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Linearized supergravity with a dynamical preferred frame

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Abstract: We study supersymmetric extension of the Einstein-aether gravitational model where local Lorentz invariance is broken down to the subgroup of spatial rotations by a vacuum expectation value of a timelike vector field called aether. Embedding aether into a chiral vector superfield, we construct the most general action which describes dynamics of linear perturbations around the Lorentz-violating vacuum and is invariant under the linearized supergravity transformations. The analysis is performed both in the off-shell non-minimal superfield formulation of supergravity and in the "on-shell" approach invoking only physical component fields. The resulting model contains a single free coupling, in addition to the standard supergravity parameters. The spectrum of physical excitations features an enhanced on-shell gravity multiplet comprising four states with helicities 2, 3/2, 3/2 and 1 propagating with superluminal velocity. The remaining excitations propagate with the speed of light. We outline the observational constraints on the model following from its low-energy phenomenology.

Keywords: Space-Time Symmetries, Supergravity Models, Classical Theories of Gravity, Superspaces

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Contents

1 Introduction and summary

The possibility to modify the laws of gravity has been the subject of an intensive theoretical research during recent decades [1–8], see [9] for review. This study has several motivations. First, it aims at solving the problems faced by the Einstein's theory of general relativity (GR) at very short distances, where it loses the predictive power because of non-renormalizability, as well as at very long — cosmological — scales where the standard paradigm leads to the cosmological constant problem. Second, phenomenological models of modified gravity can be used as proxies in the analysis of experimental data to put constraints on deviations from GR at various scales within a consistent framework. The third motivation is a deeper theoretical understanding of the principles underlying GR and the consequences implied by relaxing or replacing some of these principles.

An interesting class of modified gravity models involves violation of the local Lorentz invariance. The possibility of such violation is often attributed to the effects of quantum gravity, see [10, 11] and references therein. In particular, it has been suggested by P. Hořava [12] that the quantum theory of gravity can be rendered perturbatively renormalizable by abandoning Lorentz invariance as a fundamental symmetry at high energies. The rigorous proof of renormalizability in a version of this proposal has been given in [13, 14]. In this framework some amount of Lorentz symmetry breaking persists at all scales and at low energies the theory reduces to GR coupled to a scalar field with non-zero timelike gradient describing a preferred foliation of the spacetime [15, 16].

Hořava gravity is closely related [17–19] to the so-called Einstein-aether model [20, 21] where the effects of the dynamical preferred frame are encoded by a vector field u_m ("aether") constrained to have unit norm,[1]

$$u_m u^m = -1\,. \tag{1.1}$$

In the formal language, this vector belongs to the coset $\mathrm{SO}(3,1)/\mathrm{SO}(3)$ of the Lorentz group over the group of spatial rotations around the direction of u_m that remain unbroken. This construction is similar to the sigma-model description of non-linearly realized simmetries in particle physics. The most general action for the aether interacting with gravity and containing up to two derivatives reads,

$$S = \frac{1}{2\varkappa^2}\int d^4x\sqrt{-g}\left[R - K^{mn}{}_{sr}\nabla_m u^s \nabla_n u^r + \lambda(u_m u^m + 1)\right], \tag{1.2}$$

where λ is a Lagrange multiplier enforcing the constraint (1.1) and

$$K^{mn}{}_{sr} \equiv c_1 g^{mn} g_{sr} + c_2 \delta^m_s \delta^n_r + c_3 \delta^m_r \delta^n_s - c_4 u^m u^n g_{sr}\,. \tag{1.3}$$

The theory contains four dimensionless parameters c_i, $i = 1,2,3,4$. When the constraint (1.1) is solved explicitly and the action is written in terms of independent components of u_m, it contains non-linear derivative self-interactions of these components. This restricts the domain of validity of the model to energies below $M_* \equiv \varkappa^{-1}\sqrt{c}$, where c is the characteristic

[1] We use Latin letters from the middle of the alphabet for spacetime tensor indices; Latin letters from the beginning of the alphabet will be used for indices in the local Lorentz frame and Greek letters will be used for spinor indices. The signature of the metric is $(-,+,+,+)$; the Minkowski metric will be denoted by η_{mn}.

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value of the couplings c_i. At higher energies the model becomes strongly coupled and requires an ultraviolet (UV) completion. By analogy with sigma-models, the scale M_* can be identified with the scale of the Lorentz symmetry breaking,[2] the product $(\varkappa M_*)^2 = c$ controlling the strength of Lorentz violating effects in gravity. Phenomenology of this model has been extensively studied resulting in constraints on the couplings c_i [21–26], see [27] for review. It was also proposed to use Hořava gravity and Einstein-aether models for holographic description of strongly coupled non-relativistic systems [28–30].

It has long been envisaged that an important role at high energies can be played by supersymmetry (SUSY). In particle physics SUSY is usually considered as an extension of the Poincaré group. However, as pointed out in [31, 32], the SUSY algebra reduced by removing the boost generators closes on itself. In other words, SUSY does not necessarily require Lorentz invariance. Conversely, a general non-relativistic SUSY consisting of space- and time-translations, spatial rotations and supercharges in the spinor representation of SO(3) is equivalent to the standard SUSY algebra without boosts [33]. Remarkably, SUSY enforces emergence of Lorentz symmetry at low energies in the Standard Model, even if the high-energy theory is not Lorentz invariant [31–33]. This could explain the exquisite precision with which Lorentz invariance is satisfied in particle physics[3] [10, 11, 34].

It is natural to ask whether the local generalization of SUSY leading to the theory of supergravity (SUGRA) is also compatible with the existence of a preferred frame. Clearly, as in the case of ordinary gravity, this frame must be dynamical. The first step in answering this question was made in ref. [33] which has constructed the supersymmetric extension of the aether model in flat spacetime. In the superspace formalism, the aether is promoted to a chiral vector superfield U^c,

$$\bar{D}_{\dot{\alpha}} U^c = 0\,, \tag{1.4}$$

where $\bar{D}_{\dot{\alpha}}$ is the superspace covariant derivative.[4] The lowest component of U^c is identified with the complexified aether u^c. This choice of multiplet is motivated by the following considerations. Imposing on the superfield a constraint similar to (1.1),

$$U_c U^c = -1\,, \tag{1.5}$$

forces aether to develop a vacuum expectation value (VEV) that breaks the Lorentz symmetry. As a consequence, the latter is realized non-linearly on the perturbations around the vacuum. On the other hand, since the aether is the lowest component of the superfield, its VEV preserves SUSY which remains linearly realized.[5] This is interesting from the phenomenological perspective as it admits the mechanism of refs. [31–33] for protection

[2]It is worth stressing that, unlike the case of the usual spontaneous symmetry breaking, Lorentz invariance need not be restored above M_*. On the contrary, violation of Lorentz invariance can become increasingly more important at high energies, as it happens in Hořava gravity.

[3]Soft breaking of SUSY with superpartner masses parametrically below the scale of Lorentz violation preserves the suppression of Lorentz-violating effects in the Standard Model [31–33].

[4]We use the notations and conventions of [35] for the objects related to the spinor algebra and superspace geometry.

[5]This is different from the setup considered in the context of supersymmetric effective theory of inflation [36, 37] where not only the Lorentz group, but also SUSY is realized non-linearly.

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of Lorentz invariance in the matter sector. A different embedding (i.e. not as the lowest component of a supermultiplet) would break SUSY together with Lorentz invariance and hence lead to unsuppressed propagation of Lorentz violation to the matter fields. In addition to the aether, the superfield U^c describes its superpartner — "aetherino".

Upon an eventual soft SUSY breaking aetherino and the imaginary part of the aether acquire masses, whereas the action for the real part reduces to the flat-spacetime limit of (1.2) with a special choice of the couplings,[6]

$$c_2 + c_3 = c_4 = 0\,. \tag{1.6}$$

The coupling c_1 remains unrestricted. The analysis in flat spacetime is insufficient to decide whether SUSY constrains the parameters c_2 and c_3 separately or just their sum. For the Minkowski metric the c_2 and c_3 terms in the aether Lagrangian differ by a total derivative and only the sum $c_2 + c_3$ remains in the action after integration by parts.

In this paper we construct the interaction of the super-aether theory of ref. [33] with linearized supergravity. The latter is invariant only under the linearized version of the local Lorentz symmetry. Hence, we also need to linearize the super-aether field describing the spontaneous breaking of this symmetry. In other words, we expand the super-aether into a constant background w^c and fluctuations V^c,

$$U^c = w^c + V^c\,, \tag{1.7}$$

and keep terms up to quadratic order in V^c in the action.

The reason to focus on the linearized theory is twofold. First, this restriction allows us to bypass the complications associated to the construction of the curved superspace [35, 42–44] and use instead the transparent formalism of flat superspace. In this way we classify all possible supersymmetric terms in the Lagrangian and thereby analyze the uniqueness of the action. We find that local SUSY highly constrains the aether sector leaving only a single free parameter that is identified with c_1 in the low-energy theory, whereas the parameters c_2 and c_3 are forced to vanish. Second, this path directly leads us to the quadratic action for perturbations in components, which we use to analyze the dispersion relations of various physical modes.

We will follow two complementary approaches: the fully off-shell superfield formalism and the "on-shell" approach where one works only with physical component fields. In complete analogy with linearized gravity that is invariant both under linearized diffeomorphisms and the global Poincaré group, linearized SUGRA prossesses two sets of supersymmetries: global and local ones. The superfield formalism has the advantage of manifestly implementing the global SUSY as translations in the superspace. On the other hand, the local SUSY transformations are more complicated. They are encoded in linearized super-diffeomorphisms acting as gauge transformations on the superfields. The superspace gauge group is, however, too large. Its partial gauge fixing down to the physically relevant subgroup requires augmenting the auxiliary sector with a compensator multiplet.

[6]Soft SUSY breaking introduces corrections to these relations suppressed by the SUSY breaking scale.

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The chirality constraint that we want to impose on the aether superfield forces us to use the non-minimal off-shell formulation of SUGRA [38–40]. To see this, recall the general form of the anti-commutator of two spinor derivatives acting on a vector superfield [35],

$$\{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\}U^c = -T_{\dot\alpha\dot\beta}{}^b\mathcal{D}_bU^c - T_{\dot\alpha\dot\beta}{}^\gamma\mathcal{D}_\gamma U^c - T_{\dot\alpha\dot\beta}{}^{\dot\gamma}\bar{\mathcal{D}}_{\dot\gamma}U^c + U^bR_{\dot\alpha\dot\beta b}{}^c\,, \tag{1.8}$$

where $T_{AB}{}^C$ and $R_{ABC}{}^D$ are respectively torsion and curvature in the superspace.[7] In the minimal SUGRA all components of the torsion appearing in (1.8) vanish, whereas $R_{\dot\alpha\dot\beta bc}$ is in general non-zero [35]. This implies that the chirality constraint cannot be imposed in a covariant way as it is incompatible with (1.8). On the other hand, in the non-minimal formulation the supercovariant derivatives can be chosen such that [41, 42]

$$T_{\dot\alpha\dot\beta}{}^b = T_{\dot\alpha\dot\beta}{}^\gamma = R_{\dot\alpha\dot\beta b}{}^c = 0 \tag{1.9}$$

and therefore the covariant chirality constraint

$$\bar{\mathcal{D}}_{\dot\alpha}U^c = 0 \tag{1.10}$$

is consistent with (1.8). We classify all terms in the superspace Lagrangian that can be constructed from the linearized SUGRA and aether superfields and then fix the coefficients in front of them by imposing the invariance under super-diffeos.

In the "on-shell" approach the situation is in a sense reverse. Here the local gauge invariance is realized as simple gradient transformations of the metric and gravitino. The construction of the aether-supergravity interaction amounts to the classification of the possible aether energy-momentum tensors (EMTs) and supersymmetry currents coupled respectively to the metric perturbations and gravitino. The interaction, however, deforms global SUSY transformations and the invariance of the Lagrangian with respect to them must be checked explicitly. We work out the on-shell SUSY and perform this check perturbatively up to second order in the strength of the aether-gravity coupling.

The paper is organized as follows. In section 2 we review the formalism of linearized non-minimal SUGRA. While most of this material is standard, we present new derivations of several relations that are used in the rest of the paper. In particular, we work out in detail the gauge fixing in non-minimal linearized SUGRA and give explicit expressions for the superspace connection in terms of the gravity and compensator superfields. Supplementary material for this section is contained in appendices A and B.

In section 3 we introduce breaking of Lorentz invariance and the linearized aether superfield. We classify all inequivalent terms in the quadratic Lagrangian, derive their variations under super-diffeomorphisms and find the most general invariant superfield action. Appendix C contains technical details of this derivation.

In section 4 we derive the bosonic part of the Lagrangian in components. We first present the full off-shell action and then integrate out the auxiliary fields perturbatively in the aether coupling.

In section 5 (complemented with appendices D, E) we switch to the "on-shell" formalism and work out the most general form of the linearized aether EMT and supercurrent.

[7] Capital Latin letters $A, B, \ldots$ are used for the general superspace indices.

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This provides us with an alternative proof of the uniqueness of the aether-supergravity coupling. We reconstruct the fermionic part of the Lagrangian and derive the on-shell SUSY transformations.

In section 6 we analyze some physical implications of the model. Notably, we find that the velocity of gravitons necessarily exceeds the speed of light,[8] thereby manifesting violation of the Lorentz invariance in the low-energy theory. This is in contrast with the Standard Model matter and gauge fields which are protected by SUSY from Lorentz symmetry violation at low energies [31, 32]. The excess of graviton velocity over unity is interesting from the viewpoint of SUSY representations as it leads to an enhancement of the graviton multiplet by additional states with helicities 3/2 and 1. The dispersion relations for the correspoding modes are derived in appendix F and they are shown to match that of the graviton. We also briefly discuss the phenomenological constraints on the model.

Section 7 is devoted to conclusions.

2 Linearized non-minimal supergravity

2.1 Field content and gauge fixing

We follow [43, 44]. The basic ingredient of the linearized SUGRA is a real vector superfield H_m transforming as[9]

$$\delta_L H_{\alpha\dot{\alpha}} = \bar{D}_{\dot{\alpha}} L_\alpha - D_\alpha \bar{L}_{\dot{\alpha}} \tag{2.1}$$

under the linearized super-diffeomorphisms parameterized by the spinor superfield L_α. To understand the physical content of H_m let us decompose it in components [45],

$$c_m = H_m|\,, \qquad \chi_{\alpha\beta\dot{\beta}} = D_\alpha H_{\beta\dot{\beta}}|\,, \qquad a_m = -\frac{1}{4}D^2 H_m|\,, \tag{2.2a}$$

$$e_{mn} = -\Delta_m H_n|\,, \qquad \psi_\alpha^m = \frac{i}{16}\bar{\sigma}^{m\dot{\beta}\beta}\bar{D}^2 D_\beta H_{\alpha\dot{\beta}}|\,, \qquad d_m = \frac{1}{32}\{D^2, \bar{D}^2\}H_m|\,, \tag{2.2b}$$

where the vertical line denotes evaluation at zero fermionic coordinates $\theta = \bar{\theta} = 0$, and

$$\Delta_m H_n \equiv \frac{1}{4}\bar{\sigma}_m^{\dot{\alpha}\alpha}[\bar{D}_{\dot{\alpha}}, D_\alpha]H_n\,. \tag{2.3}$$

We see that the multiplet contains a spin-2 field e_{mn} that is identified with the perturbation of the tetrad, as well as the spin-3/2 field ψ_α^m describing gravitino. Introducing also the components of the gauge parameter L_α,

$$\xi_m \equiv \xi_m^R + i\xi_m^I = i\bar{\sigma}_m^{\dot{\alpha}\alpha}\bar{D}_{\dot{\alpha}} L_\alpha|\,, \qquad \varepsilon_\alpha = -\frac{1}{4}\bar{D}^2 L_\alpha|\,, \qquad \mu_\alpha^m = i\bar{\sigma}^{m\dot{\beta}\beta}D_\alpha \bar{D}_{\dot{\beta}} L_\beta|\,, \tag{2.4a}$$

$$\lambda_\alpha{}^\beta = -\frac{1}{4}D_\alpha \bar{D}^2 L^\beta|\,, \qquad \kappa^m = -\frac{i}{4}\sigma^m_{\alpha\dot{\alpha}}D^2\bar{D}^{\dot{\alpha}}L^\alpha|\,, \qquad \rho_\alpha = \frac{1}{16}D^2\bar{D}^2 L_\alpha|\,, \tag{2.4b}$$

[8] Superluminal propagation of signals does not lead to any inconsistencies in theories with a preferred reference frame. In particular, absence of Lorentz invariance at the fundamental level prevents from creating closed timelike curves, see e.g. discussion in [21, 23].

[9] Throughout the text D_α, $\bar{D}_{\dot{\alpha}}$ denote the covariant derivatives of the flat superspace preserving the global SUSY. They should not be confused with the derivatives covariant under local SUSY transformations that are denoted by $\mathcal{D}_\alpha$, $\bar{\mathcal{D}}_{\dot{\alpha}}$.

one obtains from (2.1) the following transformation laws:

$$\delta_L c_m = -\xi^I_m\,, \qquad \delta_L \chi_{\alpha\beta\dot\beta} = \frac{i}{2}\sigma_{m\beta\dot\beta}\mu^m_\alpha + 2\epsilon_{\alpha\beta}\bar\varepsilon_{\dot\beta}\,, \qquad \delta_L a_m = \frac{i}{2}\kappa_m\,, \tag{2.5a}$$

$$\delta_L e_{mn} = \partial_m\xi^R_n + (\sigma_{mn})^{\alpha\beta}\lambda_{\alpha\beta} - (\bar\sigma_{mn})^{\dot\alpha\dot\beta}\bar\lambda_{\dot\alpha\dot\beta} + \frac{1}{2}\eta_{mn}(\lambda^\alpha{}_\alpha - \bar\lambda_{\dot\alpha}{}^{\dot\alpha})\,, \tag{2.5b}$$

$$\delta_L\psi^m_\alpha = \partial^m\varepsilon_\alpha - \frac{i}{2}\sigma^m_{\alpha\dot\beta}\bar\rho^{\dot\beta}\,, \qquad \delta_L d_m = -\frac{1}{2}\Box\xi^I_m + \left[\frac{i}{4}(\sigma^n\bar\sigma_m)_{\alpha\gamma}\partial_n\lambda^{\alpha\gamma} + \text{h.c.}\right], \tag{2.5c}$$

where $\Box \equiv \eta^{mn}\partial_m\partial_n$. It follows from (2.5a) that the imaginary part of ξ_m and the components μ^m_α, κ_m can be chosen to impose the Wess-Zumino gauge,

$$c_m = \chi_{\alpha\beta\dot\beta} = a_m = 0\,. \tag{2.6}$$

Note that this implies the relation between the gauge parameters μ^m_α and ε_α,

$$\mu^m_\alpha = 2i\sigma^m_{\alpha\dot\beta}\bar\varepsilon^{\dot\beta}\,. \tag{2.7}$$

The remaining transformations contain infinitesimal diffeomorphisms with the parameter ξ^R_m, local Lorentz transformations parameterized by the symmetric part of $\lambda_{\alpha\beta}$ and local SUSY corresponding to ε_α. The trace part $\lambda^\alpha{}_\alpha$ and the spinor ρ_α give rise to extra symmetries: Weyl invariance and superconformal transformations. The latter symmetries are not generally present in SUGRA. They are removed by introducing a compensator.

In the minimal linearized SUGRA the compensator is chosen to be a chiral scalar superfield. However, as discussed in the Introduction, the minimal formulation does not admit a coupling to the super-aether theory. The next-to-simplest choice of the compensator, which leads to the non-minimal formulation, is a linear superfield Γ,

$$\bar D^2\Gamma = 0\,, \tag{2.8}$$

with independent components,

$$\gamma = \Gamma|\,, \qquad \bar\omega_{\dot\alpha} = \bar D_{\dot\alpha}\Gamma|\,, \qquad \phi_\alpha = D_\alpha\Gamma|\,, \tag{2.9a}$$

$$B = -\frac{1}{4}D^2\Gamma|\,, \qquad q_m \equiv q^R_m + iq^I_m = \Delta_m\Gamma|\,, \qquad \bar\nu_{\dot\alpha} = \frac{1}{4}D^2\bar D_{\dot\alpha}\Gamma|\,. \tag{2.9b}$$

It transforms under the super-diffeomorphisms as

$$\delta_L\Gamma = -\frac{n+1}{4(3n+1)}\bar D^2 D^\alpha L_\alpha + \frac{1}{4}\bar D^{\dot\alpha}D^2\bar L_{\dot\alpha}\,, \tag{2.10}$$

where $n \neq -\frac{1}{3}, 0$ is a real parameter enumerating different versions of the non-minimal SUGRA. The gauge transformations of the components read,

$$\delta_L\gamma = \frac{n+1}{3n+1}\partial_m\xi^m + \frac{n+1}{3n+1}\lambda^\alpha{}_\alpha + \bar\lambda^{\dot\alpha}{}_{\dot\alpha}\,, \qquad \delta_L\bar\omega_{\dot\alpha} = 2\bar\rho_{\dot\alpha}\,, \tag{2.11a}$$

$$\delta_L\phi_\alpha = \frac{2(n+1)}{3n+1}\rho_\alpha + \frac{n+1}{3n+1}\partial_m\mu^m_\alpha - 2i\sigma^m_{\alpha\dot\alpha}\partial_m\bar\varepsilon^{\dot\alpha}\,, \qquad \delta_L B = \frac{n+1}{3n+1}\partial_m\kappa^m\,, \tag{2.11b}$$

$$\delta_L q_m = \frac{i(n+1)}{3n+1}\partial_m\partial_n\xi^n + \frac{i(n+1)}{3n+1}\partial_m\lambda^\alpha{}_\alpha - 2i(\bar\sigma_{mn})^{\dot\alpha\dot\beta}\partial^n\bar\lambda_{\dot\alpha\dot\beta}\,, \qquad \delta_L\bar\nu_{\dot\alpha} = -2\Box\bar\varepsilon_{\dot\alpha}\,. \tag{2.11c}$$

They suggest to fix away the gauge parameters $\lambda^\alpha{}_\alpha$ and ρ_α by imposing the conditions,

$$\gamma = \frac{n+1}{3n+1} e^m{}_m\,, \tag{2.12a}$$

$$\omega_\alpha + \phi_\alpha + \frac{4in}{3n+1}\sigma^m_{\alpha\dot\alpha}\,\bar\psi^{\dot\alpha}_m = 0\,. \tag{2.12b}$$

Note that the coefficient in front of the term with gravitino in eq. (2.12b) has been chosen in such a way that the contributions with $\bar\varepsilon^{\dot\alpha}$ cancel out from the transformation of the l.h.s. when the relation (2.7) is imposed. The choice of the relative coefficients between ω_α and ϕ_α will be discussed shortly.

Further, we can use the symmetric part of $\lambda_{\alpha\beta}$ to render the tetrad symmetric,

$$e_{mn} = e_{nm} = \frac{1}{2}h_{mn}\,, \tag{2.13}$$

where h_{mn} are the perturbations of the metric. To preserve this gauge, the parameters $\lambda_{\alpha\beta}$ must be related to the linearized local translations as

$$(\sigma_{mn})^{\alpha\beta}\lambda_{\alpha\beta} - (\bar\sigma_{mn})^{\dot\alpha\dot\beta}\bar\lambda_{\dot\alpha\dot\beta} = -\frac{1}{2}\partial_m\xi^R_n + \frac{1}{2}\partial_n\xi^R_m\,. \tag{2.14}$$

This completes the gauge fixing procedure that will be used when considering the formulation of the theory in components. It leaves only the linearized diffeomorphisms and the local SUSY transformations, under which the metric perturbations and garvitino transform in the standard way,

$$\delta_L h_{mn} = \partial_m\xi^R_n + \partial_n\xi^R_m\,, \qquad \delta_L\psi^m_\alpha = \partial^m\varepsilon_\alpha\,. \tag{2.15}$$

In addition to the local super-diffeomorphisms, the linearized supergravity is invariant under the global SUSY transformations, whose coordinate-independent parameter will be denoted by ζ^α. This symmetry is manifest in the superfield formalism. In appendix A we work out the global SUSY transformations of the component fields and discuss how they are affected by the gauge fixing. In particular, we show that the special choice of coefficients in the gauge condition (2.12b) brings the SUSY transformation of the metric into the canonical form in terms of gravitino [35],[10]

$$\tilde\delta_G h_{mn} = 2i(\zeta\sigma_m\bar\psi_n + \zeta\sigma_n\bar\psi_m + \bar\zeta\bar\sigma_m\psi_n + \bar\zeta\bar\sigma_n\psi_m)\,. \tag{2.16}$$

The action of the linearized non-minimal SUGRA has the form [44],

$$S_{SG} = \int d^4x \int d^2\theta d^2\bar\theta\; \mathbb{L}_{SG}\,, \tag{2.17a}$$

with the superspace Lagrangian,

$$\begin{aligned}\mathbb{L}_{SG} = \frac{1}{\varkappa^2}\bigg[&\frac{1}{4}\Big((\partial_k H_m)^2 - (\Delta_k H_m)^2\Big) + \frac{n+1}{2n}(\partial_m H^m)^2 + \frac{n+1}{2}(\Delta_m H^m)^2 \\ &- i\frac{3n+1}{2n}\partial_m H^m\left(\Gamma - \bar\Gamma\right) + \frac{3n+1}{2}\Delta_m H^m\left(\Gamma + \bar\Gamma\right) + \frac{9n^2-1}{8n}(\Gamma^2 + \bar\Gamma^2) + \frac{(3n+1)^2}{4n}\Gamma\bar\Gamma\bigg].\end{aligned} \tag{2.17b}$$

[10]Note that our definition of gravitino differs by a factor $-1/2$ from that adopted in [35].

It is straightforward to verify that it is invariant under the transformations (2.1), (2.10). The above expressions simplify considerably for the choice $n = -1$. However, we are not going to restrict to this case as we want to study the most general coupling of the super-aether to gravity.

Using the general formula

$$\mathcal{L} = \frac{1}{32}\{\bar{D}^2, D^2\}\,\mathbb{L}| \tag{2.18}$$

that relates the component Lagrangian $\mathcal{L}$ to that in superspace one obtains the off-shell Lagrangian of non-minimal SUGRA in terms of the component fields. Its bosonic part reads,

$$\begin{aligned}\mathcal{L}^{\rm bos}_{SG} = \frac{1}{2\varkappa^2}\bigg\{&\frac{1}{4}h_{km}\Box h_{km} + \frac{n-1}{8n}\partial_k h_{km}\partial_l h_{lm} - \frac{n+1}{4(3n+1)}h\Box h\\ &+\frac{3n+1}{2n}q^I_m\partial_k h_{km} - q^I_m\partial_m h - \frac{3n+1}{2n}q^I_m q^I_m\\ &-2(n-1)d_m d_m + 2(3n+1)d_m q^R_m - \frac{3(3n+1)}{2}q^R_m q^R_m + \frac{(3n+1)^2}{2n}B\bar{B}\bigg\},\end{aligned} \tag{2.19}$$

where $h \equiv h^m_m$. After integrating out the auxiliary fields q_m, d_m and B it takes the well-known form of the linearized Einstein-Hilbert action

$$\mathcal{L}_{EH} = \frac{1}{2\varkappa^2}\left(\frac{1}{4}h_{km}\Box h^{km} + \frac{1}{2}\partial^k h_{km}\partial_l h^{lm} - \frac{1}{2}\partial_k h^{km}\partial_m h + \frac{1}{4}\partial_m h\partial^m h\right). \tag{2.20}$$

The fermionic part of the action upon elimination of the auxiliary fields is also well-known and is given by the Rarita-Schwinger Lagrangian,

$$\mathcal{L}_{RS} = \frac{4}{\varkappa^2}\,\epsilon^{klmn}\,\bar{\psi}_k\,\bar{\sigma}_l\,\partial_m\psi_n\,, \tag{2.21}$$

where ϵ^{klmn} is the totally antisymmetric tensor, $\epsilon^{0123} = 1$.

2.2 Lorentz transformations in superspace and covariant derivatives

In non-linear superspace formulation of SUGRA the most general transformation of a superfield consists of super-diffeomorphisms and local Lorentz rotations [35]. The particular realization of the linearized supergravity using the fields H_m, Γ corresponds to a (partial) gauge fixing of this symmetry linking the superspace translations and local Lorentz rotations to the single spinor superfield L_α. Thus, the transformations of a scalar superfield Ψ and a superfield with a Lorentz index Ψ^A read respectively,

$$\delta_L\Psi = l^M(L_\alpha)\,\partial_M\Psi\,, \qquad \delta_L\Psi^A = l^M(L_\alpha)\,\partial_M\Psi^A + \Psi^B\,M_B{}^A(L_\alpha)\,,$$

where capital letters denote general — vector or spinor — indices. The only non-vanishing components of the matrix $M_B{}^A$ are $M_\beta{}^\alpha$, $M^{\dot\beta}{}_{\dot\alpha}$, $M_b{}^a$ and satisfy the structural relations of the SL(2) algebra,

$$M_{\alpha\beta} = M_{\beta\alpha}\,, \qquad M_{\dot\alpha\dot\beta} = -(M_{\alpha\beta})^*\,, \qquad M_{ab} = \frac{1}{2}\bar{\sigma}^{\dot\alpha\alpha}_a\bar{\sigma}^{\dot\beta\beta}_b(\epsilon_{\dot\alpha\dot\beta}M_{\alpha\beta} - \epsilon_{\alpha\beta}M_{\dot\alpha\dot\beta})\,. \tag{2.22}$$

Our present goal is to work out the expressions for $M_A{}^B$ in terms of L_α.

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We start from the expression for the differential operator describing local superspace translations given in ref. [45],[11]

$$\hat{L} \equiv l^M(L_\alpha)\,\partial_M = -\frac{1}{4}(\bar{D}^2L^\alpha)D_\alpha - \frac{1}{4}(D^2\bar{L}_{\dot\alpha})\bar{D}^{\dot\alpha} + \frac{i}{2}(\bar{D}^{\dot\alpha}L^\alpha + D^\alpha\bar{L}^{\dot\alpha})\,\partial_{\alpha\dot\alpha}\,. \tag{2.23}$$

As discussed in [45], this form is fixed by the requirement that scalar chiral superfields should admit covariant generalizations. Covariant derivatives of a scalar must transform as spinors or vectors. In particular,

$$\delta_L(\mathcal{D}_\alpha\Psi) = \hat{L}\mathcal{D}_\alpha\Psi + (\mathcal{D}^\beta\Psi)\,M_{\beta\alpha}\,. \tag{2.24}$$

On the other hand, the covariant derivatives are related to the ordinary derivatives through the superspace vielbein $E_A{}^M$,

$$\mathcal{D}_A\Psi = E_A{}^M\,\partial_M\Psi\,. \tag{2.25}$$

At linear order the vielbein can be written as

$$E_A{}^M = E_A^{(0)\,M} - I_A{}^B\,E_B^{(0)\,M}\,, \tag{2.26}$$

where $E_A^{(0)\,M}$ is the vielbein of the flat superspace. On general grounds, the tensor $I_A{}^B$ must be linear in the SUGRA fields[12] H_b, Γ, with the precise expressions yet to be determined. Substituting eqs. (2.25), (2.26) into the l.h.s. of eq. (2.24) and using $E_A^{(0)\,M}\partial_M\Psi = D_A\Psi$ we obtain,

$$\delta_L(\mathcal{D}_\alpha\Psi) = \delta_L(D_\alpha\Psi - I_\alpha{}^BD_B\Psi) = D_\alpha\hat{L}\Psi - (\delta_LI_\alpha{}^B)D_B\Psi\,,$$

where in the second equality we neglected terms of the form $I_\alpha{}^BD_B\hat{L}\Psi$ as they are quadratic in the deviations from the flat superspace geometry. Equating the coefficients in front of different derivatives $D_A\Psi$ in eq. (2.24) we obtain a system of equations for $M_{\alpha\beta}$ and the variations of the vielbein components $I_\alpha{}^\beta$, $I_{\alpha\dot\beta}$, $I_\alpha{}^b$. To solve this system, we observe that the symmetric part of $I_{\alpha\beta}$ can be removed by a local superfield Lorentz transformation acting on the index A in eq. (2.26). In other words, its choice corresponds to a residual local Lorentz symmetry of the linearized SUGRA. To fix this symmetry completely we set $I_{\alpha\beta} + I_{\beta\alpha} = 0$. Then we obtain the unique solution,

$$M_{\alpha\beta} = \frac{1}{4}D_\alpha\bar{D}^2L_\beta + \frac{1}{8}\epsilon_{\alpha\beta}D_\gamma\bar{D}^2L^\gamma\,, \tag{2.27}$$

$$\delta_LI_\alpha{}^\beta = -\frac{1}{8}\delta_\alpha^\beta\,D_\gamma\bar{D}^2L^\gamma\,,\quad \delta_LI_{\alpha\dot\beta} = 0\,,\quad \delta_LI_\alpha{}^b = \frac{i}{2}\sigma^b_{\beta\dot\beta}D_\alpha\left(\bar{D}^{\dot\beta}L^\beta - D^\beta\bar{L}^{\dot\beta}\right)\,. \tag{2.28}$$

From (2.27) using the structural relations (2.22) we obtain the rotation matrix for Lorentz vectors,

$$M_{ab} = \frac{1}{4}(\sigma_{ab})_\beta{}^\alpha D_\alpha\bar{D}^2L^\beta + \frac{1}{4}(\bar\sigma_{ab})^{\dot\alpha}{}_{\dot\beta}\bar{D}^{\dot\beta}D^2\bar{L}_{\dot\alpha}\,. \tag{2.29}$$

Note that its lowest component reads,

$$M_{ab}| = (\sigma_{ab})^{\alpha\beta}\lambda_{\alpha\beta} - (\bar\sigma_{ab})^{\dot\alpha\dot\beta}\bar\lambda_{\dot\alpha\dot\beta}\,, \tag{2.30}$$

[11] We choose the representation of the super-diffeos that preserves real superfields.

[12] At the linearized level we do not distinguish the spacetime and Lorentz indices whenever it does not lead to confusion.

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which is precisely the matrix of local Lorentz transformations acting on the physical tetrad, see eq. (2.5b).

Equations (2.28) allow us to determine the vielbein components entering them in terms of the fields H_b, Γ. The unique combinations of these fields with the required transformation properties are,

$$I_\alpha{}^\beta = -\delta_\alpha^\beta \frac{1}{2}\bar{\Gamma}' , \qquad I_{\alpha\dot{\beta}} = 0 , \qquad I_\alpha{}^b = -iD_\alpha H^b , \tag{2.31}$$

where

$$\bar{\Gamma}' = -\frac{(3n+1)(n-1)}{4n}\bar{\Gamma} - \frac{(3n+1)(n+1)}{4n}\Gamma - \frac{(n+1)^2}{8n}\bar{D}^{\dot{\alpha}} D^\alpha H_{\alpha\dot{\alpha}} + \frac{n^2-1}{8n} D^\alpha \bar{D}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} . \tag{2.32}$$

Thus, the spinor covariant derivative of a scalar field takes the form,

$$\mathcal{D}_\alpha \Psi = \left(1 + \frac{\bar{\Gamma}'}{2}\right) D_\alpha \Psi + iD_\alpha H^b \,\partial_b \Psi . \tag{2.33}$$

The expression for $\bar{\mathcal{D}}_{\dot{\alpha}}\Psi$ is obtained by complex conjugation. Determination of the remaining components of the linearized vielbein $I_a{}^\beta$, $I_a{}^b$ requires invoking the constraints on the superspace torsion imposed in non-minimal SUGRA [42]. This analysis is performed in appendix B.

For a superfield with spinor or vector indices the covariant derivatives should be supplemented with a connection term,

$$\mathcal{D}_A \Psi^B = E_A{}^M \,\partial_M \Psi^B + (-1)^{|A||B|} \Psi^C \,\Phi_{AC}{}^B , \tag{2.34}$$

where[13]

$$|B| = \begin{cases} 0, & \text{for} \quad B = b \\ 1, & \text{for} \quad B = \beta \text{ or } \dot{\beta} \end{cases}$$

The connection coefficients $\Phi_{AC}{}^B$ obey the structural relations of SL(2) analogous to eqs. (2.22),

$$\begin{aligned} &\Phi_{A\gamma\beta} = \Phi_{A\beta\gamma} , \quad \Phi_{\alpha\dot{\gamma}\dot{\beta}} = -(\Phi_{\dot{\alpha}\gamma\beta})^* , \quad \Phi_{\dot{\alpha}\dot{\gamma}\dot{\beta}} = -(\Phi_{\alpha\gamma\beta})^* , \quad \Phi_{a\dot{\gamma}\dot{\beta}} = -(\Phi_{a\gamma\beta})^* , \\ &\Phi_{Acb} = \frac{1}{2}\bar{\sigma}_c^{\dot{\gamma}\gamma}\bar{\sigma}_b^{\dot{\beta}\beta}(\epsilon_{\dot{\gamma}\dot{\beta}}\Phi_{A\gamma\beta} - \epsilon_{\gamma\beta}\Phi_{A\dot{\gamma}\dot{\beta}}) . \end{aligned} \tag{2.35}$$

In the linearized SUGRA the connections are expressed in terms of the fields H_b, Γ. Below we will need explicit formulas for the components $\Phi_{\dot{\alpha}C}{}^B$. These are found simultaneously with the vielbein from the torsion constraints, see appendix B. The result reads,

$$\Phi_{\dot{\alpha}\beta\gamma} = -\frac{1}{8}(\bar{D}^2 D_\beta H_{\gamma\dot{\alpha}} + \bar{D}^2 D_\gamma H_{\beta\dot{\alpha}}) , \qquad \Phi_{\dot{\alpha}\dot{\beta}\dot{\gamma}} = \frac{1}{2}(\epsilon_{\dot{\alpha}\dot{\beta}}\bar{D}_{\dot{\gamma}}\Gamma + \epsilon_{\dot{\alpha}\dot{\gamma}}\bar{D}_{\dot{\beta}}\Gamma) . \tag{2.36}$$

Note that these formulas do not depend on the parameter n. The connection in the vector representation is obtained from the structural relation (2.35),

$$\Phi_{\dot{\alpha}bc} = -\frac{1}{4}(\sigma_{bc})_\alpha{}^\beta \bar{D}^2 D^\alpha H_{\beta\dot{\alpha}} - (\bar{\sigma}_{bc})^{\dot{\beta}}{}_{\dot{\alpha}} \bar{D}_{\dot{\beta}}\Gamma . \tag{2.37}$$

[13] Following [35], we assume that the fields carrying even (odd) number of spinor indices take commuting (anti-commuting) values.

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Note that under the linearized super-diffeos the connections transform as

$$\delta_L \Phi_{\dot{\alpha}C}{}^B = -\bar{D}_{\dot{\alpha}} M_C{}^B \,. \tag{2.38}$$

This is consistent with the transformations of the corresponding covariant derivatives (2.34) in the appropriate representations of SL(2).

3 Breaking Lorentz invariance

3.1 Perturbations of super-aether

We now want to generalize the linearized SUGRA to the case when Lorentz invariance is broken down to the SO(3) subgroup of spatial rotations by a vacuum expectation value (VEV) of a timelike vector field. To this end, we introduce [33] a chiral vector superfield U^a obeying the constraint (1.5). As a consequence of this constraint, the field develops a c-number VEV w^a satisfying the relations,

$$\operatorname{Re} w_a \operatorname{Re} w^a - \operatorname{Im} w_a \operatorname{Im} w^a = -1 \,, \qquad \operatorname{Re} w_a \operatorname{Im} w^a = 0 \,.$$

They imply that $\operatorname{Re} w^a$ is always timelike and $\operatorname{Im} w^a$ is spacelike. Thus, unless $\operatorname{Im} w^a = 0$, the vacuum breaks both Lorentz and rotational symmetries. In this paper we are interested in quadratic theory around a rotationally invariant vacuum, so we focus on the case of real w^a. Then there is a preferred Lorentz frame where w^a has the form,

$$w^a = (1, 0, 0, 0) \,. \tag{3.1}$$

It is important to stress that, despite the breaking of Lorentz invariance, all SUSY generators are preserved as they corresponds to translations in the superspace that leave the aether VEV invariant [31–33].

Next, we expand the super-aether field about its VEV as in eq. (1.7). As already mentioned in the Introduction, this expansion is forced on us by the fact that we work with the linearized SUGRA. Indeed, the latter is invariant under the linearized version of the local Lorentz and coordinate transformations contained in the parameter L_α, see section 2.1. Let us look at the original Einstein-aether theory (1.2) and ask what sector of it is invariant under these linearized transformations. Clearly, this sector corresponds to the quadratic part of the action expanded in metric and aether fluctuations around the background. In particular, it would be inconsistent to expand only in the metric while keeping the aether nonlinear: the latter transforms non-trivially under the local spacetime symmetries and the invariance would be lost. Returning to the SUSY case, we conclude that the requirement of invariance with respect to the linearized local SUSY requires restricting the equations of motion (action) to the linear (quadratic) order in the super-aether perturbation V^c.

The constraint (1.5) expanded to linear order translates into

$$w_a V^a = 0 \,, \tag{3.2}$$

whereas the chirality condition reads,

$$\bar{D}_{\dot{\alpha}} V^c = -w^b \Phi_{\dot{\alpha}b}{}^c \,. \tag{3.3}$$

Here we expanded the covariant derivative (2.34) to linear order both in aether perturbations and SUGRA fields. Note that the explicit form of the connection (2.37) implies that it is chiral and hence the aether perturbation V^c is linear, $\bar{D}^2 V^c = 0$. The aether perturbations transform non-trivially under super-diffeomorphisms,

$$\delta_L V^a = w^b M_b{}^a \,, \tag{3.4}$$

where $M_b{}^a$ is the matrix of Lorentz rotations (2.29).

We define the components of the aether supermultiplet as follows,

$$v^a \equiv v^{R,a} + i v^{I,a} = V^a| \,, \quad \eta^a_\alpha = \mathcal{D}_\alpha U^a| = (D_\alpha V^a + w^c \Phi_{\alpha c}{}^a)| \,, \quad G^a = -\frac{1}{4} D^2 V^a| \,. \tag{3.5}$$

Note that in the definition of the aetherino η^a_α we used the covariant spinor derivative acting on the full aether, which we expanded to the linear order in the second equality. Due to this definition, the aetherino is invariant under the linearized super-diffeos, as it follows from the transformation laws (3.4) and (2.38). The same is true also for the auxiliary field G^a due to the property $D^2 M_b{}^a = 0$ implied by the expression (2.29). Thus, we have,

$$\delta_L \eta^a_\alpha = \delta_L G^a = 0 \,. \tag{3.6a}$$

On the other hand, the lowest component v^a transforms non-trivially. From eqs. (2.30) and (2.14) we find,

$$\delta_L v^a = -\frac{1}{2} w_b \left(\partial^b \xi^{R,a} - \partial^a \xi^{R,b}\right) \tag{3.6b}$$

The transformations of the super-aether components under global SUSY are derived in appendix A.

The component fields will be used in sections 4, 5. Now our goal is to find the most general superspace action quadratic in the superfields V^a, H_a, Γ and invariant under the transformations (2.1), (2.10), (3.4).[14]

3.2 Possible terms in the Lagrangian

First we notice that the only possible term in the superpotential is the term enforcing the constraint (3.2) by means of a chiral Lagrange multiplier Λ (cf. [33]),

$$\mathcal{L}_{constr} = \int d^2\theta \, \Lambda w_a V^a + \text{h.c.} \,. \tag{3.7}$$

The combination $w_a V^a$ is chiral due to the relation (3.3) and anti-symmetry of the connection coefficient $\Phi_{\dot{\alpha} bc}$ in the last two indices. No other chiral combination can be constructed from V^a and the SUGRA fields without using the spinor derivatives $\bar{D}_{\dot{\alpha}}$. On the other hand, the terms in the superpotential that involve $\bar{D}_{\dot{\alpha}}$ can be equivalently written as contributions to the Kähler potential. For example,

$$\int d^2\theta \, \bar{D}_{\dot{\alpha}} \Gamma \bar{D}^{\dot{\alpha}} \Gamma \simeq -2 \int d^2\theta d^2\bar{\theta} \, \Gamma^2 \,,$$

[14]It is worth noting that this action cannot be found simply by a linear gauging of the super-aether action of [33]. Such gauging does not capture the terms quadratic in the SUGRA fields H_b and Γ which are present in the resulting super-aether action and are important for the analysis of its physical content, see below.

and similarly for other contributions. Here the sign $\simeq$ means 'equal up to a total derivative'. Thus, it is enough to consider the Kähler potential only.

By analogy with (1.2), we search for the action in the form

$$S = \frac{1}{\varkappa^2}\int d^4x \int d^2\theta d^2\bar{\theta}\, \tilde{\mathbb{L}}\,,$$

where the gravitational coupling with the mass dimension $[\varkappa^{-2}] = 2$ has been factored out in front of the Lagrangian. All other parameters in the Lagrangian are assumed to be dimensionless. This implies that the superspace Lagrangian $\tilde{\mathbb{L}}$ must be constructed from terms with zero mass dimension. The dimensions of the object at our disposal are

$$[H_a] = -1\,, \quad [w^a] = [V^a] = [\Gamma] = 0\,, \quad [D_\alpha] = [\bar{D}_{\dot\alpha}] = 1/2\,, \quad [\partial_a] = [\Delta_a] = 1\,. \tag{3.8}$$

Once the aether VEV w^a is included as the spurion to compensate for the breaking of the Lorentz symmetry, the Lagrangian becomes a scalar with respect to global Lorentz transformations. Besides, the Lagrangian must be real.

As our goal is to find the most general super-aether action, we proceed as follows. We first classify all inequivalent terms in the quadratic Lagrangian satisfying the above requirements. In the next subsection we will look for their linear combinations invariant under the non-linearly realized super-diffeomorphisms.

Operators quadratic in V^a. We have a single operator in this class,

$$V_a\bar{V}^a\,. \tag{3.9}$$

The combination V_aV^a and its complex conjugate can be rewritten purely in terms of the SUGRA fields H_a and Γ using the relation (3.3). Indeed,

$$\int d^2\theta d^2\bar{\theta}\; V_aV^a \simeq -\frac{1}{4}\int d^2\theta \bar{D}^2(V_aV^a) = -\frac{1}{2}\int d^2\theta\; w^bw^c\Phi_{\dot\alpha ba}\Phi^{\dot\alpha}{}_c{}^a\,.$$

As discussed above, the expression on the r.h.s. can be cast in the form of a contribution into the Kähler potential.

Operators linear in V^a. In total, there are four independent combinations,

$$w^aV^b\partial_aH_b\,, \quad w^aV^b\partial_bH_a\,, \quad w^aV^b\partial^cH^d\epsilon_{abcd}\,, \quad V^aD^2H_a\,, \tag{3.10}$$

plus their complex conjugate. From all other terms the aether perturbation can be eliminated by performing integration by parts and using (3.3). For example,

$$V^a\bar{D}^2H_a \simeq (\bar{D}^2V^a)H_a = 0\,,$$
$$w^aV^b\Delta_aH_b = w^aV^b\Big(-i\partial_a+\frac{1}{2}\bar{\sigma}_a^{\dot\alpha\alpha}\bar{D}_{\dot\alpha}D_\alpha\Big)H_b \simeq -iw^aV^b\partial_aH_b+\frac{1}{2}\bar{\sigma}_a^{\dot\alpha\alpha}w^aw^c\Phi_{\dot\alpha c}{}^bD_\alpha H_b\,,$$

and so on.

Operators without V^a. This is the most numerous group of terms. On the other hand, their role is in a sense auxiliary: as discussed below, they are needed to compensate the variation of the quadratic aether term (3.9) and do not give rise to any independent Lagrangians. We relegate their classification to appendix C.2. We find a total of 27 terms that are not equivalent to each other upon integration over the superspace.

3.3 The invariant action

The action for linearized SUGRA with broken Lorentz invariance is obtained as a linear combination of the independent operators listed in the previous subsection that is invariant under the gauge transformations (2.1), (2.10), (3.4). To find this combination, let us analyze the variations of individual operators. We start with the term quadratic in the aether perturbations,

$$\begin{aligned}\delta_L(V_a\bar{V}^a) &= w^b M_{ba}\bar{V}^a + w^b M_{ba} V^a \\ &\simeq -w^b w^c L^\beta (\sigma_{ba})_\beta{}^\gamma \left(\frac{1}{4}\bar{D}^2\Phi_{\gamma c}{}^a + i\partial_{\gamma\dot{\gamma}}\Phi^{\dot{\gamma}}{}_c{}^a\right) + \text{h.c.} \\ &= L^\beta\bigg(-\frac{i}{2}w^a w^b (\sigma_{ak})_\beta{}^\gamma \bar{D}^2 D_\gamma \partial^k H_b + \frac{i}{4}w^a w^b \bar{D}^2 D_\beta \partial_a H_b - \frac{i}{8}\bar{D}^2 D_\beta \partial_k H^k \\ &\quad - i w^a w^b \sigma_{a\beta\dot{\beta}} \bar{D}^{\dot{\beta}} \partial_b \Gamma - \frac{i}{4}\sigma_{k\beta\dot{\beta}}\bar{D}^{\dot{\beta}}\partial^k\Gamma - \frac{3}{16}\bar{D}^2 D_\beta \bar{\Gamma}\bigg) + \text{h.c.}\,,\end{aligned} \tag{3.11}$$

where in the second line we used the chirality condition (3.3). One observes that this variation is independent of V^a and is expressed exclusively in terms of the SUGRA fields H_b and Γ. The final expression is at most quadratic in the spurion field w^a. It also contains terms without the spurion altogether. The same type of terms appear in the variation of the Lorentz invariant supergravity operators. They come from the contractions of the form $w_a w^a = -1$ which arise upon substitution of the superspace connection into the second line.

On the other hand, the variations of the operators (3.10) contain contributions linear in V^a,

$$\delta_L(w^a V^b \partial_a H_b) \ni -\frac{1}{2}\bar{L}_{\dot{\beta}}\bar{\sigma}_b^{\dot{\beta}\beta} w^a D_\beta \partial_a V^b\,, \tag{3.12a}$$

$$\delta_L(w^a V^b \partial_b H_a) \ni -\frac{1}{2}\bar{L}_{\dot{\beta}}\bar{\sigma}_a^{\dot{\beta}\beta} w^a D_\beta \partial_b V^b\,, \tag{3.12b}$$

$$\delta_L(w^a V^b \partial^c H^d \epsilon_{abcd}) \ni -\frac{1}{2}\bar{L}_{\dot{\beta}}\bar{\sigma}^{d\,\dot{\beta}\beta} w^a D_\beta \partial^c V^b \epsilon_{dacb}\,, \tag{3.12c}$$

$$\delta_L(V^a D^2 H_a) \ni L_\beta\big[-4i(\sigma_{ka})_\gamma{}^\beta D^\gamma \partial^k V^a + 2iD^\beta \partial_a V^a\big]\,. \tag{3.12d}$$

It is straightforward to see that these cannot be canceled among themselves or against variations of any other operators in the Lagrangian. Thus, we conclude that the operators (3.10) do not appear in the invariant action.

The fact that there is only a single operator (3.9) with the aether fluctuation V^a which can enter the super-aether action suggests that this action must be unique. Indeed, it would be surprising to obtain independent Lorentz-violating Lagrangians that at the quadratic level would consist purely of the gravitational fields. We presently verify this intuition by explicitly solving the constraints imposed by the linearized super-diffeos.

To this aim we need to derive the variations of the 27 "auxiliary" operators from appendix C.2. This technical task is performed in appendix C.3. Let us note here that it is simplified by classifying these operators according to their transformation properties under the R-symmetry and CP. The operators with different R-charges and CP-numbers do not mix under the action of linearized super-diffeomorphisms and thus can be considered separately.

For the CP-even sector the conditions that the variation of the action under the super-diffeos cancels lead to a system of 17 linear equations for 17 unknowns — coefficients in front of the operator (3.9) and 16 operators (C.13). This system is degenerate and has a general solution with two free parameters. One of them is just the usual gravitational coupling $\varkappa^2$ and one recovers the SUGRA action (2.17) as part of the general solution. The second free parameter can be chosen as the coefficient in front of the operator (3.9) and the corresponding contribution into the action reads,

$$\begin{aligned} S_{Æ} = \frac{C}{2\varkappa^2} \int d^4x d^4\theta \Big[& V_a \bar{V}^a + i w^a w^b \partial_a H_b (\Gamma - \bar{\Gamma}) + w^a w^b \Delta_a H_b (\Gamma + \bar{\Gamma}) \\ & + \frac{1}{4} (\Delta_k H_m \Delta^k H^m - \partial_k H_m \partial^k H^m - (\Delta_m H^m)^2 + (\partial_m H^m)^2) \\ & + \frac{i}{4} \partial_m H^m (\Gamma - \bar{\Gamma}) + \frac{1}{4} \Delta_m H^m (\Gamma + \bar{\Gamma}) + \frac{3}{8} (\Gamma^2 + \bar{\Gamma}^2) \Big] , \end{aligned} \tag{3.13}$$

where C is a dimensionless coupling. This action is rather simple and its invariance under the linearized super-diffeos can be verified in a straightforward manner. Curiously, it does not depend on the choice of the parameter n labeling the off-shell realizations of the non-minimal SUGRA. Note that, along with explicitly Lorentz-violating terms in the first line, it contains contributions that are Lorentz-invariant. There is no contradiction here: these terms are needed to cancel the variations of the terms from the first line which, as pointed out above, include Lorentz-invariant contributions.

Finally, the requirement of vanishing gauge variation in the CP-odd sector leads to a system of 13 equations for only 8 unknowns which has only a trivial solution (see appendix C.3). We conclude that (3.13) combined with (2.17) gives the most general action for supersymmetric aether coupled to non-minimal linearized SUGRA with linear compensator.

4 Bosonic Lagrangian

To understand the physical consequences of the action (3.13), we compute the corresponding Lagrangian in components. We start with the bosonic part.

The components of the SUGRA fields H_m, Γ and of the super-aether V^a have been introduced in (2.2), (2.9) and (3.5), respectively. We impose the Wess-Zumino gauge (2.6) and the symmetry of the tetrad (2.13). Further, due to the relation (2.12a) fixing the Weyl invariance, the lowest component of Γ is not independent and is expressed through the trace of the metric perturbation. Applying the formula (2.18) to the superspace action (3.13) we obtain after a somewhat tedious, but straightforward calculation,

$$\begin{aligned}
\mathcal{L}^{\text{bos}}_{\text{Æ}} = \frac{C}{2\varkappa^2}\bigg\{ &-\partial_m v^a \partial_m \bar{v}^a + G^a \bar{G}^a \\
&+ v^{R,a} w^b \Big(-2\epsilon_{abkm}\partial_k d_m - \frac{1}{2}\partial_a \partial_k h_{kb} + \frac{1}{2}\partial_b \partial_k h_{ka} + \partial_b q^I_a - \partial_a q^I_b + \epsilon_{abkm}\partial_k q^R_m \Big) \\
&+ v^{I,a} w^b \Big(2\partial_a d_b - 2\partial_b d_a - \frac{1}{2}\epsilon_{abkm}\partial_k \partial_l h_{lm} - \partial_b q^R_a + \partial_a q^R_b + \epsilon_{abkm}\partial_k q^I_m \Big) \\
&- \frac{1}{8}\mathbb{D}_{ca}\bar{\mathbb{D}}_{ca} + \frac{n{+}1}{4(3n{+}1)} w^a w^b (h_{ab}\Box h - h\partial_a \partial_k h_{kb}) + \frac{1}{2} w^a w^b q^I_m (\partial_m h_{ab} - \partial_a h_{mb}) \\
&+ 2w^a w^b d_b q^R_a + \frac{1}{2} w^a w^b h_{nb}\epsilon_{nakm}\partial_k q^R_m - \frac{3}{2} d_m d_m - \frac{1}{8} h_{km}\Box h_{km} - \frac{5}{32}\partial_k h_{km}\partial_l h_{lm} \\
&- \frac{2n+1}{8(3n+1)} h\partial_k \partial_l h_{kl} + \frac{n(3n+2)}{8(3n+1)^2} h\Box h - \frac{1}{8} q^I_m \partial_k h_{km} - \frac{1}{4(3n+1)} q^I_m \partial_m h \\
&+ \frac{3}{8} q^I_m q^I_m - \frac{3}{8} q^R_m q^R_m + \frac{1}{2} d_k q^R_k \bigg\} ,
\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
\mathbb{D}_{ca} = &-w_c \Big[q_a + 2d_a - \frac{i}{2}\partial_k h_{ka} + i\frac{n}{3n+1}\partial_a h \Big] \\
&+ \eta_{ac} w^b \Big[q_b + 2d_b - \frac{i}{2}\partial_k h_{kb} + i\frac{n}{3n+1}\partial_b h \Big] \\
&+ w^b \epsilon_{bmac} \Big[iq_m - 2id_m + \frac{1}{2}\partial_k h_{km} - \frac{n}{3n+1}\partial_m h \Big] \\
&- iw^b \partial_b h_{ac} + iw^b \partial_a h_{cb} + w^b \epsilon_{bcmn}\partial_m h_{na} - w^b \epsilon_{mnac}\partial_m h_{nb} .
\end{aligned} \tag{4.2}$$

Note that the parameter C multiplies the kinetic term for the aether perturbations in (4.1) and hence must be positive to ensure the positivity of the kinetic energy. The full bosonic action of the theory is obtained by adding the standard supergravity Lagrangian (2.19).

The next step is to integrate out the auxiliary fields. Clearly, the fields B, G^a simply vanish on the equations of motion. On the other hand, the fields d_m, q_m take non-zero values. The result of integrating them out in the general case is rather cumbersome and not illuminating. For the sake of clarity, we will perform an explicit calculation under the assumption $C \ll 1$. To properly capture the mixing between the aether and gravity in the first non-trivial order in C, we canonically normalize the aether perturbations so that their leading kinetic term becomes of order 1,

$$v^{R,I}_a \mapsto \hat{v}^{R,I}_a = \sqrt{C}\, v^{R,I}_a .$$

We are interested in the contributions to the Lagrangian through order $O(C)$ in terms of the new fields. To this end, it is sufficient to find the auxiliary fields through order $O(\sqrt{C})$. However, later we will need also order $O(C)$ contributions into the fields d_m and q^R_m, so in deriving them we go one order further. We obtain,

$$d_m = \sqrt{C}\left[\frac{1}{2}w_m\partial_a\hat{v}^{I,a} - \frac{1}{2}w^a\partial_a\hat{v}^I_m - \frac{1}{4}w^b\epsilon_{bkam}\partial^k\hat{v}^{R,a}\right] - \frac{C}{8}w^aw^d\epsilon_{abcm}\partial^bh^c{}_d + O(C^{3/2}), \tag{4.3a}$$

$$q^R_m = \sqrt{C}\left[\frac{n}{3n+1}w_m\partial_a\hat{v}^{I,a} - \frac{n}{3n+1}w^a\partial_a\hat{v}^I_m - \frac{n+1}{2(3n+1)}w^b\epsilon_{bkam}\partial^k\hat{v}^{R,a}\right] - \frac{C(n+1)}{4(3n+1)}w^aw^d\epsilon_{abcm}\partial^bh^c{}_d + O(C^{3/2}), \tag{4.3b}$$

$$q^I_m = \frac{1}{2}\partial^k h_{km} - \frac{n}{3n+1}\partial_m h + \frac{\sqrt{C}n}{3n+1}\left[w_m\partial_a\hat{v}^{R,a} - w^b\partial_b\hat{v}^R_m - w^b\epsilon_{bkam}\partial^k\hat{v}^{I,a}\right] + O(C). \tag{4.3c}$$

Substituting this back into (2.19), (4.1) we arrive at,

$$\begin{aligned}\mathcal{L}^{\text{bos}}_{SG} + \mathcal{L}^{\text{bos}}_{\text{Æ}} = \frac{1}{2\varkappa^2}\Big\{&\frac{1}{4}h_{km}\Box h^{km} + \frac{1}{2}\partial^k h_{km}\partial_l h^{lm} - \frac{1}{2}\partial_k h^{km}\partial_m h + \frac{1}{4}\partial_m h\partial^m h\\ &- \partial_m\hat{v}^R_a\partial^m\hat{v}^{R,a} - \partial_m\hat{v}^I_a\partial^m\hat{v}^{I,a} + \sqrt{C}\,\hat{v}^{R,a}w^b(\partial_b\partial^k h_{ka} - \partial_a\partial^k h_{kb})\\ &- \frac{C}{4}w^aw^b(\partial_a h_{mn} - \partial_m h_{na})(\partial_b h^{mn} - \partial^m h^n{}_b) - \frac{C}{2}w^aw^b\partial_a\hat{v}^{I,m}\partial_b\hat{v}^I_m\\ &+ \frac{C}{2}(\partial_a\hat{v}^{I,a})^2 - Cw^bw^c\epsilon_{bkam}\partial^k\hat{v}^{R,a}\partial_c\hat{v}^{I,m} + O(C^{3/2})\Big\}.\end{aligned} \tag{4.4}$$

where have made a further rescaling of the fields,

$$\left(1 + \frac{C\,n}{4(3n+1)}\right)\hat{v}^R_a \mapsto \hat{v}^R_a\,, \qquad \left(1 - \frac{C\,n}{4(3n+1)}\right)\hat{v}^I_a \mapsto \hat{v}^I_a\,. \tag{4.5}$$

One notices that the parameter n has dropped from the Lagrangian and, apart from the usual gravitational coupling, the theory is described by a single dimensionless constant C. When restricted to the case of real aether, $\hat{v}^I_a = 0$, the Lagrangian (4.4) coincides with the quadratic part of the Einstein-aether Lagrangian (1.2) for the choice of couplings[15]

$$c_1 = C\,, \qquad c_2 = c_3 = c_4 = 0\,. \tag{4.6}$$

Thus we conclude that the supersymmetrization of the Einstein-aether model based on the embedding of aether into a chiral vector supermultiplet reduces the number of free parameters in the theory from four down to one.

5 Fermionic Lagrangian

In principle, the fermionic part of the Lagrangian can also be obtained from the superfield action (2.17), (3.13) by the application of eq. (2.18) and subsequent integration out of the auxiliary fields. However, we will take a different route and adopt the "on-shell" formalism where one constructs the Lagrangian directly in terms of the dynamical fields: the metric

[15] In this comparison one should recall that v^a stands for the perturbation of the aether field in the *tetrad basis*. It is related to the perturbation in the tangent space by $\delta u^m = v^m - \frac{1}{2}h^{ma}w_a$.

perturbation h_{mn}, gravitino ψ^m_α, aether perturbation v^a and aetherino η^a_α. Apart from leading more directly to the final answer, this approach will provide us with a transparent proof of uniqueness of the aether-supergravity coupling, not relying on the rather tedious classification of the superfield operators in sections 3.2, 3.3.

5.1 Gauging the super-aether action

We start with the action of super-aether in flat spacetime [33], which at the quadratic level can be written as

$$S^{\text{flat}}_{\text{Æ}} = \frac{1}{2\varkappa^2}\int d^4x\Big[-\partial_m\hat{v}^R_a\partial^m\hat{v}^{R,a} - \partial_m\hat{v}^I_a\partial^m\hat{v}^{I,a} - \frac{i}{2}\bar{\hat{\eta}}^a\bar{\sigma}^m\partial_m\hat{\eta}_a\Big], \tag{5.1}$$

where we have rescaled the aetherino in the same way as the aether perturbations, $\hat{\eta}^a_\alpha = \sqrt{C}\,\eta^a_\alpha$. This action is invariant under global spacetime translations and global SUSY transformations,

$$\delta_G\hat{v}^R_a = \frac{1}{2}(\zeta\hat{\eta}_a + \bar{\zeta}\bar{\hat{\eta}}_a), \quad \delta_G\hat{v}^I_a = -\frac{i}{2}(\zeta\hat{\eta}_a - \bar{\zeta}\bar{\hat{\eta}}_a), \quad \delta_G\hat{\eta}_{a\alpha} = 2i(\sigma^m\bar{\zeta})_\alpha\partial_m(\hat{v}^R_a + i\hat{v}^I_a), \tag{5.2}$$

where ζ^α is a coordinate independent parameter. These transformations form a closed algebra on-shell, i.e. when the fields satisfy the equations of motion,

$$\Box\hat{v}^R_a = \Box\hat{v}^I_a = \bar{\sigma}^m\partial_m\hat{\eta}_a = 0\,. \tag{5.3}$$

Coupling to supergravity can be viewed as gauging of the above symmetries [46]. At linear order in the SUGRA fields, the coupling must have the form,

$$\mathcal{L}_{\text{int}} = \frac{1}{2}\mathcal{T}^{mn}h_{mn} - \mathcal{S}^{m\alpha}\psi_{m\alpha} - \bar{\mathcal{S}}^m_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}_m\,, \tag{5.4}$$

where $\mathcal{T}^{mn}$ is the symmetric super-aether energy-momentum tensor (EMT) and $\mathcal{S}^{m\alpha}$ is the supercurrent corresponding to the invariance under (5.2) via the Noether theorem. Both the EMT and the supercurrent are conserved on-shell,

$$\partial_m\mathcal{T}^{mn} = \partial_m\mathcal{S}^{m\alpha} = 0\,. \tag{5.5}$$

Further, to allow for a consistent gauging, their SUSY transformations have to be related on-shell as [47],

$$\delta_G\mathcal{T}_{mn} = -\frac{1}{2}(\zeta\sigma_{mk}\partial^k\mathcal{S}_n + \zeta\sigma_{nk}\partial^k\mathcal{S}_m + \bar{\zeta}\bar{\sigma}_{mk}\partial^k\bar{\mathcal{S}}_n + \bar{\zeta}\bar{\sigma}_{nk}\partial^k\bar{\mathcal{S}}_m)\,, \tag{5.6a}$$

$$\delta_G\mathcal{S}_{m\alpha} = -2i(\sigma^n\bar{\zeta})_\alpha\mathcal{T}_{nm} - 4\epsilon_{mkln}(\sigma^k\partial^l\bar{\Xi}^n)_\alpha\,. \tag{5.6b}$$

Here Ξ^n_α is the spin-vector appearing as the leading correction to the on-shell SUSY transformation of gravitino. For completeness we review the derivation of eqs. (5.6) in appendix D.

The coupling (5.4) contributes into the quadratic action if the EMT and supercurrent contain terms linear in $\hat{v}^{R,I}_a$, $\hat{\eta}_{a\alpha}$. On the other hand, the Noether procedure applied to the action (5.1) yields expressions quadratic in these fields. Thus, the linear terms in the EMT and supercurrent must be of pure "improvement" type (see e.g. [48]),

$$\mathcal{T}^{mn} = \partial_k \mathcal{M}^{km\,n}\,, \qquad \mathcal{S}^{m\alpha} = \partial_k \mathcal{N}^{km\,\alpha}\,, \tag{5.7}$$

where the differentiated (spin-)tensors are anti-symmetric in the first pair of indices,

$$\mathcal{M}^{km\,n} = -\mathcal{M}^{mk\,n}\,, \qquad \mathcal{N}^{km\,\alpha} = -\mathcal{N}^{mk\,\alpha}\,. \tag{5.8}$$

It is worth stressing that (5.7) are on-shell equations and may or maynot hold off-shell. Our task now is to work out the most general form of the linearized aether EMT and the supercurrent.

Let us start with the on-shell EMT. The tensor $\mathcal{M}^{km\,n}$ must be constructed from terms that are linear in the aether perturbations $\hat{v}^R_a$ or $\hat{v}^I_a$, can contain one or several insertions of the VEV w^c and, on dimensional grounds, must include a single derivative ∂_n. Recalling the orthogonality of w^c and $\hat{v}_a$, we arrive to the following linear combination,

$$\begin{aligned}
\mathcal{M}^{km\,n} = {}& A_1(w^k\partial^m\hat{v}^{R,n} - w^m\partial^k\hat{v}^{R,n}) + A_2(w^k\partial^n\hat{v}^{R,m} - w^m\partial^n\hat{v}^{R,k}) \\
&+ A_3(w^n\partial^k\hat{v}^{R,m} - w^n\partial^m\hat{v}^{R,k}) + A_4(\eta^{kn}w^m\partial_a\hat{v}^{R,a} - \eta^{mn}w^k\partial_a\hat{v}^{R,a}) \\
&+ A_5(\eta^{kn}w^a\partial_a\hat{v}^{R,m} - \eta^{mn}w^a\partial_a\hat{v}^{R,k}) + A_6(w^kw^nw^a\partial_a\hat{v}^{R,m} - w^mw^nw^a\partial_a\hat{v}^{R,k}) \\
&+ A_7\,\epsilon^{kmna}w_a\partial_b\hat{v}^{R,b} + A_8\,\epsilon^{kmna}w^b\partial_b\hat{v}^R_a + A_9\,\epsilon^{kmab}w_a\partial^n\hat{v}^R_b \\
&+ A_{10}(\epsilon^{knab}w^m\partial_a\hat{v}^R_b - \epsilon^{mnab}w^k\partial_a\hat{v}^R_b) + A_{11}(\epsilon^{knab}w_a\partial^m\hat{v}^R_b - \epsilon^{mnab}w_a\partial^k\hat{v}^R_b) \\
&+ A_{12}(\epsilon^{knab}w_a\partial_b\hat{v}^{R,m} - \epsilon^{mnab}w_a\partial_b\hat{v}^{R,k}) + A_{13}(\eta^{kn}\epsilon^{mabc} - \eta^{mn}\epsilon^{kabc})w_a\partial_b\hat{v}^R_c \\
&+ A_{14}\,\epsilon^{kmab}w^nw_aw^c\partial_c\hat{v}^R_b + \text{terms with } \hat{v}^I_a\,,
\end{aligned} \tag{5.9}$$

where A_i, $i = 1, \ldots, 14$, are dimensionless coefficients. We show explicitly only the terms containing $\hat{v}^R_a$, the part with $\hat{v}^I_a$ has similar form with 14 more parameters $\tilde{A}_i$. In deriving this expression we omitted the terms that vanish identically upon taking the divergence and simplified the part with three vectors w^c by using the identity,[16]

$$w^kw_a\epsilon^{almn} - w^lw_a\epsilon^{amnk} + w^mw_a\epsilon^{ankl} - w^nw_a\epsilon^{aklm} + \epsilon^{klmn} = 0\,. \tag{5.10}$$

Eq. (5.9) is quite lengthy. However, the number of independent terms is drastically reduced by imposing that $\mathcal{T}^{mn}$ obtained from $\mathcal{M}^{km\,n}$ must be symmetric,[17] $\mathcal{T}^{mn} = \mathcal{T}^{nm}$. This leaves only six free parameters: A_1, A_3, A_{11} and their tilded counterparts. The resulting EMT reads,

$$\begin{aligned}
\mathcal{T}^{mn}_{\text{on-shell}} = {}& A_1(w^a\partial_a\partial^m\hat{v}^{R,n} + w^a\partial_a\partial^n\hat{v}^{R,m}) - A_3(w^n\partial^m\partial_a\hat{v}^{R,a} + w^m\partial^n\partial_a\hat{v}^{R,a}) \\
&- (A_1 - A_3)\eta^{mn}w^a\partial_a\partial_b\hat{v}^{R,b} + A_{11}(\epsilon^{mabc}w_a\partial^n\partial_b\hat{v}^R_c + \epsilon^{nabc}w_a\partial^m\partial_b\hat{v}^R_c) \\
&+ \text{terms with } \hat{v}^I_a
\end{aligned} \tag{5.11}$$

where we have omitted terms proportional to the equations of motion (5.3).

[16]To prove it one notices that the combination on the l.h.s. is totally antisymmetric in the indices $k\,l\,m\,n$. On the other hand, its contraction with the vector w^k vanishes. In four dimensions a tensor with such properties is identically zero.

[17]Note that $\mathcal{M}^{km\,n}$ itself need not, and actually cannot, be symmetric in the indices m and n.

So far, the analysis was restricted on-shell. The off-shell expression for $\mathcal{T}^{mn}$ can contain in addition to (5.11) two more terms,

$$\Delta\mathcal{T}^{mn} = -A\,(w^m\Box\hat{v}^{R,n} + w^n\Box\hat{v}^{R,m}) - \tilde{A}\,(w^m\Box\hat{v}^{I,n} + w^n\Box\hat{v}^{I,m})\,. \tag{5.12}$$

On the other hand, one obtains further constraints by requiring the off-shell invariance of the aether Lagrangian under linearized diffeomorphisms. With respect to the latter the metric perturbation h_{mn} transforms in the standard way (2.15), whereas for the aether perturbations we have (cf. eq. (3.6b)),

$$\delta_L\hat{v}^R_a = -\frac{\sqrt{C}}{2}w^b(\partial_b\xi^R_a - \partial_a\xi^R_b)\,, \qquad \delta_L\hat{v}^I_a = 0\,. \tag{5.13}$$

Assuming, as before, that C is a small parameter, the sum of the flat-space action (5.1) and the interaction term (5.4) must be invariant at order $O(\sqrt{C})$. Assuming further that all coefficients in the EMT are of order $\sqrt{C}/(2\varkappa^2)$, we get the relations,

$$A_1 = \frac{\sqrt{C}}{2\varkappa^2} + A\,, \qquad A_3 = \frac{\sqrt{C}}{2\varkappa^2} - A\,, \qquad \tilde{A}_1 = -\tilde{A}_3 = \tilde{A}\,, \qquad A_{11} = \tilde{A}_{11} = 0\,.$$

Thus, the general off-shell expression for the linearized EMT reads,

$$\begin{aligned}\mathcal{T}^{mn} = {} & \frac{\sqrt{C}}{2\varkappa^2}(w^c\partial_c\partial^m\hat{v}^{R,n} - w^n\partial^m\partial_c\hat{v}^{R,c}) \\ & + A\,(w^c\partial_c\partial^m\hat{v}^{R,n} + w^n\partial^m\partial_c\hat{v}^{R,c} - \eta^{mn}w^c\partial_c\partial_b\hat{v}^{R,b} - w^m\Box\hat{v}^{R,n}) \\ & + \tilde{A}\,(w^c\partial_c\partial^m\hat{v}^{I,n} + w^n\partial^m\partial_c\hat{v}^{I,c} - \eta^{mn}w^c\partial_c\partial_b\hat{v}^{I,b} - w^m\Box\hat{v}^{I,n}) + (m\leftrightarrow n)\,.\end{aligned} \tag{5.14}$$

We observe that the terms proportional to $\sqrt{C}$ yield, upon integration by parts, precisely the aether-metric mixing in the Lagrangian (4.4). One can check that the extra parameters A and $\tilde{A}$ correspond to non-minimal couplings of the complex aether to the Ricci tensor R_{mn} of the form,

$$R_{mn}u^m\bar{u}^n\,, \qquad R_{mn}(u^m u^n + \bar{u}^m\bar{u}^n), \qquad iR_{mn}(u^m u^n - \bar{u}^m\bar{u}^n)\,,$$

where $u^m = w^m + v^{R,m} + iv^{I,m}$ is the full non-linear aether field. Only two free parameters appear at the linearized level because the first two operators have the same expansion at the quadratic order.

Up to now, we have not imposed any restrictions due to supersymmetry. To do this, we construct the general linear supercurrent of the form (5.7) and require that it is related on-shell to the transformation of the EMT by eq. (5.6a). This will turn out to be sufficient to completely fix the form of the current and EMT: in particular, we will not need to use the second eq. (5.6b). The spin-tensor $\mathcal{N}^{km\,\beta}$ must be linear in the aetherino field and, by dimensionality, cannot contain derivatives. Guided by the form of the EMT (5.14), we consider only terms linear in w^a. Recall also that w^a and $\hat{\eta}^a_\beta$ are orthogonal. We are left with four possible operators,[18]

$$\begin{aligned}\mathcal{N}^{km\,\beta} = {} & a_1\,(w^k\hat{\eta}^{m\beta} - w^m\hat{\eta}^{k\beta}) + a_2\,(w^k\sigma^{ma}\hat{\eta}^\beta_a - w^m\sigma^{ka}\hat{\eta}^\beta_a) \\ & + a_3\,(w_a\sigma^{ak}\hat{\eta}^{m\beta} - w_a\sigma^{am}\hat{\eta}^{k\beta}) + a_4\epsilon^{kmab}w_a\hat{\eta}^\beta_b\,,\end{aligned} \tag{5.15}$$

[18]In this derivation we eliminated the term $\epsilon^k{}_{abc}w^a\sigma^{bm}\hat{\eta}^{c\beta} - (k\leftrightarrow m)$ by using the identity $\sigma^{bm} = (i/2)\epsilon^{bmnl}\sigma_{nl}$.

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with free coefficients $a_{1,\ldots,4}$. The resulting on-shell current reads,

$$\mathcal{S}^{m\beta}_{\text{on-shell}} = \Big(a_1 + \frac{a_3}{2}\Big) w^k \partial_k \hat{\eta}^{m\beta} - \Big(a_1 - \frac{a_2}{2}\Big) w^m \partial_k \hat{\eta}^{k\beta} \\ + a_2 w^k \sigma^{ma} \partial_k \hat{\eta}_a^\beta - a_3 w_a \sigma^{am} \partial_k \hat{\eta}^{k\beta} + a_4 \epsilon^{mabc} w_a \partial_b \hat{\eta}_c^\beta \,. \tag{5.16}$$

We substitute it into the relation (5.6a), use equations of motion to simplify the result and compare the coefficients in front of independent terms. We find that eq. (5.6a) can be satisfied only if

$$A = \tilde{A} = a_2 = a_3 = a_4 = 0 \,, \qquad a_1 = -\frac{\sqrt{C}}{\varkappa^2} \,.$$

We conclude that the linear supercurrent is unique and has the form,

$$\mathcal{S}^{m\beta} = -\frac{\sqrt{C}}{\varkappa^2} \big(w^c \partial_c \hat{\eta}^{m\beta} - w^m \partial_c \hat{\eta}^{c\beta}\big) \,. \tag{5.17}$$

This implies the uniqueness of the aether coupling to linear supergarvity. Notice that the above supercurrent is conserved identically (not only on-shell). Therefore, the gravitino coupling in eq. (5.4) is automatically invariant under the local SUSY transformations (2.15). We presently discuss its invariance with respect to the global SUSY.

5.2 On-shell supersymmetry

The interaction (5.4) gives rise to terms of order $O(\sqrt{C})$ in the total action. We saw that in the bosonic Lagrangian the next corrections come at order $O(C)$. In principle, the same could happen in the fermionic part. However, we now argue that this is not the case and the total fermionic Lagrangian through order $O(C)$ reads,

$$\mathcal{L}^{\text{ferm}}_{SG} + \mathcal{L}^{\text{ferm}}_{\text{Æ}} = \frac{1}{2\varkappa^2} \Big\{ 8\epsilon^{klmn} \bar{\psi}_k \bar{\sigma}_l \partial_m \psi_n - \frac{i}{2} \bar{\hat{\eta}}^k \bar{\sigma}^m \partial_m \hat{\eta}_k \\ + 2\sqrt{C}\, w^c (\psi^k \partial_c \hat{\eta}_k - \psi_c \partial_k \hat{\eta}^k) + 2\sqrt{C}\, w^c (\bar{\psi}^k \partial_c \bar{\hat{\eta}}_k - \bar{\psi}_c \partial_k \bar{\hat{\eta}}^k) + O(C^{3/2}) \Big\}. \tag{5.18}$$

To show this we need to work out the global SUSY transformations of the physical component fields. These are obtained by substituting the on-shell values of the auxiliary fields into the general formulas of appendix A.

We start with the gravity sector. The on- and off-shell transformations of the metric coincide and are given by eq. (2.16). For gravitino we use eq. (A.5), where we substitute $B = 0$ and d_m, q^R_m from eqs. (4.3). This yields,

$$\tilde{\delta}_G \psi_{m\alpha} = \frac{1}{2} (\sigma^{kn}\zeta)_\alpha \partial_k h_{nm} \\ + \sqrt{C} \Big[-\frac{i}{4} w^c (\sigma_c \bar{\sigma}_m \zeta)_\alpha \partial_b \hat{v}^{I,b} + \frac{i}{4} w^c (\sigma_b \bar{\sigma}_m \zeta)_\alpha \partial_c \hat{v}^{I,b} - \frac{i}{4} \zeta_\alpha w^c \epsilon_{cklm} \partial^k \hat{v}^{R,l} \Big] \\ - \frac{iC}{8} \zeta_\alpha w^c w^b \epsilon_{cklm} \partial^k h^l{}_b + O(C^{3/2}) \,. \tag{5.19}$$

One can check that the term of order $O(\sqrt{C})$ here matches the extra piece $\Xi_{m\alpha}$ in the on-shell transformation (5.6b) of the supercurrent (5.17).

The aether sector requires more work. First, from eq. (A.6a) one observes that to get the transformation of $\hat{v}_a^R$ to order $O(C)$ one needs to know the fermionic auxiliary field ω_α through order $O(\sqrt{C})$. In appendix E we describe how this field can be found using the superspace equations of motion. The result reads,

$$\omega_\alpha = \frac{\sqrt{C}n}{3n+1} w^b (\sigma_{bc}\hat{\eta}^c)_\alpha + O(C)\,. \tag{5.20}$$

Second, one should remember the rescaling of the aether components at order $O(C)$, eq. (4.5). In addition, it turns convenient to redefine the aetherino field as follows,

$$\left(1 - \frac{C}{4(3n+1)}\right)\hat{\eta}_{a\beta} + \frac{Cn}{2(3n+1)} P_a^b\,(\sigma_{bc}\hat{\eta}^c)_\beta \mapsto \hat{\eta}^a_\beta\,, \tag{5.21}$$

where

$$P_a^b \equiv w^b w_a + \delta_a^b \tag{5.22}$$

is the projector on the hyperplane orthogonal to w^a. Collecting everything together and substituting into eqs. (A.6) we arrive at

$$\tilde{\delta}_G \hat{v}_a^R = \frac{1}{2}\zeta\hat{\eta}_a + \sqrt{C}w^c(i\zeta\sigma_c\bar{\psi}_a - i\zeta\sigma_a\bar{\psi}_c) + \frac{Cn}{4(3n+1)} P_a^b\,\zeta\sigma_{bc}\hat{\eta}^c + \text{h.c.}\,, \tag{5.23a}$$

$$\tilde{\delta}_G \hat{v}_a^I = -\frac{i}{2}\zeta\hat{\eta}_a + \frac{iCn}{4(3n+1)} P_a^b\,\zeta\sigma_{bc}\hat{\eta}^c + \text{h.c.}\,, \tag{5.23b}$$

$$\begin{aligned}\tilde{\delta}_G \hat{\eta}_{a\beta} = {} & 2i(\sigma^m\bar{\zeta})_\beta\partial_m\hat{v}_a + i\sqrt{C}w^c(\sigma^k\bar{\zeta})_\beta[\partial_c h_{ak} - \partial_a h_{ck}] - iCw^b w^c\epsilon_{cmna}(\sigma^m\bar{\zeta})_\beta\partial_b\hat{v}^{I,n} \\ & + \frac{Cn}{2(3n+1)}\left[-iP_a^c(\sigma^m\bar{\zeta})_\beta\partial_c\bar{\hat{v}}_m + iP_a^c(\sigma_c\bar{\zeta})_\beta\partial_m\bar{\hat{v}}^m + P_a^c\epsilon_{ckmn}(\sigma^k\bar{\zeta})_\beta\partial^m\bar{\hat{v}}^n\right],\end{aligned} \tag{5.23c}$$

where we have used the identity (5.10) to simplify the $\hat{\eta}_{a\beta}$ variation. Corrections to these expressions are of order $O(C^{3/2})$.

It is now a matter of a straighforward calculation to verify that the Lagrangian given by the sum of (4.4) and (5.18) is invariant under SUSY transformations (2.16), (5.19), (5.23) through order $O(C)$. We leave it as an exercise to the reader.

We have also checked that the SUSY algebra closes on shell, up to the gauge transformations (2.15), (3.6b). Namely, the commutator of two SUSY transformations with parameters ζ_1 and ζ_2 reads,

$$[\tilde{\delta}_{G1}, \tilde{\delta}_{G2}]h_{mn} = -2Z^k\partial_k h_{mn} + \partial_m\xi_n^R + \partial_n\xi_m^R\,, \tag{5.24a}$$

$$[\tilde{\delta}_{G1}, \tilde{\delta}_{G2}]\psi_{m\alpha} = -2Z^k\partial_k\psi_{m\alpha} + \partial_m\varepsilon_\alpha + \text{e.o.m.}\,, \tag{5.24b}$$

$$[\tilde{\delta}_{G1}, \tilde{\delta}_{G2}]\hat{v}_a^R = -2Z^k\partial_k\hat{v}_a^R - \frac{\sqrt{C}}{2}w^b(\partial_b\xi_a^R - \partial_a\xi_b^R)\,, \tag{5.24c}$$

$$[\tilde{\delta}_{G1}, \tilde{\delta}_{G2}]\hat{v}_a^I = -2Z^k\partial_k\hat{v}_a^I\,, \tag{5.24d}$$

$$[\tilde{\delta}_{G1}, \tilde{\delta}_{G2}]\hat{\eta}_{a\beta} = -2Z^k\partial_k\hat{\eta}_{a\beta} + \text{e.o.m.}\,, \tag{5.24e}$$

where

$$Z^k = i\left(\zeta_1\sigma^k\bar{\zeta}_2 - \zeta_2\sigma^k\bar{\zeta}_1\right)\,, \qquad \xi_n^R = Z^k h_{kn}\,, \tag{5.25}$$

and the fermionic gauge parameter ε_α is linear in the fields $\psi_{m\alpha}$, $\hat{\eta}_{a\beta}$; we do not write its explicit expression as it is rather cumbersome. The terms denoted by "e.o.m." vanish on the fermionic equations of motion obtained from the Lagrangian (5.18). This proves that the fermionic action (5.18) is complete as any additional terms of order $O(C)$ would spoil the supersymmetry.

Notice the following peculiarity. While the component Lagrangians (4.4), (5.18) are independent of the non-minimal SUGRA parameter n, the transformations (5.23) contain n-dependent pieces. These pieces cancel among each other in the variation of the total Lagrangian. This corresponds to an additional supersymmetry of the flat-space aether action (5.1), besides the standard SUSY (5.2). Coupling to SUGRA breaks the extra SUSY and ties it to the first one. The preserved linear combination of symmetry generators is different for different n. We do not know if this leads to the dependence of the Lagrangian on n at higher orders in C or such dependence can always be eleiminated by a field redefinition. Investigating this issue goes beyond the scope of the present paper.

6 Physical implications

6.1 Particle spectrum, enhancement of graviton multiplet

Let us discuss the spectrum of modes described by the Lagrangians (4.4), (5.18). We will work in the frame where the VEV w^a is purely timelike as given by (3.1), so that the rotational symmetry is preserved. As the Lagrangian is quadratic both in space- and time-derivatives, all modes have linear dispersion relations,

$$E = s \cdot p\,, \tag{6.1}$$

where E and p are the energy and the absolute value of the mode's momentum, s is the mode's velocity. Due to invariance with respect to spatial rotations, the modes are also characterized, as in the familiar Lorentz invariant case, by the projection of the angular momentum on the direction of motion, i.e. helicity. The maximal helicity present in the spectrum is 2, which corresponds to the transverse-traceless excitations of the metric — gravitons. It is straightforward to see from (4.4) that the corresponding squared velocity is (cf. [49]),

$$s^2_{h=2} = \frac{1}{1-C} \approx 1 + C\,, \tag{6.2}$$

which differs from 1 whenever Lorentz invariance is broken ($C \neq 0$). This is in contrast with the situation for chiral and gauge SUSY multiplets which, under broad assumptions, retain unit propagation velocity even in the presence of Lorentz symmetry breaking [31, 32].

The deviation of the graviton velocity from one has interesting consequences for the structure of the gravitational supermultiplet. Indeed, consider the representation of the SUSY algebra corresponding to the dispersion relation of the form (6.1). Following the standard procedure [35] one rotates the direction of the particle momentum to align with the third axis. With this choice, the anticommutators of the supercharges take the form,

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2E \begin{pmatrix} 1+s^{-1} & 0 \\ 0 & 1-s^{-1} \end{pmatrix} \tag{6.3}$$

Note that unitarity requires the anticommutator of two conjugate operators to be non-negative. Comparing with (6.3) we conclude that in supersymmetric theories the velocity of particles is always greater or equal to one, $s \geq 1$. If $s = 1$, as it happens, in particular, in the standard Lorentz invariant case, the lower right element in the above matrix is zero implying that one pair of the supercharges vanish identically, $Q_2 = \bar{Q}_{\dot{2}} = 0$. The other pair of the supercharges Q_1, $\bar{Q}_{\dot{1}}$ describes fermionic annihilation and creation operators. Thus, starting from the state with the lowest helicity h, annihilated by Q_1, one can create a single state with helicity $(h + 1/2)$ by applying $\bar{Q}_{\dot{1}}$. As a consequence, in the Lorentz invariant case the gravitational multiplet consists of just two states with helicities $h = -2$ (graviton) and $h = -3/2$ (gravitino).[19] However, whenever $s > 1$, the anticommutator of Q_2 and $\bar{Q}_{\dot{2}}$ does not vanish and they form an independent pair of creation-annihilation operators. It implies that the multiplet must contain two additional states: one with $h = -3/2$ and another with $h = -1$. We conclude that the gravitational multiplet gets enhanced.

In the model of this paper the additional states come from the aether superfield. Indeed, its aetherino component $\hat{\eta}^m_\alpha$ carries both a spinor and a vector index and decomposes into a pair of $h = \pm 3/2$ states and two pairs of $h = \pm 1/2$ states. The above reasoning implies that the first pair is absorbed by the graviton multiplet. The aether itself, represented by $\hat{v}^{R,I}_m$, contains two pairs of $h = \pm 1$ states and a pair of $h = 0$ states. One of the $h = \pm 1$ pairs must join the graviton multiplet. We verify this by computing the velocities of the fermionic and bosonic modes in appendix F. As expected, we find that for the helicity-3/2 modes and one pair of helicity-1 modes the velocities coincide with that of gravitons,

$$s^2_{h=3/2} = s^2_{h=1,(1)} = 1 + C\,. \tag{6.4}$$

We identify these modes as belonging to the graviton multiplet. On the other hand, the remaining pair of helicity-1 modes, the helicity-1/2 modes and helicity-0 modes have unit velocities,[20]

$$s^2_{h=1,(2)} = s^2_{h=1/2} = s^2_{h=0} = 1\,. \tag{6.5}$$

Thus, apart from the graviton multiplet, the theory contains 4 bosonic degrees of freedom propagating with unit velocity that match the two pairs of $h = \pm 1/2$ fermionic states contained in $\hat{\eta}^m_\alpha$.

6.2 Comments on phenomenology

As discussed in refs. [31–33], supersymmetry suppresses a direct coupling of aether to the Standard Model fields. As a result, the non-gravitational dynamics of the Standard Model sector is essentially relativistic. In particular, the electromagnetic waves propagate with unit velocity.

Thus, all constraints on the model come from the gravitational sector. We observe that at low energies SUSY must be broken which gives mass to the imaginary part of the aether perturbations [33]. The precise value of the mass depends on the SUSY breaking pattern; nevertheless generically one expects the corresponding Compton wavelength to

[19]The states with opposite helicities +2, +3/2 appear upon the CPT conjugation.

[20]Up to possible corrections of order $O(C^2)$ that we neglect in our analysis.

be much shorter than astronomical scales. Then at these scales the imaginary part of the aether is irrelevant and the model reduces to the (linearized) Einstein-aether theory with the parameters (4.6).[21] Hence we can use the constraints on the general Einstein-aether theory [50] by restricting them to the case (4.6). Let us briefly review them.

We have seen that the velocity of gravitons, and hence gravity waves, in the super-aether model necessarily exceed unity. Detection of the gravity wave signal from the neutron star merger GW170817 [51] in coincidence with the electromagnetic counterpart [52, 53] places a stringent bound on such deviation [54]. This translates into the limit on the model parameter C,

$$C < 1.4 \times 10^{-15} \qquad \text{(GW170817)}\,. \tag{6.6}$$

This is the strongest constraint on the model to date.[22] It can be viewed as the limit on the energy scale of the Lorentz symmetry violation, $M_* \equiv \varkappa^{-1}\sqrt{C} \lesssim 10^{11}\,\text{GeV}$.

Independent, though weaker, limits come from the tests of general relativity within the Solar System and from the observations of solitary pulsars. The Solar System tests place bounds on the values of the post-Newtonian parameters α_1, α_2 describing deviations from Lorentz invariance [56],

$$|\alpha_1| \lesssim 10^{-4}\,, \qquad |\alpha_2| \lesssim 10^{-7}\,. \tag{6.7}$$

The post-Newtonian parameters for the Einstein-aether model were derived in [23]; for the choice (4.6) they reduce to

$$\alpha_1 = 0\,, \qquad \alpha_2 = -\frac{2C}{2-C}\,. \tag{6.8}$$

Hence the bound (6.7) translates into

$$C \lesssim 10^{-7} \qquad \text{(Solar System)}\,. \tag{6.9}$$

A more stringent bound

$$|\hat{\alpha}_2| < 1.6 \times 10^{-9} \tag{6.10}$$

on the analog of the parameter α_2 for strong gravitational field has been obtained in [57] by analyzing the dynamics of solitary pulsars. Strictly speaking, application of this bound to our model requires its non-linear generalization which is beyond the scope of the present work. However, due to the uniqueness of the Einstein-aether theory, this non-linear generalization must reduce to it below the SUSY breaking scale, with the SUSY origin of the theory still being encoded in the values (4.6) of the parameters. The relations between the strong- and weak-field parameters in the Einstein-aether theory have been derived in the ref. [25]. In general, they involve the sensitivities characterizing the change in the binding energies of neutron stars due to their motion with respect to the preferred frame. These depend on the masses of the stars which complicates the translation of the bound (6.10) into constraints

[21] SUSY breaking can, in principle, introduce deviations from the values (4.6) in the low-energy theory. However, these deviations are small if the SUSY breaking scale in the aether sector lies hierarchically below the scale of Lorentz symmetry breaking set by $M_* = \varkappa^{-1}\sqrt{C}$. We assume that this is the case.

[22] A very strong indirect lower bound on the graviton velocity follows from the absence of gravitational Cherenkov radiation by ultra-high energy cosmic rays [55]. However, it is automatically satisfied for superluminal gravitons.

on the model parameters. However, for the choice (4.6) the sensitivities drop out of the relation between $\hat{\alpha}_2$ and α_2 and one gets simply $\hat{\alpha}_2 = \alpha_2$. This gives the bound,

$$C < 1.6 \times 10^{-9} \qquad \text{(solitary pulsars)}\,. \tag{6.11}$$

7 Conclusions

We have constructed a linearized supergravity theory where Lorentz invariance is broken down to the subgroup of spatial rotations by a VEV of a timelike vector field. This provides a supersymmetric extension of the well-known Einstein-aether model. Our construction is based on embedding aether as the lowest component into a chiral vector superfield which ensures that aether VEV does not break SUSY. Using both the superfield formalism and the "on-shell" component-field approach, we showed that the linearized action contains, in addition to the usual SUGRA parameters, a single free dimensionless coupling. This is to be contrasted with the non-supersymmetric Einstein-aether model possessing four arbitrary couplings.

We have derived the Lagrangian in terms of the physical component fields and analyzed the spectrum of the theory. Due to breaking of Lorentz invariance, the excitations with helicity 2 (graviton) and 3/2 (gravitino) acquire the propagation velocity exceeding the speed of light. We showed that this leads to the extension of the on-shell gravity multiplet by two additional states with helicities 3/2 and 1. The extra states enter the theory as part of the aether superfield and have the same superluminal velicity as graviton and gravitino. The theory also contains one more pair of helicity ± 1 modes, two pairs of helicity $\pm 1/2$ modes and two helicity 0 modes, all propagating at the speed of light. It is worth stressing that presence of superluminal modes does not lead to physical inconsistencies as the theory features a preferred reference frame defined by the aether VEV (cf. [21, 23]).

At low energies, upon SUSY breaking, the phenomenology of the model reduces to that of the Einstein-aether theory with three out of the four couplings equal to zero. The strongest constraints on the model come from the observation of the gravitational wave signal from the neutron star merger GW170817 together with its electromagnetic counterpart that limits the deviation of the gravity wave velocity from unity. It can be translated into an upper bound on the energy scale of Lorentz violations $M_* \lesssim 10^{11}\,\text{GeV}$. Independent, though weaker constraints are imposed by the dynamics of the Solar System and pulsars.

A natural development of our work will be its generalization to the full non-linear supergravity case. This will open the way to study possible manifestations of the super-aether model in cosmology, in particular, the effects of the additional fermionic and bosonic fields present in the model on the dynamics of the early universe. We plan to address this topic in future. As another direction, it would be interesting to investigate applications of the model to the holographic description of strongly coupled non-relativistic systems.

Similarly to the non-supersymmetric Einstein-aether theory, the model presented in this paper is a valid effective theory below the scale M_*. One may wonder if and how the model can be UV completed above this scale. In particular, if the completion can be achieved along the lines of Hořava gravity [12]. We do not know the answer to this question and only note that a supersymmetric extension of Hořava gravity would need to overcome a number of important obstructions discussed in [33].

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A Global SUSY transformations in components

Global SUSY acts on a general superfield Ψ by translations in the superspace with a coordinate-independent spinor parameter ζ^α,

$$\delta_G \Psi = (\zeta^\alpha Q_\alpha + \bar{\zeta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})\Psi\,,$$

where the supercharges Q_α, $\bar{Q}_{\dot{\alpha}}$ are realized as differential operators on the superspace [35]. Let a component field ψ be defined as

$$\psi = \mathcal{O}\Psi\big|\,,$$

where $\mathcal{O}$ is an operator constructed of covariant derivatives D_α, $\bar{D}_{\dot{\alpha}}$. Then its transformation equals to,

$$\delta_G \psi = \mathcal{O}(\zeta Q + \bar{\zeta}\bar{Q})\Psi\big| = (\zeta Q + \bar{\zeta}\bar{Q})\mathcal{O}\Psi\big| = (\zeta D + \bar{\zeta}\bar{D})\mathcal{O}\Psi\big|\,,$$

where the second equality holds because the supercharges anti-commute with the covariant derivatives and the last equality holds because the difference between Q_α and D_α vanishes at zero θ, $\bar{\theta}$. Applying this formula to the component fields defined in eqs. (2.2), (2.9), (3.5), we obtain:

for the gravitational supermultiplet,

$$\delta_G c_m = \zeta\chi_m + \bar{\zeta}\bar{\chi}_m\,, \tag{A.1a}$$
$$\delta_G \chi_{\alpha m} = 2\zeta_\alpha a_m - (\sigma^n\bar{\zeta})_\alpha e_{nm} + i(\sigma^n\bar{\zeta})_\alpha \partial_n c_m\,, \tag{A.1b}$$
$$\delta_G a_m = -\bar{\zeta}\bar{\sigma}_n\sigma_m\bar{\psi}^n + i\bar{\zeta}\bar{\sigma}^n\partial_n\chi_m\,, \tag{A.1c}$$
$$\begin{aligned}\delta_G e_{mn} = &-i\zeta\partial_m\chi_n + i\bar{\zeta}\partial_m\bar{\chi}_n + i\zeta\sigma_m\bar{\psi}_n + i\zeta\sigma_n\bar{\psi}_m + i\bar{\zeta}\bar{\sigma}_m\psi_n + i\bar{\zeta}\bar{\sigma}_n\psi_m\\ &- i\eta_{mn}(\zeta\sigma_k\bar{\psi}^k + \bar{\zeta}\bar{\sigma}_k\psi^k) + \epsilon_{mnkl}(\zeta\sigma^k\bar{\psi}^l - \bar{\zeta}\bar{\sigma}^k\psi^l)\,,\end{aligned} \tag{A.1d}$$
$$\delta_G \psi^m_\alpha = \frac{i}{4}(\sigma^n\bar{\sigma}^m\zeta)_\alpha \Box c_n + \frac{1}{2}(\sigma^l\bar{\sigma}^m\sigma^{nk}\zeta)_\alpha\,\partial_k e_{nl} - \frac{i}{2}(\sigma^n\bar{\sigma}^m\zeta)_\alpha\,d_n\,, \tag{A.1e}$$
$$\delta_G d_m = \frac{1}{2}\zeta\Box\chi_m + \frac{1}{2}\bar{\zeta}\Box\bar{\chi}_m + \frac{1}{2}\zeta\sigma_n\bar{\sigma}_k\sigma_m\partial^n\bar{\psi}^k - \frac{1}{2}\bar{\zeta}\bar{\sigma}_n\sigma_k\bar{\sigma}_m\partial^n\psi^k\,; \tag{A.1f}$$

for the compensator supermultiplet,

$$\delta_G \gamma = \zeta\phi + \bar\zeta\bar\omega\,, \tag{A.2a}$$
$$\delta_G \bar\omega_{\dot\alpha} = (\zeta\sigma^m)_{\dot\alpha}\, q_m - i(\zeta\sigma^m)_{\dot\alpha}\,\partial_m\gamma\,, \tag{A.2b}$$
$$\delta_G \phi_\alpha = 2\zeta_\alpha B + (\sigma^m\bar\zeta)_\alpha\, q_m + i(\sigma^m\bar\zeta)_\alpha\,\partial_m\gamma\,, \tag{A.2c}$$
$$\delta_G B = -\bar\zeta\bar\nu + i\bar\zeta\bar\sigma^n\partial_n\phi\,, \tag{A.2d}$$
$$\delta_G q_m = i\zeta\partial_m\phi - i\bar\zeta\partial_m\bar\omega - i\bar\zeta\bar\sigma_m\sigma_k\partial^k\bar\omega + \zeta\sigma_m\bar\nu\,, \tag{A.2e}$$
$$\delta_G \bar\nu_{\dot\alpha} = -i(\bar\zeta\bar\sigma^k\sigma^m)_{\dot\alpha}\,\partial_k q_m + \bar\zeta_{\dot\alpha}\,\Box\gamma\,; \tag{A.2f}$$

for the aether supermultiplet,

$$\delta_G v_b = \zeta\eta_b + 2iw^c\zeta\sigma_m\bar\sigma_{cb}\bar\psi^m - w^c\zeta\sigma_{cb}\omega + 2iw^c\bar\zeta\bar\sigma_m\sigma_{cb}\psi^m - w^c\bar\zeta\bar\sigma_{cb}\bar\omega\,, \tag{A.3a}$$
$$\begin{aligned}\delta_G \eta_{b\alpha} = {}& 2i(\sigma^m\bar\zeta)_\alpha\,\partial_m v_b + 2\zeta_a\, G_b - 2iw^c(\sigma^{nm}\sigma_{cb}\sigma^k\bar\zeta)_\alpha\partial_n e_{mk} - 2iw^c(\sigma^k\bar\sigma_{cb}\bar\sigma^{nm}\bar\zeta)_\alpha\partial_n e_{mk}\\ &+ w^c(\sigma_{cb}\sigma^m\bar\zeta)_\alpha(\Box c_m - 2d_m + i\partial_m\bar\gamma + \bar q_m) + w^c(\sigma^m\bar\sigma_{cb}\bar\zeta)_\alpha(\Box c_m - 2d_m - i\partial_m\gamma + q_m)\,,\end{aligned} \tag{A.3b}$$
$$\begin{aligned}\delta_G G_b = {}& i\bar\zeta\bar\sigma^m\partial_m\eta_b - 2w^c\bar\zeta\partial_b\bar\psi_c + 2w^c\bar\zeta\partial_c\bar\psi_b + 2iw^c\epsilon_{cbmn}\bar\zeta\partial^m\bar\psi^n - 2w^c\bar\zeta\bar\sigma^m\sigma^n\bar\sigma_{cb}\partial_m\bar\psi_n\\ &- iw^c\bar\zeta\bar\sigma^m\sigma_{cb}\partial_m\omega + w^c\bar\zeta\bar\sigma_{cb}\bar\nu\,.\end{aligned} \tag{A.3c}$$

Note that the transformations of the super-aether components depend on the supergravity fields. This is a consequence of the dependence of the covariant chirality constraint (3.3) and the definition of aetherino (3.5) on the superspace connection.

One observes that the above transformations in general violate the gauge conditions (2.6), (2.12), (2.13). To restore the gauge, they must be supplemented by appropriate gauge transformations with the parameters depending on ζ^α and the fields. This leads to modified global SUSY transformations which we denote with $\tilde\delta_G$.

Let us find the parameters of the restoring gauge transformations. Comparing the first three of eqs. (A.1) with (2.5a) we see that to preserve the Wess-Zumino gauge we have to choose,

$$\xi^I_m = 0\,, \qquad \mu_{m\alpha} = -2i(\sigma^n\bar\zeta)_\alpha\, e_{nm}\,, \qquad \kappa_m = 2\bar\zeta\bar\sigma_n\sigma_m\bar\psi^n\,. \tag{A.4a}$$

Next, preserving the gauge conditions (2.12) requires,

$$\lambda^\alpha{}_\alpha = \frac{(3n+1)(n+1)}{4n}(\zeta\phi + \bar\zeta\bar\omega) + \frac{(3n+1)(n-1)}{4n}(\bar\zeta\bar\phi + \zeta\omega) + i(n+1)(\zeta\sigma_k\bar\psi^k + \bar\zeta\bar\sigma_k\psi^k)\,, \tag{A.4b}$$
$$\bar\rho^{\dot\alpha} = \frac{i}{2}(\bar\sigma^m\zeta)^{\dot\alpha}\partial^k e_{km} - \frac{i}{2}(\bar\sigma^m\zeta)^{\dot\alpha}\partial_m e^k{}_k - n(\bar\sigma^m\zeta)^{\dot\alpha} d_m + \frac{3n+1}{2}(\bar\sigma^m\zeta)^{\dot\alpha} q^R_m - \frac{3n+1}{2}\bar\zeta^{\dot\alpha}\bar B\,. \tag{A.4c}$$

Finally, the symmetry of the tetrad is preserved if we choose,

$$(\sigma_{mn})^{\alpha\beta}\lambda_{\alpha\beta} - (\bar\sigma_{mn})^{\dot\alpha\dot\beta}\bar\lambda_{\dot\alpha\dot\beta} = -\epsilon_{mnkl}(\zeta\sigma^k\bar\psi^l - \bar\zeta\bar\sigma^k\psi^l)\,. \tag{A.4d}$$

The remaining parameters ξ^R_m and ε^α do not need to be adjusted and describe the unconstrained gauge symmetries.

The modified global SUSY transformations are obtained by adding to (A.1), (A.2), (A.3) the gauge shifts (2.5), (2.11), (3.6) with the parameters given by (A.4). We work them out only for the physical fields e_{mn}, ψ^m_α, v_b and $\eta_{b\alpha}$. Using the expressions (A.4b), (A.4d) we find for the tetrad,

$$\tilde{\delta}_G e_{mn} = i\zeta\sigma_m\bar{\psi}_n + i\zeta\sigma_n\bar{\psi}_m + \frac{3n+1}{4}\eta_{mn}\,\zeta\left(\omega + \phi + \frac{4in}{3n+1}\sigma_k\bar{\psi}^k\right) + \text{h.c.}$$

Notice that the linear combination in brackets is the same as in the gauge condition (2.12b). Setting it to zero we eliminate the term proportional to η_{mn} and arrive to the canonical SUSY transformation of the metric (2.16). For gravitino, we use eq. (A.4c) and the symmetry of the tertrad to simplify the expression. This yields,

$$\begin{aligned}\tilde{\delta}_G\psi^m_\alpha = {} & \frac{1}{2}(\sigma^{kn}\zeta)_\alpha\partial_k h_n{}^m + i\zeta_\alpha d_m + \frac{i(n+1)}{2}(\sigma^m\bar{\sigma}^n\zeta)_\alpha\left[d_n - \frac{3n+1}{2(n+1)}q^R_m\right]\\ & + \frac{i(3n+1)}{4}(\sigma^m\bar{\zeta})_\alpha\bar{B}\,.\end{aligned} \tag{A.5}$$

Finally, for aether and aetherino upon using the gauge conditions and some simplifications we obtain,

$$\tilde{\delta}_G v_b = \zeta\eta_b + iw^c(\zeta\sigma_c\bar{\psi}_b - \zeta\sigma_b\bar{\psi}_c + \bar{\zeta}\bar{\sigma}_c\psi_b - \bar{\zeta}\bar{\sigma}_b\psi_c) - w^c(\zeta\sigma_{cb}\omega + \bar{\zeta}\bar{\sigma}_{cb}\bar{\omega})\,, \tag{A.6a}$$

$$\begin{aligned}\tilde{\delta}_G\eta_{b\alpha} = {} & 2i(\sigma^m\bar{\zeta})_\alpha\partial_m v_b + 2\zeta_\alpha G_b + iw^c(\sigma^k\bar{\zeta})_\alpha(\partial_c h_{kb} - \partial_b h_{kc})\\ & -\frac{i}{2}w^c(\sigma_c\bar{\zeta})_\alpha\partial^k h_{kb} + \frac{i}{2}w^c(\sigma_b\bar{\zeta})_\alpha\partial^k h_{kc} + \frac{in}{3n+1}w^c(\sigma_c\bar{\zeta})_\alpha\partial_b h - \frac{in}{3n+1}w^c(\sigma_b\bar{\zeta})_\alpha\partial_c h\\ & + iw^c\epsilon_{cbmn}(\sigma^m\bar{\zeta})_\alpha(2d^n - q^{R,n}) + iw^c(\sigma_c\bar{\zeta})_\alpha q^I_b - iw^c(\sigma_b\bar{\zeta})_\alpha q^I_c\,.\end{aligned} \tag{A.6b}$$

Notice that the additional terms containing the gravitational fields in (A.6a) are purely real and thus contribute only to the transformation of v^R_b. The expressions (A.5), (A.6) are used in section 5 to derive the "on-shell" SUSY algebra.

B Torsion constraints in linearized SUGRA

In this appendix we linearize the constraints on the torsion in superspace and use them to derive the expressions for the superspace connection in terms of the fields of linear non-minimal SUGRA. At the linearized level the torsion tensor is related to the connection and vielbein as follows [42],

$$\begin{aligned}T_{CB}{}^A = {} & T^{(0)}_{CB}{}^A + \Phi_{CB}{}^A - (-1)^{|B||C|}\Phi_{BC}{}^A + D_C I_B{}^A - (-1)^{|B||C|}D_B I_C{}^A\\ & + T^{(0)}_{CB}{}^D I_D{}^A - I_C{}^D T^{(0)}_{DB}{}^A + (-1)^{|B||C|}I_B{}^D T^{(0)}_{DC}{}^A\,,\end{aligned} \tag{B.1}$$

where the tensor $I_B{}^A$ describing fluctuations of the vielbein has been defined in (2.26) and $T^{(0)}_{CB}{}^A$ is the flat-superspace torsion whose only non-vanishing components are $T^{(0)}_{\gamma\dot{\beta}}{}^a =$

$T^{(0)\ a}_{\dot\beta\gamma} = 2i\sigma^a_{\gamma\dot\beta}$. The torsion (B.1) satisfies the following constraints [42],

$$T_{\gamma\beta}{}^a = T^{\dot\gamma\dot\beta a} = T_{\gamma\beta\dot\alpha} = T^{\dot\gamma\dot\beta\alpha} = 0\,, \tag{B.2a}$$

$$T_{\gamma\dot\beta}{}^a = T_{\dot\beta\gamma}{}^a = 2i\sigma^a_{\gamma\dot\beta}\,, \tag{B.2b}$$

$$T_\gamma{}^{\dot\beta}{}_{\dot\alpha} = (n-1)\delta^{\dot\beta}_{\dot\alpha}\,T_\gamma\,, \qquad T^{\dot\gamma}{}_\beta{}^\alpha = (n-1)\delta^\alpha_\beta\,\bar T^{\dot\gamma}\,, \tag{B.2c}$$

$$T_{\gamma\beta}{}^\alpha = (n+1)(\delta^\alpha_\gamma T_\beta + \delta^\alpha_\beta T_\gamma)\,, \qquad T^{\dot\gamma\dot\beta}{}_{\dot\alpha} = (n+1)(\delta^{\dot\gamma}_{\dot\alpha}\bar T^{\dot\beta} + \delta^{\dot\beta}_{\dot\alpha}\bar T^{\dot\gamma})\,, \tag{B.2d}$$

$$T_{\gamma b}{}^a = 2n\delta^a_b T_\gamma\,, \qquad T^{\dot\gamma}{}_b{}^a = 2n\delta^a_b\,\bar T^{\dot\gamma}\,, \tag{B.2e}$$

$$T_{cb}{}^a = 0\,, \tag{B.2f}$$

where the Bianchi identities imply that the superfield T_γ and its conjugate obey

$$D_\alpha T_\gamma + D_\gamma T_\alpha = 0\,, \quad \bar D_{\dot\alpha}\bar T_{\dot\gamma} + \bar D_{\dot\gamma}\bar T_{\dot\alpha} = 0\,. \tag{B.3}$$

Our strategy is to apply the constraints (B.2) to the relation (B.1) and, using the components of $I_B{}^A$ found in section 2.2, derive the equations for the remaining vielbein components and connection.

The first set of constraints (B.2a) are trivially satisfied by the vielbein (2.31) and do not provide any further information. Inserting (2.31) into (B.2b) we read off the components

$$I_a{}^b = -\delta^b_a\frac{1}{2}(\Gamma' + \bar\Gamma') - \Delta_a H^b\,. \tag{B.4}$$

From (B.2c) and (B.2e) we get respectively

$$(n-1)\delta^\alpha_\beta\,\bar T_{\dot\gamma} = \Phi_{\dot\gamma\beta}{}^\alpha + \bar D_{\dot\gamma} I_\beta{}^\alpha + 2iI_{\beta\dot\gamma}{}^\alpha\,, \tag{B.5a}$$

$$2n\delta^a_b\,\bar T_{\dot\gamma} = \Phi_{\dot\gamma b}{}^a + \bar D_{\dot\gamma} I_b{}^a - \partial_b I_{\dot\gamma}{}^a - i\sigma^a_{\delta\dot\gamma}\bar\sigma^{\dot\beta\beta}_b\,I_{\beta\dot\beta}{}^\delta\,, \tag{B.5b}$$

where we have introduced $I_{\beta\dot\beta}{}^\delta \equiv \sigma^b_{\beta\dot\beta}I_b{}^\delta$. Let us take the trace of these equations. This eliminates the connection, which is traceless, and one is left with,

$$2(n-1)\,\bar T_{\dot\gamma} = \bar D_{\dot\gamma} I_\beta{}^\beta + 2iI_{\beta\dot\gamma}{}^\beta\,, \tag{B.6a}$$

$$8n\,\bar T_{\dot\gamma} = \bar D_{\dot\gamma} I_b{}^b - \partial_b I_{\dot\gamma}{}^b + 2iI_{\beta\dot\gamma}{}^\beta\,. \tag{B.6b}$$

This system can be solved for the two unknowns $\bar T_{\dot\gamma}$, $I_{\beta\dot\gamma}{}^\beta$. Using the expressions (2.31), (B.4) we obtain,

$$I_{\beta\dot\gamma}{}^\beta = i\bar D_{\dot\gamma}\Gamma + \frac{i}{8}\bar D^2 D^\beta H_{\beta\dot\gamma}\,, \tag{B.7a}$$

$$\bar T_{\dot\gamma} = \frac{3n+1}{8n}\bar D_{\dot\gamma}\bar\Gamma + \frac{3n-1}{8n}\bar D_{\dot\gamma}\Gamma + \frac{n-1}{32n}\bar D^2 D^\alpha H_{\alpha\dot\gamma} - \frac{n+1}{16n}\bar D_{\dot\gamma} D^\alpha \bar D^{\dot\alpha} H_{\alpha\dot\alpha}\,. \tag{B.7b}$$

Note that the expression for $\bar T_{\dot\gamma}$ satisfies the condition (B.3). We now use the remaining information contained in eqs. (B.5). We take the symmetric part of eq. (B.5a),

$$2\Phi_{\dot\gamma\alpha\beta} + 2i(I_{\alpha\dot\gamma\beta} + I_{\beta\dot\gamma\alpha}) = 0\,, \tag{B.8a}$$

where we have used the symmetry of the connection in the last two indices. Multiplying eq. (B.5b) by $(\sigma_{ba})_{\alpha\beta}$ and $(\bar{\sigma}_{ba})_{\dot{\alpha}\dot{\beta}}$ and using the constitutive relation (2.35) we obtain,

$$-2\Phi_{\dot{\gamma}\alpha\beta} - i(I_{\alpha\dot{\gamma}\beta} + I_{\beta\dot{\gamma}\alpha}) = -(\sigma_{ba})_{\alpha\beta}\bar{D}_{\dot{\gamma}}I_{ba} + (\sigma_{ba})_{\alpha\beta}\partial_b I_{\dot{\gamma}a}\,, \tag{B.8b}$$

$$-2\Phi_{\dot{\gamma}\dot{\alpha}\dot{\beta}} = -(\bar{\sigma}_{ba})_{\dot{\alpha}\dot{\beta}}\bar{D}_{\dot{\gamma}}I_{ba} + (\bar{\sigma}_{ba})_{\dot{\alpha}\dot{\beta}}\partial_b I_{\dot{\gamma}a} - i\epsilon_{\dot{\alpha}\dot{\gamma}}I_{\gamma\dot{\beta}}{}^{\gamma} - i\epsilon_{\dot{\beta}\dot{\gamma}}I_{\gamma\dot{\alpha}}{}^{\gamma}\,, \tag{B.8c}$$

where we put on the r.h.s. the components that are already known. This system allows us to find the connection components $\Phi_{\dot{\gamma}\alpha\beta}$, $\Phi_{\dot{\gamma}\dot{\alpha}\dot{\beta}}$ and the symmetrized perturbation of the vielbein $I_{\alpha\dot{\gamma}\beta} + I_{\beta\dot{\gamma}\alpha}$. The results for the connections are given in eqs. (2.36) of the main text, whereas for the vielbein adding the trace part (B.7a) we obtain,

$$I_{\alpha\dot{\alpha}}{}^{\beta} = \frac{i}{2}\delta_{\alpha}^{\beta}\bar{D}_{\dot{\alpha}}\Gamma - \frac{i}{8}\bar{D}^2 D_{\alpha}H^{\beta}{}_{\dot{\alpha}}\,. \tag{B.9}$$

This completes the determination of the linearized vielbein. The connection $\Phi_{\dot{\gamma}ab}$ follows from the constitutive relation (2.35) and is given in eq. (2.37). Alternatively, it can be found from eq. (B.5b) using the expressions for the vielbein. One can check with the formulas derived above that the constraints (B.2d) are automatically satisfied. Finally, the constraint (B.2f) provides an equation for the connection components $\Phi_{cb}{}^{a}$, which we do not use in this paper.

C Calculus in superspace

C.1 Relations between superfield operators

The number of independent operators that can appear in the superfield Lagrangian for linearized SUGRA with broken Lorentz symmetry is reduced by various relations between them arising as a consequence of spinor algebra. The following properties of the superspace differential operators are used in the calculation:

commutators:

$$[\bar{D}_{\dot{\alpha}}, D^2] = 4i\partial_{\gamma\dot{\alpha}}D^{\gamma}\,, \quad [\bar{D}^2, D_{\beta}] = 4i\partial_{\beta\dot{\gamma}}\bar{D}^{\dot{\gamma}}\,, \quad [\Delta_a, \Delta_b] = 2\epsilon_{abmn}\partial^m\Delta^n\,; \tag{C.1}$$

rules for integration by parts:

$$\Psi_1 D^2\Psi_2 \simeq (D^2\Psi_1)\Psi_2\,, \quad \Psi_1\bar{D}^2\Psi_2 \simeq (\bar{D}^2\Psi_1)\Psi_2\,, \quad \Psi_1\Delta\Psi_2 \simeq (\Delta\Psi_1)\Psi_2\,, \tag{C.2}$$

where $\Psi_{1,2}$ are arbitrary superfields and the sign $\simeq$ stands for equality up to a total derivative. Using these relations one derives the identities,

$$\partial_a H^{(b}\Delta_c H^{d)} \simeq 0\,, \tag{C.3a}$$

$$\partial_a H^{(b}D^2 H^{d)} \simeq \Delta_a H^{(b}D^2 H^{d)} \simeq 0\,, \tag{C.3b}$$

$$D^2 H^{(b}\bar{D}^2 H^{d)} \simeq -2\Delta_m H^b\Delta^m H^d - 6\partial_m H^b\partial^m H^d\,, \tag{C.3c}$$

$$\Delta_{(a}H^b\Delta_{c)}H^d \simeq \partial_{(a}H^b\partial_{c)}H^d + \frac{1}{4}\eta_{ac}(\Delta_m H^b\Delta^m H^d - \partial_m H^b\partial^m H^d)\,, \tag{C.3d}$$

$$
\begin{aligned}
\Delta_a H_k \Delta_c H^k &\simeq \partial_a H_k \partial_c H^k + \frac{1}{4}\eta_{ac}(\Delta_m H_k \Delta^m H^k - \partial_m H_k \partial^m H^k)\,, && \text{(C.3e)}\\
\Delta_a H^k \Delta_k H_b &\simeq -\Delta_a H_b \Delta_k H^k + 2\partial_a H_b \partial_k H^k + \frac{1}{2}(\Delta_m H_a \Delta^m H_b - \partial_m H_a \partial^m H_b)\,, && \text{(C.3f)}\\
\Delta_a H^{[c}\Delta_b H^{d]} &\simeq -\epsilon_{abmn}\partial_m H^{[c}\Delta_n H^{d]}\,, && \text{(C.3g)}\\
\Delta_{[a} H^c \Delta_{b]} H^d &\simeq -\epsilon_{abmn}\partial_m H^c \Delta_n H^d\,, && \text{(C.3h)}\\
\epsilon_{amnk}\partial^m H_b \Delta^n H^k &\simeq -\Delta_a H_b \Delta_k H^k + \partial_a H_b \partial_k H^k + \frac{1}{4}(\Delta_m H_a \Delta^m H^b - \partial_m H_a \partial^m H_b) && \text{(C.3i)}\\
\epsilon_{amnk}\partial_b H^m \Delta^n H^k &\simeq -\epsilon_{amnk}\Delta_b H^m \partial^n H^k - 2\Delta_a H_b \Delta_k H^k + 2\partial_a H_b \partial_k H^k\\
&\quad + \frac{1}{2}(\Delta_k H_a \Delta^k H_b - \partial_k H_a \partial^k H_b)\\
&\quad + \eta_{ab}\Big((\Delta_k H^k)^2 - (\partial_k H^k)^2 - \frac{1}{4}\Delta_m H_k \Delta^m H^k + \frac{1}{4}\partial_m H_k \partial^m H^k\Big)\,. && \text{(C.3j)}
\end{aligned}
$$

Here the round (square) brackets denote symmetrization (antisymmetrization) over the corresponding indices.

C.2 Operators without aether perturbation

It is convenient to further subdivide these terms according to the number of insertions of the spurion w^a. It is straightforward to see that the maximal number of insertions is 4. Thus, we have:

4 insertions of w^a. There is a single independent operator,

$$
w^a w^b w^c w^d \partial_a H_b \partial_c H_d\,. \qquad \text{(C.4)}
$$

Two other possible operators would be

$$
w^a w^b w^c w^d \partial_a H_b \Delta_c H_d\,, \qquad w^a w^b w^c w^d \Delta_a H_b \Delta_c H_d\,.
$$

However, the first of them is a total derivative, see eq. (C.3a) in appendix C.1, whereas the second is expressed in terms of (C.4) and contributions with fewer insertions of w^a due to the relation (C.3d).

3 insertions of w^a. This group is actually empty. The operators that can be written using three w^a-insertions are

$$
w^a w^b w^c \partial_a H_b D^2 H_c\,, \qquad w^a w^b w^c \Delta_a H_b D^2 H_c
$$

and their complex conjugate. However, they vanish upon integration over the superspace, see eq. (C.3b).

2 insertions of w^a. There are in total 12 independent operators that we choose as follows,

$$
\begin{aligned}
& w^a w^b \partial_a H_b \partial_c H^c\,, && w^a w^b \partial_a H_c \partial_b H^c\,, && w^a w^b \partial_c H_a \partial^c H_b\,, && && \text{(C.5a)}\\
& w^a w^b \Delta_a H_b \Delta_c H^c\,, && w^a w^b \Delta_c H_a \Delta^c H_b\,, && && && \text{(C.5b)}\\
& w^a w^b \partial_a H_b \Delta_c H^c\,, && w^a w^b \partial_c H^c \Delta_a H_b\,, && && && \text{(C.5c)}\\
& w^a w^b \epsilon_{acde}\partial^c H^d \Delta_b H^e\,, && && && && \text{(C.5d)}\\
& w^a w^b \partial_a H_b \Gamma\,, && w^a w^b \partial_a H_b \bar{\Gamma}\,, && w^a w^b \Delta_a H_b \Gamma\,, && w^a w^b \Delta_a H_b \bar{\Gamma}\,. && \text{(C.5e)}
\end{aligned}
$$

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Other operators that can be written using two w^a insertions are

$$
\begin{aligned}
&w^a w^b \Delta_a H_c \Delta_b H^c\,, && w^a w^b \Delta_a H_c \Delta^c H_b\,, && && \text{(C.6a)}\\
&w^a w^b \partial_a H_c \Delta_b H^c\,, && w^a w^b \partial_c H_a \Delta^c H_b\,, && w^a w^b \epsilon_{acde}\partial_b H^c \partial^d H^e\,, && \text{(C.6b)}\\
&w^a w^b \epsilon_{acde}\Delta_b H^c \Delta^d H^e\,, && w^a w^b \epsilon_{acde}\Delta^c H_b \Delta^d H^e\,, && && \text{(C.6c)}\\
&w^a w^b \epsilon_{acde}\partial^c H_b \Delta^d H^e\,, && w^a w^b \epsilon_{acde}\partial_b H^c \Delta^d H^e\,, && && \text{(C.6d)}\\
&w^a w^b D^2 H_a \bar{D}^2 H_b\,. && && && \text{(C.6e)}
\end{aligned}
$$

Using the identities (C.3) one shows that the contributions of the latter operators into the Lagrangian are degenerate with the operators (C.5): for the two operators (C.6a) this is due to the relations (C.3e) and (C.3f); the terms (C.6b) vanish upon integration; the operators (C.6c) and (C.6d) are eliminated using (C.3g)–(C.3j); the operator (C.6e) is eliminated due to (C.3c).

1 insertion of w^a. There are 2 terms,

$$
w^a \partial_b H^b D^2 H_a\,, \qquad w^a D^2 H_a \Gamma\,, \tag{C.7}
$$

and their complex conjugate. One more operator

$$
w^a \partial_a H_b D^2 H^b
$$

is a total derivative, see (C.3b). Also, a replacement of the ordinary derivative ∂_a by Δ_a in the above operators does not generate new contributions due to the anti-chirality of the field $D^2 H_a$.

No insertions of w^a. 12 independent operators are,

$$
\begin{aligned}
&(\partial_a H^a)^2\,, && \partial_a H_b \partial^a H^b\,, && && && \text{(C.8a)}\\
&(\Delta_a H^a)^2\,, && \Delta_a H_b \Delta^a H^b\,, && && && \text{(C.8b)}\\
&\partial_a H^a \Delta_b H^b\,, && && && && \text{(C.8c)}\\
&\partial_a H^a \Gamma\,, && \partial_a H^a \bar{\Gamma}\,, && \Delta_a H^a \Gamma\,, && \Delta_a H^a \bar{\Gamma}\,, && \text{(C.8d)}\\
&\Gamma^2\,, && \bar{\Gamma}^2\,, && \Gamma\bar{\Gamma}\,. && && \text{(C.8e)}
\end{aligned}
$$

The five remaining combinations,

$$
\partial_a H_b \Delta^a H^b\,, \quad D^2 H_a \bar{D}^2 H^a\,, \quad \Delta_a H_b \Delta^b H^a\,, \quad \epsilon_{abcd}\Delta^a H^b \Delta^c H^d\,, \quad \epsilon_{abcd}\partial^a H^b \Delta^c H^d\,, \tag{C.9}
$$

produce degenerate contributions into the action, as it follows from eqs. (C.3a), (C.3c), (C.3f), (C.3g), (C.3i).

C.3 Transformations of the superfield operators

In this appendix we derive the gauge variations of various superfield operators that can potentially enter into the quadratic Lagrangian. The operators (3.9), (3.10) have been discussed in the main text. Here we focus on the operators from appendix C.2.

We start with the operator (C.4). Its variation reads,

$$
\delta_L (w^a w^b w^c w^d \partial_a H_b \partial_c H_d) \simeq L_\beta w^a w^b w^c w^d \bar{\sigma}_a^{\dot\beta\beta} \bar{D}_{\dot\beta} \partial_b \partial_c H_d + \text{h.c.}\,, \tag{C.10}
$$

and manifestly contains four insertions of w^a. There are no other operators whose variation would have this property and therefore (C.4) is also absent from the invariant action.

To proceed, we notice that the remaining operators split into several sectors which do not mix under the linearized super-diffeomorphisms. These sectors are characterized by the properties of the operators under the action of the R-symmetry and CP. The R-symmetry rotates the phases of the spinor derivatives,

$$D_\alpha \mapsto \mathrm{e}^{-i\varphi} D_\alpha \,, \qquad \bar{D}_{\dot\alpha} \mapsto \mathrm{e}^{i\varphi} \bar{D}_{\dot\alpha} \,, \tag{C.11}$$

with the superfields H_a, Γ, V^a and the ordinary derivatives ∂_a kept intact. Correspondingly, the operators (C.5), (C.8) have zero R-charge, whereas the R-charge of the operators (C.7) is -2. The R-charge is preserved by the super-diffeos, provided one assigns $R = -1$ to the gauge parameter L_β. This implies that if the operators (C.7) entered into the invariant action, their variations would have to cancel with each other. However, it is straightforward to see that this is impossible. We omit the operators (C.7) in what follows.

Next we turn to the properties of the operators under parity. Pure parity does not preserve the SUSY algebra and thus cannot be defined on the superspace. To be compatible with SUSY, parity must be supplemented by the charge conjugation [43]. In the Lorentz frame where the spatial components of the aether VEV vanish, see (3.1), the CP transformations have the form,[23]

$$V_i \mapsto -\bar{V}_i \tag{C.12a}$$
$$H_0 \mapsto -H_0 \,, \quad H_i \mapsto H_i \,, \quad \Gamma \mapsto \bar{\Gamma} \tag{C.12b}$$
$$\partial_0 \mapsto \partial_0 \,, \qquad \partial_i \mapsto -\partial_i \,, \tag{C.12c}$$
$$\Delta_0 \mapsto -\Delta_0 \,, \quad \Delta_i \mapsto \Delta_i \,, \tag{C.12d}$$

where $i = 1, 2, 3$ denote the spatial indices. Notice that $(V_a + \bar{V}_a)$, ∂_a transform as vectors, whereas $(V_a - \bar{V}_a)$, H_a, Δ_a are pseudo-vectors. Clearly, the SUGRA action (2.17) is CP-even. It is convenient to choose the basis of operators having definite CP quantum numbers. Out of (C.5), (C.8) we construct the following combinations:

***CP*-even:**

$$w^a w^b \partial_a H_b \partial_c H^c \,, \qquad w^a w^b \partial_a H_c \partial_b H^c \,, \qquad w^a w^b \partial_c H_a \partial^c H_b \,, \tag{C.13a}$$
$$w^a w^b \Delta_a H_b \Delta_c H^c \,, \qquad w^a w^b \Delta_c H_a \Delta^c H_b \,, \qquad w^a w^b \epsilon_{acde} \partial^c H^d \Delta_b H^e \,, \tag{C.13b}$$
$$i w^a w^b \partial_a H_b (\Gamma - \bar{\Gamma}) \,, \qquad w^a w^b \Delta_a H_b (\Gamma + \bar{\Gamma}) \,, \tag{C.13c}$$
$$(\partial_a H^a)^2 \,, \qquad \partial_a H_b \partial^a H^b \,, \qquad (\Delta_a H^a)^2 \,, \qquad \Delta_a H_b \Delta^a H^b \,, \tag{C.13d}$$
$$i \partial_a H^a (\Gamma - \bar{\Gamma}) \,, \qquad \Delta_a H^a (\Gamma + \bar{\Gamma}) \,, \qquad \Gamma^2 + \bar{\Gamma}^2 \,, \qquad \Gamma \bar{\Gamma} \,; \tag{C.13e}$$

***CP*-odd:**

$$w^a w^b \partial_a H_b \Delta_c H^c \,, \qquad w^a w^b \Delta_a H_b \partial_c H^c \,, \tag{C.14a}$$
$$w^a w^b \partial_a H_b (\Gamma + \bar{\Gamma}) \,, \qquad i w^a w^b \Delta_a H_b (\Gamma - \bar{\Gamma}) \,, \tag{C.14b}$$
$$\partial_a H^a \Delta_b H^b \,, \qquad \partial_a H^a (\Gamma + \bar{\Gamma}) \,, \qquad i \Delta_a H^a (\Gamma - \bar{\Gamma}) \,, \qquad i (\Gamma^2 - \bar{\Gamma}^2) \,. \tag{C.14c}$$

[23] Note that in this frame $V_0 = 0$ due to the constraint (3.2).

To the first group one has to add the CP-even operator (3.9). The gauge variations of operators should cancel separately within each group.

We expand the variations as linear combinations of independent contributions. For the CP-even sector the coefficients of this expansion are listed in table 1 and for the CP-odd sector in table 2. Each column in the tables corresponds to a given operator and rows to the terms in the expansion of its variation. The notations for the rows are,

$$\mathcal{O}_1 = L_\beta w^a w^b \bar{\sigma}_a^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial_b \partial_k H^k\,, \tag{C.15a}$$

$$\mathcal{O}_2 = L_\beta w^a w^b \bar{\sigma}_k^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial^k \partial_a H_b\,, \tag{C.15b}$$

$$\mathcal{O}_3 = L_\beta w^a w^b \bar{\sigma}_k^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial_a \partial_b H^k\,, \tag{C.15c}$$

$$\mathcal{O}_4 = L_\beta w^a w^b \bar{\sigma}_a^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \Box H_b\,, \tag{C.15d}$$

$$\mathcal{O}_5 = L_\beta w^a w^b (\sigma_{ak})_\gamma{}^\beta \bar{D}^2 D^\gamma \partial_b H^k\,, \tag{C.15e}$$

$$\mathcal{O}_6 = L_\beta w^a w^b (\sigma_{ak})_\gamma{}^\beta \bar{D}^2 D^\gamma \partial^k H_b\,, \tag{C.15f}$$

$$\mathcal{O}_7 = L_\beta w^a w^b \bar{D}^2 D^\beta \partial_a H_b\,, \tag{C.15g}$$

$$\mathcal{O}_8 = L_\beta w^a w^b \epsilon_{aklm} \bar{\sigma}^{k\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial_b \partial^l H^m\,, \tag{C.15h}$$

$$\mathcal{O}_9 = L_\beta w^a w^b \bar{\sigma}_a^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial_b \Gamma\,, \tag{C.15i}$$

$$\mathcal{O}_{10} = L_\beta w^a w^b \bar{\sigma}_a^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial_b \bar{\Gamma}\,, \tag{C.15j}$$

$$\mathcal{O}_{11} = L_\beta (\sigma_{kl})_\gamma{}^\beta \bar{D}^2 D^\gamma \partial^k H^l\,, \tag{C.15k}$$

$$\mathcal{O}_{12} = L_\beta \bar{\sigma}_k^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial^k \partial_l H^l\,, \tag{C.15l}$$

$$\mathcal{O}_{13} = L_\beta \bar{\sigma}_k^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \Box H^k\,, \tag{C.15m}$$

$$\mathcal{O}_{14} = L_\beta \bar{D}^2 D^\beta \partial_k H^k\,, \tag{C.15n}$$

$$\mathcal{O}_{15} = L_\beta \bar{\sigma}_k^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial^k \Gamma\,, \tag{C.15o}$$

$$\mathcal{O}_{16} = L_\beta \bar{\sigma}_k^{\dot{\beta}\beta} \bar{D}_{\dot{\beta}} \partial^k \bar{\Gamma}\,, \tag{C.15p}$$

$$\mathcal{O}_{17} = L_\beta \bar{D}^2 D^\beta \bar{\Gamma}\,. \tag{C.15q}$$

We focus only on the part of the variations proportional to L_β; the terms with $\bar{L}_{\dot{\beta}}$ are obtained by complex conjugation. For example, from the second column of table 1 one reads,

$$\delta_L(w^a w^b \partial_a H_b \partial_c H^c) \simeq \frac{1}{2}\mathcal{O}_1 + \frac{1}{2}\mathcal{O}_2 + \text{h.c.}\,,$$

where $\simeq$ stands, as usual, for 'equal up to a total derivative'.

A gauge invarinat linear combination of operators with a vector of coefficients X corresponds to a solution of the system of equations,

$$\mathcal{M} \cdot X = 0\,, \tag{C.16}$$

where $\mathcal{M}$ is the matrix of table 1 (2) for the CP-even (odd) sector respectively. In the case of CP-even operators a non-trivial solution of this system exists and is parameterized by two free variables. The corresponding invariant action is presented in the main text. For the CP-odd case eq. (C.16) has only a trivial solution, $X = 0$.

	$\delta_L(V_a\bar{V}^a)$	$\delta_L(w^aw^b\partial_aH_b\partial_cH^c)$	$\delta_L(w^aw^b\partial_aH_c\partial_bH^c)$	$\delta_L(w^aw^b\partial_cH_a\partial^cH_b)$	$\delta_L(w^aw^b\Delta_aH_b\Delta_cH^c)$	$\delta_L(w^aw^b\Delta_cH_a\Delta^cH_b)$	$\delta_L(w^aw^b\epsilon_{acde}\partial^cH^d\Delta_bH^e)$	$\delta_L(iw^aw^b\partial_aH_b(\Gamma-\bar{\Gamma}))$	$\delta_L(w^aw^b\Delta_aH_b(\Gamma+\bar{\Gamma}))$	$\delta_L((\partial_aH^a)^2)$	$\delta_L(\partial_aH_b\partial^aH^b)$	$\delta_L((\Delta_aH^a)^2)$	$\delta_L(\Delta_aH_b\Delta^aH^b)$	$\delta_L(i\partial_aH^a(\Gamma-\bar{\Gamma}))$	$\delta_L(\Delta_aH^a(\Gamma+\bar{\Gamma}))$	$\delta_L(\Gamma^2+\bar{\Gamma}^2)$	$\delta_L(\Gamma\bar{\Gamma})$
$\mathcal{O}_1$		$\frac{1}{2}$			$\frac{1}{2}$												
$\mathcal{O}_2$		$\frac{1}{2}$			$\frac{1}{2}$			$\frac{n+1}{3n+1}$	$-\frac{n+1}{3n+1}$								
$\mathcal{O}_3$			1														
$\mathcal{O}_4$				1		1											
$\mathcal{O}_5$					$-\frac{i}{4}$		$-\frac{i}{2}$										
$\mathcal{O}_6$	$-\frac{i}{2}$				$-\frac{3i}{4}$	$-2i$	$\frac{i}{2}$		$\frac{i}{2}$								
$\mathcal{O}_7$	$-\frac{i}{4}$				$-\frac{i}{4}$	$-i$		$\frac{in}{2(3n+1)}$	$\frac{i(n+1)}{4(3n+1)}$								
$\mathcal{O}_8$							i										
$\mathcal{O}_9$	$-i$							$\frac{i}{2}$	$\frac{i}{2}$								
$\mathcal{O}_{10}$								$-\frac{i}{2}$	$\frac{i}{2}$								
$\mathcal{O}_{11}$					$-\frac{i}{4}$		$-\frac{i}{2}$					$\frac{3i}{2}$	$2i$		$-\frac{i}{2}$		
$\mathcal{O}_{12}$										1		1		$\frac{n+1}{3n+1}$	$-\frac{n+1}{3n+1}$		
$\mathcal{O}_{13}$											1		1				
$\mathcal{O}_{14}$	$\frac{i}{8}$											$-\frac{i}{4}$	$-i$	$\frac{in}{2(3n+1)}$	$\frac{i(n+1)}{4(3n+1)}$		
$\mathcal{O}_{15}$	$-\frac{i}{4}$								$\frac{i}{2}$					$\frac{i}{2}$	$-\frac{3i}{2}$		i
$\mathcal{O}_{16}$														$-\frac{i}{2}$	$\frac{i}{2}$		$-\frac{i(n+1)}{3n+1}$
$\mathcal{O}_{17}$	$\frac{3}{16}$								$-\frac{1}{8}$						$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{n+1}{4(3n+1)}$

Table 1. The coefficients in the gauge variations of CP-even operators. Only non-zero entries are shown.

	$\delta_L(w^a w^b \partial_a H_b \Delta_c H^c)$	$\delta_L(w^a w^b \partial_c H^c \Delta_a H_b)$	$\delta_L(w^a w^b \partial_a H_b(\Gamma+\bar{\Gamma}))$	$\delta_L(i w^a w^b \Delta_a H_b(\Gamma-\bar{\Gamma}))$	$\delta_L(\partial_a H^a \Delta_b H^b)$	$\delta_L(\partial_a H^a(\Gamma+\bar{\Gamma}))$	$\delta_L(i\Delta_a H^a(\Gamma-\bar{\Gamma}))$	$\delta_L(i(\Gamma^2-\bar{\Gamma}^2))$
$\mathcal{O}_1$	$-\frac{i}{2}$	$\frac{i}{2}$						
$\mathcal{O}_2$	$\frac{i}{2}$	$-\frac{i}{2}$	$-\frac{i(n+1)}{3n+1}$	$-\frac{i(n+1)}{3n+1}$				
$\mathcal{O}_5$	$-\frac{1}{4}$							
$\mathcal{O}_6$		$\frac{1}{4}$		$\frac{1}{2}$				
$\mathcal{O}_7$	$\frac{3}{8}$	$-\frac{1}{8}$	$-\frac{2n+1}{2(3n+1)}$	$-\frac{n+1}{4(3n+1)}$				
$\mathcal{O}_9$			$\frac{1}{2}$	$-\frac{1}{2}$				
$\mathcal{O}_{10}$			$\frac{1}{2}$	$\frac{1}{2}$				
$\mathcal{O}_{11}$					$-\frac{1}{4}$		$-\frac{1}{2}$	
$\mathcal{O}_{12}$						$-\frac{i(n+1)}{3n+1}$	$-\frac{i(n+1)}{3n+1}$	
$\mathcal{O}_{14}$		$-\frac{1}{8}$			$\frac{3}{8}$	$-\frac{2n+1}{2(3n+1)}$	$-\frac{n+1}{4(3n+1)}$	
$\mathcal{O}_{15}$				$-\frac{1}{2}$		$\frac{1}{2}$	$\frac{3}{2}$	
$\mathcal{O}_{16}$						$\frac{1}{2}$	$\frac{1}{2}$	
$\mathcal{O}_{17}$				$\frac{i}{8}$			$-\frac{i}{2}$	$\frac{i}{2}$

Table 2. Same as table 1, but for the CP-odd operators.

D The supercurrent multiplet

Here we remind some general facts about the supercurrent multiplet. Consider a supersymmetric theory coupled to supergravity. The action can be written as

$$S = \frac{1}{\varkappa^2} S_{SG}[g_{mn}, \psi_{m\alpha}] + S_{\text{mat}}[\Phi_i; g_{mn}, \psi_{m\alpha}] + \dots\,,$$

where S_{SG} is the pure SUGRA action depending on the metric and gravitino, S_{mat} is the action of the matter sector, and Φ_i collectively denotes the matter fields (both bosons and fermions). We assume that the auxiliary fields have been integrated out, so that we are working only with the physical fields. Note that we have factored out the (inverse of) the gravitational coupling $\varkappa^{-2}$ in front of the SUGRA action. The matter action is of zeroth order in $\varkappa$. Dots denote possible extra terms proportional to positive powers of $\varkappa^2$ that appear upon integrating out the auxiliary fields.

Let us expand the action around Minkowski spacetime as a power series in the metric perturbation h_{mn} and gravitino. Further, it is convenient to rescale the fields of the gravity

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sector by introducing

$$\hat{h}_{mn} = \varkappa^{-1} h_{mn} \,, \qquad \hat{\psi}_{m\alpha} = \varkappa^{-1} \psi_{m\alpha} \,. \tag{D.1}$$

One obtains,

$$S = S^{(2)}_{SG}[\hat{h}_{mn}, \hat{\psi}_{m\alpha}] + S^{(0)}_{\rm mat}[\Phi_i] + \varkappa \int d^4x \left(\frac{1}{2} \mathcal{T}^{mn} \hat{h}_{mn} - \mathcal{S}^{m\alpha} \hat{\psi}_{m\alpha} - \bar{\mathcal{S}}^m_{\dot{\alpha}} \bar{\hat{\psi}}^{\dot{\alpha}}_m \right) + \dots \,, \tag{D.2}$$

where $S^{(2)}_{SG}$ is the quadratic part of the SUGRA action, $S^{(0)}_{\rm mat}$ is the matter action in flat spacetime, $\mathcal{T}^{mn}$ and $\mathcal{S}^{m\alpha}$ are the energy-momentum ternsor (EMT) and the supercurrent. Dots stand for terms that are of cubic and higher order in $\hat{h}_{mn}$, $\hat{\psi}_{m\alpha}$ or are proportional to $\varkappa^2$.

The action is invariant under local SUSY transformations which we also expand as power series in $\hat{h}_{mn}$, $\hat{\psi}_{m\alpha}$ (but not in the matter fields). The parameter of these transformations will be denoted by $\zeta_\alpha(x)$. Then, up to terms quadratic in $\hat{h}_{mn}$, $\hat{\psi}_{m\alpha}$, the transformations of the metric perturbation and gravitino read,

$$\delta_\zeta \hat{h}_{mn} = 2i(\bar{\zeta}\bar{\sigma}_m \hat{\psi}_n + \bar{\zeta}\bar{\sigma}_n \hat{\psi}_m) + \text{h.c.} \,, \tag{D.3a}$$

$$\delta_\zeta \hat{\psi}_{m\alpha} = \frac{1}{\varkappa} \partial_m \zeta_\alpha + \frac{1}{2} (\sigma^{kn}\zeta)_\alpha \partial_k \hat{h}_{nm} + \varkappa\, \Xi_{m\alpha}[\Phi_i] + \dots \,, \tag{D.3b}$$

where the model-dependent spin-vector $\Xi_{m\alpha}$ is constructed of matter fields and the omitted terms are of higher order in $\varkappa$. Notice that the first of these equations coincides with eq. (2.16) which has the same form irrespectively of whether ζ_α is coordinate dependent or not. Whereas the gravitino transformation is simply a combination of eqs. (2.15) and (A.5) with the identification $\varepsilon_\alpha = \zeta_\alpha$. The matter field transformations can also be written as an expansion in $\varkappa$,

$$\delta_\zeta \Phi_i = \delta^{(0)}_\zeta \Phi_i + \varkappa\, \delta^{(1)}_\zeta \Phi_i + \dots \,, \tag{D.4}$$

where the first term is the transformation in flat spacetime and the remaining terms describe corrections due to the supergravity coupling. Then the invariance of the action implies,

$$\begin{aligned} 0 = \int d^4x \bigg[&\varkappa (4\epsilon^{klmn} \Xi_k \bar{\sigma}_l \partial_m \hat{\psi}_n + \text{h.c.}) + \frac{\delta S^{(0)}_{\rm mat}}{\delta \Phi_i} (\delta^{(0)}_\zeta \Phi_i + \varkappa\, \delta^{(1)}_\zeta \Phi_i) \\ &+ \frac{\varkappa}{2} \delta_\zeta \mathcal{T}^{mn}\, \hat{h}_{mn} + \frac{\varkappa}{2} \mathcal{T}^{mn}\, \delta_\zeta \hat{h}_{mn} - \varkappa (\delta_\zeta \mathcal{S}^{m\alpha}\, \hat{\psi}_{m\alpha} + \mathcal{S}^{m\alpha}\, \delta_\zeta \hat{\psi}_{m\alpha} + \text{h.c.}) \bigg] + \dots \,, \end{aligned} \tag{D.5}$$

where we have used the explicit form of the Rarita-Schwinger Lagrangian for gravitino and took into account that the SUGRA action itself is invariant in the absence of matter. Let us consider the consequences of this equation order by order in $\varkappa$.

At the zeroth order we have the variation of the flat-space matter action. This is invariant under global SUSY with constant ζ_α. Hence, its variation is proportional to the gradient of ζ_α. The Noether theorem identifies the proportionality coefficient with the supercurrent,

$$\int d^4x\, \frac{\delta S^{(0)}_{\rm mat}}{\delta \Phi_i}\, \delta^{(0)}_\zeta \Phi_i = \int d^4x (\mathcal{S}^{m\alpha} \partial_m \zeta_\alpha + \text{h.c.}) \,.$$

We see that the gradient term in the transformation of gravitino (D.3b) precisely cancels this variation, as it should be. This cancellation happens for arbitrary configuration of the

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matter fields. On the other hand, we can restrict the fields Φ_i on shell, i.e. consider only the configurations that satisfy the flat-space equations of motion,

$$\frac{\delta S^{(0)}_{\rm mat}}{\delta \Phi_i} = 0\,. \tag{D.6}$$

Then the variation of the matter action vanishes by itself and the invariance of the remaining gravitino-matter coupling implies $\partial_m \mathcal{S}^{m\alpha} = 0$, which is nothing but the on-shell conservation of the SUSY current.

Consider now eq. (D.5) at order $O(\varkappa)$. In general, the matter fields' transformations get corrected at this order in a non-trivial model-dependent way. Thus, inferring any off-shell statements is problematic. However, the situation dramatically simplifies on shell, where the variation of the matter action vanishes due to eqs. (D.6). Restricting also to constant ζ_α, so that the part neglected in eq. (D.5) cannot produce any $O(\varkappa)$ contribution, one is left with the terms containing variations of the gravity fields, EMT and supercurrent. Substituting the explicit formulas (D.3) and equating the combinations in front of $\hat{h}_{mn}$, $\hat{\psi}_{m\alpha}$ to zero one obtains the relations (5.6) from the main text. Notice that the variations of the matter fields inside $\mathcal{T}^{mn}$ and $\mathcal{S}^{m\alpha}$ are taken at the zeroth order corresponding to the flat-space global SUSY.

To sum up, we have shown that in any supersymmetric theory the EMT and supercurrent describing the coupling of the theory to supergravity belong to the same global SUSY multiplet with on-shell transformation rules (5.6). The transformation of $\mathcal{T}^{mn}$ is completely universal, whereas that of $\mathcal{S}^{m\alpha}$ depends on the model through the spin-vector $\Xi^{m\alpha}$.

E Auxiliary field ω_α

In this appendix we derive the on-shell value of the fermionic auxiliary field ω_α that enters into the SUSY transformation of the aether. We treat the coefficient C in the super-aether action (3.13) as a small parameter and work up to terms of order $O(\sqrt{C})$. In principle, one could use the same strategy as for the bosonic sector: first derive the full off-shell fermionic Lagrangian and then find from it the equations of motion of the auxiliary fields. We find it simpler, however, to act in the reverse order: first obtain the equations of motion in terms of superfields and then project them on the appropriate components.

The superspace equations of motion are obtained by taking the variation of the sum of the actions (2.17) and (3.13) with respect to the superfields. One has to remember, however, that the superfields Γ and V^a are constrained: Γ is linear, whereas V^a satisfies the orthogonality and chirality conditions (3.2), (3.3). We implement these constraints using Lagrange multipliers. Thus, the action is supplemented with the term,

$$\begin{aligned} S_{\rm constr} = \frac{1}{\varkappa^2}\int d^4x\, d^4\theta \Big[&\bar{\Lambda}_1 \bar{D}^2\Gamma + \Lambda_1 D^2\bar{\Gamma} + \bar{\Lambda}_2 w^a V_a + \Lambda_2 w^a \bar{V}_a \\ &+ \bar{\Lambda}^{\;a}_{3\dot\beta}(\bar{D}^{\dot\beta} V_a + w^c \Phi^{\dot\beta}_{\;\;ca}) + \Lambda^{\;\beta}_{3a}(D_\beta \bar{V}^a + w^c \Phi_{\beta c}{}^{a})\Big]\,, \end{aligned} \tag{E.1}$$

where the Lagrange multipliers Λ_1, Λ_2, $\Lambda^{\;\beta}_{3a}$ are general scalar and spin-vector superfields.[24] Once this term is added, all superfields in the action can be treated as unconstrained.

[24] An alternative way to enforce the orthogonality constraint (3.2) is to use a chiral Lagrange multiplier as in eq. (3.7).

Recalling the expression (2.37) for the connection in terms of the SUGRA superfields and taking the variation with respect to $\bar{\Gamma}$, $\bar{V}^a$ we obtain,

$$\begin{aligned} & i\frac{3n+1}{2n}\partial_m H^m + \frac{3n+1}{2}\Delta_m H^m + \frac{9n^2-1}{4n}\bar{\Gamma} + \frac{(3n+1)^2}{4n}\Gamma \\ & \quad + D^2 \Lambda_1 + w^c(\sigma_{ca})_\beta{}^\gamma D_\gamma {\Lambda_3}^{a\beta} = 0\,, \end{aligned} \tag{E.2a}$$

$$w_a \Lambda_2 + D_\beta \Lambda_{3a}^\beta + \frac{\sqrt{C}}{2}\hat{V}_a = 0\,, \tag{E.2b}$$

where we have neglected terms of order $O(C)$ in the first equation and in the second equation renormalized the aether superfield canonically, $\hat{V}_a = \sqrt{C}V_a$. The neglected terms do not affect the expression for ω_α at order $O(\sqrt{C})$. We now take the covariant derivative D_α of these two equations and restrict them to the origin of spinor coordinates θ, $\bar{\theta}$. Using the definition of the component fields (2.2), (2.9), (3.5) we find,

$$-\frac{3n+1}{2n}\,\omega_\alpha + \frac{1}{2}w^c(\sigma_{ca})_{\beta\alpha} D^2 {\Lambda_3}^{a\beta}| = 0\,, \tag{E.3a}$$

$$w_a D_\alpha \Lambda_2| + \frac{1}{2}D^2 \Lambda_{3a\alpha}| + \frac{\sqrt{C}}{2}\hat{\eta}_{a\alpha} = 0\,, \tag{E.3b}$$

where we again neglected the terms of higher order in C and used the gauge conditions (2.6), (2.12b). Combining these equations to eliminate the Lagrange multipliers we arrive at the expression (5.20) from the main text.

F Velocities of elementary excitations

F.1 Bosonic modes

Here we analyze in detail the sector of modes described by the Lagrangian (4.4). We work in the frame where the aether VEV has vanishing spatial components, see eq. (3.1). In this frame the time-components of aether perturbations vanish due to the constraint (3.2), $\hat{v}_0^{R,I} = 0$.

Transverse traceless (helicity ± 2) modes are contained only in the metric. Thus, we insert the Ansatz,

$$h_{ij} = h_{ij}^{tt}\,, \qquad \partial_i h_{ij}^{tt} = h_{ii}^{tt} = 0\,, \qquad i,j = 1,2,3,$$

with all other fields vanishing. The Lagrangian becomes

$$\mathcal{L}_{h=2} = \frac{1}{2\varkappa^2}\left[-\frac{1-C}{4}h_{ij}^{tt}\ddot{h}_{ij}^{tt} + \frac{1}{4}h_{ij}^{tt}\Delta h_{ij}^{tt}\right], \tag{F.1}$$

where we use Δ to denote the spatial Laplacian, $\Delta = \partial_i \partial_i$; as we are not going to use the operator (2.3) in this appendix, this should not lead to confusion. Decomposing the field into plane waves with momentum $\mathbf{p}$ and energy E we find the dispersion relation

$$E^2 = \frac{p^2}{1-C} \approx (1+C)\,p^2\,, \tag{F.2}$$

which corresponds to the propagation velocity (6.2).

We now turn to modes with helicities ± 1. The metric perturbations are taken in the form

$$h_{00} = 0\,, \qquad h_{0i} = n_i\,, \qquad h_{ij} = \partial_i \xi_j + \partial_j \xi_i\,,$$

with all vectors being transverse,

$$\partial_i n_i = \partial_i \xi_i = \partial_i \hat{v}_i^R = \partial_i \hat{v}_i^I = 0\,.$$

The Lagrangian (4.4) in this sector reads,

$$\begin{aligned}\mathcal{L}_{h=1} = \frac{1}{2\varkappa^2}\bigg[&-\frac{1}{2}n_i\Delta n_i + \frac{1}{2}\xi_i\Delta\ddot{\xi}_i - \dot{n}_i\Delta\xi_i - \hat{v}_i^R\ddot{\hat{v}}_i^R + \hat{v}_i^R\Delta\hat{v}_i^R \\ &- \left(1-\frac{C}{2}\right)\hat{v}_i^I\ddot{\hat{v}}_i^I + \hat{v}_i^I\Delta\hat{v}_i^I - \sqrt{C}\hat{v}_i^R\dot{n}_i + \sqrt{C}\hat{v}_i^R\Delta\dot{\xi}_i + C\epsilon_{ijk}\,\hat{v}_i^R\partial_j\dot{\hat{v}}_k^I\bigg]\,,\end{aligned} \tag{F.3}$$

where ϵ_{ijk} is the 3-dimensional antisymmetric tensor, $\epsilon_{123} = 1$. In deriving this expression we kept only the leading-order terms in the gravitational part of the Lagrangian: the omitted corrections affect the dynamics of helicity-1 modes only at order $O(C^{3/2})$ or higher. Note a peculiar mixing term between the real and imaginary parts of the aether perturbations.

Varying (F.3) with respect to ξ_i and setting the gauge $\xi_i = 0$ afterwards one finds,

$$n_i = -\sqrt{C}\,\hat{v}_i^R \tag{F.4}$$

up to corrections of order[25] $O(C^{3/2})$. Substituting this into the equations for the aether perturbations we obtain,

$$-\left(1-\frac{C}{2}\right)\ddot{\hat{v}}_i^R + \Delta\hat{v}_i^R + \frac{C}{2}\epsilon_{ijk}\partial_j\dot{\hat{v}}_k^I = 0\,, \tag{F.6a}$$

$$-\left(1-\frac{C}{2}\right)\ddot{\hat{v}}_i^I + \Delta\hat{v}_i^I - \frac{C}{2}\epsilon_{ijk}\partial_j\dot{\hat{v}}_k^R = 0\,. \tag{F.6b}$$

To solve this system, we take $\hat{v}_i^{R,I}$ as a sum of circularly polarized plane waves with energy E and momentum $\boldsymbol{p}$,

$$\hat{v}_i^{R,I} = \left(e_i^{(+)} f_+^{R,I} + e_i^{(-)} f_-^{R,I}\right)\,\mathrm{e}^{-iEt+i\boldsymbol{p}\boldsymbol{x}}\,,$$

where

$$e_i^{(\pm)} = e_i^{(1)} \pm i\, e_i^{(2)}\,,$$

and the unit vectors $\boldsymbol{e}^{(1)}$, $\boldsymbol{e}^{(2)}$ form together with $\boldsymbol{e}^{(3)} \equiv \boldsymbol{p}/p$ a right-handed triad. Substituting these expressions into eqs. (F.6) and diagonalizing the resulting eigenvalue matrix we find the dispersion relations for the modes:

[25] The variation of (F.3) with respect to n_i yields the equation

$$\Delta n_i + \sqrt{C}\,\ddot{\hat{v}}_i^R = 0\,. \tag{F.5}$$

By combining this with (F.4) one could naively conclude that the velocity of the excitations described by $\hat{v}_i^R$ is equal to 1. This is true only at the zeroth order in C: equations (F.4), (F.5) are valid only up to $O(C^{3/2})$ corrections and hence do not allow to capture the $O(C)$ terms in the velocity, which we are interested in.

i) modes with $f^I_\pm = \pm i f^R_\pm \quad \Longrightarrow \quad E^2 = p^2$

ii) modes with $f^I_\pm = \mp i f^R_\pm \quad \Longrightarrow \quad E^2 = (1+C)\, p^2$

In the latter case the dispersion relation coincides with that of gravitons, see eq. (F.2), which identifies the corresponding modes as members of the gravitational supermultiplet. Note that these modes are an essential mixture of real and imaginary aether components with the admixture of metric perturbations, see eq. (F.4).

Let us comment on the consequences of SUSY breaking. As discussed in [33], it leads to the generation of mass for the imaginary part of the aether $\hat{\upsilon}^I_i$. Then the dispersion relation for the remaining component $\hat{\upsilon}^R_i$ is obtained from (F.6a) by simply dropping off the last term, which yields $E^2 = (1 + C/2)\, p^2$. We conclude that the SUSY breaking modifies the velocity of the helicity 1 modes, so that its deviation from unity is twice smaller than that for gravitons. This coincides with the result in the Einstein-aether model [49] for the choice of parameters (4.6).

Finally, we consider the helicity 0 sector. Here the Ansatz reads,

$$h_{00} = \mathcal{N}\,, \qquad h_{0i} = \partial_i \mathcal{B}\,, \qquad h_{ij} = \delta_{ij}\,\varphi + \partial_i\partial_j \mathcal{E}\,, \qquad \hat{\upsilon}^{R,I} = \partial_i \upsilon^{R,I}\,.$$

Substitution into the Lagrangian yields,

$$\begin{aligned}\mathcal{L}_{h=0} = \frac{1}{2\varkappa^2}\bigg[&\frac{3}{2}\varphi\ddot{\varphi} - \frac{1}{2}\varphi\Delta\varphi + \mathcal{N}\Delta\varphi - 2\varphi\Delta\dot{\mathcal{B}} + \varphi\Delta\ddot{\mathcal{E}} + \upsilon^R\Delta\ddot{\upsilon}^R - \upsilon^R\Delta^2\upsilon^R \\ &- \sqrt{C}\upsilon^R\Delta\dot{\varphi} - \sqrt{C}\upsilon^R\Delta\dot{\mathcal{N}} + \sqrt{C}\upsilon^R\Delta\ddot{\mathcal{B}} + \sqrt{C}\upsilon^R\Delta^2\mathcal{B} - \sqrt{C}\upsilon^R\Delta^2\dot{\mathcal{E}} \\ &+ \left(1-\frac{C}{2}\right)\upsilon^I\Delta\ddot{\upsilon}^I - \left(1-\frac{C}{2}\right)\upsilon^I\Delta^2\upsilon^I\bigg]\,,\end{aligned} \tag{F.7}$$

where we again omitted terms of order $O(C)$ in the purely gravitational part. We see right away that the mode υ^I decouples and has unit velocity. The mode associated with the real part of the aether requires a bit more work. Varying with respect to $\mathcal{N}$ and φ and then setting $\mathcal{N} = \mathcal{B} = 0$ by a gauge fixing we obtain the constraints,

$$\Delta\varphi = -\sqrt{C}\Delta\dot{\upsilon}^R\,, \qquad \Delta\ddot{\mathcal{E}} = -3\ddot{\varphi} + \Delta\varphi - \sqrt{C}\Delta\dot{\upsilon}^R\,.$$

Substituting the above expressions into the equation obtained by variation with respect to υ^R,

$$2\Delta\ddot{\upsilon}^R - 2\Delta^2\upsilon^R - \sqrt{C}\Delta\dot{\varphi} - \sqrt{C}\Delta^2\dot{\mathcal{E}} = 0\,,$$

we observe that υ^R satisfies the relativistic wave equation. Thus, this mode also has unit velocity. One can check that variation with respect to $\mathcal{B}$ and $\mathcal{E}$ does not give any new relations.

F.2 Fermionic modes

In the fermionic sector we find it more convenient to work directly with the equations of motion. From the Lagrangian (5.18) we have,

$$4\epsilon^{abcd}\bar{\sigma}_b\partial_c\psi_d - \sqrt{C}w^a\partial_b\bar{\tilde{\eta}}^b + \sqrt{C}w^b\partial_b\bar{\tilde{\eta}}^a = 0\,, \tag{F.8a}$$

$$-i\sigma^b\partial_b\bar{\tilde{\eta}}_a + 4\sqrt{C}w^b\partial_a\psi_b - 4\sqrt{C}w^b\partial_b\psi_a = 0\,. \tag{F.8b}$$

We again perform the $(3+1)$ split of all quantities into temporal and spatial components. Upon reduction to the spatial rotations, the dotted and undotted spinor indices can be identified because the fundamental representation of SU(2) is equivalent to its complex conjugate. More precisely, a spinor χ_α with lower index and its complex conjugate with upper index $\bar{\chi}^{\dot{\alpha}}$ transform in the same way and can be treated as just two-component columns. We impose the gauge $\psi_{0\alpha} = 0$ and recall that $\bar{\tilde{\eta}}_0^{\dot{\alpha}} = 0$ due to the orthogonality condition (3.2). Further, we use the explicit form of the σ-matrices [35], $\sigma_0 = \bar{\sigma}_0 = \mathbb{1}$, $\bar{\sigma}_i = -\sigma_i$, where σ_i, $i = 1, 2, 3$, are the usual Pauli matrices. Then eqs. (F.8) imply,

$$-4\epsilon_{ijk}\sigma_i\partial_j\psi_k - \sqrt{C}\partial_i\bar{\tilde{\eta}}_i = 0\,, \tag{F.9a}$$

$$-4\epsilon_{ijk}\sigma_j\dot{\psi}_k - 4\epsilon_{ijk}\partial_j\psi_k + \sqrt{C}\dot{\bar{\tilde{\eta}}}_i = 0\,, \tag{F.9b}$$

$$i\dot{\bar{\tilde{\eta}}}_i - i\sigma_j\partial_j\bar{\tilde{\eta}}_i - 4\sqrt{C}\dot{\psi}_i = 0\,, \tag{F.9c}$$

where all tensor indices run from 1 to 3. Note that the first equation (F.9a) does not contain time derivatives and thus represents a constraint reflecting the gauge invariance of the gravitino field.

Next step is to perform decomposition into plane waves. In addition, we decompose the fields in the basis of unit vectors $\boldsymbol{e}^{(r)}$, $r = 1, 2, 3$, that form a right-handed triad, $\boldsymbol{e}^{(3)}$ being aligned with the momentum (cf. section F.1). Thus we write,

$$\psi_i = \Big(\sum_r F_{(r)}\, e_i^{(r)}\Big) \mathrm{e}^{-iEt+i\boldsymbol{p}\boldsymbol{x}}\,, \qquad \bar{\tilde{\eta}}_i = \Big(\sum_r G_{(r)}\, e_i^{(r)}\Big) \mathrm{e}^{-iEt+i\boldsymbol{p}\boldsymbol{x}}\,, \tag{F.10}$$

where $F_{(r)}$, $G_{(r)}$ are spinor coefficients. Substituting into eqs. (F.9) we obtain,

$$4(\boldsymbol{\sigma}\boldsymbol{e}^{(1)})F_{(2)} - 4(\boldsymbol{\sigma}\boldsymbol{e}^{(2)})F_{(1)} - \sqrt{C}\,G_{(3)} = 0\,, \tag{F.11a}$$

$$4\big[-E(\boldsymbol{\sigma}\boldsymbol{e}^{(3)}) + p\big]F_{(2)} + 4E(\boldsymbol{\sigma}\boldsymbol{e}^{(2)})F_{(3)} - \sqrt{C}E\,G_{(1)} = 0\,, \tag{F.11b}$$

$$4\big[E(\boldsymbol{\sigma}\boldsymbol{e}^{(3)}) - p\big]F_{(1)} - 4E(\boldsymbol{\sigma}\boldsymbol{e}^{(1)})F_{(3)} - \sqrt{C}E\,G_{(2)} = 0\,, \tag{F.11c}$$

$$\big[E + (\boldsymbol{\sigma}\boldsymbol{p})\big]G_{(r)} + 4i\sqrt{C}E\,F_{(r)} = 0\,, \qquad r = 1, 2, 3\,. \tag{F.11d}$$

The system is simplified by introducing the linear combinations,

$$F_{(\pm)} = (\boldsymbol{\sigma}\boldsymbol{e}^{(2)})F_{(1)} \pm (\boldsymbol{\sigma}\boldsymbol{e}^{(1)})F_{(2)}\,, \qquad G_{(\pm)} = (\boldsymbol{\sigma}\boldsymbol{e}^{(1)})G_{(1)} \mp (\boldsymbol{\sigma}\boldsymbol{e}^{(2)})G_{(2)}\,. \tag{F.12}$$

Then the $(+)$ and $(-)$ modes decouple. In the $(+)$ sector we obtain,

$$4\big[E(\boldsymbol{\sigma}\boldsymbol{e}^{(3)}) + p\big]F_{(+)} - \sqrt{C}E\,G_{(+)} = 0\,, \tag{F.13a}$$

$$\big[E(\boldsymbol{\sigma}\boldsymbol{e}^{(3)}) - p\big]G_{(+)} - 4\sqrt{C}E\,F_{(+)} = 0\,. \tag{F.13b}$$

Mupliplying (F.13a) by the operator $\big[E(\boldsymbol{\sigma}\boldsymbol{e}^{(3)}) - p\big]$ and substituting $G_{(+)}$ from (F.13b) we arrive at a linear equation for $F_{(+)}$ which implies the dispersion relation $E^2 \approx (1+C)\,p^2$ coinciding with that of gravitons. One can show that the $(+)$ modes have helicities $\pm 3/2$. For a given momentum there are two linearly independent modes with positive energy and two modes with negative energy. Thus, in total we find four helicity $\pm 3/2$ states that belong to the graviton multiplet, as discussed in section 6.1.

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The equations in the $(-)$ sector read,

$$-4F_{(-)} - \sqrt{C}\,G_{(3)} = 0\,, \tag{F.14a}$$

$$-4[E(\boldsymbol{\sigma e}^{(3)}) + p]F_{(-)} + 8iE(\boldsymbol{\sigma e}^{(3)})F_{(3)} - \sqrt{C}E\,G_{(-)} = 0\,, \tag{F.14b}$$

$$[E(\boldsymbol{\sigma e}^{(3)}) - p]G_{(-)} - 4\sqrt{C}E\,F_{(-)} = 0\,, \tag{F.14c}$$

$$[E(\boldsymbol{\sigma e}^{(3)}) + p]G_{(3)} + 4i\sqrt{C}E(\boldsymbol{\sigma e}^{(3)})F_{(3)} = 0\,. \tag{F.14d}$$

Expressing $F_{(-)}$ and $(\boldsymbol{\sigma e}^{(3)})F_{(3)}$ from the first two equations and substituting the result back into the third and fourth ones we arrive at,

$$[E(\boldsymbol{\sigma e}^{(3)}) - p]G_{(-)} + CE\,G_{(3)} = 0\,,$$
$$(2-C)[E(\boldsymbol{\sigma e}^{(3)}) + p]G_{(3)} + CE\,G_{(-)} = 0\,.$$

We see that the mixing between $G_{(-)}$ and $G_{(3)}$ is of order $O(C)$. It contributes to the dispersion relations only at higher orders and can be neglected. Then we obtain two decoupled equations for the modes $G_{(-)}$ and $G_{(3)}$ leading to relativistic dispersion relations $E^2 = p^2$. There are two linearly independent modes with positive energy and two modes with negative energy. In total, they comprise four helicity $\pm 1/2$ states that complement the fermionic sector of the theory.

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