Light mediators in anomaly free $U(1)_X$ models. Part I. Theoretical framework

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ABSTRACT: We examine theoretical features of $U(1)_X$ extensions of the Standard Model whose quantum anomalies are canceled per generation. Similarly to other versions, the theory consists of a Two-Higgs-Doublet Model plus a scalar singlet embedded into the $SM \otimes U(1)_X$ gauge group, and introduces small modifications to the Z -boson interactions. These changes can be minimized by exclusively charging right-handed fermions under the new Abelian symmetry, and are compensated by the neutral X -boson exchange. Non-universality of fermion couplings can also be achieved by requiring one single X -charged family. In general, X gauge bosons can be separated into A' and Z' subsets, distinguished by the presence of axial-vector components in the Z' exchange. A' physics, in particular the dark photons case, is commonly simpler to constrain and therefore favored by experimental tests. Finally, the model can be UV completed both by stable χ fermions or by right-handed neutrinos. The prior case may provide cold WIMPs in the theory.

KEYWORDS: Beyond Standard Model, Higgs Physics, Neutrino Physics

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1 Introduction

The existence of dark matter (DM) has been supported by numerous astrophysical and cosmological arguments (see e.g. [1]). On the other hand, experimental and theoretical efforts for direct detection have provided strong constraints on the couplings of DM candidates to the Standard Model (SM) particles.

The simplest DM model can be obtained by adding a scalar singlet to the SM spectra [2]. Once protected by a $\mathbb{Z}_2$ symmetry, preserved after the electroweak symmetry breaking, the new boson is stable and a SM portal is induced by the Higgs exchange. In the framework of larger gauge groups, the $SM \otimes U(1)_X$ will naturally be the next step in terms of minimality. This type of extension commonly requires for the UV completion — albeit not necessarily — at least one new fermion χ [3, 4], which can be made stable by some ad hoc dark symmetry. The symmetry forbids the appearance of tree-level couplings between χ and SM fields and a DM portal is generated through the neutral bosons [5, 6], except for the photon. The portal mediated by SM bosons is commonly doubly suppressed by new gauge or Yukawa couplings and the mixing angles, and may be assumed to be negligible if compared to the contributions from the new vector boson exchange.

The occurrence where the boson associated with the new Abelian symmetry contains only vector couplings comprises a particular case, generally referred to as a *dark photon*. The dark photon interactions arise from the kinetic mixing between the new $U(1)_X$ gauge boson and the SM hypercharge, such that the correspondent vertexes are proportional to the electromagnetic couplings. Very often their searches are performed by assuming that the new vector is on-shell and then decays into $\mu^+\mu^-$ pairs [7] (see section 3). New Physics effects in the few MeV region generally require searches for e^+e^- channels facing a vast set of background events. The possible discrepancies [8–10] in the region $m_X < 2m_\mu$ motivate proposals of the simplest $U(1)_X$ SM extension containing one weakly coupled gauge boson. As summarized by the authors of ref. [11] this task must follow in accordance with some strong requirements, which goes from the absence of new electrically charged fields at low energies to the consistency of neutrino interactions with electrons and nucleons. In particular, the last argument will motivate the proposal of right-handed specific models.

The present work contains a detailed study of the basic properties of $U(1)_X$ models. We are guided by general principles as (i) introduction of a minimal set of free parameters and physical degrees of freedom (falsifiability); (ii) the fermions are accommodated in the SM representations and quantum anomalies are canceled per generation (as in the SM); (iii) the degree of fermion non-universality should explain observed discrepancies between theory predictions and experimental measurements, such as the flavor anomalies in B meson decays [12, 13] and the muon anomalous magnetic moment puzzle [14–18]. In a subsequent work we consider manifestation of New Physics in the low energy regime ($\sim$ MeV) by allowing small couplings of the new vector X to matter fields of the order $g_X \sim 10^{-4}$ to 10^{-2} . In ref. [19] one can notice, for instance, that the region 10 MeV–1 GeV still allows the presence of X with a coupling $g_X \approx 10^{-4}$ in the $U(1)_{B-L}$ case, whose strength might imply measurable effects in heavy meson physics. The same is not true in the case of $U(1)_B$ or the protophobic model. This distinction illustrates the impact of different X -hypercharge assignments albeit under the same gauge structure.

A basic SM extension is readily achieved even when non-universal X -hypercharges for leptons and quarks are considered. The first consequence of arbitrary charges is the appearance of a flavor matrix $\mathbb{F}$ in the fermion-gauge sector. Its properties are specially interesting in the case when only one generation is charged under X . Moreover, the attempt to reduce the number of free parameters may favor the choice of a chiral $U(1)_X$. Once more, if we aim to control the modification of neutrino interactions, the right-handed (RH) currents might be the appropriate option within this framework. In the scenario where only RH fermions are charged under $U(1)_X$, the constraints from quantum anomalies arise exclusively from $U(1)_X^3$, $U(1)_X^2 U(1)_Y$, $U(1)_X U(1)_Y^2$ and $SU(3)^2 U(1)_X$ currents, once $SU(2)$ does not play any role. This reduced version shows that is not possible to charge the quarks universally in order to avoid flavor changing neutral currents (FCNC) and to preserve, at the same time, the feature of anomaly cancellations per generation. Besides, the call for fermion non-universality along with the criterion to recover a consistent CKM matrix will demand the introduction of a new scalar doublet.

In summary, here we introduce a Two-Higgs-Doublet Model (2HDM) embedded in the $SM \otimes U(1)_X$ gauge group [20–22] where only the second generation of fermions is

charged under X . Our main goal is to determine the allowed region of the parameter space in comparison with similar $U(1)$ versions already presented, for instance, in [8] and [23], whose phenomenology were focused in the MeV-GeV regime. Motivated by the absence of left-handed (LH) singlets under the SM, we construct the UV completion by including in the setup a new right-handed χ_R fermion charged exclusively under $U(1)_X$. We focus on a stable fermion whose mass is generated after a symmetry breaking performed by a scalar singlet. The alternative version where a leptonic Yukawa Lagrangian generates neutrino masses is possible too and is discussed in the appendix A. Notwithstanding, we choose to investigate in detail a DM model due to the following reason — the tiny couplings governing the $\chi\chi \leftrightarrow SM$ portal can turn a Weakly Interacting Massive Particle (WIMP) overabundant in the present universe and therefore forbidden. We emphasize that Cold WIMPs (MeV) are expected to freeze-out during a pre- Big Bang nucleosynthesis (BBN) phase, such that relic density computations shall be strongly dependent on the assumptions we take for their thermal production and on the early history of the universe. Thus, our conclusions would differ depending on the specific cosmological model. In spite of this limitation, under the standard scenario presented by [24], the relic abundance constraints are expected to cover the parameter space in the opposite direction if compared with those from direct detection, which provide us with a mechanism to maximize its excluded area.

The work is divided into three sections. Section 2 introduces the general features of $U(1)_X$ theories, and motivates the variant with X -charged right-handed fermions. Section 3 discusses the experimental status for A' and Z' searches, distinguished by the presence of axial-vector couplings. The last section is devoted to our conclusions.

2 General properties of $U(1)_X$ and inclusion of right-handed fermions

In this section we determine the functional dependence of the vector and axial-vector currents on the hypercharges and fundamental constants, like v.e.v's and couplings. We see that general Abelian extensions must produce Z -like, instead of A -like, vertexes, i.e.

$$\mathcal{L} \supset \frac{x_V^f}{2} \bar{f} \gamma_\mu f X^\mu + \mathcal{L}_\chi \quad \rightarrow \quad \frac{1}{2} \bar{f} \gamma_\mu (x_V^f + x_A^f \gamma_5) f X^\mu + \mathcal{L}_\chi, \quad (2.1)$$

where x_V, x_A are non-universal and flavor violating matrices and $\mathcal{L}_\chi$ describes the possible interactions of new fermions in the theory. The eq. (2.1) illustrates the fact that the A' phenomenology comprises a subset of the X boson theory, where the axial couplings are set to zero. By keeping the complete vertex in our study, we can determine the impact of x_A on the allowed parameter space [25]. In addition, non-universality effects accommodated within the $SM \otimes U(1)_X$ model usually spread into both quark and leptonic sectors. Apart from that, the long-lived boson is commonly assumed to decay exclusively into a dark-sector, i.e. $Br(X \rightarrow \chi\chi) = 1$, which motivates experimental searches in processes with invisible final states. By allowing the gauge boson to couple to an electron-positron pair, the parameter space for X (long-lived enough to decay out of the detector) has the impact of loosening important bounds, such as those from $K \rightarrow \mu + \text{invisibles}$ [26].

Apart from the Yukawa Lagrangian and the X -hypercharge assignments, the following description and results are general in the sort of 2HDM plus scalar singlet of $SM \otimes U(1)_X$:

- Three vector fields W^μ from $SU(2)_L$, one vector B_μ^Y from $U(1)_Y$ and B_μ^X from $U(1)_X$;
- Three independent coupling constants g, g_Y, g_X and a kinetic mixing constant ϵ ;
- Three generations of weak isospin doublets:

$$(L_L)_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L, \quad (Q_L)_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L, \quad (2.2)$$

with $i = 1, 2, 3$;

- right-handed $SU(2)_L$ singlets: $\chi_R, l_{iR}, u_{iR}, d_{iR}$;
- Y hypercharges:

$$Y_L = -\frac{1}{2}; \quad Y_Q = \frac{1}{6}; \quad Y_l = -1; \quad Y_\chi = 0; \quad Y_u = \frac{2}{3}; \quad Y_d = -\frac{1}{3}; \quad (2.3)$$

- X hypercharges:

$$X_L = 0; \quad X_Q = 0; \quad X_{e2} = 1; \quad X_{\chi_R} = -1; \quad X_{u2} = -1; \quad X_{d2} = 1; \quad (2.4)$$

with the remaining RH fields uncharged.

- Higgs doublets ϕ^0, ϕ^X and singlet s :

$$Y_0 = Y_X = \frac{1}{2}; \quad X_0 = 0; \quad X_X = -1; \quad Y_s = 0 \quad X_s = 1. \quad (2.5)$$

Notice that the term $\phi_0^\dagger \phi_X s$ is allowed in the scalar potential.

- Electroweak Lagrangian is

$$\mathcal{L} = -\frac{1}{4} \mathbf{W}^{\mu\nu} \cdot \mathbf{W}_{\mu\nu} - \frac{1}{4} B^{Y\mu\nu} B_{\mu\nu}^Y - \frac{1}{4} B^{X\mu\nu} B_{\mu\nu}^X + \frac{\epsilon}{2} B^{Y\mu\nu} B_{\mu\nu}^X \quad (2.6a)$$

$$+ (D_\mu \phi^0)^\dagger (D^\mu \phi^0) + (D_\mu \phi^X)^\dagger (D^\mu \phi^X) + (D_\mu s)^\dagger (D^\mu s) - V(\phi^0, \phi^X, s) \quad (2.6b)$$

$$- \sum_{\alpha=1,2,3} \left(\sum_{\beta=1,3} \bar{L}_{\alpha L} \phi^0 Y_{\alpha\beta}^l e_{\beta R} + \bar{L}_{\alpha L} \phi^X Y_{\alpha 2R}^l e_{2R} + \text{h.c.} \right) \quad (2.6c)$$

$$- \sum_{\alpha=1,2,3} \sum_{\beta=1,3} \left(\bar{Q}_{\alpha L} \phi^0 Y_{\alpha\beta}^D d_{\beta R} + \bar{Q}_{\alpha L} \tilde{\phi}^0 Y_{\alpha\beta}^U u_{\beta R} + \text{h.c.} \right) \quad (2.6d)$$

$$- \sum_{\alpha=1,2,3} \left(\bar{Q}_{\alpha L} \phi^X Y_{\alpha 2R}^D d_{2R} + \bar{Q}_{\alpha L} \tilde{\phi}^X Y_{\alpha 2R}^U u_{2R} + \text{h.c.} \right) \quad (2.6e)$$

$$- Y_s \bar{\chi}_L \chi_R s - Y_s^* \bar{\chi}_R \chi_L s^* \quad (2.6f)$$

$$+ i \sum_{\alpha=1,2,3} \left[\bar{L}_{\alpha L} \not{D} L_{\alpha L} + \bar{Q}_{\alpha L} \not{D} Q_{\alpha L} \right. \quad (2.6g)$$

$$\left. + \bar{l}_{\alpha R} \not{D} l_{\alpha R} + \bar{d}_{\alpha R} \not{D} d_{\alpha R} + \bar{u}_{\alpha R} \not{D} u_{\alpha R} \right] + i \bar{\chi}_R \not{D} \chi_R. \quad (2.6h)$$

Anomalies. A basic prerequisite of any ultraviolet complete gauge theory is that it is free of triangle anomalies [3, 4, 27]. The following equations summarize how this criterion can be achieved for arbitrary X charges within $\text{SM} \otimes \text{U}(1)_X$ models. Again, Q and L denote quark and lepton doublets, respectively, while the rest denoted by u_R, d_R, e_R refer to the charges of right-handed fields:

$$X^3 : \quad 2 \left(\sum_L X_L^3 + 3 \sum_Q X_Q^3 \right) - \left[\sum_{l_R} X_{l_R}^3 + \sum_{\chi_R} X_{\chi_R}^3 + 3 \left(\sum_{U_R} X_{U_R}^3 + \sum_{D_R} X_{D_R}^3 \right) \right] = 0, \quad (2.7a)$$

$$YX^2 : \quad 2 \left(\sum_L Y_L X_L^2 + 3 \sum_Q Y_Q X_Q^2 \right) - \left[\sum_{l_R} Y_{l_R} X_{l_R}^2 + \sum_{\chi_R} Y_{\chi_R} X_{\chi_R}^2 + 3 \left(\sum_{U_R} Y_{U_R} X_{U_R}^2 + \sum_{D_R} Y_{D_R} X_{D_R}^2 \right) \right] = 0, \quad (2.7b)$$

$$Y^2 X : \quad \text{Same as eq. (2.7b) with } X \leftrightarrow Y, \quad (2.7c)$$

$$\text{SU}(2)^2 X : \quad \left(\sum_L X_L + 3 \sum_Q X_Q \right) = 0, \quad (2.7d)$$

$$\text{SU}(3)^2 X : \quad \left(2 \sum_Q X_Q - \sum_{U_R} X_{U_R} - \sum_{D_R} X_{D_R} \right) = 0, \quad (2.7e)$$

$$\text{grav}^2 X : \quad 2 \left(\sum_L X_L + 3 \sum_Q X_Q \right) - \left[\sum_{l_R} X_{l_R} + \sum_{\chi_R} X_{\chi_R} + 3 \left(\sum_{U_R} X_{U_R} + \sum_{D_R} X_{D_R} \right) \right] = 0. \quad (2.7f)$$

In the Standard Model the above equations are solved per generation. This property can be taken as part of the SM structure and implies that no information about a new fermion family could be found particularly through these diagrams. Here we will follow this principle. The solutions define a subset of $\text{U}(1)$ extensions and are given by¹ [3]

$$X_D = 2X_Q - X_U, \quad X_L = -3X_Q, \quad X_l = -2X_Q - X_U, \quad X_\chi = X_U - 4X_Q. \quad (2.8)$$

Hence, anomaly-free theories are defined by fixing only two free X -charges. In particular, one can take $X_U = 4X_Q$ and complete them without the claim for new fermion singlets. The resultant variants could be both universal or non-universal on flavor, and dark matter conceivable, for instance, through scalar singlets like in [2]. However, this choice would imply a nonzero charge to LH fields, which is in conflict with our preference for RH charges,

¹The solutions are obtained by solving to X_L in the quadratic expressions, with $Y_\chi = 0$, and then for the gravitational and cubic case.

as developed in section 2.2. Finally, theories with non-universality allows the option for just one generation of χ fermions. As observed by [3, 28], the same is not possible for flavor universal charges of standard fermions.

Two Higgs doublets requirement. The theory is designed to be (a) non-universal under X charges, (b) UV completed by χ fermions, and (c) minimal in its particle content. Thus, let us first assume it is possible to generate all fermion masses through only one Higgs doublet. FCNC should not appear in the scalar sector under this property. In order to construct the Yukawa Lagrangian, one has to consider solutions to the equations

$$X_{L_i} - X_{l_i} - X_0 = 0; \quad X_{Q_i} - X_{U_i} + X_0 = 0; \quad X_{Q_i} - X_{D_i} - X_0 = 0, \quad (2.9)$$

which should also satisfy eq. (2.8). In addition, the condition

$$X_{L_i} - X_{l_i} = X_{L_j} - X_{l_j}, \quad (2.10)$$

is necessary for generating masses for all charged fermions,² where the indexes denote the two non-universal X -hypercharges. Notwithstanding, the mass matrix corresponding to the above criterion can be put in a block-diagonal form, and would be inconsistent with the expected form of the mixing matrices. We must additionally fill at least two more entries by requesting the condition

$$X_{L_i} - X_{l_j} = X_{L_j} - X_{l_j}. \quad (2.11)$$

Now, both conditions combined imply

$$X_{L_i} = X_{L_j}, \quad X_{l_i} = X_{l_j}, \quad (2.12)$$

hence breaking the (a) criterion. Naturally, in the $U(1)_X$ extensions one must introduce a larger scalar sector, in comparison to the SM, due to the creation of the longitudinal polarization for the massive X_μ boson. In summary, the eq. (2.12) will state that any version containing only one Higgs doublet is necessarily universal in the fermion families. This is, for instance, the scenario selected by the authors of [3], where the analysis is centered on generation independent hypercharges and the consistency with fermion masses is achieved by introducing one Higgs doublet and one singlet. In addition to $n \geq 3$ χ_R fermions, the combination provides a number of variants dedicated to the explanation of neutrino masses.

2.1 Kinetic mixing

Once ϵ is assumed to be a small parameter, it is convenient to translate its dependence directly into the coupling constants, thus leaving the kinetic Lagrangian in a diagonal form. The task can be achieved through the field redefinition

$$B_\mu^Y \rightarrow B_\mu^Y + \epsilon B_\mu^X, \quad (2.13)$$

²Alternatively, one could create the entries $X_{L_i} - X_{l_j} = X_{L_j} - X_{l_i}$ along the diagonal ones $X_{L_i} - X_{l_i} = X_0$, which would lead to the same conclusion.

i.e., by rewriting eq. (2.6a)

$$\mathcal{L}_{\text{k.m.}} \supset -\frac{1}{2}(B_\mu^Y + \epsilon B_\mu^X)\hat{\mathcal{O}}^{\mu\nu}(B_\nu^Y + \epsilon B_\nu^X) - \frac{1}{2}B_\mu^X\hat{\mathcal{O}}^{\mu\nu}B_\nu^X + \epsilon B_\mu^X\hat{\mathcal{O}}^{\mu\nu}(B_\nu^Y + \epsilon B_\nu^X), \quad (2.14)$$

where $\hat{\mathcal{O}}^{\mu\nu} = \partial^\mu\partial^\nu - \partial^2g^{\mu\nu}$. Up to order $\mathcal{O}(\epsilon)$,

$$\mathcal{L}_{\text{k.m.}} \supset -\frac{1}{2}B_\mu^Y\hat{\mathcal{O}}^{\mu\nu}B_\nu^Y - \frac{1}{2}B_\mu^X\hat{\mathcal{O}}^{\mu\nu}B_\nu^X + \mathcal{O}(\epsilon^2), \quad (2.15)$$

i.e. the crossed terms vanishes and the mixing effect is converted into a new term in the covariant derivative:

$$D_\mu \rightarrow D_\mu = \partial_\mu - ig\mathbf{W}_\mu \cdot \boldsymbol{\tau} - ig_Y Y^p - i(\kappa Y^p + g_X X^p)B_\mu^X, \quad (2.16)$$

where, up to first order, one may write $\epsilon g_Y \equiv \kappa$.

2.2 Couplings and masses of gauge bosons

In the previous section we showed that the non-universal model must contain at least two Higgs doublets, here denoted as ϕ^0 and ϕ^X , as a necessary condition to recover the correct mass spectrum of the fermions. In addition, a singlet s is required to couple to the fermion χ_R (or to generate the mixing between the second and the remaining generations of RH neutrinos, see A) as well as to break a residual U(1) in the potential which could leave the theory with a massless pseudo-Goldstone boson at tree-level [23].

The gauge boson masses are extracted from the kinetic piece of the scalar Lagrangian once the scalars acquire a vacuum expectation value. In terms of ladder operators the covariant derivatives can be written as³

$$D_\mu\phi^p = \left[\partial_\mu - i\frac{g}{\sqrt{2}}(W^+\mathbb{I}_+ + W^-\mathbb{I}_-) - ig\tau_3 W_\mu^3 - ig_Y Y^p B_\mu^Y - i(\kappa Y^p + g_X X^p)B_\mu^X \right] \phi^p, \quad (2.17)$$

$$D_\mu s = (\partial_\mu - ig_X X^s B_\mu^X) s. \quad (2.18)$$

Once the relation $Q = T_3 + Y$ to the electric charge matrix is preserved, it follows that

$$\phi_0 = \begin{pmatrix} \varphi_0^+ \\ \frac{v_0 + H_0 + i\chi_0}{\sqrt{2}} \end{pmatrix}, \quad \phi_X = \begin{pmatrix} \varphi_X^+ \\ \frac{v_X + H_X + i\chi_X}{\sqrt{2}} \end{pmatrix}, \quad s = \frac{v_s + H_s + i\chi_s}{\sqrt{2}}. \quad (2.19)$$

The charged currents are untouched and result for the W mass $m_W = \frac{gv}{2}$, with $v^2 \equiv (v_0^2 + v_X^2)$. The neutral fields must mix and their mass matrix is extracted from the symmetric expression $\sum_{i=1}^3 (a_i W_\mu^3 + b_i B_\mu^Y + c_i B_\mu^X)^2$, whose determinant is zero (massless photon). In the $(W_\mu^3, B_\mu^Y, B_\mu^X)$ basis it is given by

$$\mathbb{M}^0 = \frac{v^2}{8} \begin{pmatrix} g^2 & -gg_Y & g(2g_X c_\beta^2 - \kappa) \\ -gg_Y & g_Y^2 & -g_Y(2g_X c_\beta^2 - \kappa) \\ g(2g_X c_\beta^2 - \kappa) & -g_Y(2g_X c_\beta^2 - \kappa) & 4[g_X^2 \frac{\bar{v}^2}{v^2} - g_X \kappa c_\beta^2] + \kappa^2 \end{pmatrix}, \quad (2.20)$$

³Note that, in the basis of eq. (2.13) and since $Y_s = 0$, the kinetic term does not enter in the interactions with the singlet scalar.

where

$$v^2 \equiv (v_0^2 + v_X^2), \quad \bar{v}^2 \equiv (v_s^2 + v_X^2), \quad c_\beta^2 = \frac{v_X^2}{v^2}. \quad (2.21)$$

The above real symmetric matrix eigenvectors define an orthonormal basis and compose the orthogonal matrix $\mathbb{V}$ which rotates the fields from the gauge to the mass basis. Although the choice of parametrization for $\mathbb{V}$ is not physical, there are options which can make the analysis simpler. Consider, for example, the choice made in terms of the three Euler angles in the usual zxz rotations by the angles (ϕ, θ, ψ) (using notation $\sin\alpha \equiv s_\alpha$ and $\cos\alpha \equiv c_\alpha$)

$$\mathbb{V} = \begin{pmatrix} c_\psi c_\phi - c_\theta s_\phi s_\psi & c_\psi s_\phi + c_\theta c_\phi s_\psi & s_\theta s_\psi \\ -s_\psi c_\phi - c_\theta s_\phi c_\psi & -s_\psi s_\phi + c_\theta c_\phi c_\psi & c_\psi s_\theta \\ s_\theta s_\phi & -s_\theta c_\phi & c_\theta \end{pmatrix}. \quad (2.22)$$

The angle θ introduces the mixing of B_μ^X with the remaining gauge fields, i.e. θ is the angle between the B_μ^X and the z -plane where the $W_\mu^3 - B_\mu^Y$ mixing occurs. All three angles can be written using the couplings, vev's and scalar charges.

One can notice from eq. (2.20) that the block (W_μ^3, B_μ^Y) has a null determinant. This substructure of $\mathbb{M}^0$ implies a zero entry in one eigenvector, which fixes one of the angles. Therefore, by taking $c_\psi = 0$; $s_\psi = -1$ and applying a phase redefinition $s_\phi \leftrightarrow c_\phi$, $c_\theta \leftrightarrow s_\theta$ we can parametrize $\mathbb{V}$ as

$$\mathbb{V} = \begin{pmatrix} s_\theta c_\phi & -s_\theta s_\phi & -c_\theta \\ s_\phi & c_\phi & 0 \\ c_\theta c_\phi & -c_\theta s_\phi & s_\theta \end{pmatrix}. \quad (2.23)$$

The minimal coupling in the covariant derivative can be presented by

$$\langle \mathbf{g}^0 | \mathbf{B}_\mu^0 \rangle \rightarrow \langle \mathbf{g}^0 | \mathbb{V}^\dagger \mathbb{V} | \mathbf{B}_\mu^0 \rangle = \langle \mathbb{V} \mathbf{g}^0 | \mathbb{V} \mathbf{B}_\mu^0 \rangle = \langle \mathbf{g} | \mathbf{B}_\mu \rangle, \quad (2.24)$$

where the vectors are defined as

$$\mathbf{B}_\mu \equiv (X_\mu, A_\mu, Z_\mu)^\dagger = \mathbb{V} \mathbf{B}_\mu^0, \quad \mathbf{g} \equiv (g_R, e\mathbb{Q}, g_Z)^\dagger = \mathbb{V} \mathbf{g}^0 \quad (2.25)$$

and

$$\mathbf{g}^0 \equiv (g\tau^3, g_Y Y, g_X X + \kappa Y)^\dagger, \quad \mathbf{B}_\mu^0 \equiv (W_\mu^3, B_\mu^Y, B_\mu^X)^\dagger. \quad (2.26)$$

The parametrization of eq. (2.23) corresponds to

$$e\mathbb{Q} = g s_\phi \tau^3 + g_Y c_\phi Y. \quad (2.27)$$

By taking g, g_Y to be the same as in the SM, the angle ϕ must correspond to the weak mixing angle, related to the electric charge by

$$g s_\phi = g_Y c_\phi = e. \quad (2.28)$$

On the other hand, the Z -couplings will be replaced by

$$g_Z = c_\theta (g c_\phi \tau_3 - g_Y s_\phi Y) + s_\theta (\kappa Y + g_X X) = c_\theta g_Z^{\text{SM}} + s_\theta (\kappa Y + g_X X), \quad (2.29)$$

where $g_Z^{\text{SM}} = \frac{g}{c_\phi}(\tau_3 - s_\phi^2 \mathbb{Q})$, thus making explicit how θ (i.e the small parameter s_θ) tunes the change in the Z interactions due to the presence of a new neutral gauge boson. Finally, the interactions with the new X_μ is governed by

$$g_R = s_\theta(g c_\phi \tau_3 - g_Y s_\phi Y) - c_\theta(\kappa Y + g_X X) = s_\theta g_Z^{\text{SM}} - c_\theta(\kappa Y + g_X X). \quad (2.30)$$

Once the parametrization of eq. (2.23) is established, the above results are general for 2HDM-like models. The fermion interactions with the new X^μ gauge boson can be determined using eq. (2.30). It contains a term proportional to the SM Z-coupling, weighted by s_θ , and its last piece will regulate the amount of flavor violation in the gauge sector, along with the gauge symmetry-mass basis rotation matrices. One can notice that, after electroweak symmetry breaking (EWSB), the chiral version with RH fields still generates the gauge boson vertexes with LH currents via the kinetic coupling κ . The gauge interactions with X-neutral fields are further suppressed by s_θ .

Using the parametrization eq. (2.23) and the choice of eq. (2.5), s_θ can be written as

$$s_\theta = [1 + (\bar{g}\lambda)^2]^{-\frac{1}{2}}, \quad (2.31)$$

where

$$\lambda = \frac{\bar{g}^2 - a_1 + [\bar{g}^2(\bar{g}^2 + 4a_2^2) - 2\bar{g}^2 a_1 + a_1^2]^{\frac{1}{2}}}{2\bar{g}^2 a_2}, \quad (2.32)$$

$$\bar{g}^2 = g_Y^2 + g^2, \quad a_1 = 4 \left[g_X^2 \frac{\bar{v}^2}{v^2} - g_X \kappa c_\beta^2 \right] + \kappa^2, \quad a_2 = 2g_X c_\beta^2 - \kappa.$$

From these definitions the neutral gauge boson masses, in terms of couplings and v.e.v's, are given by

$$m_Z^2 = \frac{v^2}{4} \frac{1}{2} \left[\bar{g}^2 + a_1 + [\bar{g}^2(\bar{g}^2 + 4a_2^2) - 2\bar{g}^2 a_1 + a_1^2]^{\frac{1}{2}} \right], \quad (2.33)$$

$$m_X^2 = \frac{v^2}{4} \frac{1}{2} \left[\bar{g}^2 + a_1 - [\bar{g}^2(\bar{g}^2 + 4a_2^2) - 2\bar{g}^2 a_1 + a_1^2]^{\frac{1}{2}} \right]. \quad (2.34)$$

Now one can verify that, if $g_X, \kappa \ll \bar{g}$

$$m_Z^2 \rightarrow \frac{v^2}{4} \bar{g}^2, \quad m_X^2 \rightarrow \frac{v^2}{4} a_1. \quad (2.35)$$

Again, since $\frac{\bar{v}^2}{v^2} \gg c_\beta^2$, it follows that $m_X^2 \rightarrow g_X^2 \bar{v}^2 + \kappa^2 \frac{v^2}{4}$. In the same limit the angle may be written in a simplified expression:

$$s_\theta \approx \frac{|2g_X c_\beta^2 - \kappa|}{\bar{g}} \left[1 - \frac{m_X^2}{m_Z^2} \right]^{-1}. \quad (2.36)$$

As mentioned before, the above description is general in 2HDM embedded in a $U(1)_X$ extension of the SM. The main feature of this study is: when the charges under X are non-universal, there must appear flavor changing neutral currents both in the interactions with Z_μ and X_μ , doubly suppressed in the first case by the factor $s_\theta g_X$. Moreover, regarding

the lepton flavor violation (LFV), the first effect of charging LH fields under X is that a number of free parameters appear, due to the matrix elements of V_L^l which rotates these fields to their mass eigenstates. Therefore, the choice for charging only one chiral fermion provides a minimal description of flavor violating processes, although it certainly does not resolve the entire combination of operators. For instance, in the case of $R_{K^{(*)}}$ anomalies in B physics, the $b \rightarrow s\mu^+\mu^-$ transition [13] get contributions from the operators $O_9 \sim (\bar{s}\gamma_\mu(1-\gamma_5)b)(\bar{l}\gamma^\mu l)$ and $O_{10} \sim (\bar{s}\gamma_\mu(1-\gamma_5)b)(\bar{l}\gamma^\mu\gamma_5 l)$, which can be generated from a particular choice for LH and RH hypercharges (see section 2.3).

Once more, we choose to charge RH fields in order to preserve SM-like (for $\kappa \sim 0$) the Z boson interactions with LH fields. From the phenomenological point of view, we also opted for charging the second generation only. Apart from the anomalous magnetic moment, both for electrons and muons, we can include the effect in the proton charge radius measured from the Lamb shift in e-hydrogen and μ -hydrogen among the most stringent constraints to the model [9, 10].

In this framework, LFV will be mediated by the currents

$$\mathcal{L} \supset c_\theta g_X X^\mu \bar{l}_{Ri}\gamma_\mu[X\delta^{i2}\delta^{j2}]l_{Rj}. \quad (2.37)$$

When e_R represents the mass eigenstates, such that $e_R = V_R^l l_R$, the term is converted to

$$\mathcal{L} \supset c_\theta g_X X^\mu \bar{e}_{Ra}\gamma_\mu(V_R^l)_{ai}[X\delta^{i2}\delta^{j2}](V_R^l)_{jk}^\dagger e_{Rk} \rightarrow c_\theta g_X X^\mu \bar{e}_{Ra}\gamma_\mu[X(V_R^l)_{a2}(V_R^l)_{k2}^*]e_{Rk}. \quad (2.38)$$

Therefore, the matrices introducing flavor violation and non-universality are given by $\mathbb{F}_{ij} = (V_R)_{i2}(V_R)_{j2}^*$. We see that only a coherent explanation about the possible alignment between flavor and mass eigenstates could confirm the assumption of small FCNC processes in the model. Some hints in different sectors of the Lagrangian could be selected, for instance, through the CKM matrix, defined as

$$(V_{CKM})_{ij} = (V_L^U)_{ik}(V_L^D)_{jk}^*, \quad (2.39)$$

with implicit summation on k . Thus, if the mass and flavor eigenstates were approximately aligned, simultaneously for U- and D-type LH quarks, all the non-diagonal (ND) elements of V_{CKM} would be suppressed compared to the diagonal ones. The inverse, however, is not true - the presence of phases could suppress ND elements of the CKM but with large terms in the above summation facing a negative interference. Phenomenologically, since the Wolfenstein parameter $\lambda \simeq 0.22$, ND CKM terms are in fact smaller. Once we do not have any particular information on the matrices components, nothing can be affirmed about interferences or suppression. The same holds, for example, for the PMNS matrix.

In the following we consider $U(1)_X$ models, at low-energies (MeV-GeV), originally proposed to explain the muon anomalous magnetic moment discrepancy and the proton charge radius discrepancy (see e.g. [29, 30]). These anomalies are often treated as a signal of lepton flavor non-universality. Thus, in the framework described above, we focus on the g_X component of the g_R coupling.

2.3 Gauge interactions with fermions

The gauge interactions with fermions are described by:

$$\mathcal{L}_{kin} \supset i \left[\bar{L}_{\alpha L} \not{D} L_{\alpha L} + \bar{Q}_{\alpha L} \not{D} Q_{\alpha L} + \bar{l}_{\alpha R} \not{D} l_{\alpha R} + \bar{d}_{\alpha R} \not{D} d_{\alpha R} + \bar{u}_{\alpha R} \not{D} u_{\alpha R} + \bar{\chi}_R \not{D} \chi_R \right], \quad (2.40)$$

with $\alpha = 1, 2, 3$. In terms of mass eigenstates the covariant derivative can be written as

$$D_\mu = \partial_\mu - ig(W^+ \mathbb{I}_+ + W^- \mathbb{I}_-) - ie \mathbb{Q} A_\mu - ig_Z Z_\mu - ig_X X_\mu \quad (2.41)$$

and

$$\begin{aligned} e\mathbb{Q} &= gs_\phi \tau^3 + g_Y c_\phi Y, \\ g_Z &= c_\theta g_Z^{\text{SM}} + s_\theta (\kappa Y + g_X X) = g_Z^I + s_\theta g_X X, \\ g_R &= s_\theta g_Z^{\text{SM}} - c_\theta (\kappa Y + g_X X) = g_R^I - c_\theta g_X X, \end{aligned} \quad (2.42)$$

where the coupling constant components proportional to the identity — i.e. those depending only on the charges assignment under the SM gauge group — have been separated and describe flavor universal vertexes:

$$g_Z^I \equiv c_\theta g_Z^{\text{SM}} + s_\theta \kappa Y, \quad g_R^I \equiv s_\theta g_Z^{\text{SM}} - c_\theta \kappa Y. \quad (2.43)$$

The charged currents occur entirely like in the SM, weighted by the CKM matrix. The new physics (NP) effects are limited to the neutral currents. The parameter s_θ introduces the size of NP contributions in comparison with the SM ones. Therefore, it can be a small parameter. Since we choose to charge RH fields only, the amount of flavor violating processes in both Z and X interactions is related to the $g_X X$ term and is therefore exclusive to RH sector, taking place in the second generation. Defining the vector of fermion fields $f = (f_1, f_2, f_3)$ and rotating the system to the mass basis, $f_R \rightarrow V_{fR} f'_R \equiv V_{fR} f_R$, the general currents depending on the X charges can be fully separated via:

$$\begin{aligned} \mathcal{L}_{kin} \supset & \left[g_R^I(u_R) \bar{u}_R \gamma^\mu u_R + g_R^I(d_R) \bar{d}_R \gamma^\mu d_R + g_R^I(l_R) \bar{l}_R \gamma^\mu l_R \right] X_\mu \\ & - c_\theta g_X \left[\bar{u}_R \mathbb{F}^U \gamma^\mu u_R + \bar{d}_R \mathbb{F}^D \gamma^\mu d_R + \bar{l}_R \mathbb{F}^l \gamma^\mu l_R \right] X_\mu. \end{aligned} \quad (2.44)$$

The matrices

$$\mathbb{F}^f \equiv V_{fR}^\dagger \mathbb{X}^f V_{fR}, \quad \text{where} \quad (\mathbb{X}^f)_{ij} \equiv X^f \delta_{2i} \delta_{2j}, \quad (2.45)$$

or

$$(\mathbb{F}^f)_{ij} = X^f (V_{fR}^\dagger)_{i2} (V_{fR})_{2j}, \quad (2.46)$$

summarize the amount of flavor violation and fermion non-universality in the model. Once more, in the scenario where flavor is aligned to the mass eigenstates, i.e. when the absolute value of diagonal elements of V_{fR} are larger than the non-diagonal ones, the flavor violating processes must favor the second generation in the final state, since it would include at least one factor of $(V_{fR})_{22}$. Moreover, the diagonal elements also fix the amount of LFV by

$$\mathbb{F}^f = X^f \begin{pmatrix} |V_{fR}|_{21}^2 & (V_{fR}^\dagger)_{12} (V_{fR})_{22} & (V_{fR}^\dagger)_{12} (V_{fR})_{23} \\ (V_{fR}^\dagger)_{22} (V_{fR})_{21} & |V_{fR}|_{22}^2 & (V_{fR}^\dagger)_{22} (V_{fR})_{23} \\ (V_{fR}^\dagger)_{32} (V_{fR})_{21} & (V_{fR}^\dagger)_{32} (V_{fR})_{22} & |V_{fR}|_{23}^2 \end{pmatrix}, \quad (2.47)$$

or, one can define,

$$|\mathbb{F}^f| \equiv X^f \begin{pmatrix} |V_{fR}|_{21}^2 & |V_{fR}|_{21}|V_{fR}|_{22} & |V_{fR}|_{21}|V_{fR}|_{23} \\ |V_{fR}|_{21}|V_{fR}|_{22} & |V_{fR}|_{22}^2 & |V_{fR}|_{22}|V_{fR}|_{23} \\ |V_{fR}|_{21}|V_{fR}|_{23} & |V_{fR}|_{22}|V_{fR}|_{23} & |V_{fR}|_{23}^2 \end{pmatrix}. \quad (2.48)$$

Due to unitarity of V_{fR} , the trace of $\mathbb{F}^f$ is equal to X^f :

$$\text{Tr}[\mathbb{F}^f] = \text{Tr}[V_{fR}^\dagger \mathbb{X}^f V_{fR}] = \text{Tr}[\mathbb{X}^f] = X^f. \quad (2.49)$$

Naturally, the closer one of the diagonal entries is to X_f , smaller the LFV is predicted by the model. Notice that, unlike the CKM matrix, $\mathbb{F}$ does not enclose all the physical processes involving RH fields and the matrices V_R must be independently present in the scalar interactions.

The Z interactions follow a similar pattern but it is doubled suppressed by $s_\theta g_X \approx g_X^2$, i.e. flavor changing and non-universality are dominated by X^μ interactions. All the non-diagonal vertexes are summarized in the second line of eq. (2.44), represented by the matrix $\mathbb{F}$, and it is useful to separate the diagonal currents in a simplified form. Here these terms will be written like

$$\mathcal{L} \supset \frac{1}{2} \bar{f} \gamma_\mu (g_V^f + g_A^f \gamma^5) f Z^\mu + \frac{1}{2} \bar{f} \gamma_\mu (x_V^f + x_A^f \gamma^5) f X^\mu, \quad (2.50)$$

such that

$$x_V^f = g_R^I(f_R) + g_R^I(f_L) - c_\theta g_X \mathbb{F}_{ii}^f, \quad (2.51)$$

$$x_A^f = g_R^I(f_R) - g_R^I(f_L) - c_\theta g_X \mathbb{F}_{ii}^f. \quad (2.52)$$

By replacing the electric charges and hypercharges:

$$x_V^U = g \frac{s_\theta}{c_\phi} \left(\frac{1}{2} - \frac{4}{3} s_\phi^2 \right) - c_\theta \kappa \frac{5}{6} - c_\theta g_X \mathbb{F}_{ii}^U, \quad (2.53a)$$

$$x_A^U = g \frac{s_\theta}{c_\phi} \left(-\frac{1}{2} \right) - c_\theta \kappa \frac{1}{2} - c_\theta g_X \mathbb{F}_{ii}^U, \quad (2.53b)$$

$$x_V^D = g \frac{s_\theta}{c_\phi} \left(-\frac{1}{2} + \frac{2}{3} s_\phi^2 \right) + c_\theta \kappa \frac{1}{6} - c_\theta g_X \mathbb{F}_{ii}^D, \quad (2.53c)$$

$$x_A^D = g \frac{s_\theta}{c_\phi} \left(\frac{1}{2} \right) + c_\theta \kappa \frac{1}{2} - c_\theta g_X \mathbb{F}_{ii}^D, \quad (2.53d)$$

$$x_V^l = g \frac{s_\theta}{c_\phi} \left(-\frac{1}{2} + 2 s_\phi^2 \right) + c_\theta \kappa \frac{3}{2} - c_\theta g_X \mathbb{F}_{ii}^l, \quad (2.53e)$$

$$x_A^l = g \frac{s_\theta}{c_\phi} \left(\frac{1}{2} \right) + c_\theta \kappa \frac{1}{2} - c_\theta g_X \mathbb{F}_{ii}^l, \quad (2.53f)$$

$$x_V^\nu = -x_A^\nu = g \frac{s_\theta}{c_\phi} \left(\frac{1}{2} \right) + c_\theta \kappa, \quad (2.53g)$$

$$x_V^\chi = x_A^\chi = c_\theta g_X. \quad (2.53h)$$

Clearly, the root for $x_A^l = 0$ results in the purely vectorial leptonic vertexes. In addition, by charging LH currents one may generate LH FCNC bi-linears in eq. (2.44) and enable the effective operators favored by the $R_{K^{(*)}}$ flavor anomalies [13].

In our model we emphasize the interactions mediated by a light X^μ ($m_X \sim 10^2$ MeV). If compared to the dark vector exchange, the effects from the remaining new fields, like $\overline{H}, \overline{H}_s, \chi_r^0, \overline{\phi}^+$ presented in section 2.4.2, are negligible due to their presence in the decoupling limit [31]. In the second part of this work, the free parameters coming from Yukawas are constrained by experimental bounds and not fixed along the analysis.

X_μ interactions with charged hadrons. In order to calculate the contribution coming from the inner X -bremsstrahlung from a charged hadron, one must first perform the transformation

$$B_\mu^Y \rightarrow B_\mu^Y + \epsilon B_\mu^X, \quad (2.54)$$

which converts the QED covariant derivative

$$D_\mu = \partial_\mu - ieqA_\mu, \quad (2.55)$$

into a minimal coupling including the vector X_μ . Note that the photon field can always be written as $A = s_\phi W^3 + c_\phi B^Y$, once the rotation in eq. (2.54) does not modify the Weinberg angle dependence in terms of the weak couplings.⁴ Thus

$$\begin{aligned} D_\mu &\stackrel{(2.54)}{\rightarrow} \partial_\mu - ieqA_\mu - ieqc_\phi \epsilon B_\mu^X \\ &= \partial_\mu - ieqA_\mu - ieqc_\phi \epsilon (\nabla_{3i}^\dagger \mathbf{B}_i) \\ &= \partial_\mu - ieqA_\mu + iqc_\phi^2 \kappa X_\mu + \mathcal{O}^2, \end{aligned} \quad (2.56)$$

where $\kappa = g_Y \epsilon$ and $c_\theta \approx 1$. The remaining terms include a Z interaction suppressed at second order in the small parameters.

2.4 Scalar potential

The scalar potential has the same features as the the given in ref. [31] (see eq. (2.26)), e.g. the absence of pseudo-Goldstone bosons ensured by the μ -dependent cubic coupling below:

$$\begin{aligned} V(\phi_0, \phi_X, s) &= \mu_0(\phi_0^\dagger \phi_0) + \mu_X(\phi_X^\dagger \phi_X) + \mu_s(s^* s) + \lambda_0(\phi_0^\dagger \phi_0)^2 + \lambda_X(\phi_X^\dagger \phi_X)^2 + \lambda_s(s^* s)^2 \\ &\quad + \lambda_3(\phi_0^\dagger \phi_0)(\phi_X^\dagger \phi_X) + \lambda_4(\phi_0^\dagger \phi_X)(\phi_X^\dagger \phi_0) + \lambda_{0s}(\phi_0^\dagger \phi_0)(s^* s) \\ &\quad + \lambda_{0X}(\phi_X^\dagger \phi_X)(s^* s) - \mu[(\phi_X^\dagger \phi_0)s + \text{h.c.}]. \end{aligned} \quad (2.57)$$

It is convenient to consider a gauge-fixing Lagrangian before we analyze the physical spectra, since it can provide useful tools for the diagonalization of the potential.

⁴In other words the resultant shift in eq. (2.56) does not depend if the rotation is performed before or after symmetry breaking.

2.4.1 R_ξ gauges

The longitudinal components of the W boson arise from the contributions of both Higgs doublets. The charged scalars will mix due to the Lagrangian

$$\mathcal{L}_S \supset \frac{i}{2}g \left[v_0(\partial\varphi_0^+ W^- - \partial\varphi_0^- W^+) + v_X(\partial\varphi_X^+ W^- - \partial\varphi_X^- W^+) \right]. \quad (2.58)$$

The gauge-fixing Lagrangian can be chosen as

$$\mathcal{L}_{\text{g.f.}} \supset -\frac{1}{\xi_W} \left(\partial W^+ - ig \frac{\xi_W}{2} (v_0 \varphi_0^+ + v_X \varphi_X^+) \right) \left(\partial W^- - ig \frac{\xi_W}{2} (v_0 \varphi_0^- + v_X \varphi_X^-) \right). \quad (2.59)$$

This Lagrangian produces, for instance, the term

$$\mathcal{L}_{\text{g.f.}} \supset -\frac{i}{2}g \left[v_0 \partial W^+ \varphi_0^- + v_X \partial W^+ \varphi_X^- \right] \quad (2.60)$$

such that, after an integration by parts, it must cancel the equivalent piece in $\mathcal{L}_S$. The remaining terms are

$$\mathcal{L}_{\text{g.f.}} \supset -\frac{1}{\xi_W} (\partial W^+) (\partial W^-) - \xi_W \frac{g^2}{4} (v_0 \varphi_0^+ + v_X \varphi_X^+) (v_0 \varphi_0^- + v_X \varphi_X^-). \quad (2.61)$$

The determinant of the mixing matrix above is obviously zero. The zero eigenvalue is linked to a physical charged scalar and the non-zero one to a Goldstone boson with mass $m_\phi^2 = \xi_W \frac{g^2 v^2}{4} = \xi_W m_W^2$. The scalars mass matrix $(\mathbb{M}_W^2)_\xi$ is given in the basis $(\varphi_0^+, \varphi_X^+)$ and can be diagonalized by matrix $\mathbb{R}_\xi^W$

$$(\mathbb{M}_W^2)_\xi \propto \begin{pmatrix} v_0^2 & v_0 v_X \\ v_0 v_X & v_X^2 \end{pmatrix}, \quad \mathbb{R}_\xi^W = \frac{1}{v} \begin{pmatrix} v_X & -v_0 \\ v_0 & v_X \end{pmatrix}, \quad (2.62)$$

such that $[\mathbb{R}_\xi^W](\mathbb{M}^2)[\mathbb{R}_\xi^W]^\top = \begin{pmatrix} 0 & 0 \\ 0 & v^2 \end{pmatrix}$ and $\begin{pmatrix} \bar{\varphi} \\ \bar{\varphi}_g \end{pmatrix} = \mathbb{R}_\xi^W \begin{pmatrix} \varphi_0^+ \\ \varphi_X^+ \end{pmatrix}$.

The Goldstone theorem implies that the matrix $\mathbb{R}_\xi^W$ must be orthogonal to the mixing matrix derived from the potential. Hence, they can be simultaneously diagonalized and, as discussed in the following section, the matrices $\mathbb{R}_\xi$ may be sufficient to diagonalize the entire system.

The construction of a gauge fixing Lagrangian for the neutral Z and X bosons is not as straightforward as in the previous example, although it is based on the same procedure. The difference comes from the the mixing matrix which contains two dependent parts on the gauge fixing parameters. In other words, there must be a set of bi-linears in the Z and X component of the following Lagrangian

$$\mathcal{L}_S = \left(\sum_k z_k \partial_\mu \chi_k \right) Z^\mu + \left(\sum_k x_k \partial_\mu \chi_k \right) X^\mu, \quad (2.63)$$

where $z_k \equiv g_Z(\chi_k)v_k$ and $x_k \equiv g_R(\chi_k)v_k$, with the index being $k \in [\chi_0, \chi_X, s]$. The masses can likewise be written as $m_Z^2 = \sum_k z_k^2$ and $m_X^2 = \sum_k x_k^2$. After introducing the gauge-fixing part, the scalars shall mix analogously to the charged case, but here with both Z

and X components containing independent ξ parameters. The mass matrices of both gauge bosons are not commuting and the total mass matrix must be diagonalized at once, such that the two non-zero eigenvalues embody the gauge-fixing parameters. Explicitly, we see that the matrix is given by

$$\mathcal{L}_{\text{g.f.}} \supset -\frac{\xi_Z}{2} \left(\sum_k z_k \chi_k \right)^2 - \frac{\xi_X}{2} \left(\sum_k x_k \chi_k \right)^2. \quad (2.64)$$

In the absence of ξ_X , for instance, the expression results in just one non-zero eigenvalue as $\mathcal{L}_{\text{g.f.}} \supset \frac{\xi_Z}{2} m_Z^2 \chi_Z^2$, and vice-versa. Therefore, we can write this part of the gauge fixing Lagrangian using two scalars χ_p and χ_a

$$\mathcal{L}_{\text{g.f.}} \supset \frac{M_p^2(\xi_Z, \xi_X)}{2} \chi_p^2 + \frac{M_a^2(\xi_Z, \xi_X)}{2} \chi_a^2, \quad (2.65)$$

such that

$$M_p^2(0, \xi_X) = \xi_X m_X^2, \quad M_p^2(\xi_Z, 0) = \xi_Z m_Z^2, \quad M_a^2(\xi_Z, 0) = M_a^2(0, \xi_X) = 0. \quad (2.66)$$

The remaining scalar is physical, whose mass is determined from the potential and will be denoted as χ_r . The matrices coming from eq. (2.64) are orthogonal to the mixing matrix coming from the potential. Finally, the gauge fixing Lagrangian can be chosen as

$$\mathcal{L}_{\text{g.f.}} \supset -\frac{1}{2\xi_Z} \left(\partial Z - \xi_Z \sum_k z_k \chi_k \right)^2 - \frac{1}{2\xi_X} \left(\partial X - \xi_X \sum_k x_k \chi_k \right)^2. \quad (2.67)$$

2.4.2 Scalar spectra

As elaborated in the previous section, the theory contains four Goldstones $\phi_g^+, \phi_g^-, \chi_p, \chi_a$, three physical (pseudo) scalars (ϕ^+, ϕ^-, χ_r) and three Higgses $(H, \bar{H}, \bar{H}_s)$, corresponding to the ten original degrees of freedom. The vacuum stability equations are extracted from the linear terms of the real neutral scalars (H_χ, H_0, H_s) and lead to the conditions:

- H_0 :

$$\mu_0 + \lambda_0 v_0^2 + \frac{\lambda_{0s}}{2} v_s^2 - \frac{\mu}{\sqrt{2}} \frac{v_s v_X}{v_0} + \frac{v_X^2}{2} (\lambda_3 + \lambda_4) = 0, \quad (2.68)$$

- H_χ :

$$\mu_\chi + \lambda_\chi v_X^2 + \frac{\lambda_{\chi s}}{2} v_s^2 - \frac{\mu}{\sqrt{2}} \frac{v_s v_0}{v_X} + \frac{v_0^2}{2} (\lambda_3 + \lambda_4) = 0, \quad (2.69)$$

- H_s :

$$\mu_s + \lambda_s v_s^2 + \frac{\lambda_{0s}}{2} v_0^2 + \frac{\lambda_{0\chi}}{2} v_X^2 - \frac{\mu}{\sqrt{2}} \frac{v_0 v_X}{v_s} = 0. \quad (2.70)$$

Mass matrix — Charged scalars. In the basis (ϕ_0^+, ϕ_χ^+) , using the vacuum stability equations (2.68), (2.69) and (2.70), the squared mass matrix of charged scalars can be written as

$$\mathbb{M}_W^2 = \lambda^+ \begin{pmatrix} v_X^2 & -v_0 v_X \\ -v_0 v_X & v_0^2 \end{pmatrix}, \quad \text{where} \quad \lambda^+ \equiv \left(\frac{\mu v_s}{\sqrt{2} v_0 v_X} - \frac{\lambda_4}{2} \right) \quad (2.71)$$

and such that

- Orthogonality: $\mathbb{M}_W^2 \cdot (\mathbb{M}_W^2)_\xi = (\mathbb{M}_W^2)_\xi \cdot \mathbb{M}_W^2 = 0$,
- Diagonalization via $\mathbb{R}_\xi^W$

$$\mathbb{R}_\xi^W \mathbb{M}_W^2 (\mathbb{R}_\xi^W)^\top = \begin{pmatrix} v^2 & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.72)$$

Therefore, there is a charged scalar $\bar{\phi}^+$ such that

$$m_+^2 = \lambda^+ v^2. \quad (2.73)$$

Neutral scalars. First, we write the mass matrix generated by the gauge-fixing part of the Lagrangian in the following form

$$\mathbb{M}_{\xi\chi}^2 = \xi_Z \left(\sum_i z_i \chi^i \right)^2 + \xi_X \left(\sum_i x_i \chi^i \right)^2, \quad (2.74)$$

where $z_i = g_Z(\chi^i) v_i$, $x_i = g_X(\chi^i) v_i$ and $i \in (0, X, s)$. On the other hand, from the potential and the vacuum stability conditions, in the basis (χ^0, χ^X, χ^s) , it follows that

$$\frac{\mathbb{M}_\chi^2}{2} = \frac{\mu}{2\sqrt{2}} \begin{pmatrix} \frac{v_s v_X}{v_0} & -v_s & -v_X \\ -v_s & \frac{v_0 v_s}{v_X} & v_0 \\ -v_X & v_0 & \frac{v_0 v_X}{v_s} \end{pmatrix}. \quad (2.75)$$

A cross-check can be performed by using the orthogonality of the above matrices. The mass of the physical scalar is given by

$$m_{\chi^r}^2 = \frac{\mu}{\sqrt{2}} \left(\frac{v_0 v_s}{v_X} + \frac{v_0 v_X}{v_s} + \frac{v_X v_s}{v_0} \right). \quad (2.76)$$

i.e. in the absence of a term breaking the residual U(1) symmetry of the potential, a massless pseudo-Goldstone boson would survive at tree-level in the model.

Once the matrices in eq. (2.74) and eq. (2.75) commute, a particular matrix $\mathbb{R}^\xi$, rotating the gauge-fixing Lagrangian, can be further applied to $\mathbb{M}_\chi$ and will result in a block-diagonal matrix, then diagonalized via a second $\mathbb{R}^V$. In other words, the total mixing matrix can be written like

$$\begin{pmatrix} \chi^p \\ \chi^a \\ \chi^r \end{pmatrix} = \mathbb{R}^\xi \mathbb{R}^V \begin{pmatrix} \chi_0 \\ \chi_X \\ \chi_s \end{pmatrix}. \quad (2.77)$$

This assertion is proved by considering two real and symmetric commuting matrices $\mathbb{A}, \mathbb{B}$. If $\mathbb{D}$ diagonalizes $\mathbb{A}$, i.e. $\mathbb{D}\mathbb{A}\mathbb{D}^\top = \mathbb{X}_A$, it follows that

$$\mathbb{D}[\mathbb{A}, \mathbb{B}]\mathbb{D}^\top = 0 \rightarrow [\bar{\mathbb{B}}, \mathbb{X}_A] = 0, \quad (2.78)$$

where $\bar{\mathbb{B}} = \mathbb{D}\mathbb{B}\mathbb{D}^\top$, or

$$(\bar{\mathbb{B}})_{ik}(\mathbb{X}_A)_{kj} - (\mathbb{X}_A)_{ik}(\bar{\mathbb{B}})_{kj} = 0 \rightarrow (\bar{\mathbb{B}})_{ik}\delta_{kj}(\mathbb{X}_A)_{jj} - \delta_{ik}(\mathbb{X}_A)_{ii}(\bar{\mathbb{B}})_{kj} = 0, \quad (2.79)$$

and finally,

$$(\bar{\mathbb{B}})_{ij}[(\mathbb{X}_A)_{ii} - (\mathbb{X}_A)_{jj}] = 0. \quad (2.80)$$

Therefore $\bar{\mathbb{B}}$ is block-diagonal whose dimension is given by the eigenvalues degree of degeneracy.

The Higgs. For the real scalars, the vacuum stability equations lead to the following matrix

$$\mathbb{M}_H^2 = \begin{pmatrix} 2\lambda_0 v_0^2 + \frac{\mu}{\sqrt{2}} \frac{v_s v_X}{v_0} & v_0 v_X (\lambda_3 + \lambda_4) - \frac{\mu v_s}{\sqrt{2}} & v_0 v_s \lambda_{0s} - \frac{\mu v_X}{\sqrt{2}} \\ v_0 v_X (\lambda_3 + \lambda_4) - \frac{\mu v_s}{\sqrt{2}} & 2\lambda_X v_X^2 + \frac{\mu}{\sqrt{2}} \frac{v_s v_0}{v_X} & v_X v_s \lambda_{Xs} - \frac{\mu v_0}{\sqrt{2}} \\ v_0 v_s \lambda_{0s} - \frac{\mu v_X}{\sqrt{2}} & v_X v_s \lambda_{Xs} - \frac{\mu v_0}{\sqrt{2}} & 2\lambda_s v_s^2 + \frac{\mu}{\sqrt{2}} \frac{v_0 v_X}{v_s} \end{pmatrix}, \quad (2.81)$$

which can be further simplified once we assume the new scales $\mu \sim v_s \gg v$. Hence $(\mathbb{M}_H)_{11} \sim (\mathbb{M}_H)_{22}$ and $(\mathbb{M}_H)_{13} \sim (\mathbb{M}_H)_{23}$, where λ_{0s} and λ_{Xs} are both positive numbers, a condition to leave the potential bounded from below. Therefore, the fields denoted by $\bar{H}, H$ and H_s have their masses given by

$$m_{\bar{H}}^2 \approx 2\lambda_0 v_0^2 - (\lambda_3 + \lambda_4)v_0 v_X + \frac{\mu v_s}{\sqrt{2}} \left(1 + \frac{v_X}{v_0}\right), \quad (2.82a)$$

$$m_H^2 \approx 2\lambda_0 v_0^2 + (\lambda_3 + \lambda_4)v_0 v_X + \frac{\mu v_s}{\sqrt{2}} \left(\frac{v_X}{v_0} - 1\right), \quad (2.82b)$$

$$m_{H_s}^2 \approx 2\lambda_s v_s^2 + \frac{\mu}{\sqrt{2}} \frac{v_0 v_X}{v_s}. \quad (2.82c)$$

Here the choice for the indexes is motivated by the region where $v_0 \sim v_X$, such that

$$m_{\bar{H}}^2 \approx \sqrt{2}\mu v_s, \quad m_H^2 \approx (2\lambda_0 + \lambda_3 + \lambda_4)v_0^2, \quad m_{H_s}^2 \approx 2\lambda_s v_s^2. \quad (2.83)$$

2.5 Yukawa Lagrangian

The choice of non-universal charges requires the inclusion of at least one additional SM-like Higgs, a necessary condition to obtain the correct fermion mass spectra. In addition, one cannot preserve the quarks neutral under X . Actually, we avoid adding a third scalar doublet by assigning the same hypercharge to two fermion generations. Following the proposal of ref. [23], we charge one generation whilst the remaining ones remain neutral. This approach reduces the flavor matrix $\mathbb{F}$ to its minimal version. Therefore, here the new Higgs doublet, charged both under X and Y , fills the second Yukawa matrix column of a

$U(1)_X$ specific to the second fermion family. The Yukawa Lagrangian in the quark sector, from the notation of ref. [31], can be given by

$$\begin{aligned}\mathcal{L}_Y = & (\bar{Q}_L)_i \mathbb{Y}_{ij}^U \phi_0^c(U_R)_j + (\bar{Q}_L)_i \mathbb{Y}_{ij}^D \phi_0(D_R)_j \\ & + (\bar{Q}_L)_i \mathbb{Y}_{i2}^U \phi_X^c(U_R)_2 + (\bar{Q}_L)_i \mathbb{Y}_{i2}^D \phi_X(D_R)_2,\end{aligned}\quad (2.84)$$

with $i \in [1, 2, 3]$ and $j \in [1, 3]$. Note that $X_U + X_D = 2X_Q$ is valid per generation.

In general, $U(1)_X$ models preserve total equivalence in their gauge structure and are distinguished by the functional form of the equations (2.53) under the kinetic and gauge couplings. In other words, the gauge sector can generically be represented by eq. (2.50) such that a particular X-hypercharge assignment will then be converted into the independent functions defining the fermion couplings of (2.53). The version we have selected, for instance, generates at tree-level the same set of vertexes as that of [23]. As in their case, we request a new singlet scalar s living at the high scale and it might generate Majorana mass terms for neutrinos. In addition, it explicitly breaks a residual global $U(1)$ in the potential. Although we aim to favor “muon-specific” processes, our X-hypercharge choice $X_{q_i} = X_{l_i} = 0$ for $i = 1, 3$, $X_s = -X_c = 1$ and $X_\mu = -X_{\nu_\mu} = 1$, will still disperse its effects into different flavors. Notwithstanding, under the same constraints, the two models are expected to produce completely independent parameter spaces. In the next part of this work we show the comparison between A' and Z' physics (see section 3) facing the same and most stringent bounds in the MeV regime, coming from the electron and muon anomalous magnetic moment [29] and the neutrino trident production [32].

The mass Lagrangian (2.84) can be written in terms of flavor vectors as

$$\mathcal{L}_{\text{mass}} \supset \frac{v}{\sqrt{2}} [\bar{u}_L (\mathbb{Y}_0^U c_\beta + \mathbb{Y}_X^U s_\beta) u_R + \bar{d}_L (\mathbb{Y}_0^D c_\beta + \mathbb{Y}_X^D s_\beta) d_R] + \text{h.c.}, \quad (2.85)$$

where $s_\beta = \frac{v_X}{v}$. Again, $\mathbb{Y}_0$ has filled the first and third columns, while $\mathbb{Y}_X$ has non-zero elements in the second one. By rotating the quark vectors as $u_R \rightarrow V_{uR} u'_R \equiv V_{uR} u_R$, $u_L \rightarrow V_{uL} u_L$, $d_R \rightarrow V_{dR} d_R$, $d_L \rightarrow V_{dL} d_L$, it follows

$$\mathcal{L}_{\text{mass}} \supset \frac{v}{\sqrt{2}} [\bar{u}_L V_{uL}^\dagger (\mathbb{Y}_0^U c_\beta + \mathbb{Y}_X^U s_\beta) V_{uR} u_R + \bar{d}_L V_{dL}^\dagger (\mathbb{Y}_0^D c_\beta + \mathbb{Y}_X^D s_\beta) V_{dR} d_R] + \text{h.c.}, \quad (2.86)$$

with real, non-negative and diagonal matrices defining the quark masses. For completeness, the interactions with neutral scalars can be put in the form

$$\mathcal{L}^h \supset \bar{u}_L (\kappa_0^U H^0 + \kappa_X^U H^X) u_R + \text{h.c.}, \quad (2.87)$$

where $\kappa_0^U \equiv V_{uL}^\dagger \mathbb{Y}_0^U V_{uR}$ and $\kappa_X^U \equiv V_{uL}^\dagger \mathbb{Y}_X^U V_{uR}$. Since there are no interactions among quarks and the singlet s , the vertices can be written as

$$\mathcal{L}^h \supset \langle \mathbf{j}^U | \mathbf{h} \rangle + \text{h.c.}, \quad (2.88)$$

with $\mathbf{j}^U \equiv (j_0^U, j_X^U, 0)$, $\mathbf{h} \equiv (H_0, H_X, H_s)$ and $j_i^U \equiv \bar{u}_L \kappa_i^U u_R$. Thus, in the mass basis,

$$\mathcal{L}^h \supset \langle \mathbb{R}^h \mathbf{j}^U | \bar{\mathbf{h}} \rangle + \text{h.c.}. \quad (2.89)$$

The matrix $\mathbb{R}^h$ rotates $\mathbb{M}_h^2$ according to section 2.4. Finally,

$$\begin{aligned} \mathcal{L}^h \supset & [(\mathbb{R}^h)_{11} j_0^U + (\mathbb{R}^h)_{12} j_X^U] \bar{H} + [(\mathbb{R}^h)_{21} j_0^U + (\mathbb{R}^h)_{22} j_X^U] H \\ & + [(\mathbb{R}^h)_{31} j_0^U + (\mathbb{R}^h)_{32} j_X^U] \bar{H}_s + \text{h.c.} . \end{aligned} \quad (2.90)$$

The same follows for down-quarks and charged leptons. For instance,

$$\mathcal{L}_{\text{mass}} \supset \frac{v}{\sqrt{2}} [\bar{l}_L (\mathbb{Y}_0^l c_\beta + \mathbb{Y}_X^l s_\beta) l_R] + \text{h.c.} , \quad (2.91)$$

The lepton vectors are rotated as $l_R \rightarrow V_{lR} l_R$, $l_L \rightarrow V_{lL} l_L$, such that $V_{lL}^\dagger (\mathbb{Y}_0^l c_\beta + \mathbb{Y}_X^l s_\beta) V_{lR}$ defines their mass matrix.

2.6 Parameter space

The multi-dimensional free parameter space in models beyond the Standard Model is commonly larger than the simple $g_X \times m_X$ planes. In order to avoid a redundant criterion for fixing these planes, it is important to consider all relations emerging in the gauge sector and connecting the remaining variables at tree level. The procedure is equivalent to the reduction of the dimension of a multi-variable set through some associated set of independent equations. In fact, a natural relation encompasses coupling constants and energy scales which, in general, may be directly fitted by the observable connected to it. In addition to that, one can also permute some of the variables. Such a replacement does not reduce the dimension of parameter space, but it might lead to a more convenient use of the model. Let us consider the SM example, which is initially described by the P set

$$P := [g, g_Y, v]. \quad (2.92)$$

After the W_3 - B mixing, the angle parameterizing the eigenvectors can be used in place of g_Y , i.e.

$$P := [g, g_Y, v] \rightarrow [g, s_w, v]. \quad (2.93)$$

In all vertexes, g_Y must be written as $g_Y(g, s_w)$ (in fact g_Y does not depend on v). Now, from the Z pole mass one can perform a fit of the parameters which eliminates, for instance, any dependence on v (i.e. $v = v(g, s_w)$). Thus,

$$P \xrightarrow{m_Z} [g, s_w]. \quad (2.94)$$

Next, once the charged currents are coupled only through the g coupling, it can be related to the Fermi constant G_F at the low energy limit, i.e. $P \rightarrow [g, s_w] \xrightarrow{G_F} [s_w]$. Finally, from the requirement that the theory must reproduce the electromagnetic interactions one last independent equation is given by

$$g s_w = e, \quad g_Y(g, s_w) c_w = e. \quad (2.95)$$

Therefore, the gauge sector of the Standard Model is fully determined. We must note that the W pole mass was not used in any of the steps presented above and it emerges as a prediction of the model.

As mentioned before, the P set is defined by the variables entering in the New Physics effective couplings and includes the mixing matrices. In our model, P is necessarily larger than in the SM but it still allows a significant reduction. Initially, it follows that

$$P := [\kappa, g, g_Y, g_X, v_X, v_0, v_s, \mathbb{F}] \quad (2.96)$$

Similarly to the previous example, the constants g, g_Y are solved in terms of the remaining elements. Since in the asymptotic limit m_Z depends only on v it might be convenient to preserve c_β in the analysis. Finally, the v_s breaking scale can be *replaced* by m_X . We end up with a five-dimensional parameter space, namely

$$P := [c_\beta, \kappa, g_X, m_X, \mathbb{F}]. \quad (2.97)$$

The kinetic mixing constant is independent and may be replaced by the new mixing angle θ . Accordingly, there must be a region for κ where the Z interactions are exactly described as in the SM, i.e. where $s_\theta = 0$.

3 A' vs. Z' gauge bosons

The full set of dark gauge bosons X_μ can be divided into two subsets, according to the presence of axial-vector currents. The first is composed by A' fields coupled exclusively to vector currents, and therefore it covers the particular case of dark photons. The second subset comprises general Z' bosons whose couplings include axial-vector components. In the following paragraphs we briefly summarize the current theoretical status as well as the results of experimental searches for the effects of these fields [8].

Dark boson searches and future experiments. From our study in the section 2.3, a general property of the vector and axial-vector couplings is that both contain universal and non-universal parts. The LEP searches [33] can primarily test possible electron couplings to dark fields by looking for recoil energy in a nucleus and therefore can be used to place bounds on the universal part. On the other hand, experiments such as *Mu3e* [34], devoted to test LFV via the decay channel $\mu \rightarrow ee^+e^-$, can place bounds on the flavor matrix⁵ of the particular model and will cover the range $10 \text{ MeV} < m_{A'} < 80 \text{ MeV}$. The BaBar collaboration has also performed A' searches [7] and their results highly constrain dark photons with mass above the di-muon threshold.

In the dark photon scenario, i.e. where the fields are neutral under X and the couplings defined exclusively by the kinetic mixing constant, one can mention the results of the KLOE experiment [35], in which the searches were performed in $\phi \rightarrow \eta X$, $\eta \rightarrow \pi^+\pi^-\pi^0$, $X \rightarrow e^+e^-$, with the dark photon mass in the range $50 \text{ MeV} < m_{A'} < 420 \text{ MeV}$. The NA48/2 collaboration [36] has covered a $9 \text{ MeV} < m_{A'} < 120 \text{ MeV}$ range in kaon decays $K \rightarrow \pi\pi^0$ and $K \rightarrow \mu\nu\pi^0$;

The Run 3 of LHCb plans to search for dark photons in the charm meson decays $D^* \rightarrow D^0 A' (A' \rightarrow e^+e^-)$ and is scheduled for 2021-2023 [37]. The DarkLight experiment [38]

⁵Matrix which summarizes the amount of non-universality and flavor violation in the model.

Figure 1. The Differential decay width $d\Gamma_{M\mu Y}$ normalized by $\Gamma_{\mu\nu}$ for $M = K, D_s$. The curves are plotted under the fixed values $(c_\beta, \kappa, F_{\mu\mu}, m_\chi) = (0.8, -4g_X, 1, 3m_X)$. In figure(b) the channel $D_s \rightarrow \tau\bar{\nu}_\tau(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)$ hides the distribution generated by $X \rightarrow \bar{\chi}\chi, 3\bar{\nu}\nu$.

will be sensitive to $10 \text{ MeV} < m_{A'} < 100 \text{ MeV}$ by e-H scattering producing on-shell dark photons. It is scheduled for 2018–2020. Similarly, the Heavy Photon Search [39] will scatter an electron beam on a Tungsten target, and is scheduled for 2020. Finally, the NA62 experiment [40] can place bounds on the light Z' couplings by measuring the rate of the rare decays $K \rightarrow \pi\bar{\nu}\nu$.

Constraints. The long-standing discrepancy between the measured and theoretically predicted muon anomalous magnetic moment [14–18] is at the level of $\sim 3.5 - 4\sigma$. Many approaches of physics beyond the SM were used to resolve this discrepancy by assuming only one new mediating particle [41–44]. In the second part of this work we will present the application of different versions of the present model under the most stringent bounds in the MeV regime. Among these processes we can mention the electron anomalous magnetic moment [29], νe scattering [45], parity non-conserving observables in Z' phenomenology, neutrino trident production [32] and the missing energy searches in $K \rightarrow \mu Y$ [26]. In figure 1 the differential decay width $d\Gamma_{M\mu Y}/\Gamma_{\mu\nu}$ for $M = K, D_s$ is presented, motivated by the work [26], with the fixed values $(c_\beta, \kappa, F_{\mu\mu}, m_\chi) = (0.8, -4g_X, 1, 3m_X)$. In (b) the differential decay width for $D_s \rightarrow \tau\bar{\nu}_\tau(\tau \rightarrow \mu\nu_\tau\bar{\nu}_\mu)$ must overshadow the $d\Gamma_{D_s\mu Y}$ normalized by $\Gamma_{D_s \rightarrow \mu\nu}$.

In order to maximize the parameter space covered in our analysis, our strategy includes DM considerations applied to a stable χ fermion, in principle lighter than Z' . In addition, we notice that lepton non-universality in the first and second families will necessarily imply a discrepancy in the proton charge radius estimated from the Lamb shift in the e-hydrogen and μ -hydrogen system [30], such that a precise measurement of such processes must correspond to one of the most severe bounds for non-universal dark boson theories.

Finally, effects of dark fields in purely leptonic processes support DM searches at future lepton colliders. The same physics may still give some effects in the leptonic meson decays such as $M \rightarrow \mu\nu ee$ for $M = K, D, D_s, B$. In the subsequent part of this work, we compute the SM branching ratios for these channels and compare them to the results from the Z' -boson exchange.

4 Conclusions

We have seen that the anomaly free $\text{SM} \otimes \text{U}(1)_X$ models may provide solutions to explain existing anomalies in the leptonic sector at low energies. In the UV complete version presented here, the quantum anomalies are canceled per generation. The chiral X-hypercharges for a single family require a second Higgs doublet and a scalar singlet in order to provide to the model a consistent fermion mass spectra. Right-handed fermions are incorporated, being constrained by the neutrino interactions, while by charging the second generation under $\text{U}(1)_X$, we find a convenient framework to explain the discrepancies involving muons. Finally, we considered X_μ bosons either as A' , or Z' gauge bosons, according to the role played by the vector-axial currents, aiming to test the common assertion that Z' physics might be disfavored by parity non-conserving effects. This work is a theoretical introduction which will be accompanied by a more complete phenomenological analysis, developed under the dark matter considerations applied to a light (MeV) and stable dark χ_R fermion. It establishes both the notation and the relations between the new parameters which might be useful for future studies.

A Possibilities for the new fermion χ_R and neutrino masses

In the version with Majorana fermions the model must include a new sterile generation such that the mass Lagrangian can be written as:

$$\mathcal{L} = -\frac{M^\chi}{2} \left[\overline{(\chi_{R1})^c} \chi_{R1} + \overline{\chi_{R1}} (\chi_{R1})^c \right] \quad (\text{A.1a})$$

$$-\frac{Y_{12}}{\sqrt{2}} \left[\overline{(\chi_{R1})^c} \chi_{R2} + \overline{(\chi_{R2})^c} \chi_{R1} \right] s - \frac{Y_{12}^*}{\sqrt{2}} \left[\overline{\chi_{R2}} (\chi_{R1})^c + \overline{\chi_{R1}} (\chi_{R2})^c \right] s^*. \quad (\text{A.1b})$$

The brackets in the Yukawa eq. (A.1b) lead to a symmetric Majorana mass matrix. The conjugated field is defined by

$$\chi_R^c = C \overline{\chi_R}^\top, \quad (\text{A.2})$$

in terms of a general realization of the charge conjugation operator C , such that

$$\chi \rightarrow \chi^c = C \overline{\chi}^\top = -\gamma^0 C \chi^* \quad (\text{A.3})$$

$$\overline{\chi} \rightarrow \overline{\chi^c} = -\chi^\top C^\dagger. \quad (\text{A.4})$$

The Lagrangian can be rewritten like

$$\mathcal{L} \supset -\frac{m^\chi}{2} \left[(\chi_{R1})^\top C^\dagger \chi_{R1} + \chi_{R1}^\dagger C \chi_{R1}^* \right] \quad (\text{A.5a})$$

$$-\frac{Y_{12}}{\sqrt{2}} \left[(\chi_{R1})^\top C^\dagger \chi_{R2} + (\chi_{R2})^\top C^\dagger \chi_{R1} \right] s - \frac{Y_{12}^*}{\sqrt{2}} \left[\chi_{R2}^\dagger C \chi_{R1}^* + \chi_{R1}^\dagger C \chi_{R2}^* \right] s^*. \quad (\text{A.5b})$$

In principle the mass matrix elements are complex numbers. However, in the minimal 2×2 version a redefinition of the $\chi_{R\alpha}$ fields can absorb their phases, leaving the final matrix real and symmetric:

$$\mathcal{L} \supset \frac{1}{2} \chi_{R\alpha}^\top C^\dagger M_{\alpha\beta} \chi_{R\beta} + \text{h.c.} \quad (\text{A.6})$$

The mass matrix arises after EWSB, given by

$$\mathbb{M} = \begin{pmatrix} m & Y_{12}v_s \\ Y_{12}v_s & 0 \end{pmatrix}, \quad (\text{A.7})$$

diagonalized via

$$(V_R^\chi)^\dagger \mathbb{M} (V_R^\chi) = \mathbb{M}^\chi, \quad (\text{A.8})$$

with V_R^χ being unitary, a criterion to give $\mathbb{M}^\chi$ with real and positive eigenvalues. The fields are rotated as $\chi_R \rightarrow V_R^\chi \chi'_R \equiv V_R^\chi \chi_R$, where in the r.h.s. the same notation is used for the mass states. In conclusion,

$$\mathcal{L}^\chi \supset \frac{m_k}{2} \chi_{Rk}^\dagger C^\dagger \chi_{Rk} + \text{h.c.} \quad (\text{A.9})$$

$k = 1, 2$, or, equivalently,

$$\mathcal{L}^\chi \supset -\frac{m_k}{2} \chi_{Rk}^c \chi_{Rk} + \text{h.c.} \quad (\text{A.10})$$

where the Majorana dark fermions

$$\chi_k = \chi_{Rk} + \chi_{Rk}^c \quad (\text{A.11})$$

have well defined mass. Note that these fields are protected to decay due to a global $U(1)^\chi$. The portal into SM is created by the scalars and by both Z and X interactions.

Neutrino masses.

$$\mathcal{L} = -\frac{1}{4} \mathbf{W}^{\mu\nu} \cdot \mathbf{W}_{\mu\nu} - \frac{1}{4} B^{Y\mu\nu} B_{\mu\nu}^Y - \frac{1}{4} B^{X\mu\nu} B_{\mu\nu}^X + \frac{\epsilon}{2} B^{Y\mu\nu} B_{\mu\nu}^X \quad (\text{A.12a})$$

$$+ (D_\mu \phi^0)^\dagger (D^\mu \phi^0) + (D_\mu \phi^X)^\dagger (D^\mu \phi^X) + (D_\mu s)^\dagger (D^\mu s) - V(\phi^0, \phi^X, s) \quad (\text{A.12b})$$

$$- \sum_{\alpha=1,2,3} \sum_{\beta=1,3} \left(\bar{L}_{\alpha L} \phi^0 Y_{\alpha\beta}^l e_{\beta R} + \bar{L}_{\alpha L} \tilde{\phi}^0 Y_{\alpha\beta}^\nu \nu_{\beta R} + \text{h.c.} \right) \quad (\text{A.12c})$$

$$- \sum_{\alpha=1,2,3} \left(\bar{L}_{\alpha L} \phi^X Y_{\alpha 2}^l e_{2R} + \bar{L}_{\alpha L} \tilde{\phi}^X Y_{\alpha 2}^\nu \nu_{2R} + \text{h.c.} \right) \quad (\text{A.12d})$$

$$- \sum_{\alpha=1,2,3} \sum_{\beta=1,3} \left(\bar{Q}_{\alpha L} \phi^0 Y_{\alpha\beta}^D d_{\beta R} + \bar{Q}_{\alpha L} \tilde{\phi}^0 Y_{\alpha\beta}^U u_{\beta R} + \text{h.c.} \right) \quad (\text{A.12e})$$

$$- \sum_{\alpha=1,2,3} \left(\bar{Q}_{\alpha L} \phi^X Y_{\alpha 2}^D d_{2R} + \bar{Q}_{\alpha L} \tilde{\phi}^X Y_{\alpha 2}^U u_{2R} + \text{h.c.} \right) \quad (\text{A.12f})$$

$$- \sum_{\alpha=1,3} Y_{\alpha 2} \left[\overline{(\nu_{R\alpha})^c} \nu_{R2} + \overline{(\nu_{R2})^c} \nu_{R\alpha} \right] s - Y_{\alpha 2}^* \left[\overline{\nu_{R2}} (\nu_{R\alpha})^c + \overline{\nu_{R\alpha}} (\nu_{R2})^c \right] s^* \quad (\text{A.12g})$$

$$- \sum_{\alpha,\beta=1,3} \frac{M_{\alpha\beta}}{2} \overline{(\nu_{R\alpha})^c} \nu_{R\beta} + \text{h.c.} \quad (\text{A.12h})$$

$$+ i \sum_{\alpha=1,2,3} \left[\bar{L}_{\alpha L} \not{D} L_{\alpha L} + \bar{Q}_{\alpha L} \not{D} Q_{\alpha L} \right. \quad (\text{A.12i})$$

$$\left. + \bar{l}_{\alpha R} \not{D} l_{\alpha R} + \bar{d}_{\alpha R} \not{D} d_{\alpha R} + \bar{u}_{\alpha R} \not{D} u_{\alpha R} + \bar{\nu}_{\alpha R} \not{D} \nu_{\alpha R} \right], \quad (\text{A.12j})$$

above, the index “c” denotes charge conjugated and is used to recover that Majorana mass terms may appear as long as the charges satisfy $X_\alpha + X_\beta = 0$. In the above variant this piece is composed by the sterile neutrinos of first and third generations. Since ν_{2R} is charged under X , the mixing with the additional fermions is inserted via the singlet s and under the condition $X_\alpha + X_2 + X_s = 0$. In this section we briefly present some general aspects of the see-saw mechanism which concludes the construction of the model.

By including three generations of RH neutrinos, after the EWSB the Dirac and Majorana mass terms can be included in the Lagrangian (see [46])

$$\mathcal{L}_\nu = -\frac{1}{2} \begin{pmatrix} \overline{\nu_L^c} & \overline{\nu_R} \end{pmatrix} \begin{pmatrix} \mathbb{M}_L & \mathbb{M}_D \\ \mathbb{M}_D & \mathbb{M}_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{h.c.}, \quad (\text{A.13})$$

which can be in this form due to $\overline{\psi^c}\psi^c = -\psi^\top C^{-1}C\overline{\psi}^\top = \overline{\psi}\psi$, where in the last step the anti-commuting property of fermion fields has been considered⁶ (see 3.52 [46]). Here the matrix $\mathbb{M}_L = 0$ once there is no Majorana mass terms for LH neutrinos. The mass matrix can be rotated by the following transformations

$$U^\dagger M U^* = \begin{pmatrix} \mathbb{D}_L & \\ & \mathbb{D}_R \end{pmatrix} \quad \text{where} \quad U = \begin{pmatrix} \mathbb{V}_L & \mathbb{V}_{LR} \\ \mathbb{V}_{RL} & \mathbb{V}_R \end{pmatrix} \quad (\text{A.14})$$

In the framework where $\mathbb{M}_D \ll \mathbb{M}_R$, the matrices $\mathbb{V}_L$ and $\mathbb{V}_R$ may be taken approximately unitary, with $\mathbb{V}_{RL}, \mathbb{V}_{LR}$ suppressed by $\approx \frac{\mathbb{M}_D}{\mathbb{M}_R}$. In summary, the light neutrinos must mix LH flavor states only, while the heavy states mix the remaining RH fields. The matrix $\mathbb{M}_R$ is extracted from the part

$$\mathcal{L} \supset - \sum_{\alpha, \beta=1,3} \frac{M_{\alpha\beta}}{2} \left[\overline{(\nu_{R\alpha})^c} \nu_{R\beta} + \overline{\nu_{R\alpha}} (\nu_{R\beta})^c \right] \quad (\text{A.15a})$$

$$- \sum_{\alpha=1,3} Y_{\alpha 2} \left[\overline{(\nu_{R\alpha})^c} \nu_{R2} + \overline{(\nu_{R2})^c} \nu_{R\alpha} \right] s + Y_{\alpha 2}^* \left[\overline{\nu_{R2}} (\nu_{R\alpha})^c + \overline{\nu_{R\alpha}} (\nu_{R2})^c \right] s^* \quad (\text{A.15b})$$

which can be presented in the basis

$$\mathcal{L} \supset \begin{pmatrix} \overline{\nu_{R1}} & \overline{\nu_{R2}} & \overline{\nu_{R3}} \end{pmatrix} \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix} \begin{pmatrix} \nu_{R1}^c \\ \nu_{R2}^c \\ \nu_{R3}^c \end{pmatrix} + \text{h.c.} \quad (\text{A.16})$$

The mass eigenstates are denoted as $\chi_{R\alpha} = (\mathbb{V}_R)_{\alpha\beta} \nu_{R\beta}$. Once the rotation occurs among ν_R 's, the fields χ are the degrees of freedom present in the interactions. As for the LH neutrinos, the χ spinors are given by

$$\chi = \chi_R + \chi_R^c. \quad (\text{A.17})$$

The case $\mathbb{M}_D \ll \mathbb{M}_R$ is valid both from the presence of small Yukawas as from the difference between the electroweak and the Majorana scales. In the first scenario, small

⁶This minus sign does not appear in mass terms from the property of hermitian conjugate of Grassmann fields.

Yukawas, which control the decay of the new mass eigenstates into SM particles (apart from the suppressed V_{LR}), will dictate how reliable it is to assume at least one generation of χ fields as a dark matter candidate.

The last part concerns neutrino interactions and a block diagonal U is assumed to rotate the system to the mass basis, such that the heavy neutrinos mixes the RH fields only, and the block $V_{\nu R}$ is approximately unitary (see [46]). The universal elements are weighted by

$$g_R^I(\nu_R) = g_Z^I(\nu_R) = 0, \quad (\text{A.18})$$

since ν_R are singlets under the SM gauge group. Thus,

$$\mathcal{L}_{kin} \supset s_\theta g_X \left[\bar{\nu}_R V_{\nu R}^\dagger \mathbb{X}^\nu V_{\nu R} \gamma^\mu \nu_R \right] Z_\mu - c_\theta g_X \left[\bar{\nu}_R V_{\nu R}^\dagger \mathbb{X}^\nu V_{\nu R} \gamma^\mu \nu_R \right] X_\mu, \quad (\text{A.19})$$

A.1 Z couplings

$$\mathcal{L} \supset \frac{1}{2} \bar{f} \gamma_\mu (g_V^f + g_A^f \gamma^5) f Z^\mu, \quad (\text{A.20})$$

$$g_V^U = g \frac{c_\theta}{c_\phi} \left(\frac{1}{2} - \frac{4}{3} s_\phi^2 \right) + s_\theta \kappa \frac{5}{6} + s_\theta g_X \mathbb{F}_{ii}^U, \quad (\text{A.21a})$$

$$g_A^U = g \frac{c_\theta}{c_\phi} \left(-\frac{1}{2} \right) + s_\theta \kappa \frac{1}{2} + s_\theta g_X \mathbb{F}_{ii}^U, \quad (\text{A.21b})$$

$$g_V^D = g \frac{c_\theta}{c_\phi} \left(-\frac{1}{2} + \frac{2}{3} s_\phi^2 \right) - s_\theta \kappa \frac{1}{6} + s_\theta g_X \mathbb{F}_{ii}^D, \quad (\text{A.21c})$$

$$g_A^D = g \frac{c_\theta}{c_\phi} \left(\frac{1}{2} \right) - s_\theta \kappa \frac{1}{2} + s_\theta g_X \mathbb{F}_{ii}^D, \quad (\text{A.21d})$$

$$g_V^l = g \frac{c_\theta}{c_\phi} \left(-\frac{1}{2} + 2s_\phi^2 \right) - s_\theta \kappa \frac{3}{2} + s_\theta g_X \mathbb{F}_{ii}^l, \quad (\text{A.21e})$$

$$g_A^l = g \frac{c_\theta}{c_\phi} \left(\frac{1}{2} \right) - s_\theta \kappa \frac{1}{2} + s_\theta g_X \mathbb{F}_{ii}^l \quad (\text{A.21f})$$

$$g_V^\nu = -g_A^\nu = g \frac{c_\theta}{c_\phi} \left(\frac{1}{2} \right) - s_\theta \kappa, \quad (\text{A.21g})$$

$$g_V^\chi = g_A^\chi = -s_\theta g_X. \quad (\text{A.21h})$$

where the index i denotes the fermion generation.

B $K^0 - \overline{K}^0$

The contribution to the mass difference of the $K_{L,S}$, given by Δm_K in the $U(1)_X$ model corresponds to the operator

$$\mathcal{H}_X = C(q^2) \bar{d} \gamma_\mu P_R s \bar{d} \gamma^\mu P_R s. \quad (\text{B.1})$$

Here the Wilson Coefficient comprises the X^μ propagator and the fermion couplings, i.e.

$$C(q^2) = -c_\theta^2 g_X^2 |\mathbb{F}_{ds}^D|^2 \frac{1}{q^2 - m_X^2} \quad (\text{B.2})$$

Figure 2. The flavor changing matrix element $|V_d|_{21}$. The region above the lines is constrained according to [47].

Following the results in [47], a bound can be imposed on $C(m_K^2)$ by claiming⁷

$$|C(m_K^2)| \leq \frac{8.8 \times 10^{-19}}{[\text{MeV}]^2} \quad (\text{B.4})$$

In the region of the parameter space where $c_\theta \sim 1$, since $X_D = 1$, it can be converted into

$$g_X^2 \frac{|V_{21}^d|^2}{m_K^2 - m_X^2} \leq \frac{8.8 \times 10^{-19}}{[\text{MeV}]^2} \quad (\text{B.5})$$

where it has been taken $|V_{32}^d| \rightarrow 0$ and neglected quartic terms in $|V_{21}^d|$. In figure 2 the bounds to four values of the coupling g_X are presented.

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⁷Note: the same bound for LH currents has been considered once

$$\langle M^0 | (\bar{a} \gamma^\mu P_L q) (\bar{a} \gamma_\mu P_L q) | \bar{M}^0 \rangle = \langle M^0 | (\bar{a} \gamma^\mu P_R q) (\bar{a} \gamma_\mu P_R q) | \bar{M}^0 \rangle = \frac{f_M^2}{3} m_M \quad (\text{B.3})$$

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