Monopoles, strings, and necklaces in $SO(10)$ and E_6

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ABSTRACT: We employ a variety of symmetry breaking patterns in $SO(10)$ and E_6 Grand Unified Theories to demonstrate the appearance of topological defects including magnetic monopoles, strings, and necklaces. We show that independent of the symmetry breaking pattern, a topologically stable superheavy monopole carrying a single unit of Dirac charge as well as color magnetic charge is always present. Lighter intermediate mass topologically stable monopoles carrying two or three quanta of Dirac charge can appear in $SO(10)$ and E_6 models respectively. These lighter monopoles as well as topologically stable intermediate scale strings can survive an inflationary epoch. We also show the appearance of a novel necklace configuration in $SO(10)$ broken to the Standard Model via $SU(4)_c \times SU(2)_L \times SU(2)_R$. It consists of $SU(4)_c$ and $SU(2)_R$ monopoles connected by flux tubes. Necklaces consisting of monopoles and antimonopoles joined together by flux tubes are also identified. Even in the absence of topologically stable strings, a monopole-string system can temporarily appear. This system decays by emitting gravity waves and we provide an example in which the spectrum of these waves is strongly peaked around 10^{-4} Hz with $\Omega_{\text{gw}} h^2 \simeq 10^{-12}$. This spectrum should be within the detection capability of LISA.

KEYWORDS: Solitons Monopoles and Instantons, Gauge Symmetry, String theory and cosmic strings

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1 Introduction

Grand Unified Theories (GUTs) such as $SU(4)_c \times SU(2)_L \times SU(2)_R$ (422, for short) [1], $SU(5)$ [2], $SO(10)$ [3, 4], and E_6 [5–7] predict the existence of topologically stable magnetic monopoles [8, 9]. The mass and the magnetic charge carried by the monopoles depends on the underlying GUT and its symmetry breaking pattern. For instance, in breaking $SU(5)$ to the Standard Model (SM) gauge group, the lightest monopole carries one unit of Dirac magnetic charge (and also color magnetic charge) [10, 11], and it weighs about ten times the GUT scale M_{GUT} . In contrast, a 422 breaking to the SM yields a stable monopole with two units of Dirac charge [12], and its mass depends on the scale of the 422 breaking which can be lower, even significantly so, than the standard GUT scale $M_{\text{GUT}} \sim 10^{16}$ GeV. Another interesting example of GUT scale and lighter monopoles comes from E_6 breaking via the trinification group $SU(3)_c \times SU(3)_L \times SU(3)_R$ (333, for short). This breaking produces a GUT scale Z_3 monopole that carries one unit of Dirac magnetic charge [13–17], as we shall verify later. The subsequent breaking of 333 to the SM gauge group yields a stable intermediate mass monopole which carries three quanta of Dirac magnetic charge [13–17].

The presence of topologically stable strings in these models depends on the Higgs fields that are employed to implement the symmetry breaking. A prime example is the appearance of Z_2 strings if $SO(10)$ is broken to the SM using only tensor representations [18]. The gauge Z_2 symmetry in this case happens to be subgroup of the Z_4 center of $SO(10)$. In supersymmetric $SO(10)$ this Z_2 is precisely equivalent to matter parity which, among other things, provides a stable cold dark matter candidate, namely the lightest sparticle.

Composite topological defects can also appear in many GUTs and some well-known examples include monopole-antimonopole pairs connected by a string (dumbbells) [19, 20], walls bounded by strings [21], and necklaces with monopoles acting as beads kept together on a string [22]. Consider, for instance, the breaking of $SO(10)$ to 422 with a **54**-plet of Higgs. This leaves unbroken a discrete symmetry, called C -parity, which interchanges the

left and right components of any representation, accompanied by charge conjugation [21]. Under C the electric charge operator $Q \rightarrow -Q$ [21, 23]. This breaking of $SO(10)$ to 422 yields Z_2 strings. However, the subsequent breaking of the 422 symmetry to the SM group necessarily breaks this C -parity, and the strings form boundaries of domain walls [21]. Such walls can be tolerated in realistic scenarios provided they are unstable and disappear before their energy density becomes the dominant component in the universe. Another well known option, if available, is to inflate away the domain walls. It is interesting to note that observation of walls bounded by strings in ^3He has been reported recently in ref. [24]. An example of a necklace made up of monopoles and antimonopoles connected by a Z_2 string is provided by the symmetry breaking $SO(10) \rightarrow SU(5) \times U(1) \rightarrow SU(5) \times Z_2$ where the last step is achieved by a Higgs **126**-plet of $SO(10)$. We will demonstrate the appearance of a new type of necklace if $SO(10)$ breaking occurs via 422.

Of great interest, of course, is the question as to whether any of these primordial topological defects exist in nature, having either survived inflation or making an appearance after the inflationary epoch. It has been recognized [25, 26] for some time that monopoles associated with an intermediate scale M_I that is comparable to H , the Hubble scale during inflation, may be present in our galaxy at an observable level. This can come about if the number of e -foldings experienced during the intermediate scale phase transition is around 25-30, rather than the 50-60 e -foldings experienced by the GUT scale phase transition. Intermediate scale cosmic strings, on the other hand, can appear either in the same way as the monopoles, or even after the end of inflation. The current bound from Cosmic Microwave Background Radiation measurements on the dimensionless string tension is given by $G\mu_s \lesssim 3.2 \times 10^{-7}$ [27], where G denotes Newton's constant and μ_s is the mass per unit length of the string. Somewhat more stringent constraints on $G\mu_s$ based on pulsar timing observations have been reported in ref. [28].

In this paper, we discuss topological defects in GUTs, with emphasis on $SO(10)$ and E_6 (see also refs. [29, 30] for a recent discussion on related topics). In section 2 we show the presence of a GUT monopole carrying one unit of Dirac magnetic charge in $SO(10)$ models, which is independent of the symmetry breaking pattern. Analogous to the $SU(5)$ case, this monopole carries some color magnetic charge. We break the $SU(4)_c \times SU(2)_L \times SU(2)_R$ symmetry to the SM in two steps and show how an intermediate mass monopole carrying two units of the Dirac charge (Schwinger monopole) emerges from a coalescence of $SU(4)_c$ and $SU(2)_R$ monopoles bound together by flux tubes in a dumbbell configuration. This symmetry breaking pattern of 422 also yields a new type of necklace configuration consisting of alternating $SU(4)_c$ and $SU(2)_R$ monopoles connected by suitable flux tubes. A variety of other configurations is also possible including a necklace made of monopoles and antimonopoles connected by a Z_2 string. In section 3 we show the presence of the GUT Dirac monopole also in E_6 models and discuss the E_6 breaking via 333, which leads to intermediate scale monopoles with three units of Dirac magnetic charge and possibly to non-superconducting stable strings. In section 4 we analyze the E_6 breaking via $SO(10) \times U(1)_\psi$ and show how unstable strings as well as stable strings or necklaces can appear. Section 5 presents a quantitative discussion of how intermediate scale monopoles, strings, and necklaces in realistic models can survive primordial inflation. In addition we

discuss how gravity waves emitted by some defects may be accessible with the space based observatory LISA. Our conclusions are summarized in section 6.

2 SO(10) breaking via $SU(4)_c \times SU(2)_L \times SU(2)_R$

We will first study the breaking of SO(10) via 422 [31]. The **210** representation of SO(10) is contained in $\mathbf{16} \times \overline{\mathbf{16}} = \mathbf{1} + \mathbf{45} + \mathbf{210}$, and so the 422 singlet in **210** comes from $(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \times$ its conjugate and $(\mathbf{4}, \mathbf{2}, \mathbf{1}) \times$ its conjugate. One combination of these singlets gives the SO(10) singlet, and the other the 422 singlet in **210**. The latter is the antisymmetric combination of these singlets and thus breaks the discrete C -parity which interchanges $SU(2)_L$ and $SU(2)_R$ and conjugates $SU(4)_c$ (C -parity, first found in ref. [21], was later called D -parity in ref. [32]). This is clear since the SO(10) singlet cannot break C , which belongs to SO(10), and thus it is bound to be the symmetric combination. On the other hand, the 422 singlet in the **54**-plet of SO(10) comes from the product $\mathbf{10} \times \mathbf{10} = \mathbf{1}_s + \mathbf{45}_a + \mathbf{54}_s$. Thus it originates from $(\mathbf{1}, \mathbf{2}, \mathbf{2}) \times (\mathbf{1}, \mathbf{2}, \mathbf{2})$ or $(\mathbf{6}, \mathbf{1}, \mathbf{1}) \times (\mathbf{6}, \mathbf{1}, \mathbf{1})$, which are both symmetric under C . One combination of them is the SO(10) singlet and the orthogonal combination is contained in **54**, and so the **54**-plet does not break the discrete symmetry C [21].

We choose here to employ a Higgs **210**-plet for the SO(10) breaking to 422 so that no strings or subsequent walls bounded by strings are generated as in ref. [21]. It is known [12] that the $(-1, -1, -1)$ element of 422 coincides with the identity, and therefore three lines, one in each of the three groups between 1 and -1 constitute a closed loop, which corresponds to a magnetic monopole. We will now show that this monopole evolves to the Dirac monopole after the electroweak symmetry breaking. It carries one unit of magnetic charge as well as some color magnetic charge. (This conclusion appears to be in disagreement with table III in ref. [30] where it is stated that the monopole is unstable.)

To make the analysis more transparent, we take the curve in $SU(4)_c$ along its $X \equiv (B-L) + 2T_c^8/3$ generator, where $T_c^8 = \text{diag}(1, 1, -2)$ in $SU(3)_c$ and B and L are the baryon and lepton number operators respectively. This choice is certainly equivalent to taking the curve along the generator $B-L$ since color is unbroken. It is easy to see that $X = \text{diag}(1, 1, -1, -1)$ in $SU(4)_c$ and the curve between 1 and -1 corresponds to a rotation by π along this generator. In $SU(2)_L$ and $SU(2)_R$, we take rotations by π along $T_L^3 = \text{diag}(1, -1)$ and $T_R^3 = \text{diag}(1, -1)$ respectively, and the overall loop therefore corresponds to a rotation by 2π along $(B-L)/2 + T_c^8/3 + T_L^3/2 + T_R^3/2 = Q + T_c^8/3$ (Q is the electric charge operator). It is clear that this rotation brings us back to the identity element. Indeed, the group element $\exp(i2\pi T_c^8/3) = \exp(2i\pi/3)$ lies in the center of $SU(3)_c$ and $\exp(2i\pi Q) = \exp(4i\pi/3)$ acting on up-type quarks or $\exp(-2i\pi/3)$ acting on down-type quarks and so the combined rotation leads to the identity element. The magnetic monopole corresponding to a rotation by 2π along the generator $Q + T_c^8/3$ is exactly the Dirac magnetic monopole as previously shown in refs. [10, 11]. Note that, in this paper, the SM was embedded in SU(5), but the argument holds for any compact group containing SU(5). The Dirac magnetic monopole, along with the ordinary magnetic field, also carries color magnetic field.

The next breaking of 422 to $SU(3)_c \times SU(2)_L \times U(1)_{B-L} \times U(1)_R$ will generate monopoles corresponding to rotations by 2π in $SU(4)_c$ and $SU(2)_R$ along X and T_R^3 respectively. This

breaking can be achieved by the vacuum expectation value (VEV) of a Higgs **45**-plet of $\text{SO}(10)$ along its $(\mathbf{1}, \mathbf{1}, \mathbf{3})$ and $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ components. The $(\mathbf{15}, \mathbf{1}, \mathbf{1})$ VEV could also be taken from a Higgs **210**-plet.

We can further break these two $\text{U}(1)$'s by the VEV of the ν^c -type SM singlet component in a Higgs **16**-plet, which leaves $X + T_R^3$ unbroken (ν^c represents right-handed neutrinos). To find the broken generator which is perpendicular to $X + T_R^3$, we must define the GUT normalized generators

$$Q_X = \frac{1}{2\sqrt{2}}X, \quad Q_R = \frac{1}{2}T_R^3. \quad (2.1)$$

Then the normalized unbroken and broken generators $\mathcal{U}$ and $\mathcal{B}$ are, respectively,

$$\begin{aligned} \mathcal{U} &= \frac{1}{\sqrt{3}}(\sqrt{2}Q_X + Q_R), \\ \mathcal{B} &= \frac{1}{\sqrt{3}}(Q_X - \sqrt{2}Q_R) = \frac{1}{2\sqrt{6}}(X - 2T_R^3). \end{aligned} \quad (2.2)$$

The smallest broken generator with integral charges so that its periodicity is 2π is $X - 2T_R^3$. A rotation along this generator by $2\pi/3$ is left unbroken by the VEV of the ν^c -type Higgs. Therefore, the generated string contains magnetic flux corresponding to a rotation by $2\pi/3$ along $(X - 2T_R^3)$. The magnetic fluxes of an $\text{SU}(4)_c$ and an $\text{SU}(2)_R$ monopole have to be rearranged in tubes with flux $(X - 2T_R^3)/3$ and Coulomb fluxes along the unbroken generator $X + T_R^3$. To this end, an $\text{SU}(4)_c$ monopole, which carries a full flux along X , sends $1/3$ of it to an $\text{SU}(2)_R$ monopole which carries a full T_R^3 flux. This latter monopole, in turn, sends $2/3$ of its flux to the other one and thus a tube is generated between them which pulls them together. The rest of the fluxes are added together to give the Coulomb flux of a doubly charged (Schwinger) monopole — see figure 1. Note that the $1/3$ of the X flux sent from the $\text{SU}(4)_c$ monopole towards the $\text{SU}(2)_R$ monopole to contribute to the tube in between cannot terminate on it but emerges as Coulomb flux from it. The same is true for the $2/3$ of T_R^3 flux sent from the $\text{SU}(2)_R$ monopole to the $\text{SU}(4)_c$ monopole. Finally, we have four fluxes (two corresponding to rotations by $4\pi/3$ and $2\pi/3$ along X , and two corresponding to rotations by $2\pi/3$ and $4\pi/3$ along T_R^3), combined together to emerge as Coulomb flux from the combined monopole. This monopole corresponds to a full (2π) rotation along $X + T_R^3$ and becomes a Schwinger monopole after the electroweak breaking. Needless to say the $\text{SU}(4)_c$ or $\text{SU}(2)_R$ monopoles can be connected by a string to their respective antimonopoles and annihilate.

Note that

$$\exp\left\{i\frac{2\pi}{3}(X - 2T_R^3)\right\} = \exp\left\{i\frac{2\pi}{3}(X + T_R^3)\right\} \times \exp(-i2\pi T_R^3) = \exp\left\{i\frac{2\pi}{3}(X + T_R^3)\right\}, \quad (2.3)$$

and thus this unbroken element belongs to the unbroken continuous subgroup, i.e. the SM group. Consequently, no unbroken discrete symmetry is left, which means that no topologically stable strings are produced since the first homotopy (fundamental) group of the vacuum manifold

$$\pi_1\left(\frac{\text{SO}(10)}{\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y}\right) = \pi_0(\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y) = \{1\}. \quad (2.4)$$

We only have dumbbells [19, 20] which can transform into Schwinger monopoles.

Figure 1. Emergence of (Schwinger) magnetic monopole with two units of Dirac charge from the symmetry breaking $SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$. This monopole also carries color magnetic charge. An $SU(4)_c$ (red) and an $SU(2)_R$ (blue) monopole are connected by a flux tube which pulls them together to form a Schwinger monopole. The magnetic flux along the tube and the Coulomb magnetic fluxes of the monopoles are indicated. Intermediate mass monopoles such as this one may survive inflation.

If we inflate away the $SU(4)_c$ and $SU(2)_R$ monopoles, we can have a network of topologically non-stable strings. After the electroweak breaking, the Higgs doublets h_u , h_d (h_u couples to the up-type quarks and h_d to the down-type ones) with $X = 0$ and $T_R^3 = 1, -1$, $T_L^3 = -1, 1$ respectively develop VEVs. As we circle a string they get a phase $-4\pi/3, 4\pi/3$ respectively. If we add to the string $1/3$ of flux along T_L^3 so that the string corresponds to a rotation by $2\pi/3$ along $X - 2T_R^3 + T_L^3$, the phases of h_u and h_d change by -2π and $+2\pi$ respectively around the string. Of course, this addition does not affect the ν^c -type VEV of the Higgs **16**-plet and also adds the minimal necessary magnetic energy on the string. For definiteness, we will assume throughout that the magnetic energy dominates over the Higgs contribution to the string energy and so these strings are superconducting [33]. We obtain left-moving and right-moving fermionic zero modes along the string via h_u , h_d which are the only Higgs fields coupling to quarks and charged leptons. Note that the ν^c -type Higgs field couples only to right-handed neutrinos and thus does not contribute to superconductivity.

Figure 2. Necklace with $SU(4)_c$ and $SU(2)_R$ monopoles from the symmetry breaking $SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, where the last step is achieved by a **126**-plet of $SO(10)$. Notation as in figure 1. We display explicitly only the Coulomb magnetic flux of two of the monopoles and the magnetic flux along one of the tubes. This necklace may survive inflation.

Now suppose that we use the $\nu^c \nu^c$ -type component of **126** to do the breaking of $X - 2T_R^3$. In this case, a rotation by $2\pi/6$ along $X - 2T_R^3$ leads to an unbroken element. This yields a string which contains magnetic flux corresponding to a rotation by $2\pi/6$ along X minus flux corresponding to a rotation by $2\pi/3$ along T_R^3 . An $SU(4)_c$ and an $SU(2)_R$ monopole are then connected by two such strings with the remaining Coulomb flux in them being $(X + T_R^3)/3$ and $2(X + T_R^3)/3$. Now if one imagines opening up one of the two strings, one finds the two monopoles connected by one string and two “loose” strings emerging from the two monopole. One can then connect these latter strings to other similar monopole-string structures in series and form necklaces [22] — see figure 2. Note that pairs of $SU(4)_c$ and $SU(2)_R$ antimonopoles connected by a string can also participate in the necklace with the $SU(4)_c$ antimonopole connected either to an $SU(4)_c$ monopole or an $SU(2)_R$ antimonopole, and the $SU(2)_R$ antimonopole connected either to an $SU(2)_R$ monopole or $SU(4)_c$ antimonopole. Also both tubes emerging from an $SU(4)_c$ monopole ($SU(2)_R$ antimonopole) can terminate on $SU(4)_c$ antimonopoles ($SU(2)_R$ monopoles). We thus see that a variety of necklaces can appear with different arrangements of $SU(4)_c$ and $SU(2)_R$ monopoles and antimonopoles.

The group element

$$\begin{aligned} \exp \left\{ i \frac{2\pi}{6} (X - 2T_R^3) \right\} &= \exp \left\{ i \frac{2\pi}{6} (X + T_R^3) \right\} \times \exp \left(-i \frac{2\pi}{2} T_R^3 \right) \\ &= \exp \left\{ i \frac{2\pi}{6} (X + T_R^3) \right\} (1, 1, -1), \end{aligned} \quad (2.5)$$

which we obtain by circling one of these strings does not belong to the SM group since its action on the SM singlet ν^c yields $\exp(i\pi) = -1$. Moreover, since $(1, 1, -1) = (-1, -1, 1)$ and $SU(2)_L$ is unbroken at this stage, this element is equivalent to the generator of the Z_2 subgroup of $U(1)_{B-L}$ [18]. Its square is then obviously equivalent to the identity, and an extra Z_2 symmetry remains unbroken. Stable Z_2 strings without monopoles on them are also present. These strings, exactly like the ones in the necklaces above, correspond to a rotation by $2\pi/6$ along $X - 2T_R^3$ and are not oriented. The necklaces are themselves Z_2 strings too.

Next let us see what happens after the electroweak symmetry breaking. Recall that the Higgs doublets h_u, h_d have $X = 0$ and $T_R^3 = 1, -1$, $T_L^3 = -1, 1$ respectively. As we go around the string they acquire a phase $-2\pi/3, +2\pi/3$ respectively. If we add to the string $-1/3$ of flux along T_L^3 such that the string corresponds to a rotation by $2\pi/6$ along $X - 2T_R^3 - 2T_L^3$, h_u, h_d remain constant around the string. Of course, this addition does not affect the $\nu^c \nu^c$ -type VEV of **126** and also adds the minimal necessary magnetic energy along the string. Thus, only the $\nu^c \nu^c$ -type component of **126** changes phase around the string. But this couples only to right-handed neutrinos and so these strings are not superconducting.

For an example of a monopole-antimonopole necklace formed with a Z_2 string, consider the following $SO(10)$ breaking pattern: $SO(10) \rightarrow SU(4)_c \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$. The first breaking, achieved by the VEVs of a **210**-plet and a **45**-plet along their **(1,1,1)** and **(1,1,3)** components respectively, produces a GUT monopole with one unit of Dirac magnetic charge, which presumably is inflated away. Of course, multiply charged monopoles may also be produced and inflated away. In particular, the doubly charged monopole coincides with the $SU(2)_R$ monopole we mentioned above since the corresponding loops in $SU(4)_c$ and $SU(2)_L$ are homotopically trivial. The second breaking, achieved by the VEV of the **(15,1,1)** component of a Higgs **45**-plet, yields an intermediate scale $SU(4)_c$ monopole which carries both $SU(3)_c$ and $U(1)_{B-L}$ magnetic fluxes. The last breaking is done by the $\nu^c \nu^c$ -type component of a Higgs **126**-plet and the $SU(4)_c$ monopoles can form, together with the antimonopoles, a necklace tied together by a Z_2 string. Namely, an $SU(4)_c$ monopole, which carries a full magnetic flux along X , rearranges its magnetic field to form two tubes with flux $(X - 2T_R^3)/6$ and a Coulomb field around it with flux $2(X + T_R^3)/3$. Since the $SU(2)_R$ monopoles are inflated away in this case, these tubes can only terminate on $SU(4)_c$ antimonopoles — see figure 3.

3 E_6 breaking via $SU(3)_c \times SU(3)_L \times SU(3)_R$

E_6 can break to the trinification group $SU(3)_c \times SU(3)_L \times SU(3)_R$ (333, for short) by the VEV of a Higgs **650**-plet, which contains two 333 singlets. One of them breaks C -parity,

Figure 3. Necklace with $SU(4)_c$ monopoles (red) and antimonopoles (green) from the symmetry breaking $SO(10) \rightarrow SU(4)_c \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, where the last step is achieved by a **126**-plet of $SO(10)$. We assume that the monopoles from the first step of symmetry breaking are inflated away. We display explicitly only the Coulomb magnetic flux of one monopole and one antimonopole and the magnetic flux along one of the tubes. This necklace may survive inflation.

but the other one does not. Note that the symmetry C , in this case, exchanges $SU(3)_L$ and $SU(3)_R$ and conjugates the representation, in which case $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$ goes to itself, while $(\mathbf{3}, \mathbf{3}, \mathbf{1})$ and $(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}})$ are interchanged. There are three 333 singlets in the product

$$\mathbf{27} \times \bar{\mathbf{27}} = \mathbf{1} + \mathbf{78} + \mathbf{650}. \quad (3.1)$$

They are the $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) \times (\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}})$, $(\mathbf{3}, \mathbf{3}, \mathbf{1}) \times (\bar{\mathbf{3}}, \bar{\mathbf{3}}, \mathbf{1})$, and $(\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \times (\mathbf{3}, \mathbf{1}, \mathbf{3})$. The sum of these three singlets gives the E_6 singlet. The other two orthogonal combinations are in **650** since **78** has no 333 singlet. They could be

$$\begin{aligned} &2(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) - (\mathbf{3}, \mathbf{3}, \mathbf{1})(\bar{\mathbf{3}}, \bar{\mathbf{3}}, \mathbf{1}) - (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}})(\mathbf{3}, \mathbf{1}, \mathbf{3}), \\ &(\mathbf{3}, \mathbf{3}, \mathbf{1})(\bar{\mathbf{3}}, \bar{\mathbf{3}}, \mathbf{1}) - (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}})(\mathbf{3}, \mathbf{1}, \mathbf{3}). \end{aligned} \quad (3.2)$$

The latter violates C . Both these singlets can acquire VEVs and thus C will be broken, and we expect that no strings or walls bounded by strings [21] associated with C are generated.

The fundamental representation of E_6 is

$$\mathbf{27} = (\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3}) + (\mathbf{3}, \mathbf{3}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{3}}) \equiv \lambda + Q + Q^c, \quad (3.3)$$

where

$$\lambda = \begin{pmatrix} h_u & e^c \\ h_d & \nu^c \\ l & N \end{pmatrix} \quad (3.4)$$

with the rows being $\bar{\mathbf{3}}$'s of $SU(3)_L$ and the columns $\mathbf{3}$'s of $SU(3)_R$, and

$$Q = \begin{pmatrix} q \\ g \end{pmatrix} \quad \text{and} \quad Q^c = (u^c, d^c, g^c), \quad (3.5)$$

which are an $SU(3)_L$ triplet and an $SU(3)_R$ antitriplet respectively.

One can very easily verify that the element $c = \left(\exp(i2\pi/3), \exp(-i2\pi/3), \exp(-i2\pi/3) \right)$ of the unbroken trinification subgroup H coincides with the identity element as it acts like the identity on the $\mathbf{27}$ -plet and, consequently, on all the representations of E_6 . The generator of the second homotopy group $\pi_2(E_6/H) = \pi_1(H) = Z_3$ of the vacuum manifold E_6/H is then a loop that connects $(1,1,1)$ with c , i.e. three curves in the three $SU(3)$'s from 1 to $\exp(i2\pi/3)$, or 1 to $\exp(-i2\pi/3)$, or 1 to $\exp(-i2\pi/3)$ respectively. Obviously, the third power of this loop is homotopically trivial, and the breaking $E_6 \rightarrow 333$ therefore generates Z_3 magnetic monopoles.

In order to understand the structure of these Z_3 monopoles, we define the generators $T_L^8 = \text{diag}(1, 1, -2)$, $T_L^3 = \text{diag}(1, -1, 0)$ of $SU(3)_L$ and $T_R^8 = \text{diag}(1, 1, -2)$, $T_R^3 = \text{diag}(1, -1, 0)$ of $SU(3)_R$. Note that we use integer elements in these definitions so that a full rotation by 2π along these generators closes a circle. We see that $(1/6)T_L^8 + (1/2)T_L^3 = \text{diag}(2/3, -1/3, -1/3)$, and a rotation by 2π along this generator brings us from 1 to $\exp(-i2\pi/3)$ in $SU(3)_L$. Similarly, a rotation by 2π along the generator $(1/6)T_R^8 + (1/2)T_R^3$ interpolates between 1 and $\exp(-i2\pi/3)$ in $SU(3)_R$. In $SU(3)_c$, we take a rotation by $2\pi/3$ along $T_c^8 = \text{diag}(1, 1, -2)$, which leads from 1 to the element $\exp(i2\pi/3)$. The generator of the first homotopy (fundamental) group $\pi_1(H) = Z_3$ of H can be represented by a 2π rotation along the generator

$$\frac{1}{3}T_c^8 + \frac{1}{6}T_L^8 + \frac{1}{2}T_L^3 + \frac{1}{6}T_R^8 + \frac{1}{2}T_R^3. \quad (3.6)$$

It is easy to check that

$$\frac{1}{6}T_L^8 + \frac{1}{2}T_L^3 + \frac{1}{6}T_R^8 + \frac{1}{2}T_R^3 = Y + \frac{1}{2}T_L^3 = Q, \quad (3.7)$$

the electric charge operator, by applying it on the various states in $\mathbf{27}$ (Y is the weak hypercharge). Finally, we see that the generator of $\pi_1(H)$ is a rotation by 2π along $T_c^8/3 + Q$, exactly as in the $SO(10)$ case. As a consequence, the Z_3 monopole in E_6 , similarly to the Z_2 monopole in $SO(10)$, carries one (Dirac) unit of ordinary magnetic flux or charge as well as color magnetic flux corresponding to the generator of the center of $SU(3)_c$. As shown in refs. [10, 11] this is the ordinary Dirac monopole also carrying color magnetic charge.

We can further break 333 to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (3221_{B-L} , for short) by giving a VEV to the N -type component of $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$ in a Higgs $\mathbf{27}$ -plet. The generator in eq. (3.6) remains in the unbroken subalgebra since

$$\frac{1}{6}(T_L^8 + T_R^8) = \frac{1}{2}(B - L). \quad (3.8)$$

The orthogonal broken generator is

$$T_L^8 - T_R^8, \quad (3.9)$$

but a rotation by $2\pi/4$ along this generator leaves N invariant and thus remains unbroken. Adding to this a rotation by $2\pi/4$ along the unbroken generator $T_L^8 + T_R^8$, we get an equivalent rotation by $2\pi/2$ along T_L^8 . This rotation corresponds to the group element $\exp(i2\pi T_L^8/2) = \text{diag}(-1, -1, 1)$ in $SU(3)_L$, which belongs to the continuous part of the unbroken subgroup 3221_{B-L} . This means that no additional discrete symmetries are left unbroken. In other words, the unbroken subgroup is precisely 3221_{B-L} .

The second homotopy group of the vacuum manifold $\pi_2(333/3221_{B-L}) = \pi_1(3221_{B-L})_{333}$, which means that it consists of the homotopically non-trivial loops in 3221_{B-L} which are trivial in 333 . The minimal loop is a 6π rotation along the generator $T_c^8/3 + Q$, and so the loop in $SU(3)_c$ becomes homotopically trivial and can be removed. Only the rotation along Q by 6π remains, which corresponds to a monopole with triple the ordinary magnetic charge and no color magnetic flux at all.

The subsequent breaking of $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ does not generate any new topological objects provided that it is done by an $SU(2)_R$ Higgs doublet, analogous to the electroweak breaking — for a detailed explanation of this fact, see ref. [34]. This breaking can be achieved by the VEV of a Higgs $\mathbf{27}$ along the ν^c -type component of it. This belongs to an $SU(2)_R$ doublet with $B - L = 1$ and generates no topological defects.

We could alternatively use for the spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ a Higgs $\overline{\mathbf{351}}'$ (contained in $\mathbf{27} \times \mathbf{27}$) with a VEV along its $(\mathbf{1}, \bar{\mathbf{6}}, \mathbf{6})$ component. In particular, we take the $SU(2)_R$ $\mathbf{3}$ -plet in the $SU(3)_R$ $\mathbf{6}$ -plet with $T_R^8 = 2$ and the $SU(2)_L$ singlet in the $\bar{\mathbf{6}}$ of $SU(3)_L$ with $T_L^8 = 4$. This is an $SU(2)_R$ triplet with $B - L = (T_R^8 + T_L^8)/3 = 2$ and has the quantum numbers of $\nu^c \nu^c$. It thus leaves the Z_2 subgroup of $U(1)_{B-L}$ unbroken. So, in this case, in addition to the two types of monopoles, we have Z_2 strings as in ref. [18]. However, there are no necklaces in this case. Note that the electroweak Higgs doublets have zero $B - L$ and thus remain constant around the string. The string is not superconducting just as the string from the Z_2 subgroup of $U(1)_{B-L}$ in the previous section.

4 E_6 breaking via $SO(10) \times U(1)_\psi$

Let us now turn to the case of $E_6 \rightarrow SO(10) \times U(1)_\psi$. This can be achieved by a Higgs $\mathbf{78}$ -plet. We can further break $SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_{\psi'}$, where $\psi' = (\chi + 5\psi)/4$, with χ corresponding to the $SU(5) \times U(1)_\chi$ subgroup of $SO(10)$. The ψ charges for the $SO(10)$ components of the $\mathbf{27}$ -plet are given in parentheses

$$\mathbf{27} = \mathbf{1}(4) + \mathbf{10}(-2) + \mathbf{16}(1), \quad (4.1)$$

while the (χ, ψ) charges of its SU(5) parts are

$$\mathbf{27} = \mathbf{1}(0, 4) + \mathbf{5}(2, -2) + \bar{\mathbf{5}}(-2, -2) + \mathbf{1}(-5, 1) + \bar{\mathbf{5}}(3, 1) + \mathbf{10}(-1, 1). \quad (4.2)$$

The breaking of $\text{SO}(10) \times \text{U}(1)_\psi \rightarrow \text{SU}(5) \times \text{U}(1)_{\psi'}$ is achieved by the VEV of $\mathbf{1}(-5, 1)$, and thus the unbroken $\text{U}(1)$ corresponds to $\psi' = (\chi + 5\psi)/4$ with the ψ' charges given as

$$\mathbf{27} = \mathbf{1}(5) + \mathbf{5}(-2) + \bar{\mathbf{5}}(-3) + \mathbf{1}(0) + \bar{\mathbf{5}}(2) + \mathbf{10}(1). \quad (4.3)$$

Note that in the definition of ψ' we divided by 4 so that the ψ' charges are the minimal integer ones (as the χ and ψ charges), so that the periodicity of $\text{U}(1)_{\psi'}$ is 2π .

The $\text{U}(1)_\psi$ intersects with $\text{SO}(10)$ in its Z_4 center. This center is generated by $-i\Gamma^{10}$, where

$$\Gamma^{10} = i^5 \Gamma^0 \Gamma^3 \Gamma^1 \Gamma^2 \Gamma^4 \Gamma^5 \Gamma^7 \Gamma^8 \Gamma^6 \Gamma^9 = \sigma^{03} \sigma^{12} \sigma^{45} \sigma^{78} \sigma^{69} \quad (4.4)$$

is the chirality operator in ten Euclidean dimensions. Here we use the notation of ref. [35], which follows the notation of ref. [36]. The $\text{SO}(10)$ **16**-plet ($\mathbf{1} + \bar{\mathbf{5}} + \mathbf{10}$) is of negative chirality and so the SU(5) singlet $\mathbf{1}$ corresponds to all σ 's being -1, the $\bar{\mathbf{5}}$ to only one of them being -1 and all others +1, and the $\mathbf{10}$ to three of them being -1 and the rest +1. So under $-i\Gamma^{10}$, $\mathbf{16} \rightarrow i\mathbf{16}$ and, consequently, $\mathbf{10} \rightarrow -\mathbf{10}$ and $\mathbf{1} \rightarrow \mathbf{1}$. Now

$$i\Gamma^{10} = i\sigma^{03}i\sigma^{12}i\sigma^{45}i\sigma^{78}i\sigma^{69} = \exp \left\{ \frac{i\pi}{2} (\sigma^{03} + \sigma^{12} + \sigma^{45} + \sigma^{78} + \sigma^{69}) \right\}. \quad (4.5)$$

It is easy to see that the sum of σ 's coincides with the χ charge since it gives -5 for the SU(5) singlet $\mathbf{1}$, -1 for the $\mathbf{10}$, and 3 for $\bar{\mathbf{5}}$. So the generator $-i\Gamma^{10}$ of the center of $\text{SO}(10)$ lies in $\text{U}(1)_\chi$ and corresponds to a rotation by $-2\pi/4$ along it.

Also, a rotation by $2\pi/4$ along ψ acts on the $\text{SO}(10)$ representations as follows: $\mathbf{16} \rightarrow i\mathbf{16}$, $\mathbf{10} \rightarrow -\mathbf{10}$, $\mathbf{1} \rightarrow \mathbf{1}$ and thus coincides with $-i\Gamma^{10}$. A rotation by $2\pi/4$ along ψ together with a rotation by $2\pi/4$ along χ is a closed loop in $\text{SO}(10) \times \text{U}(1)_\psi$. This corresponds to the smallest charge magnetic monopole generated by the breaking $E_6 \rightarrow \text{SO}(10) \times \text{U}(1)_\psi$. It has 1/4 of magnetic flux along ψ and also an $\text{SO}(10)$ flux corresponding to the inverse generator of its center $i\Gamma^{10}$. A fourfold monopole, i.e. a monopole with magnetic flux equal to four times the flux of the minimal charged monopole, corresponds to a full (2π) rotation along ψ , since a full rotation along χ is homotopically trivial in $\text{SO}(10)$.

Instead of using rotations along ψ and χ , it is more transparent to use rotation along ψ and ψ' . Note that $\psi' = (\chi + \psi)/4 + \psi$. A rotation by 2π along $(\chi + \psi)/4$ corresponds to the identity as we have just seen, and a rotation by 2π along ψ again is the identity as one can see from the ψ charges. The ψ direction has no common elements with the center of SU(5) since the ψ charges of the SU(5) singlets are 4 and 1. However, the ψ' direction has elements which coincide with the center of SU(5). Namely, $\exp(i2\pi/5)$ in $\text{U}(1)_{\psi'}$ coincides with the element $\exp(-i2\pi\bar{Y}/5)$ of the center of SU(5) with $\bar{Y} = \text{diag}(2, 2, 2, -3 - 3)$ in SU(5). It is known [37] that χ and ψ correspond to the following GUT normalized generators

$$Q_\chi = \frac{\chi}{2\sqrt{10}}, \quad Q_\psi = \frac{\psi}{2\sqrt{6}}. \quad (4.6)$$

Then the normalized generator for ψ' is

$$Q_{\psi'} = \frac{1}{4} \left(Q_\chi + \sqrt{15} Q_\psi \right) = \frac{\psi'}{2\sqrt{10}}, \quad (4.7)$$

and the orthogonal generator is

$$Q_{\chi'} = \frac{1}{4} \left(\sqrt{15} Q_\chi - Q_\psi \right) = \frac{\chi'}{2\sqrt{6}}, \quad (4.8)$$

with $\chi' = (3\chi - \psi)/4$. The χ' charges are

$$\mathbf{27} = \mathbf{1}(-1) + \mathbf{5}(2) + \bar{\mathbf{5}}(-1) + \mathbf{1}(-4) + \bar{\mathbf{5}}(2) + \mathbf{10}(-1), \quad (4.9)$$

such that the full rotation along χ' is a 2π rotation.

Note that the χ' direction has no common elements with $SU(5)$ since the charges of the $SU(5)$ singlets in $\mathbf{27}$ are -1 and -4. However, the Z_4 subgroups from ψ' and χ' coincide. Namely, a rotation by $2\pi/4$ along ψ' together with a rotation by $2\pi/4$ along χ' lead to the identity element as one can see from the ψ' , χ' charges. It is, as we will see, more convenient to use the orthogonal generators ψ' , χ' rather than ψ , χ .

What happens in the next breaking to $SU(5) \times U(1)_{\psi'}$ by the ν^c -type component of a Higgs $\mathbf{27}$ -plet, i.e. the singlet in its $SO(10)$ $\mathbf{16}$ -plet? The $U(1)_{\psi'}$ symmetry remains, of course, unbroken, but the orthogonal $U(1)_{\chi'}$ breaks to its Z_4 subgroup since the χ' charge of the ν^c -type component is -4. However, this Z_4 belongs to $U(1)_{\psi'}$ and thus the unbroken subgroup is just $SU(5) \times U(1)_{\psi'}$, which is connected, i.e. its zeroth homotopy group $\pi_0(SU(5) \times U(1)_{\psi'}) = \{1\}$. Therefore, the first homotopy (fundamental) group of the vacuum manifold

$$\pi_1 \left(\frac{E_6}{SU(5) \times U(1)_{\psi'}} \right) = \pi_0(SU(5) \times U(1)_{\psi'}) = \{1\}, \quad (4.10)$$

and no stable strings will appear at this stage.

How about the previous monopole with magnetic flux $(\chi + \psi)/4 = (\psi' + \chi')/4$? As $U(1)_{\chi'}$ breaks to its Z_4 subgroup (which belongs to $U(1)_{\psi'}$ too), the $\chi'/4$ flux of the monopole is confined to a tube and connects it to an antimonopole. We therefore obtain unstable dumbbells [19, 20] which disappear. The Coulomb $\psi'/4$ and $-\psi'/4$ fluxes of the monopole and antimonopole, of course, cancel each other. However, new monopoles appear which carry $U(1)_{\psi'}$ flux. Indeed, since the Z_5 subgroup of $U(1)_{\psi'}$ belongs to $SU(5)$, as we showed above, these monopoles correspond to a rotation by $2\pi/5$ along ψ' and also carry $SU(5)$ flux corresponding to the element $\exp(i2\pi\bar{Y}/5)$ of the center of $SU(5)$. We are left at this stage only with these ψ' monopoles. Next we can break $U(1)_{\psi'}$ by the $SO(10)$ singlet in a Higgs $\mathbf{27}$ -plet which, of course, leaves unbroken its Z_5 subgroup contained in the center of $SU(5)$. Then strings are formed with flux corresponding to a rotation by $2\pi/5$ along ψ' which connect the ψ' monopoles to antimonopoles leading them to annihilate. Thus, no topological defects survive at the end. From this point on, the story proceeds as usual with the breaking of $SU(5)$.

It is interesting to note that we could inflate away the monopoles with magnetic flux $(\psi' + \chi')/4$ and obtain a network of cosmic strings with magnetic flux $\chi'/4$, which are not

topologically stable. At the breaking of $U(1)_{\psi'}$ by the N -type component of the Higgs **27**, a $\psi'/20$ magnetic flux is added along these strings in order for the phase of N to remain constant around them. This addition certainly corresponds to the minimal necessary increase of the magnetic energy of the string. At the electroweak breaking, the phases of the Higgs fields h_u and h_d , which have $\chi' = 2, -1$ and $\psi' = -2, -3$ respectively, change around the string by $4\pi/5$ and $-4\pi/5$ respectively. If we minimally add to the string $-2/5$ of flux along $2Y$, the VEVs of h_u, h_d remain constant around it. Only the VEV of the ν^c -type Higgs field changes its phase by -2π around the string. However, this string is not superconducting. Indeed, the up-type quark masses originate from the VEV of h_u which remains constant around the string, and thus no transverse zero modes are generated along the string. The down-type quarks and charged lepton, although h_d also remains constant, could generate zero modes since the ν^c -type VEV contributes to their masses.

Recall that d^c -type quarks (e -type charged leptons) exist not only in the $\bar{\mathbf{5}}$ in the $SO(10)$ **16**-plet, but also in the $\bar{\mathbf{5}}$ in the $SO(10)$ **10**-plet, which we call D^c (E). Also, d -type quarks (e^c -type charged leptons) exist not only in the $\mathbf{10}$ in the $SO(10)$ **16**-plet but also in the $\mathbf{5}$ in the $SO(10)$ **10**-plet, which we call D (E^c). Then the masses of the down-type quarks can be schematically written as

$$\mathcal{M}_d = \begin{pmatrix} D^c, & d^c \end{pmatrix} \begin{pmatrix} N, & \alpha_{ij} h_d \\ \nu^c, & h_d \end{pmatrix} \begin{pmatrix} D \\ d \end{pmatrix}, \quad (4.11)$$

where the mass matrix is given in terms of four 3×3 blocks. Three of them are of the order of the VEVs of N, ν^c , and h_d as indicated with constant unsuppressed coefficients. The fourth is proportional to the VEV of h_d but multiplied by coefficients α_{ij} ($i, j = 1, 2, 3$) which are suppressed by powers of m_P , the Planck mass. This is due to fact that a direct trilinear Yukawa coupling is forbidden, in this case, by $U(1)_{\chi'}$ and $U(1)_{\psi'}$. The coefficients α_{ij} must then necessarily contain $U(1)_{\chi'}$ and $U(1)_{\psi'}$ violating SM singlet VEVs, i.e. $\langle N \rangle, \langle \nu^c \rangle$.

We can now apply a theorem given in ref. [38] which says that, if a particular mass matrix element remains constant around the string, we can remove from the mass matrix the row and the column that contain it when calculating the number of transverse zero modes. In our case N and h_d remain unaltered around the string, so all rows and columns can be removed and no zero modes appear. We see that the fact that ν^c changes phase around the string does not generate zero modes in this case. A very similar analysis can be done for the charged leptons by replacing D^c, d^c, D, d in eq. (4.11) by E, e, E^c, e^c respectively. We conclude that these strings are not superconducting.

We could also inflate away the monopoles with $\psi'/5$ and $SU(5)$ flux to get a network of strings with magnetic flux $\psi'/5$. Recall that the phase of N changes by 2π around such a string, while ν^c remains constant. The phases of the electroweak doublets h_u, h_d change by $(-2/5)2\pi, (-3/5)2\pi$ respectively. Adding minimally on the string $2/5$ of flux along $2Y$, we then see that h_u remains constant around the string, while the phase of h_d changes by -2π . Again, we have no zero modes from the up-quark sector. For the down-quark and charged lepton sectors, we can write mass matrices similar to the one in eq. (4.11). Then, we can remove the rows and columns which contain elements proportional to ν^c which

leaves the 3×3 matrix which is proportional to $\alpha_{ij}h_d$. This matrix also does not change phase around the string as one can see from the various charges of the product $D^c d$. Thus, no transverse zero modes exist and these strings are also not superconducting.

Now if the breaking to $SU(5) \times U(1)_{\psi'}$ is achieved by the $\nu^c \nu^c$ -type component of $\overline{\mathbf{351'}}$, the Z_8 subgroup of $U(1)_{\chi'}$ remains unbroken. But the Z_4 subgroup of it is in $U(1)_{\psi'}$, so actually the unbroken subgroup is $SU(5) \times U(1)_{\psi'} \times Z_2$. As a consequence, Z_2 strings are formed with flux $\chi'/8$. Note that these are Z_2 strings, i.e. the string and antistring coincide (they are not oriented). In this case, the $\chi'/4$ flux of the monopole with total flux $(\chi' + \psi')/4$ splits into two tubes, each with flux $\chi'/8$. The monopoles can then be connected to form necklaces which are Z_2 strings themselves. We can also have simple Z_2 strings with flux $\chi'/8$ without monopoles on them since

$$\pi_1 \left(\frac{E_6}{SU(5) \times U(1)_{\psi'} \times Z_2} \right) = \pi_0(SU(5) \times U(1)_{\psi'} \times Z_2) = Z_2. \quad (4.12)$$

Needless to say the ψ' monopoles will also appear at this stage as before.

We can further use the $SO(10)$ singlet in a Higgs **27**-plet to break $U(1)_{\psi'}$. Again, we obtain flux tubes carrying $\psi'/5$ magnetic flux which connect the ψ' monopoles and antimonopoles and lead them to annihilation. If we inflate these ψ' monopoles we obtain a network of non-superconducting strings with flux $\psi'/5$ as we have seen above. The VEV of the N -type component does not break the extra Z_2 in eq. (4.12), but merely rotates it. Actually, if we add $1/40$ of the flux along ψ' on the string with flux $\chi'/8$, the phase of the N -type component remains unchanged around it. The electroweak doublets h_u, h_d have $\chi' = 2, -1$ and $\psi' = -2, -3$ respectively and thus, around the string, their phases change by $-2\pi/5, +2\pi/5$ respectively. If we minimally add to the string $1/5$ of flux along $2Y$, the VEVs of h_u, h_d remain constant around it. In summary, these strings and necklaces survive even after the electroweak breaking but they are not superconducting.

5 Primordial monopoles, strings, and gravity waves

As previously mentioned primordial monopoles and strings can survive inflation in realistic models. Consider, for instance, the breaking of $SO(10)$ to the SM via the 422 subgroup, such that the Z_2 subgroup of the center of $SO(10)$ remains unbroken. Assume that inflation is driven by an $SO(10)$ singlet scalar field with a Higgs or Coleman-Weinberg potential and with minimal coupling to gravity [25, 39]. This model predicts that the tensor-to-scalar ratio $r \gtrsim 0.02$ [40]. In other words, the Hubble parameter H during observable inflation is estimated to be of order $10^{13} - 10^{14}$ GeV, which has important implications for primordial monopoles and strings. The GUT monopoles produced during the breaking of $SO(10)$ to 422 are inflated away, but the intermediate mass monopoles from 422 breaking at M_I may survive inflation if $M_I \sim H$. In practice, one needs about $23 - 25$ e -foldings for adequate suppression and still leave an observable number density of these intermediate mass monopoles [25, 26] — see below. By the same token the intermediate scale Z_2 cosmic strings which are produced during the breaking of 422 to the SM can also survive the inflationary epoch. In the case the Z_2 center of $SO(10)$ is broken, we do not have

topologically stable strings. However, the monopoles produced during the breaking of 422 are connected, in the next stage of symmetry breaking, by topologically non-stable strings. The monopole-string system eventually decays by emitting gravity waves which may be detectable by future experiments (see below).

Regarding E_6 , as we have shown, the symmetry breaking $E_6 \rightarrow 333$ yields a superheavy GUT monopole which is inflated away, at least in the inflationary scenarios we have mentioned here. However, analogous to the $SO(10)$ case, the triply charged intermediate mass ($\sim 10^{14}$ GeV) monopoles from the breaking of 333 to the SM may be present at an observable level in our galaxy. Other realistic examples of intermediate scale monopoles, strings and composite objects that survive an inflationary scenario can be readily constructed.

We will now give some details concerning the production and evolution of monopoles and cosmic strings as well as the gravity waves generated by topologically stable or unstable strings. The mean distance between topological defects (monopoles or strings) at production is estimated to be $\sim H^{-1}$. During inflation it acquires an extra factor e^η with η being the number of e -foldings following the generation of the defects. During the subsequent inflaton oscillations this distance is multiplied by a factor $(t_r/\tau)^{2/3}$ with t_r being the reheat time and τ the rollover time, and from reheating until the present time by another factor T_r/T_0 (T_0 is the present temperature). So all together the mean distance between defects becomes

$$H^{-1} e^\eta \left(\frac{t_r}{\tau} \right)^{2/3} \frac{T_r}{T_0}. \quad (5.1)$$

For $T_r \simeq 10^9$ GeV and for the SM spectrum, $t_r \simeq 1$ GeV $^{-1}$. For the defects to enter into the horizon until today, their present mean distance in eq. (5.1) should not be larger than the present time t_0 .

We consider the inflationary scenario with a Coleman-Weinberg potential of ref. [39] with the coupling λ_3 in eqs. (5) and (6) of this reference much smaller than λ_2 . In this case, eq. (6) of this reference reduces to

$$A = \frac{24\lambda_2^2}{64\pi^2}. \quad (5.2)$$

To be more specific, we will take as an example a particular viable realization of this scenario which appears in the fourth line of table 4 in ref. [41]. In this case, the inflationary scale is $V_0^{1/4} \simeq 1.75 \times 10^{16}$ GeV, which implies that $H \simeq 7.25 \times 10^{13}$ GeV. Also $A = 1.43 \times 10^{-14}$ corresponding to $\lambda_2 \simeq 6.14 \times 10^{-7}$. The VEV of the inflaton M in ref. [39] or v in ref. [41] is $M \simeq 29.4 m_P \simeq 7.17 \times 10^{19}$ GeV. From the formula

$$\lambda_0 = A \ln \left(\lambda_2 \frac{M^2}{H^2} \right) \quad (5.3)$$

of ref. [39], we then obtain that $\lambda_0 = 1.9 \times 10^{-13}$, and from eq. (12) of the same reference that

$$\tau \sim \frac{\pi^2}{(8\lambda_0)^{1/2}} H^{-1} \sim 1.1 \times 10^{-7} \text{ GeV}^{-1}. \quad (5.4)$$

The requirement that the defects eventually enter the horizon (i.e. they are not inflated away) gives $\eta \lesssim 67.7$. From the formula $\eta = 3c/\lambda_0$ of ref. [25], we conclude that the parameter $c \sim (M_d/M)^2 \lesssim 4.3 \times 10^{-12}$, where M_d is the breaking scale corresponding to the defects. This scale should then satisfy the inequality $M_d \lesssim 1.5 \times 10^{14}$ GeV. In the case of strings ($M_d \equiv M_s$), this gives for the dimensionless string tension $G\mu_s \simeq (M_s/m_P)^2 \lesssim 3.7 \times 10^{-9}$.

Note that the GUT scale is given by $M_{\text{GUT}} \sim \lambda_2^{1/2} M$ (see ref. [39]). For the particular example we are discussing, $M_{\text{GUT}} \sim 5.6 \times 10^{16}$ GeV. Of course, this is just an order of magnitude estimate since we do not know the precise values of the couplings a and b in the potential in eq. (5) of ref. [39] and the GUT gauge coupling constant.

For models predicting the existence of topologically stable Z_2 strings, we can employ figure 1 of ref. [28], which holds for strings surviving until the present time. We see that strings with $G\mu_s \lesssim 1.5 \times 10^{-11}$, namely $M_s \lesssim 9.45 \times 10^{12}$ GeV, are allowed by the current experimental bounds. It is important to note that topologically stable strings with $G\mu_s \gtrsim 10^{-20}$ will be possibly measurable by LISA and BBO in the future.

The number density n_m of topologically stable magnetic monopoles can be estimated as in ref. [25]. At production it is expected to be $\sim H^3$. During inflation, the monopoles are diluted by a factor $\exp(-3\eta_m)$, where η_m is the number of e -foldings from the time of monopole production until the end of inflation. During inflaton oscillations, n_m is multiplied by another factor $(t_r/\tau)^{-2}$ (this was not taken into account in ref. [25]). The final relative monopole number density is

$$r \equiv \frac{n_m}{T_r^3} \sim \left(\frac{H}{T_r}\right)^3 e^{-3\eta_m} \left(\frac{t_r}{\tau}\right)^{-2}. \quad (5.5)$$

Requiring that r does not exceed 10^{-30} (the Parker bound) [42–44] and for the numerical example discussed here, we find that $\eta_m \gtrsim 23.5$. This implies that, for topologically stable monopoles, the parameter $c_m \sim (M_m/M)^2 \gtrsim 1.5 \times 10^{-12}$, and the corresponding symmetry breaking scale $M_m \gtrsim 8.77 \times 10^{13}$ GeV.

Next let us consider $\text{SO}(10)$ broken via $\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$ without topologically stable Z_2 strings. In this case, during the breaking of $\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$ to $\text{SU}(3)_c \times \text{U}(1)_{B-L} \times \text{SU}(2)_L \times \text{U}(1)_R$, we have the formation of $\text{SU}(4)_c$ (red) and $\text{SU}(2)_R$ (blue) monopoles at a scale M_m . These monopoles are subsequently partially diluted by inflation. At a breaking scale M_s , where $\text{SU}(3)_c \times \text{U}(1)_{B-L} \times \text{SU}(2)_L \times \text{U}(1)_R$ reduces to the SM gauge group, these monopoles are connected by strings forming random walks with step about the horizon size at subsequent times. Later, the monopoles enter the horizon connected in pairs by one string segment. After this time, the monopole pairs with the string segment behave like pressureless matter. The strings eventually decay to gravity waves and the monopoles merge to form either Schwinger monopoles or simply annihilate if they are a red or blue monopole with the corresponding antimonopole.

For the analysis of this case, we follow ref. [45]. The present abundance of these gravity waves is given by combining eqs. (63) and (64) of this reference:

$$\Omega_{\text{gw}} h^2(t_0) \sim 2 \left(\frac{2}{\Gamma}\right)^{\frac{1}{2}} (G\mu_s)^{\frac{1}{2}} \left(\frac{3.9}{10.75}\right)^{\frac{4}{3}} \left(\frac{\rho_\gamma(t_0)}{\rho_c(t_0)}\right) h_0^2, \quad (5.6)$$

where $\Gamma \sim 50$, $\rho_\gamma(t_0)$ and $\rho_c(t_0)$ are the present photon and critical energy densities of the universe respectively, and $h_0 \simeq 0.7$ is the present value of the Hubble parameter in units of $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$.

As an example we take $G\mu_s \simeq 6.7 \times 10^{-14}$, which corresponds to $M_s \simeq 6.3 \times 10^{11} \text{ GeV}$. The present abundance of gravity waves, in this case, is $\Omega_{\text{gw}} h^2 \simeq 10^{-12}$. The frequency f of these waves is given by — see ref. [45] —

$$f(t_0) \sim t_H^{-1} \left(\frac{t_d}{t_{\text{eq}}} \right)^{\frac{1}{2}} \left(\frac{t_{\text{eq}}}{t_0} \right)^{\frac{2}{3}}, \quad (5.7)$$

where t_H is the time at which the monopoles enter the horizon,

$$t_d \sim (\Gamma G\mu_s)^{-1} 2t_H \quad (5.8)$$

is the decay time of the strings, and t_{eq} is the equidensity time at which matter starts dominating the universe. For the example under discussion, we find that $f \simeq 10^{-4} \text{ Hz}$ provided that $t_H \simeq 2.27 \text{ sec}$. The decay time of the string segments is then $t_d \simeq 1.35 \times 10^{12} \text{ sec} \simeq 4.28 \times 10^4 \text{ yrs}$, which is prior to matter domination at $t_{\text{eq}} = 4.7 \times 10^4 \text{ yrs}$. Consequently, the above calculation, which requires this — see e.g. eq. (65) in ref. [45] — is consistent.

Now the question arises under what circumstances the required t_H can be obtained. This will be decided by the monopole production and evolution. As explained above, the mean intermonopole distance at temperature T after reheating is given by eq. (5.1) with $\eta = \eta_m$. At horizon re-entrance of the monopoles, this distance should be equal to t_H . Of course, T in eq. (5.1) should be replaced by T_H corresponding to t_H . For $t_H \simeq 2.27 \text{ sec}$ and for the SM spectrum, we obtain $T_H \simeq 1.74 \times 10^{-4} \text{ GeV}$, which is consistently lower than the reheat temperature. Our requirement then gives for the monopoles $\eta_m \simeq 48.35$, $c_m \simeq 3.07 \times 10^{-12}$, and $M_m \simeq 1.26 \times 10^{14} \text{ GeV}$.

Summarizing, we see that if the breaking scale of 422 to $\text{SU}(3)_c \times \text{U}(1)_{B-L} \times \text{SU}(2)_L \times \text{U}(1)_R$ is about $1.26 \times 10^{14} \text{ GeV}$ and the subsequent breaking scale of this group to the SM group is $M_s \simeq 6.32 \times 10^{11} \text{ GeV}$, the monopoles re-enter the horizon after reheating at $t_H \simeq 2.3 \text{ sec}$ connected by a string segment with $G\mu_s \simeq 6.71 \times 10^{-14}$. After t_H , the monopole string structures behave like particles and the strings at t_d emit gravity waves which at present have frequency $f = 10^{-4} \text{ Hz}$ and $\Omega_{\text{gw}} h^2 = 10^{-12}$. From figure 1 in ref. [28], we see that such gravity waves will be perfectly detectable by LISA. The spectrum of these waves is expected to strongly peak around 10^{-4} Hz .

6 Conclusions

Grand Unified Theories with a unified (single) gauge coupling constant such as $\text{SU}(5)$, $\text{SO}(10)$, and E_6 all predict the existence of a topologically stable magnetic monopole that carries a single unit of Dirac magnetic charge (quantized with respect to the electron charge). This superheavy GUT scale magnetic monopole also carries color magnetic charge, and this conclusion holds independent of the symmetry breaking pattern of the underlying GUT model. In $\text{SU}(5)$, this magnetic monopole happens to be the lightest one with mass $\sim M_{\text{GUT}}/\alpha_{\text{GUT}} \sim 10^{17} \text{ GeV}$, where $\alpha_{\text{GUT}} (\sim 1/10)$ is the GUT fine structure constant.

In models such as $SO(10)$ or E_6 , where the symmetry breaking proceeds via one or more intermediate steps, magnetic monopoles can appear that carry two or three units of the Dirac magnetic charge and their masses are related to the intermediate scale. Hence they are lighter than the GUT magnetic monopole with one unit of charge. We have observed that intermediate mass ($\sim 10^{14}$ GeV or so) magnetic monopoles and cosmic strings of similar mass scale may be present in our galaxy at an observable level. We have depicted scenarios which give rise to superconducting cosmic strings as well as composite objects including a novel type of necklace that can survive inflation. The gravity waves emitted by some of these topological defects may be observable with the space based observatory LISA.

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