RECEIVED: July 28, 2017

REVISED: October 9, 2017

ACCEPTED: October 23, 2017

PUBLISHED: November 9, 2017

Electroweak baryogenesis in the $\mathbb{Z}_3$ -invariant NMSSM

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ABSTRACT: We calculate the baryon asymmetry of the Universe in the $\mathbb{Z}_3$ -invariant Next-to-Minimal Supersymmetric Standard Model where the interactions of the singlino provide the necessary source of charge and parity violation. Using the closed time path formalism, we derive and solve transport equations for the cases where the singlet acquires a vacuum expectation value (VEV) before and during the electroweak phase transition. We perform a detailed scan to show how the baryon asymmetry varies throughout the relevant parameter space. Our results show that the case where the singlet acquires a VEV during the electroweak phase transition typically generates a larger baryon asymmetry, although we expect that the case where the singlet acquires a VEV first is far more common for any model in which parameters unify at a high scale. Finally, we examine the dependence of the baryon asymmetry on the three-body interactions involving gauge singlets.

KEYWORDS: Supersymmetry Phenomenology

ARXIV EPRINT: [1706.09898](https://arxiv.org/abs/1706.09898)

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1 Introduction

A cosmological history that includes a period of inflation inevitably washes out any possible primordial baryon asymmetry in the Universe (BAU). Yet, currently we observe an asymmetry between baryons and anti-baryons, quantified by the ratio of the average baryon and entropy densities

$$Y_B = \frac{n_B}{s} = \begin{cases} 8.2\text{--}9.4 \times 10^{-11} \text{ (95\% CL)} & \text{BBN [1],} \\ 8.65 \pm 0.09 \times 10^{-11} & \text{PLANCK [2].} \end{cases} \quad (1.1)$$

Various mechanisms have been proposed to create the observed asymmetry after inflation and many of these scenarios rely on a thermodynamic phase transition that may have happened before Big Bang Nucleosynthesis (BBN). The only known cosmic phase transition that occurred before BBN is the electroweak phase transition (EWPT). The non-equilibrium condition created by EWPT is utilised by the electroweak baryogenesis (EWBG) [3, 4] mechanism to produce the baryon asymmetry. The presence of the electroweak scale suggests that tests of EWBG may be within reach [5–14]. Aesthetically attractive that EWBG features a common origin for the breaking of both the baryon and electroweak symmetry.

EWBG requires that the EWPT be strongly first order. In the Standard Model (SM) the Higgs boson is too heavy to allow for a strongly first order EWPT [15]. The order of the EWPT can be boosted by new weak scale particles that interact with the Higgs. In supersymmetry, the stops can catalyse a strongly first order EWPT [16]. Search results

from the LHC, however, impose severe constraints on such a possibility [17–23]. Even if a stop is light enough to boost the order of the EWPT, electric dipole moment (EDM) constraints render the charge and parity (CP) violating phase present in the stop-Higgs coupling¹ to be insufficient [26] to produce the observed baryon asymmetry. These issues in conjunction with the little hierarchy problem [27–34], which manifests from a combination of the Higgs mass measurement and null searches for supersymmetric particles, give strong motivation for looking at extensions to the minimal supersymmetric scenario² [35, 36].

Adding a gauge-singlet scalar superfield to the superpotential introduces extra degrees of freedom that couple to the Higgs, which can boost the strength of the phase transition [37–39] and relax the need for the stop mass to be close the EWPT scale.³ Additionally, experimental constraints on gauge singlets are not very onerous [41, 42] and there are strong motivations for considering gauge singlets beyond baryogenesis. In the case where there is a discrete $\mathbb{Z}_3$ symmetry between the singlet and Higgs sector, the Next-to-Minimal Supersymmetric Standard Model (NMSSM) is a proposed solution to the μ problem [43–55]. Further, gauge singlets naturally arise in GUTs [56] as well as in string theory [57]. Moreover, the NMSSM can accommodate a strongly first order electroweak phase transition [58–61], dark matter [62–67] and inflation simultaneously [68], and may lead to observable gravitational waves [69, 70]. Finally, the singlet can serve to boost the variation of β (where $\tan \beta$ is the ratio of the two Higgs VEVs) during the phase transition by an order of magnitude compared to the MSSM [37]. Since the BAU is proportional to $\Delta\beta$ there is a possibility that CP violating phases can be smaller, further evading EDM constraints, and still produce enough BAU.

The production of the BAU has been explored in the NMSSM via the WKB approach⁴ [73]. This approximation, however, can miss substantial “memory effects” that can result in resonant enhancements of the BAU of up to several orders of magnitude when the masses are near degenerate⁵ [76]. Non-equilibrium quantum field theory has also been used [77, 78], utilizing the closed time path (CTP) formalism [79–84] to calculate the BAU in the presence of the resonances. ref. [77], however, invokes the fast rate approximation, which can differ from more precise methods by two orders of magnitude [85] in its determination of the BAU.

In this work we derive the transport equations for the most relevant particle species in the NMSSM for the cases where the singlet acquires a VEV before and during the electroweak phase transition. We then solve them without assuming that three body Yukawa, triscalar, strong sphaleron or supergauge interactions in the Higgses and Higgsino sector are large using the semi-analytic methods described in refs. [86, 87]. We also seek to answer

¹For recent work on CP-violation in the MSSM and other MSSM extensions see [24, 25].

²Especially, when one considers the motivations for supersymmetry: radiative electroweak symmetry breaking leading to a light SM-like Higgs, dark matter, solution of the gauge hierarchy problem, gauge coupling unification in Grand Unified Theories (GUTs), and string theory.

³A singlet can also ease the little hierarchy problem [29, 40].

⁴For a derivation of the WKB approach from first principles see [71, 72].

⁵Recent work [74] using Wigner functionals for a toy model suggested the interesting possibility that the resonance might be severely dampened when the masses are exactly degenerate. A more precise treatment in [75] however seemed to indicate there was indeed a resonance.

the question as to whether three body interactions involving gauge singlets can in principle compete with resonant relaxation terms arising from CP-conserving interactions with the bubble wall. In the MSSM, such an effect provides a counter-intuitive boost to the BAU by several orders of magnitude near resonance despite these three body rates being relaxation terms. Since we are principally concerned with the plausibility of EWBG within the NMSSM being driven by interactions with the singlino, we root our results in constraints on the Higgs mass and LHC searches, performing a scan of the relevant parameter space. Finally we look at how different types of phase transitions can affect the baryon asymmetry. Whether the singlet acquires a VEV before or during the electroweak phase transition has a large impact on the determination of the BAU: if the singlet phase transition occurs before EWPT, then outside of the bubble, the singlino and Higgsinos have non-zero masses, without thermal corrections. This affects not only the CP-violating source terms and CP-conserving relaxation terms, but the thermal decay widths of the singlino. In fact, the two scenarios change the structure of the transport equations as fluctuations around the VEV are real by definition (and therefore possess no asymmetry).

The structure of this paper is as follows. In section 2 we review the NMSSM and its motivations with a particular emphasis on the structure of the effective potential. section 3 contains our derivation of the coupled transport equations for both phase transition structures mentioned above outlining our assumptions. In section 4 we discuss the input parameters of the model with particular emphasis on the thermal widths. We then solve these equations in section 5 scanning over the NMSSM parameter space. We examine the effect of both cases: the singlet acquiring a VEV before or during the EWPT. We also examine the effect of the three body gauge singlet transport coefficients in section 6, before a final discussion and conclusion in section 7.

2 The Next-to-Minimal Supersymmetric Standard Model

We calculate the baryon asymmetry within the scale invariant, $\mathbb{Z}_3$ -conserving, NMSSM defined by [35, 88]

$$W^{\text{NMSSM}} = W^{\text{MSSM}}|_{\mu=0} + \lambda \hat{S} \hat{\mathbf{H}}_u \hat{\mathbf{H}}_d + \frac{1}{3} \kappa \hat{S}^3. \quad (2.1)$$

The MSSM superpotential $W^{\text{MSSM}}|_{\mu=0}$, less the μ term, is defined in [89], and $\hat{\mathbf{H}}_{u,d}$ and $\hat{S}$ are SU(2) doublet and singlet Higgs superfields, respectively. In addition to the MSSM soft supersymmetry breaking terms (with B set to zero [89]), the scalar potential contains

$$V_{\text{soft}}^{\text{NMSSM}} = m_s^2 |S|^2 - \lambda A_\lambda S \mathbf{H}_u \mathbf{H}_d + \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.}, \quad (2.2)$$

where $\mathbf{H}_{u,d}$ and S are the scalar components of the Higgs doublet (singlet) superfields. The neutral components of these scalars acquire a non-zero VEV during electroweak symmetry breaking. Relative to the MSSM, the new terms in the Higgs potential are

$$V_{\mathbf{H}}^{\text{NMSSM}} = \lambda^2 |S|^2 (|\mathbf{H}_u|^2 + |\mathbf{H}_d|^2) \lambda^2 |\mathbf{H}_u \mathbf{H}_d|^2 + \kappa^2 |S|^4 + \kappa \lambda S^2 \mathbf{H}_u^* \mathbf{H}_d^* + \text{h.c.} \quad (2.3)$$

To boost the baryon asymmetry we assume complex values for $\lambda, \kappa, A_\lambda$ and A_κ . Once both the Higgs doublets and the singlet scalar have acquired a vacuum expectation value, one can write three re-phasing invariants that appear in the tree level potential [77, 90]

$$\phi_\lambda - \phi_\kappa, \quad \phi_\lambda + \phi_{A_\kappa}, \quad \phi_\lambda + \phi_{A_\lambda} . \quad (2.4)$$

Here ϕ_x is the complex phase associated with parameter $x \in (\lambda, \kappa, A_\kappa, A_\lambda)$. One can then use the CP-odd tadpole conditions to write two of the rephasing invariants in terms of $\phi_\lambda - \phi_\kappa$. Therefore there is in reality only a single independent rephasing invariant [91]. For more details on the NMSSM see refs. [35, 68, 89].

3 Transport equations

In this section we derive the set of coupled transport equations that govern the behaviour of number densities throughout the phase transition [92]. In the NMSSM there are two possible phase histories that qualitatively change the transport equations⁶

- Singlet first phase transition (SFPT) phase transition: the singlet acquires a VEV before the EWPT.
- Singlet spontaneous phase transition (SSPT): the singlet acquires a VEV during the EWPT.

We make use of the closed time path formalism [79, 80, 82, 84, 94] following the procedure given in [76] to derive the set of transport coefficients and CP violating source terms. We will use the usual VEV insertion approximation (VIA) which assumes that the physics responsible for producing the BAU is dominated by the region in front of the bubble wall where the Higgs VEV is small compared to the nucleation temperature and the relevant mass differences. The NMSSM includes a source of CP violation in addition to the CP violating interactions present in the MSSM due to singlino-Higgsino interactions with the space time varying vacuum. If the singlino and Higgsino masses are near degenerate in the symmetric phase there will CP violation source is expected to be resonantly enhanced [76, 95–98]. We note that the vev-insertion approach is an approach with unknown theoretical error. For example, work that included flavour oscillations showed a significant dampening of the resonance [74, 99] yet the resonance was shown to re-emerge when one uses a more consistent power counting [75]. A full systematic treatment that includes a consistent power counting, flavour oscillations and vev resummations remains a work in progress (see [74, 75, 98, 99]). For now we take our results as indicative of the dependence of the magnitude of the BAU on the Lagrangian parameters.

We ignore first and second generation quarks and squarks as well as all three generations of leptons and sleptons, invoking the assumption that the rates that connect these particle species to the rest of the transport equations are small due to the fact that the relevant Yukawa couplings are small. This is an assumption that can in some parts of

⁶This is a reduced list from the four types given in [93] as this partition is more convenient when discussing electroweak ‘baryogenesis’.

the parameter space be too rough [100] but we leave a thorough investigation of this to future work.

It has been demonstrated that assuming local equilibrium between third generation quarks and squarks holds well for large parts of the parameter space [101]. On the other hand, one has to be cautious in assuming supergauge equilibrium between the Higgs and the Higgsino. Fast supergauge interactions in the Higgs sector would lead to a suppression of the combination $\mu_{H_i} - \mu_{\tilde{H}}$ whereas three body interactions involving singlets/singlinos lead to the suppression of the combination $\mu_{H_i} + \mu_{\tilde{H}}$. If one works in the approximation that both types of rates are fast enough to set the combinations of chemical potentials given before to zero, then the baryon asymmetry vanishes. While the competition of these types of rates could indeed suppress the baryon asymmetry we take the precaution of including both rates in the transport equations rather than assuming they are large enough to result in local equilibrium relations.

As usual, number densities are defined $n \equiv n - \bar{n}$ where $\bar{n}$ is the anti-particle density. When the gauge singlet acquires a vacuum expectation value the fluctuations around this VEV are real and cannot hold any asymmetry. Therefore in a SFPT phase transition the number density of the singlet is zero. This means that there is one less transport equation for SFPT phase transitions. Finally we note that one cannot form a vector charge for the singlino, so we do not include a number density for the singlino.⁷

Under these simplifying assumptions we are able to derive a set of coupled transport equations for six charge densities for SSPT phase transitions and five for SFPT phase transitions as the number density of the singlet $n_S \equiv n_S - \bar{n}_S$ is zero. We present the more complicated SSPT case as the SFPT case can be derived from it by setting n_S to zero and modifying transport coefficients that depend on v_S . For a SFPT phase transition, the six linear combination of number densities which make up the transport equations are

$$\begin{aligned}
n_{H_1} &= n_{H_u^+} + n_{H_u^0}, \\
n_{H_2} &= n_{H_d^-} + n_{H_d^0}, \\
n_{\tilde{H}} &= n_{\tilde{H}_u^+} + n_{\tilde{H}_u^0} - n_{\tilde{H}_d^-} - n_{\tilde{H}_d^0}, \\
n_t &= n_{t_R} + n_{\tilde{t}_R}, \\
n_Q &= n_{t_L} + n_{b_L} + n_{\tilde{t}_R} + n_{\tilde{b}_L}, \\
n_S &= n_S.
\end{aligned} \tag{3.1}$$

The transport coefficients are derived using the Schwinger-Dyson equations in the closed time path formalism to relate divergences of current densities to functions of self energies. These functions of self energies can be expanded in the chemical potentials of the particles involved in each self energy interaction. We can relate chemical potentials to number densities in the usual way. Ignoring terms of $O(\mu^3)$ we can derive the relation [103]

$$\mu_x = \frac{6}{T^2} \frac{n_x}{k_x}, \tag{3.2}$$

⁷In principle one can define an axial charge as was done here for binos and $\tilde{W}^3$, but the contribution from doing so was found to be small [102].

with

$$k_x = k_x(0) \frac{c_{F,B}}{\pi^2} \int_{m/T}^{\infty} dy y \frac{e^y}{(e^y \pm 1)^2} \sqrt{y^2 - m_x^2/T^2}, \quad (3.3)$$

where $c_{F(B)} = 6(3)$ and the sign in the denominator is $\pm$ for fermions and bosons, respectively. The factors $k_i(0)$ are 2 for Dirac fermions and complex scalars and 1 for chiral fermions. The k factors of our composite number densities in eq. (3.1) are the sum of the k factors for each component. We then define the linear combinations of rates that act as coefficients of these composite number densities.

Tree level interactions with space time varying VEVs have CP conserving components known as “mass terms” typically denoted by $\Gamma_m^{(\cdot)}$ where the superscript describes the particles involved in the interaction starting with the “in” state. The full set of relevant mass terms are⁸

$$\begin{aligned} \Gamma_m^t &= \Gamma_m^{t_R, t_L} + \Gamma_m^{\tilde{t}}, \\ \Gamma_m^{\tilde{H}} &= \Gamma_m^{\tilde{H}\tilde{W}^+} + \Gamma_m^{\tilde{H}\tilde{W}^0} + \Gamma_m^{\tilde{H}\tilde{B}} + \Gamma_m^{\tilde{H}\tilde{S}}, \\ \Gamma_m^{H_u H_d} &= \Gamma_m^{H_u H_d}, \\ \Gamma_m^{H_1 S} &= \Gamma_m^{H_1 S}, \\ \Gamma_m^{H_2 S} &= \Gamma_m^{H_2 S}, \\ \Gamma_m^{\tilde{H}\tilde{S}} &= \Gamma_m^{\tilde{H}\tilde{S}}. \end{aligned} \quad (3.4)$$

Note that Higgsino-Higgsino interactions with a space-time varying singlet VEV do not contribute as the masses are exactly degenerate by definition and the resonance vanishes when masses are exactly degenerate. Tri-scalar and Yukawa interactions have a general form related to the functions $I_{F/B}(m_1, m_2, m_3)$ which are defined in ref. [85] and for completeness are also given in section A.

We define the composite transport coefficients which make up our transport equations as

$$\begin{aligned} \Gamma_Y^{tQH_1} &= \frac{12N_C y_t^2}{T^2} \left[c^2 \mathcal{I}_F(m_{t_R}, m_Q, m_{H_1}) + |s\mu e^{-i\phi_\lambda} + cA_t|^2 \mathcal{I}_B(m_{\tilde{t}_R}, m_{\tilde{Q}}, m_{H_1}) \right], \\ \Gamma_Y^{tQH_2} &= \frac{12N_C y_t^2}{T^2} \left[s^2 \mathcal{I}_F(m_{t_R}, m_Q, m_{H_2}) + |c\mu e^{-i\phi_\lambda} - sA_t|^2 \mathcal{I}_B(m_{\tilde{t}_R}, m_{\tilde{Q}}, m_{H_2}) \right], \\ \Gamma_Y^{tQ\tilde{H}} &= \frac{12N_C y_t^2}{T^2} \left[\mathcal{I}_F(m_{\tilde{H}}, m_Q, m_{\tilde{t}_R}) + \mathcal{I}_F(m_{t_R}, m_{\tilde{H}}, m_{\tilde{Q}}) \right], \\ \Gamma_Y^{H_1, H_2, S} &= \frac{12|\lambda A_\lambda|^2}{T^2} \mathcal{I}_B(m_{H_1}, m_{H_2}, m_S), \\ \Gamma_Y^{\tilde{H}H_1\tilde{S}} &= \frac{12|\lambda|^2}{T^2} \mathcal{I}_F(m_{\tilde{H}}, m_{\tilde{S}}, m_{H_1}), \\ \Gamma_Y^{\tilde{H}H_2\tilde{S}} &= \frac{12|\lambda|^2}{T^2} \mathcal{I}_F(m_{\tilde{H}}, m_{\tilde{S}}, m_{H_2}), \\ \Gamma_Y^{\tilde{H}\tilde{H}S} &= \frac{12|\lambda|^2}{T^2} \mathcal{I}_F(m_{\tilde{H}}, m_{\tilde{H}}, m_S), \end{aligned} \quad (3.5)$$

⁸These mass terms typically come in two flavours $\Gamma_m^\pm$ however the negative type is typically much larger so the positive type we ignore.

where $c \equiv \cos \alpha$, $s \equiv \sin \alpha$ and α is the Higgs mixing angle in the unbroken phase. We also need to include estimates of the four body scattering terms which in general are small but may become the dominant contribution when three body decays are kinematically suppressed. For all cases our four body scattering rate is given by $0.19|x|^2 T/(6\pi)$ where $x \in \{g_3, \lambda, \lambda A_\lambda\}$ is the appropriate coupling constant. In some regions of parameter space the BAU will be sensitive to the precise value of these four body scattering rates so future work should consider a full numerical treatment of these coefficients.

Our transport equations can then be shown to have the form

$$\begin{aligned}
 \partial_\mu t^\mu &= -\Gamma_m^t \mathcal{U}_t^m - \Gamma_Y^{tQH_1} \mathcal{U}_{tQH_1}^Y - \Gamma_Y^{tQH_2} \mathcal{U}_{tQH_2}^Y - \Gamma_Y^{tQ\tilde{H}} \mathcal{U}_{tQ\tilde{H}}^Y + \Gamma_{SS} \mathcal{U}_5 + S_t^{\mathcal{CP}}, \\
 \partial_\mu Q^\mu &= \Gamma_m^t \mathcal{U}_t^m + \Gamma_Y^{tQH_1} \mathcal{U}_{tQH_1}^Y + \Gamma_Y^{tQH_2} \mathcal{U}_{tQH_2}^Y + \Gamma_Y^{tQ\tilde{H}} \mathcal{U}_{tQ\tilde{H}}^Y - 2\Gamma_{SS} \mathcal{U}_5 - S_t^{\mathcal{CP}}, \\
 \partial_\mu H_1^\mu &= -\Gamma_m^{H_1 H_2} \mathcal{U}_{H_1 H_2}^m - \Gamma_m^{H_1 S} \mathcal{U}_{H_1 S}^m + \Gamma_Y^{tQH_1} \mathcal{U}_{tqH_1}^Y \\
 &\quad - \Gamma_Y^{H_1 H_2 S} \mathcal{U}_{H_1 H_2 S}^Y - \Gamma_Y^{\tilde{H} H_1 \tilde{S}} \mathcal{U}_{\tilde{H} H_1 \tilde{S}}^Y - \Gamma_{\tilde{V}}^{H_1 \tilde{H}} \mathcal{U}_{\tilde{V}_1}, \\
 \partial_\mu H_2^\mu &= -\Gamma_m^{H_1 H_2} \mathcal{U}_{H_1 H_2}^m - \Gamma_m^{H_2 S} \mathcal{U}_{H_2 S}^m + \Gamma_Y^{tQH_2} \mathcal{U}_{tQH_2}^Y \\
 &\quad - \Gamma_Y^{H_1 H_2 S} \mathcal{U}_{H_1 H_2 S}^Y - \Gamma_Y^{\tilde{H} H_2 \tilde{S}} \mathcal{U}_{\tilde{H} H_2 \tilde{S}}^Y - \Gamma_{\tilde{V}}^{H_2 \tilde{H}} \mathcal{U}_{\tilde{V}_2}, \\
 \partial_\mu \tilde{H}^\mu &= -\Gamma_m^{\tilde{H}} \mathcal{U}_{\tilde{H}}^m - \Gamma_Y^{\tilde{H} S} \mathcal{U}_{\tilde{H} S}^Y + \Gamma_Y^{tQ\tilde{H}} \mathcal{U}_{tQ\tilde{H}}^Y - \Gamma_Y^{\tilde{H} H_1 \tilde{S}} \mathcal{U}_{\tilde{H} H_1 \tilde{S}}^Y \\
 &\quad - \Gamma_Y^{\tilde{H} H_2 \tilde{S}} \mathcal{U}_{\tilde{H} H_2 \tilde{S}}^Y + \Gamma_{\tilde{V}}^{H_1 \tilde{H}} \mathcal{U}_{\tilde{V}_1} + \Gamma_{\tilde{V}}^{H_2 \tilde{H}} \mathcal{U}_{\tilde{V}_2} + S_{\tilde{H}\tilde{W}}^{\mathcal{CP}} - S_{\tilde{H}\tilde{S}}^{\mathcal{CP}}, \\
 \partial_\mu S^\mu &= -\Gamma_m^{H_1 S} \mathcal{U}_{H_1 S}^m - \Gamma_m^{H_2 S} \mathcal{U}_{H_2 S}^m + \Gamma_Y^{H_1 H_2 S} \mathcal{U}_{H_1 H_2 S}^Y - \Gamma_Y^{\tilde{H} S} \mathcal{U}_{\tilde{H} S}^Y.
 \end{aligned} \tag{3.6}$$

Here we have defined the combinations of chemical potentials as follows

$$\begin{aligned}
 \mathcal{U}_t^m &= \left(\frac{t}{k_t} - \frac{Q}{k_Q} \right), & \mathcal{U}_{tQH_i}^Y &= \left(\frac{t}{k_t} - \frac{Q}{k_Q} - \frac{H_i}{k_{H_i}} \right), & \mathcal{U}_{tQ\tilde{H}}^Y &= \left(\frac{t}{k_t} - \frac{Q}{k_Q} - \frac{\tilde{H}}{k_{\tilde{H}}} \right), \\
 \mathcal{U}_{H_1 H_2}^m &= \left(\frac{H_1}{k_{H_1}} + \frac{H_2}{k_{H_2}} \right), & \mathcal{U}_{H_i S}^m &= \left(\frac{H_i}{k_{H_i}} + \frac{S}{k_S} \right), & \mathcal{U}_{H_1 H_2 S}^Y &= \left(\frac{H_1}{k_{H_1}} + \frac{H_2}{k_{H_2}} + \frac{S}{k_S} \right), \\
 \mathcal{U}_{\tilde{H} H_i \tilde{S}}^Y &= \left(\frac{\tilde{H}}{k_{\tilde{H}}} + \frac{H_i}{k_{H_i}} \right), & \mathcal{U}_{\tilde{V}_i} &= \left(\frac{H_i}{k_{H_i}} - \frac{\tilde{H}}{k_{\tilde{H}}} \right), \\
 \mathcal{U}_{\tilde{H}}^m &= \frac{\tilde{H}}{k_{\tilde{H}}}, & \mathcal{U}_{\tilde{H} S}^Y &= \left(\frac{S}{k_S} \right).
 \end{aligned} \tag{3.7}$$

As usual, the axial chemical potential, $\mathcal{U}_5$, is given by $\mu_L - \mu_R$. Since only the left handed quark doublet and the right handed top is sourced, this reduces to

$$\mathcal{U}_5 = \frac{2Q}{k_Q} - \frac{t}{k_t} + \frac{9(Q+t)}{k_b}. \tag{3.8}$$

Finally, the strong sphaleron rate is taken to be $\Gamma_{SS} \approx 16\alpha_s^4 T$ [104].

4 Thermal parameters

The production of the BAU is resonantly enhanced when the masses of the singlino and Higgsino (calculated in the symmetric phase) are nearly degenerate. The width and height

of the resonance, and therefore the width of the allowed parameter space, are controlled by the thermal widths of the singlino and the Higgsino. The magnitude of the thermal widths for various particles in the NMSSM is typically dominated by gauge interactions and are proportional to the involved coupling constant squared. This means that the thermal widths for (s)quarks tend to be quite large. However, in the absence of gauge interactions, the thermal width of the singlino is expected to be small. Indeed the only two places where the thermal width of the singlino or the singlet can be broadened is through Yukawa interactions.

Let us divert more attention to the thermal width of the singlino as the thermal width of the singlet only weakly affects the BAU. The relevant Yukawa interactions involve the singlino, Higgsino and the Higgs with relevant couplings of κ and λ . From ref. [105] the thermal width that results from such Yukawa interactions is

$$\Gamma_{\tilde{g}} = b(m_H^*)0.01(\lambda^2 + \kappa^2), \quad (4.1)$$

where $b(m_H^*)$ is a function which monotonically decreases with the mass of the Higgs in the symmetric phase. For a SSPT phase transition the Higgs mass is just the Debye mass under the VIA, where we assume the physics primarily responsible for the production of the BAU occurs in the symmetric phase just ahead of the bubble wall. The Higgs mass in the symmetric phase of a SFPT phase transition gets contributions from the singlet VEV and is thus boosted. The thermal width of the singlino is therefore much smaller for a SFPT phase transition.

We estimate the remaining thermal widths, diffusion coefficients and bubble wall properties in eq. (1). For the diffusion coefficients we use the values given in [106] from which we also derive the rest of our thermal widths. For the parameter $\Delta\beta$ we note that a feature of the NMSSM is it can be an order of magnitude larger than its MSSM value [37]. However, since the BAU is linearly proportional to $\Delta\beta$, the dependence on this parameter is trivial so we just take the fiducial value of 0.05 along with a fiducial value of the CP violating phase which we set to its maximal value. It is simple to translate our results to the case where $\Delta\beta$ is large. It should be noted however, that while one has substantial freedom in the NMSSM parameter space it is an assumption of this paper that there is enough freedom to treat the phase transition parameters, including $\Delta\beta$ as essentially free parameters. It is quite possible, however, that there is a strong correlation between the singlino and Higgsino masses in the symmetric phase which determine the strength of our CP violating sources and the parameters that characterize the EWPT. Eventually to fully determine the viability of baryogenesis in the $\mathbb{Z}_3$ invariant NMSSM one needs to consider the dynamics of the phase transition and quantum transport simultaneously. For now we treat them as independent and leave this to future work.

Since we assume that $\Delta\beta$ is small we assume that the mixing angle in the symmetric phase — that is near the bubble wall — is equal to its zero temperature value evaluated at the Z boson mass. The BAU tends to get larger for small values of the bubble wall velocity which can also vary over an order of magnitude [37]. We take a moderate value.

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
D_Q	$6/T$	D_{H_u}	$110/T$	$\Gamma_{H_{u,d}}$	$0.025T$	$\Delta\beta$	0.05
$D_{\tilde{H}}$	$110/T$	D_{H_d}	$110/T$	$\Gamma_{\tilde{W}}$	$0.065T$	v_W	0.05
D_t	$6/T$	$\Gamma_{\tilde{t}}$	$0.5T$	$\Gamma_{\tilde{H}}$	$0.025T$	L_W	$5/T$
D_S	$150/T$	$\Gamma_{\tilde{B}}$	$0.04T$	Γ_S	$0.003T$	T_N	100

Table 1. The base set of parameters used for our numerical study. The diffusion constants are taken from [106]. Here $a = \{1, 0.5, 0.25\}$ for the case where the mass of the singlet is $\in [0, 0.25T], [0.25T, 0.5T], [0.5T, \infty]$, respectively, and $b = \kappa^2 + \lambda^2$.

Finally, our thermal mass from the singlino agrees with [77] and is given by

$$\Delta_T m_{\tilde{S}}^2 = \frac{|\lambda|^2 + 2|\kappa|^2}{8} T^2. \quad (4.2)$$

5 Semi-analytic solution

Neglecting the bubble wall curvature we can reduce the problem to a single dimension by solving the system in the rest frame of the bubble wall $z = |v_W t - x|$. We then use the diffusion approximation to write $\vec{\nabla} \cdot \vec{J} = \nabla^2 n$ thus reducing the problem to a set of coupled differential equations in a single space time variable. To answer the question as to whether the singlino can drive the production of the baryon asymmetry we set all CP violating phases apart from the singlino-Higgsino-Higgs interaction to zero. Transport equations in this form have a closed form analytical solution in each phase [86].

Consider the case where there is no high temperature singlet VEV. In the broken phase the solution is

$$X(z) = \sum_{i=1}^{12} x_i A_X(\alpha_i) e^{-\alpha_i z} \left(\int_0^z dy e^{-\alpha_i y} S_{\tilde{S}}^{\mathcal{CP}}(y) - \beta_i \right), \quad (5.1)$$

and in the symmetric phase we have

$$X(z) = \sum_{i=1}^{12} A_{X,s}(\gamma_i) y_i e^{\gamma_i z}, \quad (5.2)$$

where,

$$X \in \{Q, t, H_1, H_2, \tilde{H}, S, \}. \quad (5.3)$$

The derivation of $\alpha_i, \beta_i, x_i, y_i, \gamma_i$ and $A_{X,(s)}(\alpha_i(\gamma_i))$ is given in [86]. From these solutions one can then define the left handed number density $n_L(z) = Q_{1L} + Q_{2L} + Q_{3L} = 5Q + 4T$. The baryon number density, ρ_B , satisfies the equation [98, 107]

$$D_Q \rho_B''(z) - v_W \rho_B'(z) - \Theta(-z) \mathcal{R} \rho_B = \Theta(-z) \frac{n_F}{2} \Gamma_{ws} n_L(z), \quad (5.4)$$

where n_F is the number of fermion families. The relaxation parameter is given by

$$\mathcal{R} = \Gamma_{ws} \left[\frac{9}{4} \left(1 + \frac{n_{sq}}{6} \right) + \frac{3}{2} \right], \quad (5.5)$$

where n_{sq} is the number of thermally available squarks and $\Gamma_{ws} \approx 120 \alpha_W^5 T$ [104, 108, 109]. The baryon asymmetry of the Universe, Y_B is then given by

$$Y_B = -\frac{n_F \Gamma_{ws}}{2\kappa_+ D_Q S} \int_{-\infty}^0 e^{-\kappa_- x} n_L(x) dx, \quad (5.6)$$

where

$$\kappa_{\pm} = \frac{v_W \pm \sqrt{v_W^2 + 4D_Q \mathcal{R}}}{2D_Q}, \quad (5.7)$$

and the entropy is

$$S = \frac{2\pi^2}{45} g_* T^3. \quad (5.8)$$

6 Numerical results

We sample the NMSSM parameter space using `MultiNest v3.10` [110–112] and calculate the spectrum with `SOFTSUSY v3.7.2` [113, 114] by restricting the Higgs mass to 125 GeV rather than performing a global fit. The parameter space is reduced to a few dimensions by fixing the values of the soft masses for second and third generation sfermions to 6 TeV, third generation to 3 TeV and M_1 to 500 GeV. The prior distributions in the remaining free parameters are given in table 2. The parameters m_{H_u} , m_{H_d} , m_S are set by tadpole conditions and A_κ is set to get the right Higgs mass. The range of parameters are chosen with the following considerations in mind

1. Since we are considering only the CPV source that is not present in the MSSM we require the singlino and Higgsino masses (including thermal corrections) to be relatively close before EWSB and not very heavy compared to a plausible value of the nucleation temperature. This latter concern is to avoid severe Boltzmann suppression of the CPV source and the former concern is to ensure a resonant enhancement of the CPV source.
2. The singlet mass cannot be too heavy compared to the Higgs since it must catalyse a strongly first order EWPT (we assume that the stop is too heavy to perform such a role).
3. Converse to the previous consideration, a light singlet with large mixing with the standard model Higgs will be ruled out by collider constraints.

The VEV insertion approximation leads us to take the masses and degrees of freedom in the symmetric phase as it is in this phase where the total left handed number density biases unsuppressed electroweak sphalerons producing the baryon asymmetry [76]. We sanitise the results by removing points where the VEV insertion approximation is unreliable — that is, where the mass gap between the singlino or Higgsino mass (including the Debye mass) is large enough to spuriously change the sign of CP conserving relaxation terms. We also note that when the mass gap between the singlino and the Higgsino is very small the VEV insertion may become reliable [74, 75].

We also sanitise all other mass relaxation terms (e.g. Γ_m^i) by setting them to a random positive infinitesimal number when the in- and virtual-state are far from degeneracy and the naive calculation of the rate yields a negative number. This avoids the spurious case where the mass relaxation terms change sign rather than decaying to zero due to the breakdown of the VEV insertion approximation and can give a spurious boost the BAU.⁹ We set the CP violating phase to its maximal value, $\sin \phi = 1$, as the BAU is linearly proportional to the sin of this phase. The BAU is also proportional to $\Delta\beta$ which we set to a value of 0.05 as it can be it can have a range of $\sim [0.01, 0.2]$ in the NMSSM [37]. Any point in our scan that has a BAU greater than the observed value for such a CPV phase and value of $\Delta\beta$ can be interpreted as a point where the correct value of the CP violating phase is

$$\frac{\Delta\beta \sin \phi}{F} = \frac{Y_B^{\text{obs}}}{Y_B}, \quad (6.1)$$

where $F = 0.05$. For each parameter point we calculate the BAU for SFPT and SSPT if the square of the Higgs mass at $H = S = 0$ is positive when one includes the Debye mass as well as corrections from the thermal functions J_B and J_F . If the potential does not have a positive curvature at $H = S = 0$ for any reasonable range of nucleation temperature ($\lesssim 200$ GeV) the implication is that the origin of field space is a local maximum rather than a local minimum. This means that the phase transition must be either SFPT or second order for these points in the parameter space. As a GUT scale model (with soft masses on the order of the electroweak scale in magnitude) can drag $m_{H_u}^2$ to large negative values when it is evolved to the electroweak scale, it could be harder to find regions of parameter space that survive this cut; the SFPT is the more realistic case. This analysis we leave to a future study. For our prior ranges the majority of the sample does indeed survive this cut. For the points that survive this cut we calculate the baryon asymmetry. We then calculate the posterior distributions based on the mass of the Higgs and the criteria that both the singlino and Higgsino masses are less than a TeV. We colour the 1σ and 2σ credible regions in orange and blue respectively as shown in figure 1 for the SSPT. We perform a similar analysis in the SFPT case and find that generically this scenario produces a lower asymmetry as shown in figure 3. Indeed the largest value of the BAU is an order of magnitude large in the SSPT compared to the case of the SFPT. This suppression is due to the fact that the soft masses of the singlino and Higgsino are both proportional to the VEV of the singlet which tends to be quite large.

In performing the scan for SFPT we make the approximation that the singlet VEV does not change throughout the electroweak phase transition. This results in an underestimate of the baryon asymmetry if the singlet VEV is smaller in the EW symmetric phase than the EWSB phase. The reason for this is that the VEV of the singlet contributes to the masses of the particles involved in the CPV source and their contribution is usually large enough that Boltzmann suppression can become an issue. This along with the greater variability

⁹A random infinitesimal number is chosen rather than zero for the sake of the stability of our code but there is no discernible numerical difference in the BAU between setting these values to zero or a small number.

Figure 1. Two dimensional posterior probability distribution in the baryon asymmetry, produced by the maximum CP-violating phase in the SSPT (singlet simultaneous) scenario, and the thermal mass of the Higgsino (top panel) and singlino (bottom panel). We colour the 1σ and 2σ credible regions in orange and blue, respectively.

of $\Delta\beta$ really motivates future work where the dynamics of the phase transition and the calculation of the baryon asymmetry are performed simultaneously.

In figure 3 we show the range of zero temperature masses for the Higgsino and singlino against the BAU for a SFPT phase transition. There is a substantial proportion of the

A_λ	A_κ	M_2	$\tan\beta$	λ	κ	λv_S
$[-4000, 4000]$	$[0, 200]$	$[100, 1000]$	$[1.1, 5]$	$[0.001, 0.5]$	$[0.001, 0.5]$	$[200, 800]$

Table 2. Sample ranges for the scanned NMSSM parameters. All dimensionful numbers are in GeV. M_1 was fixed to 500 GeV. The modest range of $\tan\beta$ was to help satisfy flavour constraints. Sfermions were decoupled by setting the first and second generation squark masses to 6 TeV and the third generation to 3 TeV.

Figure 2. Baryon asymmetry for a benchmark point with lowest value of χ^2 for a SFPT (singlet first) phase transition with varying values of the three body rates. The horizontal axis varies the three body rates involving (s)tops and Higgs(inos). The vertical axis varies the three body rates involving singlets or singlinos.

Figure 3. Two dimensional posterior probability distribution in the baryon asymmetry, produced by the maximum CP-violating phase in the SFPT scenario, and the thermal mass of the Higgsino (top panel) and singlino (bottom panel). We colour the 1σ and 2σ credible regions in orange and blue, respectively.

Figure 4. Baryon asymmetry produced during a SFPT against the mass difference of the singlino and Higgsino masses including the Debye masses (top panel) and a contour plot of the baryon asymmetry against the same aforementioned masses (bottom panel). The resonance is the dominant feature of the plot and it appears that in the SFPT case, one needs to be near the center of the resonance to produce the observed BAU unless $\Delta\beta$ is very large. The possibility of large $\Delta\beta$ in the NMSSM leads to the interesting possibility of off resonance baryogenesis.

A_λ	A_κ	M_2	$\tan\beta$	λ	κ	λv_S
519	4.80	977	4.18	0.39	0.39	249

Table 3. Benchmark point with the lowest value of χ^2 for an assumed SFPT. Dimensionful parameters are listed in GeV. Note that neither this, nor any other point in our sample have had its zero temperature phenomenology checked in detail beyond the correct masses of standard model particles (including the Higgs) as well as some rules of thumb we explain in the text.

parameter space with a sufficiently large BAU. We find that the resonant enhancement that occurs when the masses of the singlino and Higgsino are near degenerate is the dominant predictor of a large BAU. This is clear from figure 4. Of particular interest to us is the fact that well off resonance one can still obtain a BAU that is close to an order of magnitude below the observed rate. This shortcoming in the BAU, however, can be made up for by a sufficiently large $\Delta\beta$. This possibility opens up another avenue in which the BAU can be produced within the NMSSM off resonance. As a caveat we note that as one ventures further off resonance, more skepticism should be held toward the accuracy of the result as the VEV insertion approximation is losing its reliability.

In ref. [85] it was shown that the BAU can vary by orders of magnitude with the variation of the magnitude of three body rates involving Higgs(inos) and (s)tops. The NMSSM has new three body rates involving Higgs(inos) and singlet/singlinos. We show how the BAU varies as a function of both types of three body rates for a benchmark point which has the lowest value of χ^2 (given in table 3). We introduce the factors ξ_H and ξ_S to the transport equations given in eq. (3.6) so that every three body rate Γ_Y not involving singlets or singlinos is multiplied by ξ_H , and those involving singlets or singlinos are multiplied by ξ_S . E.g.,

$$\Gamma_Y^{tQH_1} \mapsto \xi_H \Gamma_Y^{tQH_1}, \quad \text{and} \quad \Gamma_Y^{\tilde{H}H_1\tilde{S}} \mapsto \xi_S \Gamma_Y^{\tilde{H}H_1\tilde{S}}. \quad (6.2)$$

The BAU increases with the three body rates involving (s)tops and Higgs(ino) interactions. We believe the reasons are the same as that given in ref. [85]. For three body rates involving singlet (or singlino) interactions we find a different behaviour. When these rates are very small the BAU increases to a peak, similar to the 3 body rates involving (s)quarks. However, the BAU drops sharply after the peak. We explain the sharp drop by the fact that these three body rates relax the linear combination $\mu_{\tilde{H}} + \mu_H \approx 0$, whereas both the supergauge rates as well as all other triscalar and Yukawa rates conspire to relax the combination $\mu_{\tilde{H}} - \mu_H \approx 0$ [101]. The supergauge rate is typically a moderately large value, so if the singlino-Higgsino rate is also large then the result is that both $\mu_{\tilde{H}}$ and μ_H are relaxed to zero. Since the BAU is, in a fast rate approximation, proportional to μ_H [76], the BAU goes to zero as well.

7 Discussion and conclusion

Electroweak baryogenesis is an attractive paradigm for producing the BAU due to its testability. In fact, testability is an unavoidable feature of this paradigm as any physics

that is responsible for catalysing the baryon production during the EWPT must have mass scales at (or just above [9]) the weak scale and non-trivial couplings. In this paper we have examined one of the most popular extensions to the standard model — the NMSSM — and indeed we find that the scenario requires that at least some neutralinos must be relatively close to the weak scale. If the singlino and Higgsino are both light, the contribution to dark matter from a neutralino LSP tends to be smaller than the observed value. It would certainly be interesting to test whether the electroweak baryogenesis constraints derived in this paper are compatible with getting the right dark matter abundance, or whether one needs to extend the NMSSM. Apart from the constraints on the parameter space, we also examined the structure of the transport equations keeping in mind the different possibilities of how the EWPT proceeds. As usual the most striking feature is the existence of a resonant boost in the CP violating source when the masses of the singlino and Higgsino are near degenerate. In the MSSM one can also have a surprising boost to the baryon asymmetry in the case where three body rates involving stops and Higgs are large. We have new three body rates involving the Higgs, Higgsino and singlino which enhances the BAU up to a peak and then suppresses the baryon asymmetry when they get large. The suppression is quite severe when its size becomes large enough to compete with the supergauge rate involving Higgs and Higgsinos. This we put down to these two interactions creating approximate local equilibrium relations which conflict to set the baryon asymmetry to approximately zero.

On SSPT phase transitions we can make the qualitative comment that in general it is easier to get a resonance boost to the baryon asymmetry in a SSPT phase transition where the masses of the singlino and the Higgsino are just the Debye masses in the region just outside the bubble of broken electroweak phase which is primarily responsible for the BAU production. There are reasons however to be skeptical on whether SSPT would frequently occur in any GUT scale model. A full statistical analysis of this we leave to future work. SSPT phase transitions also present technical challenges. The masses of the neutralinos in the broken phase might have a larger contribution from the VEV of the singlet than the Higgs. So the space-time variation of these masses during the electroweak phase transition is very large stretching faith in the VEV insertion approximation. A full Wigner functional treatment as well as a numerical study of the phase transition would shed further light on the viability of baryogenesis in SSPT phase transitions.

Overall, the dynamics of the charge transport and the phase transition are far more correlated than in the MSSM. The masses of the singlino and Higgsino in the symmetric phase are qualitatively different if the singlet acquires a phase transition during or before the EWPT. These masses determine the strength of both the CP violating source, the associated two and three body relaxation terms and the thermal widths. Considering just these results can lead to an order of magnitude difference in the generated BAU between the SSPT and SFPT cases. The parameters that vary the soft masses also vary the effective potential. So it is likely that they alter nature of the phase transition (strongly first order, SSPT/SFPT) as well as the parameters of the phase transition including $\Delta\beta$. Considering that our work is an initial exploration of these issues, we defer the detailed study of the further details to a future paper.

Finally we conclude by noting that the relationship between masses requiring a resonance in order to produce enough BAU makes electroweak baryogenesis a fairly fine tuned

mechanism. The NMSSM has the attractive possibility of providing more paths to such a boost through producing a large enough $\Delta\beta$ which could potential make EWBG work even well off resonance. A detailed numerical study of $\Delta\beta$ in the NMSSM would shed light on how realistic this possibility is.

Acknowledgments

The authors thank Peter Athron for many elucidating discussions of the Higgs sector of the NMSSM. One of us (S.A.) thanks Angelo Monteux for a helpful exposition of cosmological parameter measurements. G.W. acknowledges Michael Ramsey-Musolf and his group for some helpful discussions of this work, and would also like to acknowledge the Australian Postgraduate Award. C.B. thanks Jose W.F. Valle for his thought-provoking questions. This work in part was supported by the ARC Centre of Excellence for Particle Physics at the Terascale.

A Transport coefficients and sources

For completeness we present the three body rates here. For a triscalar interaction one has

$$\begin{aligned} \mathcal{I}_B(m_1, m_2, m_3) = & \frac{1}{16\pi^3} \int_{m_1}^{\infty} d\omega_1 h_B(\omega_1) \\ & \times \left\{ \log \left(\frac{e^{\omega_1/T} - e^{\omega_2^+/T}}{e^{\omega_1/T} - e^{\omega_2^-/T}} \frac{e^{\omega_2^-/T} - 1}{e^{\omega_2^+/T} - 1} \right) [\Theta(m_1 - m_2 - m_3) - \Theta(m_2 - m_1 - m_3)] \right. \\ & \left. + \log \left(\frac{e^{-\omega_1/T} - e^{\omega_2^+/T}}{e^{-\omega_1/T} - e^{\omega_2^-/T}} \frac{e^{\omega_2^-/T} - 1}{e^{\omega_2^+/T} - 1} \right) \Theta(m_3 - m_2 - m_1) \right\}, \end{aligned} \quad (\text{A.1})$$

whereas the three body Yukawa rate is

$$\begin{aligned} \mathcal{I}_F(m_1, m_2, m_3) = & - (m_1^2 + m_2^2 - m_3^2) \frac{1}{16\pi^3} \int_{m_1}^{\infty} d\omega_1 h_F(\omega_1) \\ & \times \left\{ \log \left(\frac{e^{\omega_1/T} + e^{\omega_3^+/T}}{e^{\omega_1/T} + e^{\omega_3^-/T}} \frac{e^{\omega_3^-/T} - 1}{e^{\omega_3^+/T} - 1} \right) [\Theta(m_1 - m_2 - m_3) - \Theta(m_3 - m_1 - m_2)] \right. \\ & \left. + \log \left(\frac{e^{-\omega_1/T} + e^{\omega_2^+/T}}{e^{-\omega_1/T} + e^{\omega_3^-/T}} \frac{e^{\omega_3^-/T} - 1}{e^{\omega_3^+/T} - 1} \right) \Theta(m_2 - m_2 - m_3) \right\}, \end{aligned} \quad (\text{A.2})$$

where

$$h_{B/F}(x) = \frac{e^{x/T}}{(e^{x/T} \pm 1)^2}. \quad (\text{A.3})$$

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