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Weyl R^2 inflation with an emergent Planck scale

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ABSTRACT: We study inflation in Weyl gravity. The original Weyl quadratic gravity, based on Weyl conformal geometry, is a theory invariant under the Weyl symmetry of *gauged* scale transformations. In this theory the Planck scale (M) emerges as the scale where this symmetry is broken spontaneously by a geometric Stueckelberg mechanism, to Einstein-Proca action for the Weyl “photon” (of mass near M). With this action as a “low energy” broken phase of Weyl gravity, century-old criticisms of the latter (due to non-metricity) are avoided. In this context, inflation with field values above M is natural, since this is just a phase transition scale from Weyl gravity (geometry) to Einstein gravity (Riemannian geometry), where the massive Weyl photon decouples. We show that inflation in Weyl gravity coupled to a scalar field has results close to those in Starobinsky model (recovered for vanishing non-minimal coupling), with a mildly smaller tensor-to-scalar ratio (r). Weyl gravity predicts a specific, narrow range $0.00257 \leq r \leq 0.00303$, for a spectral index n_s within experimental bounds at 68%CL and e-folds number $N=60$. This range of values will soon be reached by CMB experiments and provides a test of Weyl gravity. Unlike in the Starobinsky model, the prediction for (r, n_s) is not affected by unknown higher dimensional curvature operators (suppressed by some large mass scale) since these are forbidden by the Weyl gauge symmetry.

KEYWORDS: Cosmology of Theories beyond the SM, Models of Quantum Gravity, Spontaneous Symmetry Breaking

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1 Motivation

There is a renewed interest in studying scale invariant models for physics beyond Standard Model (SM) and cosmology. This symmetry may also be present at the quantum level [1–4]. All scales (including the scale of “new physics” beyond SM) are generated spontaneously by field vev’s and this symmetry may even preserve a classical hierarchy of scales [2–11]. In cosmology there are global [12–28] or local [29–43] scale invariant alternatives to gravity with spontaneous breaking that generates the Planck scale by a non-minimal coupling.

In this paper we study inflation in a theory with (gauged) local scale invariance. The theory considered is the original Weyl gravity [44–47], based on Weyl conformal geometry [48–50]. This symmetry is also referred to as Weyl gauge symmetry and its associated gauge boson is called hereafter Weyl “photon”. This theory has *no* fundamental scale (Planck scale, etc.), forbidden by its symmetry. Weyl action is [44–47]

$$L_0 = \sqrt{g} \left\{ \frac{\xi_0}{4!} \tilde{R}^2 - \frac{1}{4q^2} F_{\mu\nu}^2 \right\} \quad (1.1)$$

Each term is Weyl gauge invariant (see later, eq. (2.4)); $\tilde{R}$ is the scalar curvature of Weyl geometry; $F_{\mu\nu}$ is the field strength of the Weyl “photon” ω_μ , with coupling q . In addition to L_0 , one can also include an independent Weyl-tensor-squared term of Weyl geometry, allowed by the symmetry and considered later. A topological Gauss-Bonnet term of Weyl geometry may be present too, not relevant here. However, higher dimensional operators ($\tilde{R}^4$, etc.) cannot be present in L_0 since there is no scale to suppress them.

As recently shown in [41, 42] scale dependence in Weyl gravity (1.1) emerges spontaneously after a geometric version of Stueckelberg breaking mechanism [52] of Weyl gauge symmetry: the dilaton, which is the Goldstone mode of this symmetry (and spin 0 mode

propagated by $\tilde{R}^2$) is absorbed by the Weyl “photon” which thus becomes massive. Denoting by R the Ricci scalar of Riemannian geometry, action (1.1) becomes [41, 42]:

$$L_0 = \sqrt{g} \left\{ -\frac{1}{2} M^2 R - \frac{3 M^2}{2 \xi_0} - \frac{1}{4} F_{\mu\nu}^2 + \frac{3}{4} q^2 M^2 \omega_\mu \omega^\mu \right\}. \quad (1.2)$$

The Planck scale M is fixed by the dilaton vev and the Weyl “photon” acquired a mass $\sim qM$ near the Planck scale (for q not too small). Therefore the Einstein-Proca action (1.2) is just a “low-energy” broken phase of Weyl gravity: it is obtained after “gauge fixing” the Weyl gauge symmetry. This involves fixing the dilaton vev which in a FRW universe is a dynamical effect [53, 54]. Given its equivalence to action (1.2), Weyl action (1.1) avoids long-held criticisms since Einstein [45] related to non-metricity effects due to ω_μ (e.g. changing of atomic lines spacing) that can be safely ignored since ω_μ decouples near Planck scale M .

Having Einstein gravity as a “low-energy” limit of Weyl action motivates us to study inflation in Weyl gravity. Moreover, the presence of $\tilde{R}^2$ in (1.1) points to similarities to successful Starobinsky inflation [55]. And since Planck scale is just a phase transition scale, field values above M are natural in Weyl gravity. This is relevant for inflation where such values are common but harder to accept in models where Planck scale is the physical cutoff.

2 Weyl gravity and inflation

To study Weyl inflation we review briefly the action of Weyl gravity coupled to a scalar field ϕ_1 , see [41, 42] for a detailed analysis.¹ The Lagrangian is

$$L = \sqrt{g} \left[\frac{\xi_0}{4!} \tilde{R}^2 - \frac{1}{4 q^2} F_{\mu\nu}^2 \right] - \frac{\sqrt{g}}{12} \xi_1 \phi_1^2 \tilde{R} + \sqrt{g} \left[\frac{1}{2} g^{\mu\nu} \tilde{D}_\mu \phi_1 \tilde{D}_\nu \phi_1 - \frac{\lambda_1}{4!} \phi_1^4 \right], \quad (2.1)$$

with couplings ξ_0, ξ_1, λ_1 ; $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ (no torsion). $\tilde{D}_\mu$ is a Weyl-covariant derivative

$$\tilde{D}_\mu \phi_1 = (\partial_\mu - 1/2 \omega_\mu) \phi_1, \quad (2.2)$$

$$\tilde{R} = R - 3 D_\mu \omega^\mu - \frac{3}{2} \omega^\mu \omega_\mu, \quad (2.3)$$

with the scalar curvature of Weyl geometry ($\tilde{R}$) related to its Riemannian counterpart (R) as above. Lagrangian (2.1) is invariant under a Weyl gauge transformation $\Omega(x)$:

$$\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}, \quad \hat{\phi}_1 = \frac{1}{\sqrt{\Omega}} \phi_1, \quad \hat{\omega}_\mu = \omega_\mu - \partial_\mu \ln \Omega, \quad \Rightarrow \quad \hat{\tilde{R}} = \frac{1}{\Omega} \tilde{R}, \quad (2.4)$$

where ω_μ is the Weyl gauge field. We also have $\sqrt{\hat{g}} = \Omega^2 \sqrt{g}$, $g \equiv |\det g_{\mu\nu}|$, $\hat{g}^{\mu\nu} = \Omega^{-1} g^{\mu\nu}$, and metric $(+, -, -, -)$. Our conventions are those of [57].

Unlike Riemannian scalar curvature (R), $\tilde{R}$ computed from the (invariant) Weyl connection transforms covariantly, eq. (2.4), due to the inverse of the metric entering its definition. With this observation, one sees the advantage of Weyl formulation (i.e. using $\tilde{R}$) instead of the Riemannian language (using R) and why L is invariant under (2.4).

¹Unlike scalars, fermions do not couple to Weyl “photon” [35, 36, 43, 56] in the absence of torsion, as here.

One “linearises” (2.1) by using an auxiliary field ϕ_0 to replace $\tilde{R}^2 \rightarrow -2\phi_0^2 \tilde{R} - \phi_0^4$ and to obtain a classically equivalent action. Indeed the equation of motion for ϕ_0 has solution $\phi_0^2 = -\tilde{R}$, which when used back in the action recovers (2.1). Given this, ϕ_0 transforms as any other scalar field. Therefore

$$L = \sqrt{g} \left[-\frac{1}{4q^2} F_{\mu\nu}^2 - \frac{1}{12} (\xi_0 \phi_0^2 + \xi_1 \phi_1^2) \tilde{R} + \frac{1}{2} g^{\mu\nu} \tilde{D}_\mu \phi_1 \tilde{D}_\nu \phi_1 - \frac{1}{4!} (\lambda_1 \phi_1^4 + \xi_0 \phi_0^4) \right]. \quad (2.5)$$

Next, we must fix the gauge which we do by a particular Weyl “gauge-fixing” transformation (2.4) of $\Omega = \rho^2/M^2$ and $\rho = (1/6)(\xi_1 \phi_1^2 + \xi_0 \phi_0^2)$ with M some scale (see later). We find

$$L = \sqrt{\hat{g}} \left[-\frac{1}{2} M^2 \hat{R} + \frac{3}{4} M^2 \hat{\omega}_\mu \hat{\omega}^\mu - \frac{1}{4q^2} \hat{F}_{\mu\nu}^2 + \frac{\hat{g}^{\mu\nu}}{2} \hat{D}_\mu \hat{\phi}_1 \hat{D}_\nu \hat{\phi}_1 - \hat{V} \right], \quad (2.6)$$

with $\hat{R}$ the Riemannian scalar curvature and

$$\hat{V} = \frac{3M^4}{2\xi_0} \left[1 - \frac{\xi_1 \hat{\phi}_1^2}{6M^2} \right]^2 + \frac{\lambda_1}{4!} \hat{\phi}_1^4. \quad (2.7)$$

Therefore, as detailed in [41, 42], the Weyl “photon” has become massive via a Stueckelberg mechanism by “absorbing” the field $\ln \rho \sim \ln \Omega$, eq. (2.4), of the radial direction in the field space ϕ_0, ϕ_1 . This is the Goldstone (dilaton) mode since under (2.4) $\ln \rho$ has a shift symmetry $\ln \rho^2 \rightarrow \ln \rho^2 - \ln \Omega$. Weyl gauge symmetry is now spontaneously broken and Einstein gravity is recovered as a “low-energy” *broken phase* of Weyl gravity (this is consistent with the fact that Riemannian-based R^2 -gravity is equivalent to Einstein gravity plus a massless mode [17] eaten here by the Weyl “photon”). The scale of breaking and the mass of Weyl “photon” is $\sim qM$, with M identified with the Planck scale. M is itself fixed by the vev of the radial direction which in a FRW universe is fixed dynamically [53, 54] following a conserved current which drives the dilaton field to a constant value. The Planck scale and the mass of ω_μ are thus determined by the dilaton vev. Fields values above M are naturally allowed here since this is just a phase transition scale in the theory.

The Weyl-covariant derivative acting on $\hat{\phi}_1$ in (2.6) is a remnant of the initial Weyl gauge symmetry, now broken; one would like to decouple $\partial_\mu \hat{\phi}_1$ from ω^μ in order to study inflation; for a “standard” kinetic term for $\hat{\phi}_1$, a field redefinition is used

$$\hat{\omega}'_\mu = \hat{\omega}_\mu - \partial_\mu \ln \cosh^2 \frac{\sigma}{M\sqrt{6}}, \quad \hat{\phi}_1 = M\sqrt{6} \sinh \left[\frac{\sigma}{M\sqrt{6}} \right] \quad (2.8)$$

to find

$$L = \sqrt{\hat{g}} \left\{ -\frac{1}{2} M^2 \hat{R} + \frac{3}{4} M^2 \cosh^2 \left[\frac{\sigma}{M\sqrt{6}} \right] \hat{\omega}'_\mu \hat{\omega}'^\mu - \frac{1}{4q^2} \hat{F}_{\mu\nu}^2 + \frac{\hat{g}^{\mu\nu}}{2} \partial_\mu \sigma \partial_\nu \sigma - V \right\}. \quad (2.9)$$

Finally, one rescales above $\hat{\omega}'_\mu \rightarrow q\hat{\omega}'_\mu$, for a canonical gauge kinetic term. The potential is

$$V = V_0 \left\{ \left[1 - \xi_1 \sinh^2 \frac{\sigma}{M\sqrt{6}} \right]^2 + \lambda_1 \xi_0 \sinh^4 \frac{\sigma}{M\sqrt{6}} \right\}, \quad V_0 = \frac{3}{2} \frac{M^4}{\xi_0}. \quad (2.10)$$

V encodes the effect of the initial presence of the Weyl “photon”. The potential has a minimum due to the non-minimal gravitational coupling $\xi_1 > 0$ and this is relevant for inflation; we assume σ as the inflaton, with V its potential.

3 Results

V of (2.10) is largely controlled by ξ_1 and the combination $\lambda_1 \xi_0$. Its minimum is at:

$$\frac{\sigma_{\min}}{M\sqrt{6}} = \ln \left[\gamma + \sqrt{1 + \gamma^2} \right], \quad \gamma = \left[\frac{\xi_1}{\xi_1^2 + \xi_0 \lambda_1} \right]^{1/2}, \quad V_{\min} = \frac{3}{2} \frac{M^4 \lambda_1}{\lambda_1 \xi_0 + \xi_1^2}. \quad (3.1)$$

The potential is shown in figure 1 in function of the field σ , for different *perturbative* values of the non-minimal coupling ξ_1 , with two fixed values of the product $(\xi_0 \lambda_1)$. Note that:

- a) For $(\lambda_1 \xi_0)$ and ξ_1 small enough, V is constant $V \approx V_0 \sim 1/\xi_0$ and controlled by ξ_0 .
- b) Inflation begins in the region $V = V_0$ and lasts a number of e-folds that depends on the width of the flat region i.e. on the position of $\sigma_{\min} \propto \gamma$. If $\lambda_1 \xi_0 \ll \xi_1^2$ then $\gamma \sim 1/\sqrt{\xi_1}$ so reducing ξ_1 will extend the flat region.
- c) From the condition the initial energy be larger than at the end of inflation, $V_0 \gg V_{\min}$ then $\lambda_1 \xi_0 \ll \xi_1^2$ and also $V_{\min} \approx 0$, as seen from eq. (2.10) and the second plot in figure 1.

Constraints on the parametric space are found from the normalization of CMB anisotropy $V_0/(24\pi^2 M^4 \epsilon_*) = \kappa_0$, $\kappa_0 \equiv 2.1 \times 10^{-9}$ [58] where ϵ_* is the slow roll parameter. With tensor-to-scalar ratio $r = 16\epsilon_*$ and $r < 0.07$ [58] then $\xi_0 = 1/(\pi^2 r \kappa_0) \geq 6.89 \times 10^8$. In conclusion we have the parametric constraints

$$\lambda_1 \xi_0 \ll \xi_1^2, \quad \xi_0 \geq 6.89 \times 10^8. \quad (3.2)$$

A large ξ_0 is always compensated by choosing an ultraweak value of $\lambda_1 \ll \xi_1^2/\xi_0 \sim 10^{-9} \xi_1^2$ so eq. (3.2) is respected for a chosen ξ_1 (note the coupling of $\tilde{R}^2$ is $1/\xi_0$ and is in the perturbative regime). We shall use these constraints to predict the spectral index n_s and r .

The potential slow-roll parameters are

$$\epsilon = \frac{M^2}{2} \left\{ \frac{V'}{V} \right\}^2 = \frac{1}{3} \frac{\sinh^2(2\tilde{\sigma}) [-\xi_1 + (\lambda_1 \xi_0 + \xi_1^2) \sinh^2 \tilde{\sigma}]^2}{[1 - 2\xi_1 \sinh^2 \tilde{\sigma} + (\lambda \xi_0 + \xi_1^2) \sinh^4 \tilde{\sigma}]^2}, \quad \tilde{\sigma} \equiv \frac{\sigma}{M\sqrt{6}}, \quad (3.3)$$

and

$$\eta = M^2 \frac{V''}{V} = \frac{V_0}{3} \frac{(\lambda_1 \xi_0 + \xi_1^2) \cosh(4\tilde{\sigma}) - (2\xi_1 + \lambda_1 \xi_0 + \xi_1^2) \cosh(2\tilde{\sigma})}{1 - 2\xi_1 \sinh^2 \tilde{\sigma} + (\tilde{\lambda} + \xi_1^2) \sinh^4 \tilde{\sigma}}. \quad (3.4)$$

For $\lambda_1 \xi_0 \ll \xi_1^2$ and $\xi_1 \ll 1$, slow roll conditions are met, $\epsilon, \eta \ll 1$, as seen from a numerical analysis. Further, the number N of e-folds is

$$N = \frac{1}{M^2} \int_{\sigma_e}^{\sigma_*} d\sigma \frac{V(\sigma)}{V'(\sigma)} = \mathcal{N}(\sigma_*) - \mathcal{N}(\sigma_e) \quad (3.5)$$

with

$$\mathcal{N}(\sigma) = c_1 \ln \cosh \frac{\sigma}{M\sqrt{6}} + c_2 \ln \left[2(\lambda_1 \xi_0 + \xi_1^2) \sinh^2 \frac{\sigma}{M\sqrt{6}} - 2\xi_1 \right] + c_3 \ln \sinh \frac{\sigma}{M\sqrt{6}} \quad (3.6)$$

Figure 1. The potential $V(\sigma)$ for two fixed values of $\lambda_1\xi_0$ and with different ξ_1 . The flat region is wide for a large range of σ , with the width controlled by $\gamma \sim 1/\sqrt{\xi_1}$ while its height is $V_0 \propto 1/\xi_0$. We have $V/V_{\min} \propto \xi_1^2/(\lambda_1\xi_0)$.

Figure 2. The spectral index n_s versus r (left plot) and n_s versus N (right plot), for models with $\lambda_1\xi_0 = 10^{-10}$ and with different ξ_1 . N varies along each curve; in the left plot, on each curve of fixed ξ_1 , the dark-blue curve has $N = 55$ and $N = 65$ at its left and right ends, respectively. The red dots correspond to $N = 60$. The curves for $\xi_1 = 10^{-3}$ and $\xi_1 = 10^{-4}$ are nearly identical (saturated bound as $\xi_1 \rightarrow 0$) in both plots. Blue (light blue) regions correspond to n_s given by $n_s = 0.9670 \pm 0.0037$ at 68 % CL (95 % CL) respectively [58]. The constraint $r < 0.07$ is easily satisfied. The value of r is very small, below that of Starobinsky case: for $N = 60$ we have $r \leq 0.00303$ (from top curve, saturated bound as $\xi_1 \rightarrow 0$) and $r > 0.00257$ from the lower bound on n_s (68% CL) with $\xi_1 = 1.6 \times 10^{-2}$.

and

$$c_1 = \frac{3}{2} \frac{\lambda_1 \xi_0 + (1 + \xi_1)^2}{\xi_1 + \lambda_1 \xi_0 + \xi_1^2}, \quad c_2 = \frac{3 \lambda_1 \xi_0}{4 \xi_1 (\xi_1 + \lambda_1 \xi_0 + \xi_1^2)}, \quad c_3 = \frac{3}{-2 \xi_1}. \quad (3.7)$$

Above $\sigma = \sigma_*$ is the value at the horizon exit. Inflation ends at $\sigma = \sigma_e$ found from $\epsilon = 1$.

Eqs. (3.3) to (3.7) are used for a numerical study of the scalar spectral index n_s , the tensor-to-scalar ratio $r = 16 \epsilon_*$ and number of e-folds N , in terms of ξ_1 and $\lambda_1 \xi_0$. We have

$$n_s = 1 + 2 \eta_* - 6 \epsilon_*, \quad (3.8)$$

where the subscript $*$ stands for $\sigma = \sigma_*$.

Before discussing numerical results, we present simple analytical results for n_s and r in the limit $\lambda_1 \xi_0 \ll \xi_1^2 \ll 1$. Therefore, we can keep only the leading term in expansion in $(\lambda_1 \xi_0)$, then expand in ξ_1 . These results depend on ξ_1 only in this approximation. We find

$$\epsilon = \frac{1}{3} \xi_1^2 \sinh^2 \frac{2\sigma}{M\sqrt{6}} + \mathcal{O}(\xi_1^3) \quad (3.9)$$

$$\eta = -\frac{2}{3} \xi_1 \cosh \frac{2\sigma}{M\sqrt{6}} + \mathcal{O}(\xi_1^2) \quad (3.10)$$

valid up to $\mathcal{O}(\lambda_1 \xi_0)$ corrections. Therefore

$$n_s = 1 - \frac{4}{3} \xi_1 \cosh \frac{2\sigma_*}{M\sqrt{6}} + \mathcal{O}(\xi_1^2) \quad (3.11)$$

The value of n_s is controlled by η in leading order $\mathcal{O}(\xi_1)$ while ϵ contribution is sub-leading $\mathcal{O}(\xi_1^2)$; hence we have a small tensor-to-scalar ratio $r = 16 \epsilon_*$. We then have

$$r = 3 (1 - n_s)^2 + \mathcal{O}(\xi_1^2) \quad (3.12)$$

This is an approximate result (valid only for smallest ξ_1) for the top curve shown in the left plot in figure 2 (this figure actually uses the exact numerical results discussed later). As an example, if $n_s = 0.968$ then $r \approx 0.003$. Increasing ξ_1 should reduce n_s of (3.11) but there is implicit ξ_1 -dependence in σ_* , computed below. First, we have

$$\mathcal{N}(\sigma) = -\frac{3}{4 \xi_1} \ln \tanh^2 \frac{\sigma}{M\sqrt{6}} + \frac{3}{4} \ln \cosh^2 \frac{\sigma}{M\sqrt{6}} + \mathcal{O}(\xi_1^2) \quad (3.13)$$

Inflation ends at $\sigma = \sigma_e$ where

$$\sinh^2 \frac{2\sigma_e}{M\sqrt{6}} = \frac{3}{\xi_1^2} + \mathcal{O}(\xi_1). \quad (3.14)$$

For example: $(\sigma_e, \xi_1) = (12.8M, 10^{-4})$; $(7.16M, 10^{-2})$. Then from (3.13), (3.14)

$$\mathcal{N}(\sigma_e) = \left[\frac{\sqrt{3}}{2} + \frac{3}{8} \ln \frac{3}{4} \right] - \frac{3}{4} \ln \xi_1 + \frac{\sqrt{3}}{4} \xi_1 + \mathcal{O}(\xi_1^2). \quad (3.15)$$

Using this result we find an iterative solution σ_* to eq. (3.5) for a fixed N :

$$\tanh^2 \frac{\sigma_*}{M\sqrt{6}} \approx 1 - \frac{4\xi_1}{3} \overline{N}, \quad (3.16)$$

where

$$\bar{N} = \left\{ N + \mathcal{N}(\sigma_e) + \frac{3}{4} \ln \left[\frac{4\xi_1}{3} (N + \mathcal{N}(\sigma_e)) \right] \right\} + \mathcal{O}(\xi_1^2). \quad (3.17)$$

For $N = 60$ then $\bar{N} \approx 64.1$ for ξ_1 between 10^{-4} and 10^{-3} ; for $N = 55$ then $\bar{N} \approx 59.06$ for the same ξ_1 range. Finally, using σ_* of (3.16) and (3.9), (3.11), we have approximate relations for n_s and $r = 16 \epsilon_*$:

$$n_s = 1 - \frac{2}{\bar{N}} + \mathcal{O}(\xi_1) \quad (3.18)$$

$$r = \frac{12}{\bar{N}^2} + \frac{16}{\bar{N}} \mathcal{O}(\xi_1) \quad (3.19)$$

which are consistent with (3.12) and accurate for $\xi \sim 10^{-3}$ for the r values considered here.

Let us now analyse the exact numerical values of n_s , r , N , eqs. (3.3) to (3.8) and compare them with the experimental data. Figure 2 summarises the main results of the paper: we presented n_s versus r (left plot) and n_s versus N (right plot), for models with parametric constraint $\lambda_1 \xi_0 = 10^{-10}$ and curves of different ξ_1 . The curves for $\xi_1 = 10^{-3}$ and $\xi_1 = 10^{-4}$ or smaller are nearly identical (saturated bound) in both plots. N varies along each curve in the plane (n_s, r) , and the range of values from $N = 55$ to $N = 60$ is shown in dark blue for each curve. Regions in blue (light blue) show the experimental values of n_s at 68% (95% CL), respectively, where $n_s = 0.9670 \pm 0.0037$ (68% CL) from Planck 2018 (TT, TE, EE + low E + lensing + BK14 + BAO) [58]. These bounds on n_s and r are comfortably respected at 68% CL for the values of ξ_1 shown.

Let us compare figure 2 to a result of Starobinsky model in which for $N = 55$ one has $n_s \approx 0.965$ and $r \approx 0.0034$ [61]. In our model, for the same N we also have $r \approx 0.00345$ which is the largest r in the model for $N = 55$ (top curve in figure 2). Therefore the values for r of the Starobinsky model (recovered for $\xi_1 \rightarrow 0$), are at the upper limit of those in our model. This is also indicated by approximate results (3.18), (3.19) which also apply to Starobinsky model but with replacement $\bar{N} \rightarrow N$. With $\bar{N} > N$ we see that for the same N , we expect a mildly larger n_s and smaller r than in Starobinsky model.

Figure 2 for $N = 60$ gives a lower bound for r from the experimental (lower) value of n_s (at 68% CL) quoted above and with $\xi_1 = 1.6 \times 10^{-2}$; there is also an upper bound on r from the smallest ξ_1 (saturated limit, top red curve) also giving the largest n_s :

$$N = 60, \quad 0.00257 \leq r \leq 0.00303. \quad (3.20)$$

Similar bounds can be extracted for different N . To conclude, a small tensor-to-scalar ratio is predicted by this model. Such value will soon be tested experimentally [62–64].

Our predictions used a hierarchy of couplings $\lambda_1 \ll \xi_1 \ll \xi_0$. This hierarchy is stable under matter (scalar) quantum corrections to (2.1) due to ultraweak λ_1 required to satisfy (3.2): we have one-loop corrections $\delta\lambda_1 \propto \lambda_1^2/\kappa$; $\delta\xi_1 \propto (\xi_1 + 1/6)\lambda_1/\kappa$ and $\delta\xi_0 \propto (\xi_1 + 1/6)^2/\kappa$, with $\kappa = (4\pi)^2$, see e.g. [60] (section 3.1). Therefore the relative λ_1 suppression factor can maintain this hierarchy since then $|\delta\lambda_1| \ll |\delta\xi_1| \ll |\delta\xi_0|$. Therefore matter quantum corrections to r and n_s are small (for the Starobinsky-Higgs model these are well below 2.5% for r and less than 1σ for n_s for minimal values of λ_1 used here [60]).

4 Weyl-tensor corrections to Weyl inflation

The analysis so far was based on L_0 of (1.1) coupled to the inflaton. A most general Weyl quadratic gravity action can contain an additional term allowed by the symmetry [41, 42]

$$L_1 = \frac{1}{\eta} \tilde{C}_{\mu\nu\rho\sigma} \tilde{C}^{\mu\nu\rho\sigma} = \frac{1}{\eta} \left[C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{3}{2} F_{\mu\nu}^2 \right], \quad (4.1)$$

where $\tilde{C}_{\mu\nu\rho\sigma}$ ($C_{\mu\nu\rho\sigma}$) denotes the Weyl tensor of Weyl (Riemannian) geometry, respectively; in the second step we used an identity to rewrite the action in the Riemannian picture.

If present, L_1 modifies our previous results for r . The analysis proceeds as before, since the transformations applied to obtain eqs. (2.9), (2.10) from eq. (2.1) leave L_1 invariant. Therefore, the new Lagrangian is that of eq. (2.9) plus L_1 . For this new Lagrangian, the corrections to r induced by the Weyl tensor were studied in [65] to which we refer the reader for details. The Weyl tensor brings a quadratic action for tensor fluctuations and a modified sound speed c_t given by $1/c_t^2 - 1 = (2H^2)/(\eta M^2)$ [65] (see eqs. (2.10), (2.22), (2.24)). With a slow roll relation $H^2 \approx V/(3M^2)$, $V \approx V_0$, the corrected tensor-to-scalar ratio r_c is then

$$r_c = r \left[1 + \frac{8}{\eta \xi_0} \right]^{1/2}, \quad (4.2)$$

where r is the value in the absence of L_1 while n_s is unchanged. The Weyl boson mass is as in eq. (2.6) but with $q^2 \rightarrow q_c^2 = q^2/(1 - 6q^2/\eta) > 0$ where q_c is the new corrected coupling of the gauge kinetic term which includes from L_1 ; also $q_c^2 < 1$ if $\eta < 0$ or $\eta > 6q^2/(1 - q^2)$.

As expected, eq. (4.2) shows that the change of r due to L_1 depends on the relative size of the perturbative couplings of the two quadratic terms, η versus $1/\xi_0$, and equals $\theta = (r_c - r)/r \approx 4/(\eta \xi_0)$ for $|\eta| \xi_0 \gg 1$. Recalling that $\xi_0 > 6.89 \times 10^8$, a relative change by θ requires $\eta \approx 6 \times 10^{-9}/\theta$. A value $r_c < r$ ($r_c > r$) corresponds to a negative (positive) η , respectively. In the limit L_1 is absent (formally $|\eta| \rightarrow \infty$, q fixed) then $r_c = r$ and $q_c = q$ which recovers our previous results. In the limit the gauge kinetic term comes solely from (4.1) i.e. L_0 contains only the $\tilde{R}^2$ term (formally $q \rightarrow \infty$, η fixed) then one has $\eta = -6q_c^2 < 0$.

A significant change θ in absolute value of an already very small tensor-to-scalar ratio needs an ultraweak $|\eta|$ which induces an instability in the theory. This is because there is a spin-two ghost (or tachyonic) state in L_1 [16] of light (mass) $^2 \propto \eta M^2$, for small $|\eta|$. Avoiding this instability below Planck scale means corrections to r from L_1 are negligible.

5 Further remarks

A potential similar to that in section 3 was encountered in a related model [34] with Weyl local symmetry. What are the differences? In our model there is no torsion (Weyl connection coefficients symmetric [41, 42]) but we have Weyl gauge symmetry and non-metricity, while in the model of [34] only torsion is present. Further, in [34] the “gauge” field (denoted $\mathcal{T}_\mu$) is emerging from the trace over the torsion and replaces our ω_μ . However, an ansatz is made

$$\mathcal{T}_\mu = \partial_\mu \phi \quad (5.1)$$

with ϕ a scalar field. Under this assumption the model is Weyl integrable i.e. Riemannian (see e.g. [59]) and non-metricity is absent. Due to (5.1) the gauge kinetic term of $\mathcal{T}_\mu$ is vanishing (no dynamics) and can be integrated out. Therefore, no geometrical Stueckelberg mass mechanism can take place. For this reason a flat (Goldstone) direction remains present in [34] and it has kinetic mixing with the inflaton. The results then depend on the dynamics of the flat direction. This has implications for inflation discussed in [34], where it is shown that a distinct field space geometry changes the slow-roll plateau, which can affect inflation. If the kinetic energy of the Goldstone is large it can dominate and a “kination” period predates the slow-roll inflation; this may have additional consequences (observable effects in the CMB on large angular scales) [34]. In our case there is no Goldstone (flat direction) left since it was eaten by the Weyl “photon” which becomes massive via Stueckelberg mechanism and eventually decouples, yet it impacts on the potential (compare (2.10) to (2.7)).²

Weyl inflation has an advantage compared to the Starobinsky model in that it cannot contain higher dimensional/curvature operators like $\tilde{R}^4/M^4$, etc. of unknown coefficients that can affect significantly the numerical predictions or the convergence of such an expansion (in powers of $\tilde{R}$), see [69] for a discussion. Unlike in the Starobinsky model, such higher dimensional operators and their corrections are forbidden by the Weyl gauge symmetry. One could think of such operators being suppressed instead by (powers of) the dilaton field (to preserve this symmetry) but this is not possible since this field was already “eaten” by the Weyl massive “photon”. Further, given the Weyl gauge symmetry, the model is allowed by black-hole physics, which is not the case of similar models of inflation with only *global* scale symmetry (global charges can be eaten by black holes which subsequently evaporate, e.g. [68]). Finally, compared to models of inflation with local scale invariance (no gauging) that have a ghost present when generating Planck scale spontaneously by the dilaton vev,³ this problem is not present in Weyl gravity action eq. (2.1).

6 Conclusions

We examined if the original Weyl quadratic gravity is suitable for inflation. This theory is based on Weyl conformal geometry and its gauged local scale symmetry (also called Weyl gauge symmetry) forbids the presence of any fundamental mass scale in the action. Its action undergoes spontaneous breaking via geometric Stueckelberg mechanism. In this way the Weyl “photon” of gauged dilatations becomes massive (mass $\sim qM$), after absorbing the Goldstone mode (compensator/dilaton) $\ln \phi_0$ which is the spin-0 mode propagated by the $\tilde{R}^2$ term in the action. The result in the broken phase is the Einstein-Proca action for the Weyl “photon” and a positive cosmological constant. If the initial action also has a non-minimal coupling ξ_1 to an additional scalar (ϕ_1), a scalar potential is found after the Stueckelberg mechanism. This potential has a minimum for non-vanishing scalar vev that

²Non-metricity effects from ω_μ are suppressed by its large mass qM , for q perturbative, not too small. Current non-metricity bounds [66, 67] are as low as TeV, but depend on the model details. The fermions in our model do not couple to Weyl “photon” [35, 36, 43] and may evade these constraints even for ultraweak q .

³See for example [37, 43] for a discussion and references.

is triggered by the gravitational effects (non-minimal coupling) and is suitable for inflation. Since the Planck scale is *emergent* as the scale where Weyl gauge symmetry is broken, the presence of field values above this scale, needed for inflation, is actually natural in Weyl gravity. Moreover, the existence of a non-zero vev of the dilaton (fixing M) is actually a dynamical effect in a FRW universe. The study is also motivated by the fact that the action involves the square of the (Weyl) scalar curvature, which points to similarities to the successful Starobinsky model.

Our analysis shows that Weyl inflation predicts a specific, small tensor-to-scalar ratio (r) within a narrow range $0.00257 \leq r \leq 0.00303$ for $N = 60$ and with n_s within 68% (CL) of the experimental value. This range of values for r will soon be tested experimentally; they are mildly smaller than those for same N in the Starobinsky model $M^2 R + R^2$ recovered in the limit of vanishing non-minimal coupling. Such value for r is also an indirect test of the presence of the Weyl gauge symmetry.

Compared to the Starobinsky model, the Weyl model of inflation has the advantage that it does not contain higher order curvature terms (e.g. effective operators R^4/M^4 , etc.) that modify the predictions or question the convergence of a series expansion in curvature; such operators are forbidden in Weyl inflation by the underlying Weyl gauge symmetry. This is because this symmetry does not allow a mass scale be present in the Weyl action to suppress such operators, while the dilaton field that could in principle suppress them (while preserving this symmetry) was already “eaten” by the massive Weyl photon. Another advantage is that the Weyl gauge symmetry of this model is also allowed by black-hole physics, unlike the models with a *global* scale symmetry, while local scale invariant models (no gauging) have a notorious ghost dilaton present, when generating the Planck scale spontaneously (by the dilaton vev). Finally, the above predictions for r and the spectral index n_s are found for values of the non-minimal coupling ξ_1 in the perturbative regime. In this respect the situation is very different from Higgs inflation where a large coupling ξ_1 is required.

Note added. While this work was being typewritten, preprint [arXiv:1906.03415](#) appeared (10 June) which analyses (section 4) inflation in this model by using the two-field basis eq. (2.5) instead of our one-field formulation, eq. (2.10). The results are consistent and complementary.

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