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Microscopic unitary description of tidal excitations in high-energy string-brane collisions

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Abstract: The eikonal operator was originally introduced to describe the effect of tidal excitations on higher-genus elastic string amplitudes at high energy. In this paper we provide a precise interpretation for this operator through the explicit tree-level calculation of generic inelastic transitions between closed strings as they scatter off a stack of parallel Dp-branes. We perform this analysis both in the light-cone gauge, using the Green-Schwarz vertex, and in the covariant formalism, using the Reggeon vertex operator. We also present a detailed discussion of the high-energy behaviour of the covariant string amplitudes, showing how to take into account the energy factors that enhance the contribution of the longitudinally polarized massive states in a simple way.

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Contents

1 Introduction

Since about 25 years transplanckian-energy gravitational scattering has been the target of numerous investigations. The original thrust was focused on the scattering among point-like particles [1, 2] or light strings [3] (see also [4]). One of its main goals was to understand how an effective curved geometry originates from studying collisions in flat D-dimensional space-time. A more ambitious aim was to find out whether and how unitarity of the S-matrix is preserved in all regimes within a consistent quantum-gravity framework.

Indeed, at sufficiently high energy and at any finite order in perturbation theory, (partial-wave) unitarity bounds are already violated even at large impact parameters. This problem was neatly solved [2, 3] by an all-loop resummation of the leading high-energy contributions: in the point particle case these exponentiate (in impact parameter space) leading to an elastic-unitarity-preserving eikonal S-matrix at leading order in the small parameter $G_D\sqrt{s}b^{3-D}$, where D is the number of non-compact space-time dimensions and G_D is the Newton constant in D dimensions

$$S(E,b) = 1 + 2i\delta(s,b) + \cdots \to \exp\left(2i\delta(s,b)\right) , \qquad \delta(s,b) \sim \hbar^{-1} G_D s b^{4-D} . \tag{1.1}$$

This small parameter controls the typical gravitational deflection angle, $\theta \sim (R_s/b)^{D-3}$.[1] In the case of string-string collisions the above c-number factorization and exponentiation fail below a certain impact parameter $b_D \gg l_s \equiv \sqrt{2\alpha'\hbar}$. [2] They can only be recovered [3, 5] at the price of promoting the conventional eikonal phase to an eikonal operator

$$\delta(s,b) \to \hat{\delta}(s,b) \sim \int_0^{2\pi}\int_0^{2\pi} \frac{d\sigma_u}{2\pi}\frac{d\sigma_d}{2\pi} : \delta(s, b + \hat{X}_u(\sigma_u, \tau=0) - \hat{X}_d(\sigma_d, \tau=0)) : , \tag{1.2}$$

where $\hat{X}_u(\sigma,\tau)$ and $\hat{X}_d(\sigma,\tau)$ are independent free bosonic fields in two dimensions containing only oscillation modes. The intuitive meaning of this operator [3, 5] is that the gravitons responsible for the transplanckian scattering are exchanged between two arbitrary points on the two colliding strings at a Lorentz contracted instant $\tau = 0$. More physically [6], the eikonal operator should ensure inelastic unitarity in the regime in which tidal forces induce substantial excitations of the incoming strings. However, in order to fully understand how unitarity works, and the precise microscopic nature of the transitions induced by such tidal forces, it would be necessary to consider, at tree level, the individual inelastic transitions whose shadow is collectively taken into account by the elastic loop amplitudes. This issue will be one of the main objectives of this paper, although we will discuss it explicitly not in the context of string-string collisions but in the slightly different context of string-brane collisions that we shall illustrate in a moment.

Understanding how unitarity is preserved at arbitrary impact parameter in string-string collisions has proven to be a much more difficult task. Nice progress was made [5, 7, 8] in the so-called stringy regime ($l_s > R_S, b$) in which one does not expect black-hole formation to occur. In the opposite regime ($R_s > l_s, b$), in which black-hole formation is expected to occur on the basis of classical collapse criteria [9–12], only a crude approximation was attempted [13]. While this approximation could reproduce semi-quantitatively the expected critical points for gravitational collapse [13–16], it has failed, so far [17, 18], to explain how unitarity is preserved beyond such critical points (i.e. in the supposed collapse regime).

In order to address such questions in an easier context we have recently turned our attention [19] to the collision of a light closed string off a stack of N parallel Dp-branes (at large N and small string coupling), where the effective metric, rather than being produced by the collision itself, should be, up to possible corrections, the known classical one generated by the branes. When the volume of the branes is compactified on a p-dimensional

[1] Here $R_s^{D-3} = \frac{8\pi\Gamma(\frac{D-1}{2})G_D\sqrt{s}}{(D-2)\pi^{\frac{D-1}{2}}}$ is the Schwarzschild radius for a mass $\sqrt{s}$ in D non-compact dimensions.

[2] We use units in which $c = 1$ and, in the following sections, we shall also set $\hbar = 1$ thus identifying l_s^2 with $2\alpha'$.

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torus, one obtains a point-like 1/2-BPS object in $9-p$ non-compact spatial dimensions with a mass proportional to the Dp-brane tension and thus very large at weak string coupling. The study of the high-energy scattering of a fundamental string on this kind of target, although it does not represent a standard black hole with a macroscopic horizon, is non trivial and the construction of an explicitly unitary string S-matrix very interesting. The S-matrix does in fact allow to test the regime of validity of the classical gravity solution as an effective description of the Dp-branes and to explore the small b regime in which the energy of the incoming string should dissipate through the creation of many open string excitations on the branes.

With these motivations in mind, in [19] we have computed the high-energy (Regge) limit of the elastic scattering of a massless closed string state, belonging to the NS-NS sector of type II string theory, on a maximally supersymmetric Dp-brane system. It turns out that also in this case the tree-level amplitude, the disk diagram, diverges with the energy of the incoming string violating unitarity at sufficiently high energy. Neglecting string-size effects unitarity is again recovered by summing the contributions to the elastic amplitude coming from surfaces with any number of boundaries. The sum of these terms exponentiates into a phase, as in the previous case of a string-string collision,

$$S(E,b) = 1 + 2\mathrm{i}\delta(s,b) + \cdots \to \exp\left(2\mathrm{i}\delta(s,b)\right) \ , \qquad \delta(s,b) \sim \frac{E\ b}{\hbar}\left(\frac{R_p}{b}\right)^{7-p} \ , \tag{1.3}$$

where $R_p^{7-p} \sim gNl_s^{7-p}$ is the characteristic scale of the geometry produced by the branes.

Taking into account string size effects, the eikonal phase becomes once more an eikonal operator which contains the bosonic oscillators of the superstring corresponding to the $8-p$ directions transverse to the world-volume of the Dp-brane and to the momentum of the fast-moving string

$$S(s,b) = \mathrm{e}^{2\mathrm{i}\hat{\delta}(s,b)} \ , \qquad \hat{\delta}(s,b) = \int_0^{2\pi} \frac{d\sigma}{2\pi} :\delta(s, b+\hat{X}(\sigma,\tau=0)): \ . \tag{1.4}$$

All calculations are done in flat Minkowski space-time with suitable conditions imposed on the closed strings by the presence of the Dp-branes, but the final results show the curved space structure generated by the presence of the Dp-branes. In particular, in the zero-slope limit $\alpha' \to 0$ one reproduces the eikonal computed with a curved space formalism in field theory.

Present derivations of the eikonal operator are somewhat indirect in that the existence and nature of such an operator is argued on the basis of the imaginary part of a higher-loop elastic amplitude which, in principle, only provides some inclusive sum over intermediate excited string states, rather than a precise microscopic description of each produced string. In previous work the eikonal operator was indeed mainly used to study the absorption of the elastic channel due to the excitation of the massive modes of the string and to identify the average excitation mass [5]. To this aim all that is required is the algebra satisfied by the modes of the bosonic fields $\hat{X}$.

The information on the high-energy string dynamics encoded in the eikonal operator is by far more detailed. Its matrix elements give in fact the asymptotic behaviour at high

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energy of the transition amplitudes between four (in the case of a string-string collision) or two (in the case of a string-brane collision) arbitrary string states, resumming to all orders the perturbative series. For the evaluation of these matrix elements it is necessary to give a precise definition of the eikonal operator by specifying the Hilbert space on which it acts, an issue that has never been clarified in the previous literature on the subject.

Although the eikonal operator is the result of an all-loop resummation, to complete its definition it is enough to consider string amplitudes at tree level. To identify the correct Hilbert space it is in fact sufficient to study the eikonal phase, the operator $\hat{\delta}(s,b)$, which gives the asymptotic behaviour at high energy of the tree-level two-point (four-point) amplitudes between arbitrary string states. In this paper we will present two independent derivations of the eikonal phase, one in the light-cone gauge and the other fully covariant, thus giving a precise meaning to the operators $\hat{X}$ in eqs. (1.2) and (1.4).

We will first derive $\hat{\delta}(s,b)$ by quantizing the string in a light-cone gauge adapted to the kinematics of the high-energy scattering, that is with the spatial direction of the light-cone chosen along the direction of the large momentum. This derivation is based on the Green-Schwarz three-string vertex and shows that the free fields $\hat{X}$ can be identified with the transverse string coordinates in the light-cone gauge.

That the eikonal operator can be interpreted as acting on the space of the physical states of the string quantized in a specific light-cone gauge is of course not unexpected. It is quite natural given the kinematics of the high-energy scattering in the Regge limit, which is characterized by the presence of one privileged direction of large momentum, and it is also clearly suggested by the original papers [3, 5]. Moreover both the eikonal operator for string-string collisions and for string-brane collisions can be derived (at large impact parameters and to first order in the string corrections, i.e. in l_s^2/b^2) by quantizing in the light-cone gauge the string sigma-model for an effective curved background. In the first case, [20–22] the relevant background is the Aichelburg-Sexl metric [23], the shock-wave generated in first approximation by one of the two colliding strings, in the second case [19] it is the Penrose limit of the extremal p-brane solutions of Type II supergravity.

Once the Hilbert space has been identified, one can proceed to consider the implications of the eikonal operator for the high-energy string dynamics. The simple way in which the dependence on the string coordinates $\hat{X}$ enters in $\hat{\delta}(s,b)$, as a shift of the impact parameter, and the absence of the worldsheet fermions ψ lead to interesting selection rules for the possible transitions. The class of states that can be reached from any given initial state in a high-energy collision can be readily identified. The form of the inelastic amplitudes, which can only involve the external polarizations and the momentum transferred, is also very constrained. Since the matrix elements are evaluated in the light-cone gauge, the initial and final state of a transition as well as the tensors that appear in the amplitudes derived from the eikonal operator are characterized only by their transformation properties with respect to the transverse SO(8) group.

The natural question then arises about what is the covariant dynamics responsible for the simple properties of the eikonal operator in the light-cone gauge. This represents the other main topic of our paper. To answer this question one needs to recall that in the Regge limit the string amplitudes are dominated by the exchange in the t-channel of the states of the leading Regge trajectory, which carry, for a given mass, the highest spin. The effect of the exchange of the whole leading Regge trajectory can be summarized by the exchange of a single effective string state, the Reggeon[3] [24–26]. The covariant dynamics captured by the eikonal operator is precisely the exponentiation of the Reggeon exchange at tree level.

The Reggeon vertex operator considerably simplifies the derivation of the Regge limit of the covariant string amplitudes and gives their high-energy behaviour directly in a neat and factorized form. For instance a four-point function reduces to the product of the Reggeon propagator and the three-point couplings of the external states to the Reggeon. Similarly a two-point function in the Dp-brane background reduces to the product of the Reggeon tadpole and the holomorphic and antiholomorphic part of the three-point couplings of the external states to the Reggeon.

This very specific factorization in the t-channel, which isolates a single coupling to a process-independent intermediate state, is the first major simplification that occurs in the covariant dynamics. The covariant equivalent of the selection rules and of the high-energy amplitudes given by the eikonal operator in the light-cone gauge are then to be found in the properties of the three-point couplings of the covariant states to the Reggeon.

The form of these couplings is restricted to be a contraction of the polarization tensors with the metric, the momentum transfer and the longitudinal polarization vectors of the massive states. The longitudinal polarization vectors appear in the asymptotic behaviour at high energy since their components increase with the energy of the massive state (as in the well-known problem of unitarity-violating amplitudes in the standard model in the absence of or for a very heavy Higgs boson). It is of course essential in order to derive the correct high-energy behaviour of a string amplitude which involves massive states to take the factors of the energy carried by the longitudinal polarizations into proper account.

We will review the derivation of the Reggeon vertex and present a detailed discussion of the evaluation of string amplitudes with massive states in the high-energy limit. The structure of these amplitudes is interesting on its own and may help to clarify the dynamics of the massive string spectrum, whose typical states transform as traceless irreducible tensors of mixed symmetry, the generic representations of the Lorentz group.

The Reggeon vertex will allow us to provide a very simple and fully covariant derivation of the eikonal phase. To achieve this we will choose a basis of physical states adapted to the kinematics of the high-energy scattering, the basis of the DDF operators [27]. Although only the SO(8) symmetry group of the space transverse to the collision axis is manifestly realized in this basis, it has the advantage that all the physical states can be easily enumerated and their couplings to the Reggeon become elementary. We will show that when expressed in this basis the tree-level scattering matrix in the Regge limit can be written in a compact operator form which coincides with the operator $\hat{\delta}(s, b)$. This covariant derivation of the eikonal phase thus leads to the identification of the modes of the free fields $\hat{X}$ in eqs. (1.2) and (1.4) with the bosonic DDF operators.

[3]We refer to this effective string state and to the corresponding vertex operator as the Reggeon, since it describes the high-energy dynamics in the Regge limit [24, 25]. It is also called the Pomeron, especially in the context of the string/gauge duality [26].

The two interpretations of the eikonal operator presented in this paper, either as an operator written in the light-cone gauge or as a covariant operator written in terms of the DDF basis, are of course connected since there is a direct correspondence between the DDF operators of the covariant string and the physical states in the light-cone gauge. The simple matrix elements of the eikonal phase as an operator in the light-cone gauge are indeed precisely the simple couplings of the DDF operators to the Reggeon.

It is interesting to understand in more detail the link between the matrix elements of the eikonal phase and the scattering amplitudes written in the basis of the covariant string states. The former contain only tensors with well-defined transformation properties with respect to the SO(8) symmetry group of the transverse directions while the latter are expressed in terms of ten-dimensional tensors and of physical polarizations characterized by their transformation properties with respect to the little group SO(9). The first step to relate the covariant amplitudes with the matrix elements of the eikonal operator is clearly to decompose the covariant tensors with respect to the transverse SO(8). Each covariant amplitude thus gives rise to several subamplitudes labeled by representations of SO(8) and expressed in terms of tensors living in the space transverse to the collision axis.

The restricted number of linearly independent amplitudes allowed at high energy results in the decoupling of a large number of covariant states. At every mass level, for a given SO(8) representation one can find the linear combinations of SO(8) components of the covariant states that do not couple to the Reggeon and the linear combinations which are produced with a specific form of the amplitude. We will show that, once written in this high-energy basis, the covariant amplitudes precisely match the matrix elements of the eikonal phase.

The discussion in this paper will be based on the string-brane system but the analysis for the case of string-string collisions would be very similar. Our main examples will thus be two-point inelastic disk amplitudes involving massive strings. In most cases we will consider explicitly only transitions from an initial massless state.

A comparison between disk amplitudes and matrix elements of the eikonal operator was attempted in two interesting previous papers [28, 29] for the states of the leading Regge trajectory. The results obtained in these works for the Regge limit of the scattering amplitudes are however correct only when the non-vanishing components of the polarization tensors are restricted to the transverse directions, since the energy factors that enhance the contribution of the longitudinal polarizations were not taken into account.

The rest of the paper is organized as follows. Section 2 is devoted to the kinematics of the high-energy scattering of a string on a collection of N Dp-branes. In section 3 we show how the operator $\hat{\delta}(s,b)$ can be derived by a systematic study of the transition amplitudes at tree level in the light-cone GS formalism, provided the light-cone direction is judiciously chosen. We also anticipate that the result can be rewritten in a covariant form using the bosonic DDF operators. In section 4 we analyse in detail the structure of the inelastic amplitudes implied by the light-cone eikonal phase, illustrating the general case with the transitions from the massless NS-NS sector to the first two massive levels of the superstring.

In section 5 we turn to a fully covariant derivation of the transition amplitudes in the Regge limit. We begin with a review of the Reggeon vertex, where we emphasize the importance of taking into account the longitudinal polarizations when counting the powers of the energy and describe the essential steps necessary to evaluate the high-energy limit of the massive string amplitudes. We then explicitly derive the Regge limit of the inelastic amplitudes from the massless NS-NS sector to the first two massive levels of the superstring. Finally we discuss our covariant derivation of the eikonal phase, which exploits the simple couplings of the DDF operators to the Reggeon.

We give in this section a complete and detailed description of the transitions to the second massive level for two main reasons. The first is that it is at this level that the first examples of holomorphic string states transforming as tensors of mixed symmetry appear. The dynamics of these states, which are generic in the string spectrum, has not been explored much in the past, although they may provide some useful lessons on the symmetries of string theory and on the consistent interactions of fields of higher spin (for more details and references see for instance [30]). The second reason is that these amplitudes neatly display all the important features of the relation between the covariant and the light-cone calculations.

This is the subject of section 6, which is devoted to the comparison between the scattering amplitudes with covariant external states and the matrix elements of the eikonal phase. We shall proceed in both directions. To relate the covariant amplitudes with the matrix elements of the eikonal phase it is sufficient to decompose the covariant tensors with respect to the transverse SO(8) and to perform a change of basis to a high-energy basis of states, characterized by their couplings to the Reggeon. To relate the matrix elements of the eikonal phase with the covariant amplitudes, it is sufficient to identify the linear combination of covariant states that corresponds to a given light-cone state. This problem is well-known and is usually addressed by studying the action of the generators of the full Lorentz group on the light-cone states. We shall address it using the DDF operators which make the connection between the light-cone and the covariant states somewhat more transparent. In section 7 we summarize our results and draw our main conclusions.

Some additional details can be found in a few appendices. In appendix A we state our conventions for describing the physical states of the RNS superstring in the old covariant quantization. In appendix B we collect some formulae for the kinematics of the high energy string-brane scattering in a convenient reference frame. In appendix C we review the DDF operators and discuss their main properties. In appendix D we discuss some properties of the polarization tensors of the massive states, in particular of those of mixed symmetry, and give the explicit expressions for the decomposition of the polarizations of the covariant states in the second massive level with respect to the transverse SO(8).

2 Kinematics of the eikonal scattering

The process that we will analyse in this paper is the scattering of a perturbative closed string state (the probe) from a stack of Dp-branes (the target) at high energy E and fixed momentum transfer t. In this section we then begin by discussing the kinematics which is relevant for a string-brane collision.

The Dp-branes are static and aligned along the first p space-like directions. We indicate the (incoming) momenta of the two states by the SO(1,9) vectors p_r^μ ($r = 1, 2$), with $p_r^2 = -m_r^2$, and their spatial part by $\vec{p}_r$ (without loss of generality we assume $\vec{p}_r$ to be orthogonal to the Dp-branes). We also set $\hat{p}_r = \vec{p}_r/|\vec{p}_r|$. The Regge limit we are interested in corresponds to taking the energies of the two closed strings very large while keeping the momentum transfer $q = p_1 + p_2$ fixed and typically small (corresponding to a large impact parameter for the collision[4]).

The identification of the left and right moving parts of the closed strings absorbed or emitted by the Dp-brane is described by a diagonal matrix $R^\mu{}_\nu$

$$R^\mu{}_\nu = \delta^\mu{}_\nu \,, \quad \mu, \nu = 0, \ldots, p \,, \qquad\qquad R^\mu{}_\nu = -\delta^\mu{}_\nu \,, \quad \mu, \nu = p+1, \ldots, 9 \,. \tag{2.1}$$

The two kinematic (Mandelstam-like) invariants characterizing this process can be chosen as follows

$$t = -(p_1 + p_2)^2 \,, \qquad\qquad s = -\frac{1}{4}(p_1 + Rp_1)^2 = -\frac{1}{4}(p_2 + Rp_2)^2 \equiv E^2 \,, \tag{2.2}$$

where in the second equation we used momentum conservation along the Neumann directions and $E > 0$ will denote, hereafter, the common energy of the incoming and outgoing closed string. Scalar products among the external momenta and the reflection matrix can be expressed in terms of the variables s, t and the masses of the external states

$$2p_1p_2 = -t + m_1^2 + m_2^2 \,, \qquad p_rRp_r = -2s + m_r^2 \,, \qquad 2p_1Rp_2 = 4s + t - m_1^2 - m_2^2 \,. \tag{2.3}$$

The physical polarizations of a massive string state can be described by introducing a basis of nine space-like polarization vectors. For a massive state with a non-zero space-like momentum $\vec{p}_r$ we first define the longitudinal polarization vector v_r

$$v_r^\mu = -\frac{m_r}{|\vec{p}_r|}\hat{t}^\mu + \frac{E_r}{|\vec{p}_r|}\frac{p_r^\mu}{m_r} = \frac{|\vec{p}_r|}{m_r}\hat{t}^\mu + \frac{E_r}{m_r}\hat{p}_r^\mu \,, \qquad v_rp_r = 0 \,, \qquad v_r^2 = 1 \,, \tag{2.4}$$

where $\hat{t}$ is the unit vector in the time direction.[5] The remaining physical polarizations are given by eight unit vectors transverse both to p_r and to v_r.

Since at high energy the longitudinal polarizations of the massive states play a special role, in the following sections we will express the covariant amplitudes in terms of the basis of physical polarizations that can be attached, in the way just described, to each state taking part in the scattering process. In order to do this it is sufficient to decompose every tensor contracted with the polarization of one of the external states along the basis of physical polarizations pertaining to that state. For instance, when the momentum q transferred to the Dp-branes is contracted with the polarization of the second external state, one can use the decomposition

$$q^\rho \equiv p_1^\rho + p_2^\rho = \frac{E(t + m_2^2 - m_1^2)}{2m_2\sqrt{E^2 - m_2^2}}v_2^\rho + \frac{t + m_2^2 - m_1^2}{2m_2^2}p_2^\rho + \bar{q}^\rho \,, \tag{2.5}$$

[4]We recall from [3, 5] that, in eikonal approximation, the typical transverse momentum carried by an individual graviton is of order $\hbar b^{-1}$, the overall momentum transfer in the collision $q \sim \theta E$ being shared among many gravitons.

[5]Some additional formulae for the kinematics in a convenient reference frame are collected in appendix B.

where $\bar{q}$ is perpendicular to both v_2 and p_2. Notice that the difference between q and $\bar{q}$ is small, $(q-\bar{q})^2 \sim t/s$ as it can been seen by using (2.4) and (2.5). We define $q_9 = q - \bar{q}$, since in the frame introduced in appendix B this vector is aligned with the ninth direction.

Similar decompositions hold for the Minkowski metric η and the reflection matrix R

$$\eta^{\rho\sigma} = -\frac{p_2^\rho}{m_2}\frac{p_2^\sigma}{m_2} + \sum_{i=1}^{8} \hat{w}_i^\rho \hat{w}_i^\sigma + v_2^\rho v_2^\sigma \equiv -\frac{p_2^\rho}{m_2}\frac{p_2^\sigma}{m_2} + \hat{\eta}^{\rho\sigma} , \tag{2.6}$$

$$\begin{aligned} R^{\rho\sigma} &= -\frac{2E^2 - m_2^2}{m_2^2}\left(\frac{p_2^\rho}{m_2}\frac{p_2^\sigma}{m_2} + v_2^\rho v_2^\sigma\right) + \sum_{i=1}^{p} \hat{w}_i^\rho \hat{w}_i^\sigma \\ &\quad - \sum_{i=p+1}^{8} \hat{w}_i^\rho \hat{w}_i^\sigma - \frac{2E\sqrt{E^2 - m_2^2}}{m_2^2}\left(\frac{p_2^\rho}{m_2} v^\sigma + v^\rho \frac{p_2^\sigma}{m_2}\right) , \end{aligned} \tag{2.7}$$

where $\hat{w}_i$, with $i = 1, \ldots, 8$, is a set of unit vectors spanning the space perpendicular to p_2 and v_2, while $\hat{\eta}$ is the metric in the space transverse to p_2.

Our main example in this paper will be the inelastic process where a massless string state of momentum p_1 is excited by the tidal forces of the Dp-brane[6] gravitational field and emerges as a massive string state of momentum p_2, with $p_2^2 = -m^2$. Setting $v_2 = v$, $m_1 = 0$ and $m_2 = m$, the high-energy limit of (2.5) reads

$$\bar{q} = q - \frac{m}{2} v \left(1 + \frac{t}{m^2}\right) - \frac{t + m^2}{2m^2} p_2 + \mathcal{O}(1/E^2) , \tag{2.8}$$

and from (2.7) we have

$$p_2 \frac{\eta + R}{2} = -p_1 \frac{\eta + R}{2} = \frac{E^2}{m^2} p_2 + \frac{E\sqrt{E^2 - m^2}}{m} v . \tag{2.9}$$

The polarizations of the massless state can be written in terms of vectors ϵ_k that satisfy both a transversality and a light-cone gauge constraint

$$\epsilon_k p_1 = \epsilon_k e^+ = 0 , \qquad k = 1 \ldots 8 . \tag{2.10}$$

At high energy it is natural to identify the light-cone vectors with the large components of the external momenta in the frame where the D-branes are at rest, thus connecting the gauge choice with physical quantities in the problem. We then define the light-cone vectors as follows

$$\sqrt{2}(e^-)^\mu = \lim_{E_1 \to \infty} \frac{p_1^\mu}{E_1} = -\lim_{E_2 \to \infty} \frac{p_2^\mu}{E_2} , \qquad (e^+)^\mu = (-(e^-)^0, \vec{e}^{\,-}) . \tag{2.11}$$

The direction e^- defines the large relative boost between the perturbative states and the D-branes, while e^+ is the complementary null direction satisfying $e^+e^- = 1$. When the polarizations ϵ_k's are contracted with q and v we have

$$\epsilon_k q = \epsilon_k p_2 \sim \bar{q}^k , \qquad \epsilon_k v \sim -\frac{\bar{q}^k}{m} , \tag{2.12}$$

where we neglected terms of order $\mathcal{O}(1/E^2)$ in the large E expansion. These relations will be useful in section 5 in order to express the covariant amplitudes in terms of tensors with non-trivial components only in the space transverse to the collision axis.

[6] We will limit ourselves to the case $p \leq 6$ so as to have an asymptotically flat region.

3 The eikonal phase from the light-cone GS vertex

The eikonal operator $\exp(2i\hat{\delta}(s,b))$ was introduced in [3, 5] to provide a manifestly unitary description of high-energy string-string collisions below a critical impact parameter b_D at which string excitations due to tidal forces become important, as briefly reviewed in the introduction. In this paper we shall focus on the eikonal operator for the high-energy scattering of a string on a collection of N Dp-branes [19], the process described in the previous section, but a very similar analysis could be performed for the former process.

Exponentiation of the eikonal phase operator $\hat{\delta}(s,b)$ was proven in [3, 5] by considering, at each loop order, the leading terms in a high-energy expansion. It provides the so-called leading eikonal operator, giving, in the present context, the leading term in an expansion in powers of $\frac{R_p}{b}$, where R_p is the scale of the curved Dp-brane background

$$R_p^{7-p} = gN\frac{(2\pi\sqrt{\alpha'})^{7-p}}{(7-p)V_{S^{8-p}}} \,, \qquad V_{S^n} = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})} \,. \tag{3.1}$$

On the other hand, the eikonal operator is supposed to resum all string (α'/b^2) corrections to the leading eikonal phase. One can also argue [7] that the leading eikonal operator gives the correct description of the high-energy scattering for any value of the impact parameter provided the string coupling g is sufficiently weak, i.e. when R_p (or R_s in the case of string-string collisions) is smaller than the string length scale l_s.

A similar exponentiation is conjectured to occur also at non-leading order in $\frac{R_p}{b}$, but the exact structure of $\hat{\delta}(s,b)$ for that case is not known.[7] For this reason, here we will limit our attention to the leading eikonal operator whose phase is given by the operator [19]

$$\hat{\delta}(s,b) = \frac{1}{4E}\int_0^{2\pi} d\sigma \; :\mathcal{A}(s,b+\hat{X}): \,, \qquad \mathcal{A}(s,b) = \int \frac{d^{8-p}\bar{q}}{(2\pi)^{8-p}} \mathrm{e}^{\mathrm{i}\bar{q}b}\mathcal{A}(s,\bar{q}) \,. \tag{3.2}$$

Here $\mathcal{A}(s,b)$ is the Fourier transform in impact parameter space of the disc amplitude in the Regge limit

$$\mathcal{A}(s,\bar{q}) = \frac{R_p^{7-p}\pi^{\frac{9-p}{2}}}{\Gamma\left(\frac{7-p}{2}\right)}\Gamma\left(-\frac{\alpha' t}{4}\right)\mathrm{e}^{-\mathrm{i}\pi\frac{\alpha' t}{4}}(\alpha' s)^{1+\frac{\alpha' t}{4}} \,, \tag{3.3}$$

$\bar{q}^2 = -t$ and $\hat{X}$ is a free bosonic field, without zero modes and evaluated at $\tau = 0$, with components only along the $8-p$ spatial directions transverse to the collision axis and to the brane

$$\hat{X}^i = \mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left(\frac{a_n^i}{n}\mathrm{e}^{\mathrm{i}n\sigma} + \frac{\bar{a}_n^i}{n}e^{-\mathrm{i}n\sigma}\right) \,, \qquad [a_n^i, a_m^j] = n\delta^{ij}\delta_{n+m,0} \,. \tag{3.4}$$

In order to fully understand how inelastic unitarity works, it is necessary to identify precisely the Hilbert space on which this operator acts. The aim of this section is to show

[7] In [19] we have computed the next-to-leading correction to the eikonal phase in the field theory limit. We plan to come back to its modification due to string-size corrections in a forthcoming paper.

that, not unexpectedly, the leading eikonal operator acts on the space of the physical states of the string quantized in the light-cone gauge, with the spatial direction of the light-cone chosen along the direction of the large momentum appearing in the process. The free fields $\hat{X}^i$ in eq. (3.4) should then be identified with the transverse string coordinates and the modes a_n^i, $\bar{a}_n^i$ with the light-cone oscillators. In section 5 we will derive the eikonal phase from the covariant dynamics and we will show that the modes a_n^i are then naturally identified with the bosonic DDF operators. The two interpretations are of course equivalent, as a consequence of the one-to-one correspondence between the DDF operators and the physical states in the light-cone gauge.

The idea of the derivation presented in this section is simple. We will use the light-cone 3-string vertex [31, 32] in the Green-Schwarz formalism, which encodes the interaction among three generic string states, to write an operator that generates all the tree-level amplitudes in the Dp-brane background with two arbitrary string states. In order to do this we first need to specialise the GS vertex to the case where only one of the three states is off-shell, while the remaining two are arbitrary on-shell states.

We then take the high-energy Regge limit, sending the energy E of the two external states to infinity while keeping finite the momentum transfer t, the momentum squared of the state exchanged between the string probe and the target. As we will see, provided the light-cone direction is aligned to the direction of the large momentum, the structure of the GS vertex considerably simplifies in the limit. The final step consists in contracting the off-shell leg with the closed string propagator and with the boundary state that describes the coupling of an arbitrary string state to a collection of Dp-branes.

In order to simplify the analysis in this section, we will consider the limit of large impact parameter b or, equivalently, of vanishing momentum transfer t. This corresponds to exchanging only the graviton between the probe and the target, since, on one hand, only the massless states contribute for small momentum transfers, and, furthermore, states with the highest spin dominate at high energy. We will show that the final result coincides with the phase of the eikonal operator in eq. (3.3) in the limit of large impact parameter b. This is sufficient to identify the oscillators that appear in the eikonal operator as the bosonic string modes in the light-cone gauge. In any case the covariant derivation of the eikonal phase discussed in section 5 will be valid for arbitrary value of the momentum transfer and not only for massless exchanges.

It is also worth noticing that the calculation is split into two independent parts, of which the first - the GS vertex - captures the emission of an off-shell graviton from the high energy probe while the second describes how the graviton propagates and then interacts with the target. The same factorized form will be displayed by the covariant amplitudes, with the GS vertex replaced by the three-point couplings of the external states to the Reggeon and the information about the target carried by the Reggeon tadpole in the given background.

Let us now start our first derivation of the eikonal phase by recalling the GS light-cone vertex. In this approach, the left moving part of the light-cone states is described by a set of bosonic oscillators A_n^i transforming as a vector of SO(8) and a set of fermionic oscillators Q_n^a transforming as a spinor of SO(8). These operators refer to (and depend from) the choice of the light-cone vectors (2.11) but we omit for simplicity a label referring to that choice. The left moving part of the spectrum is obtained by acting with the raising operators A^i_{-n} and Q^a_{-n} on a degenerate massless ground state which we indicate with $|i\rangle$ and $|\dot{a}\rangle$. The first ket represents eight states transforming as a vector of SO(8), while the second one is a spinor of SO(8) (the dot over the spinor index indicates that the vacuum chirality is opposite to that of the fermionic oscillators). In both kets we understand the eigenvalue p of the momentum. A closed string state at level n, carrying momentum p with $p^2 = -4n/\alpha'$, is given by the product of a left moving and a right moving part satisfying the level matching condition.

Following the previous literature on the subject, we indicate with α and $\bar{p}_j$ the projections of the momentum p respectively along e^+ and along the transverse space (spanned by the vectors $\hat{w}_j$)

$$\alpha_r \equiv \sqrt{\frac{\alpha'}{2}}\, 2p^{(r)}e^+ \,, \qquad \bar{p}^{(r)}_j \equiv \sqrt{\frac{\alpha'}{2}} p^{(r)}\hat{w}_j \,, \qquad r = 1, 2, 3 \,, \qquad j = 1, 2, \ldots 8 \,, \tag{3.5}$$

where the label r distinguishes the three different states that appear in the vertex. We shall treat independently the left and the right moving parts and describe the interaction in each sector by means of the vertex introduced in [31]. The full vertex can be written as a vector living in the tensor product of three closed string Hilbert spaces, one for each light-cone state involved in the interaction. The chiral part of the vertex is given by

$$|V_{GS}\rangle = \sqrt{\frac{2}{\alpha'}} \left(P_i - \alpha_1\alpha_2\alpha_3 \frac{n}{\alpha_r} N^r_n A^r_{-n,i} \right) V_b V_f |V_i\rangle + \ldots \quad , \tag{3.6}$$

where we understood the usual delta function imposing momentum conservation along the ten spacetime directions and the dots stand for terms that vanish on-shell. As we shall explain after eq. (3.19), these terms will not be relevant for the derivation of the eikonal phase at large b. The zero-mode structure $|V_i\rangle$ of the vertex is

$$\begin{aligned} |V_i\rangle = & \frac{1}{\alpha_1}|ijj\rangle + \frac{1}{\alpha_2}|jij\rangle + \frac{1}{\alpha_3}|jji\rangle + \frac{\alpha_1 - \alpha_2}{4\alpha_3}|aai\rangle + \frac{\alpha_1 - \alpha_3}{4\alpha_2}|aia\rangle \\ & + \frac{\alpha_2 - \alpha_3}{4\alpha_1}|iaa\rangle + \frac{1}{4}\gamma^{ij}_{ab}\left(|baj\rangle + |bja\rangle + |jba\rangle\right) \,, \end{aligned} \tag{3.7}$$

where the γ's are the SO(8) gamma matrices, all sums over repeated indices are understood and the kets with three indices are just the tensor product of the vector or spinor ground states for each external string state; finally these kets are normalized as in [31]

$${}_r\langle i|j\rangle_r = \delta^{ij} \,, \qquad\qquad {}_r\langle a|b\rangle_r = \frac{2}{\alpha_r}\delta^{ab} \,. \tag{3.8}$$

Most of the complexity of the vertex is in the exponentials V_b and V_f

$$V_b = \exp\left(\frac{1}{2} A^r_{-n,i} N^{rs}_{mn} A^s_{-m,i} + P_i N^r_n A^r_{-n,i} \right) \,, \tag{3.9}$$

$$V_f = \exp\left(\frac{1}{2} Q^r_{-n,a} X^{rs}_{mn} Q^s_{-m,a} - S_a \frac{n}{\alpha_r} N^r_n Q^r_{-n,a} \right) \,. \tag{3.10}$$

The operators P_i and S_a stand for the following combinations of the bosonic and fermionic zero-modes

$$P_i \equiv \sqrt{\frac{\alpha'}{2}} \left(\alpha_r \bar{p}_i^{(r+1)} - \alpha_{r+1} \bar{p}_i^{(r)} \right) , \qquad S_a \equiv \alpha_r Q_{0a}^{(r+1)} - \alpha_{r+1} Q_{0a}^{(r)} , \tag{3.11}$$

which, with the cyclic identification between $r = 4$ and $r = 1$, are independent of the choice of $r = 1, 2, 3$. Finally, the Neumann coefficients encoding the actual value of the various couplings are

$$N_{nm}^{rs} = -\frac{nm\alpha_1\alpha_2\alpha_3}{n\alpha_s + m\alpha_r} N_n^r N_m^s , \qquad X_{nm}^{rs} = \frac{n\alpha_s - m\alpha_r}{2\alpha_r\alpha_s} N_{nm}^{rs} , \tag{3.12}$$

$$N_n^r = -\frac{1}{n\alpha_{r+1}} \begin{pmatrix} -n\frac{\alpha_{r+1}}{\alpha_r} \\ n \end{pmatrix} = \frac{1}{\alpha_r n!} \frac{\Gamma\left(-n\frac{\alpha_{r+1}}{\alpha_r}\right)}{\Gamma\left(-n\frac{\alpha_{r+1}}{\alpha_r} + 1 - n\right)} . \tag{3.13}$$

In the kinematic configuration described at the beginning of this section, the above vertex drastically simplifies if one chooses the direction of the null vectors as in (2.11). In the Regge limit the energy E of the states $r = 1, 3$ is much larger than the momentum exchanged which, for a single graviton exchange, is extremely small and this is reflected in the asymptotic results for the α_i. At leading order in E, we have

$$\alpha_1 \sim \sqrt{\alpha'}\, 2E , \qquad \alpha_2 = -\sqrt{\alpha'} q_9 , \qquad \alpha_3 \sim -\sqrt{\alpha'}\, 2E , \tag{3.14}$$

where $q_9 \sim \mathcal{O}(1/E)$, see eq. (B.6). By using (3.14), this means that we have to take α_1 and α_3 large and α_2 small.[8] Also we set to zero all the oscillators of the string labeled by $r = 2$, since we wish to identify that state with the graviton exchanged between the probe and the target. Then the only surviving Neumann coefficients are N^1 and N^3, which become

$$P_i \sim \sqrt{\frac{\alpha'}{2}} \alpha_1 \bar{p}_i^{(2)} \to -\sqrt{2}\alpha' E \bar{q}_i , \qquad N_n^1 \to \frac{(-1)^{n-1}}{\alpha_1 n} , \qquad N_n^3 \to -\frac{1}{\alpha_1 n} , \tag{3.15}$$

where $\bar{q}$ is the momentum introduced[9] in (2.5). Several other simplifications occur in this limit. We first note that the second and the fifth terms in (3.7) dominate over the others implying that the two energetic external states always share one index. Then, from the second equation in (3.12) we see that X^{13} is subleading with respect to N^{13} and similarly the term proportional to S_a in V_f is subleading with respect to the one proportional to P_i in V_b. This means that all terms containing fermionic oscillators Q_a^q can be neglected in the high-energy scattering we are interested in. Finally there is a hierarchy also within V_b and within the prefactor of the full vertex (3.6): from (3.15) we see that the combination $P_i N^{r=1,3}$ is finite in the large E limit, while N^{13} vanishes in the same limit; similarly in the

[8] The states labelled here by $r = 2$ and $r = 3$ have momenta $-q$ and p_2 respectively according to the notations of section 2.

[9] With a slight abuse of language we use the same symbol to indicate both the momenta of this section and the 10D vector $\bar{q}$ orthogonal to v and p_2 introduced in the previous section, because the two objects are identical in the non-trivial 8D space.

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prefactor, the term P_i dominates over the other one. Thus, instead of the full vertex (3.6), we can use the simplified expression

$$|V_{GS}\rangle \sim \sqrt{\frac{2}{\alpha'}\frac{P_i}{\alpha_2}} \exp\left\{\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{\bar{q}_\ell}{n}\left(A^3_{-n\ell}+(-1)^n A^1_{-n\ell}\right)\right\}\left[|jij\rangle + \frac{\alpha_1-\alpha_3}{4}|aia\rangle\right] . \quad (3.16)$$

We can now easily derive the high-energy scattering amplitude describing the interaction between a string probe and a stack of Dp-branes at large distances. Schematically this process is described by

$$|W\rangle = \frac{\kappa_{10}\tau_p N}{2}\ {}_2\langle B|P\left(\kappa_{10}|V_{GS}\rangle|\widetilde{V_{GS}}\rangle\right) \sim \frac{R_p^{7-p}\pi^{\frac{9-p}{2}}}{\Gamma\left(\frac{7-p}{2}\right)}\ {}_2\langle B_0|\frac{1}{-t}\left(|V_{GS}\rangle|\widetilde{V_{GS}}\rangle\right) , \quad (3.17)$$

where $|B\rangle$ is the GS boundary state [33] for the Dp-branes written in terms of the oscillators in the Hilbert space labelled by $r = 2$ and P is the string propagator. The normalisations on the l.h.s. are the standard gravitational coupling κ_{10}, related to the string length and coupling by $2\kappa_{10}^2 = (2\pi)^7\alpha'^4 g^2$, and the tension of a single Dp brane

$$\tau_p^2 = \frac{\pi}{\kappa_{10}^2}\left(4\pi^2\alpha'\right)^{3-p} . \quad (3.18)$$

The final relation in (3.17) is obtained by implementing the high-energy and large impact parameter limits: the boundary state is truncated to its zero-mode sector $|B_0\rangle$, the string propagator reduces to the field theory one and we can use the simplified version of the 3-string vertex in eq. (3.16). The zero-mode structure gives

$$\frac{2}{\alpha'}\frac{P_h R_{hk} P_k}{\alpha_2^2(-t)} = \frac{\alpha_1^2}{\alpha_2^2}\frac{\vec{q}^2}{t} = -\frac{\alpha_1^2}{\alpha_2^2}\left(1+\frac{q_9^2}{t}\right) \sim -\frac{4E^2}{q_9^2} - \frac{4E^2}{t} , \quad (3.19)$$

where R is the reflection matrix (2.1). Since we are restricting ourselves to the contribution due to the exchange of the massless states, we have first to take the impact parameter to be large and then take the high-energy limit. Then we can neglect the first term of the final expression in (3.19) because it does not have a pole in t. For the same reason, it is possible to neglect the terms that vanish on-shell in (3.6). These contributions are proportional to $\sum_r P_r^-$, where

$$P_r^- = \frac{2}{\alpha_r}\left[\frac{\alpha'}{2}\frac{\vec{p}_r^{\,2}}{2} + \sum_{n=1}^{\infty}\left(A^{(r)}_{-n}A^{(r)}_n + nQ^{(r)}_{-n}Q^{(r)}_n\right)\right] . \quad (3.20)$$

In [31] it was shown that there are terms that vanish on-shell and depend on $\tau_0 \equiv \sum_r \alpha_r \ln|\alpha_r|$, while more recently other terms of this type were proposed in [34] (see for instance, eq. (5.1) of that paper). However, in our calculation only the string $r = 2$ is kept off-shell and in the corresponding Hilbert space we focus only on the massless sector. Thus all the extra terms in (3.6) must be proportional to $-t$, which is the momentum squared of the exchanged graviton. Again those contributions would cancel the pole in (3.20) and thus can be neglected. We then have

$$
\begin{aligned}
|W\rangle \sim \frac{R_p^{7-p}\pi^{\frac{9-p}{2}}}{\Gamma\left(\frac{7-p}{2}\right)}\frac{4E^2}{-t}\exp\left\{\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{\bar{q}_\ell}{n}(A^3_{-n\ell}+(-1)^n A^1_{-n\ell})\right\}\left[|j\rangle_1|j\rangle_3+\frac{\alpha_1}{2}|a\rangle_1|a\rangle_3\right] \\
\times\ \exp\left\{\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{\bar{q}_\ell}{n}(\bar{A}^3_{-n\ell}+(-1)^n \bar{A}^1_{-n\ell})\right\}\left[|\bar{j}\rangle_1|\bar{j}\rangle_3+\frac{\alpha_1}{2}|\bar{a}\rangle_1|\bar{a}\rangle_3\right] . \qquad (3.21)
\end{aligned}
$$

It is more natural to write this result as an operator on a single Hilbert space instead of a product of two kets in two different spaces. This can be done by taking the adjoint of the objects labelled with $r = 1$, i.e. by transforming $|i\rangle_1$ and $|a\rangle_1$ into ${}_3\langle i|$ and ${}_3\langle a|$, and $A^1_{-n\ell}$ into $(-1)^{n+1}A^3_{n\ell}$. After this the square parenthesis in (3.21) becomes just the identity operator on the zero-mode sector, as it follows from (3.8), and we can rewrite the two exponentials in terms of an auxiliary string field

$$
\hat{X}^i(\sigma) = \mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\left(\frac{A_{ni}}{n}\mathrm{e}^{\mathrm{i}n\sigma}+\frac{\bar{A}_{ni}}{n}\mathrm{e}^{-\mathrm{i}n\sigma}\right) . \qquad (3.22)
$$

So we can write (3.21) in an operator form as follows

$$
W(\bar{q}) \sim \int \frac{d\sigma}{2\pi} : \mathrm{e}^{\mathrm{i}\bar{q}\hat{X}(\sigma)} : \left(\frac{R_p^{7-p}\pi^{\frac{9-p}{2}}}{\Gamma\left(\frac{7-p}{2}\right)}\frac{4E^2}{-t}\right) , \qquad (3.23)
$$

where we included an integral over σ which is trivial when the matrix elements of W are taken between closed string states satisfying the level matching condition.

In this derivation we took into account only the contributions due to the graviton exchange and thus the result obtained captures reliably only the first term in the small t expansion. It should be possible to adapt the derivation discussed in [35] and extend our light-cone calculation of the eikonal phase to include the contribution of all the string states exchanged between the string probe and the D-branes. We will not perform this calculation here because, as already mentioned, the covariant derivation of the eikonal phase presented in section 5 does take into account the full string dynamics in the Regge limit. As we will show, the complete result can be obtained by replacing the round parenthesis in (3.23) with the Regge limit of the full elastic tree-level string amplitude $\mathcal{A}$ in eq. (3.3), not just with the graviton pole,

$$
W(\bar{q}) = \mathcal{A}(s,\bar{q}) \int \frac{d\sigma}{2\pi} : \mathrm{e}^{\mathrm{i}\bar{q}\hat{X}(\sigma)} : . \qquad (3.24)
$$

Multiplying W by $\mathrm{e}^{\mathrm{i}\bar{q}b}$ and taking the Fourier transform to write the result in terms of the impact paramenter b instead of the momentum transferred $\bar{q}$ we find

$$
W(b) = \int \frac{d^{8-p}\bar{q}}{(2\pi)^{8-p}}\, W(\bar{q})\mathrm{e}^{\mathrm{i}\bar{q}b} \equiv \int_0^{2\pi} \frac{d\sigma}{2\pi} : \mathcal{A}\left(s, b+\hat{X}\right) : , \qquad (3.25)
$$

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which coincides with eq. (3.2), the phase of the leading eikonal operator for string-brane collisions [19], after including a factor $2E$ for the relativistic normalization of the external states, $W/2E = 2\hat{\delta}$.

This analysis then shows that the exact meaning of the oscillators a_n^i in eq.(3.4) is that of the bosonic oscillators A_n^i [10] of light-cone quantization, with the light-cone axis aligned with the direction of the large momenta. As anticipated at the beginning of this section, it is also possible to recast eq. (3.25) in a covariant form (i.e one that does not depend on using a particular gauge) by exploiting the one-to-one correspondence between the light-cone states and the DDF operators [27], that we review in appendix C. It is precisely in this latter form that the eikonal phase will be given by the covariant derivation discussed in section 5.

Having identified the Hilbert space on which the eikonal operator acts, we can proceed to discuss its main properties and what information it can provide on the string dynamics in the Regge limit, which will be the subject of the next section.

4 High-energy inelastic amplitudes from the eikonal phase

We describe in this section the essential properties of the eikonal operator and what can be learned from its simple structure on the high-energy string dynamics. The discussion will be based on the light-cone RNS superstring so as to make the comparison with the covariant formalism of section 5 more direct. We shall illustrate the general case by deriving from the eikonal operator the inelastic amplitudes for transitions from the massless NS-NS sector to the first two massive levels. We will also describe a method to provide a covariant characterization of the string states excited by the tidal forces during a string-brane collision, using the DDF operators [27] of the NS sector of the superstring [36] to identify the linear combination of covariant string states that corresponds to a given light-cone state.

As shown in the previous section, the bosonic fields $\hat{X}$ in the leading eikonal operator

$$\mathrm{e}^{2\mathrm{i}\hat{\delta}(s,b)}\,, \qquad\qquad \hat{\delta}(s,b) = \frac{1}{4E}\int_0^{2\pi}\frac{d\sigma}{2\pi}\,:\mathcal{A}(s,b+\hat{X}):\,, \tag{4.1}$$

can be interpreted as the transverse string coordinates in a light-cone gauge aligned with the direction of the large momenta. The most interesting feature of this expression is that the string modes appear as a simple shift of the impact parameter b by the string position operator $\hat{X}$. This structure reflects the incoherent scattering of the individual bits of the string when $\alpha' s \gg 1$. Another feature of the eikonal operator is that it contains the light-cone modes of the bosonic fields $\hat{X}$ but not those of the fermionic fields. It is important to appreciate that since we are in the light-cone gauge the shift $b \mapsto b + \hat{X}$ describes

[10]The discussion in this section was based on the Green-Schwarz light-cone formalism but we could have also used the Ramond-Neveu-Schwarz string quantized in the light-cone gauge. Given that the eikonal operator is written only in terms of the bosonic oscillators and that the bosonic fields X of the two formalisms can be identified, we would have obtained exactly the same results.

the dynamics not only of the string excitations polarized along the directions transverse to the collision axis but also of the string excitations polarized along the longitudinal direction. This simple description of the longitudinal polarizations and the absence of the fermionic modes are a consequence of the superconformal invariance of the covariant worldsheet theory.

The matrix elements of the eikonal operator between two closed string states give the high-energy behaviour of the corresponding two-point amplitudes in the Dp-brane background. The physical information contained in these matrix elements can be most clearly displayed by first labeling the light-cone states according to their mass and their representation with respect to the transverse SO(8) and then classifying the independent contractions between the polarization tensors of the external states and the momentum transfer that can appear in the amplitudes.

The eikonal operator is the result of an all-loop resummation of string amplitudes, as it is clear from the fact that the string coupling appears in the exponent. Since in this paper we shall only consider tree-level amplitudes, it is sufficient to study the matrix elements of the eikonal phase $\hat{\delta}(s,b)$, which are related to the string scattering matrix at tree-level by $W(s,b) = 4E\,\hat{\delta}(s,b)$. To derive the scattering amplitudes it is more convenient to work in momentum space and write

$$W(s,\bar{q}) = \mathcal{A}(s,t)\int_0^{2\pi}\frac{d\sigma}{2\pi}\,:\mathrm{e}^{\mathrm{i}\bar{q}\hat{X}}:\equiv \mathcal{A}(s,t)\sum_{n,m=0}^{\infty}\Delta_{n,m}(\bar{q})\,\bar{\Delta}_{n,m}(\bar{q})\,, \tag{4.2}$$

where the operators $\Delta_{n,m}$ generate by definition all the transitions between an initial level m and a final level n. For instance the inelastic transitions from the ground state to the first two massive levels are due to the operators

$$\begin{aligned}\Delta_{1,0} &= -\sqrt{\frac{\alpha'}{2}}\,\bar{q}^i\,A^i_{-1}\,,\\ \Delta_{2,0} &= \frac{\alpha'}{4}\bar{q}^i\bar{q}^j\,A^i_{-1}A^j_{-1} - \sqrt{\frac{\alpha'}{8}}\bar{q}^i\,A^i_{-2}\,.\end{aligned} \tag{4.3}$$

Level by level we organize the light-cone string spectrum in irreducible representations of the transverse SO(8) group. The irreducible SO(8) representations are traceless tensors of type $(n_1, n_2, \ldots, n_r)$ which can be represented by Young diagrams with r rows of length n_i.[11] The polarization of a state in the representation corresponding to the Young diagram $(n_1, n_2, \ldots, n_r)$ will be written as follows

$$\omega^{(n_1,n_2,\ldots,n_r)}_{i_1\ldots i_{n_1};j_1\ldots j_{n_2};\ldots;k_1\ldots k_{n_r}}\,, \tag{4.4}$$

where the semicolons separate groups of indices in the rows of the diagram. The tensor is antisymmetric in the indices belonging to the same column and normalized, $\omega\cdot\omega = 1$. To simplify the notation we do not use the semicolon for totally antisymmetric tensors and often omit the label (1) on vectors. For instance

$$\omega^{(2,1)}_{ij;k}\,, \tag{4.5}$$

[11] Additional details on our conventions for the irreducible SO(n) tensors can be found in appendix D.

SO(8) representation	Matrix element $\langle\omega\vert\Delta_{10}\vert\epsilon\rangle$
$\vert\omega^{(2)}\rangle = \omega^{(2)}_{ij} A^i_{-1} B^j_{-\frac{1}{2}}\vert 0\rangle$	$-\sqrt{\frac{\alpha'}{2}}\,\epsilon^i\,\omega^{(2)}_{ij}\,\bar{q}^j$
$\vert\omega^{(1,1)}\rangle = \omega^{(1,1)}_{ij} A^i_{-1} B^j_{-\frac{1}{2}}\vert 0\rangle$	$\sqrt{\frac{\alpha'}{2}}\,\epsilon^i\,\omega^{(1,1)}_{ij}\,\bar{q}^j$
$\vert\omega^{(0)}\rangle = \frac{1}{\sqrt{8}}\, A^i_{-1} B^i_{-\frac{1}{2}}\vert 0\rangle$	$-\frac{\sqrt{\alpha'}}{4}\,\epsilon\bar{q}$

Table 1. Matrix elements of the eikonal phase for transitions from the massless sector to the first level.

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is a tensor antisymmetric in the couple (i,k) and satisfying the relation

$$\omega^{(2,1)}_{ij;k} + \omega^{(2,1)}_{jk;i} + \omega^{(2,1)}_{ki;j} = 0\ . \tag{4.6}$$

Let us now analyse in detail the inelastic transitions from the massless states of the NS sector. This will be sufficient to understand the general case. In the light-cone gauge the massless NS state is a vector of SO(8)

$$\vert\epsilon\rangle = \epsilon_i B^i_{-1/2}\vert 0\rangle\ . \tag{4.7}$$

Since the eikonal phase contains only the bosonic oscillators, level by level the states that can have a non-vanishing matrix element[12] with the ground state are only those created by the action on the vacuum of one $B_{-1/2}$ and any number of bosonic modes A_{-n}. The relevant states in the first level are 64 and they are displayed together with their matrix elements in table 1.

The remaining 64 NS states of the first level are

$$\vert\omega^{(1,1,1)}\rangle = \frac{1}{\sqrt{6}}\omega^{(1,1,1)}_{ijk} B^i_{-\frac{1}{2}} B^j_{-\frac{1}{2}} B^k_{-\frac{1}{2}}\vert 0\rangle\,, \qquad \vert\omega^{(1)}\rangle = \omega_i B^i_{-\frac{3}{2}}\vert 0\rangle\,, \tag{4.8}$$

and since they contain either more than one mode $B_{-\frac{1}{2}}$ or the higher mode $B_{-\frac{3}{2}}$ their matrix elements with the eikonal operator vanish. This means that the inelastic transitions from the ground state to these states are subleading in energy.

The second level contains 352 states with a non-vanishing inelastic amplitude in the Regge limit. They transform in the following SO(8) representations

$$\yng(3) \qquad \yng(2,1) \qquad 2\times\yng(1) \qquad \yng(2) \qquad \yng(1,1) \qquad \bullet \tag{4.9}$$

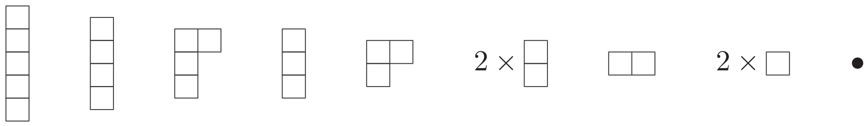
and their explicit form and matrix elements are collected in table 2. We see that the set of the representations that can be reached from the ground state in a high-energy collision comprises the same representations present at level one, created now by the action of the modes A^i_{-2}, together with two new rank three tensors and two vectors. The remaining 800

[12] As already pointed out in footnote 4, at high energy and large impact parameter the eikonal phase is controlled by soft dynamics: hence, the *in* and *out* states can be taken to have the same momentum, which we leave understood in the following equations.

SO(8) representation	Matrix element $\langle\omega\vert\Delta_{20}\vert\epsilon\rangle$
$\vert\omega^{(3)}\rangle = \frac{1}{\sqrt{2}}\omega^{(3)}_{ijk}A^i_{-1}A^j_{-1}B^k_{-\frac{1}{2}}\vert 0\rangle$	$\frac{\alpha'}{\sqrt{8}}\epsilon^i\,\omega^{(3)}_{ijk}\bar{q}^j\bar{q}^k$
$\vert\omega^{(2,1)}\rangle = \sqrt{\frac{2}{3}}\omega^{(2,1)}_{ij;k}A^i_{-1}A^j_{-1}B^k_{-\frac{1}{2}}\vert 0\rangle$	$-\frac{\alpha'}{\sqrt{6}}\epsilon^i\,\omega^{(2,1)}_{ij;k}\bar{q}^j\bar{q}^k$
$\vert\omega^{(2)}\rangle = \frac{1}{\sqrt{2}}\omega^{(2)}_{ij}A^i_{-2}B^j_{-\frac{1}{2}}\vert 0\rangle$	$\frac{\sqrt{\alpha'}}{2}\epsilon^i\,\omega^{(2)}_{ij}\bar{q}^j$
$\vert\omega^{(1,1)}\rangle = \frac{1}{\sqrt{2}}\omega^{(1,1)}_{ij}A^i_{-2}B^j_{-\frac{1}{2}}\vert 0\rangle$	$-\frac{\sqrt{\alpha'}}{2}\epsilon^i\,\omega^{(1,1)}_{ij}\bar{q}^j$
$\vert\omega^{(1)}\rangle = -\frac{\omega_i}{4\sqrt{35}}\left[8A^i_{-1}A^j_{-1}B^j_{-\frac{1}{2}} - A^j_{-1}A^j_{-1}B^i_{-\frac{1}{2}}\right]\vert 0\rangle$	$-\frac{\alpha'}{\sqrt{35}}\left(\epsilon\bar{q}\omega\bar{q} + \frac{\alpha' t}{8}\epsilon\omega\right)$
$\vert\lambda^{(1)}\rangle = \frac{\lambda_i}{4}A^j_{-1}A^j_{-1}B^i_{-\frac{1}{2}}\vert 0\rangle$	$-\frac{\alpha' t}{8}\epsilon\lambda$
$\vert\omega^{(0)}\rangle = \frac{1}{4}A^i_{-2}B^i_{-\frac{1}{2}}\vert 0\rangle$	$-\frac{\sqrt{\alpha'}}{4\sqrt{2}}\epsilon\bar{q}$

Table 2. Matrix elements of the eikonal phase for transitions from the massless sector to the second level.

states of the NS sector transform in the following representations of SO(8)



(4.10)

Their explicit form in terms of the string modes is easily derived and always involves more than one $B_{-\frac{1}{2}}$ or the higher modes $B_{-\frac{3}{2}}$ and $B_{-\frac{5}{2}}$. As in the previous case, this implies that the inelastic amplitudes for the transitions from the ground state to any of these states are subleading in energy.

It is in the second level that we find the first example of a degenerate SO(8) representation, the two vectors $|\omega^{(1)}\rangle$ and $|\lambda^{(1)}\rangle$. The degeneracy of the representation is matched by the presence in the amplitudes of two independent contractions between the momentum transfer and the polarization tensors

$$\epsilon\bar{q}\;\omega\bar{q}\,, \qquad\qquad \epsilon\omega\;t\;. \tag{4.11}$$

The basis chosen for the vectors in table 2 has the property that only the state $|\omega^{(1)}\rangle$ can be produced at large values of the impact parameter since its amplitude contains a term $\epsilon\bar{q}\;\omega\bar{q}$, without powers of t that would cancel the graviton pole.

The allowed representations and couplings for the transitions from the massless sector to the higher levels follow a similar pattern. At level l one obtains all the SO(8) representations and couplings present at level $l-1$ together with two new rank-$(l+1)$ $GL(8)$ tensors of symmetry type $(l+1)$ and $(l,1)$

(4.12)

These two tensors are not traceless and they generate a series of lower rank irreducible SO(8) tensors when the traceless and the trace part are separated. The resulting pattern of

SO(8) representations in the light-cone gauge can be compared with the pattern followed by the covariant SO(9) representations derived in the next section and displayed in eq. (5.19).

In general two string states in the SO(8) representations r_1 and r_2 can be connected by the eikonal phase if they are created by exactly the same fermionic modes and if there is at least one non-vanishing contraction between a subset (including the empty set) of the bosonic indices of their polarization tensors such that the two Young diagrams obtained from the original ones by removing the boxes corresponding to the contracted fermionic and bosonic indices consist of a single row.

Whenever an SO(8) representation r appears in a given level with multiplicity c_r, as it is the case for the two vectors in the second level, there will also be c_r linearly independent contractions of the polarization tensors of the initial and final state and the momentum transfer $\bar{q}$. The possible inequivalent couplings can be read from eq. (4.2) and have a very simple form. In the case of a transition from the massless sector they are for instance

$$\epsilon^k \omega_{ki_1\ldots 1_{n-1}} \bar{q}^{i_1} \ldots \bar{q}^{i_{n-1}} t^a \,, \qquad \epsilon^k \bar{q}_k \omega_{i_1\ldots 1_n} \bar{q}^{i_1} \ldots \bar{q}^{i_n} t^b \,, \qquad a, b \in \mathbb{N} \,. \tag{4.13}$$

The higher powers of t appear whenever we take the trace in a couple of transverse indices to decompose the $GL(8)$ tensors into irreducible SO(8) tensors.

It is worth recalling that, so far, we have only discussed the exponent appearing in the eikonal operator. When the full operator is considered also transitions produced by the repeated action of $\hat{\delta}(s, b)$ should be taken into account. Moreover, since the eikonal operator is the exponential of a normal-ordered operator, to put the exponential itself in normal-ordered form one has to apply the Baker-Campbell-Hausdorff formula. As discussed in detail in [5], this produces the exponential damping of each exclusive transition that is necessary in order to ensure unitarity.

In order to complete our discussion of the properties of the eikonal operator it remains to illustrate one more feature, the fact that not all the light-cone states obtained by acting on an initial state only with the bosonic modes have non-vanishing transition amplitudes in the Regge limit. In fact the opposite is true, as the number of the partitions of the level increases there are more and more linear combinations of light-cone states that decouple at high energy. This is a consequence of the restricted number of inequivalent amplitudes allowed by the eikonal operator, cfr. eq. (4.13).

The simplest example of this decoupling occurs at level three. The inelastic transitions from the ground state are in this case determined by the operator

$$\Delta_{3,0} = -\frac{1}{6}\left(\frac{\alpha'}{2}\right)^{\frac{3}{2}} \bar{q}^i \bar{q}^j \bar{q}^k \, A^i_{-1} A^j_{-1} A^k_{-1} + \frac{1}{4}\frac{\alpha'}{2} \bar{q}^i \bar{q}^j \, A^i_{-1} A^j_{-2} - \frac{1}{3}\sqrt{\frac{\alpha'}{2}} \bar{q}^i \, A^i_{-3} \,. \tag{4.14}$$

There are two linearly independent $(2, 1)$ tensors that can be formed using the states

$$A^i_{-1} A^j_{-2} B^k_{-\frac{1}{2}} |0\rangle \,. \tag{4.15}$$

Consider the following orthonormal basis

$$\begin{aligned} |\omega^{2,1}\rangle_1 &= \frac{1}{\sqrt{2}} \omega^{(2,1)}_{ij;k} \left(A^i_{-1} A^j_{-2} - A^j_{-1} A^i_{-2} \right) B^k_{-\frac{1}{2}} |0\rangle \,, \\ |\omega^{2,1}\rangle_2 &= \frac{1}{\sqrt{6}} \omega^{(2,1)}_{ij;k} \left(A^i_{-1} A^j_{-2} + A^j_{-1} A^i_{-2} \right) B^k_{-\frac{1}{2}} |0\rangle \,. \end{aligned} \tag{4.16}$$

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The inelastic high-energy amplitudes are

$$ {}_1\langle\omega^{2,1}|\Delta_{3,0}|\epsilon\rangle = 0\,, \qquad {}_2\langle\omega^{2,1}|\Delta_{3,0}|\epsilon\rangle = \frac{\alpha'}{2\sqrt{6}}\bar{q}^i\bar{q}^j\omega^{(2,1)}_{ij;k}\epsilon^k\,, \tag{4.17} $$

and therefore only the state $|\omega^{2,1}\rangle_2$ is produced at high energy, consistently with the fact that at level three there is only one independent contraction for tensors of type $(2,1)$.

The examples discussed so far show that the inelastic scattering amplitudes in the Regge limit can be easily derived using the eikonal operator. What is still missing in order to fully characterize a given transition is the knowledge of the linear combinations of covariant states that corresponds to the light-cone states taking part in the collision. One way of deriving this information is to study the behaviour of the light-cone states, which are labeled by their transformation properties under the transverse rotation group SO(8), with respect to the action of the full ten-dimensional Lorentz group. In this section we shall present a different method which is based on the DDF operators [27], reviewed in appendix C, since this method makes the relation between the description of the high-energy dynamics in the light-cone gauge and the covariant description discussed in the next section more direct.

The method works in the following way. Let us assume that all the covariant vertex operators in the string spectrum at level l are known and let us decompose them with respect to the transverse SO(8). Consider now a light-cone state at level l. Using the one-to-one correspondence between the light-cone modes and the DDF operators, we interpret it as a covariant state created by the operators $A_{-n,j}$ and $B_{-r,j}$. Performing then the integrals in eq. (C.2) we obtain an explicit expression for the state in terms of the modes of the worldsheet fields X^μ and ψ^μ. From this expression we can identify the linear combination of SO(8) components of the covariant vertices that corresponds to the given light-cone state. We will discuss some explicit examples of this method at the end of section 6.

5 The eikonal phase from the covariant dynamics

The string eikonal operator gives a remarkably compact description of the high-energy dynamics in the Regge limit. It constrains the class of states that can be excited in a string-string or string-brane collision as well as the form of the interaction vertices. As shown in section 3, the eikonal operator becomes very simple when written in a light-cone gauge adapted to the high-energy process under study. The aim of this section is to describe the covariant dynamics that gives rise to such a simple operator.

From the covariant point of view the dynamics simplifies because in the Regge limit the dominant interactions among strings and D-branes are those mediated by the exchange in the t-channel of the states of the leading Regge trajectory. The exchange of this set of states leads to the characteristic Regge behaviour $(\alpha' s)^{a(t)}$ displayed by the tree-level string scattering amplitudes in the limit of large energy s and fixed momentum transfer t. This universal variation of the amplitudes with a t-dependent power of the energy can be understood as due to the exchange in the t-channel of an effective string state, the Reggeon.

In the Regge limit the integrals that define the tree-level string amplitudes are dominated by the region of small worldsheet distances. This fact has the important consequence that it is possible to describe the emission of a Reggeon in terms of a local vertex operator $\mathcal{V}_R$ [24–26] and to write any scattering amplitude for a process involving two sets of states at large relative boost in a simple factorized form. Consider for instance a four-particle process with particles 1 and 2 in the initial state and particles 3 and 4 in the final state. In the Regge limit, defined as $|t'| = |p_1 + p_3|^2 \ll |s'| = |p_1 + p_2|^2$, this amplitude is

$$\mathcal{A}(s', t') \sim \Pi_R \, C_{13R}(s', t') \, C_{24R}(s', t') \,, \tag{5.1}$$

where Π_R denotes the Reggeon propagator, which is process independent, while C_{13R} and C_{24R} denote the couplings of the Reggeon to the two sets of states, which can be easily derived by evaluating the corresponding three-point functions with the vertex $\mathcal{V}_R$. Similarly, for a two-point amplitude in the background of a collection of Dp-branes, the main focus of this paper, one has

$$\mathcal{A}(s, t) \sim \Pi_R^{D_p} \, C_{12R}(s, t) \, \bar{C}_{12R}(s, t) \ . \tag{5.2}$$

In this case the Reggeon is absorbed by the D-brane and the function $\Pi_R^{D_p}$ represents the Reggeon tadpole in the D-brane background, which is again independent of the external states, while C_{12R} and $\bar{C}_{12R}$ are the holomorphic and the antiholomorphic parts of the three-point couplings of the two external states to the Reggeon.

In the following we will review the construction of the Reggeon vertex operator for the superstring. We will then apply this formalism to derive the Regge limit of string amplitudes with massive states. When evaluating the high-energy limit, it is important to remember that S-matrix elements involving the longitudinal polarizations of a massive particle contain, for purely kinematic reasons, additional powers of the energy, as shown in eq. (2.4). We will explain how these factors of the energy are taken into proper account by the contractions of the Reggeon vertex with the tensor part of the physical vertex operators of the massive string states.

Since the high-energy scattering at fixed momentum transfer is always dominated by the exchange of the Reggeon, all tree-level amplitudes will share the same Π_R or $\Pi_R^{D_p}$ and therefore the same energy dependence. The different inelastic processes will be distinguished by the couplings of the Reggeon to the external states which can be expressed as the contraction of the external polarizations with a tensor formed using the metric, the momentum transfer q and the longitudinal vector v. We will give explicit examples of these tensors for all the transitions from the massless NS sector to the first two massive levels of the superstring.

Once the high-energy dynamics has been formulated in terms of Reggeon exchange, it is possible to provide a simple covariant derivation of the eikonal phase. This derivation shows in a clear and direct way how the operator $\hat{\delta}(s, b)$ emerges from the full covariant dynamics. The covariant equivalent of the eikonal phase is the operator that associates to every couple of physical string states their three-point function with the Reggeon. While these couplings have a somewhat complex form when the external states are chosen to transform in irreducible representations of the covariant little group, they become elementary if one chooses a basis adapted to the kinematics of the high-energy scattering, the basis of the DDF operators. In this basis the tree-level scattering matrix in the Regge limit takes a compact form that, as we will show, coincides with $\hat{\delta}(s,b)$ in eq. (3.2), with the modes of the fields $\hat{X}$ identified with the bosonic DDF operators.

5.1 The Reggeon vertex operator

The general form of a string amplitude in the Regge limit displayed in eqs. (5.1)–(5.2) can be derived by first noticing that the worldsheet integral is dominated by the region of small distances [24–26] and then by analysing the energy dependence of the relevant factorization channel. The relation between the Regge limit and the limit of short worldsheet distances is a consequence of the fact that the essential dependence of a string amplitude on the external momenta is contained in the correlation function of the exponential part ($\mathrm{e}^{\mathrm{i}kX}$) of each vertex operator. For a four-particle process the dependence of this correlator on the Mandelstam variables is for instance

$$\mathrm{e}^{-\frac{\alpha' t'}{4}\ln|z|^2-\frac{\alpha' s'}{4}\ln|1-z|^2}\,, \qquad\qquad z=\frac{z_{13}z_{24}}{z_{14}z_{23}}\,. \tag{5.3}$$

When s' is large and t' finite the integral over z is dominated by a neighbourhood of the origin of size $\alpha' s'|z|^2 \leq 1$.

The class of states that contribute at leading order in energy can then be easily identified and shown to be process independent by factorizing the amplitude in the t-channel on a complete set of string states. The contribution of the states of level l are suppressed by a factor of $|z|^{2l}$, where z is the parameter controlling the factorization in the t-channel. As $|z|^2$ is of order $1/(\alpha' s)$ in the Regge limit, we require that the increase in the power of z with the mass level is compensated by an equal increase in the power of s in the three-point couplings. The final step is to show that the sum over the intermediate states together with the integral over the worldsheet are equivalent to the insertion of a single local operator, the Reggeon, in the three-point couplings with the external states.

Let us focus on the case of a two-point amplitude on the disc, but the result, as it should be clear from the discussion, is general [26]. We start from the following representation of the disc amplitude with two closed strings[13]

$$A_{12}=\frac{\alpha'}{8\pi}\int d^2z\,\langle 0|\mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)}\,\mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)}\,z^{L_0-1}\bar{z}^{L_0-1}|D_p\rangle\,, \tag{5.4}$$

where on the left there is the $\mathrm{SL}(2,\mathbb{C})$-invariant vacuum state and on the right the boundary state corresponding to the collection of N Dp-branes.[14] As usual, the vertex operators $\mathcal{V}$ factorize in a holomorphic and an antiholomorphic component and, following a standard

[13] In our conventions $d^2z = 2d\mathrm{Re}z\,d\mathrm{Im}z$.

[14] We consider here explicitly the case of a transition between states of the NS sector. The discussion for the transitions between states of the R sector is similar, with the external vertices both in the $-\frac{1}{2}$ picture. Transitions between the NS sector and the R sector are subleading in energy since at level l the highest spin that can be exchanged in the t-channel in the R sector is equal to $l+\frac{1}{2}$ and then lower than the spin $l+1$ of the states of the leading Regge trajectory.

notation, the superscripts on the vertices refer to the picture of each component. For a closed string state at level l labeled by the left and right SO(9) representations $(S,\bar{S})$ we write

$$\mathcal{V}_{(S,\bar{S})} = \frac{\kappa_{10}}{2\pi}\, V_S\, \bar{V}_{\bar{S}} = \frac{\kappa_{10}}{2\pi}\, \epsilon_{\mu_1\ldots\mu_r}\, V_S^{\mu_1\ldots\mu_r} \quad \bar{\epsilon}_{\nu_1\ldots\nu_s}\, V_{\bar{S}}^{\nu_1\ldots\nu_s} \,, \tag{5.5}$$

where we extracted from the normalization of the vertex operators an explicit factor $\frac{\kappa_{10}}{2\pi}$. The polarization tensors are normalized as $\epsilon_{\mu_1\ldots\mu_r}\epsilon^{\mu_1\ldots\mu_r} = \bar{\epsilon}_{\mu_1\ldots\mu_r}\bar{\epsilon}^{\mu_1\ldots\mu_r} = 1$ and the overall normalization of the vertices is fixed by the requirement that the corresponding state has unit norm. Finally left-right symmetric states will be simply written as $\mathcal{V}_S$.

Let us insert a complete set of string states in the picture $(-1,-1)$, labeled by a triple $(l, n_l, \bar{n}_l)$ where l is the level and n_l, $\bar{n}_l$ the set of all the remaining left and right quantum numbers needed to identify the state, including its momentum. The disc amplitude then becomes a sum of products of three-point couplings on the sphere and one-point functions on the disc with Dp-brane boundary conditions

$$A_{12} = \frac{\alpha'}{8\pi} \sum_{(l,n_l,\bar{n}_l)} \int d^2z\ (z\bar{z})^{l-1-\frac{\alpha' t}{4}} \left\langle \mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)}\, \mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)}\, \mathcal{V}^{(-1,-1)}_{l,n_l,\bar{n}_l} \right\rangle_{\mathcal{S}} \left\langle \mathcal{V}^{(-1,-1)}_{l,n_l,\bar{n}_l} \right\rangle_{D_p} . \tag{5.6}$$

In order to identify the states that give the leading contribution in the Regge limit we need to determine the scaling with the energy of the three-point correlators. A state can contribute to the Regge limit only if the power of the energy in its three-point couplings with the external states matches its level. The only operators with this property are the operator $\mathcal{Q}_l$ whose holomorphic part is

$$Q_l = \frac{1}{\sqrt{l!}}\psi^+ \left(\mathrm{i}\sqrt{\frac{2}{\alpha'}}\partial X^+ \right)^l \mathrm{e}^{\mathrm{i}kX} \,, \qquad l \geq 0 \,, \tag{5.7}$$

where $X(z,\bar{z}) = X(z) + \bar{X}(\bar{z})$ and $X^+ = e^+X$, $\psi^+ = e^+\psi$ are the components of the string fields along the light-cone directions defined in eq. (2.11). In order to see this note that there are only two possible sources for the factors of the energy in the three-point couplings. The first one is related to the contractions of the operators $\partial^r X^+$ in the vertex of the intermediate state with the exponential part of the external states

$$\sqrt{\frac{2}{\alpha'}} \frac{\partial^r X^+(z)}{\sqrt{\alpha'}E}\, \mathrm{e}^{\mathrm{i}p_1X_1(w)} \sim \partial_z^{r-1}\left(\frac{1}{z-w}\right) \mathrm{e}^{\mathrm{i}p_1X_1(w)} \,, \tag{5.8}$$

$$\sqrt{\frac{2}{\alpha'}} \frac{\partial^r X^+(z)}{\sqrt{\alpha'}E}\, \mathrm{e}^{\mathrm{i}p_2X_2(w)} \sim -\partial_z^{r-1}\left(\frac{1}{z-w}\right) \mathrm{e}^{\mathrm{i}p_2X_2(w)} \,. \tag{5.9}$$

The second possibility follows from the contractions of the operators $\partial^r X^+$ or $\partial^r\psi^+$ with the tensor part of the external states

$$\mathrm{i}\sqrt{\frac{2}{\alpha'}} \frac{\partial^r X^+(z)}{\sqrt{\alpha'}E}\, \mathrm{i}\sqrt{\frac{2}{\alpha'}} \frac{\partial^s X^\rho(w)}{\sqrt{\alpha'}E} \sim \sqrt{\frac{2}{\alpha'}}\frac{v^\rho}{m} \partial_z^{r-1}\partial_w^{s-1}\left(\frac{1}{z-w}\right)^2 \,, \tag{5.10}$$

$$\frac{\partial^r\psi^+(z)}{\sqrt{\alpha'}E}\partial^s\psi^\rho(w) \sim \sqrt{\frac{2}{\alpha'}}\frac{v^\rho}{m}\partial_z^r\partial_w^s\left(\frac{1}{z-w}\right) \,, \tag{5.11}$$

where v is the longitudinal polarization and m the mass of the state. This second class of contractions is very important for all the massive states of the string spectrum. Since in both cases one can only obtain one factor of E, all the operators that contain derivatives of X^+ higher than the first or fermionic fields in addition to ψ^+ give a subleading contribution, because the power of z in eq. (5.6) grows faster than the power of the energy in the coupling. The result of this analysis is that the class of operators that contribute to the Regge limit is universal, independent on the external states and is given by eq. (5.7).

Since all other contractions yield subleading contributions, we can then restrict the sum over the intermediate states in eq. (5.6) to the states of the leading Regge trajectory with all the polarization indices in the x^+ direction. Notice that the amplitude for their exchange has the largest power of the energy compatible with helicity conservation. For this class of states the one-point function on the disc in (5.6) is independent of l

$$\langle \mathcal{Q}_l \rangle_{D_p} = \frac{\tau_p \kappa_{10} N}{2} = \frac{1}{\kappa_{10}} \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} R_p^{7-p} \,, \tag{5.12}$$

where R_p is the scale of the Dp-brane background, given in eq. (3.1), and τ_p the tension of a single Dp-brane, given in eq. (3.18). In conclusion, we find

$$\mathcal{A}_{12} = \frac{\alpha'}{8\pi\kappa_{10}} \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} R_p^{7-p} \sum_{l=0}^{\infty} \int d^2 z \; (z\bar{z})^{l-2-\frac{\alpha' t}{4}} \; \left\langle \mathcal{V}^{(-1,-1)}_{(S_1,\bar{S}_1)} \mathcal{V}^{(0,0)}_{(S_2,\bar{S}_2)} \mathcal{Q}_l \right\rangle_{\mathcal{S}} \,. \tag{5.13}$$

It is precisely the inclusion of this infinite series of operators that gives rise to the Regge behaviour of the amplitude $(\alpha' s)^{\alpha(t)}$ [24–26], while the leading singularity in t accounts only for the graviton pole. The final step in the derivation is to express the previous sum over the three-point couplings of the intermediate states as a single three-point coupling with an effective vertex operator. This can be achieved by first summing the exponential series and then performing the integral over z. The result can be written as follows

$$\mathcal{A}_{12} = \Pi_R^{D_p}(t) \left\langle V_{S_1}^{(-1)} V_{S_2}^{(0)} V_R^{(-1)} \right\rangle \left\langle \bar{V}_{\bar{S}_1}^{(-1)} \bar{V}_{\bar{S}_2}^{(0)} \bar{V}_R^{(-1)} \right\rangle \,, \tag{5.14}$$

where the Reggeon vertex for the superstring in the -1 picture is

$$V_R^{(-1)} = \frac{\psi^+}{\sqrt{\alpha'}E} \left(\sqrt{\frac{2}{\alpha'}} \frac{\mathrm{i}\partial X^+}{\sqrt{\alpha'}E} \right)^{\frac{\alpha' t}{4}} \mathrm{e}^{-\mathrm{i}qX} \,. \tag{5.15}$$

A factor of $\sqrt{\alpha'}E$ for each string field along e^+ has been included in V_R so as to make the sphere correlator in eq. (5.14) energy independent. Then the dependence on the energy, which is universal, is included in the Reggeon tadpole which, in our case, coincides with the function $\mathcal{A}$ introduced in eq. (3.3)

$$\Pi_R^{D_p} = \frac{\pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})} R_p^{7-p} \, \Gamma\left(-\frac{\alpha' t}{4}\right) \mathrm{e}^{-\mathrm{i}\pi\frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}} \,. \tag{5.16}$$

In writing eq. (5.14), we also extracted for convenience from the three-point couplings the factor that gives the normalization of the sphere amplitude setting

$$\langle \mathcal{O} \rangle \equiv \frac{\alpha' \kappa_{10}^2}{32\pi^3} \langle \mathcal{O} \rangle_{\mathcal{S}} \,. \tag{5.17}$$

As anticipated, the two-point function on the disc has been expressed as the product of a single three-point coupling on the sphere and the Reggeon tadpole on the brane.

The OPEs with the energy-momentum tensor and with the supercurrent show that the Reggeon vertex (5.15), although it carries an off-shell momentum q, behaves as a superconformal primary of dimension one half at high energies [25] (i.e. terms in the OPEs violating the superconformal invariance exist, but are suppressed by powers of $1/E$). This guarantees that, for our purposes, the correlation functions of the Reggeon with physical string states are well-defined and in particular invariant under global superconformal transformations. Also we can use the OPE with the supercurrent to obtain the Reggeon vertex in the 0 picture

$$V_R^{(0)} = \left[-\frac{2}{\alpha'} \frac{\partial X^+ \partial X^+}{\alpha' E^2} - \mathrm{i} q \psi \frac{\psi^+ \partial X^+}{\alpha' E^2} - \frac{\alpha' t}{4} \frac{\psi^+ \partial \psi^+}{\alpha' E^2} \right] \left(\sqrt{\frac{2}{\alpha'}} \frac{\mathrm{i} \partial X^+}{\sqrt{\alpha'} E} \right)^{\frac{\alpha' t}{4} - 1} \mathrm{e}^{-\mathrm{i} q X_L} . \quad (5.18)$$

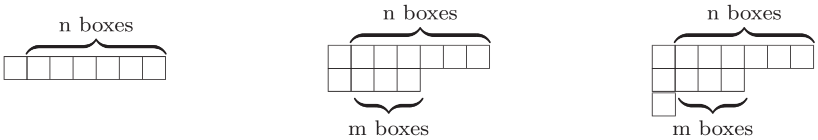
In explicit calculations of the 3-point correlators in (5.14), it is convenient to move the superghost charge from the Reggeon to the $(S_2, \bar{S}_2)$ vertex so as to have both external states in the simpler $(-1, -1)$ picture and use eq. (5.18) for the Reggeon vertex.

The factorization of the scattering amplitudes and the form of the Reggeon vertex result in some simple selection rules for the inelastic transitions in the Regge limit, which represent the covariant version of the selection rules discussed in section 4 for the eikonal phase. In order to identify the SO(9) representations that can be related by the exchange of the Reggeon and the structure of the corresponding couplings, let us first analyse the possible contractions of the fermionic fields. The first term in eq. (5.18) can only connect components of the vertex operators of the external states with the same number of fermionic fields, while the second and the third also components containing two additional fermionic fields. When the variation in the number of fermionic fields is due to the second term, one of the two additional indices of the polarization tensor must be longitudinal. When it is due to the third term, both additional indices must be longitudinal. In order to have a non-vanishing three-point coupling all the remaining fermionic fields must be contracted among the two external states, which results in the contraction of the corresponding indices of the two polarization tensors.

As for the bosonic fields, there are three types of contractions. They can be contracted between the two external states, giving the contraction of the indices of the external polarizations, or with the exponentials, giving the contraction of the polarizations with the momentum transferred q, or with the ∂X^+ operators in the Reggeon vertex, giving the contraction of the polarization with the longitudinal vector v.

As a result, two external states transforming in the SO(9) representations with Young diagrams Y_1 and Y_2 can be connected by Reggeon exchange if the following two conditions are fulfilled. First of all they should respect the selection rules described above for the fermionic indices. Moreover there should be at least one non-vanishing contraction between a subset (including the empty set) of the bosonic indices of their polarizations such that, when the boxes corresponding to the contracted bosonic indices and to the fermionic indices are removed from the original diagrams, the two resulting Young diagrams have at most two rows.

Let us apply these selection rules to the case of the inelastic transitions from the ground state to the massive states at level l. The exchange of a Reggeon in the t-channel allows to excite only the following three classes of SO(9) representations with one, two or three rows



(5.19)

For the first class there are no constraints on the indices of the massive polarization tensor and the integer n can take all the values from zero to l. For the second and the third class the integers n and m, with $n \geq m$, can take all the values from zero to $l-1$ such that $n+m \leq l-1$. The polarization index in the third row and all the polarization indices in the second row, except the first one, must be longitudinal. The values of the integers n and m that actually appear at level l depends on the physical spectrum of that level, which can be read for instance from the generating function discussed in [37].

As a final remark, let us emphasize that the discussion above is limited to tree level and therefore to the exchange of a single Reggeon. The leading high-energy contributions of the higher-genus surfaces which are resummed by the eikonal operator take into account the exchange of an arbitrary number of Reggeons. As already mentioned in section 4 this has two main consequences: on one hand one should take into account transitions mediated by multiple Reggeon exchange and therefore given by the repeated application of the previous rules, on the other hand every exclusive transition will be exponentially suppressed as required by unitarity.

5.2 Regge limit of the covariant string amplitudes with massive states

We now show how the Reggeon vertex considerably simplifies the evaluation of the Regge limit of the string amplitudes and discuss how to take into proper account the factors of the energy carried by the longitudinal polarizations of the massive states.

In the evaluation of the three-point couplings it is convenient to take both external states in the -1 picture and the Reggeon in the 0 picture. We write the three-point couplings as the product of an holomorphic and anti-holomorphic part

$$C_{(S_1,\bar{S}_1),(S_2,\bar{S}_2),(R,R)} = \frac{\kappa_{10}^3}{(2\pi)^3} C_{S_1,S_2,R} \bar{C}_{\bar{S}_1,\bar{S}_2,R} \,, \tag{5.20}$$

where

$$C_{S_1,S_2,R} = \langle V_{S_1}(p_1) V_{S_2}(p_2) V_R \rangle \,, \tag{5.21}$$

and similarly for the anti-holomorphic part. We will also isolate the tensor structure which is specific to a given process by writing

$$C_{S_1,S_2,R} = \epsilon_{\mu_1\ldots\mu_r} \zeta_{\nu_1\ldots\nu_s} T^{\mu_1\ldots\mu_r;\nu_1\ldots\nu_s}_{S_1,S_2,R} \quad . \tag{5.22}$$

Here and in the following the polarizations of the initial and of the final states will be denoted by ϵ and ζ, respectively. For compactness, we will often write directly the tensors $T_{S_1,S_2,R}$ rather then the couplings $C_{S_1,S_2,R}$. However, the contraction with the external polarizations will always be understood and we will make use of momentum conservation, of the mass-shell condition and of the physical state conditions to simplify our equations.

In the following we will derive the inelastic amplitudes for transitions from the NS-NS ground state to the first two massive levels of the string spectrum, which is equivalent to evaluating the holomorphic couplings with the Reggeon of one massless and one massive state. Once we know the holomorphic couplings, we know all the amplitudes for a given level due to the factorized form of the scattering amplitudes

$$\mathcal{A}_{g,(S,\bar{S})} = \Pi_R^{D_p} C_{g,S,R} \bar{C}_{g,\bar{S},R} \ . \tag{5.23}$$

Let us begin with the elastic amplitude. The massless vertex in the -1 picture is

$$V_g^{\mu}(p) = \psi^{\mu} \mathrm{e}^{\mathrm{i}pX} \, \mathrm{e}^{-\varphi} \ , \tag{5.24}$$

and we need to evaluate

$$T_{g,g,R}^{\mu;\rho} = \langle V_g^{\mu}(p_1) V_g^{\rho}(p_2) V_R \rangle \ . \tag{5.25}$$

In this case only the contractions of the Reggeon with the exponential part of the vertex operators give a non-vanishing result and we find

$$T_{g,g,R}^{\mu;\rho} = \eta^{\mu\rho} \ . \tag{5.26}$$

The Regge limit of the elastic amplitude is therefore

$$\mathcal{A}_{g,g} = \Pi_R^{D_p} C_{g,g,R} \bar{C}_{g,g,R} = \epsilon_{\mu\nu} \zeta^{\mu\nu} \frac{\pi^{\frac{9-p}{2}} R^{7-p}}{\Gamma\left(\frac{7-p}{2}\right)} \mathrm{e}^{-\mathrm{i}\pi \frac{\alpha' t}{4}} (\alpha' s)^{1+\frac{\alpha' t}{4}} \Gamma\left(-\frac{\alpha' t}{4}\right) \ . \tag{5.27}$$

The three-point functions for transitions to the first and to the second level will display a more interesting structure. They are given by tensors $T_{g,S,R}$ formed from the metric η and the vectors q and v and reflecting the symmetry properties of the external states. It is worth noticing that the covariant amplitudes depend on the masses of the external states also in the high energy limit, as a consequence of the contractions in (5.10) and (5.11) and the kinematic relations in (2.8) and (2.12). The masses can be eliminated and the results expressed only in terms of α' using the relation $m^2 = \frac{4l}{\alpha'}$ for states at level l.

Due to the physical state conditions, the tensors $T_{g,S,R}$ can be rewritten in terms of SO(8) tensors living in the space orthogonal to the collision axis. If the polarizations of the external states are similarly decomposed into irreducible SO(8) components, each covariant amplitude generates a class of amplitudes labeled by SO(8) representations. It is after this decomposition that the selection rules and the simple structure of the high-energy limit of the S-matrix elements predicted by the eikonal operator and discussed in section 4 become evident, as we will explain in detail in section 6.

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5.2.1 Transitions to the first level

The simplest inelastic transitions are those from the massless sector to the first massive level. The NS sector of the first massive level contains 128 physical states: a rank-2 traceless totally symmetric tensor S_2 (44 components) and a rank-3 totally antisymmetric tensor A_3 (84 components). The corresponding vertex operators in the -1 picture are

$$
\begin{aligned}
V_{S_2}^{\rho\alpha} &= \mathrm{i}\sqrt{\frac{2}{\alpha'}}\ \psi^\rho \partial X^\alpha \mathrm{e}^{\mathrm{i}pX}\ \mathrm{e}^{-\varphi}\,, \\
V_{A_3}^{\rho\alpha\gamma} &= \frac{1}{\sqrt{3!}}\ \psi^\rho\psi^\alpha\psi^\gamma \mathrm{e}^{\mathrm{i}pX}\ \mathrm{e}^{-\varphi}\,.
\end{aligned} \tag{5.28}
$$

Let us evaluate

$$
T_{g,S_2,R}^{\mu;\rho\alpha} = \langle V_g^\mu(p_1) V_{S_2}^{\rho\alpha}(p_2) V_R\rangle\,. \tag{5.29}
$$

Since this is the first example where it is important to take into account the longitudinal polarizations of a massive state, we describe this calculation in some detail. In this case not only the contraction of the Reggeon with the exponential part of the vertex operators are relevant at high energy but also the contractions with their tensor part. We find[15]

$$
T_{g,S_2,R}^{\mu;\rho\alpha} = -\sqrt{\frac{\alpha'}{2}}\left[\eta^{\mu\rho}\left(q^\alpha - \frac{2}{\alpha'}\left(1+\frac{\alpha' t}{4}\right)\frac{v^\alpha}{m}\right) + \frac{q^\mu}{m}v^\rho\left(q^\alpha - \frac{t}{2m}v^\alpha\right)\right]\,. \tag{5.30}
$$

The first term in the previous expression comes from the first term in the Reggeon vertex, contracted both with the exponentials and with the ∂X^ρ of the massive state. The second term comes from the second term in the Reggeon vertex, the $q\psi$ part contracted with the massless vertex and the $\psi^+\partial X^+$ part with the massive vertex. The third term in the Reggeon vertex clearly does not contribute to this transition. We can write

$$
T_{g,S_2,R}^{\mu;\rho\alpha} = -\sqrt{\frac{\alpha'}{2}}\left[\left(\eta^{\mu\rho} + \frac{q^\mu}{m}v^\rho\right)\left(q^\alpha - \frac{m}{2}\left(1+\frac{\alpha' t}{4}\right)v^\alpha\right) + \frac{q^\mu}{2}v^\rho v^\alpha\right]\,. \tag{5.31}
$$

We can simplify further this expression defining

$$
Q^\alpha = q^\alpha - \frac{t}{2m}v^\alpha\,, \qquad \bar{q}^\alpha = q^\alpha - \frac{m}{2}\left(1+\frac{t}{m^2}\right)v^\alpha\,, \qquad \delta_\perp^{\mu\rho} = \eta^{\mu\rho} + \frac{q^\mu}{m}v^\rho\,. \tag{5.32}
$$

When contracted with the physical polarization tensor of the massive state, the vector $\bar{q}$ coincides with the momentum transfer in the directions transverse to the collision axis, see eq. (2.8); similarly eq. (2.12) ensures that the tensor $\delta_\perp$ reduces to the Kronecker delta in the transverse directions when the first index is contracted with the massless polarization and the second with the massive one. In the following we will use the same symbol $\delta_\perp$ for both the Kronecker delta in the transverse directions and the tensor defined in eq. (5.32). By using the symmetry properties of the polarization S_2, the final result can be written as[16]

$$
T_{g,S_2,R}^{\mu;\rho\alpha} = -\sqrt{\frac{\alpha'}{2}}\left[\delta_\perp^{\mu(\rho}\bar{q}^{\alpha)} + \frac{q^\mu}{2}v^\rho v^\alpha\right]\,. \tag{5.33}
$$

[15] In this subsection m indicates the mass of the string states at the first massive level, $m^2 = 4/\alpha'$.

[16] Groups of indices are symmetrized or antisymmetrized always with weight one. For instance $A^{(\rho}B^{\alpha)} = \frac{1}{2}\left(A^\rho B^\alpha + A^\alpha B^\rho\right)$ and $A^{[\rho}B^{\alpha]} = \frac{1}{2}\left(A^\rho B^\alpha - A^\alpha B^\rho\right)$.

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The three-point coupling with the state A_3 can be derived following the same steps. In this case only the second term in the Reggeon vertex contributes and the result is

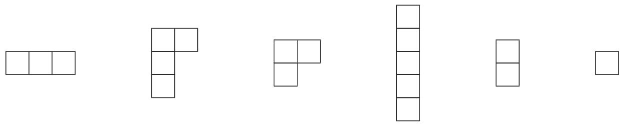
$$T_{g,A_3,R}^{\mu;\rho\alpha\gamma} = \frac{\sqrt{6}}{m}\eta^{\mu[\rho}q^{\alpha}v^{\gamma]}\,, \tag{5.34}$$

which can be rewritten, using the antisymmetry in the indices ρ, α, γ , as follows

$$T_{g,A_3,R}^{\mu;\rho\alpha\gamma} = \frac{\sqrt{6}}{m}\delta_{\perp}^{\mu[\rho}\bar{q}^{\alpha}v^{\gamma]}\,. \tag{5.35}$$

5.2.2 Transitions to the second level

The NS sector of the second massive level contains 1152 bosonic physical states in the following six irreducible representations of SO(9)


$$. \tag{5.36}$$

The corresponding normalized vertex operators in the -1 picture are [38][17]

$$\begin{aligned}
V_Z^{\rho\alpha\gamma} &= -\frac{1}{\sqrt{2}}\frac{2}{\alpha'}\partial X^{\rho}\partial X^{\alpha}\psi^{\gamma}\,\mathrm{e}^{\mathrm{i}pX}\,\mathrm{e}^{-\varphi}\,,\\
V_Y^{\rho\alpha\gamma\omega} &= \mathrm{i}\sqrt{\frac{3}{8}}\sqrt{\frac{2}{\alpha'}}\partial X^{(\rho}\psi^{\alpha)}\psi^{\gamma}\psi^{\omega}\,\mathrm{e}^{\mathrm{i}pX}\,\mathrm{e}^{-\varphi}\,,\\
V_U^{\rho\alpha\gamma} &= -\frac{1}{\sqrt{6}}\left[\frac{2}{\alpha'}\partial X^{\rho}\partial X^{\alpha}\psi^{\gamma} + 2\partial\psi^{(\rho}\psi^{\alpha)}\psi^{\gamma}\right]\,\mathrm{e}^{\mathrm{i}pX}\,\mathrm{e}^{-\varphi}\,,\\
V_X^{\rho\alpha\gamma\omega\xi} &= \frac{1}{\sqrt{5!}}\,\psi^{\rho}\psi^{\alpha}\psi^{\gamma}\psi^{\omega}\psi^{\xi}\,\mathrm{e}^{\mathrm{i}pX}\,\mathrm{e}^{-\varphi}\,,\\
V_V^{\gamma\omega} &= \frac{\mathrm{i}}{5}\sqrt{\frac{2}{7}}\sqrt{\frac{2}{\alpha'}}\left[\left(\hat{\eta}_{\rho\alpha}\partial X^{\rho}\psi^{\alpha}\right)\psi^{\gamma}\psi^{\omega} - \frac{7}{2}\left(\partial^2 X^{\gamma}\psi^{\omega} - 2\partial X^{\gamma}\partial\psi^{\omega}\right)\right]\,\mathrm{e}^{\mathrm{i}pX}\,\mathrm{e}^{-\varphi}\,,\\
V_W^{\gamma} &= \frac{1}{8\sqrt{22}}\hat{\eta}_{\alpha\rho}\left[-\frac{2}{\alpha'}\partial X^{\gamma}\partial X^{\alpha}\psi^{\rho} + 5\frac{2}{\alpha'}\partial X^{\rho}\partial X^{\alpha}\psi^{\gamma} + 11\partial\psi^{\rho}\psi^{\alpha}\psi^{\gamma}\right]\,\mathrm{e}^{\mathrm{i}pX}\,\mathrm{e}^{-\varphi}\,,
\end{aligned} \tag{5.37}$$

where $\hat{\eta}$ is the transverse metric as defined in (2.6) and the overall normalizations have been chosen so as to normalize to one the string states $\zeta^{Z}_{\rho\alpha\gamma}V_Z^{\rho\alpha\gamma}$, $\zeta^{Y}_{\rho\alpha;\gamma;\omega}V_Y^{\rho\alpha\gamma\omega}$ etc. In this subsection m indicates the mass of the string states at the second massive level, $m^2 = 8/\alpha'$.

In the second level we find the first examples of tensors of mixed symmetry in the holomorphic superstring spectrum, namely the states Y and U. The vertex V_Y describes a tensor of type $(2,1,1)$ which can be obtained from a generic tensor by first symmetrizing in $\rho\alpha$ and then antisymmetrizing in $\rho\gamma\omega$. Similarly the vertex V_U describes a tensor of type $(2,1)$ which can be obtained from a generic tensor by first symmetrizing in $\rho\alpha$ and then antisymmetrizing in $\rho\gamma$. In the list above we showed explicitly only the symmetrization in $\rho\alpha$ of the tensor part of the physical vertices, since the antisymmetrization can be left understood due to our convention of choosing the polarization tensors as manifestly antisymmetric in the column indices.

[17]In the list given in [38] there are typos in the vertices for U and V.

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The three-point couplings with one massless state, one state S of the second level and the Reggeon can be derived following the same steps as for the first level and are completely characterized by the tensors $T_{g,S,R}$. In the following we will display for each state of the second level the corresponding tensor, first in terms of the metric η and the momentum transfer q and then in terms of the transverse tensors $\delta_\perp$ and $\bar{q}$ defined in eq. (5.32).

According to the selection rules for the transitions mediated by Reggeon exchange, the totally antisymmetric rank-five tensor X is not produced at high energy in transitions from the ground state. The coupling for the state Z is

$$\sqrt{2}T^{\mu;\rho\alpha\gamma}_{g,Z,R} = \left(\eta^{\mu\gamma} + \frac{q^\mu}{m}v^\gamma\right)\left[\frac{\alpha'}{2}Q^\rho Q^\alpha - \frac{t}{2m^2}v^\rho v^\alpha\right] - \frac{1}{m}\eta^{\mu\gamma}\left(Q^\rho v^\alpha + Q^\alpha v^\rho\right) , \tag{5.38}$$

and in terms of $\delta_\perp$ and $\bar{q}$

$$\begin{aligned}\sqrt{2}T^{\mu;\rho\alpha\gamma}_{g,Z,R} &= \delta_\perp^{\mu\gamma}\left[\frac{\alpha'}{2}\bar{q}^\rho\bar{q}^\alpha + \frac{1}{m}\left(\bar{q}^\rho v^\alpha + \bar{q}^\alpha v^\rho\right) - \frac{t}{2m^2}v^\rho v^\alpha\right]\\ &\quad+\frac{\bar{q}^\mu}{m^2}\left(\bar{q}^\rho v^\alpha + \bar{q}^\alpha v^\rho\right)v^\gamma + \frac{\bar{q}^\mu}{m}v^\rho v^\alpha v^\gamma \ .\end{aligned} \tag{5.39}$$

The coupling for the state Y is

$$\sqrt{\frac{8}{3}}T^{\mu;\rho\alpha\gamma\omega}_{g,Y,R} = \sqrt{\frac{\alpha'}{2}}\frac{3}{m}\eta^{\mu[\alpha}q^\omega v^{\gamma]}Q^\rho + \sqrt{\frac{\alpha'}{2}}\frac{3}{m}\eta^{\mu[\rho}q^\omega v^{\gamma]}Q^\alpha , \tag{5.40}$$

which, after using the symmetry properties of $\zeta^Y_{\rho\alpha;\gamma;\omega}$, can be written in terms of $\delta_\perp$ and $\bar{q}$

$$\sqrt{\frac{8}{3}}T^{\mu;\rho\alpha\gamma\omega}_{g,Y,R} = \sqrt{\frac{\alpha'}{2}}\frac{4}{m}\delta_\perp^{\mu[\rho}\bar{q}^\omega v^{\gamma]}\bar{q}^\alpha + \sqrt{\frac{\alpha'}{2}}\,2\,\delta_\perp^{\mu[\rho}\bar{q}^\omega v^{\gamma]}v^\alpha \ . \tag{5.41}$$

The coupling for the state U is

$$\begin{aligned}\sqrt{6}T^{\mu;\rho\alpha\gamma}_{g,U,R} &= \left(\eta^{\mu\gamma} + \frac{q^\mu}{m}v^\gamma\right)\left[\frac{\alpha'}{2}Q^\rho Q^\alpha - v^\rho v^\alpha\frac{t}{2m^2}\right] - \eta^{\mu\gamma}\frac{1}{m}\left(Q^\rho v^\alpha + Q^\alpha v^\rho\right)\\ &\quad-\frac{1}{m}\eta^{\mu\rho}\left(q^\alpha v^\gamma - q^\gamma v^\alpha\right) + \frac{1}{m}\eta^{\mu\alpha}\left(q^\gamma v^\rho - q^\rho v^\gamma\right)\\ &\quad+\frac{t}{2m^2}\left(\eta^{\mu\alpha}v^\gamma v^\rho + \eta^{\mu\rho}v^\gamma v^\alpha - 2\eta^{\mu\gamma}v^\alpha v^\rho\right) ,\end{aligned} \tag{5.42}$$

where the first line comes from the first term in the vertex V_U and the following two lines from the second term. Using the symmetry properties of $\zeta^U_{\rho\alpha;\gamma}$ and $m^2 = 8/\alpha'$, this result can be written as follows in terms of $\delta_\perp$ and $\bar{q}$

$$\begin{aligned}\sqrt{6}T^{\mu;\rho\alpha\gamma}_{g,U,R} &= \frac{\alpha'}{4}\bar{q}^\alpha\left(\delta_\perp^{\mu\gamma}\bar{q}^\rho - \delta_\perp^{\mu\rho}\bar{q}^\gamma\right) + \frac{\alpha' m}{8}\bar{q}^\alpha\left(\delta_\perp^{\mu\gamma}v^\rho - \delta_\perp^{\mu\rho}v^\gamma\right) + \frac{\alpha' m}{8}\delta_\perp^{\mu\alpha}\left(\bar{q}^\gamma v^\rho - \bar{q}^\rho v^\gamma\right)\\ &\quad-\frac{\alpha'}{4}\bar{q}^\mu v^\alpha\left(\bar{q}^\gamma v^\rho - \bar{q}^\rho v^\gamma\right) - \frac{\alpha' t}{8}v^\alpha\left(\delta_\perp^{\mu\gamma}v^\rho - \delta_\perp^{\mu\rho}v^\gamma\right) \ .\end{aligned} \tag{5.43}$$

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Finally let us give the results for the last two vertices in Eq (5.37)

$$\sqrt{\frac{7}{2}}\,T^{\mu;\gamma\omega}_{g,V,R} = \frac{1}{2}\sqrt{\frac{\alpha'}{2}}\eta^{\mu[\gamma}q^{\omega]} - \frac{1}{m}\sqrt{\frac{2}{\alpha'}}\eta^{\mu[\gamma}v^{\omega]}\left(1+\frac{\alpha' t}{2}\right) + \frac{\alpha'}{8}q^\mu q^{[\gamma}v^{\omega]}\,, \tag{5.44}$$

$$\begin{aligned}8\sqrt{22}\,T^{\mu;\gamma}_{g,W,R} = q^\mu\left(\frac{5\alpha'}{8}Q^\gamma - \frac{v^\gamma}{2m}\right) + 5\left[\frac{9t}{2m^2}\eta^{\mu\gamma} - \frac{q^\mu v^\gamma}{m}\left(1-\frac{9t}{2m^2}\right)\right]\\ + \frac{11t}{2m^2}\left(\eta^{\mu\gamma}+\frac{q^\mu v^\gamma}{m}\right) + 11\frac{q^\mu}{m^2}\left(Q^\gamma + \frac{m}{2}v^\gamma\right)\,,\end{aligned} \tag{5.45}$$

and in terms of the transverse quantities $\delta_\perp$ and $\bar q$

$$\sqrt{\frac{7}{2}}\,T^{\mu;\gamma\omega}_{g,V,R} = \frac{\alpha'}{4}\bar q^\mu \bar q^{[\gamma}v^{\omega]} + \frac{1}{2}\sqrt{\frac{\alpha'}{2}}\delta_\perp^{\mu[\gamma}\bar q^{\omega]} - \frac{3\alpha' t}{16}\delta_\perp^{\mu[\gamma}v^{\omega]}\,, \tag{5.46}$$

$$8\sqrt{22}\,T^{\mu;\gamma}_{g,W,R} = 2\alpha'\bar q^\mu \bar q^\gamma + \frac{28t}{m^2}\delta_\perp^{\mu\gamma} + \frac{8}{m}\bar q^\mu v^\gamma\ . \tag{5.47}$$

5.3 Covariant derivation of the eikonal phase

As we have seen, the high-energy behaviour of the tree-level two-point amplitudes between arbitrary string states in the background of a collection of Dp-branes can be described in a simple and elegant way in terms of Reggeon exchange. We can summarize this dynamical information in an operator $W_R(s,q)$ defined as follows

$$W_R(s,q) = \Pi_R^{D_p}\sum_{i,\bar\imath,j,\bar\jmath}|S_i,S_{\bar\imath}\rangle\, C_{(S_i,S_{\bar\imath}),(S_j,S_{\bar\jmath}),R}\langle S_j,S_{\bar\jmath}|\,, \tag{5.48}$$

where the sum is over the complete physical spectrum of the string.

As discussed in the previous subsection, if we choose a basis of physical states which transform in irreducible representations of the covariant little group, the couplings $C_{S_i,S_j,R}$ can be expressed in terms of tensors $T_{S_i,S_j,R}$. These tensors, which are written in terms of the metric, the momentum transfer and the longitudinal polarization vector, have an interesting structure reflecting the symmetry properties of the external states. However their explicit form becomes more and more complex as the mass of the string states increases and in order to evaluate them one also needs to know level by level the covariant spectrum of the string.

There is another choice of basis, the basis provided by the DDF operators [27], which allows to easily enumerate the physical states[18] and in which the couplings to the Reggeon become elementary, although in this basis only the SO(8) symmetry group of the space transverse to the collision axis is manifestly realized. The simple couplings of the DDF operators to the Reggeon make it possible to represent the formal sum in eq. (5.48) in a compact operator form. We will show that the result coincides with $\hat\delta(s,b)$, the phase of the eikonal operator. Using the Reggeon vertex we can then derive the eikonal phase from the full covariant dynamics and identify the modes of the string coordinates X in $\hat\delta(s,b)$ with the bosonic DDF operators.

[18] Although in [36, 41, 42] only the DDF operators for the NS sector were explicitly discussed, the construction can be easily extended to the R sector by acting with the operators defined in eq.(C.2) on the Ramond ground state and taking $r\in\mathbb{Z}$ in the definition of the fermionic operators.

In order to derive the eikonal phase, we need to consider transitions between two generic NS states or two generic R states, the transitions from the NS to the R sector being subleading in energy. Let us describe explicitly the couplings of two NS states to the Reggeon. As reviewed in appendix C, a generic physical state of the NS sector at level l and carrying momentum p is generated by the action of a finite collection of bosonic and fermionic modes

$$\{A_{-n_a,i_a}, B_{-r_b,i_b}\}_{a,b}\,, \qquad l = N_a + N_b - \frac{1}{2} \qquad N_a = \sum_a n_a\,, \quad N_b = \sum_b r_b\,, \tag{5.49}$$

on a vacuum state carrying a momentum $p_T = p + (N_a + N_b)\,k$ which satisfies $p_T^2 = \frac{2}{\alpha'}$. The null vector k has the property that $kp_T = \frac{2}{\alpha'}$. Finally the GSO projection requires that N_b is a half-integer.

To represent the external states S_1 and S_2 in a string-brane collision we need two collections of modes labeled by two couples on indices (a_1, b_1) and (a_2, b_2), two momenta $p_{T,i}$ and two null vectors k_i, $i = 1, 2$. We choose the two null vectors proportional to e^+, $k_i = \lambda_i e^+$. In the high-energy limit they can be identified since the boost parameters coincide, $\lambda_i \sim \frac{\sqrt{2}}{\alpha' E}$. The transverse polarization vectors of the initial and final state will be denoted by ϵ and ζ, respectively. Consider now the coupling of two generic DDF vertices of the NS sector to the Reggeon

$$C_{S_1,S_2,R} = \langle V_{S_1}^{(-1)}(z_1) V_{S_2}^{(-1)}(z_2) V_R^{(0)}(z_3)\rangle\ . \tag{5.50}$$

Since the DDF operators do not contain X^- and ψ^-, in evaluating the correlation functions one can simplify the vertices of both the Reggeon and the external states by retaining only the following terms

$$\begin{aligned} V_R^{(0)}(z) &\sim \left(\sqrt{\frac{2}{\alpha'}}\frac{\mathrm{i}\partial X^+(z)}{\sqrt{\alpha'}E}\right)^{\frac{\alpha' t}{4}+1} \mathrm{e}^{-\mathrm{i}qX(z)}\,, \\ A_{-n,j}(z) &\sim -\mathrm{i}\sqrt{\frac{2}{\alpha'}}\oint_z dw\ (\epsilon_j)_\mu \partial X^\mu \mathrm{e}^{-\mathrm{i}nkX}\,, \\ B_{-r,j}(z) &\sim -\mathrm{i}\oint_z dw\ (\epsilon_j)_\mu \psi^\mu (\mathrm{i}k\partial X)^{\frac{1}{2}} \mathrm{e}^{-\mathrm{i}rkX}\ . \end{aligned} \tag{5.51}$$

From eq. (5.51) we see that since there are no transverse fermions in the Reggeon vertex, for a non-vanishing result the two states must be created by the action of exactly the same fermionic DDF operators. It is easy to verify that the contour integrals in the definition of the $B_{-r,j}$ reduce to the ones appearing in the scalar product between the two states and then give simply the contraction between the corresponding polarization vectors, $\epsilon\zeta$. The action of the operator $W_R(s,q)$ on the fermionic modes $B_{-r,j}$ then reduces to the action of the identity operator.

As for the bosonic modes, it is always possible to contract all of them with the exponential part of the Reggeon vertex, with the only constraint imposed by momentum conservation in the x^+ direction. The contractions with the Reggeon of the transverse bosons $\epsilon_{j_a}\partial X$ in the definition of the modes A_{-n_a,j_a} give

$$\sqrt{\frac{\alpha'}{2}}\frac{\epsilon_{j_{a_1}}\bar{q}}{w_1 - z_3}\,\mathrm{e}^{-\mathrm{i}n_{a_1}kX(w_1)}\,, \qquad\qquad -\sqrt{\frac{\alpha'}{2}}\frac{\zeta_{j_{a_2}}\bar{q}}{w_2 - z_3}\,\mathrm{e}^{\mathrm{i}n_{a_2}kX(w_2)}\,, \tag{5.52}$$

respectively for the initial and the final state. Evaluating the contour integrals the dependence on the z_i disappears, as required since all the vertices are conformal primaries. The result is then simply to replace every bosonic mode in the initial state with $-\sqrt{\frac{\alpha'}{2}}\epsilon_{i_{a_1}}\bar{q}$ and every bosonic mode in the final state with $\sqrt{\frac{\alpha'}{2}}\zeta_{j_{a_2}}\bar{q}$. A similar substitution holds for the antiholomorphic part and one should also impose the constraint

$$\sum n_{a_1} - \sum n_{a_2} - \sum \bar{n}_{\bar{a}_1} + \sum \bar{n}_{\bar{a}_2} = 0\,. \tag{5.53}$$

Finally, there are also contractions between the DDF operators whenever some of the bosonic modes that create the initial and the final state coincide. As it was the case for the fermionic modes, these contractions simply give the scalar product between the bosonic DDF operators, $\epsilon\zeta$, and clearly respect the constraint in eq. (5.53).

A similar analysis can be performed for the transitions between two generic states of the Ramond sector, taking the external states in the $-\frac{1}{2}$ picture and the Reggeon vertex in the -1 picture, and it leads to the same conclusions.

The action of $W_R(s,q)$ in the DDF basis is therefore extremely simple. It acts like the identity on the fermionic modes $B_{-r,i}$. Its action on the bosonic modes is non-trivial and consists in replacing any number of $A_{-n,i}$ satisfying the condition in eq. (5.53) with the momentum transfer $\bar{q}$. This operator can then be written as follows in terms of the DDF operators

$$W_R(s,q) = \mathcal{A}(s,t)\int_0^{2\pi}\frac{d\sigma}{2\pi}\,:\mathrm{e}^{\mathrm{i}\bar{q}X}:= \mathcal{A}(s,t)\int_0^{2\pi}\frac{d\sigma}{2\pi}\,\mathrm{e}^{\mathrm{i}\bar{q}X^<}\mathrm{e}^{\mathrm{i}\bar{q}X^>}\mathrm{e}^{\mathrm{i}\bar{q}\bar{X}^<}\mathrm{e}^{\mathrm{i}\bar{q}\bar{X}^>}\,, \tag{5.54}$$

where the integral over σ enforces the constraint in eq. (5.53) and

$$X^> = \mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{A_n}{n}\mathrm{e}^{\mathrm{i}n\sigma}\,, \qquad X^< = -\mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{A_{-n}}{n}\mathrm{e}^{-\mathrm{i}n\sigma}\,, \tag{5.55}$$

with similar expressions for $\bar{X}$. This is precisely the operator $\hat{\delta}(s,\bar{q})$ in (4.2) which, as this derivation shows, can be interpreted as a covariant operator expressed in the basis of the DDF operators.

6 The eikonal operator and the covariant amplitudes

The derivation of the eikonal phase given in the previous section shows that, although the individual amplitudes with covariant external states may have a somewhat complex tensor structure, all the dynamical information, including the longitudinally polarized states, can be summarized in a simple operator. The aim of this section is to understand in detail the relation between the scattering amplitudes of the covariant states and the matrix elements of the eikonal operator.

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The first step is to decompose the covariant tensors with respect to the SO(8) group used to define the light-cone gauge, namely the symmetry group of the space transverse to the collision axis. Each SO(9) representation then breaks into several SO(8) components whose couplings to the Reggeon have the simple structure of the basic couplings in eq. (4.13). We shall call the set of all the SO(8) components of the covariant physical states at a given level the covariant basis for that level.

To show that the covariant and the light-cone calculations precisely match it is necessary to perform a change of basis from the covariant basis to a high-energy basis. The latter is characterized by the following dynamical property: every subspace of states transforming in the same representation of SO(8) is decomposed into two orthogonal sets, containing states having a vanishing or a non-vanishing coupling to the Reggeon respectively.

Let us define the high-energy basis more precisely. Generically a given SO(8) representation r will appear in the decomposition of several covariant states and with several linearly independent couplings. If d_r is the degeneracy of the representation and c_r the number of the inequivalent couplings, there will be $d_r - c_r$ linear combinations of elements of the covariant basis that decouple at high energy. The states in the c_r-dimensional orthogonal subspace are those with a non-vanishing coupling to the Reggeon. For each SO(8) representation we choose an orthonormal basis in the $d_r - c_r$ and c_r-dimensional subspaces. The set of all these states forms the high-energy basis.

The covariant and the high-energy basis are related by a unitary transformation. Once rewritten in the high-energy basis, the dynamical information derived from the covariant amplitudes reproduces the list of SO(8) representations that can be excited at high-energy and the corresponding transition amplitudes derived from the eikonal phase. Since the high-energy basis is given explicitly in terms of linear combinations of SO(8) components of covariant string states, we obtain in this way a covariant characterization of the light-cone states that are created by the action of the eikonal phase on a given initial state.

We will perform in detail the comparison between the eikonal phase and the covariant amplitudes for the inelastic transitions from the massless NS-NS sector to the first two massive levels of the superstring, finding perfect agreement with the results derived in section 4. It is interesting to note that this precise agreement is found only after the explicit dependence of the covariant amplitudes on the masses of the external states is rewritten in terms of α' using the relation $m^2 = \frac{4l}{\alpha'}$ for states at level l.

The comparison is straightforward for the first level, since in this case there is no degeneracy in the SO(8) representations that appear in the decomposition of the covariant states and therefore the covariant and the high-energy basis coincide. The first degenerate representations appear in the second level, which will be studied in detail since it clearly exemplifies all the generic features of the relation between the covariant amplitudes and the matrix elements of the eikonal operator.

When decomposing the SO(9) representations with respect to the transverse SO(8), we will write the polarization tensors of the covariant states as products of longitudinal vectors v and of SO(8) polarization tensors ω having non vanishing components only in the transverse directions, $v^\alpha \omega_{\alpha\ldots} = 0$. We will use the following notation for the component of the polarization of a covariant state S transforming in the $(n_1, n_2, \ldots, n_r)$ representation of SO(8)

$$\zeta^{S,(n_1,n_2,\dots,n_r)} \, . \tag{6.1}$$

The decomposition with respect to the transverse SO(8) followed by the unitary transformation just described simplifies the form of the covariant amplitudes and reduces them to the matrix elements of the eikonal phase. It is also interesting to proceed in the opposite direction and to provide a covariant characterization of the light-cone states of a given matrix element. This is the well-known problem of finding the covariant representations which can be formed by combining the physical states in the light-cone gauge. As discussed at the end of section 4, a possible way to derive this information is to first identify a light-cone state with a covariant state using the DDF operators and then to find to which linear combination of SO(8) components $\zeta^{S,(n_1,n_2,\dots,n_r)}$ of physical SO(9) states it corresponds. This procedure will be illustrated with a few specific examples at the end of this section.

6.1 SO(8) decomposition of the covariant amplitudes: first massive level

The two SO(9) representations in the first level have the following decomposition with respect to the transverse SO(8)

$$\square\!\square \;\mapsto\; \square\!\square + \square + \bullet \, , \tag{6.2}$$

and

$$\begin{array}{c}\square\\[-4pt]\square\\[-4pt]\square\end{array} \;\mapsto\; \begin{array}{c}\square\\[-4pt]\square\\[-4pt]\square\end{array} + \begin{array}{c}\square\\[-4pt]\square\end{array} \, . \tag{6.3}$$

We decompose the polarization tensors into irreducible SO(8) components. For S_2 we have

$$\zeta^{S_2,(2)}_{\rho\alpha} = \omega^{(2)}_{\rho\alpha} \, , \qquad \zeta^{S_2,(1)}_{\rho\alpha} = \sqrt{2}\,\omega_{(\rho}v_{\alpha)} \, , \qquad \zeta^{S_2,(0)}_{\rho\alpha} = \frac{1}{3\sqrt{8}}\left(-\delta^{\rho\alpha}_{\perp} + 8v^{\rho}v^{\alpha}\right) \, , \tag{6.4}$$

and for A_3 we find

$$\zeta^{A_3,(1,1,1)}_{\rho\alpha\gamma} = \omega^{(1,1,1)}_{\rho\alpha\gamma} \, , \qquad \zeta^{A_3,(1,1)}_{\rho\alpha\gamma} = \sqrt{3}\,\omega^{(1,1)}_{[\rho\alpha}v_{\gamma]} \, . \tag{6.5}$$

The decompositions above can be easily derived by writing a tensor with the correct symmetry properties using ω, v and the Kronecker delta in the transverse directions $\delta_{\perp}$ and requiring that it is traceless and normalized. The couplings of the covariant states to the Reggeon are given by the tensors in eq. (5.33) and eq. (5.35). When contracted with the SO(8) polarizations we find

$$\begin{aligned}
T^{\mu,\rho\alpha}_{g,S_2,R}\zeta^{S_2,(2)}_{\rho\alpha} &= -\sqrt{\frac{\alpha'}{2}}\delta^{\mu\rho}_{\perp}\omega^{(2)}_{\rho\alpha}\bar{q}^{\alpha} \, , \\
T^{\mu,\rho\alpha}_{g,S_2,R}\zeta^{S_2,(1)}_{\rho\alpha} &= 0 \, , \\
T^{\mu,\rho\alpha}_{g,S_2,R}\zeta^{S_2,(0)}_{\rho\alpha} &= -\frac{\sqrt{\alpha'}}{4}\bar{q}^{\mu} \, ,
\end{aligned} \tag{6.6}$$

and

$$\begin{aligned}
T^{\mu,\rho\alpha\gamma}_{g,A_3,R}\zeta^{A_3,(1,1,1)}_{\rho\alpha\gamma} &= 0 \, , \\
T^{\mu,\rho\alpha\gamma}_{g,A_3,R}\zeta^{A_3,(1,1)}_{\rho\alpha\gamma} &= \sqrt{\frac{\alpha'}{2}}\delta^{\mu\rho}_{\perp}\omega^{(1,1)}_{\rho\alpha}\bar{q}^{\alpha} \, .
\end{aligned} \tag{6.7}$$

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Note that the vector and the rank-3 antisymmetric tensor of SO(8) decouple at high energy. The remaining representations and couplings are in perfect agreement with those derived using the eikonal operator and given in table 1. The states in the first massive level that can be excited in the high energy scattering of a massless NS-NS state on a stack of Dp-branes are therefore

$$S = \zeta^{\mathcal{S}_2,(2)} S_2 \,, \qquad A = \zeta^{\mathcal{A}_3,(1,1)} A_3 \,, \qquad I = \zeta^{\mathcal{S}_2,(0)} S_2 \,, \tag{6.8}$$

for a total of 64 degrees of freedom. The covariant and the high-energy basis coincide for the first level.

6.2 SO(8) decomposition of the covariant amplitudes: second massive level

We perform now the same analysis for the second level, since it neatly displays all the generic features of the relation between the covariant amplitudes and the matrix elements of the eikonal phase. The explicit expressions for the polarization tensors that correspond to the various SO(8) components of the covariant states of the second level are listed in appendix D.

The state Z in the (3) of SO(9) has the following decomposition with respect to SO(8)

$$\ydiagram{3} \;\mapsto\; \ydiagram{3} + \ydiagram{2} + \ydiagram{1} + \bullet \,. \tag{6.9}$$

The coupling of the covariant state Z to the Reggeon is given by the tensor in eq. (5.39). Contracting this tensor with the polarizations in the list in eq. (D.5) we find

$$\begin{aligned}
T^{\mu;\rho\alpha\gamma}_{g,Z,R}\,\zeta^{Z,(3)}_{\rho\alpha\gamma} &= \frac{\alpha'}{\sqrt{8}}\delta^{\mu\gamma}_{\perp}\omega^{(3)}_{\rho\alpha\gamma}\bar{q}^{\rho}\bar{q}^{\alpha} \,, \\
T^{\mu;\rho\alpha\gamma}_{g,Z,R}\,\zeta^{Z,(2)}_{\rho\alpha\gamma} &= \frac{2}{\sqrt{6}m}\delta^{\mu\gamma}_{\perp}\omega^{(2)}_{\gamma\rho}\bar{q}^{\rho} = \sqrt{\frac{\alpha'}{12}}\delta^{\mu\gamma}_{\perp}\omega^{(2)}_{\gamma\rho}\bar{q}^{\rho} \,, \\
T^{\mu;\rho\alpha\gamma}_{g,Z,R}\,\zeta^{Z,(1)}_{\rho\alpha\gamma} &= \frac{1}{2\sqrt{165}}\left(-\frac{3}{2}\alpha'\bar{q}^{\mu}\bar{q}\omega + \frac{\alpha' t}{8}\omega^{\mu}\right) \,, \\
T^{\mu;\rho\alpha\gamma}_{g,Z,R}\,\zeta^{Z,(0)}_{\rho\alpha\gamma} &= -\frac{1}{2\sqrt{11}}\frac{3}{m}\bar{q}^{\mu} \,.
\end{aligned} \tag{6.10}$$

Note the presence of two inequivalent couplings for the vector component. The state Y in the $(2,1,1)$ of SO(9) has the following decomposition with respect to SO(8)

$$\ydiagram{2,1,1} \;\mapsto\; \ydiagram{2,1,1} + \ydiagram{1,1,1} + \ydiagram{2,1} + \ydiagram{1,1} \,. \tag{6.11}$$

From the coupling of the covariant state Y to the Reggeon in eq. (5.41) we derive the

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following couplings for its SO(8) components

$$
\begin{aligned}
\sqrt{\frac{8}{3}} T_{g,Y,R}^{\mu;\rho\alpha\gamma\omega} \zeta_{\rho\alpha;\gamma;\omega}^{Y,(2,1,1)} &= 0\,, \\
\sqrt{\frac{8}{3}} T_{g,Y,R}^{\mu;\rho\alpha\gamma\omega} \zeta_{\rho\alpha;\gamma;\omega}^{Y,(1,1,1)} &= 0\,, \\
\sqrt{\frac{8}{3}} T_{g,Y,R}^{\mu;\rho\alpha\gamma\omega} \zeta_{\rho\alpha;\gamma;\omega}^{Y,(2,1)} &= \sqrt{\frac{\alpha'}{2}} \frac{4}{\sqrt{3}m} \delta_{\perp}^{\mu\gamma} \omega_{\rho\alpha;\gamma}^{(2,1)} \bar{q}^{\alpha} \bar{q}^{\rho} = \frac{\alpha'}{\sqrt{3}} \delta_{\perp}^{\mu\gamma} \omega_{\rho\alpha;\gamma}^{(2,1)} \bar{q}^{\alpha} \bar{q}^{\rho}\,, \\
\sqrt{\frac{8}{3}} T_{g,Y,R}^{\mu;\rho\alpha\gamma\omega} \zeta_{\rho\alpha;\gamma;\omega}^{Y,(1,1)} &= -\sqrt{\frac{4\alpha'}{7}} \delta_{\perp}^{\mu\alpha} \omega_{\alpha\gamma}^{(1,1)} \bar{q}^{\gamma}\,.
\end{aligned} \tag{6.12}
$$

Note that all the tensors with more than two antisymmetric indices decouple. The state U in the $(2,1)$ of SO(9) has the following decomposition with respect to SO(8)

$$
\yng(2,1) \mapsto \yng(2,1) + \yng(2) + \yng(1,1) + \yng(1)\,. \tag{6.13}
$$

Using the coupling of the covariant state U to the Reggeon in eq. (5.43) and the polarization tensors in eq. (D.9) we find

$$
\begin{aligned}
\sqrt{6} T_{g,U,R}^{\mu;\rho\alpha\gamma} \zeta_{\rho\alpha;\gamma}^{U,(2,1)} &= \frac{\alpha'}{2} \delta_{\perp}^{\mu\gamma} \omega_{\rho\alpha;\gamma}^{(2,1)} \bar{q}^{\alpha} \bar{q}^{\rho}\,, \\
\sqrt{6} T_{g,U,R}^{\mu;\rho\alpha\gamma} \zeta_{\rho\alpha;\gamma}^{U,(2)} &= -\frac{2\sqrt{2}}{m} \delta_{\perp}^{\mu\gamma} \omega_{\gamma\rho}^{(2)} \bar{q}^{\rho} = -\sqrt{\alpha'} \delta_{\perp}^{\mu\gamma} \omega_{\gamma\rho}^{(2)} \bar{q}^{\rho}\,, \\
\sqrt{6} T_{g,U,R}^{\mu;\rho\alpha\gamma} \zeta_{\rho\alpha;\gamma}^{U,(1,1)} &= 0\,, \\
\sqrt{6} T_{g,U,R}^{\mu;\rho\alpha\gamma} \zeta_{\rho\alpha;\gamma}^{U,(1)} &= -\frac{\sqrt{7}}{28} \left(3\alpha' \bar{q}^{\mu} \omega \bar{q} + \frac{5}{4} \alpha' t \omega^{\mu} \right)\,.
\end{aligned} \tag{6.14}
$$

Note the decoupling of the vector component of U, similar to the decoupling of the vector component of the state S_2 in the first level. The state V in the $(1,1)$ of SO(9) gives a two-form and a vector of SO(8). Using the coupling in eq. (5.46) we find

$$
\begin{aligned}
T_{g,V,R}^{\mu;\gamma\omega} \zeta_{\gamma\omega}^{V,(1,1)} &= \frac{\sqrt{\alpha'}}{2\sqrt{7}} \delta_{\perp}^{\mu\gamma} \omega_{\gamma\omega} \bar{q}^{\omega}\,, \\
T_{g,V,R}^{\mu;\gamma\omega} \zeta_{\gamma\omega}^{V,(1)} &= \frac{\alpha'}{4\sqrt{7}} \bar{q}^{\mu} \bar{q} \omega - \frac{3\alpha' t}{16\sqrt{7}} \omega^{\mu}\,.
\end{aligned} \tag{6.15}
$$

Finally the state W in the vector representation of SO(9) gives a vector and a scalar of SO(8). The coupling in eq. (5.47) gives

$$
\begin{aligned}
8\sqrt{22} T_{g,W,R}^{\mu;\gamma} \zeta_{\gamma}^{W,(1)} &= 2\alpha' \bar{q}^{\mu} \omega \bar{q} + \frac{28}{m^2} t \omega^{\mu}\,, \\
8\sqrt{22} T_{g,W,R}^{\mu;\gamma} \zeta_{\gamma}^{W,(0)} &= \frac{8}{m} \bar{q}^{\mu}\,.
\end{aligned} \tag{6.16}
$$

6.3 From the covariant to the high-energy basis

We now compare the SO(8) tensors and couplings given by the covariant amplitudes for the second level with those given by the eikonal phase, deriving the unitary transformation that connects the covariant and the high-energy basis.

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Let us start with the tensors of rank 3, namely the (3) and the (2, 1) representations. The (3) representation of SO(8) appears only in the covariant state Z

$$F = \zeta^{Z,(3)}\, Z\,, \tag{6.17}$$

and it is produced with the same amplitude as given by the eikonal operator, cfr. eq. (6.10) and table 2. Consider now the (2, 1) representation of SO(8). This representation appears in the covariant states Y and U and provides the simplest example of a degenerate SO(8) representation. Since there is only one independent coupling to the Reggeon, there is a linear combination of the (2, 1) components of Y and U that decouples at high energy. The new basis is easily identified as well as the corresponding couplings to the Reggeon[19]

$$\begin{aligned} H_1 &= \frac{1}{2}\left(\zeta^{Y,(2,1)}\, Y - \sqrt{3}\zeta^{U,(2,1)}\, U\right), && C_{g,H_1,R} = 0\,, \\ H_2 &= \frac{1}{2}\left(\sqrt{3}\zeta^{Y,(2,1)}\, Y + \zeta^{U,(2,1)}\, U\right), && C_{g,H_2,R} = -\frac{\alpha'}{\sqrt{6}}\epsilon^{\alpha}\omega^{(2,1)}_{\alpha\rho;\gamma}\bar{q}^{\rho}\bar{q}^{\gamma}\;. \end{aligned} \tag{6.18}$$

The coupling to the Reggeon of the state H_2, which is the one produced at high energy in the (2, 1) representation, coincides with the coupling given for this representation by the eikonal operator, cfr. table 2.

Let us now turn to the rank-two tensors. The (2) appears in Z and U and the high-energy basis is

$$\begin{aligned} S_1 &= \frac{1}{\sqrt{3}}\left(\sqrt{2}\zeta^{Z,(2)}\, Z + \zeta^{U,(2)}\, U\right), && C_{g,S_1,R} = 0\,, \\ S_2 &= \frac{1}{\sqrt{3}}\left(\zeta^{Z,(2)}\, Z - \sqrt{2}\zeta^{U,(2)}\, U\right), && C_{g,S_2,R} = \frac{\sqrt{\alpha'}}{2}\epsilon^{\mu}\omega^{(2)}_{\mu\rho}\bar{q}^{\rho}\,, \end{aligned} \tag{6.19}$$

again in agreement with the eikonal operator, cfr. table 2. The (1, 1) appears in Y and V and the high-energy basis is

$$\begin{aligned} A_1 &= \frac{1}{\sqrt{7}}\left(\zeta^{Y,(1,1)}\, Y + \sqrt{6}\zeta^{V,(1,1)}\, V\right), && C_{g,A_1,R} = 0\,, \\ A_2 &= \frac{1}{\sqrt{7}}\left(\sqrt{6}\zeta^{Y,(1,1)}\, Y - \zeta^{V,(1,1)}\, V\right), && C_{g,A_2,R} = -\frac{\sqrt{\alpha'}}{2}\epsilon^{\mu}\omega^{(1,1)}_{\mu\rho}\bar{q}^{\rho}\;. \end{aligned} \tag{6.20}$$

Similarly the SO(8) scalar appears in Z and W and the high-energy basis is

$$\begin{aligned} I_1 &= \frac{1}{\sqrt{11}}\left(\sqrt{2}\zeta^{Z,(0)}\, Z + 3\zeta^{W,(0)}\, W\right), && C_{g,I_1,R} = 0\,, \\ I_2 &= \frac{1}{\sqrt{11}}\left(3\zeta^{Z,(0)}\, Z - \sqrt{2}\zeta^{W,(0)}\, W\right), && C_{g,I_2,R} = -\frac{\sqrt{\alpha'}}{4\sqrt{2}}\epsilon_{\mu}\bar{q}^{\mu}\;. \end{aligned} \tag{6.21}$$

In both cases we find agreement with the eikonal operator, cfr. table 2. Finally the vector of SO(8) appears in Z, U, V and W. This is the first case in which there are degenerate SO(8) representations both in the covariant and in the high-energy basis, the two vectors

[19] We leave understood that the polarizations $\zeta^{S,(r)}$ in the definition of a state of the high-energy basis transforming in the representation r of SO(8) are all formed using the same transverse polarization $\omega^{(r)}$.

in table 2. Since for the vector there are two inequivalent couplings, we will find two linear combinations that decouple at high energy and two linear combinations with a non-vanishing coupling to the Reggeon, related by a unitary transformation to the states in table 2. A basis for the two-dimensional space of vectors of SO(8) that do not couple to the Reggeon is

$$\begin{aligned} B_1 &= \frac{5\sqrt{15}}{22}\zeta^{Z,(1)}\,Z + \frac{\sqrt{77}}{22}\zeta^{V,(1)}\,V + \frac{4\sqrt{2}}{22}\zeta^{W,(1)}\,W\,, \\ B_2 &= -\frac{1}{11}\sqrt{\frac{35}{3}}\zeta^{Z,(1)}\,Z + \frac{1}{2}\sqrt{\frac{11}{6}}\zeta^{U,(1)}\,U + \frac{1}{\sqrt{11}}\zeta^{V,(1)}\,V - \frac{7}{22}\sqrt{\frac{7}{2}}\zeta^{W,(1)}\,W\,. \end{aligned} \tag{6.22}$$

In the two-dimensional space of vectors of SO(8) that couple to the Reggeon we choose the basis that corresponds to the one in table 2

$$\begin{aligned} B_3 &= \frac{1}{4}\left(\sqrt{\frac{21}{11}}\zeta^{Z,(1)}\,Z + \sqrt{\frac{15}{2}}\zeta^{U,(1)}\,U - \sqrt{5}\zeta^{V,(1)}\,V - \sqrt{\frac{35}{22}}\zeta^{W,(1)}\,W\right)\,, \\ B_4 &= -\frac{1}{4}\left(\sqrt{\frac{5}{33}}\zeta^{Z,(1)}\,Z - \sqrt{\frac{7}{6}}\zeta^{U,(1)}\,U - \sqrt{7}\zeta^{V,(1)}\,V + \frac{13}{\sqrt{22}}\zeta^{W,(1)}\,W\right)\,. \end{aligned} \tag{6.23}$$

It is easy to verify that the couplings of these two vectors to the Reggeon are indeed

$$C_{g,B_3,R} = -\frac{\alpha'}{\sqrt{35}}\epsilon_\mu\left(\bar{q}^\mu\bar{q}^\gamma + \frac{\alpha' t}{8}\delta_\perp^{\mu\gamma}\right)\omega_\gamma\,, \qquad C_{g,B_4,R} = -\frac{\alpha' t}{8}\epsilon_\mu\delta_\perp^{\mu\gamma}\lambda_\gamma\,, \tag{6.24}$$

in agreement with table 2.

The analysis of the second level is now complete. The states in the second massive level that can be excited in the high-energy scattering of a massless state on a stack of Dp-branes are therefore

$$F\,, \qquad H_2\,, \qquad B_3\,, \qquad B_4\,, \qquad S_2\,, \qquad A_2\,, \qquad I_2\,, \tag{6.25}$$

in the following irreducible representations of SO(8)

$$\yng(3) \qquad \yng(2,1) \qquad \yng(1) \qquad \yng(1) \qquad \yng(2) \qquad \yng(1,1) \qquad \bullet\,, \tag{6.26}$$

for a total of 352 degrees of freedom. All the other SO(8) components of the covariant states at level two decouple from the ground state in the Regge limit. All the covariant amplitudes for the SO(8) tensors in the high-energy basis agree with the matrix elements of the eikonal phase.

6.4 From the light-cone to the covariant states

Having described in detail how to relate the covariant dynamics to the light-cone dynamics encoded in the eikonal operator, we now discuss a method to proceed in the opposite direction. This method, based on the DDF operators introduced in appendix C, allows to connect level by level the light-cone states with linear combinations of the SO(8) components of the covariant states.

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The DDF operators are in one-to-one correspondence with the light-cone states and the SO(8) symmetry rotating the transverse polarization vectors ϵ_j is the only part of the Lorentz group manifestly realized. All massive states will then appear decomposed in SO(8) representations. The fact that the DDF operators are constructed using the worldsheet fields X^μ and ψ^μ of the covariant theory will allow us to identify for any SO(8) representation in the light-cone gauge the corresponding linear combination of the SO(8) components of the covariant states.

The spectrum of the physical states of the NS sector of the superstring is generated by the action on the vacuum state in eq. (C.1) of the DDF operators in eq. (C.2), followed by the GSO projection on states with definite worldsheet fermion number. The GSO projection preserves only the states containing an odd number of $B_{-r,j}$, so the first non trivial physical state is obtained by applying the operator $B_{-\frac{1}{2},j}$ to the vacuum in eq. (C.1)

$$B_{-\frac{1}{2},j}|p_T;0\rangle = -(\epsilon_j\psi_{-\frac{1}{2}})|p_T - \frac{1}{2}k;0\rangle\,, \qquad \left(p_T-\frac{1}{2}k\right)^2 = p_T^2 - p_T k = 0\,. \tag{6.27}$$

These are the eight physical polarizations of the massless states corresponding to the covariant vertex operator in eq. (5.24) with momentum $p = p_T - k/2$.

At the first massive level we have three types of states: those created by the action of $B_{-3/2}$, those created by the action of three $B_{-1/2}$ and finally the states with one $B_{-1/2}$ oscillator and one A_{-1} oscillator. In this case the correspondence between the light-cone and the covariant states is unambiguous since there is a unique way to combine the SO(8) representations into SO(9) representations. The first two sets of states can be immediately identified with the covariant states with polarizations $\zeta^{S_2,(1)}$ and $\zeta^{A_3,(1,1,1)}$. Similarly the symmetric, antisymmetric and trace part of the states created by the action of $A_{-1,j}B_{-1/2,k}$ correspond to the covariant states with polarizations $\zeta^{S_2,(2)}$, $\zeta^{A_3,(1,1)}$ and $\zeta^{S_2,(0)}$.

Let us show how the previous identifications can be derived using the DDF operators, focusing on the states $A_{-1,j}B_{-1/2,k}$, which are the only ones in the first level produced in a high-energy collision of a massless state with a Dp-brane. This analysis will allow us to introduce all the tools that are necessary to apply the method to a general light-cone state in the higher massive levels, where the correspondence between light-cone and covariant states is not already completely fixed by the decomposition of the SO(9) representations. We have

$$|\zeta_{jk}\rangle \equiv A_{-1,j}B_{-\frac{1}{2},k}|p_T;0\rangle = -A_{-1,j}(\epsilon_k\psi_{-\frac{1}{2}})|p_T-\frac{1}{2}k;0\rangle\,, \tag{6.28}$$

where we have performed the contour integral present in the definition of the B DDF oscillator; by carrying out explicitly the integral in the definition of A we get

$$\begin{aligned}|\zeta_{jk}\rangle &= \epsilon^j_\mu\epsilon^k_\nu\left[\alpha^\mu_{-1}\psi^\nu_{-\frac{1}{2}} - (k\psi_{-\frac{1}{2}})\psi^\mu_{-\frac{1}{2}}\psi^\nu_{-\frac{1}{2}} - (k\psi_{-\frac{3}{2}})\eta^{\mu\nu} + (k\psi_{-\frac{1}{2}})(k\alpha_{-1})\eta^{\mu\nu}\right]|p_T-\frac{3}{2}k;0\rangle\,,\\ &= \left[\alpha^j_{-1}\psi^k_{-\frac{1}{2}} - (k\psi_{-\frac{1}{2}})\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}} + \delta^{jk}\left((k\psi_{-\frac{1}{2}})(k\alpha_{-1}) - (k\psi_{-\frac{3}{2}})\right)\right]|p^{(1)};0\rangle\,,\end{aligned} \tag{6.29}$$

where $p^{(1)} = p_T - \frac{3}{2}k$ and for simplicity we defined $\alpha^j_{-1} = (\epsilon_j)_\mu\alpha^\mu_{-1}$ and similarly for the ψ oscillators. This approach can obviously be used for any other light-cone state and, as explained at the end of section 4, provides an algorithm to map the light-cone spectrum in the NS formalism into the covariant spectrum. In order to make this mapping fully explicit we need however some extra work. Consider first the antisymmetric combination

$$|\zeta^a_{jk}\rangle = \frac{1}{2}\left(|\zeta_{jk}\rangle - |\zeta_{kj}\rangle\right) = \left[\frac{1}{2}\left(\alpha^j_{-1}\psi^k_{-\frac{1}{2}} - \alpha^k_{-1}\psi^j_{-\frac{1}{2}}\right) - (k\psi_{-\frac{1}{2}})\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}\right]|p^{(1)};0\rangle\ . \quad (6.30)$$

It is straightforward to show that the following state

$$|\Sigma^{[MN]}\rangle = \left[\alpha^M_{-1}\psi^N_{-\frac{1}{2}} - \alpha^N_{-1}\psi^M_{-\frac{1}{2}} + (p^{(1)}\psi_{-\frac{1}{2}})\psi^M_{-\frac{1}{2}}\psi^N_{-\frac{1}{2}}\right]|p^{(1)};0\rangle\,, \quad (6.31)$$

is spurious when the indices M, N run over the space orthogonal to the momentum p. This means that it is physical and at the same time it can be written as $G_{-1/2}$ acting on another state[20]

$$|\Sigma^{[MN]}\rangle = G_{-\frac{1}{2}}\psi^M_{-\frac{1}{2}}\psi^N_{-\frac{1}{2}}|p^{(1)};0\rangle\ . \quad (6.32)$$

Then by using (6.31) in (6.30), we can rewrite the antisymmetric state as follows

$$|\zeta^a_{jk}\rangle = \frac{1}{2}G_{-\frac{1}{2}}\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}|p^{(1)};0\rangle - \frac{1}{2}\left[\left(p_T + \frac{1}{2}k\right)\psi_{-\frac{1}{2}}\right]\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}|p^{(1)};0\rangle\ . \quad (6.33)$$

Of course we can neglect the first line because it is a spurious state; then let us focus on the combination $p_T + k/2$: it is orthogonal to both the momentum of the state and the eight polarizations ϵ_i. Then this combination must be proportional to the unit vector v^μ describing the ninth physical polarization present in the description of the massive state. It is straightforward to generalize this relation and the corresponding expression of the momentum of the massive state to the n^{th} massive level

$$v = -\frac{1}{\sqrt{2n}}\left(p_T + \left(n - \frac{1}{2}\right)k\right)\ , \qquad p^{(n)} = p_T - \left(n + \frac{1}{2}\right)k\,, \qquad p^{(n)}v = 0\ . \quad (6.34)$$

Thus we can rewrite the antisymmetric state (6.33) as

$$|\zeta^a_{jk}\rangle = \frac{1}{\sqrt{2}}\psi^v_{-\frac{1}{2}}\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}|p^{(1)};0\rangle\ . \quad (6.35)$$

where $\psi^v_{-\frac{1}{2}} = v_\mu\psi^\mu_{-\frac{1}{2}}$. In this form it is clear that ζ^a_{jk} corresponds to $\zeta^{A_3,(1,1)}$, the SO(8) part of the covariant state A_3 in eq. (5.28) and eq. (6.5), where one of the Lorentz indices is along the v direction and the remaining two are in the eight-dimensional space perpendicular to the light-cone directions $e^\pm$.

The same approach can be followed to rewrite the symmetric and the trace parts of $|\zeta_{ij}\rangle$. By using (6.34) it can be checked that

$$\begin{aligned}|\zeta^{st}_{jk}\rangle \equiv \frac{1}{2}\left(|\zeta_{jk}\rangle + |\zeta_{kj}\rangle\right) = &\left[\delta^{jk}\left(\frac{1}{2\sqrt{2}}|s_2\rangle + \frac{1}{3}|s_1\rangle\right) + \frac{1}{2}\left(\alpha^j_{-1}\psi^k_{-\frac{1}{2}} + \alpha^k_{-1}\psi^j_{-\frac{1}{2}}\right)\right.\\ &\left.- \frac{1}{6}\eta^{jk}\left(\eta_{\rho\sigma} - \frac{p^{(1)}_\rho p^{(1)}_\sigma}{(p^{(1)})^2} - 3v_\rho v_\sigma\right)\alpha^\rho_{-1}\psi^\sigma_{-\frac{1}{2}}\right]|p^{(1)};0\rangle,\end{aligned} \quad (6.36)$$

[20] It is a zero norm state that is decoupled from the physical spectrum. This kind of states were first considered in [39].

where $|s_i\rangle$ are spurious states whose explicit expression can be found at the end of the appendix A. Then eq. (6.36) becomes

$$|\zeta^{st}_{jk}\rangle = \frac{1}{2}\left[\alpha^j_{-1}\psi^k_{-\frac{1}{2}} + \alpha^k_{-1}\psi^j_{-\frac{1}{2}} - \frac{\delta^{jk}}{3}\left(\sum_{i=1}^{8}\alpha^i_{-1}\psi^i_{-\frac{1}{2}} - 2\alpha^v_{-1}\psi^v_{-\frac{1}{2}}\right)\right]|p^{(1)};0\rangle \ . \tag{6.37}$$

Separating the traceless and the trace part of the previous tensor, one can see that these states indeed correspond to the covariant SO(8) components $\zeta^{S_2,(2)}$ and $\zeta^{S_2,(0)}$ in eq. (5.28) and eq. (6.4).

A similar analysis can be performed for the higher massive levels. As an example let us consider here the two of SO(8) present at level two in the light-cone spectrum. It is not difficult to build two light-cone states transforming as tensor of type $(2,1)$ of SO(8). The first one is

$$\begin{aligned}\sqrt{2}A_{-1,\ell}|\zeta^a_{jk}\rangle =& \frac{1}{\sqrt{2}}\Big[(k\psi_{-\frac{1}{2}})\,\psi^\ell_{-\frac{1}{2}}\left(\alpha^j_{-1}\psi^k_{-\frac{1}{2}} - \alpha^k_{-1}\psi^j_{-\frac{1}{2}}\right)\\ &- \alpha^\ell_{-1}\left(\alpha^j_{-1}\psi^k_{-\frac{1}{2}} - \alpha^k_{-1}\psi^j_{-\frac{1}{2}} - 2(k\psi_{-\frac{1}{2}})\psi^j_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}\right)\Big]|p^{(2)};0\rangle\,,\end{aligned} \tag{6.38}$$

where $p^{(2)} = p_T - \frac{5}{2}k$ and, for the sake of simplicity, we assumed that $\ell \neq j \neq k$. Notice that the state (6.38) has unit norm and can be written as a linear combination of the states $|\omega^{(2,1)}\rangle$ introduced in the second line of tble 2

$$\begin{aligned}\sqrt{2}A_{-1,\ell}|\zeta^a_{jk}\rangle &= \frac{1}{\sqrt{3}}\left(|\omega^{(2,1)}_\alpha\rangle - |\omega^{(2,1)}_\beta\rangle\right)\,,\\ (\omega^{(2,1)}_\alpha)_{\ell'j'|k'} &= \frac{1}{2}\left(\delta^\ell_{\ell'}\delta^j_{j'}\delta^k_{k'} + \delta^\ell_{j'}\delta^j_{\ell'}\delta^k_{k'} - \delta^\ell_{k'}\delta^j_{j'}\delta^k_{\ell'} - \delta^\ell_{j'}\delta^j_{k'}\delta^k_{\ell'}\right)\,,\\ (\omega^{(2,1)}_\beta)_{\ell'j'|k'} &= \frac{1}{2}\left(\delta^\ell_{\ell'}\delta^k_{j'}\delta^j_{k'} + \delta^\ell_{j'}\delta^k_{\ell'}\delta^j_{k'} - \delta^\ell_{k'}\delta^k_{j'}\delta^j_{\ell'} - \delta^\ell_{j'}\delta^k_{k'}\delta^j_{\ell'}\right)\,,\end{aligned} \tag{6.39}$$

where the state $|\omega^{(2,1)}_\alpha\rangle$ is obtained from the polarization $\omega^{(2,1)}_\alpha$ and, of course, $\omega^{(2,1)}_\beta$ is just $\omega^{(2,1)}_\alpha$ with j and k exchanged.

The relation between (6.38) and the SO(8) components $\zeta^{Y,(2,1)}$ and $\zeta^{U,(2,1)}$ of the covariant states Y and U in eq. (D.7) and eq. (D.9) is far from obvious, but this state must be a linear combination of them. Again, in order to make this connection manifest, one needs to eliminate a spurious state from the expression obtained by using the DDF operators

$$|\hat{\psi}\rangle = \sqrt{2}\left[A_{-1,\ell}|\zeta^a_{jk}\rangle + \frac{1}{4}\Big(|S_{jl;k}\rangle - |S_{kl;j}\rangle\Big)\right]\,, \tag{6.40}$$

where

$$|S_{jl;k}\rangle = G_{-\frac{1}{2}}\alpha^{(\ell}_{-1}\psi^{j)}_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}|p^{(2)};0\rangle\,, \tag{6.41}$$

is explicitly given in eq. (A.12). The state $|\hat{\psi}\rangle$ has norm equal to 1. Using eqs. (6.34) for $n = 2$, one can rewrite k in (6.38) in terms of $p^{(2)}$ and v. The terms with $p^{(2)}$ in (6.40) cancel and then it is straightforward to check that

$$|\hat{\psi}\rangle = \sqrt{2}\left[-\frac{1}{2}\left(|Y_\alpha\rangle - |Y_\beta\rangle\right) - \frac{1}{12}\left(|U_\alpha\rangle - |U_\beta\rangle\right)\right]\,, \tag{6.42}$$

where the kets on the r.h.s. represent the covariant states Y and U with a specific choice of the polarization. Their explicit expressions are[21]

$$|U_\alpha\rangle = \left(\alpha^\ell_{-1}\alpha^j_{-1}\psi^k_{-\frac{1}{2}} - \alpha^k_{-1}\alpha^{(j}_{-1}\psi^{\ell)}_{-\frac{1}{2}} - 3\psi^{(\ell}_{-\frac{3}{2}}\psi^{j)}_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}\right)|p^{(2)};0\rangle = \sqrt{6}|\zeta^{U,(2,1)}_\alpha\rangle\,, \qquad (6.43)$$

and similarly for $|U_\beta\rangle$. The states $|\zeta^{U,(2,1)}_{\alpha,\beta}\rangle$ are obtained from the state, corresponding to the vertex operator V_U in eq. (5.37), with the same polarizations introduced in (6.39) and with the indices restricted to those of SO(8). Similarly

$$|Y_\alpha\rangle = \alpha^{(\ell}_{-1}\psi^{j)}_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}\psi^v_{-\frac{1}{2}}\,|p^{(2)};0\rangle = \frac{1}{\sqrt{2}}|\zeta^{Y,(2,1)}_\alpha\rangle\,, \qquad (6.44)$$

is obtained from the state corresponding to the vertex operator V_Y in (5.37) contracted with the $(2,1)$ tensor in (D.7) where the ω's are again those introduced in (6.39) and the indices are restricted to those of SO(8). It is easy to check that

$$\sqrt{2}A_{-1,\ell}|\zeta^a_{jk}\rangle = -\frac{1}{\sqrt{3}}\left(|H_{2\alpha}\rangle - |H_{2\beta}\rangle\right)\,, \qquad (6.45)$$

where H_2 is the state introduced in (6.18), while the orthogonal state within the same SO(8) representation is

$$\frac{1}{\sqrt{6}}\left[-\left(|Y_\alpha\rangle - |Y_\beta\rangle\right) + \frac{1}{2}\left(|U_\alpha\rangle - |U_\beta\rangle\right)\right] = -\frac{1}{\sqrt{3}}\left(|H_{1\alpha}\rangle - |H_{1\beta}\rangle\right)\,, \qquad (6.46)$$

where again H_1 is the state introduced in (6.18). In terms of the DDF oscillators eq. (6.46) corresponds to

$$\sqrt{\frac{2}{3}}\left(B^{(\ell}_{-\frac{3}{2}}B^{j)}_{-\frac{1}{2}}B^k_{-\frac{1}{2}} - B^{(\ell}_{-\frac{3}{2}}B^{k)}_{-\frac{1}{2}}B^j_{-\frac{1}{2}}\right)|p_T;0\rangle\,. \qquad (6.47)$$

7 Conclusions

The leading eikonal operator represents one of the rare examples of resummation of the complete perturbative series of string theory. Its simple and somewhat intuitive structure, which generalizes to an extended object the eikonal phase of a point particle, encodes a wealth of information on the high-energy string dynamics. To make this information accessible in every detail, we completed the definition of the eikonal operator showing that it acts on the Hilbert space of physical string states in the light-cone gauge, with the spatial direction determined by the collision axis. Once this is established, it is possible to evaluate and give the correct interpretation to its matrix elements which capture the asymptotic behaviour at high energy of arbitrary four-point or arbitrary two-point amplitudes, in the case of a string-string or a string-brane collision respectively.

From the covariant point of view, the high-energy dynamics described in the light-cone gauge by the eikonal operator is the multiple exchange of effective Reggeon states. Using

[21] As mentioned above, we are assuming that all indices in Y and U are different; the full expressions, including the terms necessary to ensure the tracelessness condition, can be found in (A.7)–(A.8).

the Reggeon operator we were able to provide a simple and fully covariant derivation of the eikonal phase.

We also discussed the asymptotic high-energy behaviour of the covariant string amplitudes with massive states, explaining how to take into account the longitudinal polarizations in a simple way. We illustrated our methods by calculating all the transition amplitudes from the massless NS-NS sector to the first two massive levels of the superstrings. In this way we could show in detail how the simple properties of the matrix elements of the eikonal operator emerge from the covariant amplitudes.

The covariant dynamics of the massive string spectrum is an interesting topic on its own. The second massive level provides several useful examples, including states transforming as tensors of mixed symmetry. In the Regge limit the external massive string states can be considered approximately massless and the amplitudes discussed in our paper may help to understand the consistent interactions of massless fields of higher spin.

Equipped with a detailed understanding of the leading eikonal operator, it is possible to generalise the analysis of the string high-energy scattering in several directions. An interesting problem is to derive an eikonal operator which includes the classical corrections in R_p/b. It is possible that new qualitative features arise: for instance, the shift $b \to b + \hat{X}$, which gives the string eikonal operator starting from the leading eikonal phase, might not be enough to capture the full dynamics already at the first subleading order.

Another interesting extension of our results is to study the high-energy string-brane scattering in the stringy regime $b \ll l_s$. In this case, the effects of the t-dependent phase in eq. (3.3) are not exponentially suppressed and, in order to restore unitarity, one should enlarge the Hilbert space of the eikonal operator to include also the open string oscillators.

Finally it should be possible to extend our analysis to more complicated D-brane bound states that represent the microstates of macroscopic black holes. In this case, it would be interesting to generalise the approach of [40] and see whether and under what conditions the tidal forces present in the string high-energy scattering can distinguish different microstates.

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A Conventions

In this appendix we collect our conventions for the description of type II string theories in flat space in the RNS formalism. The standard bosonic coordinates describing the embedding of the string in spacetime are indicated by $X^\mu(z, \bar{z})$, with $z = \mathrm{e}^{\tau + \mathrm{i}\sigma}$ where τ and

σ are the usual variables parametrising the (Euclidean) worldsheet. We use the greek letters $\alpha, \beta, \mu, \nu, \ldots$ for the 10-dimensional indices and a mostly plus metric $\eta = (- + \ldots +)$. When describing a massive string state, we use the capital latin letters $I, J, \ldots$ for the 9-dimensional indices that live in the space orthogonal to the momentum p^μ and the small latin letters $i, j, \ldots$ for the 8-dimensional indices that live in the subspace orthogonal to both p^μ and the longitudinal polarization v, see for instance eq. (B.4). The 2D equations of motions imply that the X^μ are a combination of a holomorphic and an anti-holomorphic part and similarly the worldsheet spinors ψ^μ have a holomorphic and an anti-holomorphic component

$$X^\mu(z,\bar z) = X^\mu(z) + \bar X^\mu(\bar z)\,, \qquad \psi^\mu(z,\bar z) = \begin{pmatrix} \bar\psi^\mu(\bar z) \\ \psi^\mu(z) \end{pmatrix} \,. \tag{A.1}$$

The OPE's of these fields are

$$X^\mu(z)X^\nu(w) \sim -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w)\,, \qquad \psi^\mu(z)\psi^\nu(w) \sim \frac{\eta^{\mu\nu}}{z-w}\,, \tag{A.2}$$

and similarly for the anti-holomorphic fields, while the corresponding mode expansions and commutation relations (in the NS sector) are

$$X^\mu(z) = q^\mu - \mathrm{i}\sqrt{\frac{\alpha'}{2}}\alpha_0^\mu \log z + \mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}z^{-n}\,, \qquad \psi^\mu(z) = \sum_{r\in\mathbb{Z}+\frac{1}{2}}\psi_r^\mu z^{-r-\frac{1}{2}}\,, \tag{A.3}$$

$$[q^\mu, \alpha_0^\nu] = \mathrm{i}\eta^{\mu\nu}\sqrt{\frac{\alpha'}{2}}\,, \qquad [\alpha_n^\mu, \alpha_m^\nu] = \eta^{\mu\nu} n\,\delta_{m+n}\,, \quad \{\psi_r^\mu, \psi_s^\nu\} = \eta^{\mu\nu}\delta_{r+s}\,,$$

where tilded (left) and untilded (right) operators are completely independent and thus (anti)commute. The left part of the Fock space is built on the vacuum state $|p;0\rangle$ defined by the following relations

$$\alpha_0^\mu|p;0\rangle = \sqrt{\frac{\alpha'}{2}}p^\mu|p;0\rangle\,, \qquad \alpha_n^\mu|p;0\rangle = \psi_r^\mu|p;0\rangle = 0\,, \qquad \text{if} \quad n\geq 1,\;\; r\geq\frac{1}{2}\,, \tag{A.4}$$

and the right part is built on a vacuum state $|p;\bar 0\rangle$ defined in a similar way. The physical NS-NS spectrum is obtained by taking the tensor product of a left and a right state annihilated separately by $L_0 - 1/2$, G_r and $\bar L_0 - 1/2$, $\bar G_r$, with $r \geq 1/2$, where

$$\begin{aligned} G_r &= \sum_{n\in\mathbb{Z}}\alpha_{-n}\psi_{r+n}\,, & L_m &= L_m^{(\alpha)} + L_m^{(\psi)}\,, \\ L_m^{(\alpha)} &= \frac{1}{2}\sum_{n\in\mathbb{Z}} :\alpha_{-n}\alpha_{n+m}:\,, & L_m^{(\psi)} &= \frac{1}{2}\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(r+\frac{m}{2}\right):\psi_{-r}\psi_{m+r}: \;. \end{aligned} \tag{A.5}$$

Finally physical states $|ph\rangle$ should obey the level-matching condition $L_0|ph\rangle = \bar L_0|ph\rangle$ and the GSO projection selects the states with an odd number of ψ oscillators both in the left and in the right part.

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For completness we list the standard $N = 1$ super-Virasoro algebra satisfied by the generators introduced above

$$\begin{aligned}\{G_r, G_s\} &= 2L_{r+s} + \frac{d}{2}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\,,\\ [L_n, G_r] &= \left(\frac{1}{2}m - r\right)G_{r+n}\,,\\ [L_n, L_m] &= (n-m)L_{n+m} + \frac{d}{8}m(m^2-1)\delta_{n+m,0}\,,\end{aligned}\tag{A.6}$$

where $d = 10$ is the space-time dimension.

We conclude this appendix, by giving the explicit expressions of the states used in section 6.4. With the definitions given above, it is straightforward to check that the states below are physical (here $p^{(2)} = p_T - \frac{5}{2}k$)

$$\begin{aligned}|Y_{(I[J)HK]}\rangle = \frac{1}{2}\Bigg[&\left(\alpha^I_{-1}\psi^J_{-\frac{1}{2}} + \alpha^J_{-1}\psi^I_{-\frac{1}{2}}\right)\psi^H_{-\frac{1}{2}}\psi^K_{-\frac{1}{2}}\\ &-\frac{\alpha_{-1,L}\psi_{-\frac{1}{2},L}}{d-3}\left(2\hat{\eta}^{IJ}\psi^H_{-\frac{1}{2}} - \hat{\eta}^{IH}\psi^J_{-\frac{1}{2}} - \hat{\eta}^{JH}\psi^I_{-\frac{1}{2}}\right)\psi^K_{-\frac{1}{2}}\\ &-\frac{\alpha_{1,L}\psi_{-\frac{1}{2},L}}{d-3}\left(\hat{\eta}^{IK}\psi^J_{-\frac{1}{2}} + \hat{\eta}^{JK}\psi^I_{-\frac{1}{2}}\right)\psi^H_{-\frac{1}{2}}\Bigg]|0,p^{(2)}\rangle\,,\end{aligned}\tag{A.7}$$

which reduces to (6.44) when all indices are different and $K = v$,

$$\begin{aligned}|U_{(I[J)K]}\rangle = \Bigg[&\frac{3}{2}\left(\psi^I_{-\frac{3}{2}}\psi^J_{-\frac{1}{2}} + \psi^J_{-\frac{3}{2}}\psi^I_{-\frac{1}{2}}\right)\psi^K_{-\frac{1}{2}} - \alpha^I_{-1}\alpha^J_{-1}\psi^K_{-\frac{1}{2}} + \frac{1}{2}\alpha^K_{-1}\left(\alpha^J_{-1}\psi^I_{-\frac{1}{2}} + \alpha^I_{-1}\psi^J_{-\frac{1}{2}}\right)\\ &-\frac{3}{2}\frac{\psi_{-\frac{3}{2},H}\psi_{-\frac{1}{2},H}}{d-2}\left(2\hat{\eta}^{IJ}\psi^K_{-\frac{1}{2}} - \hat{\eta}^{IK}\psi^J_{-\frac{1}{2}} - \hat{\eta}^{JK}\psi^I_{-\frac{1}{2}}\right)\\ &+\frac{\alpha_{-1H}\,\alpha_{-1H}}{2(d-2)}\left(2\hat{\eta}^{IJ}\psi^K_{-\frac{1}{2}} - \hat{\eta}^{KJ}\psi^I_{-\frac{1}{2}} - \hat{\eta}^{KI}\psi^J_{-\frac{1}{2}}\right)\\ &-\frac{\alpha_{-1H}\,\psi_{-\frac{1}{2},H}}{2(d-2)}\left(2\hat{\eta}^{IJ}\alpha^K_{-1} - \hat{\eta}^{KJ}\alpha^I_{-1} - \hat{\eta}^{KI}\alpha^J_{-1}\right)\Bigg]|0,p^{(2)}\rangle\,,\end{aligned}\tag{A.8}$$

which reduces to (6.43) when all indices are different, and

$$\begin{aligned}|Z_{(MNP)}\rangle = {}&\hat{\alpha}^M_{-1}\hat{\alpha}^N_{-1}\hat{\psi}^P_{-\frac{1}{2}} + \hat{\alpha}^M_{-1}\hat{\alpha}^P_{-1}\hat{\psi}^N_{-\frac{1}{2}} + \hat{\alpha}^N_{-1}\hat{\alpha}^P_{-1}\hat{\psi}^M_{-\frac{1}{2}} - \frac{1}{d+1}\\ &\times\Bigg[\hat{\eta}^{MN}\hat{\eta}^{QR}\left(\alpha_{-1,R}\alpha_{-1Q}\psi^P_{-\frac{1}{2}} + 2\alpha_{-1,R}\psi_{-\frac{1}{2},Q}\alpha^P_{-1}\right)\\ &+\hat{\eta}^{MP}\hat{\eta}^{QR}\left(\alpha_{-1,R}\alpha_{-1Q}\psi^N_{-\frac{1}{2}} + 2\alpha_{-1,R}\psi_{-\frac{1}{2},Q}\alpha^N_{-1}\right)\\ &+\hat{\eta}^{NP}\hat{\eta}^{QR}\left(\alpha_{-1,R}\alpha_{-1Q}\psi^M_{-\frac{1}{2}} + 2\alpha_{-1,R}\psi_{-\frac{1}{2},Q}\alpha^M_{-1}\right)\Bigg]|0,p^{(2)}\rangle\,,\end{aligned}\tag{A.9}$$

where $\hat{\alpha}^M_{-1} = \alpha^M_{-1} - \frac{p^M(p\alpha_{-1})}{p^2}$ and analogously for $\psi^M_{-\frac{1}{2}}$. Also the following states are physical (here $p^{(1)} = p_T - \frac{3}{2}k$)

$$
\begin{aligned}
|s_1\rangle &= \left(G_{-\frac{1}{2}} p^{(1)}\alpha_{-1} + \frac{1}{2}G_{-\frac{3}{2}}\right)|p^{(1)};0\rangle \qquad\qquad (A.10)\\
&= \left(\frac{3}{2}p^{(1)}\psi_{-\frac{3}{2}} + p^{(1)}\psi_{-\frac{1}{2}}\, p^{(1)}\alpha_{-1} + \frac{1}{2}\alpha_{-1}\psi_{-\frac{1}{2}}\right)|0,p^{(1)}\rangle\,,\\
|s_2\rangle &= -G_{-\frac{1}{2}}\left((v\psi_{-\frac{1}{2}})\,(p^{(1)}\psi_{-\frac{1}{2}}) - 2v\alpha_{-1}\right)|p^{(1)};0\rangle \qquad\qquad (A.11)\\
&= \left(2v\psi_{-\frac{3}{2}} + (v\psi_{-\frac{1}{2}})(p^{(1)}\alpha_{-1}) + (v\alpha_{-1})(p^{(1)}\psi_{-\frac{1}{2}})\right)|0,p^{(1)}\rangle\,,\\
|S_{jl;k}\rangle &= G_{-\frac{1}{2}}\alpha^{(\ell}_{-1}\psi^{j)}_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}}|0,p^{(2)}\rangle \qquad\qquad (A.12)\\
&= \left[(p^{(2)}\psi_{-\frac{1}{2}})\alpha^{(\ell}_{-1}\psi^{j)}_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}} + \psi^{(\ell}_{-\frac{3}{2}}\psi^{j)}_{-\frac{1}{2}}\psi^k_{-\frac{1}{2}} + \alpha^\ell_{-1}\alpha^j_{-1}\psi^k_{-\frac{1}{2}} - \alpha^k_{-1}\alpha^{(\ell}_{-1}\psi^{j)}_{-\frac{1}{2}}\right]|0,p^{(2)}\rangle\,.
\end{aligned}
$$

Since they are explicitly written as super-descendant, these states are spurious.

B Explicit formulae for the kinematics

In this appendix we collect some formulae relevant for the kinematics discussed in section 2 in a particular reference frame. In order to simplify the comparison with the light-cone computation discussed in section 3, it is convenient to choose the spatial momentum of the outgoing massive particle to be aligned along the same direction of $\vec{e}^{\pm}$; then we have

$$
p_2^\mu = \left(-E, 0_p; 0_{8-p}, -\sqrt{E^2-M^2}\right)\,, \qquad (B.1)
$$

where the first $p+1$ directions are parallel to the (Neumann directions of the) Dp-branes and the entries after the semicolon are along the Dirichlet directions. Then the light-cone directions defined in (2.11) read

$$
(e^+)^\mu = \frac{1}{\sqrt{2}}(-1,0,\ldots,0,1)\,, \qquad (e^-)^\mu = \frac{1}{\sqrt{2}}(1,0,\ldots,0,1)\,. \qquad (B.2)
$$

The most direct way to describe the physical polarization of massive particles is to introduce 9 vectors perpendicular to their momentum. For instance, in the case of the outgoing state (B.1) we have the unit vectors $\hat{w}^i$

$$
\hat{w}_1 = (0,1,0_{p-1};0_{8-p},0)\,,\ldots,\hat{w}_8 = (0,0_p;0_{7-p},1,0)\,, \qquad (B.3)
$$

and, as the ninth one, v^μ corresponding to the longitudinal polarization

$$
v_2^\mu = \left(\frac{\sqrt{E^2-M^2}}{M}, 0_p; 0_{8-p}, \frac{E}{M}\right)\,. \qquad (B.4)
$$

In terms of the DDF construction reviewed in appendix C, these massive state can be generated by choosing

$$
p_T = (\sinh\alpha, 0_p; 0_{8-p}, \cosh\alpha)\,, \qquad k = \mathrm{e}^{-\alpha}\sqrt{2}e^+ = \mathrm{e}^{-\alpha}(-1,0_p;0_{8-p},1)\,, \qquad (B.5)
$$

where $k^2 = 0$ and $kp_T = 1$ for any α (this parameter can be used to obtain the desired energy in (B.1)). The possible momenta of the ingoing particle take the following form

$$p_1^\mu = \left(E, 0_p; \bar{p}_1, \sqrt{E^2 - M^2} + q_9\right) , \tag{B.6}$$

$$q^{\mu=9} = \frac{t + M^2}{2\sqrt{E^2 - M^2}} , \qquad (\bar{p}_1)^2 + (q^{\mu=9})^2 = -t \equiv (p_1 + p_2)^2 , \tag{B.7}$$

where $p_1^2 = 0$. Finally ϵ_k^μ indicates the (left) part of the polarisation of the massless NS-NS state. Since we are focusing on a massless state we have eight independent polarisations. It is convenient to choose them as follows

$$\epsilon_k^\mu = \left(\frac{\bar{p}_1^k}{E + \sqrt{E^2 - M^2} + q^9}, \delta_k^i, -\frac{\bar{p}_1^k}{E + \sqrt{E^2 - M^2} + q^9}\right) . \tag{B.8}$$

This implies that we can neglect the p_1^μ part in q^μ and write p_2^μ in place of the latter

$$\epsilon_k q = \epsilon_k p_2 = \epsilon^{\mu=0}\left(E + \sqrt{E^2 - M^2}\right) \sim \bar{p}_1^k , \tag{B.9}$$

where in the final step we kept only the leading term in the high energy expansion.

We now wish to decompose the indices α and ρ of the massive states in 8D part (i, j) and the component along $v \equiv v_2$, see eq. (B.3) and (B.4). Then it is convenient to rewrite ϵ^μ in terms of their v-component and 8D part

$$\epsilon_k v = \frac{\epsilon^{\mu=0}}{M^3}\left[(E^2 - M^2)^{\frac{3}{2}} + (E^2 - M^2)E - E^2\sqrt{E^2 - M^2} - E^3\right] \sim -\frac{\bar{p}_1^k}{M} , \tag{B.10}$$

where again in the last step we implemented the high energy limit. The polarisation of the massive state (of momentum p_2) can be written in terms of tensor products of the vectors $\hat{w}_i$ and v.

C Reminder of the DDF construction

In this appendix we collect a few known results about the DDF operators and states [27] to be used in the main body of the paper. We do so by using their generalization to the NS sector of the superstring presented in [36].[22] As usual we will discuss explicitly the construction for the left movers. Identical considerations apply to the right movers.

One first introduces an auxiliary tachyon-like momentum p_T and the corresponding tachyonic state

$$|p_T; 0\rangle , \qquad p_T^2 = 1 . \tag{C.1}$$

[22] In this appendix, except for the expansion of the fermionic coordinate ψ in terms of the oscillators, we follow the notation of ref. [36] where dimensionless variables are used. In this notation the string coordinate is given in eq. (A.3) for $\alpha' = 2$.

The physical states in the NS sector of the superstring can be constructed by acting on this ground state with the DDF oscillators[23]

$$
\begin{aligned}
A_{-n,j} &= -\mathrm{i}\oint_0 dz\ (\epsilon_j)_\mu\,(\partial X^\mu + \mathrm{i}n(k\psi)\psi^\mu)\,\mathrm{e}^{-\mathrm{i}nkX(z)}\ , \\
B_{-r,j} &= \mathrm{i}\oint_0 dz\ (\epsilon_j)_\mu \left(\partial X^\mu\,(k\psi) - \psi^\mu(k\partial X) + \frac{1}{2}\psi^\mu(k\psi)\frac{(k\partial\psi)}{(k\partial X)}\right)\frac{\mathrm{e}^{-\mathrm{i}rkX(z)}}{(\mathrm{i}k\partial X)^{\frac{1}{2}}}\ ,
\end{aligned}
\tag{C.2}
$$

where n (r) is a positive integer (half-integer), k is an arbitrary null vector whose scalar product with p_T is one ($p_Tk = 1$), and ϵ_j is any one of the eight unit vectors perpendicular to $\vec{k}$ and with a trivial time component. The spectrum of the physical states of the NS sector of the superstring is generated by the action on the vacuum state in eq. (C.1) of the DDF operators in eq. (C.2), followed by the GSO projection on states with definite worldsheet fermion number. The GSO projection preserves only the states containing an odd number of $B_{-r,j}$, so the first non trivial physical state is obtained by applying the operator $B_{-\frac{1}{2},j}$ to the vacuum in eq. (C.1)

$$
B_{-\frac{1}{2},j}|p_T;0\rangle = -(\epsilon_j\psi_{-\frac{1}{2}})|p_T - \frac{1}{2}k;0\rangle\,, \qquad \left(p_T - \frac{1}{2}k\right)^2 = p_T^2 - p_Tk = 0\ . \tag{C.3}
$$

These are the eight physical polarizations of the massless states corresponding to the covariant vertex operator in eq. (5.24) with momentum $p = p_T - k/2$. We shall be interested in the massive states obtained by acting on the above massless one by applying any number of $A_{-n,i}$ DDF operators. Since each one of them carries a momentum $-nk$ the outcome will be a state of total momentum

$$
p = p_T - (n + \frac{1}{2})k\,, \qquad n = \sum n_k\,, \qquad p^2 = -2n\ . \tag{C.4}
$$

The generic (right moving component of a) state can thus be seen as a collection of photons moving in the direction of k together with a single tachyon moving in a different direction. In a convenient Lorentz frame we can take the latter to move in the opposite direction to k, in other words we can restrict the kinematics to the 2-dimensional space given by k. In this frame, defined modulo a Lorentz boost α along this axis, we can write the different vectors as follows

$$
\begin{aligned}
k &= e^{-\alpha}\,(-1, 0_p; 0_{8-p}, 1)\ , \\
p_T &= (\sinh\alpha, 0_p; 0_{8-p}, \cosh\alpha)\ , \\
p &= \frac{1}{2}\left(e^{\alpha} + 2ne^{-\alpha}, 0_p; 0_{8-p}, e^{\alpha} - 2ne^{-\alpha}\right)\ .
\end{aligned}
\tag{C.5}
$$

The latter expression coincides with the 4-momentum p_2 given in (B.1) provided we identify

$$
\sqrt{\frac{\alpha'}{2}}E = -\frac{1}{2}(e^{\alpha} + 2ne^{-\alpha})\,, \qquad \frac{\alpha'}{2}M^2 = 2n\,, \tag{C.6}
$$

the overall minus sign being there because this is an outgoing string.[24]

[23]The fermionic DDF oscillators $B_{-r,j}$ were originally constructed in refs. [41, 42].

[24]We have reinserted α' because E and M of appendix B have dimension of an energy.

We shall be interested in a high-energy process where, in the rest frame of the branes, the components of p are large for the two external, generic, closed string states. This corresponds to the limit of large boost parameter, $\alpha \gg 1$, in which the momenta carried by the "photons" of the DDF operators are, instead, very small (of order $e^{-\alpha}$). In this regime p_1, as given in (B.6), is also of the same form as in (C.5) and in the limit we have

$$p_1 \to -p_2 \to \frac{1}{2} e^{\alpha}(1, 0_p; 0_{8-p}, 1) \ . \tag{C.7}$$

If we now impose on the string coordinates in (C.2) the particular light-cone gauge corresponding to setting to zero the string oscillations of $k\partial X$ and $k\psi$, the contour integrals become simple (since $nkp = n$ for any p) and the DDF bosonic oscillators reduce to the modes of the transverse bosonic coordinates in that particular light-cone gauge. From eq. (B.4) we see that, at large α, this is precisely the gauge choice that allows us to obtain our simple eikonal operator.

D Polarizations of the massive string states

The massive string states transform in irreducible representations of the little group SO(9) and can be described in terms of irreducible tensors ζ of the Lorentz group which satisfy a transversality condition of the form $p^{\mu_i}\zeta_{\mu_1\ldots\mu_i\ldots\mu_n} = 0$ (where p is the momentum of the state) in all their indices. We restrict our discussion to the left movers and use the following standard notation for the symmetrization and the antisymmetrization of a group of n indices

$$T_{(i_1\ldots i_n)} = \frac{1}{n!} \sum_{\sigma\in S_n} T_{\sigma(i_1)\ldots\sigma(i_n)} \ , \qquad T_{[i_1\ldots i_n]} = \frac{1}{n!} \sum_{\sigma\in S_n} \mathrm{sign}(\sigma) T_{\sigma(i_1)\ldots\sigma(i_n)} \ , \tag{D.1}$$

where the sum is over all the elements σ of the symmetric group S_n.

To every irreducible tensor one can associate a Young diagram which specifies its symmetry type. We write the vertex operators and the corresponding string states as the product of a polarization tensor ζ and a polynomial in the string fields or modes with the same symmetry properties as the polarization tensor. A generic rank-n tensor can be projected onto its irreducible components by the action of the Young symmetrizer corresponding to a Young diagram with n boxes. According to our conventions the Young symmetrizer first symmetrizes the indices in the rows and then antisymmetrizes the indices in the columns. The polarization tensors are therefore totally antisymmetric in the indices corresponding to the columns of the diagram. Moreover if we denote by $\sigma_1 \ldots \sigma_k$ the indices of a given column and by ν any index of the column to its right, the polarization tensors satisfy the following identity

$$\zeta_{\ldots[\sigma_1\ldots\sigma_k\nu]\ldots} = 0 \ . \tag{D.2}$$

Since we are considering the irreducible representations of an orthogonal group, every tensor is also traceless in every couple of indices. Finally the polarization tensor of a generic string state is normalized to one

$$\zeta_{\mu_1\ldots\mu_n}\zeta^{\mu_1\ldots\mu_n} = 1 \ . \tag{D.3}$$

Using the previous identities the normalization coefficients of the massive vertex operators, as for instance those in eq. (5.37), can be easily derived.

As discussed in section 6, to relate the covariant amplitudes and the matrix elements of the eikonal phase it is necessary to decompose the covariant tensors with respect to the SO(8) symmetry group of the space transverse to the collision axis. The covariant polarization ζ^C of a state C in a given representation of the little group SO(9) then decomposes into several polarizations $\zeta^{C,(n_1,\ldots,n_r)}$ transforming as irreducible tensors of SO(8) of type $(n_1,\ldots,n_r)$. Taking into account the transversality condition satisfied by the polarization tensors ζ^C, the SO(8) components $\zeta^{C,(n_1,\ldots,n_r)}$ can be written explicitly in terms of the longitudinal vector v, the Kronecker delta $\delta_\perp$ in the transverse space and an SO(8) polarisation tensor of the required symmetry type $\omega^{(n_1,\ldots,n_r)}$ and satisfying $\omega\cdot\omega=1$.

In section 6 we gave the explicit form of the $\zeta^{C,(n_1,\ldots,n_r)}$ only for the states of the first massive level. Here we collect the formulae for the reduction of the polarization tensors of the states of the second massive level. The state Z in the (3) of SO(9) has the following decomposition with respect to SO(8)

$$\ydiagram{3} \;\mapsto\; \ydiagram{3} + \ydiagram{2} + \ydiagram{1} + \bullet\,, \tag{D.4}$$

or in terms of the polarization tensor

$$\begin{aligned}
\zeta^{Z,(3)}_{\rho\alpha\gamma} &= \omega^{(3)}_{\rho\alpha\gamma}\,,\\
\zeta^{Z,(2)}_{\rho\alpha\gamma} &= \frac{1}{\sqrt{3}}\left(\omega^{(2)}_{\rho\alpha}v_\gamma+\omega^{(2)}_{\alpha\gamma}v_\rho+\omega^{(2)}_{\gamma\rho}v_\alpha\right)\,,\\
\zeta^{Z,(1)}_{\rho\alpha\gamma} &= \frac{1}{\sqrt{330}}\left((\delta_{\perp\rho\alpha}-10v_\rho v_\alpha)\,\omega_\gamma+(\delta_{\perp\alpha\gamma}-10v_\alpha v_\gamma)\,\omega_\rho+(\delta_{\perp\gamma\rho}-10v_\gamma v_\rho)\,\omega_\alpha\right)\,,\\
\zeta^{Z,(0)}_{\rho\alpha\gamma} &= \frac{1}{\sqrt{88}}\left(\delta_{\perp\rho\alpha}v_\gamma+\delta_{\perp\alpha\gamma}v_\rho+\delta_{\perp\gamma\rho}v_\alpha-8v_\rho v_\alpha v_\gamma\right)\,.
\end{aligned}\tag{D.5}$$

The state Y in the $(2,1,1)$ of SO(9) has the following decomposition with respect to SO(8)

$$\ydiagram{2,1,1} \;\mapsto\; \ydiagram{2,1,1} + \ydiagram{1,1,1} + \ydiagram{2,1} + \ydiagram{1,1}\,. \tag{D.6}$$

We decompose the polarization tensor accordingly

$$\begin{aligned}
\zeta^{Y,(2,1,1)}_{\rho\alpha;\gamma;\omega} &= \omega^{(2,1,1)}_{\rho\alpha;\gamma;\omega}\,,\\
\zeta^{Y,(1,1,1)}_{\rho\alpha;\gamma;\omega} &= \frac{\sqrt{3}}{2}\left(\omega^{(1,1,1)}_{\rho\gamma\omega}v_\alpha+\omega^{(1,1,1)}_{\alpha[\gamma\omega}v_{\rho]}\right)\,,\\
\zeta^{Y,(2,1)}_{\rho\alpha;\gamma;\omega} &= \frac{1}{\sqrt{3}}\left(\omega^{(2,1)}_{\rho\alpha\gamma}v_\omega+\omega^{(2,1)}_{\gamma\alpha\omega}v_\rho+\omega^{(2,1)}_{\omega\alpha\rho}v_\gamma\right)\,,\\
\zeta^{Y,(1,1)}_{\rho\alpha;\gamma;\omega} &= \sqrt{\frac{2}{7}}\,3\left(\omega^{(1,1)}_{[\rho\gamma}v_{\omega]}v_\alpha-\frac{1}{6}\omega^{(1,1)}_{[\rho\gamma}\delta_{\perp\omega]\alpha}\right)\,.
\end{aligned}\tag{D.7}$$

The state U in the $(2,1)$ of SO(9) has the following decomposition with respect to SO(8)

$$\ydiagram{2,1} \;\mapsto\; \ydiagram{2,1} + \ydiagram{2} + \ydiagram{1,1} + \ydiagram{1}\,, \tag{D.8}$$

or in terms of the polarization tensor

$$
\begin{aligned}
\zeta^{U,(2,1)}_{\rho\alpha;\gamma} &= \omega^{(2,1)}_{\rho\alpha;\gamma} \,, \\
\zeta^{U,(2)}_{\rho\alpha;\gamma} &= \frac{1}{\sqrt{2}} \left(\omega^{(2)}_{\rho\alpha} v_\gamma - \omega^{(2)}_{\gamma\alpha} v_\rho \right) \,, \\
\zeta^{U,(1,1)}_{\rho\alpha;\gamma} &= \frac{1}{\sqrt{6}} \left(2\omega^{(1,1)}_{\rho\gamma} v_\alpha + \omega^{(1,1)}_{\alpha\gamma} v_\rho - \omega^{(1,1)}_{\alpha\rho} v_\gamma \right) \,, \\
\zeta^{U,(1)}_{\rho\alpha;\gamma} &= \frac{\sqrt{7}}{4} \left(\omega_\gamma v_\rho v_\alpha - \omega_\rho v_\gamma v_\alpha - \frac{1}{7}\omega_\gamma \delta_{\perp\rho\alpha} + \frac{1}{7}\omega_\rho \delta_{\perp\gamma\alpha} \right) \,. \qquad \text{(D.9)}
\end{aligned}
$$

The state V in the $(1,1)$ of SO(9) gives a two-form and a vector of SO(8). Decomposing the polarization tensor we find

$$
\zeta^{V,(1,1)}_{\gamma\omega} = \omega^{(1,1)}_{\gamma\omega} \,, \qquad \zeta^{V,(1)}_{\gamma\omega} = \frac{1}{\sqrt{2}} \left(\omega_\gamma v_\omega - \omega_\omega v_\gamma \right) \,. \qquad \text{(D.10)}
$$

Finally the state W in the vector representation of SO(9) gives a vector and a scalar of SO(8). The corresponding polarization tensors are

$$
\zeta^{W,(1)}_\gamma = \omega_\gamma \,, \qquad \zeta^{W,(0)}_\gamma = v_\gamma \,. \qquad \text{(D.11)}
$$

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