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Spontaneous symmetry breaking in fermionic random matrix model

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ABSTRACT: A fermionic random matrix model, which is a 0-dimensional version of the SYK model with replicas, is considered. The replica-off-diagonal correlation functions vanish at finite N , but we show that they do not vanish in the large N limit due to spontaneous symmetry breaking. We use the Bogoliubov quasi-averages approach to studying phase transitions. The consideration may be relevant to the study of the problem of existence of the spin glass phase in fermionic models.

KEYWORDS: Spontaneous Symmetry Breaking, $1/N$ Expansion, Random Systems

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1 Introduction

Spontaneous symmetry breaking is a common occurrence in solid state and high energy physics including the Ising model, Bose-Einstein condensation, Higgs mechanism and spin glasses [1–5]. In these cases the stable solutions of the equations, which govern the system, exhibit less symmetry than the equations themselves. One has a quantum state that is invariant under a symmetry group and which admits a nontrivial decomposition into extremal states corresponding to pure thermodynamic phases. Usually spontaneous symmetry breaking shows in the range of parameters where the phase transitions take place.

In this note a fermionic random matrix model [6–8] is considered which is a 0-dimensional version of the SYK model [9–11] with a finite number of replicas. The replica-off-diagonal correlations functions vanish at finite N , but we show that they do not vanish in the large N limit due to spontaneous symmetry breaking. We use the Bogoliubov quasi-averages approach [2, 12] to studying phase transitions.

We consider a model of fermions ψ_j^α where $j = 1, \dots, N$, $\alpha = 1, \dots, M$ with the Hamiltonian or action which are invariant under the “gauge group” transformations $\psi_j^\alpha \rightarrow \epsilon^\alpha \psi_j^\alpha$ where $\epsilon^\alpha = \pm 1$. It follows that for finite N the 2-point off-diagonal correlation function vanishes $\langle \psi_j^\alpha \psi_j^\beta \rangle_N = 0$, $\alpha \neq \beta$. One can also argue that the 2-point correlation function for finite N vanishes, since it factorizes into fermion 1-point functions which are zero. In details this argument has been considered in [13].

However, we will show that in the fermionic random matrix model in the limit $N \rightarrow \infty$ there is spontaneous symmetry breaking and the 2-point off-diagonal correlation function is

non-zero. It goes as follows. Let $f_N(A)$, $A = (A_{\alpha\beta})$ is a generating functional (the density of the free energy) for correlation functions and define the order parameter

$$Q_{\alpha\beta}^{(N)} = \left. \frac{\partial f_N(A)}{\partial A_{\alpha\beta}} \right|_{A=0} = \frac{1}{N} \sum_{j=1}^N \langle \psi_j^\alpha \psi_j^\beta \rangle_N = 0. \quad (1.1)$$

So, if N is finite, the order parameter $Q_{\alpha\beta}^{(N)} = 0$. However, if we *first* compute the large N limit, $\lim_{N \rightarrow \infty} f_N(A) = f(A)$ and *then* compute the derivative,

$$Q_{\alpha\beta} = \left. \frac{\partial f(A)}{\partial A_{\alpha\beta}} \right|_{A=0}, \quad (1.2)$$

we will find in this model that the order parameter $Q_{\alpha\beta}$ is nonvanishing. This shows the occurrence of spontaneous symmetry breaking.

Various aspects of the non-diagonal solutions of the saddle point equations and Schwarzian corrections in the SYK and related models are discussed in [6–8, 13–24].

The model considered in this note is a 0-dimensional analogue of the conformal limit of the SYK model in which nondiagonal saddle points have been studied in [19, 21] while the model investigated in [7] is an analogue of the total SYK model with replicas.

In the next section spontaneous symmetry breaking and the quasi-averages method are exposed. In section 3 the fermionic matrix model and the order parameter are discussed. In section 4 the cases $q = 2$ and $q = 4$ are considered and spontaneous breaking of symmetry is shown.

2 Setup

2.1 Spontaneous symmetry breaking and quasi-averages

A powerful method for investigation of the spontaneous symmetry breaking is the Bogoliubov method of quasi-averages, see [2] and for a recent review [12]. The quasi-averages method proceeds as follows. If, in a finite volume Λ , one has a Hamiltonian H which is invariant under a symmetry group, then one replaces $H \rightarrow H + wB$ where B is a suitable extensive operator breaking the symmetry and w is a real parameter. Define the partition function $Z_\Lambda(w) = \text{Tr} e^{-\beta(H+wB)}$, $\beta > 0$. Now we first take the infinite volume thermodynamic limit $\Lambda \rightarrow \mathbb{R}^d$ for the free energy

$$f(w) = - \lim_{\Lambda \rightarrow \mathbb{R}^d} \frac{1}{\beta|\Lambda|} \log Z_\Lambda(w) \quad (2.1)$$

and for correlation functions and then compute the limit of the free energy at $w = 0$, $f(0) = \lim_{w \rightarrow +0} f(w)$ and the same for its derivatives $f'(0)$, $f''(0)$, $\dots$ and for the correlation functions

$$\prec K \succ = \lim_{w \rightarrow +0} \lim_{\Lambda \rightarrow \mathbb{R}^d} \text{Tr}(K e^{-\beta(H+wB)}) / Z_\Lambda(w) \quad (2.2)$$

where K is some operator. It may happen that after taking the limit $w \rightarrow 0$ still there is a dependence on B in $f'(0)$, $f''(0)$, $\dots$ or in the correlation functions. In such a case one speaks of spontaneous symmetry breaking. In particular, $\prec K \succ$ is called the quasi-average of the operator K . The quantity $\prec B \succ$ can be interpreted as an order parameter in the pure phase, selected by the symmetry breaking source.

2.2 Example

There is a simple example of application of quasi-averages for describing spontaneous symmetry breaking. Consider the partition function with the Higgs potential

$$Z_N = \int_{\mathbb{R}} e^{-N(x^2 - a^2)^2} dx, \quad (2.3)$$

where $a > 0$, $N = 1, 2, 3, \dots$. There is a symmetry under transformation $x \rightarrow -x$, $\langle x \rangle_N = 0$ and moreover the free energy vanishes in the large N limit:

$$f_0 = - \lim_{N \rightarrow \infty} \frac{1}{N} \log Z_N = 0. \quad (2.4)$$

Now we add a term breaking the symmetry:

$$Z_N(w) = \int_{\mathbb{R}} e^{-NS} dx, \quad S = (x^2 - a^2)^2 + wx, \quad w > 0. \quad (2.5)$$

Then the free energy is

$$f(w) = \lim_{N \rightarrow \infty} f_N(w) = - \lim_{N \rightarrow \infty} \frac{1}{N} \log Z_N(w) = (x_0^2 - a^2)^2 + wx_0, \quad (2.6)$$

where $x_0 = x_0(w)$ is a solution of the equation

$$4x(x^2 - a^2) + w = 0 \quad (2.7)$$

with the lowest value of the action S . Therefore we obtain

$$f'(w)|_{w=0} = -a. \quad (2.8)$$

So there is a spontaneous symmetry breaking and quasi-average $\prec x \succ = -a$. Note that in this consideration the limit of large N plays a role of the thermodynamical limit.

Uniqueness of the solution $x_0 = x_0(w)$, $w > 0$, is used also when one performs the Legendre transform from the free energy $f(w)$ to the effective potential $\Gamma(\phi) = f(w) - w\phi$, $\phi = \partial_w f(w)$.

3 Fermionic random matrix model

3.1 Model

We consider a polynomial function in anticommuting variables of the form [7]

$$\mathcal{A} = \mathcal{A}_0(J) + \mathcal{A}_1(A), \quad (3.1)$$

where

$$\begin{aligned} \mathcal{A}_0(J) &= (i)^{q/2} \sum_{\alpha} \sum_{k_1 < k_2 < \dots < k_q} J_{k_1 k_2 \dots k_q} \chi_{k_1}^{\alpha} \chi_{k_2}^{\alpha} \dots \chi_{k_q}^{\alpha}, \\ \mathcal{A}_1(A) &= \frac{i}{2} \sum_{\alpha, \beta, j} \chi_j^{\alpha} A_{\alpha\beta} \chi_j^{\beta}. \end{aligned}$$

Here χ_j^α are the Grassmann hermitean variables. We consider the zero dimensional model, so the Grassmann variables χ_j^α do not depend on time τ . One has $\{\chi_j^\alpha, \chi_k^\beta\} = 0$, $k, j = 1, \dots, N$, $\alpha, \beta = 1, \dots, M$, q is an even natural number, and $\mathbf{J} = (J_{k_1 k_2 \dots k_q})$ are Gaussian random variables with zero mean $\langle J_{k_1 k_2 \dots k_q} \rangle = 0$, and variance is given by $\langle J_{k_1 k_2 \dots k_q}^2 \rangle = (q-1)! J^2 / N^{q-1}$, $J > 0$. The matrix $(A_{\alpha\beta})$ is antisymmetric with complex entries. To get correlation functions we will differentiate the generating functional over tensor type source $A_{\alpha\beta}$ and then set $A_{\alpha\beta} = 0$.

Although in this note we treat $A_{\alpha\beta}$ as auxiliary parameters in the generating functional, one can interpret them as coupling constants, such models are discussed in 0-dim case in [7] and in 1-dim in [23, 24].

The generating functional (partition function) in the fermionic random matrix model is defined as the Grassmann (Berezin) integral over anticommuting variables

$$Z_N(A) = \int d\mu(\mathbf{J}) \int d\chi e^{iA}. \quad (3.2)$$

Here $d\mu(\mathbf{J})$ is a Gaussian probability measure. Mathematical questions of superanalysis are considered in [25, 26]. The integral over Grassmann variables for $A_{\alpha\beta} = 0$ vanishes unless N is a multiple of 4, so we assume that N is a multiple of 4. Note also that $Z_N(A)$ is a polynomial at J^2 and $A_{\alpha\beta}$ because there is only a finite number of the Grassmann variables.

The free energy is defined as

$$f_N(A) = -\frac{1}{N} \log Z_N(A). \quad (3.3)$$

We are interested in correlation functions

$$Q_{\alpha\beta}^{(N)} = \frac{\partial}{\partial A_{\alpha,\beta}} f_N(A) \Big|_{A=0} = \frac{1}{N} \sum_{j=1}^N \langle \chi_j^\alpha \chi_j^\beta \rangle_N. \quad (3.4)$$

Note that $Q_{\alpha\beta}^{(N)}$ vanishes if N is finite,

$$Q_{\alpha\beta}^{(N)} = 0, \quad \alpha \neq \beta. \quad (3.5)$$

since the 2-point correlation function (3.4) factorizes into fermion 1-point functions, which vanish. Another way to get the relation (3.5) is by notice that the function $\mathcal{A}_0(J)$ is invariant under the “gauge transformation” $\chi_j^\alpha \rightarrow \epsilon_\alpha \chi_j^\alpha$ where $\epsilon_\alpha = \pm 1$. The relation (3.5) can be interpreted by saying that there is no symmetry breaking at finite N .

3.2 Quasi-averages

However if we first take the limit $N \rightarrow \infty$,

$$f(A) = \lim_{N \rightarrow \infty} f_N(A) \quad (3.6)$$

and then compute the derivative, we could get a non-zero result for 2-point off-diagonal correlation function defined as

$$Q_{\alpha\beta} = \frac{\partial f(A)}{\partial A_{\alpha\beta}} \Big|_{A=0}. \quad (3.7)$$

This is the Bogoliubov quasi-averages approach to phase transitions and spontaneous symmetry breaking. The quantity $Q_{\alpha\beta}$ is called the quasi-average correlation function or the order parameter.

The expression (3.2) can be written also by using the real variables

$$Z_N(A) = \left(\frac{N}{2\pi}\right)^{\frac{M(M-1)}{2}} \int d\Sigma dG e^{NS} \quad (3.8)$$

where

$$S = \log(\text{Pf}(\mathbf{A} + i\Sigma)) + \sum_{\alpha < \beta} \left[-\frac{J^2}{q} G_{\alpha\beta}^q + i\Sigma_{\alpha\beta} G_{\alpha\beta} \right]. \quad (3.9)$$

Here Pf is the Pfaffian, $\mathbf{G} = (G_{\alpha\beta})$ and $\Sigma = (\Sigma_{\alpha\beta})$ are antisymmetric matrices with real entries, $\mathbf{A} = (A_{\alpha\beta})$ is antisymmetric complex matrix, q is an even integer. Note the presence of the factor $\exp\{-J^2 G_{\alpha\beta}^q/q\}$ which provides the convergence of the integral. Note also that in (2.3) and (3.8) we use the different signs in front of the actions. We will show that for $q = 2, 4$ and $M = 2$ the free energy $f(\mathbf{A})$ in the limit $N \rightarrow \infty$ is equal to the action function S_0 computed at the saddle point $(G_{(0)}, \Sigma_{(0)})$, i.e. $f(\mathbf{A}) = -S_0(\mathbf{A})$. The saddle point equations read

$$(\mathbf{A} + i\Sigma)^{-1} = -\mathbf{G}, \quad \Sigma_{\alpha\beta} = -i J^2 (G_{\alpha\beta})^{q-1}. \quad (3.10)$$

Since the gradient of the action function vanishes at the saddle point we obtain the following expression for the order parameter

$$Q_{\alpha\beta} = -\frac{\partial S_0(\mathbf{A})}{\partial A_{\alpha\beta}} \Big|_{A=0} = i \left(\Sigma_{(0)}^{-1} \right)_{\alpha\beta} \Big|_{A=0} = G_{(0)\alpha\beta} \Big|_{A=0}. \quad (3.11)$$

Let us note that the order parameter could depend on the way in which $A \rightarrow 0$ in complex domain. In this note we take $M = 2$ and denote $A = A_{12} = ae^{i\varphi}$, $a > 0$. Then we can define

$$Q(\varphi) = e^{-i\varphi} \frac{\partial}{\partial a} f(ae^{i\varphi}) \Big|_{a=0}. \quad (3.12)$$

If $Q(\varphi)$ is non-zero then one has spontaneous symmetry breaking.

4 Large N asymptotics and quasi-averaging

4.1 $q = 2$ model

We start with $q = 2$, $M = 2$ case.

$$\begin{aligned} Z_N(A) &= \frac{N}{2\pi} \int_{\mathbb{R}} dy (A + iy)^N \int_{\mathbb{R}} dx \exp \left\{ N \left[-\frac{J^2}{2} x^2 + ixy \right] \right\} \\ &= \frac{N}{2\pi} \int dx dy \exp \left\{ N \left[-\frac{J^2}{2} x^2 + ixy + \ln(A + iy) \right] \right\} \end{aligned} \quad (4.1)$$

where we use the parametrization

$$(A_{\alpha\beta}) = \begin{pmatrix} 0 & A \\ -A & 0 \end{pmatrix}, \quad (G_{\alpha\beta}) = \begin{pmatrix} 0 & x \\ -x & 0 \end{pmatrix}, \quad (\Sigma_{\alpha\beta}) = \begin{pmatrix} 0 & y \\ -y & 0 \end{pmatrix}. \quad (4.2)$$

Here A is a complex parameter with small enough $|A|$. The stationary point equations are

$$-J^2 x + iy = 0 \quad (4.3)$$

$$\frac{i}{A + iy} + ix = 0 \quad (4.4)$$

The solutions to these equations have the form

$$x_1 = \frac{-A - i\sqrt{4J^2 - A^2}}{2J^2}, \quad (4.5)$$

$$y_1 = \frac{i}{2} \left(A + i\sqrt{4J^2 - A^2} \right), \quad (4.6)$$

$$x_2 = \frac{-A + i\sqrt{4J^2 - A^2}}{2J^2}, \quad (4.7)$$

$$y_2 = -\frac{i}{2} \left(-A + i\sqrt{4J^2 - A^2} \right). \quad (4.8)$$

The action at the stationary points reads

$$S_1(A) = \log \left(\frac{1}{2} \left(A - i\sqrt{4J^2 - A^2} \right) \right) + \frac{1}{4} \left(\frac{A^2}{J^2} - 2 + \frac{iA\sqrt{4J^2 - A^2}}{J^2} \right) \quad (4.9)$$

$$S_2(A) = \log \left(\frac{1}{2} \left(A + i\sqrt{4J^2 - A^2} \right) \right) + \frac{1}{4} \left(\frac{A^2}{J^2} - 2 - \frac{iA\sqrt{4J^2 - A^2}}{J^2} \right). \quad (4.10)$$

We consider these action for $A = a e^{i\varphi}$, $a > 0$ and get

$$S_1(a, \varphi) = \log \left(\frac{1}{2} \left(a e^{i\varphi} - i\sqrt{4J^2 - a^2 e^{2i\varphi}} \right) \right) + \frac{1}{4} \left(\frac{a^2 e^{2i\varphi}}{J^2} - 2 + \frac{ia e^{i\varphi} \sqrt{4J^2 - a^2 e^{2i\varphi}}}{J^2} \right) \quad (4.11)$$

$$S_2(a, \varphi) = \log \left(\frac{1}{2} \left(a e^{i\varphi} + i\sqrt{4J^2 - a^2 e^{2i\varphi}} \right) \right) + \frac{1}{4} \left(\frac{a^2 e^{2i\varphi}}{J^2} - 2 - \frac{ia e^{i\varphi} \sqrt{4J^2 - a^2 e^{2i\varphi}}}{J^2} \right). \quad (4.12)$$

In figure 1 we plot the real and imaginary parts of actions (4.11) and (4.12) for $0 < a < 2$. We see that the real part of the first solution for $0 < \varphi < \pi$ is less then the real part of the action on the second one and the second solution dominates. For $\pi < \varphi < 2\pi$ the first solution dominates.

In figure 2 we show the real parts of the actions also for $a > 2$. We see the order of dominance is changed for $a > 2$.

Figure 1. Plots of real (A) and imaginary (B) parts of the actions on the stationary point, $S_{0,i}(a, \varphi)$ for $J = 1$, different $a < 2$ and $0 < \varphi < 2\pi$. $i = 1, 2$ corresponds to the first and second stationary points and are presented by red and green color, respectively.

Figure 2. Plots for real parts of the actions at the stationary point, $ReS_{0,i}(a, \varphi)$ for $J = 1$, $0 < a < 4$ and $0 < \varphi < 2\pi$. $i = 1, 2$ corresponds to the first and second stationary points and are presented by red and green color respectively.

Therefore the free energy for $A = ae^{i\varphi}$ ($0 < a < 2$) is given by

$$f(a, \varphi) = \begin{cases} -S_2(a, \varphi) & \text{for } 0 < \varphi < \pi \\ -S_1(a, \varphi) & \text{for } \pi < \varphi < 2\pi. \end{cases} \quad (4.13)$$

Now we can evaluate the value of the order parameter Q . We have

$$\frac{\partial}{\partial a} S_1(a, \varphi) = \frac{2}{a - ie^{-i\varphi} \sqrt{4J^2 - a^2} e^{2i\varphi}} \quad (4.14)$$

$$\frac{\partial}{\partial a} S_2(a, \varphi) = \frac{2}{a + ie^{-i\varphi} \sqrt{4J^2 - a^2} e^{2i\varphi}}. \quad (4.15)$$

Now we can obtain the order parameter Q . We finally get

$$Q(\varphi) = - \begin{cases} e^{-i\varphi} \frac{\partial}{\partial a} S_2(a, \varphi) \Big|_{a=0} = \frac{i}{J}, & 0 < \varphi < \pi \\ e^{-i\varphi} \frac{\partial}{\partial a} S_1(a, \varphi) \Big|_{a=0} = -\frac{i}{J}, & \pi < \varphi < 2\pi. \end{cases} \quad (4.16)$$

The last formula shows that there is spontaneous symmetry breaking and the off-diagonal correlation function is nonvanishing.

Note that the partition function (4.1) can be written using the Hermite polynomials [7]

$$Z_N(A) = 2 \left(\frac{J}{\sqrt{2N}} \right)^N H_N \left(\frac{A}{J} \sqrt{\frac{N}{2}} \right). \quad (4.17)$$

The asymptotic behaviour of the Hermite and more general polynomials in the complex domain is evaluated in [27].

4.2 $q = 4$ model

Our starting point is

$$\begin{aligned} Z_N(A) &= \frac{N}{2\pi} \int dy \int dx (iy + A)^N \exp \left\{ N \left(-J^2 \frac{x^4}{4} + ixy \right) \right\} \\ &= \frac{N}{2\pi} J^{N/2} \int dy \int dx \exp \{ NS \}, \end{aligned} \quad (4.18)$$

where

$$S = -\frac{x^4}{4} + ixy + \log(iy + \tilde{A}) \quad (4.19)$$

and $\tilde{A} = A/\sqrt{J}$.

Here x, y are real variables and A is a complex parameter, $J > 0$ and N is a natural number.

The saddle points for $N \rightarrow \infty$ are derived from equations

$$-x^3 + iy = 0, \quad (4.20)$$

$$\frac{1}{\tilde{A} + iy} + x = 0. \quad (4.21)$$

There are 4 roots and we numerate them by index i . The values of the action (4.19) at these roots are denoted by $S_{0i}, i = 1, 2, 3, 4$. The real parts of the action on these roots are degenerate in the sense that they are pairwise equal for real A . To remove this degeneracy as in the case $q = 2$ we consider the complex $A = ae^{i\varphi}$, $a > 0$. The values of the real parts of the actions are presented in figure 3 for a particular $a = 0.1$ and $J = 1$. We see that the real part of the fourth solution for $0 < \varphi < \pi/2$ is bigger than the real parts of the action on other solutions. For $\pi/2 < \varphi < \pi$ the first solution dominates, for $\pi < \varphi < 3/2\pi$ the second solution dominates and for $3/2\pi < \varphi < 2\pi$ the third one.

The absence of degeneracy for complex A is also shown in the 3D plot in figure 4. The plot in figure 4B shows that the order of dominance is changed at $a = a_0 > 1$.

Figure 3. (A). Plots for real parts of the actions on the stationary point functions, $ReS_{0,i}(a, \varphi)$ for $J = 1$, $a = 0.1$ and $0 < \varphi < 2\pi$, $i = 1, 2, 3, 4$ corresponding to the first, second, third and fourth stationary points are presented by darker red, green, magenta and cyan color, respectively.

Calculating the first derivatives of the actions $S_{0,i}(a, \varphi)$ we get

$$\begin{aligned} \left. \frac{\partial}{\partial a} S_{0,1}(a, \varphi) \right|_{a=0} &= -\frac{(1-i)e^{i\varphi}}{\sqrt{2}} \\ \left. \frac{\partial}{\partial a} S_{0,2}(a, \varphi) \right|_{a=0} &= -\frac{(1+i)e^{i\varphi}}{\sqrt{2}} \\ \left. \frac{\partial}{\partial a} S_{0,3}(a, \varphi) \right|_{a=0} &= \frac{(1+i)e^{i\varphi}}{\sqrt{2}} \\ \left. \frac{\partial}{\partial a} S_{0,4}(a, \varphi) \right|_{a=0} &= \frac{(1-i)e^{i\varphi}}{\sqrt{2}}. \end{aligned}$$

To compute the value of the order parameter we have to take the derivative of the action at the branch with the maximal real part in the considered domain of φ . Finally we obtain

$$Q(\varphi) = \begin{cases} -\frac{(1-i)}{\sqrt{2}} & \text{for } 0 < \varphi < \pi/2 \\ \frac{(1-i)}{\sqrt{2}} & \text{for } \pi/2 < \varphi < \pi \\ \frac{(1+i)}{\sqrt{2}} & \text{for } \pi < \varphi < 3\pi/2 \\ -\frac{(1+i)}{\sqrt{2}} & \text{for } 3\pi/2 < \varphi < 2\pi. \end{cases} \quad (4.22)$$

Nonvanishing of the order parameter means that there is spontaneous symmetry breaking.

5 Conclusion

We have considered the fermionic random matrix model which is a 0-dimensional version of the SYK model with a finite number of replicas. It is shown that although the 2-point non-diagonal correlation function in the fermionic matrix model vanishes at finite N , it does not vanish in the large N limit due to spontaneous symmetry breaking. It would be interesting to study whether this phenomena may occur in other models.

Figure 4. (A) 3D plots for functions $ReS_{0,1}(a e^{i\varphi})$ (red), $ReS_{0,2}(a e^{i\varphi})$ (green), $ReS_{0,3}(a e^{i\varphi})$ (magenta) and $ReS_{0,4}(a e^{i\varphi})$ (cyan) at $J = 1$, small a and $0 < \varphi < \pi$ show that the complex $A = a e^{i\varphi}$ removes the degeneracy of the real part of the action. (B) The same plot as on (A), but for large variety of a , $0 < a < 5$.

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References

- [1] L.D. Landau, *On the theory of phase transitions*, in *Collected Papers. Volume 1*, Nauka, Moscow Russia (1969), pp. 234–252 [*Zh. Eksp. Teor. Fiz.* **7** (1937) 19] [[INSPIRE](#)].
- [2] N.N. Bogoliubov, *Lectures on quantum statistics. Volume 2: Quasi-Averages*, Gordon and Breach Science Publishers (1970), republished in *Collection of Scientific Works in Twelve Volumes: Statistical Mechanics. Volume 6*, Nauka, Moscow Russia (2007).
- [3] Y. Nambu, *Quasiparticles and Gauge Invariance in the Theory of Superconductivity*, *Phys. Rev.* **117** (1960) 648 [[INSPIRE](#)].
- [4] S. Weinberg, *The quantum theory of fields. Volume 2: Modern applications*, Cambridge University Press, Cambridge U.K. (1996).
- [5] M. Mezard, G. Parisi and M. Virasoro, *Spin Glass Theory and beyond*, World Scientific (1987).
- [6] J.S. Cotler et al., *Black Holes and Random Matrices*, *JHEP* **05** (2017) 118 [Erratum *JHEP* **09** (2018) 002] [[arXiv:1611.04650](#)] [[INSPIRE](#)].
- [7] I. Aref'eva and I. Volovich, *Notes on the SYK model in real time*, *Theor. Math. Phys.* **197** (2018) 1650 [*Teor. Mat. Fiz.* **197** (2018) 296] [[arXiv:1801.08118](#)] [[INSPIRE](#)].
- [8] P. Saad, S.H. Shenker and D. Stanford, *A semiclassical ramp in SYK and in gravity*, [arXiv:1806.06840](#) [[INSPIRE](#)].

- [9] S. Sachdev and J. Ye, *Gapless spin fluid ground state in a random, quantum Heisenberg magnet*, *Phys. Rev. Lett.* **70** (1993) 3339 [[cond-mat/9212030](#)] [[INSPIRE](#)].
- [10] A. Kitaev, *A simple model of quantum holography*, talk given at the *Entanglement in Strongly-Correlated Quantum Matter*, Santa Barbara, California, U.S.A., 6 April–2 July 2015 and online at <http://online.kitp.ucsb.edu/online/entangled15/>.
- [11] S. Sachdev, *Bekenstein-Hawking Entropy and Strange Metals*, *Phys. Rev. X* **5** (2015) 041025 [[arXiv:1506.05111](#)] [[INSPIRE](#)].
- [12] W.F. Wreszinski and V.A. Zagrebnov, *Bogoliubov quasi-averages: spontaneous symmetry breaking and algebra of fluctuations*, *Theor. Math. Phys.* **194** (2018) 157 [[arXiv:1704.00190](#)] [[INSPIRE](#)].
- [13] G. Gur-Ari, R. Mahajan and A. Vaezi, *Does the SYK model have a spin glass phase?*, *JHEP* **11** (2018) 070 [[arXiv:1806.10145](#)] [[INSPIRE](#)].
- [14] A. Georges, O. Parcollet and S. Sachdev, *Quantum fluctuations of a nearly critical Heisenberg spin glass*, *Phys. Rev. B* **63** (2001) 134406 [[cond-mat/0009388](#)].
- [15] J. Polchinski and V. Rosenhaus, *The Spectrum in the Sachdev-Ye-Kitaev Model*, *JHEP* **04** (2016) 001 [[arXiv:1601.06768](#)] [[INSPIRE](#)].
- [16] W. Fu and S. Sachdev, *Numerical study of fermion and boson models with infinite-range random interactions*, *Phys. Rev. B* **94** (2016) 035135 [[arXiv:1603.05246](#)] [[INSPIRE](#)].
- [17] J. Maldacena and D. Stanford, *Remarks on the Sachdev-Ye-Kitaev model*, *Phys. Rev. D* **94** (2016) 106002 [[arXiv:1604.07818](#)] [[INSPIRE](#)].
- [18] A.M. García-García and J.J.M. Verbaarschot, *Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model*, *Phys. Rev. D* **94** (2016) 126010 [[arXiv:1610.03816](#)] [[INSPIRE](#)].
- [19] I. Aref'eva, M. Khramtsov, M. Tikhonovskaya and I. Volovich, *Replica-nondiagonal solutions in the SYK model*, *JHEP* **07** (2019) 113 [[arXiv:1811.04831](#)] [[INSPIRE](#)].
- [20] V.V. Belokurov and E.T. Shavgulidze, *Simple rules of functional integration in the Schwarzian theory: SYK correlators*, [arXiv:1811.11863](#) [[INSPIRE](#)].
- [21] H. Wang, D. Bagrets, A.L. Chudnovskiy and A. Kamenev, *On the replica structure of Sachdev-Ye-Kitaev model*, [arXiv:1812.02666](#) [[INSPIRE](#)].
- [22] J. Kim, I.R. Klebanov, G. Tarnopolsky and W. Zhao, *Symmetry Breaking in Coupled SYK or Tensor Models*, *Phys. Rev. X* **9** (2019) 021043 [[arXiv:1902.02287](#)] [[INSPIRE](#)].
- [23] J. Maldacena and X.-L. Qi, *Eternal traversable wormhole*, [arXiv:1804.00491](#) [[INSPIRE](#)].
- [24] A.M. García-García, T. Nosaka, D. Rosa and J.J.M. Verbaarschot, *Quantum chaos transition in a two-site Sachdev-Ye-Kitaev model dual to an eternal traversable wormhole*, *Phys. Rev. D* **100** (2019) 026002 [[arXiv:1901.06031](#)] [[INSPIRE](#)].
- [25] V.S. Vladimirov and I. Volovich, *Superanalysis. I. Differential Calculus*, *Theor. Math. Phys.* **59** (1984) 317 [[INSPIRE](#)].
- [26] V.S. Vladimirov and I. Volovich, *Superanalysis. II. Integral calculus*, *Theor. Math. Phys.* **60** (1984) 743.
- [27] P. Deift, T. Kriecherbauer, K.T.-R. Mclaughlin, S. Venakides and X. Zhou, *Strong Asymptotics of Orthogonal Polynomials with Respect to Exponential Weights*, *Commun. Pure Appl. Math.* **52** (1999) 1491.