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On the Vilkovisky-DeWitt approach and renormalization group in effective quantum gravity

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ABSTRACT: The effective action in quantum general relativity is strongly dependent on the gauge-fixing and parametrization of the quantum metric. As a consequence, in the effective approach to quantum gravity, there is no possibility to introduce the renormalization-group framework in a consistent way. On the other hand, the version of effective action proposed by Vilkovisky and DeWitt does not depend on the gauge-fixing and parametrization off-shell, opening the way to explore the running of the cosmological and Newton constants as well as the coefficients of the higher-derivative terms of the total action. We argue that in the effective framework the one-loop beta functions for the zero-, two- and four-derivative terms can be regarded as exact, that means, free from corrections coming from the higher loops. In this perspective, the running describes the renormalization group flow between the present-day Hubble scale in the IR and the Planck scale in the UV.

KEYWORDS: Models of Quantum Gravity, Renormalization Group

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1 Introduction

The effective action in quantum field theory can be used for deriving the S -matrix or other physically relevant quantities. In the conventional approach, the effective action has fundamental ambiguities related to the choice of parametrization of quantum field or, in gauge theories, to the choice of the gauge-fixing condition. These ambiguities vanish on-shell, which enables one to formulate in a consistent manner the renormalizable theory. However, this situation creates serious difficulties in the effective theory. For instance, the renormalization group framework has to be constructed based on the off-shell effective action. In many cases this means that there is no way to use the renormalization group to explore the running of the relevant parameters with scale in some physical situations.

The quantum version of general relativity is non-renormalizable, but it is perfectly appropriate for the effective theory approach. The reason is that the graviton is a massless field, and the next physical degrees of freedom (coming from higher derivatives) have masses of the Planck order of magnitude $M_P \approx 10^{19}$ GeV [1]. However, in the case of quantum general relativity, the one-loop and higher-loop divergences are strongly dependent on the gauge-fixing and parametrization of the quantum metric. Owing to this, the beta functions for all relevant parameters are badly defined and there is no chance to extract the unambiguous running from the effective low-energy quantum gravity.

The on-shell conditions in the non-renormalizable theory can hardly be implemented, especially at the non-perturbative level. Thus, it is difficult to obtain the relevant information from the quantum theory. On the other hand, there is an alternative definition of the effective action, introduced by Vilkovisky [2, 3] and DeWitt [4, 5], based on the covariant formulation in the space of the quantum gauge fields. The Vilkovisky-DeWitt unique effective action is gauge- and parametrization-independent, paving the way to explore in a consistent manner the running of the parameters of the total action, including the cosmological and Newton constants and the coefficients of the higher-derivative terms.

The gauge-fixing independence of the Vilkovisky-DeWitt effective action has been proved in a general setting and was also confirmed by one-loop direct calculations [6–13]. Recently its universality has been also confirmed by an explicit calculation in an arbitrary parametrization within quantum gravity [14]. Let us stress that this verification is especially relevant in the case of conformal parametrization of the metric. One of the reasons is that there is an exception in the general proof of universality, which is mentioned in the pioneer work [2, 3]. It is known that the Vilkovisky-DeWitt effective action depends on the choice of the configuration space metric [13]. However, in the two-dimensional ($2D$) quantum gravity this metric may depend on the choice of the gauge-fixing and therefore the unique effective action becomes gauge-fixing dependent [15]. It was shown in [14] that this effect does not take place in the four-dimensional ($4D$) quantum gravity, regardless of the algebraic similarity with the $2D$ case in the conformal parametrization. The important detail is that the configuration-space metric has to be chosen as the bilinear form of the classical action in the *minimal* gauge, in a given parametrization of the quantum metric. In what follows we shall base all considerations on this assumption, which provides the universality of the Vilkovisky-DeWitt effective action.

In the present work, we shall follow the previous publication on the subject [16] and consider the renormalization group equations in the effective quantum gravity based on general relativity in the unique effective action formalism. Indeed, we expand the analysis performed in this seminal work in several directions. First of all, we stress the importance of the effective approach and discuss in more detail the corresponding area of application for the running of the parameters. Second, we extend the analysis to the higher-derivative part of the action, which is renormalized in a way similar to that of the semiclassical gravity. Third, treating the renormalization group in the effective framework, we argue that the low-energy one-loop running of the Newton constant G , cosmological constant Λ and the parameters of the higher-derivative part of the action is, in fact, non-perturbative and the corresponding beta functions can be viewed as exact.¹

The manuscript is organised as follows. In the next section 2 we briefly describe the general framework of the unique effective action in quantum general relativity. The reader can consult the parallel paper [14] for further details. In section 3 we construct, solve, and discuss the renormalization group equations for the Newton and cosmological constants in the framework of effective quantum gravity. In section 4 we extend the analysis to the higher-derivative sector of the theory and the discuss the non-perturbative aspects of the corresponding running. The perspectives of physical applications of the exact effective running are briefly described in section 5. Finally, in section 6, we draw our conclusions.

2 Vilkovisky-DeWitt effective action in quantum gravity

Let us start by formulating general definitions, valid for any gauge field theory, which we subsequently particularise for quantum gravity. As mentioned before, off the mass shell

¹We greatly appreciate the contribution of the referee of the first version of the paper [14], who gave us a valuable hint in this direction and advised to proceed this discussion independently of the parametrization-related calculations.

the effective action depends on the field parametrization. This can be readily seen by recalling that in the one-loop approximation the effective action depends on Hessian of the classical action. In fact, while the action $S(\varphi)$ is a scalar in the space of fields φ^i , its second functional derivative does not transform as a tensor. The dependence on the gauge-fixing can be understood in a similar manner, since the effective actions calculated in different gauges are related by changes of variables in the form of canonical transformations [17–19].

The problem can be addressed in a geometric framework via the introduction of a metric $\bar{G}_{ij}$ and a connection $\mathcal{T}_{ij}^k$ in the configuration space of physical fields. Accordingly, the definition of the effective action should be modified, so that it is constructed only with scalar quantities. This programme was carried out for the first time² by Vilkovisky [2, 3], who introduced the unique effective action $\Gamma(\varphi)$ through

$$\exp i\Gamma(\varphi) = \int \mathcal{D}\varphi' \mu(\varphi') \exp \left\{ i \left[S(\varphi') + \sigma^i(\varphi, \varphi') \Gamma_{,i}(\varphi) \right] \right\}, \quad (2.1)$$

where $\sigma_i(\varphi, \varphi')$ is the derivative (with respect to φ^i) of the world function [21, 22] constructed with the connection $\mathcal{T}_{ij}^k$, and $\mu(\varphi')$ is an invariant functional measure. Since $\sigma^i(\varphi, \varphi')$ behaves as a vector with respect to φ^i and as a scalar with regard to φ'^i , the effective action $\Gamma(\varphi)$ in (2.1) is reparametrization invariant and gauge independent.

The geometric objects mentioned above can be constructed from two fundamental quantities: the metric G_{ij} in the space $\mathcal{M}$ of fields and the generators R_α^i of gauge transformations. As our main concern here is on gravity theory, we assume that the field φ^i is bosonic, and that the generators R_α^i are linearly independent and form a closed algebra, with structure functions which do not depend on the fields. Introducing the metric $N_{\alpha\beta}$ on the gauge group $\mathcal{G}$ and its inverse $N^{\alpha\beta}$,

$$N_{\alpha\beta} = R_\alpha^i G_{ij} R_\beta^j, \quad N_{\alpha\lambda} N^{\lambda\beta} = \delta_\alpha^\beta, \quad (2.2)$$

one can define the projector on $\mathcal{M}/\mathcal{G}$ [2, 3, 8],

$$P_j^i = \delta_j^i - R_\alpha^i N^{\alpha\beta} R_\beta^k G_{kj}. \quad (2.3)$$

The projected metric in the space $\mathcal{M}/\mathcal{G}$ of physical fields is, therefore,

$$\bar{G}_{ij} = P_i^k G_{k\ell} P_j^\ell = G_{ij} - G_{ik} R_\alpha^k N^{\alpha\beta} R_\beta^\ell G_{\ell j}. \quad (2.4)$$

Since $N^{\alpha\beta}$ is the inverse of a differential operator, this metric contains a non-local part which arises from the constraints imposed by the gauge symmetry.

The connection $\mathcal{T}_{ij}^k$ can be obtained by requiring its compatibility with $\bar{G}_{ij}$ (see e.g. [9, 13]), and it can be written in the form [2, 3]

$$\mathcal{T}_{ij}^k = \Gamma_{ij}^k + T_{ij}^k, \quad (2.5)$$

where Γ_{ij}^k are the (local) Christoffel symbols associated to the metric G_{ij} and

$$T_{ij}^k = -2G_{(i|\ell} R_\alpha^\ell N^{\alpha\beta} \mathcal{D}_{|j)} R_\beta^k + G_{(i|\ell} R_\alpha^\ell N^{\alpha\beta} R_\beta^m (\mathcal{D}_m R_\gamma^k) N^{\gamma\delta} R_\delta^n G_{n|j)} \quad (2.6)$$

²See ref. [20] for an earlier attempt towards this in the context of non-linear σ -models.

is its non-local part. The parenthesis in the indices denote symmetrization in the pair (i, j) and $\mathcal{D}_i$ indicates the covariant derivative based on the Christoffel connection Γ_{ij}^k .

One can proceed the loop expansion of the Vilkovisky effective action (2.1),

$$\Gamma(\varphi) = S(\varphi) + \bar{\Gamma}^{(1)}(\varphi) + \bar{\Gamma}^{(2)}(\varphi) + \dots, \quad \hbar = 1, \quad (2.7)$$

which gives the one-loop contribution [2, 3]

$$\bar{\Gamma}^{(1)} = \frac{i}{2} \text{Tr} \ln G^{ik} (\mathcal{D}_k \mathcal{D}_j S - T_{kj}^\ell S_{,\ell} - \chi_{,k}^\alpha Y_{\alpha\beta} \chi_{,j}^\beta) - i \text{Tr} \ln M_\beta^\alpha, \quad (2.8)$$

where χ^α is a gauge-fixing condition, $Y_{\alpha\beta}$ is the weight functional and $M_\beta^\alpha = \chi_{,i}^\alpha R_\beta^i$ is the Faddeev-Popov ghost matrix. Compared to the standard effective action, the Hessian of the classical action has now been replaced by its covariant version, ensuring its tensor nature concerning field reparametrizations. Notice also that the non-local part of the connection (2.6) behaves as a tensor as well. The divergent part of the one-loop effective action (2.8) can be evaluated, e.g., by applying the generalized Schwinger-DeWitt technique of refs. [6, 7].

Nevertheless, the effective action defined by (2.1) cannot be viewed as a final solution aiming to the off-shell universality because of two reasons. First, it might happen that the metric G_{ij} is not uniquely defined. This issue can be solved by additional prescriptions [2, 3], as we shall comment later. The most serious obstacle, however, is the lack of one-particle irreducibility of the diagrams generated by (2.1), which may take place beyond one-loop. Indeed, it gives rise to non-local divergences at the two-loop approximation in Yang-Mills theories, as shown in ref. [10].

The modification of the Vilkovisky effective action (2.1) proposed by DeWitt [4, 5] can be viewed as a way out of this difficulty, inasmuch as it restores the one-particle irreducibility of the perturbative expansion [10, 11] (see also [12] for further discussion). For example, in [10] it was verified by explicit calculations that the aforementioned non-local divergences which appeared in original formulation (2.1) in the two-loop Yang-Mills theory are cancelled in the DeWitt approach [4, 5].

The construction introduced by DeWitt [4, 5], usually called Vilkovisky-DeWitt effective action, consists in choosing an arbitrary point φ_*^i in the configuration space and, instead of simply defining the vector-scalar quantity $\sigma^i(\varphi, \varphi')$ in terms of the mean field, one builds a system of Gaussian normal coordinates with φ_*^i , according to which the covariant Taylor expansions should be performed. The method involves defining an effective action and a mean field which depend on φ_*^i , and only in the end, after the implicit equation is solved iteratively, this point is identified to the mean field. At one-loop level, the Vilkovisky-DeWitt effective action coincides to the Vilkovisky one (2.1), therefore the eq. (2.8) also holds in this more general formalism. Since for most of the discussion in the present paper we work with one-loop results, we shall not present further technical details of the Vilkovisky-DeWitt effective action. The important point here is to stress that the formalism guarantees the one-particle irreducibility of the diagrammatic expansion.

The remaining question to deal with is the aforementioned dependence of the unique effective action and, in particular, its one-loop part (2.8), on the choice of the metric in

the space of the fields. This issue is especially relevant for quantum gravity, where there is a one-parameter family of such metrics, characterised by the parameter $\bar{a} \neq -1/4$, given by [23]

$$G^{\mu\nu,\alpha\beta} = \frac{1}{2}(\delta^{\mu\nu,\alpha\beta} + \bar{a} g^{\mu\nu} g^{\alpha\beta}), \quad \text{where} \quad \delta^{\mu\nu,\alpha\beta} = \frac{1}{2}(g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha}). \quad (2.9)$$

It was shown by explicit calculations [13] (see also [24, 25]) that the Vilkovisky-DeWitt effective action depends on the choice of $\bar{a}$. Nonetheless, already in refs. [2, 3] it was introduced a prescription to fix this ambiguity. In accordance, the field-space metric should coincide with the expression in the highest-derivative term in the bilinear part of the classical action in the minimal gauge fixing. In the parallel paper [14] we have shown that this prescription works perfectly well even under changes of the parametrization of the quantum field, which also modifies the parameter $\bar{a}$. Thus, it defines a unique off-shell effective action.

The classical action of general relativity has the form

$$S = -\frac{1}{\kappa^2} \int d^4x \sqrt{-g} (R + 2\Lambda), \quad (2.10)$$

where $\kappa^2 = 16\pi G$ and G is the Newton constant. In the effective approach, the quantum theory takes into account only the massless modes of the quantum metric [1, 26]. For this reason, these quantum effects are completely defined by the action (2.10), regardless of the presence of higher-derivative terms (to be defined below) in the full gravitational action.

The one-loop divergent part of the Vilkovisky-DeWitt effective action for the Einstein gravity was evaluated for the first time in refs. [6, 7], while the terms related to the cosmological constant were calculated in [8]. We shall skip all the calculations and refer the reader to the mentioned papers for the details. The result for the one-loop divergences is

$$\bar{\Gamma}_{\text{div}}^{(1)} = -\frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \left\{ \frac{121}{60} C^2 - \frac{151}{180} E + \frac{31}{36} R^2 + 8\Lambda R + 12\Lambda^2 \right\}, \quad (2.11)$$

where $\epsilon = (4\pi)^2(n-4)$ is the parameter of dimensional regularization and μ is the renormalization parameter. Also, $C^2 = R_{\mu\nu\alpha\beta}^2 - 2R_{\mu\nu}^2 + \frac{1}{3}R^2$ denotes the square of the Weyl tensor and $E = R_{\mu\nu\alpha\beta}^2 - 4R_{\mu\nu}^2 + R^2$ is the integrand of the Gauss-Bonnet invariant in the four-dimensional spacetime. On the classical mass shell, eq. (2.11) reduces to the divergent on-shell part of the usual effective action [27, 28]

$$\bar{\Gamma}_{\text{div}}^{(1)}|_{\text{on-shell}} = -\frac{\mu^{n-4}}{\epsilon} \int d^n x \sqrt{-g} \left\{ \frac{53}{45} E - \frac{58}{5} \Lambda^2 \right\}. \quad (2.12)$$

This expression is also gauge-fixing and parametrization independent (see e.g. [29] and references therein), but the advantage of the result (2.11) is that it is universal even off-shell. This feature opens the way for formulating consistent low-energy renormalization group equations for Λ , G [16], and for other parameters of the action, which were not included in the basic formula (2.10).

Before proceeding to the renormalization group and the effective approach to quantum gravity, let us briefly review the power counting in the quantum theory based on general

relativity. In this theory, the propagator behaves like k^{-2} and there are two kinds of vertices: the ones with two derivatives, owed to the Einstein-Hilbert term, and those with no derivative, coming with coefficient Λ . The coupling constant (parameter of the loop expansion) is κ^2 , with dimension of inverse of mass-squared. Since the quantum metric $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ is dimensionless, the power counting is especially simple. For a given p -loop diagram with n_2 vertices with two momenta and n_0 vertices with zero momenta, the superficial degree of divergence is

$$\omega = 2p - 2n_0 + 2 - d, \quad (2.13)$$

where d is the number of derivatives acting on the external lines of $h_{\mu\nu}$. This relation defines the number of derivatives $d = 2p - 2n_0 + 2$ for the logarithmically divergent diagrams with $\omega = 0$. It is easy to see that the expression (2.11) satisfies this condition; the $\mathcal{O}(\Lambda)$ -terms with $n_0 = 1$ are proportional to R , that means $d = 2$, while $\mathcal{O}(\Lambda^2)$ -terms with $n_0 = 2$ have $d = 0$.

Both at the one-loop level and in higher loops, the logarithmically divergent diagrams with $n_0 = 0$ satisfy the condition $d = 2p + 2$, which means that the maximal number of derivatives in the counterterms grows linearly with the number of loops p . In particular, for $\Lambda = 0$ the four-derivative terms in the one-loop formula (2.11) are actually exact, since they do not gain higher-loop contributions. The same concerns the $\mathcal{O}(R^3)$ -type terms at the two-loop order, and so on.

On the other hand, in the real world, these terms are practically exact even if $\Lambda \neq 0$. The reason is that the four-derivative terms gain p -loop contributions with coefficients proportional to $\Lambda\kappa^2 = \frac{\Lambda}{M_P^2}$ to the power p . In the present-day Universe, this coefficient is of the order of 10^{-120} , which is small enough to support the argument that the result can be regarded as exact. It is a direct exercise to extend this statement also to the lower-derivative terms in (2.11). We shall come back to this reasoning and use it intensively in the next two sections when discussing the renormalization group.

3 Renormalization group based on the unique effective action

One can use the result (2.11) for analyzing the renormalization group equations in the low-energy (infrared, IR) sectors of the theory. Such a construction has a direct physical sense. In the high-energy domain (UV) the theory (2.10) cannot be applied without restrictions, as it is non-renormalizable and the contributions of massive degrees of freedom, related to higher derivative terms, are supposed to modify the beta functions. However, since the quantum gravity based on general relativity is a massless theory, it makes sense to explore the renormalization group running in the IR. Assuming that the higher-derivative massive degrees of freedom have masses of the Planck order of magnitude, in most of the physically relevant situations these modes decouple [30] (see also the concrete discussion of this issue in the semiclassical gravity [31–33] and qualitative discussion in quantum gravity [34–36]), such that the running is completely defined by the action (2.10).

In other words, since the theory is massless, the quantum gravity based on general relativity can be regarded as an effective theory of quantum gravity at the energies between

the UV (Planck) scale, where the massive degrees of freedom coming from higher derivatives can become relevant, and the deep IR scale. Thus, the Vilkovisky-DeWitt unique effective action enables one to explore the scale dependence in this vast region in a gauge-fixing and parametrization independent manner.

From the classical action (2.10) and the expression for the divergences (2.11), it is easy to obtain the renormalization relations

$$\frac{1}{\kappa_0^2} = \mu^{n-4} \left[\frac{1}{\kappa^2} - \frac{8}{(4\pi)^2(n-4)} \Lambda \right], \quad \Lambda_0 = \Lambda \left[1 + \frac{2}{(4\pi)^2(n-4)} \Lambda \kappa^2 \right]. \quad (3.1)$$

The bare quantities κ_0^2 and Λ_0 are μ -independent, as it is the case for the renormalized effective action. Applying the operator $\mu \frac{d}{d\mu}$ to both sides of each of the relations (3.1), after a small algebra we arrive at the renormalization group equations

$$\mu \frac{d}{d\mu} \frac{1}{\kappa^2} = \frac{8\Lambda}{(4\pi)^2}, \quad (3.2)$$

$$\mu \frac{d\Lambda}{d\mu} = -\frac{2\Lambda^2 \kappa^2}{(4\pi)^2}, \quad (3.3)$$

which are equivalent to those obtained in [16, 25].

To solve eqs. (3.2) and (3.3), we define the dimensionless quantity $\gamma = \kappa^2 \Lambda$. Due to the uniqueness of this dimensionless combination of κ^2 and Λ , the equation for γ gets factorized,

$$\mu \frac{d\gamma}{d\mu} = -\frac{10\gamma^2}{(4\pi)^2}. \quad (3.4)$$

The solution of this equation has the standard form

$$\gamma(\mu) = \frac{\gamma_0}{1 + \frac{10}{(4\pi)^2} \gamma_0 \ln \frac{\mu}{\mu_0}}, \quad (3.5)$$

where $\gamma_0 = \gamma(\mu_0)$ and μ_0 marks a fiducial energy scale. We assume the initial values of the renormalization group trajectories of the cosmological constant $\Lambda_0 = \Lambda(\mu_0)$ and the gravitational constant $G_0 = G(\mu_0)$ as it is useful to come back from κ^2 to G at this stage.

Now, using (3.5) in (3.2) and (3.3), we obtain the final solutions

$$G(\mu) = \frac{G_0}{\left[1 + \frac{10}{(4\pi)^2} \gamma_0 \ln \frac{\mu}{\mu_0} \right]^{4/5}} \quad (3.6)$$

and

$$\Lambda(\mu) = \frac{\Lambda_0}{\left[1 + \frac{10}{(4\pi)^2} \gamma_0 \ln \frac{\mu}{\mu_0} \right]^{1/5}}, \quad (3.7)$$

which are certainly consistent with (3.5).

The solutions (3.6) and (3.7) are remarkable in several aspects. First of all, such independent solutions for the two effective charges are impossible in quantum gravity based on the usual effective action neither in quantum general relativity nor the fourth-derivative gravity, as the individual equations for $G(\mu)$ and $\Lambda(\mu)$ are completely ambiguous. In the

latter model, only the solution for the dimensionless quantity in (3.5) is gauge-fixing and parametrization independent.³ Here we have a well-defined running for the two parameters only because of the use of the Vilkovisky-DeWitt effective action.

Let us note that the unambiguous solutions for $G(\mu)$ and $\Lambda(\mu)$ exist in the superrenormalizable gravity model [35], but there are two relevant differences. The advantage of the equations and solutions of [35] is that those can be exact, in the sense of not depending on the order of the loop expansion. On the other hand, the higher-derivative models that lead to such an exact result imply the functional integration over massive degrees of freedom, which can be ghosts or healthy modes. This means that the corresponding equations are valid only in the UV for the quantum gravity energy scale, i.e., only in the trans-Planckian region. Below the Planck scale the massive degrees of freedom decouple and we are left with the quantum effects of effective quantum gravity, such as the ones of quantum general relativity (see e.g. [1], the review [26] and the recent discussion of the decoupling in gravity in [33, 36]).

On the contrary, the running described by (3.6) and (3.7) comes from the quantum effects of the purely massless degrees of freedom. Up to some extent, the running should be described by the same equations in both UV and IR. According to the general discussion which we postpone for the next section, these equations can be seen as exact, being valid in the same form even beyond the one-loop approximation.

It is clear that the physical interpretation of the solutions (3.6) and (3.7) depends on the sign of γ_0 . Since the positive sign of G is fixed by the positive definiteness of the theory, the sign of γ_0 depends on the one of Λ_0 . Due to the cosmological observations, we know that the sign of the observed cosmological constant is positive in the present-day Universe [37, 38]. For a positive γ_0 the solutions (3.6) and (3.7) indicate the asymptotic freedom in the UV. In case of a moderate cosmological constant (remember $\kappa \propto M_P^{-1}$) the value of γ_0 is very small. This implies a very weak running, that is irrelevant from the physical viewpoint. In particular, the running (3.6) and (3.7) is not essential for the cosmological constant problem between the electroweak scale and the present day, low-energy, cosmic scale.

On the other hand, at the electroweak energy scale, the early Universe probably passed through the corresponding phase transition. At that epoch, the observable value of the cosmological constant could dramatically change because of the symmetry restoration. Does this change Λ in the action (2.10)? The answer to this question is negative. Let us remember that the observable cosmological constant is a sum of the two parts: one is the vacuum parameter in the gravitational action (2.10) and another is the induced counterpart, the main part of it coming from the symmetry breaking of the Higgs potential. The main relations are (see, e.g., [39] or [40])

$$\rho_\Lambda^{\text{obs}} = \rho_\Lambda^{\text{ind}} + \rho_\Lambda^{\text{vac}}, \quad \rho_\Lambda^{\text{ind}} = \frac{\Lambda_{\text{ind}}}{8\pi G_{\text{ind}}} = -\lambda v_0^4, \quad (3.8)$$

³In quantum Einstein gravity based on the usual effective action, on the other hand, only by using the on-shell version of renormalization group it is possible to define an unambiguous equation for γ [30].

where λ is the self-coupling and v_0 the vacuum expectation value of the Higgs field. As far as $\rho_\Lambda^{\text{ind}}$ is negative and the magnitude of $\rho_\Lambda^{\text{obs}}$ is negligible, the sign of $\rho_\Lambda^{\text{vac}} = \frac{2\Lambda}{\kappa^2}$ is positive, independently of the electroweak phase transition.

Thus, we conclude that the sign of γ_0 is always positive, at least between the present-day cosmic scale in the IR and the GUT scale in the UV, where the considerations based on the Minimal Standard Model formulas, such as (3.8), may become invalid. In all this interval, the value of γ_0 is numerically small, such that the running in (3.6) and (3.7) is not physically relevant.

One can imagine a situation in which another phase transition occurs at the GUT scale (that means about 10^{14} – 10^{16} GeV), such that the new vacuum Λ between this scale and the Planck scale $M_P \approx 10^{19}$ GeV is negative. Then, the solutions (3.6) and (3.7) indicate the asymptotic freedom in the IR. Furthermore, if the cosmological constant in this energy scale interval has the order of magnitude of M_P , these solutions describe the situation of a dramatically strong running of both constants G and Λ , which are strongly decreasing in the IR. As we have learned in the previous section 2, for the values satisfying $|\Lambda| \ll M_P^2$, the higher-loop contributions cannot modify the form of the running. In any case, the construction of the corresponding model of GUT would be an interesting subject to work on in future. Here we just want to note that our results indicate this possibility.

4 Renormalization group for the fourth-derivative parameters

In order to complete the discussion, let us consider the renormalization group equations for the fourth-derivative terms in the action of gravity. To this end, we have to complement the action (2.10) with at least all those terms which are present in the expression for the divergences (2.11). According to the power counting (2.13), at p -loop order it is necessary to introduce into the action terms with up to $2p + 2$ derivatives of the metric. In this way we arrive at the well-known action of the higher-derivative quantum gravity [41],

$$S_{\text{tot}} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{\kappa^2} (R + 2\Lambda) - \frac{1}{2\lambda} C^2 + \frac{1}{2\rho} E - \frac{1}{2\xi} R^2 + \frac{1}{2\zeta} C_{\mu\nu\alpha\beta} C^{\alpha\beta}_{\cdot\cdot\rho\sigma} C^{\rho\sigma\mu\nu} \right. \\ \left. + \sum_{n=1}^N \left[\omega_{n,C} C_{\mu\nu\alpha\beta} \square^n C^{\mu\nu\alpha\beta} + \omega_{n,R} R \square^n R \right] + \mathcal{O}(R^3_{\dots}) \right\}, \quad (4.1)$$

where λ , ρ and ξ are the dimensionless parameters of the action and $N = p - 1$. The terms with more than four derivatives which contribute to the propagator of the quantum metric have the forms $R \square^n R$ and

$$C_{\mu\nu\alpha\beta} \square^n C^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} \square^n R^{\mu\nu\alpha\beta} - 2R_{\mu\nu} \square^n R^{\mu\nu} + \frac{1}{3} R \square^n R. \quad (4.2)$$

One could also include the curvature-squared higher-derivative terms of the type

$$\text{GB}_n = R_{\mu\nu\alpha\beta} \square^n R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} \square^n R^{\mu\nu} + R \square^n R \quad (4.3)$$

which represent the extended version of the four-dimensional Gauss-Bonnet topological invariant. Of course, these terms are not topological for $n \geq 1$, but can be shown (see

e.g. [41]) to be $\mathcal{O}(R_{\dots}^3)$ and therefore they do not contribute to the propagator. For the sake of simplicity we assume that the sum in (4.1) is finite, as otherwise we arrive at the non-local actions of quantum gravity (see, e.g., [42–44] for a review). In such a case, the structure of massive poles of the propagator when loop effects are taken into account is more complicated [45] and is not relevant for the present discussion. Still regarding eq. (4.1), we point out that we separated one of the possible Weyl-cubic terms $C_{\dots}^3$ from other terms of third and higher-order in curvatures, because in what follows we shall use it to discuss the two-loop effective low-energy beta function for the parameter ζ .

In the polynomial theories (4.1), the propagator can have real massive poles [41] or complex ones [46], but in both cases the natural situation is that all these massive parameters have the Planck order of magnitude [47]. Thus, in the effective approach, below the Planck scale we can completely ignore the quantum contributions of these massive degrees of freedom. The quantum effects are coming only from the massless mode, associated to the Einstein-Hilbert action (2.10).

The expression (4.1) includes the action of the fourth-derivative gravity [48],

$$S_{\text{four}} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2\lambda} C^2 + \frac{1}{2\rho} E - \frac{1}{2\xi} R^2 - \frac{1}{\kappa^2} (R + 2\Lambda) \right\}, \quad (4.4)$$

as a particular case. At the one-loop level, the power counting shows that only the terms up to the four-derivative part of the action (4.4) gain divergent contributions and, correspondingly, receive the logarithmic non-local corrections. Thus, we shall consider in details the beta functions and renormalization group equations for the remaining parameters in this sector of the total action.

The renormalization group equations for λ , ρ , and ξ were previously explored in the framework of the semiclassical theory, starting in [49] (see [50, 51] for a formal consideration and further references), and higher-derivative quantum gravity [30, 34, 52]. In the effective approach to quantum gravity based on the standard effective action one can determine only the equation for ρ since the corresponding divergence survives on-shell, see eq. (2.12), being unambiguous. On the other hand, the universality of the Vilkovisky-DeWitt effective action [2–5] makes it possible the new version of the renormalization group equations for the parameters λ and ξ . To this end, one applies the standard algorithm to the eqs. (2.11) and (4.4), from which it follows the beta functions

$$\beta_\lambda = -\frac{a_{\text{QG}}^2}{(4\pi)^2} \lambda^2, \quad a_{\text{QG}}^2 = \frac{121}{30}, \quad (4.5)$$

$$\beta_\xi = -\frac{b_{\text{QG}}^2}{(4\pi)^2} \xi^2, \quad b_{\text{QG}}^2 = \frac{31}{18}, \quad (4.6)$$

$$\beta_\rho = -\frac{c_{\text{QG}}^2}{(4\pi)^2} \rho^2, \quad c_{\text{QG}}^2 = \frac{151}{90}. \quad (4.7)$$

We have to define the lowest possible IR scale. In flat spacetime, the running produced by the quantum effects of the massless fields can be considered to occur for arbitrarily low energies. However, in the real applications (even in the low-energy cosmology) there is a natural IR cut-off, as it was described in ref. [53]. In order to understand the origin of this

cut-off, let us remember that the running of the fourth-derivative terms is related to the logarithmic form factors [54]. For the Weyl-squared term, for example, the corresponding term in the effective action reads

$$\frac{a^2}{(4\pi)^2} \int d^4x \sqrt{-g} C_{\mu\nu\alpha\beta} \ln \left(-\frac{\square}{\mu_0^2} \right) C^{\mu\nu\alpha\beta}, \quad (4.8)$$

with a^2 defined in (4.10) below; while the corresponding d'Alembert operator for weak perturbations around the cosmological (isotropic and homogeneous) background has the form (see [53] for the details)

$$\square = \partial_t^2 - 4H\partial_t - 2\dot{H} - 10H^2 + \dots, \quad (4.9)$$

where H is the Hubble parameter. The expression (4.9) shows that in the far future of the Universe, with the background becoming close to the de Sitter space, there will not be physical running of λ , because the theory is effectively massive. The cut-off (fictitious mass) parameter is defined by the relation $H \sim \sqrt{\Lambda/3}$. Numerically, this means that the running ends in the IR at the scale of the order $H_0 \approx 10^{-42} \text{ GeV}$. Between this scale and the intermediate scale defined by the neutrino masses (presumably of the order $m_\nu \approx 10^{-12} \text{ GeV}$) the running of λ , ξ and ρ is defined by the contributions of effective quantum gravity (4.5), (4.6) and (4.7) and the ones of the photon. Starting from the neutrino scale, we have to include fermion contributions. Thus, the renormalization group equation for λ is

$$\mu \frac{d\lambda}{d\mu} = -\frac{a^2}{(4\pi)^2} \lambda^2, \quad a^2 = a_{\text{QG}}^2 + \frac{1}{5} + \frac{N_f}{10}, \quad (4.10)$$

where N_f is the number of fermions. The solution of this equation has the usual form

$$\lambda(\mu) = \frac{\lambda_0}{1 + \frac{a^2}{(4\pi)^2} \lambda_0 \ln \frac{\mu}{\mu_0}}, \quad \lambda_0 = \lambda(\mu_0). \quad (4.11)$$

The remaining equations for ξ and ρ are

$$\mu \frac{d\xi}{d\mu} = -\frac{b^2}{(4\pi)^2} \xi^2, \quad b^2 = b_{\text{QG}}^2, \quad (4.12)$$

$$\mu \frac{d\rho}{d\mu} = -\frac{c^2}{(4\pi)^2} \rho^2, \quad c^2 = c_{\text{QG}}^2 + \frac{31}{90} + \frac{11}{180} N_f. \quad (4.13)$$

It is worth noting that the photon and fermion contributions to b^2 are ruled out due to the conformal invariance of these two fields in the massless versions. Another interesting point is that the contributions of effective quantum gravity to the equations for λ and ρ have the same sign of the ones related to vector and fermion fields. We remark that the same sign pattern also takes place in the scalar field theory, in fourth-derivative quantum gravity [30] (see also [34, 52] for a verification) and conformal quantum gravity [30, 55, 56]. This universality of signs probably means there are some general rules for the quantum corrections which we do not understand yet.

The solutions of eqs. (4.12) and (4.13) have the form

$$\xi(\mu) = \frac{\xi_0}{1 + \frac{b^2}{(4\pi)^2} \xi_0 \ln \frac{\mu}{\mu_0}}, \quad \xi_0 = \xi(\mu_0), \quad (4.14)$$

and

$$\rho(\mu) = \frac{\rho_0}{1 + \frac{c^2}{(4\pi)^2} \rho_0 \ln \frac{\mu}{\mu_0}}, \quad \rho_0 = \rho(\mu_0). \quad (4.15)$$

Let us make an important observation based on the discussion in section 2. Since the theory is not renormalizable by power counting, the dimensional arguments show that the higher-loop contributions to the beta functions for the parameters λ , ρ and ξ are possible only for the non-zero cosmological constant. In this case, because the coupling (parameter of the loop expansion) in the theory (2.10) is $\kappa^2 \sim M_P^{-2}$, the higher-loop corrections to the fourth-derivative terms are given by power series in the parameter $\frac{\Lambda}{M_P^2} \sim 10^{-120}$. Thus, in the real physical situations the higher-loop corrections for the dimensionless parameters λ , ρ and ξ are negligible. This is true at least until the UV energy scale defined by the electroweak phase transition. At a higher energy scale M , the value of the induced cosmological constant density (see the discussion in the previous section) is $\rho_\Lambda \propto M^4$, such that $\Lambda \approx \frac{M^4}{M_P^2}$. Then the dimensionless parameter of expansion in loops, in the framework of effective theory, is defined by the value of the ratio $\frac{\Lambda}{M_P^2} \approx \frac{M^4}{M_P^4}$. For the electroweak phase transition, $M \approx 300 \text{ GeV}$ and this parameter is about 10^{-66} . Assuming another phase transition at the GUT scale, we meet $M = M_X = 10^{14} - 10^{16} \text{ GeV}$ and the dimensionless parameter varies between 10^{-20} and 10^{-12} . All these values are certainly sufficient to claim the dominance of the one-loop effects. Therefore, the running which we have just derived, based on the effective approach to quantum general relativity within the Vilkovisky-DeWitt formalism, can be safely regarded as the exact, nonperturbative effect. Indeed, the same also concerns the running of Λ and G , which were discussed in the previous section.

Compared to other models of quantum gravity, the same level of generality can be achieved only in the polynomial [35, 41] and non-local models of quantum gravity (see the power-counting discussion in [45]), which are super-renormalizable. In both cases the beta functions for λ , ρ and ξ are not present in the published literature, and for the latter it is not clear how those functions can be derived, at least in a covariant way. Moreover, if the massive degrees of freedom in these super-renormalizable models have masses of the Planck order of magnitude, the exact running occurs only in the trans-Planckian region. On the contrary, in the case under discussion here, the exact running is an IR effect, taking place only *below* the Planck scale.

It is also worthwhile to make a comparison with the non-perturbative analysis in quantum gravity based on the functional renormalization group (see, e.g., the reviews [57, 58] and the more recent [59]). It was recently shown that in this approach the effective average action remains gauge-fixing dependent even on-shell⁴ [61]. For this reason, even within the Vilkovisky-DeWitt formulation of the off-shell effective action, regardless the last being gauge-fixing independent by construction, the effective *average* action remains gauge-fixing

⁴For a previous discussion on this subject in the context of Yang-Mills fields, see [60].

dependent in this case. No unambiguous physical predictions can be extracted from the quantum calculations in this approach.

The scheme of deriving the beta functions for λ , ρ and ξ described above resembles more the running of the vacuum action parameters in the semiclassical gravity [49, 50, 62, 63] than the renormalizable gravity [30]. The similarity with the semiclassical case is based on the fact that the running occurs in the sector of the theory which does not define the quantum effects. In the present case, this sector is related to fourth-derivative terms in the action (4.4). At higher loops one can meet the renormalization group running for the parameters of six- and higher-derivative terms in the action (4.1).

As a further illustration of the method, let us derive the two-loop beta function for the unique two-loop divergence derived until now [64–66], namely for the C^3 -term in the total action (4.1),

$$\bar{\Gamma}_{\text{div}}^{(2)} = \frac{\mu^{n-4}}{(4\pi)^4(n-4)} \frac{209}{1440} \kappa^2 \int d^n x \sqrt{-g} C_{\mu\nu\alpha\beta} C^{\alpha\beta}_{\cdot\cdot\rho\sigma} C^{\rho\sigma\mu\nu}. \quad (4.16)$$

Using the standard routine, we arrive at the beta function for the parameter ζ in the action (4.1),

$$\beta_\zeta = -\frac{a_{W3}^2}{(4\pi)^4} \kappa^2 \zeta^2, \quad a_{W3}^2 = \frac{209}{720}. \quad (4.17)$$

We shall skip the discussion of possible matter-gravity contributions in this case and restrict the consideration to the pure quantum gravity model, where the results are available. In the effective quantum theory with $\Lambda = 0$, the expression (4.17) is exact, while in the case $\Lambda \neq 0$ it gains third- and higher-loop corrections in the form of a power series in $\frac{\Lambda}{M_P^2}$. As we already discussed above, in the physically relevant situations these contributions are strongly suppressed compared to the leading two-loop term.

It is interesting to notice that the beta function (4.17) depends on G (via κ^2). This is a general feature that occurs with all the divergent terms whose number of derivatives is different than four, and it is related to the fact that the coupling κ has negative mass dimension — or, in other words, to the non-renormalizability of the theory. Here, however, the situation is different from the eq. (3.3) defining the running of the cosmological constant. In fact, the eq. (3.2) for G also depends on Λ , but it does not depend on ζ . Therefore, we can use the solution for G already established in (3.6) to determine the one of ζ . For the other massive parameters in the total action (4.1) we have a qualitatively similar picture.

All in all, the running of ζ between the H_0 scale in the IR and the Planck scale in the UV is described by the equation (4.17) and the solution has the form

$$\zeta(\mu) = \frac{\zeta_0}{1 - \frac{a_{W3}^2}{2(4\pi)^2} \frac{\zeta_0}{\Lambda_0} \left[1 - \left(1 + \frac{10}{(4\pi)^2} \gamma_0 \ln \frac{\mu}{\mu_0} \right)^{1/5} \right]}, \quad \zeta_0 = \zeta(\mu_0). \quad (4.18)$$

As in the previous cases, we have chosen the sign of the term in the action (4.1) such that the running is the asymptotic-freedom type in the UV for a positive value of the corresponding parameter. Formula (4.18) also shows that the running of ζ depends on γ_0 , thus the situation is very similar to what happens with G and Λ . Since the value

of γ_0 is very small the running is supposed to be weak; the same qualitative behaviour ought to occur for the other parameters associated to higher-order curvature terms in the total action. The last observation is that the singularity in the limit $\Lambda_0 \rightarrow 0$ in the solution (4.18) is explained because in this limit the running of G does not occur within the effective approach. In the special case $\Lambda_0 = 0$, eq. (4.17) has the standard form of solution.

5 Physical applications

Let us briefly discuss about the possible physical applications of the running of the parameters Λ , G , λ , ρ and ξ . Certainly it would be interesting to apply the solutions (3.6) and (3.7), and also the solutions for the dimensionless parameters, to both cosmology and astrophysics. Their detailed elaboration, nonetheless, is beyond the scope of the present work.

First of all, the use of the running of Λ and G requires fixing a physical identification of the scale μ from the Minimal Subtraction scheme of renormalization. In cosmology the most well-motivated identification is with the Hubble parameter, $\mu \sim H$ (see e.g. [40, 67, 68]). In astrophysics, it was originally used the identification $\mu \sim r^{-1}$ for objects like stars, galaxies and their clusters, with r being the distance from the center of the object [69–71]. Further detailed analysis led to the more intricate identification of ref. [72], which was phenomenologically successful. Nowadays, there are some publications on the systematic derivation and covariant forms of the scale identification, see e.g. [73–75], which enable one to apply the solution (3.6).

In the case of the four-derivative terms we meet the explicit non-local form factors given by eq. (4.8) and

$$\frac{b^2}{(4\pi)^2} \int d^4x \sqrt{-g} R \ln \left(-\frac{\square}{\mu_0^2} \right) R. \quad (5.1)$$

An observation concerning the cosmological applications of these two logarithmic form factors is in order. The Weyl-squared term in the action (4.4) affects the gravitational wave type cosmic perturbations, but not the background solution or density perturbations. There are no reasons why the numerical coefficient of this term should have a particularly large value. Thus, the presence of the logarithmic form factor (4.8) can give an effect of the IR running, similar to what has been previously described as a consequence of a photon effect in [53]. It is remarkable that using the unique effective action one can report on the same IR running in effective quantum gravity. At low energies the effect related to the fourth-derivative term is weak and no essential observational manifestations should be expected. At the same time, close to the Planckian scale, when the initial seeds of the tensor modes of cosmic perturbations are formed, there might be some effects of the logarithmic form factor in (4.8). This issue may deserve a detailed study, but it is beyond the scope of the present work.

On the other hand, the coefficient of the classical R^2 -term in the action (4.4) can be either unconstrained or fixed by the observational data. The last is the situation in the Starobinsky inflation [76], where one can show that this value should be as large as 5×10^8 [77]. In this case, even at the Planck scale, the effect of the form factor (5.1)

is enhanced by eight orders of magnitude. This situation is in sharp contrast with other models, including the Higgs inflation and inflaton-based models, which are otherwise equivalent to the R^2 -based model of Starobinsky. Thus, using quantum gravity we might gain a possibility to distinguish this among the other inflationary models.

It is important to stress that all these expectations become possible only because of the use of the Vilkovisky-DeWitt unique effective action. In the usual formulation of effective quantum gravity both beta functions associated to the terms C^2 and R^2 are dependent on the choice of gauge-fixing and parametrization of quantum metric [29], preventing their use in reasonable applications.

6 Conclusions

Using the effective approach and Vilkovisky-DeWitt unique effective action in quantum general relativity, we constructed the renormalization group equations for the Newton and cosmological constants and for the parameters of the fourth-derivative terms in the extended action of gravity. The part of Newton and cosmological constants has been considered earlier in [16], but our analysis is done from a different perspective. In particular, we show that in the effective approach all the mentioned one-loop beta functions can be regarded as exact, meaning they do not gain significant higher-loop corrections. The same concerns the renormalization group equation for the coefficient of the six-derivative term. This equation is derived on the basis of the two-loop divergences calculated in the well-known works [64–66] and does not require the Vilkovisky-DeWitt approach to be universal.

The one-loop equations come from the quantum effects of the purely massless modes and, therefore, are valid in both UV and IR. In the UV, the renormalization group trajectories can be used only until the scale where the massive degrees of freedom associated to higher-derivative terms become active. However, in the IR there are no restrictions except the extremely low-energy Hubble scale IR cut-off.

In this respect, the renormalization group equations under discussion strongly differ from the ones in renormalizable and superrenormalizable models of quantum gravity. In fact, those are valid only in the UV regime, usually with respect to the Planck scale.

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