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Statistical analysis of the azimuthal asymmetry in the J/ψ leptonproduction in unpolarized ep collisions

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ABSTRACT: In this paper, we study the azimuthal asymmetry in the J/ψ leptonproduction in unpolarized ep collisions. There are two independent azimuthal asymmetry modulations, namely $\cos(\psi)$ and $\cos(2\psi)$, where ψ is the azimuthal angle of the lepton scattering plane with respect to the hadron-interacting plane. We calculate the two modulations as functions of four kinematic variables, and find that they provide a very good laboratory to distinguish several models describing the heavy quarkonium production, including the color-singlet (CS) model, the nonrelativistic QCD (NRQCD). In addition, this process can be used to test the magnitude of the $^1S_0^{[8]}$ long-distance matrix element, which is extraordinarily large from various phenomenological extraction. In order to make definite conclusions, we restrict our calculation in a specific kinematic region, where the CS and CO mechanisms can be distinguished by scrutinizing the values of the $\cos(\psi)$ modulation, while the $^1S_0^{[8]}$ dominance picture can be tested by measuring the values of the $\cos(2\psi)$ modulation. Calculating their values and carrying out a meticulous statistical analysis, we find that at an integrated luminosity $\mathcal{L} = 1000\text{pb}^{-1}$, the statistical uncertainties of the two quantities are small enough to tell the three models apart. When this experiment is implemented at the future ep colliders such as the EIC, crucial information for the J/ψ production mechanism might be discovered.

KEYWORDS: Deep Inelastic Scattering (Phenomenology), QCD Phenomenology

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1 Introduction

It has not been realized that the transverse polarization and transverse momentum effects in nucleons could be significant until the first measurement of inclusive pion hadroproduction was carried out at Argonne synchrotron [1–3], the results of which were later confirmed by Fermilab [4] at a slightly higher colliding energy. In order to understand the unexpected large transverse spin asymmetries observed in those experiments, various theoretical mechanisms and new structure functions were proposed, among which are there the well-known Sivers effect [5, 6] along with a new function, the Sivers function, describing the azimuthal asymmetry of unpolarized quarks in polarized hadrons, and Collins effect [7, 8], considering the different fragmenting probabilities of transversely polarized partons to hadrons with transverse momentum along different directions. Since this century, the transverse spin effects have been studied experimentally in semi-inclusive deep-inelastic scattering (SIDIS). In 2004, HERMES [9] and COMPASS [10, 11] published their measurements of the single-spin asymmetry in the collisions of leptons off polarized proton and deuteron targets. One of the main advantages of SIDIS is that the transverse spin and transverse momentum effects are not mixed, as in hadroproduction; they result in different azimuthal asymmetry modulations. Another interesting feature of SIDIS is that, even with unpolarized targets, there also exist two types of azimuthal asymmetry modulations, namely the $\cos(\psi)$ and $\cos(2\psi)$ modulations, where ψ is the azimuthal angle for the observed final-state hadron with respect to the virtual-photon-target beam. A few years ago, HERMES [12] and COMPASS [13] collaborations presented the data for the azimuthal distributions of hadrons produced in deep inelastic scattering off unpolarized targets and found nonzero $\cos(\psi)$ and $\cos(2\psi)$ azimuthal asymmetry modulations. These modulations arise as long as the momenta of the final-state hadrons have transverse components, which may originate from either the intrinsic transverse motion of partons inside the targets, or the hard emission of the final-state hadrons. The former one was first studied by Cahn [14, 15] and thus is called the Cahn effect. Cahn effect dominates in low p_t region, while in high p_t region, the real emission plays a more important role.

The azimuthal asymmetries in unpolarized SIDIS not only provide essential information for the transverse insight of nucleons, but also open a window for the study of hadron production mechanisms, e.g. the heavy quarkonium production mechanism. Although heavy quarkonia have very simple structure, their production mechanism is still unknown. Nonrelativistic QCD (NRQCD) [16], which separates the perturbatively calculable short-distance coefficients (SDCs) from the process-independent long-distance matrix elements (LDMEs), is one of the most successful theories describing the quarkonium productions and decays. However it confronted a lot of difficulties and challenges in the past two decades. The most famous one of those is the J/ψ polarization puzzle. In the last decade, many processes have been calculated up to QCD next-to-leading order (NLO), including the J/ψ [17–27] and the η_c [28] production in e^+e^- annihilation, the J/ψ photoproduction in e^+e^- [29] and ep [30–37] collisions, the ψ [38–57], η_c [58–62], and χ_c [63–67] hadroproduction, and the J/ψ production in deep-inelastic scattering (DIS) [68]. As a noteworthy progress, the measurement of the η_c production [69, 70] stimulated a lot of theoretical works [28, 58–62, 71]. Unfortunately, neither a set of globally working LDMEs has been found, nor the importance of the color-octet (CO) mechanism has been demonstrated. We refer the readers to Reference [61] as a state-of-the-art review. Azimuthal asymmetry in the J/ψ production in DIS can serve as an alternative test of NRQCD. As we will see later, in some specific kinematic regions, the azimuthal asymmetry can distinguish different production channels. Based on statistical analysis, we propose a practical strategy to verify the CO mechanism and fix the corresponding LDMEs.

The rest of this paper is organized as follows. The analytical framework of our calculation is discussed in section 2. In section 3, we present our numerical results, according to which, the statistical analysis is given in the same section. Section 4 is a concluding remark.

2 The azimuthal asymmetry in the J/ψ leptonproduction

2.1 The calculation of the cross sections

In electron-proton collisions,

$$e(k) + p(P) \rightarrow e(k') + J/\psi(p) + X(p_X), \quad (2.1)$$

in general, can be reduced into the virtual-photon-proton fusion process as

$$\gamma^*(q) + p(P) \rightarrow J/\psi(p) + X(p_X). \quad (2.2)$$

Here, X represents the proton remnant. k , P , q , k' , p , and p_X are the momenta of the initial-state electron, proton, and virtual photon, the final-state electron, the J/ψ , and the proton remnant. Usually, we use the following invariants to describe the kinematics of this process,

$$Q^2 = -q^2, \quad W^2 = (P + q)^2, \quad z = \frac{P \cdot p}{P \cdot q}, \quad (2.3)$$

the first two of which can be replaced by the following two dimensionless invariants,

$$x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{2P \cdot q}{(P + k)^2}, \quad (2.4)$$

Figure 1. Illustration of the process.

as long as the invariant colliding energy squared, $S = (P + k)^2$, is given. Here, x is the well-known Bjorken- x , and z is named the elasticity coefficient.

To study the azimuthal effects, we adopt such frames in which the virtual photon and the proton beams are along the z and anti- z directions, respectively, as illustrated in figure 1. The azimuthal angle of the final-state J/ψ is then defined with respect to the virtual-photon-proton beams. In such frames, the physical quantities are always labelled by a superscript $\star$, in order to distinguish from those in laboratory frames. Since the laboratory frame is not concerned in this paper, we just suppress the superscript $\star$. In addition to the invariants, x , y , and z (or equivalently, Q^2 , W^2 , and z), we need two more quantities, the transverse momentum of the final-state J/ψ , p_t , and its azimuthal angle, ψ , to sufficiently describe the kinematics of the J/ψ leptonproduction process. We define two dimensionless quantities,

$$\xi = \frac{p_t^2}{S}, \quad \eta = \frac{M^2}{S}, \quad (2.5)$$

where M is the J/ψ mass, so that the SIDIS process can be described by six dimensionless variables, x , y , z , ξ , η , and ψ . Having all these kinematic variables, we can write down the hadron production cross section in unpolarized SIDIS as (see e.g. Reference [72])

$$d\sigma = \frac{\alpha}{128\pi^4} l_{\mu\nu} W_h^{\mu\nu} \frac{dx}{x} dy \frac{dz}{z} d\xi d\psi, \quad (2.6)$$

where α is the electromagnetic fine-structure constant, $W_h^{\mu\nu}$ is the hadronic tensor integrated over the phase-space of the final-state hadrons other than the J/ψ , and the normalized leptonic tensor, $l_{\mu\nu}$, is defined as

$$l_{\mu\nu} = \left(-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2} \right) + \frac{(2k - q)_\mu (2k - q)_\nu}{Q^2}. \quad (2.7)$$

If only one final-state hadron is observed, the leptonic tensor, $l_{\mu\nu}$, can be decomposed into the linear combination of four simpler tensors, factorizing all the azimuthally dependent terms into the corresponding coefficients as

$$l^{\mu\nu} = A_g \left(-g^{\mu\nu} - \frac{q^\mu q^\nu}{Q^2} \right) + A_L \epsilon_L^\mu \epsilon_L^\nu + A_{LT} (\epsilon_L^\mu \epsilon_T^\nu + \epsilon_T^\mu \epsilon_L^\nu) + A_T \epsilon_T^\mu \epsilon_T^\nu, \quad (2.8)$$

where

$$\begin{aligned}\epsilon_L &= \frac{1}{Q} \left(q + \frac{Q^2}{P \cdot q} P \right), \\ \epsilon_T &= \frac{1}{p_t} (p - \rho P - zq),\end{aligned}\tag{2.9}$$

and

$$\begin{aligned}A_g &= 1 + \frac{2(1-y)}{y^2} - \frac{2(1-y)}{y^2} \cos(2\psi), \\ A_L &= 1 + \frac{6(1-y)}{y^2} - \frac{2(1-y)}{y^2} \cos(2\psi), \\ A_{LT} &= \frac{2(2-y)}{y^2} \sqrt{1-y} \cos(\psi), \\ A_T &= \frac{4(1-y)}{y^2} \cos(2\psi).\end{aligned}\tag{2.10}$$

Here, ρ is defined as

$$\rho = \frac{p \cdot q + zQ^2}{P \cdot q} = \frac{(\xi + \eta)/z + xyz}{y},\tag{2.11}$$

where the proton mass has been neglected. Substituting equation (2.9) into equation (2.8) and taking into account the current-conserving equation,

$$q_\mu W_h^{\mu\nu} \equiv 0,\tag{2.12}$$

we can rewrite the normalized leptonic tensor in a form that is more convenient for computation as

$$l^{\mu\nu} = C_1(-g^{\mu\nu}) + C_2 \frac{P^\mu P^\nu}{S} + C_3 \frac{P^\mu p^\nu + p^\mu P^\nu}{2S} + C_4 \frac{p^\mu p^\nu}{S},\tag{2.13}$$

where

$$\begin{aligned}C_1 &= A_g, \\ C_2 &= \frac{4x}{y} A_L - 4\rho \sqrt{\frac{x}{y\xi}} A_{LT} + \frac{\rho^2}{\xi} A_T, \\ C_3 &= 4\sqrt{\frac{x}{y\xi}} A_{LT} - \frac{2}{\xi} A_T, \\ C_4 &= \frac{1}{\xi} A_T.\end{aligned}\tag{2.14}$$

If we define

$$\beta = \frac{\rho}{2} \sqrt{\frac{y}{x\xi}} = \frac{\xi + \eta + xyz^2}{2z} \sqrt{\frac{1}{xy\xi}},\tag{2.15}$$

these C_i 's can be expressed in a more compact form as

$$\begin{aligned}C_1 &= A_g, \\ C_2 &= \frac{4x}{y} (A_L - 2\beta A_{LT} + \beta^2 A_T) \\ C_3 &= 4\sqrt{\frac{x}{y\xi}} (A_{LT} + \beta A_T) \\ C_4 &= \frac{1}{\xi} A_T.\end{aligned}\tag{2.16}$$

In collinear factorization, the protons interact with the virtual photons via the on-shell partons, and thus, the hadronic tensor, $W_h^{\mu\nu}$, can be further factorized as the summation of the parton-level hadronic tensors convoluted with the parton distribution functions (PDFs). For $c\bar{c}(^3S_1^{[1]})$ production, there is only one parton-level process at QCD LO, i.e.

$$g + \gamma^* \rightarrow c\bar{c}(^3S_1^{[1]}) + g, \quad (2.17)$$

while for the production of the three color-octet states, namely $c\bar{c}(^1S_0^{[8]})$, $c\bar{c}(^3S_1^{[8]})$, and $c\bar{c}(^3P_J^{[8]})$, there are two parton-level processes at QCD LO, they are

$$\begin{aligned} g + \gamma^* &\rightarrow c\bar{c}(n) + g, \\ q(\bar{q}) + \gamma^* &\rightarrow c\bar{c}(n) + q(\bar{q}). \end{aligned} \quad (2.18)$$

Here, n represents $^1S_0^{[8]}$, $^3S_1^{[8]}$, or $^3P_J^{[8]}$. Denoting the squared amplitudes of the above processes as $W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}$, where μ and ν are the Lorentz indices for the virtual photons in the amplitude and its complex conjugate, respectively, $W_h^{\mu\nu}$ can be written as

$$W_h^{\mu\nu} = \sum_{i,n} \frac{1}{N_i} \int \frac{dX}{X} f_{i/p}(X) W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu} \langle \mathcal{O}^{J/\psi}(n) \rangle d\Phi_X, \quad (2.19)$$

where n runs over the four intermediate $c\bar{c}$ states, N_i is the synthesized initial-state averaging factor, X is the fraction of the proton momentum taken by the parton, $f_{i/p}$ is the corresponding PDF, $\langle \mathcal{O}^{J/\psi}(n) \rangle$ is the LDME for an intermediate $c\bar{c}$ state, $c\bar{c}(n)$, producing a J/ψ , and $d\Phi_X$ is defined as

$$d\Phi_X = (2\pi)^4 \delta^4(q + p_i - p - p'_i) \frac{d^3 p'_i}{(2\pi)^3 2p'_{i0}}. \quad (2.20)$$

Here, p_i and p'_i are the momenta of the initial- and final-state parton, respectively, and $p_i = XP$ has been implemented. Having dX and $d\Phi_X$ integrated over, $W_h^{\mu\nu}$ can be further simplified as (see e.g. Reference [73])

$$W_h^{\mu\nu} = \frac{2\pi}{S} \sum_{i,n} \frac{f_{i/p}(X)}{X} \frac{1}{y(1-z)} W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu} \langle \mathcal{O}^{J/\psi}(n) \rangle. \quad (2.21)$$

Having equation (2.21), equation (2.6) then leads to

$$d\sigma = \frac{\alpha}{(4\pi)^3 S} \sum_{i,n} \frac{f_{i/p}(X)}{X} l_{\mu\nu} W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu} \langle \mathcal{O}^{J/\psi}(n) \rangle \frac{dx}{x} \frac{dy}{y} \frac{dz}{z(1-z)} d\xi d\psi. \quad (2.22)$$

Note that in equation (2.22) the value of X has been fixed by the energy conservation at

$$X = x + \frac{\xi + \eta}{yz} + \frac{\xi}{y(1-z)}. \quad (2.23)$$

To write the cross section in a more compact form, we define

$$\begin{aligned}
 W_1[n] &= \alpha \sum_i \frac{f_{i/p}(X)}{X} (-g_{\mu\nu}) W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}, \\
 W_2[n] &= \alpha \sum_i \frac{f_{i/p}(X)}{X} \frac{P_\mu P_\nu}{S} W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}, \\
 W_3[n] &= \alpha \sum_i \frac{f_{i/p}(X)}{X} \frac{P_\mu p_\nu + p_\mu P_\nu}{2S} W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}, \\
 W_4[n] &= \alpha \sum_i \frac{f_{i/p}(X)}{X} \frac{p_\mu p_\nu}{S} W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}.
 \end{aligned} \tag{2.24}$$

The cross section can correspondingly be expressed as

$$d\sigma = \frac{1}{(4\pi)^3 S} \sum_n \sum_{i=1}^4 C_i W_i[n] \langle \mathcal{O}^{J/\psi}(n) \rangle \frac{dx}{x} \frac{dy}{y} \frac{dz}{z(1-z)} d\xi d\psi. \tag{2.25}$$

Since $W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}$ can be easily evaluated via Feynman diagrams, we have been equipped with all the elements needed in equation (2.22) for calculating the J/ψ leptoproduction cross sections.

2.2 The calculation of the azimuthal asymmetry modulations

Since $W_{i+\gamma^* \rightarrow c\bar{c}(n)+i}^{\mu\nu}$ and the tensor basis in equation (2.13) are all restricted in the hadronic plane, i.e. they are independent of the azimuthal angle ψ , the azimuthal asymmetry modulations appear only in the coefficients in the leptonic tensor expansions, namely in C_i 's. In any case, the cross section can be written in the following form,

$$d\sigma = \sigma_0 [1 + A_{\cos\psi} \cos(\psi) + A_{\cos 2\psi} \cos(2\psi)] dx dy dz d\xi d\psi. \tag{2.26}$$

Therein, $A_{\cos\psi}$ and $A_{\cos 2\psi}$ are two independent azimuthal asymmetry modulations. They are functions of x , y , z , and ξ . Explicitly, they can be accessed via the following equations,

$$\begin{aligned}
 A_{\cos\psi} &= \frac{2 \int_0^{2\pi} d\psi \cos(\psi) \frac{d\sigma}{dx dy dz d\xi}}{\int_0^{2\pi} d\psi \frac{d\sigma}{dx dy dz d\xi}}, \\
 A_{\cos 2\psi} &= \frac{2 \int_0^{2\pi} d\psi \cos(2\psi) \frac{d\sigma}{dx dy dz d\xi}}{\int_0^{2\pi} d\psi \frac{d\sigma}{dx dy dz d\xi}}.
 \end{aligned} \tag{2.27}$$

Since the four intermediate $c\bar{c}$ states contribute independently, one can study the azimuthal asymmetry originated from them separately. In this sense, the cross section can be written in the following form as

$$d\sigma = \sum_n \sigma_0^n [1 + A_{\cos\psi}^n \cos(\psi) + A_{\cos 2\psi}^n \cos(2\psi)] dx dy dz d\xi d\psi. \tag{2.28}$$

In the next section as we present their numerical results, we just omit the superscript n .

Sometimes, it is useful to study the integrated cross sections, and the azimuthal asymmetry modulations should be redefined accordingly. When we observe the azimuthal asymmetry modulations in a specific kinematic region, *e.g.* D , where D is a 4-dimensional area in the x - y - z - ξ space, the corresponding azimuthal asymmetry modulations in the region D , $A_{\cos\psi}^D$ and $A_{\cos 2\psi}^D$, are defined according to the following equation,

$$d\sigma^D = \sigma_0^D [1 + A_{\cos\psi}^D \cos(\psi) + A_{\cos 2\psi}^D \cos(2\psi)] d\psi. \quad (2.29)$$

They can also be expressed in the following explicit form as

$$\begin{aligned} A_{\cos\psi}^D &= \frac{2 \int_0^{2\pi} d\psi \cos(\psi) \int_D dx dy dz d\xi \frac{d\sigma}{dx dy dz d\xi d\psi}}{\int_0^{2\pi} d\psi \int_D dx dy dz d\xi \frac{d\sigma}{dx dy dz d\xi d\psi}}, \\ A_{\cos 2\psi}^D &= \frac{2 \int_0^{2\pi} d\psi \cos(2\psi) \int_D dx dy dz d\xi \frac{d\sigma}{dx dy dz d\xi d\psi}}{\int_0^{2\pi} d\psi \int_D dx dy dz d\xi \frac{d\sigma}{dx dy dz d\xi d\psi}}. \end{aligned} \quad (2.30)$$

3 Numerical results and phenomenological analysis

In our numerical calculation, we adopt the following parameter choices. The charm quark mass, m_c , is fixed at one half of the J/ψ mass, which is approximated to $M = 3.0$ GeV. The colliding energy is chosen as $\sqrt{S} = 318$ GeV to accord with the HERA experiment. To evaluate the parton distributions in protons, we employ the GRV PDF given in Reference [74]. Therein, the factorization scale is set to be $\mu_f = \sqrt{Q^2 + p_t^2} = \sqrt{S(xy + \xi)}$.

With the above parameter choices, we present the values of $A_{\cos\psi}$ in figures 2, 3, 4, and those of $A_{\cos 2\psi}$ in figures 5, 6, 7, for individual $c\bar{c}$ states. The ξ dependence of the azimuthal asymmetry modulations are presented for any combination of the following parameter choices, $x = 0.005, 0.05, 0.5$, $y = 0.1, 0.4, 0.7$, and $z = 0.3, 0.6, 0.9$. The value of ξ ranges from 0.00001 to 0.001, corresponding to $p_t \approx 1$ GeV to $p_t \approx 10$ GeV.

As an inspiring result, the values of the azimuthal asymmetry modulations for the four $c\bar{c}$ states are remarkably distinguished in some of these kinematic regions. To tell the CS and CO channels apart, we can measure $A_{\cos\psi}$ at $x > 0.05$, and $z \sim 0.9$, where the values of $A_{\cos\psi}$ for all the three CO states lie around 0, while those for the CS state is as large as 1. If, as argued in some papers, the J/ψ production is dominated by the CS channel, the value of $A_{\cos\psi}$ in this region should coincide with the solid curve presented in the plot at the R.H.S. column in figure 4. However, this strategy is not feasible because in large- x region, the cross sections are so small that no enough events can be produced in this region to perform reasonable analysis. Fortunately, this problem can be amended by including the small- x region, where although the CS and $^3S_1^{[8]}$ channels cannot be distinguished, the CS and CO mechanisms are still distinguishable as the $^3S_1^{[8]}$ channel is greatly suppressed in J/ψ lepton production. For this reason, we perform our analysis in the region $x > 0.001$ and $0.75 < z < 0.9$, in which the value of Q^2 is generally larger than 4 GeV^2 and the validity of the perturbative expansion is guaranteed.

Another interesting feature of our numerical results is that the behaviour of the $^1S_0^{[8]}$ channel is manifestly different from the other channels in the kinematic region, $x \sim 0.005$,

Figure 2. The $\cos(\psi)$ modulation as functions of ξ for $x = 0.005$. The upper, mid, and lower rows correspond to $y = 0.1, 0.4$, and 0.7 , respectively, while the left, mid, and right columns correspond to $z = 0.3, 0.6$, and 0.9 , respectively. The solid, dotted, dashed, dashdotted curves correspond to the results for the ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ states, respectively.

Figure 3. The $\cos(\psi)$ modulation as functions of ξ for $x = 0.05$. The upper, mid, and lower rows correspond to $y = 0.1, 0.4$, and 0.7 , respectively, while the left, mid, and right columns correspond to $z = 0.3, 0.6$, and 0.9 , respectively. The solid, dotted, dashed, dashdotted curves correspond to the results for the ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ states, respectively.

Figure 4. The $\cos(\psi)$ modulation as functions of ξ for $x = 0.5$. The upper, mid, and lower rows correspond to $y = 0.1, 0.4$, and 0.7 , respectively, while the left, mid, and right columns correspond to $z = 0.3, 0.6$, and 0.9 , respectively. The solid, dotted, dashed, dashdotted curves correspond to the results for the ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ states, respectively.

Figure 5. The $\cos(2\psi)$ modulation as functions of ξ for $x = 0.005$. The upper, mid, and lower rows correspond to $y = 0.1, 0.4$, and 0.7 , respectively, while the left, mid, and right columns correspond to $z = 0.3, 0.6$, and 0.9 , respectively. The solid, dotted, dashed, dashdotted curves correspond to the results for the ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ states, respectively.

Figure 6. The $\cos(2\psi)$ modulation as functions of ξ for $x = 0.05$. The upper, mid, and lower rows correspond to $y = 0.1, 0.4$, and 0.7 , respectively, while the left, mid, and right columns correspond to $z = 0.3, 0.6$, and 0.9 , respectively. The solid, dotted, dashed, dashdotted curves correspond to the results for the ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ states, respectively.

Figure 7. The $\cos(2\psi)$ modulation as functions of ξ for $x = 0.5$. The upper, mid, and lower rows correspond to $y = 0.1, 0.4$, and 0.7 , respectively, while the left, mid, and right columns correspond to $z = 0.3, 0.6$, and 0.9 , respectively. The solid, dotted, dashed, dashdotted curves correspond to the results for the ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, and ${}^3P_J^{[8]}$ states, respectively.

$y \sim 0.1$, and $z \sim 0.9$, where the value of $A_{\cos 2\psi}$ for $^1S_0^{[8]}$ is almost -1, while that for all the other three states is roughly 0. Since most of the extractions of the CO LDMEs lead to the same picture, i.e. the $^1S_0^{[8]}$ LDME is at least one order of magnitude greater than the other two CO LDMEs, which however is not consistent with the NRQCD velocity scaling rule, the measurement of $A_{\cos 2\psi}$, which can test the $^1S_0^{[8]}$ dominance picture, may have a remarkable impact on the determination of the LDMEs. If the J/ψ leptonproduction is also dominated by the $^1S_0^{[8]}$ channel, we should observe the value of $A_{\cos 2\psi}$ at almost -1 in this kinematic region. Since at larger value of x , the cross sections are very small, we can carry out the analysis in the same kinematic region as in the above paragraph, namely $x > 0.001$ and $0.75 < z < 0.9$.

In order to draw up a practical strategy for the experimental measurement, we need to calculate the number of events assuming a luminosity, and analyse the systematic uncertainties. For this purpose, we need to specify the value of the renormalization scale μ_r , the strong coupling constant α_s , and electromagnetic fine structure constant α . The value of μ_r in our calculation is given by $\mu_r = \sqrt{S(xy + \xi)}$. The running of α_s follows the following equation,

$$\alpha_s(\mu_r) = \frac{12\pi}{(33 - 2n_f)\log(\mu_r^2/\Lambda_{QCD}^2)}, \quad (3.1)$$

where at $n_f = 5$, Λ_{QCD} is given by $\Lambda_{QCD} = 0.226 \text{ GeV}$. It is easy to verify that at the Z_0 boson mass $M_Z \approx 91 \text{ GeV}$, the value of α_s is approximately 0.130. The electromagnetic fine structure constant $\alpha = 1/137$ is adopted.

In the following, we carry out our study in the kinematic region, say $x > 0.001$ and $0.75 < z < 0.9$. Due to the limit of the capability of the detectors, we further constrain the value of y and ξ in the region $0.04 < y < 0.6$ and $\xi > 0.00001$. In the following, we denote this kinematic region as D . This configuration corresponds to $Q^2 > 4 \text{ GeV}^2$, $60 \text{ GeV} < W < 240 \text{ GeV}$, and $p_t > 1 \text{ GeV}$. The lower bound of X can be easily obtained as $X_{\min} \approx 0.0013$, which is a moderate value to neglect the gluon saturation effects.

Integrating over x , y , z , and ξ in the region D , we obtain the cross sections contributed from the four channels as

$$d\sigma^D|_n = \sigma_0^D|_n \left[1 + A_{\cos\psi}^D|_n \cos(\psi) + A_{\cos 2\psi}^D|_n \cos(2\psi) \right] d\psi, \quad (3.2)$$

where

$$\begin{aligned} \sigma_0^D|_{\text{CS}} &= 5.06 \times 10^1 \times \frac{\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle}{M^3} \text{ pb}, \\ \sigma_0^D|_{^1S_0^{[8]}} &= 4.18 \times 10^3 \times \frac{\langle \mathcal{O}^{J/\psi}(^1S_0^{[1]}) \rangle}{M^3} \text{ pb}, \\ \sigma_0^D|_{^3S_1^{[8]}} &= 1.08 \times 10^2 \times \frac{\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle}{M^3} \text{ pb}, \\ \sigma_0^D|_{^3P_J^{[8]}} &= 6.56 \times 10^4 \times \frac{\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle}{M^5} \text{ pb}, \end{aligned} \quad (3.3)$$

and

$$\begin{aligned}
 A_{\cos\psi}^D \Big|_{\text{CS}} &= 0.561, & A_{\cos 2\psi}^D \Big|_{\text{CS}} &= -0.0658, \\
 A_{\cos\psi}^D \Big|_{1S_0^{[8]}} &= -0.225, & A_{\cos 2\psi}^D \Big|_{1S_0^{[8]}} &= -0.281, \\
 A_{\cos\psi}^D \Big|_{3S_1^{[8]}} &= 0.531, & A_{\cos 2\psi}^D \Big|_{3S_1^{[8]}} &= -0.0555, \\
 A_{\cos\psi}^D \Big|_{3P_J^{[8]}} &= -0.346, & A_{\cos 2\psi}^D \Big|_{3P_J^{[8]}} &= 0.0220.
 \end{aligned} \tag{3.4}$$

As expected, the CS and CO channels can be well separated by measuring $A_{\cos\psi}^D$. From equation (3.4) we can see that the values of $A_{\cos\psi}^D$ for the $1S_0^{[8]}$ and $3P_J^{[8]}$ channels are quite different from that for the CS one, while the $3S_1^{[8]}$ and CS channels are not distinguishable by observing $A_{\cos\psi}^D$. Although the short-distance coefficient for the $3S_1^{[8]}$ channel is almost twice of that for the CS one, the cross section for this channel, however, is negligible due to a much smaller LDME which is suppressed by two orders of magnitude relative to the CS one. As a result, the experiment can distinguish the CS and the CO contribution. In addition, the $1S_0^{[8]}$ channel can be distinguished by studying $A_{\cos 2\psi}^D$.

Before utilizing results in experiment, we should first study the theoretical uncertainties brought about by the choices of the scales. When choosing $\mu_r = \mu_f = 0.5\sqrt{S(xy + \xi)}$, we obtain the following results,

$$\begin{aligned}
 \sigma_0^D \Big|_{\text{CS}} &= 6.76 \times 10^1 \times \frac{\langle \mathcal{O}^{J/\psi}(3S_1^{[1]}) \rangle}{M^3} \text{ pb}, & \sigma_0^D \Big|_{1S_0^{[8]}} &= 5.64 \times 10^3 \times \frac{\langle \mathcal{O}^{J/\psi}(1S_0^{[1]}) \rangle}{M^3} \text{ pb}, \\
 \sigma_0^D \Big|_{3S_1^{[8]}} &= 1.45 \times 10^2 \times \frac{\langle \mathcal{O}^{J/\psi}(3S_1^{[8]}) \rangle}{M^3} \text{ pb}, & \sigma_0^D \Big|_{3P_J^{[8]}} &= 8.86 \times 10^4 \times \frac{\langle \mathcal{O}^{J/\psi}(3P_0^{[8]}) \rangle}{M^5} \text{ pb},
 \end{aligned} \tag{3.5}$$

and

$$\begin{aligned}
 A_{\cos\psi}^D \Big|_{\text{CS}} &= 0.558, & A_{\cos 2\psi}^D \Big|_{\text{CS}} &= -0.0652, \\
 A_{\cos\psi}^D \Big|_{1S_0^{[8]}} &= -0.224, & A_{\cos 2\psi}^D \Big|_{1S_0^{[8]}} &= -0.290, \\
 A_{\cos\psi}^D \Big|_{3S_1^{[8]}} &= 0.527, & A_{\cos 2\psi}^D \Big|_{3S_1^{[8]}} &= -0.0548, \\
 A_{\cos\psi}^D \Big|_{3P_J^{[8]}} &= -0.348, & A_{\cos 2\psi}^D \Big|_{3P_J^{[8]}} &= 0.0214.
 \end{aligned} \tag{3.6}$$

For $\mu_r = \mu_f = 2\sqrt{S(xy + \xi)}$, we obtain

$$\begin{aligned}
 \sigma_0^D \Big|_{\text{CS}} &= 3.85 \times 10^1 \times \frac{\langle \mathcal{O}^{J/\psi}(3S_1^{[1]}) \rangle}{M^3} \text{ pb}, \\
 \sigma_0^D \Big|_{1S_0^{[8]}} &= 3.17 \times 10^3 \times \frac{\langle \mathcal{O}^{J/\psi}(1S_0^{[1]}) \rangle}{M^3} \text{ pb}, \\
 \sigma_0^D \Big|_{3S_1^{[8]}} &= 8.21 \times 10^1 \times \frac{\langle \mathcal{O}^{J/\psi}(3S_1^{[8]}) \rangle}{M^3} \text{ pb}, \\
 \sigma_0^D \Big|_{3P_J^{[8]}} &= 4.96 \times 10^4 \times \frac{\langle \mathcal{O}^{J/\psi}(3P_0^{[8]}) \rangle}{M^5} \text{ pb},
 \end{aligned} \tag{3.7}$$

and

$$\begin{aligned}
 A_{\cos\psi}^D \Big|_{\text{CS}} &= 0.564, & A_{\cos 2\psi}^D \Big|_{\text{CS}} &= -0.0660, \\
 A_{\cos\psi}^D \Big|_{1S_0^{[8]}} &= -0.227, & A_{\cos 2\psi}^D \Big|_{1S_0^{[8]}} &= -0.275, \\
 A_{\cos\psi}^D \Big|_{3S_1^{[8]}} &= 0.533, & A_{\cos 2\psi}^D \Big|_{3S_1^{[8]}} &= -0.0556, \\
 A_{\cos\psi}^D \Big|_{3P_J^{[8]}} &= -0.346, & A_{\cos 2\psi}^D \Big|_{3P_J^{[8]}} &= 0.0221.
 \end{aligned} \tag{3.8}$$

The cross sections change considerably when the scales vary. Interestingly, the azimuthal asymmetry modulations stay almost unchanged. This is because both σ_0 and the azimuthally dependent cross sections change along with the scales, and their effects cancel when computing their ratios. To this end, we do not concern the effects of the scale variation in the following of this paper. In addition to the undetermined values of the scales, QCD corrections can also affect our results. Fortunately, many studies [35, 68] reveal that just slight corrections appear at QCD NLO in the J/ψ production in ep collisions, which might not change the values of the azimuthal asymmetry modulations significantly. Hence, the following analysis is based on only the results obtained with the default setting of the scales.

On the experiment side, the J/ψ events are collected in bins of ψ . The azimuthal modulations thus can be extracted by fitting the data to equation (2.29), where linear regression is always used as a standard technique. Since the quantities that we are interested in are ratios, most of the systematic uncertainties are cancelled when doing the data analysis. As a result, we consider only the statistical uncertainty in the following. In our analysis, the range, $[0, 2\pi)$, of ψ is divided into 12 equidistant bins, namely $[0, \pi/6)$, $\dots$, $[11\pi/6, 2\pi)$. The integral over ψ of the three modulations, 1, $\cos(\psi)$, and $\cos(2\psi)$, in each bin are calculated, and make up three vectors each of which consists of 12 elements. Explicitly, they are

$$\begin{aligned}
 v_1 &= \left(\frac{\pi}{6}, \dots, \frac{\pi}{6} \right), \\
 v_{\cos\psi} &= \left(\frac{1}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}, -1 + \frac{\sqrt{3}}{2}, \frac{1}{2} - \frac{\sqrt{3}}{2}, -\frac{1}{2}, \right. \\
 &\quad \left. -\frac{1}{2}, \frac{1}{2} - \frac{\sqrt{3}}{2}, -1 + \frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}, \frac{1}{2} \right), \\
 v_{\cos 2\psi} &= \left(\frac{\sqrt{3}}{4}, 0, -\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}, 0, \frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{4}, 0, -\frac{\sqrt{3}}{4}, -\frac{\sqrt{3}}{4}, 0, \frac{\sqrt{3}}{4} \right).
 \end{aligned} \tag{3.9}$$

It is easy to verify that the three vectors are linearly independent and not correlated.

If the J/ψ leptoproduction is dominated by the CS mechanism, the integrated cross section in the region D can be expressed as

$$d\sigma^D \Big|_{\text{CS}} = 2.17 [1 + 0.561\cos(\psi) - 0.0658\cos(2\psi)] d\psi \text{ pb}, \tag{3.10}$$

where

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.16 \text{ GeV}^3 \tag{3.11}$$

is employed [75]. Assuming the integrated luminosity at the future ep collider is $\mathcal{L} = 10^3 \text{ pb}^{-1}$, we can construct the similar vector, as in the above paragraph, for this distribution as

$$v_d = (1683, 1582, 1361, 1035, 691, 466, \\ 466, 691, 1036, 1361, 1582, 1683), \quad (3.12)$$

each element of which is expectation of the number of events collected in the corresponding bin. This number, denoted as v_d^i , obey Gaussian distribution, the standard error of which can be obtained as the square root of v_d^i . Denoting the probability density function of the number of events in each bin as $f_i(N_i)$, $i = 1, \dots, 12$, we fit N_i to the following equation,

$$N_i = \mathcal{A}v_1^i + \mathcal{B}v_{\cos\psi}^i + \mathcal{C}v_{\cos 2\psi}^i. \quad (3.13)$$

We can then obtain the expectation values of $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$, for each combination of the values of N_i . These expectation values thus have a probability density $\prod_i f_i(N_i)$. Then we can calculate the expectation value of the coefficients in equation (3.2) for each combination of N_i , and the probability density function with respect to σ_0 , $A_{\cos\psi}$, and $A_{\cos 2\psi}$. It is easy to demonstrate that $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ also obey Gaussian distribution, and their expectation values and standard errors are $\overline{\mathcal{A}} = 2.17 \times 10^3$, $\sigma_{\mathcal{A}} = 18.6$, $\overline{\mathcal{B}} = 1.22 \times 10^3$, $\sigma_{\mathcal{B}} = 26.2$, and $\overline{\mathcal{C}} = -0.143 \times 10^3$, $\sigma_{\mathcal{C}} = 27.5$, respectively. Apparently, the expectation values obtained here agree with equation (3.10). The uncertainties for $A_{\cos\psi}$ and $A_{\cos 2\psi}$ can be obtained as

$$\Delta A_{\cos\psi} = 0.013. \quad (3.14)$$

If the CO parts are not negligible, we need to sum over their contributions. Employing the LDMEs obtained in References [55, 60], which are given below,

$$\begin{aligned} \langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle &= (0.645 \pm 0.405) \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle &= (0.78 \pm 0.34) \times 10^{-2} \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle &= (1.0 \pm 0.1) \times 10^{-2} \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle &= (3.8 \pm 0.1) \times 10^{-2} \text{ GeV}^5, \end{aligned} \quad (3.15)$$

the cross section can be expressed as

$$d\sigma^D \Big|_{NRQCD} = 12.7_{-1.4}^{+1.1} \left[1 - 0.246_{-0.031}^{+0.055} \cos(\psi) - 0.0155_{-0.0160}^{+0.0126} \cos(2\psi) \right]. \quad (3.16)$$

Note that, here, the uncertainties are obtained by extracting the maximum and minimum values of σ_0^D , $A_{\cos\psi}$, and $A_{\cos 2\psi}$, respectively, from those given by tentatively making use of all the combinations of the LDME boundary values. In this procedure, all the correlation effects are not concerned, for this reason, the errors in equation (3.16) are actually overestimated. The values of $A_{\cos\psi}$ can be read from equation (3.16) as $A_{\cos\psi} = -0.246_{-0.031}^{+0.055}$. It is easy to see that this number is well separated from that led to by the CS model in the sense of the uncertainty given in equation (3.16) and equation (3.14).

In the rest of this paper, we will demonstrate that the azimuthal asymmetry in the J/ψ leptonproduction can also distinguish the LDMEs that are consistent with the $^1S_0^{[8]}$ dominance picture. Taking those given in Reference [47] as an example, say

$$\begin{aligned}\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle &= 1.16 \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle &= (8.9 \pm 0.98) \times 10^{-2} \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle &= (0.3 \pm 0.12) \times 10^{-2} \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle &= (1.26 \pm 0.47) \times 10^{-2} \text{ GeV}^5.\end{aligned}\tag{3.17}$$

The cross section for the J/ψ production in the region D can be expressed as

$$d\sigma^D \Big|_{\text{Chao}} = 19.4_{-2.8}^{+2.8} [1 - 0.158_{-0.021}^{+0.015} \cos(\psi) - 0.203_{-0.021}^{+0.021} \cos(2\psi)] d\psi \text{ pb.} \tag{3.18}$$

Using the same analysis strategy, we can obtain

$$A_{\cos 2\psi} \Big|_{\text{Chao}} = -0.203 \pm 0.021 \pm 0.004, \tag{3.19}$$

where the first error corresponds to the uncertainty originating from those of the LDMEs and the second one is the statistical error. Since with the LDMEs given in equation (3.15), the value of $A_{\cos 2\psi}$ is $-0.0155_{-0.0160}^{+0.0126}$, it is obvious that one can distinguish the two sets of LDMEs by scrutinize the value of $A_{\cos 2\psi}$.

Another widely used set of the LDMEs is given in Reference [45] by global fit of the J/ψ yield data at e^+e^- , ep , hadron colliders. Their values of the LDMEs are given by

$$\begin{aligned}\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle &= 1.32 \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle &= (3.04 \pm 0.35) \times 10^{-2} \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle &= (0.168 \pm 0.046) \times 10^{-2} \text{ GeV}^3, \\ \langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle &= (-0.908 \pm 0.161) \times 10^{-2} \text{ GeV}^5,\end{aligned}\tag{3.20}$$

with which the cross section for the J/ψ production in the region D can be expressed as

$$d\sigma^D \Big|_{\text{Butenschoen}} = 4.74_{-0.98}^{+0.98} [1 + 0.249_{-0.090}^{+0.137} \cos(\psi) - 0.325_{-0.031}^{+0.047} \cos(2\psi)] d\psi \text{ pb.} \tag{3.21}$$

The value of $A_{\cos 2\psi}$ and its uncertainty can be obtained as

$$A_{\cos 2\psi} \Big|_{\text{Butenschoen}} = -0.325_{-0.031}^{+0.047} \pm 0.009, \tag{3.22}$$

which can also be distinguished from the LDMEs given in equation (3.15) and equation (3.17).

The above analyses demonstrate that the azimuthal asymmetry in the J/ψ leptonproduction provides a perfect laboratory to distinguish different mechanisms describing the J/ψ production.

4 Summary

In this paper, we calculated the azimuthal asymmetry modulations in the J/ψ lepton production, namely $A_{\cos\psi}$ and $A_{\cos 2\psi}$ defined in equation (2.27), as functions of x , y , z , and ξ . By scrutinizing their behaviours, we find that the CS and CO mechanisms can be well distinguished through the measurement of the values of $A_{\cos\psi}$. Further, the $^1S_0^{[8]}$ dominance picture can also be tested by measuring the values of $A_{\cos 2\psi}$. Restricted in the region, $0.001 < x < 1$, $0.04 < y < 0.6$, $0.75 < z < 0.9$, and $p_t > 1 \text{ GeV}$, we carried out the calculation of the differential cross sections with respect to the azimuthal angle for three models, and found that they lead to clearly different values of the azimuthal asymmetry modulations. Having applied rigorous statistical analysis, we found that at the integrated luminosity $\mathcal{L} = 1000 \text{ pb}^{-1}$, the statistical uncertainties of $A_{\cos\psi}$ and $A_{\cos 2\psi}$ are small enough to tell the three models apart. As a conclusion, the azimuthal asymmetry in the J/ψ lepton production provides a good laboratory for the study of the quarkonium production mechanisms. We suggest that this experiment be implemented at the future ep colliders such as the EIC.

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Note added. When our calculation was finished and the manuscript was being prepared for submission, we noticed an independent interesting paper [76] about the same topic. They calculated the same partonic processes as we did in this paper. However, there are differences between the two papers. Reference [76] employed the transverse-momentum-dependent (TMD) PDFs, while our calculations are carried out in collinear factorization. They utilize the azimuthal asymmetry to study the TMD PDFs while we consider it as an opportunity to distinguish different J/ψ production mechanisms. For this purpose, we performed an elaborative statistical analysis. They focused on $A_{\cos 2\psi}$ while our paper calculated not only $A_{\cos 2\psi}$ but also $A_{\cos\psi}$.

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