Published for SISSA by Springer

Received: *June 13, 2023*
Revised: *September 9, 2023*
Accepted: *October 9, 2023*
Published: *October 24, 2023*

JHEP10(2023)147

On the decay mode $\Lambda_b \to X_s\gamma$

P. Colangelo, F. De Fazio and F. Loparco

Istituto Nazionale di Fisica Nucleare, Sezione di Bari,
Via Orabona 4, Bari I-70126, Italy

E-mail: pietro.colangelo@ba.infn.it, fulvia.defazio@ba.infn.it, francesco.loparco1@ba.infn.it

Abstract: We study the inclusive $H_b \to X_s\gamma$ decay with H_b a beauty baryon, in particular Λ_b, employing an expansion in the heavy quark mass at $\mathcal{O}(m_b^{-3})$ at leading order in α_s, keeping the dependence on the hadron spin. For a polarized baryon we compute the distribution $\frac{d^2\Gamma}{dy\, d\cos\theta_P}$, with $y = 2E_\gamma/m_b$, E_γ the photon energy and θ_P the angle between the baryon spin vector and the photon momentum in the H_b rest-frame. We discuss the correlation between the baryon and photon polarization, and show that effects of physics beyond the Standard Model can modify the photon polarization asymmetry. We also discuss a method to treat the singular terms in the photon energy spectrum obtained by the OPE.

Keywords: Bottom Quarks, Nonperturbative Effects, Rare Decays

ArXiv ePrint: 2306.02748

https://doi.org/10.1007/JHEP10(2023)147

Contents

1 Introduction

The processes induced at the quark level by the $b \to s\gamma$ transition are recognized as a powerful testground of the Standard Model (SM) [1–3]. They occur at loop-level in SM and are sensitive to heavy particle exchanges. Upon integration of the heavy quanta, an effective Hamiltonian is obtained in terms of local operators and Wilson coefficients [4, 5]. Physics beyond the Standard Model induces new operators with respect to the SM ones and modifies the Wilson coefficients, hence measurements of various observables tightly constrain operators and coefficients. This allows to probe the SM and select the possible extensions [6–13].

The radiative $b \to s$ transition has been intensively analyzed in theory.[1] Strong experimental efforts have also been devoted since the first observation of the $B \to K^*(892)\gamma$ mode [15]. Several measurements of exclusive processes are now available, namely $\mathcal{B}(B^+ \to K^{*+}(892)\gamma)$ and $\mathcal{B}(B^0 \to K^{*0}(892)\gamma)$, and the rates of $B \to (K_1(1400), K_2^*(1430), K_3^*(1780))\gamma$, $B \to K\eta\gamma$, $B_s \to \phi(1020)\gamma$ [16, 17]. Time-dependent CP asymmetries for decaying neutral mesons have been investigated [16]. For baryons, the rate and the photon polarization of $\Lambda_b \to \Lambda\gamma$ have been measured [18, 19], and an upper bound has been put to $\mathcal{B}(\Xi_b^- \to \Xi^-\gamma)$ [20]. For such exclusive processes, the hadronic uncertainties are related to the form factors parametrizing the matrix elements of the local operators at $q^2 = 0$, with q the photon momentum [21].

[1] A detailed discussion and a list of references can be found in [14].

JHEP10(2023)147

Among the inclusive $H_b \to X_s\gamma$ modes the prime example is $\bar{B} \to X_s\gamma$. Here we focus on H_b a baryon, in particular on $\Lambda_b \to X_s\gamma$. The peculiarity of the inclusive modes consists in the possibility, invoking quark-hadron duality, of exploiting a well defined theoretical framework based on controlled expansions in QCD quantities $\alpha_s(m_b)$ and $1/m_b$ (the heavy quark expansion, HQE) to compute decay rates, decay distributions and moments of the distributions. At the leading order in the heavy quark expansion the partonic result is recovered, NLO terms involve nonperturbative corrections. In particular, the combination of Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET) [22] allows to express the inclusive decay widths as an expansion in the inverse heavy quark mass. Based on this approach, inclusive semileptonic modes of hadrons comprising a single heavy quark are exploited to access fundamental parameters, the heavy quark masses and elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. Input quantities are the hadronic matrix elements of local operators, starting from the kinetic energy and the chromomagnetic operators, which are defined in the following. Differential distributions together with other observables can also be described. In $B \to X_s\gamma$, direct CP asymmetries sensitive to new physics (NP) effects can be studied upon accounting for long-distance contributions [23, 24].

There are several issues in inclusive modes induced by the $b \to s\gamma$ transition needing to be considered. The actual expansion parameter is the inverse of the energy released in the process. Such energy is $\mathcal{O}(m_b)$ in a portion of the phase-space, but in some regions its inverse is no longer small. Signals about the reliability of the method show up as singularities in differential distributions. This occurs in the calculation of perturbative corrections to the spectra, where Sudakov terms appear.[2] Singular terms appear at higher orders in the HQE in form of the delta distribution and its derivatives, the argument of which vanishes in the regions corresponding to the endpoints of the differential distributions determined in the partonic kinematics, different from the hadronic kinematics. The gap between the two borders is governed by nonperturbative physics responsible of bound state effects. Such effects can be related to the Fermi motion of the heavy quark in the decaying hadron, and can be accounted for introducing a shape function which encodes information on the distribution of the b quark residual momentum in the hadron [27–29]. The same function enters in the description of different inclusive modes, and it affects the photon energy spectrum in $H_b \to X_s\gamma$. In case of B, the moments of the shape function have been constrained using measurements [16, 30].

Another source of uncertainty in inclusive $b \to s\gamma$ processes are the resolved photon contributions [31], related to the photon couplings different from the effective weak interaction vertex. The most important operators giving rise to these contributions are O_2 and O_8 (in the notation specified in the following section). Such effects appear at $\mathcal{O}(1/m_b)$ and produce contributions to the total decay width which are not described by the HQE [32, 33]. They can be expressed in terms of subleading shape functions [31]. The resolved photon contributions include a nonperturbative term proportional to the matrix element μ_G^2 of the

[2] Sudakov terms (referred to as Sudakov shoulders) located in the middle of the phase-space can also be present when singularities from real and virtual perturbative corrections do not compensate each other [25, 26].

JHEP10(2023)147

chromomagnetic operator, related to the gluon-photon penguin mechanism, for which the short distance scale is $1/m_c$ rather than $1/m_b$ [34–37]. This is estimated to be small in $\bar{B} \to X_s\gamma$. For Λ_b the matrix element of the chromomagnetic operator vanishes, therefore the comparison of the inclusive Λ_b and B radiative decay widths provides a way to shed light on the role of such corrections.

In radiative modes the photon spectrum can be measured above an energy threshold. For $\bar{B}$ the HFLAV collaboration quotes $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.49 \pm 0.19) \times 10^{-4}$ for $E_\gamma > 1.6$ GeV, with both charged and neutral mesons included in the average. The SM result is $\mathcal{B}(\bar{B} \to X_s\gamma) = (3.36 \pm 0.23) \times 10^{-4}$ for the same threshold [5]. A global fit including all data on the photon energy spectrum in $\bar{B} \to X_s\gamma$ has been performed by the SIMBA collaboration [30].

The interest for inclusive $\Lambda_b \to X_s\gamma$ relies on the possibility in baryon modes to investigate observables sensitive to the spin of the decaying hadron. This is important for the planned new lepton facilities, since heavy baryons with a b-quark produced from Z^0 and top-quark decays are expected to have a sizable polarization, as observed at LEP [38–40]. The application of HQE to baryons requires new information, namely the operator matrix elements for specified hadron spin. Such matrix elements have been analyzed in [41]. Moreover, the leading and subleading shape functions are different for different hadrons, namely for Λ_b and B, and require dedicated considerations.

In the present study we focus on two issues. The first one is the dependence on the heavy baryon spin in a double differential decay distribution, considering hadronic matrix elements at $\mathcal{O}(1/m_b^3)$, for the leading operator in the SM effective weak Hamiltonian and for a single new physics (NP) operator, studying the correlations between the baryon and photon polarization. The second one is a way to treat the singular terms in the inclusive photon spectrum to reconstruct the Λ_b leading shape function, a method which can be systematically applied when higher order terms in the heavy quark expansion are computed.

The plan of the paper is the following. Section 2 includes the $b \to s\gamma$ low-energy Hamiltonian with SM operators and operators obtained in extensions of the Standard Model. In section 3 we describe the application of the HQE to the inclusive $H_b \to X_s\gamma$ process with H_b a baryon, in particular Λ_b, keeping the dependence on the baryon spin. In section 4 we investigate the correlation between the photon and Λ_b polarizations. A treatment of the singular terms in the double differential decay rate is discussed in section 5. Details are collected in the appendices. In the last section we present our conclusions and the perspectives for further progress.

2 $b \to s\gamma$ effective Hamiltonian

The low-energy Hamiltonian governing the $\Delta B = -1$, $\Delta S = 1$ $b \to s\gamma$ transition can be written as

$$H_{\text{eff}}^{b\to s\gamma} = -\,4\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_i \left[C_i(\mu)O_i + C_i'(\mu)O_i'\right] \, , \tag{2.1}$$

with $i = 1, \dots 8$ and $i = 15, \dots 20$. G_F is the Fermi constant and V_{jk} are elements of the CKM matrix. Doubly Cabibbo-suppressed terms proportional to $V_{ub}V_{us}^*$ have been neglected in (2.1). The effective Hamiltonian comprises the magnetic penguin operators

$$O_7 = \frac{e}{16\pi^2}[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)\, b]F_{\mu\nu}\,, \tag{2.2}$$

$$O_8 = \frac{g_s}{16\pi^2}\left[\bar{s}_\alpha\sigma^{\mu\nu}\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}(m_s P_L + m_b P_R) b_\beta\right] G^a_{\mu\nu}\,, \tag{2.3}$$

with $P_{R,L} = \frac{1\pm\gamma_5}{2}$ helicity projectors, α, β colour indices, λ^a the Gell-Mann matrices. $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the electromagnetic and gluonic field strengths, e and g_s the electromagnetic and strong coupling constants, m_b and m_s the b and s quark mass. The Hamiltonian also comprises the current-current operators $O_{1,2}$,

$$O_1 = (\bar{s}_\alpha\gamma^\mu P_L\, c_\beta)(\bar{c}_\beta\gamma_\mu P_L\, b_\alpha)\,, \tag{2.4}$$

$$O_2 = (\bar{s}\gamma^\mu P_L\, c)(\bar{c}\gamma_\mu P_L\, b), \tag{2.5}$$

and the QCD penguin operators $O_{i=3,\ldots 6}$,

$$O_3 = (\bar{s}\gamma^\mu P_L\, b)\sum_q(\bar{q}\gamma^\mu P_L\, q)\,, O_4 = (\bar{s}_\alpha\gamma^\mu P_L\, b_\beta)\sum_q(\bar{q}_\beta\gamma^\mu P_L\, q_\alpha)\,, \tag{2.6}$$

$$O_5 = (\bar{s}\gamma^\mu P_L\, b)\sum_q(\bar{q}\gamma^\mu P_R\, q)\,, O_6 = (\bar{s}_\alpha\gamma^\mu P_L\, b_\beta)\sum_q(\bar{q}_\beta\gamma^\mu P_R\, q_\alpha). \tag{2.7}$$

The sum in (2.6)–(2.7) runs over the flavours $q = u, d, s, c, b$. The remaining operators, absent in SM, are analogous to the QCD penguins but have a scalar or tensor structure [42]:

$$\begin{aligned}
O^q_{15} &= (\bar{s}P_R b)\sum_q(\bar{q}P_R q)\,, & O^q_{16} &= (\bar{s}_\alpha P_R b_\beta)\sum_q(\bar{q}_\beta P_R q_\alpha)\,, \\
O^q_{17} &= (\bar{s}P_R b)\sum_q(\bar{q}P_L q)\,, & O^q_{18} &= (\bar{s}_\alpha P_R b_\beta)\sum_q(\bar{q}_\beta P_L q_\alpha)\,, \\
O^q_{19} &= (\bar{s}\sigma^{\mu\nu}P_R b)\sum_q(\bar{q}\sigma_{\mu\nu}P_R q)\,, & O^q_{20} &= (\bar{s}_\alpha\sigma^{\mu\nu}P_R b_\beta)\sum_q(\bar{q}_\beta\sigma_{\mu\nu}P_R q_\alpha)\,.
\end{aligned} \tag{2.8}$$

The primed operators have opposite chirality with respect to the unprimed ones.

In SM the process $b \to s\gamma$ is described by photon penguin diagrams, with the photon coupled either to the intermediate fermion or to the $W^\pm$, giving rise to the magnetic operator O_7. This is the only operator contributing at lowest order in QCD. The renormalization group evolution to the scale $\mu_b \simeq \mathcal{O}(m_b)$ also involves the magnetic gluon penguin operator O_8 and the operators $O_{1,\ldots 6}$. Their mixing into O_7 generates large logarithms producing a strong enhancement of the rate. The anomalous dimension matrix governing the mixing turns out to be regularization scheme dependent. One can get rid of such a dependence defining an effective coefficient $C_7^{\rm eff}(\mu_b)$ which includes contributions of $O_{1,\ldots 6}$ [43]. In this way O_7 turns out to be the dominant contribution to $b \to s\gamma$, with the SM Wilson coefficients known at NNLO in QCD [14, 44]. Extensions of the SM can also induce the operators O^q_{15}–O^q_{20} and the primed operators in the low-energy Hamiltonian.

In this paper we work at the leading order in α_s so that the only operator mediating the $b \to s\gamma$ transition is O_7 in SM and possibly O_7' beyond SM, the effect of $O^{(\prime)}_{1,\ldots 6}$ being

JHEP10(2023)147

included in the effective coefficients. Therefore, we consider the effective Hamiltonian at the scale μ_b consisting of only two operators,

$$H_{\text{eff}}^{b\to s\gamma} = -\,4\,\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_7^{\text{eff}} O_7 + C_7^{\prime\text{eff}} O_7^\prime \right\}. \tag{2.9}$$

We do not consider operators from NLO electroweak corrections.

$H_{\text{eff}}^{b\to s\gamma}$ can be recast in a way suitable for the heavy quark expansion:

$$H_{\text{eff}}^{b\to s\gamma} = -\,4\,\frac{G_F}{\sqrt{2}} \lambda_t \,\frac{e}{16\pi^2} \sum_{i=7,7^\prime} C_i^{\text{eff}}\, J_{\mu\nu}^i\, F^{\mu\nu}\,, \tag{2.10}$$

where $\lambda_t = V_{tb} V_{ts}^*$, $J_{\mu\nu}^i = [\bar{s}\sigma_{\mu\nu}(m_s(1-P_i) + m_b P_i)\, b]$ and $P_i = P_R$ for $i = 7$, $P_i = P_L$ for $i = 7^\prime$. In the next section we compute the inclusive width $\Gamma(H_b \to X_s\gamma)$ using the Hamiltonian (2.10), as done for the semileptonic modes in [41].

3 Inclusive decay width

To describe the inclusive mode $H_b(p,s) \to X_s(p_X)\gamma(q,\epsilon)$ we preliminarily define

$$\mathcal{F}^{MN} = \mathcal{F}^{\mu\nu\mu^\prime\nu^\prime} = \sum_\epsilon 4q^\nu\, q^{\nu^\prime}\, \epsilon^\mu \epsilon^{*\mu^\prime} = -4q^\nu\, q^{\nu^\prime}\, g^{\mu\mu^\prime}, \tag{3.1}$$

where q and ϵ are the photon momentum and polarization four-vector, respectively, using the compact notation $M = \mu\,\nu$, $N = \mu^\prime\,\nu^\prime$. To obtain the results specifying the photon polarization we also define

$$\begin{aligned} \mathcal{F}_+^{MN} &= 4q^\nu\, q^{\nu^\prime}\, \epsilon_+^\mu \epsilon_+^{*\mu^\prime}\,, \\ \mathcal{F}_-^{MN} &= 4q^\nu\, q^{\nu^\prime}\, \epsilon_-^\mu \epsilon_-^{*\mu^\prime}\,, \end{aligned} \tag{3.2}$$

where

$$\epsilon_\pm = \mp\,\frac{1}{\sqrt{2}}\,(0,1,\pm i,0)\,. \tag{3.3}$$

The differential inclusive decay width can be written as

$$d\Gamma = [dq]\,\frac{G_F^2|\lambda_t|^2}{8m_{H_b}}\,\frac{\alpha}{\pi^2} \sum_{i,j=7,7^\prime} C_i^{\text{eff}*} C_j^{\text{eff}}\; W_{MN}^{ij} \mathcal{F}^{MN}, \tag{3.4}$$

with $[dq] = \frac{d^3q}{(2\pi)^3 2q^0}$. By the optical theorem, the hadronic tensor W_{MN}^{ij} is related to the discontinuity of the forward scattering amplitude

$$T_{MN}^{ij} = i \int d^4x\, e^{-i\,q\cdot x} \langle H_b(p,s)|T[J_M^{i\dagger}(x)\, J_N^j(0)]|H_b(p,s)\rangle \tag{3.5}$$

across the cut corresponding to the process $H_b(p,s) \to X_s(p_X)\gamma(q,\epsilon)$:

$$W_{MN}^{ij} = \frac{1}{\pi}\text{Im}\, T_{MN}^{ij}. \tag{3.6}$$

The range of the invariant mass p_X^2 of the states produced in B and Λ_b decays (with $p_X = p - q$) is $p_X^2 \in [m_{K^*}^2, m_B^2]$ and $p_X^2 \in [m_\Lambda^2, m_{\Lambda_b}^2]$, respectively. For $m_b \to \infty$, p_X^2 is

JHEP10(2023)147

almost always large enough to exploit the short distance limit $x \to 0$ in eq. (3.5), thus allowing a computation of T^{ij} and W^{ij} by an OPE with expansion parameter $\frac{1}{m_b}$ [45, 46]. The first term of the expansion describes the free beauty quark decay, the partonic result. The expansion is valid in the largest part of the phase-space, it fails in the region with small p_X^2, therefore a reliable computation of the decay width and of moments of the decay distributions can be carried out. For spectra, the result obtained by the short distance OPE needs to be smeared: in section 5 we discuss a way to implement the smearing.

The procedure for the derivation of the OPE for eq. (3.5) is summarized in appendix A. Using the definition

$$\tilde{\mathcal{T}}^{ij} = T^{ij}_{MN}\mathcal{F}^{MN} \tag{3.7}$$

we obtain the expression:

$$\begin{aligned}\sum_{i,j=7,7'} C_i^{\text{eff}*}\, C_j^{\text{eff}}\, \tilde{\mathcal{T}}^{ij} \,=\, &\left[(m_b^2+m_s^2)\,(|C_7^{\text{eff}}|^2+|C_7'^{\,\text{eff}}|^2)+4\,m_b\,m_s\,\text{Re}[C_7^{\text{eff}}\,C_7'^{\,\text{eff}*}]\right]\tilde{T}\\ &+(m_b^2-m_s^2)\,(|C_7^{\text{eff}}|^2-|C_7'^{\,\text{eff}}|^2)\,\tilde{S},\end{aligned} \tag{3.8}$$

where

$$\begin{aligned}\tilde{T} \,&=\, 16\,m_H\,(v\cdot q)^2\sum_{n=1}^{4}\left(\frac{m_b}{\Delta_0}\right)^n\,\tilde{T}_n,\\ \tilde{S} \,&=\, 16\,m_H\,(v\cdot q)\,(q\cdot s\,)\sum_{n=1}^{4}\left(\frac{m_b}{\Delta_0}\right)^n\,\tilde{S}_n,\end{aligned} \tag{3.9}$$

with v and Δ_0 defined in appendix A. The four terms in (3.9) involve the hadronic parameters $\hat{\mu}_\pi^2$, $\hat{\mu}_G^2$, $\hat{\rho}_D^3$ and $\hat{\rho}_{LS}^3$ defined in the same appendix, and read:

$$\tilde{T}_1 \,=\, 1+\frac{5}{6\,m_b^2}\,[\hat{\mu}_\pi^2-\hat{\mu}_G^2]-\frac{2}{3m_b^3}\,[\hat{\rho}_D^3+\hat{\rho}_{LS}^3], \tag{3.10}$$

$$\tilde{S}_1 \,=\, 1+\frac{1}{4m_b^2}\,[\hat{\mu}_\pi^2+\hat{\mu}_G^2]+\frac{1}{6m_b^3}\,\hat{\rho}_D^3, \tag{3.11}$$

$$\tilde{T}_2 \,=\, \frac{7v\cdot q}{3m_b^2}\,\hat{\mu}_\pi^2+\frac{1}{3m_b^2}(4m_b-5v\cdot q)\hat{\mu}_G^2+\frac{2}{3m_b^3}\left[(4m_b-3v\cdot q)\,\hat{\rho}_D^3+(2m_b-3v\cdot q)\hat{\rho}_{LS}^3\right], \tag{3.12}$$

$$\tilde{S}_2 \,=\, \frac{7v\cdot q}{3m_b^2}\,\hat{\mu}_\pi^2+\frac{1}{m_b^2}(2m_b-v\cdot q)\hat{\mu}_G^2+\frac{2}{3m_b^3}(4m_b-v\cdot q)\,\hat{\rho}_D^3, \tag{3.13}$$

$$\tilde{T}_3 \,=\, \frac{4}{3m_b^2}(v\cdot q)^2\hat{\mu}_\pi^2+\frac{4}{3m_b^3}(m_b-v\cdot q)(v\cdot q)(2\hat{\rho}_D^3+\hat{\rho}_{LS}^3), \tag{3.14}$$

$$\tilde{S}_3 \,=\, \frac{4}{3m_b^2}(v\cdot q)^2\hat{\mu}_\pi^2+\frac{8}{3m_b^3}(m_b-v\cdot q)(v\cdot q)\,\hat{\rho}_D^3, \tag{3.15}$$

$$\tilde{T}_4 \,=\, \frac{8}{3m_b^3}(m_b-v\cdot q)(v\cdot q)^2\hat{\rho}_D^3, \tag{3.16}$$

$$\tilde{S}_4 \,=\, \frac{8}{3m_b^3}(m_b-v\cdot q)(v\cdot q)^2\hat{\rho}_D^3. \tag{3.17}$$

Using these results, from eq. (3.4) the distribution $\frac{d^2\Gamma}{dy\,d\cos\theta_P}$ in the photon energy $y = 2\,E_\gamma/m_b$ and $\cos\theta_P$ can be computed, θ_P being the angle between the photon momentum

$\vec{q}$ and the H_b spin vector $\vec{s}$ in the H_b rest frame. Upon integration, the photon energy spectrum, the $\cos\theta_P$ distribution and the decay width are obtained.

In the H_b rest frame $v \cdot q = E_\gamma = \frac{m_b}{2}\, y$, and the distribution comprises two terms:

$$\frac{d^2\Gamma}{dy\, d\cos\theta_P} = \tilde{\Gamma}_1 + \cos\theta_P\, \tilde{\Gamma}_2\,. \tag{3.18}$$

Integrating (3.18) on $\cos\theta_P$ one has $\tilde{\Gamma}_1 = \frac{1}{2}\frac{d\Gamma}{dy}$ and the photon energy spectrum

$$\begin{aligned}
\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = &\left[1 - \frac{\hat{\mu}_\pi^2}{2\, m_b^2} - \frac{\hat{\mu}_G^2}{2\, m_b^2}\,\frac{3+5\, z}{1-z} - \frac{10\, \hat{\rho}_D^3}{3\, m_b^3}\,\frac{1+z}{1-z}\right]\delta(1-z-y) \\
&+\left[\frac{\hat{\mu}_\pi^2}{2\, m_b^2}\,(1-z) - \frac{\hat{\mu}_G^2}{6\, m_b^2}\,(3+5\, z) - \frac{4\, \hat{\rho}_D^3}{3\, m_b^3}\,(1+2\, z) + \frac{2\, \hat{\rho}_{LS}^3}{3\, m_b^3}\,(1+z)\right]\delta'(1-z-y) \\
&+\left[\frac{\hat{\mu}_\pi^2}{6\, m_b^2}\,(1-z)^2 - \frac{\hat{\rho}_D^3}{3\, m_b^3}\,(1-z)\,(1+2\, z) + \frac{\hat{\rho}_{LS}^3}{6\, m_b^3}\,(1-z^2)\right]\delta''(1-z-y) \\
&-\frac{\hat{\rho}_D^3}{18\, m_b^3}\,(1-z)^2\,(1+z)\,\delta'''(1-z-y)\,,
\end{aligned} \tag{3.19}$$

where $z = \frac{m_s^2}{m_b^2}$ and

$$\Gamma_0 = \frac{\alpha\, G_F^2\, |\lambda_t|^2}{32\, \pi^4}\, m_b^5\, (1-z)^3\, \left[|C_+^{\rm eff}|^2 + |C_+^{\prime\, \rm eff}|^2\right]\,, \tag{3.20}$$

with

$$C_+^{\rm eff} = C_7^{\rm eff} + \sqrt{z}\, C_7^{\prime\, \rm eff}\,, \qquad\qquad C_+^{\prime\, \rm eff} = \sqrt{z}\, C_7^{\rm eff} + C_7^{\prime\, \rm eff}\,.$$

$\tilde{\Gamma}_2$ is given by:

$$\begin{aligned}
\frac{2}{\Gamma_0}\tilde{\Gamma}_2 = &-\frac{|C_+^{\rm eff}|^2 - |C_+^{\prime\, \rm eff}|^2}{|C_+^{\rm eff}|^2 + |C_+^{\prime\, \rm eff}|^2} \\
&\times\left\{\left[1 - \frac{13\, \hat{\mu}_\pi^2}{12\, m_b^2} - \frac{3\, \hat{\mu}_G^2}{4\, m_b^2}\,\frac{5+3\, z}{1-z} - \frac{\hat{\rho}_D^3}{6\, m_b^3}\,\frac{31+9\, z}{1-z}\right]\delta(1-z-y)\right. \\
&+\left[\frac{\hat{\mu}_\pi^2}{2\, m_b^2}\,(1-z) - \frac{\hat{\mu}_G^2}{2\, m_b^2}\,(3+z) - \frac{2\, \hat{\rho}_D^3}{m_b^3}\,(1+z)\right]\delta'(1-z-y) \\
&+\left[\frac{\hat{\mu}_\pi^2}{6\, m_b^2}\,(1-z)^2 - \frac{\hat{\rho}_D^3}{3\, m_b^3}\,(1-z)\,(1+2\, z)\right]\delta''(1-z-y) \\
&\left.-\frac{\hat{\rho}_D^3}{18\, m_b^3}\,(1-z)^2\,(1+z)\,\delta'''(1-z-y)\right\}.
\end{aligned} \tag{3.21}$$

The $\cos\theta_P$ distribution also comprises two terms:

$$\frac{d\Gamma(H_b \to X_s\, \gamma)}{d\cos\theta_P} = A + B\, \cos\theta_P\,, \tag{3.22}$$

with

$$A = \frac{1}{2}\Gamma(H_b \to X_s\,\gamma)\,, \tag{3.23}$$

$$B = -\frac{\Gamma_0}{2}\frac{|C_+^{\rm eff}|^2 - |C_+^{\prime\,\rm eff}|^2}{|C_+^{\rm eff}|^2 + |C_+^{\prime\,\rm eff}|^2}\left[1 - \frac{13\,\hat\mu_\pi^2}{12\,m_b^2} - \frac{3\,\hat\mu_G^2}{4\,m_b^2}\frac{5+3\,z}{1-z} - \frac{\hat\rho_D^3}{6\,m_b^3}\frac{31+9\,z}{1-z}\right]. \tag{3.24}$$

The inclusive $H_b \to X_s\,\gamma$ decay width is given by

$$\Gamma(H_b \to X_s\,\gamma) = \Gamma_0\left[1 - \frac{\hat\mu_\pi^2}{2\,m_b^2} - \frac{\hat\mu_G^2}{2\,m_b^2}\frac{3+5\,z}{1-z} - \frac{10\,\hat\rho_D^3}{3\,m_b^3}\frac{1+z}{1-z}\right]. \tag{3.25}$$

The SM result is recovered for $C_7^\prime \to 0$. At $\mathcal{O}(1/m_b^2)$ and for $m_s = 0$ eqs. (3.25) and (3.19) agree with the SM expressions obtained in [47, 48]. At $\mathcal{O}(1/m_b^3)$ they agree with the expressions in [49] substituting

$$\begin{aligned}
\hat\mu_\pi^2 &\to -\left(\lambda_1 + \frac{\mathcal{T}_1 + 3\,\mathcal{T}_3}{m_b}\right)\\
\hat\mu_G^2 &\to 3\left(\lambda_2 + \frac{\mathcal{T}_3 + 3\,\mathcal{T}_4}{m_b}\right) - \frac{\rho_1 + 3\,\rho_2}{m_b}\\
\hat\rho_D^3 &\to \rho_1\\
\hat\rho_{LS}^3 &\to 3\,\rho_2\,.
\end{aligned} \tag{3.26}$$

For $m_s = 0$ the $\mathcal{O}(1/m_b^4)$ corrections to the decay width have been computed in [50]. For Λ_b, the distribution (3.22) has been computed at $\mathcal{O}(1/m_b^2)$ [51], we agree with such a result.

4 Photon polarization

The photon polarization in the $b \to s\gamma$ transition can be measured in radiative beauty baryon decays. In SM the photon polarization asymmetry A_P, measuring the relative abundance of left-handed with respect to right-handed photons, is predicted $A_P \simeq -1$. Deviations from this result would hint physics beyond the Standard Model. The photon polarization has been studied for the exclusive [52–57] and inclusive beauty baryon decays [51], the experimental feasibility at LHCb has also been scrutinized [58].

The correlation between the photon polarization and the initial b-baryon polarization is particularly relevant, and it can be obtained considering the A_P dependence on $\cos\theta_P$. We refer to Λ_b, however our results can be applied also to other beauty baryons. Preliminary experimental analyses for Σ_b are reported in [20].

To present the results for the photon polarizations $\epsilon_\pm$ we write the double differential width as

$$\frac{d^2\Gamma_\pm}{dy\,d\cos\theta_P} = \tilde\Gamma_{\pm,1} + \tilde\Gamma_{\pm,2}\,\cos\theta_P\,, \tag{4.1}$$

where $\tilde\Gamma_{\pm,1} = \frac{1}{2}\frac{d\Gamma_\pm}{dy}$. The energy spectrum for definite photon helicities has the expression

$$\frac{d\Gamma_+}{dy} = R_{+,1}\,\frac{d\Gamma}{dy} = \frac{|C_+^{\prime\,\rm eff}|^2}{|C_+^{\rm eff}|^2 + |C_+^{\prime\,\rm eff}|^2}\,\frac{d\Gamma}{dy}\,, \tag{4.2}$$

$$\frac{d\Gamma_-}{dy} = R_{-,1}\,\frac{d\Gamma}{dy} = \frac{|C_+^{\rm eff}|^2}{|C_+^{\rm eff}|^2 + |C_+^{\prime\,\rm eff}|^2}\,\frac{d\Gamma}{dy}\,. \tag{4.3}$$

$\tilde{\Gamma}_{\pm,2}$ are given by

$$\tilde{\Gamma}_{+,2} = R_{+,2}\,\tilde{\Gamma}_2 = -\frac{|C'^{\rm eff}_+|^2}{|C^{\rm eff}_+|^2 - |C'^{\rm eff}_+|^2}\,\tilde{\Gamma}_2\,, \tag{4.4}$$

$$\tilde{\Gamma}_{-,2} = R_{-,2}\,\tilde{\Gamma}_2 = \frac{|C^{\rm eff}_+|^2}{|C^{\rm eff}_+|^2 - |C'^{\rm eff}_+|^2}\,\tilde{\Gamma}_2\,. \tag{4.5}$$

In SM (for $C'^{\rm eff}_7 \to 0$) we have:

$$\frac{d\Gamma^{\rm SM}_+}{dy} = \frac{z}{1+z}\,\frac{d\Gamma^{\rm SM}}{dy}\,, \tag{4.6}$$

$$\frac{d\Gamma^{\rm SM}_-}{dy} = \frac{1}{1+z}\,\frac{d\Gamma^{\rm SM}}{dy}\,, \tag{4.7}$$

and

$$\tilde{\Gamma}^{\rm SM}_{+,2} = -\frac{z}{1-z}\,\tilde{\Gamma}^{\rm SM}_2\,, \tag{4.8}$$

$$\tilde{\Gamma}^{\rm SM}_{-,2} = \frac{1}{1-z}\,\tilde{\Gamma}^{\rm SM}_2. \tag{4.9}$$

In SM for $m_s = 0$ only the polarization ϵ_- contributes. Indeed, the operator O_7 produces a right-handed b quark; the massless s quark has fixed helicity and, due to the angular momentum conservation, the b and s quarks have spins aligned to the photon spin, in the opposite direction.

For polarized photons the $\cos\theta_P$ distributions have the form

$$\frac{d\Gamma_\pm(H_b \to X_s\,\gamma)}{d\cos\theta_P} = A_\pm + B_\pm\,\cos\theta_P\,, \tag{4.10}$$

where

$$\begin{aligned} A_\pm &= \frac{\Gamma_\pm(H_b \to X_s\,\gamma)}{2} = R_{\pm,1}\,A\,,\\ B_\pm &= R_{\pm,2}\,B\,, \end{aligned} \tag{4.11}$$

and $R_{\pm,1(2)}$ defined in eqs. (4.2)–(4.5). The decay widths to polarized photons are given by

$$\Gamma_\pm(H_b \to X_s\,\gamma) = R_{\pm,1}\,\Gamma(H_b \to X_s\,\gamma). \tag{4.12}$$

The photon polarization asymmetry is defined as

$$A_P(\cos\theta_P) = \frac{\frac{d\Gamma_+}{d\cos\theta_P} - \frac{d\Gamma_-}{d\cos\theta_P}}{\frac{d\Gamma_+}{d\cos\theta_P} + \frac{d\Gamma_-}{d\cos\theta_P}}\,. \tag{4.13}$$

In SM the photon polarization asymmetry is $A_P(\cos\theta_P) \simeq -1$ for almost all $\cos\theta_P$, it increases only for $\cos\theta_P \to 1$, see figure 1. Physics beyond SM can produce a sizable effect. For a quantitative insight on the possible deviation from SM, we consider ranges for $C^{\rm NP}_7 = C^{\rm eff}_7 - (C^{\rm eff}_7)^{\rm SM}$ and $C'^{\rm eff}_7$, assuming that both coefficients are real, exploiting the results of a global fit of the $b \to s$ transitions [59]. Using $m_b = 4.62\,{\rm GeV}$, $m_s = 0.150\,{\rm GeV}$, $\hat{\mu}^2_\pi(\Lambda_b) = 0.5\,{\rm GeV}^2$, $\hat{\rho}^3_D(\Lambda_b) = 0.17\,{\rm GeV}^3$, $\hat{\mu}^2_G(\Lambda_b) = \hat{\rho}^3_{LS}(\Lambda_b) = 0$, and varying $C'^{\rm eff}_7/C^{\rm eff}_7 \in [-0.3, 0.3]$ we obtain the asymmetry shown in figure 2. In the same figure we plot $A_P(\cos\theta_P)$ versus $C'^{\rm eff}_7/C^{\rm eff}_7$ for selected values of $\cos\theta_P$. A deviation of the polarization asymmetry from the SM value can be obtained, with the largest effect for $\cos\theta_P \simeq 1$.

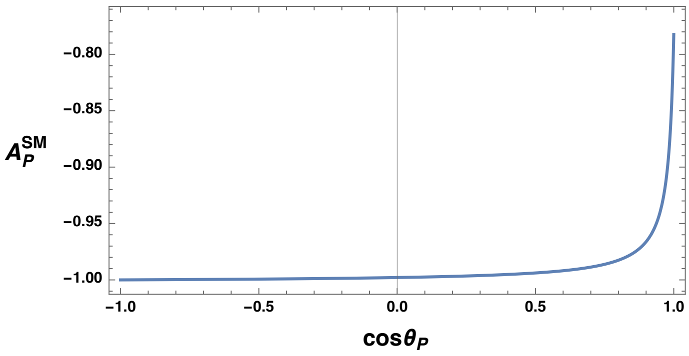


Figure 1. Photon polarization asymmetry eq. (4.13) versus $\cos\theta_P$ in SM.

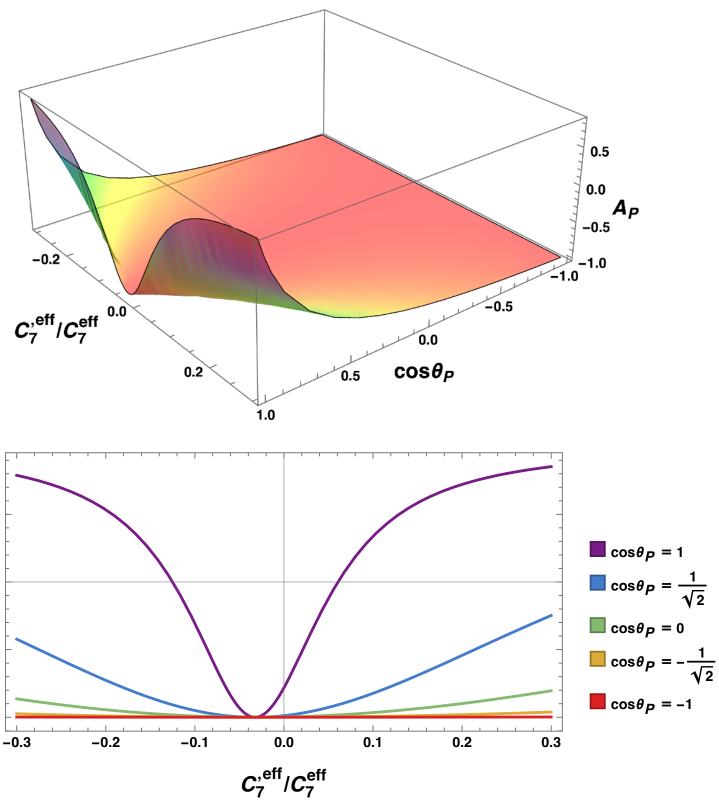


Figure 2. Photon polarization asymmetry eq. (4.13) varying $\cos\theta_P$ and $C_7^{\prime\,\mathrm{eff}}/C_7^{\mathrm{eff}}$ (top panel) and projected for several values of $\cos\theta_P$ (bottom panel).

5 Treatment of the singular terms

The spectrum obtained by the short distance OPE does not account for the Fermi motion of the b quark due to soft interactions with the light degrees of freedom in the hadron. For decays of a beauty hadron H_b to light partons the relevant scales are m_b, $\sqrt{m_b\Lambda_{\mathrm{QCD}}}$ and Λ_{QCD} [60]. The "shape function" region is the kinematic region where the hadronic invariant mass $p_X^2 \sim \mathcal{O}(m_b\Lambda_{\mathrm{QCD}})$. Suitable effective field theory must be used in the various regions: QCD is first matched to the soft collinear effective theory (SCET) [61–64], followed by matching to HQET [60, 65]. In the shape function region the short distance OPE is replaced by a twist expansion, with the infinite set of power corrections resummed into nonperturbative functions.[3]

At leading order in the HQE there is a single shape function defined in HQET. Considering the process $H_b \to X_s \gamma$, one defines the spectral function $S_s(y)$ [28]

$$S_s(y) = \langle \delta \left[1 - y - z + \frac{2}{m_b}(v - \hat{q}) \cdot iD \right] \rangle, \tag{5.1}$$

with $\hat{q} = q/m_b$. For a generic operator $\mathcal{O}$ the matrix element in (5.1) is defined as

$$\langle \mathcal{O} \rangle = \frac{\langle H_b(v) | \bar{h}_v \mathcal{O} h_v | H_b(v) \rangle}{\langle H_b(v) | \bar{h}_v h_v | H_b(v) \rangle}, \tag{5.2}$$

with h_v the HQET field with velocity v. Introducing the vector $n_\mu + \delta n_\mu = 2(v - \hat{q})_\mu|_{y=1-z}$, with $n^2 = 0$, $v \cdot n = 1$, and $n \cdot \delta n \sim O(\Lambda_{\mathrm{QCD}}/m_b)$ in the shape function region [67], and defining $k_+ = n \cdot k$ for a vector k, we have

$$S_s(y) = \langle \delta \left[1 - y - z + \frac{iD_+}{m_b} \right] \rangle. \tag{5.3}$$

$S_s(y)$ can be expressed as

$$S_s(y) = \int dk_+ \delta \left(1 - y - z + \frac{k_+}{m_b} \right) [f(k_+) + \mathcal{O}(m_b^{-1})]. \tag{5.4}$$

The function

$$f(k_+) = \langle \delta(iD_+ - k_+) \rangle \tag{5.5}$$

is the leading shape function.[4] The photon energy spectrum is given by the convolution [28]

$$\frac{d\Gamma}{dy} = \int dk_+ f(k_+) \frac{d\Gamma^*}{dy}. \tag{5.6}$$

In $\frac{d\Gamma^*}{dy}$ the b quark mass m_b is replaced by $m_b^* = m_b + k_+$, an exact substitution at tree level. For k_+ in the range $k_+ \in [-m_b, m_{H_b} - m_b]$, replacing $m_b \to m_b^*$ in the variable y, we find (for $m_s = 0$ to simplify the discussion) that $y \to \frac{2E_\gamma}{(m_b + k_+)}$. Therefore, for $k_+^{\max} = m_{H_b} - m_b$ the maximum photon energy is $E_\gamma = \frac{m_{H_b}}{2}$, the physical endpoint.

The shape function provides an interpretation of the singular terms in the photon energy spectrum obtained in the previous sections. The distribution in (3.19) can be written as

$$\frac{1}{\Gamma} \frac{d\Gamma}{dy} = \sum_{n=0}^{3} \frac{M_n}{n!} \delta^{(n)}(1 - z - y), \tag{5.7}$$

[3]The literature on the shape function, on its properties and on the RGE evolution is wide, a list of references is in [66].

[4]Perturbative corrections to the shape function and to its moments are discussed in [60, 65].

with Γ in eq. (3.25) and $M_{0,\ldots 3}$ given by

$$\begin{aligned}
M_0 &= 1\,, \\
M_1 &= \frac{\hat\mu_\pi^2}{2\,m_b^2}\,(1-z) - \frac{\hat\mu_G^2}{6\,m_b^2}\,(3+5\,z) - \frac{4\,\hat\rho_D^3}{3\,m_b^3}\,(1+2\,z) + \frac{2\,\hat\rho_{LS}^3}{3\,m_b^3}\,(1+z)\,, \\
M_2 &= \frac{\hat\mu_\pi^2}{3\,m_b^2}\,(1-z)^2 - \frac{2\hat\rho_D^3}{3\,m_b^3}\,(1-z)\,(1+2\,z) + \frac{\hat\rho_{LS}^3}{3\,m_b^3}\,(1-z^2)\,, \\
M_3 &= -\frac{\hat\rho_D^3}{3\,m_b^3}\,(1-z)^2\,(1+z)\,.
\end{aligned} \tag{5.8}$$

At $\mathcal{O}(1/m_b^2)$ eqs. (5.8) agree with the expressions obtained in [28]. They can be considered as the first few terms of the infinite sum [27–29, 46, 67]

$$S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!}\,\delta^{(n)}(1-z-y)\,. \tag{5.9}$$

As pointed out in [28], a feature of eqs. (5.8) is that each moment M_n has an expansion in powers of $1/m_b$ starting at the same order of the moment,

$$M_n = \sum_{k=n}^{\infty} \frac{M_{n,k}}{m_b^k}\,. \tag{5.10}$$

Analogously, the double differential distribution (3.18) can be written as

$$\frac{d^2\Gamma}{dy\,d\cos\theta_P} = A\left[\sum_{n=0}^{3} \frac{M_n}{n!}\,\delta^{(n)}(1-z-y)\right] + B\left[\sum_{n=0}^{3} \frac{M_n^{\theta_P}}{n!}\,\delta^{(n)}(1-z-y)\right]\cos\theta_P\,, \tag{5.11}$$

with A, B in (3.23), (3.24). The moments M_n in eq. (3.19) can be considered as resulting from the expansion of the function (5.9), and the terms

$$\begin{aligned}
M_0^{\theta_P} &= 1 \\
M_1^{\theta_P} &= \frac{\hat\mu_\pi^2}{2\,m_b^2}\,(1-z) - \frac{\hat\mu_G^2}{2\,m_b^2}\,(3+z) - \frac{2\,\hat\rho_D^3}{m_b^3}\,(1+z) \\
M_2^{\theta_P} &= \frac{\hat\mu_\pi^2}{3\,m_b^2}\,(1-z)^2 - \frac{2\,\hat\rho_D^3}{3\,m_b^3}\,(1-z)\,(1+2\,z) \\
M_3^{\theta_P} &= -\frac{\hat\rho_D^3}{3\,m_b^3}\,(1-z)^2\,(1+z)
\end{aligned} \tag{5.12}$$

as deriving from the function

$$S_s^{\theta_P}(y) = \sum_{n=0}^{\infty} \frac{M_n^{\theta_P}}{n!}\,\delta^{(n)}(1-z-y)\,. \tag{5.13}$$

The factors M_n in (5.9) are related to the moments of the photon energy spectrum

$$\langle y^k\rangle = \frac{1}{\Gamma}\int_0^{1-z} dy\,y^k\,\frac{d\Gamma}{dy}\,. \tag{5.14}$$

JHEP10(2023)147

	LO	$O(1/m_b^2)$	$O(1/m_b^3)$
$\langle y\rangle$	0.999	1.011	1.008
$\langle y^2\rangle$	0.998	1.029	1.023
σ_y^2	0	0.008	0.007

Table 1. First moments of the photon energy spectrum at LO, $O(1/m_b^2)$, $O(1/m_b^3)$.

Indeed, using (5.7) we have:

$$\langle y^k\rangle = \sum_{n=0}^{\infty} \frac{M_n}{n!} \int_0^{1-z} dy\, y^k\, \delta^{(n)}(1-z-y) = \sum_{j=0}^{k} \binom{k}{j} (1-z)^{k-j} M_j\,, \tag{5.15}$$

and

$$\langle y\rangle = (1-z) + M_1\,, \tag{5.16}$$

$$\langle y^2\rangle = (1-z)^2 + 2\,(1-z)\,M_1 + M_2\,, \tag{5.17}$$

$$\sigma_y^2 = \langle y^2\rangle - \langle y\rangle^2 = M_2 - M_1^2\,. \tag{5.18}$$

Such results imply that

$$\langle y\rangle = (1-z)\left[1 + \frac{\hat\mu_\pi^2}{2\,m_b^2} - \frac{\hat\mu_G^2}{6\,m_b^2}\,\frac{3+5\,z}{1-z} - \frac{4\,\hat\rho_D^3}{3\,m_b^3}\,\frac{1+2\,z}{1-z} + \frac{2\,\hat\rho_{LS}^3}{3\,m_b^3}\,\frac{1+z}{1-z}\right], \tag{5.19}$$

$$\langle y^2\rangle = (1-z)^2\left[1 + \frac{4\,\hat\mu_\pi^2}{3\,m_b^2} - \frac{\hat\mu_G^2}{3\,m_b^2}\,\frac{3+5\,z}{1-z} - \frac{10\,\hat\rho_D^3}{3\,m_b^3}\,\frac{1+2\,z}{1-z} + \frac{5\,\hat\rho_{LS}^3}{3\,m_b^3}\,\frac{1+z}{1-z}\right], \tag{5.20}$$

$$\sigma_y^2 = (1-z)^2\left[\frac{\hat\mu_\pi^2}{3\,m_b^2} - \frac{2\,\hat\rho_D^3}{3\,m_b^3}\,\frac{1+2\,z}{1-z} + \frac{\hat\rho_{LS}^3}{3\,m_b^3}\,\frac{1+z}{1-z}\right]. \tag{5.21}$$

After the substitutions in (3.26), the above expressions agree with those given in [68] for $z=0$.

Table 1 contains numerical results for the first moments, increasing the order in $1/m_b$ and using the parameters in section 4. The moments of the measured photon energy spectrum can be used to determine the HQET parameters, as for B mesons. Baryons have the advantage that the $\cos\theta_P$ distribution (3.22) can also be exploited.

The Fermi motion of the heavy quark in the hadron has the effect of smearing the spectrum. Indeed, at the various orders in the $1/m_b$ expansion, the photon energy spectrum obtained by the local OPE corresponds to a monochromatic line. At the leading order the line is placed at $y = 2E_\gamma/m_b = 1-z$, the next terms correspond to a displacement of this position. The convolution with the shape function $f(k_+)$ in (5.6) produces the smearing. The shape function is a nonperturbative quantity, it must be determined by methods as lattice QCD or QCD sum rules starting from the moments, or it must be modelized to reproduce the experimental photon spectrum [26, 69–72]. In the latter case, the uncertainty connected to the functional dependence is usually estimated using different model functions, or varying the model parameters.

JHEP10(2023)147

Instead of parametrizing the shape function, we proceed considering that, if an infinite number of terms is included, the sum eq. (5.7) gives the spectral function $S_s(y)$ in (5.9). In the sum the first term corresponds to a monocromatic line at the zero of the δ-function, with $\langle y\rangle = (1-z)$ and $\sigma_y^2 = 0$, the leading order results in eqs. (5.16)–(5.18). We observe that the Dirac delta can be represented as

$$\delta(1-z-y) = \lim_{\sigma_y \to 0} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{(b-y)^2}{2\sigma_y^2}} \tag{5.22}$$

with $b = (1-z) = \langle y\rangle_{LO}$. We can fix σ_y^2 at each order in $1/m_b$, starting from $1/m_b^2$. For $m_b \to \infty$ the limit $\sigma_y \to 0$ reproduces the partonic result.

We represent the spectral function $S_s(y)$ by the substitution

$$S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!}\, \delta^{(n)}(1-z-y) \to S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!} (-1)^n \frac{d^n}{dy^n} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{(b-y)^2}{2\sigma_y^2}} . \tag{5.23}$$

Using the representation of the Hermite polynomials

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2} , \tag{5.24}$$

the substitution gives

$$S_s(y) = \frac{1}{\sqrt{2\pi}\,\sigma_y} \sum_{n=0}^{\infty} \frac{M_n}{n!} \left(-\frac{1}{\sqrt{2}\sigma_y}\right)^n e^{-\frac{(b-y)^2}{2\,\sigma_y^2}} H_n\left(\frac{b-y}{\sqrt{2}\,\sigma_y}\right) . \tag{5.25}$$

Notice that, denoting by $\langle y^k\rangle_{\mathcal{N}}$ the moments computed using this expression for the spectral function $S_s(y)$,

$$\langle y^k\rangle_{\mathcal{N}} = \int_0^{y_{\max}} dy\, y^k\, S_s(y) , \tag{5.26}$$

in the limit $\sigma_y \to 0$ one obtains

$$\lim_{\sigma_y \to 0} \langle y^k\rangle_{\mathcal{N}} = \langle y^k\rangle \tag{5.27}$$

with $\langle y^k\rangle$ in (5.15). This is shown in appendix C.

The ansatz eq. (5.25) has many advantages with respect to other representations of the shape function or of the spectral function, based on a choice of a functional representation able to reproduce the photon spectrum, with parameters set by the first computed moments M_n. Indeed, such representations generally do not guarantee that higher moments are reproduced. Moreover, in such models the moments M_n generally increase with the order n [73]. To cure such features, in [74] the shape function is expressed using a complete set of orthonormal functions. In particular, the normalized Legendre polynomials are considered in the range $[-1,+1]$, and a function mapping the range $[-1,1]$ into $[0,+\infty)$ is chosen to represent the shape function $f(\omega)$ in the definition having support $\omega \in [0,+\infty)$.

Remarkably, with the ansatz in eq. (5.25), by construction $S_s(y)$ can include all moments M_n once they are computed. Moreover, less singular terms in the expansion of the moments are not discarded. Each M_n starts at $\mathcal{O}(1/m_b^n)$ and depends on the matrix elements of the HQET operators of increasing dimension,

$$\mathcal{M}_{\mu_1\dots\mu_n} = \langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle \tag{5.28}$$

of $\mathcal{O}(\Lambda^n_{\rm QCD})$, maintaining a hierarchical ordering.

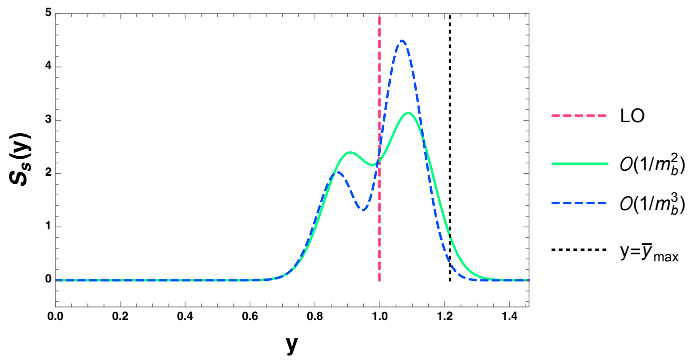
We point out the main differences with the method proposed in [74] and adopted in [30]. In that approaches it is assumed that the leading shape function is positive, hence it is expressed as the square of the sum of orthonormal functions, choosing in particular the normalized Legendre polynomials. The positivity assumption is not necessary in the ansatz (5.25). In such expression the expansion in Hermite polynomials is not arbitrary, since it comes from the replacement of the Dirac delta by the normal distribution with standard deviation σ_y (5.22). This produces a different result from the single gaussian function in e.g. [67, 75]. Indeed, the derivatives of the exponential in eq. (5.23) produce coefficients with a non trivial dependence on y, which are resummed giving the Hermite polynomials in (5.25). This produces the asymmetry of the shape function with respect to the point $y = b$. While b is fixed to the LO result for $\langle y\rangle$, σ_y can be determined at an arbitrary order in the $1/m_b$ expansion; eq. (5.21) satisfies by construction the condition $\lim_{m_b\to\infty}\sigma_y = 0$, recovering the monochromatic spectrum in the limit. In (5.25) the Hermite polynomials are not weighted by new unkown coefficients to be fitted, but by the computed moments M_n.

For an analysis based on the ansatz (5.25), in figure 3 we show the spectral function obtained at LO, $\mathcal{O}(1/m_b^2)$ and $\mathcal{O}(1/m_b^3)$. In the same figure we plot the shape function obtained from (5.4) at $\mathcal{O}(1/m_b^2)$ and $\mathcal{O}(1/m_b^3)$. As a consequence of broadening the spectrum through the substitution in (5.23), there is a tail exceeding the physical endpoint $\bar{y}_{\rm max} \simeq \frac{m_{\Lambda_b}}{m_b}(1-z)$, and a tail in the shape function exceeding $k_+^{\rm max} = m_{\Lambda_b} - m_b$. This is a spurious effect of the truncation. When higher orders in the HQE are included, the area below such tails approaches to zero. Indeed, denoting this area by $\Delta\langle y^0\rangle_n = \int_{\bar{y}_{\rm max}}^\infty dy\, \frac{1}{\Gamma}\frac{d\Gamma}{dy}$ computed at $\mathcal{O}(1/m_b^n)$, we numerically find $\frac{\Delta\langle y^0\rangle_3}{\Delta\langle y^0\rangle_2} \simeq \mathcal{O}\left(\frac{1}{m_b}\right)$. Increasing the order in the HQ expansion $\Delta\langle y^0\rangle_n$ reduces to zero, so that the physical endpoint is reached.

6 Conclusions

The HQE has been exploited to compute the inclusive decay width induced by the $b \to s\gamma$ transition for a beauty baryon, in particular Λ_b. The differential width in the rescaled photon energy $y = \frac{2E_\gamma}{m_b}$ and in $\cos\theta_P$ allows to construct new observables with respect to mesons. The calculation has been carried out at $\mathcal{O}(1/m_b^3)$ for non-vanishing strange quark mass, using the baryon matrix elements determined in [41]. Physics beyond the Standard Model represented by the operator O_7' is found to affect the photon polarization asymmetry. For the singular terms appearing in the spectrum as δ-distribution and its derivatives we have proposed a treatment that can be systematically improved including higher order terms in the expansion.

Progress in the next studies will be achieved considering the full Hamiltonian (2.1), the resolved photon contribution and the subleading shape functions for b-baryons. This is



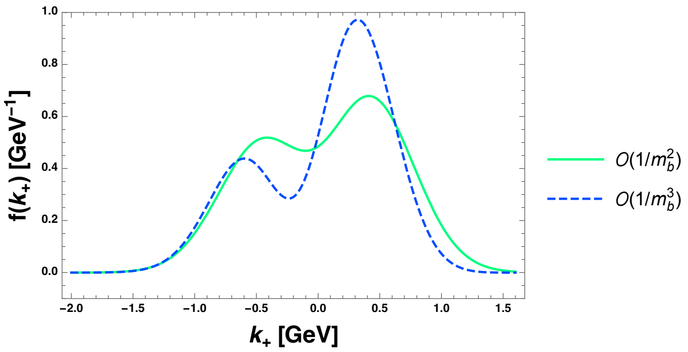


Figure 3. Spectral function $S_s(y)$ (top panel) and shape function $f(k_+)$ (bottom panel) obtained using the ansatz eq. (5.25) up to $n = 3$.

an important step forward, in view of the wealth of new information which can be gained on SM and on the possible extensions analyzing the beauty baryon rare radiative decay modes together with the polarization effects.

Acknowledgments

We thank Andrzej J. Buras and Gil Paz for discussions. This research has been carried out within the INFN project (Iniziativa Specifica) QFT-HEP.

A Derivation of the OPE

The OPE for the expression (3.5) can be constructed expressing the hadron momentum $p = m_H v$, with v the four-velocity, in terms of m_b and of a residual momentum k: $p = m_b v + k$. The QCD b quark field is rescaled

$$b(x) = e^{-i\, m_b v\cdot x} b_v(x), \tag{A.1}$$

and the QCD field $b_v(x)$ satisfies the equation of motion

$$b_v(x) = \left(P_+ + \frac{i\not{D}}{2m_b}\right) b_v(x)\,, \tag{A.2}$$

with velocity projector $P_+ = \frac{1+\not{v}}{2}$. Expressed in terms of $b_v(x)$ eq. (3.5) becomes:

$$T^{ij}_{MN} = i \int d^4x \, e^{i\,(m_b v - q)\cdot x} \langle H_b(v,s)| T[\hat{J}^{i\dagger}_M(x)\, \hat{J}^j_N(0)] |H_b(v,s)\rangle \,. \tag{A.3}$$

$\hat{J}^i$ contains the field b_v. The heavy quark expansion is obtained from

$$T^{ij}_{MN} = \langle H_b(v,s)| \bar{b}_v(0) \bar{\Gamma}^i_M S_s(p_X) \Gamma^j_N b_v(0) |H_b(v,s)\rangle \,, \tag{A.4}$$

with $\bar{\Gamma}^i_M = \gamma^0 \Gamma^{i\dagger}_M \gamma^0$ and $\Gamma^7_M = \sigma_{\mu\nu}\,(m_b\, P_R + m_s\, P_L)$, $\Gamma^{7\prime}_M = \sigma_{\mu\nu}\,(m_b\, P_L + m_s\, P_R)$. $S_s(p_X)$ is the s quark propagator. Replacing $k \to iD$, with D the QCD covariant derivative, the s quark propagator can be expanded:

$$S_s(p_X) = S_s^{(0)} - S_s^{(0)}(i\not{D})S_s^{(0)} + S_s^{(0)}(i\not{D})S_s^{(0)}(i\not{D})S_s^{(0)} + \dots \tag{A.5}$$

where $S_s^{(0)} = \frac{1}{m_b \not{v} - \not{q} - m_s}$. Defining $p_s = m_b v - q$, $\mathcal{P} = \not{p}_s + m_s$ and $\Delta_0 = p_s^2 - m_s^2$, the expansion at order $1/m_b^3$ is given by:

$$\begin{aligned}
\frac{1}{\pi}\mathrm{Im}\, T^{ij}_{MN} = {}& \frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0}\langle H_b(v,s)|\bar{b}_v[\bar{\Gamma}^i_M \mathcal{P}\Gamma^j_N] b_v|H_b(v,s)\rangle \\
& -\frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^2}\langle H_b(v,s)|\bar{b}_v[\bar{\Gamma}^i_M \mathcal{P}\gamma^{\mu_1}\mathcal{P}\Gamma^j_N](iD_{\mu_1}) b_v|H_b(v,s)\rangle \\
& +\frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^3}\langle H_b(v,s)|\bar{b}_v[\bar{\Gamma}^i_M \mathcal{P}\gamma^{\mu_1}\mathcal{P}\gamma^{\mu_2}\mathcal{P}\Gamma^j_N](iD_{\mu_1})(iD_{\mu_2}) b_v|H_b(v,s)\rangle \\
& -\frac{1}{\pi}\mathrm{Im}\frac{1}{\Delta_0^4}\langle H_b(v,s)|\bar{b}_v[\bar{\Gamma}^i_M \mathcal{P}\gamma^{\mu_1}\mathcal{P}\gamma^{\mu_2}\mathcal{P}\gamma^{\mu_3}\mathcal{P}\Gamma^j_N](iD_{\mu_1})(iD_{\mu_2})(iD_{\mu_3}) b_v|H_b(v,s)\rangle \,.
\end{aligned} \tag{A.6}$$

This expression involves the H_b matrix elements of QCD operators of increasing dimension,

$$\begin{aligned}
&\langle H_b(v,s)|\bar{b}_v[\bar{\Gamma}^i_M \mathcal{P}\gamma^{\mu_1}\dots\gamma^{\mu_n}\mathcal{P}\Gamma^j_N](iD_{\mu_1})\dots(iD_{\mu_n}) b_v|H_b(v,s)\rangle = \\
&\qquad \mathrm{Tr}\left[(\bar{\Gamma}^i_M \mathcal{P}\gamma^{\mu_1}\dots\gamma^{\mu_n}\mathcal{P}\Gamma^j_N)_{ba}\langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle\right] \,,
\end{aligned} \tag{A.7}$$

with a, b Dirac indices. The hadronic matrix elements

$$(\mathcal{M}_{\mu_1\dots\mu_n})_{ab} = \langle H_b(v,s)|(\bar{b}_v)_a(iD_{\mu_1})\dots(iD_{\mu_n})(b_v)_b|H_b(v,s)\rangle \tag{A.8}$$

can be expressed in terms of a set of nonperturbative parameters, the number of which increases with the operator dimension. At $\mathcal{O}(1/m_b^3)$ the following matrix elements are required:

$$\langle H_b(v,s)|\bar{b}_v(iD)^2 b_v|H_b(v,s)\rangle = -2m_H\,\hat{\mu}_\pi^2 \tag{A.9}$$

$$\langle H_b(v,s)|\bar{b}_v(iD_\mu)(iD_\nu)(-i\sigma^{\mu\nu}) b_v|H_b(v,s)\rangle = 2m_H\,\hat{\mu}_G^2 \tag{A.10}$$

$$\langle H_b(v,s)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD^\mu) b_v|H_b(v,s)\rangle = 2m_H\,\hat{\rho}_D^3 \tag{A.11}$$

$$\langle H_b(v,s)|\bar{b}_v(iD_\mu)(iv\cdot D)(iD_\nu)(-i\sigma^{\mu\nu}) b_v|H_b(v,s)\rangle = 2m_H\,\hat{\rho}_{LS}^3 \,. \tag{A.12}$$

A procedure to compute $\mathcal{M}_{\mu_1\dots\mu_n}$ has been exploited for B meson for $n = 4$ [76] and $n = 5$ [77], introducing additional parameters with respect to those in (A.9)–(A.12). For a heavy baryon, the dependence on the spin four-vector s_μ is specified in (A.8). The matrix elements in the expansion at $\mathcal{O}(1/m_b^3)$ keeping the s_μ dependence have been parametrized in [41] and are used in the present study.

JHEP10(2023)147

B Factorization formula in the endpoint region and shape functions

The local OPE exploited in this paper holds in the kinematic region where the hadronic invariant mass p_X^2 is $\mathcal{O}(m_b^2)$. In the region where $p_X^2 \sim \mathcal{O}(m_b \Lambda_{\rm QCD})$, the endpoint or shape function region, the theoretical treatment is different according to which of the operators in the effective Hamiltonian one considers. In our analysis we have focused on the dipole operators $O_7^{(\prime)}$ since they are the only terms contributing to lowest order in QCD. In the SM the contribution of O_7 to the correlation function (3.5) at the endpoint obeys a factorization formula

$$d\Gamma^{77} \sim H \cdot J \otimes S + \frac{1}{m_b} \sum_i H \cdot J \otimes s_i + \frac{1}{m_b} \sum_i H \cdot J_i \otimes S + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_b^2}\right) . \tag{B.1}$$

H denote hard functions and J and j_i are jet functions computed perturbatively, with H of $\mathcal{O}(1)$. S and s_i are the shape functions, which are nonperturbative. The function S entering in (B.1) at leading order is the shape function defined in HQET.

When the other operators in (3.5) are considered, a more involved factorization formula holds at $\mathcal{O}(1/m_b)$ [31]. The most important operators are O_2 and O_8, and the pairing of the operators in (3.5) produces different effects. In particular, the resolved photon contribution (RPC) mentioned in the Introduction appears as single RPC from the pairing of O_8 and O_2 among themselves and with O_7. Double RPC arise from the pairing of O_8 and O_2 among themselves [31]. For such contributions the leading term is $\mathcal{O}(\alpha_s)$: this justifies their neglect in the present analysis.

C Moments of the photon energy spectrum

To obtain eq. (5.27) we use the representation of the Hermite polynomials

$$H_n(x) = n! \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^m}{m!(n-2m)!} (2x)^{n-2m} . \tag{C.1}$$

The moments $\langle y^k \rangle_{\mathcal{N}}$ are given by:

$$\langle y^k \rangle_{\mathcal{N}} = \sum_{j=0}^{k} \binom{k}{j} b^{k-j} \sum_{n=0}^{\infty} M_n \, (-\sqrt{2}\, \sigma_y)^{j-n} \, \Phi_{j,n} \, , \tag{C.2}$$

where

$$\begin{aligned} \Phi_{j,n} = {} & \frac{1}{\sqrt{\pi}} \sum_{m=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^m}{m!\,(n-2\,m)!} \, 2^{n-2\,m} \\ & \times \frac{1}{2} \left[\gamma\left(\frac{j+n-2\,m+1}{2}, x_{\rm max}^2\right) - (-1)^{j+n-2\,m+1} \gamma\left(\frac{j+n-2\,m+1}{2}, x_{\rm min}^2\right)\right] . \end{aligned} \tag{C.3}$$

The parameters b and σ_y are defined in section 5, and $x_{\rm max(min)}$ are $x_{\rm max} = \frac{b}{\sqrt{2}\,\sigma_y} > 0$, $x_{\rm min} = \frac{b - y_{\rm max}}{\sqrt{2}\,\sigma_y} < 0$. $\gamma(n,z)$ is the lower incomplete Euler function with the condition

$\mathrm{Re}(j+n-2\,m) > -1$ always satisfied in our case. $\Phi_{j,n}$ depends on σ_y only through $x_{\max(\min)}$. Since $\lim_{\sigma_y\to 0} x_{\max(\min)} = +(-)\infty$, we have $\lim_{\sigma_y\to 0}\Phi_{j,n} = 0$ for $n \neq j$. Consequently, for $\sigma_y \to 0$ we obtain that $\langle y^k\rangle_{\mathcal{N}}$ is given by the $n = j$ terms in (C.2),

$$\langle y^k\rangle_{\mathcal{N}} = \sum_{j=0}^{k} \binom{k}{j} b^{k-j}\, M_j\, \Phi_{j,j}\,. \tag{C.4}$$

Since $\lim_{\sigma_y\to 0}\Phi_{j,j} = 1$, eq. (5.15) is recovered.

References

[1] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *On the Weak Radiative Decays (Effects of Strong Interactions at Short Distances)*, *Phys. Rev. D* **18** (1978) 2583 [*Erratum ibid.* **19** (1979) 2815] [INSPIRE].

[2] S. Bertolini, F. Borzumati and A. Masiero, *QCD Enhancement of Radiative b Decays*, *Phys. Rev. Lett.* **59** (1987) 180 [INSPIRE].

[3] B. Grinstein, R.P. Springer and M.B. Wise, *Effective Hamiltonian for Weak Radiative B Meson Decay*, *Phys. Lett. B* **202** (1988) 138 [INSPIRE].

[4] G. Cella, G. Curci, G. Ricciardi and A. Vicere, *The $b \to s\gamma$ decay revisited*, *Phys. Lett. B* **325** (1994) 227 [hep-ph/9401254] [INSPIRE].

[5] M. Misiak et al., *Updated NNLO QCD predictions for the weak radiative B-meson decays*, *Phys. Rev. Lett.* **114** (2015) 221801 [arXiv:1503.01789] [INSPIRE].

[6] B. Grinstein and M.B. Wise, *Weak Radiative B Meson Decay as a Probe of the Higgs Sector*, *Phys. Lett. B* **201** (1988) 274 [INSPIRE].

[7] W.-S. Hou and R.S. Willey, *Effects of Charged Higgs Bosons on the Processes $b \to s\gamma$, $b \to sg^*$ and $b \to s\ell^+\ell^-$*, *Phys. Lett. B* **202** (1988) 591 [INSPIRE].

[8] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A Complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model*, *Nucl. Phys. B* **477** (1996) 321 [hep-ph/9604387] [INSPIRE].

[9] L.L. Everett, G.L. Kane, S. Rigolin, L.-T. Wang and T.T. Wang, *Alternative approach to $b \to s\gamma$ in the uMSSM*, *JHEP* **01** (2002) 022 [hep-ph/0112126] [INSPIRE].

[10] F. Borzumati, C. Greub and Y. Yamada, *Beyond leading order corrections to $\bar{B} \to X_s\gamma$ at large* $\tan(\beta)$*: The Charged Higgs contribution*, *Phys. Rev. D* **69** (2004) 055005 [hep-ph/0311151] [INSPIRE].

[11] A.J. Buras, A. Poschenrieder, M. Spranger and A. Weiler, *The Impact of universal extra dimensions on $B \to X_s\gamma$, $B \to X_s$ gluon, $B \to X_s\mu^+\mu^-$, $K_L \to \pi^0 e^+e^-$ and ϵ'/ϵ*, *Nucl. Phys. B* **678** (2004) 455 [hep-ph/0306158] [INSPIRE].

[12] M. Blanke, A.J. Buras, A. Poschenrieder, C. Tarantino, S. Uhlig and A. Weiler, *Particle-Antiparticle Mixing, ϵ_K, $\Delta\Gamma_q$, A_{SL}^q, $A_{CP}(B_d \to \psi K_S)$, $A_{CP}(B_s \to \psi\phi)$ and $B \to X_{s,d}\gamma$ in the Littlest Higgs Model with T-Parity*, *JHEP* **12** (2006) 003 [hep-ph/0605214] [INSPIRE].

[13] M. Misiak and M. Steinhauser, *Weak radiative decays of the B meson and bounds on $M_{H^\pm}$ in the Two-Higgs-Doublet Model*, Eur. Phys. J. C **77** (2017) 201 [arXiv:1702.04571] [INSPIRE].

[14] A. Buras, *Gauge Theory of Weak Decays*, Cambridge University Press (2020) [DOI:10.1017/9781139524100] [INSPIRE].

[15] CLEO collaboration, *Evidence for penguins: First observation of $B \to K^*(892)\gamma$*, Phys. Rev. Lett. **71** (1993) 674 [INSPIRE].

[16] Heavy Flavor Averaging Group collaboration, *Averages of b-hadron, c-hadron, and τ-lepton properties as of 2021*, Phys. Rev. D **107** (2023) 052008 [arXiv:2206.07501] [INSPIRE].

[17] Particle Data collaboration, *Review of Particle Physics*, Prog. Theor. Exp. Phys. **2022** (2022) 083C01 [INSPIRE].

[18] LHCb collaboration, *First Observation of the Radiative Decay $\Lambda_b^0 \to \Lambda\gamma$*, Phys. Rev. Lett. **123** (2019) 031801 [arXiv:1904.06697] [INSPIRE].

[19] LHCb collaboration, *Measurement of the photon polarization in $\Lambda_b^0 \to \Lambda\ \gamma$ decays*, Phys. Rev. D **105** (2022) L051104 [arXiv:2111.10194] [INSPIRE].

[20] LHCb collaboration, *Search for the radiative $\Xi_b^- \to \Xi^-\gamma$ decay*, JHEP **01** (2022) 069 [arXiv:2108.07678] [INSPIRE].

[21] P. Colangelo, C.A. Dominguez, G. Nardulli and N. Paver, *Radiative $B \to K^*\gamma$ transition in QCD*, Phys. Lett. B **317** (1993) 183 [hep-ph/9308264] [INSPIRE].

[22] M. Neubert, *Heavy quark symmetry*, Phys. Rep. **245** (1994) 259 [hep-ph/9306320] [INSPIRE].

[23] A.L. Kagan and M. Neubert, *Direct CP violation in $B \to X_s\gamma$ decays as a signature of new physics*, Phys. Rev. D **58** (1998) 094012 [hep-ph/9803368] [INSPIRE].

[24] M. Benzke, S.J. Lee, M. Neubert and G. Paz, *Long-Distance Dominance of the CP Asymmetry in $B \to X_{s,d} + \gamma$ Decays*, Phys. Rev. Lett. **106** (2011) 141801 [arXiv:1012.3167] [INSPIRE].

[25] S. Catani and B.R. Webber, *Infrared safe but infinite: Soft gluon divergences inside the physical region*, JHEP **10** (1997) 005 [hep-ph/9710333] [INSPIRE].

[26] F. De Fazio and M. Neubert, *$B \to X_u \ell \bar{\nu}_\ell$ decay distributions to order α_s*, JHEP **06** (1999) 017 [hep-ph/9905351] [INSPIRE].

[27] M. Neubert, *QCD based interpretation of the lepton spectrum in inclusive $\bar{B} \to X_u$ lepton anti-neutrino decays*, Phys. Rev. D **49** (1994) 3392 [hep-ph/9311325] [INSPIRE].

[28] M. Neubert, *Analysis of the photon spectrum in inclusive $B \to X_s\gamma$ decays*, Phys. Rev. D **49** (1994) 4623 [hep-ph/9312311] [INSPIRE].

[29] I.I.Y. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, *On the motion of heavy quarks inside hadrons: Universal distributions and inclusive decays*, Int. J. Mod. Phys. A **9** (1994) 2467 [hep-ph/9312359] [INSPIRE].

[30] SIMBA collaboration, *Precision Global Determination of the $B \to X_s\gamma$ Decay Rate*, Phys. Rev. Lett. **127** (2021) 102001 [arXiv:2007.04320] [INSPIRE].

[31] M. Benzke, S.J. Lee, M. Neubert and G. Paz, *Factorization at Subleading Power and Irreducible Uncertainties in $\bar{B} \to X_s\gamma$ Decay*, JHEP **08** (2010) 099 [arXiv:1003.5012] [INSPIRE].

[32] J.F. Donoghue and A.A. Petrov, *Is $B \to X_s\gamma$ equal to $b \to s\gamma$? Spectator contributions to rare inclusive B decays*, Phys. Rev. D **53** (1996) 3664 [hep-ph/9510227] [INSPIRE].

JHEP10(2023)147

[33] S.J. Lee, M. Neubert and G. Paz, *Enhanced Non-local Power Corrections to the $\bar{B} \to X_s\gamma$ Decay Rate*, Phys. Rev. D **75** (2007) 114005 [hep-ph/0609224] [INSPIRE].

[34] M.B. Voloshin, *Large $O(m_c^{-2})$ nonperturbative correction to the inclusive rate of the decay $B \to X_s\gamma$*, Phys. Lett. B **397** (1997) 275 [hep-ph/9612483] [INSPIRE].

[35] Z. Ligeti, L. Randall and M.B. Wise, *Comment on nonperturbative effects in $\bar{B} \to X_s\gamma$*, Phys. Lett. B **402** (1997) 178 [hep-ph/9702322] [INSPIRE].

[36] A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei, *Comment on nonperturbative $O(1/m_c^2)$ corrections to $\Gamma(\bar{B} \to X_s\gamma)$*, Phys. Rev. D **56** (1997) 3151 [hep-ph/9702380] [INSPIRE].

[37] G. Buchalla, G. Isidori and S.J. Rey, *Corrections of order $\Lambda^2_{\mathrm{QCD}}/m_c^2$ to inclusive rare B decays*, Nucl. Phys. B **511** (1998) 594 [hep-ph/9705253] [INSPIRE].

[38] ALEPH collaboration, *Measurement of the Λ_b polarization in Z decays*, Phys. Lett. B **365** (1996) 437 [INSPIRE].

[39] OPAL collaboration, *Measurement of the average polarization of b baryons in hadronic Z^0 decays*, Phys. Lett. B **444** (1998) 539 [hep-ex/9808006] [INSPIRE].

[40] DELPHI collaboration, *Λ_b polarization in Z^0 decays at LEP*, Phys. Lett. B **474** (2000) 205 [INSPIRE].

[41] P. Colangelo, F. De Fazio and F. Loparco, *Inclusive semileptonic Λ_b decays in the Standard Model and beyond*, JHEP **11** (2020) 032 [*Erratum ibid.* **12** (2022) 098] [arXiv:2006.13759] [INSPIRE].

[42] F. Borzumati, C. Greub, T. Hurth and D. Wyler, *Gluino contribution to radiative B decays: Organization of QCD corrections and leading order results*, Phys. Rev. D **62** (2000) 075005 [hep-ph/9911245] [INSPIRE].

[43] A.J. Buras, M. Misiak, M. Munz and S. Pokorski, *Theoretical uncertainties and phenomenological aspects of $B \to X_s\gamma$ decay*, Nucl. Phys. B **424** (1994) 374 [hep-ph/9311345] [INSPIRE].

[44] M. Misiak, *Radiative Decays of the B Meson: a Progress Report*, Acta Phys. Pol. B **49** (2018) 1291 [INSPIRE].

[45] J. Chay, H. Georgi and B. Grinstein, *Lepton energy distributions in heavy meson decays from QCD*, Phys. Lett. B **247** (1990) 399 [INSPIRE].

[46] I.I.Y. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, *QCD predictions for lepton spectra in inclusive heavy flavor decays*, Phys. Rev. Lett. **71** (1993) 496 [hep-ph/9304225] [INSPIRE].

[47] A.F. Falk, M.E. Luke and M.J. Savage, *Nonperturbative contributions to the inclusive rare decays $B \to X_s\gamma$ and $B \to X_s l^+ l^-$*, Phys. Rev. D **49** (1994) 3367 [hep-ph/9308288] [INSPIRE].

[48] A. Kapustin and Z. Ligeti, *Moments of the photon spectrum in the inclusive $B \to X_s\gamma$ decay*, Phys. Lett. B **355** (1995) 318 [hep-ph/9506201] [INSPIRE].

[49] C.W. Bauer, *Corrections to moments of the photon spectrum in the inclusive decay $B \to X_s\gamma$*, Phys. Rev. D **57** (1998) 5611 [*Erratum ibid.* **60** (1999) 099907] [hep-ph/9710513] [INSPIRE].

[50] T. Mannel and K.K. Vos, *Reparametrization Invariance and Partial Re-Summations of the Heavy Quark Expansion*, JHEP **06** (2018) 115 [arXiv:1802.09409] [INSPIRE].

JHEP10(2023)147

[51] M. Gremm, F. Kruger and L.M. Sehgal, *Angular distribution and polarization of photons in the inclusive decay $\Lambda_b \to X_s\gamma$*, Phys. Lett. B **355** (1995) 579 [hep-ph/9505354] [INSPIRE].

[52] T. Mannel and S. Recksiegel, *Flavor changing neutral current decays of heavy baryons: The Case $\Lambda_b \to \Lambda\gamma$*, J. Phys. G **24** (1998) 979 [hep-ph/9701399] [INSPIRE].

[53] C.-S. Huang and H.-G. Yan, *Exclusive rare decays of heavy baryons to light baryons: $\Lambda_b \to \Lambda\gamma$ and $\Lambda_b \to \Lambda\ell^+\ell^-$*, Phys. Rev. D **59** (1999) 114022 [*Erratum ibid.* **61** (2000) 039901] [hep-ph/9811303] [INSPIRE].

[54] G. Hiller and A. Kagan, *Probing for new physics in polarized Λ_b decays at the Z*, Phys. Rev. D **65** (2002) 074038 [hep-ph/0108074] [INSPIRE].

[55] P. Colangelo, F. De Fazio, R. Ferrandes and T.N. Pham, *FCNC B_s and Λ_b transitions: Standard model versus a single universal extra dimension scenario*, Phys. Rev. D **77** (2008) 055019 [arXiv:0709.2817] [INSPIRE].

[56] L. Oliver, J.-C. Raynal and R. Sinha, *Note on new interesting baryon channels to measure the photon polarization in $b \to s\gamma$*, Phys. Rev. D **82** (2010) 117502 [arXiv:1007.3632] [INSPIRE].

[57] T. Blake, T. Gershon and G. Hiller, *Rare b hadron decays at the LHC*, Ann. Rev. Nucl. Part. Sci. **65** (2015) 113 [arXiv:1501.03309] [INSPIRE].

[58] L.M. García Martín et al., *Radiative b-baryon decays to measure the photon and b-baryon polarization*, Eur. Phys. J. C **79** (2019) 634 [arXiv:1902.04870] [INSPIRE].

[59] A. Paul and D.M. Straub, *Constraints on new physics from radiative B decays*, JHEP **04** (2017) 027 [arXiv:1608.02556] [INSPIRE].

[60] C.W. Bauer and A.V. Manohar, *Shape function effects in $B \to X_s\gamma$ and $B \to X_u\ell\bar{\nu}$ decays*, Phys. Rev. D **70** (2004) 034024 [hep-ph/0312109] [INSPIRE].

[61] C.W. Bauer, S. Fleming, D. Pirjol and I.W. Stewart, *An Effective field theory for collinear and soft gluons: Heavy to light decays*, Phys. Rev. D **63** (2001) 114020 [hep-ph/0011336] [INSPIRE].

[62] C.W. Bauer and I.W. Stewart, *Invariant operators in collinear effective theory*, Phys. Lett. B **516** (2001) 134 [hep-ph/0107001] [INSPIRE].

[63] M. Beneke, A.P. Chapovsky, M. Diehl and T. Feldmann, *Soft collinear effective theory and heavy to light currents beyond leading power*, Nucl. Phys. B **643** (2002) 431 [hep-ph/0206152] [INSPIRE].

[64] M. Beneke and T. Feldmann, *Multipole expanded soft collinear effective theory with non-Abelian gauge symmetry*, Phys. Lett. B **553** (2003) 267 [hep-ph/0211358] [INSPIRE].

[65] S.W. Bosch, B.O. Lange, M. Neubert and G. Paz, *Factorization and shape function effects in inclusive B meson decays*, Nucl. Phys. B **699** (2004) 335 [hep-ph/0402094] [INSPIRE].

[66] P. Gambino et al., *Challenges in semileptonic B decays*, Eur. Phys. J. C **80** (2020) 966 [arXiv:2006.07287] [INSPIRE].

[67] T. Mannel and M. Neubert, *Resummation of nonperturbative corrections to the lepton spectrum in inclusive $B \to X_q\ell\bar{\nu}_\ell$ decays*, Phys. Rev. D **50** (1994) 2037 [hep-ph/9402288] [INSPIRE].

[68] D. Benson, I.I. Bigi and N.G. Uraltsev, *On the photon energy moments and their 'bias' corrections in $B \to X_s + \gamma$*, Nucl. Phys. B **710** (2005) 371 [hep-ph/0410080] [INSPIRE].

[69] B.O. Lange, M. Neubert and G. Paz, *Theory of charmless inclusive B decays and the extraction of V_{ub}*, Phys. Rev. D **72** (2005) 073006 [hep-ph/0504071] [INSPIRE].

[70] J.R. Andersen and E. Gardi, *Inclusive spectra in charmless semileptonic B decays by dressed gluon exponentiation*, JHEP **01** (2006) 097 [hep-ph/0509360] [INSPIRE].

[71] P. Gambino, P. Giordano, G. Ossola and N.G. Uraltsev, *Inclusive semileptonic B decays and the determination of* $|V_{ub}|$, JHEP **10** (2007) 058 [arXiv:0707.2493] [INSPIRE].

[72] U. Aglietti, F. Di Lodovico, G. Ferrera and G. Ricciardi, *Inclusive measure of* $|V_{ub}|$ *with the analytic coupling model*, Eur. Phys. J. C **59** (2009) 831 [arXiv:0711.0860] [INSPIRE].

[73] T. Mannel, *Inclusive Semi-Leptonic B Decays*, in proceedings of the *Pushing the Limits of the Theoretical Physics*, Mainz, Germany, 8–12 May 2023 https://indico.mitp.uni-mainz.de/event/341/.

[74] Z. Ligeti, I.W. Stewart and F.J. Tackmann, *Treating the b quark distribution function with reliable uncertainties*, Phys. Rev. D **78** (2008) 114014 [arXiv:0807.1926] [INSPIRE].

[75] G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, *Leptonic Decay of Heavy Flavors: A Theoretical Update*, Nucl. Phys. B **208** (1982) 365 [INSPIRE].

[76] B.M. Dassinger, T. Mannel and S. Turczyk, *Inclusive semi-leptonic B decays to order* $1/m_b^4$, JHEP **03** (2007) 087 [hep-ph/0611168] [INSPIRE].

[77] T. Mannel, S. Turczyk and N.G. Uraltsev, *Higher Order Power Corrections in Inclusive B Decays*, JHEP **11** (2010) 109 [arXiv:1009.4622] [INSPIRE].

JHEP10(2023)147