Mimetic Einstein-Cartan-Sciama-Kibble (ECSK) gravity

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ABSTRACT: In this paper, we formulate the Mimetic theory of gravity in first-order formalism for differential forms, i.e., the mimetic version of Einstein-Cartan-Sciama-Kibble (ECSK) gravity. We consider different possibilities on how torsion is affected by Weyl transformations and discuss how this translates into the interpolation between two different Weyl transformations of the spin connection, parameterized with a zero-form parameter λ . We prove that regardless of the type of transformation one chooses, in this setting torsion remains as a non-propagating field. We also discuss the conservation of the mimetic stress-energy tensor and show that the trace of the total stress-energy tensor is not null but depends on both, the value of λ and spacetime torsion.

KEYWORDS: Classical Theories of Gravity, Cosmology of Theories beyond the SM

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1 Introduction

General Relativity (GR) is a classical field theory describing the gravitational interaction through the Einstein field equations. Remarkably, it has proven success in a wide range of phenomena [1], including black-holes as realistic astrophysical objects [2] and the existence of gravitational waves [3–5]. Another significant development of GR lies in the context of cosmology, where extending Einstein’s field equations, by the inclusion of an early inflationary stage and a cold dark matter contribution, is in good agreement with observational data [6]. This fact is somehow dramatic since we know very little about the nature of dark matter, apart from the fact that it does not appear to interact with the electromagnetic field. For this reason, the problem of identifying dark matter candidates attracts attention from both cosmology and particle physics. There are several dark matter candidates: weakly interacting massive particles, sterile neutrinos, axions, cold massive halo objects, and primordial black holes [7–9]. Nevertheless, despite many experiments to directly detect and study dark matter particles being actively undertaken, none have succeeded.

Given the difficulties for the standard cosmology models to describe the nature of dark matter (See, for instance, [10]), there has been a popular trend for considering modified gravity models [11–16]. Most of these models attempt to account for all observations without invoking supplemental non-baryonic matter, but observational constraints make it hard to create a consistent model of modified gravity still compatible with GR’s principles.

Another exciting direction is to consider GR beyond the limits of Riemannian geometry. The canonical model generalizing GR is the Einstein-Cartan-Sciama-Kibble (ECSK) gravity. This modification came firstly with the works of Elie Cartan in 1922, before the discovery of spin. However, Cartan’s model did not bring much attention until the late 1950s,

where Sciama and Kibble rediscovered Cartan’s results [17, 18]. The main feature of ECSK gravity is that it accounts for spacetime torsion [19]. In ECSK, quantum mechanical spin acts as the source of torsion,¹ in the same way as energy is a source of curvature [21, 22]. Consequently, torsion could have been relevant in the early universe because of its extremely high fermion-densities [23–30]. Another interesting observation in standard ECSK is that the torsion two-form does not propagate in a vacuum, and it interacts very weakly with Standard Model fermions (See Chap. 8.4 of ref. [31] and refs. [32–34]). Thus, torsion might potentially be a component of dark matter [35–40].

More recently, in [41, 42], Chamseddine and Mukhanov have considered a different approach for addressing dark matter with the Mimetic gravity theory. This model shows that the conformal degree of freedom of the gravitational field becomes dynamical even in the absence of matter. This extra degree of freedom corresponds to the mimetic field’s energy density, which mimics the stress-energy tensor of extra pressureless dust without needing dark matter particles. Moreover, ref. [43] discusses whether mimetic cosmology can create late-time acceleration and the inflationary stage of the universe. Besides, many authors have considered different aspects of mimetic gravity during the last years with exciting results. Some examples are black holes [44–54], black strings [55], and brane-world scenarios [56–61], among others. For a more exhaustive survey, see [62–103] and references therein.

In this paper, we pursue constructing a mimetic theory of gravity in first-order formalism for differential forms. For a sufficiently general family of Weyl transformations for the affine connection, the resulting theory is equivalent to “mimicking” the ECSK model. For the construction, we assume that the vierbein one-form $e^a(x)$ and the spin connection one-form $\omega^{ab}(x)$ describe independent metric and affine properties of the spacetime. We consider diverse ways of generalizing Weyl transformations for a Riemann-Cartan geometry with independent vierbein and spin connection, following a similar approach to [104]. Remarkably, none of these generalizations generate a propagating torsion field. Therefore, in this setting, only a non-vanishing spin tensor can be a source of torsion. When using scalar fields, non-minimal couplings and higher derivatives terms seem to be the way of creating a propagating torsion (See, for instance, [105, 106]). When considering generalized conformal invariance on Riemann-Cartan geometry, the stress-energy tensor’s trace, which usually vanishes for conformally invariant theories of gravity, has a non-zero value depending on torsion and on a parameter which characterizing Weyl transformations for the spin connection.

This paper organization is the following. In section 2, we summarize the main aspects of mimetic gravity. In section 3, we revisit ECSK gravity, and we give a brief description of Cartan’s first order formalism for differential forms. In section 4, we introduce some useful mathematical tools to describe conformal structures in Cartan’s first-order formalism. In section 5, we derive the equations of mimetic gravity in first-order formalism, show how these correspond to the mimetic ECSK model, and analyze the conservation law for the

¹It is important to stress that with spin, we refer only to intrinsic quantum mechanical spin, and we should not confuse this with the angular momentum density. This natural confusion has already led to some mistakes in the literature, see ref. [20].

mimetic stress-energy tensor. Finally, in section 6, we discuss the stress-energy tensor's trace and its dependency on torsion and conformal parameter λ . The paper concludes in 7 with a summary and comments regarding some possible cosmological applications.

2 Review of mimetic gravity

Mimetic Gravity was first introduced by A. Chamseddine and V. Mukhanov as a theory of gravity which naturally exhibits conformal symmetry as internal degree of freedom [41]. Let M_4 be a four dimensional spacetime and let us consider a physical metric $g_{\mu\nu}$, with Lorentz signature $(-, +, +, +)$, depending on an auxiliary metric $\bar{g}_{\mu\nu}$ and a scalar field ϕ , namely

$$g_{\mu\nu} = -\bar{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \bar{g}_{\mu\nu} . \quad (2.1)$$

The metric $g_{\mu\nu}$ is invariant with respect to Weyl transformations of the auxiliary metric $\bar{g}_{\mu\nu}$, i.e., it remains unchanged after rescaling

$$\bar{g}_{\mu\nu} \rightarrow \Omega^2(x) \bar{g}_{\mu\nu} . \quad (2.2)$$

Additionally, it follows from (2.1) that

$$g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi = -1 . \quad (2.3)$$

The resultant new degree of freedom associated with the transformation (2.1) represents the longitudinal mode of gravity which is excited even in the absence of any matter field configurations.

The canonical action of GR is rewritten by considering the physical metric $g_{\mu\nu}$ as function of the scalar field ϕ and the auxiliary metric $\bar{g}_{\mu\nu}$

$$S = \frac{1}{c} \int d^4x \sqrt{-g(\bar{g}_{\mu\nu}, \phi)} \left[\frac{1}{\kappa_4} \left(\frac{1}{2} R(\bar{g}_{\mu\nu}, \phi) - \Lambda \right) + L_m \right] , \quad (2.4)$$

where $\kappa_4 = \frac{8\pi G}{c^4}$, R is the Ricci scalar constructed from $g_{\mu\nu}$ and L_m stands for the matter Lagrangian. The action (2.4) is invariant under Weyl transformations because it only depends on $g_{\mu\nu}$ which is conformally invariant under (2.2). The resulting dynamics can be directly obtained by starting from the variation of (2.4) with respect to the physical metric $g_{\mu\nu}$, then expressing $\delta g_{\mu\nu}$ in terms of $\delta \bar{g}_{\mu\nu}$ and $\delta \phi$ and assuming that the last two are independent. Thus, field equations read

$$G^{\mu\nu} - \kappa_4 \mathcal{T}^{\mu\nu} + (G - \kappa_4 \mathcal{T}) g^{\lambda\mu} g^{\sigma\nu} \partial_\lambda \phi \partial_\sigma \phi = 0 , \quad (2.5)$$

$$\nabla_\mu [(G - \kappa_4 \mathcal{T}) \partial^\mu \phi] = 0 , \quad (2.6)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu}$ is the Einstein tensor, $\mathcal{T}_{\mu\nu}$ the energy momentum tensor, and G , $\mathcal{T}$ denote their respective traces. The dynamics given in (2.5) and (2.6) departs from pure GR.

In ref. [107], an equivalent formulation of Mimetic Gravity has been proposed where, instead of introducing ϕ through the reparametrization (2.4), the physical metric $g_{\mu\nu}$ is

directly used together with a constrained scalar field, enforcing (2.3) through a Lagrange multiplier. Taking the trace in (2.5), direct calculation shows

$$(G - \kappa_4 \mathcal{T}) \left(1 + g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \right) = 0. \quad (2.7)$$

This last equation is automatically satisfied by the constraint (2.3) even for $G \neq \kappa_4 \mathcal{T}$. From this point of view, even in absence of matter, the gravitational field equations have non-trivial solutions for the conformal mode. To understand this extra degree of freedom, let us rewrite eq. (2.5) as

$$G^{\mu\nu} = \kappa_4 (\mathcal{T}^{\mu\nu} + \bar{\mathcal{T}}^{\mu\nu}), \quad (2.8)$$

where

$$\bar{\mathcal{T}}^{\mu\nu} = \left(\mathcal{T} - \frac{G}{\kappa_4} \right) g^{\mu\alpha} g^{\nu\beta} \partial_\alpha \phi \partial_\beta \phi. \quad (2.9)$$

Now compare this expression with the energy momentum tensor for a perfect fluid

$$\mathcal{T}^{\mu\nu} = \frac{1}{c^2} (\rho + p) u^\mu u^\nu - p g^{\mu\nu}, \quad (2.10)$$

where ρ is the energy density, p is the pressure and u^μ is the four-velocity which satisfies $\frac{1}{c^2} u^\lambda u_\lambda = -1$. Setting $p = 0$ and making the following identification

$$\rho = \left(\mathcal{T} - \frac{G}{\kappa_4} \right), \quad (2.11)$$

$$u^\mu = c g^{\mu\alpha} \partial_\alpha \phi, \quad (2.12)$$

the energy momentum tensor (2.10) becomes equivalent to $\bar{\mathcal{T}}^{\mu\nu}$. Thus, the extra degree of freedom mimics the potential motions of dust with energy density $\left(\mathcal{T} - \frac{G}{\kappa_4} \right)$, and the scalar field plays the role of a velocity potential. In absence of matter this energy density is proportional to $G = 4\Lambda - R$, which does not vanish for generic solutions. Note that the normalization condition for the four velocity u^μ and the conservation law for $\bar{\mathcal{T}}^{\mu\nu}$, are equivalent to (2.3) and (2.6), respectively.

3 ECSK gravity and first order formalism

So far we have used Greek indices $\mu, \nu, \dots$ to denote tensor components in the coordinate basis. From now on, we use lower case Latin indices $a, b, \dots$ for tensors defined in Lorentz (orthonormal) basis. We denote by $\Omega^p(M_4)$ to the set of differential p -forms defined over M_4 .

At a particular point $P \in M_4$, the vierbein components $e^a{}_\mu(x)$ are determined through the relation

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu, \quad (3.1)$$

where η_{ab} is the Minkowski metric. In terms of $e^a{}_\mu(x)$ we define the vierbein $e^a = e^a{}_\mu(x) dx^\mu$ as the set of one-forms $\Omega^1(M_4)$. The vierbein contains all the information on the metric. Moreover, the affine properties of geometry are described by the one-form spin connection

$\omega^{ab} = \omega^{ab}_{\mu}(x)dx^{\mu}$, which is anti-symmetric provided the metricity condition $\nabla_{\lambda}g_{\mu\nu} = 0$. Additionally, it relates to the affine connection $\Gamma^{\lambda}_{\mu\nu}$ through the vierbein postulate

$$\partial_{\mu}e^a_{\nu} + \omega^a_{b\mu}e^b_{\nu} - \Gamma^{\lambda}_{\mu\nu}e^a_{\lambda} = 0. \quad (3.2)$$

The covariant derivative of the vierbein is defined as the two-form torsion $T^a = De^a$ where

$$De^a = de^a + \omega^a_b \wedge e^b. \quad (3.3)$$

Unlike $d^2 = 0$, higher order covariant derivatives of the vierbein does not vanish. In fact, direct calculation shows $DT^a = R^a_b \wedge e^b$ where

$$R^{ab} = d\omega^{ab} + \omega^a_c \wedge \omega^{cb}, \quad (3.4)$$

is the Lorentz two-form curvature which transforms covariantly under local Lorentz transformations.

The spin connection can also be decomposed in a torsion-free part $\dot{\omega}^{ab}$ satisfying

$$de^a + \dot{\omega}^a_b \wedge e^b = 0, \quad (3.5)$$

and a second rank anti-symmetric one-form κ^{ab} , usually called the contorsion or contortion. An important observation is that $\dot{\omega}^{ab}$ is completely determined in terms of the vierbein. Therefore, all the affine degree of freedom are encoded into the contorsion

$$\kappa^{ab} = \omega^{ab} - \dot{\omega}^{ab}, \quad (3.6)$$

and consequently $T^a = \kappa^a_b \wedge e^b$. In terms of this splitting, the Lorentz curvature is given by

$$R^{ab} = \mathring{R}^{ab} + \mathring{D}\kappa^{ab} + \kappa^a_c \wedge \kappa^{cb}, \quad (3.7)$$

where $\mathring{R}^{ab} = d\dot{\omega}^{ab} + \dot{\omega}^a_c \wedge \dot{\omega}^{cb}$ is the Riemann curvature two-form and $\mathring{D}$ stands for the covariant derivative with respect to the torsion-free part of the connection $\dot{\omega}^{ab}$.

There are many theories leading to non-vanishing torsion; see for instance refs. [33, 36–39, 105, 108–115], and for the most general case, Poincaré Gauge Theory, see [21, 116–120]). However, the closest to standard General Relativity is Einstein-Cartan-Kibble-Sciama (ECSK) gravity [121–127]. In this framework, geometry also depends of the spin tensor of matter, but its physical effects would be noticeable only for densities much higher than nuclear density [128]. Moreover, it has been shown that torsion prevents cosmological singularities [129–134], gives rise to a new universe from a collapse [23, 135, 136], and introduce an effective ultraviolet cutoff in a quantum field theory for fermions [128].

In differential form language, the ECSK four-form Lagrangian is given by

$$\mathcal{L}_{\text{ECSK}} = \mathcal{L}_G(e, \omega) + \mathcal{L}_M(e, \omega, \varphi), \quad (3.8)$$

where

$$\mathcal{L}_G = \frac{1}{4\kappa_4}\epsilon_{abcd}\left(R^{ab} - \frac{\Lambda}{3!}e^a \wedge e^b\right) \wedge e^c \wedge e^d, \quad (3.9)$$

is the four-form Lagrangian for geometry and $\mathcal{L}_M$ is the four-form Lagrangian for matter fields. Up to boundary terms, variation of the action functional $S = \frac{1}{c} \int_{M_4} \mathcal{L}_{\text{ECSK}}$ reads

$$\delta S = \frac{1}{c} \int_{M_4} \frac{1}{\kappa_4} \left(\frac{1}{2} \delta \omega^{ab} \wedge \mathcal{W}_{ab} + \mathcal{E}_d \wedge \delta e^d \right), \quad (3.10)$$

with

$$\mathcal{E}_d = \epsilon_{abcd} \left(\frac{1}{2} R^{ab} - \frac{\Lambda}{3!} e^a \wedge e^b \right) \wedge e^c - \kappa_4 * \mathcal{T}_d, \quad (3.11)$$

$$\mathcal{W}_{ab} = \epsilon_{abcd} T^c \wedge e^d - \kappa_4 * \sigma_{ab}. \quad (3.12)$$

Here, the relations

$$\delta_e \mathcal{L}_M^{(4)} = - * \mathcal{T}_d \wedge \delta e^d, \quad (3.13)$$

$$\delta_\omega \mathcal{L}_M^{(4)} = - \frac{1}{2} \delta \omega^{ab} \wedge * \sigma_{ab}, \quad (3.14)$$

implicitly define the stress-energy one-form $\mathcal{T}^a = \mathcal{T}^a_\mu dx^\mu$ and the spin tensor 1-form $\sigma^{ab} = \sigma^{ab}_\mu dx^\mu$. Note also that $* : \Omega^p(M_d) \rightarrow \Omega^{d-p}(M_d)$ denotes the Hodge dual operator.

A usual practice when studying this system's phenomenology is to pack all the torsional terms (coming from the Lorentz curvature (3.7)) to create an extra stress-energy tensor in (3.11).

4 Conformal Riemann-Cartan structure

To characterize conformal structures in differential forms language, let us introduce an operator $I_{a_1 \dots a_q} : \Omega^p(M_d) \rightarrow \Omega^{p-q}(M_d)$ defined in four dimensional spacetime $(-, +, +, +)$ by [105]

$$I_{a_1 \dots a_q} = - (-1)^{p(p-q)} * e_{a_1} \wedge \dots \wedge e_{a_q} \wedge *. \quad (4.1)$$

The case $q = 1$ that gives

$$I_a = - * (e_a \wedge *), \quad (4.2)$$

is of particular relevance because it satisfies useful properties. It satisfies the Leibniz rule for differential forms and, together with D , the operator I_a defines another important object $\mathcal{D}_a : \Omega^p(M_d) \rightarrow \Omega^p(M_d)$ via the anti-commutator

$$\mathcal{D}_a = \{I_a, D\} = I_a D + D I_a. \quad (4.3)$$

The operators $\mathcal{D}$, I_a and D , form an open superalgebra where the two-forms curvature and torsion play the role of structure parameters (See [106]).

$$\{I_a, D\} = \mathcal{D}_a, \quad (4.4)$$

$$\{I_a, I_b\} = 0, \quad (4.5)$$

$$\{D, D\} = 2D^2, \quad (4.6)$$

$$[I_a, \mathcal{D}_b] = -T^c_{ab} I_c, \quad (4.7)$$

$$[D, \mathcal{D}_b] = D^2 I_b - I_b D^2, \quad (4.8)$$

$$[\mathcal{D}_a, \mathcal{D}_b] = I_{ab} D^2 + D^2 I_{ab} + I_a D^2 I_b - I_b D^2 I_a - (D T^c_{ab} \wedge I_c + T^c_{ab} \mathcal{D}_c). \quad (4.9)$$

The Weyl transformation

$$g_{\mu\nu} = \exp(2\varepsilon) \bar{g}_{\mu\nu}, \quad (4.10)$$

that relates the spacetime and the auxiliary manifolds supposes implicitly that a local mapping $\gamma : M_4 \rightarrow \bar{M}_4$ has been chosen in such a way that the same coordinates x^μ can be used for $P \in M_4$ and $\bar{P} = \gamma(P) \in \bar{M}_4$. This means that a coordinate transformation $x'^\mu = x'^\mu(x^\nu)$ in M_4 induces the same transformation in $\bar{M}_4$ and thus, tensors or forms defined on these manifolds transforms with the same Jacobian matrices. This fact allows us to find the relation between the vielbeins associated with these metrics, which by definition satisfy

$$g_{\mu\nu} = e_\mu^a e_\nu^a \eta_{ab}, \quad (4.11)$$

$$\bar{g}_{\mu\nu} = \bar{e}_\mu^a \bar{e}_\nu^a \eta_{ab}. \quad (4.12)$$

Indeed, mixing these expressions together with (4.10), it is direct to see that

$$e^a = \exp(\varepsilon) \bar{e}^a. \quad (4.13)$$

From the vierbein $e^a(x)$, it is possible to define “structure parameters” $\mathcal{C}_{ab}{}^c(x)$ satisfying a generalized Maurer-Cartan equation.

$$de^c = -\frac{1}{2} \mathcal{C}_{ab}{}^c e^a \wedge e^b. \quad (4.14)$$

This parameter allow us to solve the torsion-free part of the spin connection

$$\dot{\omega}_{ab} = \frac{1}{2} (\mathcal{C}_{abc} + \mathcal{C}_{cba} - \mathcal{C}_{cab}) e^c. \quad (4.15)$$

Using eqs. (4.13), (4.14) and (4.15), we find

$$\dot{\omega}_{ab} = \bar{\omega}_{ab} + \bar{e}_a \xi_b - \xi_a \bar{e}_b, \quad (4.16)$$

where

$$\xi^a = \bar{\Gamma}^a d\varepsilon, \quad (4.17)$$

and

$$\bar{\Gamma}_a = -\bar{*}(\bar{e}_a \wedge \bar{*}). \quad (4.18)$$

Note that $\bar{*}$ denotes $\bar{e}^a$ -vierbein dependence in the Hodge dual operator. In this way, eq. (4.16) characterizes the Weyl transformation associated to the torsion free part of the spin connection.

Notice that we have no information yet on the Weyl transformation of the contorsion κ^{ab} . This is due to the fact that in the context of Riemann-Cartan geometry, e^a and κ^{ab} are completely independent degrees of freedom. Therefore, there are multiple possible choices on how κ^{ab} should transform under a Weyl transformation. An important family of choices can be parameterized as

$$\bar{e}^a \rightarrow e^a = \exp(\varepsilon) \bar{e}^a, \quad (4.19)$$

$$\bar{\kappa}_{ab} \rightarrow \kappa_{ab} = \bar{\kappa}_{ab} + (\lambda - 1) \theta_{ab}, \quad (4.20)$$

$$\bar{\omega}_{ab} \rightarrow \omega_{ab} = \bar{\omega}_{ab} + \lambda \theta_{ab}, \quad (4.21)$$

where λ is a parameter $0 \leq \lambda \leq 1$ and $\theta^{ab} = -\theta^{ba}$ corresponds to the 1-form

$$\theta^{ab} = \bar{e}^a \xi^b - \xi^a \bar{e}^b. \quad (4.22)$$

The case $\lambda = 1$ implies,

$$\kappa_{ab} = \bar{\kappa}_{ab}, \quad (4.23)$$

$$\omega_{ab} = \bar{\omega}_{ab} + \theta_{ab}, \quad (4.24)$$

which is the “canonical case”: the full spin connection changes as the torsionless case, and the contorsion is left untouched by the Weyl transformation. The most “exotic” case corresponds to $\lambda = 0$

$$\kappa_{ab} = \bar{\kappa}_{ab} - \theta_{ab}, \quad (4.25)$$

$$\omega_{ab} = \bar{\omega}_{ab}, \quad (4.26)$$

where the spin connection is left untouched by the Weyl transformation and the contorsion absorbs the transformation.

It is clear that the torsionless condition is preserved only for the $\lambda = 1$ case. In fact, the Lorentz curvature and torsion change under the generalized Weyl transformation (4.19)–(4.21) as

$$\bar{T}_a \rightarrow T_a = \exp(\varepsilon) \bar{T}_a + (\lambda - 1) \bar{e}_a \wedge d \exp(\varepsilon), \quad (4.27)$$

$$\bar{R}^{ab} \rightarrow R^{ab} = \bar{R}^{ab} + \lambda \bar{D} \theta^{ab} + \lambda^2 \theta^a_c \wedge \theta^{cb}. \quad (4.28)$$

5 Mimetic ECSK gravity

Mimetic transformations are a particular choice of Weyl transformations. Let us consider the auxiliary vierbein $\bar{e}^a$ and spin connection $\bar{\omega}^{ab}$ 1-forms, and a scalar field $\phi(x)$. In terms of operators $\bar{\mathbb{I}}_a$ and ϕ , let us define a zero-form Lorentz vector

$$\bar{Z}_a = \bar{\mathbb{I}}_a d\phi, \quad (5.1)$$

and the scalar

$$\bar{Z}^2 = -\eta_{ab} \bar{Z}^a \bar{Z}^b. \quad (5.2)$$

The generalized mimetic vierbein e^a , contorsion κ^{ab} , and spin connection ω^{ab} are defined by

$$\bar{e}^a \rightarrow e^a = \bar{Z} \bar{e}^a, \quad (5.3)$$

$$\bar{\kappa}_{ab} \rightarrow \kappa_{ab} = \bar{\kappa}_{ab} + (\lambda - 1) \theta_{ab}, \quad (5.4)$$

$$\bar{\omega}_{ab} \rightarrow \omega_{ab} = \bar{\omega}_{ab} + \lambda \theta_{ab}, \quad (5.5)$$

where the one-form θ^{ab} is given in (4.22) with

$$\xi^a = \frac{1}{\bar{Z}} \bar{\mathbb{I}}^a d\bar{Z}. \quad (5.6)$$

Notice that I_a and $\bar{I}_a$ operator relate each other by

$$I_a = \frac{1}{\bar{Z}} \bar{I}_a, \quad (5.7)$$

and consequently

$$Z_a = \frac{1}{\bar{Z}} \bar{Z}_a, \quad (5.8)$$

so the constraint (2.3) reads

$$Z^2 = -\eta_{ab} Z^a Z^b = 1. \quad (5.9)$$

5.1 Mimetic field equations

To construct the mimetic version of ECSK theory, let us consider the Lagrangian (3.8), with the vierbein e^a and ω^{ab} in terms of the auxiliary variables $\bar{e}^a$, $\bar{\omega}^{ab}$ and ϕ as in eqs. (5.3)–(5.5). A priori, it would seem that different choices of λ would lead us to different dynamics. In particular, the canonical and exotic choices $\lambda = 1$ and $\lambda = 0$ seem to lead to completely different theories. However, nothing is further from truth. The dynamics of the generalized mimetic theory is the same regardless of the choice of λ . Since

$$e^a = \bar{Z} \bar{e}^a, \quad (5.10)$$

$$\omega_{ab} = \bar{\omega}_{ab} + \lambda (\bar{e}_a \xi_b - \xi_a \bar{e}_b), \quad (5.11)$$

we have that the functional variations of the vierbein and the spin connection are given by

$$\delta_{\bar{\omega}} e^d = 0, \quad (5.12)$$

$$\delta_{\bar{e}} e^d = \delta_{\bar{e}} \bar{Z} \bar{e}^d + \bar{Z} \delta \bar{e}^d, \quad (5.13)$$

$$\delta_{\phi} e^d = \delta_{\phi} \bar{Z} \bar{e}^d, \quad (5.14)$$

and

$$\delta_{\bar{\omega}} \omega_{ab} = \delta \bar{\omega}_{ab}, \quad (5.15)$$

$$\delta_{\bar{e}} \omega_{ab} = \lambda (\delta \bar{e}_a \xi_b - \xi_a \delta \bar{e}_b) + \lambda (\bar{e}_a \delta_{\bar{e}} \xi_b - \delta_{\bar{e}} \xi_a \bar{e}_b), \quad (5.16)$$

$$\delta_{\phi} \omega_{ab} = \lambda (\bar{e}_a \delta_{\phi} \xi_b - \delta_{\phi} \xi_a \bar{e}_b). \quad (5.17)$$

Notice that we need special care when performing the functional variation $\delta \bar{Z}^a$. In fact, from definition (5.1), it is clear that $\bar{Z} = \bar{Z}(\bar{e}, \partial\phi)$. Therefore, we have to consider independent variations of $\bar{Z}^a$ with respect to both, the vierbein $\bar{e}^a(x)$ and the scalar field $\phi(x)$. Since

$$\delta \bar{Z} = -\frac{1}{\bar{Z}} \bar{Z}^a \delta \bar{Z}_a, \quad (5.18)$$

it is possible to prove that

$$\delta_{\bar{e}} \bar{Z} = \bar{Z}^2 Z_a Z_b I^a (\delta \bar{e}^b), \quad (5.19)$$

$$\delta_{\phi} \bar{Z} = -\bar{Z} Z^a I_a d\phi. \quad (5.20)$$

Replacing (5.19)–(5.20) into (5.13)–(5.14) we get the expressions

$$\delta_{\bar{\omega}} e^d = 0, \quad (5.21)$$

$$\delta_{\bar{e}} e^d = \bar{Z} \left[Z_a Z_b \Gamma^a \left(\delta \bar{e}^b \right) e^d + \delta \bar{e}^d \right], \quad (5.22)$$

$$\delta_{\phi} e^d = -e^d Z^a \Gamma_a d\phi. \quad (5.23)$$

Up to boundary terms, variation of (3.8) reads

$$\delta_{\bar{e}} \mathcal{L}_{\text{ECSK}}^{(4)} = \frac{1}{\kappa_4} \left[\frac{1}{2} \delta_{\bar{e}} \omega^{ab} \wedge \mathcal{W}_{ab} + \mathcal{E}_d \wedge \delta_{\bar{e}} e^d \right] = 0, \quad (5.24)$$

$$\delta_{\phi} \mathcal{L}_{\text{ECSK}}^{(4)} = \frac{1}{\kappa_4} \left[\frac{1}{2} \delta_{\phi} \omega^{ab} \wedge \mathcal{W}_{ab} + \mathcal{E}_d \wedge \delta_{\phi} e^d \right] = 0, \quad (5.25)$$

$$\delta_{\bar{\omega}} \mathcal{L}_{\text{ECSK}} = \frac{1}{2} \frac{1}{\kappa_4} \delta_{\bar{\omega}} \omega^{ab} \wedge \mathcal{W}_{ab} = 0, \quad (5.26)$$

where the three-forms $\mathcal{E}_a$ and $\mathcal{W}_{ab}$ are given in (3.11) and (3.12) respectively. Since $\delta_{\bar{\omega}} \omega_{ab} = \delta \bar{\omega}_{ab}$, eq. (5.26) implies $\delta \bar{\omega}^{ab} \wedge \mathcal{W}_{ab} = 0$ and consequently

$$\mathcal{W}_{ab} = 0, \quad (5.27)$$

just as in the standard ECSK model. Inserting $\mathcal{W}_{ab} = 0$ in the equations of motion, we are left with

$$\delta_{\bar{e}} \mathcal{L}_{\text{ECSK}}^{(4)} = \frac{1}{\kappa_4} \mathcal{E}_d \wedge \delta_{\bar{e}} e^d = 0, \quad (5.28)$$

$$\delta_{\phi} \mathcal{L}_{\text{ECSK}}^{(4)} = \frac{1}{\kappa_4} \mathcal{E}_d \wedge \delta_{\phi} e^d = 0. \quad (5.29)$$

From here, using the expressions (5.22) and (5.23), and integrating by parts in Γ^a and d , we get the set of mimetic ECSK field equations

$$\mathcal{E}_d - Z_a Z_d \Gamma^a (\mathcal{E}_m \wedge e^m) = 0, \quad (5.30)$$

$$d \left[Z^a \Gamma_a (\mathcal{E}_d \wedge e^d) \right] = 0, \quad (5.31)$$

$$\mathcal{W}_{ab} = 0. \quad (5.32)$$

It is remarkable that they do not depend on the choice of λ . For the mimetic theory, all the choices of Weyl transformations for the contorsion lead to the same dynamics.

In order to study the equivalence of these equations written using tensors, it is useful to consider Hodge duality between p -forms and $(d-p)$ -forms. For a three-form $\mathcal{E}_d$ in four dimensions, we have

$$\mathcal{E}_d = \mathcal{E}_{md} * e^m. \quad (5.33)$$

It is straightforward to prove that

$$\mathcal{E}_q \wedge e^q = -\mathcal{E}^p_p v_{(4)}, \quad (5.34)$$

$$\Gamma^m v_{(4)} = *e^m, \quad (5.35)$$

where $v_{(4)}$ denotes the volume form in four-dimensions. Replacing these relations into the field equations (5.30)–(5.32) it is possible to write them as

$$\mathcal{E}_d - * (Z_d \mathcal{E}^p_p d\phi) = 0, \quad (5.36)$$

$$-d * [\mathcal{E}^p_p d\phi] = 0, \quad (5.37)$$

$$\mathcal{W}_{ab} = 0. \quad (5.38)$$

Remarkably, eqs. (5.36)–(5.37) have the same form as eqs. (2.5)–(2.6) but in terms of the full Lorentz curvature (3.7) instead of just the Riemannian piece. Note that eq. (5.38) is the standard field equation for torsion in terms of the spin tensor of matter.

5.2 Conservation laws

Both ideas, mimetic gravity and nonvanishing torsion introduce subtle features regarding conservation laws. On the mimetic side, let us observe that eq. (5.36) implies an extra mimetic stress-energy one-form $\bar{\mathcal{T}}_a = \bar{\mathcal{T}}_{ab} e^b$ corresponding to

$$\bar{\mathcal{T}}_d = \frac{1}{\kappa_4} Z_d \mathcal{E}^p_p d\phi. \quad (5.39)$$

Now, when using the covariant derivative $D = d + \omega$, conservation of (5.39) does not hold $D * \bar{\mathcal{T}}_d \neq 0$. In fact,

$$\begin{aligned} D * \bar{\mathcal{T}}_d &= \frac{1}{\kappa_4} D (Z_d * [\mathcal{E}^p_p d\phi]) \\ &= \frac{1}{\kappa_4} (D Z_d \wedge * [\mathcal{E}^p_p d\phi] + Z_d \wedge D * [\mathcal{E}^p_p d\phi]) \\ &= \frac{1}{\kappa_4} (\mathcal{E}^p_p D Z_d \wedge * d\phi + Z_d \wedge d * [\mathcal{E}^p_p d\phi]). \end{aligned} \quad (5.40)$$

In addition, using eq. (5.37), we have

$$\begin{aligned} D * \bar{\mathcal{T}}_d &= \frac{1}{\kappa_4} \mathcal{E}^p_p D Z_d \wedge * d\phi \\ &= \frac{1}{\kappa_4} \mathcal{E}^p_p Z^a \mathcal{D}_a Z_d v_{(4)}. \end{aligned} \quad (5.41)$$

Applying (4.7) to $Z_a = I_a d\phi$, one finds

$$\mathcal{D}_a Z_d - \mathcal{D}_d Z_a = -T^c_{ad} Z_c, \quad (5.42)$$

and therefore

$$\begin{aligned} D * \bar{\mathcal{T}}_d &= \frac{1}{\kappa_4} \mathcal{E}^p_p \left(Z^a \mathcal{D}_d Z_a - T_{abd} Z^a Z^b \right) v_{(4)} \\ &= \frac{1}{\kappa_4} \mathcal{E}^p_p \left(\frac{1}{2} I_d d(Z^a Z_a) - T_{abd} Z^a Z^b \right) v_{(4)}. \end{aligned} \quad (5.43)$$

Finally using eq. (5.9), we arrive to

$$\begin{aligned} D * \bar{\mathcal{T}}_d &= -\frac{1}{\kappa_4} \mathcal{E}^p_p T_{abd} Z^a Z^b v_{(4)} \\ &= -\bar{\mathcal{T}}^{ab} T_{abd} v_{(4)}. \end{aligned} \quad (5.44)$$

The result $D * \bar{\mathcal{T}}_d \neq 0$ may seem strange, but it is not an anomaly. This behavior is usual when considering nonvanishing torsion, especially in the context of standard ECSK theory. In order to clarify this point, we must observe that an authentic conservation law for a current one-form $J = J_\mu dx^\mu = J_a e^a$, takes the following form

$$* (d * J) = 0. \quad (5.45)$$

Consequently, the generalized Stokes theorem leads us to the integral on the spacetime boundary

$$\int_{\partial M_4} * J = 0. \quad (5.46)$$

It is essential to note that regardless of the torsional background, the conservation relation (5.45) corresponds to

$$* (d * J) = -I_a \mathring{D} J^a = 0, \quad (5.47)$$

making use of $\mathring{D}$ and not of D . Thus, in the context of standard ECSK, to write a “conservation law” we use eq. (3.7) to pack all the torsional terms together in the effective torsional stress-energy one form

$$\mathcal{T}_d^{(T)} = -\frac{1}{2\kappa_4} * \left[\left(\epsilon_{abcd} \mathring{D} \kappa^{ab} + \epsilon_{abcd} \kappa^a_e \wedge \kappa^{eb} \right) \wedge e^c \right], \quad (5.48)$$

where κ^{ab} corresponds to the contorsion one-form. Using (5.48), the ECSK field equation (3.11) is recast as

$$\frac{1}{2} \epsilon_{abcd} \mathring{R}^{ab} \wedge e^c - \frac{\Lambda}{3!} \epsilon_{abcd} e^a \wedge e^b \wedge e^c = \kappa_4 * \left(\mathcal{T}_d + \mathcal{T}_d^{(T)} \right). \quad (5.49)$$

This means that a “conservation law” should be written as

$$\mathring{D} * \left(\mathcal{T}_d + \mathcal{T}_d^{(T)} \right) = 0, \quad (5.50)$$

again making use of $\mathring{D}$ instead of D . In conclusion, in the ECSK theory torsion plays the role of a new Riemannian curvature source.

In the mimetic context, applying the steps (5.39)–(5.41) for $\mathring{D}$ instead of D , one finds

$$\mathring{D} * \bar{\mathcal{T}}_d = 0, \quad (5.51)$$

for the effective mimetic stress-energy tensor $\bar{\mathcal{T}}_d$, without resorting to an effective torsional stress-energy tensor, in contrast with ECSK theory obeying eq. (5.50). This fact reflects the structure of the mimetic field equations (5.36)–(5.38). In them, only the spin tensor of matter is the torsion source, and there is no torsion- ϕ interaction. This lack of interaction leads to eq. (5.51) since ϕ and torsion do not interchange energy or momentum.

Of course, it is important to remark that even when a stress-energy one form $\mathcal{T}_a = \mathcal{T}_{ab} e^b$ obeys $\mathring{D} * \mathcal{T}_d = 0$, $\mathcal{T}_a$ by itself is not an authentic conserved current one form in the sense of eq. (5.46). Regardless of the torsional background, to construct a real conserved current one form, we must consider $J = \zeta^a \mathcal{T}_a$, where $\vec{\zeta} = \zeta^\mu \partial_\mu$ corresponds to a Killing vector.

6 The trace of the stress-energy tensor, torsion and λ

For the mimetic theory dynamics, the choice of the parameter λ for the Weyl transformations (5.3)–(5.5) seems to be irrelevant. However, it does not mean that the parameter is meaningless. In fact, it is related with the value of the trace of stress-energy tensor of matter when its Lagrangian has conformal symmetry.

Let us consider a matter Lagrangian $\mathcal{L}_M$ obeying conformal symmetry by itself

$$\mathcal{L}_M(e^a, \omega^{ab}, \varphi) = \mathcal{L}_M \left(\Omega e^a, \omega^{ab} + \frac{\lambda}{\Omega} (e^a \Gamma^b - e^b \Gamma^a) d\Omega, \frac{1}{\Omega^\alpha} \varphi \right), \quad (6.1)$$

where $\Omega = \exp(\varepsilon)$. In the standard torsionless case, it would lead to a traceless on-shell stress-energy tensor. This is no longer true in the current context of non-vanishing torsion. In fact, under an infinitesimal Weyl transformation $\Omega = 1 + \varepsilon$, the field content of the matter Lagrangian changes according to

$$\delta_\varepsilon e^a = \varepsilon e^a, \quad (6.2)$$

$$\delta_\varepsilon \omega^{ab} = \lambda \theta^{ab}, \quad (6.3)$$

$$\delta_\varepsilon \varphi = -\alpha \varepsilon \varphi, \quad (6.4)$$

with θ^{ab} is given in (4.22). Moreover, an arbitrary variation of $\mathcal{L}_M(e, \omega, \varphi)$ reads

$$\delta \mathcal{L}_M = - * \mathcal{T}_d \wedge \delta e^d + \frac{1}{2} * \sigma_{ab} \wedge \delta \omega^{ab} + \Phi \delta \varphi + d \left(\mathcal{B}_a^{(2)} \wedge \delta e^a + \mathcal{B}_{ab}^{(2)} \wedge \delta \omega^{ab} + \mathcal{B}^{(3)} \delta \varphi \right), \quad (6.5)$$

where Φ denotes the variation of $\mathcal{L}_M$ with respect to φ and $\mathcal{B}_a$, $\mathcal{B}_{ab}$, and $\mathcal{B}$ are boundary terms. Therefore, demanding invariance of $\mathcal{L}_M$ under the infinitesimal Weyl transformations (6.2)–(6.4) we obtain

$$\varepsilon e^d \wedge * \mathcal{T}_d + \lambda * \sigma_{ab} \wedge e^a \xi^b - \alpha \varepsilon \Phi \varphi + d \left(\mathcal{B}_a^{(2)} \wedge \varepsilon e^a + 2\lambda \mathcal{B}_{ab}^{(2)} \wedge e^a \xi^b - \alpha \varepsilon \mathcal{B}^{(3)} \varphi \right) = 0. \quad (6.6)$$

It is straightforward to prove the identity

$$* \sigma_{ab} \wedge e^a \xi^b = -d \left[\varepsilon \Gamma^b (e^a \wedge * \sigma_{ab}) \right] + \varepsilon d \Gamma^b (e^a \wedge * \sigma_{ab}), \quad (6.7)$$

and therefore, integrating (6.6), to conclude that

$$\int_{M_4} \mathcal{H}^{(4)} + \int_{\partial M_4} \mathcal{U}^{(3)} = 0, \quad (6.8)$$

where

$$\mathcal{H}^{(4)} = \varepsilon \left[e^d \wedge * \mathcal{T}_d + \lambda d \Gamma^b (e^a \wedge * \sigma_{ab}) - \alpha \Phi \varphi \right] \quad (6.9)$$

and

$$\mathcal{U}^{(3)} = \varepsilon \mathcal{B}_a^{(2)} \wedge e^a + \lambda \left[2\mathcal{B}_{ab}^{(2)} \wedge e^a \xi^b - \varepsilon \Gamma^b (e^a \wedge * \sigma_{ab}) \right] - \alpha \varepsilon \mathcal{B}^{(3)} \varphi. \quad (6.10)$$

Since ε is local and arbitrary parameter, the integrals over the bulk and boundary must vanish independently. Neglecting boundary contributions, from (6.9) we are left with

$$e^d \wedge * \mathcal{T}_d + \lambda d \Gamma^b (e^a \wedge * \sigma_{ab}) - \alpha \Phi \varphi = 0. \quad (6.11)$$

Note that in differential forms language the trace of the stress-energy tensor is written as $\mathcal{T} = - * (e^d \wedge * \mathcal{T}_d)$. In this way, taking the Hodge dual in (6.11) we find

$$\mathcal{T} - \lambda * d\Gamma^b (e^a \wedge * \sigma_{ab}) + \alpha * \Phi \varphi = 0. \quad (6.12)$$

Using (4.2) it is possible to probe that $\Gamma^b (e^a \wedge * \sigma_{ab}) = * e^b \sigma^a_{ab}$. Furthermore, in a theory as ECSK or its current mimetic version, imposing the on-shell condition implies for one hand $\Phi = 0$, and on the other hand, equation (3.12) allows to solve the components of the spin tensor in terms of the components of torsion. In fact

$$\sigma^a_{ba} = \frac{2}{\kappa_4} T^a_{ab}, \quad (6.13)$$

and replacing (6.13) in eq. (6.12) on-shell, it leads us to the trace value

$$\mathcal{T} = \frac{2\lambda}{\kappa_4} d^\dagger I_a T^a, \quad (6.14)$$

with the four-dimensional coderivative operator $d^\dagger$ given by $d^\dagger = *d*$. This way, we can see that the stress-energy tensor for a Weyl transformation invariant Lagrangian is not always traceless. It is traceless only when the torsion vanishes or when the Weyl transformation invariance is associated to the case $\lambda = 0$, i.e. when the spin connection remains untouched by the transformation.

7 Summary & comments

In summary, we have developed the closest version of mimetic gravity in first-order formalism, i.e., the mimetic version of ECSK gravity theory. Dynamics in this theory is described by the following equations of motion

$$\mathcal{E}_d - \kappa_4 * \bar{\mathcal{T}}_d = 0, \quad (7.1)$$

$$d * [\mathcal{E}^p_p d\phi] = 0, \quad (7.2)$$

$$\mathcal{W}_{ab} = 0, \quad (7.3)$$

where $\mathcal{E}_a$, $\mathcal{W}_{ab}$, and $\bar{\mathcal{T}}_d$ are given in eqs. (3.11), (3.12), and (5.39) respectively. By construction, the system obeys the following conditions:

$$\mathring{D} * \bar{\mathcal{T}}_d = 0, \quad (7.4)$$

$$Z^2 = 1. \quad (7.5)$$

These equations reduce to the standard mimetic gravity equation when torsion T^a vanishes.

We have considered different possibilities of how torsion is affected by Weyl transformation (2.1), all of them mapped by the parameter λ . The torsionless part of the spin connection $\hat{\omega}^{ab}$ has a definite Weyl transformation (4.15), obtained from purely metric properties. However, we have the freedom to choose the contorsion κ^{ab} transformation. The possibilities (4.19)–(4.21) range from a non-changing contorsion, to a contorsion that

changes in such a way that it leaves the full spin connection invariant. Regardless of the type of transformation under consideration, dynamics enforce torsion to remain a non-propagating field, as in standard ECSK.

An interesting application of this model is in the context of cosmology.² Let us consider an explicit solution of equation (7.2). For the sake of simplicity, during this section we take $c = 1$ and $\Lambda = 0$. Additionally, it is convenient to work in synchronous coordinates where the metric adopts the form

$$ds^2 = -d\tau^2 + \gamma_{ij}dx^i dx^j, \quad (7.6)$$

with γ_{ij} the spatial section of the metric $g_{\mu\nu}$ (See [138, Chap.11]). As discussed in [41], we take the scalar field to be the same as the hypersurfaces of constant time, namely

$$\phi(x^\mu) = \tau, \quad (7.7)$$

which naturally satisfies (7.5). In these coordinates, eq. (7.2) reads

$$\partial_0(\sqrt{\gamma}(G - \mathcal{T})) = 0, \quad (7.8)$$

and consequently

$$G - \mathcal{T} = \frac{\mathcal{C}(x^i)}{\sqrt{\gamma}}. \quad (7.9)$$

Here $\mathcal{C}$ is an integration τ -constant, depending only on the spatial coordinates x^i . For a flat Friedmann Universe, the metric γ_{ij} corresponds to

$$\gamma_{ij} = a^2(\tau) \delta_{ij}, \quad (7.10)$$

and therefore (7.8) leads to

$$G - \mathcal{T} = \frac{\mathcal{C}(x^i)}{a^3(\tau)}. \quad (7.11)$$

Therefore, the scalar field coming from the conformal degree of freedom mimics a dark matter source. However, torsion is also present, and it behaves as an additional dark matter source in $G = -R$. Let us split the Ricci scalar as

$$R = \mathring{R} + R(\kappa), \quad (7.12)$$

$$R(\kappa) = 2\mathring{\nabla}_\mu \kappa^{\mu\nu}{}_\nu + \kappa^{\mu\gamma}{}_\mu \kappa_{\gamma\nu}{}^\nu - \kappa^\mu{}_{\gamma\nu} \kappa^{\gamma\nu}{}_\mu, \quad (7.13)$$

where $\kappa^{\mu\nu}{}_\lambda$ is the contortion, $\mathring{R}$ is the torsionless Ricci scalar and $\mathring{\nabla}$ is the covariant derivative associated with the Christoffel symbol. Thus, using $\mathring{G} = -\mathring{R}$, we get

$$G = \mathring{G} - R(\kappa). \quad (7.14)$$

In order to evaluate eq. (7.13) explicitly, let us consider a spin tensor distribution σ_{ab} which may be relevant at cosmological scales. Such spin tensor distribution has been considered, for instance, in [40] where the Ansatz for the torsion tensor reads

$$T_{\lambda\mu\nu} = [X(\tau)(g_{\lambda\nu}g_{\mu\rho} - g_{\lambda\mu}g_{\nu\rho}) - 2\sqrt{g}Y(\tau)\epsilon_{\lambda\mu\nu\rho}]u^\rho, \quad (7.15)$$

²By the way, almost simultaneous with this article, another interesting cosmological application of mimetic theories of gravity involving non-vanishing torsion has been presented in [137].

where $u^\rho = \delta_0^\rho$ in synchronous coordinates, and X and Y are arbitrary functions of time. These configurations can arise from a dark matter candidates, such as dark/Elko spinors, see for instance [139–145]. The non-vanishing components of (7.15) are

$$T_{ij0} = -T_{i0j} = X\gamma_{ij}u^0 = a^2X\delta_{ij}, \quad (7.16)$$

$$T_{kij} = -2\sqrt{\gamma}Y\epsilon_{kij}u^0 = -2a^3Y\epsilon_{kij}. \quad (7.17)$$

Since

$$\kappa_{\mu\nu\lambda} = \frac{1}{2}(T_{\nu\mu\lambda} - T_{\mu\nu\lambda} + T_{\lambda\mu\nu}), \quad (7.18)$$

we can evaluate eq. (7.13) and eqs. (7.11)–(7.14) to arrive to

$$\mathring{G} - \mathcal{T} = \frac{\mathcal{C}}{a^3} + 3! [X(\mathcal{S} + 3\mathcal{H}) + X^2 + Y^2], \quad (7.19)$$

where $\mathcal{C} = \mathcal{C}(x^i)$, $\mathcal{S}(\tau) = \frac{\dot{X}}{X}$ and $\mathcal{H}(\tau) = \frac{\dot{a}}{a}$. From here the role of the spin-tensor as dark matter becomes evident.

It is interesting to observe that in mimetic ECSK theory, there are three dark matter species. The first is the dark matter by itself, with a non-vanishing spin tensor. The second arises from isolating the conformal mode in a covariant way, and the last comes from the torsional degrees of freedom. In particular, from eq. (7.19) one sees that dark matter arises as a direct sum of contributions coming from mimetic dark matter and torsion separately. This is no longer the case when consider more elaborated mimetic theories containing nonminimal couplings of scalar fields with geometry (See [105]). In such a scenario, a complex interplay between ϕ and propagating torsion creates dark matter effects.

This behavior is due to a general result. In ref. [105], we proved that for the Horndeski Lagrangian on a Riemann-Cartan geometry, the nonminimal couplings with the scalar field and second-order terms in the Lagrangian (e.g, a $\square\phi$ term) are generic sources of torsion. The same will happen in the mimetic context. Observing how the structure of the mimetic equations of motion came to be (see, for instance, eqs. (5.24)–(5.26)), it is clear that the mimetic mapping to a new theory preserves the propagating/non-propagating torsion character of the original theory. In both cases, propagating and non-propagating, nonvanishing torsion influences cosmic evolution. In a non-propagating context, torsion and ϕ do not interchange energy during the cosmic evolution, as we can see from the former analysis. However, when considering theories with nonminimal couplings or second-order terms in the Lagrangian, in general, ϕ and torsion's evolution get completely interrelated; see, for instance, refs. [36, 37, 112, 114].

The non-propagating torsion's simplicity may give the false impression of it being uninteresting in a cosmological setting. That would be far from the truth; for instance, a non-propagating torsion can straightforwardly explain the Hubble parameter tension [40].

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