False vacuum decay in kink scattering

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ABSTRACT: In this work we consider kink-antikink and antikink-kink collisions in a modified ϕ^4 model with a false vacuum characterized by a dimensionless parameter ϵ. The usual ϕ^4 model is recovered for $\epsilon = 0$. We investigate the $\epsilon \ll 1$ regime where the kink in the presence of false vacuum can be understood as a small deformation of the standard kink for the ϕ^4 model. We show that the attractive interaction between the kink-antikink pair leads to a rich scattering pattern, in some cases delaying considerably the false vacuum decay.

Contents

1 Introduction

False vacua play an important role in fundamental physics, in particular in the context of inflationary models [1–3], electroweak phase transition and baryogenesis [4–8], bubble collisions [9–15], signals for the possible metastability of Higgs vacuum [16–18] and dark energy models [19, 20]. Soliton solutions in nonlinear field theory describe localized energy concentrations which are able to propagate without modifying their shape. They have large applicability in condensed matter physics [21], optics [22], quantum field theory [23], nuclear physics [24–26] and cosmology [27–30]. The simplest soliton solutions are the kink and antikink in $(1, 1)$-dimensions.

Kink scattering processes in nonintegrable models have intricate structures. This has been studied not only in the simple ϕ^4 model [31–40], but also in potentials of higher self-interaction [41–48] and non-polynomial potentials [49–55]. Further interesting directions of investigation include the interaction of a kink with an impurity [56–58], kinks in models of two scalar fields [59–62], multi-kink collisions [63–65], boundary scattering [66, 67] and models with generalized dynamics [68].

Embedded in higher dimensional spaces, kinks give rise to domain walls and branes [69]. In some cyclic universe scenarios, domain walls were considered as a possibility to generate the Big Bang conditions [70–73], where planar symmetry is assumed and the dynamics is effectively $(1, 1)$ dimensional. Solitons are also studied in the context of bubble collisions in the early universe. In the limit of a high nucleation rate per unit four-volume in comparison to H^4 (where H is the Hubble expansion rate), one can neglect the spacial curvature of the universe and consider the bubbles as in flat spacetime [74]. In this regime, collisions of two bubbles have $SO(2, 1)$ symmetry and are described by a $(1, 1)$ dimensional wave equation.

The mechanism of false vacuum decay in scalar field theories is well understood [75, 76]. In particular, for an asymmetric double well potential, domain wall collisions with planar and $SO(2, 1)$ symmetry were considered in ref. [77]. Domain wall collision with such high degree of symmetry can be described as collisions between a kink and an antikink in $(1, 1)$

dimensions. In this background the effects of small initial planar [77] and nonplanar [78] small fluctuations were investigated. However, ref. [77] only provided some examples for the output of background collisions, with no description of their dependence either with initial velocity or with the parameter controlling the difference in potential energy between the two wells.

The present work intends to fill this gap, with the aim of investigating the kink scattering and the corresponding false vacuum evolution, considering two situations: i) the false vacuum occupying the unbounded region outside the area defined by the kink-antikink pair; i) the false vacuum initially occupying the restricted region between the antikink-kink pair. We consider symmetric kink-antikink and antikink-kink collisions, with the pair initially at equal distances from the center of mass and initial velocities with same modulus. We show how the attractive interaction between the kink and antikink is the main factor affecting the structure of the scattering and, in some cases, delaying considerably the complete decay of the false vacuum even in a restricted region around the center of mass of the kink-antikink pair.

In the next section we review some of the main properties of the ϕ^4 model. The numerical results are presented in the section 3, and we conclude in the section 4.

2 The model

We consider the action in $(1, 1)$-dimensions in Minkowski spacetime with a Lagrangian with standard dynamics

$$S = \int dt dx \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \tag{2.1}$$

and a modified ϕ^4 potential

$$V(\phi) = \frac{\lambda}{4} \left(\phi^2 - \frac{M^2}{\lambda} \right)^2 - \frac{1}{2} \epsilon \frac{M^3}{\sqrt{\lambda}} \phi, \tag{2.2}$$

where ϵ is a dimensionless self-coupling parameter. From here on we will consider $\epsilon \geq 0$ without loosing generality. For $\epsilon \ll 1$ the potential has a local minimum (the false vacuum) $\phi_+ \approx -M/\sqrt{\lambda}$, a local maximum at $\phi = 0$ and a global minimum (the true vacuum) $\phi_- \approx M/\sqrt{\lambda}$. Here we are interested in the $|\epsilon| \ll 1$ limit, where the presence of the linear term in eq. (2.2) leads to a small force on the ϕ^4 kink that favors the expansion of the true vacuum region [79]. For the sake of definiteness, we take $\lambda = M^2 = 2$. In this case the equation of motion is given by

$$\ddot{\phi} - \phi'' - 2\phi + 2\phi^3 - \epsilon = 0 \tag{2.3}$$

Static solutions can be found for $\epsilon = 0$, given by the usual ϕ^4 kink $\phi_K(x) = \tanh(x)$ and antikink $\phi_{\bar{K}}(x) = -\tanh(x)$. These solutions, of minimal energy, can also be obtained by first-order BPS equations [80, 81]. Applying a Lorentz boost, a travelling kink with velocity v and initial position x_0 is obtained as $\phi_K(x, v, t) = \tanh(\gamma(x - x_0 - vt))$, with

$\gamma = 1/\sqrt{1 - v^2}$. Linear stability analysis around the static kink solution with $\phi(x,t) = \phi_K(x) + \cos(\omega t)\eta(x,t)$ leads to a Schrödinger-like equation:

$$-\eta'' + (-2 + 6\phi_K^2)\eta = \omega^2\eta. \tag{2.4}$$

This equation results in the eigenvalues [31] $\omega_0^2 = 0$ (the zero-mode or translational mode) and $\omega_1^2 = 3$ (the vibrational mode), followed by a continuum of states with $\omega_k^2 = 4 + k^2$. The existence of vibrational modes is important for the comprehension of the complex behavior of kink-antikink scattering. One such effect is known as two-bounces, and it is characterized by a collision process where the initial translational energy is stored in the vibrational mode of the kink-antikink pair during an amount of time. As a result, the pair, after being scattered, oscillates around the contact point and retrocedes for a second collision. This mechanism was described in the ref. [33] as an exchange of energy between the translational and vibrational modes. Despite applicable in the present case, we remark that there are some known exceptions to this mechanism: in the ϕ^6 model, two-bounces occur even in the absence of a vibrational mode [41]; the presence of more than one vibrational mode can result in the suppression of two-bounce windows [51]. Also, quasinormal modes can store energy during a collision in the ϕ^4 model [40].

Now back to our problem for $\epsilon \neq 0$, as far as we know, there are no explicit solutions of the equation of motion. In particular, corrections ϕ_n to the kink field in nth order in ϵ were presented in ref. [79], and are described in terms of the basis of eigenfunctions $\{\eta_n\}$.

3 Numerical results

The kink scattering processes have a strong dependence on whether the initial region with false vacuum is finite or not. In the following we will consider separately kink-antikink and antikink-kink collisions.

3.1 False vacuum in an unbounded interval: kink-antikink collisions

First of all let us consider a symmetric kink-antinkink collision. We take as initial conditions a ϕ^4 kink with velocity v and a ϕ^4 antikink with velocity $-v$:

$$\phi(x,0) = \phi_K(x + x_0, v, 0) - \phi_K(x - x_0, -v, 0) - 1, \tag{3.1}$$

$$\dot\phi(x,0) = \dot\phi_K(x + x_0, v, 0) - \dot\phi_K(x - x_0, -v, 0). \tag{3.2}$$

We fixed $x_0 = 15$ as the initial kink position, i.e., the kink solution centered at $-x_0$ and the antikink at x_0. Note that the initial condition is only a solution for $\epsilon = 0$. We apply this even for $\epsilon \ll 1$ considering that the system will relax by emitting scalar radiation during their free propagation toward the collision region. To solve eq. (2.3) we use a pseudospectral method on a grid with 2048 nodes with periodic boundary conditions for ϕ and $\dot\phi$ and we set the grid boundary at $x_{\max} = 200$. A sympletic method with the Dirichlet condition imposed at $x = \pm x_{\max}$ was also applied to double check our numerical results. We used a 4th order finite-difference method with spatial step $\delta x = 0.09$ and a 6th order symplectic integrator with time step $\delta t = 0.04$.
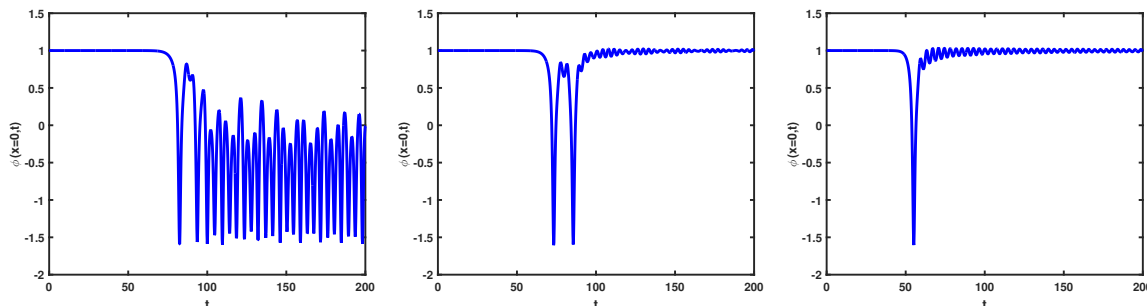
Figure 1. Kink-antikink collisions for the ϕ^4 model, corresponding to $\epsilon = 0$. Scalar field at the center of mass, $\phi(0,t)$, showing a) the formation of bion state (for $v = 0.17$), b) two-bounce collision (for $v = 0.1926$) and c) one-bounce collision (for $v = 0.26$).

We start our analysis by reviewing some aspects of the $\epsilon = 0$ case, corresponding to the ϕ^4 theory, and well described in the literature [31–40]. Figures 1a–1c depict three profiles of scalar field at the center of mass as a function of time, $\phi(0,t)$, for different values of the initial velocity. Initially the point $x = 0$ is at the vacuum $\phi = 1$. For small velocities, the output of the scattering process is a bion state — see an example in figure 1a — where the pair radiates with the scalar field oscillating without a recognizable pattern until, in the long run, finishing at the vacuum $\phi = -1$. Velocities above a critical velocity $v_{\text{crit}} \sim 0.26$ show an inelastic scattering, where the output field retains the initial value $\phi = 1$ after one bounce, as in the example of figure 1c. We could characterize this inelastic scattering as having $N_b = 1$ bounces. For smaller velocities with $v \lesssim v_{\text{crit}}$ there are some velocity windows where the scalar field presents two-bounces ($N_b = 2$) during the collision process, as shown in figure 1b.

The structure of such two-bounce windows is better visualized in figure 2a, which shows the number of bounces as a function of the initial velocity. As shown in figure 1a, bion states have a large number of oscillations of $\phi(x,0)$. Large values of N_b are not directly represented in figure 2, but there one represent bion states as making frontier with the two-bounce windows. Each two-bounce window is labeled by an integer m, the number of oscillations of $\phi(0,t)$ between the bounces. For instance, in figure 1b we have $m = 1$, meaning that this collision belongs to the first two-bounce window from figure 2a.

Now we turn to the effect of ϵ on kink scattering. Figures 2b–2d show the number of bounces as a function of the initial velocity for several values of $\epsilon \neq 0$. They show the inelastic scattering behavior with 1-bounce for $v > v_{\text{crit}}$, but with v_{crit} growing with ϵ. This is a direct effect of the tendency of the false vacuum to decay. One example of 1-bounce collision is presented in figure 3a. We observed an effect that appears only for $\epsilon \neq 0$, that is the lack of bounces or even bion states for $v < v^*$: the kink-antikink pair has not enought energy to encounter at the center of mass before the false vacuum decay. An example of this effect is shown in figure 3b. Also, 2-bounce windows are still present, but are thinner in velocity for larger values of ϵ. One example of this is presented in the figure 3c.

The behavior of the kink-antikink pair after the collision process can be understood aproximating the static kink-antikink profile by
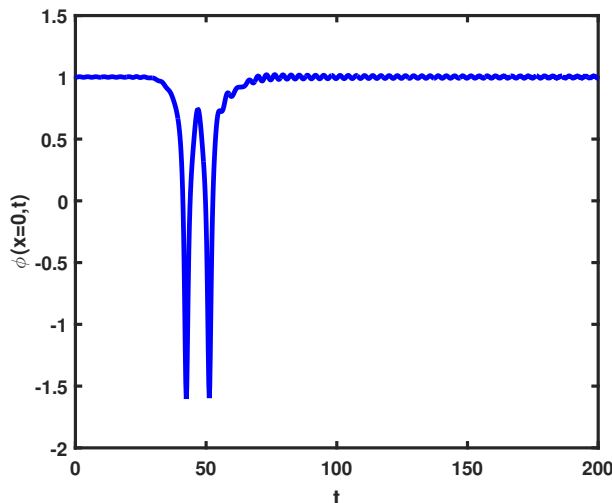
$$\phi(x) \approx \phi_K(x + X) - \phi_K(x - X) - \frac{M}{\sqrt{\lambda}}, \tag{3.3}$$
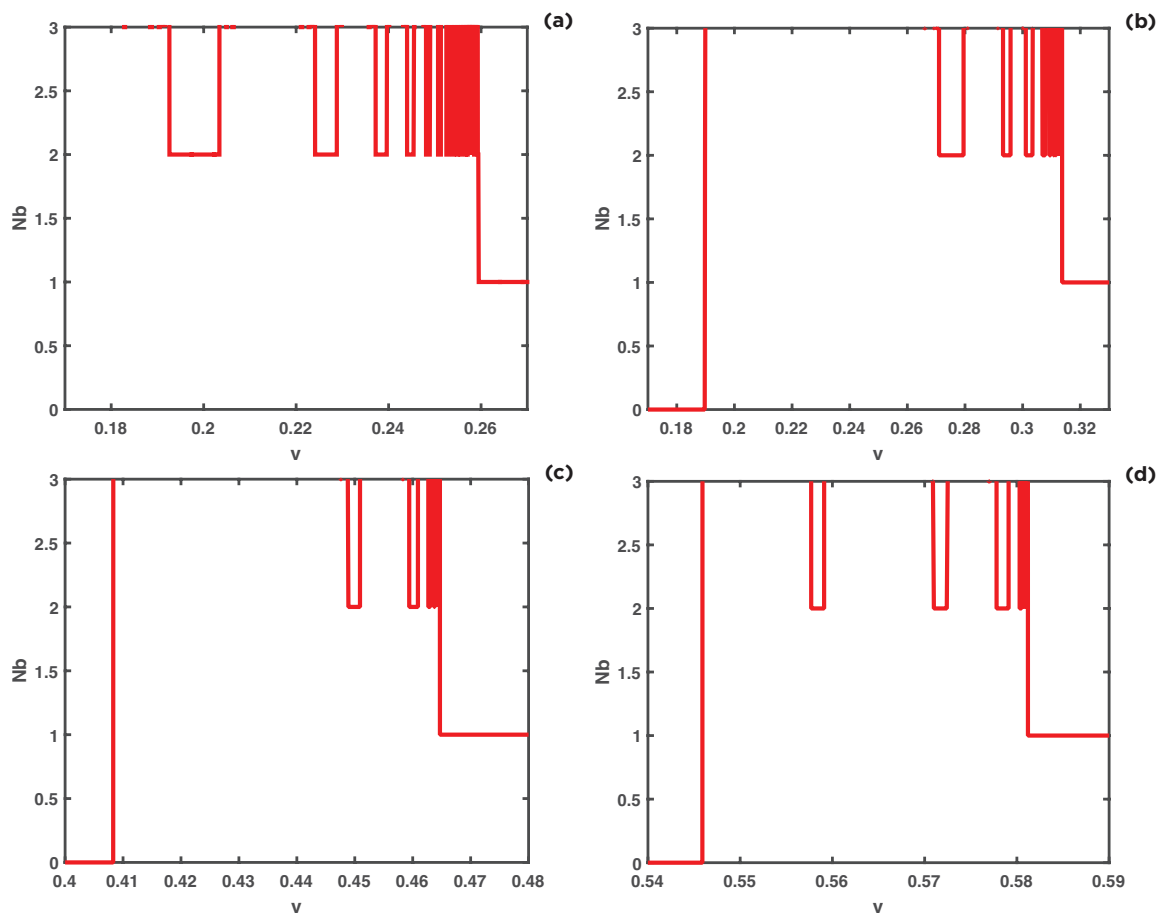
Figure 2. False vacuum in an unbounded interval: kink-antikink collisions. Number of bounces versus initial velocity v for (a) $\epsilon = 0$, (b) $\epsilon = 0.001$, (c) $\epsilon = 0.005$ and (d) $\epsilon = 0.01$.
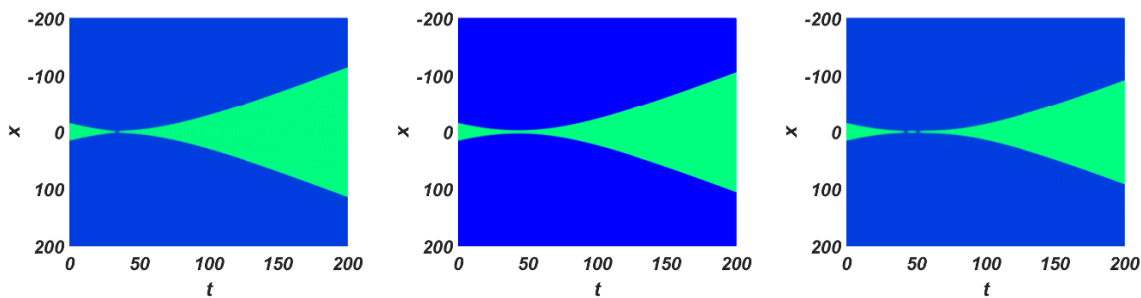
Figure 3. False vacuum in an unbounded interval: kink-antikink collision for $\epsilon = 0.01$ with (a) $v = 0.59$, (b) $v = 0.54$ and (c) $v = 0.558$. Compare with diagram of figure 2d. Blue represents the false vacuum $\phi = -1$ and the true vacuum $\phi = +1$ is represented in green.

Figure 4. False vacuum in an unbounded interval: kink-antikink collision showing a zero order ($m = 0$) two-bounce for $\epsilon = 0.01$ with $v = 0.558$.

where X is the translational collective coordinate. The total energy of the kink-antikink system for large values of X is given by [82] $E_{K\bar{K}} = 2M_K - 2(\Delta\rho)X$, where $M_K = 2\sqrt{2}M^3/(3\lambda)$ is the classical kink mass and $\Delta\rho = \epsilon M^4/\lambda$ is the energy density difference between the two vacua. The energy is reduced for even larger values of X, meaning that soon after the scattering the kink-antikink pair separate at the velocity of light, as confirmed in figures 3a–c.

Another interesting aspect of kink-antikink collisions with the growing of ϵ is the appearing of a two-bounce window of zero order for $\epsilon \gtrsim 0.008$. An example of such effect is presented in figure 4. There one can see that there is no oscillations of $\phi(x = 0, t)$ between the bounces, meaning that $m = 0$. The appearing of zero order two-bounce windows was reported before in models with degenerate vacua in the context of boundary scattering [66].

3.2 False vacuum in a bounded interval: antikink-kink collisions

We take as initial condition a ϕ^4 antikink with velocity v and a ϕ^4 kink with velocity $-v$:

$$\phi(x, 0) = -\phi_K(x + x_0, v, 0) + \phi_K(x - x_0, -v, 0) + 1, \tag{3.4}$$

$$\dot{\phi}(x, 0) = -\dot{\phi}_K(x + x_0, v, 0) + \dot{\phi}_K(x - x_0, -v, 0). \tag{3.5}$$

The numerical procedure was the same used for kink-antikink collision. In this case, the attractive antikink-kink interaction and the tendency of the false vacuum to decay favor the confinement of the antikink-kink and its subsequent extinction. This can be explained using collective coordinates, following the same procedure described above [82] for kink-antikink scatering. The antikink-kink profile is aproximated by

$$\phi(x) \approx \phi_K(x + X) - \phi_K(x - X) + \frac{M}{\sqrt{\lambda}}. \tag{3.6}$$

For large values of X the energy of the antikink-kink pair is given by $E_{\bar{K}K} = 2M_K + 2(\Delta\rho)X$. This means that energy favors the reduction of X, in a net confining effect.
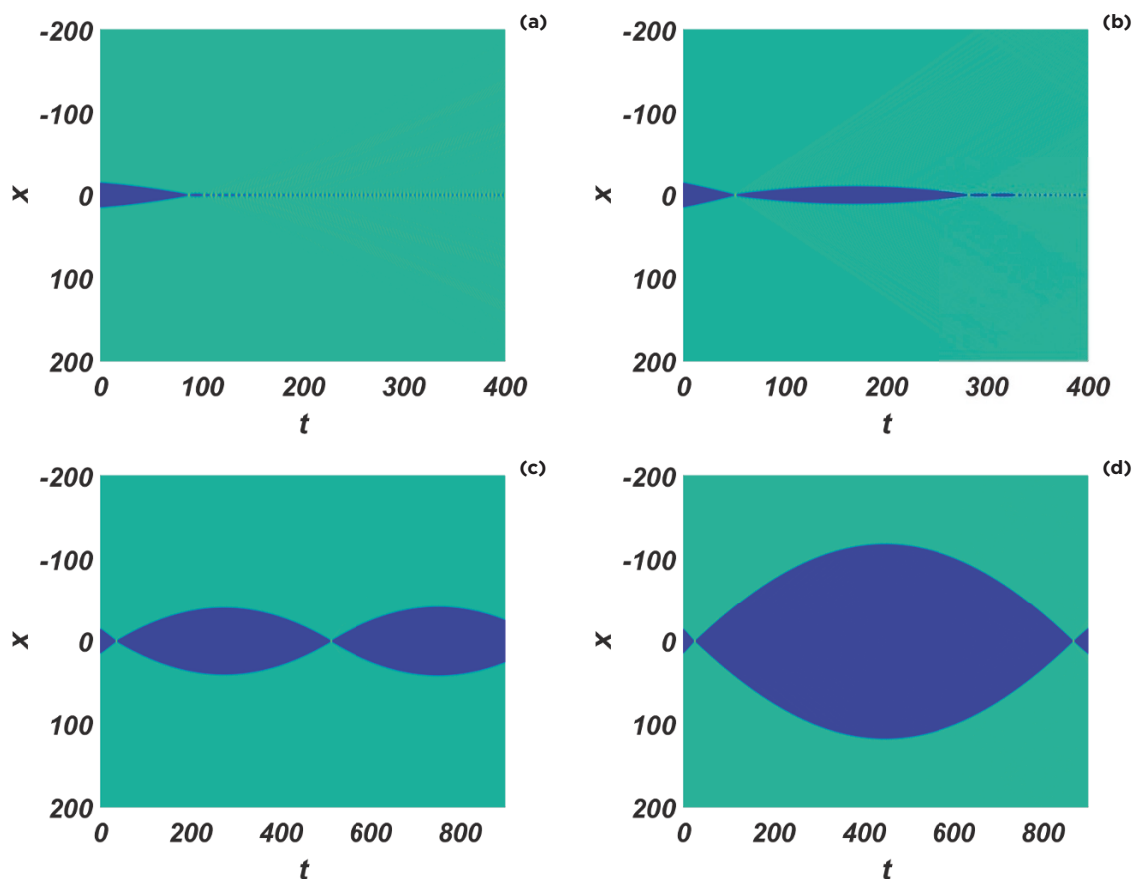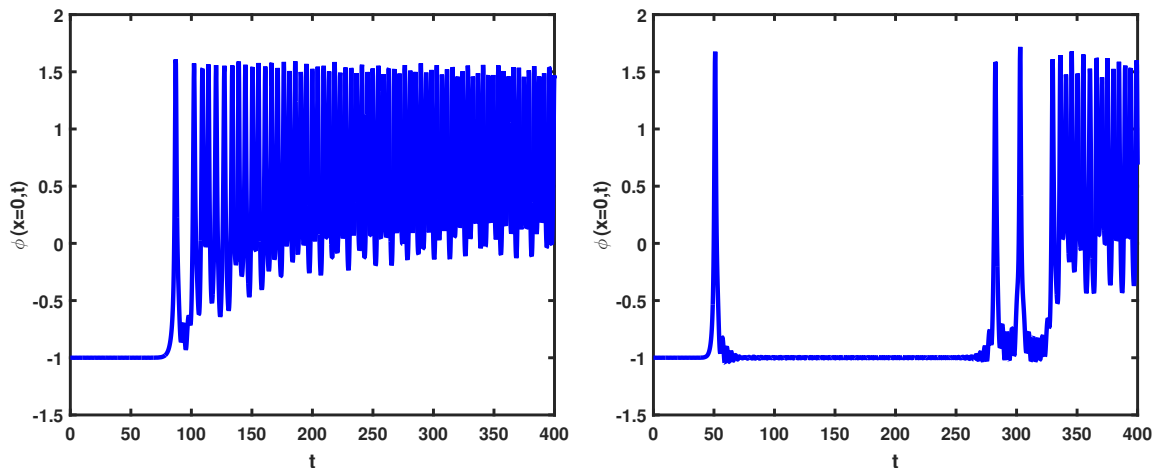
Figure 5. False vacuum in a bounded interval: antikink-kink collision for $\epsilon = 0.001$ and with (a) $v = 0.1$ (upper left), (b) $v = 0.25$ (uper right), (c) $v = 0.4$ (lower left), (d) $v = 0.6$ (lower right). The figures show slices in the plane (x, t) of the scalar field. Blue color represents the false vacuum $\phi = -1$ and the true vacuum $\phi = +1$ is represented in green.

The numerical simulations show that the complete process of vacuum decay has long duration and that the pattern of oscillations is intrincate. The output of mechanism is a two-step process: firstly the field bounces around the false vacuum with low frequency. Then the antikink-kink pair radiates and the field oscillates in a high-frequency, approaching in the long run to the true vacuum state. This is showed in figures 5a–5d for $\epsilon = 0.001$. Surprisingly larger initial velocities are not more effective in inducing the process of vacuum decay. On the contrary, with the growing of v the low-frequency oscillation lasts even more before starting the decaying process of the false vacuum. It seems that, with false vacuum, larger initial velocities excite the vibrational state of the kink and antikink in a way that does not occurs for the degenerate vacuum, and the antikink-kink pair can bounce several times and be even more departed during such low-frequency oscillations (compare figures 5a–5b).

The relation between high- and low-frequency oscillations is more evident in a plot of the scalar field at the center of mass, $\phi(0, t)$, as shown in figures 6a-b. There one can

Figure 6. False vacuum in a bounded interval: antikink-kink collision. Scalar field at the center of mass, $\phi(0, t)$ as a function of time for $\epsilon = 0.001$ with (a) $v = 0.1$ and (b) $v = 0.25$, corresponding to figures 5a-b.

see that, for a too low initial velocity, the field is not even able to bounce and the high-frequency process already takes place, as can be see in figure 6a for $v = 0.1$. As an example for larger velocities we take $v = 0.25$ as in figure 6b. There one can see three low-frequency oscillations around the false vacuum $\phi = -1$ before the field jumps and acquires the high-frequency oscillations that approach the true vacuum $\phi = +1$. As seen in figures 5c–5d, a too larger initial velocity would require a much longer running time of simulations to see the final high-frequency decay of the false vacuum. For the values $x_0 = 15$ considered in the simulations, the pattern of collisions described here is valid up to $\epsilon \sim 5 \times 10^{-3}$. Larger values of ϵ leads to a non-recognizable pattern, meaning that the nonlinear coupling for antikink-kink collisions is already too large.

An important issue is the possibility of oscillon formation connected to particle production due to vacuum decay. In an integrable theory, the particle and kink sectors are disconnected: that is, the kink-antikink pair scatter with at most a phase shift with no particle production. One knows that the integrable sine-Gordon model is dual to the massive Thirring model. In particular, the bound state of two massive Thirring fermions at strong coupling corresponds to a breather in the sine-Gordon model at small coupling [83]. Also, in the quantized breather to lowest order, the energy levels of the quantized breather corresponds to an integer number of particles, leading to the interpretation that the breather is a particle in the sine-Gordon model [84]. Then one expects that a breather-like profile could be a bridge between particle and soliton solutions in nonintegrable theories. This reasoning was used in the ϕ^4 theory for the creation kink-antikink pairs and radiation after the scattering of small identical wave pulses [85]. The opposite process, that is, particle production in the wake of the colliding walls needs a better discussion of the breather-analog for nonintegrable theories, the oscillon. The oscillon is an extremely long-lived state considered as a bound state of ϕ-particles with a finite binding energy [86]. When quantized, the extremely stable classical oscillon solution has its lifetime shortened due to
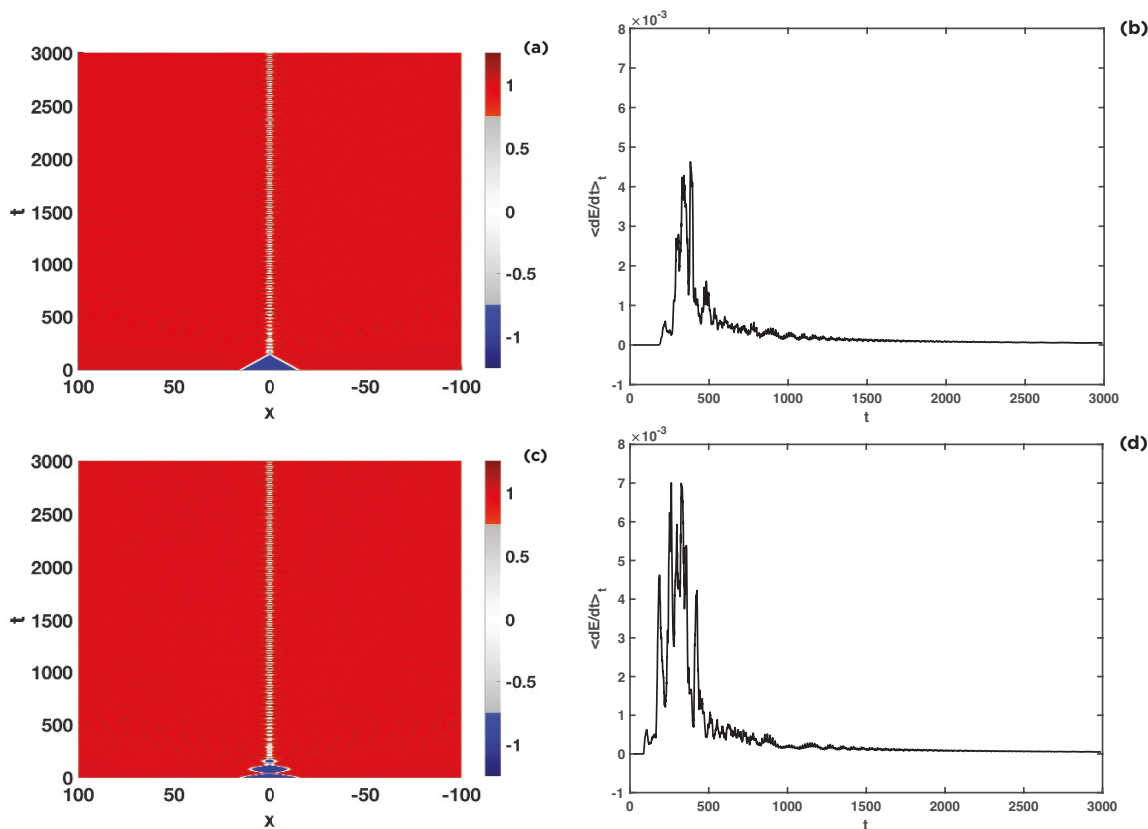
Figure 7. (Left) $\phi(x,t)$ showing long-lived oscillations after the scattering. (Right) Corresponding time-averaged energy-emission rate $< dE/dt >_t$. The parameters chosed are $\epsilon = 0$ (upper figures) and $\epsilon = 10^{-2}$ (lower figures). In both cases we fixed $v = 0.1$.

the emission of ϕ-particles [87]. This means that the production of particles is connected in our classical setup by the production of oscillons. Our numerical simulations for the false vacuum in a bounded interval suggests the tendency of oscillon production in the long-term during the high-frequency oscillations of the scalar field at the center of mass, as showed in the figures 7a and 7c. There one can see the long-lived oscillations after the scattering for several values of ϵ.

The fact that the existence of false vacuum in a limited region induces energy dissipation is evident from the reduction of the maximum distance between antikink and kink with time, as shown for $\epsilon = 0.01$ in figures 5a and 5b. The emission of radiation is quantitatively explained by the coupling of the vibrational (normal) mode with the states of the continuum. As a result of this coupling in nonlinear theories, an oscillating kink produces radiation with twice the frequency of the vibrational state. For example, for the kink of the ϕ^4 model ($\epsilon = 0$), the rate of energy loss by the discrete mode, averaged over a period, is given by [88] $dE/dt \sim t^{-2}$, meaning an amplitude of the vibrational mode decaying as $A(t) \sim t^{-1/2}$. If the vibrational mode has low energy such that the second harmonic is bellow the continuum threshold, radiation is emitted by higher harmonics [66]. Once the oscillon is formed, the decay rate is considerably smaller, with [89] $dE/dt \sim \exp(-B/E)$.

In general, even for $\epsilon \neq 0$, energy conservation results in the following expression for the energy density ρ: $d\rho/dt = \partial_x(\phi'\dot{\phi})$. Integrating the former expression in a box close to the scattered region and averaging in time results in the average emitted rate of energy (radiated power) $< dE/dt >_t$. The figures 7b and 7d present $< dE/dt >_t$ for several values of ϵ. There one sees that radiation is emitted in bursts connected with the bouncings, and that the radiated power grows with ϵ. We also verified that the radiated power grows for lower values of λ. After a transition, the decay rate is reduced considerably, but it is not suppressed, being very small with an almost constant rate for low energies, as expected for an oscillon. Despite the long-run oscillations described here still have not a single frequency (which characterize an oscillon in a strictly way), the behavior of low radiated power and long lifetime of the formed oscillatory state approaching the true vacuum with low amplitude, suggests the existence of a mechanism of oscillon formation.

We remark that we have confirmed with very good precision, for all cases considered, the conservation of total energy in the simulation box. For instance, when the false vacuum is in a limited region (antikink-kink collisions in our setup), and fixing $\lambda = 1$, we have the following results before the bounces: for the case $\epsilon = 0$, energy is conserved better than 1 part in 10^9. The increasing of ϵ is acompanied with energy conservation with less precision. For the worst case analyzed, $\epsilon = 0.01$, this means better 1 part in 10^6. For the complete scattering process, for the case $\epsilon = 0$, energy is conserved better than 1 part in 10^7. For $\epsilon = 0.01$, this means better than 1 part in 10^4.

4   Conclusions

In this work we have studied the effect of kink scattering in the false vacuum decay. Kink-antikink collisions are characterized by two critical velocities, v^* (not present for degenerate vacua) and v_{crit}. For $0 < v < v^*$ we have inelastic scattering of the pair without contact. For $v^* < v < v_{\text{crit}}$ the scalar field at the center of mass $\phi(x, 0)$ can show bion or two-bounce around the true vacuum. In a diagram in velocities we have the usual structure of bion states and a sequence of two-bounce windows accumulating around v_{crit}. For $\epsilon \gtrsim \bar{\epsilon}$ there is the appearance of zero-order two-bounce windows. For $v > v_{\text{crit}}$ we have inelastic scattering, and $\phi(x, 0)$ presents one-bounce around the true vacuum. Larger choices of initial position of the kink-antikink pair up to $x_0 = 25$ mean more time for the false vacuum to decay during the free propagation of the kink-antikink pair, leading to an increasing of v^* and v_{crit}.

It is relatively easier for a finite region in a true vacuum to expand through the false vacuum region, as described here in kink-antikink collisions. This is evident for low initial velocities, where the kink-antikink pair scatters at non-null distance. In comparison, it demands considerably more time for a finite region in false vacuum to decay completely to the true vacuum. We showed this for antikink-kink collisions. That is, the attractive kink-antikink interaction favors the formation of a long-lived bion state that inhibits the vacuum decay. The antikink-kink collisions were described in a two-step process with a low-frequency oscillation around the false vacuum followed by a high-frequency oscillation tending to the true vacuum in the long run. This two-step process resembles the behavior

of what is known in the literature as false n-bounce-windows. In the end such bounce windows must be false since the scalar field will not remain in the initial state. We also investigated the effect of increasing the gap parameter ϵ. For kink-antikink collisions we have the increasing of both velocities v^* and v_{crit}.

For antikink-kink collisions the center of mass belongs to the false vacuum region and we have a complex mechanism for decay of the false vacuum: a low-frequency fast decaying bouncing state, followed by another long-lived high frequency bion state that radiates continuously, suggesting a transition in the long run to an oscillon state. For antikink-kink collisions, the more notable effect of the increasing of ϵ is the reduction of the maximum separation of the antikink-kink pair during the low-frequency oscillations. This shows the stronger tendency of the false vacuum to decay. The final state is the same, with the field oscillating fastly around the true vacuum and radiating energy in a low rate. We interpret that the radiation rate suffered a changing connected to a transition of energy from the vibrational state of the antikink-kink pair to an oscillon configuration. A smooth transformation from oscillational normal modes to oscillons was verified recently [90] in a modified ϕ^4 theory with an additional potential that vanishes asymptotically. In the present work such transformation is induced by the false vacuum decay and occurs nonadiabatically. Depending on the initial velocity, the route to the oscillon formation is preceded by a two-step process, where sharp emission of radiation sincronized to the bouncings of the scalar field is folowed by a bion solution with an almost constant emission rate.

Acknowledgments

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