Three-point functions of higher-spin spinor current multiplets in $\mathcal{N} = 1$ superconformal theory

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ABSTRACT: In this paper, we study the general form of three-point functions of conserved current multiplets $S_{\alpha(k)} = S_{(\alpha_1 \dots \alpha_k)}$ of arbitrary rank in four-dimensional $\mathcal{N} = 1$ superconformal theory. We find that the correlation function of three such operators $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle$ is fixed by the superconformal symmetry up to a single complex coefficient though the precise form of the correlator depends on the values of k and l . In addition, we present the general structure of mixed correlators of the form $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle$ and $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$, where L is the flavour current multiplet and $J_{\gamma\dot{\gamma}}$ is the supercurrent.

KEYWORDS: Conformal Field Theory, Superspaces

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1 Introduction

It is well known that in (super)conformal field theory the general form of two- and three-point functions of conserved currents is fully determined by the (super)conformal symmetry and conservation laws up to finitely many independent coefficients. In the non-supersymmetric case, the systematic study of the two- and three-point functions of conserved currents was presented in [1, 2] building on the earlier results obtained in refs. [3–12].¹ The approach of [1, 2] was later extended to superconformal field theories in diverse dimensions in [14–24].

The most important conserved currents in conformal field theory are the energy-momentum tensor and vector currents. In the supersymmetric case, they are embedded in special multiplet of conserved currents. The energy-momentum tensor is, therefore, replaced with the supercurrent [25] (see also [26–30]) and vector currents are replaced with flavour current multiplets [31]. Three-point functions of these current multiplets have been extensively studied. In general, (super)conformal field theories also possess higher-spin conserved currents. In the case of three-dimensional conformal field theory, it was proven (under certain assumptions) by Maldacena and Zhiboedov in [32] that all correlation functions of higher-spin currents are equal to the ones in a free theory. The theorem of Maldacena and Zhiboedov was later generalised by Stanev [33] and by Alba and Diab [34, 35] to the four- (and higher-) dimensional case. One can view these results as the analog of the Coleman-Mandula theorem [36] for conformal field theories.

¹Parity violating structures in three-dimensional conformal field theory were not considered in [1, 2]. They were later found in [13].

We believe that the analysis of [32–35] has some limitations. First, the authors of [32–35] considered only bosonic symmetric traceless currents. However, in supersymmetric theories conserved currents form supermultiplets consisting of both bosonic and fermionic component currents. Second, the results of [32–35] are proven under certain assumptions, the main being the existence of only one conserved current of spin two which is the energy-momentum tensor. In [32] it was shown that in three-dimensional conformal field theory the existence of a half-integer higher-spin conserved current implies the existence of one more conserved current of spin two. This means that in supersymmetric conformal field theory possessing higher-spin currents the assumptions of [32] might be violated. It is likely that the same conclusion also holds in four dimensions. Therefore, in the supersymmetric case it is unclear if higher-spin current multiplets exist only in free theory. In any case, regardless whether a theory is free or not, it is interesting to understand the general structure of their correlation functions.

In the non-supersymmetric case, the general structure of the three-point functions of conserved bosonic, vector currents of arbitrary spin was determined by Stanev [37] and Zhiboedov [38], see also [39] for similar results in the embedding formalism. The aim of this paper is to make first steps towards determining the three-point functions of conserved higher-spin currents in the $\mathcal{N} = 1$ supersymmetric case in four dimensions. In fact, we will be interested in multiplets which do not contain vector currents. To start with, it is worth reminding the reader of the known conformal current multiplets.

In the case of $\mathcal{N} = 1$ supersymmetry, there are three types of conformal current multiplets depending on the corresponding superfield Lorentz type $(k/2, l/2)$. These multiplets were explicitly described in flat [40] and curved [41] backgrounds.

- Given two positive integers k and l , the conformal current multiplet $J_{\alpha(k)\dot{\alpha}(l)} := J_{\alpha_1 \dots \alpha_k \dot{\alpha}_1 \dots \dot{\alpha}_l} = J_{(\alpha_1 \dots \alpha_k)(\dot{\alpha}_1 \dots \dot{\alpha}_l)}$ obeys the conservation equations

$$D^\beta J_{\alpha(k-1)\beta\dot{\alpha}(l)} = 0, \quad \bar{D}^{\dot{\beta}} J_{\alpha(k)\dot{\alpha}(l-1)\dot{\beta}} = 0. \quad (1.1)$$

This superfield has dimension $(2 + \frac{1}{2}(k+l))$ and its $U(1)_R$ charge is equal to $\frac{1}{3}(k-l)$, see [41] for the technical details. If $k = l$, the supercurrent $J_{\alpha(k)\dot{\alpha}(k)}$ is neutral with respect to the R -symmetry group $U(1)_R$, and therefore it is consistent to restrict $J_{\alpha(k)\dot{\alpha}(k)}$ to be real. The $k = l = 1$ case corresponds to the ordinary conformal supercurrent [25]. The case $k = l > 1$ was first discussed in [42].

- If $k > 0$ and $l = 0$, the conformal current multiplet $S_{\alpha(k)}$ obeys the conservation equation

$$D^\beta S_{\beta\alpha_1 \dots \alpha_{k-1}} = 0, \quad \bar{D}^2 S_{\alpha(k)} = 0. \quad (1.2)$$

The case $k = 1$ was first considered in [19], where it was shown that the spinor supercurrent S_α naturally originates from the reduction of the conformal $\mathcal{N} = 2$ supercurrent [43] to $\mathcal{N} = 1$ superspace.

- Finally, the $k = l = 0$ case corresponds to the flavour current multiplet [31], $L = \bar{L}$, constrained by

$$D^2 L = 0, \quad \bar{D}^2 L = 0. \quad (1.3)$$

Do all of these multiplets occur in superconformal field theories?

The conformal supercurrent $J_{\alpha\dot{\alpha}}$ exists in every $\mathcal{N} = 1$ superconformal field theory. Flavour current multiplets exist in every superconformal field theory possessing an internal symmetry Lie group. The spinor current multiplet S_α exists in every $\mathcal{N} = 2$ superconformal field theory realised in terms of $\mathcal{N} = 1$ superfields. Explicit realisations in terms of free conformal scalar multiplets are known for the conformal current higher-spin multiplets $J_{\alpha(k)\dot{\alpha}(k)}$ and $J_{\alpha(k+1)\dot{\alpha}(k)}$, with $k > 1$, both in Minkowski [44] and anti-de Sitter [45] superspace.

Higher-spin current multiplets $J_{\alpha(k)\dot{\alpha}(l)}$ and $S_{\alpha(k)}$ may be realised in terms of the on-shell chiral field strengths $W_{\alpha(k)}$ and massless antichiral scalar $\bar{\Phi}$ constrained as

$$\bar{D}_{\dot{\beta}} W_{\alpha(k)} = 0, \quad D^\beta W_{\beta\alpha(k-1)} = 0, \quad (1.4a)$$

$$D_\beta \bar{\Phi} = 0, \quad \bar{D}^2 \bar{\Phi} = 0. \quad (1.4b)$$

These realisations are as follows [45–47]:

$$J_{\alpha(k)\dot{\alpha}(l)} = W_{\alpha(k)} \bar{W}_{\dot{\alpha}(l)}, \quad S_{\alpha(k)} = W_{\alpha(k)} \bar{\Phi}. \quad (1.5)$$

Here W_α , $W_{\alpha(2)}$ and $W_{\alpha(3)}$ are the gauge-invariant field strengths describing the on-shell vector, gravitino and linearised supergravity multiplets, respectively. The chiral superfields $W_{\alpha(k)}$ for $k > 3$ are the on-shell gauge-invariant field strengths corresponding to the massless higher-spin gauge multiplets [48, 49] (see section 6.9 in [50] for a review). Choosing $k = l = 1$ in (1.5) gives the supercurrent of the free $\mathcal{N} = 1$ vector multiplet, $J_{\alpha\dot{\alpha}} = W_\alpha \bar{W}_{\dot{\alpha}}$, and the spinor supercurrent of the free $\mathcal{N} = 2$ vector multiplet, $S_\alpha = W_\alpha \bar{\Phi}$.

The off-shell gauge-invariant models for massless higher-spin multiplets proposed in [48, 49] are not superconformal, although the corresponding on-shell field strengths $W_{\alpha(k)}$ constrained by (1.4a) furnish irreducible representations of the superconformal group, see [50] for the technical details. At the moment we are not aware of explicit realisations of the current multiplets $S_{\alpha(k)}$ for $k > 1$ in superconformal field theories. Nevertheless, it is of interest to understand the most general structure of three-point correlation functions of such current multiplets, which are compatible with the superconformal symmetry and conservation laws.

In this paper we will restrict ourselves to four-dimensional $\mathcal{N} = 1$ superconformal field theory and to higher-spin current multiplets carrying only undotted or dotted indices, $S_{\alpha(k)}$ and its conjugate $\bar{S}_{\dot{\alpha}(k)}$. We will refer to them as “higher-spin spinor current multiplets”. For $k = 1$ the multiplet S_α is just a spinor current multiplet. This case was studied in our previous work [51]. Three-point correlation functions of more general higher-spin current multiplets require additional study and will be explored elsewhere.

The main result of this paper is the most general structure of the three-point function

$$\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle \quad (1.6)$$

for arbitrary integers k and ℓ . We showed that in all cases eq. (1.6) is fixed by the superconformal symmetry and the conservation equations (1.2) up to a single overall coefficient. However, we found that one has to distinguish two cases depending on values of k and ℓ , see section 5 for details. Additionally, we also studied the mixed correlators

$$\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle, \quad \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle, \quad (1.7)$$

where L is the flavour current multiplet and $J_{\gamma\dot{\gamma}}$ is the supercurrent.

The paper is organised as follows. A brief review of the two- and three-point building blocks for correlation functions is given in section 2. In section 3 we study a three-point function of higher-spin spinor current multiplets with the flavour current multiplet. We show that this three-point function is fixed by the superconformal symmetry up to two independent real coefficients. In the special case $k = 1$, our result coincides with that found in our previous work [51]. In section 4 we consider a three-point function of higher-spin spinor current multiplets with the supercurrent. In this case the general form of the correlation function is fixed up to three independent real coefficients. In the special case $k = 1$, our result coincides with that found in [51]. Finally, in section 5 we compute three-point correlator involving just the conserved higher-spin spinor current multiplets with arbitrary number of dotted and undotted indices. We found that its general structure is fixed up to a single complex coefficient.

2 Superconformal building blocks

This section contains a concise summary of two and three-point superconformal building blocks in 4D $\mathcal{N} = 1$ superspace, which are important for our subsequent analysis. These superconformal structures were introduced in [14, 15], and later generalised to arbitrary $\mathcal{N}$ in [16] (see also [19] for a review). A review of the general structure of two- and three-point correlation functions of primary operators is also given. The presentation of this section closely follows [51]. Our 4D notation and conventions are those of [50].

2.1 Infinitesimal superconformal transformations

We denote the $\mathcal{N} = 1$ Minkowski superspace by $\mathbb{M}^{4|4}$. It is parametrised by coordinates $z^A = (x^a, \theta^\alpha, \bar{\theta}_{\dot{\alpha}})$, where $a = 0, 1, \dots, 3$, $\alpha, \dot{\alpha} = 1, 2$. Infinitesimal superconformal transformations

$$\delta z^A = \xi z^A, \quad (2.1)$$

are generated by conformal Killing real supervector fields [19, 50]

$$\xi = \bar{\xi} = \xi^a(z) \partial_a + \xi^\alpha(z) D_\alpha + \bar{\xi}_{\dot{\alpha}}(z) \bar{D}^{\dot{\alpha}} \quad (2.2)$$

obeying the equation

$$[\xi, D_\alpha] \propto D_\beta. \quad (2.3)$$

As a result, the spinor parameters are expressed in terms of the vector ones

$$\xi^\alpha = -\frac{i}{8} \bar{D}_{\dot{\beta}} \xi^{\dot{\beta}\alpha}, \quad \bar{D}_{\dot{\beta}} \xi^\alpha = 0. \quad (2.4)$$

The latter satisfy

$$D_{(\alpha}\xi_{\beta)\dot{\beta}} = \bar{D}_{(\dot{\alpha}}\xi_{\beta\dot{\beta})} = 0, \quad (2.5)$$

which leads to the standard conformal Killing equation

$$\partial_a \xi_b + \partial_b \xi_a = \frac{1}{2} \eta_{ab} \partial_c \xi^c. \quad (2.6)$$

The general solution to eq. (2.5) was given in [50] for $\mathcal{N} = 1$ and in [16] for $\mathcal{N} > 1$. These conformal Killing supervector fields span a Lie superalgebra which is isomorphic to $\text{su}(2, 2|1)$. For the purpose of this paper, it suffices to consider the relation

$$[\xi, D_\alpha] = -(D_\alpha \xi^\beta) D_\beta = \hat{\omega}_\alpha{}^\beta(z) D_\beta - (2\bar{\sigma}(z) - \sigma(z)) D_\alpha. \quad (2.7)$$

The superfield parameters $\hat{\omega}_{\alpha\beta}(z)$ and $\sigma(z)$ are expressed in terms of $\xi^A = (\xi^a, \xi^\alpha, \bar{\xi}_{\dot{\alpha}})$ as follows

$$\hat{\omega}_{\alpha\beta}(z) = -D_{(\alpha}\xi_{\beta)}, \quad \sigma(z) = \frac{1}{6} (D_\alpha \xi^\alpha + 2\bar{D}^{\dot{\alpha}} \bar{\xi}_{\dot{\alpha}}) \quad (2.8)$$

and can be explicitly found using the components of the conformal Killing supervector, see refs. [16, 19, 50] for detail. Due to their action on the covariant derivative (2.7), the z -dependent parameters $\hat{\omega}_{\alpha\beta}(z)$ and $\sigma(z)$ can be thought of as the parameters of special local Lorentz and scale transformations, respectively. These z -dependent parameters, along with ξ , appear in the superconformal transformation law of a primary tensor superfield, see subsection 2.4.

2.2 Two-point structures

Let z_1 and z_2 be two different points in superspace. In 4D superconformal theories, all building blocks for the two- and three-point correlation functions are composed of the two-point structures:

$$x_{12}^a = -x_{21}^a = x_{1-}^a - x_{2+}^a + 2i\theta_2\sigma^a\bar{\theta}_1, \quad (2.9a)$$

$$\theta_{12} = \theta_1 - \theta_2, \quad \bar{\theta}_{12} = \bar{\theta}_1 - \bar{\theta}_2, \quad (2.9b)$$

where $x_\pm^a = x^a \pm i\theta\sigma^a\bar{\theta}$. In spinor notation, we write

$$(x_{12})^{\dot{\alpha}\alpha} = (\tilde{\sigma}_a)^{\dot{\alpha}\alpha} x_{12}^a, \quad (2.10a)$$

$$(x_{21})_{\alpha\dot{\alpha}} = (\sigma_a)_{\alpha\dot{\alpha}} x_{21}^a = -(\sigma_a)_{\alpha\dot{\alpha}} x_{12}^a = -\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}(x_{12})^{\dot{\beta}\beta}, \quad (2.10b)$$

$$(x_{12}^\dagger)^{\dot{\alpha}\alpha} = -(x_{21})^{\dot{\alpha}\alpha}. \quad (2.10c)$$

Note that $(x_{12})^{\dot{\alpha}\alpha}(x_{21})_{\alpha\dot{\beta}} = x_{12}^2 \delta^{\dot{\alpha}}_{\dot{\beta}}$. We sometimes employ matrix-like conventions of [15, 19] where the spinor indices are not explicitly written:

$$\psi = (\psi^\alpha), \quad \tilde{\psi} = (\psi_\alpha), \quad \bar{\psi} = (\bar{\psi}^{\dot{\alpha}}), \quad \tilde{\bar{\psi}} = (\bar{\psi}_{\dot{\alpha}}), \quad (2.11a)$$

$$x = (x_{\alpha\dot{\alpha}}), \quad \tilde{x} = (x^{\dot{\alpha}\alpha}). \quad (2.11b)$$

Since $x^2 \equiv x^a x_a = -\frac{1}{2}\text{tr}(\tilde{x}x)$, it follows that $\tilde{x}^{-1} = -x/x^2$. The notation ‘ $\tilde{x}_{12}$ ’ means that $\tilde{x}_{12}$ is antichiral with respect to z_1 and chiral with respect to z_2 . That is,

$$D_{(1)\alpha}\tilde{x}_{12} = 0, \quad \bar{D}_{(2)\dot{\alpha}}\tilde{x}_{12} = 0, \quad (2.12)$$

where $D_{(1)\alpha}$ and $\bar{D}_{(1)\dot{\alpha}}$ are the superspace covariant spinor derivatives acting on the point z_1 . Similarly, $D_{(2)\alpha}$ and $\bar{D}_{(2)\dot{\alpha}}$ act on the point z_2 . Explicitly, they are given by

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i(\sigma^a)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \frac{\partial}{\partial x^a}, \quad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha (\sigma^a)_{\alpha\dot{\alpha}} \frac{\partial}{\partial x^a}. \quad (2.13)$$

The superconformal transformation laws of the two-point structures are given by

$$\delta x_{12}^{\dot{\alpha}\alpha} = -\left(\hat{\omega}^{\dot{\alpha}}_{\dot{\beta}}(z_1) - \delta^{\dot{\alpha}}_{\dot{\beta}} \bar{\sigma}(z_1)\right) x_{12}^{\dot{\beta}\alpha} - x_{12}^{\dot{\alpha}\beta} \left(\hat{\omega}_\beta^\alpha(z_2) - \delta_\beta^\alpha \sigma(z_2)\right) \quad (2.14a)$$

$$\delta \theta_{12}^\alpha = 2(\bar{\sigma}(z_1) - \sigma(z_1)) \theta_{12}^\alpha - i \hat{\eta}_{\dot{\beta}}(z_1) x_{12}^{\dot{\beta}\alpha} - \theta_{12}^\beta (\hat{\omega}_\beta^\alpha(z_2) - \delta_\beta^\alpha \sigma(z_2)), \quad (2.14b)$$

with $\hat{\eta}_\alpha(z) := \frac{1}{2} D_\alpha \sigma(z)$. Let us define

$$\mathcal{I}_{\alpha\dot{\alpha}}(x_{2\bar{1}}) = \frac{(x_{2\bar{1}})_{\alpha\dot{\alpha}}}{(x_{12}^2)^{\frac{1}{2}}} \in \text{SL}(2, \mathbb{C}). \quad (2.15)$$

With the aid of (2.14a), it follows that x_{12}^2 and $\mathcal{I}_{\alpha\dot{\alpha}}(x_{2\bar{1}})$ transform covariantly under superconformal transformations:

$$\delta x_{12}^2 = 2(\bar{\sigma}(z_1) + \sigma(z_2)) x_{12}^2, \quad (2.16a)$$

$$\delta \mathcal{I}_{\alpha\dot{\alpha}}(x_{2\bar{1}}) = \mathcal{I}_{\alpha\dot{\gamma}}(x_{2\bar{1}}) \hat{\omega}^{\dot{\gamma}}_{\dot{\alpha}}(z_1) + \hat{\omega}^\gamma_\alpha(z_2) \mathcal{I}_{\gamma\dot{\alpha}}(x_{2\bar{1}}). \quad (2.16b)$$

We also note several useful differential identities:

$$D_{(1)\alpha}(x_{2\bar{1}})^{\dot{\beta}\beta} = 4i\delta_\alpha^\beta \bar{\theta}_{12}^{\dot{\beta}}, \quad \bar{D}_{(1)\dot{\alpha}}(x_{12})^{\dot{\beta}\beta} = 4i\delta_{\dot{\alpha}}^{\dot{\beta}} \theta_{12}^\beta, \quad (2.17a)$$

$$D_{(1)\alpha}\left(\frac{1}{x_{2\bar{1}}^2}\right) = -\frac{4i}{x_{2\bar{1}}^2} (\tilde{x}_{2\bar{1}}^{-1})_{\alpha\dot{\beta}} \bar{\theta}_{12}^{\dot{\beta}}, \quad \bar{D}_{(1)\dot{\alpha}}\left(\frac{1}{x_{12}^2}\right) = -\frac{4i}{x_{12}^2} (\tilde{x}_{12}^{-1})_{\alpha\dot{\alpha}} \theta_{12}^\alpha, \quad (2.17b)$$

Here and throughout, we assume that the space points are not coincident, $x_1 \neq x_2$.

2.3 Three-point structures

Given three superspace points z_1, z_2 and z_3 , we have the following three-point structures $\mathbf{Z}_1, \mathbf{Z}_2$ and $\mathbf{Z}_3$, with $\mathbf{Z}_1 = (\mathbf{X}_1^a, \boldsymbol{\Theta}_1^\alpha, \bar{\boldsymbol{\Theta}}_1^{\dot{\alpha}})$:

$$\mathbf{X}_1 = \tilde{x}_{21}^{-1} \tilde{x}_{23} \tilde{x}_{13}^{-1}, \quad \tilde{\boldsymbol{\Theta}}_1 = i(\tilde{x}_{21}^{-1} \bar{\theta}_{12} - \tilde{x}_{31}^{-1} \bar{\theta}_{13}), \quad \tilde{\bar{\boldsymbol{\Theta}}}_1 = i(\theta_{12} \tilde{x}_{12}^{-1} - \theta_{13} \tilde{x}_{13}^{-1}), \quad (2.18a)$$

$$\mathbf{X}_2 = \tilde{x}_{32}^{-1} \tilde{x}_{31} \tilde{x}_{21}^{-1}, \quad \tilde{\boldsymbol{\Theta}}_2 = i(\tilde{x}_{32}^{-1} \bar{\theta}_{23} - \tilde{x}_{12}^{-1} \bar{\theta}_{21}), \quad \tilde{\bar{\boldsymbol{\Theta}}}_2 = i(\theta_{23} \tilde{x}_{23}^{-1} - \theta_{21} \tilde{x}_{21}^{-1}), \quad (2.18b)$$

$$\mathbf{X}_3 = \tilde{x}_{13}^{-1} \tilde{x}_{12} \tilde{x}_{32}^{-1}, \quad \tilde{\boldsymbol{\Theta}}_3 = i(\tilde{x}_{13}^{-1} \bar{\theta}_{31} - \tilde{x}_{23}^{-1} \bar{\theta}_{32}), \quad \tilde{\bar{\boldsymbol{\Theta}}}_3 = i(\theta_{31} \tilde{x}_{31}^{-1} - \theta_{32} \tilde{x}_{32}^{-1}). \quad (2.18c)$$

Since (2.18b) and (2.18c) are obtained through cyclic permutations of superspace points, it suffices to study the properties of (2.18a). Let us also define

$$\bar{\mathbf{X}}_1 = \mathbf{X}_1^\dagger = -\tilde{x}_{31}^{-1} \tilde{x}_{32} \tilde{x}_{12}^{-1}. \quad (2.19)$$

Similar relations hold for $\bar{\mathbf{X}}_2, \bar{\mathbf{X}}_3$.

The structures $\mathbf{Z}_i$ transform as tensors at the point z_i , $i = 1, 2, 3$. For instance, the transformation law of (2.18a) reads

$$\delta \mathbf{X}_{1\alpha\dot{\alpha}} = \left(\hat{\omega}_{\alpha}{}^{\beta}(z_1) - \delta_{\alpha}{}^{\beta} \sigma(z_1) \right) \mathbf{X}_{1\beta\dot{\alpha}} + \mathbf{X}_{1\alpha\dot{\beta}} \left(\hat{\bar{\omega}}^{\dot{\beta}}{}_{\dot{\alpha}}(z_1) - \delta^{\dot{\beta}}{}_{\dot{\alpha}} \bar{\sigma}(z_1) \right), \quad (2.20a)$$

$$\delta \Theta_{1\alpha} = \hat{\omega}_{\alpha}{}^{\beta}(z_1) \Theta_{1\beta} + (\sigma(z_1) - 2\bar{\sigma}(z_1)) \Theta_{1\alpha}. \quad (2.20b)$$

We list several properties of $\mathbf{Z}$'s which will be useful later (see [15] for details):

$$\mathbf{X}_1^2 = \frac{x_{23}^2}{x_{21}^2 x_{13}^2}, \quad \bar{\mathbf{X}}_1^2 = \frac{x_{32}^2}{x_{31}^2 x_{12}^2}, \quad (2.21a)$$

$$\bar{\mathbf{X}}_{1\alpha\dot{\alpha}} = \mathbf{X}_{1\alpha\dot{\alpha}} + i \mathbf{P}_{1\alpha\dot{\alpha}}, \quad \mathbf{P}_{1\alpha\dot{\alpha}} = -4 \Theta_{1\alpha} \bar{\Theta}_{1\dot{\alpha}}, \quad \mathbf{P}_1^2 = -8 \Theta_1^2 \bar{\Theta}_1^2, \quad (2.21b)$$

$$\frac{1}{\bar{\mathbf{X}}_1^2} = \frac{1}{\mathbf{X}_1^2} - 2i \frac{(\mathbf{P}_1 \cdot \mathbf{X}_1)}{(\mathbf{X}_1^2)^2}, \quad (\mathbf{P}_1 \cdot \mathbf{X}_1) = -\frac{1}{2} \mathbf{P}_1^{\dot{\alpha}\alpha} \mathbf{X}_{1\alpha\dot{\alpha}}, \quad (2.21c)$$

and hence, $\bar{\mathbf{X}}$ is not an independent variable for it can be expressed in terms of $\mathbf{X}$, Θ , $\bar{\Theta}$. The variables $\mathbf{Z}$ with different labels are related to each other via the identities

$$\tilde{x}_{13} \mathbf{X}_3 \tilde{x}_{31} = -\bar{\mathbf{X}}_1^{-1} = \frac{\tilde{\mathbf{X}}_1}{\bar{\mathbf{X}}_1^2}, \quad \tilde{x}_{13} \bar{\mathbf{X}}_3 \tilde{x}_{31} = -\mathbf{X}_1^{-1} = \frac{\tilde{\mathbf{X}}_1}{\mathbf{X}_1^2}, \quad (2.22a)$$

$$\frac{x_{31}^2}{x_{13}^2} \tilde{x}_{13} \tilde{\Theta}_3 = -\mathbf{X}_1^{-1} \tilde{\Theta}_1, \quad \frac{x_{13}^2}{x_{31}^2} \tilde{\Theta}_3 \tilde{x}_{31} = \tilde{\Theta}_1 \bar{\mathbf{X}}_1^{-1}. \quad (2.22b)$$

Eqs. (2.20) and (2.22) imply that

$$\frac{\mathbf{X}_1^2}{\bar{\mathbf{X}}_1^2} = \frac{\mathbf{X}_2^2}{\bar{\mathbf{X}}_2^2} = \frac{\mathbf{X}_3^2}{\bar{\mathbf{X}}_3^2}, \quad (2.23)$$

and this combination is a superconformal invariant.

2.4 Correlation functions of primary superfields

A tensor superfield $\mathcal{O}^{\mathcal{A}}(z)$ is called primary, if it is characterised by the following infinitesimal superconformal transformation law

$$\begin{aligned} \delta \mathcal{O}^{\mathcal{A}}(z) = & -\xi \mathcal{O}^{\mathcal{A}}(z) + (\hat{\omega}^{\alpha\beta}(z) M_{\alpha\beta} + \hat{\bar{\omega}}^{\dot{\alpha}\dot{\beta}}(z) \bar{M}_{\dot{\alpha}\dot{\beta}}) \mathcal{O}^{\mathcal{A}}(z) \\ & - 2(q\sigma(z) + \bar{q}\bar{\sigma}(z)) \mathcal{O}^{\mathcal{A}}(z). \end{aligned} \quad (2.24)$$

In the above, ξ is the superconformal Killing vector, while $\hat{\omega}^{\alpha\beta}(z)$ and $\sigma(z)$ are the z -dependent parameters associated with ξ , see eq. (2.8). The superscript ' $\mathcal{A}$ ' collectively denotes the undotted and dotted spinor indices on which the Lorentz generators $M_{\alpha\beta}$ and $\bar{M}_{\dot{\alpha}\dot{\beta}}$ act. The weights q and $\bar{q}$ are such that $(q + \bar{q})$ is the scale dimension and $(q - \bar{q})$ is proportional to the $U(1)$ R -symmetry charge of the superfield $\mathcal{O}^{\mathcal{A}}$.

Various primary superfields, including conserved current multiplets, are subject to certain differential constraints. These constraints need to be taken into account when

computing correlation functions. It proves beneficial to make use of these conformally covariant operators [15]: $\mathcal{D}_{\bar{A}} = (\partial/\partial \mathbf{X}_3^a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}})$ and $\mathcal{Q}_{\bar{A}} = (\partial/\partial \mathbf{X}_3^a, \mathcal{Q}_\alpha, \bar{\mathcal{Q}}^{\dot{\alpha}})$ defined by

$$\begin{aligned}\mathcal{D}_\alpha &= \frac{\partial}{\partial \Theta_3^\alpha} - 2i(\sigma^a)_{\alpha\dot{\alpha}} \bar{\Theta}_3^{\dot{\alpha}} \frac{\partial}{\partial \mathbf{X}_3^a}, & \bar{\mathcal{D}}^{\dot{\alpha}} &= \frac{\partial}{\partial \bar{\Theta}_3^{\dot{\alpha}}}, \\ \mathcal{Q}_\alpha &= \frac{\partial}{\partial \Theta_3^\alpha}, & \bar{\mathcal{Q}}^{\dot{\alpha}} &= \frac{\partial}{\partial \bar{\Theta}_3^{\dot{\alpha}}} + 2i\Theta_{3\alpha}(\bar{\sigma}^a)^{\dot{\alpha}\alpha} \frac{\partial}{\partial \mathbf{X}_3^a},\end{aligned}\quad (2.25)$$

$$[\mathcal{D}_{\bar{A}}, \mathcal{Q}_{\bar{B}}] = 0,$$

from which we can derive these anti-commutation relations

$$\{\mathcal{D}^\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\} = 2i(\tilde{\sigma}^a)^{\dot{\alpha}\alpha} \frac{\partial}{\partial \mathbf{X}^a}, \quad \{\mathcal{Q}^\alpha, \bar{\mathcal{Q}}^{\dot{\alpha}}\} = -2i(\tilde{\sigma}^a)^{\dot{\alpha}\alpha} \frac{\partial}{\partial \mathbf{X}^a}. \quad (2.26)$$

One can further prove the following differential identities:

$$D_{(1)\alpha} t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3) = -\frac{i}{x_{1\bar{3}}^2} (x_{1\bar{3}})_{\alpha\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3), \quad (2.27a)$$

$$\bar{D}_{(1)\dot{\alpha}} t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3) = -\frac{i}{x_{3\bar{1}}^2} (x_{3\bar{1}})_{\alpha\dot{\alpha}} \mathcal{D}^\alpha t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3), \quad (2.27b)$$

$$D_{(2)\alpha} t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3) = \frac{i}{x_{2\bar{3}}^2} (x_{2\bar{3}})_{\alpha\dot{\alpha}} \bar{\mathcal{Q}}^{\dot{\alpha}} t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3), \quad (2.27c)$$

$$\bar{D}_{(2)\dot{\alpha}} t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3) = \frac{i}{x_{3\bar{2}}^2} (x_{3\bar{2}})_{\alpha\dot{\alpha}} \mathcal{Q}^\alpha t(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3). \quad (2.27d)$$

In accordance with the general prescription of [14–16], the two-point function of a primary superfield $\mathcal{O}^A$ with its conjugate $\bar{\mathcal{O}}^B$ is expressed in terms of $\mathcal{I}$, see eq. (2.15):

$$\langle \mathcal{O}^A(z_1) \bar{\mathcal{O}}^B(z_2) \rangle = C_{\mathcal{O}} \frac{\mathcal{I}^{AB}(x_{1\bar{2}}, x_{2\bar{1}})}{(x_{1\bar{2}}^2)^{\bar{q}} (x_{2\bar{1}}^2)^q}, \quad (2.28)$$

with $C_{\mathcal{O}}$ being a normalisation constant.

The three-point function of primary superfields $\Phi^{\mathcal{A}_1}, \Psi^{\mathcal{A}_2}$ and $\Pi^{\mathcal{A}_3}$ has the following general expression [14–16]:

$$\langle \Phi^{\mathcal{A}_1}(z_1) \Psi^{\mathcal{A}_2}(z_2) \Pi^{\mathcal{A}_3}(z_3) \rangle = \frac{\mathcal{I}^{\mathcal{A}_1 \mathcal{B}_1}(x_{1\bar{3}}, x_{3\bar{1}}) \mathcal{I}^{\mathcal{A}_2 \mathcal{B}_2}(x_{2\bar{3}}, x_{3\bar{2}})}{(x_{1\bar{3}}^2)^{\bar{q}_1} (x_{3\bar{1}}^2)^{q_1} (x_{2\bar{3}}^2)^{\bar{q}_2} (x_{3\bar{2}}^2)^{q_2}} H_{\mathcal{B}_1 \mathcal{B}_2}^{\mathcal{A}_3}(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3), \quad (2.29)$$

where the functional form of the tensor $H_{\mathcal{B}_1 \mathcal{B}_2}^{\mathcal{A}_3}$ is highly constrained by the superconformal symmetry:

- (i) It possesses the homogeneity property

$$\begin{aligned}H_{\mathcal{B}_1 \mathcal{B}_2}^{\mathcal{A}_3}(\lambda \bar{\lambda} \mathbf{X}, \lambda \Theta, \bar{\lambda} \bar{\Theta}) &= \lambda^{2a} \bar{\lambda}^{2\bar{a}} H_{\mathcal{B}_1 \mathcal{B}_2}^{\mathcal{A}_3}(\mathbf{X}, \Theta, \bar{\Theta}), \\ a - 2\bar{a} &= \bar{q}_1 + \bar{q}_2 - q_3, \\ \bar{a} - 2a &= q_1 + q_2 - \bar{q}_3,\end{aligned}\quad (2.30)$$

This condition guarantees that the correlation function has the correct transformation law under the superconformal group. By construction, eq. (2.29) has the correct transformation properties at the points z_1 and z_2 . The above homogeneity property implies that it also transforms correctly at z_3 . The tensor $H_{\mathcal{B}_1 \mathcal{B}_2}^{\mathcal{A}_3}$ has dimension $(a + \bar{a})$.

- (ii) If any of the superfields Φ , Ψ and Π obey differential equations (such as conservation laws in the case of conserved current multiplets), then $H_{\mathcal{B}_1\mathcal{B}_2}^{\mathcal{A}_3}$ is constrained by certain differential equations too. The latter may be derived using (2.27).
- (iii) If any (or all) of the superfields Φ , Ψ and Π coincide, then $H_{\mathcal{B}_1\mathcal{B}_2}^{\mathcal{A}_3}$ obeys certain constraints, as a consequence of the symmetry under permutations of superspace points. As an example,

$$\langle \Phi^{\mathcal{A}}(z_1) \Phi^{\mathcal{B}}(z_2) \Pi^{\mathcal{C}}(z_3) \rangle = (-1)^{\epsilon(\Phi)} \langle \Phi^{\mathcal{B}}(z_2) \Phi^{\mathcal{A}}(z_1) \Pi^{\mathcal{C}}(z_3) \rangle, \quad (2.31)$$

where $\epsilon(\Phi)$ denotes the Grassmann parity of $\Phi^{\mathcal{A}}$. Note that under permutations of any two superspace points, the three-point building blocks transform as

$$\mathbf{X}_{3\alpha\dot{\alpha}} \xrightarrow{1\leftrightarrow 2} -\bar{\mathbf{X}}_{3\alpha\dot{\alpha}}, \quad \Theta_{3\alpha} \xrightarrow{1\leftrightarrow 2} -\Theta_{3\alpha}, \quad (2.32a)$$

$$\mathbf{X}_{3\alpha\dot{\alpha}} \xrightarrow{2\leftrightarrow 3} -\bar{\mathbf{X}}_{2\alpha\dot{\alpha}}, \quad \Theta_{3\alpha} \xrightarrow{2\leftrightarrow 3} -\Theta_{2\alpha}, \quad (2.32b)$$

$$\mathbf{X}_{3\alpha\dot{\alpha}} \xrightarrow{1\leftrightarrow 3} -\bar{\mathbf{X}}_{1\alpha\dot{\alpha}}, \quad \Theta_{3\alpha} \xrightarrow{1\leftrightarrow 3} -\Theta_{1\alpha}. \quad (2.32c)$$

The above conditions fix the functional form of $H_{\mathcal{B}_1\mathcal{B}_2}^{\mathcal{A}_3}$ (and, therefore, the three-point function under consideration) up to a few arbitrary constants.

The goal of this paper is to study how $\mathcal{N} = 1$ superconformal symmetry imposes constraints on the general structure of two- and three-point correlation functions involving a higher-spin conserved spinor current multiplet $S_{\alpha(k)}$ and its conjugate $\bar{S}_{\dot{\alpha}(k)}$. Here the complex superfield $S_{\alpha(k)} = S_{\alpha_1 \dots \alpha_k} = S_{(\alpha_1 \dots \alpha_k)}$ is a symmetric rank- k spinor, subject to the following conservation conditions [41]

$$D^{\beta} S_{\beta\alpha_1 \dots \alpha_{k-1}} = 0 \quad (2.33a)$$

$$\bar{D}^2 S_{\alpha(k)} = 0. \quad (2.33b)$$

Making use of the transformation law (2.24), it can be shown that eqs. (2.33) are consistent with superconformal invariance provided $S_{\alpha(k)}$ is a primary superfield with weight $(q, \bar{q}) = (1 + \frac{k}{2}, 1)$ and dimension $2 + \frac{k}{2}$.

Let us first consider the two-point correlation function of $S_{\alpha(k)}$ and its conjugate $\bar{S}_{\dot{\alpha}(k)}$. Adapting the general prescription (2.28) to this case leads to

$$\langle S_{\alpha(k)}(z_1) \bar{S}_{\dot{\alpha}(k)}(z_2) \rangle = i^k A \frac{(x_{1\bar{2}})_{(\alpha_1 \dot{\alpha}_1} \dots (x_{1\bar{2}})_{\alpha_k \dot{\alpha}_k)}}{x_{1\bar{2}}^2 (x_{2\bar{1}}^2)^{k+1}}, \quad (2.34)$$

where A is a real coefficient. Using the identities (2.17), along with

$$D_{(1)}^{\alpha} \left[\frac{(x_{1\bar{2}})_{\alpha\dot{\alpha}}}{(x_{2\bar{1}}^2)^p} \right] = -4i(p-2) \frac{\bar{\theta}_{12\dot{\alpha}}}{(x_{2\bar{1}}^2)^p}, \quad (2.35)$$

one may verify that the correlator (2.34) respects the conservation conditions

$$\begin{aligned} D_{(1)}^{\alpha_1} \langle S_{\alpha(k)}(z_1) \bar{S}_{\dot{\alpha}(k)}(z_2) \rangle &= \bar{D}_{(1)}^2 \langle S_{\alpha(k)}(z_1) \bar{S}_{\dot{\alpha}(k)}(z_2) \rangle = 0, \\ \bar{D}_{(2)}^{\dot{\alpha}_1} \langle S_{\alpha(k)}(z_1) \bar{S}_{\dot{\alpha}(k)}(z_2) \rangle &= D_{(2)}^2 \langle S_{\alpha(k)}(z_1) \bar{S}_{\dot{\alpha}(k)}(z_2) \rangle = 0, \end{aligned} \quad (2.36)$$

at non-coincident points $z_1 \neq z_2$.

In the case of $k = 1$, the expression (2.34) is in agreement with [15, 19]. Various three-point functions involving S_{α} and its conjugate $\bar{S}_{\dot{\alpha}}$ have been studied in detail in [51].

3 Correlator $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle$

In $\mathcal{N} = 1$ superconformal field theory, the $U(1)$ flavour current multiplet is a primary real superfield $L = \bar{L}$, subject to the conservation equation

$$D^2 L = \bar{D}^2 L = 0. \quad (3.1)$$

Its superconformal transformation law is

$$\delta L = -\xi L - 2(\sigma + \bar{\sigma})L, \quad (3.2)$$

which means that L has weights $(q, \bar{q}) = (1, 1)$ and dimension 2.

In our recent work [51], we found that the three-point correlator with two spinor current insertions and a flavour current multiplet has two linearly independent functional structures with real coefficients c_1 and d_1 :

$$\langle \bar{S}_{\dot{\alpha}}(z_1) S_{\beta}(z_2) L(z_3) \rangle = \frac{(x_{3\bar{1}})^{\dot{\alpha}} (x_{2\bar{3}})^{\dot{\beta}}}{(x_{1\bar{3}}^2)^2 x_{3\bar{1}}^2 x_{2\bar{3}}^2 (x_{3\bar{2}}^2)^2} H_{\alpha\dot{\beta}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \quad (3.3a)$$

where

$$H_{\alpha\dot{\beta}} = ic_1 \frac{\mathbf{X}_{\alpha\dot{\beta}}}{\mathbf{X}^4} + \frac{d_1}{2\mathbf{X}^6} (\mathbf{X}^2 \mathbf{P}_{\alpha\dot{\beta}} - 4(\mathbf{P} \cdot \mathbf{X}) \mathbf{X}_{\alpha\dot{\beta}}). \quad (3.3b)$$

Here we adopt the notation in which $\mathbf{X}^k \equiv (\mathbf{X}^2)^{k/2}$.

A natural extension is to consider the three-point function $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle$, with $k = 1, 2, \dots$ ² In accordance with (2.29), we start with the ansatz

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle &= \frac{(x_{3\bar{1}})^{(\dot{\alpha}_1} \dots (x_{3\bar{1}})^{\dot{\alpha}_k} (x_{2\bar{3}})^{\beta_k} (x_{2\bar{3}})^{(\alpha_1} \dots (x_{2\bar{3}})^{\alpha_k)} (x_{3\bar{2}}^2)^{\dot{\beta}_k}}{(x_{1\bar{3}}^2)^{k+1} x_{3\bar{1}}^2 x_{2\bar{3}}^2 (x_{3\bar{2}}^2)^{k+1}} \\ &\times H_{\beta_1 \dots \beta_k \dot{\beta}_1 \dots \dot{\beta}_k}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \end{aligned} \quad (3.4)$$

where the tensor $H_{\beta_1 \dots \beta_k \dot{\beta}_1 \dots \dot{\beta}_k}$ has the symmetry property $H_{(\beta_1 \dots \beta_k)(\dot{\beta}_1 \dots \dot{\beta}_k)} = H_{\beta(k)\dot{\beta}(k)}$. It is also characterised by the homogeneity property

$$H_{\beta(k)\dot{\beta}(k)}(\lambda \bar{\lambda} \mathbf{X}, \lambda \boldsymbol{\Theta}, \bar{\lambda} \bar{\boldsymbol{\Theta}}) = \lambda^{-(k+2)} \bar{\lambda}^{-(k+2)} H_{\beta(k)\dot{\beta}(k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}), \quad (3.5)$$

and hence, its dimension is $-(k+2)$. Taking the complex conjugate of the correlator (3.4),

$$\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle^* = \langle \bar{S}_{\dot{\alpha}(k)}(z_2) S_{\alpha(k)}(z_1) L(z_3) \rangle, \quad (3.6)$$

leads to the reality constraint on $H_{\beta(k)\dot{\beta}(k)}$:

$$\bar{H}_{\beta(k)\dot{\beta}(k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) = H_{\beta(k)\dot{\beta}(k)}(-\bar{\mathbf{X}}, -\boldsymbol{\Theta}, -\bar{\boldsymbol{\Theta}}). \quad (3.7)$$

The conservation equations of the higher-spin spinor current and flavour current multiplets result in

$$\bar{D}_{(1)}^{\dot{\alpha}_1} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle = 0, \quad D_{(1)}^2 \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle = 0, \quad (3.8a)$$

$$D_{(2)}^{\alpha_1} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle = 0, \quad \bar{D}_{(2)}^2 \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle = 0, \quad (3.8b)$$

$$D_{(3)}^2 \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle = 0, \quad \bar{D}_{(3)}^2 \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle = 0. \quad (3.8c)$$

²We can also consider $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(\ell)}(z_2) L(z_3) \rangle$ with $k \neq \ell$. However, this correlator carries a non-trivial R -symmetry charge and, hence, vanishes.

With the use of identities (2.27), conditions (3.8a) and (3.8b) are translated to

$$\mathcal{D}^{\beta_1} H_{\beta_1 \dots \beta_k \dot{\beta}(k)} = 0, \quad (3.9a)$$

$$\bar{\mathcal{D}}^2 H_{\beta(k) \dot{\beta}(k)} = 0, \quad (3.9b)$$

$$\bar{\mathcal{Q}}^{\dot{\beta}_1} H_{\beta(k) \dot{\beta}_1 \dots \dot{\beta}_k} = 0, \quad (3.9c)$$

$$\mathcal{Q}^2 H_{\beta(k) \dot{\beta}(k)} = 0. \quad (3.9d)$$

Imposing differential constraints (3.8c) is more complicated. We will take care of (3.8c) at the end.

The problem of computing higher-spin correlator (3.4) is thus reduced to determining the most general form of $H_{\beta(k) \dot{\beta}(k)}$ satisfying the constraints (3.5), (3.7), (3.8c) and (3.9).

Since $H_{\beta(k) \dot{\beta}(k)}$ is Grassmann even, the most general expansion we can write is

$$\begin{aligned} H_{\beta(k) \dot{\beta}(k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) &= A_{\beta(k) \dot{\beta}(k)}^{(1)}(\mathbf{X}) + A_{\beta(k) \dot{\beta}(k)}^{(2)}(\mathbf{X}) \boldsymbol{\Theta}^2 + A_{\beta(k) \dot{\beta}(k)}^{(3)}(\mathbf{X}) \bar{\boldsymbol{\Theta}}^2 \\ &+ A_{\beta(k) \dot{\beta}(k)}^{(4)}(\mathbf{X}) \boldsymbol{\Theta}^2 \bar{\boldsymbol{\Theta}}^2 + B_{\beta(k) \dot{\beta}(k), \gamma \dot{\gamma}}(\mathbf{X}) \boldsymbol{\Theta}^\gamma \bar{\boldsymbol{\Theta}}^{\dot{\gamma}}. \end{aligned} \quad (3.10)$$

Constraints (3.9b) and (3.9d) immediately tell us that

$$A_{\beta(k) \dot{\beta}(k)}^{(2)} = A_{\beta(k) \dot{\beta}(k)}^{(3)} = A_{\beta(k) \dot{\beta}(k)}^{(4)} = 0, \quad (3.11)$$

which leaves us with

$$H_{\beta(k) \dot{\beta}(k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) = A_{\beta(k) \dot{\beta}(k)}^{(1)}(\mathbf{X}) + B_{\beta(k) \dot{\beta}(k), \gamma \dot{\gamma}}(\mathbf{X}) \boldsymbol{\Theta}^\gamma \bar{\boldsymbol{\Theta}}^{\dot{\gamma}}, \quad (3.12a)$$

where we have redenoted $A_{\beta(k) \dot{\beta}(k)} = A_{\beta(k) \dot{\beta}(k)}^{(1)}$. Since $A_{\beta(k) \dot{\beta}(k)}$ and $B_{\beta(k) \dot{\beta}(k), \gamma \dot{\gamma}}$ can be constructed using only tensors $\mathbf{X}_{\alpha\dot{\alpha}}$, $\varepsilon_{\alpha\beta}$, $\varepsilon_{\dot{\alpha}\dot{\beta}}$, it is not hard to list all possible independent structures consistent with the homogeneity property (3.5) by performing irreducible decompositions into symmetric and antisymmetric parts in dotted and undotted indices:

$$A_{\beta(k) \dot{\beta}(k)} = \frac{a_1}{\mathbf{X}^{2k+2}} \mathbf{X}_{(\beta_1 \dot{\beta}_1} \dots \mathbf{X}_{\beta_k) \dot{\beta}_k)}, \quad (3.12b)$$

$$\begin{aligned} B_{\beta(k) \dot{\beta}(k), \gamma \dot{\gamma}} &= \frac{b_1}{\mathbf{X}^{2k+4}} \mathbf{X}_{(\gamma \dot{\gamma}} \mathbf{X}_{\beta_1 \dot{\beta}_1} \dots \mathbf{X}_{\beta_k) \dot{\beta}_k)} \\ &+ \frac{b_2}{\mathbf{X}^{2k+2}} \varepsilon_{\gamma(\beta_1} \varepsilon_{\dot{\gamma}(\dot{\beta}_1} \mathbf{X}_{\beta_2 \dot{\beta}_2} \dots \mathbf{X}_{\beta_k) \dot{\beta}_k)}. \end{aligned} \quad (3.12c)$$

Here a_1, b_1, b_2 are arbitrary complex coefficients. To continue it is convenient to introduce auxiliary commuting complex variables $(u^\alpha, \bar{w}^{\dot{\alpha}})$, with the property $u^\alpha u_\alpha = \bar{w}_{\dot{\alpha}} \bar{w}^{\dot{\alpha}} = 0$. Given a tensor superfield $T_{\alpha(k) \dot{\alpha}(l)}$, we can associate to it the following index-free superfield

$$T_{(k,l)}(u, \bar{w}) := u^{\alpha_1} \dots u^{\alpha_k} \bar{w}^{\dot{\alpha}_1} \dots \bar{w}^{\dot{\alpha}_l} T_{\alpha_1 \dots \alpha_k \dot{\alpha}_1 \dots \dot{\alpha}_l}, \quad (3.13)$$

which is homogeneous of degree (k, l) in the variables $u^\alpha, \bar{w}^{\dot{\alpha}}$. Making use of these auxiliary variables and their corresponding partial derivatives $(\partial/\partial u^\alpha, \partial/\partial \bar{w}^{\dot{\alpha}})$ allows us to also convert the conformally covariant derivatives into index-free operators, for example:

$$\mathcal{D}_{(-1,0)} := \mathcal{D}^\alpha \frac{\partial}{\partial u^\alpha}, \quad \mathcal{D}_{(-1,0)}^2 = 0, \quad (3.14)$$

$$\bar{\mathcal{Q}}_{(0,-1)} := \bar{\mathcal{Q}}^{\dot{\alpha}} \frac{\partial}{\partial \bar{w}^{\dot{\alpha}}}, \quad \bar{\mathcal{Q}}_{(0,-1)}^2 = 0. \quad (3.15)$$

These nilpotent operators decrease the degree of homogeneity in u^α and $\bar{w}^{\dot{\alpha}}$.

Using the notation introduced, eq. (3.12) turns into

$$H_{(k,k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{w}) = \frac{a_1}{\mathbf{X}^{2k+2}} \mathbf{X}_{(1,1)}^k + \frac{1}{2(k+1)} \left\{ (b_1 - (k+1)b_2) \frac{(\mathbf{P} \cdot \mathbf{X}) \mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} \right. \\ \left. + (kb_1 + (k+1)b_2) \frac{(\mathbf{P} \cdot \boldsymbol{\mathcal{K}}) \mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+4}} \right\}, \quad (3.16)$$

where we have defined

$$\mathbf{X}_{(1,1)} := u^\alpha \bar{w}^{\dot{\alpha}} \mathbf{X}_{\alpha\dot{\alpha}}, \quad \boldsymbol{\mathcal{K}}_{\gamma\dot{\gamma}} := u^\alpha \bar{w}^{\dot{\alpha}} \mathbf{X}_{\alpha\dot{\gamma}} \mathbf{X}_{\gamma\dot{\alpha}}, \quad (3.17)$$

$$(\mathbf{P} \cdot \mathbf{X}) := 2\boldsymbol{\Theta}^\alpha \bar{\boldsymbol{\Theta}}^{\dot{\alpha}} \mathbf{X}_{\alpha\dot{\alpha}}, \quad (\mathbf{P} \cdot \boldsymbol{\mathcal{K}}) := 2\boldsymbol{\Theta}^\alpha \bar{\boldsymbol{\Theta}}^{\dot{\alpha}} \boldsymbol{\mathcal{K}}_{\alpha\dot{\alpha}}. \quad (3.18)$$

Throughout the paper, we will also often employ the following shorthand notation

$$(u \cdot \boldsymbol{\Theta}) = u^\alpha \boldsymbol{\Theta}_\alpha, \quad (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) = \bar{w}^{\dot{\alpha}} \bar{\boldsymbol{\Theta}}_{\dot{\alpha}}, \\ (\boldsymbol{\Theta} \mathbf{X} \bar{w}) = \boldsymbol{\Theta}^\alpha \mathbf{X}_{\alpha\dot{\alpha}} \bar{w}^{\dot{\alpha}}, \quad (u \mathbf{X} \bar{\boldsymbol{\Theta}}) = u^\alpha \mathbf{X}_{\alpha\dot{\alpha}} \bar{\boldsymbol{\Theta}}^{\dot{\alpha}}. \quad (3.19)$$

The differential constraints (3.9a) and (3.9c) now read

$$\mathcal{D}_{(-1,0)} H_{(k,k)} = 0, \quad (3.20)$$

$$\bar{\mathcal{Q}}_{(0,-1)} H_{(k,k)} = 0. \quad (3.21)$$

A useful observation is that

$$\mathcal{D}_{(-1,0)} \left(\frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+2}} \right) = \bar{\mathcal{Q}}_{(0,-1)} \left(\frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+2}} \right) = 0. \quad (3.22)$$

This means that a_1 is an independent coefficient, since the first term in (3.16) already satisfies conservation equations. It can be explicitly checked that

$$\mathcal{D}_{(-1,0)} H_{(k,k)} = -(k+1)b_2 \left\{ (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+2}} + 2i\bar{\boldsymbol{\Theta}}^2 (\boldsymbol{\Theta} \mathbf{X} \bar{w}) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+4}} \right\}, \quad (3.23)$$

and

$$\bar{\mathcal{Q}}_{(0,-1)} H_{(k,k)} = -(k+1)b_2 \left\{ (u \cdot \boldsymbol{\Theta}) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+2}} + 2i\boldsymbol{\Theta}^2 (u \mathbf{X} \bar{\boldsymbol{\Theta}}) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+4}} \right\}, \quad (3.24)$$

which imply that the conservation laws (3.20) and (3.21) hold provided we set

$$b_2 = 0. \quad (3.25)$$

In deriving (3.23) and (3.24), the following easily verified identities have been used:

$$\mathcal{D}^\alpha \mathbf{X}_{\gamma\dot{\gamma}} = -4i \delta^\alpha_\gamma \bar{\boldsymbol{\Theta}}_{\dot{\gamma}}, \quad \bar{\mathcal{Q}}^{\dot{\alpha}} \mathbf{X}_{\gamma\dot{\gamma}} = -4i \delta^{\dot{\alpha}}_{\dot{\gamma}} \boldsymbol{\Theta}_\gamma, \quad (3.26a)$$

$$\mathcal{D}^\alpha \mathbf{X}_{(1,1)}^k = -4ik (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) u^\alpha \mathbf{X}_{(1,1)}^{k-1}, \quad (3.26b)$$

$$\bar{\mathcal{Q}}^{\dot{\alpha}} \mathbf{X}_{(1,1)}^k = -4ik(u \cdot \Theta) \bar{w}^{\dot{\alpha}} \mathbf{X}_{(1,1)}^{k-1}, \quad (3.26c)$$

$$\mathcal{D}_{(-1,0)} \mathbf{X}_{(1,1)}^k = -4ik(k+1)(\bar{w} \cdot \bar{\Theta}) \mathbf{X}_{(1,1)}^{k-1}, \quad (3.26d)$$

$$\bar{\mathcal{Q}}_{(0,-1)} \mathbf{X}_{(1,1)}^k = -4ik(k+1)(u \cdot \Theta) \mathbf{X}_{(1,1)}^{k-1}, \quad (3.26e)$$

$$\mathcal{D}^{\alpha} \left(\frac{1}{\mathbf{X}^{2p}} \right) = -4ip \frac{\bar{\Theta}_{\dot{\alpha}} \mathbf{X}^{\dot{\alpha}\alpha}}{\mathbf{X}^{2p+2}}, \quad \bar{\mathcal{Q}}^{\dot{\alpha}} \left(\frac{1}{\mathbf{X}^{2p}} \right) = -4ip \frac{\Theta_{\alpha} \mathbf{X}^{\dot{\alpha}\alpha}}{\mathbf{X}^{2p+2}}, \quad (3.26f)$$

$$\mathcal{D}^{\alpha} \mathbf{P}_{\gamma\dot{\gamma}} = 4\delta^{\alpha}_{\gamma} \bar{\Theta}_{\dot{\gamma}}, \quad \bar{\mathcal{Q}}^{\dot{\alpha}} \mathbf{P}_{\gamma\dot{\gamma}} = 4\delta^{\dot{\alpha}}_{\dot{\gamma}} \Theta_{\gamma}. \quad (3.26g)$$

At this stage, we are left with two unconstrained complex parameters, a_1 and b_1 . The next task is to check if the flavour current conservation equations (3.8c) are satisfied. Checking conservation laws on z_3 requires more work as there are no identities that would allow differential operators acting on the z_3 dependence to pass through the prefactor of (3.4). Following the procedure carried out in [51], let us first express (3.4) as

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle &= \frac{1}{k_1} \mathcal{I}_{\beta(k)\dot{\alpha}(k)}(x_{3\bar{1}}) \mathcal{I}_{\alpha(k)\dot{\beta}(k)}(x_{2\bar{3}}) H^{\dot{\beta}(k)\beta(k)}(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3), \\ k_1 &:= (x_{\bar{1}3}^2)^{(k+2)/2} x_{\bar{3}1}^2 x_{2\bar{3}}^2 (x_{3\bar{2}}^2)^{(k+2)/2}. \end{aligned} \quad (3.27)$$

Here the $\mathcal{I}$ -operators are the higher-spin extensions of (2.15). Specifically, we define

$$\mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x_{3\bar{1}}) := \frac{1}{(x_{\bar{1}3}^2)^{k/2}} (x_{3\bar{1}})_{(\alpha_1(\dot{\alpha}_1 \dots (x_{3\bar{1}})_{\alpha_k} \dot{\alpha}_k))}. \quad (3.28)$$

Its inverse will be denoted by

$$\bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(\tilde{x}_{\bar{1}3}) := \frac{1}{(x_{\bar{1}3}^2)^{k/2}} (x_{\bar{1}3})^{(\dot{\alpha}_1(\alpha_1 \dots (x_{\bar{1}3})^{\dot{\alpha}_k} \alpha_k))}. \quad (3.29)$$

These operators satisfy

$$\mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x_{3\bar{1}}) \bar{\mathcal{I}}^{\dot{\alpha}(k)\gamma(k)}(\tilde{x}_{\bar{1}3}) = \delta_{(\alpha_1}^{\gamma_1} \dots \delta_{\alpha_k)}^{\gamma_k), \quad (3.30a)$$

$$\bar{\mathcal{I}}^{\dot{\gamma}(k)\gamma(k)}(\tilde{x}_{\bar{1}3}) \mathcal{I}_{\gamma(k)\dot{\alpha}(k)}(x_{3\bar{1}}) = \delta_{(\dot{\alpha}_1}^{\dot{\gamma}_1} \dots \delta_{\dot{\alpha}_k)}^{\dot{\gamma}_k)}. \quad (3.30b)$$

It should be kept in mind that the spinor indices can be raised or lowered in accordance with (2.10b). This allows us to write

$$\mathcal{I}_{\dot{\alpha}(k)}^{\alpha(k)}(x_{3\bar{1}}) = \varepsilon^{\alpha_1\beta_1} \dots \varepsilon^{\alpha_k\beta_k} \mathcal{I}_{\beta(k)\dot{\alpha}(k)}(x_{3\bar{1}}), \quad (3.31)$$

$$\bar{\mathcal{I}}_{\alpha(k)}^{\dot{\alpha}(k)}(\tilde{x}_{\bar{1}3}) = \varepsilon_{\alpha_1\beta_1} \dots \varepsilon_{\alpha_k\beta_k} \bar{\mathcal{I}}^{\dot{\alpha}(k)\beta(k)}(\tilde{x}_{\bar{1}3}), \quad (3.32)$$

By rearranging the operators in the three-point function and taking into account their Grassmann parities, one may express the left-hand side of (3.27) as

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle &= (-1)^k \langle S_{\alpha(k)}(z_2) L(z_3) \bar{S}_{\dot{\alpha}(k)}(z_1) \rangle \\ &= \frac{(-1)^k}{k_2} \mathcal{I}_{\alpha(k)\dot{\gamma}(k)}(x_{2\bar{1}}) \tilde{H}^{\dot{\gamma}(k)}_{\dot{\alpha}(k)}(\mathbf{X}_1, \Theta_1, \bar{\Theta}_1), \\ k_2 &:= (x_{\bar{1}2}^2)^{(k+2)/2} x_{2\bar{1}}^2 x_{\bar{1}3}^2 x_{3\bar{1}}^2, \end{aligned} \quad (3.33)$$

for some function $\tilde{H}^{\dot{\gamma}(k)}_{\dot{\alpha}(k)}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1)$. Comparing eqs. (3.27) and (3.33) allows us to relate H and $\tilde{H}$

$$\begin{aligned} \tilde{H}^{\dot{\gamma}(k)}_{\dot{\alpha}(k)}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1) &= \frac{(-1)^k k_2}{k_1} \bar{\mathcal{I}}^{\dot{\gamma}(k)\beta(k)}(\tilde{\mathbf{X}}_1) \mathcal{I}_{\beta(k)\dot{\beta}(k)}(x_{1\bar{3}}) \mathcal{I}_{\alpha(k)\dot{\alpha}(k)}(x_{3\bar{1}}) \\ &\quad \times H^{\dot{\beta}(k)\alpha(k)}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \end{aligned} \quad (3.34)$$

where we have used the identity

$$\bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(\tilde{x}_{1\bar{2}}) \mathcal{I}_{\alpha(k)\dot{\beta}(k)}(x_{2\bar{3}}) = \bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(\tilde{\mathbf{X}}_1) \mathcal{I}_{\alpha(k)\dot{\beta}(k)}(x_{1\bar{3}}), \quad (3.35a)$$

with

$$\bar{\mathcal{I}}^{\dot{\alpha}(k)\alpha(k)}(\tilde{\mathbf{X}}) = \frac{1}{\tilde{\mathbf{X}}^k} \bar{\mathbf{X}}^{(\dot{\alpha}_1(\alpha_1) \dots \dot{\alpha}_k(\alpha_k))}. \quad (3.35b)$$

In order to compute the expression (3.34), we further note that

$$\begin{aligned} \mathcal{I}_{\dot{\alpha}(k)}^{\alpha(k)}(x_{3\bar{1}}) \bar{\mathcal{I}}^{\dot{\beta}(k)}(\tilde{x}_{3\bar{1}}) \mathbf{X}_{3(\alpha_1(\dot{\beta}_1) \dots \mathbf{X}_{3\alpha_k(\dot{\beta}_k))} \\ = (-1)^k \frac{\bar{\mathbf{X}}_{1(\beta_1(\dot{\alpha}_1) \dots \bar{\mathbf{X}}_{1\beta_k(\dot{\alpha}_k))}}{\bar{\mathbf{X}}_1^{2k} (x_{1\bar{3}}^2 x_{3\bar{1}}^2)^{k/2}}, \end{aligned} \quad (3.36a)$$

$$(\mathbf{P}_3 \cdot \mathbf{X}_3) = \frac{(\mathbf{P}_1 \cdot \mathbf{X}_1)}{\mathbf{X}_1^2 \bar{\mathbf{X}}_1^2 x_{1\bar{3}}^2 x_{3\bar{1}}^2}, \quad \frac{1}{\mathbf{X}_3^2} = \bar{\mathbf{X}}_1^2 x_{1\bar{3}}^2 x_{3\bar{1}}^2. \quad (3.36b)$$

Direct computation yields

$$\begin{aligned} \tilde{H}^{\dot{\gamma}(k)}_{\dot{\alpha}(k)}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1) \\ = \frac{a_1}{\mathbf{X}_1^2} \delta_{(\dot{\alpha}_1}^{\dot{\gamma}_1} \dots \delta_{\dot{\alpha}_k)}^{\dot{\gamma}_k)} + \frac{b_1}{\mathbf{X}_1^4} \left(\boldsymbol{\Theta}_1^\lambda \bar{\boldsymbol{\Theta}}_1^\lambda \mathbf{X}_{1\lambda\dot{\lambda}} \delta_{(\dot{\alpha}_1}^{(\dot{\gamma}_1} \dots \delta_{\dot{\alpha}_k)}^{\dot{\gamma}_k)} - \frac{k}{k+1} \boldsymbol{\Theta}_1^\lambda \bar{\boldsymbol{\Theta}}_1^{(\dot{\gamma}_1} \mathbf{X}_{1\lambda(\dot{\alpha}_1} \delta_{\dot{\alpha}_2}^{\dot{\gamma}_2} \dots \delta_{\dot{\alpha}_k)}^{\dot{\gamma}_k)} \right), \end{aligned} \quad (3.37)$$

which can again be written in index-free notation as

$$\begin{aligned} \tilde{H}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1; \bar{v}, \bar{w}) \\ = \bar{v}^{\dot{\alpha}_1} \dots \bar{v}^{\dot{\alpha}_k} \bar{w}_{\dot{\gamma}_1} \dots \bar{w}_{\dot{\gamma}_k} \tilde{H}^{\dot{\gamma}(k)}_{\dot{\alpha}(k)}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1) \\ = \frac{a_1}{\mathbf{X}_1^2} (\bar{v} \cdot \bar{w})^k + \frac{b_1}{2\mathbf{X}_1^4} (\bar{v} \cdot \bar{w})^{k-1} \left\{ (\mathbf{P}_1 \cdot \mathbf{X}_1) (\bar{v} \cdot \bar{w}) + \frac{k}{2(k+1)} \mathbf{P}_1^{\dot{\alpha}\alpha} \mathbf{X}_{1\alpha\dot{\beta}} \bar{v}^{\dot{\beta}} \bar{w}_{\dot{\alpha}} \right\}, \end{aligned} \quad (3.38)$$

with $(\bar{v} \cdot \bar{w}) = \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{v}^{\dot{\alpha}} \bar{w}^{\dot{\beta}}$. We now observe that

$$\begin{aligned} \langle S_{\alpha(k)}(z_2) L(z_3) \bar{S}_{\dot{\alpha}(k)}(z_1) \rangle &= [\text{relabel } z_2 \rightarrow z_1, z_3 \rightarrow z_2, z_1 \rightarrow z_3] \\ &= \langle S_{\alpha(k)}(z_1) L(z_2) \bar{S}_{\dot{\alpha}(k)}(z_3) \rangle \\ &= \frac{1}{k_3} \mathcal{I}_{\alpha(k)\dot{\gamma}(k)}(x_{1\bar{3}}) \tilde{H}^{\dot{\gamma}(k)}_{\dot{\alpha}(k)}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \\ k_3 &:= (x_{3\bar{1}}^2)^{(k+2)/2} x_{3\bar{2}}^2 x_{2\bar{3}}^2 x_{1\bar{3}}^2. \end{aligned} \quad (3.39)$$

As a consequence of (2.27), the conservation conditions on the flavour current multiplet are equivalent to

$$\mathcal{Q}^2 \tilde{H}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3; \bar{v}, \bar{w}) = 0, \quad \bar{\mathcal{Q}}^2 \tilde{H}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3; \bar{v}, \bar{w}) = 0. \quad (3.40)$$

It is straightforward to show that (3.38) indeed satisfies (3.40) for arbitrary a_1 and b_1 .

The final step is to impose the reality constraint, (3.7), which requires

$$\begin{aligned} \bar{a}_1 &= (-1)^k a_1, & \bar{b}_1 &= (-1)^{k+1} b_1, \\ \implies a_1 &= i^k A, & b_1 &= i^{k-1} B, \end{aligned} \quad (3.41)$$

where A and B are real. Thus, the higher-spin correlator (3.4) is fixed by the $\mathcal{N} = 1$ superconformal symmetry up to two independent, real coefficients A and B .

As a result, the final form of (3.4) proves to be

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle &= \frac{(x_{3\bar{1}})_{(\dot{\alpha}_1}^{\beta_1} \dots (x_{3\bar{1}})_{\dot{\alpha}_k}^{\beta_k} (x_{2\bar{3}})_{(\alpha_1}^{\dot{\beta}_1} \dots (x_{2\bar{3}})_{\alpha_k}^{\dot{\beta}_k}}{(x_{1\bar{3}}^2)^{k+1} x_{31}^2 x_{23}^2 (x_{32}^2)^{k+1}} \\ &\times H_{\beta_1 \dots \beta_k \dot{\beta}_1 \dots \dot{\beta}_k}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \end{aligned} \quad (3.42a)$$

where the tensor $H_{\beta_1 \dots \beta_k \dot{\beta}_1 \dots \dot{\beta}_k}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3)$ has the compact expression

$$\begin{aligned} H_{(k,k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{w}) &= u^{\beta_1} \dots u^{\beta_k} \bar{w}^{\dot{\beta}_1} \dots \bar{w}^{\dot{\beta}_k} H_{\beta(k) \dot{\beta}(k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) \\ &= i^{k-1} \left(iA \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+2}} + \frac{B}{2(k+1)} \left\{ \frac{(\mathbf{P} \cdot \mathbf{X}) \mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} + k \frac{(\mathbf{P} \cdot \boldsymbol{\kappa}) \mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+4}} \right\} \right). \end{aligned} \quad (3.42b)$$

Setting $k = 1$, direct comparison with (3.3) shows that we have an agreement provided the parameters are related in the following way:

$$c_1 = A, \quad d_1 = -\frac{B}{4}. \quad (3.43)$$

In a more general setting, one may also consider

$$\langle \bar{S}'_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) L(z_3) \rangle, \quad (3.44)$$

in which case the only difference with (3.42a) is that the superfield $\bar{S}'_{\dot{\alpha}(k)}$ is not the complex conjugate of $S_{\alpha(k)}$, i.e. $(S_{\alpha(k)})^* \neq \bar{S}'_{\dot{\alpha}(k)}$. It still obeys the conservation conditions $\bar{D}^{\dot{\alpha}_1} \bar{S}'_{\dot{\alpha}_1 \dots \dot{\alpha}_k} = D^2 \bar{S}'_{\dot{\alpha}(k)} = 0$. Since the reality constraint (3.7) no longer holds, the correlator (3.44) is determined up to two *complex* coefficients, a_1 and b_1 .

4 Correlator $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$

We turn to constructing the three-point function of the higher-spin spinor current multiplets with the supercurrent, namely $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$.³

The $\mathcal{N} = 1$ supercurrent [25] is described by a primary real superfield $J_{\alpha\dot{\alpha}} = \bar{J}_{\alpha\dot{\alpha}}$ obeying the conservation law

$$D^\alpha J_{\alpha\dot{\alpha}} = \bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = 0. \quad (4.1)$$

³The correlator $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(\ell)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$ with $k \neq \ell$ vanishes since it carries a non-trivial R -symmetry charge.

Its superconformal transformation is

$$\delta J_{\alpha\dot{\alpha}} = -\xi J_{\alpha\dot{\alpha}} + (\hat{\omega}_\alpha{}^\beta \delta_\alpha^{\dot{\beta}} + \bar{\hat{\omega}}_{\dot{\alpha}}{}^{\dot{\beta}} \delta_\alpha^\beta) J_{\beta\dot{\beta}} - 3(\sigma + \bar{\sigma}) J_{\alpha\dot{\alpha}} . \quad (4.2)$$

As a consequence of the reality constraint and conservation law, $J_{\alpha\dot{\alpha}}$ has weights $(q, \bar{q}) = (\frac{3}{2}, \frac{3}{2})$ and dimension 3.

The three-point function of the type $\langle \bar{S}_{\dot{\alpha}}(z_1) S_\alpha(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$ was constructed in [51]. It is determined up to three independent, real coefficients:

$$\langle \bar{S}_{\dot{\alpha}}(z_1) S_\alpha(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle = \frac{(x_{3\bar{1}})_{\beta\dot{\alpha}} (x_{2\bar{3}})_{\alpha\dot{\beta}}}{(x_{1\bar{3}}^2)^2 (x_{3\bar{2}}^2)^2 x_{\bar{3}1}^2 x_{2\bar{3}}^2} H^{\dot{\beta}\beta}{}_{\gamma\dot{\gamma}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3) , \quad (4.3a)$$

with

$$\begin{aligned} H^{\dot{\beta}\beta}{}_{\gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) &= \frac{1}{2}(d_2 + d_3) \frac{1}{\mathbf{X}^2} \delta_\gamma^\beta \delta_{\dot{\gamma}}^{\dot{\beta}} + \frac{1}{2}(d_3 + d_4) \frac{1}{(\mathbf{X}^2)^2} \mathbf{X}^{\dot{\beta}\beta} \mathbf{X}_{\gamma\dot{\gamma}} \\ &+ \frac{1}{2} d_4 \frac{i}{(\mathbf{X}^2)^2} \mathbf{X}^{\dot{\beta}\beta} \mathbf{P}_{\gamma\dot{\gamma}} + \frac{1}{2} d_3 \frac{i}{(\mathbf{X}^2)^2} \mathbf{P}^{\dot{\beta}\beta} \mathbf{X}_{\gamma\dot{\gamma}} \\ &- d_2 \frac{i}{(\mathbf{X}^2)^2} (\mathbf{P} \cdot \mathbf{X}) \delta_\gamma^\beta \delta_{\dot{\gamma}}^{\dot{\beta}} - d_3 \frac{i}{(\mathbf{X}^2)^3} (\mathbf{P} \cdot \mathbf{X}) \mathbf{X}^{\dot{\beta}\beta} \mathbf{X}_{\gamma\dot{\gamma}} . \end{aligned} \quad (4.3b)$$

Let us consider $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle$. Using the prescription (2.29), the correlator takes the form

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle &= \frac{(x_{3\bar{1}})_{(\dot{\alpha}_1}{}^{\beta_1} \dots (x_{3\bar{1}})_{\dot{\alpha}_k)}{}^{\beta_k} (x_{2\bar{3}})_{(\alpha_1}{}^{\dot{\beta}_1} \dots (x_{2\bar{3}})_{\alpha_k)}{}^{\dot{\beta}_k}}{(x_{1\bar{3}}^2)^{k+1} x_{\bar{3}1}^2 x_{2\bar{3}}^2 (x_{3\bar{2}}^2)^{k+1}} \\ &\times H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3) . \end{aligned} \quad (4.4)$$

Following similar analysis as in section 3, the tensor $H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}$ is required to satisfy these properties:

- *Homogeneity:*

$$H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\lambda \bar{\lambda} \mathbf{X}, \lambda \boldsymbol{\Theta}, \bar{\lambda} \bar{\boldsymbol{\Theta}}) = \lambda^{-(k+1)} \bar{\lambda}^{-(k+1)} H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) , \quad (4.5)$$

- *Reality:*

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle^* &= \langle \bar{S}_{\dot{\alpha}(k)}(z_2) S_{\alpha(k)}(z_1) J_{\gamma\dot{\gamma}}(z_3) \rangle , \\ \implies \bar{H}_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) &= H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(-\bar{\mathbf{X}}, -\boldsymbol{\Theta}, -\bar{\boldsymbol{\Theta}}) . \end{aligned} \quad (4.6)$$

- *Conservation conditions:*

$$\mathcal{D}^{\beta_1} H_{\beta_1 \dots \beta_k \dot{\beta}(k), \gamma\dot{\gamma}} = 0 , \quad \bar{\mathcal{D}}^2 H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}} = 0 , \quad (4.7a)$$

$$\bar{\mathcal{Q}}^{\dot{\beta}_1} H_{\beta(k)\dot{\beta}_1 \dots \dot{\beta}_k, \gamma\dot{\gamma}} = 0 , \quad \mathcal{Q}^2 H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}} = 0 . \quad (4.7b)$$

Furthermore, the conservation equations of the supercurrent, eq. (4.1), demand that the following be satisfied:

$$D_{(3)}^\gamma \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle = 0, \quad \bar{D}_{(3)}^{\dot{\gamma}} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle = 0. \quad (4.8)$$

First of all, the fact that $H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}}$ is Grassmann even and obeys the differential constraints $\bar{D}^2 H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}} = 0$ and $\mathcal{Q}^2 H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}} = 0$, imply that it has the general form

$$H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) = A_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}}(\mathbf{X}) + B_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma},\delta\dot{\delta}}(\mathbf{X}) \boldsymbol{\Theta}^\delta \bar{\boldsymbol{\Theta}}^{\dot{\delta}}. \quad (4.9a)$$

We then write all possible independent structures consistent with (4.5). This is done by decomposing $A_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}}$ and $B_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma},\delta\dot{\delta}}$ in terms of their irreducible components

$$A_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}} = \frac{a_1}{\mathbf{X}^{2k+2}} \mathbf{X}_{(\gamma\dot{\gamma}} \mathbf{X}_{\beta_1\dot{\beta}_1} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)} + \frac{a_2}{\mathbf{X}^{2k}} \varepsilon_{\gamma(\beta_1} \varepsilon_{\dot{\gamma}(\dot{\beta}_1} \mathbf{X}_{\beta_2\dot{\beta}_2} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)}, \quad (4.9b)$$

$$B_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma},\delta\dot{\delta}} = \frac{b_1}{\mathbf{X}^{2k+4}} \mathbf{X}_{(\gamma\dot{\gamma}} \mathbf{X}_{\delta\dot{\delta}} \mathbf{X}_{\beta_1\dot{\beta}_1} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)} + \frac{b_2}{\mathbf{X}^{2k+2}} \varepsilon_{\gamma(\delta} \varepsilon_{\dot{\gamma}(\dot{\delta}} \mathbf{X}_{\beta_1\dot{\beta}_1} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)} + \frac{b_3}{\mathbf{X}^{2k+2}} \varepsilon_{\delta(\gamma} \varepsilon_{\dot{\delta}(\dot{\gamma}} \mathbf{X}_{\beta_1\dot{\beta}_1} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)} + \frac{b_4}{\mathbf{X}^{2k+2}} \varepsilon_{\gamma\delta} \varepsilon_{\dot{\gamma}\dot{\delta}} \mathbf{X}_{(\beta_1\dot{\beta}_1} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)} + \frac{b_5}{\mathbf{X}^{2k+2}} \varepsilon_{\delta(\gamma} \varepsilon_{\dot{\gamma}(\dot{\delta}} \mathbf{X}_{\beta_1\dot{\beta}_1} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)} + \frac{b_6}{\mathbf{X}^{2k}} \varepsilon_{\gamma(\beta_1} \varepsilon_{|\delta|\beta_2} \varepsilon_{\dot{\gamma}(\dot{\beta}_1} \varepsilon_{|\dot{\delta}|\dot{\beta}_2} \mathbf{X}_{\beta_3\dot{\beta}_3} \cdots \mathbf{X}_{\beta_k)\dot{\beta}_k)}, \quad (4.9c)$$

where a_1, a_2 and $b_1, \dots, b_6$ are arbitrary complex coefficients. Before imposing conservation conditions (4.7), it is useful to contract the indices of $H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}}$ with auxiliary variables. We thus introduce

$$\begin{aligned} H_{(k,k),\gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{w}) &:= u^{\beta_1} \cdots u^{\beta_k} \bar{w}^{\dot{\beta}_1} \cdots \bar{w}^{\dot{\beta}_k} H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}} \\ &= \frac{\tilde{a}_1}{\mathbf{X}^{2k+2}} \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{(1,1)}^k + \frac{\tilde{a}_2}{\mathbf{X}^{2k+2}} \mathcal{K}_{\gamma\dot{\gamma}} \mathbf{X}_{(1,1)}^{k-1} \\ &\quad + \frac{\boldsymbol{\Theta}^\delta \bar{\boldsymbol{\Theta}}^{\dot{\delta}}}{\mathbf{X}^{2k+4}} \left\{ \tilde{b}_1 \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^k + \tilde{b}_2 \mathbf{X}_{\gamma\dot{\delta}} \mathbf{X}_{\delta\dot{\gamma}} \mathbf{X}_{(1,1)}^k + \tilde{b}_3 \mathcal{K}_{\gamma\dot{\gamma}} \mathbf{X}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^{k-1} \right. \\ &\quad \left. + \tilde{b}_4 \mathcal{K}_{\delta\dot{\delta}} \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{(1,1)}^{k-1} + \tilde{b}_5 \mathcal{K}_{\delta\dot{\gamma}} \mathbf{X}_{\gamma\dot{\delta}} \mathbf{X}_{(1,1)}^{k-1} + \tilde{b}_6 \mathcal{K}_{\gamma\dot{\gamma}} \mathcal{K}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^{k-2} \right\}, \end{aligned} \quad (4.10)$$

where $\mathcal{K}_{\gamma\dot{\gamma}}$ is defined the same way as in (3.17), $\mathcal{K}_{\gamma\dot{\gamma}} = u^\alpha \bar{w}^{\dot{\alpha}} \mathbf{X}_{\alpha\dot{\gamma}} \mathbf{X}_{\gamma\dot{\alpha}}$. In this basis, the (tilde) coefficients are related to those in (4.9c) by the rule

$$\tilde{a}_1 = \frac{1}{k+1} a_1 - a_2, \quad \tilde{a}_2 = \frac{k}{k+1} a_1 + a_2, \quad (4.11a)$$

$$\tilde{b}_1 = -\frac{k-1}{(k+1)(k+2)} b_1 - \frac{2k+1}{(k+1)^2} (b_2 + b_3) - b_4 + \frac{k^2 + k + 1}{(k+1)^2} b_5 + b_6, \quad (4.11b)$$

$$\tilde{b}_2 = \frac{1}{k+2}b_1 + \frac{1}{k+1}(b_2 + b_3) + b_4 - b_5, \quad (4.11c)$$

$$\tilde{b}_3 = \frac{2k}{(k+1)(k+2)}b_1 + \frac{2k}{(k+1)^2}b_2 - \frac{k^2}{(k+1)^2}(b_3 + b_5) - b_6, \quad (4.11d)$$

$$\tilde{b}_4 = \frac{2k}{(k+1)(k+2)}b_1 - \frac{k^2}{(k+1)^2}b_2 + \frac{2k}{(k+1)^2}b_3 - \frac{k^2}{(k+1)^2}b_5 - b_6, \quad (4.11e)$$

$$\tilde{b}_5 = \frac{k(k+2)}{(k+1)^2}b_5, \quad (4.11f)$$

$$\tilde{b}_6 = \frac{k(k-1)}{(k+1)^2} \left(\frac{k+1}{k+2}b_1 + b_2 + b_3 + b_5 \right) + b_6. \quad (4.11g)$$

As in the previous section, the first order constraints in (4.7) are equivalent to

$$\mathcal{D}_{(-1,0)}H_{(k,k),\gamma\dot{\gamma}} = 0, \quad \bar{\mathcal{Q}}_{(0,-1)}H_{(k,k),\gamma\dot{\gamma}} = 0. \quad (4.12)$$

Let us first look at the condition $\mathcal{D}_{(-1,0)}H_{(k,k),\gamma\dot{\gamma}} = 0$. With the help of identities (3.26) and considering only the terms linear in $\bar{\Theta}_{\dot{\alpha}}$, we find the following constraints on the coefficients

$$\begin{aligned} k\tilde{a}_1 + \tilde{a}_2 &= -\frac{i}{4}(k\tilde{b}_1 + \tilde{b}_3 - \tilde{b}_4 + \tilde{b}_6), & k\tilde{a}_1 &= -\frac{i}{4}(-k\tilde{b}_2 + \tilde{b}_3 + \tilde{b}_5 + \tilde{b}_6), \\ (k-1)\tilde{a}_2 &= -\frac{i}{4}((k-1)\tilde{b}_3 - 2\tilde{b}_6). \end{aligned} \quad (4.13)$$

After some lengthy calculations, it can be shown that the terms cubic in the Grassmann variables (that is, $\sim \bar{\Theta}^2\Theta^\delta$) result in

$$k(\tilde{b}_1 + \tilde{b}_2) + \tilde{b}_3 + \tilde{b}_5 + \tilde{b}_6 = 0, \quad (k-1)(2\tilde{b}_3 + \tilde{b}_4 + 2\tilde{b}_5) + (k-3)\tilde{b}_6 = 0. \quad (4.14)$$

In a similar way, one can compute $\bar{\mathcal{Q}}_{(0,-1)}H_{(k,k),\gamma\dot{\gamma}}$ and show that the terms linear in Θ lead to the requirements

$$\begin{aligned} k\tilde{a}_1 + \tilde{a}_2 &= -\frac{i}{4}(k\tilde{b}_1 + \tilde{b}_3 - \tilde{b}_4 - k\tilde{b}_5 + \tilde{b}_6), & k\tilde{a}_1 &= -\frac{i}{4}(-k\tilde{b}_2 + \tilde{b}_3 - k\tilde{b}_5 + \tilde{b}_6), \\ (k-1)\tilde{a}_2 &= -\frac{i}{4}((k-1)(\tilde{b}_3 + \tilde{b}_5) - 2\tilde{b}_6), \end{aligned} \quad (4.15)$$

while the terms proportional to $\Theta^2\bar{\Theta}^{\dot{\delta}}$ give

$$k(\tilde{b}_1 + \tilde{b}_2) + \tilde{b}_3 + \tilde{b}_6 = 0, \quad (k-1)(2\tilde{b}_3 + \tilde{b}_4 + \tilde{b}_5) + (k-3)\tilde{b}_6 = 0. \quad (4.16)$$

The system of equations (4.13)–(4.16) turn out to be consistent and can be solved in terms of three independent coefficients which we choose to be $\tilde{a}_2, \tilde{b}_1$ and $\tilde{b}_3$:

$$\begin{aligned} \tilde{b}_5 &= 0, \\ \tilde{a}_1 &= -\frac{1}{4k} \left(4(k-1)\tilde{a}_2 + i(k\tilde{b}_1 + (k+1)\tilde{b}_3) \right), \\ \tilde{b}_2 &= \frac{1}{2k} \left(4i(k-1)\tilde{a}_2 - 2k\tilde{b}_1 - (k+1)\tilde{b}_3 \right), & \tilde{b}_4 &= 2i(k-3)\tilde{a}_2 - \frac{1}{2}(k+1)\tilde{b}_3, \\ \tilde{b}_6 &= \frac{1}{2}(k-1)(\tilde{b}_3 - 4i\tilde{a}_2). \end{aligned} \quad (4.17)$$

Keeping in mind the relations (4.11), one may rewrite (4.17) in the basis (4.9). Choosing the three independent coefficients to be b_1, b_2, b_4 , we find that

$$\begin{aligned} b_5 &= 0, & b_6 &= 0, \\ a_1 &= \frac{i}{4} \left(\frac{k+3}{k+1} b_1 + \frac{k+2}{(k+1)^2} b_2 + b_4 \right), & a_2 &= -\frac{ik}{4(k+1)} b_4, \\ b_3 &= \frac{k+3}{k+2} b_1 + \frac{1}{k+1} b_2. \end{aligned} \quad (4.18)$$

It remains to impose conservation equations on the supercurrent $J_{\gamma\dot{\gamma}}$, eq. (4.8). As in section 3, this can be done by appropriately rearranging the operators in the correlator and transforming the tensor $H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}$. Let us rewrite (4.4) in terms of the $\mathcal{I}$ -operators:

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle &= \frac{1}{k_1} \mathcal{I}_{\beta(k)\dot{\alpha}(k)}(x_{3\bar{1}}) \mathcal{I}_{\alpha(k)\dot{\beta}(k)}(x_{2\bar{3}}) H^{\dot{\beta}(k)\beta(k)}_{\gamma\dot{\gamma}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \\ k_1 &:= (x_{1\bar{3}}^{-2})^{(k+2)/2} (x_{3\bar{2}}^{-2})^{(k+2)/2} x_{3\bar{1}}^{-2} x_{2\bar{3}}^{-2}. \end{aligned} \quad (4.19)$$

On the other hand, the same correlator can be written as

$$\begin{aligned} &\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle \\ &= (-1)^k \langle S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \bar{S}_{\dot{\alpha}(k)}(z_1) \rangle \\ &= \frac{(-1)^k}{k_2} \mathcal{I}_{\alpha(k)\dot{\rho}(k)}(x_{2\bar{1}}) \mathcal{I}_{\gamma\dot{\delta}}(x_{3\bar{1}}) \mathcal{I}_{\delta\dot{\gamma}}(x_{1\bar{3}}) \tilde{H}^{\dot{\rho}(k), \delta\dot{\delta}}_{\dot{\alpha}(k)}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1), \\ k_2 &:= (x_{1\bar{2}}^{-2})^{(k+2)/2} (x_{1\bar{3}}^{-2})^{(k+2)/2} (x_{3\bar{1}}^{-2})^{(k+2)/2} x_{2\bar{1}}^{-2}. \end{aligned} \quad (4.20)$$

Thus, knowing H , the tensor $\tilde{H}$ can be computed using the formula

$$\begin{aligned} &\tilde{H}^{\dot{\mu}(k)}_{\dot{\alpha}(k), \delta\dot{\delta}}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1) \\ &= -\frac{k_2}{k_1} \bar{\mathcal{I}}^{\dot{\mu}(k)\mu(k)}(\bar{\mathbf{X}}_1) \left[\mathcal{I}_{\dot{\alpha}(k)}^{\beta(k)}(x_{3\bar{1}}) \bar{\mathcal{I}}_{\mu(k)}^{\dot{\beta}(k)}(\tilde{x}_{3\bar{1}}) \mathcal{I}_{\delta}^{\gamma}(x_{3\bar{1}}) \bar{\mathcal{I}}_{\dot{\delta}}^{\dot{\gamma}}(\tilde{x}_{3\bar{1}}) \right] \\ &\quad \times H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \end{aligned} \quad (4.21)$$

where the identity (3.35a) has been used.

We now substitute the explicit form of $H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}$ into (4.21). This is given by (4.9), but with $b_5 = b_6 = 0$, in accordance with the constraints (4.18). After some tedious calculations and making use of (3.36), along with the identities

$$\mathcal{I}_{\alpha\dot{\alpha}}(x_{1\bar{3}}) \bar{\boldsymbol{\Theta}}_3^{\dot{\alpha}} = \left(\frac{x_{3\bar{1}}^{-2}}{\bar{\mathbf{X}}_1^2} \right)^{\frac{1}{2}} \frac{\bar{\boldsymbol{\Theta}}_{1\alpha}^I}{x_{1\bar{3}}^{-2}}, \quad \boldsymbol{\Theta}_3^{\alpha} \mathcal{I}_{\alpha\dot{\alpha}}(x_{3\bar{1}}) = \left(\frac{x_{1\bar{3}}^{-2}}{\mathbf{X}_1^2} \right)^{\frac{1}{2}} \frac{\boldsymbol{\Theta}_{1\dot{\alpha}}^I}{x_{3\bar{1}}^{-2}}, \quad (4.22a)$$

$$\boldsymbol{\Theta}_{1\dot{\alpha}}^I := \boldsymbol{\Theta}_1^{\alpha} \mathcal{I}_{\alpha\dot{\alpha}}(-\mathbf{X}_1) = -\frac{\boldsymbol{\Theta}_1^{\alpha} \mathbf{X}_{1\alpha\dot{\alpha}}}{(\mathbf{X}_1^2)^{\frac{1}{2}}}, \quad \bar{\boldsymbol{\Theta}}_{1\alpha}^I := \mathcal{I}_{\alpha\dot{\alpha}}(\bar{\mathbf{X}}_1) \bar{\boldsymbol{\Theta}}_1^{\dot{\alpha}} = \frac{\bar{\mathbf{X}}_{1\alpha\dot{\alpha}}}{(\bar{\mathbf{X}}_1^2)^{\frac{1}{2}}} \bar{\boldsymbol{\Theta}}_1^{\dot{\alpha}}, \quad (4.22b)$$

we obtain the following expression

$$\begin{aligned} &\tilde{H}_{\delta\dot{\delta}}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1; \bar{v}, \bar{w}) \\ &= \bar{v}^{\dot{\alpha}_1} \dots \bar{v}^{\dot{\alpha}_k} \bar{w}_{\dot{\mu}_1} \dots \bar{w}_{\dot{\mu}_k} \tilde{H}^{\dot{\mu}(k)}_{\dot{\alpha}(k), \delta\dot{\delta}}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1) \\ &= \frac{(\bar{v} \cdot \bar{w})^{k-2}}{4(k+1)^2} \left\{ i \left[(k+1)(k+3)b_1 + (k+2)b_2 + (k+1)^2 b_4 \right] (\bar{v} \cdot \bar{w})^2 \frac{\mathbf{X}_{1\delta\dot{\delta}}}{\mathbf{X}_1^4} \right. \end{aligned}$$

$$\begin{aligned}
 & +i \left[k(k+3)b_1 + \frac{k(k+2)}{k+1}b_2 \right] (\bar{v} \cdot \bar{w}) \frac{\mathbf{V}_{1\delta\dot{\delta}}}{\mathbf{X}_1^4} + \left[(k-1)b_1 + \frac{k(k+2)}{k+1}b_2 \right] (\bar{v} \cdot \bar{w})^2 \frac{\mathbf{P}_{1\delta\dot{\delta}}}{\mathbf{X}_1^4} \\
 & + 4(k+1)b_1 (\bar{v} \cdot \bar{w})^2 \frac{(\mathbf{P}_1 \cdot \mathbf{X}_1) \mathbf{X}_{1\delta\dot{\delta}}}{\mathbf{X}_1^6} + \left[8kb_1 + \frac{4k(k+2)}{k+1}b_2 \right] (\bar{v} \cdot \bar{w}) \frac{(\mathbf{P}_1 \cdot \mathbf{X}_1) \mathbf{V}_{1\delta\dot{\delta}}}{\mathbf{X}_1^6} \\
 & - \left[2k(k-1)b_1 - \frac{2k(k+2)}{k+1}b_2 \right] (\bar{v} \cdot \bar{w}) \frac{(\mathbf{P}_1 \cdot \mathbf{V}_1) \mathbf{X}_{1\delta\dot{\delta}}}{\mathbf{X}_1^6} \\
 & + \left[k(k+3)b_1 + \frac{k(k+2)}{k+1}b_2 \right] (\bar{v} \cdot \bar{w}) \frac{\mathbf{P}_1^{\dot{\gamma}\gamma} \mathbf{V}_{1\delta\dot{\gamma}} \mathbf{X}_{1\gamma\dot{\delta}}}{\mathbf{X}_1^6} \\
 & - \left[4k(k-1)b_1 + \frac{2k(k-1)(k+2)}{k+1}b_2 \right] \frac{(\mathbf{P}_1 \cdot \mathbf{V}_1) \mathbf{V}_{1\delta\dot{\delta}}}{\mathbf{X}_1^6} \Big\} . \tag{4.23}
 \end{aligned}$$

We recall that b_1, b_2 and b_4 are independent. We have also defined $\mathbf{V}_{\alpha\dot{\alpha}} := \mathbf{X}_{\alpha\dot{\beta}} \bar{v}_{\dot{\alpha}} \bar{w}^{\dot{\beta}}$.

Going back to eq. (4.20) and relabelling superspace points, we see that

$$\begin{aligned}
 & \langle S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \bar{S}_{\dot{\alpha}(k)}(z_1) \rangle \\
 & = [\text{relabel } z_2 \rightarrow z_1, z_3 \rightarrow z_2, z_1 \rightarrow z_3] \\
 & = \langle S_{\alpha(k)}(z_1) J_{\gamma\dot{\gamma}}(z_2) \bar{S}_{\dot{\alpha}(k)}(z_3) \rangle \tag{4.24} \\
 & = -\frac{1}{k_3} \mathcal{I}_{\alpha(k)\dot{\mu}(k)}(x_{1\bar{3}}) \bar{\mathcal{I}}_{\gamma}^{\dot{\delta}}(\tilde{x}_{3\bar{2}}) \mathcal{I}_{\dot{\gamma}}^{\delta}(x_{3\bar{2}}) \tilde{H}^{\dot{\mu}(k)}_{\dot{\alpha}(k), \delta\dot{\delta}}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1) , \\
 & k_3 := (x_{\bar{3}1}^2)^{(k+2)/2} (x_{\bar{3}2}^2)^{(k+2)/2} (x_{\bar{2}3}^2)^{(k+2)/2} x_{\bar{1}3}^2 .
 \end{aligned}$$

By virtue of eq. (2.27), the conservation law of $J_{\gamma\dot{\gamma}}$ requires that the following equations must hold:

$$\mathcal{Q}^{\delta} \tilde{H}_{\delta\dot{\delta}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3; \bar{v}, \bar{w}) = 0, \quad \bar{\mathcal{Q}}^{\dot{\delta}} \tilde{H}_{\delta\dot{\delta}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3; \bar{v}, \bar{w}) = 0 . \tag{4.25}$$

It is not hard to verify that (4.23) satisfies (4.25) for an arbitrary choice of complex coefficients b_1, b_2, b_4 .

The last step is to impose the reality condition (4.6):

$$\bar{H}_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) = H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(-\bar{\mathbf{X}}, -\boldsymbol{\Theta}, -\bar{\boldsymbol{\Theta}}) . \tag{4.26}$$

Here $\bar{H}_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}})$ means that we are taking the complex conjugate of the expression (4.9). This implies that

$$b_1 = i^k A, \quad b_2 = i^k B, \quad b_4 = i^k C , \tag{4.27}$$

where A, B, C are real coefficients.

The correlator (4.4) is thus fixed up to three real coefficients. It is given by

$$\begin{aligned}
 \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle & = \frac{(x_{3\bar{1}})_{(\dot{\alpha}_1)^{\beta_1}} \dots (x_{3\bar{1}})_{(\dot{\alpha}_k)^{\beta_k}} (x_{2\bar{3}})_{(\alpha_1)^{\dot{\beta}_1}} \dots (x_{2\bar{3}})_{(\alpha_k)^{\dot{\beta}_k}}}{(x_{\bar{1}3}^2)^{k+1} x_{\bar{3}1}^2 x_{\bar{2}3}^2 (x_{\bar{3}2}^2)^{k+1}} \\
 & \times H_{\beta(k)\dot{\beta}(k), \gamma\dot{\gamma}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3) . \tag{4.28a}
 \end{aligned}$$

For convenience, here we rewrite explicitly the form of $H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3)$ in the basis (4.11)

$$\begin{aligned}
 H_{(k,k),\gamma\dot{\gamma}}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{w}) &:= u^{\beta_1} \dots u^{\beta_k} \bar{w}^{\dot{\beta}_1} \dots \bar{w}^{\dot{\beta}_k} H_{\beta(k)\dot{\beta}(k),\gamma\dot{\gamma}} \\
 &= \frac{\tilde{a}_1}{\mathbf{X}^{2k+2}} \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{(1,1)}^k + \frac{\tilde{a}_2}{\mathbf{X}^{2k+2}} \mathcal{K}_{\gamma\dot{\gamma}} \mathbf{X}_{(1,1)}^{k-1} \\
 &\quad + \frac{\boldsymbol{\Theta}^\delta \bar{\boldsymbol{\Theta}}^{\dot{\delta}}}{\mathbf{X}^{2k+4}} \left\{ \tilde{b}_1 \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^k + \tilde{b}_2 \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^k + \tilde{b}_3 \mathcal{K}_{\gamma\dot{\gamma}} \mathbf{X}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^{k-1} \right. \\
 &\quad \left. + \tilde{b}_4 \mathcal{K}_{\delta\dot{\delta}} \mathbf{X}_{\gamma\dot{\gamma}} \mathbf{X}_{(1,1)}^{k-1} + \tilde{b}_6 \mathcal{K}_{\gamma\dot{\gamma}} \mathcal{K}_{\delta\dot{\delta}} \mathbf{X}_{(1,1)}^{k-2} \right\}, \tag{4.28b}
 \end{aligned}$$

where

$$\begin{aligned}
 \tilde{a}_1 &= \frac{i^{k+1}}{4(k+1)^3} \left((k+1)(k+3)A + (k+2)B + (k+1)^3C \right), \\
 \tilde{a}_2 &= \frac{i^{k+1}k}{4(k+1)^3} \left((k+1)(k+3)A + (k+2)B \right), \\
 \tilde{b}_1 &= -\frac{i^k}{(k+1)^3} \left((k+1)(3k+1)A + (2k+1)(k+2)B + (k+1)^3C \right), \\
 \tilde{b}_2 &= \frac{i^k}{(k+1)^2} \left(2(k+1)A + (k+2)B + (k+1)^2C \right), \\
 \tilde{b}_3 &= -\frac{i^k k}{(k+1)^3} \left((k^2-1)A - (k+2)B \right), \\
 \tilde{b}_4 &= \frac{i^k}{(k+1)^3} \left(4k(k+1)A - k(k+2)(k-1)B \right), \\
 \tilde{b}_6 &= \frac{i^k k(k-1)}{(k+1)^3} \left(2(k+1)A + (k+2)B \right). \tag{4.28c}
 \end{aligned}$$

Setting $k=1$ and comparing (4.28) with (4.3), we see that we have an agreement with the parameters related by

$$d_2 = -\frac{3B}{16}, \quad d_3 = -\frac{A}{2}, \quad d_4 = -\frac{1}{8}(3B + 4(A+C)). \tag{4.29}$$

In a more general setting, one may also consider

$$\langle \bar{S}'_{\dot{\alpha}(k)}(z_1) S_{\alpha(k)}(z_2) J_{\gamma\dot{\gamma}}(z_3) \rangle. \tag{4.30}$$

The only difference with (4.28a) is that now $(S_{\alpha(k)})^* \neq \bar{S}'_{\dot{\alpha}(k)}$. Since one can no longer impose the reality constraint (4.6), the correlator (4.30) is determined up to three *complex* coefficients, i.e. b_1, b_2 and b_4 .

5 Correlator $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle$

We consider a three-point function of the higher-spin spinor current multiplets in the form $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle$.⁴ Its general form is given by

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle &= \frac{(x_{3\bar{1}})_{(\dot{\alpha}_1}^{\alpha_1} \dots (x_{3\bar{1}})_{\dot{\alpha}_k}^{\alpha_k} (x_{2\bar{3}})_{(\beta_1}^{\dot{\beta}_1} \dots (x_{2\bar{3}})_{\beta_{k+l}}^{\dot{\beta}_{k+l}}}{(x_{1\bar{3}}^2)^{k+1} x_{31}^2 x_{23}^2 (x_{32}^2)^{k+l+1}} \\ &\quad \times H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \end{aligned} \quad (5.1)$$

with $k, l = 1, 2, \dots$. The tensor $H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}$ is subject to the following constraints:

- *Homogeneity:*

$$H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\lambda \bar{\lambda} \mathbf{X}, \lambda \boldsymbol{\Theta}, \bar{\lambda} \bar{\boldsymbol{\Theta}}) = \lambda^{-(k+2)} \bar{\lambda}^{-(k+2)} H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}), \quad (5.2)$$

- *Symmetry under $z_1 \leftrightarrow z_3$:*

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle &= (-1)^{k+l+kl} \langle \bar{S}_{\dot{\gamma}(l)}(z_3) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\alpha}(k)}(z_1) \rangle \\ &= (-1)^{k+l+kl} \langle \bar{S}_{\dot{\alpha}(l)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(k)}(z_3) \rangle \Big|_{z_1 \leftrightarrow z_3, \dot{\alpha} \leftrightarrow \dot{\gamma}}, \end{aligned} \quad (5.3)$$

where we have taken into account the Grassmann parities of the superfields.

- *Conservation conditions:*

$$\mathcal{D}^{\alpha_1} H_{\alpha_1 \dots \alpha_k \dot{\beta}(k+l),\dot{\gamma}(l)} = 0, \quad \bar{\mathcal{D}}^2 H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)} = 0, \quad (5.4a)$$

$$\bar{\mathcal{Q}}^{\dot{\beta}_1} H_{\alpha(k)\dot{\beta}_1 \dots \dot{\beta}_{k+l},\dot{\gamma}(l)} = 0, \quad \mathcal{Q}^2 H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)} = 0. \quad (5.4b)$$

Upon imposing (5.4), it follows from the symmetry property (5.3) that the conservation conditions at z_3 are automatically satisfied,

$$\bar{D}_{(3)}^{\dot{\gamma}_1} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle = 0, \quad D_{(3)}^2 \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle = 0. \quad (5.5)$$

The constraints $\bar{\mathcal{D}}^2 H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)} = 0$ and $\mathcal{Q}^2 H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)} = 0$ tell us that $H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}$, being Grassmann even, can be expressed in the form

$$H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) = A_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}) + B_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l),\delta\dot{\delta}}(\mathbf{X}) \boldsymbol{\Theta}^\delta \bar{\boldsymbol{\Theta}}^{\dot{\delta}}. \quad (5.6a)$$

Let us then contract the indices using auxiliary variables and list all possible independent structures consistent with (5.2). The independent tensor structures can be constructed by performing decomposition of $A_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X})$ and $B_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l),\delta\dot{\delta}}(\mathbf{X})$ into the irreducible components. We obtain

$$\begin{aligned} &H_{(k,k+l,l)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{v}, \bar{w}) \\ &= u^{\alpha_1} \dots u^{\alpha_k} \bar{w}^{\dot{\beta}_1} \dots \bar{w}^{\dot{\beta}_{k+l}} \bar{v}^{\dot{\gamma}_1} \dots \bar{v}^{\dot{\gamma}_l} \left\{ \frac{a_1}{\mathbf{X}^{2k+2}} \varepsilon_{\dot{\gamma}_1 \dot{\beta}_1} \dots \varepsilon_{\dot{\gamma}_l \dot{\beta}_l} \mathbf{X}_{\alpha_1 \dot{\beta}_{l+1}} \dots \mathbf{X}_{\alpha_k \dot{\beta}_{k+l}} \right. \end{aligned}$$

⁴The correlator $\langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(m)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle$ with $m \neq k+l$ vanishes because it carries a non-trivial R -symmetry charge.

$$\begin{aligned}
 & + \frac{\Theta^\delta \bar{\Theta}^{\dot{\delta}}}{X^{2k+2}} \left(\frac{b_1}{X^2} \varepsilon_{\dot{\gamma}_l \dot{\delta}} \varepsilon_{\dot{\gamma}_1 \dot{\beta}_1} \cdots \varepsilon_{\dot{\gamma}_{l-1} \dot{\beta}_{l-1}} X_{(\dot{\delta} \dot{\beta}_l} X_{\alpha_1 \dot{\beta}_{l+1}} \cdots X_{\alpha_k) \dot{\beta}_{k+l}} \right. \\
 & + \frac{b_2}{X^2} \varepsilon_{\dot{\beta}_l \dot{\delta}} \varepsilon_{\dot{\gamma}_1 \dot{\beta}_1} \cdots \varepsilon_{\dot{\gamma}_{l-1} \dot{\beta}_{l-1}} X_{(\dot{\delta} \dot{\gamma}_l} X_{\alpha_1 \dot{\beta}_{l+1}} \cdots X_{\alpha_k) \dot{\beta}_{k+l}} \\
 & \left. + b_3 \varepsilon_{\delta \alpha_1} \varepsilon_{\dot{\delta} \dot{\beta}_1} \varepsilon_{\dot{\gamma}_1 \dot{\beta}_2} \cdots \varepsilon_{\dot{\gamma}_l \dot{\beta}_{l+1}} X_{\alpha_2 \dot{\beta}_{l+2}} \cdots X_{\alpha_k \dot{\beta}_{k+l}} \right) \Big\} \\
 & = a_1 (\bar{v} \cdot \bar{w})^l \frac{X_{(1,1)}^k}{X^{2k+2}} + b_1 (\bar{v} \cdot \bar{w})^{l-1} (\Theta X \bar{w}) (\bar{v} \cdot \bar{\Theta}) \frac{X_{(1,1)}^k}{X^{2k+4}} \\
 & + b_2 (\bar{v} \cdot \bar{w})^{l-1} (\Theta X \bar{v}) (\bar{w} \cdot \bar{\Theta}) \frac{X_{(1,1)}^k}{X^{2k+4}} \\
 & + \left(b_3 - \frac{k}{k+1} b_2 \right) (\bar{v} \cdot \bar{w})^l (u \cdot \Theta) (\bar{w} \cdot \bar{\Theta}) \frac{X_{(1,1)}^{k-1}}{X^{2k+2}} .
 \end{aligned} \tag{5.6b}$$

All the coefficients above are complex. Looking at the conformally covariant structures in (5.6b) after contraction with the auxiliary spinors, it might appear that there are more admissible structures which can be added to the list, namely

$$\begin{aligned}
 & (\bar{v} \cdot \bar{w})^l (\Theta X \bar{w}) (u X \bar{\Theta}) \frac{X_{(1,1)}^{k-1}}{X^{2k+2}} , \quad (\bar{v} \cdot \bar{w})^l (\Theta X \bar{\Theta}) \frac{X_{(1,1)}^k}{X^{2k+4}} , \\
 & (\bar{v} \cdot \bar{w})^{l-1} (\Theta X \bar{w}) (\bar{w} \cdot \bar{\Theta}) (u X \bar{v}) \frac{X_{(1,1)}^{k-1}}{X^{2k+4}} .
 \end{aligned} \tag{5.7}$$

However, these structures prove to be linearly dependent of the others. More precisely, one can prove that

$$\begin{aligned}
 & (\bar{v} \cdot \bar{w})^l (\Theta X \bar{w}) (u X \bar{\Theta}) \frac{X_{(1,1)}^{k-1}}{X^{2k+2}} \\
 & = (\bar{v} \cdot \bar{w})^{l-1} \left[(\Theta X \bar{w}) (\bar{v} \cdot \bar{\Theta}) - (\Theta X \bar{v}) (\bar{w} \cdot \bar{\Theta}) \right] \frac{X_{(1,1)}^k}{X^{2k+4}} \\
 & + (\bar{v} \cdot \bar{w})^l (u \cdot \Theta) (\bar{w} \cdot \bar{\Theta}) \frac{X_{(1,1)}^{k-1}}{X^{2k+2}} ,
 \end{aligned} \tag{5.8a}$$

$$\begin{aligned}
 & (\bar{v} \cdot \bar{w})^l (\Theta X \bar{\Theta}) \frac{X_{(1,1)}^k}{X^{2k+4}} \\
 & = (\bar{v} \cdot \bar{w})^{l-1} \left[(\Theta X \bar{w}) (\bar{v} \cdot \bar{\Theta}) - (\Theta X \bar{v}) (\bar{w} \cdot \bar{\Theta}) \right] \frac{X_{(1,1)}^k}{X^{2k+4}} ,
 \end{aligned} \tag{5.8b}$$

$$\begin{aligned}
 & (\bar{v} \cdot \bar{w})^{l-1} (\Theta X \bar{w}) (\bar{w} \cdot \bar{\Theta}) (u X \bar{v}) \frac{X_{(1,1)}^{k-1}}{X^{2k+4}} \\
 & = (\bar{v} \cdot \bar{w})^{l-1} \frac{X_{(1,1)}^{k-1}}{X^{2k+2}} \left[(\Theta X \bar{v}) \frac{X_{(1,1)}}{X^2} - (\bar{v} \cdot \bar{w}) (u \cdot \Theta) \right] (\bar{w} \cdot \bar{\Theta}) .
 \end{aligned} \tag{5.8c}$$

Next, we impose conservation laws (5.4). The first condition, $\mathcal{D}^{\alpha_1} H_{\alpha_1 \dots \alpha_k \dot{\beta}(k+l), \dot{\gamma}(l)} = 0$, is equivalent to requiring

$$\mathcal{D}_{(-1,0,0)} H_{(k,k+l,l)}(X, \Theta, \bar{\Theta}; u, \bar{v}, \bar{w}) = 0 , \quad \mathcal{D}_{(-1,0,0)} := \mathcal{D}^\alpha \frac{\partial}{\partial u^\alpha} . \tag{5.9}$$

Keeping in mind the relations (3.26) and further noting that

$$\mathcal{D}_{(-1,0,0)}\left(\frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+2}}\right) = \mathcal{D}_{(-1,0,0)}\left((\Theta \mathbf{X} \bar{w})(\bar{v} \cdot \bar{\Theta}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}}\right) = 0, \quad (5.10)$$

it can be easily checked that (5.9) gives rise to

$$b_3 = 0. \quad (5.11)$$

The second conservation condition reads

$$\bar{\mathcal{Q}}_{(0,-1,0)} H_{(k,k+l,l)} = 0, \quad \bar{\mathcal{Q}}_{(0,-1,0)} := \bar{\mathcal{Q}}_{\dot{\alpha}} \frac{\partial}{\partial \bar{w}^{\dot{\alpha}}}. \quad (5.12)$$

Using the fact that $b_3 = 0$, we obtain

$$\begin{aligned} 0 &= \bar{\mathcal{Q}}_{(0,-1,0)} H_{(k,k+l,l)} \\ &= [4il(k+1)a_1 - (k+1)b_1 - (k+l+1)b_2] \\ &\quad \times \left\{ (\bar{v} \cdot \bar{w})^{l-1} (\Theta \mathbf{X} \bar{v}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} - \frac{k}{k+1} (\bar{v} \cdot \bar{w})^l (u \cdot \Theta) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+2}} \right\} \\ \implies b_2 &= \frac{k+1}{k+l+1} (4il a_1 - b_1). \end{aligned} \quad (5.13)$$

Imposing conservation laws thus leaves us with only two independent coefficients, which we choose to be a_1 and b_1 .

We turn to analysing the algebraic constraint (5.3). Under the exchange $z_1 \leftrightarrow z_3$, the three-point function possesses the property

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle &= (-1)^{k+l+kl} \langle \bar{S}_{\dot{\gamma}(l)}(z_3) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\alpha}(k)}(z_1) \rangle \\ &= (-1)^{k+l+kl} \langle \bar{S}_{\dot{\alpha}(l)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(k)}(z_3) \rangle \Big|_{z_1 \leftrightarrow z_3, \dot{\alpha} \leftrightarrow \dot{\gamma}}. \end{aligned} \quad (5.14)$$

As per usual, upon expressing the first line of (5.14) in terms of the $\mathcal{I}$ -operators, we have

$$\begin{aligned} &\frac{(-1)^{k+l}}{k_1} \mathcal{I}_{\dot{\alpha}(k)}^{\alpha(k)}(x_{3\bar{1}}) \bar{\mathcal{I}}_{\beta(k+l)}^{\dot{\beta}(k+l)}(\tilde{x}_{32}) H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3) \\ &= (-1)^{kl} \frac{1}{k_2} \mathcal{I}_{\dot{\gamma}(l)}^{\rho(l)}(x_{1\bar{3}}) \bar{\mathcal{I}}_{\beta(k+l)}^{\dot{\rho}(k+l)}(\tilde{x}_{12}) \tilde{H}_{\rho(l)\dot{\rho}(k+l),\dot{\alpha}(k)}(\mathbf{X}_1, \Theta_1, \bar{\Theta}_1), \\ k_1 &:= (x_{1\bar{3}}^2)^{(k+2)/2} x_{3\bar{1}}^2 x_{2\bar{3}}^2 (x_{3\bar{2}}^2)^{(k+l+2)/2}, \\ k_2 &:= x_{1\bar{3}}^2 (x_{3\bar{1}}^2)^{(l+2)/2} x_{2\bar{1}}^2 (x_{1\bar{2}}^2)^{(k+l+2)/2}, \end{aligned} \quad (5.15)$$

which then yields the relation

$$\begin{aligned} &\tilde{H}_{\rho(l)\dot{\rho}(k+l),\dot{\alpha}(k)}(\mathbf{X}_1, \Theta_1, \bar{\Theta}_1) \\ &= (-1)^{kl+l} \frac{k_2}{k_1} \mathcal{I}_{\beta(k+l)\dot{\rho}(k+l)}(\bar{\mathbf{X}}_1) \\ &\quad \times \mathcal{I}_{\dot{\alpha}(k)}^{\alpha(k)}(x_{3\bar{1}}) \bar{\mathcal{I}}_{\beta(k+l)}^{\dot{\beta}(k+l)}(\tilde{x}_{31}) \bar{\mathcal{I}}_{\rho(l)}^{\dot{\gamma}(l)}(\tilde{x}_{31}) H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}_3, \Theta_3, \bar{\Theta}_3). \end{aligned} \quad (5.16)$$

On the other hand, from the second line of (5.14), it follows that

$$\tilde{H}_{\rho(l)\dot{\rho}(k+l),\dot{\alpha}(k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}) = H_{\rho(l)\dot{\rho}(k+l),\dot{\alpha}(k)}(-\bar{\mathbf{X}}, -\boldsymbol{\Theta}, -\bar{\boldsymbol{\Theta}}), \quad (5.17)$$

where (2.32c) has been used.

By virtue of eqs. (5.6b), (5.11) and (5.13), the right-hand side of (5.17) takes the form

$$\begin{aligned} & H_{(l,k+l,k)}(-\bar{\mathbf{X}}, -\boldsymbol{\Theta}, -\bar{\boldsymbol{\Theta}}; u, \bar{v}, \bar{w}) \\ &= u^{\rho(l)} \bar{w}^{\dot{\rho}(k+l)} \bar{v}^{\dot{\alpha}(k)} H_{\rho(l)\dot{\rho}(k+l),\dot{\alpha}(k)}(-\bar{\mathbf{X}}, -\boldsymbol{\Theta}, -\bar{\boldsymbol{\Theta}}) \\ &= (-1)^l \left\{ a_1 (\bar{v} \cdot \bar{w})^k \frac{\bar{\mathbf{X}}_{(1,1)}^l}{\bar{\mathbf{X}}^{2l+2}} - b_1 (\bar{v} \cdot \bar{w})^{k-1} (\boldsymbol{\Theta} \bar{\mathbf{X}} \bar{w}) (\bar{v} \cdot \bar{\boldsymbol{\Theta}}) \frac{\bar{\mathbf{X}}_{(1,1)}^l}{\bar{\mathbf{X}}^{2l+4}} \right. \\ &\quad + \frac{(\bar{v} \cdot \bar{w})^{k-1}}{k+l+1} (b_1 - 4i k a_1) \left[(l+1) (\boldsymbol{\Theta} \bar{\mathbf{X}} \bar{v}) (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) \frac{\bar{\mathbf{X}}_{(1,1)}^l}{\bar{\mathbf{X}}^{2l+4}} \right. \\ &\quad \left. \left. - l (\bar{v} \cdot \bar{w}) (u \cdot \boldsymbol{\Theta}) (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) \frac{\bar{\mathbf{X}}_{(1,1)}^{l-1}}{\bar{\mathbf{X}}^{2l+2}} \right] \right\}. \end{aligned} \quad (5.18)$$

Direct computation of (5.16) gives rise to

$$\begin{aligned} & \tilde{H}_{(l,k+l,k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{v}, \bar{w}) \\ &= u^{\rho(l)} \bar{w}^{\dot{\rho}(k+l)} \bar{v}^{\dot{\alpha}(k)} \frac{(-1)^{kl+l}}{\mathbf{X}^2 \bar{\mathbf{X}}^{2l}} \\ &\quad \times \left\{ a_1 \varepsilon_{\dot{\alpha}_1 \dot{\rho}_1} \dots \varepsilon_{\dot{\alpha}_k \dot{\rho}_k} \bar{\mathbf{X}}_{\rho_1 \dot{\rho}_{k+1}} \dots \bar{\mathbf{X}}_{\rho_l \dot{\rho}_{k+l}} \right. \\ &\quad + \frac{b_1}{\mathbf{X}^2} \boldsymbol{\Theta}^\delta \bar{\boldsymbol{\Theta}}^\delta \varepsilon_{\dot{\alpha}_1 \dot{\rho}_1} \dots \varepsilon_{\dot{\alpha}_k \dot{\rho}_k} \mathbf{X}_{|\delta| \dot{\rho}_{k+1}} \bar{\mathbf{X}}_{\rho_1 |\delta|} \bar{\mathbf{X}}_{\rho_2 \dot{\rho}_{k+2}} \dots \bar{\mathbf{X}}_{\rho_l \dot{\rho}_{k+l}} \\ &\quad - \frac{1}{\mathbf{X}^2} \frac{(4i l a_1 - b_1)}{k+l+1} \left[(k+1) \varepsilon_{\dot{\alpha}_1 \dot{\rho}_1} \dots \varepsilon_{\dot{\alpha}_k \dot{\rho}_k} \mathbf{X}^{\delta \delta} \boldsymbol{\Theta}_{|\delta|} \bar{\boldsymbol{\Theta}}_{\dot{\rho}_{k+1}} \bar{\mathbf{X}}_{\rho_1 \dot{\rho}_{k+2}} \dots \bar{\mathbf{X}}_{\rho_{l-1} \dot{\rho}_{k+l}} \bar{\mathbf{X}}_{\rho_l |\delta|} \right. \\ &\quad \left. \left. - k \bar{\mathbf{X}}_{\rho_1 \dot{\rho}_1} \dots \bar{\mathbf{X}}_{\rho_l \dot{\rho}_l} \boldsymbol{\Theta}^\delta \bar{\boldsymbol{\Theta}}^\delta \mathbf{X}_{|\delta| \dot{\alpha}_1} \varepsilon_{|\delta| \dot{\rho}_{l+1}} \varepsilon_{\dot{\alpha}_2 \dot{\rho}_{l+2}} \dots \varepsilon_{\dot{\alpha}_k \dot{\rho}_{k+l}} \right] \right\}. \end{aligned} \quad (5.19)$$

In order to compare this result with eq. (5.18) we will use eq. (2.21) to express $\mathbf{X}$ in terms of $\bar{\mathbf{X}}$. This gives

$$\begin{aligned} & \tilde{H}_{(l,k+l,k)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{v}, \bar{w}) \\ &= (-1)^{kl+l} \left\{ a_1 (\bar{v} \cdot \bar{w})^k \frac{\bar{\mathbf{X}}_{(1,1)}^l}{\bar{\mathbf{X}}^{2l+2}} + (4i a_1 + b_1) (\bar{v} \cdot \bar{w})^{k-1} (\boldsymbol{\Theta} \bar{\mathbf{X}} \bar{w}) (\bar{v} \cdot \bar{\boldsymbol{\Theta}}) \frac{\bar{\mathbf{X}}_{(1,1)}^l}{\bar{\mathbf{X}}^{2l+4}} \right. \\ &\quad - \frac{(\bar{v} \cdot \bar{w})^{k-1}}{k+l+1} (b_1 + 4i(k+1)a_1) \left[(l+1) (\boldsymbol{\Theta} \bar{\mathbf{X}} \bar{v}) (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) \frac{\bar{\mathbf{X}}_{(1,1)}^l}{\bar{\mathbf{X}}^{2l+4}} \right. \\ &\quad \left. \left. - l (\bar{v} \cdot \bar{w}) (u \cdot \boldsymbol{\Theta}) (\bar{w} \cdot \bar{\boldsymbol{\Theta}}) \frac{\bar{\mathbf{X}}_{(1,1)}^{l-1}}{\bar{\mathbf{X}}^{2l+2}} \right] \right\}. \end{aligned} \quad (5.20)$$

Equating (5.18) and (5.20), it can be seen that the coefficients are constrained by

$$\begin{aligned} a_1 &= (-1)^{kl} a_1, \\ b_1 &= (-1)^{kl+1} (4ia_1 + b_1), \quad b_1 - 4ika_1 = (-1)^{kl+1} (b_1 + 4i(k+1)a_1). \end{aligned} \quad (5.21)$$

As a result, we have two scenarios to consider. The first case is when both k and l are odd. Here $a_1 = 0$ and b_1 is an arbitrary complex parameter. The other case is when at least one of them is even, for which both parameters are now non-vanishing but related by $a_1 = \frac{ib_1}{2}$.

The three-point correlator of the higher-spin spinor current multiplets is thus determined up to a single complex coefficient $b_1 \equiv B$. It has the following structure

$$\begin{aligned} \langle \bar{S}_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}_{\dot{\gamma}(l)}(z_3) \rangle &= \frac{(x_{3\bar{1}})^{\alpha_1} \dots (x_{3\bar{1}})^{\alpha_k} (x_{2\bar{3}})_{\beta_1}^{\dot{\beta}_1} \dots (x_{2\bar{3}})_{\beta_{k+l}}^{\dot{\beta}_{k+l}}}{(x_{1\bar{3}}^2)^{k+1} x_{\bar{3}1}^2 x_{2\bar{3}}^2 (x_{\bar{3}2}^2)^{k+l+1}} \\ &\times H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}(\mathbf{X}_3, \boldsymbol{\Theta}_3, \bar{\boldsymbol{\Theta}}_3), \end{aligned} \quad (5.22a)$$

where the functional form of $H_{\alpha(k)\dot{\beta}(k+l),\dot{\gamma}(l)}$ depends on the values of k and l . There are two different cases to consider:

- $(k, l) = (odd, odd)$

$$\begin{aligned} H_{(k,k+l,l)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{v}, \bar{w}) &= B(\bar{v} \cdot \bar{w})^{l-1} \left\{ (\boldsymbol{\Theta} \mathbf{X} \bar{w})(\bar{v} \cdot \bar{\boldsymbol{\Theta}}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} \right. \\ &\quad \left. + \frac{(\bar{w} \cdot \bar{\boldsymbol{\Theta}})}{k+l+1} \left[(k+1)(\boldsymbol{\Theta} \mathbf{X} \bar{v}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} - k(\bar{v} \cdot \bar{w})(u \cdot \boldsymbol{\Theta}) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+2}} \right] \right\}. \end{aligned} \quad (5.22b)$$

- $(k, l) = (even, even), (odd, even), (even, odd)$

$$\begin{aligned} H_{(k,k+l,l)}(\mathbf{X}, \boldsymbol{\Theta}, \bar{\boldsymbol{\Theta}}; u, \bar{v}, \bar{w}) &= B(\bar{v} \cdot \bar{w})^{l-1} \left\{ \frac{i}{2}(\bar{v} \cdot \bar{w}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+2}} + (\boldsymbol{\Theta} \mathbf{X} \bar{w})(\bar{v} \cdot \bar{\boldsymbol{\Theta}}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} \right. \\ &\quad \left. + \frac{2l+1}{k+l+1}(\bar{w} \cdot \bar{\boldsymbol{\Theta}}) \left[(k+1)(\boldsymbol{\Theta} \mathbf{X} \bar{v}) \frac{\mathbf{X}_{(1,1)}^k}{\mathbf{X}^{2k+4}} - k(\bar{v} \cdot \bar{w})(u \cdot \boldsymbol{\Theta}) \frac{\mathbf{X}_{(1,1)}^{k-1}}{\mathbf{X}^{2k+2}} \right] \right\}. \end{aligned} \quad (5.22c)$$

In a more general setting, one may also consider

$$\langle \bar{S}'_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}''_{\dot{\gamma}(l)}(z_3) \rangle, \quad (5.23)$$

where the correlator now does not satisfy (5.3) under the exchange $z_1 \leftrightarrow z_3$. As a result, one has to explicitly check the conservation equation at z_3 , that is

$$\bar{D}_{(3)}^{\dot{\gamma}_1} \langle \bar{S}'_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}''_{\dot{\gamma}(l)}(z_3) \rangle = 0, \quad D_{(3)}^2 \langle \bar{S}'_{\dot{\alpha}(k)}(z_1) S_{\beta(k+l)}(z_2) \bar{S}''_{\dot{\gamma}(l)}(z_3) \rangle = 0. \quad (5.24)$$

Making use of the relation (5.15) and the expression of $\tilde{H}_{\rho(l)\dot{\rho}(k+l),\dot{\alpha}(k)}(\mathbf{X}_1, \boldsymbol{\Theta}_1, \bar{\boldsymbol{\Theta}}_1)$ which has been computed previously in (5.20), it can be shown that (5.24) holds. Hence, the correlator (5.23) is determined up to two *complex* coefficients, a_1 and b_1 .

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