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Improved anatomy of ε'/ε in the Standard Model

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Abstract: We present a new analysis of the ratio ε'/ε within the Standard Model (SM) using a formalism that is manifestly independent of the values of leading $(V-A)\otimes(V-A)$ QCD penguin, and EW penguin hadronic matrix elements of the operators Q_4, Q_9, and Q_{10}, and applies to the SM as well as extensions with the same operator structure. It is valid under the assumption that the SM exactly describes the data on CP-conserving $K\to\pi\pi$ amplitudes. As a result of this and the high precision now available for CKM and quark mass parameters, to high accuracy ε'/ε depends only on two non-perturbative parameters, $B_6^{(1/2)}$ and $B_8^{(3/2)}$, and perturbatively calculable Wilson coefficients. Within the SM, we are separately able to determine the hadronic matrix element $\langle Q_4\rangle_0$ from CP-conserving data, significantly more precisely than presently possible with lattice QCD. Employing $B_6^{(1/2)} = 0.57\pm 0.19$ and $B_8^{(3/2)} = 0.76\pm 0.05$, extracted from recent results by the RBC-UKQCD collaboration, we obtain $\varepsilon'/\varepsilon = (1.9\pm 4.5)\times 10^{-4}$, substantially more precise than the recent RBC-UKQCD prediction and $2.9\,\sigma$ below the experimental value $(16.6\pm 2.3)\times 10^{-4}$, with the error being fully dominated by that on $B_6^{(1/2)}$. Even discarding lattice input completely, but employing the recently obtained bound $B_6^{(1/2)}\le B_8^{(3/2)}\le 1$ from the large-N approach, the SM value is found more than $2\,\sigma$ below the experimental value. At $B_6^{(1/2)} = B_8^{(3/2)} = 1$, varying all other parameters within one sigma, we find $\varepsilon'/\varepsilon = (8.6\pm 3.2)\times 10^{-4}$. We present a detailed anatomy of the various SM uncertainties, including all sub-leading hadronic matrix elements, briefly commenting on the possibility of underestimated SM contributions as well as on the impact of our results on new physics models.

Keywords: CP violation, Kaon Physics, QCD

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Contents

1 Introduction

One of the important actors of the 1990s in particle physics was the ratio ε'/ε that measures the size of the direct CP violation in $K_L \to \pi\pi$ relative to the indirect CP violation described by ε_K. In the Standard Model (SM), ε' is governed by QCD penguins but receives also an important destructively interfering contribution from electroweak penguins that is generally much more sensitive to new physics (NP) than the QCD penguin contribution. Reviews on ε'/ε can be found in [1–5].

A long-standing challenge in making predictions for ε'/ε within the SM and its extensions has been the strong interplay of QCD penguin contributions and electroweak penguin contributions to this ratio. In the SM, QCD penguins give a positive contribution and electroweak penguins a negative one. In order to obtain a useful prediction for ε'/ε, the relevant contributions of the QCD penguin and electroweak penguin operators must be know accurately.

As far as short-distance contributions (Wilson coefficients of QCD and electroweak penguin operators) are concerned, they have been known already for more than twenty years at the NLO level [6–11] and present technology could extend them to the NNLO level if necessary. First steps in this direction have been taken in [12–14].

The situation with hadronic matrix elements is another story and even if significant progress on their evaluation has been made over the last 25 years, the present status is clearly not satisfactory as we will discuss below. But, already in 1993, an approach has been proposed in [10] which, as far as ε'/ε is concerned, avoids direct calculation of some of the most difficult hadronic matrix elements. It assumes that the real parts of the isospin amplitudes A_0 and A_2, which exhibit the $\Delta I = 1/2$ rule, are fully described by SM dynamics and their experimental values are used to determine to a very good approximation hadronic matrix elements of all $(V-A)\otimes(V-A)$ operators, among them the so-called Q_4 QCD penguin operator. While not as important as the $(V-A)\otimes(V+A)$ QCD penguin and electroweak penguin operators, Q_6 and Q_8, the operator Q_4 has been known since the early days of analyses of ε'/ε [9, 10, 15] to be responsible for a significant part of the suppression of this ratio. In the presence of a partial cancellation of the positive contribution of Q_6 to ε'/ε by the one of Q_8, an accurate determination of the contribution from Q_4 and from the electroweak penguin operators Q_9 and Q_{10} to ε'/ε by means of CP-conserving data was an important virtue of our approach.

Another virtue of our approach is based on the fact that in the SM the amplitudes $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ originate already at tree-level. Similar to the observables used for tree-level determination of CKM parameters, also relevant for ε'/ε, they are expected to be only marginally affected by NP contributions. Whether NP could contribute to $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ at some level is an interesting question, to which we will return briefly in section 6. But, for the time being we assume that they are fully dominated by SM dynamics.

With the contribution of $(V-A)\otimes(V-A)$ operators being determined from the data on $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ it was possible to write down an analytic formula for ε'/ε that incorporated all NLO QCD and QED corrections and summarised the remaining dominant hadronic uncertainty in terms of two parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ that parametrise the relevant matrix elements of the dominant operators Q_6 and Q_8 and have to be calculated using a non-perturbative framework like lattice QCD or the large-N approach [16, 17]. They cannot be extracted from CP-conserving data as their contributions to $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ are marginal at $\mu \approx m_c$ used in the approach of [10]. In fact one of the reasons for choosing the value $\mu = m_c$ was to eliminate them from the determination of the matrix elements of $(V-A)\otimes(V-A)$ operators from the CP-conserving data.

Over the last twenty years the basic formula for ε'/ε of [10] has been improved [2, 18, 19] due to the increased accuracy in the value of the QCD coupling and other input parameters, like the values of m_t and m_s. We refer to [2, 18, 19], where useful information on our approach can be found. The most recent version of our analytic formula has been presented in [20, 21].

One new aspect of the present paper is the realisation that under the assumption that NP contributions to $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ are negligible, the leading contributions of $(V-A)\otimes(V-A)$ operators to ε'/ε can be entirely expressed in terms of their Wilson coefficients. Furthermore, we derive a formula for ε'/ε which under the above assumption can be used in any extension of the SM in which the operator structure is the same as in the SM. NP enters only through the modified values of the Wilson coefficients and the dominant non-perturbative uncertainties are contained in

$$B_6^{(1/2)}, \qquad B_8^{(3/2)}, \qquad q \equiv \frac{z_+(\mu)\langle Q_+(\mu)\rangle_0}{z_-(\mu)\langle Q_-(\mu)\rangle_0}\,. \tag{1.1}$$

The ratio q, involving matrix elements of current-current operators $Q_\pm$ and their Wilson coefficients $z_\pm$, enters the determination of the contribution of $(V-A)\otimes(V-A)$ operators from CP-conserving data and its range will be estimated in section 2. But for $0\le q\le 0.1$ obtained from QCD lattice and large-N approaches the dependence of ε'/ε on q is very weak.

As far as the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are concerned, $B_6^{(1/2)}=B_8^{(3/2)}=1$ in the large-N limit of QCD. The study of $1/N$ corrections to the large-N limit indicated that $B_8^{(3/2)}$ is suppressed below unity [22], but no clear-cut conclusion has been reached in that paper on $B_6^{(1/2)}$. Moreover, the precise amount of suppression of $B_8^{(3/2)}$ could not be calculated in this approach. Fortunately, in the meantime significant progress has been achieved in the case of the matrix element $\langle Q_8\rangle_2$ by the RBC-UKQCD lattice collaboration [23], which allowed to determine $B_8^{(3/2)}$ to be [21]

$$B_8^{(3/2)}(m_c) = 0.76\pm 0.05 \qquad \text{(RBC-UKQCD)}, \tag{1.2}$$

in agreement with large-N expectations [22, 24], but with higher precision.

But also some progress on $B_6^{(1/2)}$ has been made, both by lattice QCD and the large-N approach. In particular, very recently the RBC-UKQCD lattice collaboration [25] presented their first result for the matrix element $\langle Q_6\rangle_0$ from which one can extract (see below and [24])

$$B_6^{(1/2)}(m_c) = 0.57\pm 0.19 \qquad \text{(RBC-UKQCD)}. \tag{1.3}$$

This low value of $B_6^{(1/2)}$ is at first sight surprising and as it is based on a numerical simulation one could wonder whether it is the result of a statistical fluctuation. But the very recent analysis in the large-N approach in [24] gives strong support to the values in (1.2) and (1.3). In fact, in this analytic approach one can demonstrate explicitly the suppression of both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ below their large-N limit $B_6^{(1/2)}=B_8^{(3/2)}=1$ and derive a conservative upper bound on both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ which reads [24]

$$B_6^{(1/2)} \le B_8^{(3/2)} < 1 \qquad (\text{large-}N). \tag{1.4}$$

While one finds $B_8^{(3/2)}(m_c) = 0.80 \pm 0.10$, the result for $B_6^{(1/2)}$ is less precise but there is a strong indication that $B_6^{(1/2)} < B_8^{(3/2)}$ in agreement with (1.2) and (1.3). For further details, see [24] and section 3 below.

Employing the lattice results of (1.2) and (1.3), in our numerical analysis we find

$$\varepsilon'/\varepsilon = (1.9 \pm 4.5) \times 10^{-4}\,, \tag{1.5}$$

consistent with, but significantly more precise than the result obtained recently by the RBC-UKQCD lattice collaboration [25],

$$(\varepsilon'/\varepsilon)_{\rm SM} = (1.4 \pm 7.0) \times 10^{-4}\,. \tag{1.6}$$

This is even more noteworthy considering the fact that our result comprises also uncertainties from isospin corrections and CKM parameters which were not considered in the error estimate of [25]. Our result differs with close to $3\,\sigma$ significance from the experimental world average from NA48 [26] and KTeV [27, 28] collaborations,

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}\,, \tag{1.7}$$

suggesting evidence for new physics in K decays.

But even discarding the lattice results, varying all input parameters, we find at the bound $B_6^{(1/2)} = B_8^{(3/2)} = 1$,

$$(\varepsilon'/\varepsilon)_{\rm SM} = (8.6 \pm 3.2) \times 10^{-4}\,, \tag{1.8}$$

still $2\,\sigma$ below the experimental data. We consider this bound conservative since employing the lattice value in (1.2) and $B_6^{(1/2)} = B_8^{(3/2)} = 0.76$, instead of (1.8), one obtains $(6.0 \pm 2.4) \times 10^{-4}$.

This already shows that with the rather precise value of $B_8^{(3/2)}$ from lattice QCD, the final result for ε'/ε dominantly depends on the value of $B_6^{(1/2)}$ and both lattice QCD [25] and the large-N approach [24] indicate that the SM value of ε'/ε is significantly below the data.

The two main goals of the present paper are:

- Derivation of a new version of our formula for ε'/ε which could also be used beyond the SM and which appears to be more useful than its variants presented by us in the past.

- Demonstration that our approach provides a substantially more accurate prediction for ε'/ε in the SM than it is presently possible within lattice QCD and that the upper bound in (1.8) is rather conservative.

It should be stressed that assuming dominance of SM dynamics in CP-conserving data, our determination of the contributions of $(V-A)\otimes(V-A)$ operators to ε'/ε is basically independent of the non-perturbative approach used. The RBC-UKQCD lattice collaboration calculates these contributions directly and we will indeed identify a significant difference between their estimate of the Q_4 contribution to ε'/ε and ours.

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Our paper is organised as follows. In section 2, we derive the analytic formula for ε'/ε in question using the strategy of [10] but improving on it. Using this formula, we present a new analysis of ε'/ε within the SM exhibiting its sensitivity to the precise value of $B_6^{(1/2)}$ and the weak dependence on q. In section 3, we perform the anatomy of uncertainties affecting ε'/ε and present the prediction of ε'/ε in the SM, including a discussion of its $B_6^{(1/2)}$ dependence. In section 4, we extract from the lattice-QCD results of [25] the values of the most important hadronic matrix elements and compare them with ours. This allows us to identify the main origin of the difference between (1.5) and (1.6). In particular, we point out an approximate correlation between the contribution of the Q_4 operator to ε'/ε and the value of $\mathrm{Re}A_0$ valid in any non-perturbative approach. In section 5, we investigate if thus far neglected SM contributions could bring our result for ε'/ε into agreement with the experimental findings. A brief general discussion of the impact of possible NP contributions to $\mathrm{Re}A_{0,2}$ and $\mathrm{Im}A_{0,2}$ and of the implications of our results for NP models is given in section 6. The summary of our observations and an outlook are presented in section 7. In appendix A, we discuss the sub-leading contributions to our prediction for ε'/ε and in appendix B, for completeness, an updated analytic formula for ε'/ε in the SM is presented in the form used in several of our papers in the past (e.g. [21]) that is equivalent to the one derived in section 2, but exhibits the m_t, α_s, m_s and m_d dependences more explicitly.

2 Basic formulae

2.1 Effective Hamiltonian

We use the effective Hamiltonian for $\Delta S = 1$ transitions of [6–11]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\, V_{ud}V_{us}^* \sum_{i=1}^{10} \big(z_i(\mu) + \tau\, y_i(\mu)\big)\, Q_i(\mu)\,, \qquad \tau \equiv -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}\,. \tag{2.1}$$

The contributing operators are given as follows:

Current-Current:

$$Q_1 = (\bar s_\alpha u_\beta)_{V-A}\,(\bar u_\beta d_\alpha)_{V-A} \qquad Q_2 = (\bar s u)_{V-A}\,(\bar u d)_{V-A} \tag{2.2}$$

QCD-Penguins:

$$Q_3 = (\bar s d)_{V-A} \sum_{q=u,d,s,c,b} (\bar q q)_{V-A} \qquad Q_4 = (\bar s_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar q_\beta q_\alpha)_{V-A} \tag{2.3}$$

$$Q_5 = (\bar s d)_{V-A} \sum_{q=u,d,s,c,b} (\bar q q)_{V+A} \qquad Q_6 = (\bar s_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar q_\beta q_\alpha)_{V+A} \tag{2.4}$$

Electroweak Penguins:

$$Q_7 = \frac{3}{2}\,(\bar s d)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar q q)_{V+A} \qquad Q_8 = \frac{3}{2}\,(\bar s_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar q_\beta q_\alpha)_{V+A} \tag{2.5}$$

$$Q_9 = \frac{3}{2}\,(\bar s d)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar q q)_{V-A} \qquad Q_{10} = \frac{3}{2}\,(\bar s_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar q_\beta q_\alpha)_{V-A} \tag{2.6}$$

	$\alpha_s(M_Z) = 0.1179$	$\alpha_s(M_Z) = 0.1185$	$\alpha_s(M_Z) = 0.1191$
z_1	–0.4036	–0.4092	–0.4150
z_2	1.2084	1.2120	1.2157
y_3	0.0275	0.0280	0.0285
y_4	–0.0555	–0.0563	–0.0571
y_5	0.0054	0.0052	0.0050
y_6	–0.0849	–0.0867	–0.0887
y_7/α	–0.0404	–0.0403	–0.0402
y_8/α	0.1207	0.1234	0.1261
y_9/α	–1.3936	–1.3981	–1.4027
y_{10}/α	0.4997	0.5071	0.5146

Table 1. $\Delta S = 1$ Wilson coefficients at $\mu = m_c = 1.3\,\text{GeV}$ for three values of $\alpha_s(M_Z)$ and $m_t = 163\,\text{GeV}$ in the NDR-$\overline{\text{MS}}$ scheme.

Here, α, β denote colour indices and e_q denotes the electric quark charges reflecting the electroweak origin of $Q_7, \ldots, Q_{10}$. Finally, $(\bar{s}d)_{V-A} \equiv \bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha$.

The Wilson coefficients z_i and y_i have been calculated at the NLO level more than twenty years ago [10, 11], and some pieces of NNLO corrections are also available [12–14]. In table 1, we collect values for $z_{1,2}$ and y_i at $\mu = m_c$, used in our approach, for three values of $\alpha_s(M_Z)$ and $m_t = 163\,\text{GeV}$, in the NDR-$\overline{\text{MS}}$ scheme.

2.2 Basic formula for ε'/ε

Our starting expression is formula (8.16) of [29] which we recall here in our notation[1]

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}\left[\frac{\text{Im}A_0}{\text{Re}A_0}(1-\Omega_{\text{eff}}) - \frac{\text{Im}A_2}{\text{Re}A_2}\right], \tag{2.7}$$

where [29]

$$\omega_+ = a\,\frac{\text{Re}A_2}{\text{Re}A_0} = (4.53 \pm 0.02) \times 10^{-2}, \qquad a = 1.017, \qquad \Omega_{\text{eff}} = (6.0 \pm 7.7) \times 10^{-2}. \tag{2.8}$$

Here a and Ω_{eff} summarise isospin breaking corrections and include strong isospin violation ($m_u \neq m_d$), the correction to the isospin limit coming from $\Delta I = 5/2$ transitions and electromagnetic corrections [29, 30]. The amplitudes $\text{Re}A_{0,2}$ are then extracted from the branching ratios on $K \to \pi\pi$ decays in the isospin limit. Their values are given in (2.39) below. In the limit $a = 1$ and $\Omega_{\text{eff}} = 0$ formula (2.7) reduces to the one used in [25], where all isospin breaking corrections except electroweak penguin contributions have been set to zero.

The quantity Ω_{eff} includes, in addition to other isospin breaking corrections, electroweak penguin contributions that are then not included in $\text{Im}A_0$. Here we prefer to

[1] In order to simplify the notation we denote $\text{Re}(\varepsilon'/\varepsilon)$ simply by ε'/ε, which is real to an excellent approximation. The latter is a model-independent consequence of the experimentally known values of the (strong) phases of ε' and ε.

include these contributions to $\mathrm{Im}A_0$ and therefore, instructed by the authors of [29], we remove them from $\Omega_{\rm eff}$. However, we keep in $\Omega_{\rm eff}$ their term Δ_0 in the limit of $\alpha = 0$. Using table 4 of [29], we then obtain the modified $\Omega_{\rm eff}$:

$$\hat{\Omega}_{\rm eff} = (14.8 \pm 8.0) \times 10^{-2}\,. \tag{2.9}$$

As the second term in (2.7) is an isospin breaking effect by itself, strictly speaking in this term the parameter a should be set to unity if we want to remove higher order isospin breaking corrections. In addition, in order to remove the effects of $\hat{\Omega}_{\rm eff} \neq 0$ in electroweak penguin contributions to $\mathrm{Im}A_0$, we write

$$\mathrm{Im}A_0 = (\mathrm{Im}A_0)^{\rm QCDP} + b\,(\mathrm{Im}A_0)^{\rm EWP}, \qquad b = \frac{1}{a\,(1-\hat{\Omega}_{\rm eff})} \tag{2.10}$$

with the first term including the contributions from Q_{3-6} and the second from Q_{7-10}. Except for the tiny corrections due to $a \neq 1$, this procedure is equivalent to multiplying the coefficients y_{3-6} by $(1-\hat{\Omega}_{\rm eff})$ leaving y_{7-10} unchanged.

Our final basic formula which we will use in what follows then reads

$$\frac{\varepsilon'}{\varepsilon} = -\,\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\,(1-\hat{\Omega}_{\rm eff}) - \frac{1}{a}\,\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right], \tag{2.11}$$

with (ω_+, a), $\hat{\Omega}_{\rm eff}$ and $\mathrm{Im}A_0$ given in (2.8), (2.9) and (2.10), respectively. $\mathrm{Im}A_2$ contains only contributions of the electroweak penguin operators Q_{7-10}.

The crucial theory task for a precision SM prediction is to determine the real and imaginary parts of the (strong-)isospin amplitudes

$$A_I \equiv \langle(\pi\pi)_I|\mathcal{H}_{\rm eff}|K\rangle \tag{2.12}$$

entering (2.11) in terms of the Wilson coefficients and hadronic matrix elements of the operators in the weak Hamiltonian (2.1).

2.3 Hadronic matrix elements

The hadronic matrix elements of the operators Q_i entering the isospin amplitudes,

$$\langle Q_i\rangle_I \equiv \langle(\pi\pi)_I\,|Q_i|\,K\rangle\,, \qquad I = 0, 2\,, \tag{2.13}$$

generally depend on the scale μ and on the renormalisation scheme used for the operators. These two dependencies are cancelled by those present in the coefficients $C_i(\mu)$ so that the effective Hamiltonian and the resulting amplitudes do not depend on μ and on the scheme used to renormalise the operators. We will work exclusively in the NDR-$\overline{\rm MS}$ scheme and for scales $\mu \le m_c$, although in [10] also extensive discussion of scales above m_c can be found.

For $\mu \le m_c$, when the charm quark has been integrated out, only seven of the operators listed above are independent of each other. Eliminating then Q_4, Q_9 and Q_{10} in terms of the remaining seven operators results in the following important relations in the isospin

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limit [10]:

$$\langle Q_4\rangle_0 = \langle Q_3\rangle_0 + \langle Q_2\rangle_0 - \langle Q_1\rangle_0\,, \tag{2.14}$$

$$\langle Q_9\rangle_0 = \frac{3}{2}\langle Q_1\rangle_0 - \frac{1}{2}\langle Q_3\rangle_0\,, \tag{2.15}$$

$$\langle Q_{10}\rangle_0 = \langle Q_2\rangle_0 + \frac{1}{2}\langle Q_1\rangle_0 - \frac{1}{2}\langle Q_3\rangle_0\,, \tag{2.16}$$

$$\langle Q_9\rangle_2 = \langle Q_{10}\rangle_2 = \frac{3}{2}\langle Q_1\rangle_2\,, \tag{2.17}$$

where we have employed

$$\langle Q_1\rangle_2 = \langle Q_2\rangle_2\,. \tag{2.18}$$

As stressed in [10], in the NDR-$\overline{\mathrm{MS}}$ scheme the relation (2.14) receives an $\mathcal{O}(\alpha_s)$ correction due to the presence of evanescent operators which have to be taken into account when using Fierz identities in its derivation. The other relations above do not receive such corrections. The complete expression for $\langle Q_4\rangle_0$ in the NDR-$\overline{\mathrm{MS}}$ scheme reads [10]

$$\langle Q_4\rangle_0 = \langle Q_3\rangle_0 + \langle Q_2\rangle_0 - \langle Q_1\rangle_0 - \frac{\alpha_s}{4\pi}\left(\langle Q_6\rangle_0 + \langle Q_4\rangle_0 - \frac{1}{3}\langle Q_3\rangle_0 - \frac{1}{3}\langle Q_5\rangle_0\right), \tag{2.19}$$

which of course then has to be solved for $\langle Q_4\rangle_0$. However, due to the partial cancellation between the matrix elements $\langle Q_4\rangle_0$ and $\langle Q_6\rangle_0$, and the smallness of the matrix elements of Q_3 and Q_5, this correction affects the determination of $\langle Q_4\rangle_0$ by at most few percent and can be neglected. This procedure is supported both by the results on hadronic matrix elements RBC-UKQCD collaboration [25] and the large-N approach [24].

Setting the contribution of Q_3 to zero[2] and using the operators

$$Q_\pm = \frac{1}{2}(Q_2 \pm Q_1)\,, \tag{2.20}$$

the formulae (2.14)–(2.17) read

$$\langle Q_4\rangle_0 = 2\,\langle Q_-\rangle_0\,, \tag{2.21}$$

$$\langle Q_9\rangle_0 = \frac{3}{2}\left(\langle Q_+\rangle_0 - \langle Q_-\rangle_0\right), \tag{2.22}$$

$$\langle Q_{10}\rangle_0 = \frac{3}{2}\langle Q_+\rangle_0 + \frac{1}{2}\langle Q_-\rangle_0\,, \tag{2.23}$$

$$\langle Q_9\rangle_2 = \langle Q_{10}\rangle_2 = \frac{3}{2}\langle Q_+\rangle_2\,, \tag{2.24}$$

which reduces the number of independent $(V-A)\otimes(V-A)$ matrix elements entering $\mathrm{Re}A_{0,2}$ and $\mathrm{Im}A_{0,2}$ to three. On the other hand, to an excellent approximation the amplitudes $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ at $\mu = m_c$ are fully described by the operators Q_- and Q_+, so that we can write

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}}\,V_{ud}V_{us}^*\left(z_+\langle Q_+\rangle_0 + z_-\langle Q_-\rangle_0\right), \tag{2.25}$$

$$\mathrm{Re}A_2 = \frac{G_F}{\sqrt{2}}\,V_{ud}V_{us}^*\,z_+\langle Q_+\rangle_2\,. \tag{2.26}$$

[2]In our numerical analysis below, all operators will be taken into account.

Introducing the ratio

$$q \equiv \frac{z_+(\mu)\langle Q_+(\mu)\rangle_0}{z_-(\mu)\langle Q_-(\mu)\rangle_0}\,, \qquad z_\pm = z_2 \pm z_1\,, \tag{2.27}$$

allows us to express the ratios involving only $(V-A)\otimes(V-A)$ operators that will enter our basic formula for ε'/ε as follows:

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{V-A} = \mathrm{Im}\tau\,\frac{[4y_4 - b(3y_9 - y_{10})]}{2(1+q)z_-} + \mathrm{Im}\tau\, b\,\frac{3q(y_9+y_{10})}{2(1+q)z_+}\,, \tag{2.28}$$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{V-A} = \mathrm{Im}\tau\,\frac{3(y_9+y_{10})}{2z_+}\,. \tag{2.29}$$

Besides the CKM ratio τ, the first ratio depends only on Wilson coefficients and the single hadronic ratio q to which we will return below. On the other hand the second ratio is free from hadronic uncertainties, being fully determined by the Wilson coefficients z_+, y_9, y_{10} and by τ.

The remaining contributions to $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$ are due to $(V-A)\otimes(V+A)$ operators and are dominated by the operators Q_6 and Q_8, respectively. We find this time

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_6 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_6\,\frac{\langle Q_6\rangle_0}{\mathrm{Re}A_0}\,, \tag{2.30}$$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_8 = -\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\lambda_t\, y_8^{\mathrm{eff}}\,\frac{\langle Q_8\rangle_2}{\mathrm{Re}A_2}\,. \tag{2.31}$$

Contributions from Q_3 and Q_5 are very suppressed but can and have been included in our numerical error estimate. (See appendix A.) We have also taken into account the small effect of $\langle Q_7\rangle_2$, for which a relatively precise lattice prediction exists [23], through the substitution

$$y_8 \;\to\; y_8^{\mathrm{eff}} \equiv y_8 + p_{72}\, y_7 \tag{2.32}$$

which is included in writing (2.31). Here $p_{72} \equiv \langle Q_7\rangle_2/\langle Q_8\rangle_2 = 0.222$ for central values of [23]. (In our numerics, we have added the corresponding errors linearly and attribute a 15% uncertainty to this contribution.)

The matrix elements of the Q_6 and Q_8 operators are conveniently parametrised by

$$\langle Q_6(\mu)\rangle_0 = -\,4h\left[\frac{m_{\mathrm{K}}^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi)\, B_6^{(1/2)}\,, \tag{2.33}$$

$$\langle Q_8(\mu)\rangle_2 = \sqrt{2}h\left[\frac{m_{\mathrm{K}}^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi\, B_8^{(3/2)}\,, \tag{2.34}$$

with [31, 32]

$$B_6^{(1/2)} = B_8^{(3/2)} = 1 \tag{2.35}$$

in the large-N limit. As had been demonstrated in [10], $B_6^{(1/2)}$ and $B_8^{(3/2)}$ exhibit a very weak scale dependence. The dimensionful parameters entering (2.33), (2.34) are given by [33, 34]

$$m_K = 497.614\,\mathrm{MeV}, \qquad F_\pi = 130.41(20)\,\mathrm{MeV}, \qquad \frac{F_K}{F_\pi} = 1.194(5)\,, \tag{2.36}$$

$$m_s(m_c) = 109.1(2.8)\,\mathrm{MeV}, \qquad m_d(m_c) = 5.44(19)\,\mathrm{MeV}\,. \tag{2.37}$$

In [34], the light quark masses are presented at a scale of 2 GeV, and we have evolved them to $\mu = m_c = 1.3\,\mathrm{GeV}$ with the help of the renormalisation group equation. For the comparison with lattice results below, we also need their values at $\mu = 1.53\,\mathrm{GeV}$, which are found to be

$$m_s(1.53\,\mathrm{GeV}) = 102.3(2.7)\,\mathrm{MeV}, \qquad m_d(1.53\,\mathrm{GeV}) = 5.10(17)\,\mathrm{MeV}\,. \tag{2.38}$$

Below, we will neglect the tiny errors on m_K, F_K, and F_π.

It should be emphasised that the overall factor h in (2.33), (2.34) depends on the normalisation of the amplitudes $A_{0,2}$. In [10] and recent papers of the RBC-UKQCD collaboration [23, 35] $h = \sqrt{3/2}$ is used whereas in most recent phenomenological papers [4, 17, 20, 21], $h = 1$. Correspondingly, the experimental values quoted for $A_{0,2}$ differ by this factor. To facilitate comparison with [10] and the RBC-UKQCD collaboration results [23, 25, 35], we will set $h = \sqrt{3/2}$ in the present paper and consequently the experimental numbers to be used are

$$\mathrm{Re}A_0 = 33.22(1) \times 10^{-8}\,\mathrm{GeV}\,, \qquad \mathrm{Re}A_2 = 1.479(3) \times 10^{-8}\,\mathrm{GeV}\,, \tag{2.39}$$

which display the $\Delta I = 1/2$ rule

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \equiv \frac{1}{\omega} = 22.46\,. \tag{2.40}$$

We also note that while equation (2.33) is identical to (5.10) in [10], the definition of $B_8^{(3/2)}$ in the present paper differs from [10] [cf (5.18) there]. This is to ensure that $B_6^{(1/2)} = 1$ and $B_8^{(3/2)} = 1$ both correctly reproduce the large-N limit of QCD. In contrast, (5.18) in [10] was based on the so-called vacuum insertion approximation, in which additional terms appear in the normalisation of $B_8^{(3/2)}$. Such terms misrepresent the large-N limit of QCD. With our conventions, $1/N$ corrections in (2.33) and (2.34) are represented by the departure of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from unity. They have been investigated in [22] and very recently in [24] with the result summarised in (1.4). We refer to this paper for further details.

We now turn to the parameter q which enters (2.28). We first note that, like $B_6^{(1/2)}$ and $B_8^{(3/2)}$, it is nearly renormalisation-scale independent. Its value can be estimated in the large-N approach [17]; as this approach correctly accounts for the bulk of the experimental value of $\mathrm{Re}A_0$, the ensuing estimate can be considered a plausible one. In the large-N limit, corresponding to $\mu = 0$, one finds first $\langle Q_+(0)\rangle_0 / \langle Q_-(0)\rangle_0 = 1/3$. Using the meson evolution in [17] up to $\mu = 1.0\,\mathrm{GeV}$ and then quark evolution up to $\mu = m_c$, multiplying the result by $z_+(m_c)/z_-(m_c)$, we obtain $q \approx 0.1$. On the other hand the results of the RBC-UKQCD collaboration [25] are consistent with a value of zero ($q = 0.029 \pm 0.087$).

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	value range	comment
$B_6^{(1/2)}$	0.57 ± 0.19	eq. (1.3) and surrounding discussion
$B_8^{(3/2)}$	0.76 ± 0.05	eq. (1.2) and surrounding discussion
q	0.05 ± 0.05	see (2.27), (2.41)
$B_8^{(1/2)}$	1.0 ± 0.2	defined in eq. (A.4)
p_{72}	0.222 ± 0.033	eq. (2.32) and surrounding discussion
p_3	0 ± 0.5	see appendix A
p_5	0 ± 0.5	see appendix A
p_{70}	$0 \pm 1/3$	see appendix A
$\mathrm{Im}\lambda_t$	$(1.4 \pm 0.1) \times 10^{-4}$	see text
$m_t(m_t)$	$(163 \pm 3)\,\mathrm{GeV}$	calculated from pole mass value [33]
$m_s(m_c)$	$(109.1 \pm 2.8)\,\mathrm{GeV}$	value from [33], evolved
$m_d(m_c)$	$(5.4 \pm 1.9)\,\mathrm{GeV}$	value from [33], evolved
$\alpha_s(M_Z)$	0.1185 ± 0.0006	from [33]
s_W^2	0.23126	$\overline{\mathrm{MS}}$ scheme value from [33]
$\hat{\Omega}_{\text{eff}}$	$(14.8 \pm 8.0) \times 10^{-2}$	from [29]
y_3–y_{10}	$y_i \times (1 \pm 0.1)$	see Text

Table 2. Input parameter ranges, grouped into: hadronic matrix elements, parametric, isospin breaking and NNLO. The (numerically unimportant) ratios p_{72}, p_3, p_5, p_{70} are defined in appendix A). The remaining parameters (F_π, F_K, m_K, V_{ud}, V_{us}, α_{em}, G_F, ε_K) are fixed at their central values.

As the large-N approach gives $\mathrm{Re}A_0$ below the data while [25] above it, we expect the true value of q at $\mu = m_c$ to lie between these two estimates and will take q in the range

$$0 \leq q \leq 0.1\,. \tag{2.41}$$

We consider this a credible range, but already mention that our phenomenological results below would change very little even if we enlarged this range by a factor of a few: q is simply too small to introduce a large error on ε'/ε.

Our input parameters including sub-leading hadronic parameters defined in appendix A are collected in table 2. Regarding $\mathrm{Im}\lambda_t$, we choose a central value between the UTfit [36] and CKMfitter [37] determinations and an error slightly larger than that obtained from either fit. This is to account for the very small errors on V_{ud} and V_{us}, which we fix to PDG central values [33]. The Wilson coefficients in table 1 come with an additional uncertainty from unknown higher-order corrections. In particular the threshold corrections at m_c can be substantial even at NNLO. This can for example be seen in the perturbative convergence of ε_K [14, 38]. We use a scale variation to establish the typical size of higher order corrections and estimate a 10% uncertainty for each Wilson coefficient y_3–y_{10} of table 1.

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2.4 Convenient formula for ε'/ε

Before turning to quantitative phenomenology, in order to make easier connection with the phenomenological literature and aid discussion of our results, we summarise the discussion so far in a concise formula (derived first in [10]) for ε'/ε that exhibits the sensitivity to the two most important hadronic matrix elements $B_6^{(1/2)}$ and $B_8^{(3/2)}$ transparently.

Using the effective Hamiltonian (2.1) and the experimental data for ω, $\mathrm{Re}A_0$ and ε_K, we find

$$\frac{\varepsilon'}{\varepsilon} = \mathrm{Im}\lambda_{\mathrm{t}} \cdot \left[a\big(1-\hat{\Omega}_{\mathrm{eff}}\big)\, P^{(1/2)} - P^{(3/2)} \right], \tag{2.42}$$

where

$$P^{(1/2)} = \sum P_i^{(1/2)} = r \sum y_i \langle Q_i \rangle_0 \,, \tag{2.43}$$

$$P^{(3/2)} = \sum P_i^{(3/2)} = \frac{r}{\omega} \sum y_i \langle Q_i \rangle_2 \,, \tag{2.44}$$

with

$$r = \frac{G_F\, \omega}{2\,|\varepsilon_K|\,\mathrm{Re}A_0}\,. \tag{2.45}$$

In (2.43) and (2.44) the sums run over all contributing operators. Therefore in $P^{(1/2)}$ in the case of EWP contributions we have to take into account the correction $b \neq 1$ defined in (2.10).

Writing then

$$P^{(1/2)} = a_0^{(1/2)} + a_6^{(1/2)}\, B_6^{(1/2)}\,, \tag{2.46}$$

$$P^{(3/2)} = a_0^{(3/2)} + a_8^{(3/2)}\, B_8^{(3/2)}\,, \tag{2.47}$$

with the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ taken at $\mu = m_c$ and using the expressions (2.28)–(2.34) we find:

$$a_0^{(1/2)} = r_1 \left[\frac{[4y_4 - b(3y_9 - y_{10})]}{2(1+q)z_-} + b\, \frac{3q(y_9 + y_{10})}{2(1+q)z_+} \right] + r_2\, b\, y_8\, \frac{\langle Q_8 \rangle_0}{\mathrm{Re}A_0}\,, \tag{2.48}$$

$$a_6^{(1/2)} = r_2\, y_6\, \frac{\langle Q_6 \rangle_0}{B_6^{(1/2)} \mathrm{Re}A_0}\,, \tag{2.49}$$

$$a_0^{(3/2)} = r_1\, \frac{3(y_9 + y_{10})}{2z_+}\,, \tag{2.50}$$

$$a_8^{(3/2)} = r_2\, y_8^{\mathrm{eff}}\, \frac{\langle Q_8 \rangle_2}{B_8^{(3/2)} \mathrm{Re}A_2}\,, \tag{2.51}$$

where

$$r_1 = \frac{\omega}{\sqrt{2}|\varepsilon_K|} \frac{1}{V_{ud} V_{us}^*}\,, \qquad r_2 = \frac{\omega}{2|\varepsilon_K|} G_F\,, \tag{2.52}$$

and $\langle Q_6 \rangle_0$, $\langle Q_8 \rangle_0$, and $\langle Q_8 \rangle_2$ are given in (2.33), (A.4) and (2.34), respectively. The second term in (2.48) proportional to q amounts at most to a 2% correction and could be safely neglected. y_8^{eff} is defined in (2.32). $a_0^{(1/2)}$ and $a_0^{(3/2)}$ receive further small corrections which can be extracted from the expressions in appendix A. Apart from that, the coefficients

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$\alpha_s(M_Z)$	m_t [GeV]	$a_0^{(1/2)}$	$a_6^{(1/2)}$	$a_0^{(3/2)}$	$a_8^{(3/2)}$
	160	$-2.93(12)$	17.23	-0.82	6.96
0.1179	163	$-2.90(12)$	17.25	-0.84	7.27
	166	$-2.87(12)$	17.26	-0.85	7.58
	160	$-2.95(12)$	17.61	-0.82	7.13
0.1185	163	$-2.92(12)$	17.63	-0.84	7.44
	166	$-2.89(12)$	17.64	-0.85	7.76
	160	$-2.98(12)$	18.00	-0.82	7.31
0.1191	163	$-2.95(12)$	18.02	-0.84	7.62
	166	$-2.92(12)$	18.03	-0.85	7.95

Table 3. The coefficients $a_i^{(1/2)}$ and $a_i^{(3/2)}$ in the NDR-$\overline{\text{MS}}$ scheme for different values of $\alpha_s(M_Z)$ and m_t. The uncertainty shown for $a_0^{(1/2)}$ only includes the variation of q.

$a_i^{(1/2)}$ and $a_i^{(3/2)}$ depend only on q, α_s, m_t, and the renormalisation scheme considered. The dependencies on α_s and m_t are given in the NDR-$\overline{\text{MS}}$ scheme in table 3.

In summary the ratio ε'/ε is governed by the following four contributions:

i) The contribution of $(V-A)\otimes(V-A)$ operators to $P^{(1/2)}$ is represented by the first term in (2.46). As seen in (2.48) this term is governed by the operator Q_4 and includes also small contributions from $(V-A)\otimes(V-A)$ electroweak penguin operators. We find that this term is *negative* and only weakly dependent on q. Also the dependences on α_s and renormalisation scheme (see [10]) are weak. These weak dependences originate from the fact that in our approach the matrix elements entering the first term in $P^{(1/2)}$ cancel out. The weak dependence on m_t results from the contributions of sub-leading electroweak penguin operators and is exhibited in the formulae in appendix B. As pointed out in [10], the suppression of ε'/ε through $a_0^{(1/2)}$ increases with increasing $\text{Re}A_0$, a feature which in the next section will help us to partly understand the result in (1.6).

ii) The contribution of $(V-A)\otimes(V+A)$ QCD penguin operators to $P^{(1/2)}$ is given by the second term in (2.46). This contribution is large and *positive* and is dominated by the operator Q_6. The coefficient $a_6^{(1/2)}$ depends sensitively on α_s, but as in the last two decades the precision on α_s increased, this uncertainty is small in 2015 as can be seen from table 3.

iii) The contribution of the $(V-A)\otimes(V-A)$ electroweak penguin operators Q_9 and Q_{10} to $P^{(3/2)}$ is represented by the first term in $P^{(3/2)}$. As in the case of the contribution i), the matrix elements contributing to $a_0^{(3/2)}$ cancel out in the SM. Consequently, the scheme and α_s dependences of $a_0^{(3/2)}$ are weak. As seen in (2.50) the sizable m_t-dependence of $a_0^{(3/2)}$ results from the corresponding dependence of y_9+y_{10} but again the precision on m_t increased by much in the last two decades. $a_0^{(3/2)}$ contributes *positively* to ε'/ε.

iv) The contribution of the $(V-A)\otimes(V+A)$ electroweak penguin operators Q_7 and Q_8 to $P^{(3/2)}$ is represented by the second term in (2.47). This contribution is dominated by Q_8 and depends sensitively on m_t and α_s. It contributes *negatively* to ε'/ε.

The competition between these four contributions is the reason why it is difficult to predict ε'/ε precisely. In this context, one should appreciate the virtue of our approach: the contributions i) and iii) can be determined rather precisely by CP-conserving data so that the dominant uncertainty in our approach in predicting ε'/ε resides in the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$.

3 Prediction for ε'/ε in the SM

3.1 Prediction for ε'/ε and discussion

We begin our analysis by employing the lattice values in (1.2) and (1.3). Varying all parameters within their input ranges and combining the resulting variations in ε'/ε in quadrature, we obtain:

$$(\varepsilon'/\varepsilon)_{\text{SM}} = (1.9 \pm 4.5) \times 10^{-4}. \tag{3.1}$$

Comparing to the experimental result $(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$ (average of NA48 [26] and KTeV [27, 28]), we observe a discrepancy of $2.9\,\sigma$ significance.

A detailed error budget is given in table 4. It is evident that the error is dominated by the hadronic parameter $B_6^{(1/2)}$. Uncertainties from higher-order corrections are still significant yet small if compared to the deviation from the experimental value. All other individual errors are below 10^{-4}, with the third most important uncertainty coming from the isospin breaking parameter $\hat{\Omega}_{\text{eff}}$, at a level of 0.7×10^{-4} and about six times smaller than the error due to $B_6^{(1/2)}$. If matrix elements are taken from a lattice calculation, the m_s dependence is only an artifact of our parametrisation in terms of $B_6^{(1/2)}$ and $B_8^{(3/2)}$. Therefore including the m_s variation in our error estimate for ε'/ε leads to a slight, but negligible, overestimate of the total error. At the same time, the small m_s dependence we do find in the final result shows that this is no longer a relevant source of uncertainty in non-lattice approaches (like the large-N approach in particular) in which $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are directly calculated.

At this stage it is important to emphasise that the results for $B_8^{(3/2)}$ and $B_6^{(1/2)}$ in (1.2) and (1.3) receive strong support from the large-N approach as recently demonstrated in [24]. In particular the smallness of the matrix element $\langle Q_6\rangle_0$ with respect to $\langle Q_8\rangle_2$ is the result of the chiral suppression of $\langle Q_6\rangle_0$, signalled by $F_K - F_\pi$ in (2.33). As seen in (2.34) no such suppression is present in $\langle Q_8\rangle_2$. But in addition it is possible to demonstrate that both $B_6^{(1/2)}$ and $B_8^{(3/2)}$ are below unity as given in (1.4). Moreover, while $B_8^{(3/2)} = 0.8 \pm 0.1$ is found in this approach, the values of $B_6^{(1/2)}$ are in the ballpark of the lattice result and consequently give a strong support for $B_6^{(1/2)} < B_8^{(3/2)}$ as indicated by the lattice data. But as present calculations by lattice QCD and in [24] are not precise enough, at this moment, we cannot exclude that $B_6^{(1/2)}$ could be as large as $B_8^{(3/2)}$ and this leads conservatively to the bound in (1.4).

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quantity	error on ε'/ε	quantity	error on ε'/ε
$B_6^{(1/2)}$	4.1	$m_d(m_c)$	0.2
NNLO	1.6	q	0.2
$\hat{\Omega}_{\rm eff}$	0.7	$B_8^{(1/2)}$	0.1
p_3	0.6	$\mathrm{Im}\lambda_t$	0.1
$B_8^{(3/2)}$	0.5	p_{72}	0.1
p_5	0.4	p_{70}	0.1
$m_s(m_c)$	0.3	$\alpha_s(M_Z)$	0.1
$m_t(m_t)$	0.3		

Table 4. Error budget, ordered from most important to least important. Each line shows the variation from the central value of our ε'/ε prediction, in units of 10^{-4}, as the corresponding parameter is varied within its input range, all others held at central values.

For these reasons it is instructive to consider other values of the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ than those obtained by RBC-UKQCD collaboration which are, however, consistent with the large-N bound in (1.4). Of particular interest is the choice $B_6^{(1/2)} = B_8^{(3/2)} = 1$ which corresponds to the saturation of this bound and the choice in which the bound on $B_6^{(1/2)}$ is saturated when $B_8^{(3/2)}$ is fixed to the central lattice value in (1.2). Using the same input for the remaining parameters, we find

$$(\varepsilon'/\varepsilon)_{\rm SM} = (8.6 \pm 3.2) \times 10^{-4}, \qquad (B_6^{(1/2)} = B_8^{(3/2)} = 1), \tag{3.2}$$

$$(\varepsilon'/\varepsilon)_{\rm SM} = (6.0 \pm 2.4) \times 10^{-4}, \qquad (B_6^{(1/2)} = B_8^{(3/2)} = 0.76). \tag{3.3}$$

We observe that even for these values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ the SM predictions for ε'/ε are significantly below the data. This is an important result as it shows that even if the value of $B_6^{(1/2)}$ from lattice calculations would move up in the future, the SM would face difficulty in reproducing the data provided the large-N bound in (1.4) is respected.

With these results at hand, we are in the position to summarise the present picture of the estimate of ε'/ε in the SM:

- First, parametric uncertainties decreased by much since the analyses of ε'/ε around the year 2000. This includes the uncertainty in $\mathrm{Im}\lambda_t$ which is presently about $\pm 7\%$ and is irrelevant in the estimate in (3.1) but plays some role when ε'/ε is larger. Also the improvement on m_s should be appreciated, entailing that the uncertainty on m_s no longer is an issue.

- Second, the previously sizeable uncertainty due to $B_8^{(3/2)}$ has become sub-dominant, much smaller for example than the one due to isospin violation. This is thanks to impressive progress on the lattice [23], which confirms large-N estimates employed in our previous papers, but with far smaller uncertainty.

- Third, the present analysis further increased the effectiveness of our framework, leading to a situation in which a single parameter $B_6^{(1/2)}$ is playing the decisive role in the answer to the question whether ε'/ε in the SM can be reconciled with the data or not. The new finding both by the lattice QCD and large-N approach that $B_6^{(1/2)}$ is below unity narrowed significantly the range for ε'/ε in the SM in our framework.

This picture clearly indicates the emergence of a new anomaly in K physics. As this anomaly is strictly correlated in our framework with the value of $B_6^{(1/2)}$, this parameter must be a priority for future non-perturbative calculations for flavour physics. Fortunately, it is accessible by first-principle lattice-QCD calculations. Systematic improvement is hence possible. (See also comparison with lattice below.) Progress on isospin violation will also be important.

But already now, the results presented here motivate further scrutiny of the SM prediction as well as searching for viable beyond-SM explanations. We will briefly discuss both directions, in sections 5 and 6, respectively.

Last but not least, the great reduction in parametric and hadronic uncertainties, made effective through our formalism, and good prospects on $B_6^{(1/2)}$, may make a more precise measurement of ε'/ε in the future worthwhile.

3.2 Discussion of $B_6^{(1/2)}$ dependence

The domination of our error estimate by the uncertainty on $B_6^{(1/2)}$ leads us to investigate the dependence of ε'/ε on $B_6^{(1/2)}$ in more detail.

There is a hierarchy in the four contributions discussed in the previous section with ii) being most important followed by iv), i) and iii). For central values of input parameters we find

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4}\left[\frac{\mathrm{Im}\lambda_{\rm t}}{1.4\cdot 10^{-4}}\right]\left[a\,(1-\hat{\Omega}_{\rm eff})\left(-4.1(8)+24.7\,B_6^{(1/2)}\right)+1.2(1)-10.4\,B_8^{(3/2)}\right], \quad (3.4)$$

with the four terms corresponding to the four contributions in question. The first number in brackets comprise the uncertainties of the sub-leading hadronic parameters q, p_3, p_5, p_{70} and $B_8^{(1/2)}$, while the second number in brackets is due to the uncertainty in p_{72}. This assignment of uncertainties will simplify the comparison with (4.1), even though it does not strictly follow our formalism. Furthermore, a remark on error correlations is in order. Due to implementing the constraints from CP-conserving data, correlations between the different contributions to ε'/ε are introduced. However, as the initial correlations of the hadronic matrix elements determined on the lattice are not available, we refrain from incorporating them into our analysis.

It should be noted that the term representing Q_6 penguin operator involves the product $a(1-\hat{\Omega}_{\rm eff})B_6^{(1/2)}$. Therefore, effectively isospin breaking corrections lower the value of $B_6^{(1/2)}$ by 0.866, implying in the case of $B_6^{(1/2)}=0.57$ an effective value of 0.49.

In figure 1, we show ε'/ε as a function of $B_6^{(1/2)}$ for different values of $B_8^{(3/2)}$:

$$B_8^{(3/2)} = 0.7 \text{ (blue)}, \qquad B_8^{(3/2)} = 0.8 \text{ (red)}, \quad (3.5)$$

$$B_8^{(3/2)} = 0.9 \text{ (green)}, \qquad B_8^{(3/2)} = 1 \text{ (brown)}. \quad (3.6)$$

The vertical band represents central value and error on $B_6^{(1/2)}$ from (1.3), the horizontal band the experimental world average on ε'/ε. The black region on each line is excluded by

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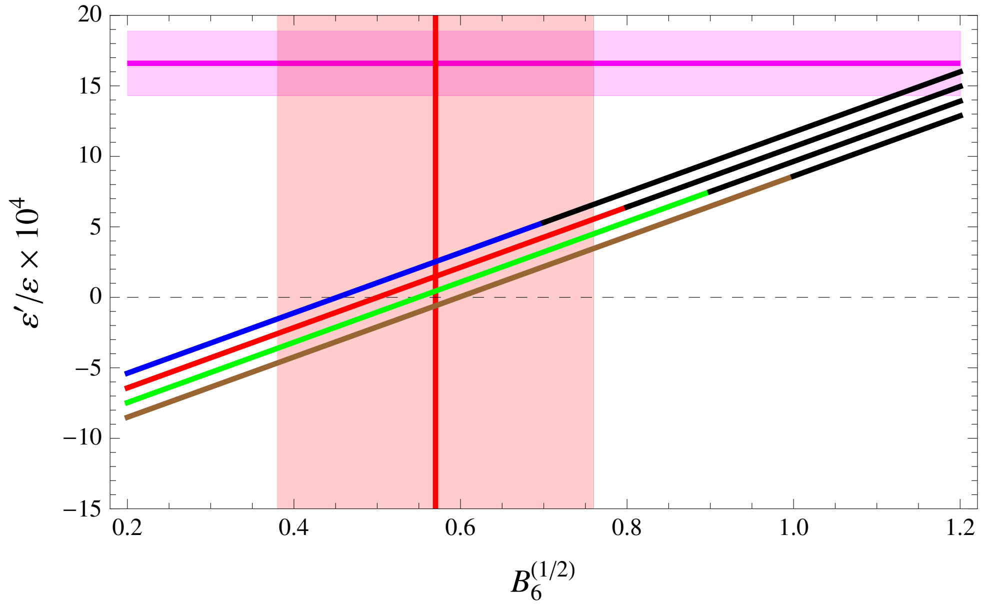

Figure 1. ε'/ε as a function of $B_6^{(1/2)}$. For further explanation see the text.

the bound (1.4). We observe that the experimental value of ε'/ε can only be reproduced in the SM far outside the RBC-UKQCD range and then only for values $B_6^{(1/2)} > B_8^{(3/2)}$ and $B_6^{(1/2)} > 1$ in variance with the bound (1.4).

We finally observe that even if the bound $B_6^{(1/2)} \le B_8^{(3/2)}$ is violated, but the bound $B_6^{(1/2)} \le 1$ is respected, the SM cannot quite reach the experimental data. Indeed, employing this unlikely hypothesis, we find this time

$$(\varepsilon'/\varepsilon)_{\rm SM} = (11.1 \pm 3.2) \times 10^{-4}, \qquad (B_6^{(1/2)} = 1.0, \quad B_8^{(3/2)} = 0.76). \tag{3.7}$$

4 Comparison with RBC-UKQCD lattice QCD

4.1 Preliminaries

The results for ε'/ε presented in [23, 25] can be summarised by a formula analogous to (3.4),

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4} \left[\frac{{\rm Im}\lambda_{\rm t}}{1.4 \cdot 10^{-4}}\right] \left[-6.5(3.2) + 25.3\, B_6^{(1/2)} + 1.2(8) - 10.2\, B_8^{(3/2)}\right]. \tag{4.1}$$

In deriving this formula, we used the value of the matrix element $\langle Q_6 \rangle$ given in [25] for $\mu = 1.53\,\text{GeV}$:

$$\langle Q_6(\mu)\rangle_0 = -\,0.379(97)(83)\,\text{GeV}^3 \qquad \text{(RBC-UKQCD)} \tag{4.2}$$

with the first error being statistical and the second systematic. Using (2.33), we find (see also [24])

$$B_6^{(1/2)}(\mu = 1.53\,\text{GeV}) = 0.57 \pm 0.19\,, \tag{4.3}$$

and consequently (1.3). The value of $B_6^{(1/2)}$ is significantly lower than its upper limit from the large-N approach in (1.4) [24] and the values for $B_6^{(1/2)}$ used in many papers until now. This is the central reason why the lattice result is substantially below the data.

Using (2.34) and comparing to the corresponding matrix element in [23] one extracts [21]

$$B_8^{(3/2)}(3\,\mathrm{GeV}) = 0.75 \pm 0.05, \qquad B_8^{(3/2)}(m_c) = 0.76 \pm 0.05, \tag{4.4}$$

which displays the very weak μ dependence mentioned above.

Setting $B_6^{(1/2)} = 0.57 \pm 0.19$ and $B_8^{(3/2)} = 0.76 \pm 0.05$, we indeed obtain the result in (1.6).

Comparing formulae (3.4) and (4.1), we observe the following differences:

- In [25], $a = 1$ and $\hat{\Omega}_{\rm eff} = 0$ have been employed.
- The main difference for fixed $B_6^{(1/2)}$ and $B_8^{(3/2)}$ is found in the first term in (4.1). Not only is the error in this term much larger than in our formula but also is this term significantly larger than found by us.
- Also striking is the sizeable error in the third term which is very small in our case.

Let us then have a closer look at the contribution of the Q_4 operator in order to clarify the reason for this difference.

4.2 Contribution of Q_4 and ReA_0

Using the formulae of the previous section, we readily find

$$\langle Q_4(m_c)\rangle_0 = \frac{2\sqrt{2}}{(1+q)z_-}\,\frac{\mathrm{Re}A_0}{G_F V_{ud} V_{us}^*}\,. \tag{4.5}$$

For $q = 0.05$, using the experimental value of ReA_0, we obtain

$$\langle Q_4(m_c)\rangle_0 = 0.22(1)\,\mathrm{GeV}^3. \tag{4.6}$$

On the other hand in [25] $q \approx 0$ and

$$\mathrm{Re}A_0 = 4.62(0.95)(0.27) \times 10^{-7}\,\mathrm{GeV}, \tag{4.7}$$

the central value of which is roughly 40% larger than the experimental value in (2.39). From (4.5) we now find

$$\langle Q_4(m_c)\rangle_0 = 0.31(7)\,\mathrm{GeV}^3\,. \tag{4.8}$$

This value agrees with the one given in [25]:

$$\langle Q_4(1.53\,\mathrm{GeV})\rangle_0 = 0.271(93)(60)\,\mathrm{GeV}^3 \qquad \text{(RBC-UKQCD)}. \tag{4.9}$$

But what is striking is the high precision obtained for this matrix element in our approach and still large uncertainty in the lattice result. It should also be noted that the contribution of the Q_4 operator to ε'/ε is in the present lattice result comparable to the one of Q_8 and can be even larger than the latter one, which is not possible in our approach.

4.3 Electroweak contribution

On the other hand the electroweak penguin contribution to ε'/ε is similar because the lattice value for $\mathrm{Re}A_2$ agrees well with experiment [23]. Using the lattice result $B_8^{(3/2)} = 0.76{\pm}0.05$, we find

$$(\varepsilon'/\varepsilon)_{\mathrm{EWP}} = -\,(6.7 \pm 0.5) \times 10^{-4}\,, \tag{4.10}$$

which can also be obtained from the last two terms in (3.4). This result compares well with [23]

$$(\varepsilon'/\varepsilon)_{\mathrm{EWP}} = -\,(6.6 \pm 1.0) \times 10^{-4}\,, \qquad \text{(RBC-UKQCD)}, \tag{4.11}$$

although our error is substantially smaller.

5 Can the large observed ε'/ε be made consistent with the SM?

Given the significant discrepancy between our SM prediction and the experimental result, we first consider possible missing or underestimated contributions in the SM.

5.1 Missing chromomagnetic contributions

In our discussion (and much of the literature) the chromomagnetic penguin Q_{8g} (and also its electromagnetic counterpart $Q_{7\gamma}$) have been tacitly dropped. It is straightforward to extend the formalism to include Q_{8g}, which being pure $\Delta I = 1/2$ impacts only on $\mathrm{Im}A_0$. While $y_{7\gamma}$ is small compared to the leading electroweak penguin coefficients, precluding any effect, the coefficient y_{8g} is sizeable. The status of the hadronic matrix element $\langle Q_{8g}\rangle_0$ is rather uncertain. A calculation at leading non-vanishing order in the chiral quark model [39] gave

$$\langle Q_{8g}\rangle_0 = -\,h\,\frac{1}{16\,\pi^2}\,\frac{11}{2}\,\frac{m_s}{m_s+m_d}\,\frac{F_K^2}{F_\pi^3}\,m_K^2 m_\pi^2\,B_{8g}\,, \tag{5.1}$$

(recall $h=\sqrt{3/2}$ in our normalisation) with $B_{8g}=1$, obtaining an upward shift of about 0.3×10^{-4} on ε'/ε. Due to uncertainties from unknown higher orders and $1/N$ corrections, an ad-hoc range $1 \le B_{8g} \le 4$ was advocated in [40] for setting bounds on new physics. For $C_{8g}(m_c) \approx -\,0.185$ and central values of our other input parameters, the resultant shift is in the range

$$\Delta\,\frac{\varepsilon'}{\varepsilon}\bigg|_{Q_{8g}} = (0.2\ldots0.7)\times10^{-4}. \tag{5.2}$$

At the upper end of the range, while still being insufficient to explain the tension between theory and experiment, the contribution becomes competitive with some of the larger sub-leading uncertainties. Although a chromomagnetic contribution has never been seriously considered as a sizable SM contribution, the possibility cannot be fully excluded. A more definite conclusion would be desirable and will require the computation of $\langle Q_{8g}\rangle_0$ in the large-N approach or on the lattice.

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5.2 Missing low-energy contributions

Two of the largest terms in our error budget concern low-energy physics: hadronic matrix elements in the isospin limit, as well as corrections to the isospin limit. In [41, 42] it has been pointed out that for approaches that do not include final-state interactions, analyticity suggests extra positive contributions to the value of $B_6^{(1/2)}$ and negative corrections to the value of $B_8^{(3/2)}$, both of which would raise ε'/ε. (See however [43].) If we naively apply the correction factors of [41, 42] to typical large-N values $B_6^{(1/2)} = 0.6$ and $B_8^{(3/2)} = 0.8$, an increase of ε'/ε to 7.8×10^{-4} results, still well below the data. (Employing the lattice-inspired central values in our error estimate, $B_6^{(1/2)} = 0.57$ and $B_8^{(3/2)} = 0.76$, results in a very similar value $\varepsilon'/\varepsilon = 8.5 \times 10^{-4}$.) While a complete non-perturbative calculation should account for the full matrix elements including final-state interactions, the issue of final-state interactions may not yet be completely under control[3] and certainly deserves further study.

Another type of long-distance corrections is isospin breaking, both due to electromagnetism and $m_u \neq m_d$. This is parametrised by the two parameters $\hat{\Omega}_{\text{eff}}$ and a. The latter only affects the overall normalisation and cannot bring the SM into agreement with data. Explaining the measured ε'/ε due to the former would require a value of opposite sign and an order of magnitude larger than the value obtained in [29, 30]. Nevertheless, given the profound implications of the ε'/ε anomaly, this issue deserves further scrutiny, and also lattice-QCD studies should take these corrections into account.

5.3 Missing higher-order corrections to the Wilson coefficients

Higher-order corrections to the Wilson coefficients will also have an impact on the theory prediction of ε'/ε. While it seems highly unlikely that they can bring the SM prediction into agreement with experiment, it is still instructive to discuss them in slightly more detail. In our analyses we fixed the renormalisation scale to $\mu = m_c$ in the three-flavour theory. Hence the computation of the Wilson coefficients involves several steps, which start with matching at the weak scale and end with integrating out the charm quark at $\mu = m_c$. The intermediate steps involve the renormalisation group evolution of Q_1–Q_{10} and integrating out the bottom quark.

The weak-scale matching corrections are known at NNLO for the electroweak penguin [12] as well as the current-current and QCD penguins [44], albeit in a different renormalisation scheme for the later two. The respective scheme transformation is given in [13], where the relevant anomalous dimensions for the NNLO evolution of Q_1–Q_6 can also be found. For these operators the matching corrections at $\mu = m_b$ are also known [14], yet all other matching corrections and anomalous dimension matrices are currently known only at NLO.

In particular the unknown matching corrections at $\mu = m_c$ could be sizeable [38] since the strong coupling is growing rapidly in this region. For this reason we estimated higher-order corrections by varying the matching scale around $\mu = m_c$, and used the three-

[3] For instance, the final-state phase shifts obtained in [25] are not in good agreement with the values extracted from experiment. We thank Chris Sachrajda for discussion.

flavour renormalisation group running to determine the Wilson coefficients at $\mu = m_c$. The resulting residual scale dependence is typically in the ball park of 10% for y_3–y_{10}, but substantially smaller for z_+ and z_-. Using this procedure, only the uncertainties in y_6, and to a lesser extent y_8, have a significant impact on the error budget of ϵ'/ϵ.

The partially known NNLO corrections to y_8 are quite large [12] and decrease the SM prediction for ε'/ε. Accordingly, only the NNLO corrections to y_6 could arguably lead to a significant enhancement of ϵ'/ϵ, but our error estimate shows that a 10% increase in y_6 results only in a 1.2×10^{-4} increase in the SM prediction. Bringing the SM prediction close to the experimental value would require a very large higher-order correction to y_6 which would cast serious doubts on the convergence of the perturbative series in our approach. If this was indeed the case, we would have to perform our analysis in a four-flavour setup, i.e. above the charm scale, which would also require new calculations of matrix elements on the lattice.

6 BSM physics in ε'/ε

Not having been able to identify a plausible way to reconcile our prediction with the data (other than attributing it to a large statistical fluctuation somewhere), we turn to a discussion of physics Beyond the Standard Model (BSM) in ε'/ε. We first note that (2.11), reproduced here for convenience:

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\,(1-\hat{\Omega}_{\mathrm{eff}}) - \frac{1}{a}\,\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right], \tag{6.1}$$

remains intact in the presence of new physics, which can be classified by which of $\mathrm{Re}A_{0,2}$ and $\mathrm{Im}A_{0,2}$ is affected.

6.1 BSM physics in $\mathrm{Re}A_{0,2}$

Noting that the RBC-UKQCD prediction [25] of $\mathrm{Re}A_0$ exceeds the experimental determination, while the large-N method exhibits a deficit [17], we define the ratio

$$H = \frac{(\mathrm{Re}A_0)_{\mathrm{SM}}}{(\mathrm{Re}A_0)_{\mathrm{EXP}}}, \tag{6.2}$$

which takes the central value $H = 1.4$ and $H = 0.7$ in [25] and [17], respectively. In other words, we are considering a scenario where the experimental value of $\mathrm{Re}A_0$ is a sum of the SM contribution and a BSM contribution. We cannot presently exclude that such a sub-leading part of $\mathrm{Re}A_0$ comes from NP, a possibility investigated in [20]. As we have seen there is a strong correlation between $\mathrm{Re}A_0$ and the matrix element of Q_4 and consequently there is an effect on ε'/ε. We stress that the denominators in (2.11) are always the true (experimental) values including any BSM contributions. It is the *numerator* term $\mathrm{Im}A_0$ that is affected through the correlation of hadronic matrix elements.

Our formalism can easily be adapted to this case; one merely needs to multiply the $V-A$ term given in (2.28) by a factor of H. In this fashion, the denominators in the ratios (2.28) are corrected for their BSM "contamination" and the theoretical SM expressions are again

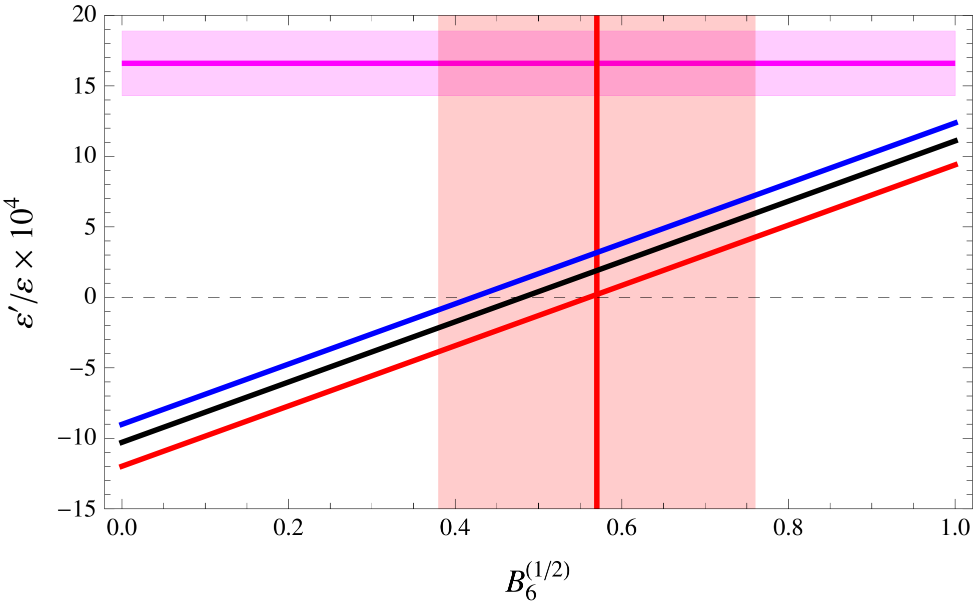

Figure 2. ε'/ε as a function of $B_6^{(1/2)}$, for three values of H defined in the text.

valid. Note that the ratio (2.30) is not modified. In figure 2, we plot ε'/ε as a function of $B_6^{(1/2)}$ for $H = 0.7$ (blue), $H = 1.0$ (black), and $H = 1.4$ (red). We see that taking the RBC-UKQCD central value for $\text{Re}A_0$ to be the true SM prediction, the agreement between theory and data for ε'/ε is worsened — and compensating for this requires even larger values of $B_6^{(1/2)}$ than in the SM. Conversely, taking the large-N central value at face value one observes a slight improvement (reduction) of the tension in ε'/ε by means of an upward shift. But in both cases, the effect is not huge, dwarfed by the uncertainty in $B_6^{(1/2)}$, and reconciling theory and experiment still requires $B_6^{(1/2)} > 1$. We conclude that CP-conserving data does not favour a scenario of BSM in $\text{Re}A_0$, although there is sizable room for it. A similar discussion could be given for NP in $\text{Re}A_2$.

6.2 BSM physics in $\text{Im}A_{0,2}$

The result obtained in our paper that ε'/ε in the SM is significantly below the experimental data has an impact on various NP models. This is in particular the case for models in which there is a strong correlation between ε'/ε and the branching ratios for rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. Such a correlation has been stressed first in [45] and investigated in many papers since then. See [46] and references to earlier literature therein.

In several models, like littlest Higgs model with T-parity (LHT) [47], and generally Z-models with new FCNCs, only in left-handed currents [20, 48], enhancement of the branching ratio for $K_L \to \pi^0\nu\bar{\nu}$ is significantly constrained by ε'/ε because in these models such an enhancement is correlated with the suppression of ε'/ε with respect to the SM. This is also the case of $K^+ \to \pi^+\nu\bar{\nu}$ but as $K^+ \to \pi^+\nu\bar{\nu}$ receives in addition to imaginary parts of the relevant amplitudes also the real parts, this correlation is much less pronounced. Therefore in such models in order to have large enhancements of $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to$

$\pi^+\nu\bar{\nu}$ the SM prediction for ε'/ε must be above the data, which is certainly not favoured by our analysis.

Therefore, in these models the agreement with the data for ε'/ε can generally be obtained only with strongly suppressed branching ratio for $K_L \to \pi^0\nu\bar{\nu}$. In the case of $K^+ \to \pi^+\nu\bar{\nu}$ this suppression is not required but significant departures from the SM are not allowed. The recent analysis within the LHT model [49] shows this explicitly.

Now, in the models just described NP enters ε'/ε only through $\mathrm{Im}A_2$ and the presence of only left-handed FCNCs implies uniquely the strict correlation between ε'/ε and $K_L \to \pi^0\nu\bar{\nu}$ mentioned above. But as shown in [50] in the presence of both left-handed and right-handed FCNCs it is possible to arrange these couplings without significant fine-tuning so that the enhancement of ε'/ε required to fit data implies automatically the enhancement of $K_L \to \pi^0\nu\bar{\nu}$ and to lesser extent of $K^+ \to \pi^+\nu\bar{\nu}$. An explicit example of a model with tree-level Z exchanges contributing to ε'/ε, $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ can be found in [50].

In Z' models the situation can in principle be different even if ε'/ε is only modified through $\mathrm{Im}A_2$ because flavour diagonal quark couplings to Z' could have proper signs so that ε'/ε and $K_L \to \pi^0\nu\bar{\nu}$ can be simultaneously enhanced in models with only left-handed flavour violating Z' couplings. As pointed out in [51] some 331 models have this property.

Another route towards the enhancement of ε'/ε, less studied in the literature, are Z' tree level exchanges with flavour universal diagonal couplings to quarks. In this case $\mathrm{Im}A_2$ is not modified and NP enters only $\mathrm{Im}A_0$ through QCD penguin contributions. As demonstrated in [20, 50] also in this model ε'/ε and $K \to \pi\nu\bar{\nu}$ can be simultaneously enhanced. Moreover, this can be achieved with only left-handed FCNCs. If the ε'/ε anomaly will be confirmed and future data on rare decays will exhibit such enhancements, models of this kind and the ones mentioned in previous paragraph will be favoured.

Clearly there are other possibilities involving new operators, like supersymmetric models [40, 52, 53], Randall-Sundrum models [54], or left-right models [5], but this is another story which requires further study.

7 Summary and outlook

Motivated by the recent results on $K \to \pi\pi$ amplitudes from the RBC-UKQCD collaboration, we gave another look to the ratio ε'/ε within the SM. The main result of our analysis is the identification of a possible new anomaly in flavour physics, this time in K physics. This was possible because:

- Improved results for the parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ became available through recent lattice-QCD studies by the RBC-UKQCD collaboration that are supported by the large-N approach which provides upper bounds on these parameters.

- We employed a formalism that is manifestly independent of the values of leading $(V-A)\otimes(V-A)$ QCD penguin, and EW penguin hadronic matrix elements of the operators Q_4, Q_9, and Q_{10}. In this manner a prediction for ε'/ε could be made that is more precise than presently possible by direct lattice-QCD simulations.

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In this context, we have presented a new analytic formula for ε'/ε in terms of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ that is valid also in SM extensions with the same operator structure. This formula depends on the Wilson coefficients of the contributing operators which are model dependent while $B_6^{(1/2)}$ and $B_8^{(3/2)}$, related to long-distance dynamics, are independent of NP contributions. Thus our formula can be used for models such as the models with constrained MFV, 3-3-1 models and littlest Higgs models. We have also provided an update of a formula for ε'/ε in which NP enters directly through the shifts in basic one-loop functions.

Our analysis emphasises the correlation between the amplitude $\mathrm{Re}A_0$ and the contribution of the Q_4 operator to ε'/ε given in (4.5). As the central value of $\mathrm{Re}A_0$ in [25] is by 40% above the data, this calculation overestimates the contribution of Q_4 to ε'/ε making it smaller. Assuming that $\mathrm{Re}A_0$ is fully described by SM dynamics, we could improve the accuracy of its estimate by roughly an order of magnitude, as seen in (4.6) and (4.9).

We have extracted from [25] the value of $B_6^{(1/2)}$ obtained by the RBC-UKQCD collaboration to find that it is significantly lower than unity. In fact this is the main reason for the low value of ε'/ε found in that paper. On the other hand, should the values of $B_6^{(1/2)}$ and $B_8^{(3/2)}$ eventually turn out to be close to the upper bound from the large-N approach [24], significantly larger values of ε'/ε are found, although still roughly by a factor of two below the data.

Our improved anatomy of ε'/ε clearly demonstrates that the SM has potential difficulties in describing the data for ε'/ε. However, there are several open questions that have to be answered before one can be fully confident that NP is at work here. Answering them would also allow us to give a better estimate of the room left for particular NP models.

Our analysis shows that the next most important issues that have to be clarified are as follows:

- The value of $B_6^{(1/2)}$ should be determined with an accuracy of at least 10%. Figure 1 demonstrates this need clearly, but also a higher precision on $B_8^{(3/2)}$ would be beneficial.

- The values of the Wilson coefficients y_i at the NNLO level. First steps in this direction have been taken in [12, 13].

- Improved calculations of isospin breaking effects, represented in our formula in (2.11) by the parameters a and $\hat{\Omega}_{\text{eff}}$.

- The role of electromagnetic corrections to the hadronic matrix elements, as emphasised already in [10]. Without these corrections there remains some uncertainty due to the renormalisation scheme used for operators.

- Precise theoretical predictions of $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$ within the SM, which would tell us to which degree our assumption of neglecting NP contributions in these amplitudes is justified.

- Finally, our understanding of the role of final-state interactions in ε'/ε, see [4] and references therein, should be improved.

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Our present results could be affected to some extent by the future finding that some part of the amplitude ReA_0 does not come from the SM dynamics but NP. As figure 2 shows, if ReA_0 in the SM is below the experimental value, as indicated by the large-N approach [17], the suppression of ε'/ε by the Q_4 operator is smaller, implying a larger value of ε'/ε. On the other hand if ReA_0 in the SM is above the experimental value as presently seen in lattice data, the role of Q_4 will be enhanced and consequently ε'/ε smaller. But as seen in the error budget of table 4, this effect is by far less important than the sensitivity to $B_6^{(1/2)}$.

In view of the tendency of ε'/ε in the SM to be significantly below the data, it is exciting that in the coming years LHC might tell us what this physics could be. But also independent studies of ε'/ε in various extensions of the SM could select those extensions of the SM in which ε'/ε could be enhanced over its SM value. In fact first phenomenological implications of our results on new physics models have been presented in [50, 55]. In any case it appears that ε'/ε could soon become again a leading light in flavour physics.

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A Subleading contributions to Im$A_{0,2}$ and related operator matrix elements.

Including operators with small Wilson coefficients and colour-suppressed hadronic matrix element, the isospin ratios (2.28) and (2.30) receive the following corrections:

$$\Delta\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{V-A} = \mathrm{Im}\tau\,\frac{p_3\big(2(y_3+y_4)-b\,(y_9+y_{10})\big)}{2(1+q)z_-}\,, \tag{A.1}$$

$$\Delta\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)_{6} = -\,\frac{G_F}{\sqrt{2}}\,\mathrm{Im}\,\lambda_t\,\frac{p_5\,y_5\,\langle Q_6\rangle_0+b\,(y_8+p_{70}\,y_7)\,\langle Q_8\rangle_0}{\mathrm{Re}A_0}\,, \tag{A.2}$$

where we have defined

$$p_3 = \frac{\langle Q_3 \rangle_0}{\langle Q_- \rangle_0}\,, \qquad p_5 = \frac{\langle Q_5 \rangle_0}{\langle Q_6 \rangle_0}\,, \qquad p_{70} = \frac{\langle Q_7 \rangle_0}{\langle Q_8 \rangle_0}\,. \tag{A.3}$$

All three ratios are formally at least $1/N$-suppressed and multiplied by small Wilson coefficients. Note that in (2.31) we have already included the $y_7 \langle Q_7 \rangle_2$ contribution into $y_8^{\rm eff}$; eq. (2.31) then does not receive an additional correction. We also define $B_8^{(1/2)}$ through

$$\langle Q_8(\mu) \rangle_0 = 2h \left[\frac{m_{\rm K}^2}{m_s(\mu) + m_d(\mu)} \right]^2 F_\pi \, B_8^{(1/2)}\,. \tag{A.4}$$

This convention deviates from (5.12) in [10] but is motivated by the result [24]

$$B_8^{(1/2)} = 1 + \mathcal{O}\left(\frac{1}{N}\right) = 1.1 \pm 0.1\,, \tag{A.5}$$

strongly supported by the lattice result for $\langle Q_8(\mu) \rangle_0$ in [25].

To keep our phenomenological formulae simple and central values transparent, we set the central values of p_3, p_5, and p_{70} to zero and allow generous error ranges that comprise both the intervals expected from large-N counting and those computed in [25]. For the ratio p_{72} defined below (2.32), which is also $1/N$-suppressed and plays a very minor role numerically, we take the central value from [25] and, conservatively, attribute a 100% error to it. We furthermore employ $B_8^{(1/2)} = 1.0 \pm 0.2$, also derived from [25]. All input rages are summarised in table 2. In this treatment, we tend to overestimate our error on ε'/ε, but as shown in the body of the paper, this has a very minor impact on our predictions.

B Analytic formula for ε'/ε

The expression (2.42) can be put into a formula that is more useful for numerical evaluations as it shows explicitly the dependence on m_t and m_s. The most recent version of it has been presented in [20], but we update it here due to the change of some input parameters entering the formulae for hadronic matrix elements and a different treatment of isospin breaking corrections. We then have

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} = {\rm Im}\lambda_t \cdot F_{\varepsilon'}(x_t)\,, \tag{B.1}$$

where

$$F_{\varepsilon'}(x_t) = P_0 + P_X\, X_0(x_t) + P_Y\, Y_0(x_t) + P_Z\, Z_0(x_t) + P_E\, E_0(x_t)\,. \tag{B.2}$$

The first term is dominated by QCD-penguin contributions, the next three terms by electroweak penguin contributions. The last term expresses the m_t dependence from contribution of QCD penguin operators and is totally negligible. The x_t dependent functions are

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	$\alpha_s(M_Z) = 0.1179$			$\alpha_s(M_Z) = 0.1185$			$\alpha_s(M_Z) = 0.1191$		
i	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$	$r_i^{(0)}$	$r_i^{(6)}$	$r_i^{(8)}$
0	-3.392	15.293	1.271	-3.421	15.624	1.231	-3.451	15.967	1.191
X_0	0.655	0.029	0.	0.655	0.030	0.	0.655	0.031	0.
Y_0	0.451	0.114	0.	0.449	0.116	0.	0.447	0.118	0.
Z_0	0.406	-0.022	-13.434	0.420	-0.022	-13.649	0.435	-0.023	-13.872
E_0	0.229	-1.760	0.652	0.228	-1.788	0.665	0.226	-1.816	0.678

Table 5. The coefficients $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ of formula (B.7) in the NDR-$\overline{\text{MS}}$ scheme for three values of $\alpha_s(M_Z)$.

given as follows

$$X_0(x_t) = \frac{x_t}{8}\left[\frac{x_t+2}{x_t-1} + \frac{3x_t-6}{(x_t-1)^2}\log x_t\right], \tag{B.3}$$

$$Y_0(x_t) = \frac{x_t}{8}\left[\frac{x_t-4}{x_t-1} + \frac{3x_t}{(x_t-1)^2}\ln x_t\right], \tag{B.4}$$

$$\begin{aligned} Z_0(x_t) = &-\frac{1}{9}\ln x_t + \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t-1)^3} + \\ &+ \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t-1)^4}\ln x_t\,, \end{aligned} \tag{B.5}$$

$$E_0(x_t) = -\frac{2}{3}\ln x_t + \frac{x_t^2(15-16x_t+4x_t^2)}{6(1-x_t)^4}\ln x_t + \frac{x_t(18-11x_t-x_t^2)}{12(1-x_t)^3}, \tag{B.6}$$

where $x_t = m_t^2/M_W^2$.

The coefficients P_i are given by

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8\,, \tag{B.7}$$

where we have defined

$$R_6 \equiv B_6^{(1/2)}\left[\frac{114.54\,\text{MeV}}{m_s(m_c)+m_d(m_c)}\right]^2, \tag{B.8}$$

$$R_8 \equiv B_8^{(3/2)}\left[\frac{114.54\,\text{MeV}}{m_s(m_c)+m_d(m_c)}\right]^2. \tag{B.9}$$

The coefficients $r_i^{(0)}$, $r_i^{(6)}$ and $r_i^{(8)}$ comprise information on the Wilson-coefficient functions of the $\Delta S = 1$ weak effective Hamiltonian at NLO and incorporate the values of the matrix elements of those operators that we could extract by imposing the experimental values of ReA_0 and ReA_2. Their numerical values are given in the NDR-$\overline{\text{MS}}$ renormalisation scheme for $\mu = m_c$ and three values of $\alpha_s(M_Z)$ in table 5.

References

[1] S. Bertolini, M. Fabbrichesi and J.O. Eeg, *Theory of the CP-violating parameter epsilon-prime/epsilon*, Rev. Mod. Phys. **72** (2000) 65 [hep-ph/9802405] [INSPIRE].

[2] A.J. Buras and M. Jamin, *ε'/ε at the NLO: 10 years later*, JHEP **01** (2004) 048 [hep-ph/0306217] [INSPIRE].

[3] A. Pich, *ε'/ε in the standard model: Theoretical update*, hep-ph/0410215 [INSPIRE].

[4] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich and J. Portoles, *Kaon Decays in the Standard Model*, Rev. Mod. Phys. **84** (2012) 399 [arXiv:1107.6001] [INSPIRE].

[5] S. Bertolini, J.O. Eeg, A. Maiezza and F. Nesti, *New physics in ε' from gluomagnetic contributions and limits on Left-Right symmetry*, Phys. Rev. **D 86** (2012) 095013 [arXiv:1206.0668] [INSPIRE].

[6] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Effective Hamiltonians for $\Delta S = 1$ and $\Delta B = 1$ nonleptonic decays beyond the leading logarithmic approximation*, Nucl. Phys. **B 370** (1992) 69 [INSPIRE].

[7] A.J. Buras, M. Jamin, M.E. Lautenbacher and P.H. Weisz, *Two loop anomalous dimension matrix for $\Delta S = 1$ weak nonleptonic decays. 1. $O(\alpha_S^2)$*, Nucl. Phys. **B 400** (1993) 37 [hep-ph/9211304] [INSPIRE].

[8] A.J. Buras, M. Jamin and M.E. Lautenbacher, *Two loop anomalous dimension matrix for $\Delta S = 1$ weak nonleptonic decays. 2. $O(\alpha\alpha_S)$*, Nucl. Phys. **B 400** (1993) 75 [hep-ph/9211321] [INSPIRE].

[9] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *ε'/ε at the Next-to-leading order in QCD and QED*, Phys. Lett. **B 301** (1993) 263 [hep-ph/9212203] [INSPIRE].

[10] A.J. Buras, M. Jamin and M.E. Lautenbacher, *The Anatomy of ε'/ε beyond leading logarithms with improved hadronic matrix elements*, Nucl. Phys. **B 408** (1993) 209 [hep-ph/9303284] [INSPIRE].

[11] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, *The $\Delta S = 1$ effective Hamiltonian including next-to-leading order QCD and QED corrections*, Nucl. Phys. **B 415** (1994) 403 [hep-ph/9304257] [INSPIRE].

[12] A.J. Buras, P. Gambino and U.A. Haisch, *Electroweak penguin contributions to nonleptonic $\Delta F = 1$ decays at NNLO*, Nucl. Phys. **B 570** (2000) 117 [hep-ph/9911250] [INSPIRE].

[13] M. Gorbahn and U. Haisch, *Effective Hamiltonian for non-leptonic $|\Delta F| = 1$ decays at NNLO in QCD*, Nucl. Phys. **B 713** (2005) 291 [hep-ph/0411071] [INSPIRE].

[14] J. Brod and M. Gorbahn, *ϵ_K at Next-to-Next-to-Leading Order: The Charm-Top-Quark Contribution*, Phys. Rev. **D 82** (2010) 094026 [arXiv:1007.0684] [INSPIRE].

[15] G. Buchalla, A.J. Buras and M.K. Harlander, *The Anatomy of ε'/ε in the Standard Model*, Nucl. Phys. **B 337** (1990) 313 [INSPIRE].

[16] W.A. Bardeen, A.J. Buras and J.M. Gerard, *The $K \to \pi\pi$ Decays in the Large-N Limit: Quark Evolution*, Nucl. Phys. **B 293** (1987) 787 [INSPIRE].

[17] A.J. Buras, J.-M. Gérard and W.A. Bardeen, *Large-N Approach to Kaon Decays and Mixing 28 Years Later: $\Delta I = 1/2$ Rule, $\hat{B}_K$ and ΔM_K*, Eur. Phys. J. **C 74** (2014) 2871 [arXiv:1401.1385] [INSPIRE].

[18] A.J. Buras, M. Jamin and M.E. Lautenbacher, *A 1996 analysis of the CP-violating ratio ε'/ε*, Phys. Lett. **B 389** (1996) 749 [hep-ph/9608365] [INSPIRE].

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[19] S. Bosch et al., *Standard model confronting new results for ε'/ε*, Nucl. Phys. **B 565** (2000) 3 [hep-ph/9904408] [INSPIRE].

[20] A.J. Buras, F. De Fazio and J. Girrbach, *$\Delta I = 1/2$ rule, ε'/ε and $K \to \pi\nu\bar{\nu}$ in $Z'(Z)$ and G' models with FCNC quark couplings*, Eur. Phys. J. **C 74** (2014) 2950 [arXiv:1404.3824] [INSPIRE].

[21] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, *$K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ in the Standard Model: status and perspectives*, JHEP **11** (2015) 033 [arXiv:1503.02693] [INSPIRE].

[22] T. Hambye, G.O. Kohler, E.A. Paschos, P.H. Soldan and W.A. Bardeen, *$1/N_c$ corrections to the hadronic matrix elements of Q_6 and Q_8 in $K \to \pi\pi$ decays*, Phys. Rev. **D 58** (1998) 014017 [hep-ph/9802300] [INSPIRE].

[23] T. Blum et al., *$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit*, Phys. Rev. **D 91** (2015) 074502 [arXiv:1502.00263] [INSPIRE].

[24] A.J. Buras and J.-M. Gerard, *Upper Bounds on ε'/ε Parameters $B_6^{(1/2)}$ and $B_8^{(3/2)}$ from Large-N QCD and other News*, arXiv:1507.06326 [INSPIRE].

[25] RBC, UKQCD collaboration, Z. Bai et al., *Standard Model Prediction for Direct CP-violation in $K \to \pi\pi$ Decay*, Phys. Rev. Lett. **115** (2015) 212001 [arXiv:1505.07863] [INSPIRE].

[26] NA48 collaboration, J.R. Batley et al., *A Precision measurement of direct CP-violation in the decay of neutral kaons into two pions*, Phys. Lett. **B 544** (2002) 97 [hep-ex/0208009] [INSPIRE].

[27] KTeV collaboration, A. Alavi-Harati et al., *Measurements of direct CP-violation, CPT symmetry and other parameters in the neutral kaon system*, Phys. Rev. **D 67** (2003) 012005 [*Erratum ibid.* **D 70** (2004) 079904] [hep-ex/0208007] [INSPIRE].

[28] KTeV collaboration, E.T. Worcester, *The Final Measurement of Epsilon-prime/Epsilon from KTeV*, arXiv:0909.2555 [INSPIRE].

[29] V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, *Isospin breaking in $K \to \pi\pi$ decays*, Eur. Phys. J. **C 33** (2004) 369 [hep-ph/0310351] [INSPIRE].

[30] V. Cirigliano, A. Pich, G. Ecker and H. Neufeld, *Isospin violation in ϵ'*, Phys. Rev. Lett. **91** (2003) 162001 [hep-ph/0307030] [INSPIRE].

[31] A.J. Buras and J.M. Gérard, *$1/N$ Expansion for Kaons*, Nucl. Phys. **B 264** (1986) 371 [INSPIRE].

[32] A.J. Buras and J.M. Gérard, *Isospin Breaking Contributions to ε'/ε*, Phys. Lett. **B 192** (1987) 156 [INSPIRE].

[33] Particle Data Group collaboration, K.A. Olive et al., *Review of Particle Physics*, Chin. Phys. **C 38** (2014) 090001 [INSPIRE].

[34] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, Eur. Phys. J. **C 74** (2014) 2890 [arXiv:1310.8555] [INSPIRE].

[35] T. Blum et al., *Lattice determination of the $K \to (\pi\pi)_{I=2}$ Decay Amplitude A_2*, Phys. Rev. **D 86** (2012) 074513 [arXiv:1206.5142] [INSPIRE].

[36] UTfit collaboration http://www.utfit.org.

[37] J. Charles et al., *Current status of the Standard Model CKM fit and constraints on $\Delta F = 2$ New Physics*, Phys. Rev. **D 91** (2015) 073007 [arXiv:1501.05013] [INSPIRE].

[38] J. Brod and M. Gorbahn, *Next-to-Next-to-Leading-Order Charm-Quark Contribution to the CP-violation Parameter ε_K and ΔM_K*, Phys. Rev. Lett. **108** (2012) 121801 [arXiv:1108.2036] [INSPIRE].

[39] S. Bertolini, M. Fabbrichesi and E. Gabrielli, *The Relevance of the dipole Penguin operators in ε'/ε*, Phys. Lett. **B 327** (1994) 136 [hep-ph/9312266] [INSPIRE].

[40] A.J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, *Connections between ε'/ε and rare kaon decays in supersymmetry*, Nucl. Phys. **B 566** (2000) 3 [hep-ph/9908371] [INSPIRE].

[41] E. Pallante and A. Pich, *Strong enhancement of ε'/ε through final state interactions*, Phys. Rev. Lett. **84** (2000) 2568 [hep-ph/9911233] [INSPIRE].

[42] E. Pallante and A. Pich, *Final state interactions in kaon decays*, Nucl. Phys. **B 592** (2001) 294 [hep-ph/0007208] [INSPIRE].

[43] A.J. Buras, M. Ciuchini, E. Franco, G. Isidori, G. Martinelli and L. Silvestrini, *Final state interactions and ε'/ε: a critical look*, Phys. Lett. **B 480** (2000) 80 [hep-ph/0002116] [INSPIRE].

[44] C. Bobeth, M. Misiak and J. Urban, *Photonic penguins at two loops and m_t-dependence of $BR(B \to X_s \ell^+\ell^-)$*, Nucl. Phys. **B 574** (2000) 291 [hep-ph/9910220] [INSPIRE].

[45] A.J. Buras and L. Silvestrini, *Upper bounds on $K \to \pi\nu\bar{\nu}$ and $K_L \to \pi^0 e^+e^-$ from ε'/ε and $K_L \to \mu^+\mu^-$*, Nucl. Phys. **B 546** (1999) 299 [hep-ph/9811471] [INSPIRE].

[46] A.J. Buras and J. Girrbach, *Towards the Identification of New Physics through Quark Flavour Violating Processes*, Rept. Prog. Phys. **77** (2014) 086201 [arXiv:1306.3775] [INSPIRE].

[47] M. Blanke, A.J. Buras, B. Duling, S. Recksiegel and C. Tarantino, *FCNC Processes in the Littlest Higgs Model with T-Parity: a 2009 Look*, Acta Phys. Polon. **B 41** (2010) 657 [arXiv:0906.5454] [INSPIRE].

[48] A.J. Buras, F. De Fazio and J. Girrbach, *The Anatomy of Z' and Z with Flavour Changing Neutral Currents in the Flavour Precision Era*, JHEP **02** (2013) 116 [arXiv:1211.1896] [INSPIRE].

[49] M. Blanke, A.J. Buras and S. Recksiegel, *Quark flavour observables in the Littlest Higgs model with T-parity after LHC Run 1*, arXiv:1507.06316 [INSPIRE].

[50] A.J. Buras, D. Buttazzo and R. Knegjens, *$K \to \pi\nu\bar{\nu}$ and ε'/ε in Simplified New Physics Models*, arXiv:1507.08672 [INSPIRE].

[51] A.J. Buras, F. De Fazio and J. Girrbach-Noe, *Z-Z' mixing and Z-mediated FCNCs in $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ Models*, JHEP **08** (2014) 039 [arXiv:1405.3850] [INSPIRE].

[52] A. Masiero and H. Murayama, *Can ε'/ε be supersymmetric?*, Phys. Rev. Lett. **83** (1999) 907 [hep-ph/9903363] [INSPIRE].

[53] A.J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, *ε'/ε and rare K and B decays in the MSSM*, Nucl. Phys. **B 592** (2001) 55 [hep-ph/0007313] [INSPIRE].

[54] O. Gedalia, G. Isidori and G. Perez, *Combining Direct and Indirect Kaon CP-violation to Constrain the Warped KK Scale*, Phys. Lett. **B 682** (2009) 200 [arXiv:0905.3264] [INSPIRE].

[55] M. Blanke, A.J. Buras and S. Recksiegel, *Quark flavour observables in the Littlest Higgs model with T-parity after LHC Run 1*, arXiv:1507.06316 [INSPIRE].