Probing anomalous $\gamma\gamma\gamma Z$ couplings through γZ production in $\gamma\gamma$ collisions at the CLIC

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ABSTRACT: We have estimated the sensitivity to the anomalous couplings of the $\gamma\gamma\gamma Z$ vertex in the $\gamma\gamma \rightarrow \gamma Z$ scattering of the Compton backscattered photons at the CLIC. Both polarized and unpolarized collisions at the e^+e^- energies 1500 GeV and 3000 GeV are addressed, and anomalous contributions to helicity amplitudes are derived. The differential and total cross sections are calculated. We have obtained 95% C.L. exclusion limits on the anomalous quartic gauge couplings (QGCs). They are compared with corresponding bounds derived for the $\gamma\gamma\gamma Z$ couplings via γZ production at the LHC. The constraints on the anomalous QGCs are one to two orders of magnitude more stringent than at the HL-LHC. The partial-wave unitarity constraints on the anomalous couplings are examined. It is shown that the unitarity is not violated in the region of the anomalous QGCs studied in the paper.

KEYWORDS: Phenomenological Models

ARXIV EPRINT: [2108.04478](https://arxiv.org/abs/2108.04478)

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1 Introduction

In our previous paper [1] we probed the anomalous quartic gauge couplings (QGCs) in the $\gamma\gamma \rightarrow \gamma\gamma$ process at the Compact Linear Collider (CLIC) [2, 3]. Both the unpolarized and polarized light-by-light scatterings were considered, and the bounds on QGCs were obtained. The neutral anomalous quartic couplings are of particular interest. The anomaly interactions γZZZ , $\gamma\gamma ZZ$, and $\gamma\gamma\gamma Z$ at the LHC were analyzed in [4]–[9]. The LHC experimental bounds on QGCs were presented by the CMS [10] and ATLAS [11] collaborations (see also [14]). The bounds on the anomalous $\gamma\gamma\gamma Z$ vertex can be also derived from the constraints on the $\mathcal{B}(Z \rightarrow \gamma\gamma\gamma)$ branching ratio obtained at the LEP [12] and LHC [13]. As for e^+e^- colliders, they may operate in $e\gamma$ and $\gamma\gamma$ modes [15, 16]. The bounds on QGCs in e^+e^- , $e\gamma$ and $\gamma\gamma$ collisions were given in [17]–[22]. In particular, the limits on the quartic couplings for the vertex $\gamma\gamma\gamma Z$ were derived in [23] using LEP 2 data for the reactions $e^+e^- \rightarrow \gamma\gamma\gamma, \gamma\gamma Z$. A similar analysis for the exclusive γZ production with intact protons at the LHC was done in [24]. The search for virtual SUSY effects in the process $\gamma\gamma \rightarrow \gamma Z$ at high energies was presented in [25].

As one can see, the anomalous $\gamma\gamma\gamma Z$ vertex urgently needs to be examined in high energy e^+e^- collisions. That is why, in the present paper we study the process (see figure 1)

$$\gamma(p_1, \mu) + \gamma(p_2, \nu) \rightarrow \gamma(p_3, \rho) + Z(p_4, \alpha), \quad (1.1)$$

where p_1, p_2, p_3, p_4 are boson momenta, μ, ν, ρ, α are boson Lorentz indices, and ingoing particles are real polarized photons generated at the CLIC by the laser Compton backscattering [26]–[29]. Our main goal is to derive bounds on anomaly couplings for the vertex

Figure 1. The process $\gamma + \gamma \rightarrow \gamma + Z$.

$\gamma\gamma\gamma Z$ which can be reached at the CLIC using both polarized and unpolarized photon beams. The great potential of the CLIC in probing new physics is well-known [30]–[32]. Let us underline that a physical potential of a linear high energy e^+e^- collider may be significantly enhanced, provided the polarized beams are used [33, 34].

Let λ_e be the helicity of the initial electron beam, while λ_0 be the helicity of the ingoing laser photon beam. In our calculations, we will consider two sets of these helicities, with opposite sign of λ_e ,

$$\begin{aligned} (\lambda_e^{(1)}, \lambda_0^{(1)}; \lambda_e^{(2)}, \lambda_0^{(2)}) &= (0.8, 1; 0.8, 1), \\ (\lambda_e^{(1)}, \lambda_0^{(1)}; \lambda_e^{(2)}, \lambda_0^{(2)}) &= (-0.8, 1; -0.8, 1), \end{aligned} \quad (1.2)$$

where the superscripts 1 and 2 enumerate the beams. We will work in the effective field theory framework. Previously effective Lagrangians were used in [35]–[38] for examining the $\gamma\gamma\gamma Z$ interaction in the $Z \rightarrow \gamma\gamma\gamma$ decay, as well as in [39], [23], and [24]. Anomalous quartic gauge couplings (QGCs) are induced at the dimension-six level already. However, they are not independent of anomalous trilinear gauge couplings. That is why, in our paper, we study anomalous QGCs which enter the effective Lagrangian at dimension-eight without contributing to anomalous trilinear gauge interactions.

The paper is organized as follows. In the next section, the effective Lagrangian is described, and Feynman rules for the anomalous $\gamma\gamma\gamma Z$ vertex are presented. The helicity amplitudes are studied in section 3. In section 4, both differential and total cross sections for the process (1.1) are calculated, and bounds on the QGCs are given. In section 5, unitarity constraints on anomalous quartic couplings are obtained. In appendix A, polarization tensors for the vertex $\gamma\gamma\gamma Z$ are listed. The explicit expressions for the anomalous contributions to the helicity amplitudes are given in appendix B. Some formulas for Wigner’s d -function are collected in appendix C. Finally, in section 6, we summarize our results and give conclusions.

2 Effective Lagrangian

It is appropriate to describe the anomalous $\gamma\gamma\gamma Z$ interaction by means of an effective Lagrangian. Given parity is conserved and gauge invariance is valid, there are only two independent operators with dimension 8. Following [35, 36], we take the Lagrangian

$$\mathcal{L}_{\gamma\gamma\gamma Z} = g_1 O_1 + g_2 O_2, \quad (2.1)$$

with the operators

$$O_1 = F^{\rho\mu} F^{\alpha\nu} \partial_\rho F_{\mu\nu} Z_\alpha, \quad O_2 = F^{\rho\mu} F_\mu^\nu \partial_\rho F_{\alpha\nu} Z^\alpha, \quad (2.2)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The operators $O_{1,2}$ arise from a $SU(2) \times U(1)_Y$ effective Lagrangian with two operators like $B_{\mu\nu} B^{\nu\nu} B_{\rho\sigma} B^{\rho\sigma}$, four operators like $W_{\mu\nu} W^{\nu\nu} W_{\rho\sigma} W^{\rho\sigma}$, and four operators like $B_{\mu\nu} B^{\nu\nu} W_{\rho\sigma} W^{\rho\sigma}$, where W_μ and B_μ are the $SU(2)$ and hypercharge gauge fields, respectively [40]. We consider only CP conserving operators hence the dual field strength tensors $\tilde{W}_{\mu\nu}$ and $\tilde{B}_{\mu\nu}$ are not used. The coupling $g_{1,2}$ are linear combinations of ten coefficients of dimension-eight operators mentioned above. Note that certain combinations of these coefficients must obey so-called positivity constraints [41]–[43].

As one can see, this Lagrangian contains no derivatives of the Z boson field (correspondingly, no p_4 in the momentum space), that simplifies a derivation of Feynman rules for the $\gamma\gamma\gamma Z$ vertex.

Some authors use the Lagrangian [38]

$$\mathcal{L}_{\gamma\gamma\gamma Z}^{(N)} = G_1 \bar{O}_1 + G_2 \bar{O}_2, \quad (2.3)$$

with the operators

$$\bar{O}_1 = F^{\mu\nu} F_{\mu\nu} F^{\rho\sigma} Z_{\rho\sigma}, \quad \bar{O}_2 = F^{\mu\nu} F_{\nu\rho} F^{\rho\sigma} Z_{\sigma\mu}, \quad (2.4)$$

where $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, or the Lagrangian [24]

$$\mathcal{L}_{\gamma\gamma\gamma Z}^{(B)} = \zeta O + \tilde{\zeta} \tilde{O}, \quad (2.5)$$

with the operators

$$O = F^{\mu\nu} F_{\mu\nu} F^{\rho\sigma} Z_{\rho\sigma}, \quad \tilde{O} = F^{\mu\nu} \tilde{F}_{\mu\nu} F^{\rho\sigma} \tilde{Z}_{\rho\sigma}, \quad (2.6)$$

where $\tilde{F}_{\mu\nu} = (1/2) \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$, and $\tilde{Z}_{\mu\nu} = (1/2) \epsilon^{\mu\nu\rho\sigma} Z_{\rho\sigma}$.

Using integration by parts and equations of motion, one can easily obtain the following relations between two bases for the effective Lagrangian [38],

$$\bar{O}_1 = -8O_1, \quad \bar{O}_2 = 2(O_2 - O_1). \quad (2.7)$$

We also have the relations [24]

$$O = \bar{O}_1 = -8O_1, \quad \tilde{O} = 4\bar{O}_2 - 2\bar{O}_1 = 8(O_2 + O_1). \quad (2.8)$$

The above listed equations enable us to relate anomalous coupling in eqs. (2.1), (2.3), and (2.5). In particular, we find

$$g_1 = 8(\tilde{\zeta} - \zeta), \quad g_2 = 8\tilde{\zeta}. \quad (2.9)$$

The Feynman rules for the effective anomalous vertex, resulting from the Lagrangian (2.1), are given by [36]

$$\begin{aligned}
 P^{\mu\nu\rho\alpha} = \mathcal{P}\{ & g_1[(p_1 \cdot p_2)(p_2 \cdot p_3)g^{\mu\nu}g^{\rho\alpha} - (p_1 \cdot p_3)p_2^\mu p_1^\nu g^{\rho\alpha} \\
 & - (p_1 \cdot p_3)p_1^\nu p_2^\alpha g^{\mu\rho} + p_2^\mu p_1^\nu p_1^\rho p_3^\alpha] \\
 & + g_2[-(p_1 \cdot p_2)(p_1 \cdot p_3)g^{\mu\alpha}g^{\nu\rho} + (p_2 \cdot p_3)p_1^\nu p_1^\alpha g^{\mu\rho} \\
 & - (p_2 \cdot p_3)p_1^\nu p_1^\rho g^{\mu\alpha} + (p_2 \cdot p_3)p_1^\nu p_2^\alpha g^{\mu\rho} + 2(p_2 \cdot p_3)p_2^\mu p_1^\rho g^{\nu\alpha} \\
 & - (p_1 \cdot p_3)p_2^\rho p_1^\alpha g^{\mu\nu} + p_3^\mu p_1^\nu p_2^\rho p_1^\alpha]\}, \quad (2.10)
 \end{aligned}$$

where $\mathcal{P}$ denotes possible permutations $(p_1, \mu) \leftrightarrow (p_2, \nu) \leftrightarrow (p_3, \rho)$, and all momenta in the $\gamma\gamma\gamma Z$ vertex are assumed to be incoming ones. Correspondingly, the polarization tensor is equal to

$$P_{\mu\nu\rho\alpha}(p_1, p_2, p_3) = g_1 \sum_{i=1}^4 P_{\mu\nu\rho\alpha}^{(1,i)}(p_1, p_2, p_3) + g_2 \sum_{i=1}^7 P_{\mu\nu\rho\alpha}^{(2,i)}(p_1, p_2, p_3). \quad (2.11)$$

Electromagnetic gauge invariance results in equations $p_1^\mu P_{\mu\nu\rho\alpha} = p_2^\nu P_{\mu\nu\rho\alpha} = p_3^\rho P_{\mu\nu\rho\alpha} = 0$. Note that terms proportional to $p_1^\mu, p_2^\nu, p_3^\rho$ are omitted in (2.10), since they do not contribute to the matrix element, see eq. (3.5) below. Explicit expressions for the tensors $P_{\mu\nu\rho\alpha}^{(1,i)}$ and $P_{\mu\nu\rho\alpha}^{(2,i)}$ are presented in appendix A. To calculate helicity amplitudes for the process (1.1), one has to make the replacement $p_3 \rightarrow -p_3$ in the Feynman rules for the $\gamma\gamma\gamma Z$ vertex given by eqs. (2.10), (2.11), and (A.1)–(A.11).

3 Helicity amplitudes

We work in the c.m.s. of the colliding real photons, $\vec{p}_1 + \vec{p}_2 = 0$, where the momenta are given by

$$\begin{aligned}
 p_1^\mu &= (p, 0, 0, p), \\
 p_2^\mu &= (p, 0, 0, -p), \\
 p_3^\mu &= (k, 0, k \sin \theta, k \cos \theta), \\
 p_4^\mu &= (E, 0, -k \sin \theta, -k \cos \theta). \quad (3.1)
 \end{aligned}$$

Here $E = \sqrt{k^2 + m_Z^2}$, with m_Z being the mass of the Z boson. The Mandelstam variables of the process (1.1) are

$$\begin{aligned}
 s &= (p_1 + p_2)^2 = 4p^2, \\
 t &= (p_1 - p_3)^2 = -2pk(1 - \cos \theta), \\
 u &= (p_2 - p_3)^2 = -2pk(1 + \cos \theta), \quad (3.2)
 \end{aligned}$$

where θ is a scattering angle in the c.m.s. Note that $s + t + u = m_Z^2$.

In the chosen system the polarization vectors are equal to

$$\begin{aligned}
 \varepsilon_\mu^+(p_1) &= \varepsilon_\mu^-(p_2) = \frac{1}{\sqrt{2}}(0, 1, i, 0), \\
 \varepsilon_\mu^-(p_1) &= \varepsilon_\mu^+(p_2) = \frac{1}{\sqrt{2}}(0, 1, -i, 0), \\
 \varepsilon_\mu^+(p_3) &= \varepsilon_\mu^-(p_4) = \frac{1}{\sqrt{2}}(0, 1, i \cos \theta, -i \sin \theta), \\
 \varepsilon_\mu^-(p_3) &= \varepsilon_\mu^+(p_4) = \frac{1}{\sqrt{2}}(0, 1, -i \cos \theta, i \sin \theta), \\
 \varepsilon_\mu^0(p_4) &= \frac{1}{m_Z}(k, 0, -E \sin \theta, -E \cos \theta).
 \end{aligned} \tag{3.3}$$

They obey the orthogonality condition $\varepsilon_\mu^\lambda(k)k^\mu = 0$. Correspondingly, we get the helicity vectors of the final photon and Z boson,

$$\begin{aligned}
 \varepsilon_\mu^{*+}(p_3) &= \varepsilon_\mu^{*-}(p_4) = \frac{1}{\sqrt{2}}(0, 1, -i \cos \theta, i \sin \theta), \\
 \varepsilon_\mu^{*-}(p_3) &= \varepsilon_\mu^{*+}(p_4) = \frac{1}{\sqrt{2}}(0, 1, i \cos \theta, -i \sin \theta), \\
 \varepsilon_\mu^{*0}(p_4) &= \varepsilon_\mu^0(p_4) = \frac{1}{m_Z}(k, 0, -E \sin \theta, -E \cos \theta).
 \end{aligned} \tag{3.4}$$

The matrix element of the process (1.1) with the definite helicities of the incoming and outgoing bosons can be written as

$$M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(p_1, p_2, p_3) = P_{\mu\nu\rho\alpha}(p_1, p_2, p_3) \varepsilon_\mu^{\lambda_1}(p_1) \varepsilon_\nu^{\lambda_2}(p_2) \varepsilon_\rho^{*\lambda_3}(p_3) \varepsilon_\alpha^{*\lambda_4}(p_4), \tag{3.5}$$

where the polarization tensor $P_{\mu\nu\rho\alpha}$ is given by eq. (2.11). We have calculated the anomalous helicity amplitudes, and present their explicit expressions in appendix B. Using these expressions, we obtain the unpolarized amplitude squared

$$\sum_{\lambda_1 \dots \lambda_4} |M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 = \frac{1}{4} [g_1^2(3A + 2B) - 4g_1 g_2(A + B) + 4g_2^2(A + B)], \tag{3.6}$$

where

$$A = s^2 t^2 + t^2 u^2 + u^2 s^2, \quad B = stu m_Z^2. \tag{3.7}$$

With a help of relations (2.9), we get from (3.6) the differential cross section

$$\begin{aligned}
 \frac{d\sigma_{\gamma\gamma \rightarrow \gamma Z}}{d\Omega} &= \frac{\beta}{64\pi^2 s} \frac{1}{4} \sum_{\lambda_1 \dots \lambda_4} |M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 \\
 &= \frac{\beta}{16\pi^2 s} [(3\zeta^2 + 3\tilde{\zeta}^2 - 2\zeta\tilde{\zeta})A + 2(\zeta^2 + \tilde{\zeta}^2)B],
 \end{aligned} \tag{3.8}$$

where $\beta = 1 - m_Z^2/s$, in a full agreement with eq. (2.3) in [24].

To estimate a SM background, we take analytical expressions for the SM helicity amplitudes from appendix A in [25]. Both W boson loops [44–48] and charged

fermion loops [44–46, 49, 50] contribute to these amplitudes. As shown in [25], for $s > (250 \text{ GeV})^2$ the dominant SM amplitudes $A_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$ are the W -loop non-flip amplitudes $A_{++++}^W(s, t, u)$ and $A_{+-+-}^W(s, t, u) = A_{-+ -+}^W(s, u, t)$. Almost negligible are $A_{++00}^W(s, t, u)$ and $A_{+-+0}^W(s, t, u) = A_{+0-+}^W(s, u, t)$. The rest are even smaller. The fermion-loop amplitudes are comparable only to very small W -loop amplitudes [25]. Similar properties of the SM helicity amplitudes are also valid for the process $\gamma\gamma \rightarrow \gamma\gamma$ [51, 52].

Another possible background comes from the SM process $\gamma\gamma \rightarrow \gamma l^+ l^-$ where the invariant mass of the lepton pair, $m_{l^+ l^-}$, is close to the Z boson mass m_Z . We have obtained the cross section of the process to be of order 10^{-3} fb for $|m_{l^+ l^-} - m_Z| < 10 \text{ GeV}$. So, this background can be safely ignored.

4 Numerical results

The differential cross section of the process $\gamma\gamma \rightarrow \gamma Z$ depends on spectra of the Compton backscattered (CB) photons $f_{\gamma/e}(x_i)$, their helicities $\xi(E_\gamma^{(i)}, \lambda_0)$ ($i = 1, 2$), and helicity amplitudes [1, 20],

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} = & \frac{\beta}{128\pi s} \int_{x_{1\min}}^{x_{1\max}} \frac{dx_1}{x_1} f_{\gamma/e}(x_1) \int_{x_{2\min}}^{x_{2\max}} \frac{dx_2}{x_2} f_{\gamma/e}(x_2) \\ & \times \left\{ \left[1 + \xi(E_\gamma^{(1)}, \lambda_0^{(1)}) \right] \left[1 + \xi(E_\gamma^{(2)}, \lambda_0^{(2)}) \right] \sum_{\lambda_3 \lambda_4} |M_{++\lambda_3 \lambda_4}|^2 \right. \\ & + \left[1 + \xi(E_\gamma^{(1)}, \lambda_0^{(1)}) \right] \left[1 - \xi(E_\gamma^{(2)}, \lambda_0^{(2)}) \right] \sum_{\lambda_3 \lambda_4} |M_{+-\lambda_3 \lambda_4}|^2 \\ & + \left[1 - \xi(E_\gamma^{(1)}, \lambda_0^{(1)}) \right] \left[1 + \xi(E_\gamma^{(2)}, \lambda_0^{(2)}) \right] \sum_{\lambda_3 \lambda_4} |M_{-+\lambda_3 \lambda_4}|^2 \\ & \left. + \left[1 - \xi(E_\gamma^{(1)}, \lambda_0^{(1)}) \right] \left[1 - \xi(E_\gamma^{(2)}, \lambda_0^{(2)}) \right] \sum_{\lambda_3 \lambda_4} |M_{--\lambda_3 \lambda_4}|^2 \right\}, \quad (4.1) \end{aligned}$$

where $\lambda_3 = +, -, \lambda_4 = +, -, 0$, $x_1 = E_\gamma^{(1)}/E_e$ and $x_2 = E_\gamma^{(2)}/E_e$ are the energy fractions of the CB photon beams, $x_{1\min} = p_\perp^2/E_e^2$, $x_{2\min} = p_\perp^2/(x_1 E_e^2)$, and $p_\perp$ is the transverse momentum of the outgoing particles. Note that $\sqrt{s x_1 x_2}$ is the invariant energy of the backscattered photons. The explicit expressions for $f_{\gamma/e}(x_i)$ and $\xi(E_\gamma^{(i)}, \lambda_0)$ can be found in [1].

The differential cross sections are shown in figures 2, 3 as functions of the invariant mass of the γZ system. We have imposed the cut on the rapidity of the final bosons, $|\eta| < 2.5$, and considered the region $m_{\gamma Z} > 250 \text{ GeV}$. As one can see, the anomalous cross sections dominate the SM one for $m_{\gamma Z} > 600 \text{ GeV}$. The effect is more pronounced for the collision energy $\sqrt{s} = 3000 \text{ GeV}$, especially as $m_{\gamma Z}$ grows. Note that for $\sqrt{s} = 3000 \text{ GeV}$ the differential cross sections depend weakly on electron beam helicity λ_e . In figures 4, 5 the total cross sections are presented depending on $m_{\gamma Z, \min}$, minimal invariant mass of two outgoing bosons. The anomalous contribution dominates both the interference one and

Figure 2. The differential cross sections for the process $\gamma\gamma \rightarrow \gamma Z$ as functions of the invariant mass of the outgoing bosons for the CLIC energy $\sqrt{s} = 1500$ GeV. The left, middle and right panels correspond to the electron beam helicities $\lambda_e = 0.8, -0.8$, and 0 , respectively. On each plot the curves denote (from the top downwards) the differential cross sections for the couplings $g_1 = 10^{-12} \text{ GeV}^{-4}$, $g_2 = 0$, and $g_1 = 0$, $g_2 = 10^{-12} \text{ GeV}^{-4}$, the anomalous contributions for the same values of couplings, the SM cross section.

Figure 3. The same as in figure 2, but for the e^+e^- collider energy $\sqrt{s} = 3000$ GeV, and coupling sets $g_1 = 10^{-13} \text{ GeV}^{-4}$, $g_2 = 0$, and $g_1 = 0$, $g_2 = 10^{-13} \text{ GeV}^{-4}$.

Figure 4. The total cross sections for the process $\gamma\gamma \rightarrow \gamma Z$ as functions of the minimal invariant mass of the outgoing bosons for the e^+e^- collider energy $\sqrt{s} = 1500$ GeV. The left, middle and right panels correspond to the electron beam helicities $\lambda_e = 0.8, -0.8$, and 0 , respectively. On each plot the curves denote (from the top downwards) the total cross sections for the couplings $g_1 = 10^{-12} \text{ GeV}^{-4}$, $g_2 = 0$, and $g_1 = 0$, $g_2 = 10^{-12} \text{ GeV}^{-4}$, the anomalous contributions for the same values of couplings, the SM cross section.

SM cross section. The ratio of the total cross section to the SM one grows with an increase of $m_{\gamma Z}$, being more than one order of magnitude at large $m_{\gamma Z}$.

The knowledge of the total cross sections and planned CLIC integrated luminosities [34] enables us to calculate the exclusion regions for the QGCs. In our study we consider leptonic (electrons and muons) decays of the Z boson. Let $s(b)$ be the total number of signal (background) events, and δ the percentage systematic error. The number of events is defined as $\sigma \times L \times \mathcal{B}(Z \rightarrow e, \mu)$. The exclusion significance is given by [53]

$$S_{\text{excl}} = \sqrt{2 \left[s - b \ln \left(\frac{b + s + x}{2b} \right) - \frac{1}{\delta^2} \ln \left(\frac{b - s + x}{2b} \right) - (b + s - x) \left(1 + \frac{1}{\delta^2 b} \right) \right]}, \quad (4.2)$$

where

$$x = \sqrt{(s + b)^2 - 4\delta^2 s b^2 / (1 + \delta^2 b)}. \quad (4.3)$$

We define the regions $S_{\text{excl}} \leq 1.645$ as a regions that can be excluded at the 95% C.L. in the process $\gamma\gamma \rightarrow \gamma Z$ at the CLIC. To reduce the SM background, we impose the cut $m_{\gamma Z} > 1000$ GeV, in addition to the bound $|\eta| < 2.5$. The expected integrated luminosity at the CLIC can be found, for instance, in [34].

It is worth considering the unpolarized case first. One can obtain from eq. (3.6) that the anomalous contribution to the unpolarized total cross section is proportional to the coupling combination $3g_1^2 - 4g_1g_2 + 4g_2^2$, provided terms proportional to $m_Z^2/s \ll 1$ are

Figure 5. The same as in figure 4, but for the e^+e^- collider energy $\sqrt{s} = 3000$ GeV, and coupling sets $g_1 = 10^{-13} \text{ GeV}^{-4}$, $g_2 = 0$, and $g_1 = 0$, $g_2 = 10^{-13} \text{ GeV}^{-4}$.

λ_e		0	-0.8	0.8
$L, \text{ fb}^{-1}$		2500	2000	500
$ g_1 , \text{ GeV}^{-4}$ ($g_2 = 0$)	$\delta = 0\%$	4.19×10^{-14}	6.25×10^{-14}	4.42×10^{-14}
	$\delta = 5\%$	5.32×10^{-14}	7.91×10^{-14}	5.38×10^{-14}
	$\delta = 10\%$	6.81×10^{-14}	1.02×10^{-13}	6.78×10^{-14}
$ g_2 , \text{ GeV}^{-4}$ ($g_1 = 0$)	$\delta = 0\%$	3.61×10^{-14}	5.47×10^{-14}	4.53×10^{-14}
	$\delta = 5\%$	4.63×10^{-14}	6.94×10^{-14}	5.51×10^{-14}
	$\delta = 10\%$	5.91×10^{-14}	8.87×10^{-14}	6.94×10^{-14}

Table 1. The 95% C.L. exclusion limits on the anomalous quartic couplings g_1 and g_2 for the collision energy $\sqrt{s} = 1500$ GeV, and the cut $m_{\gamma Z} > 1000$ GeV.

neglected in it. In such a case, the exclusion regions are ellipses in the plane $(g_1 - g_2)$ rotated clockwise through the angle $0.5 \arctan 8 \simeq 41.4^\circ$ around the origin. It is clear that our process is slightly more sensitive to the coupling g_2 rather than to g_1 . Our 95% C.L. exclusion regions for anomalous QGCs for the unpolarized process $\gamma\gamma \rightarrow \gamma Z$ at the CLIC are shown in figures 6, 7. The results are presented for $\delta = 0$, $\delta = 5\%$, and $\delta = 10\%$.

In tables 1, 2 we show the exclusion bounds on the couplings g_1 and g_2 for three values of the electron beam helicity λ_e and corresponding integrated luminosity L . Let us underline that this time we did not neglect the terms proportional to m_Z^2 , both for unpolarized and for polarized reactions. As one can see, the best bound on the couplings $g_{1,2}$ is approximately $5 \times 10^{-15} \text{ GeV}^{-4}$ for the e^+e^- energy $\sqrt{s} = 3000$ GeV and electron beam helicity $\lambda_e = 0.8$.

Figure 6. The 95% C.L. exclusion regions for the couplings g_1, g_2 in the unpolarized reaction $\gamma\gamma \rightarrow \gamma Z$ at the CLIC with the systematic errors $\delta = 0\%$ (black ellipse), $\delta = 5\%$ (blue ellipse), and $\delta = 10\%$ (red ellipse). The inner regions of the ellipses are inaccessible. The collision energy is $\sqrt{s} = 1500$ GeV, the integrated luminosity is $L = 2500 \text{ fb}^{-1}$. The cut on the outgoing photon invariant mass $m_{\gamma\gamma} > 1000$ GeV was imposed.

Figure 7. The same as in figure 6, but for $\sqrt{s} = 3000$ GeV and $L = 5000 \text{ fb}^{-1}$.

λ_e		0	-0.8	0.8
L, fb^{-1}		5000	4000	1000
$ g_1 , \text{GeV}^{-4}$ ($g_2 = 0$)	$\delta = 0\%$	5.98×10^{-15}	7.14×10^{-15}	5.13×10^{-15}
	$\delta = 5\%$	1.33×10^{-14}	1.73×10^{-14}	7.79×10^{-15}
	$\delta = 10\%$	1.85×10^{-14}	2.39×10^{-14}	1.04×10^{-14}
$ g_2 , \text{GeV}^{-4}$ ($g_1 = 0$)	$\delta = 0\%$	5.18×10^{-15}	6.62×10^{-15}	5.19×10^{-15}
	$\delta = 5\%$	1.16×10^{-14}	1.60×10^{-14}	7.87×10^{-15}
	$\delta = 10\%$	1.62×10^{-14}	2.21×10^{-14}	1.05×10^{-14}

Table 2. The same as in table 1, but for the energy $\sqrt{s} = 3000 \text{ GeV}$ and different values of the integrated luminosities.

Recently, the bounds on the anomalous quartic couplings for the vertex $\gamma\gamma\gamma Z$ were obtained via γZ production with intact protons in the forward region at the LHC [24]. To examine this process, the effective Lagrangian (2.5) was used with the anomalous couplings $\zeta, \tilde{\zeta}$. Both for integrated luminosity 300 fb^{-1} and high luminosity 3000 fb^{-1} sensitivities were found to be similar, $\zeta, \tilde{\zeta} \sim 1 \times 10^{-13}$ at the 95% C.L. Taking into account the relations between couplings $\zeta, \tilde{\zeta}$ and our couplings g_1, g_2 , (2.9), we expect that the sensitivities of $g_1, g_2 \sim 8 \times 10^{-13}$ can be reached at the LHC (HL-LHC). These values should be compared with the CLIC bounds in tables 1 and 2. Note that the expected sensitivity from the $Z \rightarrow \gamma\gamma\gamma$ decay search at the LHC [13] is approximately three orders of magnitude smaller than that obtained in [24].

5 Unitarity constraints on anomalous quartic couplings

The anomalous contribution to the total cross section rises as s^3 . Thus, the contribution of the effective operators in (2.1) may lead to unitarity violation at high energies. That is why we need to study bounds imposed by partial-wave unitarity. The partial-wave expansion of the helicity amplitude in the center-of-mass system was derived in [54] and used in a number of papers [55–59]. It looks like

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, \theta, \varphi) = 16\pi \sum_J (2J+1) \sqrt{(1+\delta_{\lambda_1\lambda_2})(1+\delta_{\lambda_3\lambda_4})} \times e^{i(\lambda-\mu)\phi} d_{\lambda\mu}^J(\theta) T_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s), \quad (5.1)$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$, $\theta(\phi)$ is the polar (azimuth) scattering angle, and $d_{\lambda\mu}^J(\theta)$ is the Wigner (small) d -function [60]. Relevant formulas for the d -functions are given in appendix C. In our case λ, μ are even numbers, $\lambda, \mu = 0, \pm 2$ (see below). If we choose the plane $(x-z)$ as a scattering plane, then $\phi = 0$ in (5.1). Parity conservation means that

$$T_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s) = (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4} T_{-\lambda_1-\lambda_2-\lambda_3-\lambda_4}^J(s). \quad (5.2)$$

Partial-wave unitarity in the limit $s \gg (m_1 + m_2)^2$ requires that

$$|T_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s)| \leq 1. \quad (5.3)$$

Using orthogonality of the d -functions (C.2), we find the partial-wave amplitude

$$T_{\lambda_1\lambda_2\lambda_3\lambda_4}^J(s) = \frac{1}{32\pi} \frac{1}{\sqrt{(1+\delta_{\lambda_1\lambda_2})(1+\delta_{\lambda_3\lambda_4})}} \int_{-1}^1 M_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, z) d_{\lambda\mu}^J(z) dz. \quad (5.4)$$

Here and in what follows, $z = \cos\theta$. Note that $M_{\lambda_1\lambda_2\lambda_3\lambda_4} = g_1 M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1)} + g_2 M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(2)}$, and the helicity amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1,2)}$ are given in appendix B.

5.1 Unitarity bounds on coupling g_1 ($g_2 = 0$)

To obtain a unitarity bound on the coupling g_1 , we put $g_2 = 0$. Let us note that due to eq. (5.2), it is sufficient to examine the helicity amplitudes with $\lambda_1 = +1$ only. Moreover, it is enough to consider four amplitudes, $M_{++++}^{(1)}$, $M_{+--+}^{(1)}$, $M_{+--+}^{(1)}$, and $M_{++--}^{(1)}$, since the rest are suppressed by small factor $m_Z/\sqrt{s}$ or zero.

1. $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$, then $\lambda = \mu = 0$. The helicity amplitude is given by the first of equations (B.2),

$$M_{++++}(s, z) = g_1 M_{++++}^{(1)}(s) = -\frac{g_1}{4} s(s - m_Z^2). \quad (5.5)$$

Using eqs. (C.7)–(C.8), we find that the partial-wave amplitude with $J = 0$ is the only non-zero amplitude,

$$T_{++++}^0(s) = -\frac{g_1}{128\pi} s(s - m_Z^2) \int_{-1}^1 d_{00}^0(z) dz = -\frac{g_1}{64\pi} s(s - m_Z^2). \quad (5.6)$$

Correspondingly, we obtain from (5.3), (5.6)

$$|g_1| \leq 64\pi [s(s - m_Z^2)]^{-1}. \quad (5.7)$$

2. $\lambda_1 = -\lambda_2 = \lambda_3 = -\lambda_4 = 1$, then $\lambda = \mu = 2$. According to (B.2),

$$M_{+--+}(s, z) = g_1 M_{+--+}^{(1)}(s, z) = -\frac{g_1}{4} \frac{s^3}{s - m_Z^2} \left(\frac{1+z}{2}\right)^2. \quad (5.8)$$

It follows from (5.4), (C.5) that

$$\begin{aligned} T_{+--+}^J(s) &= -\frac{g_1}{128\pi} \frac{s^3}{s - m_Z^2} \int_{-1}^1 \left(\frac{1+z}{2}\right)^2 d_{22}^J(z) dz = -\frac{g_1}{128\pi} \frac{s^3}{s - m_Z^2} \\ &\quad \times \int_{-1}^1 \left(\frac{1+z}{2}\right)^4 {}_2F_1\left(2 - J, J + 3; 1; \frac{1-z}{2}\right) dz. \end{aligned} \quad (5.9)$$

Let $(1 - z)/2 = x$, then $(1 + z)/2 = 1 - x$, and we find

$$\begin{aligned} T_{+--+}^J(s) &= -\frac{1}{64\pi} \frac{g_1 s^3}{s - m_Z^2} \int_0^1 (1 - x)^4 {}_2F_1(2 - J, J + 3; 1; x) dx \\ &= -\frac{3g_1}{8\pi\Gamma(4 + J)\Gamma(3 - J)} \frac{s^3}{s - m_Z^2}, \end{aligned} \quad (5.10)$$

where we used formulas 2.21.1.5 and 2.21.1.6 in [61]. Thus, only three partial-waves amplitudes, $T_{+-+}^0(s)$, $T_{+-+}^1(s)$, and $T_{+-+}^2(s)$, are non-zero. The most important for us is $T_{+-+}^0(s)$, since it results in the strongest constraint on the coupling g_1 ,

$$|g_1| \leq 32\pi(s - m_Z^2)s^{-3}. \quad (5.11)$$

3. $\lambda_1 = -\lambda_2 = -\lambda_3 = \lambda_4 = 1$, then $\lambda = 2$, $\mu = -2$, and we have

$$M_{+--+}(s, z) = g_1 M_{+--+}^{(1)}(s, z) = -\frac{g_1}{4} \frac{s^3}{s - m_Z^2} \left(\frac{1-z}{2} \right)^2. \quad (5.12)$$

Using eq. (C.6) and first relation in (C.4), after substitutions $(1+z)/2 = x$, $(1-z)/2 = 1-x$, we reduce this case to the previous one. As a result, we come again to the upper bound (5.11).

4. $\lambda_1 = \lambda_2 = -\lambda_3 = -\lambda_4 = 1$, then $\lambda = \mu = 0$, and

$$M_{++--}(s, z) = g_1 M_{++--}^{(1)}(s, z) = -\frac{g_1}{8} \frac{s^3}{s - m_Z^2} (3 + z^2). \quad (5.13)$$

Only two partial-waves amplitudes, $T_{++--}^0(s)$ and $T_{++--}^2(s)$, are non-zero,

$$T_{++--}^0(s) = -\frac{5g_1}{192\pi} \frac{s^3}{s - m_Z^2}, \quad T_{++--}^2(s) = -\frac{g_1}{960\pi} \frac{s^3}{s - m_Z^2}. \quad (5.14)$$

The strongest bound on g_1 comes from unitarity constraint on $T_{++--}^0(s)$,

$$|g_1| \leq \frac{192\pi}{5} (s - m_Z^2)s^{-3}. \quad (5.15)$$

5.2 Unitarity bounds on coupling g_2 ($g_1 = 0$)

To derive a unitarity bound on the coupling g_2 , we take $g_1 = 0$. It is sufficient to consider three amplitudes, $M_{++++}^{(2)}$, $M_{+--+}^{(2)}$, and $M_{+--+}^{(2)}$. The rest are suppressed by small factor $m_Z/\sqrt{s}$ or zero.

1. $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$, then $\lambda = \mu = 0$. The helicity amplitude is given by the first of equations (B.4)

$$M_{++++}(s, z) = g_2 M_{++++}^{(2)}(s) = \frac{g_2}{2} s(s - m_Z^2). \quad (5.16)$$

As a result, we get

$$|g_2| \leq 32\pi[s(s - m_Z^2)]^{-1}. \quad (5.17)$$

2. $\lambda_1 = -\lambda_2 = \lambda_3 = -\lambda_4 = 1$, then $\lambda = \mu = 2$, and we find from (B.4)

$$M_{+--+}(s, z) = g_2 M_{+--+}^{(2)}(s, z) = \frac{g_2}{2} \frac{s^3}{s - m_Z^2} \left(\frac{1+z}{2} \right)^2. \quad (5.18)$$

We follow the derivation of eq. (5.11) and come to the inequality

$$|g_2| \leq 16\pi(s - m_Z^2)s^{-3}. \quad (5.19)$$

	1 operator ($g_2 = 0$ or $g_1 = 0$)		2 operators ($g_1 g_2 > 0$)		2 operators ($g_1 g_2 < 0$)	
g_1	$32\pi s^{-2}$	$20(1.2) \text{ TeV}^{-4}$	$\frac{192}{5} s^{-2}$	$24(1.5) \text{ TeV}^{-4}$	$32\pi s^{-2}$	$20(1.2) \text{ TeV}^{-4}$
g_2	$16\pi s^{-2}$	$10(0.6) \text{ TeV}^{-4}$	$\frac{176}{5} s^{-2}$	$22(1.4) \text{ TeV}^{-4}$	$16\pi s^{-2}$	$10(0.6) \text{ TeV}^{-4}$

Table 3. Unitarity constraints on the anomalous couplings when just one coupling is non-zero (second and third columns), and when both couplings are non-vanishing (fourth and fifth columns for the couplings of the same sign, sixth and seventh columns for the couplings of opposite signs). The numerical values of the bounds are given for the collision energy $\sqrt{s} = 1500(3000) \text{ GeV}$.

3. $\lambda_1 = -\lambda_2 = -\lambda_3 = \lambda_4 = 1$, then $\lambda = 2$, $\mu = -2$, and we obtain

$$M_{+--+}(s, z) = g_2 M_{+--+}^{(2)}(s, z) = \frac{g_2}{2} \frac{s^3}{s - m_Z^2} \left(\frac{1 - z}{2} \right)^2. \quad (5.20)$$

Using the first relation in (C.4) and eq. (C.6), we can reduce this case to the previous one to get eq. (5.19).

4. $\lambda_1 = \lambda_2 = -\lambda_3 = -\lambda_4 = 1$. The helicity amplitude $M_{+--+}^{(2)}(s, z) = 0$.

5.3 Unitarity bounds on couplings g_1 and g_2

Now we consider a general case with $g_1, g_2 \neq 0$. Note that $M_{++++}^{(2)} = -2M_{++++}^{(1)}$, $M_{+--+}^{(2)} = -2M_{+--+}^{(1)}$, and $M_{+--+}^{(2)} = -2M_{+--+}^{(1)}$. Correspondingly, $M_{++++} = (g_1 - 2g_2)M_{++++}^{(1)}$, etc. From formulas derived in two previous subsections we immediately get the following bound on a linear combination of g_1 and g_2 ,

$$|g_1 - 2g_2| \leq 32\pi(s - m_Z^2)s^{-3}. \quad (5.21)$$

Let us underline that $M_{+--+}(s, z) = g_1 M_{+--+}^{(1)}(s, z)$. It means that inequality (5.15) holds for a general case ($g_1, g_2 \neq 0$). It enables us to obtain constraints separately on each coupling. If the couplings g_1, g_2 have the same sign, then

$$|g_1| \leq \frac{192\pi}{5}(s - m_Z^2)s^{-3}, \quad |g_2| \leq \frac{176\pi}{5}(s - m_Z^2)s^{-3}. \quad (5.22)$$

If the signs of the couplings g_1, g_2 are opposite, we obtain

$$|g_1| \leq 32\pi(s - m_Z^2)s^{-3}, \quad |g_2| \leq 16\pi(s - m_Z^2)s^{-3}. \quad (5.23)$$

The bounds on the couplings g_1, g_2 along with their numerical values are collected in table 3. We have taken into account that $m_Z^2/s \ll 1$ for the CLIC energies.

To summarize, in spite of the fact that the anomalous contribution to the total cross section is proportional to s^3 , the unitarity is not violated in the region of the anomalous QGCs presented in tables 1, 2.

6 Conclusions

In the present paper, the CLIC discovery potential for exclusive γZ production in the scattering of the Compton backscattered photons at the e^+e^- collision energies 1500 GeV and 3000 GeV is studied. We have shown that such a process provides an opportunity of searching for the anomalous quartic neutral gauge couplings for the $\gamma\gamma\gamma Z$ vertex at the CLIC. Both unpolarized and polarized initial electron beams are examined. To describe the anomalous quartic gauge couplings we used the effective Lagrangian which conserves gauge invariance. Although quartic gauge couplings are already induced at the dimension-six level, we considered the effective Lagrangian with CP conserving dimension-eight operators without contributing to anomalous trilinear gauge interactions.

We have derived the explicit expressions for the anomalous contributions to the helicity amplitudes of the process $\gamma\gamma \rightarrow \gamma Z$. After that the differential and total cross sections are calculated depending on $m_{Z\gamma}$, the invariant mass of the γZ system. It is shown that the anomalous contribution dominates both the interference and SM cross sections. Moreover, the ratio of the total cross section to the SM one grows with the increase of $m_{Z\gamma}$, being more approximately one order of magnitude at large $m_{Z\gamma}$.

It enabled us to obtain the exclusion regions for the anomalous couplings with the systematic errors of 0%, 5%, and 10%. We have considered the Z boson decay into leptons (electron and muons). For both couplings, $g_{1,2}$, the best bounds are equal to approximately $4.4 \times 10^{-14} \text{ GeV}^{-4}$ and $5.1 \times 10^{-15} \text{ GeV}^{-4}$, for the e^+e^- energies 1500 GeV and 3000 GeV, respectively. They are achieved when electron beam helicity is equal to 0.8. We have checked that the unitarity is not violated in the region of the couplings considered in the paper. Our best bound on the anomalous couplings for the collision energy 3000 GeV is roughly two orders of magnitude stronger than the limits which can be reached at the LHC and HL-LHC. This points to a great potential of the CLIC and other future leptonic colliders to probe the anomalous $\gamma\gamma\gamma Z$ couplings.

A Components of polarization tensor

Here we present explicit expressions for components of the polarization tensor (2.11). They are the following

$$P_{\mu\nu\rho\alpha}^{(1.1)} = (p_1 \cdot p_2)[(p_1 \cdot p_3) + (p_2 \cdot p_3)]g_{\mu\nu}g_{\rho\alpha} + (p_1 \cdot p_3)[(p_1 \cdot p_2) + (p_2 \cdot p_3)]g_{\mu\rho}g_{\nu\alpha} \\ + (p_2 \cdot p_3)[(p_1 \cdot p_2) + (p_1 \cdot p_3)]g_{\nu\rho}g_{\mu\alpha}, \quad (\text{A.1})$$

$$P_{\mu\nu\rho\alpha}^{(1.2)} = -\{[(p_1 \cdot p_2) + (p_1 \cdot p_3)]p_{3\nu}p_{2\rho}g_{\mu\alpha} + [(p_1 \cdot p_2) + (p_2 \cdot p_3)]p_{3\mu}p_{1\rho}g_{\nu\alpha} \\ + [(p_1 \cdot p_3) + (p_2 \cdot p_3)]p_{2\mu}p_{1\nu}g_{\rho\alpha}\}, \quad (\text{A.2})$$

$$P_{\mu\nu\rho\alpha}^{(1.3)} = -[(p_1 \cdot p_2)(p_{1\rho} + p_{2\rho})p_{3\alpha}g_{\mu\nu} + (p_1 \cdot p_3)(p_{1\nu} + p_{3\nu})p_{2\alpha}g_{\mu\rho} \\ + (p_2 \cdot p_3)(p_{2\mu} + p_{3\mu})p_{1\alpha}g_{\nu\rho}], \quad (\text{A.3})$$

$$P_{\mu\nu\rho\alpha}^{(1.4)} = p_{2\mu}p_{1\nu}(p_{1\rho} + p_{2\rho})p_{3\alpha} + p_{3\mu}(p_{1\nu} + p_{3\nu})p_{1\rho}p_{2\alpha} \\ + p_{3\nu}p_{2\rho}(p_{2\mu} + p_{3\mu})p_{1\alpha}, \quad (\text{A.4})$$

and

$$P_{\mu\nu\rho\alpha}^{(2.1)} = -2[(p_1 \cdot p_3)(p_2 \cdot p_3)g_{\mu\nu}g_{\rho\alpha} + (p_1 \cdot p_2)(p_2 \cdot p_3)g_{\mu\rho}g_{\nu\alpha} + (p_1 \cdot p_2)(p_1 \cdot p_3)g_{\nu\rho}g_{\mu\alpha}], \quad (\text{A.5})$$

$$P_{\mu\nu\rho\alpha}^{(2.2)} = (p_1 \cdot p_2)[p_{3\mu}p_{3\alpha}g_{\nu\rho} + p_{3\nu}p_{3\alpha}g_{\mu\rho}] + (p_1 \cdot p_3)[p_{2\mu}p_{2\alpha}g_{\nu\rho} + p_{2\rho}p_{2\alpha}g_{\mu\nu}] + (p_2 \cdot p_3)[p_{1\nu}p_{1\alpha}g_{\mu\rho} + p_{1\rho}p_{1\alpha}g_{\mu\nu}], \quad (\text{A.6})$$

$$P_{\mu\nu\rho\alpha}^{(2.3)} = -2[(p_1 \cdot p_2)p_{3\mu}p_{3\nu}g_{\rho\alpha} + (p_1 \cdot p_3)p_{2\mu}p_{2\rho}g_{\nu\alpha} + (p_2 \cdot p_3)p_{1\nu}p_{1\rho}g_{\mu\alpha}], \quad (\text{A.7})$$

$$P_{\mu\nu\rho\alpha}^{(2.4)} = (p_1 \cdot p_2)[p_{3\mu}p_{1\alpha}g_{\nu\rho} + p_{3\nu}p_{2\alpha}g_{\mu\rho}] + (p_1 \cdot p_3)[p_{2\mu}p_{1\alpha}g_{\nu\rho} + p_{2\rho}p_{3\alpha}g_{\mu\nu}] + (p_2 \cdot p_3)[p_{1\nu}p_{2\alpha}g_{\mu\rho} + p_{1\rho}p_{3\alpha}g_{\mu\nu}], \quad (\text{A.8})$$

$$P_{\mu\nu\rho\alpha}^{(2.5)} = 2\{(p_1 \cdot p_2)[p_{3\mu}p_{2\rho}g_{\nu\alpha} + p_{3\nu}p_{1\rho}g_{\mu\alpha}] + (p_1 \cdot p_3)[p_{2\mu}p_{3\nu}g_{\rho\alpha} + p_{2\rho}p_{1\nu}g_{\mu\alpha}] + (p_2 \cdot p_3)[p_{3\mu}p_{1\nu}g_{\rho\alpha} + p_{2\mu}p_{1\rho}g_{\nu\alpha}]\}, \quad (\text{A.9})$$

$$P_{\mu\nu\rho\alpha}^{(2.6)} = -\{(p_1 \cdot p_2)[p_{3\mu}p_{2\alpha}g_{\nu\rho} + p_{3\nu}p_{1\alpha}g_{\mu\rho}] + (p_1 \cdot p_3)[p_{2\mu}p_{3\alpha}g_{\nu\rho} + p_{2\rho}p_{1\alpha}g_{\mu\nu}] + (p_2 \cdot p_3)[p_{1\nu}p_{3\alpha}g_{\mu\rho} + p_{1\rho}p_{2\alpha}g_{\mu\nu}]\}, \quad (\text{A.10})$$

$$P_{\mu\nu\rho\alpha}^{(2.7)} = -(p_{2\mu}p_{3\nu}p_{1\rho} + p_{3\mu}p_{1\nu}p_{2\rho})(p_{1\alpha} + p_{2\alpha} + p_{3\alpha}). \quad (\text{A.11})$$

Note that the last tensor does not contribute to the matrix element (3.5), since it is proportional to $p_{4\alpha}$. One can directly check that

$$\begin{aligned} p_1^\mu \sum_{i=1}^4 P_{\mu\nu\rho\alpha}^{(1,i)} &= p_2^\nu \sum_{i=1}^4 P_{\mu\nu\rho\alpha}^{(1,i)} = p_3^\rho \sum_{i=1}^4 P_{\mu\nu\rho\alpha}^{(1,i)} = 0, \\ p_1^\mu \sum_{i=1}^7 P_{\mu\nu\rho\alpha}^{(2,i)} &= p_2^\nu \sum_{i=1}^7 P_{\mu\nu\rho\alpha}^{(2,i)} = p_3^\rho \sum_{i=1}^7 P_{\mu\nu\rho\alpha}^{(2,i)} = 0. \end{aligned} \quad (\text{A.12})$$

B Anomalous helicity amplitudes

In accordance with eq. (2.11), any anomalous helicity amplitude is the sum of two terms,

$$M_{\lambda_1\lambda_2\lambda_3\lambda_4} = g_1 M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1)} + g_2 M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(2)}. \quad (\text{B.1})$$

There are $2^3 \times 3 = 24$ helicity amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1)}$ and, correspondingly, 24 amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(2)}$ for the process (1.1). Bose-Einstein statistics and parity invariance demand that there exist nine independent helicity amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1)}$ with $\lambda_1 = +1$, six for transverse Z and three for longitudinal Z . Our calculations resulted in the following helicity amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(1)}$ with $\lambda_1 = +1$

$$\begin{aligned} M_{++++}^{(1)}(s, t, u) &= \frac{1}{4}s(t+u), \\ M_{+++-}^{(1)}(s, t, u) &= 0, \\ M_{++-+}^{(1)}(s, t, u) &= \frac{1}{2}\frac{tu}{t+u}m_Z^2, \\ M_{+-++}^{(1)}(s, t, u) &= \frac{1}{2}\frac{s(t^2+tu+u^2)}{t+u}, \\ M_{+--+}^{(1)}(s, t, u) &= \frac{1}{4}\frac{tu}{t+u}m_Z^2, \end{aligned}$$

$$\begin{aligned}
 M_{+-+}^{(1)}(s, t, u) &= \frac{1}{4} \frac{su^2}{t+u}, \\
 M_{+++0}^{(1)}(s, t, u) &= 0, \\
 M_{++-0}^{(1)}(s, t, u) &= \frac{i}{2\sqrt{2}} \sqrt{stu} \frac{t-u}{t+u} m_Z, \\
 M_{+-+0}^{(1)}(s, t, u) &= -\frac{i}{2\sqrt{2}} \frac{u\sqrt{stu}}{t+u} m_Z.
 \end{aligned} \tag{B.2}$$

Three more amplitudes $M_{+\lambda_2\lambda_3\lambda_4}^{(1)}$ can be obtained by exchanging Mandelstam variables t and u [25, 37],

$$\begin{aligned}
 M_{+--+}^{(1)}(s, t, u) &= M_{+-+}^{(1)}(s, u, t) = \frac{1}{4} \frac{st^2}{t+u}, \\
 M_{+---}^{(1)}(s, t, u) &= M_{+-++}^{(1)}(s, u, t) = \frac{1}{4} \frac{tu}{(t+u)} m_Z^2, \\
 M_{+--0}^{(1)}(s, t, u) &= M_{+-+0}^{(1)}(s, u, t) = -\frac{i}{2\sqrt{2}} \frac{t\sqrt{stu}}{t+u} m_Z.
 \end{aligned} \tag{B.3}$$

Nine independent helicity amplitudes $M_{\lambda_1\lambda_2\lambda_3\lambda_4}^{(2)}$ with $\lambda_1 = +1$ are

$$\begin{aligned}
 M_{++++}^{(2)}(s, t, u) &= -\frac{1}{2} s(t+u), \\
 M_{++++-}^{(2)}(s, t, u) &= 0, \\
 M_{+++-}^{(2)}(s, t, u) &= 0, \\
 M_{++--}^{(2)}(s, t, u) &= 0, \\
 M_{+-++}^{(2)}(s, t, u) &= -\frac{1}{2} \frac{tu}{t+u} m_Z^2, \\
 M_{+-+-}^{(2)}(s, t, u) &= -\frac{1}{2} \frac{su^2}{t+u}, \\
 M_{+++0}^{(2)}(s, t, u) &= 0, \\
 M_{++-0}^{(2)}(s, t, u) &= 0, \\
 M_{+-+0}^{(2)}(s, t, u) &= \frac{i}{\sqrt{2}} \frac{u\sqrt{stu}}{t+u} m_Z.
 \end{aligned} \tag{B.4}$$

The other three helicity amplitudes $M_{+\lambda_2\lambda_3\lambda_4}^{(2)}$ are given by

$$\begin{aligned}
 M_{+--+}^{(2)}(s, t, u) &= M_{+-+}^{(2)}(s, u, t) = -\frac{1}{4} \frac{st^2}{t+u}, \\
 M_{+---}^{(2)}(s, t, u) &= M_{+-++}^{(2)}(s, u, t) = -\frac{1}{2} \frac{tu}{t+u} m_Z^2, \\
 M_{+--0}^{(2)}(s, t, u) &= M_{+-+0}^{(2)}(s, u, t) = \frac{i}{\sqrt{2}} \frac{t\sqrt{stu}}{t+u} m_Z.
 \end{aligned} \tag{B.5}$$

Note that all amplitudes $M_{\lambda_1\lambda_2\lambda_30}^{(1,2)}$ are equal to zero in the limit $m_Z = 0$.

The amplitudes with $\lambda_1 = -1$ can be obtained from constraints imposed by parity invariance [25, 37],

$$M_{-\lambda_2\lambda_3\lambda_4}^{(1,2)}(s, t, u) = (-1)^{1-\lambda_4} M_{+\lambda_2-\lambda_3-\lambda_4}^{(1,2)}(s, t, u). \quad (\text{B.6})$$

Note that we have directly calculated all 48 helicity amplitudes using eq. (3.5). Our calculations show that relations (B.3), (B.5), and (B.6) really hold.

C Wigner's d -function and relevant formulas

Wigner's d -functions [60] are related to the Jacobi polynomials $P_n^{(\alpha, \beta)}(z)$ with nonnegative α, β [62],

$$d_{\lambda\mu}^J(z) = \left[\frac{(J+\lambda)!(J-\lambda)!}{(J+\mu)!(J-\mu)!} \right]^{1/2} \left(\frac{1-z}{2} \right)^{(\lambda-\mu)/2} \left(\frac{1+z}{2} \right)^{(\lambda+\mu)/2} \times P_{J-\lambda}^{(\lambda-\mu, \lambda+\mu)}(z), \quad (\text{C.1})$$

where $z = \cos \theta$. The d -functions obey the orthogonality condition [62]

$$\int_{-1}^1 d_{\lambda\lambda'}^J(z) d_{\lambda\lambda'}^{J'}(z) dz = \frac{2}{2J+1} \delta_{JJ'}. \quad (\text{C.2})$$

In its turn, the Jacobi polynomial is related to the hypergeometric function [62],

$$P_n^{(\rho, \sigma)}(z) = \frac{\Gamma(\rho+1+n)}{\Gamma(\rho+1)n!} {}_2F_1\left(-n, \rho+\sigma+n+1; \rho+1; \frac{1-z}{2}\right). \quad (\text{C.3})$$

Note that $P_n^{(\alpha, \beta)}(-z) = (-1)^n P_n^{(\beta, \alpha)}(z)$, and, correspondingly,

$$d_{\lambda\mu}^J(-z) = (-1)^{J-\lambda} d_{\mu-\lambda}^J(z), \quad d_{\lambda\mu}^J(z) = (-1)^{\lambda-\mu} d_{-\lambda-\mu}^J(z). \quad (\text{C.4})$$

In particular, we get

$$d_{22}^J(z) = \left(\frac{1+z}{2} \right)^2 {}_2F_1\left(2-J, J+3; 1; \frac{1-z}{2}\right), \quad (\text{C.5})$$

$$d_{2-2}^J(z) = (-1)^J \left(\frac{1-z}{2} \right)^2 {}_2F_1\left(2-J, J+3; 1; \frac{1+z}{2}\right). \quad (\text{C.6})$$

In the simplest case, $\lambda = \mu = 0$, we find

$$d_{00}^J(z) = P_J(z), \quad (\text{C.7})$$

where $P_J(z)$ being the Legendre polynomial. Using table integral 7.231.1 in [63], we derive the following formula

$$\int_{-1}^1 z^m P_J(z) dz = \frac{1}{2} \left[1 + (-1)^J \right] (-1)^{J/2} \frac{\Gamma\left(\frac{J-m}{2}\right) \Gamma\left(\frac{1+m}{2}\right)}{\Gamma\left(-\frac{m}{2}\right) \Gamma\left(\frac{J+m+3}{2}\right)}, \quad (\text{C.8})$$

with integer J and even number $m \geq 0$. To obtain unitarity constraints on the anomalous couplings, we need integral (C.8) with $m = 0, 2$.

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References

- [1] S.C. İnan and A.V. Kisselev, *Probing anomalous quartic $\gamma\gamma\gamma\gamma$ couplings in light-by-light collisions at the CLIC*, *Eur. Phys. J. C* **81** (2021) 664.
- [2] CLIC STUDY Group, *CLIC 2008 Parameters*, CERN-OPEN-2008-021 [CLIC-NOTE-764].
- [3] CLIC and CLICDP collaborations, *Updated baseline for a staged Compact Linear Collider*, CERN-2016-004 [[arXiv:1608.07537](https://arxiv.org/abs/1608.07537)] [[INSPIRE](#)].
- [4] E. Chapon, C. Royon and O. Kepka, *Probing $WW\gamma\gamma$ and $ZZ\gamma\gamma$ quartic anomalous couplings with 10 pb^{-1} at the LHC*, [arXiv:0908.1061](https://arxiv.org/abs/0908.1061) [[INSPIRE](#)].
- [5] E. Chapon, C. Royon and O. Kepka, *Anomalous quartic $WW\gamma\gamma$, $ZZ\gamma\gamma$, and trilinear WW gamma couplings in two-photon processes at high luminosity at the LHC*, *Phys. Rev. D* **81** (2010) 074003 [[arXiv:0912.5161](https://arxiv.org/abs/0912.5161)] [[INSPIRE](#)].
- [6] R.S. Gupta, *Probing Quartic Neutral Gauge Boson Couplings using diffractive photon fusion at the LHC*, *Phys. Rev. D* **85** (2012) 014006 [[arXiv:1111.3354](https://arxiv.org/abs/1111.3354)] [[INSPIRE](#)].
- [7] I. Şahin and B. Şahin, *Anomalous quartic $ZZ\gamma\gamma$ couplings in γp collision at the LHC*, *Phys. Rev. D* **86** (2012) 115001 [[arXiv:1211.3100](https://arxiv.org/abs/1211.3100)] [[INSPIRE](#)].
- [8] A. Senol, *Anomalous quartic $WW\gamma\gamma$ and $ZZ\gamma\gamma$ couplings in γp collision at the LHC*, *Int. J. Mod. Phys. A* **29** (2014) 1450148 [[arXiv:1311.1370](https://arxiv.org/abs/1311.1370)] [[INSPIRE](#)].
- [9] O.J.P. Éboli, M.C. Gonzalez-Garcia and S.M. Lietti, *Bosonic quartic couplings at CERN LHC*, *Phys. Rev. D* **69** (2004) 095005 [[hep-ph/0310141](https://arxiv.org/abs/hep-ph/0310141)] [[INSPIRE](#)].
- [10] CMS collaboration, *Measurement of the cross section for electroweak production of a Z boson, a photon and two jets in proton-proton collisions at $\sqrt{s} = 13\text{ TeV}$ and constraints on anomalous quartic couplings*, *JHEP* **06** (2020) 076 [[arXiv:2002.09902](https://arxiv.org/abs/2002.09902)] [[INSPIRE](#)].
- [11] ATLAS collaboration, *Measurements of $Z\gamma$ and $Z\gamma\gamma$ production in pp collisions at $\sqrt{s} = 8\text{ TeV}$ with the ATLAS detector*, *Phys. Rev. D* **93** (2016) 112002 [[arXiv:1604.05232](https://arxiv.org/abs/1604.05232)] [[INSPIRE](#)].
- [12] L3 collaboration, *Search for anomalous $Z \rightarrow \gamma\gamma\gamma$ events at LEP*, *Phys. Lett. B* **345** (1995) 609 [[INSPIRE](#)].
- [13] ATLAS collaboration, *Search for new phenomena in events with at least three photons collected in pp collisions at $\sqrt{s} = 8\text{ TeV}$ with the ATLAS detector*, *Eur. Phys. J. C* **76** (2016) 210 [[arXiv:1509.05051](https://arxiv.org/abs/1509.05051)] [[INSPIRE](#)].
- [14] L. Schoeffel, C. Baldenegro, H. Hamdaoui, S. Hassani, C. Royon and M. Saimpert, *Photon-photon physics at the LHC and laser beam experiments, present and future*, *Prog. Part. Nucl. Phys.* **120** (2021) 103889.
- [15] I.F. Ginzburg, G.L. Kotkin, V.G. Serbo and V.I. Telnov, *Colliding γe and $\gamma\gamma$ beams based on the single-pass e^+e^- colliders (of VLEPP type)*, *Nucl. Instrum. Meth.* **205** (1983) 47 [[INSPIRE](#)].
- [16] I.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo and V.I. Telnov, *Colliding γe and $\gamma\gamma$ beams based on single-pass e^+e^- accelerators II. Polarization effects, monochromatization improvement*, *Nucl. Instrum. Meth.* **219** (1984) 5.

- [17] O.J.P. Éboli, M.C. Gonzalez-Garcia and S.F. Novaes, *Quartic anomalous couplings in e gamma colliders*, *Nucl. Phys. B* **411** (1994) 381 [[hep-ph/9306306](#)] [[INSPIRE](#)].
- [18] S. Atağ and I. Şahin, *Anomalous quartic WW gamma gamma and $ZZ\gamma\gamma$ couplings in $e\gamma$ collision with initial beams and final state polarizations*, *Phys. Rev. D* **75** (2007) 073003 [[hep-ph/0703201](#)] [[INSPIRE](#)].
- [19] O.J.P. Éboli and J.K. Mizukoshi, *Probing anomalous quartic couplings in $e\gamma$ and $\gamma\gamma$ colliders*, *Phys. Rev. D* **64** (2001) 075011 [[hep-ph/0105238](#)] [[INSPIRE](#)].
- [20] I. Şahin, *Anomalous Quartic $WW\gamma\gamma$ and $WWZ\gamma$ Couplings Through W^+W^-Z Production in $\gamma\gamma$ Colliders*, *J. Phys. G* **36** (2009) 075007 [[arXiv:0807.4777](#)] [[INSPIRE](#)].
- [21] M. Köksal, *Anomalous quartic $ZZ\gamma\gamma$ couplings at the CLIC*, *Eur. Phys. J. Plus* **130** (2015) 75 [[arXiv:1402.3773](#)] [[INSPIRE](#)].
- [22] M. Köksal, V. Ari and A. Senol, *Search for anomalous quartic $ZZ\gamma\gamma$ couplings in photon-photon collisions*, *Adv. High Energy Phys.* **2016** (2016) 8672391 [[arXiv:1606.04433](#)] [[INSPIRE](#)].
- [23] A. Gutiérrez-Rodríguez, C.G. Honorato, J. Montañó and M.A. Pérez, *Limits on the quartic couplings $Z\gamma\gamma\gamma$ and $ZZ\gamma\gamma$ from e^+e^- colliders*, *Phys. Rev. D* **89** (2014) 034003 [[arXiv:1304.7410](#)] [[INSPIRE](#)].
- [24] C. Baldenegro, S. Fichet, G. von Gersdorff and C. Royon, *Probing the anomalous $\gamma\gamma\gamma Z$ coupling at the LHC with proton tagging*, *JHEP* **06** (2017) 142 [[arXiv:1703.10600](#)] [[INSPIRE](#)].
- [25] G.J. Gounaris, J. Layssac, P.I. Porfyriadis and F.M. Renard, *The $\gamma\gamma \rightarrow \gamma Z$ process at high-energies and the search for virtual SUSY effects*, *Eur. Phys. J. C* **10** (1999) 499 [[hep-ph/9904450](#)] [[INSPIRE](#)].
- [26] M. Krämer, J.H. Kühn, M.L. Stong and P.M. Zerwas, *Prospects of measuring the parity of Higgs particles*, *Z. Phys. C* **64** (1994) 21 [[hep-ph/9404280](#)] [[INSPIRE](#)].
- [27] R. Brinkmann et al., *An Interaction region for gamma-gamma and gamma-electron collisions at TESLA/SBLC*, *Nucl. Instrum. Meth. A* **406** (1998) 13 [[hep-ex/9707017](#)] [[INSPIRE](#)].
- [28] V.I. Telnov, *Physics goals and parameters of photon colliders*, *Int. J. Mod. Phys. A* **13** (1998) 2399 [[hep-ex/9802003](#)] [[INSPIRE](#)].
- [29] V.I. Telnov, *Electron photon interactions in high-energy beam production and cooling*, in proceedings of the 15th Advanced ICFA Beam Dynamics Workshop on Quantum Aspects of Beam Physics, Monterey, CA, U.S.A., 4–9 January 1998, P. Chen ed., Singapore, World Scientific (1998), pp. 173–189 [[hep-ex/9805002](#)] [[INSPIRE](#)].
- [30] D. Dannheim et al., *CLIC e^+e^- Linear Collider Studies*, in proceedings of the 2013 Community Summer Study on the Future of U.S. Particle Physics: Snowmass on the Mississippi (CSS2013), Minneapolis, MN, U.S.A., 29 July–6 August 2013, [arXiv:1208.1402](#) [[INSPIRE](#)].
- [31] J. de Blas et al., *The CLIC Potential for New Physics*, in *CERN Yellow report: Monographs* **3**, CERN, Geneva Switzerland (2018) [[CERN-2018-009-M](#)] [[arXiv:1812.02093](#)] [[INSPIRE](#)].
- [32] R. Franceschini, *Beyond the Standard Model physics at CLIC*, *Int. J. Mod. Phys. A* **35** (2020) 2041015 [[arXiv:1902.10125](#)] [[INSPIRE](#)].
- [33] G.A. Moortgat-Pick et al., *The Role of polarized positrons and electrons in revealing fundamental interactions at the linear collider*, *Phys. Rept.* **460** (2008) 131 [[hep-ph/0507011](#)] [[INSPIRE](#)].

- [34] R. Franceschini, P. Roloff, U. Schnoor and A. Wulzer, *The Compact Linear e^+e^- Collider (CLIC): Physics Potential*, [arXiv:1812.07986](#) [[INSPIRE](#)].
- [35] M. Stöhr and J. Hořejší, *Effective Lagrangian for the Z boson decay into photons*, *Phys. Rev. D* **49** (1994) 3775 [[INSPIRE](#)].
- [36] J. Hořejší and M. Stöhr, *One loop induced effective on-shell $Z\gamma\gamma\gamma$ interactions*, *Z. Phys. C* **64** (1994) 407 [[INSPIRE](#)].
- [37] E.W.N. Glover and A.G. Morgan, *Z boson decay into photons*, *Z. Phys. C* **60** (1993) 175 [[INSPIRE](#)].
- [38] J. Novotný, M. Stöhr and J. Hořejší, *P odd effective $Z\gamma\gamma\gamma$ interactions*, *Z. Phys. C* **67** (1995) 313 [[INSPIRE](#)].
- [39] M. Baillargeon, F. Boudjema, E. Chopin and V. Lafage, *New physics with three photon events at LEP*, *Z. Phys. C* **71** (1996) 431 [[hep-ph/9506396](#)] [[INSPIRE](#)].
- [40] S. Fichet and G. von Gersdorff, *Anomalous gauge couplings from composite Higgs and warped extra dimensions*, *JHEP* **03** (2014) 102 [[arXiv:1311.6815](#)] [[INSPIRE](#)].
- [41] G.N. Remmen and N.L. Rodd, *Consistency of the Standard Model Effective Field Theory*, *JHEP* **12** (2019) 032 [[arXiv:1908.09845](#)] [[INSPIRE](#)].
- [42] C. Zhang and S.-Y. Zhou, *Positivity bounds on vector boson scattering at the LHC*, *Phys. Rev. D* **100** (2019) 095003 [[arXiv:1808.00010](#)] [[INSPIRE](#)].
- [43] Q. Bi, C. Zhang and S.-Y. Zhou, *Positivity constraints on aQGC: carving out the physical parameter space*, *JHEP* **06** (2019) 137 [[arXiv:1902.08977](#)] [[INSPIRE](#)].
- [44] M. Baillargeon and F. Boudjema, *Contribution of the bosonic loops to the three photon decay of the Z*, *Phys. Lett. B* **272** (1991) 158 [[INSPIRE](#)].
- [45] F.-X. Dong, X.-D. Jiang and X.-J. Zhou, *Partial decay width of the Z into three photons via the W loops*, *Phys. Rev. D* **46** (1992) 5074 [[INSPIRE](#)].
- [46] M.-Z. Yang and X.-J. Zhou, *The Decay of the Z boson into three photons via W boson loops*, *Phys. Rev. D* **52** (1995) 5018 [[INSPIRE](#)].
- [47] G. Jikia and A. Tkabladze, *Photon-photon scattering at the photon linear collider*, *Phys. Lett. B* **323** (1994) 453 [[hep-ph/9312228](#)] [[INSPIRE](#)].
- [48] G. Jikia and A. Tkabladze, *γZ pair production at the photon linear collider*, *Phys. Lett. B* **332** (1994) 441 [[hep-ph/9312274](#)] [[INSPIRE](#)].
- [49] J.J. van der Bij and E.W.N. Glover, *Z Boson Production and Decay via Gluons*, *Nucl. Phys. B* **313** (1989) 237 [[INSPIRE](#)].
- [50] M.L. Laursen, K.O. Mikaelian and M.A. Samuel, *Z^0 decay into three gluons*, *Phys. Rev. D* **23** (1981) 2795 [[INSPIRE](#)].
- [51] G.J. Gounaris, P.I. Porfyriadis and F.M. Renard, *Light by light scattering at high-energy: A Tool to reveal new particles*, *Phys. Lett. B* **452** (1999) 76 [Erratum *ibid.* **513** (2001) 431] [Erratum *ibid.* **464** (1999) 350] [[hep-ph/9812378](#)] [[INSPIRE](#)].
- [52] G.J. Gounaris, P.I. Porfyriadis and F.M. Renard, *The $\gamma\gamma \rightarrow \gamma\gamma$ process in the standard and SUSY models at high-energies*, *Eur. Phys. J. C* **9** (1999) 673 [[hep-ph/9902230](#)] [[INSPIRE](#)].
- [53] Y.-J. Zhang and J.-F. Shen, *Probing anomalous tqh couplings via single top production in associated with the Higgs boson at the HE-LHC and FCC-hh*, *Eur. Phys. J. C* **80** (2020) 811 [[INSPIRE](#)].

- [54] M. Jacob and G.C. Wick, *On the General Theory of Collisions for Particles with Spin*, *Annals Phys.* **7** (1959) 404 [*Annals Phys.* **281** (2000) 774] [[INSPIRE](#)].
- [55] G.J. Gounaris, J. Laysaac and M. Renard, *Unitarity constraints for transverse gauge bosons at LEP and supercolliders*, *Phys. Lett. B* **332** (1994) 146 [[hep-ph/9311370](#)] [[INSPIRE](#)].
- [56] T. Corbett, O.J.P. Éboli and M.C. Gonzalez-Garcia, *Unitarity Constraints on Dimension-six Operators II: Including Fermionic Operators*, *Phys. Rev. D* **96** (2017) 035006 [[arXiv:1705.09294](#)] [[INSPIRE](#)].
- [57] G. Perez, M. Sekulla and D. Zeppenfeld, *Anomalous quartic gauge couplings and unitarization for the vector boson scattering process $pp \rightarrow W^+W^+jjX \rightarrow \ell^+\nu_\ell\ell^+\nu_\ell jjX$* , *Eur. Phys. J. C* **78** (2018) 759 [[arXiv:1807.02707](#)] [[INSPIRE](#)].
- [58] E. da Silva Almeida, O.J.P. Éboli and M.C. Gonzalez-Garcia, *Unitarity constraints on anomalous quartic couplings*, *Phys. Rev. D* **101** (2020) 113003 [[arXiv:2004.05174](#)] [[INSPIRE](#)].
- [59] J.-C. Yang, Y.-C. Guo, C.-X. Yue and Q. Fu, *Constraints on anomalous quartic gauge couplings via $Z\gamma jj$ production at the LHC*, *Phys. Rev. D* **104** (2021) 035015 [[arXiv:2107.01123](#)] [[INSPIRE](#)].
- [60] E.P. Wigner, *Group theory: and its application to the quantum mechanics of atomic spectra*, Academic Press, New York NY U.S.A. (1959).
- [61] A.P. Prudnikov, Y.A. Brychkov and O.I. Marichev, *Integrals and series, in More special functions. Volume 3*, Gordon & Breach Science Publishers, New York NY U.S.A. (1989).
- [62] D.A. Varshalovich, A.N. Moskalev and V.K. Khersonskii, *Quantum theory of angular momentum*, World Scientific, Singapore (1988).
- [63] I.S. Gradshteyn and I.M. Ryzhik, *Table of integrals, series, and products*, 7th edition, Academic Press, New York NY U.S.A. (2007).