}$$

where $Q^{(0)}$ is another harmonic function in $\mathbb{E}^3$. $k \neq 0$ will, in general, give rise to non-asymptotically flat metrics and, therefore, we will set it to zero.

3.3.2 Solution D2

The second solution available in the literature was found in ref. [29] in $\mathbb{E}^4$ and it was used to construct a dyonic instanton solution of Heterotic supergravity in [30]. The generalization of this solution to the case of arbitrary HK metrics is straightforward. In order to show this, let us first rewrite eq. (3.22) as

$$\begin{aligned} d\star_\sigma dZ_A - g\,\epsilon^{ABC}\,Z_B\,d\star_\sigma \hat{A}^C - 2g\,\epsilon^{ABC}(\star_\sigma \hat{A}^B)\wedge dZ_C \\ +g^2\left(Z_B\hat{A}^B\wedge(\star_\sigma\hat{A}^A) - Z_A\,\hat{A}^B\wedge(\star_\sigma\hat{A}^B)\right) = 0\,. \end{aligned} \tag{3.35}$$

Let us keep just one of the Z_A functions active, say Z_3. Substituting the 't Hooft ansatz eq. (3.12) into eq. (3.35), we get the following conditions

$$\bar{\eta}^1{}_{mn}\partial_n P\,\partial_m Z_3 = \bar{\eta}^2{}_{mn}\partial_n P\,\partial_m Z_3 = 0\,, \tag{3.36}$$

$$\nabla^2 Z_3 - 2\,Z_3\,(\partial \log P)^2 = 0\,, \tag{3.37}$$

which are solved by

$$Z_3 = \frac{\xi_2}{gP}\,, \tag{3.38}$$

where ξ_2 is an arbitrary constant.

Notice that when the HK metric is a GH metric and the vector fields do not depend on the isometric coordinate η, eq. (3.38) is a particular case of eq. (3.32) given by the choice of harmonic function $Q(\vec{x}) = \xi_2\,x^3$.

3.4 The 1-form ω

Finally, let us consider eq. (2.11). When the HK metric is a GH metric of the form eq. (3.26), we can always write the 1-form ω as

$$\omega = \omega_5(d\eta + \chi) + \breve{\omega}\,, \tag{3.39}$$

and then eq. (2.11) takes the following form:

$$\begin{aligned} d\breve{\omega} + \omega_5 d\chi - H \star_{(3)} d\omega_5 &= \frac{\sqrt{3}}{g}\left(\frac{Z_B\varphi^B}{H}d\chi + Z_B\breve{F}^B\right) \\ &= \frac{\sqrt{3}}{2g}\left[\frac{Z_B\varphi^B}{H}d\chi - H\star_{(3)} d\left(\frac{Z_B\varphi^B}{H}\right)\right. \\ &\qquad\left. + \star_{(3)}\left(\varphi^B\,\breve{\mathfrak{D}}Z_B - Z_B\breve{\mathfrak{D}}\varphi^B\right)\right]\,, \end{aligned} \tag{3.40}$$

where we have made use of the Bogomol'nyi equation (3.30) in order to rewrite the r.h.s. of the equation. The integrability condition of this equation is[12]

$$H\,d\star_{(3)} d\left(\omega_5 - \frac{\sqrt{3}}{2g}\frac{Z_B\varphi^B}{H}\right) = 0\,, \tag{3.42}$$

so that

$$\omega_5 = M + \frac{\sqrt{3}}{2g}\frac{Z_B\varphi^B}{H}\,, \tag{3.43}$$

where M is another harmonic function in $\mathbb{E}^3$. Finally, substituting eq. (3.43) back in eq. (3.40), we arrive at the following equation for ω:

$$\star_{(3)}\, d\breve{\omega} = HdM - MdH + \frac{\sqrt{3}}{2g}\left(\varphi^B\,\breve{\mathfrak{D}}Z_B - Z_B\breve{\mathfrak{D}}\varphi^B\right)\,. \tag{3.44}$$

[12]To derive this equation, we use that

$$d\star_{(3)}\left(\varphi^B\,\breve{\mathfrak{D}}Z_B - Z_B\breve{\mathfrak{D}}\varphi^B\right) = \varphi^B\,\breve{\mathfrak{D}}\star_{(3)}\breve{\mathfrak{D}}Z_B = 0\,, \tag{3.41}$$

as a consequence of the Bogomol'nyi equation (3.30).

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This is the equation that will have to be solved if we use the D1 solution for the gauge fields or if we use the D2 solution over a GH space.

For the D2 solution eq. (3.38) over a generic HK space, it is natural to try an ansatz of the form

$$\omega = \bar{\eta}^3{}_{mn}\partial_n\Omega\, v^m\,. \tag{3.45}$$

Then, using eqs. (3.19) and (3.45), we find that eq. (2.11) reduces to

$$\begin{aligned}2\,\bar{\eta}^3{}_{[n|p}\nabla_{m]}\partial_p\Omega + \frac{1}{2}\bar{\eta}^3{}_{mn}\nabla^2\Omega = \frac{\sqrt{3}\xi}{g^2 P}\Big\{&\bar{\eta}^3{}_{[n|p}\nabla_{m]}\partial_p \log P \\ &+\bar{\eta}^3{}_{[m|p}\,\partial_p \log P\,\partial_{n]}\log P - \frac{1}{2}\bar{\eta}^3{}_{mn}\,(\partial \log P)^2\Big\}\,,\end{aligned} \tag{3.46}$$

which is solved by

$$\Omega = -\frac{\sqrt{3}\xi_2}{2g^2}P^{-1}\,. \tag{3.47}$$

Summarizing, for the solutions D1 and D2 of the gauge fields, the 1-form ω is given by

$$\begin{aligned}&\text{D1 \& D2 on GH space:} \qquad \omega = \left(M + \frac{\sqrt{3}}{2g}\frac{Z_B\,\varphi^B}{H}\right)(d\eta+\chi) + \breve{\omega}\,,\\ &\text{D2 on general HK space:} \qquad \omega = \frac{\sqrt{3}\,\xi_2}{2g^2P^2}\,\bar{\eta}^3{}_{mn}\,\partial_n P\, v^m\,.\end{aligned} \tag{3.48}$$

with $\breve{\omega}$ satisfying eq. (3.44).

3.5 Dimensional reduction to $d = 4$

When the HK metric is a GH metric taking the form eq. (3.26) in the coordinate system adapted to the triholomorphic isometry and (quite naturally) none of the physical fields depends on the isometric coordinate η, it is possible to perform a standard Kaluza-Klein reduction of the solution to $d = 4$ along that direction to obtain a solution of the SU(2)-gauged ST[2, 6] model of $\mathcal{N} = 2, d = 4$ SEYM. The matter fields of this theory are vector fields $A^\Lambda{}_\mu$, $\Lambda = 0, 1, \cdots, 6$ and complex scalars Z^i, $i = 1, \ldots, 6$ parametrizing the coset space

$$\frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{SO}(2)} \times \frac{\mathrm{SO}(2,5)}{\mathrm{SO}(2)\times\mathrm{SO}(5)}\,. \tag{3.49}$$

The interactions are determined by the cubic prepotential

$$\mathcal{F} = -\frac{1}{3!}\frac{d_{ijk}\mathcal{X}^i\mathcal{X}^j\mathcal{X}^k}{\mathcal{X}^0}\,, \tag{3.50}$$

where the constant, fully symmetric, tensor d_{ijk} is related to the tensor C_{IJK} of the 5-dimensional theory by

$$d_{ijk} = 6C_{i-1\,j-1\,k-1}\,, \qquad i, j, k = 1, \ldots, 6\,. \tag{3.51}$$

The index 1 corresponds to the 5-dimensional 0 and the 4-dimensional 0 is associated to the Kaluza-Klein vector of the dimensional reduction fomr 5 to 4 dimensions.

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In this model, the SU(2) gauge group acts on the complex scalars and vector fields with indices $4, 5, 6$. Furthermore, the $+$ and $-$ combinations defined in the 5-dimensional case now correspond to

$$A^{\pm}{}_{\mu} \equiv A^{2}{}_{\mu} \pm A^{3}{}_{\mu}\,. \tag{3.52}$$

A solution-generating technique to construct directly the timelike supersymmetric solutions of $\mathcal{N} = 2, d = 4$ SEYM theories was found in refs. [6, 9, 11], but the procedure turns out to be completely equivalent to the construction of timelike supersymmetric solutions with an additional triholomorphic isometry in the auxiliary HK space (sometimes called *base space*) in $d = 5$.

The formulae relating the 4- and 5-dimensional fields were given in full generality in ref. [14]. Here, we particularize those formulae for the dyonic configurations considered in this paper.[13]

We find that the 4-dimensional Einstein-frame metric takes the standard form of the timelike supersymmetric solutions of $\mathcal{N} = 2, d = 4$ SEYM theories

$$ds^2_{(4)} = e^{2U}\,(dt + \breve{\omega})^2 - e^{-2U}\,dx^i dx^i\,, \tag{3.53}$$

where the metric function e^{-2U} is given by

$$e^{-2U} = \sqrt{f^{-3}H - (\omega_5 H)^2} = \sqrt{\frac{27}{2} Z_0 \tilde{Z}_+ Z_- H - (\omega_5 H)^2}\,, \tag{3.54}$$

and the 1-form $\breve{\omega}$ is the solution to eq. (3.44).

The 4-dimensional vector fields are given by[14]

$$A^0_{(4)} = \frac{1}{2\sqrt{2}}\left[-e^{4U} H^2 \omega_5\,(dt + \breve{\omega}) + \chi\right]\,, \tag{3.55}$$

$$A^1_{(4)} = \frac{1}{6\sqrt{2}} \frac{e^{4U} H f^{-3}}{Z_0}\,(dt + \breve{\omega})\,, \tag{3.56}$$

$$A^{\pm}_{(4)} = \frac{1}{3\sqrt{2}} \frac{e^{4U} H f^{-3} Z_+}{Z_{\pm} \tilde{Z}_+}\,(dt + \breve{\omega})\,, \tag{3.57}$$

$$A^A_{(4)} = \frac{-e^{4U} H}{3\sqrt{2}} \left(\frac{f^{-3} Z_A}{\tilde{Z}_+ Z_-} + \frac{\sqrt{3}\omega_5 \varphi^A}{2g}\right)(dt + \breve{\omega}) + \frac{1}{g}\breve{A}^A\,, \tag{3.58}$$

where the φ^A are defined in eq. (3.28).

[13] It is worth mentioning that there are many purely magnetic solutions of the Bogomol'nyi equations (and, hence, of the selfduality equations) which are, by definition, non-Abelian BPS magnetic monopoles. They were found by Protogenov in ref. [36] and all of them can and have been used to construct regular 4-dimensional black-hole solutions with non-Abelian fields in these theories [6, 8, 9, 11, 14]. However, not all these magnetic monopoles correspond to regular instantons in 5 dimensions and, therefore, they cannot be used to construct regular 5-dimensional black holes. Here we are solving directly the selfduality equation (2.9) and it is guaranteed (it is tautological) that, if they are defined on a GH space and are independent of η, they give solutions of the Bogomol'nyi equations which correspond to regular instantons.

[14] We have added a subindex $_{(4)}$ to distinguish them from the 5-dimensional ones.

Finally, the six 4-dimensional scalars are given by[15]

$$Z^1 = -\frac{1}{3\, Z_0\, H}\left(\omega_5 H - ie^{-2U}\right)\,, \tag{3.59}$$

$$Z^+ = -\frac{2}{3\, \tilde{Z}_+\, H}\left(\omega_5 H - ie^{-2U}\right)\,, \tag{3.60}$$

$$Z^- = -\frac{2\, Z_+}{3\, \tilde{Z}_+\, Z_-\, H}\left(\omega_5 H - ie^{-2U}\right)\,, \tag{3.61}$$

$$Z^A = \frac{2\, Z_A}{3\, \tilde{Z}_+\, Z_-\, H}\left(\omega_5 H - ie^{-2U}\right) + \frac{\varphi^A}{g\, H}\,. \tag{3.62}$$

The Kaluza-Klein scalar of the $5 \to 4$ compactification is a particular combination of these 6 complex scalars and it is given by

$$\ell^2 = e^{-4U} H^{-2} f^2\,. \tag{3.63}$$

4 Rotating black holes

In the previous section, out of the many solutions that can be obtained by using the techniques explained in section 2, we have selected two more restricted classes characterized by dyonic Yang-Mills fields of two different kinds that we have labeled D1 and D2. These two classes of solutions still depend on functions and building blocks which must satisfy certain differential equations and conditions which do not determine them completely. In this section we are going to make some particular choices of these building blocks adequate to find single-center black-hole solutions.[16] First of all, although we can use any HK metric for the solutions of section 3 based on the dyonic instanton D2, we are going to restrict ourselves to GH metrics, eq. (3.26), and, in particular, to spherically-symmetric GH metrics of the form

$$d\sigma^2 = H^{-1}(d\eta + \chi)^2 + H\left(dr^2 + r^2 d\Omega^2_{(2)}\right)\,, \quad r^2 = x^i x^i\,, \quad d\Omega^2_{(2)} = d\theta^2 + \sin\theta^2 d\phi^2\,, \tag{4.1}$$

where H only depends on the radial coordinate r and where the coordinate η is compact and has period $\eta \sim \eta + 2\pi\ell_s$, ℓ_s being a length scale that we take to be the string length.

Since H is a function of r harmonic in $\mathbb{E}^3$, the most general choice of H and the corresponding χ are locally given by

$$H = a_H + \frac{b_H}{r}\,, \qquad \chi = b_H \cos\theta\, d\phi\,, \tag{4.2}$$

where a_H, b_H are two integration constants to be determined.

As it stands, for generic values of b_H, this metric has an undesirable feature: it has a Dirac-Misner string. Fortunately, it can be eliminated from the metric (4.1) by taking

$$b_H = n\ell_s/2\,, \qquad n \in \mathbb{Z}\,, \tag{4.3}$$

[15] Note that we use superscripts to label the 4-dimensional scalars to distinguish them from the Z-functions which instead have subindices.

[16] Multicenter solutions have been considered in ref. [18].

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and covering the HK manifold with two patches. For the time being, though, we will just study the metric locally in one of those patches.

Furthermore, when $a_H = 0$, the compactification to 4 dimensions is singular (observe that the KK scalar in (3.63) blows up), which means that the solutions with $a_H = 0$ only makes sense in 5 dimensions. When $a_H \neq 0$, however, the asymptotic radius of the internal direction is finite and therefore the solution is effectively 4-dimensional. We will deal with these two possibilities separately.

Once the HK metric has been chosen, we have to specify the magnetic part of the non-Abelian vector fields, $\hat{A}^A$, which is given in terms of the harmonic function P of the 't Hooft ansatz by eq. (3.12). Again, if P depends only on the radial coordinate, it must be given by

$$P = 1 + \frac{\lambda^{-2}}{r}\,, \tag{4.4}$$

where λ^{-2} measures the instanton size.

This choice automatically determines (up to a proportionality constant) the electric part of the non-Abelian vectors for solution D2. For the solution D1, the harmonic function Q still has to be specified, but if it is only a function of the radial coordinate, it has to be proportional to P

$$Q = \xi_1 P\,, \tag{4.5}$$

for some constant ξ_1.[17]

Hence, for the two classes of solutions D1 and D2, we have

$$Z_A = \begin{cases} -\delta_{A\,i}\,\dfrac{\xi_1}{g\,(1+\lambda^2 r)\,r}\,\dfrac{x^i}{r}\,, & \text{(D1)} \\[2ex] \delta_{A\,3}\,\dfrac{\lambda^2 \xi_2 r}{g\,(1+\lambda^2 r)}\,. & \text{(D2)} \end{cases} \tag{4.6}$$

For the harmonic functions $Z^{(0)}_{0,+,-}$, we take

$$Z^{(0)}_{0,+,-} = a_{0,+,-} + \frac{b_{0,+,-}}{r}\,, \tag{4.7}$$

and the complete functions $Z_0, \tilde{Z}_+$ and Z_- appearing in the metric eq. (2.5) read

$$Z_0 = a_0 + \frac{b_0}{r} - \frac{1}{18g^2}\frac{1}{r\,(a_H r + b_H)\,(1+\lambda^2 r)^2}\,, \tag{4.8}$$

$$\begin{aligned} Z_- &= a_- + \frac{b_-}{r}\,, \\ \tilde{Z}_+ &= \begin{cases} a_+ + \dfrac{b_+}{r} - \dfrac{\xi_1^2}{g^2 r\,(a_- r + b_-)\,(1+\lambda^2 r)^2}\,, & \text{(D1)} \\[2ex] a_+ + \dfrac{b_+}{r} - \dfrac{\xi_2^2 \lambda^4 r^3}{g^2\,(a_- r + b_-)\,(1+\lambda^2 r)^2}\,. & \text{(D2)} \end{cases} \end{aligned} \tag{4.9}$$

[17]The additive constant in Q is irrelevent.

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Since we have restricted ourselves to GH spaces, the 1-form ω takes the form eq. (3.39) with ω_5 given by eq. (3.43) and with $\breve{\omega}$ implicitly determined by eq. (3.48). Choosing the harmonic function M as

$$M = a_M + \frac{b_M}{r}\,, \tag{4.10}$$

one finds

$$\omega_5 = \begin{cases} a_M + \dfrac{b_M}{r} + \dfrac{\sqrt{3}\xi_1}{2g^2}\dfrac{1}{r\,(a_H r + b_H)\,(1+\lambda^2 r)^2}\,, & \text{(D1)} \\[2ex] a_M + \dfrac{b_M}{r} - \dfrac{\sqrt{3}\xi_2}{2g^2}\dfrac{\lambda^2 r\cos\theta}{(a_H r + b_H)\,(1+\lambda^2 r)^2}\,, & \text{(D2)} \end{cases} \tag{4.11}$$

$$\breve{\omega} = \begin{cases} (a_H b_M - a_M b_H)\cos\theta d\phi\,, & \text{(D1)} \\[2ex] \left[(a_H b_M - a_M b_H)\cos\theta - \dfrac{\sqrt{3}\xi_2}{2g^2}\dfrac{\lambda^2 r\sin^2\theta}{(1+\lambda^2 r)^2}\right] d\phi\,. & \text{(D2)} \end{cases} \tag{4.12}$$

At this point, the solutions are fully specified, up to the choice of integration constants. This choice is constrained by requirements of regularity, asymptotic flatness etc., which demand a closer, case by case, study.

In particular, as we have already mentioned, the $a_H = 0$ and $a_H \neq 0$ cases correspond to asymptotically-flat 5 and 4-dimensional solutions, respectively. It is natural to analyze them separately.

4.1 5-dimensional black holes ($a_H = 0$)

When $a_H = 0$, the change of variables $\rho^2 = 4b_H r$ brings the metric eq. (4.1) to the form

$$d\sigma^2 = d\rho^2 + \frac{\rho^2}{4}\left(d\Psi^2 + d\phi^2 + d\theta^2 + d\phi^2 + 2\cos\theta d\Psi d\phi\right) \equiv d\rho^2 + \rho^2\, d\Omega^2_{(3)/n}\,, \tag{4.13}$$

where we have introduced the angular coordinate $\Psi = 2\eta/(n\ell_s)$ whose period is $\Psi \sim \Psi + 4\pi/n$ and where $d\Omega^2_{(3)/n}$ is the metric of the lens space $S^3/\mathbb{Z}_n$. From now on we discuss the $n = 1$ case, for which the above metric is that of $\mathbb{E}^4$ and the 5-dimensional spacetime metric of the solution eq. (2.5) can be cast in the form

$$ds^2 = \left(\mathcal{Z}_0\tilde{\mathcal{Z}}_+\mathcal{Z}_-\right)^{-2/3}(dt+\omega)^2 - \left(\mathcal{Z}_0\tilde{\mathcal{Z}}_+\mathcal{Z}_-\right)^{1/3}\left(d\rho^2 + \rho^2 d\Omega^2_{(3)}\right)\,, \tag{4.14}$$

where we have defined

$$\mathcal{Z}_0 \equiv Z_0/a_0\,, \qquad \mathcal{Z}_- \equiv Z_-/a_-\,, \qquad \tilde{\mathcal{Z}}_+ \equiv \tilde{Z}_+/\tilde{a}_+\,, \tag{4.15}$$

$\tilde{a}_+$ being the asymptotic value of $\tilde{Z}_+$, which is given by

$$\tilde{a}_+ = \begin{cases} a_+\,, & \text{(D1)} \\[2ex] a_+ - \dfrac{\xi_2^2}{a_- g^2}\,. & \text{(D2)} \end{cases} \tag{4.16}$$

and where we have imposed an asymptotic flatness condition on a_0, a_- and $\tilde{a}_+$, namely

$$\frac{27}{2}a_0\tilde{a}_+a_- = 1\,. \tag{4.17}$$

This condition, together with the expressions of the scalars in terms of the Z functions eq. (3.8) allows us to write a_0, a_- and $\tilde{a}_+$ in terms of the two moduli of this theory ϕ_∞, k_∞:

$$a_0 = \frac{1}{3} e^{\phi_\infty} k_\infty^{-2/3}\,, \qquad \tilde{a}_+ = \frac{1}{3} k_\infty^{4/3}\,, \qquad a_- = \frac{2}{3} e^{-\phi_\infty} k_\infty^{-2/3}\,. \tag{4.18}$$

We can, therefore, eliminate these three integration constants in the functions that appear in the metric, which, with the definitions

$$\kappa^2 \equiv 4 b_H \lambda^{-2}\,, \qquad \tilde{\xi}_1 \equiv 4 b_H \xi_1\,, \tag{4.19}$$

now take the form

$$\mathcal{Z}_0 = 1 + \frac{\mathcal{Q}_0}{\rho^2} + \frac{2\, e^{-\phi_\infty} k_\infty^{2/3}}{3g^2} \frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2}\,, \tag{4.20}$$

$$\mathcal{Z}_- = 1 + \frac{\mathcal{Q}_-}{\rho^2}\,, \tag{4.21}$$

$$\tilde{\mathcal{Z}}_+ = \begin{cases} 1 + \dfrac{\tilde{\mathcal{Q}}_+}{\rho^2} + \dfrac{9\tilde{\xi}_1^2\, e^{\phi_\infty} k_\infty^{-2/3}}{2g^2} \dfrac{\rho^4 + \rho^2\left(\mathcal{Q}_- + 2\kappa^2\right) + \kappa^2\left(\kappa^2 + 2\mathcal{Q}_-\right)}{\mathcal{Q}_-\left(\rho^2 + \mathcal{Q}_-\right)\left(\rho^2+\kappa^2\right)^2}\,, & \text{(D1)} \\[2ex] 1 + \dfrac{\tilde{\mathcal{Q}}_+}{\rho^2} + \dfrac{9\xi_2^2\, e^{\phi_\infty} k_\infty^{-2/3}}{2g^2} \dfrac{\rho^4\left(\mathcal{Q}_- + 2\kappa^2\right) + \rho^2\kappa^2\left(2\mathcal{Q}_- + \kappa^2\right) + \mathcal{Q}_-\kappa^4}{\left(\rho^2 + \mathcal{Q}_-\right)\left(\rho^2+\kappa^2\right)^2}\,. & \text{(D2)} \end{cases} \tag{4.22}$$

Here we have introduced new constants $\mathcal{Q}_0$, $\tilde{\mathcal{Q}}_+$ and $\mathcal{Q}_-$ whose relation with the parameters of the harmonic functions b_0, b_+, b_-, b_H in eqs. (4.7) is

$$\mathcal{Q}_0 = \frac{4b_H}{a_0}\left(b_0 - \frac{1}{18g^2}\right)\,, \qquad \mathcal{Q}_- = \frac{4b_H b_-}{a_-}\,, \qquad \tilde{\mathcal{Q}}_+ = \begin{cases} \dfrac{4b_H}{\tilde{a}_+}\left(b_+ - \dfrac{\xi_1^2}{g^2 b_-}\right)\,, & \text{(D1)} \\[2ex] \dfrac{4b_H b_+}{\tilde{a}_+}\,, & \text{(D2)} \end{cases} \tag{4.23}$$

Finally, setting $a_M = 0$ in eqs. (4.11) and (4.12), we find

$$\omega = \begin{cases} \left[\dfrac{\mathcal{J} + \frac{\sqrt{3}\tilde{\xi}_1}{2g^2}}{\rho^2} - \dfrac{\sqrt{3}\tilde{\xi}_1}{2g^2}\dfrac{\rho^2 + 2\kappa^2}{\left(\rho^2+\kappa^2\right)^2}\right](d\Psi + \cos\theta d\phi)\,, & \text{(D1)} \\[2ex] \dfrac{\mathcal{J}}{\rho^2}(d\Psi + \cos\theta d\phi) - \dfrac{\sqrt{3}\,\xi_2\,\kappa^2\rho^2}{2g^2(\rho^2+\kappa^2)^2}(d\phi + \cos\theta d\Psi)\,, & \text{(D2)} \end{cases} \tag{4.24}$$

where we have defined a new constant

$$\mathcal{J} = 4 b_M b_H^2\,. \tag{4.25}$$

We have replaced some integration constants by physical quantities and we have also made some redefinitions. Then, at this point, the solutions we have constructed depend on the independent constants

$$\phi_\infty\,,\ \ k_\infty\,,\ \ g\,,\ \ \kappa\,,\ \ \mathcal{J}\,,\ \ \mathcal{Q}_0\,,\ \ \tilde{\mathcal{Q}}_+\,,\ \ \mathcal{Q}_-\,,\ \ \text{and}\ \ \tilde{\xi}_1\ \ \text{or}\ \ \xi_2\,.$$

The first three of these have a clear physical meaning. They are moduli of the solutions: asymptotic values of two scalars and Yang-Mills coupling constant. There is another modulus of the solutions: the asymptotic value of the gauge-invariant combination $\sqrt{\ell_A \ell_A}$, that we can denote by v. Only for the D2 solution it has a non-trivial value:

$$v = \frac{\xi_2}{a_- g} = \frac{3}{2g}\xi_2 e^{\phi_\infty} k_\infty^{2/3} \,, \qquad \text{(D2)} \tag{4.26}$$

which allows us to replace ξ_2 by $\frac{2}{3} g v e^{-\phi_\infty} k_\infty^{-2/3}$.

Our next task will be to compute the conserved charges of the solution in terms of the rest of the integration constants.

Charges of the solution. It is well-known that the presence of Chern-Simons terms in field strengths or actions leads to the possibility of defining different notions of charge, see for instance refs. [37, 38]. In the theories under consideration, they induce the occurrence of $F \wedge F$ terms in the equations of motion of the vector fields, as can be seen in eq. (2.3), which in differential-form language takes the form:

$$-\mathfrak{D}(a_{IJ} \star F^J) + \frac{1}{\sqrt{3}} C_{IJK} F^J \wedge F^K + g k_{I\,x} \mathfrak{D}\phi^x = 0 \,. \tag{4.27}$$

The $F \wedge F$ terms vanish when all the vector fields are purely electric and static, but they give non-vanishing contributions at infinity when they are non-static or magnetic (instantonic, for instance). Therefore, one gets different results in the calculation of a charge, depending on whether one includes these terms in the definition or not.

Let us study the different possibilities.

If we couple the supergravity action to a 0-brane that couples electrically to the vector field A^I, the equations of motion eq. (4.27) are modified by a 1-form current J_I^S as follows[18]

$$-\mathfrak{D}(a_{IJ} \star F^J) + \frac{1}{\sqrt{3}} C_{IJK} F^J \wedge F^K + g k_{I\,x} \mathfrak{D}\phi^x = \star J_I{}^S \,, \tag{4.28}$$

and we can compute the so-called *brane-source charges*, $\mathcal{Q}_I^S$, by integrating both sides over some spatial 4-volume (such as a $t =$ constant hypersurface):

$$\mathcal{Q}_I^S \equiv \int_{V^4} \star J_I{}^S = \int_{V^4} \left\{ -\mathfrak{D}(a_{IJ} \star F^J) + \frac{1}{\sqrt{3}} C_{IJK} F^J \wedge F^K + g k_{I\,x} \mathfrak{D}\phi^x \right\} . \tag{4.29}$$

In general, this charge is not conserved, $d \star J_I{}^S \neq 0$, because the l.h.s of eq. (4.28) is not closed. However, in the ungauged directions, the Killing vectors $k_I{}^x$ vanish, the gauge-covariant derivative becomes an ordinary exterior derivative and the $F \wedge F$ terms are a closed (but not exact) 4-form and

$$\mathcal{Q}_I^S = \frac{1}{2\pi^2} \int_{V^4} \left\{ -d(a_{IJ} \star F^J) + \frac{1}{\sqrt{3}} C_{IJK} F^J \wedge F^K \right\} , \tag{4.30}$$

is a conserved charge.

[18] Apart from the overall factor of $16\pi G_N^{(5)}$, that we are ignoring here, typically the 1-form current J_I^S will come multiplied by different combinations of asymptotic vallues of the scalars and other constants that we will ignore in our discussion.

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Since each of the two terms that appear in the above integral for the Abelian directions are closed 4-forms, we can use them separately to define other possible conserved charges. In particular, using only the terms with second derivatives in the volume integral gives the so-called *Maxwell charges*

$$\mathcal{Q}_I^M \equiv -\frac{1}{2\pi^2}\int_{V^4} d(a_{IJ}\star F^J)\,. \tag{4.31}$$

For our supergravity model (see the action eq. (3.5)) and applying Stokes theorem, we have

$$\mathcal{Q}_0^M = -\frac{1}{6\pi^2}\int_{\partial V_4} e^{2\phi}k^{-4/3}\star F^0\,, \tag{4.32}$$

$$\mathcal{Q}_+^M = -\frac{1}{6\pi^2}\int_{\partial V_4}\left\{\frac{1}{4}k^{8/3}\left(1+2e^{-\phi}k^{-2}\ell^B\ell^B\right)^2\star F^+ + e^{-2\phi}k^{-4/3}\ell^B\ell^B\star F^- \right.$$
$$\left. + e^{-\phi}k^{2/3}\left(1+2e^{-\phi}k^{-2}\ell^B\ell^B\right)\ell^A\star F^A\right\}\,, \tag{4.33}$$

$$\mathcal{Q}_-^M = -\frac{1}{6\pi^2}\int_{\partial V_4} e^{-2\phi}k^{-4/3}\star\left(F^- + \ell^B\ell^B F^+ + 2\ell^A F^A\right)\,, \tag{4.34}$$

where ∂V_4 is the boundary of V_4. The relation between the Maxwell charges and the brane-source charges is[19]

$$\mathcal{Q}_0^S = \mathcal{Q}_0^M + \frac{1}{12\sqrt{3}\pi^2}\int_{V_4}\left(F^+\wedge F^- - F^A\wedge F^A\right)\,, \tag{4.35}$$

$$\mathcal{Q}_\pm^S = \mathcal{Q}_\pm^M + \frac{1}{6\sqrt{3}\pi^2}\int_{V_4} F^0\wedge F^\mp\,. \tag{4.36}$$

By direct computation, we find that, for the solutions described in this section, the Maxwell charges have the following values

$$\mathcal{Q}_0^M = \frac{2}{\sqrt{3}}e^{\phi_\infty}k_\infty^{-2/3}\mathcal{Q}_0^\infty\,, \tag{4.37}$$

$$\mathcal{Q}_+^M = \begin{cases}\frac{1}{\sqrt{3}}k_\infty^{4/3}\tilde{\mathcal{Q}}_+^\infty & \text{(D1)}\\ \frac{1}{\sqrt{3}}k_\infty^{4/3}\tilde{\mathcal{Q}}_+ & \text{(D2)}\end{cases}\,, \tag{4.38}$$

$$\mathcal{Q}_-^M = \begin{cases}\frac{2}{\sqrt{3}}e^{-\phi_\infty}k_\infty^{-2/3}\mathcal{Q}_-^\infty & \text{(D1)}\\ \frac{2}{\sqrt{3}}e^{-\phi_\infty}k_\infty^{-2/3}\left(1+2v^2e^{-\phi_\infty}k_\infty^{-2}\right)\mathcal{Q}_-^\infty & \text{(D2)}\end{cases}\,, \tag{4.39}$$

where we have defined $\mathcal{Q}_0^\infty$, $\tilde{\mathcal{Q}}_+^\infty$ and $\mathcal{Q}_-^\infty$ as

$$\mathcal{Q}_0^\infty \equiv \lim_{\rho\to\infty}\rho^2\left(\mathcal{Z}_0-1\right) = \mathcal{Q}_0 + \frac{2e^{-\phi_\infty}k_\infty^{2/3}}{3g^2}\,, \tag{4.40}$$

[19]Observe that, in general, we cannot apply Stokes' theorem since they are closed but not exact 4-forms.

$$
\begin{aligned}
\tilde{\mathcal{Q}}_+^\infty &\equiv \lim_{\rho\to\infty} \rho^2 \left(\tilde{\mathcal{Z}}_+ - 1\right) = \begin{cases} \tilde{\mathcal{Q}}_+ \left(1 + \dfrac{9\tilde{\xi}_1^2 e^{\phi_\infty} k_\infty^{-2/3}}{2g^2 \tilde{\mathcal{Q}}_+ \mathcal{Q}_-}\right) & \text{(D1)} \\ \tilde{\mathcal{Q}}_+ + 2v^2 e^{-\phi_\infty} k_\infty^{-2} \left(\mathcal{Q}_- + 2\kappa^2\right) & \text{(D2)} \end{cases} , \\
\mathcal{Q}_-^\infty &\equiv \lim_{\rho\to\infty} \rho^2 \left(\mathcal{Z}_- - 1\right) = \mathcal{Q}_- \,.
\end{aligned} \tag{4.41}
$$

Observe that the difference between $\mathcal{Q}_i^\infty$ and the constants $\mathcal{Q}_i$ is always a shift by some quantity. This behavior is characteristic of systems which have delocalized sources (such as those introduced by the non-Abelian fields) that can contribute at infinity. In this respect, the shift in $\mathcal{Q}_0$ corresponds to the contribution of the instanton to this kind of charge, already observed in refs. [2, 4, 20, 25]. The new shifts in $\tilde{\mathcal{Q}}_+$, which are proportional respectively to $\tilde{\xi}_1^2$ and v^2 (or equivalently to ξ_2^2), are due to the dyonic nature of the instanton.

The integrals of the $F \wedge F$ terms are

$$
\frac{1}{2\pi^2} \int_{V_4} F^0 \wedge F^+ = 12 e^{-\phi_\infty} k_\infty^{-2/3} \frac{\beta}{1+\beta} \mathcal{Q}_- \,, \tag{4.42}
$$

$$
\frac{1}{2\pi^2} \int_{V_4} F^0 \wedge F^- = \begin{cases} 6k_\infty^{4/3} \dfrac{\beta}{1+\beta} \tilde{\mathcal{Q}}_+^\infty \,, & \text{(D1)} \\ 6k_\infty^{4/3} \dfrac{\beta}{1+\beta} \tilde{\mathcal{Q}}_+ \,, & \text{(D2)} \end{cases} \tag{4.43}
$$

$$
\frac{1}{2\pi^2} \int_{V_4} F^+ \wedge F^- = \begin{cases} 12 e^{\phi_\infty} k_\infty^{-2/3} \dfrac{\beta}{1+\beta} \left(1 + \dfrac{9 e^{\phi_\infty} k_\infty^{-2/3} \tilde{\xi}_1^2}{2g^2 \tilde{\mathcal{Q}}_+ \mathcal{Q}_-}\right) \mathcal{Q}_0 \,, & \text{(D1)} \\ 12 e^{\phi_\infty} k_\infty^{-2/3} \dfrac{\beta}{1+\beta} \mathcal{Q}_0 \,, & \text{(D2)} \end{cases} \tag{4.44}
$$

$$
\frac{1}{2\pi^2} \int_{V_4} F^A \wedge F^A = \begin{cases} \dfrac{8}{g^2} + 54 e^{2\phi_\infty} k_\infty^{-4/3} \mathcal{Q}_0 \dfrac{\beta}{1+\beta} \dfrac{\tilde{\xi}_1^2}{g^2 \tilde{\mathcal{Q}}_+ \mathcal{Q}_-} \,, & \text{(D1)} \\ \dfrac{8}{g^2} \,, & \text{(D2)} \end{cases} \tag{4.45}
$$

where the constant β is defined by

$$
\beta = \frac{4\tilde{\mathcal{J}}^2}{\mathcal{Q}_0 \tilde{\mathcal{Q}}_+ \mathcal{Q}_- - 4\tilde{\mathcal{J}}^2} \,, \quad \text{with} \quad \tilde{\mathcal{J}} = \begin{cases} \mathcal{J} + \dfrac{\sqrt{3}\tilde{\xi}_1}{2g^2} \,, & \text{(D1)} \\ \mathcal{J} \,, & \text{(D2)} \end{cases} \tag{4.46}
$$

and $\mathcal{J}$ is related to the angular momenta of the solutions, as we will see below. The shift in $\tilde{\mathcal{J}}$ is due to the contribution of the non-Abelian field to the angular momentum at the horizon.

Eq. (4.45) indicates that our solutions include a dyonic deformation of the BPST instanton [39]. The electric part of this dyonic configuration will be characterized later on. Notice that the instanton number in the D1 case is not quantized since the integral $\int_{V_4} F^A \wedge F^A$ has a second contribution due to the fact that the gauge fields do not vanish at the horizon.

Taking all these results into account, we find that the brane-source charges are given by

$$\mathcal{Q}_0^S = \frac{2}{\sqrt{3}} e^{\phi_\infty} k_\infty^{-2/3} \left(1 + \frac{\beta}{1+\beta}\right) \mathcal{Q}_0 \,, \tag{4.47}$$

$$\mathcal{Q}_+^S = \begin{cases} \dfrac{1}{\sqrt{3}} k_\infty^{4/3} \left(1 + \dfrac{\beta}{1+\beta}\right) \tilde{\mathcal{Q}}_+^\infty \,, & \text{(D1)} \\ \dfrac{1}{\sqrt{3}} k_\infty^{4/3} \left(1 + \dfrac{\beta}{1+\beta}\right) \tilde{\mathcal{Q}}_+ \,, & \text{(D2)} \end{cases} \tag{4.48}$$

$$\mathcal{Q}_-^S = \begin{cases} \dfrac{2}{\sqrt{3}} e^{-\phi_\infty} k_\infty^{-2/3} \left(1 + \dfrac{\beta}{1+\beta}\right) \mathcal{Q}_- \,, & \text{(D1)} \\ \dfrac{2}{\sqrt{3}} e^{-\phi_\infty} k_\infty^{-2/3} \left(1 + 2v^2 e^{-\phi_\infty} k_\infty^{-2} + \dfrac{\beta}{1+\beta}\right) \mathcal{Q}_- \,. & \text{(D2)} \end{cases} \tag{4.49}$$

In order to characterize the electric part of the non-Abelian dyonic configuration, we can integrate the gauge-invariant quantity $\ell^A \star F^A$ over a S^3 and take the $\rho \to \infty$ limit or that in which it goes to zero. For the solution D1, the profile of the fields is such that $\int_{S^3} \ell^A \star F^A$ vanishes when computed in the $\rho \to \infty$ limit since they fall off to zero too fast, but in the $\rho \to 0$ limit it does not. The opposite is true for the D2 solution. Thus, we find[20]

$$\frac{1}{2\pi^2} \int_{S_0^3} \ell^A \star F^A = 9\sqrt{3} \frac{\tilde{\xi}_1^2 e^{2\phi_\infty}}{g^2} \left(\frac{\mathcal{Q}_0^2}{\tilde{\mathcal{Q}}_+ \mathcal{Q}_-^4} \right)^{1/3} \,, \tag{4.50}$$

$$\mathcal{Q}_{\text{D2}} = \frac{1}{2\pi^2} \int_{S_\infty^3} \ell^A \star F^A = 9\sqrt{3} \frac{\xi_2^2 e^{2\phi_\infty}}{g^2} \left(\kappa^2 + \tilde{\mathcal{Q}}_+^\infty + \mathcal{Q}_-^\infty \right) \,. \tag{4.51}$$

It is worth mentioning that the while the interpretation of $\int_{S_0^3} \ell^A \star F^A$ as a charge is not very rigorous, the quantity that we have denoted by $\mathcal{Q}_{D_2}$ does have a charge interpretation. It is the charge (up to moduli factors) associated of the unbroken U(1) vector field [30].

Mass and angular momenta. The mass and the two independent angular momenta of the solution can be found by examining the asymptotic behavior of the metric eq. (4.14) in a suitable coordinate system. Thus, to this aim, it is convenient to introduce a new set of coordinates $(\tilde{t}, \tilde{\rho}, \tilde{\theta}, \tilde{\phi}_+, \tilde{\phi}_-)$ related to the previous one by the following coordinate transformation

$$\tilde{t} = t \,, \quad \tilde{\rho} = \rho \left(\mathcal{Z}_0 \tilde{\mathcal{Z}}_+ \mathcal{Z}_- \right)^{1/3} \,, \quad \tilde{\theta} = \frac{\theta}{2} \,, \quad \tilde{\phi}_\pm = \frac{\Psi \pm \phi}{2} \,. \tag{4.52}$$

In terms of these new coordinates, the asymptotic expansion of eq. (4.14) for large values of $\tilde{\rho}$ reads

$$\begin{aligned} ds^2 \sim & \left(1 - \frac{8G_N^{(5)}\mathcal{M}}{3\pi\tilde{\rho}^2}\right) d\tilde{t}^2 + \frac{4\mathcal{J}_+}{\tilde{\rho}^2} \cos^2\tilde{\theta} \, d\tilde{t} \, d\tilde{\phi}_+ + \frac{4\mathcal{J}_-}{\tilde{\rho}^2} \sin^2\tilde{\theta} \, dt \, d\tilde{\phi}_- \\ & - \left(1 + \frac{8G_N^{(5)}\mathcal{M}}{3\pi\tilde{\rho}^2}\right) d\tilde{\rho}^2 - \tilde{\rho}^2 \left(d\tilde{\theta}^2 + \cos^2\tilde{\theta} \, d\tilde{\phi}_+^2 + \sin^2\tilde{\theta} \, d\tilde{\phi}_-^2 \right) \,, \end{aligned} \tag{4.53}$$

[20] $\tilde{\xi}_1/g$ has dimensions of length squared, as a charge, while ξ_2/g is dimensionless.

where $\mathcal{M}$, the ADM mass of the solution, is given by

$$\mathcal{M} = \frac{\pi}{4G_N^{(5)}}\left(\mathcal{Q}_0^{\infty} + \tilde{\mathcal{Q}}_+^{\infty} + \mathcal{Q}_-^{\infty}\right) , \tag{4.54}$$

and $\mathcal{J}_\pm$ are the two independent angular momenta of the solution

$$\mathcal{J}_\pm = \begin{cases} \mathcal{J} \,, & \text{(D1)} \\ \mathcal{J} \mp \dfrac{\sqrt{3}\kappa^2\xi_2}{2g^2} \,. & \text{(D2)} \end{cases} \tag{4.55}$$

Properties of the solution. Let us list here the main properties of these solutions:

- There is a regular horizon located at $\rho = 0$. Hence, they describe supersymmetric, rotating, asymptotically-flat black holes. The induced metric at the horizon is

$$-\,ds^2_{\rm H} = \frac{\left(\mathcal{Q}_0\tilde{\mathcal{Q}}_+\mathcal{Q}_-\right)^{1/3}}{4}\left[\frac{1}{1+\beta}\left(d\Psi + \cos\theta d\phi\right)^2 + d\Omega^2_{(2)}\right] . \tag{4.56}$$

  Therefore, the horizon is a squashed 3-sphere and the squashing parameter β is given by eq. (4.46).

  In the D1 solution, β vanishes when both the total angular momentum $\mathcal{J}$ and the parameter $\tilde{\xi}_1$ vanish. Therefore, there can be squashing even for vanishing total angular momentum due to the contribution of the dyonic field to the angular momentum at the horizon (related to $\tilde{\xi}_1$).

  In the D2 solution the squashing parameter can vanish even when there is angular momentum ($\mathcal{J}_\pm = \mp\frac{\sqrt{3}\kappa^2\xi_2}{2g^2}$) because there is a delocalized source of angular momentum in the dyonic non-Abelian field.

- The Bekenstein-Hawking entropy is given in terms of the near-horizon charges by

$$S_{\rm BH} = \frac{A_{\rm H}}{4G_N^{(5)}} = \frac{\pi^2}{2G_N^{(5)}}\sqrt{\mathcal{Q}_0\tilde{\mathcal{Q}}_+\mathcal{Q}_- - 4\tilde{\mathcal{J}}^2} \,. \tag{4.57}$$

  Rewriting this expression in terms of the brane-source of Maxwell charges is very difficult or would result in a very complicated expression.

- These black holes can be seen as non-Abelian generalizations of the 5-dimensional supersymmetric black holes of ref. [40] (with the BMPV black hole [41] as a particular case). The non-Abelian interactions play an important role here, particularly in solution D2. They are the essential ingredient that allow us to describe an asymptotically flat, supersymmetric, rotating black hole with two different angular momenta, something that has not appeared so far in the literature. Furthermore, as a consequence of the interactions between electric and magnetic non-Abelian sources, the 2-form $d\omega$ is no longer anti-selfdual, as can be seen at the level of eq. (2.11). This property, which does not hold here, was thought to be crucial to construct regular supersymmetric rotating black holes in five dimensions [42], although the analysis carried out in that reference did not include non-Abelian fields.

- Even though the black holes are spinning, there is no ergosurface. This is expected for supersymmetric solutions because the existence of ergosurfaces was shown to be incompatible with supersymmetry in ref. [43].

- The presence of closed timelike curves (CTCs) is a quite common feature of these kind of metrics. This problem has been studied with special emphasis in the context of the microstate geometries program [44, 45]. It turns out that the condition that guarantees the spacetime is free of closed timelike curves reduces to the positivity in the whole spacetime of certain function.[21] In our case, we must demand

$$\mathcal{Z}_0\tilde{\mathcal{Z}}_+\mathcal{Z}_-H-(\omega_5 H)^2-\left(\frac{\breve{\omega}_\phi}{r\sin\theta}\right)^2\geq 0\,. \tag{4.58}$$

  In general, this is a complicated problem that has to be studied in a case by case basis for particular values of the physical constants. Often, however, it is enough to study this condition in the $\rho\to 0$ and $\rho\to\infty$ limits, in which case it is equivalent to the positivity of the horizon area $A_{\rm H}$ and to the positivity of the ADM mass $\mathcal{M}$, respectively. In the case at hands, we have checked numerically that if this is the case, then eq. (4.58) can be satisfied without imposing more constraints on the parameters.

- In the D1 solution, the instanton size κ remains a modulus with arbitrary value while the parameter $\tilde{\xi}_1$ appears, as we have seen, non-linearly in the Maxwell and brane-source charges. It also contributes to some quantities computed at the horizon such as the angular momentum $\tilde{\mathcal{J}}$ and the entropy.

- In the D2 solution, however, the instanton size κ is no longer a free parameter since it can be fixed for instance in terms of the electric charge of the dyon by using eq. (4.51). As we have already seen, the non-Abelian fields of this solution also contribute to the total angular momentum in an asymmetric way, giving rise to different components of the angular momentum in different planes. Indeed, we can also use eq. (4.55) to fix the instanton size in terms of the combination of angular momenta $\Delta\mathcal{J}\equiv\mathcal{J}_+-\mathcal{J}_-$ as follows

$$\kappa^2=-\frac{g^2\Delta\mathcal{J}}{\sqrt{3}\xi_2}=-\frac{\sqrt{3}}{2v}e^{\phi_\infty}k_\infty^{2/3}g\,\Delta\mathcal{J}\,. \tag{4.59}$$

Globally smooth solution. The family of solutions D2 includes a gobally regular and horizonless solution that does not require the addition of localized brane sources for the choice of charges $\mathcal{Q}_0=\tilde{\mathcal{Q}}_+=\mathcal{Q}_-=\mathcal{J}=0$. In this case, the $\mathcal{Z}$ functions now take the simpler form

$$\mathcal{Z}_0=1+\frac{2\,e^{-\phi_\infty}k_\infty^{2/3}}{3\,g^2}\frac{\rho^2+2\kappa^2}{(\rho^2+\kappa^2)^2}\,, \tag{4.60}$$

$$\mathcal{Z}_-=1\,, \tag{4.61}$$

[21] Recently in [23], there has been some progress to reduce eq. (4.58) to an algebraic relation, simplifying the task of constructing explicit solutions. Although the results of [23] only apply strictly to a special class of smooth horizonless solutions, we expect a similar analysis may also work for more general configurations.

$$\tilde{\mathcal{Z}}_+ = 1 + \frac{9\xi_2^2\, e^{\phi_\infty} k_\infty^{-2/3}}{2g^2} \frac{\left(2\rho^2 + \kappa^2\right)\kappa^2}{\left(\rho^2 + \kappa^2\right)^2}\,, \tag{4.62}$$

and the 1-form ω becomes just

$$\omega = -\frac{\sqrt{3}\,\xi_2\,\kappa^2\rho^2}{2g^2(\rho^2+\kappa^2)^2}\left(d\phi + \cos\theta d\Psi\right)\,. \tag{4.63}$$

This 5-dimensional solution describes the heterotic dyonic instanton constructed in ref. [30] compactified on a T^5. It can also be seen as a rotating generalization of the instantonic solution considered in ref. [20]. The solution is characterized by two non-vanishing asymptotic charges

$$\mathcal{Q}_0^\infty = \frac{2e^{-\phi_\infty} k_\infty^{2/3}}{3g^2}\,, \tag{4.64}$$

$$\tilde{\mathcal{Q}}_+^\infty = \frac{9\, e^{\phi_\infty} k_\infty^{-2/3}\,\xi_2^2\,\kappa^2}{g^2}\,, \tag{4.65}$$

and by only one independent angular momentum

$$\mathcal{J}_+ = -\mathcal{J}_- = -\frac{\sqrt{3}\,\xi_2\,\kappa^2}{2\,g^2}\,. \tag{4.66}$$

Finally, the mass of the solution eq. (4.54) reduces to

$$\mathcal{M} = \frac{\pi}{4G_N^{(5)}}\left(\mathcal{Q}_0^\infty + \tilde{\mathcal{Q}}_+^\infty\right) = \frac{\pi}{4G_N^{(5)}}\left(\frac{2e^{-\phi_\infty} k_\infty^{2/3}}{3g^2} + \frac{9\, e^{\phi_\infty} k_\infty^{-2/3}\,\xi_2^2\,\kappa^2}{g^2}\right)\,. \tag{4.67}$$

4.2 4-dimensional black holes ($a_H \neq 0$)

The 4-dimensional metric of these solutions is

$$ds^2_{(4)} = e^{2U}\left(dt + \breve{\omega}\right)^2 - e^{-2U}\left(dr^2 + r^2 d\Omega^2_{(2)}\right)\,, \tag{4.68}$$

where the 1-form $\breve{\omega}$ is given in eq. (4.12). The metric function e^{-2U} is

$$e^{-2U} = \sqrt{\frac{27}{2} Z_0 \tilde{Z}_+ Z_- H - (\omega_5 H)^2}\,, \tag{4.69}$$

where

$$Z_0 = a_0\left(1 + \frac{q_0}{r} + \frac{2}{9a_0 a_H g^2} F\left(r; q_H, \lambda^{-2}\right)\right)\,, \tag{4.70}$$

$$Z_- = a_-\left(1 + \frac{q_-}{r}\right)\,, \tag{4.71}$$

$$\tilde{Z}_+ = \begin{cases} \tilde{a}_+\left(1 + \dfrac{\tilde{q}_+}{r} + \dfrac{4\xi_1^2}{\tilde{a}_+ a_- g^2} F\left(r; q_-, \lambda^{-2}\right)\right), & \text{(D1)} \\ \tilde{a}_+\left(1 + \dfrac{\tilde{q}_+}{r} + \dfrac{\xi_2^2}{\tilde{a}_+ a_- g^2} \dfrac{(r+q_-)\left(1+2\lambda^2 r\right) + q_-\lambda^4 r^2}{(r+q_-)\left(1+\lambda^2 r\right)^2}\right), & \text{(D2)} \end{cases} \tag{4.72}$$

$$H = a_H \left(1 + \frac{q_H}{r}\right) , \tag{4.73}$$

$$\omega_5 = \begin{cases} a_M \left(1 + \frac{q_M}{r} - \frac{2\sqrt{3}\,\xi_1}{a_M a_H g^2} F\left(r; q_H, \lambda^{-2}\right)\right) , & \text{(D1)} \\ a_M \left(1 + \frac{q_M}{r} - \frac{\sqrt{3}\,\xi_2}{2a_M a_H g^2} \frac{\lambda^2 r \cos\theta}{(r+q_H)\,(1+\lambda^2 r)^2}\right) , & \text{(D2)} \end{cases} \tag{4.74}$$

where we have introduced the function

$$F\left(r; q_1, q_2\right) \equiv \frac{(r+q_1)\,(r+2q_2) + q_2^2}{4q_1\,(r+q_1)\,(r+q_2)^2} . \tag{4.75}$$

The relation between the constants $q_0, \tilde{q}_+, q_-, q_H$ and q_M and the original parameters of the harmonic functions is

$$q_0 = \frac{1}{a_0}\left(b_0 - \frac{1}{18 b_H g^2}\right) , \qquad q_- = \frac{b_-}{a_-} , \qquad q_H = \frac{b_H}{a_H} , \tag{4.76}$$

and

$$\tilde{q}_+ = \begin{cases} \frac{1}{\tilde{a}_+}\left(b_+ - \frac{\xi_1^2}{a_- g^2}\right) , & \text{(D1)} \\ \frac{b_+}{\tilde{a}_+} , & \text{(D2)} \end{cases} , \qquad q_M = \begin{cases} \frac{1}{a_M}\left(b_M + \frac{\sqrt{3}\xi_1}{2g^2 b_H}\right) , & \text{(D1)} \\ \frac{b_M}{a_M} , & \text{(D2)} \end{cases} , \tag{4.77}$$

where $\tilde{a}_+$ is again given by eq. (4.16). We have implicitly assumed the finiteness of several constants which appear in the denominators of these expressions. In most cases, this is demanded by asymptotic flatness, but we will have to take this fact into account at certain points.

These 4-dimensional solutions depend on the parameters

$$a_0, \tilde{a}_+, a_-, a_H, a_M, \lambda, g, q_0, \tilde{q}_+, q_-, q_H, q_M, \text{ and } \xi_1 \text{ or } \xi_2 . \tag{4.78}$$

As already mentioned, not all of these parameters are independent because they have to satisfy certain relations demanded by asymptotic flatness and the standard normalization of the metric at spatial infinity. These conditions are:

1. The vanishing of NUT charge. This condition demands that[22]

$$a_M b_H = a_H\, b_M , \tag{4.79}$$

[22] This condition is also equivalent to imposing that the integrability condition of eq. (3.44) is also satisfied at the pole. One may wonder if there is a fundamental reason to demand this since, after all, the Z functions as well as the GH function H are singular at that point. Leaving aside the requirement of asymptotic flatness and the wire singularities characteristic of Taub-NUT geometries [1], the main reason to impose the vanishing of the NUT charge is that we do not know of any string theory configuration (source) that can account for it. It was also argued in ref. [31] that the vanishing of the NUT charge is a necessary condition to for the solution to be globally supersymmetric.

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which guarantees that $\breve{\omega}$ vanishes asymptotically, see eq. (4.12). This equation can be satisfied in two ways:

(a) We can just set $a_M = b_M = 0$.

(b) If $a_M \neq 0$, we have to fix the integration constant q_M in terms of q_H (and ξ_1 in the D1 case) as follows

$$q_M = \begin{cases} q_H + \dfrac{\sqrt{3}\xi_1}{2a_M a_H q_H g^2}\,, & \text{(D1)} \\ q_H\,. & \text{(D2)} \end{cases} \tag{4.80}$$

Either way, when there is no NUT charge, the 1-form $\breve{\omega}$ is given by

$$\breve{\omega} = \begin{cases} 0\,, & \text{(D1)} \\ -\dfrac{\sqrt{3}\xi_2}{2g^2}\dfrac{\lambda^2 r \sin^2\theta}{(1+\lambda^2 r)^2}\,d\phi\,. & \text{(D2)} \end{cases} \tag{4.81}$$

Therefore, as already observed in ref. [18], the solution D1 is static, but the solution D2 describes a supersymmetric, asymptotically-flat, rotating black hole.

2. At spatial infinity, the metric function e^{2U} must take a constant value that is conventially taken to be 1, i.e.,

$$\lim_{r\to\infty} e^{2U} \to 1\,, \qquad \Rightarrow \qquad \frac{27}{2}a_0\tilde{a}_+ a_- a_H - (a_M a_H)^2 = 1\,, \tag{4.82}$$

which allow us to rewrite $a_0, \tilde{a}_+, a_-, a_H$ and a_M in terms of only four independent parameters. For future convenience, we choose these parameters to be $e^{\phi_\infty}, k_\infty, \ell_\infty$ and f_∞.[23] The relation between these constants is

$$\begin{aligned} a_0 &= \frac{1}{3}e^{\phi_\infty}k_\infty^{-2/3}f_\infty^{-1}\,, & a_- &= \frac{2}{3}e^{-\phi_\infty}k_\infty^{-2/3}f_\infty^{-1}\,, & \tilde{a}_+ &= \frac{1}{3}k_\infty^{4/3}f_\infty^{-1}\,, \\ a_H &= \frac{f_\infty}{\ell_\infty}\,, & a_M &= \pm\frac{\ell_\infty}{f_\infty}\sqrt{\frac{1-\ell_\infty f_\infty^2}{\ell_\infty f_\infty^2}}\,, \end{aligned} \tag{4.83}$$

with $\ell_\infty f_\infty^2 \leq 1$. Together with the quotient ξ_2/g, these four constants completely determine the asymptotic values of the 4-dimensional scalars which are given in eqs. (3.63)–(3.62). Defining

$$Z^x_\infty = v_x\, e^{i\gamma_x}\,, \qquad x = 0, +, -, A\,, \tag{4.84}$$

we find,

$$v_0 = e^{-\phi_\infty}k_\infty^{2/3}\ell_\infty^{1/2}f_\infty^{-1}\,, \qquad v_+ = 2k_\infty^{-4/3}\ell_\infty^{1/2}f_\infty^{-1}\,, \tag{4.85}$$

[23] Recall that ℓ_∞ is the asymptotic value of the Kaluza-Klein scalar that measures the radius of the circle of the $5\to 4$ compactification. f_∞ is the asymptotic value of the 5-dimensional metric function f, which is given in eq. (3.7) and no longer has to be equal to 1.

$$v_- = \begin{cases} e^{\phi_\infty} k_\infty^{2/3} \ell_\infty^{1/2} f_\infty^{-1} \,, & \text{(D1)} \\ e^{\phi_\infty} k_\infty^{2/3} \ell_\infty^{1/2} f_\infty^{-1} \left(1 + \dfrac{9\, e^{\phi_\infty} f_\infty^2 \xi_2^2}{2\, g^2\, k_\infty^{2/3}}\right) \,, & \text{(D2)} \end{cases} \tag{4.86}$$

$$v_1 = v_2 = 0\,, \qquad v_3 = \begin{cases} 0\,, & \text{(D1)} \\ \dfrac{2\, e^{\phi_\infty} \ell_\infty^{1/2} \xi_2}{g\, k_\infty^{2/3} f_\infty}\,, & \text{(D2)} \end{cases} \tag{4.87}$$

$$\tan^2 \gamma_x = \frac{\ell_\infty f_\infty^2}{1 - \ell_\infty f_\infty^2}\,, \qquad \forall x\,. \tag{4.88}$$

Let us now rewrite the solution replacing the integration constants $a_0, a_-, a_M, \tilde{a}_+, a_H$ by the physical parameters $e^{\phi_\infty}, k_\infty, \ell_\infty$ and f_∞. We get[24]

$$Z_0 = \frac{e^{\phi_\infty} k_\infty^{-2/3}}{3\, f_\infty} \left(1 + \frac{q_0}{r} + \frac{2\, e^{-\phi_\infty} k_\infty^{2/3} \ell_\infty}{3 g^2} F\left(r; q_H, \lambda^{-2}\right)\right), \tag{4.89}$$

$$Z_- = \frac{2\, e^{-\phi_\infty} k_\infty^{-2/3}}{3\, f_\infty} \left(1 + \frac{q_-}{r}\right), \tag{4.90}$$

$$\tilde{Z}_+ = \begin{cases} \dfrac{k_\infty^{4/3}}{3\, f_\infty} \left(1 + \dfrac{\tilde{q}_+}{r} + \dfrac{18\, e^{\phi_\infty} f_\infty^2 \xi_1^2}{k_\infty^{2/3} g^2} F\left(r; q_-, \lambda^{-2}\right)\right), & \text{(D1)} \\ \dfrac{k_\infty^{4/3}}{3\, f_\infty} \left(1 + \dfrac{\tilde{q}_+}{r} + \dfrac{9\, e^{\phi_\infty} f_\infty^2 \xi_2^2}{2\, k_\infty^{2/3} g^2} \dfrac{(r+q_-)\left(1+2\lambda^2 r\right) + q_- \lambda^4 r^2}{(r+q_-)\left(1+\lambda^2 r\right)^2}\right), & \text{(D2)} \end{cases} \tag{4.91}$$

$$H = \frac{f_\infty}{\ell_\infty} \left(1 + \frac{q_H}{r}\right), \tag{4.92}$$

$$\omega_5 = \begin{cases} \pm \dfrac{\ell_\infty}{f_\infty} \sqrt{\dfrac{1 - \ell_\infty f_\infty^2}{\ell_\infty f_\infty^2}} \left(1 + \dfrac{q_H}{r}\right) + \dfrac{\sqrt{3} \ell_\infty \xi_1}{2\, g^2\, f_\infty\, q_H r} - \dfrac{2\sqrt{3}\, \ell_\infty\, \xi_1}{g^2\, f_\infty} F\left(r; q_H, \lambda^{-2}\right), & \text{(D1)} \\ \pm \dfrac{\ell_\infty}{f_\infty} \sqrt{\dfrac{1 - \ell_\infty f_\infty^2}{\ell_\infty f_\infty^2}} \left(1 + \dfrac{q_H}{r}\right) - \dfrac{\sqrt{3}\, \ell_\infty\, \xi_2}{2\, g^2\, f_\infty} \dfrac{\lambda^2 r \cos\theta}{(r+q_H)\left(1+\lambda^2 r\right)^2}\,. & \text{(D2)} \end{cases} \tag{4.93}$$

At this point, the solutions depend on

$$e^{\phi_\infty}, k_\infty, \ell_\infty, f_\infty, g, q_0, \tilde{q}_+, q_-, q_H, \lambda, \xi_1, \text{or } \xi_2\,. \tag{4.94}$$

The first 5 of these and ξ_2 are moduli (asymptotic values of scalar fields and gauge coupling constant). The 4 qs will be interpreted as near-horizon charges and we still have to find the physical meaning of λ and ξ_1. Let us study the charges of these solutions.

Charges of the solutions. The black hole solutions that we have constructed are electrically charged with respect to the 4 non-trivial Abelian vectors $A^0_{(4)}, A^1_{(4)}, A^\pm_{(4)}$ in eqs. (3.55)–(3.57) and magnetically charged only with respect to the KK vector $A^0_{(4)}$. Therefore, these

[24] The $a_M = 0$ case can be smoothly recovered in the $f_\infty \to 1/\sqrt{\ell_\infty}$ limit.

dyonic black holes have 5 conserved (Abelian) charges: 4 electric and 1 magnetic. Not all these charges are independent, though, as a consequence of the zero-NUT-charge condition, but this is something that depends on the definition of charges being used. For the near-horizon charges, eq. (4.80) leaves only 4 independent charges in the D2 case, since $q_M = q_H$. In the D1 case, however, the near-horizon charge associated to the function ω_5 is given by $q_H + \sqrt{\frac{\ell_\infty f_\infty}{1-\ell_\infty f_\infty^2}\frac{\sqrt{3}\xi_1}{2 q_H g^2}}$ and therefore it is not fixed since ξ_1 is a free parameter. This strongly suggests that the quantity $\xi_1/\left(q_H g^2\right)$ could be interpreted as the electric charge of the non-Abelian dyon.

We can now compute the asymptotic charges $q_0^\infty, \tilde{q}_+^\infty, q_-^\infty, q_H^\infty, q_M^\infty$, defined as

$$q_0^\infty = \lim_{r\to\infty} r\left(\frac{Z_0}{a_0} - 1\right) = q_0 + \frac{e^{-\phi_\infty} k_\infty^{2/3} \ell_\infty}{6\, g^2\, q_H}\,, \tag{4.95}$$

$$q_-^\infty = \lim_{r\to\infty} r\left(\frac{Z_-}{a_-} - 1\right) = q_-\,, \tag{4.96}$$

$$\tilde{q}_+^\infty = \lim_{r\to\infty} r\left(\frac{\tilde{Z}_+}{\tilde{a}_+} - 1\right) = \begin{cases} \tilde{q}_+ + \dfrac{9\, e^{\phi_\infty} k_\infty^{-2/3}\, \xi_1^2}{2\, g^2\, f_\infty^2\, q_-}\,, & \text{(D1)} \\[2ex] \tilde{q}_+ + \dfrac{9\, e^{\phi_\infty} k_\infty^{-2/3}\, \xi_2^2}{2\, g^2\, f_\infty^2}\left(q_- + 2\lambda^{-2}\right)\,, & \text{(D2)} \end{cases} \tag{4.97}$$

$$q_H^\infty = \lim_{r\to\infty} r\left(\frac{H}{a_H} - 1\right) = q_H\,, \tag{4.98}$$

$$q_M^\infty = \lim_{r\to\infty} r\left(\frac{\omega_5}{a_M} - 1\right) = q_H^\infty = q_H\,. \tag{4.99}$$

In both the D1 and D2 cases, we find only 4 independent charges as a consequence of eq. (4.80).

Mass and angular momentum. Comparing the asymptotic expansion of the metric eq. (4.68) with

$$ds^2_{(4)} \sim \left(1 - \frac{2G_N^{(4)}\mathcal{M}}{r}\right) dt^2 + \frac{4\mathcal{J}\sin^2\theta}{r} dt d\phi - \left(1 - \frac{2G_N^{(4)}\mathcal{M}}{r}\right) dr^2 - r^2 d\Omega^2_{(2)}\,, \tag{4.100}$$

we find that the ADM mass $\mathcal{M}$ and the angular momentum $\mathcal{J}$ of the solution are given by

$$\mathcal{M} = \frac{q_0^\infty + q_+^\infty + q_- + q_H - 4\left(1 - \ell_\infty f_\infty^2\right) q_H}{4\,\ell_\infty f_\infty^2\, G_N^{(4)}}\,, \tag{4.101}$$

$$\mathcal{J} = \begin{cases} 0\,, & \text{(D1)} \\[1ex] -\dfrac{\sqrt{3}\,\xi_2}{4\, g^2 \lambda^2}\,. & \text{(D2)} \end{cases} \tag{4.102}$$

In the D2 case we can use this last equation to fix the instanton size λ in terms of angular momentum and the moduli of the solution:

$$\lambda^{-2} = -\frac{4g^2}{\sqrt{3}\xi_2}\mathcal{J}\,. \qquad \text{(D2)} \tag{4.103}$$

Properties of the solutions

- The D1 solution is an asymptotically-flat, static solution characterized by the independent physical parameters

$$e^{\phi_\infty}, k_\infty, \ell_\infty, f_\infty, g, q_0, \tilde{q}_+, q_-, q_H, \xi_1, \lambda\,. \tag{4.104}$$

  The first 5 are moduli and the next 5 can be interpreted as near-horizon charges. The parameter λ characterizing the instanton size can be interpreted as non-Abelian hair.

- The D2 solution is an asymptotically-flat, rotating solution characterized by the independent physical parameters

$$e^{\phi_\infty}, k_\infty, \ell_\infty, f_\infty, g, \xi_2, q_0, \tilde{q}_+, q_-, q_H, \mathcal{J}\,. \tag{4.105}$$

  The first 6 are the moduli of the solution and the next 4 are the near-horizon charges. Finally, $\mathcal{J}$ is the angular momentum. As already discussed, in this solution the instanton size λ gets fixed in terms of $\mathcal{J}$ and some of the moduli by eq. (4.103).

- Both solutions have a spherical horizon at $r = 0$ with area

$$A_{\mathrm{H}} = 4\pi\sqrt{q_0\, q_+\, q_-\, q_H - (q_M\, q_H)^2}\,, \tag{4.106}$$

  as long as this quantity is real and finite, i.e., if $q_0\, q_+\, q_-\, q_H > (q_M\, q_H)^2$. The angular momentum of the D2 solution does not modify the shape of the horizon because $\lim_{r\to 0}\breve{\omega} \to 0$, contrary to what we found in the 5-dimensional case.

- If, on top of having a regular horizon, the condition

$$\frac{27}{2} Z_0 \tilde{Z}_+ Z_- H > (\omega_5 H)^2 \quad \text{if} \quad r \geq 0\,, \tag{4.107}$$

  is satisfied everywhere,[25] so that the metric function $e^{-2U} \neq 0$, these solutions will describe, respectively, a static (D1) and a rotating (D2) asymptotically-flat black hole.

5 Discussion

5.1 Summary of the results

In this paper we have presented a general class of supersymmetric solutions of 4- and 5-dimensional SU(2)-gauged supergravities with 8 supercharges (that is: $\mathcal{N} = 1, d = 5$ and $\mathcal{N} = 2, d = 4$ supergravities) using the techniques developed in refs. [7, 14, 32].

The novel aspect of our solutions is the addition of delocalized sources of charge through non-trivial dyonic Yang-Mills fields. As we have seen, they play a fundamental role in our

[25] We have checked numerically that it can be satisfied.

analysis since the angular momentum of the solutions (specially in 4 dimensions) is related to the electric-type charges introduced by these fields.

In 5 dimensions, we have found two non-Abelian generalizations of the well-known BMPV black hole. One of these (D2) has two independent angular momenta, which is a feature that has not been observed in the literature before for supersymmetric asymptotically-flat black holes.

We have also constructed 4-dimensional asymptotically-flat black holes which can be seen as the non-Abelian counterparts of the heterotic black holes studied in ref. [46]. One of these has in fact a non-vanishing angular momentum and it is regular, being this the first example (up to our knowledge) of this type. Actually, a "no-go" theorem had been proven in ref. [31] in the context of ungauged $\mathcal{N} = 2, d = 4$ supergravity. Indeed, these theories have Abelian vector fields only, and, with a single center, dyonic fields do not give rise to angular momentum and any other sources of angular momentum give rise to singularities. The rotating 4-dimensional solution that we have constructed overcomes these problems because of the delocalized nature of the dyonic non-Abelian fields, which do give rise to angular momentum.

5.2 Future directions

Since the theories that we have considered here can be obtained from toroidal compactification of the 10-dimensional Heterotic supergravity, these solutions can be easily uplifted to 10 dimensions as it has been recently done in ref. [25]. Heterotic supergravity, however, does not capture the complete set of first-order α'-corrections, since it is well-known that the effective action of the Heterotic Superstring [27] also contains terms which are quadratic in the curvature of the torsionful spin connection. In some cases, it can be argued that the corrections introduced by those terms are small enough to be ignored, but, as shown in refs. [2, 4], sometimes it is possible to compute them exactly (to that order, at least) if the results of ref. [3] can be applied to the particular supergravity solution under consideration.

Thus, it is natural to consider the embedding in Heterotic Superstring theory of these solutions and the α' corrections of these solutions. Work in these directions is in progress [47].

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