$\mathcal{N} = (8, 0)$ AdS vacua of three-dimensional supergravity

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ABSTRACT: We give a classification of fully supersymmetric chiral $\mathcal{N} = (8, 0)$ AdS₃ vacua in general three-dimensional half-maximal gauged supergravities coupled to matter. These theories exhibit a wealth of supersymmetric vacua with background isometries given by the supergroups OSp(8|2, $\mathbb{R}$), F(4), SU(4|1, 1), and OSp(4*|4), respectively. We identify the associated embedding tensors and the structure of the associated gauge groups. We furthermore compute the mass spectra around these vacua. As an off-spin we include results for a number of $\mathcal{N} = (7, 0)$ vacua with supergroups OSp(7|2, $\mathbb{R}$) and G(3), respectively. We also comment on their possible higher-dimensional uplifts.

KEYWORDS: AdS-CFT Correspondence, Extended Supersymmetry, Supergravity Models

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1 Introduction

Supersymmetric Anti-de Sitter (AdS) backgrounds of string theory and supergravity are of central importance in the holographic AdS/CFT correspondence [1–3]. Within higher-dimensional supergravity, these correspond to supersymmetric solutions of the form $\text{AdS}_D \times \mathcal{M}$, and may give rise after consistent truncation to a D -dimensional gauged supergravity with a stationary point in its scalar potential. The AdS_D solution of the lower-dimensional theory with all scalars constant located at the stationary point then corresponds to the higher-dimensional $\text{AdS}_D \times \mathcal{M}$ solution.

A systematic approach to the classification of such backgrounds may start directly from a classification of supersymmetric AdS_D backgrounds in D -dimensional gauged supergravity. These supergravities are determined by the choice of a constant embedding tensor which encodes the gauge structure and couplings of the theories [4–6]. Rather than searching AdS vacua within a given theory, one may instead determine the most general embedding tensor such that the resulting theory admits a supersymmetric AdS_D vacuum, thereby determining the relevant D -dimensional theories together with their solutions. For half-maximal supergravities in $D \geq 4$ dimensions, such an analysis has been performed in refs. [7–11], where the general gauging admitting a fully supersymmetric AdS vacuum has been determined and analyzed.

AdS_3 vacua have so far escaped a similar classification. This is mostly due to the fact that the structure of gauged supergravity theories and their solutions in three space-time dimensions is very rich. Already the maximal ($\mathcal{N} = 16$) gauged supergravity in three dimensions offers a plethora of fully supersymmetric AdS_3 vacua [12]. This is in marked contrast to higher dimensions, where there is a single maximally supersymmetric AdS vacuum in $D = 7$ [13] and $D = 5$ [14], together with a one-parameter family of maximally supersymmetric AdS_4 vacua [15, 16]. Similarly, many AdS_3 vacua have been identified in theories with $\mathcal{N} = 9$ and $\mathcal{N} = 10$ supersymmetry [17, 18]. The wealth of three-dimensional structures is based on the particular properties of three-dimensional gauge and gravitational theories. The gravitational (super-)multiplet in three dimensions is non-propagating which allows for the construction of a gravitational Chern-Simons action for any AdS_3 supergroup [19]. Further coupling to scalar matter offers ample possibilities due to the on-shell duality between scalar and gauge fields in three dimensions. Again, the possible structures are most conveniently encoded in terms of a properly constrained embedding tensor [4, 20, 21]. Finally, the AdS_3 isometry group $\text{SO}(2, 2) \sim \text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$ is not simple, but a product of two factors. Consequently, the supergroup of AdS_3 background isometries in general factors into a direct product of simple supergroups $\mathcal{G}_L \times \mathcal{G}_R$ for which there are various options [22, 23]. The supercharges accordingly split

Supergroup $\mathcal{G}$	$\text{OSp}(8 2, \mathbb{R})$	$F(4)$	$SU(4 1, 1)$	$\text{OSp}(4^* 4)$
Even($\mathcal{G}$)	$\text{SO}(8) \times \text{Sp}(2, \mathbb{R})$	$\text{SO}(7) \times \text{SL}(2, \mathbb{R})$	$U(4) \times \text{SU}(1, 1)$	$\text{SO}^*(4) \times \text{USp}(4)$
$G^{R\text{-sym}}$	$\text{SO}(8)$	$\text{SO}(7)$	$U(1) \times \text{SO}(6)$	$\text{SO}(5) \times \text{SO}(3)$
Supercharges	$\mathbf{8}_s$	$\mathbf{8}$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$(\mathbf{4}, \mathbf{2})$

Table 1. Supergroups $\mathcal{G}$ with $\text{SL}(2, \mathbb{R})$ factor and eight supercharges [22, 23]. We also list their even part $G_{\text{even}} \simeq \text{SL}(2, \mathbb{R}) \times G^{R\text{-sym}}$ and their R -symmetry group $G^{R\text{-sym}}$. The supercharges are given in representations of $G^{R\text{-sym}}$.

into $\mathcal{N} = (p, q)$ charges transforming under $\mathcal{G}_R$ and $\mathcal{G}_L$, respectively. As a result, there is an immense number of half-maximal $\mathcal{N} = 8$ AdS vacua in three dimensions.

In this paper, we take a first step towards their classification, by determining all chiral $\mathcal{N} = (8, 0)$ AdS₃ vacua within half-maximal $D = 3$ gauged supergravity. For these vacua, the background isometries build a supergroup $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R$, where the simple supergroup $\mathcal{G}_R$ is chosen among the options listed in table 1. Some such vacua have recently appeared in ref. [24] as particular type IIA AdS₃ compactifications on a six sphere $S^6 = \text{SO}(7)/\text{SO}(6)$ fibered over an interval, preserving the exceptional supergroup $F(4)$. We give a classification of the half-maximal $D = 3$ gauged supergravities admitting an $\mathcal{N} = (8, 0)$ AdS₃ vacuum. These theories couple the non-propagating $\mathcal{N} = 8$ supergravity multiplet to n scalar multiplets, realizing different gauge groups embedded into the $\text{SO}(8, n)$ isometry group of ungauged supergravity. We restrict to $n \leq 8$ matter multiplets.¹ For each of the supergroups of table 1, we identify the possible three-dimensional supergravities, characterized by their gauge groups which we list in table 2, together with the external global symmetry group G_{ext} preserved by the vacuum. We also indicate in this table, which of these theories admit a free parameter, entering in particular the structure constants and the scalar potential. In the main body of this paper, we perform the explicit analysis of consistency conditions on the embedding tensor, leading to this classification. We moreover compute for every vacuum the associated mass spectrum, organized into supermultiplets of $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R$.

As a by-product of our constructions, we also identify a number of AdS₃ vacua with $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ supersymmetry, respectively. Some of the latter with $G(3)$ superisometry group have also emerged in the type IIA compactifications of ref. [24] with fluxes responsible for the breaking of the R -symmetry group down to G_2 . We list our findings in table 3. Let us recall that there is no three-dimensional supergravity theory with $\mathcal{N} = 7$ local supersymmetries and non-trivial matter content [25]. As a consequence, $\mathcal{N} = (7, 0)$ vacua can only be realized within half-maximal $\mathcal{N} = 8$ theories with $1/8$ of supersymmetry spontaneously broken at the vacuum. Indeed, we find that both our $\mathcal{N} = (7, 0)$ vacua live in half-maximal theories which also admit a fully supersymmetric $\mathcal{N} = (8, 0)$ vacuum, cf. figures 1 and 2 below.

The rest of this paper is organized as follows. In section 2, we review the structure of half-maximal $D = 3$ gauged supergravities, specifically their embedding tensors and the set of algebraic constraints imposed onto the embedding tensors in order to ensure consistency

¹This choice is discussed in section 7.

$n \leq 8$ matter multiplets	Gauge group	G_{ext}	free parameter
$\mathcal{G}_R = \text{OSp}(8 2, \mathbb{R})$			
$n_+ + n_-$	$\text{SO}(8, n_+) \times \text{SO}(n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$	ξ
$n_+ + 4$	$\text{SO}(8, n_+) \times \text{SO}(4)$	$\text{SO}(n_+) \times \text{SO}(4)$	
$n_+ + 4$	$\text{SO}(8, n_+) \times \text{SO}(3)$	$\text{SO}(n_+) \times \text{SO}(3)$	
$n_+ + 6$	$\text{SO}(8, n_+) \times \text{U}(3)$	$\text{SO}(n_+) \times \text{U}(3)$	
$n_+ + 7$	$\text{SO}(8, n_+) \times G_2$	$\text{SO}(n_+) \times G_2$	
8	$\text{SO}(8) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$	—
8	$\text{GL}(8)$	—	
$\mathcal{G}_R = \text{F}(4)$			
$n_+ + n_-$	$\text{SO}(7, n_+) \times \text{SO}(1, n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$	ξ
$n_+ + 3$	$\text{SO}(7, n_+) \times \text{SO}(1, 3)$	$\text{SO}(n_+) \times \text{SO}(3)$	
$n \geq 7$	$\text{GL}(7) \times \text{SO}(1, n-7)$	$\text{SO}(n-7)$	
n	$\text{SO}(7) \times \text{SO}(n)$	$\text{SO}(n)$	ξ
4	$\text{SO}(7) \times \text{SO}(4)$	$\text{SO}(4)$	
4	$\text{SO}(7) \times \text{SO}(3)$	$\text{SO}(3)$	
6	$\text{SO}(7) \times \text{U}(3)$	$\text{U}(3)$	
7	$\text{SO}(7) \times G_2$	G_2	
8	$\text{SO}(7) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$	
$\mathcal{G}_R = \text{SU}(4 1, 1)$			
$n_+ + n_-$	$\text{SO}(6, n_+) \times \text{SO}(2, n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$	ξ
$n_+ + 2m$	$\text{SO}(6, n_+) \times \text{U}(m, 1)$	$\text{SO}(n_+) \times \text{U}(m)$	
$n_+ + 2$	$\text{SO}(6, n_+) \times \text{SO}(2, 1)$	$\text{SO}(n_+)$	
$n_+ + 2$	$\text{SO}(6, n_+) \times \text{SO}(2, 2)$	$\text{SO}(n_+) \times \text{U}(1)$	
$6 \leq n < 8$	$\text{GL}(6) \times \text{SO}(2, n-6)$	$\text{SO}(n-6)$	
8	$\text{GL}(6) \times \text{SO}(2, 1)$	$\text{U}(1)$	ξ
6	$\text{GL}(6) \times \text{SO}(2, 2)$	$\text{SO}(2)$	
n	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(n)$	$\text{SO}(n)$	ξ
4	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(4)$	$\text{SO}(4)$	
4	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(3)$	$\text{SO}(3)$	
6	$\text{SO}(6) \times \text{SO}(2) \times \text{U}(3)$	$\text{U}(3)$	
7	$\text{SO}(6) \times \text{SO}(2) \times G_2$	G_2	
8	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$	
8	$\text{U}(4, 4)$	$\text{U}(4)$	
8	$\text{SL}(2) \times \text{Sp}(4, \mathbb{R})$	$\text{U}(1)$	
$2 + n_-$	$\text{U}(4, 1) \times \text{SO}(n_-)$	$\text{SO}(n_-) \times \text{U}(1)$	
$\mathcal{G}_R = \text{OSp}(4^* 4)$			
$n_+ + n_-$	$\text{SO}(5, n_+) \times \text{SO}(3, n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$	ξ
$n \geq 4$	$\text{SO}(5, n-4) \times G_{2(2)}$	$\text{SO}(n-4) \times \text{SO}(3)$	
$n \geq 5$	$\text{GL}(5) \times \text{SO}(3, n-5)$	$\text{SO}(n-5)$	
$n \geq 3$	$\text{GL}(3) \times \text{SO}(5, n-3)$	$\text{SO}(n-3)$	
8	$\text{GL}(5) \times \text{GL}(3)$	—	
n	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(n)$	$\text{SO}(n)$	ξ
4	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(4)$	$\text{SO}(4)$	
4	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(3)$	$\text{SO}(3)$	
6	$\text{SO}(5) \times \text{SO}(3) \times \text{U}(3)$	$\text{U}(3)$	
7	$\text{SO}(5) \times \text{SO}(3) \times G_2$	G_2	
8	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$	
8	$\text{Sp}(2, 2) \times \text{SO}(3)$	$\text{USp}(4)$	

Table 2. $\mathcal{N} = (8, 0)$ AdS_3 vacua preserving a global $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R \times G_{\text{ext}}$ symmetry.

$\mathcal{N}$	$n \leq 8$ matter multiplets	Gauge group	G_{ext}	free parameter
$\mathcal{G}_R = \text{OSp}(7 2, \mathbb{R})$				
$(7, 0)$	8	$\text{SO}(7) \times \text{SO}(8)$	—	
$\mathcal{G}_R = \mathbf{G}(3)$				
$(7, 1)$	n	$\text{SO}(n, 1) \times \mathbf{G}_2$	$\text{SO}(n)$	
$(7, 0)$	7	$\text{SO}(7) \times \mathbf{G}_2$	—	

Table 3. $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ AdS_3 vacua preserving a global $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R \times G_{\text{ext}}$ symmetry.

of the gauging and the existence of a supersymmetric AdS_3 vacuum. In the following two sections 3 and 4, we then turn to the analysis and solutions of these constraints. The computation is organized by choice of supergroup from table 1 together with the inequivalent embeddings of the desired AdS_3 R -symmetry group $\text{SO}(p) \times \text{SO}(q)$ into the isometry group $\text{SO}(8, n)$ of ungauged $D = 3$ supergravity. For every solution, we list the explicit form of the embedding tensor, determine the gauge group of the $D = 3$ theory, and compute the mass spectrum of the $\mathcal{N} = (8, 0)$ vacuum. In section 5, we collect some partial results on AdS_3 vacua with $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ supersymmetry. In section 6, we raise and answer the question which of these AdS_3 vacua can in fact be further embedded as vacua in a maximal ($\mathcal{N} = 16$) three-dimensional supergravity. This translates into a couple of additional algebraic constraints to be imposed onto the embedding tensor, which we check for all our vacua. Finally, in section 7 we summarize our findings in tables 22–24, and discuss possible generalizations, in particular the possible higher-dimensional origin of these vacua.

2 Gauged supergravities in three dimensions

In this section, we recall some relevant facts about the three-dimensional half-maximal gauged supergravities. Their gauge structure is most conveniently encoded in a constant embedding tensor subject to a set of algebraic constraints. We spell out the conditions for supersymmetric AdS_3 vacua and give the general formulas for the mass spectra around these vacua.

2.1 Lagrangian

Half-maximal gauged supergravities in three dimensions have been constructed in refs. [21, 26] by deforming the half-maximal ungauged theory of ref. [27]. This ungauged theory

contains an $\mathcal{N} = 8$ supergravity multiplet composed of a dreibein e_μ^α and eight Rarita-Schwinger fields ψ_μ^A , where μ and α denote respectively the curved and flat spacetime indices, and A is the index of the spinorial representation of the Minkowski R -symmetry $\text{SO}(8)$. The matter fields combine into n copies of the $\mathcal{N} = 8$ scalar multiplet, each one composed of eight scalars and eight spin-1/2 fermions, transforming in the vectorial and cospinorial representations of $\text{SO}(8)$, respectively. The scalar matter forms an $\text{SO}(8, n)/(\text{SO}(8) \times \text{SO}(n))$ coset space sigma model. In the following, the indices $I, J, \dots$ and $r, s, \dots$ denote the vectorial indices of $\text{SO}(8)$ and $\text{SO}(n)$, respectively, which we combine into $\text{SO}(8, n)$ vector indices $\mathcal{M} = \{I, r\}$. The $\text{SO}(8, n)$ invariant tensor is defined as

$$\eta_{\mathcal{M}\mathcal{N}} = \begin{pmatrix} -\delta_{IJ} & 0 \\ 0 & \delta_{rs} \end{pmatrix}, \quad (2.1)$$

and the generators of $\mathfrak{so}(8, n)$, in the chosen vectorial representation, are given by

$$(L^{\mathcal{MN}})_K{}^{\mathcal{L}} = 2\delta_K^{[\mathcal{M}}\eta^{\mathcal{N}]\mathcal{L}}. \quad (2.2)$$

Then, $\{L^{IJ}, L^{rs}\}$ form the generators of $\mathfrak{so}(8) \oplus \mathfrak{so}(n)$ while the coset is parametrized by the $8n$ scalar fields ϕ_{Ir} through the $\text{SO}(8, n)$ matrix

$$\mathcal{V} = e^{\phi_{Ir}L^{Ir}}. \quad (2.3)$$

Finally the spin-1/2 fermions are denoted by $\chi_{\dot{A}r}$, with the index $\dot{A}$ of the cospinorial representation of $\text{SO}(8)$.

The gauging of the theory is described using the embedding tensor formalism [4, 5]. We briefly review its main features in this context. Gauging amounts to promoting a subgroup $G_0 \subset \text{SO}(8, n)$ to a local symmetry, in such a way that the local supersymmetry remains preserved. The embedding of $\mathfrak{g}_0$ in $\mathfrak{so}(8, n)$ is given by the embedding tensor $\Theta_{\mathcal{MN}|\mathcal{PQ}}$, so that the gauge group generators $X_{\mathcal{MN}}$ are

$$X_{\mathcal{MN}} = \Theta_{\mathcal{MN}|\mathcal{PQ}} L^{\mathcal{PQ}}. \quad (2.4)$$

The embedding tensor $\Theta_{\mathcal{MN}|\mathcal{PQ}}$ is antisymmetric in $[\mathcal{MN}]$ and $[\mathcal{PQ}]$, and symmetric under exchange of the two pairs (in order to allow for an action principle of the gauged theory). It is thus contained in the symmetric tensor product of two adjoint representations of $\text{SO}(8, n)$ and may accordingly be decomposed into its irreducible parts:

$$\Theta_{\mathcal{MN}|\mathcal{PQ}} \subset \mathbf{1} \oplus \boxed{} \oplus \boxed{\begin{smallmatrix} & \\ & \end{smallmatrix}} \oplus \boxed{\begin{smallmatrix} & \\ & \end{smallmatrix}}, \quad (2.5)$$

where each box represents a vector representation $\mathbf{8} + \mathbf{n}$ of $\text{SO}(8, n)$. With this group-theoretical representation, the constraint on $\Theta_{\mathcal{MN}|\mathcal{PQ}}$ that ensures supersymmetry of the gauged theory takes a simple form [21]:

$$\mathbb{P}_{\boxplus} \Theta_{\mathcal{MN}|\mathcal{PQ}} = 0, \quad (2.6)$$

i.e. one has to project out the “Weyl-tensor” type representation.² This constraint is often called the “linear constraint”. It can be explicitly solved by parametrizing $\Theta_{\mathcal{MN}|\mathcal{PQ}}$ as

$$\Theta_{\mathcal{MN}|\mathcal{PQ}} = \theta_{\mathcal{MN}\mathcal{PQ}} + 2(\eta_{\mathcal{M}[\mathcal{P}}\theta_{\mathcal{Q}]\mathcal{N}} - \eta_{\mathcal{N}[\mathcal{P}}\theta_{\mathcal{Q}]\mathcal{M}}) + 2\eta_{\mathcal{M}[\mathcal{P}}\eta_{\mathcal{Q}]\mathcal{N}}\theta, \quad (2.7)$$

where $\theta_{\mathcal{MN}\mathcal{PQ}}$ is totally antisymmetric and $\theta_{\mathcal{MN}}$ is symmetric and traceless.

To ensure that this embedding defines a proper gauge group, the embedding tensor $\Theta_{\mathcal{MN}|\mathcal{PQ}}$ must be invariant under transformations of $\mathfrak{g}_0$ itself. Explicitly, this reads

$$0 = (X_{\mathcal{RS}})_{\mathcal{M}}{}^{\mathcal{K}}\Theta_{\mathcal{KN}|\mathcal{PQ}} + (X_{\mathcal{RS}})_{\mathcal{N}}{}^{\mathcal{K}}\Theta_{\mathcal{MK}|\mathcal{PQ}} + (X_{\mathcal{RS}})_{\mathcal{P}}{}^{\mathcal{K}}\Theta_{\mathcal{MN}|\mathcal{KQ}} + (X_{\mathcal{RS}})_{\mathcal{Q}}{}^{\mathcal{K}}\Theta_{\mathcal{MN}|\mathcal{PK}}. \quad (2.8)$$

Since $X_{\mathcal{MN}}$ is defined in terms of the embedding tensor (2.4), this condition gives rise to a set of equations bilinear in $\Theta_{\mathcal{MN}|\mathcal{PQ}}$, referred to as the “quadratic constraints”. For later use, we rewrite these constraints in the parametrization (2.7) after contraction with an antisymmetric parameter $\Lambda_{\mathcal{RS}}$

$$\begin{cases} 0 = -\Lambda_{\mathcal{RS}}\theta^{\mathcal{RSK}\mathcal{L}}\theta_{\mathcal{K}(\mathcal{M}}\eta_{\mathcal{N})\mathcal{L}} + 2\theta^{\mathcal{RS}}\theta_{\mathcal{R}(\mathcal{M}}\Lambda_{\mathcal{N})\mathcal{S}} + 2\theta\theta^{\mathcal{R}}{}_{(\mathcal{M}}\Lambda_{\mathcal{N})\mathcal{R}}, \\ 0 = -\Lambda_{\mathcal{RS}}\theta^{\mathcal{RSK}\mathcal{L}}\theta_{\mathcal{K}[\mathcal{N}\mathcal{PQ}}\eta_{\mathcal{M}]\mathcal{L}} + 2\theta^{\mathcal{SK}}\theta_{\mathcal{K}[\mathcal{N}\mathcal{PQ}}\Lambda_{\mathcal{M}]\mathcal{S}} \\ \quad + 2\Lambda_{\mathcal{RS}}\theta^{\mathcal{R}}{}_{[\mathcal{M}}\theta^{\mathcal{S}}{}_{\mathcal{N}\mathcal{PQ}]} + 2\theta\theta^{\mathcal{K}}{}_{[\mathcal{N}\mathcal{PQ}}\Lambda_{\mathcal{M}]\mathcal{K}}. \end{cases} \quad (2.9a) \quad (2.9b)$$

In terms of $\text{SO}(8, n)$ representations, the quadratic constraints (2.9a), (2.9b) can be shown to transform according to

$$\begin{array}{c} \square\square \oplus \begin{array}{c} \square\square \\ \square \end{array} \oplus \begin{array}{c} \square \\ \square \\ \square \end{array} \oplus \begin{array}{c} \square\square \\ \square \\ \square \\ \square \end{array} \end{array}. \quad (2.10)$$

Any solution to these constraints defines a viable gauging.

The gauging procedure then follows the standard scheme, introducing covariant derivatives with vectors $A_{\mu}{}^{\mathcal{MN}}$, and associated field-strengths $F_{\mu\nu}{}^{\mathcal{MN}}$. These fields are not present in the ungauged Lagrangian, but may be defined on-shell upon dualizing the Noether currents of the global $\text{SO}(8, n)$ symmetry. In the gauged theory they couple with a Chern-Simons term with the gauge parameter g , and do not carry propagating degrees of freedom.

Their minimal coupling to scalars breaks supersymmetry, and necessitates the introduction of new terms to the Lagrangian, specifically fermionic mass terms, and a scalar potential. Before presenting the full Lagrangian, it is useful to introduce the so-called T -tensor, that encodes these additional terms in the Lagrangian. We define $T_{\mathcal{MN}|\mathcal{PQ}} = \mathcal{V}_{\mathcal{M}}{}^{\mathcal{I}}\mathcal{V}_{\mathcal{N}}{}^{\mathcal{J}}\mathcal{V}_{\mathcal{P}}{}^{\mathcal{K}}\mathcal{V}_{\mathcal{Q}}{}^{\mathcal{L}}\Theta_{\mathcal{I}\mathcal{J}|\mathcal{K}\mathcal{L}}$, with the coset representative (2.3), or equivalently

$$\begin{cases} T_{\mathcal{MN}\mathcal{PQ}} = \mathcal{V}_{\mathcal{M}}{}^{\mathcal{I}}\mathcal{V}_{\mathcal{N}}{}^{\mathcal{J}}\mathcal{V}_{\mathcal{P}}{}^{\mathcal{K}}\mathcal{V}_{\mathcal{Q}}{}^{\mathcal{L}}\theta_{\mathcal{I}\mathcal{J}\mathcal{K}\mathcal{L}}, \\ T_{\mathcal{MN}} = \mathcal{V}_{\mathcal{M}}{}^{\mathcal{I}}\mathcal{V}_{\mathcal{N}}{}^{\mathcal{J}}\theta_{\mathcal{I}\mathcal{J}}, \\ T = \theta. \end{cases} \quad (2.11)$$

²In ref. [26] a stronger condition has been applied by projecting the embedding tensor on its totally antisymmetric and trace parts. This explains the extra pieces in our fermion mass terms, given in eq. (2.13) below.

The full Lagrangian is then

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4}eR + \frac{1}{2}\varepsilon^{\mu\nu\rho}\bar{\psi}_\mu^A\mathcal{D}_\nu\psi_\rho^A + \frac{1}{4}e\mathcal{P}_\mu^{Ir}\mathcal{P}^{\mu Ir} - \frac{1}{2}ie\bar{\chi}^{\dot{A}r}\gamma^\mu\mathcal{D}_\mu\chi^{\dot{A}r} - \frac{1}{2}e\mathcal{P}_\mu^{Ir}\bar{\chi}^{\dot{A}r}\Gamma_{A\dot{A}}^I\gamma^\mu\gamma^\nu\psi_\nu^A \\
 & - \frac{1}{4}\varepsilon^{\mu\nu\rho}\Theta_{\mathcal{MN}|\mathcal{PQ}}A_\mu^{\mathcal{MN}}\left(\partial_\nu A_\rho^{\mathcal{PQ}} + \frac{1}{3}g\Theta_{\mathcal{RS}|\mathcal{UV}}f^{\mathcal{PQ},\mathcal{RS}}{}_{\mathcal{XY}}A_\nu^{\mathcal{UV}}A_\rho^{\mathcal{XY}}\right) \\
 & + \frac{1}{2}eA_1^{AB}\bar{\psi}_\mu^A\gamma^{\mu\nu}\psi_\nu^B + ieA_2^{A\dot{A}r}\bar{\chi}^{\dot{A}r}\gamma^\mu\psi_\mu^A + \frac{1}{2},eA_3^{\dot{A}r\dot{B}s}\bar{\chi}^{\dot{A}r}\chi^{\dot{B}s} - eV.
 \end{aligned} \tag{2.12}$$

Here, $e = \sqrt{|\det g_{\mu\nu}|}$, R denotes the Ricci scalar, and $f_{\mathcal{MN},\mathcal{PQ}}^{\mathcal{KL}}$ describe the structure constants of $\mathfrak{so}(8, n)$. We refer to appendix A for further details, particularly for the supersymmetry transformations. The last four terms in eq. (2.12) are the most relevant ones for the following as they carry the fermionic mass matrices and scalar potential characteristic for the given gauging. Explicitly, the fermionic mass tensors $A_{1,2,3}$ are given by

$$\begin{cases}
 A_1^{AB} = -\frac{1}{48}\Gamma_{AB}^{IJKL}T_{IJKL} - \frac{1}{4}\delta^{AB}(T_{II} - 4T), \\
 A_2^{A\dot{A}r} = -\frac{1}{12}\Gamma_{A\dot{A}}^{IJK}T_{IJKr} - \frac{1}{2}\Gamma_{A\dot{A}}^IT_{Ir}, \\
 A_3^{\dot{A}r\dot{B}s} = \frac{1}{48}\delta^{rs}\Gamma_{\dot{A}\dot{B}}^{IJKL}T_{IJKL} + \frac{1}{2}\Gamma_{\dot{A}\dot{B}}^{IJ}T_{IJrs} - 2\delta^{\dot{A}\dot{B}}\delta^{rs}T - 2\delta^{\dot{A}\dot{B}}T_{rs} + \frac{1}{4}\delta^{\dot{A}\dot{B}}\delta^{rs}T_{II},
 \end{cases} \tag{2.13}$$

as functions of the T -tensor (2.11) and products of the $\text{SO}(8)$ Γ -matrices $\Gamma_{A\dot{A}}^I$, while the scalar potential V is given by

$$\begin{aligned}
 V = & -\frac{1}{4}A_1^{AB}A_1^{AB} + \frac{1}{8}A_2^{A\dot{A}r}A_2^{A\dot{A}r} \\
 = & -\frac{1}{48}\left(T_{IJKL}T_{IJKL} + \frac{1}{4!}\varepsilon^{IJKLMNPQ}T_{IJKL}T_{MNPQ} - 2T_{IJKr}T_{IJKr}\right) \\
 & - \frac{1}{8}(T_{II} - 4T)^2 + \frac{1}{4}T_{Ir}T_{Ir}.
 \end{aligned} \tag{2.14}$$

We finally note that the quadratic constraints (2.9) imply the relation

$$-\frac{1}{4}A_1^{AC}A_1^{BC} + \frac{1}{8}A_2^{A\dot{C}r}A_2^{B\dot{C}r} = \frac{1}{8}\delta^{AB}V, \tag{2.15}$$

for the fermionic mass tensors, often referred to as a supersymmetric Ward identity.

2.2 $\mathcal{N} = (p, q)$ supersymmetry

In the following, we will classify and analyze supersymmetric AdS vacua of the Lagrangian (2.12). As a first condition, the existence of an AdS vacuum necessitates an extremal point of the scalar potential (2.14), i.e.

$$\begin{aligned}
 0 = \delta_\phi V = & -\frac{1}{48}\delta\phi_{Ir}\left[4T_{IJKL}T_{JKLr} + \frac{1}{3}\varepsilon^{IJKLMNPQ}T_{MNPQ}T_{JKLr} + 12T_{IJKs}T_{JKrs}\right. \\
 & \left. - 24\left(T_{JJ}T_{Ir} - T_{Is}T_{sr} - T_{IJ}T_{Jr} - 4TT_{Ir}\right)\right].
 \end{aligned} \tag{2.16}$$

The precise amount of preserved supersymmetry can be read off from the eigenvalues of the gravitino mass matrix A_1^{AB} . In units of the AdS length $\ell^2 = 2/|V_0|$, the condition for $\mathcal{N} = (p, q)$ supersymmetry takes the form

$$\begin{cases} A_1^{ab} = \delta^{ab} \ell^{-1}, \\ A_1^{\dot{a}b} = -\delta^{\dot{a}b} \ell^{-1}, \end{cases} \quad (2.17)$$

and all other components vanishing, where we have split the index A according to $A = \{a, \dot{a}, i\}$ with $a \in \llbracket 1, p \rrbracket$, $\dot{a} \in \llbracket p+1, p+q \rrbracket$ and $i \in \llbracket p+q+1, 8 \rrbracket$. Together with eq. (2.15), this implies

$$\begin{cases} A_2^{a\dot{A}r} = 0, \\ A_2^{\dot{a}A r} = 0. \end{cases} \quad (2.18)$$

For a vacuum with $p+q=8$, preserving all supercharges, these conditions together with eq. (2.13) further imply that $T_{IJKr} = T_{Ir} = 0$ at the vacuum. From this, it follows that the vacuum condition (2.16) is automatically satisfied. Moreover, for an $\mathcal{N} = (8, 0)$ vacuum, the tensor T_{IJKL} has to be anti-selfdual at the vacuum.

Around a supersymmetric AdS vacuum, the matter content of the theory (2.12) organizes into supermultiplets of the associated supergroup that extends the spacetime isometry group. As the AdS_3 isometry group $\text{SO}(2, 2) \simeq \text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$ is not simple, the corresponding supergroup in general is a direct product $\mathcal{G}_L \times \mathcal{G}_R$ of two simple supergroups, whose even parts are isomorphic to the products $\text{SL}(2, \mathbb{R})_{L,R} \times \text{G}_{L,R}^{R\text{-sym}}$, of the AdS factor $\text{SL}(2, \mathbb{R})_{L,R}$ with the respective R -symmetry groups $\text{G}_{L,R}^{R\text{-sym}}$. These supergroups have been classified in ref. [22], and further analyzed in ref. [23]. Supersymmetry in three dimensions is thus factorizable and admits the decomposition $\mathcal{N} = (p, q)$, where p and q are the number of fermionic generators of $\mathcal{G}_R$ and $\mathcal{G}_L$ respectively. In the following, we will mainly focus on chiral $\mathcal{N} = (8, 0)$ vacua, for which the even part of the supergroup is of the form

$$\text{G}_{\text{even}} = \text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R \times \text{G}_R^{R\text{-sym}}. \quad (2.19)$$

The relevant simple supergroups have been given in table 1 above, along with their even subgroups and the representations of supercharges. In particular, the different R -symmetry groups are of the form $\text{G}^{R\text{-sym}} \simeq \text{SO}(p) \times \text{SO}(q)$, with $p+q=8$ and $p \neq 4$. The supercharges originally transform in the spinor representation of $\text{SO}(8)$ chirally embedded according to

$$\text{SO}(8) \subset \text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n). \quad (2.20)$$

According to table 1, they remain irreducible $A \rightarrow A$ (as a real representation) when $\text{SO}(8)$ is broken down to the R -symmetry group $\text{G}^{R\text{-sym}}$. For the vector representation of $\text{SO}(8)$, this leaves two options. Depending on performing or not a triality rotation of $\text{SO}(8)$ before embedding the R -symmetry group, the vector decomposes as

$$\begin{cases} \text{(i)} : I \rightarrow I, \end{cases} \quad (2.21a)$$

$$\begin{cases} \text{(ii)} : I \rightarrow \{i, \alpha\}, \quad i \in \llbracket 1, p \rrbracket, \quad \alpha \in \llbracket p+1, 8 \rrbracket, \end{cases} \quad (2.21b)$$

i.e. either (i) it remains irreducible, or (ii) it decomposes into the vector of $\text{SO}(p) \times \text{SO}(q)$. In the first case, it is the cospinor of $\text{SO}(8)$ which decomposes into the vector of $\text{SO}(p) \times \text{SO}(q)$,

while it stays irreducible in the second case. For $\text{OSp}(8|2, \mathbb{R})$, i.e. $p = 8$, the two options are equivalent. The breaking (2.21) determines how the vector of $\text{SO}(8, n)$ transforms under the R -symmetry group, thus the transformation of the various components of the embedding tensor (2.7).

For the following analysis of vacua it is convenient to discuss the two cases in (2.21) separately, and we will do so in sections 3 and 4, respectively.

2.3 Vacua and spectra

In this paper, we will classify and analyze AdS_3 vacua preserving $\mathcal{N} = (8, 0)$ supersymmetry for a general choice of the embedding tensor. Without loss of generality, we will search for vacua at the scalar origin $\mathcal{V} = \mathbb{1}$, since any extremal point located at a different $\mathcal{V}_0$ can be mapped into an extremal point at the scalar origin of the theory with embedding tensor rotated by $\mathcal{V}_0^{-1}$ [28, 29]. We thus simultaneously solve the quadratic constraints (2.9a) and (2.9b) together with the extremality condition (2.16) (evaluated at $T_{\mathcal{MN}|\mathcal{PQ}} = \Theta_{\mathcal{MN}|\mathcal{PQ}}$) and the supersymmetry condition (2.17). For every AdS_3 vacuum, we then determine the associated gauge group and compute the mass spectrum of fluctuations.

The analysis is simplified by the symmetries of the desired vacuum. For a given choice of supergroup $\mathcal{G}_R$, we first parametrize $\Theta_{\mathcal{MN}|\mathcal{PQ}}$ as a singlet of the R -symmetry group $\text{G}_R^{\text{R-sym}}$, cf. eq. (2.19). In general, this group may be embedded in different ways into the compact $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$, such that the representation of the supercharges branches into the relevant representation collected in table 1. All $\text{G}_R^{\text{R-sym}}$ admit a chiral embedding into $\text{SO}(8) \subset \text{SO}(8) \times \text{SO}(n)$ according to either of the options from (2.21), while for sufficiently large n , the group $\text{G}_R^{\text{R-sym}}$ or one of its factors may also admit a diagonal embedding into $\text{SO}(8) \times \text{SO}(n)$, as we will see in the following. With the proper parametrization of $\Theta_{\mathcal{MN}|\mathcal{PQ}}$, we then solve the eqs. (2.9a), (2.9b), (2.16) and (2.17) to identify the vacua. The gauge group of the theory is deduced from the algebra satisfied by the generators (2.4). At the vacuum, it is spontaneously broken down to its compact subgroup.

For a given vacuum, the spectrum is entirely determined by the embedding tensor $\Theta_{\mathcal{MN}|\mathcal{PQ}}$. In our conventions, the mass matrices for the spin $s = 1/2, 1, 3/2$ fields are given by [21]

$$\begin{cases} M_{(1/2)}^{\dot{A}r\dot{B}s} = \frac{1}{2} A_3^{\dot{A}r\dot{B}s}, \\ M_{(1)Ir|Js} = \frac{1}{2} \Theta_{Ir|Js}, \\ M_{(3/2)}^{AB} = \frac{1}{2} A_1^{AB}, \end{cases} \quad (2.22)$$

with the tensors A_1, A_3 are defined in eq. (2.13).³ As for the scalars, their mass matrix is

³We give here the expression of the spin-1/2 fermions mass matrix after projecting out the goldstini. This is sufficient for most of the following, since we are mainly dealing with fully supersymmetric vacua. See ref. [21] for the complete expression.

given by the second order variation of the scalar potential (2.14) which yields

$$\begin{aligned}
 M_{(0) Lr, Ms}^2 = & \delta_{LM} \left(\frac{1}{3} \theta_{IJKr} \theta_{IJKs} - \theta_{IJrp} \theta_{IJsp} + 2 \theta_{II} \theta_{rs} - 8 \theta \theta_{rs} - 2 \theta_{Ir} \theta_{Is} - 2 \theta_{pr} \theta_{ps} \right) \\
 & + \delta_{rs} \left(\frac{1}{3} \theta_{IJKL} \theta_{IJKM} - \theta_{IJLp} \theta_{IJMp} + \frac{1}{36} \theta_{IJKR} \varepsilon^{IJKRMNPQ} \theta_{LNPQ} \right. \\
 & \quad \left. + 2 \theta_{II} \theta_{LM} - 2 \theta_{IL} \theta_{IM} - 2 \theta_{Mp} \theta_{Lp} - 8 \theta \theta_{LM} \right) \\
 & - 2 \theta_{ILMp} \theta_{Irsp} + 2 \theta_{ILsp} \theta_{IMrp} + \frac{1}{12} \theta_{IJKN} \varepsilon^{IJKNPQLM} \theta_{PQrs} \\
 & - \frac{1}{9} \theta_{IJKr} \varepsilon^{IJKLMNPQ} \theta_{NPQs} + 4 \theta_{Lr} \theta_{Ms} - 4 \theta_{Ls} \theta_{Mr} - 4 \theta_{LM} \theta_{rs}. \quad (2.23)
 \end{aligned}$$

We normalize all masses by the AdS length $\ell = \sqrt{2}/\sqrt{|V_0|}$. The spectrum is then most conveniently given in terms of the corresponding conformal dimensions $\Delta_{(s)}$, which allows the identification of the supermultiplets. In $D = 3$, the conformal dimensions are related to the normalized masses through [30–35]

$$\begin{cases} \Delta_{(0)} (\Delta_{(0)} - 2) = (m_{(0)} \ell)^2, \\ \Delta_{(1)} = 1 + |m_{(1)} \ell|, \end{cases} \quad \text{and} \quad \begin{cases} \Delta_{(1/2)} = 1 \pm m_{(1/2)} \ell, \\ \Delta_{(3/2)} = 1 + |m_{(3/2)} \ell|. \end{cases} \quad (2.24)$$

Upon projecting out the Goldstone scalars and goldstini, the spectrum organizes into $\mathcal{G}$ supermultiplets. As the supergroup $\mathcal{G} = \mathcal{G}_L \times \mathcal{G}_R$ is not simple, each Δ decomposes itself as $\Delta = \Delta_L + \Delta_R$, with conformal dimensions $\Delta_{L,R}$ associated to the representations of $\mathcal{G}_{L,R}$. The spacetime spin s is identified as $s = \Delta_R - \Delta_L$.

3 Solutions with irreducible vector embedding (2.21a)

We are now in position to start analyzing the consequences of the algebraic equations (2.9a) and (2.9b). We will do this analysis separately for all the supergroups given in table 1, with the two possible embeddings of the R -symmetry groups $\text{SO}(p) \times \text{SO}(q)$ according to eq. (2.21). In this section, we will consider the case of an irreducible vector embedding (2.21a). We restrict to $n \leq 8$ matter multiplets.

3.1 Constraining the embedding tensor

As explained above, upon implementing eq. (2.18) in eq. (2.13) for $\mathcal{N} = (8, 0)$, the remaining possibly non-vanishing components of the embedding tensor are *a priori* given by

$$\{ \theta_{IJ}, \theta_{rs}, \theta = \kappa, \theta_{IJKL}^-, \theta_{IJrs}, \theta_{I pqr}, \theta_{p qrs} \}, \quad (3.1)$$

where θ_{IJKL}^- is anti-selfdual. The fact that the embedding tensor is singlet under the respective R -symmetry group, embedded according to eq. (2.21a), further restricts these components as

$$\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{rs}, \theta = \kappa, \theta_{IJKL}^- = \xi^{\dot{A}\dot{B}} \Gamma_{\dot{A}\dot{B}}^{IJKL}, \theta_{IJrs}, \theta_{I pqr}, \theta_{p qrs} \}, \quad (3.2)$$

with traceless $\xi^{\dot{A}\dot{B}}$ of signature (p, q) (only non-vanishing for $p \neq 8$). The first quadratic constraint (2.9a) with free indices chosen as $(\mathcal{M}, \mathcal{N}) = (I, J)$ is then identically satisfied. Choosing the free indices as $(\mathcal{M}, \mathcal{N}) = (I, r)$ gives rise to the equations

$$\begin{cases} \theta_{rp} \theta_{ps} + \kappa \theta_{rs} + \lambda (\kappa - \lambda) \delta_{rs} = 0, & (3.3a) \\ \lambda \theta_{IJrs} + \theta_{IJrp} \theta_{ps} = 0, & (3.3b) \\ \lambda \theta_{Ipqr} + \theta_{Ipqs} \theta_{sr} = 0. & (3.3c) \end{cases}$$

Eq. (3.3a) determines the eigenvalues of the matrix θ_{rs} to be

$$\theta_+ = -\lambda, \quad \theta_- = \lambda - \kappa. \quad (3.4)$$

Accordingly, we choose a basis in which θ_{rs} is diagonal, split the indices r into $\{r_+, r_-\}$ and denote by $n_{\pm}$ the multiplicities of these eigenvalues. Tracelessness of $\theta_{\mathcal{MN}}$ implies that

$$(8 + n_+ - n_-) \lambda = -n_- \kappa. \quad (3.5)$$

If we now set all $\theta_{\mathcal{MN}\mathcal{PQ}} = 0$, all remaining quadratic constraints are satisfied. Up to an arbitrary overall scaling factor, this yields an embedding tensor of the form

$$\begin{aligned} \theta_{IJ} &= n_- \delta_{IJ}, & \theta &= -(8 + n_+ - n_-), \\ \theta_{r_+s_+} &= -n_- \delta_{r_+s_+}, & \theta_{r_-s_-} &= (8 + n_+) \delta_{r_-s_-}, \end{aligned} \quad (3.6)$$

with all other components vanishing. The gauge group, defined via eq. (2.4) by this embedding tensor, is $\text{SO}(8, n_+) \times \text{SO}(n_-)$. The vacuum breaks this group down to its compact subgroup $\text{SO}(8) \times \text{SO}(n_+) \times \text{SO}(n_-)$. Together with the preserved $\mathcal{N} = (8, 0)$ supersymmetries and the AdS symmetries, the isometry group of this vacuum thus is $\text{OSp}(8|2, \mathbb{R})$. Following the discussion in section 2.3 we may compute the spectrum around this vacuum. The result is collected in table 4(a), organized into $\text{OSp}(8|2, \mathbb{R})$ supermultiplets with the conformal dimensions obtained via eq. (2.24).

It remains to analyze how the solution (3.6) can be extended to non-vanishing $\theta_{\mathcal{MN}\mathcal{PQ}}$. The first quadratic constraint (2.9a) with free indices chosen as $(\mathcal{M}, \mathcal{N}) = (r, s)$ together with eqs. (3.3b), (3.3c) gives rise to the equations

$$\begin{cases} (2\lambda - \kappa) \theta_{pqr_+s_-} = 0, \\ (2\lambda - \kappa) \theta_{IJrs_-} = 0, \\ (2\lambda - \kappa) \theta_{Ipqr_-} = 0, \end{cases} \quad (3.7)$$

respectively. These equations could be simultaneously solved by choosing $\kappa = 2\lambda$. However, with eq. (3.5) this choice implies that in fact $\kappa = \lambda = 0$, resulting in $\theta = 0 = \theta_{\mathcal{MN}}$. As a consequence, both tensors A_1^{AB} and $A_2^{\dot{A}\dot{B}r}$ from eq. (2.13) vanish at the vacuum, inducing a vanishing potential (2.14) and thus a Minkowski vacuum, which is beyond the scope of the present analysis. In all the following we thus assume that $\kappa \neq 2\lambda$. Eqs. (3.7) then imply that, after complete resolution of the first quadratic constraint (2.9a), the solution (3.6) can be extended to potentially non-vanishing components

$$\left\{ \theta_{IJKL}^- = \xi^{\dot{A}\dot{B}} \Gamma_{\dot{A}\dot{B}}^{IJKL}, \theta_{IJr_+s_+}, \theta_{Ip_+q_+r_+}, \theta_{p_+q_+r_+s_+}, \theta_{p_-q_-r_-s_-} \right\}. \quad (3.8)$$

The remaining quadratic constraints restricting these components follow from evaluating eq. (2.9b). For readability, we defer the full set of constraint equations to appendix B, and in the following subsections treat each of the four possible supergroups separately.

3.2 $\text{OSp}(8|2, \mathbb{R})$

3.2.1 Chiral embedding

The R -symmetry group in this case is $\text{SO}(8)$. Let us first assume that it is chirally embedded into the first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$. Since the embedding tensor must be singlet under this group, its possible non-vanishing components within $\theta_{\mathcal{M}\mathcal{N}\mathcal{P}\mathcal{Q}}$ further reduce from eq. (3.8) to

$$\{\theta_{p+q+r+s+}, \theta_{p-q-r-s-}\}. \quad (3.9)$$

The second quadratic constraint is then reduced to two non-trivial equations, given by eqs. (B.4c) and (B.5a). They take the explicit form

$$\begin{cases} (2\lambda - \kappa)\theta_{pqrs+} = 0, & (3.10a) \\ \lambda\theta_{l-[q-r-s-}\theta_{p-]u-v-l-} = n_-(2\lambda - \kappa)(\delta_{u-[p-}\theta_{q-r-s-]v-} - \delta_{v-[p-}\theta_{q-r-s-]u-}). & (3.10b) \end{cases}$$

Following the discussion after eq. (3.7), we restrict to the case $\kappa \neq 2\lambda$, after which the first equation implies that the only non-vanishing components of θ_{pqrs} is $\theta_{p-q-r-s-}$. Next, we solve the remaining equation (3.10b) by considering each value of $n_- \geq 4$ separately. Since θ_{pqrs} does not enter in the mass formulas (2.22), (2.23), the spectra of all the resulting theories are still given by table 4(a). But, we will find in the following that non-vanishing θ_{pqrs} generically reduces the factor $\text{SO}(n_-)$ of the gauge group to a subgroup K_0 , such that the n_- in table 4(a) is to be replaced by the corresponding representation of K_0 .

$n_- = 4$. Both sides of eq. (3.10b) identically vanish, such that the general solution admits a non-vanishing $\theta_{p-q-r-s-} = \xi \varepsilon_{p-q-r-s-}$, with a free parameter ξ . The full solution then extends eq. (3.6) to

$$\begin{aligned} \theta_{IJ} &= 4\delta_{IJ}, & \theta &= -(4 + n_+), & \theta_{p-q-r-s-} &= (12 + n_+)\xi \varepsilon_{p-q-r-s-}, \\ \theta_{r+s+} &= -4\delta_{r+s+}, & \theta_{r-s-} &= (8 + n_+)\delta_{r-s-}. \end{aligned} \quad (3.11)$$

For $|\xi| \neq 1$ the gauge group is $\text{SO}(8, n_+) \times \text{SO}(4)$, as in the $\xi = 0$ case. On the other hand, when ξ takes a critical value $\xi = \pm 1$, the gauge group reduces to $\text{SO}(8, n_+) \times \text{SO}(3)_{\pm}$, i.e. $\text{SO}(4) = \text{SO}(3)_+ \times \text{SO}(3)_-$ is broken down to one of its chiral factors.

$n_- = 5$. Setting $\theta_{p-q-r-s-} = \varepsilon_{p-q-r-s-t-}\xi^{t-}$, eq. (3.10b) shows that non-vanishing ξ^{t-} implies that $\kappa = 2\lambda$, thus $\kappa = \lambda = 0$ and the vacuum is not AdS.

$n_- = 6$. Setting $\theta_{p-q-r-s-} = \frac{1}{2}\varepsilon_{p-q-r-s-u-v-}\xi^{u-v-}$, eq. (3.10b) leads to

$$(2\lambda - \kappa)\xi_{p-q-} = -\frac{1}{8}\varepsilon_{p-q-k-l-m-n-}\xi^{k-l-}\xi^{m-n-}. \quad (3.12)$$

In particular, this implies that

$$\xi_{p-r-}\xi_{r-q-} = \frac{\delta_{p-q-}}{4!2(2\lambda - \kappa)} \left(\varepsilon_{k-l-m-n-r-s-} \xi^{k-l-} \xi^{m-n-} \xi^{r-s-} \right). \quad (3.13)$$

For $\kappa \neq 2\lambda$ this implies that there is a basis such that⁴

$$\xi_{p-q-} = -(2\lambda - \kappa) \begin{pmatrix} -\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}, \quad \text{where } \sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (3.14)$$

The full solution is then given by

$$\begin{aligned} \theta_{IJ} &= 6\delta_{IJ}, \quad \theta = -(2 + n_+), \quad \theta_{r_+s_+} = -6\delta_{r_+s_+}, \quad \theta_{r_-s_-} = (8 + n_+)\delta_{r_-s_-}, \\ \theta_{p-q-r-s-} &= \frac{1}{2} \varepsilon_{p-q-r-s-u-v-} \xi^{u-v-}, \quad \xi_{u-v-} = -(14 + n_+) \begin{pmatrix} -\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}. \end{aligned} \quad (3.15)$$

The gauge group in this case is $\text{SO}(8, n_+) \times \text{U}(3)$, i.e. due to the presence of a non-vanishing $\theta_{p-q-r-s-}$, the $\text{SO}(6)$ factor is reduced to $\text{U}(3)$ compared to solution (3.6).

$n_- = 7$. Setting $\theta_{p-q-r-s-} = \varepsilon_{p-q-r-s-u-v-w-} \xi^{u-v-w-}$, eq. (3.10b) leads to

$$\varepsilon_{m-n-p-q-r-s-[u-} \xi_{v-w-]q-} \xi_{m-n-p-} = (2\lambda - \kappa) (\delta_{r-[u-} \xi_{v-w-]s-} - \delta_{s-[u-} \xi_{v-w-]r-}). \quad (3.16)$$

For $\kappa \neq 2\lambda$, this implies

$$\xi^{m-r-s-} \xi^{n-r-s-} = -\frac{\Lambda}{14(2\lambda - \kappa)} \delta^{m-n-}, \quad (3.17)$$

with the constant Λ given by

$$\Lambda = \left(\varepsilon^{p-q-r-s-u-v-w-} \xi^{k-p-q-} \xi^{k-r-s-} \xi^{u-v-w-} \right). \quad (3.18)$$

This equation is solved by choosing ξ_{m-r-s-} proportional to the $\text{G}_2 \subset \text{SO}(7)$ invariant 3-form ω_{m-r-s-} .⁵ The full embedding tensor is then given by

$$\begin{aligned} \theta_{IJ} &= 7\delta_{IJ}, \quad \theta_{r_+s_+} = -7\delta_{r_+s_+}, \quad \theta_{r_-s_-} = (8 + n_+)\delta_{r_-s_-}, \quad \theta = -(1 + n_+), \\ \theta_{p-q-r-s-} &= \frac{15 + n_+}{12} \varepsilon_{p-q-r-s-u-v-w-} \omega^{u-v-w-}. \end{aligned} \quad (3.19)$$

The gauge group is $\text{SO}(8, n_+) \times \text{G}_2$, i.e. compared to solution (3.6) the group $\text{SO}(7)$ is reduced to G_2 .

⁴The global sign of ξ_{p-q-} is fixed by the choice of convention for the Levi-Civita tensor $\varepsilon_{p-q-k-l-m-n-}$. Here we chose $\varepsilon_{123456} = 1$.

⁵We choose the normalisation of ω so that $\omega_{u-v-w-} \omega_{u-v-w-} = 42$.

Δ_L	Δ_R	Δ	s	SO(8)	SO(n_+)	SO(n_-)
$5/4$	$3/4$	2	$-1/2$	$\mathbf{8}_c$	$\mathbf{n}_+$	$\mathbf{1}$
	$1/4$	$3/2$	-1	$\mathbf{8}_v$	$\mathbf{n}_+$	$\mathbf{1}$
$1/4$	$3/4$	1	$1/2$	$\mathbf{8}_c$	$\mathbf{1}$	$\mathbf{n}_-$
	$1/4$	$1/2$	0	$\mathbf{8}_v$	$\mathbf{1}$	$\mathbf{n}_-$

(a) $G = \text{SO}(8, n_+) \times \text{SO}(n_-)$.

Δ_L	Δ_R	Δ	s	SO(8)
$3/2$	$5/2$	4	1	$\mathbf{1}$
	2	$7/2$	$1/2$	$\mathbf{8}_s$
	$3/2$	3	0	$\mathbf{28}$
	1	$5/2$	$-1/2$	$\mathbf{56}_s$
	$1/2$	2	-1	$\mathbf{35}_c$

(b) $G = \text{GL}(8)$.

Table 4. Mass spectra for the $\text{OSp}(8|2, \mathbb{R})$ solutions with (a) chiral and (b) diagonal embedding of the R -symmetry group. The spectrum organizes into multiplets of $\text{OSp}(8|2, \mathbb{R})$, given in eq. (5.3) and table 7 of ref. [23], respectively. For non-vanishing θ_{pqrs} , the factor $\text{SO}(n_-)$ in (a) reduces to a subgroup K_0 , and the representation $\mathbf{n}_-$ is replaced by the corresponding representation of K_0 .

$\mathbf{n}_- = \mathbf{8}$. In this case, eq. (3.5) implies $\kappa = 0$ and eq. (3.10b) is solved by a self-dual θ_{pqrs} :

$$\theta_{pqrs} = \theta_{pqrs}^+ = \Gamma_{ab}^{pqrs} \xi^{ab}, \quad (3.20)$$

with SO(8) Γ -matrices Γ_{ab}^{pqrs} and traceless ξ^{ab} subject to the equation

$$\xi^{ac} \xi^{bc} - \frac{1}{8} \delta^{ab} \xi^{cd} \xi^{cd} = \frac{\lambda}{2} \xi^{ab}. \quad (3.21)$$

This implies that the eigenvalues of ξ_{ab} are

$$\xi^+ = \frac{\lambda}{4} \frac{8-p}{(4-p)}, \quad \xi^- = -\frac{\lambda}{4} \frac{p}{(4-p)}, \quad (3.22)$$

with multiplicity p and $8-p$ respectively and $p \neq 4$.⁶ The full embedding tensor then takes the form:

$$\begin{aligned} \theta_{IJ} &= 8 \delta_{IJ}, \quad \theta_{rs} = 8 \delta_{rs}, \quad \theta = 0, \quad \theta_{pqrs} = \Gamma_{ab}^{pqrs} \xi^{ab}, \\ \text{where } \xi^{ab} &= \frac{2}{4-p} \text{diag}\{\underbrace{8-p, \dots, 8-p}_p, \underbrace{-p, \dots, -p}_{8-p}\}, \quad p \neq 4. \end{aligned} \quad (3.23)$$

There is an analogous solution for anti-selfdual choice of θ_{pqrs} . The gauge group is $\text{SO}(8) \times \text{SO}(8-p) \times \text{SO}(p)$, i.e. $\text{SO}(8)$ is reduced to $\text{SO}(8-p) \times \text{SO}(p)$ compared to solution (3.6).

3.2.2 Diagonal embedding

For $n = 8$ matter multiplets, the R -symmetry group $\text{SO}(8)$ alternatively allows a diagonal embedding as

$$\text{SO}(8) = \text{SO}(8)_{\text{diag}} \subset \text{SO}(8) \times \text{SO}(8) \subset \text{SO}(8, 8). \quad (3.24)$$

⁶The case $p = 4$ implies $\kappa = 2\lambda$ thus again leading to a Minkowski vacuum.

Moreover, there are inequivalent diagonal embeddings according to possible triality rotations in the two $\text{SO}(8)$ factors. In this case the condition of being singlet under the R -symmetry group reduces the possible components of the embedding tensor from eq. (3.8) to

$$\left\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \theta = \kappa, \theta_{IJr_+s_+} \right\}, \quad (3.25)$$

with either $(n_+, n_-) = (8, 0)$ or $(n_+, n_-) = (0, 8)$. For vanishing $\theta_{IJr_+s_+}$, we are back to solution (3.6). A non-vanishing $\theta_{IJr_+s_+}$ on the other hand implies $(n_+, n_-) = (8, 0)$ and from eq. (3.5) we deduce that $\lambda = 0$. We are then left with the following surviving components

$$\{\theta = \kappa, \theta_{IJrs}\}, \quad (3.26)$$

where we suppress the $+$ subscript. The remaining equations of the quadratic constraints are given by eqs. (B.2b), (B.3a), (B.3b) and (B.4c):

$$\begin{cases} \theta_{usM}[I\theta_{JK}]_{rs} + \kappa \delta_M[I\theta_{JK}]_{ru} = 0, \\ \Lambda_{uv} [\theta_{uvL}[I\theta_J]_{Lrs} + 2\kappa \theta_{IJu}[r\delta_s]_v] = 0, \\ \Lambda_{MN} [\theta_{MNP}[r\theta_s]_{pIJ} + 2\kappa \theta_{rsM}[I\delta_J]_N] = 0, \\ \theta_{MLu}[p\theta_{qr}]_{IL} + \kappa \delta_u[p\theta_{qr}]_{IM} = 0. \end{cases} \quad (3.27)$$

An $\text{SO}(8)_{\text{diag}}$ singlet in θ_{IJrs} can be parametrized as

$$\theta_{IJrs} = \rho \delta_{r[I} \delta_{J]s}. \quad (3.28)$$

Eqs. (3.27) fix $\rho = -2\kappa$, thus leading to an embedding tensor

$$\theta = -\frac{1}{2}, \quad \theta_{IJrs} = \delta_{r[I} \delta_{J]s}. \quad (3.29)$$

The gauge group induced by this tensor is $\text{GL}(8)$. The spectrum around this vacuum is given in table 4(b).

Upon triality rotation of the second factor in eq. (3.24), the singlet in θ_{IJrs} would alternatively be given by

$$\theta_{IJrs} = \rho \Gamma_{rs}^{IJ}. \quad (3.30)$$

This however does not lead to a non-trivial solution of eqs. (3.27).

3.3 F(4)

3.3.1 Chiral embedding

We now consider the supergroup $\text{F}(4)$, with R -symmetry group $\text{SO}(7)$. We first assume that $\text{SO}(7)$ is entirely embedded into the first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$ according to eq. (2.21a), such that the potentially non-vanishing components of the embedding tensor reduce from eq. (3.8) to

$$\left\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \theta = \kappa, \theta_{p_+q_+r_+s_+}, \theta_{p_-q_-r_-s_-}, \right. \\ \left. \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}} \right\}, \quad \text{with traceless } \xi^{\dot{A}\dot{B}} \propto \text{diag}\{-7, \underbrace{1, \dots, 1}_7\}. \quad (3.31)$$

Δ_L	Δ_R	Δ	s	SO(7)	G_{ext}
	$4/3$	$13/6$	$1/2$	1	n
$5/6$	$5/6$	$5/3$	0	8	n
	$1/3$	$7/6$	$-1/2$	7	n

Table 5. Mass spectrum for F(4) solutions with chiral embedding (2.21a). The gauge groups are of the form $\text{SO}(7) \times G_{\text{ext}}$.

Note that a non-vanishing $\xi^{\dot{A}\dot{B}}$ is required in order to realize the breaking of R -symmetry from $\text{SO}(8)$ to $\text{SO}(7)$, whereas for vanishing $\xi^{\dot{A}\dot{B}}$ we are back to the case $\text{OSp}(8|2, \mathbb{R})$ discussed in section 3.2 above.

The remaining quadratic constraints for θ_{IJKL}^- and θ_{pqrs} are not coupled. For θ_{IJKL}^- , they are given by eqs. (B.1a) and (B.2b):

$$\begin{cases} \Lambda_{MN} \left[\theta_{MNP[I}^- \theta_{JKL]P}^- + 2(2\lambda - \kappa) \delta_{M[I}^- \theta_{JKL]N}^- \right] = 0, & (3.32a) \\ (\theta_{rs} + (\kappa - \lambda) \delta_{rs}) \theta_{IJKL}^- = 0. & (3.32b) \end{cases}$$

The second line shows that non-vanishing $\xi^{\dot{A}\dot{B}}$ requires $n_+ = 0$, thus

$$n = n_- . \quad (3.33)$$

We are left with eq. (3.32a), which fixes the proportionality constant in $\xi^{\dot{A}\dot{B}}$ to be

$$\xi^{\dot{A}\dot{B}} = \frac{(8+n)\lambda}{24n} \text{diag}\{-7, \underbrace{1, \dots, 1}_7\}. \quad (3.34)$$

Setting $\theta_{pqrs} = 0$ solves all remaining equations, in which case the embedding tensor is given by

$$\begin{aligned} \theta_{IJ} &= n \delta_{IJ}, & \theta_{rs} &= 8 \delta_{rs}, & \theta &= -(8-n), \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{(8+n)}{24} \text{diag}\{-7, \underbrace{1, \dots, 1}_7\}, \end{aligned} \quad (3.35)$$

up to an arbitrary overall scaling factor. The associated gauge group is $\text{SO}(7) \times \text{SO}(n)$. The spectrum is given in table 5, organized into supermultiplets of F(4).

Finally, we consider the possibility of non-vanishing θ_{pqrs} . The equations for θ_{pqrs} are the same as in section 3.2 above, and so are the solutions, with the only difference that eq. (3.33) restricts to vanishing n_+ . As in the $\text{OSp}(8|2, \mathbb{R})$ case, a non-vanishing θ_{pqrs} does not affect the spectrum of the theories which is still given by table 5. Rather, it will restrict the $\text{SO}(n)$ factor of the gauge group to some subgroup. Without repeating the details of the derivation, in the rest of this section, we simply list the different solutions for non-vanishing θ_{pqrs} , organized by the different values for n .

$n = 4$.

$$\begin{aligned} \theta_{IJ} &= 4\delta_{IJ}, & \theta_{rs} &= 8\delta_{rs}, & \theta &= -4, & \theta_{pqrs} &= 12\xi\varepsilon_{pqrs}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{1}{2}\text{diag}\{-7, \underbrace{1, \dots, 1}_7\}. \end{aligned} \quad (3.36)$$

If $|\xi| \neq 1$, the gauge group is $\text{SO}(7) \times \text{SO}(4)$, as in the case $\xi = 0$. Otherwise, the gauge group is $\text{SO}(7) \times \text{SO}(3)_{\pm}$, i.e. $\text{SO}(4) = \text{SO}(3)_+ \times \text{SO}(3)_-$ is reduced to one of its chiral factors.

$n = 6$.

$$\begin{aligned} \theta_{IJ} &= 6\delta_{IJ}, & \theta_{rs} &= 8\delta_{rs}, & \theta &= -2, & \theta_{pqrs} &= \frac{1}{2}\varepsilon_{pqrsuv}\xi^{uv}, & \xi_{uv} &= -14 \begin{pmatrix} -\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{7}{12}\text{diag}\{-7, \underbrace{1, \dots, 1}_7\}, \end{aligned} \quad (3.37)$$

where σ was introduced in eq. (3.14). The gauge group is $\text{SO}(7) \times \text{U}(3)$.

$n = 7$.

$$\begin{aligned} \theta_{IJ} &= 7\delta_{IJ}, & \theta_{rs} &= 8\delta_{rs}, & \theta &= -1, & \theta_{pqrs} &= \frac{5}{4}\varepsilon_{pqrsuvw}\omega^{uvw}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{5}{8}\text{diag}\{-7, \underbrace{1, \dots, 1}_7\}, \end{aligned} \quad (3.38)$$

where the G_2 invariant three-form ω^{uvw} was introduced in eq. (3.19). The gauge group is $\text{SO}(7) \times G_2$.

$n = 8$.

$$\begin{aligned} \theta_{IJ} &= 8\delta_{IJ}, & \theta_{rs} &= 8\delta_{rs}, & \theta &= 0, & \theta_{pqrs} &= \Gamma_{ab}^{pqrs}\tilde{\xi}^{ab}, \\ \text{with } \tilde{\xi}^{ab} &= \frac{2}{4-p}\text{diag}\{\underbrace{8-p, \dots, 8-p}_p, \underbrace{-p, \dots, -p}_{8-p}\}, & p &\neq 4, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{2}{3}\text{diag}\{-7, \underbrace{1, \dots, 1}_7\}, \end{aligned} \quad (3.39)$$

with products of $\text{SO}(8)$ Γ -matrices Γ_{ab}^{pqrs} . There is an analogous solution for anti-selfdual choice of θ_{pqrs} . The gauge group is $\text{SO}(7) \times \text{SO}(8-p) \times \text{SO}(p)$.

3.3.2 Diagonal embedding

Similar to eq. (3.24), the R -symmetry group $\text{SO}(7)$ could in principle be embedded diagonally into $\text{SO}(8, n)$, but unlike for $\text{OSp}(8|2, \mathbb{R})$ this does not give any new solution.

Δ_L	Δ_R	Δ	s	$\text{SO}(6) \times \text{U}(1)$	G_{ext}
1	$3/2$	$5/2$	$1/2$	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	$\mathbf{n}$
	1	2	0	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{n}$
	$1/2$	$3/2$	$-1/2$	$\mathbf{6}^0$	$\mathbf{n}$

Table 6. Mass spectrum for the $\text{SU}(4|1,1)$ solutions with chiral embedding (2.21a). The gauge groups are of the form $\text{SO}(6) \times \text{U}(1) \times G_{\text{ext}}$.

3.4 $\text{SU}(4|1,1)$

3.4.1 Chiral embedding

The supergroup $\text{SU}(4|1,1)$ has an R -symmetry group $\text{SO}(6) \times \text{SO}(2)$. We first consider its chiral embedding into the first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8,n)$. The components of the embedding tensor are then given by

$$\left\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \theta = \kappa, \theta_{p+q+r_+s_+}, \theta_{p-q-r_-s_-} \right. \\ \left. \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}} \right\}, \quad \text{with traceless } \xi^{\dot{A}\dot{B}} \propto \text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}. \quad (3.40)$$

The non-vanishing equations of the second quadratic constraint are eqs. (B.1), (B.2b), (B.3a), (B.3b), (B.4c), (B.5a) and (B.5c). Eq. (B.2b) in particular implies

$$(\theta_{rs} + (\kappa - \lambda) \delta_{rs}) \theta_{IJKL}^- = 0. \quad (3.41)$$

As in the $\text{F}(4)$ case, θ_{IJKL}^- is thus non vanishing only if

$$n = n_- . \quad (3.42)$$

We are thus left with the same parametrization and set of equations as in the $\text{F}(4)$ case. The proportionality constant in $\xi^{\dot{A}\dot{B}}$ is fixed by the remaining quadratic constraints to be

$$\xi^{\dot{A}\dot{B}} = \frac{(8+n)\lambda}{16n} \text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}. \quad (3.43)$$

Setting $\theta_{pqrs} = 0$ solves all remaining equations, in which case the full embedding tensor is given by

$$\theta_{IJ} = n \delta_{IJ}, \quad \theta_{rs} = 8 \delta_{rs}, \quad \theta = -(8-n), \\ \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, \quad \text{with } \xi^{\dot{A}\dot{B}} = \frac{(8+n)}{16} \text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}, \quad (3.44)$$

up to an arbitrary overall scaling factor. The associated gauge group is $\text{SO}(6) \times \text{SO}(2) \times \text{SO}(n)$. The spectrum is given in table 6, organized into supermultiplets of $\text{SU}(4|1,1)$.

In the remainder of this section, in analogy to the $\text{OSp}(8|2, \mathbb{R})$ case, we list the different solutions for non-vanishing θ_{pqrs} , organized by the different values for n . The spectrum of these theories is still given by table 6.

$n = 4$.

$$\begin{aligned} \theta_{IJ} &= 4\delta_{IJ}, & \theta_{rs} &= 8\delta_{rs}, & \theta &= -4, & \theta_{pqrs} &= 12\xi\epsilon_{pqrs}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{3}{4}\text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}. \end{aligned} \quad (3.45)$$

If $|\xi| \neq 1$ the gauge group is $\text{SO}(6) \times \text{SO}(2) \times \text{SO}(4)$, as in the case $\xi = 0$. Otherwise, the gauge group is $\text{SO}(6) \times \text{SO}(2) \times \text{SO}(3)_{\pm}$, i.e. $\text{SO}(4) = \text{SO}(3)_+ \times \text{SO}(3)_-$ is reduced to one of its chiral factors.

$n = 6$.

$$\begin{aligned} \theta_{IJ} &= 6\delta_{IJ}, \quad \theta_{rs} = 8\delta_{rs}, \quad \theta = -2, \quad \theta_{pqrs} = \frac{1}{2}\epsilon_{pqrsuv}\xi^{uv}, \quad \xi_{uv} = -14 \begin{pmatrix} -\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, \quad \text{with } \xi^{\dot{A}\dot{B}} = \frac{7}{8}\text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}, \end{aligned} \quad (3.46)$$

where σ was introduced in eq. (3.14). The gauge group is $\text{SO}(6) \times \text{SO}(2) \times \text{U}(3)$.

$n = 7$.

$$\begin{aligned} \theta_{IJ} &= 7\delta_{IJ}, \quad \theta_{rs} = 8\delta_{rs}, \quad \theta = -1, \quad \theta_{pqrs} = \frac{5}{4}\epsilon_{pqrsuvw}\omega^{uvw}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, \quad \text{with } \xi^{\dot{A}\dot{B}} = \frac{15}{16}\text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}, \end{aligned} \quad (3.47)$$

where the G_2 invariant three-form ω^{uvw} was introduced in eq. (3.19). The gauge group is $\text{SO}(6) \times \text{SO}(2) \times \text{G}_2$.

$n = 8$.

$$\begin{aligned} \theta_{IJ} &= 8\delta_{IJ}, \quad \theta_{rs} = 8\delta_{rs}, \quad \theta = 0, \quad \theta_{pqrs} = \Gamma_{ab}^{pqrs}\tilde{\xi}^{ab}, \\ \text{where } \tilde{\xi}^{ab} &= \frac{2}{4-p}\text{diag}\{\underbrace{8-p, \dots, 8-p}_p, \underbrace{-p, \dots, -p}_{8-p}\}, \quad p \neq 4, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL}\xi^{\dot{A}\dot{B}}, \quad \text{with } \xi^{\dot{A}\dot{B}} = \text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}, \end{aligned} \quad (3.48)$$

with products of $\text{SO}(8)$ Γ -matrices Γ_{ab}^{pqrs} . There is an analogous solution for anti-selfdual choice of θ_{pqrs} . The gauge group is $\text{SO}(6) \times \text{SO}(2) \times \text{SO}(8-p) \times \text{SO}(p)$.

3.4.2 Diagonal embedding

Alternatively, the R -symmetry group $\text{SO}(6) \times \text{SO}(2)$ or one of its factors can be diagonally embedded into $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$ for $n \geq 2$. The non-vanishing components of the

Δ_L	Δ_R	Δ	s	$\text{SO}(6) \times \text{U}(1)$	$\text{SO}(6) \times \text{U}(1)$
$3/2$	2	$7/2$	$1/2$	$\mathbf{1}^{+4} \oplus \mathbf{1}^{-4}$	$\mathbf{4}^Q$
	$3/2$	3	0	$\bar{\mathbf{4}}^{+3} \oplus \mathbf{4}^{-3}$	$\mathbf{4}^Q$
	1	$5/2$	$-1/2$	$\mathbf{6}^{+2} \oplus \mathbf{1}^0 \oplus \mathbf{1}^0 \oplus \mathbf{6}^{-2}$	$\mathbf{4}^Q$
	$1/2$	2	-1	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{4}^Q$

Table 7. Mass spectrum for the $\text{SU}(4|1,1)$ solution with diagonal embedding (2.21a) with gauge group $\text{U}(4,4)$.

embedding tensor are given by

$$\left\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{r+s+} = -\lambda \delta_{r+s+}, \theta_{r-s-} = (\lambda - \kappa) \delta_{r-s-}, \theta = \kappa, \right. \\ \left. \theta_{IJKL}^-, \theta_{IJr+s+}, \theta_{p+q+r+s+}, \theta_{p-q-r-s-} \right\}. \quad (3.49)$$

The set of non-trivial constraints which follow from the second quadratic constraint is the same as the one given for the chiral embedding. The new solutions are listed below. They are all defined in terms of the matrices

$$\xi^{\dot{A}\dot{B}} = \text{diag}\{-6, -6, \underbrace{2, \dots, 2}_6\}, \quad \tilde{\xi}^{\dot{A}\dot{B}} = \begin{pmatrix} \sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (3.50)$$

defined in accordance with the embedding (2.21a). The 2×2 matrix σ has been introduced in eq. (3.14).

$n_+ = 8$ (1). The first new solution with $n_+ = 8$ and $\text{SO}(6) \times \text{SO}(2)$ entirely embedded in $\text{SO}(8) \times \text{SO}(8) \subset \text{SO}(8,8)$ has the form

$$\theta = -16, \quad \theta_{pqrs} = \Gamma_{\dot{A}\dot{B}}^{pqrs} \xi^{\dot{A}\dot{B}}, \quad \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, \quad \theta_{IJrs} = \pm 4 \Gamma_{\dot{A}\dot{B}}^{IJ} \tilde{\xi}^{\dot{A}\dot{B}} \Gamma_{\dot{C}\dot{D}}^{rs} \tilde{\xi}^{\dot{C}\dot{D}}. \quad (3.51)$$

The gauge group is $\text{U}(4,4)$. The spectrum is given in table 7.

$n_+ = 8$ (2). The second solution with $n_+ = 8$ and $\text{SO}(6) \times \text{SO}(2)$ entirely embedded in $\text{SO}(8) \times \text{SO}(8) \subset \text{SO}(8,8)$ is

$$\theta = -16, \quad \theta_{pqrs} = \Gamma_{\dot{A}\dot{B}}^{pqrs} \xi^{\dot{A}\dot{B}}, \quad \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, \\ \theta_{IJrs} = 32 \delta_{I[r} \delta_{s]J} - \Gamma_{\dot{A}\dot{B}}^{IJrs} \xi^{\dot{A}\dot{B}} + 8 \Gamma_{\dot{A}\dot{B}}^{IJ} \tilde{\xi}^{\dot{A}\dot{B}} \Gamma_{\dot{C}\dot{D}}^{rs} \tilde{\xi}^{\dot{C}\dot{D}}. \quad (3.52)$$

The gauge group is $\text{SL}(2) \times \text{Sp}(4, \mathbb{R})$. The spectrum is given in table 8.

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{U}(1)$
2	3	5	1	$\mathbf{1}^0 \oplus \mathbf{1}^0$	Q
	$5/2$	$9/2$	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1} \oplus \mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	Q
	2	4	0	$\mathbf{6}^{+2} \oplus \mathbf{15}^0 \oplus \mathbf{15}^0 \oplus \mathbf{6}^{-2}$	Q
	$3/2$	$7/2$	$-1/2$	$\bar{\mathbf{4}}^{+3} \oplus \mathbf{20}^{+1} \oplus \bar{\mathbf{20}}^{-1} \oplus \mathbf{4}^{-3}$	Q
	1	3	-1	$\bar{\mathbf{10}}^{+2} \oplus \mathbf{10}^{-2}$	Q

Table 8. Mass spectrum for the $\text{SU}(4|1,1)$ solution with diagonal embedding (2.21a) with gauge group $\text{SL}(2) \times \text{Sp}(4, \mathbb{R})$.

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{SO}(n_-)^{\text{U}(1)}$
$3/2$	2	$7/2$	$1/2$	$\mathbf{1}^{+4} \oplus \mathbf{1}^{-4}$	$\mathbf{1}^Q$
	$3/2$	3	0	$\bar{\mathbf{4}}^{+3} \oplus \mathbf{4}^{-3}$	$\mathbf{1}^Q$
	1	$5/2$	$-1/2$	$\mathbf{6}^{+2} \oplus \mathbf{1}^0 \oplus \mathbf{1}^0 \oplus \mathbf{6}^{-2}$	$\mathbf{1}^Q$
	$1/2$	2	-1	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{1}^Q$
1	$3/2$	$5/2$	$1/2$	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	$\mathbf{n}_-^0$
	1	2	0	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{n}_-^0$
	$1/2$	$3/2$	$-1/2$	$\mathbf{6}^0$	$\mathbf{n}_-^0$

Table 9. Mass spectrum for the $\text{SU}(4|1,1)$ solution with diagonal embedding (2.21a) with gauge group $\text{SO}(n_-) \times \text{U}(4,1)$.

$\mathbf{n}_+ = \mathbf{2}$. Finally, there is a solution with $n_+ = 2$ and only the factor $\text{SO}(2)$ embedded in $\text{SO}(8) \times \text{SO}(2 + n_-) \subset \text{SO}(8, 2 + n_-)$, given by

$$\begin{aligned}
 \theta_{IJ} &= \frac{n_-}{2} \delta_{IJ}, & \theta_{r-s-} &= 5 \delta_{r-s-}, & \theta_{r+s+} &= -\frac{n_-}{2} \delta_{r+s+}, & \theta &= n_- - 8, \\
 \theta_{IJKL}^- &= \frac{1}{2} \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \theta_{IJr+s+} &= \pm 4 \Gamma_{\dot{A}\dot{B}}^{IJ} \tilde{\xi}^{\dot{A}\dot{B}} \sigma_{r+s+},
 \end{aligned} \tag{3.53}$$

where σ was introduced in eq. (3.14). The gauge group is $\text{SO}(n_-) \times \text{U}(4,1)$. The spectrum is given in table 9.

3.5 $\text{OSp}(4^*|4)$

3.5.1 Chiral embedding

We now turn to the last possible supergroup. $\text{OSp}(4^*|4)$ has R -symmetry group $\text{SO}(5) \times \text{SO}(3)$ and we first consider its chiral embedding according to eq. (2.21a) into the first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$. The potentially non-vanishing components of the embedding tensor are then given by

$$\left\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{r+s+} = -\lambda \delta_{r+s+}, \theta_{r-s-} = (\lambda - \kappa) \delta_{r-s-}, \theta = \kappa, \theta_{p+q+r+s+}, \theta_{p-q-r-s-}, \right. \\
 \left. \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}} \right\}, \quad \text{with traceless } \xi^{\dot{A}\dot{B}} \propto \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}. \tag{3.54}$$

Δ_L	Δ_R	Δ	s	$\text{SO}(6) \times \text{U}(1)$	G_{ext}
	2	$7/2$	$1/2$	(1, 3)	n
$3/2$	$3/2$	3	0	(4, 2)	n
	1	$5/2$	$-1/2$	(5, 1)	n

Table 10. Mass spectrum for the $\text{OSp}(4^*|4)$ solution with chiral embedding (2.21a). The gauge groups are of the form $\text{SO}(5) \times \text{SO}(3) \times \text{G}_{\text{ext}}$.

Again, the second quadratic constraint (2.9b) implies that θ_{IJKL}^- is non-vanishing only if

$$n = n_- , \quad (3.55)$$

which we will assume in the following. The only difference with the case F(4) is the signature of $\xi^{\dot{A}\dot{B}}$ in θ_{IJKL}^- , which is fixed by the $\text{SO}(5) \times \text{SO}(3)$ invariance.

Setting $\theta_{pqrs} = 0$ solves all remaining equations, in which case the full embedding tensor is given by

$$\begin{aligned} \theta_{IJ} &= n \delta_{IJ}, & \theta_{rs} &= 8 \delta_{rs}, & \theta &= -(8 - n), \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{(8+n)}{8} \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}, \end{aligned} \quad (3.56)$$

up to an arbitrary overall scaling factor. The gauge group is $\text{SO}(5) \times \text{SO}(3) \times \text{SO}(n)$. The spectrum is given in table 10 organized into supermultiplets of $\text{OSp}(4^*|4)$.

In the remainder of this section, in analogy to the $\text{OSp}(8|2, \mathbb{R})$ case, we list the different solutions for non-vanishing θ_{pqrs} , organized by the different values for n . The spectrum of these theories is still given by table 10, with non-trivial θ_{pqrs} only reducing the $\text{SO}(n)$ factor of the gauge group.

$n = 4$.

$$\begin{aligned} \theta_{IJ} &= 4 \delta_{IJ}, & \theta_{rs} &= 8 \delta_{rs}, & \theta &= -4, & \theta_{pqrs} &= 12 \xi \varepsilon_{pqrs}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{3}{2} \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}. \end{aligned} \quad (3.57)$$

If $|\xi| \neq 1$ the gauge group is $\text{SO}(5) \times \text{SO}(3) \times \text{SO}(4)$, as in the case $\xi = 0$. Otherwise, the gauge group is reduced to $\text{SO}(5) \times \text{SO}(3) \times \text{SO}(3)_{\pm}$, after factorizing $\text{SO}(4) = \text{SO}(3)_+ \times \text{SO}(3)_-$.

$n = 6$.

$$\begin{aligned} \theta_{IJ} &= 6 \delta_{IJ}, & \theta_{rs} &= 8 \delta_{rs}, & \theta &= -2, & \theta_{pqrs} &= \frac{1}{2} \varepsilon_{pqrsuv} \xi^{uv}, & \xi_{uv} &= -14 \begin{pmatrix} -\sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{7}{4} \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}, \end{aligned} \quad (3.58)$$

where σ was introduced in eq. (3.14). The gauge group is $\text{SO}(5) \times \text{SO}(3) \times \text{U}(3)$.

$n = 7$.

$$\begin{aligned} \theta_{IJ} &= 7 \delta_{IJ}, & \theta_{rs} &= 8 \delta_{rs}, & \theta &= -1, & \theta_{pqrs} &= \frac{5}{4} \varepsilon_{pqrsuvw} \omega^{uvw}, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= \frac{15}{8} \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}, \end{aligned} \quad (3.59)$$

where the G_2 invariant three-form ω^{uvw} was introduced in eq. (3.19). The gauge group is $SO(5) \times SO(3) \times G_2$.

$n = 8$.

$$\begin{aligned} \theta_{IJ} &= 8 \delta_{IJ}, & \theta_{rs} &= 8 \delta_{rs}, & \theta &= 0, & \theta_{pqrs} &= \Gamma_{ab}^{pqrs} \tilde{\xi}^{ab}, \\ \text{where } \tilde{\xi}^{ab} &= \frac{2}{4-p} \text{diag}\{\underbrace{8-p, \dots, 8-p}_p, \underbrace{-p, \dots, -p}_{8-p}\}, & p &\neq 4, \\ \theta_{IJKL}^- &= \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, & \text{with } \xi^{\dot{A}\dot{B}} &= 2 \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}, \end{aligned} \quad (3.60)$$

with products of $SO(8)$ Γ -matrices Γ_{ab}^{pqrs} . There is an analogous solution for anti-selfdual choice of θ_{pqrs} . The gauge group is $SO(5) \times SO(3) \times SO(8-p) \times SO(p)$.

3.5.2 Diagonal embedding

Alternatively, for $n \geq 3$ we may consider embedding of the R -symmetry group $SO(5) \times SO(3)$ of $OSp(4^*|4)$ or one of its factors diagonally into $SO(8) \times SO(n) \subset SO(8, n)$. The potentially non-vanishing components of the embedding tensor are

$$\begin{aligned} &\left\{ \theta_{IJ} = \lambda \delta_{IJ}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \theta = \kappa, \right. \\ &\left. \theta_{IJKL}^-, \theta_{IJr_+s_+}, \theta_{p+q+r_+q_+} \right\}. \end{aligned} \quad (3.61)$$

The non vanishing equations given by the second quadratic constraint are the same as in the $SU(4|1, 1)$ case. There is only one new solution, for $n_+ = 8$ and $SO(5) \times SO(3)$ entirely embedded into $SO(8) \times SO(8) \subset SO(8, 8)$, given by

$$\begin{aligned} \theta &= -\frac{1}{2}, \theta_{pqrs} = \Gamma_{\dot{A}\dot{B}}^{pqrs} \xi^{\dot{A}\dot{B}}, \theta_{IJKL}^- = \Gamma_{\dot{A}\dot{B}}^{IJKL} \xi^{\dot{A}\dot{B}}, \text{ with } \xi^{\dot{A}\dot{B}} = \frac{1}{16} \text{diag}\{-5, -5, -5, \underbrace{3, \dots, 3}_5\}, \\ \theta_{IJrs} &= -\frac{1}{4} \Gamma_{\dot{\alpha}\dot{\beta}}^{IJ} \Gamma_{\dot{\alpha}\dot{\beta}}^{rs}, \quad \dot{\alpha}, \dot{\beta} \in \{1, 2, 3\}, \end{aligned} \quad (3.62)$$

where the sum in the last line runs over the $SO(3)$ vector according to the decomposition of the cospinor $\dot{A}$ into the vector of $SO(3) \times SO(5)$. The gauge group is $Sp(2, 2) \times SO(3)$. The spectrum is given in table 11.

4 Solutions with reducible vector embedding (2.21b)

We now consider the second possible embedding (2.21b) of the R -symmetry $SO(p) \times SO(q)$ into $SO(8, n)$ according to which the $SO(8)$ vector breaks into the vector of $SO(p) \times SO(q)$. We denote it as

$$I \longrightarrow \{i, \alpha\}, \quad i \in \llbracket 1, p \rrbracket, \quad \alpha \in \llbracket p+1, 8 \rrbracket. \quad (4.1)$$

Δ_L	Δ_R	Δ	s	$\text{SO}(5) \times \text{SO}(3)$	$\text{SO}(5)$
2	$5/2$	$9/2$	$1/2$	$(\mathbf{1}, \mathbf{4})$	4
	2	4	0	$(\mathbf{4}, \mathbf{3})$	4
	$3/2$	$7/2$	$-1/2$	$(\mathbf{1}, \mathbf{2}) \oplus (\mathbf{5}, \mathbf{2})$	4
	1	3	-1	$(\mathbf{4}, \mathbf{1})$	4

Table 11. Mass spectrum for the $\text{OSp}(4^*|4)$ solution for diagonal embedding (2.21a) with gauge group $\text{Sp}(2, 2) \times \text{SO}(3)$.

The only relevant supergroups are $\text{F}(4)$, $\text{SU}(4|1, 1)$ and $\text{OSp}(4^*|4)$, as for $\text{OSp}(8|2, \mathbb{R})$ both embeddings (2.21a), (2.21b) are equivalent. In this case, the potentially non-vanishing components of the embedding tensor are then of the form

$$\left\{ \theta_{ij} = \lambda \delta_{ij}, \theta_{\alpha\beta} = \frac{\theta_{rr} - p\lambda}{q} \delta_{\alpha\beta}, \theta_{rs}, \theta \equiv \kappa, \theta_{IJKL}^-, \theta_{IJrs}, \theta_{Ipqr}, \theta_{pqrs} \right\}, \quad (4.2)$$

with anti-selfdual θ_{IJKL}^- . The fact, that the embedding tensor is singlet under the respective R -symmetry group will pose further constraints on these components that we shall evaluate case by case in the following.

With the parametrization (4.2), the first quadratic constraint (2.9a), with free index values $(\mathcal{M}, \mathcal{N}) = (i, \alpha)$ and depending on the different Λ parameters, gives the following set of independent equations:

$$\begin{cases} (8\lambda - \theta_{rr})(\theta_{rr} - (p - q)\lambda - q\kappa) = 0, & (4.3a) \end{cases}$$

$$\begin{cases} (8\lambda - \theta_{rr}) \theta_{ij\alpha\beta}^- = 0, & (4.3b) \end{cases}$$

$$\begin{cases} (8\lambda - \theta_{rr}) \theta_{\alpha ijk}^- = 0, & (4.3c) \end{cases}$$

$$\begin{cases} (8\lambda - \theta_{rr}) \theta_{i\alpha\beta\gamma}^- = 0. & (4.3d) \end{cases}$$

Eq. (4.3a) leaves two options, imposing either of the two factors to vanish. Setting $\theta_{rr} = 8\lambda$ however implies that $\theta_{IJ} = \lambda \delta_{IJ}$, i.e. we go back to the case of an irreducible vector embedding (2.21a) with the parametrization (3.2) carried out in section 3.⁷ We thus set for the rest of this section

$$\theta_{rr} = (p - q)\lambda + q\kappa, \quad (4.4)$$

such that eqs. (4.3) together with the anti-selfduality of θ_{IJKL}^- imply that

$$\theta_{IJKL}^- = 0. \quad (4.5)$$

⁷Strictly speaking, $\theta_{IJ} = \lambda \delta_{IJ}$ could also be realized with an embedding (2.21a) in which case the breaking (4.1) of the vector representation would only be visible on the components of $\theta_{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}}$. However, the remaining quadratic constraints rule out this possibility.

We now focus on the remaining components of the first quadratic constraint (2.9a). With indices $(\mathcal{M}, \mathcal{N}) = (i, r)$ and $(\mathcal{M}, \mathcal{N}) = (\alpha, r)$, it gives the independent equations:

$$\begin{cases} \theta_{rp} \theta_{ps} + \kappa \theta_{rs} + \lambda (\kappa - \lambda) \delta_{rs} = 0, & (4.6a) \\ \lambda \theta_{ijrs} + \theta_{ijrp} \theta_{ps} = 0, & (4.6b) \\ \lambda \theta_{ipqr} + \theta_{ipqs} \theta_{sr} = 0. & (4.6c) \\ (\kappa - \lambda) \theta_{\alpha\beta rs} + \theta_{\alpha\beta rp} \theta_{ps} = 0, & (4.6d) \\ (\kappa - \lambda) \theta_{\alpha pqr} + \theta_{\alpha pqs} \theta_{sr} = 0, & (4.6e) \\ \lambda \theta_{\alpha irs} + \theta_{\alpha irp} \theta_{ps} = 0, & (4.6f) \\ (\kappa - \lambda) \theta_{\alpha irs} + \theta_{\alpha irp} \theta_{ps} = 0. & (4.6g) \end{cases}$$

As in eq. (3.3a) above, eq. (4.6a) implies that we can take θ_{rs} to be a diagonal matrix with eigenvalues

$$\theta_+ = -\lambda, \quad \theta_- = \lambda - \kappa, \quad (4.7)$$

with multiplicities that, as above, we denote as $n_{\pm}$. Tracelessness of $\theta_{\mathcal{MN}}$ then implies that

$$(p - q + n_+ - n_-) \lambda = -(n_- + q) \kappa. \quad (4.8)$$

Finally, the component $(\mathcal{M}, \mathcal{N}) = (r, s)$ of the first quadratic constraint is unchanged compared to the previous section (see eqs. (3.7)) and together with eqs. (4.6) it implies that the only potentially non-vanishing components of the embedding tensor are

$$\left\{ \theta_{ij} = \lambda \delta_{ij}, \theta_{\alpha\beta} = -(\lambda - \kappa) \delta_{\alpha\beta}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \quad (4.9) \right.$$

$$\left. \theta = \kappa, \theta_{ijr_+s_+}, \theta_{\alpha\beta r_-s_-}, \theta_{ip_+q_+r_+}, \theta_{\alpha p_-q_-r_-}, \theta_{p_+q_+r_+s_+}, \theta_{p_-q_-r_-s_-} \right\}. \quad (4.10)$$

In the following, we solve the second quadratic constraint (2.9b) in this parametrization for the different supergroups with chiral and diagonal embeddings, respectively. Again, for readability we defer the full set of constraint equations to appendix B.

4.1 F(4)

4.1.1 Chiral embedding

We first consider the supergroup F(4), for which $p = 7$ and $q = 1$, so that the index splitting (4.1) gives $i \in \llbracket 1, 7 \rrbracket$ and $\alpha = 8$. We first assume that the R -symmetry group $\text{SO}(7)$ is entirely embedded into the first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$. The potentially non-vanishing components of the embedding tensor are

$$\left\{ \theta_{ij} = \lambda \delta_{ij}, \theta_{88} = -(\lambda - \kappa), \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \quad (4.11) \right.$$

$$\left. \theta = \kappa, \theta_{8p_-q_-r_-}, \theta_{p_+q_+r_+s_+}, \theta_{p_-q_-r_-s_-} \right\}. \quad (4.12)$$

Setting all $\theta_{\mathcal{MNPQ}}$ to zero always gives a solution to the remaining quadratic constraints. This yields an embedding tensor of the form

$$\begin{aligned}\theta_{ij} &= (1 + n_-) \delta_{ij}, & \theta_{88} &= -(7 + n_+), & \theta &= -(6 + n_+ - n_-), \\ \theta_{r_+s_+} &= -(1 + n_-) \delta_{r_+s_+}, & \theta_{r_-s_-} &= (7 + n_+) \delta_{r_-s_-}.\end{aligned}\quad (4.13)$$

The gauge group is $\text{SO}(7, n_+) \times \text{SO}(1, n_-)$. The spectrum is given in table 12(a).

If $\theta_{\mathcal{MNPQ}}$ is not vanishing, the set of equations given by the second quadratic constraint is composed of eqs. (B.3a), (B.3c), (B.4), (B.5a) and (B.5b). Eq. (B.4c) implies

$$(2\lambda - \kappa) \theta_{pqr s_+} = 0, \quad (4.14)$$

which implies that $\theta_{pqr s}$ has $\theta_{p-q-r-s_-}$ as only non-vanishing components. We are then left with the following set of independent equations:

$$\begin{cases} (2\lambda - \kappa) \theta_{p-q-r-s_-} = -3 \theta_{8u_-[p-q-} \theta_{r-s_-]u_-8}, \\ 3 \theta_{8s_-[p-q-} \theta_{r-]u_-v-s_-} \\ -\theta_{8s_-u_-v-} \theta_{p-q-r-s_-} = 3(2\lambda - \kappa) (\theta_{8u_-[p-q-} \delta_{r-]v_-} - \theta_{8v_-[p-q-} \delta_{r-]u_-}), \\ \theta_{t-u_-v_-[p-} \theta_{q-r-s_-]t_-} = (2\lambda - \kappa) \delta_{u_-[p-} \theta_{q-r-s_-]v_-} - (2\lambda - \kappa) \delta_{v_-[p-} \theta_{q-r-s_-]u_-} \\ \quad + \theta_{8u_-v_-[p-} \theta_{q-r-s_-]8}. \end{cases} \quad (4.15)$$

It has a non-vanishing solution for $n_- = 3$ only, which gives the total embedding tensor

$$\begin{aligned}\theta_{ij} &= 4 \delta_{ij}, & \theta_{88} &= -(7 + n_+), & \theta_{r_+s_+} &= -4 \delta_{r_+s_+}, & \theta_{r_-s_-} &= (7 + n_+) \delta_{r_-s_-}, \\ \theta &= -(3 + n_+), & \theta_{8p-q-r-} &= \xi \varepsilon_{p-q-r-}.\end{aligned}\quad (4.16)$$

The gauge group is $\text{SO}(7, n_+) \times \text{SO}(1, 3)$, as in the $\xi = 0$ case (4.13). The spectrum is also the same as given in table 12(a).

4.1.2 Diagonal embedding

The R -symmetry group could also be embedded diagonally into $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$ for $n \geq 7$. There is only one new solution for $n_+ = 7$, given by

$$\begin{aligned}\theta_{ij} &= (n_- + 1) \delta_{ij}, & \theta_{88} &= -14, & \theta_{r_+s_+} &= -(n_- + 1) \delta_{r_+s_+}, & \theta_{r_-s_-} &= 14 \delta_{r_-s_-}, \\ \theta &= (n_- - 13), & \theta_{ijr_+s_+} &= 2(n_- + 15) \delta_{i[r_+} \delta_{s_+]j}.\end{aligned}\quad (4.17)$$

The gauge group is $\text{GL}(7) \times \text{SO}(1, n_-)$ and the spectrum is given in table 12(b).

4.2 $\text{SU}(4|1, 1)$

4.2.1 Chiral embedding

For the supergroup $\text{SU}(4|1, 1)$, $p = 6$ and $q = 2$ and the index splitting (4.1) gives $i \in \llbracket 1, 6 \rrbracket$ and $\alpha \in \{7, 8\}$. The R -symmetry group $\text{SO}(6) \times \text{SO}(2)$ could first be embedded into the

Δ_L	Δ_R	Δ	s	SO(7)	SO(n_+)	SO(n_-)
$1/3$	$4/3$	$5/3$	1	1	1	n_-
	$5/6$	$7/6$	$1/2$	8	1	n_-
	$1/3$	$2/3$	0	7	1	n_-
$4/3$	$4/3$	$8/3$	0	1	n_+	1
	$5/6$	$13/6$	$-1/2$	8	n_+	1
	$1/3$	$5/3$	-1	7	n_+	1

(a) $\text{SO}(7, n_+) \times \text{SO}(1, n_-)$.

Δ_L	Δ_R	Δ	s	SO(7)	SO(n_-)
$1/3$	$4/3$	$5/3$	1	1	n_-
	$5/6$	$7/6$	$1/2$	8	n_-
	$1/3$	$2/3$	0	7	n_-
$5/3$	$8/3$	$13/3$	1	1	1
	$13/6$	$23/6$	$1/2$	8	1
	$5/3$	$10/3$	0	7 $\oplus$ 21	1
	$7/6$	$17/6$	$-1/2$	48	1
	$2/3$	$7/3$	-1	27	1

(b) $\text{GL}(7) \times \text{SO}(1, n_-)$.

Table 12. Mass spectra for the F(4) solutions with (a) chiral and (b) diagonal embeddings (2.21b).

first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$. The potentially non-vanishing components of the embedding tensor are then given by

$$\left\{ \theta_{ij} = \lambda \delta_{ij}, \theta_{\alpha\beta} = -(\lambda - \kappa) \delta_{\alpha\beta}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \right. \quad (4.18)$$

$$\left. \theta = \kappa, \theta_{\alpha\beta r_-s_-} = \sigma_{\alpha\beta} \Omega_{r_-s_-}, \theta_{p_+q_+r_+s_+}, \theta_{p_-q_-r_-s_-} \right\}, \quad (4.19)$$

where $\Omega_{r_-s_-}$ is an anti-symmetric matrix and σ has been introduced in eq. (3.14). A first solution is given by setting all $\theta_{\mathcal{MN}\mathcal{P}\mathcal{Q}} = 0$. The embedding tensor then has the form

$$\begin{aligned} \theta_{ij} &= (2 + n_-) \delta_{ij}, & \theta_{\alpha\beta} &= -(6 + n_+) \delta_{\alpha\beta}, & \theta &= -(4 + n_+ - n_-), \\ \theta_{r_+s_+} &= -(2 + n_-) \delta_{r_+s_+}, & \theta_{r_-s_-} &= (6 + n_+) \delta_{r_-s_-}. \end{aligned} \quad (4.20)$$

The gauge group is $\text{SO}(6, n_+) \times \text{SO}(2, n_-)$. The spectrum is given in table 13(a).

Let us now turn to the second quadratic constraint (2.9b) with a non-vanishing $\theta_{\mathcal{MN}\mathcal{P}\mathcal{Q}}$. It reduces to eqs. (B.2b), (B.3a), (B.3b), (B.4c), (B.5a) and (B.5c). From eq. (B.4c), we find

$$(2\lambda - \kappa) \theta_{pqrs_+} = 0, \quad (4.21)$$

Δ_L	Δ_R	Δ	s	$\text{SO}(6) \times \text{U}(1)$	$\text{SO}(n_+)$	$\text{SO}(n_-)$
$1/2$	$3/2$	2	1	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	$\mathbf{1}$	$\mathbf{n}_-$
	1	$3/2$	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{1}$	$\mathbf{n}_-$
	$1/2$	1	0	$\mathbf{6}^0$	$\mathbf{1}$	$\mathbf{n}_-$
$3/2$	$3/2$	3	0	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	$\mathbf{n}_+$	$\mathbf{1}$
	1	$5/2$	$-1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{n}_+$	$\mathbf{1}$
	$1/2$	2	-1	$\mathbf{6}^0$	$\mathbf{n}_+$	$\mathbf{1}$

 (a) $\text{SO}(6, n_+) \times \text{SO}(2, n_-)$.

Δ_L	Δ_R	Δ	s	$(\text{SO}(6) \times \text{SU}(m))^{\text{U}(1)}$	$\text{SO}(n_+)$	$\text{U}_{\text{ext}}(1)$
1	2	3	1	$(\mathbf{1}, m)^{+4} \oplus (\mathbf{1}, \bar{m})^{-4}$	$\mathbf{1}$	0
	$3/2$	$5/2$	$1/2$	$(\mathbf{4}, m)^{+3} \oplus (\bar{\mathbf{4}}, \bar{m})^{-3}$	$\mathbf{1}$	0
	1	2	0	$(\mathbf{6}, m)^{+2} \oplus (\mathbf{6}, \bar{m})^{-2}$	$\mathbf{1}$	0
	$1/2$	$3/2$	$-1/2$	$(\bar{\mathbf{4}}, m)^{+1} \oplus (\mathbf{4}, \bar{m})^{-1}$	$\mathbf{1}$	0
	0	1	-1	$(\mathbf{1}, m)^0 \oplus (\mathbf{1}, \bar{m})^0$	$\mathbf{1}$	0
$3/2$	$3/2$	3	0	$(\mathbf{1}, \mathbf{1})^{+2} \oplus (\mathbf{1}, \mathbf{1})^{-2}$	$\mathbf{n}_+$	0
	1	$5/2$	$-1/2$	$(\mathbf{4}, \mathbf{1})^{+1} \oplus (\bar{\mathbf{4}}, \bar{m})^{-1}$	$\mathbf{n}_+$	0
	$1/2$	2	-1	$(\mathbf{6}, \bar{m})^0$	$\mathbf{n}_+$	0

 (b) $\text{SO}(6, n_+) \times \text{U}(m, 1)$.

Table 13. Mass spectra for $\text{SU}(4|1, 1)$ solutions for chiral embedding (2.21b) with (a) $\theta_{\mathcal{MNPQ}} = 0$ and (b) $\theta_{\mathcal{MNPQ}} \neq 0$. These are the multiplets of $\text{SU}(|1, 1)$ given in table 1 of ref. [23] for $m = n_-/2$.

which implies once again that the only non-vanishing components of θ_{pqrs} are $\theta_{p-q-r-s_-}$. The other equations are then equivalent to the following set of independent equations:

$$\begin{cases} (2\lambda - \kappa) \theta_{p-q-r-s_-} = -3 \Omega_{[p-q} \Omega_{r-s_-]}, & (4.22a) \end{cases}$$

$$\begin{cases} \theta_{p-q-r-s_-} \Omega_{s_-u_-} = 3(2\lambda - \kappa) \Omega_{[p-q} \delta_{r-]u_-}, & (4.22b) \end{cases}$$

$$\begin{cases} \theta_{u-v-l_-[p-} \theta_{q-r-s_-]l_-} = (2\lambda - \kappa) [\theta_{u-[q-r-s_-} \delta_{p-]v_-} - \theta_{v-[q-r-s_-} \delta_{p-]u_-}]. & (4.22c) \end{cases}$$

As Ω_{p-q-} is antisymmetric, there exists a basis where it has the form

$$\begin{pmatrix} f_1 \sigma & & \\ & \ddots & \\ & & f_m \sigma \end{pmatrix} \quad \text{if } n_- = 2m, \text{ and } \begin{pmatrix} f_1 \sigma & & \\ & \ddots & \\ & & f_m \sigma \\ & & & 0 \end{pmatrix} \quad \text{if } n_- = 2m + 1, \quad (4.23)$$

with scalar functions $f_1, \dots, f_m$, and σ from eq. (3.14). With this parametrization, eqs. (4.22) imply that

$$\begin{cases} f_i f_j^2 = (2\lambda - \kappa)^2 f_i, \\ f_i f_j f_k^2 = (2\lambda - \kappa)^2 f_i f_j, \end{cases} \quad i, j, k \in \llbracket 1, m \rrbracket, i \neq j \neq k, \quad (4.24)$$

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{SO}(n_+)$	$\text{U}_{\text{ext}}(1)$
$\frac{1}{2} + \frac{ \xi }{2}$	$\frac{3}{2} + \frac{ \xi }{2}$	$2 + \xi $	1	$\mathbf{1}^{+4} \oplus \mathbf{1}^{-4}$	$\mathbf{1}$	Q
	$1 + \frac{ \xi }{2}$	$\frac{3}{2} + \xi $	$1/2$	$\mathbf{4}^{+3} \oplus \bar{\mathbf{4}}^{-3}$	$\mathbf{1}$	Q
	$\frac{1}{2} + \frac{ \xi }{2}$	$1 + \xi $	0	$\mathbf{6}^{+2} \oplus \mathbf{6}^{-2}$	$\mathbf{1}$	Q
	$\frac{ \xi }{2}$	$\frac{1}{2} + \xi $	$-1/2$	$\bar{\mathbf{4}}^{+1} \oplus \mathbf{4}^{-1}$	$\mathbf{1}$	Q
	$-\frac{1}{2} + \frac{ \xi }{2}$	$ \xi $	-1	$\mathbf{1}^0 \oplus \mathbf{1}^0$	$\mathbf{1}$	Q
$3/2$	$3/2$	3	0	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	n_+	0
	1	$5/2$	$-1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	n_+	0
	$1/2$	2	-1	$\mathbf{6}^0$	n_+	0

 (a) $1 \leq |\xi|$.

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{SO}(n_+)$	$\text{U}_{\text{ext}}(1)$
$\frac{1}{2} + \frac{ \xi }{2}$	$\frac{3}{2} + \frac{ \xi }{2}$	$2 + \xi $	1	$\mathbf{1}_{+1} \oplus \mathbf{1}^{-2}$	$\mathbf{1}$	Q
	$1 + \frac{ \xi }{2}$	$\frac{3}{2} + \xi $	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{1}$	Q
	$\frac{1}{2} + \frac{ \xi }{2}$	$1 + \xi $	0	$\mathbf{6}^0$	$\mathbf{1}$	Q
$\frac{1}{2} - \frac{ \xi }{2}$	$\frac{3}{2} - \frac{ \xi }{2}$	$2 - \xi $	1	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	$\mathbf{1}$	Q
	$1 - \frac{ \xi }{2}$	$\frac{3}{2} - \xi $	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{1}$	Q
	$\frac{1}{2} - \frac{ \xi }{2}$	$1 - \xi $	0	$\mathbf{6}^0$	$\mathbf{1}$	Q
$3/2$	$3/2$	3	0	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	n_+	0
	1	$5/2$	$-1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	n_+	0
	$1/2$	2	-1	$\mathbf{6}^0$	n_+	0

 (b) $|\xi| < 1$.

Table 14. Mass spectra for the $\text{SU}(4|1, 1)$ solutions for chiral embedding (2.21b) with $\theta_{\mathcal{MNPQ}} \neq 0$ and $n_- = 2$. The gauge group is $\text{SO}(6, n_+) \times \text{SO}(2, 1)$ for $|\xi| = 1$ and $\text{SO}(6, n_+) \times \text{SO}(2, 2)$ otherwise.

if $n_- = 2m$, whereas they imply that all f_i vanish for $n_- = 2m + 1$. Thus, for $\kappa \neq 2\lambda$ there are solutions with non-vanishing $\theta_{\mathcal{MNPQ}}$ only if $n_- = 2m$ is even, and they are given by

$$\begin{aligned}
 \theta_{ij} &= (2 + n_-) \delta_{ij}, \quad \theta_{\alpha\beta} = -(6 + n_+) \delta_{\alpha\beta}, \quad \theta_{r+s+} = -(2 + n_-) \delta_{r+s+}, \quad \theta_{r-s-} = (6 + n_+) \delta_{r-s-}, \\
 \theta &= -(4 + n_+ - n_-), \quad \theta_{\alpha\beta p-q-} = \sigma_{\alpha\beta} \Omega_{p-q-}, \quad \theta_{p-q-r-s-} = -\frac{3}{8+n} \Omega_{[p-q-} \Omega_{r-s-]}, \\
 \text{with } \Omega_{p-q-} &= (8+n) \begin{pmatrix} \sigma & & \\ & \ddots & \\ & & \sigma \end{pmatrix}.
 \end{aligned} \tag{4.25}$$

The gauge group is $\text{SO}(6, n_+) \times \text{SO}(2, 1)$ if $n_- = 2$ and $\text{SO}(6, n_+) \times \text{U}(n_-/2, 1)$ otherwise. The spectrum is given in table 13(b).

In the case $n_- = 2$, the previous solution admits a continuous parameter ξ , given by

$$\begin{aligned}\theta_{ij} &= 4\delta_{ij}, \quad \theta_{\alpha\beta} = -(6+n_+)\delta_{\alpha\beta}, \quad \theta_{r_+s_+} = -4\delta_{r_+s_+}, \quad \theta_{r_-s_-} = (6+n_+)\delta_{r_-s_-}, \\ \theta &= -(2+n_+), \quad \theta_{\alpha\beta p_-q_-} = (10+n_+)\xi\sigma_{\alpha\beta}\sigma_{p_-q_-}.\end{aligned}\quad (4.26)$$

For $|\xi| \neq 1$, the gauge group enhances from $\text{SO}(6, n_+) \times \text{SO}(2, 1)$ to $\text{SO}(6, n_+) \times \text{SO}(2, 2)$. The spectra are given in table 14. Although all masses are continuous functions of the parameter ξ , the conformal dimensions are assigned differently for $|\xi| < 1$ and $|\xi| \geq 1$, respectively (within the values allowed by eq. (2.24)), in order to combine the different fields into supermultiplets of $\text{SU}(4|1, 1)$.

4.2.2 Diagonal embedding

For a diagonal embedding of $\text{SO}(6) \times \text{SO}(2)$ in $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$, there is one new solution for $n_+ = 6$, depending on a free parameter ξ :

$$\begin{aligned}\theta_{ij} &= (n_- + 2)\delta_{ij}, \quad \theta_{\alpha\beta} = -12\delta_{\alpha\beta}, \quad \theta_{r_+s_+} = -(n_- + 2)\delta_{r_+s_+}, \quad \theta_{r_-s_-} = 12\delta_{r_-s_-}, \\ \theta &= (n_- - 10), \quad \theta_{\alpha\beta r_-s_-} = (14+n_-)\xi\sigma_{\alpha\beta}\sigma_{r_-s_-}, \quad \theta_{ijr_+s_+} = 2(14+n_-)\delta_{i[r_+}\delta_{s_+]j}.\end{aligned}\quad (4.27)$$

In the case $n_- = 2$, if $|\xi| \neq 1$ the gauge group is $\text{GL}(6) \times \text{SO}(2, 1)$ for $|\xi| = 1$ and enhances to $\text{GL}(6) \times \text{SO}(2, 2)$ otherwise. The spectra are given in table 15. Again, the conformal dimensions are assigned differently for $|\xi| < 1$ and $|\xi| \geq 1$, respectively (within the values allowed by eq. (2.24)), in order to combine the different fields into supermultiplets of $\text{SU}(4|1, 1)$. For $0 \leq n_- < 2$, the gauge group is $\text{GL}(6) \times \text{SO}(2, n_-)$ with the spectrum given in table 16.

4.3 $\text{OSp}(4^*|4)$

4.3.1 Chiral embedding

We now consider the last possible supergroup, $\text{OSp}(4^*|4)$. Here $p = 5$, and $q = 3$, and the index splitting (4.1) is given by $i \in \llbracket 1, 5 \rrbracket$ and $\alpha \in \{6, 7, 8\}$. Let us first assume that $\text{SO}(5) \times \text{SO}(3)$ is entirely embedded into the first factor of $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$. The potentially non-vanishing components of the embedding tensor are then given by

$$\left\{ \theta_{ij} = \lambda\delta_{ij}, \quad \theta_{\alpha\beta} = -(\lambda - \kappa)\delta_{\alpha\beta}, \quad \theta_{r_+s_+} = -\lambda\delta_{r_+s_+}, \right. \quad (4.28)$$

$$\left. \theta_{r_-s_-} = (\lambda - \kappa)\delta_{r_-s_-}, \quad \theta = \kappa, \quad \theta_{p_+q_+r_+s_+}, \quad \theta_{p_-q_-r_-s_-} \right\}. \quad (4.29)$$

The second quadratic constraint (2.9b) reduces to eqs. (B.4c) and (B.5a), of which the former implies

$$(2\lambda - \kappa)\theta_{pqr u_+} = 0 = (2\lambda - \kappa)\theta_{pqr u_-}. \quad (4.30)$$

Thus, all the components of θ_{pqrs} vanish. The solution of the embedding tensor then is given by

$$\begin{aligned}\theta_{ij} &= (3+n_-)\delta_{ij}, & \theta_{\alpha\beta} &= -(5+n_+)\delta_{\alpha\beta}, & \theta &= -(2+n_+-n_-), \\ \theta_{r_+s_+} &= -(3+n_-)\delta_{r_+s_+}, & \theta_{r_-s_-} &= (5+n_+)\delta_{r_-s_-}.\end{aligned}\quad (4.31)$$

The gauge group is $\text{SO}(5, n_+) \times \text{SO}(3, n_-)$ and the spectrum is given in table 17.

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{U}_{\text{ext}}(1)$
$\frac{1}{2} + \frac{ \xi }{2}$	$\frac{3}{2} + \frac{ \xi }{2}$	$2 + \xi $	1	$\mathbf{1}^{+4} \oplus \mathbf{1}^{-4}$	Q
	$1 + \frac{ \xi }{2}$	$\frac{3}{2} + \xi $	$1/2$	$\mathbf{4}^{+3} \oplus \bar{\mathbf{4}}^{-3}$	Q
	$\frac{1}{2} + \frac{ \xi }{2}$	$1 + \xi $	0	$\mathbf{6}^{+2} \oplus \mathbf{6}^{-2}$	Q
	$\frac{ \xi }{2}$	$\frac{1}{2} + \xi $	$-1/2$	$\bar{\mathbf{4}}^{+1} \oplus \mathbf{4}^{-1}$	Q
	$-\frac{1}{2} + \frac{ \xi }{2}$	$ \xi $	-1	$\mathbf{1}^0 \oplus \mathbf{1}^0$	Q
2	3	5	1	$\mathbf{1}^0$	0
	$5/2$	$9/2$	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	0
	2	4	0	$\mathbf{6}^{+2} \oplus \mathbf{15}^0 \oplus \mathbf{6}^{-2}$	0
	$3/2$	$7/2$	$-1/2$	$\bar{\mathbf{20}}^{+1} \oplus \mathbf{20}^{-1}$	0
	1	3	-1	$\mathbf{20}'^0$	0

 (a) $1 \leq |\xi|$.

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{U}_{\text{ext}}(1)$
$\frac{1}{2} + \frac{ \xi }{2}$	$\frac{3}{2} + \frac{ \xi }{2}$	$2 + \xi $	1	$\mathbf{1}_{+1} \oplus \mathbf{1}^{-2}$	Q
	$1 + \frac{ \xi }{2}$	$\frac{3}{2} + \xi $	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	Q
	$\frac{1}{2} + \frac{ \xi }{2}$	$1 + \xi $	0	$\mathbf{6}^0$	Q
$\frac{1}{2} - \frac{ \xi }{2}$	$\frac{3}{2} - \frac{ \xi }{2}$	$2 - \xi $	1	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	Q
	$1 - \frac{ \xi }{2}$	$\frac{3}{2} - \xi $	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	Q
	$\frac{1}{2} - \frac{ \xi }{2}$	$1 - \xi $	0	$\mathbf{6}^0$	Q
2	3	5	1	$\mathbf{1}^0$	0
	$5/2$	$9/2$	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	0
	2	4	0	$\mathbf{6}^{+2} \oplus \mathbf{15}^0 \oplus \mathbf{6}^{-2}$	0
	$3/2$	$7/2$	$-1/2$	$\bar{\mathbf{20}}^{+1} \oplus \mathbf{20}^{-1}$	0
	1	3	-1	$\mathbf{20}'^0$	0

 (b) $|\xi| < 1$.

Table 15. Mass spectra for the $\text{SU}(4|1,1)$ solutions with diagonal embedding (2.21b). The gauge group is $\text{GL}(6) \times \text{SO}(2,1)$ for $|\xi| = 1$ and $\text{GL}(6) \times \text{SO}(2,2)$ otherwise.

4.3.2 Diagonal embedding

Finally, for $n \geq 3$, there could be solutions when the entire R -symmetry group $\text{SO}(5) \times \text{SO}(3)$ or one of its factors is embedded diagonally in $\text{SO}(8) \times \text{SO}(n) \subset \text{SO}(8, n)$. The

Δ_L	Δ_R	Δ	s	$\text{SO}(6)^{\text{U}(1)}$	$\text{SO}(n_-)$
$1/2$	$3/2$	2	1	$\mathbf{1}^{+2} \oplus \mathbf{1}^{-2}$	$\mathbf{n_-}$
	1	$3/2$	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	$\mathbf{n_-}$
	$1/2$	1	0	$\mathbf{6}^0$	$\mathbf{n_-}$
2	3	5	1	$\mathbf{1}^0$	1
	$5/2$	$9/2$	$1/2$	$\mathbf{4}^{+1} \oplus \bar{\mathbf{4}}^{-1}$	1
	2	4	0	$\mathbf{6}^{+2} \oplus \mathbf{15}^0 \oplus \mathbf{6}^{-2}$	1
	$3/2$	$7/2$	$-1/2$	$\mathbf{20}^{+1} \oplus \mathbf{20}^{-1}$	1
	1	3	-1	$\mathbf{20}'^0$	1

Table 16. Mass spectrum for the $\text{SU}(4|1,1)$ solution with diagonal embedding (2.21b) and gauge group $\text{GL}(6) \times \text{SO}(2, n_-)$.

Δ_L	Δ_R	Δ	s	$\text{SO}(5) \times \text{SO}(3)$	$\text{SO}(n_+)$	$\text{SO}(n_-)$
1	2	3	1	$(\mathbf{1}, \mathbf{3})$	1	$\mathbf{n_-}$
	$3/2$	$5/2$	$1/2$	$(\mathbf{4}, \mathbf{2})$	1	$\mathbf{n_-}$
	1	2	0	$(\mathbf{5}, \mathbf{1})$	1	$\mathbf{n_-}$
2	2	4	0	$(\mathbf{1}, \mathbf{3})$	$\mathbf{n_+}$	1
	$3/2$	$7/2$	$-1/2$	$(\mathbf{4}, \mathbf{2})$	$\mathbf{n_+}$	1
	1	3	-1	$(\mathbf{5}, \mathbf{1})$	$\mathbf{n_+}$	1

Table 17. Mass spectrum for the $\text{OSp}(4^*|4)$ solution for the chiral embedding (2.21b) with gauge group $\text{SO}(5, n_+) \times \text{SO}(3, n_-)$.

potentially non-vanishing components of the embedding tensor are

$$\left\{ \theta_{ij} = \lambda \delta_{ij}, \theta_{\alpha\beta} = -(\lambda - \kappa) \delta_{\alpha\beta}, \theta_{r_+s_+} = -\lambda \delta_{r_+s_+}, \theta_{r_-s_-} = (\lambda - \kappa) \delta_{r_-s_-}, \right. \quad (4.32)$$

$$\left. \theta = \kappa, \theta_{ijr_+s_+}, \theta_{\alpha\beta r_-s_-}, \theta_{p_+q_+r_+s_+}, \theta_{p_-q_-r_-s_-} \right\}. \quad (4.33)$$

Evaluating the quadratic constraints with this parametrization gives rise to a number of new solutions for the different embeddings which we list in the following.

$\text{SO}(5) \times \text{SO}(3)$. If $\text{SO}(5) \times \text{SO}(3)$ is embedded diagonally, there is one new solution for $n_- = 3$ and $n_+ = 5$. It is given by

$$\begin{aligned} \theta_{ij} &= -6 \delta_{ij}, & \theta_{\alpha\beta} &= 10 \delta_{\alpha\beta}, & \theta_{r_+s_+} &= 6 \delta_{r_+s_+}, & \theta_{r_-s_-} &= -10 \delta_{r_-s_-}, \\ \theta &= 4, & \theta_{ijr_+s_+} &= -32 \delta_{i[r_+} \delta_{s_+]j}, & \theta_{\alpha\beta r_-s_-} &= 32 \delta_{\alpha[r_-} \delta_{s_-]\beta}. \end{aligned} \quad (4.34)$$

The gauge group is $\text{GL}(5) \times \text{GL}(3)$ and the spectrum is given in table 18(a).

SO(5). If SO(5) is embedded diagonally and SO(3) chirally, there is one new solution, given for $n_+ = 5$ by

$$\begin{aligned}\theta_{ij} &= (n_- + 3)\delta_{ij}, & \theta_{\alpha\beta} &= -10\delta_{\alpha\beta}, & \theta_{r_+s_+} &= -(n_- + 3)\delta_{r_+s_+}, & \theta_{r_-s_-} &= 10\delta_{r_-s_-}, \\ \theta &= (n_- - 7), & \theta_{ijr_+s_+} &= 2(n_- + 13)\delta_{i[r_+}\delta_{s_+]j}.\end{aligned}\quad (4.35)$$

The gauge group is $\text{GL}(5) \times \text{SO}(3, n_-)$ and the spectrum is given in table 18(b).

SO(3) (1). Finally, if SO(3) is embedded diagonally and SO(5) chirally, there are two new solutions. The first one requires $n_- = 3$ and is given by

$$\begin{aligned}\theta_{ij} &= -6\delta_{ij}, & \theta_{\alpha\beta} &= (n_+ + 5)\delta_{\alpha\beta}, & \theta_{r_+s_+} &= 6\delta_{r_+s_+}, & \theta_{r_-s_-} &= -(n_+ + 5)\delta_{r_-s_-}, \\ \theta &= (n_+ - 1), & \theta_{\alpha\beta r_-s_-} &= 2(n_+ + 11)\delta_{\alpha[r_-}\delta_{s_-]\beta}.\end{aligned}\quad (4.36)$$

The gauge group is $\text{GL}(3) \times \text{SO}(5, n_+)$ and the spectrum is given in table 18(c).

SO(3) (2). The second solution for diagonal embedding of SO(3) requires $n_- = 4$ and is given by

$$\begin{aligned}\theta_{ij} &= -14\delta_{ij}, & \theta_{\alpha\beta} &= 2(n_+ + 5)\delta_{\alpha\beta}, & \theta_{r_+s_+} &= 14\delta_{r_+s_+}, & \theta_{r_-s_-} &= -2(n_+ + 5)\delta_{r_-s_-}, \\ \theta &= 2(n_+ - 2), & \theta_{\alpha\beta\bar{r}_-\bar{s}_-} &= 2(n_+ + 12)\delta_{\alpha[\bar{r}_-}\delta_{\bar{s}_-]\beta}, \\ \theta_{\alpha\beta\bar{0}\bar{r}_-} &= -(n_+ + 12)\varepsilon_{\alpha\beta\bar{r}_-}, & \theta_{\bar{0}\bar{p}-\bar{q}-\bar{r}_-} &= -(n_+ + 12)\varepsilon_{\bar{p}-\bar{q}-\bar{r}_-},\end{aligned}\quad (4.37)$$

where the index r_- has been split into $r_- \rightarrow \{\bar{0}, \bar{r}_-\}$, $\bar{r}_- \in \{1, 2, 3\}$. The gauge group is $\text{SO}(5, n_+) \times \text{G}_{2(2)}$ and the spectrum is given in table 18(d).

5 Some $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ vacua

Having given an exhaustive classification of $\mathcal{N} = (8, 0)$ vacua for $n \leq 8$, in this section we present some partial analysis of vacua with $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ supersymmetries, respectively. The relevant supergroups with $\mathcal{N} = 7$ supercharges are $\text{OSp}(7|2, \mathbb{R})$ and $\text{G}(3)$, whose R -symmetry subgroups are $\text{SO}(7)$ and G_2 , respectively. The supergroup with one supercharge is $\text{OSp}(1|2, \mathbb{R})$.

In these cases, the constraints (2.17) and (2.18) get replaced by

$$\mathcal{N} = (7, 1) : \quad \begin{cases} A_1^{ab} = \ell^{-1} \delta^{ab}, \\ A_1^{88} = -\ell^{-1}, \\ A_2^{a\dot{A}r} = 0 = A_2^{8\dot{A}r}, \end{cases} \quad \mathcal{N} = (7, 0) : \quad \begin{cases} A_1^{ab} = \ell^{-1} \delta^{ab}, \\ A_1^{88} \neq \pm \ell^{-1}, \\ A_2^{a\dot{A}r} = 0, \end{cases} \quad (5.1)$$

respectively, where the index A splits according to $A = \{a, 8\}$ with $a \in \llbracket 1, 7 \rrbracket$. For $\mathcal{N} = (7, 1)$ the potentially non-vanishing components of the embedding tensor are thus given by

$$\{\theta_{IJ}, \theta_{rs}, \theta, \theta_{IJKL}, \theta_{IJrs}, \theta_{Ipqr}, \theta_{pqrs}\}, \quad (5.2)$$

where θ_{IJKL} now also has a selfdual contribution, unlike the $\mathcal{N} = (8, 0)$ case of eq. (3.2). For $\mathcal{N} = (7, 0)$ vacua, the potentially non-vanishing components of the embedding tensor are

$$\{\theta_{IJ}, \theta_{rs}, \theta_{Ir}, \theta, \theta_{IJKL}, \theta_{IJKr}, \theta_{IJrs}, \theta_{Ipqr}, \theta_{pqrs}\}, \quad (5.3)$$

again with a θ_{IJKL} which is not restricted to anti-selfdual tensors. In the rest of this section, we present our findings for such vacua.

Δ_L	Δ_R	Δ	s	$\text{SO}(5) \times \text{SO}(3)$
2	3	5	1	$(\mathbf{1}, \mathbf{5})$
	$5/2$	$9/2$	$1/2$	$(\mathbf{4}, \mathbf{4})$
	2	4	0	$(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{5}, \mathbf{3})$
	$3/2$	$7/2$	$-1/2$	$(\mathbf{4}, \mathbf{2})$
	1	3	-1	$(\mathbf{1}, \mathbf{1})$
3	4	7	1	$(\mathbf{1}, \mathbf{1})$
	$7/2$	$13/2$	$1/2$	$(\mathbf{4}, \mathbf{2})$
	3	6	0	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{5}, \mathbf{3})$
	$5/2$	$11/2$	$-1/2$	$(\mathbf{16}, \mathbf{2})$
	2	5	-1	$(\mathbf{14}, \mathbf{1})$

 (a) $\text{GL}(5) \times \text{GL}(3)$.

Δ_L	Δ_R	Δ	s	$\text{SO}(5) \times \text{SO}(3)$	$\text{SO}(n_-)$
1	2	3	1	$(\mathbf{1}, \mathbf{3})$	n_-
	$3/2$	$5/2$	$1/2$	$(\mathbf{4}, \mathbf{2})$	n_-
	1	2	0	$(\mathbf{5}, \mathbf{1})$	n_-
3	4	7	1	$(\mathbf{1}, \mathbf{1})$	1
	$7/2$	$13/2$	$1/2$	$(\mathbf{4}, \mathbf{2})$	1
	3	6	0	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{5}, \mathbf{3})$	1
	$5/2$	$11/2$	$-1/2$	$(\mathbf{16}, \mathbf{2})$	1
	2	5	-1	$(\mathbf{14}, \mathbf{1})$	1

 (b) $\text{GL}(5) \times \text{SO}(3, n_-)$.

Δ_L	Δ_R	Δ	s	$\text{SO}(5) \times \text{SO}(3)$	$\text{SO}(n_+)$
2	3	5	1	$(\mathbf{1}, \mathbf{5})$	1
	$5/2$	$9/2$	$1/2$	$(\mathbf{4}, \mathbf{4})$	1
	2	4	0	$(\mathbf{1}, \mathbf{3}) \oplus (\mathbf{5}, \mathbf{3})$	1
	$3/2$	$7/2$	$-1/2$	$(\mathbf{4}, \mathbf{2})$	1
	1	3	-1	$(\mathbf{1}, \mathbf{1})$	1
2	2	4	0	$(\mathbf{1}, \mathbf{3})$	n_+
	$3/2$	$7/2$	$-1/2$	$(\mathbf{4}, \mathbf{2})$	n_+
	1	3	-1	$(\mathbf{5}, \mathbf{1})$	n_+

 (c) $\text{GL}(3) \times \text{SO}(5, n_+)$.

Δ_L	Δ_R	Δ	s	$\text{SO}(5) \times \text{SO}(3)$	$\text{SO}(3)$	$\text{SO}(n_+)$
$3/2$	$5/2$	4	1	$(\mathbf{1}, \mathbf{4})$	2	1
	2	$7/2$	$1/2$	$(\mathbf{4}, \mathbf{3})$	2	1
	$3/2$	3	0	$(\mathbf{5}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{4})$	2	1
	1	$5/2$	$-1/2$	$(\mathbf{4}, \mathbf{1})$	2	1
2	2	4	0	$(\mathbf{1}, \mathbf{3})$	1	n_+
	$3/2$	$7/2$	$-1/2$	$(\mathbf{4}, \mathbf{2})$	1	n_+
	1	3	-1	$(\mathbf{5}, \mathbf{1})$	1	n_+

 (d) $\text{SO}(5, n_+) \times \text{G}_{2(2)}$.

 Table 18. Mass spectra for the $\text{OSp}(4^*|4)$ solutions with diagonal embedding (2.21b).

Δ_L	Δ_R	Δ	s	SO(7)
$3/2$	3	$9/2$	$3/2$	1
	$5/2$	4	1	7
	2	$7/2$	$1/2$	21
	$3/2$	3	0	1 $\oplus$ 35
	1	$5/2$	$-1/2$	7 $\oplus$ 35
	$1/2$	2	-1	21

Table 19. Mass spectrum for the $\text{OSp}(7|2, \mathbb{R})$ solution with $\mathcal{N} = (7, 0)$ supersymmetries. The gauge group is $\text{SO}(8) \times \text{SO}(7)$.

5.1 $\text{OSp}(7|2, \mathbb{R})$

For $\text{OSp}(7|2, \mathbb{R})$, a pair of $\mathcal{N} = (7, 0)$ vacua is given by the embedding tensor

$$\begin{aligned}
 \theta_{IJ} &= 56 \delta_{IJ}, \quad \theta_{rs} = 56 \delta_{rs}, \quad \theta_{Ir} = \pm 32 \sqrt{3} \delta_{Ir}, \quad \theta = 0, \quad \theta_{IJKL} = -6 \Gamma_{AB}^{IJKL} \xi^{AB}, \\
 \theta_{IJKr} &= \mp 4 \sqrt{3} \Gamma_{AB}^{IJKr} \xi^{AB}, \quad \theta_{IJrs} = -8 \Gamma_{AB}^{IJrs} \xi^{AB}, \quad \theta_{Ipqr} = \mp \frac{16}{\sqrt{3}} \Gamma_{AB}^{Ipqr} \xi^{AB}, \\
 \theta_{pqrs} &= -\frac{32}{3} \Gamma_{AB}^{pqrs} \xi^{AB}, \quad \text{with } \xi^{AB} = \text{diag} \left\{ -7, \underbrace{1, \dots, 1}_7 \right\},
 \end{aligned} \tag{5.4}$$

with $n = 8$. The gauge group is $\text{SO}(8) \times \text{SO}(7)$, which at the vacuum is spontaneously broken down to a diagonal $\text{SO}(7)$ subgroup. The spectrum is given in table 19.

Closer inspection shows that these embedding tensors may be related to the embedding tensor of eq. (3.23) (with selfdual θ_{pqrs}) by an $\text{SO}(8, 8)$ rotation of the form

$$\mathcal{V}_{\mathcal{M}}^{\mathcal{I}}(\phi) = \begin{pmatrix} \cosh(\phi) \mathbb{1}_8 & \sinh(\phi) \mathbb{1}_8 \\ \sinh(\phi) \mathbb{1}_8 & \cosh(\phi) \mathbb{1}_8 \end{pmatrix}, \tag{5.5}$$

with

$$\phi = \phi_{\pm} = \ln \left(7 \pm 4 \sqrt{3} \right) / 2. \tag{5.6}$$

In view of our discussion in section 2.3, we have thus identified three vacua which all belong to the same three-dimensional theory. To illustrate this structure, we evaluate the scalar potential (2.14) on the 1-scalar truncation (5.5) to $\text{SO}(7)$ singlets, which takes the form

$$V(\phi) = \frac{16}{9} \left(-250 + 105 \cosh(2\phi) - 150 \cosh(4\phi) + 7 \cosh(6\phi) \right), \tag{5.7}$$

that is sketched in figure 1. It exhibits the fully symmetric $\mathcal{N} = (8, 0)$ vacuum at the scalar origin $\phi = 0$, together with the two $\mathcal{N} = (7, 0)$ vacua at $\phi = \phi_{\pm}$ from eq. (5.6). This potential may be cast into the following form

$$V(\phi) = -2 W(\phi)^2 + \frac{1}{8} \left(\frac{dW}{d\phi}(\phi) \right)^2, \tag{5.8}$$

Figure 1. Potential for the 1-scalar truncation (5.5) of the theory with gauge group $\text{SO}(8) \times \text{SO}(7)$. The vacuum at the scalar origin $\phi = 0$ preserves full $\mathcal{N} = (8, 0)$ supersymmetry, while the two vacua at $\phi_{\pm} = \ln(7 \pm 4\sqrt{3})/2$ spontaneously break supersymmetry down to $\mathcal{N} = (7, 0)$.

in terms of a real superpotential

$$W(\phi) = \frac{2}{3} (3 - 28 \cosh(2\phi) + \cosh(4\phi)), \quad (5.9)$$

which shares its stationary points with $V(\phi)$.

5.2 G(3)

For the supergroup $\text{G}(3)$, we present vacua with both, $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ supersymmetry, respectively.

$\mathcal{N} = (7, 1)$. The embedding tensor

$$\begin{aligned} \theta_{ij} &= (n+1) \delta_{ij}, & \theta_{88} &= -7, & \theta_{rs} &= 7 \delta_{rs}, & \theta &= (n-6), \\ \theta_{ijkl} &= -\frac{(8+n)}{12} \varepsilon_{ijklxyz} \omega^{xyz}, \end{aligned} \quad (5.10)$$

with $i \in \llbracket 1, 7 \rrbracket$ and the G_2 -invariant 3-form ω^{xyz} from eq. (3.19), describes an $\mathcal{N} = (7, 1)$ vacuum within an $\text{SO}(n, 1) \times \text{G}_2$ gauged theory whose gauge group at the vacuum is broken down to its compact $\text{SO}(n) \times \text{G}_2$ subgroup. Its spectrum is given in table 20.

$\mathcal{N} = (7, 0)$. For $n = 7$, a pair of $\text{G}(3)$ solutions with $\mathcal{N} = (7, 0)$ supersymmetry is given by the embedding tensors

$$\begin{aligned} \theta_{ij} &= 127 \delta_{ij}, \quad \theta_{88} = 7, \quad \theta_{rs} = 128 \delta_{rs}, & \theta_{ir} &= \pm 90 \sqrt{2} \delta_{ir}, & \theta_{8r} &= 0, \quad \theta = -1, \\ \theta_{ijkl} &= 75 \tilde{\omega}_{ijkl}, & \theta_{8ijk} &= -135 \omega_{ijk}, & \theta_{ijk r} &= \pm 90 \sqrt{2} \tilde{\omega}_{ijk r}, & \theta_{8ijr} &= \mp 90 \sqrt{2} \omega_{ijr}, \\ \theta_{ijrs} &= 180 \tilde{\omega}_{ijrs}, & \theta_{8irs} &= -120 \omega_{irs}, & \theta_{ipqr} &= \pm 660 \sqrt{2} \tilde{\omega}_{ipqr}, & \theta_{8pqr} &= \mp 80 \sqrt{2} \omega_{pqr}, \\ \theta_{pqrs} &= \frac{575}{2} \tilde{\omega}_{pqrs}, \end{aligned} \quad (5.11)$$

Δ_L	Δ_R	Δ	s	G_2	$SO(n)$
$5/4$	$3/4$	2	$-1/2$	1	n
$5/4$	$1/4$	$3/2$	-1	1	n
$3/4$	$3/4$	$3/2$	0	7	n
$3/4$	$1/4$	1	$-1/2$	7	n

Table 20. Mass spectrum for the $G(3)$ solution with $\mathcal{N} = (7, 1)$ supersymmetry. The gauge group is $SO(n, 1) \times G_2$.

Δ_L	Δ_R	Δ	s	G_2
$7/4$	$13/4$	5	$3/2$	1
	$11/4$	$9/2$	1	7
	$9/4$	4	$1/2$	$7 \oplus 14$
	$7/4$	$7/2$	0	$1 \oplus 7 \oplus 27$
	$5/4$	3	$-1/2$	$7 \oplus 27$
	$3/4$	$5/2$	-1	14

Table 21. Mass spectrum for the $G(3)$ solution with $\mathcal{N} = (7, 0)$ supersymmetry. The gauge group is $SO(7) \times G_2$.

with the index $i \in [1, 7]$, the G_2 invariant three-form ω^{kmn} from eq. (3.19) and its dual defined by $\tilde{\omega}_{ijkl} \equiv \frac{1}{6} \varepsilon_{ijklmnp} \omega^{mnp}$. The gauge group is $SO(7) \times G_2$, broken down at the vacuum to a diagonal G_2 subgroup. The spectrum is given in table 21.

The embedding tensors (5.11) turn out to be related to the previously found $\mathcal{N} = (8, 0)$ solution (3.38) by an $SO(8, 7)$ rotation of the form

$$\mathcal{V}_{\mathcal{M}}^{\mathcal{I}}(\phi) = \begin{pmatrix} \cosh(\phi) \mathbb{1}_7 & 0 & \sinh(\phi) \mathbb{1}_7 \\ 0 & 1 & 0 \\ \sinh(\phi) \mathbb{1}_7 & 0 & \cosh(\phi) \mathbb{1}_7 \end{pmatrix}. \quad (5.12)$$

Starting from eq. (3.38) (up to a change of basis) at the origin $\phi = 0$, the tensors (5.11) are obtained at $\phi_{\pm} = \ln(3 \pm 2\sqrt{2})$. In view of our discussion in section 2.3, we have thus identified three vacua which all belong to the same three-dimensional theory. To illustrate this structure, we evaluate the scalar potential (2.14) on the 1-scalar truncation (5.12) to G_2 singlets, which takes the form

$$V(\phi) = \frac{25}{1024} \left(-15163 + 3416 \cosh(\phi) - 3640 \cosh(2\phi) + 2856 \cosh(3\phi) - 6860 \cosh(4\phi) \right. \\ \left. + 952 \cosh(5\phi) + 56 \cosh(6\phi) - 56 \cosh(7\phi) + 7 \cosh(8\phi) \right), \quad (5.13)$$

that is sketched in figure 2. It exhibits the fully symmetric $\mathcal{N} = (8, 0)$ vacuum at the scalar origin $\phi = 0$, together with the two $\mathcal{N} = (7, 0)$ vacua at $\phi_{\pm} = \ln(3 \pm 2\sqrt{2})$.

Figure 2. Potential for the 1-scalar truncation (5.12) of the theory with gauge group $\text{SO}(7) \times \text{G}_2$. The vacuum at the scalar origin $\phi = 0$ preserves full $\mathcal{N} = (8, 0)$ supersymmetry, while the two vacua at $\phi_{\pm} = \ln(3 \pm 2\sqrt{2})$ spontaneously break supersymmetry down to $\mathcal{N} = (7, 0)$.

Let us finally note, that the scalar potential (5.13) admits a “fake” superpotential

$$W(\phi) = -\frac{5}{32} (33 + 84 \cosh(\phi) - 56 \cosh(2\phi) + 28 \cosh(3\phi) + 7 \cosh(4\phi)), \quad (5.14)$$

in terms of which it may be written as

$$V(\phi) = -2W(\phi)^2 + \frac{1}{7} \left(\frac{dW}{d\phi}(\phi) \right)^2. \quad (5.15)$$

However, the $\mathcal{N} = (7, 0)$ vacua are not stationary points of $W(\phi)$.

6 Embedding into the maximal theory

For a given supersymmetric vacuum, identified as a solution of eqs. (2.9a), (2.9b), (2.16) and (2.17), the embedding tensor defines the gauge group generators according to eq. (2.4) from which we have determined the specific half-maximal theory that fulfils all the requirements. It is an interesting question to ask, which of the vacua identified in this analysis can actually be embedded into a maximally supersymmetric ($\mathcal{N} = 16$) three-dimensional supergravity, thus spontaneously breaking half of the supersymmetries of the theory. For $D = 4$ supergravities, the analogous question has been addressed in ref. [36].

To answer this question, we first recall some of the basic structures of maximal $D = 3$ supergravities [4, 20]. In this case, the scalar sector describes an $\text{E}_{8(8)}/\text{SO}(16)$ coset space sigma model and the embedding tensor, which defines the gauge group generators within $\text{E}_{8(8)}$ in analogy to eq. (2.4), transforms in the $\mathbf{1} \oplus \mathbf{3875}$ representation of $\text{E}_{8(8)}$. The maximal theory can be truncated to a half-maximal subsector upon truncating the coset space as

$$\text{E}_{8(8)}/\text{SO}(16) \longrightarrow \text{SO}(8, 8)/(\text{SO}(8) \times \text{SO}(8)). \quad (6.1)$$

Under $\text{SO}(8,8)$, the embedding tensor of the maximal theory decomposes as

$$\mathbf{1} \oplus \mathbf{3875} \longrightarrow \mathbf{1} \oplus \mathbf{135} \oplus \mathbf{1820} \oplus \mathbf{1920}_c, \quad (6.2)$$

of which the first three parts reproduce the embedding tensor (2.5), (2.6) of the half-maximal theory, whereas the last part drops out in the projection to the half-maximal theory. Specifically, splitting the $\text{E}_{8(8)}$ generators according to

$$\left\{ L^{[\mathcal{MN}]}, Y^{\mathcal{A}} \right\}, \quad \mathcal{M} \in \llbracket 1, 16 \rrbracket, \quad \mathcal{A} \in \llbracket 1, 128 \rrbracket, \quad (6.3)$$

into $\text{SO}(8,8)$ and its orthogonal complement (transforming in the spinor representation $\mathbf{128}_s$ of $\text{SO}(8,8)$), an embedding tensor of the maximal theory triggered by the first three terms in eq. (6.2) takes the form

$$\begin{cases} \Theta_{\mathcal{MN}|\mathcal{PQ}} = \theta_{\mathcal{MN}\mathcal{PQ}} + 2 \left(\eta_{\mathcal{M}[\mathcal{P}} \theta_{\mathcal{Q}]\mathcal{N}} - \eta_{\mathcal{N}[\mathcal{P}} \theta_{\mathcal{Q}]\mathcal{M}} \right) + 2 \eta_{\mathcal{M}[\mathcal{P}} \eta_{\mathcal{Q}]\mathcal{N}} \theta, \\ \Theta_{\mathcal{A}|\mathcal{B}} = -\theta \eta_{\mathcal{AB}} + \frac{1}{96} \Gamma_{\mathcal{AB}}^{\mathcal{MN}\mathcal{PQ}} \theta_{\mathcal{MN}\mathcal{PQ}}, \end{cases} \quad (6.4)$$

where $\Gamma_{\mathcal{AB}}^{\mathcal{MN}\mathcal{PQ}}$ denotes the four-fold product of $\text{SO}(8,8)$ Γ -matrices. Introducing covariant derivatives

$$D_\mu = \partial_\mu - g A_\mu^{\mathcal{MN}} \Theta_{\mathcal{MN}|\mathcal{PQ}} L^{[\mathcal{PQ}]} - g A_\mu^{\mathcal{A}} \Theta_{\mathcal{A}|\mathcal{B}} Y^{\mathcal{B}}, \quad (6.5)$$

in the maximal theory, the gauge group is thus given by an extension of the gauge group of the half-maximal theory by the additional generators $X_{\mathcal{A}} = \Theta_{\mathcal{A}|\mathcal{B}} Y^{\mathcal{B}}$. We may now address the following question: given an embedding tensor (2.7) of the half-maximal $\text{SO}(8,8)$ theory, satisfying the quadratic constraints (2.9a), (2.9b), does the associated embedding tensor (6.4) satisfy the quadratic constraints of the maximal theory and thereby define a consistent gauging of the maximal theory? By consistency of the truncation, the $\mathcal{N} = (8,0)$ vacuum of the half-maximal theory then turns into a vacuum of some maximal gauged supergravity, breaking half of the supersymmetries spontaneously.

To answer this question, we recall that the quadratic constraints of the maximal theory transform in the

$$\mathbf{3875} \oplus \mathbf{147250} \quad (6.6)$$

under $\text{E}_{8(8)}$. Breaking these under $\text{SO}(8,8)$ and restricting to the representations that can actually appear in the symmetric tensor product of two half-maximal embedding tensors shows that in order to define a maximal $\mathcal{N} = 16$ gauging, the components of the embedding tensor (6.4) must satisfy additional constraints transforming as $\mathbf{35} \oplus \mathbf{6435}_c$, i.e.

$$\begin{array}{c} \square \square \oplus \begin{array}{c} \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \\ \square \end{array}, \end{array} \quad (6.7)$$

anti-sd

where the last term refers to the anti-selfdual contribution in the 8-fold antisymmetric tensor. These additional conditions may be worked out explicitly in analogy to eq. (2.8)

and take the following form

$$\begin{cases} 0 = 48 \theta_{\mathcal{M}\mathcal{N}} + \theta_{\mathcal{M}}^{\mathcal{P}\mathcal{Q}\mathcal{R}} \theta_{\mathcal{N}\mathcal{P}\mathcal{Q}\mathcal{R}} - \frac{1}{16} \eta_{\mathcal{M}\mathcal{N}} \theta^{\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}} \theta_{\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}}, \\ 0 = \Gamma_{\dot{A}\dot{B}}^{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}} \theta_{\mathcal{K}\mathcal{L}\mathcal{M}\mathcal{N}} \theta_{\mathcal{P}\mathcal{Q}\mathcal{R}\mathcal{S}}. \end{cases} \quad (6.8)$$

Let us note that the first equation has the same representation content $\square\square$ as the constraint

$$\theta \theta_{\mathcal{M}\mathcal{N}} + \theta_{\mathcal{M}}^{\mathcal{K}} \theta_{\mathcal{K}\mathcal{N}} - \frac{1}{16} \eta_{\mathcal{M}\mathcal{N}} \theta^{\mathcal{K}\mathcal{L}} \theta_{\mathcal{K}\mathcal{L}} = 0, \quad (6.9)$$

obtained from proper contraction of the constraints (2.9b) of the $\mathcal{N} = 8$ theory. Furthermore, the choice of anti-selfduality (vs. selfduality) in eq. (6.7) is a pure convention here, depending on the embedding of $\text{SO}(8, 8)$ into $\text{E}_{8(8)}$.

To summarize, an embedding tensor of the half-maximal theory which in addition to the quadratic constraints (2.9a), (2.9b) of the half-maximal theory satisfies the additional constraints (6.8) defines a consistent maximal three-dimensional supergravity. The half-maximal theory is recovered upon truncation (6.1). Vacua of the half-maximal theory then give rise to vacua within the maximal theory. It is a straightforward task to check the additional constraints (6.8) for all the vacua we have identified in this paper. For theories with a free parameter these constraints single out specific values of the parameter. We collect the results of the possible embedding into the maximal theory for every vacuum in the summarizing tables 22–24.

7 Summary and outlook

In this paper, we have presented a classification of $\mathcal{N} = (8, 0)$ AdS_3 vacua in half-maximal $D = 3$ supergravities. Analyzing the consistency constraints on the embedding tensor, we have determined the full set of possible gauge groups embedded in the $\text{SO}(8, n)$ isometry group of ungauged $D = 3$ supergravity, for $n \leq 8$. There are four classes of such vacua according to the different superisometry groups $\text{OSp}(8|2, \mathbb{R})$, $\text{F}(4)$, $\text{SU}(4|1, 1)$, and $\text{OSp}(4^*|4)$, respectively. For each of the vacua, we have determined the explicit embedding tensor, the gauge group embedded into the $\text{SO}(8, n)$ isometry group of ungauged $D = 3$ supergravity, and the physical mass spectrum, organized in terms of supermultiplets. We summarize our results in tables 22, 23. For all the vacua identified, we have furthermore determined if and under which conditions on the free parameters, the half-maximal theories admit an embedding into a maximal ($\mathcal{N} = 16$) supergravity, with the gauge group enhanced by additional generators according to eqs. (6.4), (6.5) above.

In table 24, we collect our findings of $\mathcal{N} = (7, 1)$ and $\mathcal{N} = (7, 0)$ AdS_3 vacua. We have shown that the latter vacua are realized in half-maximal theories that also admit fully supersymmetric $\mathcal{N} = (8, 0)$ AdS_3 vacua as different stationary points in their scalar potential, cf. figures 1, 2 above. In particular, this indicates the existence of domain wall solutions interpolating between $\mathcal{N} = (8, 0)$ and $\mathcal{N} = (7, 0)$ AdS_3 vacua. It would be very interesting to generalize our findings to a classification of general fully supersymmetric $\mathcal{N} = (p, 8 - p)$ vacua of the half-maximal theories. In this case, the classification will

be organized by products of smaller supergroups, which presumably leaves even more possibilities for the potential embeddings of their bosonic parts into $\text{SO}(8, n)$.

Throughout, we have restricted the analysis to theories with $n \leq 8$ matter multiplets. In this range, our classification is exhaustive. For general n , we expect the classification to straightforwardly extend to theories with chiral embedding of the R -symmetry group into the first factor of the compact $\text{SO}(8) \times \text{SO}(n)$ invariance group. Indeed, we have identified various families of theories labelled by integers n_+ , n_- which are defined for arbitrary (unbounded) values of these integers. For diagonal embedding of the R -symmetry group on the other hand, one may expect new patterns to arise, since with increasing n , also the number of possible distinct embeddings of $\text{SO}(8)$ into $\text{SO}(n)$ increases. In particular, there are maximal embeddings for arbitrarily high values of n . For $\mathcal{N} = (4, 4)$ vacua, many theories based on different such embedding patterns have been identified in ref. [37].

An immediate question to address for the vacua identified in this paper is the existence and the structure of their moduli spaces as submanifolds of the scalar target space $\text{SO}(8, n)/(\text{SO}(8) \times \text{SO}(n))$. In the holographic context, this relates to the exactly marginal operators of the putative holographically dual conformal field theories.

Probably the most outstanding issue about these vacua and theories concerns their possible higher-dimensional origin. More precisely, it would be very interesting to embed the half-maximal $D = 3$ supergravities identified in this paper as consistent truncations into ten- or eleven-dimensional supergravities, such that in particular the AdS_3 vacua would uplift to full $D = 11$ or IIB solutions, subject to the constraints from higher-dimensional classifications and no-go results [38–40]. For example, it would be interesting to see if the type IIA compactifications with exceptional supersymmetry of ref. [24] can be embedded into consistent truncations to $D = 3$ supergravities constructed in this paper. In particular, for the solution with $\text{F}(4)$ supersymmetry, table 23 offers several candidates with gauge group factors $\text{SO}(7)$ potentially realized as the isometry group of the round six sphere $S^6 = \text{SO}(7)/\text{SO}(6)$. For the theories with free continuous parameter, it would be very interesting to identify its possible higher-dimensional origin.

A systematic approach for higher-dimensional uplifts builds on the reformulation of the higher-dimensional supergravities as exceptional field theories based on the group $\text{SO}(8, n)$ [41]. In this framework, consistent truncations are described as generalized Scherk-Schwarz reductions, leaving the task of solving the consistency equations for suitable Scherk-Schwarz twist matrices. An apparent obstacle to a standard geometrical uplift of many of the theories collected in tables 22–24 is the rank and the size of their gauge groups which do not admit a geometric realization as the isometry group of a 7- or 8-dimensional internal manifold. It remains to be seen if the rich structure of three-dimensional supergravity hints at some more general reduction mechanisms specific to three-dimensional theories. In this context, it may be advantageous to exploit the possibility of embedding the $D = 3$ half-maximal theory into maximal higher-dimensional supergravities via their formulation as an $\text{E}_{8(8)}$ exceptional field theory [42] upon suitable generalization of the methods developed in ref. [43]. Another interesting option to explore is the possible existence of Scherk-Schwarz twist matrices realising these gauged supergravities while explicitly violating the section constraints, although the higher-dimensional interpretation of such a construction remains somewhat mysterious.

Finally, it would be very interesting to extend the present analysis to include AdS_3 solutions with less supersymmetry and relate to known solutions and structures such as refs. [44–51].

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$n \leq 8$	Gauge group	G_{ext}	Param.	Embedding max.	Spectrum	$\Theta_{MN PQ}$
$\mathcal{G}_R = \text{OSp}(8 2, \mathbb{R})$						
$n_+ + n_-$	$\text{SO}(8, n_+) \times \text{SO}(n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$		$(n_+, n_-) = (8, 0), (0, 8)$	Table 4(a)	Eq. (3.6)
$n_+ + 4$	$\text{SO}(8, n_+) \times \text{SO}(4)$	$\text{SO}(n_+) \times \text{SO}(4)$	ξ	$n_+ = 4$ and $\xi = \pm 2$	Table 4(a)	Eq. (3.11)
$n_+ + 4$	$\text{SO}(8, n_+) \times \text{SO}(3)$	$\text{SO}(n_+) \times \text{SO}(3)$		—	Table 4(a)	Eq. (3.11)
$n_+ + 6$	$\text{SO}(8, n_+) \times \text{U}(3)$	$\text{SO}(n_+) \times \text{U}(3)$		$n_+ = 2$	Table 4(a)	Eq. (3.15)
$n_+ + 7$	$\text{SO}(8, n_+) \times \text{G}_2$	$\text{SO}(n_+) \times \text{G}_2$		$n_+ = 1$	Table 4(a)	Eq. (3.19)
8	$\text{SO}(8) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$		—	Table 4(a)	Eq. (3.23)
8	$\text{GL}(8)$	—		$\checkmark$	Table 4(b)	Eq. (3.29)
$\mathcal{G}_R = \text{SU}(4 1, 1)$						
$n_+ + n_-$	$\text{SO}(6, n_+) \times \text{SO}(2, n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$		$(n_+, n_-) = (2, 6)$	Table 13(a)	Eq. (4.20)
$n_+ + 2m$	$\text{SO}(6, n_+) \times \text{U}(m, 1)$	$\text{SO}(n_+) \times \text{U}(m)$		$(n_+, m) = (4, 2)$	Table 13(b)	Eq. (4.25)
$n_+ + 2$	$\text{SO}(6, n_+) \times \text{SO}(2, 1)$	$\text{SO}(n_+)$		—	Table 14	Eq. (4.26)
$n_+ + 2$	$\text{SO}(6, n_+) \times \text{SO}(2, 2)$	$\text{SO}(n_+) \times \text{U}(1)$	ξ	$n_+ = 6$ and $\xi = \pm 2$	Table 14	Eq. (4.26)
$6 \leq n < 8$	$\text{GL}(6) \times \text{SO}(2, n-6)$	$\text{SO}(n-6)$		—	Table 16	Eq. (4.27)
8	$\text{GL}(6) \times \text{SO}(2, 1)$	$\text{U}(1)$		—	Table 15	Eq. (4.27)
6	$\text{GL}(6) \times \text{SO}(2, 2)$	$\text{SO}(2)$	ξ	$\xi = \pm 3$	Table 15	Eq. (4.27)
n	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(n)$	$\text{SO}(n)$		—	Table 6	Eq. (3.44)
4	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(4)$	$\text{SO}(4)$	ξ	—	Table 6	Eq. (3.45)
4	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(3)$	$\text{SO}(3)$		—	Table 6	Eq. (3.45)
6	$\text{SO}(6) \times \text{SO}(2) \times \text{U}(3)$	$\text{U}(3)$		—	Table 6	Eq. (3.46)
7	$\text{SO}(6) \times \text{SO}(2) \times \text{G}_2$	G_2		—	Table 6	Eq. (3.47)
8	$\text{SO}(6) \times \text{SO}(2) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$		$p = 2$	Table 6	Eq. (3.48)
8	$\text{U}(4, 4)$	$\text{U}(4)$		$\checkmark$	Table 7	Eq. (3.51)
8	$\text{SL}(2) \times \text{Sp}(4, \mathbb{R})$	$\text{U}(1)$		$\checkmark$	Table 8	Eq. (3.52)
$2 + n_-$	$\text{U}(4, 1) \times \text{SO}(n_-)$	$\text{SO}(n_-) \times \text{U}(1)$		—	Table 9	Eq. (3.53)

Table 22. Theories with n matter multiplets admitting an $\mathcal{N} = (8, 0)$ AdS₃ vacuum which preserves a global $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R \times G_{\text{ext}}$ symmetry. References to the explicit embedding tensors and the associated mass spectra are given. The fourth column indicates if the embedding tensor possesses a free parameter, the fifth column indicates, if possible, the condition for the theory to admit an embedding into maximal ($\mathcal{N} = 16$) $D = 3$ supergravity.

$n \leq 8$	Gauge group	G_{ext}	Param.	Embedding max.	Spectrum	$\Theta_{\mathcal{MN} \mathcal{PQ}}$
$\mathcal{G}_R = \mathbf{F}(4)$						
$n_+ + n_-$	$\text{SO}(7, n_+) \times \text{SO}(1, n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$		$(n_+, n_-) = (1, 7)$	Table 12(a)	Eq. (4.13)
$n_+ + 3$	$\text{SO}(7, n_+) \times \text{SO}(1, 3)$	$\text{SO}(n_+) \times \text{SO}(3)$	ξ	—	Table 12(a)	Eq. (4.16)
$n \geq 7$	$\text{GL}(7) \times \text{SO}(1, n-7)$	$\text{SO}(n-7)$		—	Table 12(b)	Eq. (4.17)
n	$\text{SO}(7) \times \text{SO}(n)$	$\text{SO}(n)$		—	Table 5	Eq. (3.35)
4	$\text{SO}(7) \times \text{SO}(4)$	$\text{SO}(4)$	ξ	—	Table 5	Eq. (3.36)
4	$\text{SO}(7) \times \text{SO}(3)$	$\text{SO}(3)$		—	Table 5	Eq. (3.36)
6	$\text{SO}(7) \times \text{U}(3)$	$\text{U}(3)$		—	Table 5	Eq. (3.37)
7	$\text{SO}(7) \times \text{G}_2$	G_2		—	Table 5	Eq. (3.38)
8	$\text{SO}(7) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$		$p=1$	Table 5	Eq. (3.39)
$\mathcal{G}_R = \text{OSp}(4^* 4)$						
$n_+ + n_-$	$\text{SO}(5, n_+) \times \text{SO}(3, n_-)$	$\text{SO}(n_+) \times \text{SO}(n_-)$		$(n_+, n_-) = (3, 5)$	Table 17	Eq. (4.31)
$n \geq 4$	$\text{SO}(5, n-4) \times \text{G}_{2(2)}$	$\text{SO}(n-4) \times \text{SO}(3)$		$n=8$	Table 18(d)	Eq. (4.37)
$n \geq 5$	$\text{GL}(5) \times \text{SO}(3, n-5)$	$\text{SO}(n-5)$		—	Table 18(b)	Eq. (4.35)
$n \geq 3$	$\text{GL}(3) \times \text{SO}(5, n-3)$	$\text{SO}(n-3)$		$n=8$	Table 18(c)	Eq. (4.36)
8	$\text{GL}(5) \times \text{GL}(3)$	—		—	Table 18(a)	Eq. (4.34)
n	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(n)$	$\text{SO}(n)$		—	Table 10	Eq. (3.56)
4	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(4)$	$\text{SO}(4)$	ξ	—	Table 10	Eq. (3.57)
4	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(3)$	$\text{SO}(3)$		—	Table 10	Eq. (3.57)
6	$\text{SO}(5) \times \text{SO}(3) \times \text{U}(3)$	$\text{U}(3)$		—	Table 10	Eq. (3.58)
7	$\text{SO}(5) \times \text{SO}(3) \times \text{G}_2$	G_2		—	Table 10	Eq. (3.59)
8	$\text{SO}(5) \times \text{SO}(3) \times \text{SO}(8-p) \times \text{SO}(p)$	$\text{SO}(8-p) \times \text{SO}(p)$		$p=3$	Table 10	Eq. (3.60)
8	$\text{Sp}(2, 2) \times \text{SO}(3)$	$\text{USp}(4)$		$\checkmark$	Table 11	Eq. (3.62)

Table 23. Theories with n matter multiplets admitting an $\mathcal{N} = (8, 0)$ AdS₃ vacuum which preserves a global $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R \times G_{\text{ext}}$ symmetry. References to the explicit embedding tensors and the associated mass spectra are given. The fourth column indicates if the embedding tensor possesses a free parameter, the fifth column indicates, if possible, the condition for the theory to admit an embedding into maximal ($\mathcal{N} = 16$) $D = 3$ supergravity.

$\mathcal{N}$	$n \leq 8$	Gauge group	G_{ext}	Param.	Embedding max.	Spectrum	$\Theta_{\mathcal{MN} \mathcal{PQ}}$
$\mathcal{G}_R = \text{OSP}(7 2, \mathbb{R})$							
$(7, 0)$	8	$\text{SO}(7) \times \text{SO}(8)$	—	—	Table 19	Eq. (5.4)	
$\mathcal{G}_R = \text{G}(3)$							
$(7, 1)$	n	$\text{SO}(n, 1) \times \text{G}_2$	$\text{SO}(n)$	$n = 8$	Table 20	Eq. (5.10)	
$(7, 0)$	7	$\text{SO}(7) \times \text{G}_2$	—	—	Table 21	Eq. (5.11)	

Table 24. Theories with n matter multiplets admitting an $\mathcal{N} = (7, q)$ AdS₃ vacuum which preserves a global $\text{SL}(2, \mathbb{R})_L \times \mathcal{G}_R \times \text{G}_{\text{ext}}$ symmetry. References to the explicit embedding tensors and the associated mass spectra are given. The fifth column indicates if the embedding tensor possesses a free parameter, the sixth column indicates, if possible, the condition for the theory to admit an embedding into maximal ($\mathcal{N} = 16$) $D = 3$ supergravity.

A Notations and supersymmetry transformation

We list here the conventions for the gauging and the supersymmetry transformations in the Lagrangian (2.12), following refs. [20, 26]. The coset representative $\mathcal{V}(\phi)$ defined in eq. (2.3) transforms as

$$\mathcal{V}(\phi') = g \mathcal{V}(\phi) h^{-1}(\phi), \quad (\text{A.1})$$

with $g \in \text{SO}(8, n)$ and $h(\phi) \in \text{SO}(8) \times \text{SO}(n)$. For the ungauged theory, its coupling to fermions is described in terms of the Cartan-Maurer form

$$\mathcal{V}^{-1} \partial_\mu \mathcal{V} = \frac{1}{2} Q_\mu^{IJ} L^{IJ} + \frac{1}{2} Q_\mu^{rs} L^{rs} + P_\mu^{Ir} L^{Ir}. \quad (\text{A.2})$$

Once the theory is gauged, the covariantization gives

$$\mathcal{V}^{-1} \mathcal{D}_\mu \mathcal{V} = \mathcal{V}^{-1} \partial_\mu \mathcal{V} + A_\mu^{\mathcal{MN}} \mathcal{V}^{-1} \Theta_{\mathcal{MN}|\mathcal{PQ}} L^{\mathcal{MN}} \mathcal{V} = \frac{1}{2} \mathcal{Q}_\mu^{IJ} L^{IJ} + \frac{1}{2} \mathcal{Q}_\mu^{rs} L^{rs} + \mathcal{P}_\mu^{Ir} L^{Ir}. \quad (\text{A.3})$$

The full covariant derivatives of the fermions then read

$$\begin{cases} \mathcal{D}_\mu \psi_\nu^A = \partial_\mu \psi_\nu^A + \frac{1}{4} \omega_\mu^{\alpha\beta} \gamma_{\alpha\beta} \psi_\nu^A + \frac{1}{4} \mathcal{Q}_\mu^{IJ} \Gamma_{AB}^{IJ} \psi_\nu^B, \\ \mathcal{D}_\mu \chi^{\dot{A}r} = \partial_\mu \chi^{\dot{A}r} + \frac{1}{4} \omega_\mu^{\alpha\beta} \gamma_{\alpha\beta} \chi^{\dot{A}r} + \frac{1}{4} \mathcal{Q}_\mu^{IJ} \Gamma_{\dot{A}\dot{B}}^{IJ} \chi^{\dot{B}r} + \mathcal{Q}_\mu^{rs} \chi^{\dot{A}s}, \end{cases} \quad (\text{A.4})$$

with the spin-connection $\omega_\mu^{\alpha\beta}$, the three-dimensional γ matrices γ^α in flat spacetime and $\gamma^{\alpha\beta} = \gamma^{[\alpha} \gamma^{\beta]}$. Finally, the full supersymmetry variations are

$$\begin{cases} \mathcal{V}^{-1} \delta \mathcal{V} = L^{Ir} \bar{\varepsilon}^A \Gamma_{A\dot{A}}^I \chi^{\dot{A}r}, \\ \delta e_\mu^\alpha = i \bar{\varepsilon}^A \gamma^\alpha \psi_\mu^A, \\ \delta A_\mu^{\mathcal{MN}} = -\frac{1}{2} \mathcal{V}^{\mathcal{MN}}_{IJ} \bar{\varepsilon}^A \Gamma_{AB}^{IJ} \psi_\mu^B + i \mathcal{V}^{\mathcal{MN}}_{Ir} \bar{\varepsilon}^A \Gamma_{A\dot{A}}^I \gamma_\mu \chi^{\dot{A}r}, \\ \delta \chi^{\dot{A}r} = \frac{i}{2} \Gamma_{A\dot{A}} \gamma^\mu \varepsilon^A \mathcal{P}_\mu^{Ir} + g A_2^{A\dot{A}r} \varepsilon^A, \\ \delta \psi_\mu^A = \mathcal{D}_\mu \varepsilon^A + i g A_1^{AB} \gamma_\mu \varepsilon^B, \end{cases} \quad (\text{A.5})$$

with supersymmetry parameter ε^A and $\gamma^\mu = e^\mu_\alpha \gamma^\alpha$.

B Second quadratic constraint

We list here the full set of independent equations contained in the second quadratic constraint (2.9b) with parametrization (3.2) for the various values of the free indices.

$$(\mathcal{MN}\mathcal{PQ}) = (IJKL).$$

$$\begin{cases} \Lambda_{MN} [\theta_{MNP[I\theta_{JKL}]P} + 4\theta_{M[I\theta_{JKL}]N} - 2\kappa \delta_{M[I\theta_{JKL}]N}] = 0, \end{cases} \quad (\text{B.1a})$$

$$\begin{cases} \theta_{uvP[I\theta_{JKL}]P} = 0. \end{cases} \quad (\text{B.1b})$$

$$(\mathcal{MN}\mathcal{PQ}) = (IJKr).$$

$$\begin{cases} 3\theta_{uvs[I\theta_{JK}]rs} - \theta_{uvrL}\theta_{IJKL} = 0, \end{cases} \quad (\text{B.2a})$$

$$\begin{cases} -3\theta_{usM[I\theta_{JK}]rs} - 3\delta_{M[I\theta_{JK}]rs}\theta_{su} + 3\theta_{M[I\theta_{JK}]ru} - 3\kappa \delta_{M[I\theta_{JK}]ru} \\ + \theta_{urML}\theta_{IJKL} + (\theta_{ur} + \kappa \delta_{ur}) \theta_{IJKM} - \delta_{ur} \theta_{ML}\theta_{IJKL} = 0. \end{cases} \quad (\text{B.2b})$$

$$(\mathcal{MNPQ}) = (IJrs).$$

$$\left\{ \begin{array}{l} \Lambda_{uv} [\theta_{uwp}[I\theta_J]_{prs} - \theta_{uvL}[I\theta_J]_{Lrs} - \theta_{uwp}[r\theta_s]_{pIJ} \\ + 2\theta_{up}\theta_{IJp}[r\delta_s]_v + 2\theta_{IJu}[r\theta_s]_v - 2\kappa\theta_{IJu}[r\delta_s]_v] = 0, \end{array} \right. \quad (\text{B.3a})$$

$$\Lambda_{MN} [\theta_{MNL}[I\theta_J]_{Lrs} + \theta_{MNP}[r\theta_s]_{pIJ} + 4\theta_{M[I\theta_J]_{Nrs}} - 2\kappa\delta_{M[I\theta_J]_{Nrs}}] = 0, \quad (\text{B.3b})$$

$$\theta_{Mup}[I\theta_J]_{prs} - \theta_{Mup}[r\theta_s]_{pIJ} + \delta_{M[I\theta_J]_{rsp}}\theta_{pu} + \kappa\delta_{M[I\theta_J]_{rsu}} - \theta_{M[I\theta_J]_{rsu}} = 0. \quad (\text{B.3c})$$

$$(\mathcal{MNPQ}) = (Ipqr).$$

$$\left\{ \begin{array}{l} \Lambda_{uv} [\theta_{Iuv}\theta_{spqr} - \theta_{uvIL}\theta_{Lpqr} + 3\theta_{Is[pq}\theta_r]_{uvs} - 3\theta_{IL[pq}\theta_r]_{uvL} \\ - 6\theta_{vs}\theta_{Is[pq}\delta_r]_u + 6\theta_{Iv[pq}\theta_r]_u + 6\kappa\theta_{Iv[pq}\delta_r]_u] = 0, \end{array} \right. \quad (\text{B.4a})$$

$$\Lambda_{uv} [-\theta_{MNIL}\theta_{Lpqr} + 3\theta_{Is[pq}\theta_r]_{sMN} + 4\theta_{MI}\theta_{Npqr} - 2\kappa\delta_{MI}\theta_{Npqr}] = 0, \quad (\text{B.4b})$$

$$\left\{ \begin{array}{l} \theta_{MIus}\theta_{spqr} - 3\theta_{Is[pq}\theta_r]_{usM} - 3\theta_{IL[pq}\theta_r]_{uML} + \delta_{MI}\theta_{us}\theta_{spqr} + \kappa\delta_{MI}\theta_{upqr} \\ - \theta_{MI}\theta_{upqr} - 3\theta_{ML}\theta_{IL[pq}\delta_r]_u + 3\kappa\theta_{IM[pq}\delta_r]_u + 3\theta_{IM[pq}\theta_r]_u = 0. \end{array} \right. \quad (\text{B.4c})$$

$$(\mathcal{MNPQ}) = (pqrs).$$

$$\left\{ \begin{array}{l} \Lambda_{uv} [\theta_{uvl}[p\theta_{qrs}]_l - \theta_{Luv}[p\theta_{qrs}]_L - 2\delta_{u[p}\theta_{qrs}]_l\theta_{lv} - 2\theta_{u[p}\theta_{qrs}]_v - 2\delta_{u[p}\theta_{qrs}]_v] = 0, \end{array} \right. \quad (\text{B.5a})$$

$$\left\{ \begin{array}{l} -\theta_{MLu}[p\theta_{qrs}]_L + \theta_{Mlu}[p\theta_{qrs}]_l + \delta_{u[p}\theta_{qrs}]_L\theta_{ML} - \kappa\delta_{u[p}\theta_{qrs}]_M - 2\theta_{u[p}\theta_{qrs}]_M = 0, \end{array} \right. \quad (\text{B.5b})$$

$$\left\{ \begin{array}{l} \theta_{MNI}[p\theta_{qrs}]_l = 0. \end{array} \right. \quad (\text{B.5c})$$

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References

- [1] J.M. Maldacena, *The large N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [[hep-th/9711200](#)] [[INSPIRE](#)].
- [2] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)] [[INSPIRE](#)].
- [3] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [[hep-th/9905111](#)] [[INSPIRE](#)].
- [4] H. Nicolai and H. Samtleben, *Maximal gauged supergravity in three-dimensions*, *Phys. Rev. Lett.* **86** (2001) 1686 [[hep-th/0010076](#)] [[INSPIRE](#)].
- [5] B. de Wit, H. Samtleben and M. Trigiante, *On Lagrangians and gaugings of maximal supergravities*, *Nucl. Phys. B* **655** (2003) 93 [[hep-th/0212239](#)] [[INSPIRE](#)].
- [6] J. Schön and M. Weidner, *Gauged N = 4 supergravities*, *JHEP* **05** (2006) 034 [[hep-th/0602024](#)] [[INSPIRE](#)].
- [7] J. Louis and H. Triendl, *Maximally supersymmetric AdS₄ vacua in N = 4 supergravity*, *JHEP* **10** (2014) 007 [[arXiv:1406.3363](#)] [[INSPIRE](#)].
- [8] J. Louis and S. Lüster, *Supersymmetric AdS₇ backgrounds in half-maximal supergravity and marginal operators of (1,0) SCFTs*, *JHEP* **10** (2015) 120 [[arXiv:1506.08040](#)] [[INSPIRE](#)].
- [9] J. Louis, H. Triendl and M. Zagermann, *N = 4 supersymmetric AdS₅ vacua and their moduli spaces*, *JHEP* **10** (2015) 083 [[arXiv:1507.01623](#)] [[INSPIRE](#)].

- [10] P. Karndumri and J. Louis, *Supersymmetric AdS_6 vacua in six-dimensional $N = (1, 1)$ gauged supergravity*, *JHEP* **01** (2017) 069 [[arXiv:1612.00301](#)] [[INSPIRE](#)].
- [11] S. Lüst, P. Ruter and J. Louis, *Maximally supersymmetric AdS solutions and their moduli spaces*, *JHEP* **03** (2018) 019 [[arXiv:1711.06180](#)] [[INSPIRE](#)].
- [12] T. Fischbacher, H. Nicolai and H. Samtleben, *Vacua of maximal gauged $D = 3$ supergravities*, *Class. Quant. Grav.* **19** (2002) 5297 [[hep-th/0207206](#)] [[INSPIRE](#)].
- [13] M. Pernici, K. Pilch and P. van Nieuwenhuizen, *Gauged maximally extended supergravity in seven-dimensions*, *Phys. Lett. B* **143** (1984) 103 [[INSPIRE](#)].
- [14] M. Günaydin, L.J. Romans and N.P. Warner, *Compact and noncompact gauged supergravity theories in five-dimensions*, *Nucl. Phys. B* **272** (1986) 598 [[INSPIRE](#)].
- [15] B. de Wit and H. Nicolai, *$N = 8$ supergravity*, *Nucl. Phys. B* **208** (1982) 323 [[INSPIRE](#)].
- [16] G. Dall’Agata, G. Inverso and M. Trigiante, *Evidence for a family of $SO(8)$ gauged supergravity theories*, *Phys. Rev. Lett.* **109** (2012) 201301 [[arXiv:1209.0760](#)] [[INSPIRE](#)].
- [17] A. Chatrabhuti and P. Karndumri, *Vacua and RG flows in $N = 9$ three dimensional gauged supergravity*, *JHEP* **10** (2010) 098 [[arXiv:1007.5438](#)] [[INSPIRE](#)].
- [18] A. Chatrabhuti and P. Karndumri, *Vacua of $N = 10$ three dimensional gauged supergravity*, *Class. Quant. Grav.* **28** (2011) 125027 [[arXiv:1011.5355](#)] [[INSPIRE](#)].
- [19] A. Achúcarro and P.K. Townsend, *A Chern-Simons action for three-dimensional anti-de Sitter supergravity theories*, *Phys. Lett. B* **180** (1986) 89 [[INSPIRE](#)].
- [20] H. Nicolai and H. Samtleben, *Compact and noncompact gauged maximal supergravities in three-dimensions*, *JHEP* **04** (2001) 022 [[hep-th/0103032](#)] [[INSPIRE](#)].
- [21] B. de Wit, I. Herger and H. Samtleben, *Gauged locally supersymmetric $D = 3$ nonlinear σ -models*, *Nucl. Phys. B* **671** (2003) 175 [[hep-th/0307006](#)] [[INSPIRE](#)].
- [22] W. Nahm, *Supersymmetries and their representations*, *Nucl. Phys. B* **135** (1978) 149 [[INSPIRE](#)].
- [23] M. Günaydin, G. Sierra and P.K. Townsend, *The unitary supermultiplets of $d = 3$ anti-de Sitter and $d = 2$ conformal superalgebras*, *Nucl. Phys. B* **274** (1986) 429 [[INSPIRE](#)].
- [24] G. Dibitetto, G. Lo Monaco, A. Passias, N. Petri and A. Tomasiello, *AdS_3 solutions with exceptional supersymmetry*, *Fortsch. Phys.* **66** (2018) 1800060 [[arXiv:1807.06602](#)] [[INSPIRE](#)].
- [25] B. de Wit, A.K. Tollsten and H. Nicolai, *Locally supersymmetric $D = 3$ nonlinear σ -models*, *Nucl. Phys. B* **392** (1993) 3 [[hep-th/9208074](#)] [[INSPIRE](#)].
- [26] H. Nicolai and H. Samtleben, *$N = 8$ matter coupled AdS_3 supergravities*, *Phys. Lett. B* **514** (2001) 165 [[hep-th/0106153](#)] [[INSPIRE](#)].
- [27] N. Marcus and J.H. Schwarz, *Three-dimensional supergravity theories*, *Nucl. Phys. B* **228** (1983) 145 [[INSPIRE](#)].
- [28] G. Dibitetto, A. Guarino and D. Roest, *Charting the landscape of $N = 4$ flux compactifications*, *JHEP* **03** (2011) 137 [[arXiv:1102.0239](#)] [[INSPIRE](#)].
- [29] G. Dall’Agata and G. Inverso, *On the vacua of $N = 8$ gauged supergravity in 4 dimensions*, *Nucl. Phys. B* **859** (2012) 70 [[arXiv:1112.3345](#)] [[INSPIRE](#)].
- [30] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys. B* **556** (1999) 89 [[hep-th/9905104](#)] [[INSPIRE](#)].
- [31] M. Henningson and K. Sfetsos, *Spinors and the AdS/CFT correspondence*, *Phys. Lett. B* **431** (1998) 63 [[hep-th/9803251](#)] [[INSPIRE](#)].

- [32] W. Mück and K.S. Viswanathan, *Conformal field theory correlators from classical scalar field theory on AdS_{d+1}* , *Phys. Rev. D* **58** (1998) 041901 [[hep-th/9804035](#)] [[INSPIRE](#)].
- [33] W. Mück and K.S. Viswanathan, *Conformal field theory correlators from classical field theory on anti-de Sitter space. 2. Vector and spinor fields*, *Phys. Rev. D* **58** (1998) 106006 [[hep-th/9805145](#)] [[INSPIRE](#)].
- [34] W.S. l'Yi, *Correlators of currents corresponding to the massive p form fields in AdS/CFT correspondence*, *Phys. Lett. B* **448** (1999) 218 [[hep-th/9811097](#)] [[INSPIRE](#)].
- [35] A. Volovich, *Rarita-Schwinger field in the AdS/CFT correspondence*, *JHEP* **09** (1998) 022 [[hep-th/9809009](#)] [[INSPIRE](#)].
- [36] G. Dibitetto, A. Guarino and D. Roest, *How to halve maximal supergravity*, *JHEP* **06** (2011) 030 [[arXiv:1104.3587](#)] [[INSPIRE](#)].
- [37] H. Nicolai and H. Samtleben, *Kaluza-Klein supergravity on $AdS_3 \times S^3$* , *JHEP* **09** (2003) 036 [[hep-th/0306202](#)] [[INSPIRE](#)].
- [38] U. Gran, J.B. Gutowski and G. Papadopoulos, *On supersymmetric anti-de-Sitter, de-Sitter and Minkowski flux backgrounds*, *Class. Quant. Grav.* **35** (2018) 065016 [[arXiv:1607.00191](#)] [[INSPIRE](#)].
- [39] S. Beck, U. Gran, J. Gutowski and G. Papadopoulos, *All Killing superalgebras for warped AdS backgrounds*, *JHEP* **12** (2018) 047 [[arXiv:1710.03713](#)] [[INSPIRE](#)].
- [40] A.S. Haupt, S. Lautz and G. Papadopoulos, *A non-existence theorem for $N > 16$ supersymmetric AdS_3 backgrounds*, *JHEP* **07** (2018) 178 [[arXiv:1803.08428](#)] [[INSPIRE](#)].
- [41] O. Hohm, E.T. Musaev and H. Samtleben, *$O(d+1, d+1)$ enhanced double field theory*, *JHEP* **10** (2017) 086 [[arXiv:1707.06693](#)] [[INSPIRE](#)].
- [42] O. Hohm and H. Samtleben, *Exceptional field theory. III. $E_{8(8)}$* , *Phys. Rev. D* **90** (2014) 066002 [[arXiv:1406.3348](#)] [[INSPIRE](#)].
- [43] E. Malek, *Half-maximal supersymmetry from exceptional field theory*, *Fortsch. Phys.* **65** (2017) 1700061 [[arXiv:1707.00714](#)] [[INSPIRE](#)].
- [44] N. Kim, *AdS_3 solutions of IIB supergravity from D3-branes*, *JHEP* **01** (2006) 094 [[hep-th/0511029](#)] [[INSPIRE](#)].
- [45] J.P. Gauntlett, O.A.P. Mac Conamhna, T. Mateos and D. Waldram, *Supersymmetric AdS_3 solutions of type IIB supergravity*, *Phys. Rev. Lett.* **97** (2006) 171601 [[hep-th/0606221](#)] [[INSPIRE](#)].
- [46] E. D'Hoker, J. Estes, M. Gutperle and D. Krym, *Exact half-BPS flux solutions in M-theory. I: local solutions*, *JHEP* **08** (2008) 028 [[arXiv:0806.0605](#)] [[INSPIRE](#)].
- [47] E. Ó Colgáin, J.-B. Wu and H. Yavartanoo, *Supersymmetric $AdS_3 \times S^2$ M-theory geometries with fluxes*, *JHEP* **08** (2010) 114 [[arXiv:1005.4527](#)] [[INSPIRE](#)].
- [48] Y. Lozano, N.T. Macpherson, J. Montero and E. Ó Colgáin, *New $AdS_3 \times S^2$ T-duals with $N = (0, 4)$ supersymmetry*, *JHEP* **08** (2015) 121 [[arXiv:1507.02659](#)] [[INSPIRE](#)].
- [49] C. Couzens, C. Lawrie, D. Martelli, S. Schäfer-Nameki and J.-M. Wong, *F-theory and AdS_3/CFT_2* , *JHEP* **08** (2017) 043 [[arXiv:1705.04679](#)] [[INSPIRE](#)].
- [50] L. Wulff, *All symmetric $AdS_{n>2}$ solutions of type-II supergravity*, *J. Phys. A* **50** (2017) 495402 [[arXiv:1706.02118](#)] [[INSPIRE](#)].
- [51] L. Eberhardt, *Supersymmetric AdS_3 supergravity backgrounds and holography*, *JHEP* **02** (2018) 087 [[arXiv:1710.09826](#)] [[INSPIRE](#)].