RECEIVED: September 5, 2017

REVISED: October 17, 2017

ACCEPTED: October 30, 2017

PUBLISHED: November 17, 2017

Testing minimal flavor violation in leptoquark models of the $R_{K^{(*)}}$ anomaly

Daniel Aloni, Avital Dery, Claudia Frugiuele and Yosef Nir

Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot, 7610001 Israel

E-mail: daniel.aloni@weizmann.ac.il, avital.dery@weizmann.ac.il, claudia.frugiuele@weizmann.ac.il, yosef.nir@weizmann.ac.il

ABSTRACT: The $R_{K^{(*)}}$ anomaly can be explained by tree level exchange of leptoquarks. We study the consequences of subjecting these models to the principle of minimal flavor violation (MFV). We consider MFV in the linear regime, and take the charged lepton Yukawa matrix to be the only spurion that violates lepton flavor universality. We find that a combination of constraints from a variety of processes — $b \rightarrow s\mu\mu$, $b \rightarrow s\tau\tau$, $b \rightarrow s\nu\nu$, $b\bar{b} \rightarrow \tau\tau$ and $b \rightarrow c\tau\nu$ — excludes MFV in these models.

KEYWORDS: Phenomenological Models

ARXIV EPRINT: [1708.06161](https://arxiv.org/abs/1708.06161)

Contents

1	Introduction	1
2	MFV for leptoquarks	3
3	Minimal lepton flavor violation (MLFV)	4
3.1	$B \rightarrow K^{(*)}\ell^+\ell^-$	5
3.2	$B \rightarrow K^{(*)}\nu\bar{\nu}$	6
3.3	Summary of MLFV	7
4	Minimal quark flavor violation (MQFV)	8
4.1	Back to $R_{K^{(*)}}$	9
4.2	$b\bar{b} \rightarrow \tau^+\tau^-$	10
4.3	$R_{D^{(*)}}$	11
4.4	Summary of MQFV	12
5	Conclusions	12
A	$B_s - \bar{B}_s$ mixing	14
B	LHC direct searches for U_1^μ	15
C	Perturbative unitarity	16
D	$pp \rightarrow \mu^+\mu^-$	17
E	$R_{\tau/K}$ and $R_{D^{(*)}}$	18

1 Introduction

Within the Standard Model (SM), lepton flavor universality (LFU) is respected by the weak interactions. Consequently, LFU is predicted to hold — up to (calculable) phase-space effects — in processes where the Yukawa interactions are negligible. Hints of violation of LFU have, however, been observed by the LHCb experiment in $B \rightarrow K^{(*)}\ell^+\ell^-$ decays. While LFU implies that the ratios

$$R_{K^{(*)},[a,b]} = \frac{\int_a^b dq^2 [d\Gamma(B \rightarrow K^{(*)}\mu^+\mu^-)/dq^2]}{\int_a^b dq^2 [d\Gamma(B \rightarrow K^{(*)}e^+e^-)/dq^2]} \quad (1.1)$$

(q^2 is the invariant dilepton mass-squared) should be very close to unity, the measurements give [1, 2]

$$\begin{aligned} R_{K,[1,6]\text{GeV}^2} &= 0.745_{-0.074}^{+0.090} \pm 0.036, \\ R_{K^*,[1.1,6.0]\text{GeV}^2} &= 0.69_{-0.07}^{+0.11} \pm 0.05, \\ R_{K^*,[0.045,1.1]\text{GeV}^2} &= 0.66_{-0.07}^{+0.11} \pm 0.03, \end{aligned} \quad (1.2)$$

which stand in a $2.2 - 2.6\sigma$ discrepancy with the SM predictions.

The discrepancy, if not a statistical fluctuation, requires new degrees of freedom. In this work we focus on new physics models where heavy new bosons contribute to $b \rightarrow s\mu^+\mu^-$ transitions at tree level. Such new bosons can be $\text{SU}(3)_C$ singlets or triplets. We focus on the latter class, i.e. on leptoquark models [3]. More specifically, we consider simplified models where a single leptoquark representation is added to the SM fields.

There are eight leptoquark representations that have couplings to down-type quarks and to charged leptons. The $R_{K^{(*)}}$ measurements suggest that the integration out of these leptoquarks generate an effective four-fermi operator of the form

$$C_{bs\mu\mu}(\bar{s}_L\gamma^\mu b_L)(\bar{\mu}_L\gamma_\mu \mu_L). \quad (1.3)$$

Accordingly, the eight leptoquark representations can be divided to three groups:

- One of the scalar leptoquark representations,

$$S(3, 1)_{-1/3}, \quad (1.4)$$

couples down quarks to neutrinos and up quarks to the charged leptons, so it does not generate (at tree level) the operator of eq. (1.3).

- The couplings of three of the scalar and one of the vector leptoquark representations,

$$S'(3, 1)_{-4/3}, \quad D(3, 2)_{+7/6}, \quad D'(3, 2)_{+1/6}, \quad V^\mu(3, 2)_{-5/6}, \quad (1.5)$$

involve right-handed fields and thus cannot explain the anomaly.

- The remaining one scalar and two vector leptoquark representations,

$$T(3, 3)_{-1/3}, \quad U_3^\mu(3, 3)_{+2/3}, \quad U_1^\mu(3, 1)_{+2/3}, \quad (1.6)$$

are viable candidates to explain the $R_{K^{(*)}}$ anomaly [4–9].

In this work, we thus focus on the three simplified models of eq. (1.6).

The requirement that the contribution of the leptoquarks to $R_{K^{(*)}}$ breaks LFU implies that the leptoquark couplings have a non-trivial flavor structure. In particular, they must break the accidental $\text{SU}(3)_Q \times \text{SU}(3)_L$ global symmetry of the gauge interactions. Generic breaking would lead to unacceptably large contributions to various flavor changing processes. This situation is the specific realization of the new physics flavor puzzle [10] in the leptoquark framework. Thus, the $R_{K^{(*)}}$ measurements provide an opportunity to test

the various ideas that have been proposed to solve this puzzle [5, 11–13]. Arguably the simplest, and the most easily falsifiable of these is the principle of minimal flavor violation (MFV) [14]. In this work we ask whether the leptoquark models that explain the $R_{K^{(*)}}$ anomaly can be MFV (see [5] for related work).

Within the MFV framework, various flavor changing processes are related to each other. For example, the $b \rightarrow s\mu^+\mu^-$ transition relevant to $R_{K^{(*)}}$ is related to the $b \rightarrow s\tau^+\tau^-$ and the $b \rightarrow d\mu^+\mu^-$ transitions. We ask whether the MFV relations exclude some or all of the three otherwise-viable leptoquark models.

The plan of this paper goes as follows. In section 2 we present the principles of applying MFV on leptoquark couplings. In section 3 we obtain the viable lepton flavor representations for leptoquarks, and exclude some of the gauge representations that would be viable if MFV were not imposed. In section 4 we test the various quark flavor representations against experimental constraints. We present our conclusions in section 5. Several additional phenomenological constraints are discussed in appendices: $B_s - \bar{B}_s$ mixing (appendix A), direct LHC searches (appendix B), perturbative unitarity (appendix C), $pp \rightarrow \mu\mu$ (appendix D), and $\tau \rightarrow s\bar{u}\nu$ (appendix E).

2 MFV for leptoquarks

In this section we discuss in more detail the implementation of MFV in leptoquark models [15]. In the absence of Yukawa couplings, the SM acquires an accidental non-Abelian global symmetry,

$$\begin{aligned} G_{\text{flavor}} &= \text{SU}(3)_q^3 \times \text{SU}(3)_\ell^2, \\ \text{SU}(3)_q^3 &= \text{SU}(3)_Q \times \text{SU}(3)_U \times \text{SU}(3)_D, \\ \text{SU}(3)_\ell^2 &= \text{SU}(3)_L \times \text{SU}(3)_E. \end{aligned} \quad (2.1)$$

The Yukawa couplings,

$$\mathcal{L}_{\text{Yukawa}} = \bar{Q}Y^D D\phi + \bar{Q}Y^U U\tilde{\phi} + \bar{L}Y^E E\phi + \text{h.c.}, \quad (2.2)$$

break $G_{\text{flavor}} \rightarrow \text{U}(1)_B \times \text{U}(1)_e \times \text{U}(1)_\mu \times \text{U}(1)_\tau$. Thus, the three Yukawa matrices can be taken as spurions with the following transformation properties under G_{flavor} :

$$Y^U(3, \bar{3}, 1, 1, 1), \quad Y^D(3, 1, \bar{3}, 1, 1), \quad Y^E(1, 1, 1, 3, \bar{3}). \quad (2.3)$$

Imposing MFV on the SM extended with leptoquark fields means that we assign the leptoquark fields with well-defined transformation properties under G_{flavor} and require the following:

- All terms made of SM fields, leptoquark fields and the Yukawa spurions are formally invariant under G_{flavor} .

One subtlety relates to the definition of minimal lepton flavor violation. We consider the case that the only spurion that breaks $\text{SU}(3)_\ell^2$ is Y^E . If one takes into account the fact

that neutrinos are massive, additional spurions may play a role. For example, if neutrino masses arise from a seesaw mechanism with three heavy SM-singlet fermions N , then G_{flavor} is extended by an $\text{SU}(3)_N$ factor, and both M_N , the mass matrix of these fermions, and Y^N , the neutrino Yukawa matrix, break the flavor symmetry. Taking Y^E to be the only leptonic spurion is equivalent to assuming that the seesaw scale is higher than the scale at which the leptoquark couplings are set. Moreover, if this scenario holds in Nature, it explains why lepton flavor violation (e.g., $\mu \rightarrow e\gamma$ [16]) has not been observed except in neutrino oscillations.

We are interested in leptoquarks that generate the effective four-fermi operator of eq. (1.3). Thus, the $\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ invariant operator must involve the leptoquark field, the quark doublet fields Q_i and the lepton doublet fields L_j . Since our starting point is the anomaly in $b \rightarrow s\mu^+\mu^-$ transitions, we work in the down and charged lepton mass basis. Hence the quark doublets are $Q_{d,s,b}$ and the lepton doublets are $L_{e,\mu,\tau}$. In this basis, the three Yukawa spurions have the form

$$\begin{aligned} Y^D &= \lambda_d \equiv \text{diag}(y_d, y_s, y_b), \\ Y^U &= V^\dagger \lambda_u \equiv V_{\text{CKM}}^\dagger \times \text{diag}(y_u, y_c, y_t), \\ Y^E &= \lambda_e \equiv \text{diag}(y_e, y_\mu, y_\tau). \end{aligned} \quad (2.4)$$

To have a predictive framework for processes that involve the third generation fermions (in particular the b -quark and the τ -lepton), we make two assumptions:

1. The spurions related to Y^D and Y^E are small enough to keep the leptoquark couplings perturbative.
2. Terms that are higher power in Y^F ($F = U, D, E$) are suppressed compared to lower powers.

The first assumption can be satisfied in the models that we consider for leptoquark masses not much heavier than a few TeV. A quantitative analysis is given in appendix C.

The second assumption means that we do not consider MFV in the nonlinear regime [17]. The implications of relaxing this assumption are briefly discussed in section 5. The only case where we include spurions that are quadratic (or higher order) in the Yukawa couplings is when the leading contribution to flavor changing couplings arises from the operator

$$O^{Q_U}(8, 1, 1, 1, 1) \equiv Y^U Y^{U\dagger}. \quad (2.5)$$

In the down mass basis, and neglecting y_c and y_u , it has the form

$$(O^{Q_U})_{ji} = y_t^2 V_{tj}^* V_{ti}. \quad (2.6)$$

3 Minimal lepton flavor violation (MLFV)

For the sake of concreteness we continue by considering a specific model out of the three — that is the $T(3, 3)_{-1/3}$ model — but at this stage the lessons drawn are common to all

three. The leptoquark couplings of T have the form

$$\mathcal{L}_{\text{Yukawa}}^T = \lambda_{\alpha j} \bar{Q}_j^c \epsilon (T_a^\dagger \tau_a) L_\alpha + \text{h.c.}, \quad (3.1)$$

where $\epsilon = i\tau_2$, and τ_a are Pauli matrices in $\text{SU}(2)_L$. Integrating out T , we obtain the following EFT Lagrangian:

$$\mathcal{L}_T^{\text{EFT}} = \frac{\lambda_{\alpha j} \lambda_{\beta i}^*}{M_T^2} (\bar{L}_\beta \tau_a \epsilon^T Q_i^c) (\bar{Q}_j^c \epsilon \tau_a L_\alpha). \quad (3.2)$$

3.1 $B \rightarrow K^{(*)} \ell^+ \ell^-$

The relevant leptoquark models generate, among others, operators of the following form:

$$C_{bs\mu\mu}^{\text{NP}} (\bar{s}_L \gamma^\mu b_L) (\bar{\mu}_L \gamma_\mu \mu_L) + C_{bs\tau\tau}^{\text{NP}} (\bar{s}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \tau_L), \quad (3.3)$$

with

$$C_{bs\ell\ell}^{\text{NP}} = \frac{\lambda_{\ell b}^* \lambda_{\ell s}}{M_T^2}. \quad (3.4)$$

We consider the experimental data, $\text{BR}(B^+ \rightarrow K^+ \tau^+ \tau^-) < 2.25 \times 10^{-3}$ [18], and $\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-) = (4.4 \pm 0.3) \times 10^{-7}$ [19], which give

$$R_{\tau/\mu} \equiv \frac{\text{BR}(B^+ \rightarrow K^+ \tau^+ \tau^-)}{\text{BR}(B^+ \rightarrow K^+ \mu^+ \mu^-)} < 5 \times 10^3. \quad (3.5)$$

We now examine various possibilities for the representation of T under $\text{SU}(3)_\ell^2$ and their predictions for $\lambda_{\alpha i}$ and, consequently, for $C_{bs\mu\mu}$ and $C_{bs\tau\tau}$.

- $\text{SU}(3)_\ell^2$ -singlet:

$$T(1, 1)_{\text{SU}(3)_\ell^2} \implies \lambda = 0. \quad (3.6)$$

The reason is that no combination of Y^E 's transforms as $(\bar{3}, 1)_{\text{SU}(3)_\ell^2}$. Thus this MFV model cannot account for the $R_{K^{(*)}}$ anomaly.

- $\text{SU}(3)_L$ -anti-triplet:

The spurion must transform as $(1 + 8, 1)_{\text{SU}(3)_\ell^2}$ and thus

$$T(\bar{3}, 1) \implies \lambda \propto (1 + Y^E Y^{E\dagger}). \quad (3.7)$$

Given the smallness of the lepton Yukawa couplings, we expect that the leading contribution is lepton-flavor universal and thus cannot account for the $R_{K^{(*)}}$ anomaly.

It could, however, be that the singlet contribution is negligibly small for some reason, and the octet contribution dominates. In the case of octet-spurion dominance, $\lambda \propto Y^E Y^{E\dagger}$, we have

$$\frac{C_{bs\tau\tau}^{\text{NP}}}{C_{bs\mu\mu}^{\text{NP}}} = \frac{y_\tau^4}{y_\mu^4} = 8 \times 10^4. \quad (3.8)$$

Taking into account that the $\mathcal{O}(0.25)$ deviation of R_K from unity comes from the interference of the SM and leptoquark amplitudes, we find that eq. (3.8) implies $R_{\tau/\mu} \sim 10^8$, strongly violating the experimental upper bound of eq. (3.5). We conclude that having a leptoquark transform as $(\bar{3}, 1)$ under $\text{SU}(3)_L \times \text{SU}(3)_E$ is excluded.

- $SU(3)_E$ -anti-triplet:

$$T(1, \bar{3})_{SU(3)_\ell^2} \implies \lambda \propto Y^{E\dagger}. \quad (3.9)$$

For all such models, we have the ratio between the T -mediated amplitudes given by

$$\frac{C_{bs\tau\tau}^{\text{NP}}}{C_{bs\mu\mu}^{\text{NP}}} = \frac{y_\tau^2}{y_\mu^2} = 2.8 \times 10^2. \quad (3.10)$$

Thus, these models predict

$$R_{\tau/\mu} \sim 1.2 \times 10^3, \quad (3.11)$$

a factor of 4 below the present bound.

3.2 $B \rightarrow K^{(*)}\nu\bar{\nu}$

In the previous subsection, we proved that the only viable lepton flavor representation is $(1, \bar{3})_{SU(3)_\ell^2}$. In this subsection we use the experimental data on $B \rightarrow K^{(*)}\nu\bar{\nu}$ to exclude some of these models.

Experiments put the upper bounds $\text{BR}(B^+ \rightarrow K^+\nu\bar{\nu}) < 1.6 \times 10^{-5}$ [20] and $\text{BR}(B^+ \rightarrow K^{*+}\nu\bar{\nu}) < 4.0 \times 10^{-5}$ [21]. Thus,

$$R_{\nu/\mu}^{(*)} \equiv \frac{\text{BR}(B^+ \rightarrow K^{(*)+}\nu\bar{\nu})}{\text{BR}(B^+ \rightarrow K^{(*)+}\mu^+\mu^-)} \lesssim 40. \quad (3.12)$$

The relevant leptoquark models generate, among others, operators of the following form:

$$C_{bs\mu\mu}(\bar{s}_L\gamma^\mu b_L)(\bar{\mu}_L\gamma_\mu\mu_L) + C_{bs\nu_\tau\nu_\tau}(\bar{s}_L\gamma^\mu b_L)(\bar{\nu}_\tau\gamma_\mu\nu_\tau). \quad (3.13)$$

The SM predicts [22, 23]

$$C_{bs\nu\nu}^{\text{SM}}/C_{bs\mu\mu}^{\text{SM}} = -1.49. \quad (3.14)$$

(Note that $C_{bs\nu\nu}^{\text{SM}}$ is the value for a single flavor, and thus the SM prediction is $R_{\nu/\mu} \sim 6.6$.)

The $R_{K^{(*)}}$ anomaly requires

$$C_{bs\mu\mu}^{\text{NP}}/C_{bs\mu\mu}^{\text{SM}} = -0.12. \quad (3.15)$$

We now obtain the ratio $C_{bs\nu_\tau\nu_\tau}^{\text{NP}}/C_{bs\mu\mu}^{\text{NP}}$ for each of T , U_3^μ and U_1^μ , and the resulting prediction for $R_{\nu/\mu}$:

$$R_{\nu/\mu} \sim \frac{2|C_{bs\nu\nu}^{\text{SM}}|^2 + |C_{bs\nu_\tau\nu_\tau}^{\text{NP}} + C_{bs\nu\nu}^{\text{SM}}|^2}{|C_{bs\mu\mu}^{\text{NP}} + C_{bs\mu\mu}^{\text{SM}}|^2}. \quad (3.16)$$

Note that for $R_{\tau/\mu}$ we had to consider only the lepton flavor representation of the leptoquark. In contrast, for $R_{\nu/\mu}$, the result depends also on the Lorentz and $SU(2) \times U(1)$ representation and is thus different among the three models.

- $T(3, 3)_{-1/3}$:

$$\frac{C_{bs\nu_\tau\nu_\tau}^T}{C_{bs\mu\mu}^T} = -\frac{y_\tau^2}{2y_\mu^2} \implies R_{\nu/\mu} \sim 3.1 \times 10^2. \quad (3.17)$$

- $U_3^\mu(3,3)_{+2/3}$:

$$\frac{C_{bs\nu_\tau\nu_\tau}^{U_3}}{C_{bs\mu\mu}^{U_3}} = -\frac{2y_\tau^2}{y_\mu^2} \implies R_{\nu/\mu} \sim 5.6 \times 10^3. \quad (3.18)$$

- $U_1^\mu(3,1)_{+2/3}$:

$$\frac{C_{bs\nu_\tau\nu_\tau}^{U_1}}{C_{bs\mu\mu}^{U_1}} = 0 \implies R_{\nu/\mu} \sim 8.5. \quad (3.19)$$

We conclude that, for the $(1, \bar{3})_{\text{SU}(3)_\ell^2}$ representation, the T and U_3^μ models are excluded by the upper bound on $R_{\nu/\mu}$. On the other hand, the U_1^μ models predict this ratio to be a factor of 4.7 below the present bound (or, equivalently, 1.3 above the SM prediction).

3.3 Summary of MLFV

There are four classes of MLFV models for leptoquarks that can a-priori (that is, without imposing MLFV) generate the operator of eq. (1.3):

- Models where it does not couple to the leptons. These are the models where the leptoquark transforms as $(1, 1)_{\text{SU}(3)_\ell^2}$.
- Models where the couplings are lepton-flavor-universal to a good approximation. This is the case if the leptoquark transforms as $(\bar{3}, 1)_{\text{SU}(3)_\ell^2}$ and the leading spurion is a lepton flavor singlet.
- Models where the leptoquark couplings are quadratic in the lepton-Yukawa. This is the case if the leptoquark transforms as $(\bar{3}, 1)_{\text{SU}(3)_\ell^2}$ and the leading spurion is a lepton flavor octet.
- Models where the leptoquark couplings are linear in the lepton-Yukawa. These is the case if the leptoquark transforms as $(1, \bar{3})_{\text{SU}(3)_\ell^2}$.

Only the latter class is good for explaining the $R_{K^{(*)}}$ anomaly (without violating the $R_{\tau/\mu}$ bound). This MFV classification is common to all three viable leptoquark models: $T(3, 3)_{-1/3}$, $U_3^\mu(3, 3)_{+2/3}$, and $U_1^\mu(3, 1)_{+2/3}$. However, additional processes put further constraints:

- The upper bounds on $\text{BR}(B \rightarrow K^{(*)}\nu\bar{\nu})$ exclude the MFV- T and MFV- U_3^μ models.
- In appendix A we show that the MFV- T model is excluded also by the upper bound on new physics contribution to $B_s - \bar{B}_s$ mixing.

We conclude that the only model that is not excluded by the above consideration is the U_1^μ model in the $(1, \bar{3})_{\text{SU}(3)_\ell^2}$ representation.

To make further progress, we need to consider the $\text{SU}(3)_q^3$ representation of the leptoquark, which we do in the next section.

4 Minimal quark flavor violation (MQFV)

We now consider the possible $SU(3)_q^3$ representations of the U_1^μ leptoquark. For simplicity, from here on we omit the sub-index 1 and the Lorentz super-index μ and denote the Lorentz-vector in the $(3,1)_{+2/3}$ gauge representation simply by U .

- $SU(3)_q^3$ -singlet:

$$U(1,1,1)_{SU(3)_q^3} \implies \lambda = 0. \quad (4.1)$$

The reason is that no combination of Y^U 's and Y^D 's transforms as $(3,1,1)_{SU(3)_q^3}$.

We conclude that U must transform as a triplet under $SU(3)_q^3$ and as an anti-triplet under $SU(3)_\ell^2$. If indeed U transforms as a quark-flavor-triplet and lepton-flavor-anti-triplet, then there are nine U -flavor states, that can be denoted as

$$U_{\alpha i}, \quad \alpha = e, \mu, \tau, \quad i = d, s, b \text{ or } u, c, t. \quad (4.2)$$

The 9×9 mass-squared matrix of the U flavor states transforms as either 1 or $1 + 8$ under each of the five $SU(3)$'s. Given the smallness of all Yukawa couplings except y_t , and the smallness of $|V_{ts}|$ and $|V_{td}|$, the 9×9 mass-squared matrix is near diagonal, so that we can call the nine U mass-eigenstates by the same names as the flavor states, namely $U_{\alpha i}$. Furthermore, given our assumption of small $SU(3)_\ell^2$ spurions, the masses are lepton-flavor universal to a good approximation. As concerns the quark-flavor, in some cases the b -states (t -states) are separated by $\mathcal{O}(y_t^2)$ from the s - and d -states (u - and c -states), but in any case there is no hierarchy.

In what follows we denote the couplings of $U_{\alpha i} \overline{Q}_j L_\beta$ as $\lambda_{\alpha\beta ij}$:

$$\mathcal{L}_{\text{Yukawa}}^U = \lambda_{\alpha\beta ij} \overline{Q}_j \gamma_\mu L_\beta U_{\alpha i}^\mu + \text{h.c.} \quad (4.3)$$

As argued above, the only viable lepton flavor representation is $(1, \bar{3})_{SU(3)_\ell^2}$ and thus

$$\lambda_{\alpha\beta ij} \propto (Y^{E\dagger})_{\alpha\beta} = \delta_{\alpha\beta} y_\beta, \quad (4.4)$$

where the second equality applies in the charged lepton mass basis.

There are three possible $SU(3)_q^3$ representation. We denote the three models by $U_{Q,U,D}$ in correspondence to the flavor group — $SU(3)_{Q,U,D}$ — under which they transform as a triplet:

- $U_Q(3,1,1)_{SU(3)_q^3}$:

The required spurion transforms as $(1+8,1,1)_{SU(3)_q^3}$ and thus

$$\lambda_{\alpha\beta ij} \propto (x_{1/8} \mathbf{1} + Y^U Y^{U\dagger})_{ji} = x_{1/8} \delta_{ji} + y_t^2 V_{tj}^* V_{ti}. \quad (4.5)$$

- $U_U(1,3,1)_{SU(3)_q^3}$:

The required spurion transforms as $(3, \bar{3}, 1)_{SU(3)_q^3}$ and thus

$$\lambda_{\alpha\beta ij} \propto Y_{ji}^U = V_{tj}^* y_t \delta_{it}. \quad (4.6)$$

Model	[SU(3)] ⁵ -rep	$\lambda_{\alpha\alpha ij}$
$(U_Q)_{\alpha i}$	$(3, 1, 1, 1, \bar{3})$	$by_\alpha(x_{1/8}\delta_{ij} + y_t^2 V_{tj}^* V_{ti})$
$(U_U)_{\alpha i}$	$(1, 3, 1, 1, \bar{3})$	$b\delta_{it}y_\alpha y_t V_{tj}^*$
$(U_D)_{\alpha i}$	$(1, 1, 3, 1, \bar{3})$	$b\delta_{ib}y_\alpha y_b(x_{1/8}\delta_{jb} + y_t^2 V_{tj}^* V_{tb})$

Table 1. The $\lambda_{\alpha\beta ij}$ couplings in the down and charged lepton mass basis for the three flavor representations.

- $U_D(1, 1, 3)_{\text{SU}(3)_q^3}$:

The required spurion transforms as $(3, 1, \bar{3})_{\text{SU}(3)_q^3}$ and thus

$$\lambda_{\alpha\beta ij} \propto [(x_{1/8}\mathbf{1} + O^{Q_U})Y^D]_{ji} = y_b\delta_{ib}(x_{1/8}\delta_{jb} + y_t^2 V_{tj}^* V_{tb}). \quad (4.7)$$

To summarize, we present the couplings that play a role in our framework in table 1.

4.1 Back to $R_{K^{(*)}}$

Given the couplings in table 1, we can now translate the $R_{K^{(*)}}$ requirement,

$$C_{bs\mu\mu} = \sum_i \frac{\lambda_{\mu\mu ib}^* \lambda_{\mu\mu is}}{M_{\mu i}^2} \sim \frac{10^{-3}}{\text{TeV}^2}, \quad (4.8)$$

into a constraint on the model parameters.

- U_U :

$$C_{bs\mu\mu} = \frac{|b|^2 y_\mu^2 y_t^2 V_{tb} V_{ts}^*}{M_{\mu t}^2} \implies \frac{b}{M_U} \sim \frac{263}{\text{TeV}}. \quad (4.9)$$

- U_Q :

$$C_{bs\mu\mu} = \frac{|b|^2 y_\mu^2 y_t^2 V_{tb} V_{ts}^*}{M_{\mu i}^2} (2\mathcal{R}e(x_{1/8}) + y_t^2 |V_{tb}|^2). \quad (4.10)$$

In order that to have destructive interference with the SM amplitude, we need $2\mathcal{R}e(x_{1/8}) + y_t^2 |V_{tb}|^2 > 0$, namely (assuming that $x_{1/8}$ is real)

$$x_{1/8} > -0.5. \quad (4.11)$$

- U_D :

$$C_{bs\mu\mu} = \frac{|b|^2 y_\mu^2 y_b^2 y_t^2 V_{tb} V_{ts}^*}{M_{\mu b}^2} (x_{1/8}^* + y_t^2 |V_{tb}|^2). \quad (4.12)$$

In order that to have destructive interference with the SM amplitude, we need $x_{1/8} + y_t^2 |V_{tb}|^2 > 0$, namely

$$x_{1/8} > -1. \quad (4.13)$$

Figure 1. Constraints in the $M_U - |\lambda_{bb\tau\tau}|$ plane in the U_U model: the 1σ allowed range from $R_{K^{(*)}}$, the region excluded by the high- p_T $pp \rightarrow \tau\tau$ search [24], and the region excluded by LHC direct searches (see appendix B).

4.2 $b\bar{b} \rightarrow \tau^+\tau^-$

Within the MFV models that we study, the requirement that the leptoquarks contribute to the Wilson coefficient of the operator of eq. (1.3), namely to the $b \rightarrow s\mu^+\mu^-$ decay, implies that they contribute also to $b\bar{b} \rightarrow \ell^+\ell^-$ scattering processes [24]. MFV suggests that the largest contribution will be to the final $\tau^+\tau^-$ state. This contribution is constrained by the LHC searches for the $\tau^+\tau^-$ signature.

In ref. [24], the results of the ATLAS searches [25, 26] have been recast into bounds on vector leptoquarks mass and coupling:

$$C_{bb\tau\tau}^U = \frac{|\lambda_{\tau\tau bb}|^2}{M_U^2} \lesssim 2.6 \text{ (4.0) TeV}^{-2}, \quad (4.14)$$

where the stronger (weaker) bound applies in case that $M_U > 2 \text{ TeV}$ ($M_U \lesssim 2 \text{ TeV}$), which is above (within) the LHC direct reach. The bound for $M_U \lesssim 2 \text{ TeV}$ is not constant and is slightly weaker than 4 TeV^{-2} below 1 TeV, which is anyway excluded by the LHC direct searches (see appendix B). In figure 1 we present the excluded region for $M_U < 2 \text{ TeV}$, compared to the 1σ allowed region to fit the R_K anomaly.

Within our models, we have [see eq. (A.5)] $C_{bs\tau\tau} = 0.28 \text{ TeV}^{-2}$. We thus require

$$C_{bb\tau\tau}^U / C_{bs\tau\tau}^U \lesssim 9.3(14.3) \text{ for } M_U > (\lesssim) 2 \text{ TeV}. \quad (4.15)$$

The MFV prediction for $C_{bb\tau\tau}$ depends on the quark flavor representation:

- U_U :

$$\frac{C_{bb\tau\tau}^U}{C_{bs\tau\tau}^U} = \frac{V_{tb}}{V_{ts}} \sim 25, \quad (4.16)$$

which is excluded.

- U_Q :

$$\frac{C_{bb\tau\tau}^U}{C_{bs\tau\tau}^U} = \frac{(x_{1/8} + y_t^2 |V_{tb}|^2)^2}{y_t^2 V_{tb} V_{ts}^* (2\mathcal{R}e(x_{1/8}) + y_t^2 |V_{tb}|^2)}. \quad (4.17)$$

In the region allowed by $R_{K^{(*)}}$, $x_{1/8} > -0.5$ [see eq. (4.11)], the function $(1 + x_{1/8})^2 / (1 + 2x_{1/8})$ has a minimum value of 1, and consequently

$$\frac{C_{bb\tau\tau}^U}{C_{bs\tau\tau}^U} \gtrsim \frac{V_{tb}}{V_{ts}} \sim 25, \quad (4.18)$$

which is excluded.

- U_D :

$$\frac{C_{bb\tau\tau}^U}{C_{bs\tau\tau}^U} = \frac{x_{1/8} + y_t^2 |V_{tb}|^2}{y_t^2 V_{tb} V_{ts}^*}. \quad (4.19)$$

eqs. (4.13) and (4.15) imply a narrow allowed window:

$$-1 < x_{1/8} \lesssim -0.6(-0.4). \quad (4.20)$$

We conclude that, within the MFV framework, the combined constraints from $R_{K^{(*)}}$ and $b\bar{b} \rightarrow \tau^+\tau^-$ exclude the U_U and U_Q scenarios, and leave the U_D model as the only viable one.

4.3 $R_{D^{(*)}}$

In addition to the measurements of $R_{K^{(*)}}$, there are hints of violation of LFU in $B \rightarrow D^{(*)}\tau\nu$ decay. Consider the ratios

$$R_{D^{(*)}} \equiv \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu)}{\Gamma(B \rightarrow D^{(*)}\ell\nu)}, \quad (\ell = e, \mu). \quad (4.21)$$

The combination of measurements by BaBar [27, 28], Belle [29–32] and LHCb [33] reads [34]¹

$$R_D = 0.403 \pm 0.047, \quad R_D^{\text{SM}} = 0.310 \pm 0.017. \quad \rho = -0.23. \quad (4.22)$$

The SM predicts the following values [35–38]:

$$R_D^{\text{SM}} = 0.300 \pm 0.008, \quad R_D^{*\text{SM}} = 0.252 \pm 0.003. \quad (4.23)$$

Thus, there is a deviation from the SM prediction at $\sim 4\sigma$, with $B \rightarrow D^{(*)}\tau\nu$ enhanced with respect to $B \rightarrow D^{(*)}\ell\nu$.

Within the MFV framework, the leptoquarks that generate the effective term of eq. (1.3), relevant to $b \rightarrow s\mu\mu$ decays, generate also the term

$$C_{bc\tau\nu}(\bar{c}_L\gamma^\mu b_L)(\bar{\tau}_L\gamma_\mu\nu_{\tau L}). \quad (4.24)$$

In contrast to the $b \rightarrow s\mu\mu$ and other processes discussed so far, the $b \rightarrow c\tau\nu_\tau$ decay is a quark-flavor changing *charged current* process. We have

$$C_{bc\tau\nu}^U = \sum_i \frac{\lambda_{\tau\tau ib}^*}{M_{\tau i}^2} \sum_j \lambda_{\tau\tau ij} V_{cj}. \quad (4.25)$$

¹See also online update at <http://www.slac.stanford.edu/xorg/hfag>.

The data require $C_{bc\tau\nu}^{\text{NP}} \sim 0.17 \text{ TeV}^{-2}$. Together with the $R_{K^{(*)}}$ constraint, we need

$$C_{bc\tau\nu}^{\text{NP}}/C_{bs\mu\mu}^{\text{NP}} \sim -170. \quad (4.26)$$

In the U_D model, we have the following prediction for $C_{bc\tau\nu}$:

$$C_{bc\tau\nu}^U = \frac{|b|^2 x_{1/8} y_\tau^2 y_b^2 V_{cb}}{M_{\tau b}^2} \left(x_{1/8}^* + y_t^2 |V_{tb}|^2 \right). \quad (4.27)$$

Thus,

$$\frac{C_{bc\tau\nu}^U}{C_{bs\mu\mu}^U} = \frac{y_\tau^2 V_{cb}}{y_\mu^2 V_{ts}^*} x_{1/8} \sim +(110 - 280), \quad (4.28)$$

where the range corresponds to $-1 < x_{1/8} \lesssim -0.4$. The U_D model predicts a strong *suppression* (by at least 10%) of $R_{D^{(*)}}$ from the SM prediction and is thus excluded. In fact, it will remain excluded even if $R_{D^{(*)}}$ turns out to be consistent with the SM prediction (with experimental uncertainties no larger than the present ones) as long as $R_{K^{(*)}}$ is substantially suppressed compared to the SM.

We discuss additional aspects of $R_{D^{(*)}}$ within the MFV framework, independent of $R_{K^{(*)}}$, in appendix E.

4.4 Summary of MQFV

There are three classes of MQFV for leptoquarks that can generate the operator of eq. (1.3):

- The U_U model in the $(1, 3, 1)_{\text{SU}(3)_q^3}$ representation. It is excluded by a combination of the $R_{K^{(*)}}$ and $b\bar{b} \rightarrow \tau^+\tau^-$ measurements.
- The U_Q model in the $(3, 1, 1)_{\text{SU}(3)_q^3}$ representation. It is excluded by a combination of the $R_{K^{(*)}}$ and $b\bar{b} \rightarrow \tau^+\tau^-$ measurements.
- The U_D model in the $(1, 1, 3)_{\text{SU}(3)_q^3}$ representation. It is excluded by a combination of the $R_{K^{(*)}}$, $b\bar{b} \rightarrow \tau^+\tau^-$ and $R_{D^{(*)}}$ measurements.

We conclude that all MFV models considered by us are excluded.

5 Conclusions

The $R_{K^{(*)}}$ anomaly can be accounted for in models where there is a significant contribution to the $b \rightarrow s\mu^+\mu^-$ transition from the tree level exchange of leptoquarks. The pattern of deviations from lepton flavor universality (LFU) allows three simplified models, each with a single new leptoquark field:

- A Lorentz scalar, $\text{SU}(2)_L$ triplet: $T(3, 3)_{-1/3}$;
- A Lorentz vector, $\text{SU}(2)_L$ triplet: $U_3^\mu(3, 3)_{+2/3}$;
- A Lorentz vector, $\text{SU}(2)_L$ singlet: $U_1^\mu(3, 1)_{+2/3}$.

Model	$R_{\nu/\mu}$	$\frac{\Gamma_{bb \rightarrow \tau\tau}}{\Gamma_{bb \rightarrow \tau\tau}^{\text{exp}}}$	$R_{D^*}/R_{D^*}^{\text{SM}}$
Experiment	< 40	< 1	1.23 ± 0.07
$U_3^\mu(3, 3)_{+2/3}$	5600		
$T(3, 3)_{-1/3}$	310		
$U_{1U}^\mu(3, 1)_{+2/3}$	8.5	3.2	
$U_{1Q}^\mu(3, 1)_{+2/3}$	8.5	$\gtrsim 3.2$	
$U_{1D}^\mu(3, 1)_{+2/3}$	8.5	$< 1^*$	$\lesssim 0.9$

Table 2. MFV-predictions of simplified leptoquark models that account for the $R_{K^{(*)}}$ anomaly. $R_{\nu/\mu}$ is discussed in section 3.2, $\Gamma_{bb \rightarrow \tau\tau}$ in section 4.2, and $R_{D^{(*)}}$ in section 4.3. A super-index * means that consistency with the observable applies for a small range of the parameter $x_{1/8}$.

Since the Yukawa couplings of these fields constitute new flavor parameters, they provide an opportunity to test various ideas for the flavor structure of new physics. In this work, we tested the idea of minimal flavor violation (MFV).

The need to break LFU, while keeping the $b \rightarrow s\tau^+\tau^-$ rates within bounds, implies that the new leptoquarks have to transform as $(1, \bar{3})$ under the $SU(3)_L \times SU(3)_E$ lepton flavor group. On the other hand, without considering additional constraints, the representation under the quark flavor group $SU(3)_Q \times SU(3)_U \times SU(3)_D$ can be any of the three triplets, $(3, 1, 1)$, $(1, 3, 1)$ or $(1, 1, 3)$.

MFV relates the measured $B \rightarrow K^{(*)}\mu^+\mu^-$ rates to various other processes, such as $B \rightarrow K^{(*)}\nu\bar{\nu}$, $b\bar{b} \rightarrow \tau^+\tau^-$ and $b \rightarrow c\tau\nu$. We summarize our use of these relations to test MFV in table 2. Additional measurements ($B_s - \bar{B}_s$ mixing, direct leptoquark searches, $pp \rightarrow \mu^+\mu^-$, $\tau \rightarrow s\bar{u}\nu$) and considerations (perturbative unitarity) which are relevant to leptoquark models that aim to explain the $R_{K^{(*)}}$ anomaly, are discussed in appendices.

Before we state our conclusions, let us repeat the ingredients of the models that we consider:

1. Simplified models, with a single leptoquark representation;
2. The leptoquark contribution to the $b \rightarrow s\mu^+\mu^-$ transition occurs at tree level;
3. The only spurion that breaks the lepton flavor symmetry is the charged lepton Yukawa matrix.
4. MFV is in the linear regime (higher powers in the spurions are suppressed compared to lower ones).

Thus, we should bear in mind the following caveats:

1. Nature might have more than one leptoquark representation at play (see, e.g., [39]);
2. Additional leptoquark representations might play a role via loop, rather than tree level contributions (see, e.g., [6, 40–42]);

3. Minimal lepton flavor violation might involve neutrino-related spurions (see, e.g., [43]);
4. The relations between third generation spurions and the lighter generation spurions are modified if MFV is in the nonlinear regime.

Most of our conclusions hold, however, in generic such extensions of our framework. For example, even with tree level contribution to $B \rightarrow K^{(*)}\mu\mu$, the MFV framework predicts that the third generation couplings of the leptoquarks are close to the perturbative limit. If the contribution is suppressed by an additional loop factor, then these couplings will be pushed to non-perturbative values. As another example, if we allow neutrino-related spurions to play a significant role in lepton flavor conserving processes, it will be hard to avoid too large contributions to lepton flavor changing ones, such as $\mu \rightarrow e\gamma$ [44].

Our conclusions do not hold, however, if MLFV is in the nonlinear regime. In this case, the strict relations between τ and μ couplings do not hold. Specifically, the bounds from $R_{\tau/\mu}$, $R_{\nu/\mu}$, $b\bar{b} \rightarrow \tau^+\tau^-$ and $B_s - \bar{B}_s$ mixing cannot be strictly applied. Yet, for some of the constraints, fine-tuned cancelations between the linear term and the higher order ones are needed to satisfy the constraints, which goes against the spirit of MFV. Order one modifications of the linear MFV prediction can, however, bring U_1^μ models into consistency with the $\Gamma_{b\bar{b} \rightarrow \tau\tau}$ and R_{D^*} constraints. In fact, the phenomenology of models of nonlinear minimal flavor violation [17] is similar to that of $U(2)$ models, which have been shown to be viable candidates to explain the $R_{K^{(*)}}$ anomaly [12]

We find that all models are excluded by a combination of $b \rightarrow s\mu^+\mu^-$, $b \rightarrow s\tau^+\tau^-$ and the processes presented in the table. Note that for vector-leptoquark models, constraints from loop diagrams are sensitive to the UV completion of the model. It is thus important that we exclude these models based on tree level processes alone. (In appendix A we consider a loop process, $B_s - \bar{B}_s$ mixing, but we confront it with only scalar leptoquark models.)

We conclude that if the $R_{K^{(*)}}$ anomaly is experimentally established, then minimal flavor violation in the linear regime will be excluded.

Acknowledgments

YN is the Amos de-Shalit chair of theoretical physics. YN is supported by grants from the Israel Science Foundation (grant number 394/16), the United States-Israel Binational Science Foundation (BSF), Jerusalem, Israel (grant number 2014230), the I-CORE program of the Planning and Budgeting Committee and the Israel Science Foundation (grant number 1937/12), and the Minerva Foundation.

A $B_s - \bar{B}_s$ mixing

Leptoquarks which contribute at tree level to $b \rightarrow s\mu\mu$ contribute also via box diagrams to $B_s - \bar{B}_s$ mixing amplitude M_{12}^s . The modification of the SM prediction for M_{12}^s is parameterized as follows:

$$M_{12}^s = |\Delta_s| e^{i\phi_s^\Delta} \times M_{12}^{\text{SM},s}. \quad (\text{A.1})$$

Fitting the mixing amplitude to the experimental ranges of Δm_{B_s} , $\Delta \Gamma_{B_s}$ and a_{SL}^s , gives [45]

$$|\Delta_s| = 1.05_{-0.13}^{+0.14}, \quad \phi_s^\Delta = (1.5_{-2.4}^{+2.3})^\circ. \quad (\text{A.2})$$

Requiring that the contribution from the scalar leptoquark T is within $||\Delta_s| - 1| \lesssim 0.25$ gives [46]:

$$\frac{|\lambda_{\tau b} \lambda_{\tau s}|^2}{M_T^2} \lesssim ||\Delta_s| - 1| \frac{192\pi^2 \Delta m_{B_s}}{f_{B_s}^2 m_{B_s}} \sim 2.0 \times 10^{-2} \text{ TeV}^{-2}, \quad (\text{A.3})$$

where we used $\Delta m_{B_s} = 1.17 \times 10^{-11} \text{ GeV}$, $m_{B_s} = 5.37 \text{ GeV}$ and $f_{B_s} \sim 0.23 \text{ GeV}$.

The $R_{K^{(*)}}$ anomaly requires (see e.g. [47, 48])

$$C_{bs\mu\mu} = \frac{\lambda_{\mu b} \lambda_{\mu s}}{M_T^2} \sim 10^{-3} \text{ TeV}^{-2}. \quad (\text{A.4})$$

In the viable models, $\lambda_{\tau b}^* \lambda_{\tau s} = (y_\tau/y_\mu)^2 \lambda_{\mu b}^* \lambda_{\mu s}$, so that

$$C_{bs\tau\tau} = \frac{\lambda_{\tau b} \lambda_{\tau s}}{M_T^2} \sim 0.28 \text{ TeV}^{-2}. \quad (\text{A.5})$$

eqs. (A.3) and (A.5) can be simultaneously satisfied only for $M_T \lesssim 0.5 \text{ TeV}$.

The members of the third generation T -triplet of charges $-4/3$, $-1/3$ and $+2/3$ decay into, respectively, τb , νt and τt . The latter has branching ratio 1 which leads to an exclusion of 850 GeV [49]. The recast [50] of the SUSY CMS analysis [51] for the $t\bar{t}\nu\nu$ topology leads to an even stronger bound of 1.07 TeV. Thus, $M_T \lesssim 0.5 \text{ TeV}$, as required by the Δm_{B_s} constraint, is excluded by LHC direct searches. We conclude that Δm_{B_s} constraints exclude the MFV- T model as a possible explanation of the $R_{K^{(*)}}$ anomaly.

As concerns the case of vector leptoquarks, their contribution to $B_s - \bar{B}_s$ mixing is divergent. The divergence comes from the $k_\mu k_\nu$ term in their propagator, $i[(k_\mu k_\nu)/M_U^2 - g_{\mu\nu}]/(k^2 - M_U^2)$. Ref. [46] suggests that a conservative bound can be obtained by considering the contribution of the $g_{\mu\nu}$ term only. The numerical factor of the mixing amplitude is four times larger than in the scalar case, and the resulting bound on the mass is therefore two times stronger, $M_U \lesssim 0.25 \text{ TeV}$, which is excluded by the direct searches. Yet, this bound is model dependent. To relax the bound by a factor of $\mathcal{O}(7)$ (see appendix C), the contributions from the terms that we omitted should cancel with those that we took into account to the two percent level.

B LHC direct searches for U_1^μ

The production cross section $\sigma(pp \rightarrow U_1^\mu U_1^{\mu\dagger})$ for vector leptoquarks is considered in [47]. As already discussed in section 4, MFV implies that the nine flavor states are almost degenerate. This represents then an important and distinctive feature of our framework.

Each of the nine flavor-states has a branching ratio of 50% to decay into a specific charged lepton and a jet, and a branching ratio of 50% to decay into a neutrino plus jet. For each final state topology searched for at the LHC, we define

$$\sigma(pp \rightarrow ff) = \sigma(pp \rightarrow U_1^\mu U_1^{\mu\dagger}) N_f, \quad (\text{B.1})$$

with

$$N_f = \sum_{\alpha i} \text{BR}(U_{\alpha i} \rightarrow f)^2. \quad (\text{B.2})$$

We ignore the mixed final states $pp \rightarrow ff'$ depending on $\sigma(pp \rightarrow U_1^\mu U_1^{\mu\dagger}) \sum_{\alpha i} \text{BR}(U_{\alpha i} \rightarrow f) \times \text{BR}(U_{\alpha i}^\dagger \rightarrow f')$. In almost all cases, the decay is prompt. The only possible exceptions are the decays of U_{eq} , with $q = u$ or d , where the decay might have a displaced vertex.

The strongest bound come from the $U_1^\mu \rightarrow eb$ search [52]:

$$M_{eb} \gtrsim 1.5 \text{ TeV}. \quad (\text{B.3})$$

The $U_{\tau b}$ state is the one related to both the perturbative unitarity bound, discussed in appendix C, and to the $pp \rightarrow \tau\tau$ bound, discussed in section 4.2. It decays with branching ratios of 50% into τb and 50% into νt , thus leading to 25% of the events with $b\bar{b}\tau\tau$ final state and 25% of the events with $t\bar{t}\nu\nu$ final state. These final states are constrained by, respectively, the search for third generation leptoquarks and a recast [50] of the CMS SUSY search [51]:

$$M_{\tau b} \gtrsim 1.0 \text{ TeV}, \quad M_{\nu t} \gtrsim 1.2 \text{ TeV}. \quad (\text{B.4})$$

(The latter bound is significantly stronger than the reach of the dedicated leptoquark search for the final state $t\bar{t}\nu\nu$ [53] where the current limit is below 1 TeV.)

C Perturbative unitarity

Perturbative unitarity requires for the leptoquark vector-singlet case of U_1^μ [47]

$$|\lambda_{\tau\tau bb}|^2 < 4\pi. \quad (\text{C.1})$$

Requiring that the $R_{K^{(*)}}$ anomaly is accounted for by U_1^μ gives eq. (4.8). MFV relates $\sum_i \lambda_{\mu\mu ib}^* \lambda_{\mu\mu is}$ to $|\lambda_{\tau\tau bb}|^2$. Consequently, the combination of eqs. (C.1) and (4.8) leads to an upper bound on the leptoquark mass, in particular on $M_{\tau b}$, which can then be compared to the direct lower bound presented in eq. (B.3) or, allowing for mass splitting within the U_1^μ multiplet, eq. (B.4).

- U_U

$$|\lambda_{\tau\tau bb}|^2 \sim 7 \frac{M_{U_U}^2}{\text{TeV}^2} < 4\pi \implies M_{U_U} < 1.3 \text{ TeV}, \quad (\text{C.2})$$

which is excluded by eq. (B.3) but not by eq. (B.4). Thus, a mass splitting larger than 200 GeV between $U_{\tau b}$ and U_{eb} would be required to avoid the direct bounds and fulfill perturbative unitarity.

- U_Q

$$|\lambda_{\tau\tau bb}|^2 \sim \frac{7(1+x_{1/8})^2}{(1+2x_{1/8})} \frac{M_{U_Q}^2}{\text{TeV}^2} < 4\pi \implies M_{U_Q} < \frac{1.3\sqrt{1+2x_{1/8}}}{1+x_{1/8}} \text{ TeV} < 1.3 \text{ TeV}, \quad (\text{C.3})$$

where, for the last inequality, we take into account that the correct sign of the $R_{K^{(*)}}$ anomaly requires $x_{1/8} > -0.5$. The situation is then similar to the U_U case.

- U_D

$$|\lambda_{\tau\tau bb}|^2 \sim 7(1 + x_{1/8}) \frac{M_{U_D}^2}{\text{TeV}^2} < 4\pi \implies M_{U_D} < \frac{1.3}{1 + x_{1/8}} \text{TeV}, \quad (\text{C.4})$$

which is allowed by direct LHC searches for $x_{1/8} < -0.24$.

We conclude that, assuming quasi-degeneracy within the U_1^μ multiplet, the combination of perturbative unitarity and LHC direct searches excludes the U_U and U_Q flavor models.

D $pp \rightarrow \mu^+ \mu^-$

MFV models that contribute to $C_{bs\mu\mu}$ of eq. (1.3), generate also the terms

$$C_{ss\mu\mu}(\overline{s_L}\gamma^\mu s_L)(\overline{\mu_L}\gamma_\mu \mu_L) + C_{dd\mu\mu}(\overline{d_L}\gamma^\mu d_L)(\overline{\mu_L}\gamma_\mu \mu_L). \quad (\text{D.1})$$

These terms contribute to $pp \rightarrow \mu^+ \mu^-$. Ref. [54] obtains from the experimental measurements the following bounds, which hold for leptoquark heavy enough that its effect on $pp \rightarrow \mu^+ \mu^-$ is captured by EFT:

$$C_{dd\mu\mu} < 0.023 \text{ TeV}^{-2}, \quad C_{ss\mu\mu} < 0.15 \text{ TeV}^{-2}. \quad (\text{D.2})$$

Together with the $R_{K^{(*)}}$ constraints, these bounds imply

$$C_{dd\mu\mu}^U / C_{bs\mu\mu}^U \lesssim 23, \quad C_{ss\mu\mu}^U / C_{bs\mu\mu}^U \lesssim 150. \quad (\text{D.3})$$

- U_U :

$$C_{dd\mu\mu}^U / C_{bs\mu\mu}^U = |V_{td}|^2 / V_{ts}^*, \quad C_{ss\mu\mu}^U / C_{bs\mu\mu}^U = V_{ts}. \quad (\text{D.4})$$

The U_U contribution to $pp \rightarrow \mu\mu$ is negligibly small.

- U_Q :

$$\frac{C_{dd\mu\mu}^U}{C_{bs\mu\mu}^U} = \frac{C_{ss\mu\mu}^U}{C_{bs\mu\mu}^U} = \frac{|x_{1/8}|^2}{y_t^2 V_{tb} V_{ts}^* (2\mathcal{R}e(x_{1/8}) + y_t^2 |V_{tb}|^2)}. \quad (\text{D.5})$$

Thus, for the U_Q model, the $pp \rightarrow \mu\mu$ bound forbids $x_{1/8}$ outside the window $-0.4 \lesssim x_{1/8} \lesssim +2.4$.

- U_D :

$$\frac{C_{dd\mu\mu}^U}{C_{bs\mu\mu}^U} = \frac{|V_{tb} V_{td}|^2}{V_{tb} V_{ts}^* (x_{1/8}^* + y_t^2 |V_{tb}|^2)}, \quad \frac{C_{ss\mu\mu}^U}{C_{bs\mu\mu}^U} = \frac{V_{tb}^* V_{ts}}{x_{1/8}^* + y_t^2 |V_{tb}|^2}. \quad (\text{D.6})$$

Thus U_D contribution to $pp \rightarrow \mu\mu$ is negligibly small, except for a small region which is excluded, $-1 < x_{1/8} \lesssim -0.99$.

E $R_{\tau/K}$ and $R_{D^{(*)}}$

Here we discuss two tests of lepton flavor universality by charged current decays. We reconsider $R_{D^{(*)}}$, discussed above in section 4.3, and we add another test, the $R_{\tau/K}$ ratio. We show that neither the U_U model nor the U_Q model can account for the $R_{D^{(*)}}$ anomaly, independently of the $R_{K^{(*)}}$ anomaly.

The $R_{\tau/K}$ ratio is defined via

$$R_{\tau/K} \equiv \frac{\Gamma(\tau \rightarrow K \nu_\tau)}{\Gamma(K \rightarrow \mu \nu_\mu)}. \quad (\text{E.1})$$

ref. [55] translates the experimental measurements of the rates into the following bound (see their table 2):

$$\frac{|C_{su\tau\nu}^{\text{SM}} + C_{su\tau\nu}^{\text{NP}}|}{|C_{su\mu\nu}^{\text{SM}} + C_{su\mu\nu}^{\text{NP}}|} = 0.986 \pm 0.007, \quad (\text{E.2})$$

where $C_{sul\nu}$ is the Wilson coefficient of the term

$$C_{sul\nu}(\overline{u_L}\gamma^\mu s_L)(\overline{\ell_L}\gamma_\mu \nu_{\ell L}). \quad (\text{E.3})$$

The MFV models that we discuss contribute mainly to $C_{su\tau\nu}$:

$$C_{su\tau\nu}^U = \sum_i \frac{\lambda_{\tau\tau is}^*}{M_{\tau i}^2} \sum_j \lambda_{\tau\tau ij} V_{uj}. \quad (\text{E.4})$$

We now obtain the predictions of the three U_1^μ MFV-models for $C_{su\tau\nu}$ and, where relevant, for $C_{bc\tau\nu}$.

- U_U :

$$C_{su\tau\nu}^U = C_{bc\tau\nu}^U = 0. \quad (\text{E.5})$$

The U_U model predicts

$$R_{\tau/K} = R_{\tau/K}^{\text{SM}}, \quad R_{D^{(*)}} = R_{D^{(*)}}^{\text{SM}}. \quad (\text{E.6})$$

Thus, the U_U model cannot account for the $R_{D^{(*)}}$ anomaly *independently* of the $R_{K^{(*)}}$ anomaly.

- U_Q :

$$C_{su\tau\nu}^U = \frac{|bx_{1/8}|^2 y_\tau^2 V_{us}}{M_{\tau s}^2}, \quad C_{bc\tau\nu}^U = \frac{|bx_{1/8}|^2 y_\tau^2 V_{cb}}{M_{\tau i}^2}. \quad (\text{E.7})$$

Thus

$$C_{su\tau\nu}^U / C_{bc\tau\nu}^U = V_{us} / V_{cb} = C_{su\tau\nu}^{\text{SM}} / C_{bc\tau\nu}^{\text{SM}}, \quad (\text{E.8})$$

and the U_Q model predicts

$$\frac{R_{\tau/K}}{R_{\tau/K}^{\text{SM}}} = \frac{R_{D^{(*)}}}{R_{D^{(*)}}^{\text{SM}}}, \quad (\text{E.9})$$

which is strongly excluded.

- U_D :

$$C_{su\tau\nu} = \frac{|b|^2 x_{1/8} y_\tau^2 y_b^2 y_t^2 V_{ts} V_{tb}^* V_{ub}}{M_{\tau b}^2}, \quad (\text{E.10})$$

which is negligibly small. The U_D model predicts

$$R_{\tau/K} \approx R_{\tau/K}^{\text{SM}}. \quad (\text{E.11})$$

The combination of the $R_{D^{(*)}}$ and $R_{K^{(*)}}$ constraints on U_D was discussed in section 4.3. An analysis of the U_D model with regard to $R_{D^{(*)}}$, independently of $R_{K^{(*)}}$, was carried out in ref. [56].

The analysis of $R_{\tau/\pi} \equiv \Gamma(\tau \rightarrow \pi \nu_\tau)/\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu)$ goes along similar lines.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] LHCb collaboration, *Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **113** (2014) 151601 [[arXiv:1406.6482](https://arxiv.org/abs/1406.6482)] [[INSPIRE](#)].
- [2] LHCb collaboration, *Test of lepton universality with $B^0 \rightarrow K^{*0} \ell^+ \ell^-$ decays*, *JHEP* **08** (2017) 055 [[arXiv:1705.05802](https://arxiv.org/abs/1705.05802)] [[INSPIRE](#)].
- [3] G. Hiller and M. Schmaltz, *R_K and future $b \rightarrow s \ell \ell$ physics beyond the Standard Model opportunities*, *Phys. Rev. D* **90** (2014) 054014 [[arXiv:1408.1627](https://arxiv.org/abs/1408.1627)] [[INSPIRE](#)].
- [4] G. Hiller and I. Nisandzic, *R_K and R_{K^*} beyond the Standard Model*, *Phys. Rev. D* **96** (2017) 035003 [[arXiv:1704.05444](https://arxiv.org/abs/1704.05444)] [[INSPIRE](#)].
- [5] R. Alonso, B. Grinstein and J. Martin Camalich, *Lepton universality violation and lepton flavor conservation in B -meson decays*, *JHEP* **10** (2015) 184 [[arXiv:1505.05164](https://arxiv.org/abs/1505.05164)] [[INSPIRE](#)].
- [6] D. Bečirević, N. Košnik, O. Sumensari and R. Zukanovich Funchal, *Palatable leptoquark scenarios for lepton flavor violation in exclusive $b \rightarrow s \ell_1 \ell_2$ modes*, *JHEP* **11** (2016) 035 [[arXiv:1608.07583](https://arxiv.org/abs/1608.07583)] [[INSPIRE](#)].
- [7] A. Crivellin, D. Müller and T. Ota, *Simultaneous explanation of $R(D^{(*)})$ and $b \rightarrow s \mu^+ \mu^-$: the last scalar leptoquarks standing*, *JHEP* **09** (2017) 040 [[arXiv:1703.09226](https://arxiv.org/abs/1703.09226)] [[INSPIRE](#)].
- [8] A.K. Alok, B. Bhattacharya, A. Datta, D. Kumar, J. Kumar and D. London, *New physics in $b \rightarrow s \mu^+ \mu^-$ after the measurement of R_{K^*}* , [arXiv:1704.07397](https://arxiv.org/abs/1704.07397) [[INSPIRE](#)].
- [9] S. Fajfer and N. Košnik, *Vector leptoquark resolution of R_K and $R_{D^{(*)}}$ puzzles*, *Phys. Lett. B* **755** (2016) 270 [[arXiv:1511.06024](https://arxiv.org/abs/1511.06024)] [[INSPIRE](#)].
- [10] Y. Nir, *Flavor physics: past, present, future*, *Phys. Scripta* **T 158** (2013) 014005 [[INSPIRE](#)].
- [11] L. Calibbi, A. Crivellin and T. Ota, *Effective field theory approach to $b \rightarrow s \ell \ell^{(\prime)}$, $B \rightarrow K^{(*)} \nu \bar{\nu}$ and $B \rightarrow D^{(*)} \tau \nu$ with third generation couplings*, *Phys. Rev. Lett.* **115** (2015) 181801 [[arXiv:1506.02661](https://arxiv.org/abs/1506.02661)] [[INSPIRE](#)].
- [12] D. Buttazzo, A. Greljo, G. Isidori and D. Marzocca, *B -physics anomalies: a guide to combined explanations*, [arXiv:1706.07808](https://arxiv.org/abs/1706.07808) [[INSPIRE](#)].

- [13] J.M. Cline and J. Martin Camalich, *B decay anomalies from non-Abelian local horizontal symmetry*, *Phys. Rev. D* **96** (2017) 055036 [[arXiv:1706.08510](#)] [[INSPIRE](#)].
- [14] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: an effective field theory approach*, *Nucl. Phys. B* **645** (2002) 155 [[hep-ph/0207036](#)] [[INSPIRE](#)].
- [15] S. Davidson and S. Descotes-Genon, *Minimal flavour violation for leptiquarks*, *JHEP* **11** (2010) 073 [[arXiv:1009.1998](#)] [[INSPIRE](#)].
- [16] A. Crivellin, D. Müller, A. Signer and Y. Ulrich, *Correlating lepton flavour (universality) violation in B decays with $\mu \rightarrow e\gamma$ using leptiquarks*, [arXiv:1706.08511](#) [[INSPIRE](#)].
- [17] A.L. Kagan, G. Perez, T. Volansky and J. Zupan, *General minimal flavor violation*, *Phys. Rev. D* **80** (2009) 076002 [[arXiv:0903.1794](#)] [[INSPIRE](#)].
- [18] BABAR collaboration, J.P. Lees et al., *Search for $B^+ \rightarrow K^+ \tau^+ \tau^-$ at the BaBar experiment*, *Phys. Rev. Lett.* **118** (2017) 031802 [[arXiv:1605.09637](#)] [[INSPIRE](#)].
- [19] PARTICLE DATA GROUP collaboration, C. Patrignani et al., *Review of particle physics*, *Chin. Phys. C* **40** (2016) 100001 [[INSPIRE](#)].
- [20] BABAR collaboration, J.P. Lees et al., *Search for $B \rightarrow K^{(*)} \nu \bar{\nu}$ and invisible quarkonium decays*, *Phys. Rev. D* **87** (2013) 112005 [[arXiv:1303.7465](#)] [[INSPIRE](#)].
- [21] BELLE collaboration, O. Lutz et al., *Search for $B \rightarrow h^{(*)} \nu \bar{\nu}$ with the full Belle $\Upsilon(4S)$ data sample*, *Phys. Rev. D* **87** (2013) 111103 [[arXiv:1303.3719](#)] [[INSPIRE](#)].
- [22] A.J. Buras, J. Girrbach-Noe, C. Niehoff and D.M. Straub, *$B \rightarrow K^{(*)} \nu \bar{\nu}$ decays in the Standard Model and beyond*, *JHEP* **02** (2015) 184 [[arXiv:1409.4557](#)] [[INSPIRE](#)].
- [23] L.-S. Geng, B. Grinstein, S. Jäger, J. Martin Camalich, X.-L. Ren and R.-X. Shi, *Towards the discovery of new physics with lepton-universality ratios of $b \rightarrow s \ell \ell$ decays*, [arXiv:1704.05446](#) [[INSPIRE](#)].
- [24] D.A. Faroughy, A. Greljo and J.F. Kamenik, *Confronting lepton flavor universality violation in B decays with high- p_T tau lepton searches at LHC*, *Phys. Lett. B* **764** (2017) 126 [[arXiv:1609.07138](#)] [[INSPIRE](#)].
- [25] ATLAS collaboration, *A search for high-mass resonances decaying to $\tau^+ \tau^-$ in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, *JHEP* **07** (2015) 157 [[arXiv:1502.07177](#)] [[INSPIRE](#)].
- [26] ATLAS collaboration, *Search for minimal supersymmetric Standard Model Higgs bosons H/A and for a Z' boson in the $\tau\tau$ final state produced in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector*, *Eur. Phys. J. C* **76** (2016) 585 [[arXiv:1608.00890](#)] [[INSPIRE](#)].
- [27] BABAR collaboration, J.P. Lees et al., *Evidence for an excess of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ decays*, *Phys. Rev. Lett.* **109** (2012) 101802 [[arXiv:1205.5442](#)] [[INSPIRE](#)].
- [28] BABAR collaboration, J.P. Lees et al., *Measurement of an excess of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ decays and implications for charged Higgs bosons*, *Phys. Rev. D* **88** (2013) 072012 [[arXiv:1303.0571](#)] [[INSPIRE](#)].
- [29] BELLE collaboration, M. Huschle et al., *Measurement of the branching ratio of $\bar{B} \rightarrow D^{(*)} \tau^- \bar{\nu}_\tau$ relative to $\bar{B} \rightarrow D^{(*)} \ell^- \bar{\nu}_\ell$ decays with hadronic tagging at Belle*, *Phys. Rev. D* **92** (2015) 072014 [[arXiv:1507.03233](#)] [[INSPIRE](#)].

- [30] BELLE collaboration, Y. Sato et al., *Measurement of the branching ratio of $\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau$ relative to $\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}_\ell$ decays with a semileptonic tagging method*, *Phys. Rev. D* **94** (2016) 072007 [[arXiv:1607.07923](#)] [[INSPIRE](#)].
- [31] A. Abdesselam et al., *Measurement of the τ lepton polarization in the decay $\bar{B} \rightarrow D^*\tau^-\bar{\nu}_\tau$* , [arXiv:1608.06391](#) [[INSPIRE](#)].
- [32] BELLE collaboration, S. Hirose et al., *Measurement of the τ lepton polarization and $R(D^*)$ in the decay $\bar{B} \rightarrow D^*\tau^-\bar{\nu}_\tau$* , *Phys. Rev. Lett.* **118** (2017) 211801 [[arXiv:1612.00529](#)] [[INSPIRE](#)].
- [33] LHCb collaboration, *Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\mu^-\bar{\nu}_\mu)$* , *Phys. Rev. Lett.* **115** (2015) 111803 [*Addendum ibid.* **115** (2015) 159901] [[arXiv:1506.08614](#)] [[INSPIRE](#)].
- [34] Y. Amhis et al., *Averages of b -hadron, c -hadron and τ -lepton properties as of summer 2016*, [arXiv:1612.07233](#) [[INSPIRE](#)].
- [35] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, *Eur. Phys. J. C* **77** (2017) 112 [[arXiv:1607.00299](#)] [[INSPIRE](#)].
- [36] S. Fajfer, J.F. Kamenik and I. Nisandzic, *On the $B \rightarrow D^*\tau\bar{\nu}_\tau$ sensitivity to new physics*, *Phys. Rev. D* **85** (2012) 094025 [[arXiv:1203.2654](#)] [[INSPIRE](#)].
- [37] D. Bigi, P. Gambino and S. Schacht, *$R(D^*)$, $|V_{cb}|$ and the heavy quark symmetry relations between form factors*, [arXiv:1707.09509](#) [[INSPIRE](#)].
- [38] S. Jaiswal, S. Nandi and S.K. Patra, *Extraction of $|V_{cb}|$ from $B \rightarrow D^{(*)}\ell\nu_\ell$ and the Standard Model predictions of $R(D^{(*)})$* , [arXiv:1707.09977](#) [[INSPIRE](#)].
- [39] I. Doršner, S. Fajfer, D.A. Faroughy and N. Košnik, *Saga of the two GUT leptoquarks in flavor universality and collider searches*, [arXiv:1706.07779](#) [[INSPIRE](#)].
- [40] M. Bauer and M. Neubert, *Minimal leptoquark explanation for the $R_{D^{(*)}}$, R_K and $(g-2)_g$ anomalies*, *Phys. Rev. Lett.* **116** (2016) 141802 [[arXiv:1511.01900](#)] [[INSPIRE](#)].
- [41] D. Bečirević and O. Sumensari, *A leptoquark model to accommodate $R_K^{\text{exp}} < R_K^{\text{SM}}$ and $R_{K^*}^{\text{exp}} < R_{K^*}^{\text{SM}}$* , *JHEP* **08** (2017) 104 [[arXiv:1704.05835](#)] [[INSPIRE](#)].
- [42] Y. Cai, J. Gargalionis, M.A. Schmidt and R.R. Volkas, *Reconsidering the one leptoquark solution: flavor anomalies and neutrino mass*, *JHEP* **10** (2017) 047 [[arXiv:1704.05849](#)] [[INSPIRE](#)].
- [43] C.-W. Chiang, X.-G. He, J. Tandean and X.-B. Yuan, *$R_{K^{(*)}}$ and related $b \rightarrow s\ell\bar{\ell}$ anomalies in minimal flavor violation framework with Z' boson*, [arXiv:1706.02696](#) [[INSPIRE](#)].
- [44] A. Dery, A. Efrati, G. Hiller, Y. Hochberg and Y. Nir, *Higgs couplings to fermions: 2HDM with MFV*, *JHEP* **08** (2013) 006 [[arXiv:1304.6727](#)] [[INSPIRE](#)].
- [45] J. Charles et al., *Current status of the Standard Model CKM fit and constraints on $\Delta F = 2$ new physics*, *Phys. Rev. D* **91** (2015) 073007 [[arXiv:1501.05013](#)] [[INSPIRE](#)].
- [46] S. Davidson, D.C. Bailey and B.A. Campbell, *Model independent constraints on leptoquarks from rare processes*, *Z. Phys. C* **61** (1994) 613 [[hep-ph/9309310](#)] [[INSPIRE](#)].
- [47] L. Di Luzio and M. Nardecchia, *What is the scale of new physics behind the B -flavour anomalies?*, *Eur. Phys. J. C* **77** (2017) 536 [[arXiv:1706.01868](#)] [[INSPIRE](#)].
- [48] G. D'Amico et al., *Flavour anomalies after the R_{K^*} measurement*, *JHEP* **09** (2017) 010 [[arXiv:1704.05438](#)] [[INSPIRE](#)].

- [49] CMS collaboration, *Search for third-generation scalar leptoquarks and heavy right-handed neutrinos in final states with two tau leptons and two jets in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *JHEP* **07** (2017) 121 [[arXiv:1703.03995](#)] [[INSPIRE](#)].
- [50] B. Diaz, M. Schmaltz and Y.-M. Zhong, *The leptoquark hunter's guide: pair production*, *JHEP* **10** (2017) 097 [[arXiv:1706.05033](#)] [[INSPIRE](#)].
- [51] CMS collaboration, *Search for new physics in the all-hadronic final state with the M_{T2} variable*, *CMS-PAS-SUS-16-036*, CERN, Geneva Switzerland, (2016).
- [52] ATLAS collaboration, *A search for B-L R-parity-violating scalar tops in $\sqrt{s} = 13$ TeV pp collisions with the ATLAS experiment*, *ATLAS-CONF-2017-036*, CERN, Geneva Switzerland, (2017).
- [53] ATLAS collaboration, *Searches for scalar leptoquarks in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, *Eur. Phys. J. C* **76** (2016) 5 [[arXiv:1508.04735](#)] [[INSPIRE](#)].
- [54] A. Greljo and D. Marzocca, *High- p_T dilepton tails and flavor physics*, *Eur. Phys. J. C* **77** (2017) 548 [[arXiv:1704.09015](#)] [[INSPIRE](#)].
- [55] A. Pich, *Precision tau physics*, *Prog. Part. Nucl. Phys.* **75** (2014) 41 [[arXiv:1310.7922](#)] [[INSPIRE](#)].
- [56] M. Freytsis, Z. Ligeti and J.T. Ruderman, *Flavor models for $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$* , *Phys. Rev. D* **92** (2015) 054018 [[arXiv:1506.08896](#)] [[INSPIRE](#)].