

RECEIVED: June 11, 2020

REVISED: August 18, 2020

ACCEPTED: September 18, 2020

PUBLISHED: October 16, 2020

A connection between flavour anomaly, neutrino mass, and axion

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ABSTRACT: We propose a minimal model in which the flavour anomaly in the $b \rightarrow s\mu^+\mu^-$ transition is connected to the breaking of Peccei-Quinn (PQ) symmetry. The flavour anomaly is explained from new physics contribution by introducing one generation of heavy quark and heavy lepton which are vector-like under the standard model (SM) gauge group but charged under a local $U(1)_X$ group. They mix with the SM quarks and leptons, inducing flavour-changing Z' couplings, which generates the $b \rightarrow s\mu^+\mu^-$ anomaly at tree level. On the other hand the new fermions are chiral under the global Peccei-Quinn(PQ) symmetry. The pseudo-Goldstone boson coming from the spontaneous breaking of the PQ symmetry becomes an axion, solving the strong CP problem and providing a cold dark matter candidate. The same symmetry prevents the right-handed neutrino from having a Majorana mass term. But the introduction of a neutrino-specific Higgs doublet allows neutrino to have Dirac mass term without fine-tuning problem. The model shows an interplay between axion, neutrino, dark matter, and flavour physics.

KEYWORDS: Beyond Standard Model, Gauge Symmetry, Global Symmetries, Heavy Quark Physics

ARXIV EPRINT: [2006.02050](https://arxiv.org/abs/2006.02050)

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1 Introduction

Although the standard model (SM) has passed the experimental tests successfully for decades, there are hints that suggest more fundamental theory beyond the SM. These include the existence of dark matter (DM), the neutrino mass and mixing, and the strong CP problem. In addition, there are some tantalizing anomalies in the B decay data from the Belle and the LHCb experiments, which may also require new physics (NP) beyond the SM. In this paper we consider a NP model which addresses the above problems of the SM simultaneously.

The most popular solution of the strong CP problem is the introduction of axion which is a pseudo-Goldstone boson coming from the breaking of the global Peccei-Quinn (PQ) symmetry, $U(1)_{\text{PQ}}$. In the paper [1] we suggested a NP model where the neutrino mass is generated from the breaking of PQ symmetry, thereby the explanation of the neutrino masses and axion can be unified. In the model two Higgs doublets are introduced, the SM-like Higgs doublet (Φ_2) couples to the quarks and charged leptons, whereas the new Higgs doublet (Φ_1) couples solely to the neutrino sector. In this neutrino-specific two Higgs doublet model (ν THDM), the Higgs Φ_1 and the right-handed neutrinos ν_{iR} ($i = 1, 2, 3$) as well as S whose phase is the main component of the axion are charged under the PQ symmetry [2, 3]. The PQ symmetry prohibits the right-handed neutrino from having mass term, making the type-I seesaw mechanism not effective. But it allows the light neutrino to have Dirac-type mass without fine-tuning problem. There is a simple seesaw-like relation for the vacuum expectation value (VEV) of Φ_1 :

$$v_1 \approx \frac{\mu v_2 v_S}{M_{\Phi_1}^2}, \tag{1.1}$$

where $v_2 \equiv \sqrt{2}\langle\Phi_2^0\rangle$, $v_S \equiv \sqrt{2}\langle S\rangle$, M_{Φ_1} is the mass scale of Φ_1 , and μ is the coupling constant of trilinear interaction, $\mu\Phi_1^\dagger\Phi_2S$. For $M_{\Phi_1} \sim v_S \sim 10^{12}$ GeV, $v_2 \simeq 246$ GeV, and $\mu \sim 1$ GeV, we get $v_1 \sim 0.1$ eV, which renders the neutrino mass, $m_\nu (= y_\nu v_1/\sqrt{2})$, at

sub-eV scale when neutrino Yukawa coupling is of order one, $y_\nu \sim \mathcal{O}(1)$. By introducing vector-like heavy quarks or additional Higgs doublets, we can introduce either KSVZ-type or DFSZ-type axions [1].

The flavour-changing neutral current (FCNC) processes are very sensitive probes of NP [4–7]. Especially the quark-level process, $b \rightarrow s\mu^+\mu^-$, has been drawing much interest during recent years due to discrepancies between the experimental measurements and the theoretical predictions. A measurement of particular interest is the ratio of branching fractions,

$$R_{K^{(*)}}[q_{\min}^2 < q^2 < q_{\max}^2] = \frac{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\Gamma[B \rightarrow K^{(*)}\mu^+\mu^-]}{dq^2} dq^2}{\int_{q_{\min}^2}^{q_{\max}^2} \frac{d\Gamma[B \rightarrow K^{(*)}e^+e^-]}{dq^2} dq^2}, \quad (1.2)$$

where q^2 is the dilepton mass squared. In the SM the gauge interactions of all three charged leptons are identical, and the ratio is predicted to be 1 up to small corrections related to the lepton mass. The experimental reports are

$$\begin{aligned} R_K[1.1 \text{ GeV}^2 < q^2 < 6.0 \text{ GeV}^2] &= 0.846_{-0.054-0.014}^{+0.060+0.016} && \text{(LHCb [8])}, \\ R_{K^*}[1.1 \text{ GeV}^2 < q^2 < 6.0 \text{ GeV}^2] &= 0.69_{-0.07}^{+0.11} \pm 0.05, && \text{(LHCb [9])}, \\ R_{K^*}[0.1 \text{ GeV}^2 < q^2 < 8.0 \text{ GeV}^2] &= 0.90_{-0.21}^{+0.27} \pm 0.10, && \text{(Belle [10])}, \end{aligned} \quad (1.3)$$

which show violation of the lepton flavour universality (LFU). Combining with other $b \rightarrow s\ell^+\ell^-$ and $b \rightarrow s\gamma$ observables, the global fits show the SM is disfavoured with large significance [11–17]. For example, for the scenario $C_9^{\mu,\text{NP}} = -C_{10}^{\mu,\text{NP}}$ which we will take in this paper, the pull with respect to the SM is 5.2σ with the best fit value [11],

$$C_9^{\mu,\text{NP}} = -C_{10}^{\mu,\text{NP}} = -0.46. \quad (1.4)$$

The 1σ and 2σ regions are $[-0.56, -0.37]$ and $[-0.66, -0.28]$, respectively [11].

Here the effective Hamiltonian for the $C_{9(10)}$ is defined as

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{4\pi} V_{ts}^* V_{tb} \sum_{\ell=e,\mu,\tau} \bar{s}_L \gamma_\mu b_L (C_9^\ell \bar{\ell} \gamma^\mu \ell + C_{10}^\ell \bar{\ell} \gamma^\mu \gamma_5 \ell). \quad (1.5)$$

The anomaly can be explained in numerous NP models. Among them, many models [18–40] show an interesting interplay between the flavour physics and dark matter in which the WIMP cold DM (CDM) is related to the mechanism explaining the flavour anomaly or contributes to the $b \rightarrow s\mu^+\mu^-$ process through loop.

In this paper we present a NP model which shows connection between the $b \rightarrow s\mu^+\mu^-$ anomaly, the axion, and the neutrino mass. Since the axion is a good candidate for CDM, the model can address the dark matter candidate, the strong CP problem, the neutrino mass, and the flavour problem at the same time. In the model super-heavy scalar S , vector-like quark doublets $Q_{L,R}$, and lepton doublets $L_{L,R}$ are introduced. They are charged under the global $U(1)_{\text{PQ}}$ symmetry. Thereby the pseudo-scalar component of S becomes an axion of the KSVZ-type [41, 42].

	Scalars				Fermions									
	Φ_1	Φ_2	ϕ	S	q_{iL}	u_{iR}	d_{iR}	ℓ_{iL}	e_{iR}	ν_{iR}	Q_L	Q_R	L_L	L_R
$SU(3)_C$	1	1	1	1	3	3	3	1	1	1	3	3	1	1
$SU(2)_L$	2	2	1	1	2	1	1	2	1	1	2	2	2	2
$U(1)_Y$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	0	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$U(1)_X$	0	0	1	0	0	0	0	0	0	0	-1	-1	-1	-1
$U(1)_{PQ}$	$\frac{1}{2}$	$-\frac{1}{2}$	1	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	0	-1

Table 1. The particles and their charges under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X \times U(1)_{PQ}$ in our model.

The vector-like fermions are also charged under a local $U(1)_X$ symmetry. The SM fields are neutral under $U(1)_X$. The gauge boson Z' of the $U(1)_X$ can still interact with the SM fermions through the mixing with the heavy vector-like fermions after the gauge symmetry is spontaneously broken by the VEV of $U(1)_X$ -charged scalar ϕ . This allows tree-level diagrams to produce FCNC processes such as $b \rightarrow s\ell^+\ell^-$ and $B_s - \bar{B}_s$ mixing.

The paper is organised as follows: we introduce our model in the next section. In section 3 we investigate the axion properties in our model and compare with those in the original KSVZ model. We also briefly review the neutrino mass generation. In section 4 we resolve the flavour problem by introducing the NP $b \rightarrow s\mu^+\mu^-$ transition at tree-level. We discuss the constraints. In section 5 we conclude the paper.

2 The model

The new particles in the model as well as the SM ones are shown in table 1 with their representations under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and their charges under the local $U(1)_X$ symmetry and the global $U(1)_{PQ}$ symmetry.

The scalar potential is in the form

$$\begin{aligned}
 V = & \mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 - \mu_S^2 S^* S - \mu_\phi^2 \phi^* \phi - (\mu \Phi_1^\dagger \Phi_2 S + \text{h.c.}) \\
 & + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) \\
 & + \lambda_\phi (\phi^* \phi)^2 + \lambda_{1\phi} (\Phi_1^\dagger \Phi_1) (\phi^* \phi) + \lambda_{2\phi} (\Phi_2^\dagger \Phi_2) (\phi^* \phi) \\
 & + \lambda_S (S^* S)^2 + \lambda_{1S} (\Phi_1^\dagger \Phi_1) (S^* S) + \lambda_{2S} (\Phi_2^\dagger \Phi_2) (S^* S) + \lambda_{\phi S} (\phi^* \phi) (S^* S). \quad (2.1)
 \end{aligned}$$

The scalar potential contains the μ -term, $(-\mu \Phi_1^\dagger \Phi_2 S + \text{h.c.})$, which plays an essential role in the generation of the neutrino mass [1]. To keep the hierarchy between the PQ scale ($\sim 10^{12}$ GeV), the $U(1)_X$ scale ($\sim 10^5$ GeV), the EW scale (~ 100 GeV), and the neutrino mass scale (~ 0.1 eV) from the radiative corrections, we need to suppress the corresponding mixing parameters $\lambda_{\phi S}$, $\lambda_{1(2)S}$, $\lambda_{1(2)\phi}$. The smallness of these parameters is technically natural due to the extended Poincaré symmetry [1, 43].

The Yukawa interactions are

$$\mathcal{L}_\Phi = -y_{ij}^u \bar{q}_{iL} \tilde{\Phi}_2 u_{jR} - y_{ij}^d \bar{q}_{iL} \Phi_2 d_{jR} - y_{ij}^e \bar{\ell}_{iL} \Phi_2 e_{jR} - y_{ij}^\nu \bar{\ell}_{iL} \tilde{\Phi}_1 \nu_{jR} + \text{h.c.} \quad (2.2)$$

$$\mathcal{L}_\phi = -\lambda_i^q \bar{q}_{iL} Q_R \phi - \lambda_i^\ell \bar{\ell}_{iL} L_R \phi + \text{h.c.} \quad (2.3)$$

$$\mathcal{L}_S = -y_Q \bar{Q}_L Q_R S - y_L \bar{L}_L L_R S + \text{h.c.}, \quad (2.4)$$

where $i, j = 1, 2, 3$ are generation indices, and $\tilde{\Phi}_{1(2)} = i\sigma^2 \Phi_{1(2)}^*$. We assign the PQ charges in such a way that only the terms in the interactions (2.1), (2.2), (2.3), and (2.4) are allowed. The interactions are already invariant under the SM gauge group and $U(1)_X$. We notice that all the quarks and charged-leptons are coupled to the Higgs doublet Φ_2 , whereas the neutrinos are coupled to the Higgs doublet Φ_1 . It is also noted that the right-handed neutrinos do not have the Majorana mass terms because they are charged under $U(1)_{\text{PQ}}$. The Q 's and L 's are vector-like under both the SM gauge and $U(1)_X$ gauge symmetries. Therefore gauge anomalies cancel in our model.

3 The axion and the Dirac neutrino

To explain the flavour anomaly in $b \rightarrow s\mu^+\mu^-$ transition, we need to fix the properties of the heavy vector-like quarks and leptons. A minimal model for the scenario (1.4) is to introduce $SU(2)_L$ -doublet vector-like quarks and leptons, $Q_{L,R}$ and $L_{L,R}$:

$$Q_{L,R} = \begin{pmatrix} U_{L,R} \\ D_{L,R} \end{pmatrix}, \quad L_{L,R} = \begin{pmatrix} N_{L,R} \\ E_{L,R} \end{pmatrix}. \quad (3.1)$$

As a consequence they have a definite model-dependent predictions for the axion properties, such as axion couplings to the photons and to the other SM particles.

We decompose the scalar fields as

$$\Phi_i = \begin{pmatrix} G_i^+ \\ \frac{1}{\sqrt{2}}(v_i + h_i + ia_i) \end{pmatrix}, \quad \phi = \frac{1}{\sqrt{2}}(v_\phi + h_\phi + ia_\phi), \quad S = \frac{1}{\sqrt{2}}(v_S + h_S + ia_S), \quad (3.2)$$

where $i = 1, 2$. From the charge assignments in table 1, the Nambu-Goldstone fields read [44]

$$\begin{aligned} a_Y &= \frac{1}{v}(v_1 a_1 + v_2 a_2), \\ a_X &= a_\phi, \\ a_{\text{PQ}} &= \frac{1}{2}v_1 a_1 - \frac{1}{2}v_2 a_2 + v_\phi a_\phi + v_S a_S, \end{aligned} \quad (3.3)$$

where $v \equiv \sqrt{v_1^2 + v_2^2}$, a_Y (a_X) is eaten by the longitudinal components of Z (Z') gauge boson, a_{PQ} corresponds to the axion, and a_{PQ} is not normalized for convenience. We see that a_{PQ} is not orthogonal to a_Y and a_X , while $a_Y \perp a_X$. We can orthogonalise it to get the physical axion field [44],

$$a'_{\text{PQ}} = a_{\text{PQ}} - (a_{\text{PQ}}, a_X)a_X - (a_{\text{PQ}}, a_Y)a_Y = X_1 v_1 a_1 + X_2 v_2 a_2 + v_S a_S, \quad (3.4)$$

where $(a_{PQ}, a_X) = v_\phi$ and $(a_{PQ}, a_Y) = (v_1^2 - v_2^2)/2v$ are “inner products”, and $X_1 = v_2^2/v^2$ and $X_2 = -v_1^2/v^2$ are *effective PQ charges* of Φ_1 and Φ_2 . The normalized axion field is

$$a = \frac{1}{v_{PQ}}(X_1 v_1 a_1 + X_2 v_2 a_2 + v_S a_S), \quad (3.5)$$

where $v_{PQ} = [(X_1 v_1)^2 + (X_2 v_2)^2 + v_S^2]^{1/2}$.

Alternatively we can identify the axion by defining [45],¹

$$\Phi_1 = \frac{v_1}{\sqrt{2}} e^{iX_1 a/v_{PQ}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \Phi_2 = \frac{v_2}{\sqrt{2}} e^{iX_2 a/v_{PQ}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \phi = \frac{v_\phi}{\sqrt{2}} e^{iX_\phi a/v_{PQ}}, \quad S = \frac{v_S}{\sqrt{2}} e^{iX_S a/v_{PQ}}, \quad (3.6)$$

where $a \rightarrow a + \alpha v_{PQ}$ under $U(1)_{PQ}$ transformation. We can determine the X 's as in [1]:

$$X_1 = \frac{v_2^2}{v^2}, \quad X_2 = -\frac{v_1^2}{v^2}, \quad X_\phi = 0, \quad (3.7)$$

where we take the normalization, $X_S = 1$. They satisfy $-X_1 + X_2 + X_S = 0$ as required by the trilinear term in (2.1). The canonical normalization for the kinetic energy term of a is obtained with

$$v_{PQ} = \left(\sum_{i=1,2,\phi,S} X_i^2 v_i^2 \right)^{1/2}. \quad (3.8)$$

As can be seen from (2.2), (2.3), and (2.4), the other PQ-charges are related as follows:

$$\begin{aligned} -X_{q_L} - X_2 + X_{u_R} &= 0, & -X_{q_L} + X_2 + X_{d_R} &= 0, \\ -X_{\ell_L} + X_2 + X_{e_R} &= 0, & -X_{\ell_L} - X_1 + X_{\nu_R} &= 0, \\ -X_{q_L} + X_{Q_R} &= 0, & -X_{\ell_L} + X_{L_R} &= 0, \\ -X_{Q_L} + X_{Q_R} + 1 &= 0, & -X_{L_L} + X_{L_R} + 1 &= 0. \end{aligned} \quad (3.9)$$

By expanding (3.6) we can also write the axion a as

$$a = \frac{1}{v_{PQ}} \sum_{i=1,2,\phi,S} X_i v_i a_i, \quad (3.10)$$

where a_i 's ($i = 1, 2, \phi, S$) are the pseudo-scalar components of scalar fields Φ_1, Φ_2, ϕ, S whose VEVs are v_i .

The PQ-current

$$\begin{aligned} J_{PQ}^\mu &= v_{PQ} \partial^\mu a - \frac{X_2}{2} \sum_i \bar{u}_i \gamma^\mu \gamma_5 u_i + \frac{X_2}{2} \sum_i \bar{d}_i \gamma^\mu \gamma_5 d_i + \frac{X_2}{2} \sum_i \bar{e}_i \gamma^\mu \gamma_5 e_i \\ &\quad - \frac{X_1}{2} \sum_i \bar{\nu}_i \gamma^\mu \gamma_5 \nu_i + \frac{1}{2} \bar{Q} \gamma^\mu \gamma_5 Q + \frac{1}{2} \bar{L} \gamma^\mu \gamma_5 L, \end{aligned} \quad (3.11)$$

¹The axion field can be identified also by explicit orthogonalization [44].

satisfies anomaly equation:

$$\partial_\mu J_{\text{PQ}}^\mu = -\frac{Ng_s^2}{16\pi^2}G_{\mu\nu}^a\tilde{G}^{a\mu\nu} - \frac{Ee^2}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (3.12)$$

where $G_{\mu\nu}^a(F_{\mu\nu})$ is the gluon (electromagnetic) field strength tensor and $\tilde{G}_{\mu\nu}^a(\tilde{F}_{\mu\nu})$ is its dual tensor. We obtain

$$N = 1, \quad E = \frac{8}{3}. \quad (3.13)$$

We note that the QCD and the electromagnetic anomalies from the SM fermions cancel and the non-trivial contributions come entirely from the new heavy fermions. In this model the axion domain wall number, given by

$$N_{\text{DW}} = 2N = 2, \quad (3.14)$$

is different from that of the KSVZ (DFSZ) model where $N_{\text{DW}} = 1(6)$ [41, 42, 46, 47].

The axion mass is

$$m_a = \frac{f_\pi m_\pi}{f_a} \frac{\sqrt{z}}{1+z} \simeq 5.7 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right), \quad (3.15)$$

where $m_\pi \simeq 135 \text{ MeV}$ is the neutral pion mass, $f_\pi \simeq 92.3 \text{ MeV}$ is the pion decay constant, $f_a = v_{\text{PQ}}/N_{\text{DW}}$ is the axion decay constant, and $z = m_u/m_d \simeq 0.472$ is the light quarks mass ratio. The axion interaction with the photon can be written in the form

$$\mathcal{L}_{a\gamma\gamma} = -\frac{g_{a\gamma\gamma}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}a = g_{a\gamma\gamma}\mathbf{E} \cdot \mathbf{B}a, \quad (3.16)$$

where

$$g_{a\gamma\gamma} = \frac{\alpha_{\text{em}}}{2\pi f_a} \left(\frac{E}{N} - \frac{2}{3} \frac{4+z}{1+z} \right) \quad \text{with} \quad \frac{E}{N} = \frac{8}{3}. \quad (3.17)$$

The axion-photon coupling (3.17) agrees with that of the DFSZ model *not* the KSVZ model. The axion interaction with fermions can be written in the form,

$$\mathcal{L}_{\text{aff}} = \frac{C_f}{2f_a} \partial_\mu a \bar{f} \gamma^\mu \gamma_5 f. \quad (3.18)$$

The tree-level axion coupling to electrons and neutrinos are

$$C_e = \frac{X_2}{2}, \quad C_\nu = -\frac{X_1}{2}, \quad (3.19)$$

which can be compared with the KSVZ model where $C_e = C_\nu = 0$. Since X_2 is tiny in our model, the loop-induced C_e is much larger [44] as in the case of KSVZ model. Making use of the fact that it is DFSZ-type in $a\gamma\gamma$ coupling and KSVZ-type in aee coupling, our model can be tested in experiments. The tree-level C_ν is of order unity, and may also be probed at future neutrino oscillation experiments [48].

Now we briefly review the mechanism for the neutrino mass generation suggested in [1]. The tree-level diagram shown in figure 1 generates the neutrino mass. The red (black)

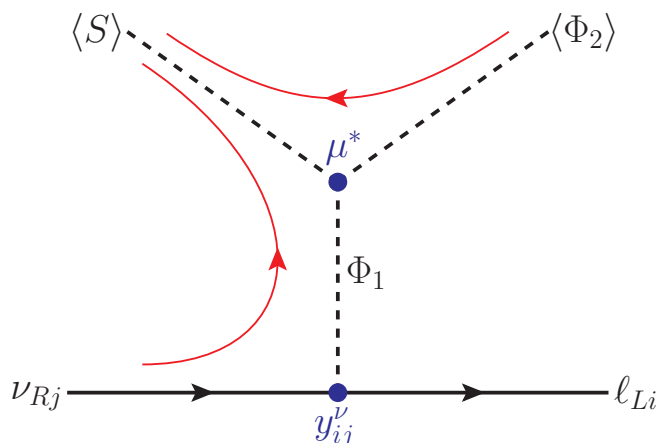


Figure 1. Feynman diagram for the neutrino mass generation. The red (black) arrows represent the flow of $U(1)_{PQ}$ (lepton number) current.

arrows represent the flow of $U(1)_{PQ}$ (lepton number) current when we set $X_{\ell_L} = 0$. The Yukawa interaction and the μ -term generates the Dirac neutrino masses after the S and Φ_2 fields get VEVs:

$$m_{ij}^\nu = \frac{y_{ij}^\nu \mu v_2 v_S}{2M_{\Phi_1}^2} \sim 0.1 y_{ij}^\nu \left(\frac{\mu}{1 \text{ GeV}} \right) \left(\frac{v_2}{246 \text{ GeV}} \right) \left(\frac{v_S}{10^{12} \text{ GeV}} \right) \left(\frac{10^{12} \text{ GeV}}{M_{\Phi_1}} \right)^2 \text{ eV}. \quad (3.20)$$

The seesaw-like formula shows that the neutrino masses are $\mathcal{O}(0.1)$ eV for $y_{ij}^\nu \sim 1$, when $\mu \sim 1 \text{ GeV}$, $v_S \sim M_{\Phi_1} \sim 10^{12} \text{ GeV}$. Other studies on the link between $U(1)_{PQ}$ symmetry and neutrino/flavour can be found in [49–63].

4 The $b \rightarrow s\mu^+\mu^-$ transition

The new colored fermions $Q_{L,R}$ induce the QCD anomaly, making a an axion candidate to solve the strong CP problem. The heavy quarks (heavy leptons) can also mix with the SM quarks (leptons). The mixing can generate $Z' - b - s$ ($Z' - \mu - \mu$) vertex at tree-level, which can induce $b \rightarrow s\mu^+\mu^-$ transition at tree-level.

In table 1 we introduced heavy vector-like quarks $Q_{L,R}$ and leptons $L_{L,R}$ in the same representation under the SM gauge group with q_L and ℓ_L , respectively. The heavy vector-like fermions $Q_L(L_L)$ can mix with the left-handed quarks (leptons) $q_{iL}(\ell_{iL})$ when ϕ gets VEV, v_ϕ . Since the Z' couples only to the left-handed quarks and leptons, we can achieve the scenario in (1.4).

In the CKM basis where the SM Yukawa couplings are diagonal, we can write (2.3) as

$$\mathcal{L} = -(\bar{u}_{iL} V_{ij}^{\text{CKM}} \lambda_j^q U_R + \bar{d}_{iL} \lambda_i^q D_R) \phi + \text{h.c.}, \quad (4.1)$$

where we assumed the quark mixing represented by the CKM matrix V_{CKM} arises only in the up-quark sector. When we include the new heavy quark, the down-type quark mass

matrix in the CKM basis has off-diagonal components,

$$\mathcal{L}_{\text{mass}} = -(\bar{d}_L, \bar{s}_L, \bar{b}_L, \bar{D}_L) \begin{pmatrix} m_d & 0 & 0 & \frac{1}{\sqrt{2}}\lambda_d v_\phi \\ 0 & m_s & 0 & \frac{1}{\sqrt{2}}\lambda_s v_\phi \\ 0 & 0 & m_b & \frac{1}{\sqrt{2}}\lambda_b v_\phi \\ 0 & 0 & 0 & M_Q \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \\ D_R \end{pmatrix} + \text{h.c.}, \quad (4.2)$$

where $M_Q \equiv M_D = M_U = y_Q v_S / \sqrt{2}$, and $(\lambda_d, \lambda_s, \lambda_b) \equiv (\lambda_1^q, \lambda_2^q, \lambda_3^q)$. The corresponding up-quark mass matrix is in the same form with replacement (m_d, m_s, m_b) and λ_i^q by (m_u, m_c, m_t) and $(V_{\text{CKM}}\lambda^q)_i$, respectively. We note that the flavour mixing between the SM sector and Q is controlled by a single column matrix $\lambda^q \equiv (\lambda_d, \lambda_s, \lambda_b)^T$. The mass matrix (4.2) can be diagonalized by biunitary transformation:

$$\mathcal{V}_L^{d\dagger} \mathcal{M}_d \mathcal{V}_R^d \simeq \text{diag}(m_d, m_s, m_b, M_Q), \quad (4.3)$$

where $\mathcal{M}_d \equiv (4 \times 4 \text{ mass matrix})$ in (4.2). We obtain approximately

$$\mathcal{V}_R^d \simeq \text{diag}(1, 1, 1, 1),$$

$$\mathcal{V}_L^d \simeq \begin{pmatrix} 1 & 0 & 0 & \frac{\lambda_d v_\phi}{\sqrt{2}M_Q} \\ 0 & 1 & 0 & \frac{\lambda_s v_\phi}{\sqrt{2}M_Q} \\ 0 & 0 & 1 & \frac{\lambda_b v_\phi}{\sqrt{2}M_Q} \\ -\frac{\lambda_d^* v_\phi}{\sqrt{2}M_Q} & -\frac{\lambda_s^* v_\phi}{\sqrt{2}M_Q} & -\frac{\lambda_b^* v_\phi}{\sqrt{2}M_Q} & 1 \end{pmatrix}. \quad (4.4)$$

The relations (4.3) and (4.4) are correct up to $\mathcal{O}(\lambda_{\text{max}} v_\phi / \sqrt{2}M_Q)^2$ with $\lambda_{\text{max}} = \max(\lambda_d, \lambda_s, \lambda_b)$.

Then the effective $Z' - s - b$ vertex for $b \rightarrow s$ transition is

$$\Lambda_L^{sb} Z'^\alpha \bar{s}_L \gamma_\alpha b_L + \text{h.c.} \equiv g_X \left(\mathcal{V}_L^d \right)_{42}^* \left(\mathcal{V}_L^d \right)_{43} Z'^\alpha \bar{s}_L \gamma_\alpha b_L + \text{h.c.}, \quad (4.5)$$

where g_X is the $U(1)_X$ gauge coupling constant and

$$\Lambda_L^{sb} = \frac{g_X \lambda_s \lambda_b^* v_\phi^2}{2M_Q^2}. \quad (4.6)$$

This result is consistent with the one in [18, 64] where diagrammatic method was used. The Z' boson does not couple directly to μ either, and the effective $Z' - \mu - \mu$ vertex is obtained by a similar procedure:

$$\Lambda_L^{\mu\mu} Z'^\alpha \bar{\mu}_L \gamma_\alpha \mu_L + \text{h.c.}, \quad (4.7)$$

where

$$\Lambda_L^{\mu\mu} = \frac{g_X |\lambda_\mu|^2 v_\phi^2}{2M_L^2}. \quad (4.8)$$

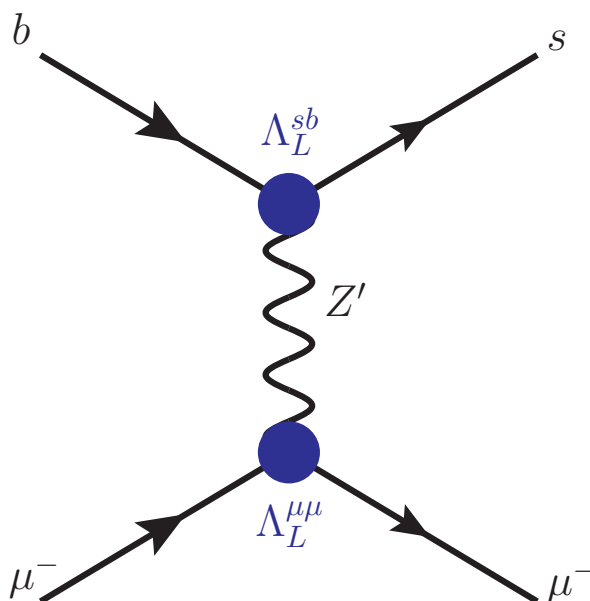


Figure 2. Feynman diagram for $b \rightarrow s\mu^+\mu^-$.

The prediction of Wilson coefficients C_9, C_{10} in the model can be made simply by integrating the mediating Z' gauge boson out in figure 2:

$$C_9 = -C_{10} = -\frac{\pi}{\sqrt{2}G_F\alpha_{\text{em}}} \frac{1}{V_{ts}^*V_{tb}} \frac{\Lambda_L^{sb}\Lambda_L^{\mu\mu}}{m_{Z'}^2}. \quad (4.9)$$

By defining $x_i \equiv \lambda_i v_\phi / \sqrt{2}M_Q$, $x_\mu \equiv \lambda_\mu v_\phi / \sqrt{2}M_L$, and using the relation $m_{Z'} = g_X v_\phi$, the above results can be rewritten as

$$C_9 = -C_{10} = -\frac{\pi}{\sqrt{2}G_F\alpha_{\text{em}}V_{ts}^*V_{tb}} \frac{x_{sb}|x_\mu|^2}{v_\phi^2} \approx -0.43 \left(\frac{x_{sb}}{-0.001} \right) \left| \frac{x_\mu}{0.5} \right|^2 \left(\frac{0.6 \text{ TeV}}{v_\phi} \right)^2, \quad (4.10)$$

where $x_{sb} \equiv x_s x_b^*$. We can explain (1.4) with relatively low $v_\phi (= 0.6 \text{ TeV})$ when $x_{sb} = -0.001$ and $x_\mu = 0.5$.

The effective vertex (4.5) also generates $B_s - \bar{B}_s$ mixing at tree-level, which turns out to be the most stringent constraint in our model [18, 65]. In our model the effective Hamiltonian for the mixing is

$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \frac{4G_F}{\sqrt{2}} (V_{ts}^*V_{tb})^2 [C_{sb}^{LL} (\bar{s}_L \gamma_\mu b_L)(\bar{s}_L \gamma^\mu b_L) + \text{h.c.}]. \quad (4.11)$$

The Wilson coefficient C_{sb}^{LL} is constrained by the mass difference ΔM_s of the B_s system. The experimental measurement [66]

$$\Delta M_s^{\text{exp}} = (17.757 \pm 0.021) \text{ ps}^{-1} \quad (4.12)$$

is smaller than the SM prediction² [65]

$$\Delta M_s^{\text{SM}} = (19.9 \pm 1.5) \text{ ps}^{-1}. \quad (4.13)$$

For the NP model which interferes with the SM constructively, as in our case when we take the coupling constants to be real, the constraint is very severe. The NP contributes to C_{sb}^{LL} through the Z' -exchanging tree-level diagram with its effective vertex in (4.5):

$$C_{sb}^{LL} = \frac{1}{4\sqrt{2}G_F(V_{ts}^*V_{tb})^2} \frac{(\Lambda_L^{sb})^2}{m_{Z'}^2} = \frac{1}{4\sqrt{2}G_F(V_{ts}^*V_{tb})^2} \frac{x_{sb}^2}{v_\phi^2}. \quad (4.14)$$

Then we get 2σ upper bound

$$\frac{x_{sb}}{v_\phi} \leq 2.05 \times 10^{-3} \text{ TeV}^{-1}. \quad (4.15)$$

The LHC experiments can also give constraints on the couplings [67]. For example, the parameter region which explains the $b \rightarrow s\mu^+\mu^-$ anomaly in the model,

$$\mathcal{L} \supset g_* Z'_\mu \bar{d}_{iL} (V_{\text{CKM}})_{ij} \gamma^\mu d_{jL} + g_* Z'_\mu \bar{\ell}_{iL} \gamma^\mu \ell_{iL}, \quad (4.16)$$

is already ruled out by the 13 TeV ATLAS $pp \rightarrow \mu^+\mu^-$ analysis with 36 fb^{-1} data at 95% CL [67]. Since Z' in our model couples to the SM fermions through mixing, the dimuon production cross section at the LHC is suppressed by the mixing factor, $|x_{d(u)}|^4 |x_\mu|^4$, where x_i ($i = d, u, s$) is defined below (4.9) and $x_j = (V_{\text{CKM}} \lambda^q)_j v_\phi / \sqrt{2} M_Q$ ($j = u, c, t$). The mixing factor is constrained to be $|x_d|^4 |x_\mu|^4 \ll |x_{ds}|^2 |x_\mu|^4 \lesssim 5 \times 10^{-8}$ for $v_\phi = 1 \text{ TeV}$ (see below). So we can safely neglect the constraint from the LHC in our model.

It turns out the strongest constraint comes from the neutrino trident production, which reads [68]

$$\frac{m_{Z'}}{|\Lambda_L^{\mu\mu}|} \gtrsim 0.47 \text{ TeV}. \quad (4.17)$$

Using $m_{Z'} = g_X v_\phi$, the above bound translates into

$$|x_\mu| \lesssim \left(\frac{v_\phi}{\text{TeV}} \right)^{1/2}. \quad (4.18)$$

We note that the Z' mass, $m_{Z'}$, enters C_9 , C_{sb}^{LL} , the LHC searches, and the neutrino trident production only in the combination $m_{Z'}/g_X = v_\phi$. Therefore $m_{Z'}$ itself is not directly constrained. The perturbativity, however, constrains $g_X \lesssim 4\pi$ or equivalently $m_{Z'} \lesssim 4\pi v_\phi$, for example $m_{Z'} \lesssim 6.3 \text{ TeV}$ for $v_\phi = 0.5 \text{ TeV}$. We also assume $m_{Z'}$ is at least the electroweak scale, in which case we can integrate out the Z' and use the effective Hamiltonian (1.5).

In figure 3 we show the region which can explain the $b \rightarrow s\mu^+\mu^-$ puzzle in $(v_\phi, |x_{sb}|)$ plane. The solid red line is the central value (1.4) to solve the $b \rightarrow s\mu^+\mu^-$ puzzle. The

²The prediction uses only tree-level inputs for the CKM parameters.

dashed (dotted) lines represent 1σ (2σ) region. The gray region is disfavoured by the $B_s - \bar{B}_s$ mixing. We fixed $x_\mu = 0.5$, which satisfies the constraint (4.18), $|x_\mu| \lesssim 0.7$ when $v_\phi \gtrsim 0.5$ TeV.

For $v_\phi = 0.6$ TeV, $|x_{sb}| = 10^{-3}$, and $|\lambda_s \lambda_b^*| = 1$, we get $M_Q \approx 13$ TeV. This heavy quark mass is much smaller than the PQ-breaking scale, and therefore the Yukawa coupling should be very small $y_Q \sim 10^{-8}$. Since the symmetry is enhanced in the $y_Q \rightarrow 0$ limit,³ the small y_Q is technically natural.

For the vector-like heavy lepton, we get $M_L = 0.85$ TeV for $v_\phi = 0.6$ TeV, $|x_\mu| = 0.5$, $|\lambda_\mu| = 1$. The vector-like lepton can be much lighter than the vector-like quark. The current lower limit on M_L is about 460 GeV [69]. It can be discovered at HL-LHC collider with $\sqrt{s} = 14$ TeV and 3 ab^{-1} pp collisions, if the mass is less than about 900 GeV [70].

Up to now we considered only the mixing between the 2nd and 3rd generation quarks, implicitly neglecting the 1st generation. Although it is irrelevant to our analysis, let us estimate the upper bound of the 1st and the 2nd generation quark mixing to see the possible flavour structure of the couplings ($\lambda_d, \lambda_s, \lambda_b$) which mix the vector-like and the SM quarks. As in the case of the 2nd and the 3rd generation mixing, the 1st and the 2nd generation mixing parameter is also strongly constrained by the corresponding mass difference of the neutral meson system. Here the relevant effective Hamiltonian for the $K^0 - \bar{K}^0$ mixing is

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \frac{x_{ds}^2}{v_\phi^2} (\bar{d}_L \gamma_\mu s_L) (\bar{d}_L \gamma_\mu s_L) + \text{h.c.}, \quad (4.19)$$

where $x_{ds} \equiv x_d x_s^*$. Since the long distance contribution to the mass difference, Δm_K , is important, we will be satisfied with the order of magnitude estimate. In the ‘vacuum insertion approximation’, we obtain [71]

$$\Delta m_K \approx \frac{2f_K^2 m_K}{3} \frac{\text{Re}(x_{ds}^2)}{v_\phi^2}, \quad (4.20)$$

$f_K \approx 114.4$ MeV is the kaon decay constant, and $m_K \approx 497.6$ MeV is its mass. Conservatively we require the above NP contribution should be less than the experimental measurement, $\Delta m_K^{\text{exp}} = 3.48 \times 10^{-12}$ MeV. Assuming x_{ds} is real, we get

$$\frac{x_{ds}}{v_\phi} \lesssim 9 \times 10^{-4} \text{ TeV}^{-1}, \quad (4.21)$$

which should be compared with (4.15). If we include the SM contribution, the above ratio is further constrained, and it is natural to assume the hierarchical structure: $\lambda_d \ll \lambda_s \ll \lambda_b$. Since we assume the NP affects only the muon coupling in this paper, we expect a similar hierarchy in the lepton sector couplings: $\lambda_e \ll \lambda_\mu \ll \lambda_\tau$. These hierarchies in the quark and lepton sector naturally lead the violation of the lepton flavour universality to solve the $b \rightarrow s\mu^+\mu^-$ puzzle, while avoiding the stringent constraints related to the 1st generation experiments.’

Since $Q_L(L_L)$ has the same quantum number with the SM $q_L(\ell_L)$ and $Q_R(L_R)$ does not mix with the SM $u_R, d_R(e_R, \nu_R)$, the SM Z boson coupling to the SM fermions are

³The PQ charges of $Q_{L,R}$ can be arbitrary.

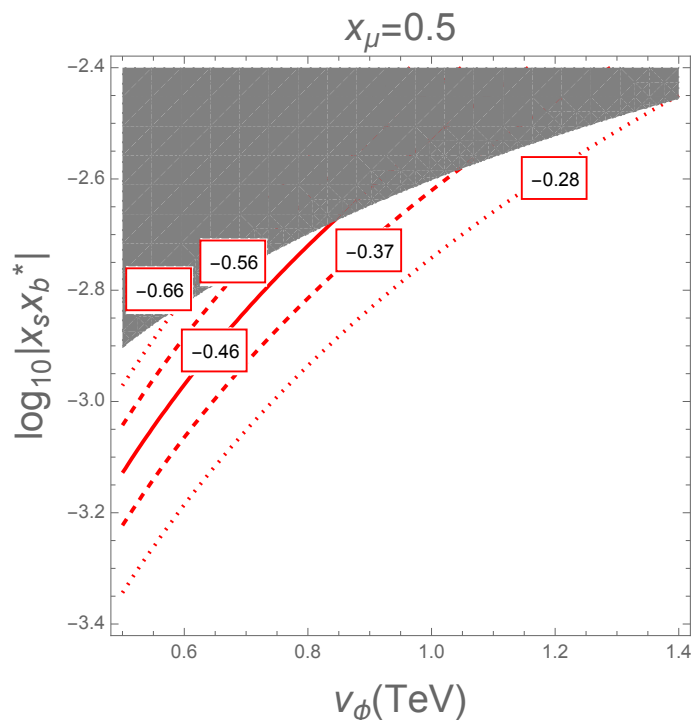


Figure 3. The solid red line is the central value (1.4) to solve the $b \rightarrow s\mu^+\mu^-$ puzzle. The dashed (dotted) line represents 1σ (2σ) region. The gray region is disfavoured by the $B_s - \bar{B}_s$ mixing. We fixed $x_\mu = 0.5$.

flavour-diagonal. Therefore the Z -mediated FCNC interactions are not generated, which makes the constraint from the Z boson interactions mild. For other constraints for the model, we refer the reader to [18] where possible constraints are considered in detail.

5 Conclusions

There are several clues which suggest new physics beyond the standard model of particle physics: dark matter, neutrino mass, strong CP problem, and possibly the flavour anomaly in $b \rightarrow s\mu^+\mu^-$ transition. We have considered a minimal model which can address the above hints simultaneously.

The neutrino mass is obtained when neutrino-specific Higgs-doublet gets a VEV, v_1 [1–3]. The value v_1 is due to the breaking of the PQ symmetry as shown in figure 1, is related to the high energy scale via seesaw-like relation (3.20), and is naturally small. Since the PQ symmetry does not allow the right-handed neutrino mass term, the neutrinos are Dirac-type.

The axion is KSVZ-type, but is distinguished from the original KSVZ model in that the axion-photon coupling is that of DFSZ-type axion model and the axion-neutrino coupling is sizable. The heavy quarks carrying PQ charges are also at 10 TeV scale without causing fine-tuning problem, and also can be tested in near future experiments.

The heavy quarks and heavy leptons also carry the charges of $U(1)_X$ gauge symmetry. The $U(1)_X$ -gauge-boson-mediating tree-level diagram generates the quark-level $b \rightarrow s\mu^+\mu^-$ transition, which can explain the flavour puzzle. We considered the constraint from the existing experiments including $B_s - \bar{B}_s$ mixing.

In the model, the neutrino mass and the axion are generated by the breaking the PQ-symmetry. The PQ-charged heavy quarks which generate the QCD anomaly also induce the flavour-changing Z' couplings. The axion is also a good candidate for the cold dark matter. Therefore our model shows an interplay between neutrino mass, flavour physics, axion, and dark matter.

Acknowledgments

This work was supported in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT), Grant No. NRF-2018R1A2A3075605.

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