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AdS_5 black strings in the stu model of FI-gauged N = 2 supergravity

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Abstract: We analytically construct asymptotically AdS_5 black string solutions starting from the four-dimensional domain wall black hole of [1]. It is shown that its uplift gives a black string in $d = 5$ minimal gauged supergravity, with momentum along the string. Applying instead the residual symmetries of $N = 2$, $d = 4$ Fayet-Iliopoulos-gauged supergravity discovered in [2] to the domain wall seed leads, after uplifting, to a dyonic black string that interpolates between AdS_5 and $AdS_3 \times H^2$ at the horizon. A Kaluza-Klein reduction of the latter along an angular Killing direction ϕ followed by a duality transformation yields, after going back to five dimensions, a black string with both momentum along the string and rotation along ϕ. This is the first instance of using solution-generating techniques in gauged supergravity to add rotation to a given seed. These solutions all have constant scalar fields. As was shown in [3], the construction of supersymmetric static magnetic black strings in the FI-gauged stu model amounts to solving the $SO(2,1)$ spinning top equations, which descend from an inhomogeneous version of the Nahm equations. We are able to solve these in a particular case, which leads to a generalization of the Maldacena-Nuñez solution.

Keywords: Black Holes, Black Holes in String Theory, Supergravity Models

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Contents

1 Introduction

Exact solutions to Einstein's field equations and their supergravity generalizations have been playing, and continue to play, a crucial role in many important developments in general relativity, black hole physics, integrable systems, string theory and quantum gravity. Being highly nonlinear, coupled partial differential equations, these are notoriously difficult to solve, sometimes even in presence of a high degree of symmetry, for instance in supergravity where one has typically many other fields in addition to the metric. For this reason, solution generating techniques have become a very powerful tool to generate new solutions from a given seed. The basic idea is to reduce the action (if one has sufficiently many commuting Killing vector fields) to three dimensions, where all vector fields can be dualized to scalars, so that one ends up with a nonlinear sigma model coupled to gravity. The target space symmetries can then be used to obtain new solutions to the field equations by starting from a known one.[1] In the case of four-dimensional Einstein-Maxwell gravity, for instance, one gets a sigma model whose scalars parametrize the Bergmann space $\mathrm{SU}(2,1)/\mathrm{S}(\mathrm{U}(1,1) \times \mathrm{U}(1))$ [6, 7].

In gauged supergravity, supersymmetry requires a potential for the moduli (except for flat gaugings), that generically breaks the target space symmetries. For the simple example quoted above, the addition of a cosmological constant breaks three of the eight $\mathrm{SU}(2,1)$ symmetries, corresponding to the generalized Ehlers and the two Harrison transformations. This leaves a semidirect product of a one-dimensional Heisenberg group and a translation group $\mathbb{R}^2$ as residual symmetry [8], that cannot be used to generate new solutions.

Recently however, elaborating on earlier work [9], the authors of [2] developed a solution generating technique for $N = 2$, $d = 4$ Fayet-Iliopoulos (FI)-gauged supergravity as

[1] Two of the many notable examples are the most general rotating black hole solution of five-dimensional $N = 4$ superstring vacua constructed by Cvetič and Youm [4], or the black Saturn found in [5].

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well, which essentially involves the stabilization of the symplectic vector of gauge couplings (FI parameters) under the action of the U-duality symmetry of the ungauged theory. One of the main goals of the present paper is to provide an explicit application of the method introduced in [2]. Namely, we start from the supersymmetric domain wall black hole in the stu model of $N = 2$, $d = 4$ FI-gauged supergravity, with prepotential $F = -X^1X^2X^3/X^0$, constructed in [1]. We then act on it with one of the symmetry transformations of [2] and lift the resulting configuration to five dimensions. This leads to a dyonic black string in minimal $N = 2$, $d = 5$ gauged supergravity, with momentum along the string.[2] In the next step, one performs a Kaluza-Klein reduction along an angular Killing direction ϕ followed by another duality transformation. After going back to $d = 5$ one gets a black string with both momentum and rotation. This is the first instance of using solution-generating techniques in gauged supergravity to add rotation to a given seed.

Black strings that interpolate between AdS_5 at infinity and $\mathrm{AdS}_3 \times \Sigma$ near the horizon (where Σ denotes a two-dimensional space of constant curvature), are interesting in their own right, since they provide a holographic realization of so-called RG flows across dimensions, from a four-dimensional CFT to a CFT_2 in the IR. A first example for the gravity dual of such a scenario was given in [11], and subsequently Maldacena and Nuñez [12] found a string theory realization in terms of D3-branes wrapping holomorphic Riemann surfaces. Since then, many papers on this subject appeared, cf. [13–21] for an (incomplete) list of references.

The remainder of this paper is organized as follows: in the next section, we briefly introduce the stu model of $N = 2$, $d = 5$ FI-gauged supergravity and the r-map which relates it to the corresponding four-dimensional model. We also determine the residual duality symmetries of the latter along the recipe of [2], and review the domain wall black hole seed solution constructed in [1]. In section 3, this configuration is lifted to $d = 5$, and it is shown that the result is a black string with momentum along the string. Subsequently, in 4, we apply a duality transformation to the seed of [1] to get, after uplifting, a dyonic black string. This is then Kaluza-Klein reduced along an angular Killing direction ϕ in section 5, duality-transformed and lifted back to generate a black string with both momentum and rotation. Finally, 6 contains the construction of supersymmetric static magnetic black strings generalizing the Maldacena-Nuñez solution, by solving the $\mathrm{SO}(2,1)$ spinning top equations. Moreover, we make some comments on a possible inclusion of hypermultiplets. We conclude in section 7 with a discussion of our results. A summary of the supersymmetry variations of the five-dimensional theory is relegated to an appendix.

2 The stu model of FI-gauged $N = 2$ supergravity

The bosonic Lagrangian of $N = 2$, $d = 5$ FI-gauged supergravity coupled to n_{v} vector multiplets is given by [22][3]

$$e^{-1}\mathscr{L}^{(5)} = \frac{R}{2} - \frac{1}{2}\mathcal{G}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - \frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J\mu\nu} + \frac{e^{-1}}{48}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda - g^2V_5\,, \tag{2.1}$$

[2] Notice that the idea of relating black strings in five-dimensional gauged supergravity to black holes in 4d gauged supergravity goes back to [10].

[3] The indices $I, J, \ldots$ range from 1 to $n_{\mathrm{v}}+1$, while $i, j, \ldots = 1, \ldots, n_{\mathrm{v}}$.

where the scalar potential reads

$$V_5 = V_I V_J \left(\frac{9}{2} \mathcal{G}^{ij} \partial_i h^I \partial_j h^J - 6 h^I h^J \right) . \tag{2.2}$$

Here, V_I are FI constants, ∂_i denotes a partial derivative with respect to the real scalar field ϕ^i, and $h^I = h^I(\phi^i)$ satisfy the condition

$$\mathcal{V} \equiv \frac{1}{6} C_{IJK} h^I h^J h^K = 1 . \tag{2.3}$$

The stu model is defined by the symmetric tensor with only nontrivial component $C_{123} = 1$, up to permutations. In this case, the functions $h^I = h^I(\phi^1, \phi^2)$ are given by

$$h^1 = e^{-\frac{\phi^1}{\sqrt{6}} - \frac{\phi^2}{\sqrt{2}}} , \qquad h^2 = e^{\frac{2\phi^1}{\sqrt{6}}} , \qquad h^3 = e^{-\frac{\phi^1}{\sqrt{6}} + \frac{\phi^2}{\sqrt{2}}} , \tag{2.4}$$

and therefore the constraint (2.3) is satisfied.

Below we will need the relationship between (2.1) and four-dimensional $N = 2$ FI-gauged supergravity, with bosonic Lagrangian

$$e^{-1} \mathscr{L}^{(4)} = \frac{R}{2} - g_{I\bar{J}} \partial_\mu z^I \partial^\mu \bar{z}^{\bar{J}} + \frac{1}{4} I_{\Lambda\Sigma} F^{\Lambda\mu\nu} F^\Sigma_{\mu\nu} + \frac{1}{4} R_{\Lambda\Sigma} F^{\Lambda\mu\nu} \star F^\Sigma_{\mu\nu} - V_4(z, \bar{z}) , \tag{2.5}$$

where $R_{\Lambda\Sigma} = \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma}$, $I_{\Lambda\Sigma} = \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma}$, and $\mathcal{N}$ is the period matrix that determines the couplings of the scalars to the vector fields. $\mathcal{N}$ is determined by a homogeneous function F of degree two, called the prepotential. The scalar potential can be written in the symplectically covariant form [23]

$$V_4 = g^{I\bar{J}} D_I \mathcal{L} D_{\bar{J}} \bar{\mathcal{L}} - 3 \mathcal{L} \bar{\mathcal{L}} , \tag{2.6}$$

with $\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle \equiv \mathcal{G}^t \Omega \mathcal{V}$, where $\mathcal{V}$ denotes the covariantly holomorphic symplectic section, $\mathcal{G} = (g^\Lambda, g_\Lambda)^t$ is the symplectic vector of FI parameters and D the Kähler-covariant derivative. More details can be found e.g. in [2, 23–25].

If one reduces the action (2.1) to four dimensions using the r-map [26][4]

$$\mathrm{d}s_5^2 = e^{\frac{\phi}{\sqrt{3}}} \mathrm{d}s_4^2 + e^{-\frac{2}{\sqrt{3}}\phi} (\mathrm{d}z + K_\mu \mathrm{d}x^\mu)^2 , \qquad A^I = B^I (\mathrm{d}z + K_\mu \mathrm{d}x^\mu) + C^I_\mu \mathrm{d}x^\mu , \tag{2.7}$$

$$\begin{aligned} z^I &= B^I + i e^{-\frac{\phi}{\sqrt{3}}} h^I , \qquad e^{\mathcal{K}} = \frac{1}{8} e^{\sqrt{3}\phi} , \qquad g V_I = \frac{g_I}{3\sqrt{2}} , \\ g_{I\bar{J}} &= \frac{1}{2} e^{\frac{2\phi}{\sqrt{3}}} G_{IJ} , \qquad F^\Lambda_{\mu\nu} = \frac{1}{\sqrt{2}} (K_{\mu\nu}, C^I_{\mu\nu}) , \end{aligned} \tag{2.8}$$

one ends up with the model with cubic prepotential

$$F = -\frac{X^1 X^2 X^3}{X^0} , \tag{2.9}$$

and FI parameters

$$\mathcal{G} = (0, 0, 0, 0, 0, g_1, g_2, g_3)^t . \tag{2.10}$$

[4] We apologize for using the same greek indices $\mu, \nu, \ldots$ both in five and four dimensions, but the meaning should be clear from the context.

The theory (2.5) enjoys a residual symmetry that is determined by evaluating the stabilizer of the U-duality group acting in the symplectic representation on the vector $\mathcal{G}$ of the couplings of the theory [2]. In the present case the embedding of $\mathrm{SL}(2,\mathbb{R})^3$ in $\mathrm{Sp}(8,\mathbb{R})$ can be found in [2] and the vector $\mathcal{G}$ is given by (2.10). With a slight loss of generality we impose $g_1 = g_2 = g_3 \equiv g$. Then the stabilizer algebra reads

$$T(a_1,a_2) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -a_1-a_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -a_1 & -a_2 & a_1+a_2 \\ 0 & 0 & a_1+a_2 & -a_2 & 0 & 0 & 0 & 0 \\ 0 & a_1+a_2 & 0 & -a_1 & 0 & 0 & 0 & 0 \\ 0 & -a_2 & -a_1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2.11}$$

a two-dimensional abelian nilpotent subalgebra of order three of $\mathrm{sp}(8,\mathbb{R})$.

The solution generating technique of [2] consists in the transformation of the seed configuration $(\mathcal{V},\mathcal{G},\mathcal{Q}) \mapsto (S\mathcal{V},\mathcal{G},S\mathcal{Q})$, where S is an element of the stabilizer group $\mathcal{S}_{\mathcal{G}}$. Here, $\mathcal{Q} = (p^\Lambda, q_\Lambda)^t$ is the symplectic vector of magnetic and electric charges that enters the field strengths as follows: for a static solution of the type that we shall consider below,

$$\mathrm{d}s^2 = -e^{2U}\mathrm{d}t^2 + e^{-2U}\left(\mathrm{d}Y^2 + e^{2\psi}(\mathrm{d}\theta^2 + \sinh^2\theta\mathrm{d}\phi^2)\right), \tag{2.12}$$

the Maxwell equations are solved by

$$F^\Lambda_{tY} = \frac{1}{2}e^{2(U-\psi)}I^{\Lambda\Sigma}(R_{\Sigma\Gamma}p^\Gamma - q_\Sigma), \qquad F^\Lambda_{\theta\phi} = \frac{p^\Lambda}{2}\sinh\theta, \tag{2.13}$$

and the corresponding dual field strengths

$$F_{\Lambda\mu\nu} = R_{\Lambda\Sigma}F^\Sigma_{\mu\nu} - \frac{1}{2}I_{\Lambda\Sigma}e\epsilon_{\mu\nu\rho\sigma}F^{\Sigma\rho\sigma} \tag{2.14}$$

are

$$F_{\Lambda tY} = \frac{1}{2}e^{2(U-\psi)}\left(I_{\Lambda\Sigma}p^\Sigma + R_{\Lambda\Gamma}I^{\Gamma\Omega}R_{\Omega\Sigma}p^\Sigma - R_{\Lambda\Gamma}I^{\Gamma\Omega}q_\Omega\right), \qquad F_{\Lambda\theta\phi} = \frac{q_\Lambda}{2}\sinh\theta.$$

The charge vector $\mathcal{Q}$ is therefore completely determined by the $(\theta\phi)$-components of F^Λ and F_Λ.

In [1], various BPS solutions to the theory (2.5) were constructed, by using the recipe of [27], where all timelike supersymmetric backgrounds of $N=2$, $d=4$ FI-gauged supergravity are classified.[5] In particular, for the prepotential $F = -X^1X^2X^3/X^0$, the solution reads[6]

$$\mathrm{d}s^2 = -4b^2\mathrm{d}t^2 + \frac{1}{b^2}\frac{y\mathrm{d}y^2}{cy+2gp} + \frac{y^3}{b^2}(\mathrm{d}\theta^2 + \sinh^2\theta\mathrm{d}\phi^2), \tag{2.15}$$

[5] For a classification of the null case cf. [28].

[6] To translate between (2.5) and the conventions of [1] take $g_{\mathrm{CK}} \to g/2$ and $G_{\mathrm{CK}} \to \frac{1}{8\pi}$.

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where

$$b^4 = \frac{8g_1g_2g_3y^{\frac{9}{2}}}{H^0(cy+2gp)^{\frac{3}{2}}}\,, \qquad H^0 = \frac{2q_0}{3g^2p^2y^{\frac{3}{2}}}(cy+2gp)^{\frac{1}{2}}(cy-gp)+h^0\,, \tag{2.16}$$

with field strengths and scalars

$$\begin{aligned} F^0 &= 4\mathrm{d}t\wedge\mathrm{d}(H^0)^{-1}\,, \qquad F^I = \frac{p^I}{2}\sinh\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\,, \\ z^I &= i\tau^I = i\frac{\sqrt{g_1g_2g_3}}{\sqrt{2}g_I}\frac{\sqrt{H^0}y^{\frac{3}{4}}}{(cy+2gp)^{\frac{1}{4}}}\,, \end{aligned} \tag{2.17}$$

and the magnetic charges are constrained by $g_Ip^I \equiv gp = 2/3$ (no summation over I)[7] for $I = 1,2,3$. This field configuration preserves two real supercharges, while the near-horizon limit is a half-BPS attractor point $\mathrm{AdS}_2\times\mathrm{H}^2$. The range of the parameters is $q_0 < 0$, $p^I > 0$, $c > 0$, $h^0 > 3|q_0|c^{3/2}/2$ and $g_I > 0$. (2.15) has a horizon in $y = 0$ and asymptotes to a curved domain wall (whose worldvolume is an open Einstein static universe) for $y\to\infty$.

The (θ,ϕ)-components of the dual field strengths are given by

$$F_{0\theta\phi} = \frac{q_0}{2}\,, \qquad F_{I\theta\phi} = 0\,. \tag{2.18}$$

From this, together with (2.17), one can easily read off the charge vector $\mathcal{Q}$.

3 Black string with momentum along ∂_z

The r-map (2.7), (2.8) can be used to uplift the field configuration (2.15), (2.17) to five dimensions. If we define $y =: r^2$, the resulting metric in $d = 5$ reads

$$\mathrm{d}s^2 = 2\left(\frac{1}{H^0b}\right)^{\frac{2}{3}}\left(\frac{1}{b^2}\frac{4r^4\mathrm{d}r^2}{cr^2+2gp}+\frac{r^6}{b^2}(\mathrm{d}\theta^2+\sinh^2\theta\mathrm{d}\phi^2)\right)+\frac{1}{4}(H^0b)^{\frac{4}{3}}\left(\mathrm{d}z^2-\frac{8\sqrt{2}}{H^0}\mathrm{d}t\mathrm{d}z\right)\,, \tag{3.1}$$

with

$$b^4 = \frac{8g_1g_2g_3r^9}{H^0(cr^2+2gp)^{\frac{3}{2}}}\,, \qquad H^0 = \frac{2q_0}{3g^2p^2r^3}(cr^2+2gp)^{\frac{1}{2}}(cr^2-gp)+h^0\,. \tag{3.2}$$

The fluxes and the scalars are given by

$$F^I = \frac{p^I}{\sqrt{2}}\sinh\theta\mathrm{d}\theta\wedge\mathrm{d}\phi\,, \qquad h^I = \frac{(g_1g_2g_3)^{1/3}}{g_I}\,. \tag{3.3}$$

The configuration (3.1), (3.3) satisfies the BPS equations (A.1) with Killing spinor

$$\epsilon = Y(r)(1+i\Gamma^{32})(1-\Gamma^1)\epsilon_0\,, \tag{3.4}$$

where ϵ_0 is a generic constant Dirac spinor. Since ϵ_0 is subject to a double projection, the solution is one quarter BPS. It has a horizon in $r = 0$, where the spacetime becomes $\mathrm{AdS}_3\times\mathrm{H}^2$, while asymptotically it approaches what is commonly termed magnetic [29] AdS_5.

[7] This follows from eq. (2.23) of [1] with $\kappa = -1$, by taking into account that $p^I_{\mathrm{here}} = 8\pi p^I_{\mathrm{CK}}$.

The solution (3.1), (3.3) was first constructed in [30], by using the results of [31], where all supersymmetric backgrounds of minimal gauged supergravity in five dimensions were classified. It describes a gravitational wave propagating along a magnetic black string, which can be seen by introducing the new coordinates

$$\rho = \frac{\ell^2 (H^0)^{1/3} b^{4/3}}{\sqrt{2} r^3}\,, \qquad u = \frac{8^{1/4} z}{\ell^{1/2} c^{3/4}}\,, \qquad v = -\frac{32 t}{\ell^{1/2} c^{3/4} 8^{1/4}}\,, \tag{3.5}$$

where ℓ is defined by

$$\frac{2}{\ell^2} = (g_1 g_2 g_3)^{2/3}\,, \tag{3.6}$$

such that the cosmological constant is $\Lambda = -6\ell^{-2}$. The metric (3.1) becomes

$$\mathrm{d}s^2 = \frac{\ell^2 \mathrm{d}\rho^2}{\rho^2 h^2} + \frac{\ell^4}{\rho^2}(\mathrm{d}\theta^2 + \sinh^2\theta \mathrm{d}\phi^2) + \frac{\ell^2}{\rho^2} h^{3/2}\left(H^0 \mathrm{d}u^2 + \mathrm{d}u\mathrm{d}v\right)\,, \tag{3.7}$$

with

$$h = 1 - \frac{\rho^2}{3\ell^2}\,. \tag{3.8}$$

In the new radial coordinate ρ, the wave profile reads[8]

$$H^0 = \frac{3 q_0 c^{3/2}}{2 h^{3/2}}\left(1 - \frac{\rho^2}{2\ell^2}\right) + h^0\,. \tag{3.9}$$

In the near-horizon limit $\rho \to \sqrt{3}\ell$, (3.7) approaches

$$\mathrm{d}s^2 = (\mathrm{d}u + l\hat{\rho}\mathrm{d}v)^2 + l^2\left(\frac{\mathrm{d}\hat{\rho}^2}{4\hat{\rho}^2} - \hat{\rho}^2 \mathrm{d}v^2\right) + \frac{\ell^2}{3}(\mathrm{d}\theta^2 + \sinh^2\theta \mathrm{d}\phi^2)\,, \tag{3.10}$$

where we eliminated the constant h^0 by shifting $v \to v - h^0 u$, and subsequently rescaled

$$u \to \frac{2u}{|q_0|^{1/2} c^{3/4}}\,, \qquad v \to (|q_0|/2)^{1/2}(3c)^{3/4}\ell v\,. \tag{3.11}$$

Moreover we defined

$$\hat{\rho} = \left(\sqrt{3} - \frac{\rho}{\ell}\right)^{3/2}\,, \qquad l = \frac{2\ell}{3}\,. \tag{3.12}$$

Note that the AdS_3 factor in (3.10) is written as a Hopf-like fibration over AdS_2, in which ∂_v is a null direction.

The momentum L_z along the string can be computed as a Komar integral associated to the Killing vector $\xi = \partial_u$,

$$L_z = \frac{1}{16\pi G_5}\oint_{\partial V} dS^{\mu\nu}\nabla_\mu \xi_\nu\,, \tag{3.13}$$

where ∂V is the boundary of a spacelike hypersurface V of constant v. ∂V is defined by $\rho = \text{const.} \to 0$. The oriented measure on ∂V reads

$$dS^{\mu\nu} = (\zeta^\mu n^\nu - n^\mu \zeta^\nu)\sqrt{\sigma}\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}u\,, \tag{3.14}$$

[8]To get (3.7) from (4.21) of [30], set $k = -1$ there (which implies $\mathcal{P} = 0$) and $\mathcal{H}_2 = 1$. Then the Heun equation (4.25) of [30] boils down to a simple differential equation that is solved by (3.9).

where

$$n = \frac{\rho}{\ell h^{3/4} H^{0\,1/2}} (\partial_u - 2H^0 \partial_v) \tag{3.15}$$

denotes the unit normal to V, while $\zeta = -\rho h \ell^{-1} \partial_\rho$ and σ is the determinant of the induced metric on ∂V,

$$\sqrt{\sigma} = \frac{\ell^5}{\rho^3} h^{3/4} H^{0\,1/2} \sinh\theta \,. \tag{3.16}$$

This leads to

$$L_z = \frac{L \ell q_0 c^{3/2} (\mathfrak{g} - 1)}{16 G_5} \,. \tag{3.17}$$

Here $\mathfrak{g} = 2, 3, \dots$ denotes the genus of the Riemann surface to which H^2 is compactified, and L is the period of u in (3.10).

The Bekenstein-Hawking entropy of the solution (3.7) is given by[9]

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{hor}}}{4G_5} = \frac{\ell^2 L \pi (\mathfrak{g} - 1)}{3G_5} \,. \tag{3.18}$$

It would be very interesting to generalize (3.7), for instance by allowing for a nontrivial dependence of the wave profile on the 'angular' coordinates θ, ϕ. According to the governing equations for the wave profile (cf. (4.22) and (4.26) of [30]) this is possible, and a Kaluza-Klein reduction of such a solution to four dimensions would lead to a static black hole with dipole- or higher multipole charges. Work in this direction is in progress.

4 Dyonic black string with momentum

As we said, the results of [2] can be used to generate a new configuration from the seed solution (2.15) and the respective fluxes and scalars (2.17). It would be interesting to fix the parameters of the symmetry transformation $T(a_1, a_2)$ in (2.11) to get a vanishing graviphoton and therefore a static metric in five dimensions. However, as the following calculation shows, this results to be impossible. Starting form the fluxes[10]

$$\mathcal{Q} = (0, p, p, p, -|q_0|, 0, 0, 0)^t \,, \tag{4.1}$$

the action of $U = e^{T(a_1, a_2)}$, with $T(a_1, a_2)$ given by (2.11), generates

$$\mathcal{Q}' = \begin{pmatrix} 0 \\ p \\ p \\ p \\ -|q_0| + (a_1 a_2 + a_1^2 + a_2^2) p \\ -a_1 p \\ -a_2 p \\ (a_1 + a_2) p \end{pmatrix} \,. \tag{4.2}$$

[9] To compute the horizon area, we took a section of constant $\hat{\rho}$ and v in (3.10).

[10] Note that $q_0 = -|q_0|$. Moreover, if we choose $g_1 = g_2 = g_3 \equiv g$, the constraint $g_I p^I = gp$ satisfied by the seed implies $p^1 = p^2 = p^3 = p$.

The scalars acquire a constant axion,

$$z^{1\prime} = z^1 + a_1 \,, \qquad z^{2\prime} = z^2 + a_2 \,, \qquad z^{3\prime} = z^3 - a_1 - a_2 \,. \tag{4.3}$$

Looking at (4.2) one may think that the graviphoton can be set to zero by choosing properly the parameters a_1, a_2. This is however not the case, as can be seen from the field strengths in presence of axions, that read

$$F^\Lambda = \frac{S^\Lambda b^2}{y^{5/2}(cy+2gp)^{1/2}} \mathrm{d}t \wedge \mathrm{d}y + \frac{p^\Lambda}{2} \sinh\theta \mathrm{d}\theta \wedge \mathrm{d}\phi \,, \tag{4.4}$$

with

$$S^\Lambda = \frac{|q_0|}{\tau^1\tau^2\tau^3}(1, a_1, a_2, -a_1-a_2)^t \,, \qquad p^\Lambda = (0,p,p,p)^t \,. \tag{4.5}$$

The uplifted metric is still (3.1). The field strengths and scalars in five dimensions become respectively

$$F^I = \frac{2\sqrt{2}b^2 S^I}{r^4(cr^2+2gp)^{1/2}} \mathrm{d}t \wedge \mathrm{d}r + \frac{p^I}{\sqrt{2}} \sinh\theta \mathrm{d}\theta \wedge \mathrm{d}\phi \,, \qquad h^I = 1 \,, \tag{4.6}$$

where now

$$S^I = \frac{|q_0|}{\tau^1\tau^2\tau^3}(a_1, a_2, -a_1-a_2)^t \,, \qquad p^I = (p,p,p)^t \,, \tag{4.7}$$

so we have generated two additional electric charge parameters. Note that the metric remains untouched by the duality rotation. This solution describes again a flow between magnetic AdS_5 and $\mathrm{AdS}_3 \times \mathrm{H}^2$, and preserves the same amount of supersymmetry as before, as can be easily seen by using the Killing spinor equations following from (A.1).

5 Dyonic black string with both momentum and rotation

We now want to generate a rotating black string by applying the same technique as above. The starting point is the seed metric (3.1), with fluxes and scalars (4.6). After a Kaluza-Klein reduction to four dimensions along the angular Killing direction ∂_ϕ, one gets

$$\begin{aligned}
\mathrm{d}s^2 &= \sinh\theta \left(\frac{\sqrt{2}(cr^2+2gp)^{1/2}}{g^3 r^2} \mathrm{d}r^2 + \frac{(cr^2+2gp)^{3/2}}{2\sqrt{2}g^3} \mathrm{d}\theta^2 + 2r^3 \left(\frac{H^0}{4\sqrt{2}} \mathrm{d}z - \mathrm{d}t \right) \mathrm{d}z \right) , \\
A^\Lambda &= \left(0, \frac{4q_I}{H^0} \mathrm{d}t\right) , \qquad z^I = \frac{p^I}{\sqrt{2}} \cosh\theta + i \frac{(cr^2+2gp)^{1/2} \sinh\theta}{\sqrt{2} g_I} \,,
\end{aligned} \tag{5.1}$$

where $q_3 = -q_1 - q_2$. Applying the duality transformation (2.11) leads to a configuration with a nonvanishing graviphoton,

$$\mathcal{Q} = (0,0,0,0,0,q_1,q_2,-q_1-q_2) \mapsto \mathcal{Q}' = (0,0,0,0,\omega,q_1,q_2,-q_1-q_2)^t \,, \tag{5.2}$$

where $\omega = -a_1(2q_1 + q_2) - a_2(q_1 + 2q_2)$, and the scalar fields acquire a real constant part as in (4.3). Lifting the solution back to $d = 5$ gives

$$\begin{aligned} \mathrm{d}s^2 &= \frac{2\mathrm{d}r^2}{g^2 r^2} + \frac{cr^2 + 2gp}{2g^2}\mathrm{d}\Omega^2_{\mathrm{H}^2} + \frac{2\sqrt{2}gr^3}{(cr^2+2gp)^{1/2}}\left(\frac{H^0}{4\sqrt{2}}\mathrm{d}z - \mathrm{d}t\right)\mathrm{d}z \\ &\quad + 4\sqrt{2}\omega\frac{cr^2+2gp}{g^2H^0}\sinh^2\theta\left(\mathrm{d}\phi + \frac{2\sqrt{2}\omega}{H^0}\mathrm{d}t\right)\mathrm{d}t\,, \end{aligned} \tag{5.3}$$

$$A^I = \frac{p^I}{\sqrt{2}}\cosh\theta\mathrm{d}\phi + \frac{4}{H^0}\left(q_I + \omega s^I + \frac{\omega p^I}{\sqrt{2}}\cosh\theta\right)\mathrm{d}t\,, \qquad h^I = 1\,, \tag{5.4}$$

with $s^I = (a_1, a_2, -a_1 - a_2)$ and H^0 was defined in (3.2).

The near-horizon limit $r \to 0$ of (5.3) leads to the metric

$$\mathrm{d}s^2 = \frac{|q_0|}{3p}(\mathrm{d}z - \hat{\rho}\mathrm{d}t)^2 + \frac{l^2\mathrm{d}\hat{\rho}^2}{4\hat{\rho}^2} - \frac{|q_0|}{3p}\hat{\rho}^2\mathrm{d}t^2 + \frac{p}{g}\left[\mathrm{d}\theta^2 + \sinh^2\theta(\mathrm{d}\phi + 2\hat{\rho}\omega\mathrm{d}t)^2\right]\,, \tag{5.5}$$

where

$$\hat{\rho} \equiv \frac{3\sqrt{gp}}{|q_0|}r^3\,. \tag{5.6}$$

(5.5) represents a deformation of (3.10), i.e., of $\mathrm{AdS}_3 \times \mathrm{H}^2$.

For $r \to \infty$ (5.3) approaches again magnetic AdS_5, as can be easily shown by using some simple coordinate transformations.

It would be interesting to check whether the solution (5.3), (5.4) is still BPS, or, more generally, if the solution-generating technique based on [2] preserves supersymmetry.

6 Solutions with running scalars

In the appendix of [3] the problem of finding one quarter magnetic BPS strings with running scalars is reduced to solve a system of three first-order differential equations. The metric is given by[11]

$$\mathrm{d}s^2 = e^{2V}(-\mathrm{d}t^2 + \mathrm{d}z^2) + e^{2W}(\mathrm{d}u^2 + \mathrm{d}\Omega^2_\kappa)\,, \tag{6.1}$$

with

$$\mathrm{d}\Omega^2_\kappa = \mathrm{d}\theta^2 + \frac{\sin^2\sqrt{\kappa}\theta}{\kappa}\mathrm{d}\phi^2\,, \qquad \kappa = 0, \pm 1\,, \tag{6.2}$$

$$e^{2V} = (x^1x^2x^3)^{-\frac{1}{3}}e^{-g\int(x^1+x^2+x^3)\mathrm{d}u}\,, \qquad e^{2W} = (x^1x^2x^3)^{\frac{2}{3}}\,, \tag{6.3}$$

and the $x^I(u)$ define the scalar fields according to $h^I = x^I/(x^1x^2x^3)^{1/3}$. They are determined by the system[12]

$$\begin{aligned} y^{1\prime} &= y^2y^3 + Q^1\,, \\ y^{2\prime} &= y^1y^3 + Q^2\,, \\ y^{3\prime} &= y^1y^2 + Q^3\,, \end{aligned} \tag{6.4}$$

[11] In this section we choose $V_I = \frac{1}{3}$ in (2.2) without loss of generality.
[12] In the following we set $g = 1$.

where a prime denotes a derivative w.r.t. the radial coordinate u and

$$\begin{aligned} y^1 &= x^1 + x^2 - x^3 \,, \\ y^2 &= x^1 - x^2 - x^3 \,, \\ y^3 &= x^1 - x^2 + x^3 \,. \end{aligned} \tag{6.5}$$

The fluxes are purely magnetic, i.e., $F^I_{\theta\phi} = \sqrt{\kappa} q^I \sin\sqrt{\kappa}\theta$, and the parameters Q^I are defined by

$$\begin{aligned} Q^1 &= -\kappa(q^1 + q^2 - q^3) \,, \\ Q^2 &= -\kappa(q^1 - q^2 - q^3) \,, \\ Q^3 &= -\kappa(q^1 - q^2 + q^3) \,. \end{aligned} \tag{6.6}$$

(6.4) can be derived from an inhomogeneous version of the SU(2) Nahm equations [32–34]

$$\frac{\mathrm{d}T^I}{\mathrm{d}u} = \epsilon^{IJK}\left[T^J, T^K\right] + S^I \tag{6.7}$$

(where the T^I and S^I take values in the Lie algebra su(2)) by setting $T^I = y^I\sigma^I$, $S^I = Q^I\sigma^I$ (no summation over I), and the σ^I denote Pauli matrices. Notice that for $Q^I = 0$ this leads to the Ercolani-Sinha solution [35], which is given in terms of elliptic functions. (6.4) can be written as

$$y^{I\prime} = C^I{}_{JK} y^J y^K + Q^I \,, \tag{6.8}$$

and thus its symmetries are determined by the transformations that leave the tensor $C^I{}_{JK}$ invariant, $T^{-1}CTT = C$. Unfortunately this implies $T = 1$. The discrete symmetry group of (6.8), which is easily shown to be $(\mathbb{Z}_2)^6 \times \mathbb{Z}_3$, is not very useful for generating new solutions from known ones.

The system (6.8) is equivalent to the SO(2, 1) spinning top equations, which are given by

$$\begin{aligned} I_1\omega_1' &= (I_2 - I_3)\omega_2\omega_3 + M_1 \,, \\ I_2\omega_2' &= -(I_3 - I_1)\omega_3\omega_1 + M_2 \,, \\ I_3\omega_3' &= (I_1 - I_2)\omega_1\omega_2 + M_3 \,, \end{aligned} \tag{6.9}$$

where I_K are the principal moments of inertia and M_K represents an external torque. If we set

$$\omega_1 = \lambda_1 y^1 \,, \qquad \omega_2 = \lambda_2 y^2 \,, \qquad \omega_3 = \lambda_3 y^3 \,, \tag{6.10}$$

where

$$\lambda_1^2 = \frac{-I_2 I_3}{(I_3 - I_1)(I_1 - I_2)} \,, \qquad \lambda_2^2 = \frac{I_3 I_1}{(I_1 - I_2)(I_2 - I_3)} \,, \qquad \lambda_3^2 = \frac{-I_1 I_2}{(I_2 - I_3)(I_3 - I_1)} \,,$$

(6.9) reduces to (6.8). Here we assumed (without loss of generality) $I_1 > I_2 > I_3 > 0$. Then all λ_K^2 are positive. Note that, as compared to Euler's equations, there is a minus sign on the r.h.s. of the second of (6.9), which is the reason for the terminology SO(2, 1). In fact, in the untorqued case $M_K = 0$, the eqs. (6.9) can be derived from the Hamiltonian

$$H = \frac{\ell_1^2}{2I_1} - \frac{\ell_2^2}{2I_2} + \frac{\ell_3^2}{2I_3}\,, \tag{6.11}$$

by using the Poisson brackets

$$[\ell_I, \ell_J] = \epsilon_{IJ}{}^K \ell_K\,, \tag{6.12}$$

as well as $\ell'_I = [\ell_I, H]$. In (6.12), $\epsilon_{123} = 1$ and the indices of the Levi-Civita tensor are raised with the Minkowski metric $\eta = \mathrm{diag}(1, -1, 1)$. W.r.t. the Euler top, (6.11) has one kinetic term that comes with the 'wrong' sign.

In [3] a particular solution to (6.4) is found. We will now show a different way to integrate these equations. Imposing $Q^1 = Q^2 = 0$, i.e., $q^2 = 0$ and $q^1 = q^3$, and defining $y_\pm = y^1 \pm y^2$ one finds that

$$y_\pm = k_\pm e^{\int y^3 \mathrm{d}u}\,, \qquad y^{3\prime} = \frac{1}{4}\left(k_+^2 e^{2\int y^3 \mathrm{d}u} - k_-^2 e^{-2\int y^3 \mathrm{d}u}\right) + Q^3\,, \tag{6.13}$$

where $Q^3 = -2\kappa q^1$ and $k_\pm$ are integration constants. The Dirac-type charge quantization condition $3gV_I q^I = 1$ of [3] implies $2gq^1 = 1$, and thus $Q^3 = -\kappa$. Introducing a new radial coordinate $y = -\int y^3 \mathrm{d}u$, the last equation becomes

$$y'' = \frac{1}{4}\left(k_-^2 e^{2y} - k_+^2 e^{-2y}\right) + \kappa\,, \tag{6.14}$$

which can be integrated once to give[13]

$$y' = \frac{\mathrm{d}y}{\mathrm{d}u} = -y^3 = -\sqrt{\frac{1}{4}\left(k_-^2 e^{2y} + k_+^2 e^{-2y}\right) + 2\kappa y}\,. \tag{6.15}$$

This leads to the metric

$$\mathrm{d}s^2 = (x^1x^2x^3)^{-\frac{1}{3}} e^{\int \frac{x^1+x^2+x^3}{y^3}\mathrm{d}y}(-\mathrm{d}t^2 + \mathrm{d}z^2) + (x^1x^2x^3)^{\frac{2}{3}}\left(\frac{\mathrm{d}y^2}{(y^3)^2} + \mathrm{d}\Omega_\kappa^2\right)\,, \tag{6.16}$$

where

$$\begin{aligned} x^1 &= \frac{1}{4}\left(k_+e^{-y} + k_-e^y + \sqrt{k_+^2 e^{-2y} + k_-^2 e^{2y} + 8\kappa y}\right)\,, \\ x^2 &= \frac{k_-}{2}e^y\,, \\ x^3 &= \frac{1}{4}\left(-k_+e^{-y} + k_-e^y + \sqrt{k_+^2 e^{-2y} + k_-^2 e^{2y} + 8\kappa y}\right)\,. \end{aligned} \tag{6.17}$$

In what follows we assume $k_- > 0$. Then asymptotically for $y \to \infty$ the geometry becomes (magnetic) AdS_5,

$$\mathrm{d}s^2 = \frac{2e^{2y}}{k_-}(-\mathrm{d}t^2 + \mathrm{d}z^2) + \mathrm{d}y^2 + \frac{k_-^2 e^{2y}}{4}\mathrm{d}\Omega_\kappa^2\,. \tag{6.18}$$

[13]The plus sign corresponds to an unphysical solution with negative scalars h^I. A possible additive integration constant can be eliminated by shifting y.

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For generic integration constants $k_\pm$ the metric becomes singular at a certain point and the solution does not have a horizon. However, in the case $\kappa = -1$, if $k_\pm$ are related by[14]

$$k_- = e^{-a}\sqrt{4a+2}\,, \qquad k_+ = -e^{a}\sqrt{4a-2}\,, \tag{6.19}$$

where a denotes an arbitrary parameter, there is a horizon for $y = a$, where the solution approaches $\mathrm{AdS}_3 \times \mathrm{H}^2$, as we will show below. In the cases $\kappa = 0, 1$ a similar reasoning cannot be done, and the metric has no event horizon.

(6.19) are real for $a \geq 1/2$. We note that, if (6.19) holds, one has

$$y^3(y) = \sqrt{2a\cosh(2(a-y)) - \sinh(2(a-y)) - 2y}\,, \tag{6.20}$$

and thus

$$\frac{\mathrm{d}(y^3)^2}{\mathrm{d}y} = 2\cosh(2(a-y)) - 4a\sinh(2(a-y)) - 2 \geq 2(e^{2(y-a)} - 1) \geq 0\,. \tag{6.21}$$

y^3 is always well-defined for $y \geq a$ and becomes zero at the horizon, $y^3(a) = 0$. The scalar fields (6.17) are positive in the whole range $a \leq y < \infty$.

The value $a = 1/2$ is special since it corresponds to the limit in which $x^1 = x^3$, i.e., $\phi^2 = 0$. This truncation leads to the solution of [12] with two nonzero and equal fluxes. Indeed, one easily verifies that ϕ^1 and the function W appearing in the metric (6.1) satisfy the equation

$$e^{2W + \frac{\phi^1}{\sqrt{6}}} = e^{2W - \frac{2\phi^1}{\sqrt{6}}} + \frac{\sqrt{6}W + 2\phi^1}{2\sqrt{6}} + \frac{1}{4}\,, \tag{6.22}$$

which is precisely eq. (17) of [12]. The black strings defined by (6.16), (6.17) represent thus generalizations of the Maldacena-Nuñez solution. Note that also the latter was not known analytically up to now.

The near-horizon limit $y \to a$ can be obtained from the expansion

$$\begin{aligned}
\frac{x^1 + x^2 + x^3}{y^3} &= \sqrt{\frac{1+2a}{2a}}\frac{1}{y-a} + O((y-a)^0)\,, \\
\frac{(y^3)^3}{x^1x^2x^3} &= 32a\sqrt{\frac{2a}{1+2a}}(y-a)^3 + O((y-a)^4)\,, \\
x^1x^2x^3 &= \sqrt{\frac{1+2a}{32}} + O((y-a))\,.
\end{aligned} \tag{6.23}$$

Introducing the new radial coordinate

$$\hat{u}^2 = (y-a)^{-\sqrt{\frac{1+2a}{2a}}}\,, \tag{6.24}$$

the metric (6.16) becomes for $\hat{u} \to 0$

$$\mathrm{d}s^2 = \frac{1}{\hat{u}^2}\left[-\mathrm{d}t^2 + \mathrm{d}z^2 + \frac{\mathrm{d}\hat{u}^2}{(2+4a)^{2/3}}\right] + \left(\frac{1+2a}{32}\right)^{1/3}\mathrm{d}\Omega^2_{-1}\,, \tag{6.25}$$

which is $\mathrm{AdS}_3 \times \mathrm{H}^2$.

[14] The other possibility $k_\pm = e^{\pm a}\sqrt{4a \mp 2}$ is related to (6.19) by the $\mathbb{Z}_2$ symmetry $x^1 \leftrightarrow x^3$ and corresponds to negative h^I.

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The central charge of the two-dimensional SCFT dual to the near-horizon configuration is given by [12, 26]

$$c = \frac{3R_{\mathrm{AdS}_3}}{2G_3} = \frac{6\pi(\mathfrak{g}-1)R_{\mathrm{AdS}_3}R^2_{\Sigma_\mathfrak{g}}}{G_5}\,, \tag{6.26}$$

where $\mathfrak{g} = 2, 3, \ldots$ is the genus of the Riemann surface $\Sigma_\mathfrak{g}$ to which H^2 is compactified. The values of the curvature radii are

$$R_{\mathrm{AdS}_3} = \frac{1}{(2+4a)^{1/3}}\,, \qquad R^2_{\Sigma_\mathfrak{g}} = \frac{(1+2a)^{1/3}}{2^{5/3}}\,, \tag{6.27}$$

with

$$2a = \sqrt{1+\left(\frac{k_+k_-}{2}\right)^2}\,. \tag{6.28}$$

For the truncation $\phi^2 = 0$, which means $k_+ = 0$, one has $R_{\mathrm{AdS}_3} = 2^{-2/3}$, i.e. the value found in [12].

Using (6.27), the central charge can be written in the form

$$c = \frac{6\pi(\mathfrak{g}-1)}{4G_5} = 3N^2(\mathfrak{g}-1)\,, \tag{6.29}$$

where we used the AdS/CFT dictionary $N^2 = \pi/(2G_5 g^3)$ (with $g = 1$). Near the conformal boundary the scalar fields behave like

$$\frac{2\phi^1}{\sqrt{6}} \sim 2Q^3 y e^{-2y}\,, \qquad \sqrt{2}\phi^2 \sim -\frac{k_+}{k_-}e^{-2y}\,, \tag{6.30}$$

and thus are read in the dual SCFT as an insertion and an expectation value of an operator of scaling dimension $\Delta = 2$. The relevant deformation of the dual superpotential relative to ϕ^1 is described in [12], while ϕ^2 is a marginal deformation of two-dimensional $N = (4,4)$ SYM theory. Thus, the solution does not describe the gravity dual of 2d $N = (2,2)^*$ SYM [36]. The constant a represents the physical scale of the energy in the renormalization group flow at which the IR fixed point appears, but which being a CFT is independent of the energy scale.

6.1 Inclusion of hypermultiplets

It is worthwhile to note that with running hyperscalars the BPS equations, (5.17) of [26], can be simplified to a system for which the number of equations equals the number of the scalar fields in the model. The idea is basically the same as that of [3]: introducing a new radial coordinate R by $\mathrm{d}R = e^{-\psi}\mathrm{d}r$ and the rescaled scalars $y^I = e^{\psi-2T}h^I$, where $\psi(r)$ and $T(r)$ are metric functions defined in eq. (3.1) of [26] and r denotes the radial coordinate used there, the system (5.17) of [26] boils down to

$$\begin{aligned}
\psi &= \int 9\kappa g^2\mathcal{L}_y\mathrm{d}R\,, \qquad e^{3\psi-6T} = \frac{1}{6}C_{IJK}y^Iy^Jy^K\,,\\
y^{I\prime} - 9g^2\kappa(\mathcal{L}_y y^I &- \mathcal{Q}^x P^x_J G^{IJ}_y) - p^I = 0\,,\\
q^{u\prime} &= -\frac{9}{2}\kappa g^2 h^{uv}\partial_v\mathcal{L}_y\,,
\end{aligned} \tag{6.31}$$

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where

$$\mathcal{L}_y = \mathcal{Q}^x P_I^x y^I\,, \qquad G_y^{IJ} = -C^{IJK} C_{KLM} y^L y^M + 2y^I y^J\,, \qquad C^{IJK} \equiv \delta^{IL}\delta^{JM}\delta^{KN} C_{LMN}\,.$$

Even if the complete integration of these equations in a particular model remains a hard task, this partial integration can be considered as a first step towards the solution. A numerical and asymptotic analysis of this type of models can be found in [37],[15] where a particular truncation of $N = 8$, $d = 5$ gauged supergravity is studied.

7 Conclusions

In this paper, we used the residual symmetries of $N = 2$, $d = 4$ Fayet-Iliopoulos-gauged supergravity discovered in [2] to add an electric charge density and rotation to five-dimensional black strings that asymptote to AdS_5. This is the first instance of using solution-generating techniques in gauged supergravity to add rotation to a given seed, and opens the possibility to construct in a similar way many solutions hitherto unknown, which are potentially interesting particularly in an AdS/CFT context.

The rotating string (5.3) interpolates between magnetic AdS_5 at infinity and a deformation of $\mathrm{AdS}_3 \times \mathrm{H}^2$ near the horizon. This deformation implies that the CFT_2, to which the dual four-dimensional CFT flows in the IR, has less symmetry. We did not check explicitely how many supercharges are preserved by the near-horizon metric (5.5), but we would expect that the rotation breaks at least some of the original supersymmetries.

We also constructed static magnetic BPS black strings with running scalars in the FI-gauged stu model. It was shown that this amounts to solving the $\mathrm{SO}(2,1)$ spinning top equations, which descend from an inhomogeneous version of the Nahm equations. We were able to solve these in a particular case, which leads to a generalization of the Maldacena-Nuñez solution. Moreover, we computed the central charge of the CFT_2 dual to the near-horizon configuration. From the behaviour of the two bulk scalar fields ϕ^1 and ϕ^2 near the conformal boundary we saw that they correspond in the dual SCFT to an insertion and an expectation value of an operator of scaling dimension $\Delta = 2$. The relevant deformation of the dual superpotential relative to ϕ^1 is described in [12], while ϕ^2 is a marginal deformation of two-dimensional $N = (4, 4)$ SYM theory. Thus, our solution does not describe the gravity dual of 2d $N = (2, 2)^*$ SYM [36].

We hope to come back in particular to further applications of the duality transformations of [2] in the near future.

A Supersymmetry variations

The supersymmetry variations of the gravitino ψ_μ and the gauginos λ_i in $N = 2$, $d = 5$ FI-gauged supergravity coupled to vector multiplets read [22]

$$\begin{aligned} \delta\psi_\mu &= \left(D_\mu + \frac{i}{8} h_I ({\Gamma_\mu}^{\nu\rho} - 4{\delta_\mu}^\nu \Gamma^\rho) F^I_{\nu\rho} + \frac{1}{2} g \Gamma_\mu h^I V_I\right)\epsilon\,, \\ \delta\lambda_i &= \left(\frac{3}{8}\Gamma^{\mu\nu} F^I_{\mu\nu}\partial_i h_I - \frac{i}{2}\mathcal{G}_{ij}\Gamma^\mu \partial_\mu \phi^j + \frac{3i}{2} g V_I \partial_i h^I\right)\epsilon\,, \end{aligned} \tag{A.1}$$

[15] Cf. also [38].

where

$$D_\mu \epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\Gamma_{ab} - \frac{3i}{2} g V_I A^I_\mu \right) \epsilon\,. \tag{A.2}$$

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