

# Proper TMD factorization for quarkonia production: $pp \rightarrow \eta_{c,b}$ as a study case

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**ABSTRACT:** Quarkonia production in different high-energy processes has recently been proposed in order to probe gluon transverse-momentum-dependent parton distribution and fragmentation functions (TMDs in general). However, no proper factorization theorems have been derived for the discussed processes, but rather just ansatzs, whose main assumption is the factorization of the two soft mechanisms present in the process: soft-gluon radiation and the formation of the bound state. In this paper it is pointed out that, at low transverse momentum, these mechanisms are entangled and thus encoded in a new kind of non-perturbative hadronic quantities beyond the TMDs: the TMD shape functions. This is illustrated by deriving the factorization theorem for the process  $pp \rightarrow \eta_{c,b}$  at low transverse momentum.

**KEYWORDS:** Effective Field Theories, Perturbative QCD, Renormalization Group

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**1 Motivation**

Gluons, together with quarks, are the fundamental constituents of nucleons. They generate almost all their mass and carry about half of their momentum. However, their three-dimensional (3D) structure is still unknown, as well as their contribution to the nucleon spin. The 3D structure of hadrons in momentum space is parametrized by the so-called Transverse-Momentum-Dependent Parton Distribution/Fragmentation functions (TMDPDFs/TMDFFs, TMDs in general) [1]. These are the 3D generalization of the one-dimensional PDFs and FFs, where a dependence on the transverse momentum of the partons is also allowed. They include as well the correlations between this transverse momentum and the spins of the considered parton and its parent hadron.

Constraining gluon TMDs is a crucial step in our understanding of nucleon 3D structure, and with that also our understanding of confinement in QCD and the structure of ordinary matter in general. In fact, the study of gluon TMDs in particular, and the gluon content of the nucleons in general, is one of the main motivations that is pushing forward the design of the Electron-Ion Collider in the US [2] and fixed-target experiments at the LHC at CERN [3–6].

In the last years a huge step forward has been made in the quark TMDs sector, obtaining their proper definition and properties, and their connection with observable cross-sections in terms of robust factorization theorems [7–11]. This, together with new higher-order perturbative calculations (see e.g. [12–16]), has allowed the phenomenological analyses to enter a new precision stage (see e.g. [17–23]). However, for gluons the situation is very different. Even if their proper definition is currently known [24], the processes where they can be probed are less clean compared to the ones which are used to access quark TMDs.

All and all, quarkonium production seems the most promising way to probe gluon TMDs. Indeed, there has been a growing interest lately, with numerous proposals based on tree-level ansatzs for TMD factorization for quarkonium production [25–42] and even several next-to-leading-order (NLO) calculations [43–46].

The caveat shared by all these attempts, however, is that all of them assume the decoupling of the two soft mechanisms present in the processes: the soft physics underlying

the formation of the quarkonium bound state and the soft gluon resummation. Indeed, these two soft phenomena cannot be factorized when their relevant scales are comparable, i.e. when  $q_T \sim m_Q v$ , being  $q_T$  the measured transverse momentum and  $m_Q v$  the typical momentum of a heavy quark of mass  $m_Q$  and velocity  $v$  inside the quarkonium state. For  $b\bar{b}$  states one typically has  $v^2 \sim 0.1$ , while for  $c\bar{c}$  one has  $v^2 \sim 0.3$ , so then  $m_b v \sim 1.3$  GeV and  $m_c v \sim 0.7$  GeV. Roughly speaking, when the transverse momentum is in the non-perturbative region around and below  $\Lambda_{\text{QCD}}$ , then the factorization ansatz made in all previous analyses is not accurate. And it is precisely this region which is claimed to be sensitive to gluon TMDs and where they can potentially be probed.

In this paper the process  $pp \rightarrow \eta_{c,b} + X$  is considered as an example of quarkonium production process, and the factorization theorem at low transverse momentum is derived. It turns out that the cross-section is not given only in terms of gluon TMDs, but there is an additional new non-perturbative hadronic quantity which encodes the two mentioned soft processes together: the TMD shape function. Using the newly derived factorization theorem, the hard part of the process is obtained at one-loop, which is an essential ingredient for future phenomenological analyses since it allows the resummation of large logarithms at higher logarithmic orders. Moreover, the obtention of a hard factor free from infrared divergences represents a non-trivial consistency check of the newly derived factorization theorem.

## 2 Factorization theorem for $pp \rightarrow \eta_{c,b} + X$ at low $q_T$

The differential cross section for  $\eta_Q$  ( $Q = c, b$ ) hadro-production is given by

$$d\sigma = \frac{1}{2s} \frac{d^3q}{(2\pi)^3 2E_q} \int d^4\xi e^{-iq\cdot\xi} \sum_X \langle PS_A, \bar{P}S_B | \mathcal{O}^\dagger(\xi) | X \eta_Q \rangle \langle \eta_Q X | \mathcal{O}(0) | PS_A, \bar{P}S_B \rangle, \quad (2.1)$$

where  $s = (P + \bar{P})^2$ . The effective operator which mediates this process within an effective theory which combines both soft-collinear effective theory (SCET) [47–52] and non-relativistic QCD (NRQCD) [53] degrees of freedom can be written as<sup>1</sup>

$$\mathcal{O}(\xi) = -2q^2 \sum_n C_H^{(n)}(-q^2; \mu^2) \left[ \psi^\dagger(\xi) \Gamma_{\mu\nu}^{(n)} K_{aa'}^{[1,8]} \chi(\xi) \right] \left[ B_{\bar{n}\perp}^{\mu,b}(\xi) \mathcal{Y}_{\bar{n}}^{\dagger ba}(\xi) \mathcal{Y}_n^{a'c}(\xi) B_{n\perp}^{\nu,c}(\xi) \right], \quad (2.2)$$

where the sum runs over the different states  $n$  which can contribute to the process. In the usual spectroscopic notation,  $n$  stands for  $^{2S+1}L_J^{[i]}$ , with  $S$  the spin,  $L$  the angular momentum and  $J$  the total angular momentum, and  $i = 1(8)$  for singlet (octet).  $C_H^{(n)}$  are the spin-independent matching coefficients for each state  $n$ , which integrate out the hard scale of the process,  $q^2 = M^2$ .  $a, a', b, c$  are the gauge group indexes, while  $\Gamma_{\mu\nu}^{(n)}$  is a matrix which encodes the Lorentz structure of the partonic process for the creation of the state  $n$ .  $K_{aa'}$  is a color matrix, with  $K_{aa'}^{[1]} = \delta_{aa'}$  for color-singlet states and  $K_{aa'}^{[8]} = t_{aa'}^A$  for color-octet states.  $\chi$  and  $\psi$  are the spinors describing the  $Q\bar{Q}$  state. The  $B_{n(\bar{n})}^{\perp\mu}$  operators,

<sup>1</sup>A vector  $a^\mu$  is decomposed as  $a^\mu = \bar{n} \cdot a \frac{n^\mu}{2} + n \cdot a \frac{\bar{n}^\mu}{2} + a_\perp^\mu = a^+ \frac{n^\mu}{2} + a^- \frac{\bar{n}^\mu}{2} + a_\perp^\mu$ , with  $n = (1, 0, 0, 1)$ ,  $\bar{n} = (1, 0, 0, -1)$ ,  $n^2 = \bar{n}^2 = 0$  and  $n \cdot \bar{n} = 2$ . We denote  $a_T = |\mathbf{a}_\perp|$ , i.e.  $a_\perp^2 = -a_T^2 < 0$ .

which stand for gauge invariant gluon fields, are given by

$$B_{n\perp}^\mu = B_{n\perp}^{\mu,a} t^a = \frac{1}{g} [W_n^\dagger i D_n^{\perp\mu} W_n] = \frac{1}{\bar{n} \cdot \mathcal{P}} i \bar{n}_\alpha g_{\perp\beta}^\mu W_n^\dagger F_n^{\alpha\beta} W_n \frac{1}{\bar{n} \cdot \mathcal{P}} i \bar{n}_\alpha g_{\perp\beta}^\mu t^a (\mathcal{W}_n^\dagger)^{ab} F_n^{\alpha\beta,b}. \quad (2.3)$$

The collinear and soft Wilson lines are the path-ordered exponentials

$$\begin{aligned} W_n(\xi) &= P \exp \left[ ig \int_{-\infty}^0 ds \bar{n} \cdot A_n^a(\xi + \bar{n}s) t^a \right], \\ Y_n(\xi) &= P \exp \left[ ig \int_{-\infty}^0 ds n \cdot A_s^a(\xi + ns) t^a \right]. \end{aligned} \quad (2.4)$$

Wilson lines with calligraphic typography are in the adjoint representation, i.e., the color generators are given by  $(t^a)^{bc} = -if^{abc}$ . In order to guarantee gauge invariance among regular and singular gauges, transverse gauge links need to be added (as described in [54, 55]).

In the case of  $\eta_Q$  production, NRQCD formalism dictates that the operator for the state  $^1S_0^{[1]}$  is the leading one in the power counting in the velocity  $v$  [53]. Thus from now on we will consider only that contribution. Moreover, the production of color-octet states will potentially spoil TMD factorization due to the presence of uncanceled Glauber gluons [56–58]. The Lorentz structure  $\Gamma_{\mu\nu}^{^1S_0}$  is fixed by requiring the effective operator to give the same tree-level amplitude as in full QCD for the production of a pseudoscalar  $\eta_Q$  in the configuration  $^1S_0^{[1]}$ :

$$\Gamma_{\mu\nu}^{^1S_0} = \frac{i\pi\alpha_s 2\sqrt{2}}{N_c \sqrt{M^5}} \epsilon_{\perp\mu\nu}, \quad (2.5)$$

where  $\epsilon_{\perp}^{\mu\nu} = \epsilon^{\alpha\beta\mu\nu} n^\alpha \bar{n}^\beta / (n \cdot \bar{n})$ , with  $\epsilon_{\perp}^{12} = 1$ .

The effective Lagrangian for this process is given by the combination of both SCET and NRQCD effective Lagrangians. This means that (anti)collinear modes are decoupled from soft and ultrasoft modes, while the latter are coupled among them (through the NRQCD Lagrangian). One can thus decompose the final state as the following product of states:

$$|X\eta_Q\rangle = |X_n\rangle \otimes |X_{\bar{n}}\rangle \otimes |X_s\eta_Q\rangle, \quad (2.6)$$

where  $X_{n,\bar{n},s}$  are the collinear, anticollinear and soft modes of the unobserved final states. Notice that the state  $\eta_Q$  cannot be decoupled from  $X_s$ . A similar decomposition applies to the initial state, considering proton A to be collinear and proton B anticollinear:

$$|PS_A, \bar{P}S_B\rangle = |PS_A\rangle \otimes |\bar{P}S_B\rangle. \quad (2.7)$$

Using the decompositions in modes (2.6) and (2.7) the cross section is written as

$$\begin{aligned} d\sigma &= \frac{1}{2s} \frac{d^3q}{(2\pi)^3 2E_q} 4M^4 H(M^2, \mu^2) \Gamma_{\rho\sigma}^* \Gamma_{\mu\nu} \int d^4\xi e^{-iq\xi} \\ &\times \sum_{X_n} \langle PS_A | B_{n\perp}^{\sigma,c'}(\xi) | X_n \rangle \langle X_n | B_{n\perp}^{\nu,c}(0) | PS_A \rangle \\ &\times \sum_{X_{\bar{n}}} \langle \bar{P}S_B | B_{\bar{n}\perp}^{\rho,b'}(\xi) | X_{\bar{n}} \rangle \langle X_{\bar{n}} | B_{\bar{n}\perp}^{\mu,b}(0) | \bar{P}S_B \rangle \\ &\times \sum_{X_s} \langle 0 | [\mathcal{Y}_n^{\dagger c' a'} \mathcal{Y}_{\bar{n}}^{a' b'} \chi^\dagger \psi](\xi) | X_s \eta_Q \rangle \langle \eta_Q X_s | [\mathcal{Y}_{\bar{n}}^{\dagger ba} \mathcal{Y}_n^{ac} \psi^\dagger \chi](0) | 0 \rangle, \end{aligned} \quad (2.8)$$

where  $H(M^2, \mu^2) = |C_H(-q^2; \mu^2)|^2$ . The Lorentz structure  $\Gamma_{\mu\nu} \equiv \Gamma_{\mu\nu}^{1S_0}$  is kept for simplicity. This result needs to be Taylor expanded in order to extract the leading contribution with a homogenous power counting. The produced quarkonium is hard with momentum  $q \sim M(1, 1, \lambda)$ , where  $\lambda$  is a small parameter parametrizing the relative strength of the momentum components of different modes,  $\lambda \sim q_T/M$ . In the exponent  $e^{-iq\xi}$  in (2.1) one then has  $\xi \sim M(1, 1, 1/\lambda)$ . In addition, the scalings of the derivatives of the collinear, anticollinear and soft terms are the same as their respective momentum scalings. Given this, the obtained leading term in the Taylor expansion of the cross section is

$$\begin{aligned}
 d\sigma &= \frac{1}{2s} \frac{d^3q}{(2\pi)^3 2E_q} \frac{4M^4 H(M^2, \mu^2)}{(N_c^2 - 1)^2} \Gamma_{\rho\sigma}^* \Gamma_{\mu\nu} \int d^4\xi e^{-iq\xi} \\
 &\times \langle PS_A | B_{n\perp}^{\sigma,c}(\xi^-, \xi_\perp) B_{n\perp}^{\nu,c}(0) | PS_A \rangle \langle \bar{P}S_B | B_{\bar{n}\perp}^{\rho,b}(\xi^+, \xi_\perp) B_{\bar{n}\perp}^{\mu,b}(0) | \bar{P}S_B \rangle \\
 &\times \langle 0 | \left[ \mathcal{Y}_n^{\dagger ca'} \mathcal{Y}_{\bar{n}}^{a'b} \chi^\dagger \psi \right] (\xi_\perp) a_{\eta_Q}^\dagger a_{\eta_Q} \left[ \mathcal{Y}_{\bar{n}}^{\dagger ba} \mathcal{Y}_n^{ac} \psi^\dagger \chi \right] (0) | 0 \rangle + \mathcal{O}(\lambda), \quad (2.9)
 \end{aligned}$$

where we have also used the fact that (anti)collinear matrix elements are diagonal in color, and the completeness relations

$$\begin{aligned}
 \sum_{X_n} |X_n\rangle \langle X_n| &= 1, \quad \sum_{X_{\bar{n}}} |X_{\bar{n}}\rangle \langle X_{\bar{n}}| = 1, \\
 \sum_{X_s} |X_s \eta_Q\rangle \langle X_s \eta_Q| &= a_{\eta_Q}^\dagger \sum_{X_s} |X_s\rangle \langle X_s| a_{\eta_Q} = a_{\eta_Q}^\dagger a_{\eta_Q}. \quad (2.10)
 \end{aligned}$$

Performing standard algebraic manipulations and dropping the suppressed terms, the cross-section can be written as:

$$\begin{aligned}
 \frac{d\sigma}{dy d^2q_\perp} &= \frac{4M^4 H(M^2, \mu^2)}{2sM^2(N_c^2 - 1)} \Gamma_{\rho\sigma}^* \Gamma_{\mu\nu} (2\pi) \int d^2\mathbf{k}_{n\perp} d^2\mathbf{k}_{\bar{n}\perp} d^2\mathbf{k}_{s\perp} \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_{n\perp} - \mathbf{k}_{\bar{n}\perp} - \mathbf{k}_{s\perp}) \\
 &\times J_n^{(0)\sigma\nu}(x_A, \mathbf{k}_{n\perp}, S_A; \mu; \delta_n) J_{\bar{n}}^{(0)\rho\mu}(x_B, \mathbf{k}_{\bar{n}\perp}, S_B; \mu; \delta_{\bar{n}}) S_{\eta_Q}^{(0)} \left[ 1 S_0^{[1]} \right] (\mathbf{k}_{s\perp}; \mu; \delta_n, \delta_{\bar{n}}), \quad (2.11)
 \end{aligned}$$

where  $x_{A,B} = \sqrt{\tau} e^{\pm y}$ ,  $\tau = (M^2 + q_T^2)/s$  and  $y$  is the rapidity of the produced  $\eta_Q$ . The *pure* collinear matrix elements  $J_{n(\bar{n})}^{(0)}$  and the bare TMD shape function  $S_Q^{(0)}$  (TMDShF from now on) are defined as

$$\begin{aligned}
 J_n^{(0)\mu\nu} &= \frac{x_A P^+}{2} \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}x_A \xi^- P^+ - \xi_\perp \mathbf{k}_{n\perp})} \langle PS_A | B_{n\perp}^{\mu,a}(\xi^-, \xi_\perp) B_{n\perp}^{\nu,a}(0) | PS_A \rangle, \\
 J_{\bar{n}}^{(0)\mu\nu} &= \frac{x_B \bar{P}^-}{2} \int \frac{d\xi^+ d^2\xi_\perp}{(2\pi)^3} e^{-i(\frac{1}{2}x_B \xi^+ \bar{P}^- - \xi_\perp \mathbf{k}_{\bar{n}\perp})} \langle \bar{P}S_B | B_{\bar{n}\perp}^{\mu,a}(\xi^+, \xi_\perp) B_{\bar{n}\perp}^{\nu,a}(0) | \bar{P}S_B \rangle, \\
 S_{\eta_Q}^{(0)} \left[ 1 S_0^{[1]} \right] &= \frac{1}{N_c^2 - 1} \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{i\xi_\perp \mathbf{k}_{s\perp}} \langle 0 | \left[ \mathcal{Y}_n^{\dagger ab} \mathcal{Y}_{\bar{n}}^{bc} \chi^\dagger \psi \right] (\xi_\perp) a_{\eta_Q}^\dagger a_{\eta_Q} \left[ \mathcal{Y}_{\bar{n}}^{\dagger cd} \mathcal{Y}_n^{da} \psi^\dagger \chi \right] (0) | 0 \rangle. \quad (2.12)
 \end{aligned}$$

Notice that the spurious contribution of the soft momentum modes in the naively calculated collinear matrix elements, denoted  $J_{n(\bar{n})}$  (the so-called ‘‘zero-bin’’ in the SCET nomenclature), should be subtracted, in order to avoid their double counting.

Both the collinear matrix elements and the bare TMDShF in (2.11) have been written with a dependence on  $\delta_{n(\bar{n})}$ , which stand for generic rapidity regulators. These divergences

cancel in the full combination of the three matrix elements. However a different soft function needs to be invoked in order to properly define the gluon TMDs:

$$S = \frac{1}{N_c^2 - 1} \int \frac{d^2 \boldsymbol{\xi}_\perp}{(2\pi)^2} e^{i \boldsymbol{\xi}_\perp \cdot \mathbf{k}_{s\perp}} \langle 0 | \left[ \mathcal{Y}_n^{\dagger ab} \mathcal{Y}_{\bar{n}}^{bc} \right] (\boldsymbol{\xi}_\perp) \left[ \mathcal{Y}_{\bar{n}}^{\dagger cd} \mathcal{Y}_n^{da} \right] (0) | 0 \rangle . \quad (2.13)$$

This soft function can be split in rapidity space to all orders in perturbation theory as [15]

$$\tilde{S}(\xi_T; \mu; \delta_n, \delta_{\bar{n}}) = \tilde{S}_-(\xi_T; \mu; \delta_n) \tilde{S}_+(\xi_T; \mu; \delta_{\bar{n}}) . \quad (2.14)$$

With these pieces, the gluon TMDPDFs are defined as [24]

$$\begin{aligned} \tilde{G}_{g/A}^{\mu\nu}(x_A, \boldsymbol{\xi}_\perp, S_A; \zeta_A, \mu) &= \tilde{J}_n^{(0)\mu\nu}(x_A, \boldsymbol{\xi}_\perp, S_A; \mu; \delta_n) \tilde{S}_-(\xi_T; \mu; \delta_n) , \\ \tilde{G}_{g/B}^{\mu\nu}(x_B, \boldsymbol{\xi}_\perp, S_B; \zeta_B, \mu) &= \tilde{J}_{\bar{n}}^{(0)\mu\nu}(x_B, \boldsymbol{\xi}_\perp, S_B; \mu; \delta_{\bar{n}}) \tilde{S}_+(\xi_T; \mu; \delta_{\bar{n}}) , \end{aligned} \quad (2.15)$$

where  $\zeta_{A,B}$  are auxiliary energy scales which arise when the rapidity divergences are cancelled in each TMD, and the twiddle labels the functions in coordinate space. The chosen rapidity regulator is arbitrary, however the auxiliary energy scales  $\zeta_A$  and  $\zeta_B$  in the TMDs are bound together by  $\zeta_A \zeta_B = q^4 = M^4$ . We emphasize that gluon TMDs so defined are free from rapidity divergences, i.e., they have well-behaved evolution properties and can be extracted from experimental data.

Given the definitions in (2.15), the factorized cross-section for proton-proton collisions at low  $q_T$  is finally written as

$$\begin{aligned} \frac{d\sigma}{dy d^2 q_\perp} &= \frac{4M^4 H(M^2, \mu^2)}{2sM^2(N_c^2 - 1)} \Gamma_{\rho\sigma}^* \Gamma_{\mu\nu} (2\pi) \int d^2 \mathbf{k}_{n\perp} d^2 \mathbf{k}_{\bar{n}\perp} d^2 \mathbf{k}_{s\perp} \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_{n\perp} - \mathbf{k}_{\bar{n}\perp} - \mathbf{k}_{s\perp}) \\ &\times G_{g/A}^{\sigma\nu}(x_A, \mathbf{k}_{n\perp}, S_A; \zeta_A, \mu) G_{g/B}^{\rho\mu}(x_B, \mathbf{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu) S_{\eta Q} \left[ {}^1S_0^{[1]} \right] (\mathbf{k}_{s\perp}; \mu) , \end{aligned} \quad (2.16)$$

where we have defined, for convenience, the TMDShF free from rapidity divergences as

$$\tilde{S}_{\eta Q} \left[ {}^1S_0^{[1]} \right] (\xi_T; \mu) = \frac{\tilde{S}_{\eta Q}^{(0)} \left[ {}^1S_0^{[1]} \right] (\xi_T; \mu)}{\tilde{S}(\xi_T; \mu)} . \quad (2.17)$$

Given that there are no other vectors available, the TMDShF, as the soft function, depends on the modulus  $\xi_T$ .

The factorization theorem in (2.16) is the main result of this letter. It contains 3 non-perturbative hadronic quantities at low transverse momentum: two gluon TMDPDFs, and the newly defined TMDShF. Thus, the phenomenological extraction of gluon TMDs from quarkonium production processes is still possible, i.e., a robust factorization theorem can potentially be obtained like in this particular case of  $\eta_{c,b}$  hadro-production. However one also needs to model and extract the involved TMDShFs for the relevant angular/color configurations.

Notice that, while the factorized cross-section in (2.16) contains all the (un)polarized gluon TMDs, the TMDShF  $\tilde{S}_{\eta Q} \left[ {}^1S_0^{[1]} \right]$  is spin independent. In particular, if unpolarized

proton collisions are considered, which is relevant e.g. for the LHC, one can parametrize the gluon TMD  $G_{g/A}^{\mu\nu}$  in momentum space as

$$G_{g/A}^{\mu\nu}(x_A, \mathbf{k}_\perp, S_A; \zeta_A, \mu) \equiv \frac{1}{2} \left[ -g_\perp^{\mu\nu} f_1^g(x_A, k_T^2; \zeta_A, \mu) + \frac{k_T^{\mu\nu}}{M_p^2} h_1^{\perp g}(x_A, k_T^2; \zeta_A, \mu) \right], \quad (2.18)$$

where  $M_p$  is the mass of the proton and  $k_T^{\mu\nu}$  is a symmetric traceless tensor of rank 2 [59]:

$$k_T^{\mu\rho} = k_\perp^\mu k_\perp^\rho + \frac{1}{2} \mathbf{k}_\perp^2 g_\perp^{\mu\rho}. \quad (2.19)$$

The function  $f_1^g$  is the TMDPDF for unpolarized gluons in an unpolarized proton, while  $h_1^{\perp g}$  parametrizes linearly polarized gluons inside an unpolarized proton. The parametrization in position space reads

$$\tilde{G}_{g/A}^{\mu\nu}(x_A, \mathbf{b}_\perp, S_A; \zeta_A, \mu) \equiv \frac{1}{2} \left[ -g_\perp^{\mu\nu} \tilde{f}_1^g(x_A, b_T^2; \zeta_A, \mu) - \frac{M_p^2}{2} b_T^{\mu\nu} \tilde{h}_1^{\perp g(2)}(x_A, b_T^2; \zeta_A, \mu) \right], \quad (2.20)$$

where the Fourier transform of the functions and their moments follow the conventions in [59].

Inserting the decomposition (2.18) in (2.16), one obtains the factorized cross section for collisions of unpolarized protons:

$$\frac{d\sigma}{dy d^2q_\perp} = \sigma_0(\mu) H(M^2, \mu^2) \left[ \mathcal{C}[f_1^g f_1^g S_Q] - \mathcal{C}[w_{UU} h_1^{\perp g} h_1^{\perp g} S_{Q\bar{Q}}] \right], \quad (2.21)$$

where  $S_Q$  stands for  $S_{\eta_Q} [1 S_0^{[1]}]$  and the Born-level cross-section is

$$\sigma_0 = \frac{(4\pi\alpha_s)^2 \pi}{N_c^2 (N_c^2 - 1) s M^3}. \quad (2.22)$$

The convolutions  $\mathcal{C}$  are defined in general as:

$$\begin{aligned} \mathcal{C}[w f f S_Q] &\equiv \int d^2\mathbf{p}_{\perp a} d^2\mathbf{p}_{\perp b} d^2\mathbf{k}_\perp \delta^2(\mathbf{p}_{\perp a} + \mathbf{p}_{\perp b} + \mathbf{k}_\perp - \mathbf{q}_\perp) \\ &\times w(p_{\perp a}, p_{\perp b}) f(x_A, \mathbf{p}_{T a}^2; \zeta_A, \mu) f(x_B, \mathbf{p}_{T b}^2; \zeta_B, \mu) S_Q(k_T^2; \mu), \end{aligned} \quad (2.23)$$

and the transverse momentum weight  $w_{UU}$  for the contribution of linearly polarized gluon TMDs is

$$w_{UU} = \frac{p_{\perp a}^{\mu\nu} p_{\perp b \mu\nu}}{2M_p^4}. \quad (2.24)$$

Given this, the two Fourier transforms are

$$\begin{aligned} \mathcal{C}[f_1^g f_1^g S_Q] &= \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T \cdot \mathbf{q}_T} \tilde{f}_1^g(x_A, b_T^2; \zeta_A, \mu) \tilde{f}_1^g(x_B, b_T^2; \zeta_B, \mu) S_Q(b_T^2; \mu) \\ &= \frac{1}{2\pi} \int_0^{+\infty} db_T b_T J_0(b_T q_T) \tilde{f}_1^g(x_A, b_T^2; \zeta_A, \mu) \tilde{f}_1^g(x_B, b_T^2; \zeta_B, \mu) S_Q(b_T^2; \mu), \end{aligned} \quad (2.25)$$

and

$$\begin{aligned} \mathcal{C}[w_{UU} h_1^{\perp g} h_1^{\perp g} S_Q] &= \frac{M_p^4}{16} \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T \cdot \mathbf{q}_T} \mathbf{b}_T^4 \tilde{h}_1^{\perp g(2)}(x_A, b_T^2; \zeta_A, \mu) \tilde{h}_1^{\perp g(2)}(x_B, b_T^2; \zeta_B, \mu) S_Q(b_T^2; \mu) \\ &= \frac{M_p^4}{32\pi} \int_0^{+\infty} db_T \mathbf{b}_T^5 J_0(b_T q_T) \tilde{h}_1^{\perp g(2)}(x_A, b_T^2; \zeta_A, \mu) \tilde{h}_1^{\perp g(2)}(x_B, b_T^2; \zeta_B, \mu) S_Q(b_T^2; \mu). \end{aligned} \quad (2.26)$$

### 3 Calculation of the hard part at NLO

The calculation of the hard part of the process not only provides a necessary ingredient to perform the resummation of large logarithms to get more reliable results, but it is also a test of the newly derived factorization theorem.

A necessary condition for the factorized cross-section to be correct, is that it has to exactly reproduce all the infrared physics of the cross-section in full QCD, order by order in perturbation theory. In other words, the hard factor should turn out to be a finite quantity, just an expansion in  $\alpha_s$ .

It is worth emphasizing that the hard part will only come from the miss-match of virtual diagrams in the full theory and the factorized expression, since by construction it only depends on the hard scale  $M$ , and diagrams with real gluons will have a dependence on the transverse momentum, which is a lower scale. Thus one just needs to compute the virtual part of the cross-section in QCD and then subtract the virtual parts of the two gluon TMDs and the TMDShF.

Let us start with the virtual part of the cross-section up to  $\mathcal{O}(\alpha_s)$  which, after renormalization (i.e. after removing ultraviolet divergences), in coordinate space is [60]

$$\left. \frac{d\sigma}{\sigma_0} \right|_v = \delta(1-x_A)\delta(1-x_B) + \frac{\alpha_s}{2\pi} \left[ C_F \frac{\pi^2}{v} - 2 \left( \frac{C_A}{\varepsilon_{\text{IR}}^2} + \frac{1}{\varepsilon_{\text{IR}}} \left( \frac{\beta_0}{2} + C_A \ln \frac{\mu^2}{M^2} \right) - C_A \ln^2 \frac{\mu^2}{M^2} - C_A \frac{\pi^2}{6} + 2B_{1S_0}^{[1]} \right) \right] \delta(1-x_A)\delta(1-x_B), \quad (3.1)$$

with

$$B_{1S_0}^{[1]} = C_F \left( -5 + \frac{\pi^2}{4} \right) + C_A \left( 1 + \frac{5\pi^2}{12} \right). \quad (3.2)$$

The virtual contribution of the renormalized TMDPDF in coordinate space can be obtained e.g. from [24]:

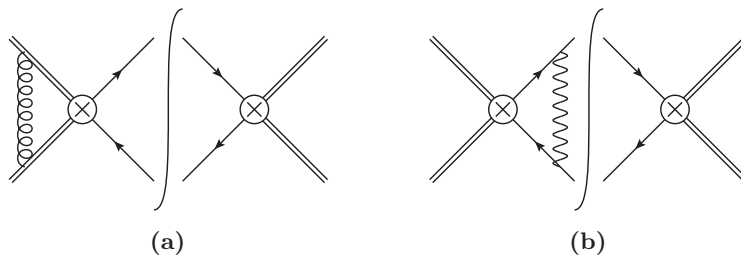
$$\left. \tilde{f}_1^g \right|_v = \delta(1-x) + \frac{\alpha_s}{2\pi} \left[ -\frac{C_A}{\varepsilon_{\text{IR}}^2} - \frac{1}{\varepsilon_{\text{IR}}} \left( \frac{\beta_0}{2} + C_A \ln \frac{\mu^2}{M^2} \right) \right] \delta(1-x). \quad (3.3)$$

The one-loop virtual part of the renormalized TMDShF, defined in (2.17), is given by the virtual diagrams of the bare TMDShF in figure 1 and the ones of the soft function. On one hand, diagram 1a (and its crossed one) gives exactly the same as the corresponding one for the soft function  $S$ , and thus it is cancelled in  $\tilde{S}_Q$ . On the other, diagram 1b (and its crossed one) is analogous to the one found in the long-distance matrix element (LDME)  $\langle 0 | \chi^\dagger \psi a_{\eta_Q}^\dagger a_{\eta_Q} \psi^\dagger \chi | 0 \rangle$ , and can thus be obtained e.g. from [60, 61]. Putting everything together, the result is:

$$\left. \tilde{S}_Q \right|_v = 1 + \frac{\alpha_s}{2\pi} C_F \frac{\pi^2}{v}. \quad (3.4)$$

Notice that the heavy-quark self-energy vanishes on the energy shell (see e.g. [53]). In addition, there are no interactions at this order between the heavy quarks (soft) and the soft gluons from the soft Wilson lines [62]. In fact, the gluon connecting the two soft





**Figure 1.** Relevant Feynman diagrams for the virtual part of the TMDShF. Double solid lines stand for soft Wilson lines, while single lines for heavy quarks. The gluon in diagram 1a is soft, while the one in diagram 1b, denoted with a wavy line, is ultrasoft. Corresponding crossed diagrams should be added.

Wilson lines in diagram 1a is soft, while the one connecting the heavy quarks in diagram 1b is ultrasoft. This is the reason why the authors in [44, 46] get to the misleading conclusion that the factorization ansatz they propose for this process is justified, i.e., that the cross-section is given in terms of two (subtracted) gluon TMDs and the local LDME, which they claim is completely factorized from the soft function at low transverse momentum. Indeed, it turns out that, at one-loop, the virtual part of the TMDShF  $S_Q$  is given by the virtual part of the local LDME. However this fact does not hold for higher-orders. The TMDShF at low transverse momentum is a genuine non-perturbative quantity.

Finally, subtracting to the virtual part of the cross-section in full QCD the virtual part of two gluon TMDs and the virtual part of the TMDShF, we obtain the hard part up to  $\mathcal{O}(\alpha_s)$ :

$$H = 1 + \frac{\alpha_s}{2\pi} \left[ -C_A \ln^2 \frac{\mu^2}{M^2} - C_A \frac{\pi^2}{6} + 2B_{1S_0}^{[1]} \right], \quad (3.5)$$

As expected, this coefficient turns out to be free from infrared divergences, which means that the derived factorization theorem properly reproduces the infrared part of the cross-section in full QCD at one loop. This constitutes a non-trivial consistency check. In addition, this coefficient is a necessary ingredient for the resummation of large logarithms at higher orders, allowing for precise phenomenological studies in the near future.

## 4 Conclusions

By applying the effective field theory approach, a proper factorization theorem for  $\eta_{c,b}$  hadro-production at low transverse momentum is derived, finding a new kind of non-perturbative hadronic quantity: the TMD shape function (TMDShF). This matrix element encodes the two soft mechanisms present in the process, the formation of the heavy-quark bound state and the soft-gluon radiation, which were assumed to factorize in all previous works in the literature.

In general, there are as many TMDShFs for a given process as relevant angular/color Fock states within NRCQD power counting. Simply stated, they could be considered the TMD extensions of the well-known LDMEs.

Quarkonium production processes can thus be used to access gluon TMDs, but the phenomenology is more involved as compared to quark TMDs in, e.g., Drell-Yan or semi-inclusive deep-inelastic scattering processes, since it requires in addition the parametrization of several TMDShFs.

These findings can straightforwardly be applied to other quarkonia production processes, for instance in lepton-hadron collisions (like  $ep \rightarrow J/\psi$ ) or electron-positron annihilation (like  $e^+e^- \rightarrow J/\psi\pi$ ). This is left for a future effort.

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