Boundary theories for dilaton supergravity in 2D

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ABSTRACT: The $\text{osp}(2, \mathbb{N})$ -BF formulation of dilaton supergravity in two dimensions is considered. We introduce a consistent class of asymptotic conditions preserved by the extended superreparametrization group of the thermal circle at infinity. In the $N = 1$ and $N = 2$ cases the phase space foliation in terms of orbits of the super-Virasoro group allows to formulate suitable integrability conditions for the boundary terms that render the variational principle well-defined. Once regularity conditions are imposed, requiring trivial holonomy around the contractible cycle the asymptotic symmetries are broken to some subsets of exact isometries. Different coadjoint orbits of the asymptotic symmetry group yield different types of boundary dynamics; we find that the action principle can be reduced to either the extended super-Schwarzian theory, consistent with the dynamics of a non-vanishing Casimir function, or to superparticle models, compatible with bulk configurations whose Casimir is zero. These results are generalized to $\mathcal{N} \geq 3$ by making use of boundary conditions consistent with the loop group of $\text{OSp}(2, \mathbb{N})$. Appropriate integrability conditions permit to reduce the dynamics of dilaton supergravity to a particle moving on the $\text{OSp}(2, \mathbb{N})$ group manifold. Generalizations of the boundary dynamics for $\mathcal{N} > 2$ are obtained once bulk geometries are supplemented with super- AdS_2 asymptotics.

KEYWORDS: 2D Gravity, Extended Supersymmetry, Field Theories in Lower Dimensions

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1 Introduction

Gauge symmetry is generically linked to the redundancy in the variables used to describe a field theory. This association is nevertheless not entirely true in theories with boundaries: part of the gauge degrees of freedom becomes dynamical on these boundaries. The working example of this realization is the fact that three-dimensional Chern-Simons theories in presence of a boundary can be expressed as a WZW model in two dimensions [1, 2]. Since then, this relation has been extended to topological theories of gravity [3–9] and more recently to theories with local degrees of freedom [10–13]. Deepening further this program can help to understand the role played by boundary modes in the context of quantum gravity. From a more general perspective, this can shed light on the origin of the holographic behavior of gravity [14, 15].

Here we focus on the study of boundary dynamics associated with two-dimensional BF theories based on the gauge group $\mathrm{OSp}(2, \mathbb{N})$. These lagrangians are supersymmetric extensions of the so-called Jackiw-Teitelboim (JT) model [16, 17]. The motivation for this is twofold. On the one hand, the addition of local supersymmetry is a natural way to generalize the results of [18], where the asymptotic dynamics was analyzed for (bosonic) higher spin extensions of the JT theory. On the other hand, it is known that supersymmetric BF models provide gravitational duals for supersymmetric extensions of the SYK model in the low temperature regime [19–22]. In this sense, the main purpose of this paper is to explore the asymptotic dynamics of two-dimensional dilaton supergravity and study its connection with models placed at the conformal boundary of the spacetime; namely the super-Schwarzian theory and superconformal quantum mechanics [23–25]. This analysis yields a consistent setup that extends and complements previous analyses in the $N = 1$ case [26] and more recent ones in the second order formulation of dilaton supergravity [27, 28] for $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetries.

Working with BF models allows to introduce a generalized notion of Euclidean black holes: they can be regarded as gauge fields with trivial holonomies along the thermal cycle. In two dimensions, this condition adds restrictions on the set of configurations with given asymptotics. Specifically, restricting the phase space to globally well-defined black holes produces an explicit breaking of the asymptotic symmetry group. Notably, this effect has been observed in dual descriptions controlled by the Schwarzian theory [8, 29, 30].

In this work, we propose two different sets of boundary conditions with fluctuating dilaton multiplets. The first one reproduces the $SO(N)$ -extended superreparametrization group around the thermal circle at infinity, $SDiff(S^1)$. The second one is preserved by the loop group of $OSp(2, \mathbb{N})$. For the former set of asymptotic conditions, the reduced action principle can be determined by two different boundary models related to specific coadjoint orbits of $SDiff(S^1)$. Concretely, we show that the asymptotic dynamics described by the extended super-Schwarzian theory is consistent with a nonvanishing Casimir function while the dynamics compatible with zero Casimir entails the existence of a supersymmetric particle model at the boundary. In both cases, however, the phase space is written in terms of orbits of the super-Virasoro group, which is only accessible for $N = 1$ and $N = 2$. This is due to quadratic contributions of affine $\mathfrak{so}(N)$ currents in the infinitesimal transformation laws $N \geq 3$.¹ In order to access the asymptotic dynamics for $N \geq 3$, we make use of the second set of boundary conditions, consistent with the loop group of $OSp(2, \mathbb{N})$, which allows to reformulate the problem in terms of a particle moving on the group manifold. Further constraints on the latter reduced model are introduced when requiring to have asymptotically super- AdS_2 solutions in the bulk. This procedure gives rise to supersymmetric extensions of particle models without self-interacting potentials and extended super-Schwarzian theories with $\mathcal{N} > 2$.

The paper is organized as follows. In the next section, we present the BF formulation of minimal dilaton supergravity in two dimensions with gauge group $OSp(2, 1)$. In section 3, a consistent set of asymptotic conditions is proposed together with a superspace formulation that describes in a super-covariant way the action of the asymptotic symmetries on the fields. The latter approach turns out to be particularly useful in order to foliate the phase space of the theory in terms of the adjoint and coadjoint actions of the super-Virasoro group. Using this prescription, we demand suitable boundary conditions that render the variational principle well-defined. This is discussed in section 4. Section 5 is devoted to analyze regularity conditions on solutions describing Euclidean black holes. It is shown that configurations with trivial holonomies are solutions of the supersymmetric Hill equation, which must obey (anti)periodic boundary conditions. In section 6, the reduced action is obtained by evaluating the variational principle on the constraint surface, which for our set of asymptotic conditions makes the bulk contribution vanish. Therefore, the regularized action is only determined by a boundary term that leads — after restricting to regular solutions — to either the $\mathcal{N} = 1$ super-Schwarzian theory, or to a supersymmetric particle model. In section 7, we present asymptotic conditions extending the previous results but mainly focusing on the $\mathcal{N} = 2$ case, arriving at analogous conclusions. Section 8 deals with the boundary dynamics of $\mathcal{N} > 2$ extended dilaton supergravity, where supersymmetric extensions of conformal mechanics and Schwarzian theories are found. In section 9 we go beyond the highest-weight ansatz to consider more general supersymmetric asymptotically AdS_2 boundary conditions. Finally, section 10 is dedicated to discuss possible applications and extensions of this work. Conventions associated to the explicit representation of $\mathfrak{osp}(2, \mathbb{N})$ generators can be found in appendix A.

¹This kind of non-linearities also appears in the case of extended AdS supergravity in 3D [31].

Before starting we mention that we work in Euclidean signature throughout the whole paper. Thus, when speaking about “time” we always mean “Euclidean time”.

2 $\mathcal{N} = 1$ dilaton supergravity in two dimensions

The first order action for the minimal supersymmetric extension of the JT model [32, 33] reads²

$$I = \frac{k}{4\pi} \int_{\mathcal{M}} \left[X^a \left(de_a + \epsilon_{ab} \omega e^b - \frac{1}{4} \bar{\psi} \gamma_a \psi \right) + X d\omega + \frac{1}{2} X \left(\epsilon_{ab} e^a e^b + \frac{1}{2} \bar{\psi} \gamma_5 \psi \right) + \bar{\lambda} D\psi + \frac{1}{2} \bar{\lambda} e^a \gamma_a \psi \right] \quad (2.1)$$

where the manifold $\mathcal{M}$ has the topology of a disk, endowed with a time τ identified as $\tau \sim \tau + \beta$ and a non-compact direction $r > 0$.

The field content of this theory is the following: X^a is a pair of Lagrange multipliers enforcing the supercovariant torsion constraint, X stands for the dilaton field, λ^α is the dilatino field, e^a corresponds to the one-form zweibein, ω is the dualized spin connection and ψ^α is the gravitino one-form. The covariant derivative acting on the gravitino is defined as

$$D\psi = d\psi + \frac{1}{2} \omega \gamma_5 \psi. \quad (2.2)$$

The indices are contracted with the flat metric δ_{ab} and $\epsilon_{01} = -1$, where $a, b = 0, 1$. The Majorana conjugate is defined as $\bar{\psi}_\alpha = \psi^\beta C_{\beta\alpha}$, where $C_{\alpha\beta} = \epsilon_{\alpha\beta}$ ($\epsilon_{+-} = -1$) is the charge conjugation matrix. The matrices γ_a satisfy the two-dimensional Clifford algebra, $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$, and $\gamma_5 = -\gamma_0\gamma_1$, then $\gamma_5^2 = -\mathbb{I}_2$.

It is possible to show that action (2.1), apart from being invariant under local Lorentz transformations and conformal boosts, is invariant under the following local supersymmetry transformations

$$\delta e^a = -\frac{1}{2} \bar{\psi} \gamma^a \epsilon \quad \delta \omega = \frac{1}{2} \bar{\psi} \gamma_5 \epsilon \quad \delta \psi = D\epsilon + \frac{1}{2} e^a \gamma_a \epsilon \quad (2.3)$$

$$\delta X^a = -\frac{1}{2} \bar{\lambda} \gamma^a \epsilon \quad \delta X = -\frac{1}{2} \bar{\lambda} \gamma_5 \epsilon \quad \delta \lambda = \frac{1}{2} X^a \gamma_a \epsilon - \frac{1}{2} X \gamma_5 \epsilon. \quad (2.4)$$

Minimal dilaton supergravity (2.1) does not propagate local degrees of freedom. This property is made manifest by writing the action in terms of a BF-theory for $\text{OSp}(2, 1)$ [34]. Its bulk action reads

$$I = \frac{k}{2\pi} \int_{\mathcal{M}} \text{Str} [\mathbf{X} (d\mathbf{A} + \mathbf{A}^2)] \quad (2.5)$$

where the zero-form dilaton multiplet $\mathbf{X} = X^I J_I + \lambda^\alpha Q_\alpha$ and the one-form connection $\mathbf{A} = A^I J_I + \psi^\alpha Q_\alpha$, are valued in the $\mathfrak{osp}(2, 1)$ algebra

$$[J_I, J_J] = \epsilon_{IJK} J^K \quad [J_I, Q_\alpha] = \frac{1}{2} (\Gamma_I)^\beta_\alpha Q_\beta \quad \{Q_\alpha, Q_\beta\} = -\frac{1}{2} (C\Gamma^I)_{\alpha\beta} J_I. \quad (2.6)$$

²The form of the action implies that we have adopted the convention $(\psi_1 \psi_2)^* = \psi_1 \psi_2$ for the complex conjugate of the product of two real Grassmann variables.

Here, the Γ -matrices $\Gamma_I = (\gamma_a, -\gamma_5)$, satisfy the Clifford algebra in three dimensions, $\{\Gamma_I, \Gamma_J\} = 2\eta_{IJ}$, where the metric is given by $\eta_{IJ} = \text{diag}(1, 1, -1)$ and the orientation is chosen as $\epsilon_{012} = -1$. The components that go along the $\mathfrak{so}(2, 1)$ generators collect the bosonic content of the model, such that $X^I = (X^a, X)$ and $A_I = (e_a, \omega)$. The nonvanishing components of the supertrace are³

$$\text{Str}[J_I J_J] = \frac{1}{2}\eta_{IJ} \quad \text{Str}[Q_\alpha, Q_\beta] = \frac{1}{2}C_{\alpha\beta}. \quad (2.7)$$

Let us review some of the features associated with the action (2.5). The field equations for the BF-theory generically read as follows

$$\mathbf{F} = 0 \quad d\mathbf{X} + [\mathbf{A}, \mathbf{X}] = 0. \quad (2.8)$$

In the case we are dealing with, the field strength $\mathbf{F} = d\mathbf{A} + \mathbf{A}^2$ associated to $\mathbf{A}$, is explicitly given by

$$\mathbf{F} = \left(R^I - \frac{1}{4}\bar{\psi}\Gamma^I\psi \right) J_I + \nabla\psi^\alpha Q_\alpha \quad (2.9)$$

where the $\mathfrak{so}(2, 1)$ curvature two-form R^I and the covariant derivative ∇ are, respectively,

$$R^I = dA^I + \frac{1}{2}\epsilon^{IJK}A_J A_K \quad \nabla\psi = d\psi + \frac{1}{2}A^I\Gamma_I\psi. \quad (2.10)$$

A crucial feature of formulating the theory as (2.5) is that the gauge invariance of the model becomes manifest. Indeed, it is straightforward to prove that under the action of the gauge group, $\mathbf{A}$ and $\mathbf{X}$ transform in the adjoint and coadjoint representations, respectively, which leaves (2.5) unchanged. The form of the infinitesimal gauge transformations, generated by a Lie-algebra-parameter Λ , then read

$$\delta_\Lambda \mathbf{A} = d\Lambda + [\mathbf{A}, \Lambda] \quad \delta_\Lambda \mathbf{X} = [\mathbf{X}, \Lambda]. \quad (2.11)$$

From the transformation rule of the gauge field in (2.11), it is possible to conclude that the dilaton equation of motion can be written as $\delta_\mathbf{X}\mathbf{A}|_{\text{on-shell}} = 0$. Therefore, the on-shell value of the dilaton $\mathbf{X}$ turns out to be the stabilizer of the gauge field $\mathbf{A}$.

It is important to mention that diffeomorphisms are a distinctive class of gauge symmetries. These are spanned by $\Lambda = \xi^\sigma \mathbf{A}_\sigma$, whose infinitesimal group action (2.11) is expressed as Lie derivatives acting on $\mathbf{A}_\mu$ and $\mathbf{X}$, up to transformations of the form

$$\delta_{\text{trivial}}\mathbf{A}_\mu = \chi_\mu \frac{\delta I}{\delta \mathbf{X}} \quad \delta_{\text{trivial}}\mathbf{X} = -\chi_\mu \frac{\delta I}{\delta \mathbf{A}_\mu} \quad (2.12)$$

where $\chi_\mu = \frac{2\pi}{k}\xi^\sigma \epsilon_{\sigma\mu}$. The latter type of transformations are indeed trivial symmetries of the action, no matter what parameter χ_μ is being used (see, e.g., [35]).

It is reassuring to verify that gauge transformations (2.11) generated by $\Lambda = \epsilon^\alpha Q_\alpha$,

$$\delta_\epsilon A^I = -\frac{1}{2}\bar{\psi}\Gamma^I\epsilon \quad \delta_\epsilon X^I = -\frac{1}{2}\bar{\lambda}\Gamma^I\epsilon \quad \delta_\epsilon\psi = \nabla\epsilon \quad \delta_\epsilon\lambda = \frac{1}{2}X^I\Gamma_I\epsilon \quad (2.13)$$

lead to the ones given in (2.3) and (2.4).

³Explicit matrix representations for the $\mathfrak{osp}(2, \mathbb{N})$ generators are provided in appendix A.

Finally, we note that BF models admit the definition of a conserved quantity called (super) Casimir function, which is defined as

$$\mathcal{C} \equiv -\frac{1}{2} \text{Str}(\mathbf{X}^2). \quad (2.14)$$

This is a gauge invariant object that satisfies the conservation equation $d\mathcal{C} = 0$ by virtue of the dilaton equation of motion $\delta_{\mathbf{X}} \mathbf{A}|_{\text{on-shell}} = 0$.

3 Asymptotic structure

We are interested in studying the set of superdiffeomorphisms preserving the form of the fall-off of the fields at infinity, which corresponds to a subset of gauge transformations modulo trivial terms proportional to the equations of motion (2.12). More precisely, we proceed to find the symmetries of a class of configurations that asymptotically satisfies the conditions

$$\delta_{\Lambda} \mathbf{A}_{\mu} = \mathcal{O}(\mathbf{A}_{\mu}) \quad \delta_{\Lambda} \mathbf{X} = \mathcal{O}(\mathbf{X}) \quad (3.1)$$

where Λ is the Lie-superalgebra-parameter that generates the infinitesimal transformations associated to the asymptotic symmetries.

3.1 Fall-off conditions

Here, we present a set of boundary conditions for the theory, that besides of containing all the bosonic solutions of interest with non-trivial dilaton [36–38], turns out to be preserved by $\text{SDiff}(S^1)$ at infinity. The fall-off of the fields $\mathbf{A}$ and $\mathbf{X}$, written in the Coussaert-Henneaux-van Driel gauge [4], reads as follows

$$\mathbf{A}[\mathcal{L}, \psi] = b^{-1}(d+a)b + \mathcal{O}(r^{-2}) \quad \mathbf{X}[y, \rho] = b^{-1}xb + \mathcal{O}(r^{-2}) \quad (3.2)$$

where $b = \exp[\log(r)L_0]$. The radially independent gauge field a reads

$$a = \left[L_1 + \mathcal{L}L_{-1} + \psi G_{-\frac{1}{2}} \right] d\tau \quad (3.3)$$

and the dilaton x is given by

$$x = yL_1 - y'L_0 + \left(\frac{1}{2}y'' + y\mathcal{L} + \psi\rho \right) L_{-1} + \rho G_{\frac{1}{2}} - (\rho' - y\psi) G_{-\frac{1}{2}}. \quad (3.4)$$

We have expressed $\mathfrak{osp}(2, 1)$ in terms of the basis $\{L_{-1}, L_0, L_1, G_{-\frac{1}{2}}, G_{\frac{1}{2}}\}$,⁴ such that

$$[L_m, L_n] = (m - n) L_{m+n} \quad [L_m, G_p] = \left(\frac{m}{2} - p \right) G_{m+p} \quad \{G_p, G_q\} = -2L_{p+q} \quad (3.5)$$

with $m, n = \pm 1, 0$ and $p, q = \pm \frac{1}{2}$. The dynamical fields $\mathcal{L}$ and y are arbitrary functions of time τ , while ψ and ρ correspond to arbitrary Grassmann-valued functions of τ .

⁴For the change of basis see appendix A.

The gauge parameter that maintains the asymptotic form of the configurations (3.3), (3.4) is given by

$$\Lambda = \mathbf{X}[\xi, \epsilon, \mathcal{L}, \psi] \quad (3.6)$$

where ξ and ϵ represent superdiffeomorphisms in the thermal circle at infinity. Under these asymptotic symmetries, the fields transform as

$$\delta_{(\xi, \epsilon)} \mathcal{L} = 2\mathcal{L}\xi' + \mathcal{L}'\xi + \frac{1}{2}\xi''' + 3\psi\epsilon' + \psi'\epsilon \quad (3.7)$$

$$\delta_{(\xi, \epsilon)} \psi = \frac{3}{2}\psi\xi' + \psi'\xi - \mathcal{L}\epsilon - \epsilon'' \quad (3.8)$$

$$\delta_{(\xi, \epsilon)} y = \xi y' - \xi' y + 2\epsilon\rho \quad (3.9)$$

$$\delta_{(\xi, \epsilon)} \rho = \frac{1}{2}\epsilon y' - \epsilon' y - \frac{1}{2}\xi'\rho + \xi\rho'. \quad (3.10)$$

Note that equations (3.7) and (3.8) correspond to infinitesimal transformations of a super-Virasoro coadjoint orbit with representatives $\mathcal{L}$ and ψ , while (3.9) and (3.10) reflect that y and ρ transform in the adjoint representation of the super-Virasoro group [39–41].

It is worth mentioning that the asymptotic form of $\mathbf{X}$, given in (3.4), allows to incorporate fluctuating dilaton and dilatino without spoiling the superreparametrization group at infinity. This is because (3.4) was chosen to have the same form of the gauge parameter (3.6) that in turn preserves the asymptotic behaviour of the gauge connection (3.3). This fact is reminiscent of the procedure introduced in [42], that in the context of three-dimensional higher spin gravity permits to include a generic choice of chemical potentials without breaking the W -symmetry at infinity.

3.2 Asymptotic field equations

The field equations for the gauge field are asymptotically satisfied for the set of asymptotic conditions (3.2), i.e., the strength field associated to the gauge field (3.3) vanishes at infinity

$$\mathbf{F} = \mathcal{O}(r^{-2}). \quad (3.11)$$

The remaining equations of motion dictate that the dilaton and dilatino acquire asymptotically the role of stabilizers of the dynamical fields $\mathcal{L}$ and ψ , namely

$$\delta_{(y, \rho)} \mathcal{L} = 0 \quad \delta_{(y, \rho)} \psi = 0. \quad (3.12)$$

By virtue of these two equations of motion, the *off-shell* Casimir function (2.14)

$$\mathcal{C} = \mathcal{L}y^2 + \frac{1}{2}yy'' - \frac{1}{4}(y')^2 + 3y\psi\rho + 2\rho\rho' + \mathcal{O}(r^{-1}) \quad (3.13)$$

defines an asymptotically conserved quantity, i.e., $\partial_\tau \mathcal{C} = \mathcal{O}(r^{-1})$.

3.3 Superspace formulation

For later purposes we will describe the realization of the asymptotic symmetry group on the dynamical variables in the superspace formalism (see, e.g., [43] and references therein).

Superspace is defined by a pair of coordinates (τ, θ) where θ denotes a Grassmann variable. Superdiffeomorphisms correspond to coordinate transformations $\tilde{\theta} = \tilde{\theta}(\tau, \theta)$ and $\tilde{\tau} = \tilde{\tau}(\tau, \theta)$ that preserve the constraint

$$D\tilde{\tau} = \tilde{\theta}D\tilde{\theta} \quad (3.14)$$

where the supercovariant derivative $D \equiv \partial_\theta + \theta\partial_\tau$ must transform as $D = D\tilde{\theta}\tilde{D}$. A parametrization that infinitesimally solves restriction (3.14) is given by

$$\tilde{\tau} = \tau - \xi(\tau) + \theta\epsilon(\tau) \quad \tilde{\theta} = \theta + \epsilon(\tau) - \frac{1}{2}\theta\xi'(\tau). \quad (3.15)$$

Let us translate the field content of the asymptotic conditions (3.2) into superfields. Quantities $\mathcal{L}$ and ψ can be combined into a single superfield Ψ defined as

$$\Psi(\tau, \theta) \equiv \psi(\tau) + \theta\mathcal{L}(\tau), \quad (3.16)$$

which has conformal weight $\frac{3}{2}$. The finite transformation of Ψ under superdiffeomorphism $\tilde{\theta}$ is given by

$$\tilde{\Psi}(\tilde{\tau}, \tilde{\theta}) = (D\tilde{\theta})^{-3} \left[\Psi(\tau, \theta) - S(\tilde{\tau}, \tilde{\theta}; \tau, \theta) \right] \quad (3.17)$$

with $S(\tilde{\tau}, \tilde{\theta}; \tau, \theta)$ the super-Schwarzian [44]

$$S(\tilde{\tau}, \tilde{\theta}; \tau, \theta) = \frac{D^4\tilde{\theta}}{D\tilde{\theta}} - \frac{2D^2\tilde{\theta}D^3\tilde{\theta}}{(D\tilde{\theta})^2}. \quad (3.18)$$

Expression (3.17) defines the coadjoint action of the super-Virasoro group that reproduces its associativity. Indeed, by applying the supertransformation $\tilde{\theta}(\tau, \theta)$ with $\tau = \tau(\hat{\tau}, \hat{\theta})$ and $\theta = \theta(\hat{\tau}, \hat{\theta})$ on $\tilde{\Psi}$, we recover the form of (3.17) since the super-Schwarzian (3.18) satisfies the following cocycle condition

$$S(\tilde{\tau}, \tilde{\theta}; \hat{\tau}, \hat{\theta}) = (\hat{D}\tilde{\theta})^3 S(\tilde{\tau}, \tilde{\theta}; \tau, \theta) + S(\tau, \theta; \hat{\tau}, \hat{\theta}). \quad (3.19)$$

Associativity of the group translates into closure of the algebra for linearized superdiffeomorphisms. Defining $\delta\Psi$ as the difference between of $\tilde{\Psi}$ and Ψ at the same point (τ, θ) and using (3.15) on (3.17) yields

$$\delta_\chi\Psi = \frac{3}{2}D^2\chi\Psi + \frac{1}{2}D\chi D\Psi + \chi D^2\Psi + \frac{1}{2}D^5\chi \quad (3.20)$$

where $\chi(\tau, \theta) = \xi(\tau) - 2\theta\epsilon(\tau)$, which is in agreement with the infinitesimal asymptotic symmetry transformations (3.7) and (3.8).

The dilaton multiplet is characterized by the fields y and ρ . In terms of superfields, this can be organized into the following supervector

$$Y(\tau, \theta) \equiv y(\tau) - 2\theta\rho(\tau). \quad (3.21)$$

Specifically, Y corresponds to an adjoint vector of the super-Virasoro group, then

$$\tilde{Y}(\tilde{\tau}, \tilde{\theta}) = (D\tilde{\theta})^2 Y(\tau, \theta). \quad (3.22)$$

The infinitesimal action of the group on Y , $\delta Y = \tilde{Y}(\tau, \theta) - Y(\tau, \theta)$, is then given by

$$\delta_\chi Y = -D^2 \chi Y + \frac{1}{2} D \chi D Y + \chi D^2 Y \quad (3.23)$$

which reduces to the transformations (3.9) and (3.10) obtained from the asymptotic analysis.

4 Variational principle

A well-defined action principle for dilaton supergravity is given by the following functional

$$I = \frac{k}{2\pi} \int \text{Str} [\mathbf{X} (d\mathbf{A} + \mathbf{A}^2)] + I_B. \quad (4.1)$$

In order to ensure that the variational principle attains an extremum, the action (2.5) is supplemented with a surface term I_B , that evaluated at $r \rightarrow \infty$ becomes

$$\delta I_B = -\frac{k}{2\pi} \oint d\tau \text{Str} [\mathbf{X} \delta \mathbf{A}_\tau] \quad (4.2)$$

where $\mathbf{A}_\tau$ stands for the time component of the one-form $\mathbf{A}$. The variation of the action (4.1) is then a bulk term proportional to the equations of motion, that in consequence vanishes on-shell.

4.1 Phase space of dilaton supergravity and integrability condition

The boundary term (4.2) is a priori non-integrable and then one must provide suitable integrability conditions at the boundary. Implementing the asymptotic conditions (3.3), (3.4) into the variation of I_B (4.2) yields

$$\delta I_B = \frac{k}{2\pi} \oint d\tau (y \delta \mathcal{L} - 2\rho \delta \psi) = \frac{k}{2\pi} \int d\tau d\theta Y \delta \Psi \quad (4.3)$$

where Y and Ψ are given in (3.16) and (3.21), respectively. In the above equation, we have introduced Berezin integrals defined by $\int d\theta = 0$ and $\int d\theta \theta = 1$.

In order to obtain the value of the boundary term I_B , one has to be able to integrate the expression (4.3). To accomplish this task, we need to single out a family of fields (Y, Ψ) respecting our boundary conditions. A choice that is consistent with the asymptotic symmetry group corresponds to picking a set of fields (y_0, Ψ_0) and act with a superreparametrization Θ to construct

$$Y[\Theta; y_0] = (D\Theta)^{-2} y_0 \quad \Psi[\Theta; \Psi_0] = (D\Theta)^3 \Psi_0 + S(T, \Theta; \tau, \theta) \quad (4.4)$$

where T is the coordinate defined through the relation $DT = \Theta D\Theta$. Therefore, the phase space of $\mathcal{N} = 1$ dilaton supergravity is now defined in terms of coadjoint orbits of the super-Virasoro group characterized by a representative (y_0, Ψ_0) and a superdiffeomorphism Θ . However, in order to maintain the phase space dimension, it is necessary to impose an

additional restriction among these new variables. Here, we will define Θ as the transformation that brings the dilaton superfield Y into its constant value y_0 . Furthermore, we demand as an integrability condition

$$\delta y_0 = 0 \quad (4.5)$$

meaning that y_0 is a constant without variation, which introduces a new coupling constant into the system. This condition has been used previously in the bosonic case [18, 38]. In applications to the (super) SYK model in the low temperature regime, $1/y_0$ is the variance associated with the random quartic interaction [18].

Let us further describe phase space defined by (4.4) and (4.5), by studying the equations of motion associated to it. Let us first note that

$$y_0^2 D_\Theta \Psi_0 = \mathcal{C} + \theta y \delta_{(y,\rho)} \psi \quad (4.6)$$

with $D_\Theta = \partial_\Theta + \Theta \partial_T$ and $\mathcal{C}$ is the asymptotic value of the Casimir function defined in (3.13). On-shell, the latter equation can be easily solved using that $\mathcal{C}$ is a constant of motion and that $\delta_{(y,\rho)} \psi$ vanishes, giving $\Psi_0 \approx y_0^{-2} \Theta \mathcal{C} + \psi_0$ with ψ_0 a fermionic constant.⁵

Summarizing, the superdiffeomorphism Θ is such that, when the field equations hold, it transforms the conserved quantities of the system, y_0 and $(\mathcal{C}, \psi_0)$ into the fields Y and Ψ , respectively. This is reminiscent of the Hamilton-Jacobi theory, where through a canonical transformation, the dynamics of the system is defined in terms of coordinates that are constants of motion. In the present case, the superreparametrization Θ plays the role of a canonical transformation.

4.2 Boundary term

Let us turn to the variation of I_B . Considering the family of fields (4.4) subject to the integrability condition (4.5), the variation of the boundary action (4.3) becomes

$$\delta I_B = \frac{ky_0}{2\pi} \int d\tau d\theta [\delta (D_\Theta \Psi_0) + 2D\delta\Theta \Psi_0 + (D_\Theta)^{-2} \delta S] . \quad (4.7)$$

The third term in the above expression is dismissed since it is a total super derivative that vanishes due to the periodicity conditions on the superfield. For the second term, we use the on-shell condition $\Psi_0 \approx \Theta y_0^{-2} \mathcal{C} + \psi_0$, which transforms it into

$$2\mathcal{C} \int d\tau d\theta \Theta D\delta\Theta = \mathcal{C}\delta \int d\tau d\theta \Theta D\Theta = \mathcal{C}\delta\beta . \quad (4.8)$$

For a generic constant value of the Casimir $\mathcal{C}$, the latter term vanishes by demanding that β is a state-independent function. Another way of dealing with this term is considering configurations with $\mathcal{C} = 0$, and as will be seen in section 6.2, this assumption turns out to be consistent with a boundary dynamics controlled by the action of the $\mathcal{N} = 1$ superparticle.

The first term in (4.7) defines a local expression for I_B . By *pulling out* the delta from it, the boundary term reads

$$I_B = \frac{ky_0}{2\pi} \int d\tau d\theta D_\Theta \Psi_0 . \quad (4.9)$$

⁵The expression $A \approx B$ means that A and B are equal provided the field equations are satisfied.

Having the boundary term I_B to render the action well-defined allows us to compute the value of $I = I_{\text{bulk}} + I_B$ for classical configurations. Once the equations of motion hold, I_{bulk} vanishes and the on-shell action value is fully determined by I_B , then

$$I_{\text{on-shell}} = \frac{k\beta}{2\pi y_0} \mathcal{C} \quad (4.10)$$

This expression is in agreement with the one obtained in the bosonic case [18, 38].

5 Regularity conditions: holonomy and super-Hill equation

Euclidean black holes are smooth geometries as long as the time period is identified with the inverse of the Hawking temperature. In a gauge formulation, this requires the holonomy $H_C[\mathbf{A}]$ to be trivial along a contractible cycle C . For $\mathfrak{osp}(2, 1)$, the previous condition takes the form⁶

$$H_C[\mathbf{A}] = \mathcal{P}e^{-\oint \mathbf{A}_\mu dx^\mu} = \Gamma_\pm = \begin{pmatrix} \pm \mathbb{I}_{2 \times 2} & 0_{2 \times 1} \\ 0_{1 \times 2} & 1 \end{pmatrix}. \quad (5.1)$$

Different signs in (5.1) are related to the possible spinorial representations; the center group element Γ_+ is consistent with fermions satisfying Ramond (periodic) boundary conditions, while Γ_- is associated to fermions obeying Neveu-Schwarz (anti-periodic) boundary conditions.⁷

In order to evaluate (5.1), we consider the most general solution satisfying (3.3) and (3.4). These are configurations of the form $\mathbf{A} = b^{-1}(d+a)b$, where $b = \exp[\log(r)L_0]$ and $a = a_\tau d\tau$, with

$$a_\tau = L_1 + \mathcal{L}(\tau)L_{-1} + \psi(\tau)G_{-\frac{1}{2}}. \quad (5.2)$$

Thus, in this case the holonomy condition becomes $H_C[a] = \Gamma_\pm$ and C is chosen to be the thermal circle $0 < \tau \leq \beta$. Since we can always write the radial independent connection as $a = u du^{-1}$, the holonomy is reduced to $H_C[a] = u(\beta)u(0)^{-1}$. Regularity of the searched-for solutions requires to find a group element u satisfying the boundary condition $u(\beta) = \Gamma_\pm u(0)$. To do so, let us note that u can be written in terms of two independent three-dimensional row vectors, $\varphi_{(1)}$ and $\varphi_{(2)}$, as

$$u = \begin{pmatrix} -\varphi_{(2)}' \\ \varphi_{(2)} \\ \varphi_{(1)} \end{pmatrix} \quad (5.3)$$

where $\varphi_{(1)}$ and $\varphi_{(2)}$ are independent solutions to the super-Hill equation

$$(D^3 + \Psi)\Phi = 0 \quad (5.4)$$

with $\Phi \equiv \varphi_{(2)} + \theta\varphi_{(1)}$ and Ψ given in (3.16).

When using the fact that (5.4) is covariant under the super-Virasoro group [46, 47] and that Φ respects the boundary conditions for the group element, one can always perform

⁶See [45] for an application of this condition in the context of hypersymmetric black holes in 3D.

⁷Note that Γ_- anticommutes with the fermionic generators of $\mathfrak{osp}(2, 1)$.

a superdiffeomorphism and study solutions of (5.4) for a constant superfield Ψ_* . Note that finding this superfield means that we are able to obtain solutions for a collection of densities Ψ connected to Ψ_* by a superdiffeomorphism. More precisely, Ψ belongs to a super-Virasoro coadjoint orbit defined as quotient space between $\text{SDiff}(S^1)$ modulo the transformations leaving Ψ_* fixed. We characterize this orbit as follows. The solution of (5.4) for a constant Ψ_* states that Φ is of the form $e^{\pm i\sqrt{\mathcal{L}_*}\tau}$. The boundary conditions on the group element $u(\beta) = \Gamma_{\pm} u(0)$ demand

$$\mathcal{L}_* = \frac{4\pi^2 n^2}{\beta^2} \quad (5.5)$$

where n is an arbitrary (half-)integer for (anti)periodic boundary conditions. In the Neveu-Schwarz sector the fermionic field ψ_* has to vanish and the little group associated to Ψ_* is a $2n$ -cover of $\text{OSp}(2, 1)$. Thus, in the simplest case $n = \frac{1}{2}$, the superfield Ψ stands for an element of the coadjoint orbit $\text{SDiff}(S^1)/\text{OSp}(2, 1)$.

In the Ramond sector, there is not any condition on the fermionic field ψ_* . Generically, the isotropy group of Ψ_* is $\text{U}(1)$. Then, Ψ is a representative of the orbit $\text{SDiff}(S^1)/\text{U}(1)$. For the specific case $\mathcal{L}_* = 0$ the little group becomes a graded extension of $\text{U}(1)$, while for $\psi_* = 0$, the little group corresponds to a $2n$ -cover of $\text{OSp}(2, 1)$.

In order to make contact with the Hawking temperature we will restrict, unless otherwise stated, to the Neveu-Schwarz sector and for definiteness we set $n = \frac{1}{2}$.

6 Boundary dynamics

As concluded in section 4, the variational principle for dilaton supergravity dictates that the bulk action has to be supplemented by the boundary term (4.9). In this section it is shown that, for the considered class of configurations, this boundary term actually controls the boundary dynamics of the theory. For doing this, we will gradually reduce the action in two steps:

- (i) Evaluate the theory on the constraint surface, and make use of the asymptotic conditions (3.2).
- (ii) Restrict the phase space to the one of regular solutions.

6.1 $\mathcal{N} = 1$ super-Schwarzian action

In order to proceed with the proposed steps it is necessary to write the action in a Hamiltonian form, which can be done by splitting the field components in space and time. Thus, the variational principle (4.1) becomes

$$I = -\frac{k}{2\pi} \int d\tau dr \text{Str}[\mathbf{X}\mathbf{A}'_r + \mathbf{A}_\tau \mathcal{G}] + \frac{k}{2\pi} \oint d\tau \text{Str}[xa_\tau] + I_B \quad (6.1)$$

where

$$\mathcal{G} = \partial_r \mathbf{X} + [\mathbf{A}_r, \mathbf{X}]. \quad (6.2)$$

Note that the second term of (6.1) has been written in terms of the radial-independent fields x and a_τ , by virtue of the asymptotic conditions (3.2).

Let us apply step (i). The variation of (6.1) with respect to the Lagrange multiplier A_τ implies the constraint $\mathcal{G} = 0$. Its solution is generically given by

$$\mathbf{X} = G^{-1}x_0G \quad (6.3)$$

with $x_0 = x_0(\tau)$ and G being a group element that satisfies $\mathbf{A}_r = G^{-1}\partial_r G$. Once the constraint is satisfied, the remaining bulk term appearing in (6.1) reduces to

$$-\frac{k}{2\pi} \oint d\tau \text{Str}[x_0 g' g^{-1}] \quad (6.4)$$

where we used that, according to (3.2), the group element G can be asymptotically factorized as $G = g(\tau)b(r)$. The group element g can be determined from the fact that, for this class of configurations, the gauge connection is asymptotically flat, then $a_\tau = g^{-1}g'$. Additionally, one can also conclude that $x_0 = gxg^{-1}$. By taking in consideration the above, (6.4) can be written as

$$-\frac{k}{2\pi} \oint d\tau \text{Str}[xa_\tau] \quad (6.5)$$

which is exactly cancelled with the second term in (6.1). Hence, for the set of asymptotic conditions (3.2), the value of the action on the constraint surface is solely determined by the boundary term I_B given in (4.9). By virtue of (3.17), the action (6.1) reduces to

$$I_{\text{red}}[\Theta; \Psi] = \frac{ky_0}{2\pi} \int d\tau d\theta (D\Theta)^{-2} [\Psi - S(T, \Theta; \tau, \theta)] . \quad (6.6)$$

Here, the bulk inherited quantity Ψ plays the role of a source for the superreparametrization field Θ . At first glance, the reduced theory is invariant under the whole group of superdiffeomorphisms, whose infinitesimal transformation law for Ψ is given in (3.20) and the one for Θ reads

$$\delta_\chi \Theta = \chi \partial_\tau \Theta + \frac{1}{2} D\chi D\Theta \quad (6.7)$$

However, we still have to implement the step (ii), which is performed as follows below.

From the analysis made in section 5 [with $n = \frac{1}{2}$ in (5.5)], regular solutions are given by elements whose representative

$$\Psi_* = \theta \frac{\pi^2}{\beta^2} \quad (6.8)$$

belongs to a super-Virasoro coadjoint orbit. In turn, the symmetry of (6.6) is broken to the set of configurations satisfying $\delta_\chi \Psi_* = 0$, where χ generates the $\mathfrak{osp}(2, 1)$ algebra.

The boundary equations of motion that follow from (6.6) can be obtained by taking its variation with respect to the superreparametrization fields $\{T, \Theta\}$. These are given by $\delta_Y \Psi = 0$, implying Casimir conservation $\partial_\tau \mathcal{C} = 0$, consistently with the variational principle.

In order to make contact with a more familiar representation of the boundary action (6.6), let us rewrite it in terms of the inverse superreparametrizations $\tau(T, \Theta)$, $\theta(T, \Theta)$. By means of the identity,

$$S(T, \Theta; \tau, \theta) = -(D\Theta)^3 S(\tau, \theta; T, \Theta) , \quad (6.9)$$

we find that the boundary degrees of freedom are described by the $\mathcal{N} = 1$ *super-Schwarzian theory*

$$I_{\text{red}}[\theta] = \frac{ky_0}{2\pi} \int dT d\Theta \left[\frac{\pi^2}{\beta^2} \theta (D_\Theta \theta)^3 + S(\tau, \theta; T, \Theta) \right]. \quad (6.10)$$

This action was shown to describe the Goldstone modes associated to the spontaneous superreparametrization symmetry breaking of the super-SYK model at the low temperature regime [19]. In the second order formulation of minimal dilaton supergravity, (6.10) has been also obtained in [27] using a different regularization procedure than the one described here.

6.2 $\mathcal{N} = 1$ superparticle

In this subsection, we will show that for the class of configurations with a vanishing Casimir, the boundary dynamics of dilaton supergravity is rather governed by the $\mathcal{N} = 1$ superparticle model. In order to obtain the corresponding action principle we recast the reparametrization field Θ in terms of a superfield $K = q - \theta\zeta$ of conformal dimension $-\frac{1}{2}$, such that $D\Theta = K^{-1}$, where q and ζ are commuting and anticommuting functions, respectively. In fact, by virtue of (6.7), the induced infinitesimal transformation law for K is given by

$$\delta_\chi K = -\frac{1}{2}\chi'K + \chi K' + \frac{1}{2}D\chi DK \quad (6.11)$$

as expected for its conformal weight. Considering the above, the parametrization for the dynamical variables (4.4) becomes

$$Y[K; y_0] = K^2 y_0 \quad \Psi[K; \Psi_0] = K^{-3} \Psi_0 - K^{-1} DK'. \quad (6.12)$$

By replacing this change of variables in (4.3) and taking under consideration the integrability condition (4.5), the variation of the boundary term is now given by

$$\delta I_B = \frac{ky_0}{2\pi} \int d\tau d\theta \left[\delta (K^{-1} \Psi_0) - 2\Psi_0 \delta[\log(K)] - K^2 \delta(K^{-1} DK') \right]. \quad (6.13)$$

The third term of (6.13) is a total derivative and thus is dismissed. The second term cannot be integrated for arbitrary Ψ_0 . When restricting to configurations satisfying $\Psi_0 = 0$ we can get rid of this term. This is, in fact, the only possible choice consistent with supersymmetry (recall that Ψ_0 is an odd superfield, while K is even). In that case, only the first term contributes to I_B , obtaining that its local expression is given by

$$I_B = \frac{ky_0}{2\pi} \int d\tau d\theta K^{-1} \Psi_0. \quad (6.14)$$

The on-shell value of the bulk action for classical configurations is only given by the boundary term, which under the assumption $\Psi_0 = 0$ identically vanishes

$$I_{\text{on-shell}} = 0. \quad (6.15)$$

The residual dynamics is obtained by using the steps (i) and (ii) described at the beginning of this section. The action on the constraint surface gives

$$I_{\text{red}}[K; \Psi] = \frac{ky_0}{2\pi} \int d\tau d\theta (K^2 \Psi_* - K' DK) \quad (6.16)$$

which, at first sight, seems to be the superfield form of the $\mathcal{N} = 1$ superconformal mechanics action without a self-interacting potential [25]

$$I_{\text{red}}[q, \zeta; \mathcal{L}_*, \psi_*] = \frac{ky_0}{2\pi} \oint d\tau [\mathcal{L}_* q^2 + 2\psi_* q \zeta - (q')^2 + \zeta \zeta'] . \quad (6.17)$$

The action (6.16) is invariant under $\text{SDiff}(S^1)$. This is because of the presence of the external source Ψ_* , which possesses conformal weight $\frac{3}{2}$. However, regular bulk configurations must respect the integrability condition $\Psi_0 = 0$. This restricts us to the Ramond sector of the holonomy operator (5.1), whose little group is graded $U(1)$ with representatives $\mathcal{L}_* = \psi_* = 0$. Hence, the boundary model effectively describes the free $\mathcal{N} = 1$ superparticle.

Let us study the dynamics of the model. Taking the variation of (6.16) with respect to K yields the equations of motion

$$K \Psi_* + DK' = 0 \quad (6.18)$$

which, by virtue of (6.12), directly imply $\Psi_0 = 0$ or similarly $\mathcal{C} = 0$. This fact ensures that the parametrization of the boundary phase space in terms of the superdensity K (6.12) is consistent with the variational principle.

Finally, we mention that $\mathcal{C} = 0$ coincides with the existence condition of BPS-like states in the context of graded Poisson sigma models [48]. Therefore, bulk configurations with unbroken supersymmetries necessarily reduce their dynamics to a free superparticle model.

7 Extended case

7.1 Extended dilaton supergravity

Dilaton supergravity in 2D can be easily generalized in order to accommodate N gravitini ψ_A corresponding to Majorana spinors. This can be achieved by enlarging the gauge group of the BF action (2.5) to $\text{OSp}(2, \mathbb{N})$, where it is necessary to introduce additional $\text{SO}(N)$ gauge fields Y^{AB} and C^{AB} . In this case, $\mathbf{X}$ and $\mathbf{A}$, given by

$$\mathbf{X} = X^I J_I + \lambda_A^\alpha Q_\alpha^A + \frac{1}{2} Y^{AB} T_{AB} \quad (7.1)$$

$$\mathbf{A} = A^I J_I + \psi_A^\alpha Q_\alpha^A + \frac{1}{2} C^{AB} T_{AB} \quad (7.2)$$

are evaluated on the $\mathfrak{osp}(2, N)$ algebra:

$$[J_I, J_J] = \epsilon_{IJK} J^K \quad [J_I, Q_\alpha^A] = \frac{1}{2} (\Gamma_I)^\beta_\alpha Q_\beta^A \quad (7.3)$$

$$[Q_\alpha^A, T^{BC}] = 2\delta^{A[B} Q_\alpha^{C]} \quad [T^{AB}, T^{CD}] = 4\delta^{[A[C} T^{D]B]} \quad (7.4)$$

$$\{Q_\alpha^A, Q_\beta^B\} = -\frac{1}{2} \delta^{AB} (C\Gamma^I)_{\alpha\beta} J_I - \frac{1}{4} C_{\alpha\beta} T^{AB} . \quad (7.5)$$

The capital latin letters $A, B, C, D, \dots$, that label the generators $T_{AB} = -T_{BA}$ of the $\text{SO}(N)$ subgroup, run from 1 to N .

We choose the supertrace whose nonvanishing components are given by (see appendix A)

$$\text{Str}[J_I J_J] = \frac{1}{2} \eta_{IJ} \quad \text{Str}[Q_\alpha^A Q_\beta^B] = \frac{1}{2} C_{\alpha\beta} \delta^{AB} \quad \text{Str}[T^{AB} T^{CD}] = 4 \delta^{A[C} \delta^{D]B}. \quad (7.6)$$

Taking into consideration the above definitions, one obtains that the field strength associated with the gauge field $\mathbf{A}$ reads

$$\mathbf{F} = \left(R^I - \frac{1}{4} \bar{\psi}_A \Gamma^I \psi_A \right) J_I + \nabla \psi_A^\alpha Q_\alpha^A + \frac{1}{2} F^{AB} T_{AB}. \quad (7.7)$$

Here, the curvature R^I is given in the first equation of (2.10), while the covariant derivative acting on spinors and the field strength associated to C are given by

$$\nabla \psi_A^\alpha = d\psi_A^\alpha + \frac{1}{2} A^I (\Gamma_I)^\alpha_\beta \psi_A^\beta + C_{AB} \psi_B^\alpha \quad (7.8)$$

$$F^{AB} = dC^{AB} + C^{AC} C^{CB} - \frac{1}{2} \bar{\psi}^{[A} \psi^{B]} \quad (7.9)$$

respectively. By replacing (7.1), (7.2) into (2.5) and then using the supertrace (7.6), the action for the extended supergravity theory becomes

$$I = \frac{k}{2\pi} \int X^I \left[\left(R_I - \frac{1}{4} \bar{\psi}_A \Gamma_I \psi_A \right) + \bar{\lambda}_A \nabla \psi_A + Y_{AB} F^{AB} \right]. \quad (7.10)$$

As in the $\mathcal{N} = 1$ case, the field equations (2.8) impose that the dilaton $\mathbf{X}$ must be the stabilizer of the flat gauge connection $\mathbf{A}$.

Action (7.10) is invariant under the following local extended supersymmetry transformations

$$\delta A^I = -\frac{1}{2} \bar{\psi}_A \Gamma^I \epsilon_A \quad \delta X^I = -\frac{1}{2} \bar{\lambda}_A \Gamma^I \epsilon_A \quad (7.11)$$

$$\delta C^{AB} = -\frac{1}{2} \bar{\psi}^{[A} \epsilon^{B]} \quad \delta Y^{AB} = -\frac{1}{2} \bar{\lambda}^{[A} \epsilon^{B]} \quad (7.12)$$

$$\delta \psi_A = \nabla \epsilon_A \quad \delta \lambda_A = \frac{1}{2} X^I \Gamma_I \epsilon_A + Y_{AB} \epsilon_B \quad (7.13)$$

which can be obtained from (2.11) by considering the gauge parameter $\eta = \epsilon_A^\alpha Q_\alpha^A$.

7.2 Asymptotic conditions

The objective of this subsection is to study asymptotic configurations that are left invariant under the group of extended superdiffeomorphisms on the circle. Such group comprises diffeomorphisms $\xi(\tau)$, local supersymmetry transformations $\epsilon^A(\tau)$ and internal $\text{SO}(N)$ gauge symmetries $\mu^{AB}(\tau)$. The analysis of the asymptotic symmetries is performed by imposing the condition (2.11) modulo trivial gauge transformations of the form (2.12).

The configurations of interest can be expressed as in (3.2) where the gauge field a and the radial independent dilaton x are given by

$$a = \left[L_1 + \left(\mathcal{L} + \frac{1}{2} \mathcal{P}^{AB} \mathcal{P}^{AB} \right) L_{-1} + \psi^A G_{-\frac{1}{2}}^A + \frac{1}{2} \mathcal{P}^{AB} T^{AB} \right] d\tau \quad (7.14)$$

$$x = y L_1 - y' L_0 + \left[\frac{1}{2} y'' + y \left(\mathcal{L} + \frac{1}{2} \mathcal{P}^{AB} \mathcal{P}^{AB} \right) + \psi^A \rho^A \right] L_{-1} \\ + \frac{1}{2} (\lambda^{AB} + y \mathcal{P}^{AB}) T^{AB} + \rho^A G_{\frac{1}{2}}^A - (\rho^{A'} - y \psi^A + \mathcal{P}^{AB} \rho^B) G_{-\frac{1}{2}}^A. \quad (7.15)$$

This set of fall-off conditions is a consistent extension of the minimal supersymmetric case (3.3) and (3.4). Here, we have considered the following basis for the $\mathfrak{osp}(2, \mathbb{N})$ algebra (see appendix A)

$$[L_m, L_n] = (m - n) L_{m+n} \quad [L_m, G_p^A] = \left(\frac{m}{2} - p \right) G_{m+p}^A \quad (7.16)$$

$$[G_p^A, T^{BC}] = 2\delta^{A[B} G_p^{C]} \quad [T^{AB}, T^{CD}] = 4\delta^{[A|[C} T^{D][B]} \quad (7.17)$$

$$\{G_p^A, G_q^B\} = -2L_{p+q} \delta^{AB} - (p - q) T^{AB}, \quad (7.18)$$

with $m, n = \pm 1, 0$ and $p, q = \pm \frac{1}{2}$. As explained in section 3, the Lie-algebra-parameter

$$\Lambda = \mathbf{X}[\xi, \epsilon_A, \mu_{AB}] \quad (7.19)$$

singles out the gauge transformations that keep invariant the asymptotic behavior (7.14) and (7.15) as long as the transformation laws of the fields take the following form

$$\delta_{(\xi, \epsilon, \mu)} \mathcal{L} = 2\mathcal{L}\xi' + \mathcal{L}'\xi + \frac{1}{2}\xi''' + 3\psi^A \epsilon^{A'} + \psi^{A'} \epsilon^A - \mathcal{P}^{AB} \mu^{AB'} \quad (7.20)$$

$$\delta_{(\xi, \epsilon, \mu)} \psi^A = \frac{3}{2} \psi^A \xi' + \psi^{A'} \xi - \mu^{AB} \psi^B - \mathcal{L} \epsilon^A - \epsilon^{A''} - \mathcal{P}^{AB'} \epsilon^B - 2\mathcal{P}^{AB} \epsilon^{B'} \\ - \left(\mathcal{P}^{AB} \mathcal{P}^{BE} + \frac{1}{2} \mathcal{P}^{BC} \mathcal{P}^{BC} \delta^{AE} \right) \epsilon^E \quad (7.21)$$

$$\delta_{(\xi, \epsilon, \mu)} \mathcal{P}^{AB} = \mathcal{P}^{AB} \xi' + \mathcal{P}^{AB'} \xi + 2\mathcal{P}^{[A|C} \mu^{C|B]} + \mu^{AB'} + 2\psi^{[A} \epsilon^{B]} \quad (7.22)$$

$$\delta_{(\xi, \epsilon, \mu)} y = y' \xi - y \xi' - 2\rho_A \epsilon^A \quad (7.23)$$

$$\delta_{(\xi, \epsilon, \mu)} \rho_A = \rho'_A \xi - \frac{1}{2} \rho_A \xi' + \frac{1}{2} y' \epsilon_A - y \epsilon'_A + \rho_B \mu_{BA} + \lambda_{AB} \epsilon_B \quad (7.24)$$

$$\delta_{(\xi, \epsilon, \mu)} \lambda_{AB} = \xi \lambda'_{AB} - y \mu'_{AB} + 2\lambda_{C[A} \mu_{B]C} + 2\rho_{[A} \epsilon'_{B]} - 2\rho'_{[A} \epsilon_{B]} + 2\rho_C \mathcal{P}_{C[A} \epsilon_{B]} \\ - 2\epsilon_C \mathcal{P}_{C[A} \rho_{B]} + 2\rho_C \epsilon_C \mathcal{P}_{AB}. \quad (7.25)$$

This set of infinitesimal transformations defines the action of the $\text{SO}(N)$ -extended super-Virasoro algebra [49, 50] on the dynamical fields. In the generic case, the transformations are quadratic in the affine $\mathfrak{so}(N)$ currents $\mathcal{P}^{AB}$. However, for $\mathcal{N} = 2$ the non-linear terms appearing in $\delta\psi^A$ and $\delta\lambda_{AB}$ cancel out each other, leaving the transformation laws linear.

As was shown in section 6, the boundary dynamics of the $\mathcal{N} = 1$ case is controlled by a finite superdiffeomorphism Θ . This is a property associated to the asymptotic symmetry group. Due to the non-linearities present in (7.21), the corresponding group structure is

not completely understood for generic N . Since only the $\text{SO}(2)$ -extended super-Virasoro algebra reproduces linear transformations in the fields, in the remainder of this section we will restrict ourselves to that case only.

Even though we do not have a representation for finite transformations with $N > 2$, one can use the relation between dilaton gravity and the action of the particle on a group manifold [18, 51, 52] to find suitable boundary theories for the breaking of the $\text{SO}(N)$ -extended superreparametrization groups. This connection will be studied in section 8.

7.3 $\mathcal{N} = 2$ superspace formulation

Hereafter, we will make use of the extended superspace formalism to represent the field content and the action of the asymptotic symmetry group. In accordance with [53], we will denote the supercoordinates as the set $z = (\tau, \theta_1, \theta_2)$. The corresponding covariant derivative $D_A = \theta_A \partial_\tau + \partial_A$ satisfies

$$\{D_A, D_B\} = 2\delta_{AB}\partial_\tau. \quad (7.26)$$

The requirement that D_A transforms covariantly, i.e., $D_A = (D_A \tilde{\theta}_B) \tilde{D}_B$ leads to the following constraints

$$D_A \tilde{\tau} = \tilde{\theta}_B D_A \tilde{\theta}_B \quad (D_A \tilde{\theta}_C)(D_B \tilde{\theta}_C) = \delta_{AB} \left(\partial_\tau \tilde{\tau} + \tilde{\theta}_C \partial_\tau \tilde{\theta}_C \right). \quad (7.27)$$

An infinitesimal superreparametrization can be written as $\delta\tau = -\chi + \frac{1}{2}\theta_A D_A \chi$ and $\delta\theta_A = -\frac{1}{2}D_A \chi$, with $\chi = \xi - 2\theta_B \epsilon_B + \theta_A \theta_B \mu_{AB}$. It is then found that $\tilde{\theta}_A$ transforms as

$$\delta\tilde{\theta}_A = \chi \partial_\tau \tilde{\theta}_A + \frac{1}{2} (D_B \chi) D_B \tilde{\theta}_A. \quad (7.28)$$

The field content can be arranged in the following superfields

$$\Psi = \theta_1 \theta_2 \mathcal{L} + \epsilon_{AB} (\psi_A \theta_B - \frac{1}{2} \mathcal{P}_{AB}) \quad (7.29)$$

$$Y = y - 2\theta_B \rho_B + \theta_A \theta_B \lambda_{AB} \quad (7.30)$$

so that it reproduces the action of the asymptotic symmetry group. In the case of Ψ , it can be seen that its finite transformation corresponds to the coadjoint action of the $\mathcal{N} = 2$ super-Virasoro group [41, 46, 47, 54] on a quasi-primary field of conformal weight one. The coadjoint action is dictated by

$$\tilde{\Psi}(\tilde{z}) = \Omega^{-1} [\Psi(z) - S^{(2)}(\tilde{z}; z)] \quad S^{(2)}(\tilde{z}; z) = \frac{1}{2} \left(\frac{D_1 D_2 \Omega}{\Omega} - \frac{3}{2} \frac{D_1 \Omega D_2 \Omega}{\Omega^2} \right) \quad (7.31)$$

where Ω stands for the determinant of $D_A \tilde{\theta}_B$ and $S^{(2)}$ is the generalization of the Schwarzian derivative. Infinitesimally, (7.31) reproduces the transformations of the fields (7.20), (7.21), (7.22) for the $N = 2$ case by means of (7.28). Furthermore, under the same group action, Y transforms as

$$\tilde{Y}(\tilde{z}) = \Omega Y(z), \quad (7.32)$$

from which it is recognized that Y is a vector superfield. Noteworthy, by virtue of (7.28), the infinitesimal version of (7.32) reproduces (7.23), (7.24), (7.25).

7.4 Action principle for $\mathcal{N} = 2$ dilaton supergravity

It is necessary to provide a well-defined variational principle for (7.10) associated to the set of extended asymptotic conditions (3.2). As before, the enhanced action, given by the bulk term (7.10) plus a boundary term I_B , has an extremum as long as

$$\delta I_B = -\frac{k}{2\pi} \oint d\tau \text{Str} [\mathbf{X} \delta \mathbf{A}_\tau] = \frac{k}{2\pi} \oint d\tau (y \delta \mathcal{L} - \lambda^{AB} \delta \mathcal{P}^{AB} - 2\rho^A \delta \psi^A) . \quad (7.33)$$

By using the superfields defined in (7.29) and (7.30) we can express the above integral as

$$\delta I_B = \frac{k}{2\pi} \int d\tau d\theta_1 d\theta_2 Y \delta \Psi \quad (7.34)$$

where Berezin integration is defined by $\int d\theta_1 d\theta_2 \theta_1 \theta_2 = 1$ and $\int d\theta_1 d\theta_2 = 0$.

7.4.1 Phase space

Analogously to $\mathcal{N} = 1$, we parametrize the phase space by means of $\mathcal{N} = 2$ super-Virasoro coadjoint orbits. Then, the superfields Y and Ψ can be written as

$$Y = \Omega_\Theta^{-1} y_0 \quad \Psi = \Omega_\Theta \Psi_0 + S(T, \Theta_A; z) \quad (7.35)$$

where $\Omega_\Theta = \det(D_A \Theta_B)$ and T is the coordinate associated to the constraint $D_C T = \Theta_A D_C \Theta_A$. Here, we choose Θ_A to be the reparametrization that brings Y into a constant y_0 . In what follows we will assume that y_0 is a fixed constant without variation, i.e., $\delta y_0 = 0$.

Before analyzing the boundary term that makes the variational principle well-defined, let us discuss the relation between Ψ_0 and the conserved charges when the field equations are satisfied. If one takes two successive supercovariant derivatives to Ψ_0 with respect to Θ_2 and Θ_1 , we find that

$$y_0^2 D_{\Theta_1} D_{\Theta_2} \Psi_0 = -\mathcal{C} - \theta_A (y \delta_{(y,\rho,\lambda)} \psi_A + 2\rho_B \delta_{(y,\rho,\lambda)} \mathcal{P}_{AB}) - \frac{1}{2} \theta_A \theta_B (y \delta_{(y,\rho,\lambda)} \mathcal{P}_{AB})' \quad (7.36)$$

where $\mathcal{C}$ is the asymptotic value of the Casimir, that by virtue of its definition in (2.14) is given by

$$\mathcal{C} = \mathcal{L} y^2 + 3y \psi_A \rho_A + (2\rho_A \rho_B - y \lambda_{AB}) \mathcal{P}_{AB} + \frac{1}{2} y y'' - \frac{1}{4} (y')^2 + 2\rho_A \rho'_A - \frac{1}{2} \lambda_{AB} \lambda_{AB} . \quad (7.37)$$

On-shell, equation (7.36) can be solved to obtain

$$\Psi_0 \approx y_0^{-2} \Theta_1 \Theta_2 \mathcal{C} + \epsilon_{AB} \left(\psi_A^{(0)} \Theta_B - \frac{1}{2} \mathcal{P}_{AB}^{(0)} \right) \quad (7.38)$$

where $\psi_A^{(0)}$ and $\mathcal{P}_{AB}^{(0)}$ are two extra possible on-shell constants. We can again conclude that Θ_A brings Ψ to the frame where its components are defined in terms of the constants of motion $(\mathcal{C}, \psi_A^{(0)}, \mathcal{P}_{AB}^{(0)})$.

7.4.2 Boundary term

Let us go back to (7.34) using the reduced phase space defined by (7.35). Plugging in these expressions, it is found that

$$\delta I_B = \frac{ky_0}{2\pi} \int dz \left[\delta \Psi_0 + \Omega_\Theta^{-1} \delta \Omega_\Theta \Psi_0 + \Omega_\Theta^{-1} \delta S^{(2)} \right]. \quad (7.39)$$

Here, the last term is a total derivative that vanishes due to the periodicity of the fields. In order to provide a local expression for I_B we need to deal with the integrability problem present in the second term of (7.39). Imposing the on-shell condition (7.38), this term becomes

$$\int dz \Omega_\Theta^{-1} \delta \Omega_\Theta \Psi_0 \approx y_0^{-2} \mathcal{C} \int dz \Theta_1 \Theta_2 D_{\Theta_A} \delta \Theta_A. \quad (7.40)$$

We must express the latter integral in terms of T and Θ_A . Since the Berezinian between z and (T, Θ_A) is the identity, the right-hand side of (7.40) is zero since β is a state independent function. Bear in mind that this derivation is only valid for the case $\mathcal{C} \neq 0$. As it will be explained in section 7.6, the case $\mathcal{C} = 0$ needs an special treatment and in fact the term in (7.40) will yield a new contribution to I_B . Indeed, it will be proved that this enhancement leads to the action of $\mathcal{N} = 2$ superconformal mechanics.

The remaining term in (7.39) defines the value of the boundary term

$$I_B = \frac{ky_0}{2\pi} \int d\tau d\theta_1 d\theta_2 \Psi_0. \quad (7.41)$$

The value of the on-shell action is again given by the boundary term I_B . In terms of the Casimir function this value takes the same the form as in (4.10).

7.5 Regularity conditions: extended super-Hill equation

The asymptotic symmetry breaking exhibited in the case with $\mathcal{N} = 1$ replicates in the model with extended supersymmetries. For seeing this, let us proceed to find the class of configurations satisfying trivial holonomy around the thermal circle C . The generalization of the center group element to $\text{OSp}(2, \mathbb{N})$ is given by

$$\Gamma_\pm = \begin{pmatrix} \pm \mathbb{I}_{2 \times 2} & 0_{2 \times N} \\ 0_{N \times 2} & \mathbb{I}_{N \times N} \end{pmatrix}. \quad (7.42)$$

Thus, from $a = udu^{-1}$ one translates the trivial holonomy condition $H_C[a] = \Gamma_\pm$ to the boundary condition for the group element $u(\beta) = \Gamma_\pm u(0)$.

In order to determine u for the case at hand, we have to consider the expression of the gauge connection a (7.14) and the matrix representation of the $\text{OSp}(2, 2)$ generators (see appendix A). It is then found that, in terms of 3 independent 4-dimensional row vectors $\varphi_{(1)}^A$ and $\varphi_{(2)}$, the group element reads

$$u = \begin{pmatrix} -\varphi_{(2)}' \\ \varphi_{(2)} \\ \varphi_{(1)A} \end{pmatrix} \quad (7.43)$$

where $\varphi_{(1)A}$ and $\varphi_{(2)}$ are solutions to the $\mathcal{N} = 2$ super-Hill equation

$$(D_1 D_2 + \Psi) \Phi = 0. \quad (7.44)$$

Here $\Phi \equiv \varphi_{(2)} + \theta_A \varphi_{(1)A} - \theta_1 \theta_2 \mathcal{P} \varphi_{(2)}$, with $\mathcal{P} = \frac{1}{2} \epsilon_{AB} \mathcal{P}_{AB}$ and Ψ given in (7.29). Thus, the components of Φ must satisfy the following equations

$$\varphi_{(2)}'' + (\mathcal{L} + \mathcal{P}^2) \varphi_{(2)} - \psi_A \varphi_{(1)A} = 0 \quad (7.45)$$

$$\varphi_{(1)A}' + \mathcal{P} \epsilon_{AB} \varphi_{(1)B} + \psi_A \varphi_{(2)} = 0. \quad (7.46)$$

For solving (7.44), we will use the fact that the extended super-Hill equation is covariant under the group of extended superreparametrizations [46, 47], one is then allowed to take the system to the rest frame, where the supercurrent $\Psi = \Psi_*$ is constant. In the Neveu-Schwarz sector, $u(\beta) = \Gamma_- u(0)$, it is found that ψ_*^A must vanish and thus the above equations decouple. It follows that

$$\mathcal{L}_* = \frac{4\pi^2}{\beta^2} (n^2 - m^2) \quad \mathcal{P}_* = \frac{2\pi m}{\beta} \quad (7.47)$$

where n and m are restricted to be half-integers and integers, respectively. In the case $m = 0$, the little group of Ψ_* is given by a $2n$ -cover of $\text{OSp}(2, 2)$. In consequence, for $n = \frac{1}{2}$, Ψ corresponds to an element of the coadjoint orbit with $\text{OSp}(2, 2)$ as little group [46, 54].

In the Ramond sector, $\Gamma_+ = \mathbb{I}_{4 \times 4}$, the solutions to the equations (7.45) and (7.46) are of the form $\exp(i\lambda_{\pm}\tau)$, with $\lambda_+ = \frac{2\pi n}{\beta}$ and $\lambda_- = \frac{2\pi m}{\beta}$ with n and m arbitrary integers. Making use of the *body and soul* decomposition $\lambda_{\pm} = \lambda_b^{\pm} + \lambda_s^{\pm} \psi_*^1 \psi_*^2$ in (7.45) and (7.46) we find that $\lambda_b^{\pm}$ and $\lambda_s^{\pm}$ satisfy

$$\lambda_b^+ = \pm \sqrt{\mathcal{L}_* + \mathcal{P}_*^2}, \quad \lambda_b^- = \pm \mathcal{P}_*, \quad [\mathcal{P}_* - (2(\lambda_b^{\pm})^2 - \mathcal{L}_* - 2\mathcal{P}_*^2) \lambda_b^{\pm} \lambda_s^{\pm}] \psi_*^1 \psi_*^2 = 0. \quad (7.48)$$

The cases $\mathcal{L}_* = 0$ and $\mathcal{L}_* = -\mathcal{P}_*^2$ deserve special attention. The case $\mathcal{L}_* = 0$ necessarily implies that $\mathcal{P}_* = 0$, without any restrictions on the fermions ψ_*^A . For $\psi_*^1 = 0$ (or $\psi_*^2 = 0$), the corresponding isotropy group is graded $U(1)$. If $\mathcal{L}_* = -\mathcal{P}_*^2$ (with $\mathcal{P}_* \neq 0$), the boundary conditions imply that $\psi_*^1 \psi_*^2 = 0$. This case can be represented by (7.47) setting $n = 0$ (and m integer). Coadjoint orbits in this case are again characterized by graded $U(1)$.

For the generic case, $\mathcal{L}_* \neq 0$ and $\mathcal{L}_* \neq -\mathcal{P}_*^2$, one finds that

$$\lambda_+ = \pm \left(\sqrt{\mathcal{L}_* + \mathcal{P}_*^2} + \frac{\mathcal{P}_* \psi_*^1 \psi_*^2}{\mathcal{L}_* \sqrt{\mathcal{L}_* + \mathcal{P}_*^2}} \right) \quad \lambda_- = \pm \left(\mathcal{P}_* - \frac{\psi_*^1 \psi_*^2}{\mathcal{L}_*} \right). \quad (7.49)$$

If $\psi_*^1 \psi_*^2 = 0$, we recover (7.47) (with n integer). There are no additional conditions on ψ_*^A , therefore the superfield Ψ_* is preserved only by $U(1)$ transformations. In the case of $\psi_*^A = 0$ and $\mathcal{P}_* = 0$, the isotropy group of Ψ_* is given by a $2n$ -cover of $\text{OSp}(2, 2)$.

7.6 $\mathcal{N} = 2$ super-Schwarzian theory

As in the $\mathcal{N} = 1$ case, shown in section 6, the variational principle for the $\mathcal{N} = 2$ case evaluated on the constraint surface is also determined by the boundary term I_B , given by (7.41). After using (7.31) to express Ψ_0 in terms of Ψ , we find

$$I_{\text{red}} = \frac{ky_0}{2\pi} \int d\tau d\theta_1 d\theta_2 \Omega_{\Theta}^{-1} \left[\Psi - S^{(2)}(T, \Theta_A; z) \right]. \quad (7.50)$$

The equations of motion are obtained varying with respect to $\{T, \Theta_A\}$, conditions that imply the conservation of the boundary Casimir, $\partial_\tau \mathcal{C} = 0$, being in complete agreement with the bulk dynamics. Additionally, the action (7.50) is invariant under the following transformations

$$\delta\Psi = \partial_\tau \chi \Psi + \chi \partial_\tau \Psi + \frac{1}{2} D_A \chi D_A \Psi + \frac{1}{2} D_1 D_2 \partial_\tau \chi \quad (7.51)$$

$$\delta\Theta_A = \chi \partial_\tau \Theta_A + \frac{1}{2} D_B \chi D_B \Theta_A. \quad (7.52)$$

In order to fully describe the boundary dynamics of regular black holes geometries we must evaluate (7.51) and (7.52) in $\Psi = \Psi_*$ determined by (7.47). These configurations are invariant under a single cover of $\text{OSp}(2, 2)$ for the values $n = \frac{1}{2}$ and $m = 0$, therefore we choose $\Psi_* = \frac{\pi^2}{\beta^2} \theta_1 \theta_2$.

Similarly to what was done in section 6, it is useful to express (7.50) in terms of $z(T, \Theta_A)$. To achieve this, one can use the expression of the super-Schwarzian function transforming under superreparametrizations

$$S^{(2)}(T, \Theta_A; z) = -\Omega_\Theta S^{(2)}(z; T, \Theta_A) \quad (7.53)$$

such that

$$I_{\text{red}} = \frac{ky_0}{2\pi} \int dT d\Theta_1 d\Theta_2 \left[\frac{\pi^2}{\beta^2} \theta_1 \theta_2 \Omega_z + S^{(2)}(z; T, \Theta_A) \right], \quad (7.54)$$

where Ω_z is the determinant associated to the inverse of $D_A \Theta_B$. Thus, we have obtained that the boundary dynamics — for bulk configurations with a nonvanishing Casimir — turns out to be controlled by the $\mathcal{N} = 2$ super-Schwarzian action, whose form was first proposed in [19]. It has been argued in [27], that through a different regularization method, the second order formulation of $\mathcal{N} = (2, 2)$ dilaton supergravity yields the zero temperature version of (7.54).

7.7 $\mathcal{N} = 2$ superconformal mechanics

Here, we will study the boundary dynamics associated to the class of configurations that possesses a vanishing Casimir function. In this case the bulk action is shown to reduce to the one of $\mathcal{N} = 2$ superconformal mechanics, whose space of solutions corresponds to the orbit of bulk BPS states. The latter fact is in fully agreement with the observation made in section 6.2 for the $\mathcal{N} = 1$ case.

Let us proceed by writing the determinant of the extended superdiffeomorphism Θ_A as $\Omega_\Theta = K^{-2}$, where $K = q - \theta_A \zeta_A + \theta_1 \theta_2 u$ is an even superfield of conformal dimension $-\frac{1}{2}$. In terms of K , the phase space (7.35) becomes

$$Y = K^2 y_0 \quad \Psi = K^{-2} (\Psi_0 - K D_1 D_2 K). \quad (7.55)$$

For these configurations, the variation of I_B is given by

$$\delta I_B = \frac{ky_0}{2\pi} \int dz [\delta \Psi_0 - 2\delta [\log(K)] \Psi_0]. \quad (7.56)$$

In order to integrate the second term it is necessary to impose an integrability condition on Ψ_0 . Since Ψ_0 stands for an even superfield, one could consider the set of configurations satisfying

$$\Psi_0 \approx g, \quad (7.57)$$

with g a constant without variation. From (7.38), it can be seen that, in terms of supercoordinates Θ_A , this means that the asymptotic value of the Casimir function (7.37) vanishes and $\mathcal{P} = -g$. Within this set of solutions, the second term in (7.56) can be easily integrated yielding

$$-\frac{ky_0}{\pi}g \int dz \log(K). \quad (7.58)$$

Thus, the action principle $I_{\text{bulk}} + I_B$ is well defined for configurations that satisfy (7.57). We then must evaluate the action on the constraint surface, which implies that its value is only given by I_B . Using (7.55) to express Ψ_0 in terms of Ψ and K , and integrating by parts, we find that

$$I_{\text{red}}[K; \Psi] = \frac{ky_0}{2\pi} \int d\tau d\theta_1 d\theta_2 [K^2\Psi - D_1 K D_2 K - 2g \log(K)]. \quad (7.59)$$

The action of the reduced theory (7.59) corresponds to the one of $\mathcal{N} = 2$ superconformal mechanics, that besides of being endowed with an inverse square potential, it contains a harmonic-like term with coupling constant g . This latter term ensures the existence of a ground state at quantum level [23–25].⁸

Due to the presence of the external source Ψ , the action (7.59) is seemingly invariant under the entire $\text{SDiff}(S^1)$ group with $\mathcal{N} = 2$. Nonetheless, consistency with integrability condition (7.57) imposes that $\mathcal{L}_* = 0$, $\psi_*^A = 0$, $\mathcal{P}_* = -g$. By virtue of regularity (see section 7.5) the latter representative is only consistent with periodic boundary conditions (taking Γ_+ in (7.42), which in turn, is determined by (7.47) with $n = m$). This fact breaks the $\mathcal{N} = 2$ $\text{SDiff}(S^1)$ symmetry down to a graded $U(1)$ symmetry and additionally imposes the following quantization condition on the coupling constant

$$g = -\frac{2\pi m}{\beta}. \quad (7.60)$$

On-shell, the boundary dynamics of (7.59) encodes the set of configurations considered in this section. Indeed, the equation of motion that follows from the action principle reads

$$K\Psi_* + D_1 D_2 K + \frac{2\pi m}{\beta} K^{-1} = 0 \quad (7.61)$$

which is nothing but condition (7.57). Thus, as anticipated at the beginning of this subsection, solutions of the boundary model (7.59) are mapped into ground states of the bulk theory, that according to [48] correspond to bulk BPS-like states.

⁸Note that the action (7.59) can also be recovered in the context of nonlinear realizations of the $\mathcal{N} = 2$ super-Virasoro group [55].

8 Particle on a supergroup manifold

So far, we have successfully determined the boundary dynamics associated to $\mathcal{N} = 1$ and $\mathcal{N} = 2$ dilaton supergravity equipped with the set of asymptotic conditions introduced in sections 3.1 and 7.2, respectively. The crucial point in these derivations has been to find consistent integrability conditions between the fields $\mathbf{X}$ and $\mathbf{A}$ such that the boundary term required for the variational principle,

$$\delta I_B = -\frac{k}{2\pi} \oint d\tau \text{Str} [\mathbf{X} \delta \mathbf{A}_\tau] \quad (8.1)$$

yields a local expression for I_B . In order to deal with the integrability of (8.1) for $\mathcal{N} > 2$, we will consider the set of asymptotic conditions consistent with the loop group of $\text{OSp}(2, \mathbb{N})$, and then impose an appropriate boundary condition between $\mathbf{A}$ and $\mathbf{X}$. It will be also shown that the above treatment allows to reduce the dynamics of the theory to the one of a particle moving in a group manifold.

In a second stage, we restrict ourselves to asymptotically AdS_2 supergeometries (7.14) containing a fluctuating dilaton multiplet. This is achieved by applying a suitable Hamiltonian reduction that yields some constraints on the particle model. Finally, we consider geometries representing the orbit of regular black hole solutions. These reductions not only agree with the results obtained in the previous sections for $N = 1$ and $N = 2$ cases but also extend them to $N > 2$, which lead to a generic class of super-Schwarzian actions for generic $\text{OSp}(2, \mathbb{N})$ dilaton supergravity models.

8.1 Integrability condition and regularized action principle

We parameterize our asymptotic conditions as in (3.2),

$$\mathbf{A} = b^{-1}(d + a)b + \mathcal{O}(r^{-2}) \quad \mathbf{X} = b^{-1}xb + \mathcal{O}(r^{-2}). \quad (8.2)$$

We assume that the radial-independent fields a and x satisfy the following integrability condition

$$a_\tau = f_\tau x \quad (8.3)$$

with f_τ an arbitrary function that is assumed to play the role of a one-form component. It must be noted that the integrability condition (8.3) has been previously used for JT gravity [51] and for suitable bosonic extensions thereof [18, 52].

The solution space of x constraining by (8.3) reduces the dilaton equation $\delta_{\mathbf{X}} \mathbf{A} = 0$ to $\partial_\tau x = 0$. Combining the above to the fact that the gauge connection is flat $a_\tau = g^{-1} \partial_\tau g$, we find that the group element g obeys the following condition

$$\partial_\tau (f_\tau^{-1} g^{-1} \partial_\tau g) = 0 \quad (8.4)$$

which turns out to be the equation of motion for a particle in a group manifold. The variational principle that leads to equations (8.4) will be obtained by regularizing the BF model through the integration of (8.1). For this, we make use of the asymptotic

conditions (8.2) together with (8.3). Thus, by assuming that f_τ has a fixed zero mode, it follows that the integrated expression for I_B is given by

$$I_B = -\frac{k}{4\pi} \oint d\tau f_\tau \text{Str}(x^2). \quad (8.5)$$

The integrability conditions (8.3) play the same role as boundary conditions on canonical variables. In a theory with n canonical pairs (q^i, p_i) one has to impose n boundary conditions to achieve a consistent variation principle. Here, we also imposed n conditions on n pairs of boundary variables (a, x) , but we managed to do better: the conditions (8.3) contain an arbitrary function f_τ that fluctuates freely except for the zero mode.

We recall that the full action principle consists of the BF bulk piece (2.5) plus the boundary term I_B (8.5). Nonetheless, as shown in section 6, once the theory is evaluated on the constraint surface, the action becomes the boundary term I_B by virtue of the asymptotic conditions (8.2). Therefore, from (8.3) one obtains that the action reduces to

$$I_{\text{red}}[g] = -\frac{ky_0}{4\pi} \oint df \text{Str} [(g^{-1} \partial_f g)^2]. \quad (8.6)$$

Here, we have used that $f_\tau = \frac{1}{y_0} \partial_\tau f$ ensuring that f_τ has a fixed zero mode. The set of symmetries, making $I_{\text{red}}[g]$ invariant, corresponds to left and right multiplications

$$g \mapsto h_L g h_R^{-1} \quad (8.7)$$

where h_L and h_R are constant elements of $\text{OSp}(2, \mathbb{N})$. The action on the right is the residual symmetry subjects to the integrability condition (8.3). On the other hand, left symmetry appears after imposing that $a_\tau d\tau$ is a flat connection defined by a left invariant Maurer-Cartan form. In the next section we will see how the right symmetry is lost after restricting the phase space to asymptotically AdS spaces, and that the left symmetry, which is the only one that persists, implements the $\text{OSp}(2, \mathbb{N})$ symmetry in the super-Schwarzian theory at the boundary.

8.2 Hamiltonian reduction: extended super-Schwarzian action

The purpose of this subsection is to constrict the configurations to be asymptotically AdS_2 supergeometries, which results in the following form of the gauge field

$$a = \left[L_1 + \left(\mathcal{L} + \frac{1}{2} \mathcal{P}^{AB} \mathcal{P}^{AB} \right) L_{-1} + \psi^A G_{-\frac{1}{2}}^A + \frac{1}{2} \mathcal{P}^{AB} T^{AB} \right] df. \quad (8.8)$$

Subsequently, the Hamiltonian reduction is performed by considering the Gauss decomposition of an arbitrary $\text{OSp}(2, \mathbb{N})$ group element⁹

$$g = \exp \left[h(f) L_1 + \eta_A(f) G_{\frac{1}{2}}^A \right] \exp \left[2q(f) L_0 + \frac{1}{2} \sigma_{AB}(f) T^{AB} \right] \exp \left[k(f) L_{-1} + \zeta_A(f) G_{-\frac{1}{2}}^A \right] \quad (8.9)$$

⁹See appendix A for an explicit matrix representation of g .

whose associated Maurer-Cartan form is assumed to be equal to the gauge field encoding the AdS₂ supergeometries (8.8), i.e. $a_f = g^{-1}\partial_f g$. This equality leads to the set of relations

$$\mathcal{L} = -q'' - (q')^2 + \zeta_B \zeta_B' - \frac{1}{2}(M_{AB}')^2 \quad (8.10)$$

$$\mathcal{P}_{AB} = M_{CA} M_{CB}' + \zeta_A \zeta_B \quad (8.11)$$

$$\psi_A = \zeta_A' + q' \zeta_A + M_{CA} M_{CB}' \zeta_B \quad (8.12)$$

where

$$\zeta_A = -e^q \eta_B' M_{BA} \quad k = -q' \quad e^{-2q} = h' + \eta_B \eta_B' \quad M = e^{\frac{1}{2}\sigma_{AB} m_{AB}} \quad (8.13)$$

with $(m^{AB})^C_D = 2\delta^{C[A}\delta_D^{B]}$ and primes denote differentiation with respect to f . At this point, one can notice that the condition $g^{-1}\partial_f g = L_1 + \dots$ fixes the right symmetry (h_R) to be generated only by L_1 . One can completely fix this residual freedom by absorbing h_R in a redefinition of the function $h(f)$ appearing in (8.9) and thus the right action is not a symmetry of the reduced action principle anymore.

By replacing (8.8) in (8.6), the reduced action becomes

$$I_{\text{red}} = \frac{ky_0}{2\pi} \int df \mathcal{L}. \quad (8.14)$$

Then, we proceed to compute $\mathcal{L}$ in terms of the fields defining the Gauss decomposition (8.9). One can readily see from (8.10) that in terms of the field (q, ζ_A, M_{AB}) the reduced model corresponds to the action of a free superparticle related to the coadjoint orbit passing through $\mathcal{L}_* = \mathcal{P}_*^{AB} = \psi_*^A = 0$. On the other hand, solving the constraints (8.13) in terms of the variables (h, η_A, M_{AB}) we are able to find a generalization of the super-Schwarzian action. In this case, (8.10) takes the following form

$$2\mathcal{L} = \frac{(h'' + \eta_B \eta_B'')'}{h' + \eta_A \eta_A'} - \frac{3}{2} \left(\frac{h'' + \eta_B \eta_B''}{h' + \eta_A \eta_A'} \right)^2 + \frac{2\eta_B' \eta_B''}{h' + \eta_A \eta_A'} + \frac{2\eta_B' \eta_C'}{h' + \eta_A \eta_A'} M_{BE} M_{CE}' - (M_{AB}')^2 \quad (8.15)$$

which turns out to be an explicit representation of the extended super-Schwarzian as it is invariant under global $\text{OSp}(2, \mathbb{N})$ transformations inherited from the left action (8.7). Infinitesimally, this corresponds to $\delta g = \lambda_L g$ that in terms of the fields h, η_A, M_{AB} reads

$$\delta h = \xi_+ + \xi_- h + \xi_0 h^2 - \epsilon_-^A \eta_A h + \epsilon_+^A \eta_A \quad (8.16)$$

$$\delta \eta_A = \frac{1}{2} \xi_- \eta_A + \xi_0 \eta_A h + \epsilon_-^A h - \epsilon_-^B \eta_B \eta_A - \epsilon_+^A - \frac{1}{2} \rho_{AB} \eta_B \quad (8.17)$$

$$\delta M_{AB} = -\xi_0 \eta_A \eta_C M_{CB} + \epsilon_-^C \eta_A M_{CB} - \epsilon_-^A \eta_C M_{CB} - \frac{1}{2} \rho_{AC} M_{CB} \quad (8.18)$$

with Lie-algebra-parameter $\lambda_L = \xi_m L_m + \epsilon_p^A G_p^A + \frac{1}{2} \rho_{AB} T_{AB}$.

Until now we have constructed the super-Schwarzian theory at zero temperature. To introduce finite temperature, we need to provide a map in such a way that the connection a in (8.8) lie on the orbit of configurations with trivial holonomy, $\mathcal{L}_* = \frac{\pi^2}{\beta^2}$ and $\psi_*^A =$

$\mathcal{P}_*^{AB} = 0$. This restriction is ensured by introducing new fields $\theta(f)$ and $\nu_A(f)$ through the following map

$$h(f) = \tan(\theta(f)/2) \quad \eta_A(f) = \sqrt{h'(f)} \nu_A(f) \quad (8.19)$$

where $\nu_A(f+2\pi) = \nu_A(f)$ and θ satisfies $\theta(f+2\pi) = \theta(f) + 2\pi$ (for definiteness we assume that $\beta = 2\pi$). We can prove that this is the right map by studying the perturbations of the Lagrangian (8.15) around the saddle solution

$$\theta(f) = f + \varepsilon(f) \quad \nu_A(f) \quad M_{AB} = \delta_{AB} + \sigma_{AB}(f) \quad (8.20)$$

with ε , ν_A and σ_{AB} small deviations. Keeping up to quadratic terms, we find

$$\begin{aligned} \mathcal{L} = & \frac{1}{4} + \frac{1}{2}(\varepsilon' + \varepsilon''') + \frac{1}{4}(\varepsilon')^2 - \frac{1}{2}\varepsilon'\varepsilon''' - \frac{3}{4}(\varepsilon'')^2 \\ & + \frac{3}{2}\nu'_A\nu''_A + \frac{1}{2}\nu_A\nu'''_A - \frac{1}{4}\nu_A\nu'_A - \frac{1}{2}(\sigma'_{AB})^2 + \dots \end{aligned} \quad (8.21)$$

Recalling that $\beta = 2\pi$, the above relation shows that (8.20) is a perturbation of the point $\mathcal{L} = \pi^2/\beta^2$. Furthermore, they vanish for $\mathfrak{osp}(2, \mathbb{N})$ transformations

$$\varepsilon = \varepsilon_n e^{inf} \quad \nu_A = \nu_A^p e^{ipf} \quad \sigma_{AB} = \sigma_{AB}^0 \quad (8.22)$$

with $n = 0, \pm 1$, $p = \pm \frac{1}{2}$ and σ_{AB}^0 a constant $\mathfrak{so}(N)$ Lie algebra element. This last step tells us that map (8.19) brings the system to the orbit associated to the Neveu-Schwarz sector $[\Gamma_- \text{ for } N = 1 \text{ and } N = 2 \text{ in (7.42)}]$.

9 Beyond the highest-weight ansatz

Usually, the gauge connection a is considered to have highest-weight form

$$a = L_1 + Q \quad (9.1)$$

where

$$[L_{-1}, Q] = 0. \quad (9.2)$$

For example, it can be easily shown that the connections (3.3) and (7.14) fulfill the highest-weight condition (9.2). The main motivation for this choice is that the leading terms in the metric are fixed, which simplifies the analysis considerably.

In [38] more general boundary conditions for JT gravity were considered that went beyond the highest weight ansatz allowing all fields in the $\mathfrak{sl}(2)$ basis to fluctuate. In this section, we show that one can consider more general sets of AdS_2 boundary conditions also in the supergravity context.

Note, that the construction of the boundary action and the integrability conditions of section 8.1 do not rely on the highest-weight ansatz and thus can be used to address the consistency of the variation principle for the boundary conditions described below.

In this section, we shall only analyze the asymptotic symmetries. The full analysis of boundary theories shall be postponed to some future work.

9.1 Some field redefinitions

Our strategy is quite simple. First we define for each set of boundary conditions the set of large gauge transformations that preserve the form of asymptotic fields. Then we make some redefinitions of the fields and the transformation parameters, so that the action of asymptotic symmetries coincides with a representation of a known algebra. This algebra is then identified with the asymptotic symmetry transformations for that boundary conditions. Of course, the success of this procedure depends on our ability to find the right redefinitions. The method is thus non-generic. It works for the cases that we list below. In this subsection we present some general redefinitions that we write for any $\mathfrak{osp}(2, \mathbb{N})$ model.

For our purposes it is enough to analyze the connection variables only. Let us write the asymptotic field a [see (8.2)] in components as

$$a = [\mathcal{L}^m(\tau)L_m + \psi_A^\alpha(\tau)G_\alpha^A + \frac{1}{2}\mathcal{S}_{AB}(\tau)T_{AB}] d\tau. \quad (9.3)$$

Note that in contrast to (3.3) and (7.14), all fields in the $\mathfrak{osp}(2, \mathbb{N})$ basis of the gauge connection (9.3) are allowed to vary.

Now, let us rescale the fermionic fields, $\psi_A^+ \equiv (\mathcal{L}^+)^{-1/2} \psi_A^{\frac{1}{2}}$, $\psi_A^- \equiv (\mathcal{L}^+)^{1/2} \psi_A^{-\frac{1}{2}}$ and define

$$\psi_A \equiv \psi_A^- - \psi_A^{+'} - \mathcal{J}\psi_A^+ - \mathcal{P}_{AC}\psi_C^+ \quad (9.4)$$

$$\mathcal{P}_{AB} \equiv \mathcal{S}_{AB} + \psi_A^+\psi_B^+ \quad (9.5)$$

$$\mathcal{L} \equiv \mathcal{T} - \mathcal{J}^2 - \mathcal{J}' - 2\psi_A^+\psi_A^- + \psi_A^+\psi_A^{+'} + \mathcal{P}_{AB}\psi_A^+\psi_B^+ - \frac{1}{2}\mathcal{P}_{AB}\mathcal{P}_{AB} \quad (9.6)$$

where

$$\mathcal{T} \equiv \mathcal{L}^+\mathcal{L}^- \quad \mathcal{J} \equiv -\frac{1}{2}\partial_\tau \ln \mathcal{L}^+ + \frac{1}{2}\mathcal{L}^0. \quad (9.7)$$

In the absence of fermions and $\mathrm{SO}(N)$ gauge fields, $\mathcal{L}$ in (9.6) reduces to the mass function defined in [38].

We also write in components the parameter of asymptotic symmetry transformations

$$\lambda = \xi^m L_m + \epsilon_A^\alpha G_\alpha^A + \frac{1}{2}\nu_{AB}T_{AB} \quad (9.8)$$

that is assumed to depend on τ only. The components of a are transformed as

$$\delta_\lambda \mathcal{L}^0 = \xi^{0'} + 2\mathcal{L}^+\xi^- - 2\mathcal{L}^-\xi^+ - 2\psi_A^{\frac{1}{2}}\epsilon_A^{-\frac{1}{2}} - 2\psi_A^{-\frac{1}{2}}\epsilon_A^{\frac{1}{2}} \quad (9.9)$$

$$\delta_\lambda \mathcal{L}^\pm = \xi^{\pm'} \mp \mathcal{L}^0 \xi^\pm \pm \mathcal{L}^\pm \xi^0 - 2\psi_A^{\pm\frac{1}{2}}\epsilon_A^{\pm\frac{1}{2}} \quad (9.10)$$

$$\delta_\lambda \psi_A^{\pm\frac{1}{2}} = \epsilon_A^{\pm\frac{1}{2}'} \mp \frac{1}{2}\mathcal{L}^0 \epsilon_A^{\pm\frac{1}{2}} \pm \mathcal{L}^\pm \epsilon_A^{\mp\frac{1}{2}} \pm \frac{1}{2}\xi^0 \psi_A^{\pm\frac{1}{2}} \mp \xi^\pm \psi_A^{\mp\frac{1}{2}} + \mathcal{P}_{AB}\epsilon_B^{\pm\frac{1}{2}} - \nu_{AB}\psi_B^{\pm\frac{1}{2}} \quad (9.11)$$

$$\delta_\lambda \mathcal{P}_{AB} = \nu_{AB}' - \psi_A^{\frac{1}{2}}\epsilon_B^{-\frac{1}{2}} + \psi_B^{\frac{1}{2}}\epsilon_A^{-\frac{1}{2}} + \psi_A^{-\frac{1}{2}}\epsilon_B^{\frac{1}{2}} - \psi_B^{-\frac{1}{2}}\epsilon_A^{\frac{1}{2}} + \mathcal{P}_{AC}\nu_{CB} - \mathcal{P}_{BC}\nu_{CA}. \quad (9.12)$$

It is useful to introduce new transformation parameters

$$\xi = (\mathcal{L}^+)^{-1} \xi^+ \quad (9.13)$$

$$\epsilon_A = (\mathcal{L}^+)^{-1/2} \epsilon_A^{\frac{1}{2}} - \psi_A^+ \xi \quad (9.14)$$

$$\mu_{AB} = \nu_{AB} - \xi \mathcal{S}_{AB} + \psi_A^+ \epsilon_B - \psi_B^+ \epsilon_A. \quad (9.15)$$

After these redefinitions we reproduce the transformation rules (7.20)–(7.22) for quantities $\mathcal{L}$, ψ_A , $\mathcal{P}_{AB}$ for the highest-weight asymptotic conditions.

9.2 Asymptotic symmetries for $\mathfrak{osp}(2, 1)$ BF gravity

9.2.1 Loop group boundary condition

Our first example corresponds to the case when all components of the boundary connection a are allowed to vary. This form of boundary conditions is preserved by arbitrary gauge transformations with the parameters $\xi^m(\tau)$, $\epsilon^\alpha(\tau)$ that depend just on the boundary coordinate τ . The corresponding asymptotic algebra is just the algebra of one-dimensional $\mathfrak{osp}(2, 1)$ gauge transformations or the current algebra of $\mathfrak{osp}(2, 1)$. This very simple example has a rather important property. The asymptotic symmetry algebra obtained contains two independent one-dimensional supergenerators, i.e. we have an $\mathcal{N} = 2$ boundary supersymmetry. Unfortunately, we were not able to get such an extended supersymmetry in less trivial examples.

9.2.2 Superconformal boundary condition

Alternatively, let us consider the boundary conditions $\mathcal{L}^0 = 0$, $\mathcal{L}^+ = 1$ and $\psi^{\frac{1}{2}} = 0$ in (9.4) and (9.6). In terms of $\mathcal{L} = \mathcal{L}^-$ and $\psi = \psi^-$ the gauge connection (9.3) reduces to (3.3).

The gauge parameter preserving these boundary conditions is

$$\lambda = \xi L_1 - \xi' L_0 + \left(\frac{1}{2} \xi'' + \mathcal{L} \xi + \psi \epsilon \right) L_{-1} + \epsilon G_{\frac{1}{2}} + (-\epsilon' + \xi \psi) G_{\frac{1}{2}}. \quad (9.16)$$

The transformations (9.10), (9.11) of $\mathcal{L}$ and ψ take the form (3.7) and (3.8), thus giving the superconformal symmetry algebra that has been considered in detail above.

9.2.3 Warped superconformal boundary condition

Let us set $\mathcal{L}^0 = \psi^{\frac{1}{2}} = 0$ while allowing $\mathcal{L}^\pm$ and $\psi^- \equiv \psi$ to vary. Through the conditions $\delta \mathcal{L}^0 = \delta \psi^{\frac{1}{2}} = 0$, we can express the parameters ξ^- and $\epsilon^{-\frac{1}{2}}$ in terms of ξ^0 , ξ and ϵ

$$\xi^- = (\mathcal{L}^+)^{-1} \left[-\frac{1}{2} \xi^{0'} + (\mathcal{L} + \mathcal{J}' + \mathcal{J}^2) \xi + \psi \epsilon \right] \quad (9.17)$$

$$\epsilon^{-\frac{1}{2}} = (\mathcal{L}^+)^{-1/2} (-\epsilon' + \mathcal{J} \epsilon + \psi \xi). \quad (9.18)$$

We have the following transformation rules

$$\delta \mathcal{J} = \mathcal{J} \xi' + \mathcal{J}' \xi - \frac{1}{2} \xi^{0'} - \frac{1}{2} \xi'' \quad (9.19)$$

$$\delta \mathcal{T} = 2\mathcal{T} \xi' + \mathcal{T}' \xi - \mathcal{J} \xi^{0'} - \frac{1}{2} \xi^{0''} + 3\psi \epsilon' + \psi' \epsilon \quad (9.20)$$

$$\delta \psi = -\epsilon'' - \mathcal{L} \epsilon + \frac{3}{2} \psi \xi' + \psi' \xi. \quad (9.21)$$

Note that $\mathcal{J}$ is unaffected by the supersymmetry, and ψ transforms as (3.8), while $\mathcal{L} = \mathcal{T} - \mathcal{J}^2 - \mathcal{J}'$ transforms exactly as in (3.7). The algebra of $\mathcal{J}$, $\mathcal{T}$, ψ corresponds to the

warped superconformal algebra (see e.g. [56]) with a twist term. For vanishing fermions it reduces to the algebra (6.28) in [38].

Using as generators Fourier modes for the functions $\mathcal{J}, \mathcal{L}, \psi$ with normalizations $L_n = k/(2\pi) \oint e^{in\tau} \mathcal{L} d\tau$ (and similarly for J_n and ψ_n), shifting $\lambda := \xi^0 + \xi'$, and converting the transformation behavior above into commutation relations yields the (untwisted) warped superconformal algebra,

$$[L_n, L_m] = (n - m) L_{n+m} + \frac{k}{2} n^3 \delta_{n+m,0} \quad (9.22)$$

$$[L_n, \psi_m] = (n/2 - m) \psi_{n+m} \quad (9.23)$$

$$[L_n, J_m] = -m J_{n+m} \quad (9.24)$$

$$\{\psi_n, \psi_m\} = -L_{n+m} + ik n^2 \delta_{n+m,0} \quad (9.25)$$

$$[\psi_n, J_m] = 0 \quad (9.26)$$

$$[J_n, J_m] = \frac{k}{2} n \delta_{n+m,0}. \quad (9.27)$$

9.2.4 All $\mathcal{L}^m$ vary while $\psi^{\frac{1}{2}} = 0$

The condition $\delta\psi^{\frac{1}{2}} = 0$ leads to an expression for ξ^+ through ξ and σ ,

$$\epsilon^{-\frac{1}{2}} = (\mathcal{L}^+)^{-1/2} (-\epsilon' + \mathcal{J}\epsilon + \psi\xi). \quad (9.28)$$

Thus, we have the following transformation rules for gauges fields

$$\delta\mathcal{L}^0 = \xi^{0'} + 2\alpha - 2\mathcal{T}\xi - 2\psi\epsilon \quad (9.29)$$

$$\delta\mathcal{J} = -\frac{1}{2}\xi'' + \mathcal{J}\xi' + \alpha + (-\mathcal{T} + \mathcal{J}')\xi - \psi\epsilon \quad (9.30)$$

$$\delta\mathcal{T} = \mathcal{T}\xi' - 2\mathcal{T}\mathcal{J}\xi + \alpha' + 2\mathcal{J}\alpha + 2\psi\epsilon' - 2\mathcal{J}\psi\epsilon \quad (9.31)$$

$$\delta\psi = -\epsilon'' - \mathcal{L}\epsilon + \frac{3}{2}\xi'\psi + \psi'\xi \quad (9.32)$$

where $\alpha = \mathcal{L}^+\xi^-$.

To remove non-linearities in $\delta\mathcal{T}$ we redefine the parameter α as $\alpha = -\frac{1}{2}\lambda' + \mathcal{T}\xi + \psi\epsilon$, yielding

$$\delta\mathcal{J} = \mathcal{J}\xi' + \mathcal{J}'\xi - \frac{1}{2}\lambda' - \frac{1}{2}\xi'' \quad (9.33)$$

$$\delta\mathcal{T} = 2\mathcal{T}\xi' + \mathcal{T}'\xi - \mathcal{J}\lambda' - \frac{1}{2}\lambda'' + 3\psi\epsilon' + \psi'\epsilon \quad (9.34)$$

$$\delta\psi = -\epsilon'' - \mathcal{L}\epsilon + \frac{3}{2}\xi'\psi + \psi'\xi. \quad (9.35)$$

The parameter ξ^0 only appears in $\delta\mathcal{L}^0$ which now reads

$$\delta\mathcal{L}^0 = \xi^{0'} - \lambda'. \quad (9.36)$$

The algebra is linear and closed on $(\mathcal{J}, \mathcal{T}, \mathcal{L}^0, \psi)$. It is an $\mathcal{N} = 1$ super Virasoro plus a $\mathfrak{u}(1)$ current algebra plus translations on $\mathcal{L}^0$. Except for the translations, this algebra is the same as we had in the previous section for the warped conformal case.

9.3 Asymptotic symmetries for $\mathfrak{osp}(2, 2)$ BF gravity

9.3.1 Loop group boundary condition

By allowing all components in the connection to fluctuate, exactly as we did in section 9.2.1 for $N = 1$, one obtains that the current algebra of $\mathfrak{osp}(2, 2)$ plays the role of an asymptotic symmetry algebra. Again, the doubled number of one-dimensional supersymmetries is the main lesson here.

9.3.2 Extended superconformal boundary condition

Let us impose the boundary conditions $\mathcal{L}^0 = 0$, $\mathcal{L}^+ = 1$ and $\psi_A^{\frac{1}{2}} = 0$. Stability of these conditions under the gauge transformations implies the following relations on the gauge parameter components

$$\xi^0 = -\xi' \quad (9.37)$$

$$\xi^- = \frac{1}{2}\xi'' + \left(\mathcal{L} + \frac{1}{2}\mathcal{P}_{AB}\mathcal{P}_{AB}\right)\xi + \psi_A\epsilon_A \quad (9.38)$$

$$\epsilon_A^{-\frac{1}{2}} = \psi_A\xi - \epsilon'_A - \mathcal{P}_{AB}\epsilon_B \quad (9.39)$$

so that

$$\delta\mathcal{L} = \frac{1}{2}\xi''' + 2\xi'\mathcal{L} + \mathcal{L}'\xi + 3\psi_A\epsilon'_A + \psi'_A\epsilon_A - \mathcal{P}_{AB}\mu'_{AB} \quad (9.40)$$

$$\delta\psi_A = \frac{3}{2}\xi'\psi_A + \xi\psi'_A - \mu_{AB}\psi_B - \epsilon''_A - \mathcal{L}\epsilon_A - \mathcal{P}'_{AB}\epsilon_B - 2\mathcal{P}_{AB}\epsilon'_B \quad (9.41)$$

$$\delta\mathcal{P}_{AB} = \mu'_{AB} + \xi'\mathcal{P}_{AB} + \mathcal{P}'_{AB}\xi + 2\mathcal{P}_{C[A}\mu_{B]C} + 2\psi_{[A}\epsilon_{B]}. \quad (9.42)$$

Equations (9.41) and (7.21) coincide for $N = 2$. Furthermore, $\mathcal{L}$, ψ_A and $\mathcal{P}_{AB}$ generate one copy of the $\text{SO}(2)$ -extended super-Virasoro algebra.

9.3.3 Extended warped superconformal boundary condition

Now, we relax the restriction on $\mathcal{L}^+$ and require $\mathcal{L}^0 = \psi^{\frac{1}{2}} = 0$ leaving $\mathcal{L}$ as defined in (9.6) with (9.7), $\mathcal{J} = -\frac{1}{2}\partial_\tau \ln \mathcal{L}^+$, $\psi_A = \psi_A^-$ and $\mathcal{P}_{AB} = \mathcal{S}_{AB}$ to vary on the boundary. The above conditions imply the following relations on the parameters

$$\xi^- = (\mathcal{L}^+)^{-1} \left[-\frac{1}{2}\xi^{0'} + \left(\mathcal{L} + \mathcal{J}' + \mathcal{J}^2 + \frac{1}{2}\mathcal{P}_{AB}\mathcal{P}_{AB} \right) \xi + \psi_A\epsilon_A \right] \quad (9.43)$$

$$\epsilon_A^{-\frac{1}{2}} = (\mathcal{L}^+)^{-1/2} [\psi_A\xi - \epsilon'_A + \mathcal{J}\epsilon_A - \mathcal{P}_{AB}\epsilon_B] . \quad (9.44)$$

The transformation of $\mathcal{J}$, $\mathcal{L}$ and ψ_A read (using again $\lambda := \xi^0 + \xi'$)

$$\delta\mathcal{J} = -\frac{1}{2}\lambda' + \mathcal{J}\xi' + \mathcal{J}'\xi \quad (9.45)$$

$$\delta\mathcal{L} = \frac{1}{2}\xi''' + 2\xi'\mathcal{L} + \mathcal{L}'\xi + 3\psi_A\epsilon'_A + \psi'_A\epsilon_A - \mathcal{P}_{AB}\mu'_{AB} \quad (9.46)$$

$$\delta\psi_A = \frac{3}{2}\xi'\psi_A + \xi\psi'_A - \mu_{AB}\psi_B - \epsilon''_A - \mathcal{L}\epsilon_A - \mathcal{P}'_{AB}\epsilon_B - 2\mathcal{P}_{AB}\epsilon'_B \quad (9.47)$$

$$\delta\mathcal{P}_{AB} = \mu'_{AB} + \xi'\mathcal{P}_{AB} + \mathcal{P}'_{AB}\xi + 2\mathcal{P}_{C[A}\mu_{B]C} + 2\psi_{[A}\epsilon_{B]}. \quad (9.48)$$

Dilaton supergravity theory	Boundary fields	Boundary theory
$\mathfrak{osp}(2, 1)$ -BF	Θ	$\mathcal{N} = 1$ super-Schwarzian
	K	$\mathcal{N} = 1$ free superparticle
$\mathfrak{osp}(2, 2)$ -BF	Θ_A	$\mathcal{N} = 2$ super-Schwarzian
	K	$\mathcal{N} = 2$ SCQM
$\mathfrak{osp}(2, \mathbb{N})$ -BF	(h, η_A, M_{AB})	$\mathcal{N} = N$ super-Schwarzian
	(q, ζ_A, M_{AB})	$\mathcal{N} = N$ free superparticle

Table 1. Boundary dynamics for (extended) dilaton supergravities in 2D.

The transformation of $\mathcal{J}$ is equal to the one for the warped superconformal case (9.19), meaning that $\mathcal{J}$ is also unaffected by the $\text{SO}(2)$ gauge transformations. The algebra of $\mathcal{J}, \mathcal{L}, \psi_A, \mathcal{P}_{AB}$ corresponds to the extended warped superconformal algebra.

10 Concluding remarks

We have provided a consistent set of asymptotic conditions for two-dimensional dilaton supergravity preserved by the $\text{SO}(N)$ -extended group of reparametrizations of the super-circle. This fall-off was shown to be relaxed enough in order to permit the fluctuation of the dilaton multiplet without spoiling the superreparametrization symmetry at infinity. Through a detailed analysis of the asymptotic dynamics of the model, we have established that the boundary theory strongly depends on the coadjoint orbits associated to the super-Virasoro algebra. When considering orbits associated to black hole solutions, the boundary field corresponds to a superdiffeomorphism, whose dynamics is controlled by the extended super-Schwarzian theory. On the other hand, the orbits where the representative vanishes — as BPS states — lead to a dynamics described by (super)conformal quantum mechanics for $\mathcal{N} \leq 2$, and the extended free superparticle for $\mathcal{N} > 2$. The results are summarized in table 1.

These are the first steps towards the understanding of the relation between quantum dilaton supergravity and its corresponding supersymmetric dual. In connection with recent results, we would like to extend the analysis of [57, 58] by tracing the relation between Wilson lines and entanglement entropy in the supersymmetric BF formulation with the corresponding boundary observables. Furthermore, it would be interesting to understand the consequences of the different asymptotic dynamics in the factorization problem of quantum dilaton supergravity. One important step here is to expand the analysis of [59] to the supersymmetric case with Lorentzian signature.

As shown along this work, the group structure of the asymptotic symmetries associated to the theory allows to determine in a consistent manner its boundary dynamics. Thus, for instance, in view of the results obtained in [36, 37], it would be interesting to explore whether it is possible to obtain a reparametrization theory from generalized models of dilaton (super)gravity [60]. Another interesting aspect to explore is related to non-perturbative local quantum triviality of bosonic [61] and supersymmetric [62] dilaton theories in 2D. The

absence of any local quantum effects suggests that these theories are fully equivalent to some effective quantum theories at the boundary.

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A Matrix representation of $\mathbf{OSp}(2, \mathbb{N})$

The $\mathbf{osp}(2, \mathbb{N})$ algebra is given by

$$\begin{aligned} [J_I, J_J] &= \epsilon_{IJK} J^K & [J_I, Q_\alpha^A] &= \frac{1}{2} (\Gamma_I)^\beta_\alpha Q_\beta^A \\ [Q_\alpha^A, T^{BC}] &= 2\delta^{A[B} Q_\alpha^{C]} & [T^{AB}, T^{CD}] &= 4\delta^{[A|[C} T^{D]B]} \\ \{Q_\alpha^A, Q_\beta^B\} &= -\frac{1}{2} \delta^{AB} (C\Gamma^I)_{\alpha\beta} J_I - \frac{1}{4} C_{\alpha\beta} T^{AB} \end{aligned} \quad (\text{A.1})$$

where capital letters $A, B, C, D, \dots$ run from 1 to N . Here, the Γ -matrices satisfy the Clifford algebra in three dimensions; $\{\Gamma_I, \Gamma_J\} = 2\eta_{IJ}$, where the metric is chosen as $\eta_{IJ} = \text{diag}(1, 1, -1)$ and $\epsilon_{012} = -1$. The generators of the $\text{SO}(N)$ subgroup were denoted as T^{AB} . The charge conjugation matrix is given by $C_{\alpha\beta} = \epsilon_{\alpha\beta}$, where $\epsilon_{+-} = -1$.

The invariant bilinear metric associated to $\mathbf{osp}(2, \mathbb{N})$ is given by the supertrace, whose nonvanishing components are chosen as

$$\text{Str}[J_I J_J] = \frac{1}{2} \eta_{IJ} \quad \text{Str}[Q_\alpha^A Q_\beta^B] = \frac{1}{2} C_{\alpha\beta} \delta^{AB} \quad \text{Str}[T^{AB} T^{CD}] = 4\delta^{A[C} \delta^{D]B} \quad (\text{A.2})$$

which is identically satisfied by the spinorial representation of $\mathbf{osp}(2, \mathbb{N})$;

$$J_I = \left(\begin{array}{c|c} \frac{1}{2} (\Gamma_I)^\alpha_\beta & 0_{2 \times N} \\ \hline 0_{N \times 2} & 0_{N \times N} \end{array} \right) \quad Q_\alpha^A = \left(\begin{array}{c|c} 0_{2 \times 2} & \frac{1}{2} \delta_\alpha^\beta \delta_C^A \\ \hline -\frac{1}{2} \delta^{AB} C_{\alpha\gamma} & 0_{N \times N} \end{array} \right) \quad T^{AB} = \left(\begin{array}{c|c} 0_{2 \times 2} & 0_{2 \times N} \\ \hline 0_{N \times 2} & 2\delta^{C[A} \delta_D^{B]} \end{array} \right). \quad (\text{A.3})$$

An explicit matrix representation for the Γ -matrices is given by

$$\Gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \Gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \Gamma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (\text{A.4})$$

In order to perform the analysis of the asymptotic structure of the theory it is useful to express $\mathfrak{osp}(2, \mathbb{N})$ as follows

$$[L_m, L_n] = (m - n) L_{m+n} \quad [L_m, G_p^A] = \left(\frac{m}{2} - p\right) G_{m+p}^A \quad (\text{A.5})$$

$$[G_p^A, T^{BC}] = 2\delta^{A[B} G_p^{C]} \quad [T^{AB}, T^{CD}] = 4\delta^{[A|[C} T^{D][B]} \quad (\text{A.6})$$

$$\{G_p^A, G_q^B\} = -2L_{p+q}\delta^{AB} - (p - q)T^{AB} \quad (\text{A.7})$$

where the map of the generators is given by

$$L_0 = J_1 \quad L_{-1} = J_2 - J_0 \quad L_1 = J_2 + J_0 \quad (\text{A.8})$$

$$G_{-\frac{1}{2}}^A = 2Q_+^A \quad G_{\frac{1}{2}}^A = 2Q_-^A. \quad (\text{A.9})$$

The matrix representation in this basis reads

$$L_{-1} = \left(\begin{array}{c|c} \begin{matrix} 0 & -1 \\ 0 & 0 \end{matrix} & 0_{2 \times N} \\ \hline 0_{N \times 2} & 0_{N \times N} \end{array} \right) \quad L_0 = \left(\begin{array}{c|c} \begin{matrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{matrix} & 0_{2 \times N} \\ \hline 0_{N \times 2} & 0_{N \times N} \end{array} \right) \quad L_1 = \left(\begin{array}{c|c} \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} & 0_{2 \times N} \\ \hline 0_{N \times 2} & 0_{N \times N} \end{array} \right) \quad (\text{A.10})$$

$$G_{-\frac{1}{2}}^A = \left(\begin{array}{c|c} \begin{matrix} 0_{2 \times 2} & \delta_+^\beta \delta_C^A \\ \hline -\delta^{AB} C_{+\gamma} & 0_{N \times N} \end{matrix} & \\ \hline \end{array} \right) \quad G_{\frac{1}{2}}^A = \left(\begin{array}{c|c} \begin{matrix} 0_{2 \times 2} & \delta_-^\beta \delta_C^A \\ \hline -\delta^{AB} C_{-\gamma} & 0_{N \times N} \end{matrix} & \\ \hline \end{array} \right). \quad (\text{A.11})$$

The nonvanishing components of the supertrace are now given by

$$\text{Str}[L_{-1}L_1] = \text{Str}[L_1L_{-1}] = -1 \quad \text{Str}[L_0^2] = \frac{1}{2} \quad (\text{A.12})$$

$$\text{Str}\left[G_{-\frac{1}{2}}^A G_{\frac{1}{2}}^B\right] = -\text{Str}\left[G_{\frac{1}{2}}^A G_{-\frac{1}{2}}^B\right] = -2\delta^{AB}. \quad (\text{A.13})$$

The group element in (8.9) and its inverse can be explicitly computed by using the representation (A.10) and (A.11), which read

$$g = \begin{bmatrix} e^q & -ke^q & e^q \zeta_B \\ he^q & e^{-q} - khe^q + M^{CD} \eta_C \zeta_D & e^q h \zeta_B + \eta^C M_{CB} \\ -e^q \eta^A & e^q k \eta^A + M^{AC} \zeta_C & -e^q \eta^A \zeta_B + M^A_B \end{bmatrix} \quad (\text{A.14})$$

and

$$g^{-1} = \begin{bmatrix} e^{-q} - khe^q + M^{CD} \eta_C \zeta_D & ke^q & -e^q k \eta_B - M_{BC} \zeta^C \\ -he^q & e^q & -e^q \eta_B \\ e^q h \zeta^A + M^{CA} \eta_C & -e^q \zeta^A & e^q \zeta^A \eta_B + M_B^A \end{bmatrix} \quad (\text{A.15})$$

respectively.

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