RECEIVED: August 2, 2018

REVISED: September 15, 2018

ACCEPTED: September 26, 2018

PUBLISHED: October 2, 2018

Neutrino masses and gauged $U(1)_\ell$ lepton number

We-Fu Chang^{a,b} and John N. Ng^b

^a*Department of Physics, National Tsing Hua University,
101 Sec. 2, KuangFu Rd., Hsinchu 300, Taiwan*

^b*TRIUMF Theory Group,
4004 Wesbrook Mall, Vancouver, B.C. V6T2A3, Canada*

E-mail: wfchang@phys.nthu.edu.tw, misery@triumf.ca

ABSTRACT: We investigate the tree-level neutrino mass generation in the gauged $U(1)_\ell$ lepton model recently proposed by us [1]. With the addition of one Standard Model (SM) singlet, $\phi_1 (Y = 0, \ell = 1)$, and one SM triplet scalar, $T (Y = -1, \ell = 0)$, realistic lepton masses can be accommodated. The resulting magnitude of neutrino mass is given by $\sim v_t^3/v_L^2$, where v_t and v_L are the vacuum expectation values of T and ϕ_1 , respectively, and it is automatically of the inverse see-saw type. Since v_L is the lepton number violation scale we take it to be high, i.e. $O \gtrsim (\text{TeV})$. Moreover, the induced lepton flavor violating processes and the phenomenology of the peculiar triplet are studied. An interesting bound, $0.1 \lesssim v_t \lesssim 24.1 \text{ GeV}$, is obtained when taking into account the neutrino mass generation, $Br(\mu \rightarrow e\gamma)$, and the limits from oblique parameters, ΔS and ΔT . Collider phenomenology of the SM triplets is also discussed.

KEYWORDS: Anomalies in Field and String Theories, Beyond Standard Model, Gauge Symmetry, Neutrino Physics

ARXIV EPRINT: [1807.09439](https://arxiv.org/abs/1807.09439)

Contents

1	Introduction	1
2	$U(1)_\ell$ anomalies cancelations and new fields	3
3	Lepton masses for 1 generation	6
4	3-generation lepton masses	8
4.1	Charged lepton mass matrix	8
4.2	Neutral lepton mass matrix	9
5	Neutrino oscillations and data fitting	10
6	$\mu \rightarrow e\gamma$, and a_μ	12
7	Triplets at colliders	15
7.1	Decays of the triplet	16
7.2	Triplet production at hadron colliders	18
7.3	Triplet pair productions at the e^+e^- machine	19
8	EW precision, ΔS and ΔT from the exotic fermions and scalars	20
8.1	Tree-level ρ -parameter	20
8.2	Loop corrections	21
9	Higgs to 2γ	22
10	Conclusions	23

1 Introduction

It is now generally accepted that neutrino oscillation data indicate that at least two of the three active neutrinos have nonvanishing masses. This cannot be accommodated in the minimal Standard Model (SM) without adding new degrees of freedom such as two or more SM right-handed neutrinos. However, neutrino masses can be generated by the addition of the Weinberg operator [2], O_5 . This nonrenormalizable dimension five operator takes the form of $O_5 = \frac{y}{\Lambda} \ell_L \ell_L H H$, where H is the SM Higgs field, ℓ_L denotes a SM lefthanded lepton doublet, y is a free dimensionless parameter, and Λ is an unknown high scale. After H takes on a vacuum expectation value $v \simeq 247$ GeV, the electroweak symmetry is spontaneously broken, and we get a neutrino mass $m_\nu \sim \frac{yv^2}{\Lambda}$. Since data indicate that $m_\nu \lesssim 1$ eV, depending on the value of y , the scale Λ can range from 1 to 10^{11} TeV. This elegant way of generating neutrino masses using only SM fields comes with the price of

nonrenormalizability. Furthermore, it reinforces the idea that the SM is an effective theory and the neutrino masses call for its extension.

Neutrino mass generated from the Weinberg operator is of the Majorana type, and it has lepton number $\ell = 2$ provided the conventional lepton number assignments that all SM charged leptons e, μ, τ and their associated neutrinos ν_e, ν_μ, ν_τ have $\ell = 1$ and all other SM fields carry $\ell = 0$ are assumed. Also the anti-leptons have $\ell = -1$. This is a natural consequence if lepton number is a $U(1)_\ell$ symmetry. Thus, the SM is largely invariant under this symmetry with a very small breaking by the Weinberg operator. However, the nature of this symmetry is unknown. Usually, the total lepton number is taken to be a global symmetry that is broken at a very high scale $\Lambda \gtrsim 10^{12} \text{ GeV}$ by two or more SM singlet righthanded neutrinos N_R with Majorana masses of $O(\Lambda)$. Integrating them out gives rise to the Weinberg operator, and this is the celebrated type I seesaw mechanism [3–7]. Doing so raises the question of the origin of the Majorana mass bestowed to N_R . One can add a Majorana mass for N_R by hand. However, our current understanding is that masses of fermions are generated by the Higgs mechanism. It is interesting to also to apply this to $U(1)_\ell$. Doing so will lead to the existence of a Goldstone boson in the physical spectrum which can act as a candidate for dark radiation [8, 9].

Moreover, it is phenomenologically and theoretically interesting to investigate the possibility of a gauged $U(1)_\ell$ and study the spontaneously broken gauge theory. There are several possibilities. One can gauge the total lepton number as in [10].¹ One can also gauge a combination of lepton generation number such as $L_\mu - L_\tau$ [16, 17]. In ref. [1], hereafter referred to as (I), we gauged each lepton family with the usual lepton number assignments for them. Of the just mentioned three examples only the second one is anomaly-free with only the SM fields. Gauging the total lepton will require extra leptons with very exotic lepton charges such as $\ell = 3$ to cancel the anomalies from $U(1)_\ell$. In (I), the extra anomalies cancelations require two extra pairs of vector-like $SU(2)$ doublet leptons with eigenvalues $\ell = 1, 0$ for each family. We also did not include any singlet N_R field, and the Weinberg operator is generated radiatively at 1-loop. The principal source of lepton number violation comes from a SM singlet scalar with $\ell = 2$ which picks up a vacuum expectation value.

In this paper, we study a different mechanism of neutrino mass generation in the gauged lepton number scheme introduced in (I). The extra leptons presented before is sufficient to generate neutrino masses with the aid of a SM triplet scalar T and a SM singlet scalar ϕ_1 . T has $\ell = 0$ whereas ϕ_1 is given $\ell = 1$, with both fields being Higgsed. This naturally leads to an inverse seesaw mechanism (ISM) [18–20] for active neutrino mass. The novel feature here is that we do not add by hand any SM singlet leptons to implement ISM as is commonly done. The required leptons are dictated by anomaly cancelations. Details will be given in section 3. Since the physics involved with the gauge new gauge boson Z_ℓ and the extra leptons are the same as in (I), we will not repeat their phenomenology here. Instead, we focus on neutrino physics and the phenomenology of T . We find that T has interesting different signatures at high energy colliders from previous studies of $l = 2$ Higgs triplets [21–25], which are commonly employed in the type-II see-saw model [26–31]. For a recent review see [32].

¹For other constructions in conjunction with gauged baryon number see [11–15].

Field	SU(2)	Y	ℓ
$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	2	$-\frac{1}{2}$	1
e_R	1	-1	1
$L_{1L} = \begin{pmatrix} N_{1L} \\ E_{1L} \end{pmatrix}$	2	$-\frac{1}{2}$	-1
E_{1R}	1	-1	-1
$L_{2R} = \begin{pmatrix} N_{2R} \\ E_{2R} \end{pmatrix}$	2	$-\frac{1}{2}$	0
E_{2L}	1	-1	0

Table 1. Lepton fields for anomalies free solution.

We organize the paper as follows. The next section we present our anomalies solution for completeness. Then we discuss lepton mass generation for one generation to illustrate the physics. This is followed by a realistic 3-generation study. Section 4 gives fits to the neutrino oscillation data. Constraints from charged lepton flavor changing neutral currents are given in section 5. Important electroweak precision constraints are studied in section 6. The productions of different new triplet scalars at the LHC and CLIC are examined in section 7. Our conclusions are given in section 8.

2 $U(1)_\ell$ anomalies cancelations and new fields

We extend the SM gauged group by adding a $U(1)_\ell$ and is explicitly given as $G = SU(2) \times U(1)_Y \times U(1)_\ell$. All SM leptons have the conventional value of $\ell = 1$. We will concentrate on one family. This can be trivially extended for all 3 SM families.

The new anomaly coefficients are

$$\mathcal{A}_1([SU(2)]^2 U(1)_\ell) = -1/2, \quad (2.1a)$$

$$\mathcal{A}_2([U(1)_Y]^2 U(1)_\ell) = 1/2, \quad (2.1b)$$

$$\mathcal{A}_3([U(1)_Y][U(1)_\ell]^2) = 0, \quad (2.1c)$$

$$\mathcal{A}_4([U(1)_\ell]^3) = -1, \quad (2.1d)$$

$$\mathcal{A}_5(U(1)_\ell) = -1, \quad (2.1e)$$

where $\mathcal{A}_5$ stands for the lepton-graviton anomaly. While new chiral leptons are introduced to cancel eq. (2.1), one also needs to make sure that the SM anomalies of $\mathcal{A}_6([SU(2)]^2 U(1)_Y)$, $\mathcal{A}_7([U(1)_Y]^3)$, and $\mathcal{A}_8(U(1)_Y)$ are canceled. It is easy to check that the new leptons in table 1 cancel the above anomalies. Since the pair of new leptons are vectorlike, the SM anomalies $\mathcal{A}_6([SU(2)]^2 U(1)_Y)$, $\mathcal{A}_7([U(1)_Y]^3)$, and $\mathcal{A}_8(U(1)_Y)$ cancelations are not affected.

Field	SU(2)	Y	ℓ
H	2	$\frac{1}{2}$	0
T	3	-1	0
ϕ_1	1	0	1

Table 2. Scalar fields content.

The minimal set of scalar fields, by utilizing the triplet scalar for neutrino mass generation, can be obtained by examining the gauge invariant set of lepton bilinears that can be formed from the above fields. They are given in table 2. where all H, ϕ_1 , and T develop non-zero VEVs.

The Yukawa interactions are

$$\begin{aligned}
 \mathcal{L}_Y = & f_1 \bar{l}_L L_{2R} \phi_1 + f_2 \bar{e}_R E_{2L} \phi_1 + f_3 \bar{L}_{1L} L_{2R} \phi_1^* + f_4 \bar{E}_{1R} E_{2L} \phi_1^* \\
 & + h_1 \bar{l}_L e_R H + h_2 \bar{L}_{1L} E_{1R} H + h_3 \bar{L}_{2R} E_{2L} H \\
 & + y_1 \bar{l}_L^c T^\dagger L_{1L} + \frac{y_2}{2} \bar{L}_{2R}^c T^\dagger L_{2R} + h.c.
 \end{aligned} \tag{2.2}$$

where all the generation indices are suppressed. The full gauge invariant and renormalizable scalar potential reads,

$$\begin{aligned}
 V = & -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_L^2 |\phi_1|^2 + \lambda_L |\phi_1|^4 \\
 & -\mu_t^2 \text{Tr}(T^\dagger T) + \lambda_t [\text{Tr}(T^\dagger T)]^2 \\
 & + \lambda_1 (H^\dagger H) \text{Tr}(T^\dagger T) + \lambda_2 (H^\dagger H) |\phi_1|^2 + \lambda_3 \text{Tr}(T^\dagger T) |\phi_1|^2 \\
 & + \lambda_4 \text{Tr}(T^\dagger T T^\dagger T) + \lambda_5 \det T^\dagger T + \lambda_6 H^\dagger T T^\dagger H \\
 & -\sqrt{2} \kappa H^T (i\tau_2) T (i\tau_2) H + h.c.
 \end{aligned} \tag{2.3}$$

where we have used the bi-doublet form for T as below²

$$T = \begin{pmatrix} T_0 & \frac{1}{\sqrt{2}} T_- \\ \frac{1}{\sqrt{2}} T_- & T_{--} \end{pmatrix}. \tag{2.4}$$

The following conditions must hold ($\lambda_{4t} = \lambda_4 + \lambda_t$)

$$\lambda_H, \lambda_L, \lambda_{4t} > 0, \quad \lambda_1 > -2\sqrt{\lambda_H \lambda_{4t}}, \quad \lambda_2 > -2\sqrt{\lambda_H \lambda_L}, \quad \lambda_3 > -2\sqrt{\lambda_L \lambda_{4t}}, \tag{2.5}$$

so as to ensure that the potential is bounded from below.

After SSB,

$$\langle H \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \phi_1 \rangle = \frac{v_L}{\sqrt{2}}, \quad \langle T \rangle = \frac{v_t}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.6}$$

²If a doublet, D , transforms under SU(2) as $D \rightarrow U_2 D$, then $T \rightarrow U_2 T U_2^T$.

and the fields can be expanded around their VEVs as

$$H = \begin{pmatrix} H_+ \\ \frac{v + \Re H_0 + i\Im H_0}{\sqrt{2}} \end{pmatrix}, \quad \phi_1 = \frac{v_L + \Re \Phi + i\Im \Phi}{\sqrt{2}}, \quad T = \begin{pmatrix} \frac{v_t + \Re T_0 + i\Im T_0}{\sqrt{2}} & \frac{1}{\sqrt{2}} T_- \\ \frac{1}{\sqrt{2}} T_- & T_{--} \end{pmatrix}. \quad (2.7)$$

And the minimal condition for the scalar potential become

$$v \left(-\mu_H^2 + \lambda_H v^2 + \frac{\lambda_1}{2} v_t^2 + \frac{\lambda_2}{2} v_L^2 - \kappa v_t \right) = 0, \quad (2.8)$$

$$v_L \left(-\mu_L^2 + \lambda_L v_L^2 + \frac{\lambda_2}{2} v^2 + \frac{\lambda_3}{2} v_t^2 \right) = 0, \quad (2.9)$$

$$v_t \left(-\mu_t^2 + \lambda_t v_t^2 + \frac{\lambda_1}{2} v^2 + \frac{\lambda_3}{2} v_L^2 + \lambda_4 v_t^2 \right) - \frac{1}{2} \kappa v^2 = 0. \quad (2.10)$$

Note that $\lambda_{5,6}$ do not come into play here. From the above equations, the tree-level mass squared for T_- and $\Re T_0$ are

$$M_{T_-}^2 = \frac{\kappa v^2}{2v_t} + \frac{\lambda_6}{4} v^2, \quad M_{\Re T_0}^2 = \frac{\kappa v^2}{2v_t} + \frac{1}{2} (\mu_t^2 + \lambda_t v_t^2 + \lambda_4 v_t^2). \quad (2.11)$$

From phenomenology we expect that $v_t \ll v$ (see section 8) and before scalar mixings considerations we have

$$M_{T_-} \simeq \left(\frac{\kappa}{v_t} \right)^{\frac{1}{2}} \frac{v}{\sqrt{2}}, \quad M_{\Re T_0} \simeq M_{T_-} \sqrt{1 + \frac{v_t \mu_t^2}{\kappa v^2}} \quad (2.12)$$

which are above a TeV if κ takes a phenomenologically interesting value around the electroweak scale. However, in general, κ is a free parameter.

The scalar potential after SSB gives a small mass splitting between T^- and T^{--} . The mass squared difference can be worked out to be

$$M_{T^{--}}^2 - M_{T^-}^2 = v^2 \frac{\lambda_6}{4} - v_t^2 \left(\lambda_4 - \frac{\lambda_5}{2} \right), \quad (2.13)$$

which is $\sim \mathcal{O}(v^2)$ provided λ_6 is not much smaller than $\lambda_{4,5}$. Therefore, it is a good approximation to assume that T^- and T^{--} are degenerate. However, we should keep in mind the mass splitting could be about the Fermi scale.

Similarly, ignoring the contribution from v_t , we have

$$-\mu_H^2 + \lambda_H v^2 + \frac{1}{2} \lambda_2 v_L^2 \simeq 0, \quad (2.14)$$

$$-\mu_L^2 + \lambda_L v_L^2 + \frac{1}{2} \lambda_2 v^2 \simeq 0. \quad (2.15)$$

Since we expect that $v_L \gg v$, i.e. lepton symmetry breaking to be above the Fermi scale, we obtain

$$v_L \simeq \sqrt{\frac{\mu_L^2}{\lambda_L}}, \quad M_{\phi_1} \simeq \sqrt{2} \mu_L, \quad v^2 \simeq \frac{1}{\lambda_H} \left(\mu_H^2 - \frac{\lambda_2}{4\lambda_L} M_{\phi_1}^2 \right). \quad (2.16)$$

$T_0^* T_- W_\mu^+$	:	$ig_2(p_- - \bar{p}_0)_\mu$	$T_+ T_{--} W_\mu^+$	:	$ig_2(p_{--} - p_+)_\mu$
$T_+ T_0 W_\mu^-$	:	$ig_2(p_0 - p_+)_\mu$	$T_{++} T_- W_\mu^-$	:	$ig_2(p_- - p_{++})_\mu$
$T_+ T_- P_\mu$	:	$-ie(p_- - p_+)_\mu$	$T_{++} T_{--} P_\mu$	:	$-2ie(p_{--} - p_{++})_\mu$
$T_+ T_- Z_\mu$	:	$i \frac{g_2}{c_W} (s_W^2)(p_- - p_+)_\mu$	$T_{++} T_{--} Z_\mu$	:	$i \frac{g_2}{c_W} (-1 + 2s_W^2)(p_{--} - p_{++})_\mu$
$T_0^* T_0 Z_\mu$	:	$i \frac{g_2}{c_W} (p_0 - \bar{p}_0)_\mu$	$\Re T_0 \Im T_0 Z_\mu$	:	$\frac{g_2}{c_W} (p_{\Im T_0} - p_{\Re T_0})_\mu$

Table 3. Couplings of gauge bosons to triplet fields.

Thus, it is also required to have $|\lambda_2| \ll \lambda_L$. As expected, there will be mixing among the three neutral scalars $\mathbf{H} = (\Re H_0, \Re T_0, \Re \Phi)$. They are related to the physical states $\mathbf{h} = (h_{SM}, t_0, \phi_0)$ via the usual unitary rotation given by

$$\mathbf{h} = \mathbf{U}_h \cdot \mathbf{H}. \quad (2.17)$$

Details of this transformation are not important for this study and we will not present them.

For completeness, we discuss the imaginary parts of the scalar fields. $\Im \Phi$ is the would-be Goldstone for the gauge boson Z_ℓ . Moreover, the would-be Goldstone bosons eaten by $W^\pm, Z$, the physical singly charged scalars, $h^\pm$, and the pseudoscalar, A_0 , can be identified as:

$$\begin{aligned} G_\pm &= \frac{vH_\pm - \sqrt{2}v_t T_\pm}{\sqrt{v^2 + 2v_t^2}} \simeq H_\pm, & G^0 &= \frac{v\Im H_0 - 2v_t \Im T_0}{\sqrt{v^2 + 4v_t^2}} \simeq \Im H_0, \\ h_\pm &= \frac{\sqrt{2}v_t H_\pm + vT_\pm}{\sqrt{v^2 + 2v_t^2}} \simeq T_\pm, & A_0 &= \frac{2v_t \Im H_0 + v\Im T_0}{\sqrt{v^2 + 4v_t^2}} \simeq \Im T_0. \end{aligned} \quad (2.18)$$

Since $v_t \ll v$ from the electroweak precision studies (see section 8), it is a good approximation to treat $T_\pm$ and $\Im T_0$ as the physical states. Being the only degree of freedom with two units of electric charge, $T_{\pm\pm}$ are the physical scalars.

Since the symmetry G forbids T from coupling to two SM fermions simultaneously, its gauge interactions become the most relevant for phenomenology. From the G -covariant derivative we obtain the Feynman rules for its triple couplings to gauge bosons, displayed in table 3, where P stands for the photon, and all the momenta are incoming.

3 Lepton masses for 1 generation

The physics of how the new leptons affect the SM charged leptons is best seen in the one family scenario. In the basis $\{e, E_1, E_2\}$, the Dirac mass matrix is

$$\mathcal{M}_C = \frac{v_L}{\sqrt{2}} \times \begin{pmatrix} h_1 \epsilon_v & 0 & f_1 \\ 0 & h_2 \epsilon_v & f_3 \\ f_2^* & f_4^* & h_3 \epsilon_v \end{pmatrix}, \quad (3.1)$$

where $\epsilon_v = \frac{v}{v_L} \ll 1$. In general the electron will mix with $E_{1,2}$ and the mixing depends on f_1 and f_2 . In that case, the charged-current interaction of the SM leptons could deviate

from the canonical SM ($V - A$) form due to their mixings with L_{2R} and E_{2L} . Moreover, the SM gauge couplings are flavor non-diagonal. Physically, this mixing must be very small and we can take the limiting case of $f_1 = f_2 = 0$ and eliminate the mixing of the electron with the new charged leptons.³ In general, we can write the physical mass eigenstates $\mathcal{E}'_\alpha = (e, E_-, E_+)$ where $\alpha = 1, 2, 3$ as

$$\mathcal{E}'_{L/R} = V_{L/R} \cdot \mathcal{E}_{L/R}, \quad (3.2)$$

where $V_{L/R}$ is the left-handed/right-handed unitary matrix that diagonalizes the charged lepton mass matrix so that $V_L^\dagger \cdot M_C \cdot V_R = \text{diag}\{m_e, M_-^E, M_+^E\}$. For the limiting case of $f_1 = f_2 = 0$ and $f = f_3 = f_4(1 + \delta)$ with $|\delta| \ll 1$, the mass eigenvalues can be worked out to be

$$m_e = h_1 \frac{\epsilon_v v_L}{\sqrt{2}}, \quad M_\pm^E \simeq \pm \frac{f_4 v_L}{\sqrt{2}} \left(1 + \frac{\delta}{2}\right) + \frac{\epsilon_v v_L}{\sqrt{2}} \frac{h_2 + h_3}{2}. \quad (3.3)$$

One can see that the leading mass splitting between E_+ and E_- , apart from the phase convention, comes from the SM Higgs Yukawa interaction, $h_{2,3}$, and to a very good approximation,

$$V_{L/R} \simeq V^B \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (3.4)$$

In the basis $\{\nu_L, N_{1L}, N_{2R}^c\}$, the neutrino mass matrix is

$$\mathcal{M}_N = \frac{v_L}{\sqrt{2}} \times \begin{pmatrix} 0 & y_1 \epsilon_t & f_1^* \\ y_1 \epsilon_t & 0 & f_3^* \\ f_1^* & f_3^* & y_2 \epsilon_t \end{pmatrix} \quad (3.5)$$

and $\epsilon_t = \frac{v_t}{v_L} < \epsilon_v$. Again, we consider the case that $f_1 \ll 1$ and $y_1 \sim y_2 = y$. The eigenvalues can be worked out to be around $(y\epsilon_t/f)^3$, $-1 + (y\epsilon_t/2f)$, and $1 + (y\epsilon_t/2f)$ in units of $f v_L/\sqrt{2}$. It is natural to identify the first term as the mass of the active neutrino. For $y v_t \sim 0.1 \text{ GeV}$ and $f v_L \sim 3 \text{ TeV}$, the resulting active neutrino mass is about $(y v_t)^3/(f v_L)^2 \sim 0.1 \text{ eV}$. From electroweak precision measurements we expect $v_t \lesssim \mathcal{O}(1) \text{ GeV}$. We see that the desired neutrino mass can be obtained without much tuning of the Yukawa couplings.

Notice that the neutrino mass matrix given in eq. (3.5) is of the inverse seesaw type [18, 19], and a review can be found in [20]. The novel feature here is that we do not require ad hoc addition of the SM singlet leptons. The additional leptons are dictated by anomaly cancelation and are SM doublets.

³Theoretically, these two Yukawa couplings can not be forbidden by any U(1) or Z_N charge assignment in this model. However, one can obtain the desired Yukawa hierarchy in the split-fermion model if the 5D wave-functions centers are in the order of $e_R, l_L, L_{1L}, L_{2R}; E_{1R}, E_{2L}$, where “,” and “;” mean large and small separations in between two adjacent wave functions, respectively, along the fifth dimension, see [33–35] for some other examples of achieving the hierarchical 4D Yukawa.

4 3-generation lepton masses

One can extend the above to the realistic 3-generation case. Without losing any generality, we can start with the basis that the Yukawa couplings for $\overline{N_{2R}}N_{1L}$ are diagonal. And we can go to the basis where the SM charged leptons are in their mass eigenstates by bi-unitary transformation among the e_R and e_L . Similarly, we have the freedom to start with diagonal (3×3) $\bar{E}_{1R}E_{2L}$ and $\bar{E}_{2R}E_{1L}$ sub-matrices.

4.1 Charged lepton mass matrix

For simplicity, let's consider that $f_{1,2} = 0$, $f_{3,4} \sim f$, and the heavy charged lepton are roughly degenerate. Then, in the basis $(\mathbf{e}, \mathbf{E}_1, \mathbf{E}_2)$ where each entry is a 3-vector in family space, the most general (9×9) mass matrix for charged leptons looks like

$$\mathcal{M}_C = \frac{v_L}{\sqrt{2}} \begin{pmatrix} h_1 \epsilon_v & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & h_2 \epsilon_v & f \cdot \mathbf{1} + \delta_1 \\ \mathbf{0} & f \cdot \mathbf{1} + \delta_2 & h_3 \epsilon_v \end{pmatrix} \quad (4.1)$$

where h_1 and $\delta_{1,2}$ are 3×3 diagonal matrices and $\mathbf{1}$ is the unit matrix. For convenience, $\delta_{1,2}$ which encodes the small splitting of the heavy charged leptons are separated out from the leading term. One can first perform a rotation among the heavy charged leptons by $U = \mathbf{V}_B$, which is a (9×9) generalization of eq. (3.4). Then the small perturbation can be separated from the leading order mass eigenvalues,

$$\begin{aligned} U^T \mathcal{M}_C U &= \mathcal{M}_C^{(0)} + \Delta \mathcal{M}_C \\ \mathcal{M}_C^{(0)} &= \begin{pmatrix} \text{diag}(m_e, m_\mu, m_\tau) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\frac{v_L}{\sqrt{2}} [f \cdot \mathbf{1} + \frac{1}{2}(\delta_1 + \delta_2)] & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{v_L}{\sqrt{2}} [f \cdot \mathbf{1} + \frac{1}{2}(\delta_1 + \delta_2)] \end{pmatrix}, \\ \Delta \mathcal{M}_C &= \frac{v_L}{2\sqrt{2}} \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (h_2 + h_3) & (h_2 - h_3 + \delta_1 - \delta_2) \\ \mathbf{0} & (h_2 - h_3 - \delta_1 + \delta_2) & (h_2 + h_3) \end{pmatrix}. \end{aligned} \quad (4.2)$$

One can further diagonalize the diagonal (3×3) block, $(h_2 + h_3)$, by a bi-unitary transformation such that $V_L^\dagger \cdot (h_2 + h_3) \cdot V_R = h_{\text{diag}}$. Or by using

$$U_R = U \cdot \text{diag}\{\mathbf{1}, V_R, V_R\}, \quad U_L = U \cdot \text{diag}\{\mathbf{1}, V_L, V_L\}, \quad (4.3)$$

so that

$$\begin{aligned} U_L^\dagger \mathcal{M}_C U_R &= \text{diag}\{\mathbf{M}_e, \mathbf{M}_-, \mathbf{M}_+\} + \Delta \mathcal{M}'_C \\ \mathbf{M}_e &= \text{diag}(m_e, m_\mu, m_\tau), \quad \mathbf{M}_\pm = \pm \frac{v_L}{\sqrt{2}} \left[f \cdot \mathbf{1} + \frac{1}{2}(\delta_1 + \delta_2 \pm h_{\text{diag}}) \right], \\ \Delta \mathcal{M}'_C &= \frac{v_L}{2\sqrt{2}} \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & V_L^\dagger \cdot (h_2 - h_3 + \delta_1 - \delta_2) \cdot V_R \\ \mathbf{0} & V_L^\dagger \cdot (h_2 - h_3 - \delta_1 + \delta_2) \cdot V_R & \mathbf{0} \end{pmatrix}. \end{aligned} \quad (4.4)$$

It is clear that the 6 heavy charged leptons will form 3 nearly degenerate pairs. And as in the 1-generation case, the mass splitting for each pair is mainly controlled by $h_{2,3}$. Moreover, they decouple from the SM charged leptons.

4.2 Neutral lepton mass matrix

Using the notation of the charged leptons and factor out the common mass, we write the general (9×9) neutrino mass matrix as

$$\mathcal{M}_N = \frac{fv_L}{\sqrt{2}} \widetilde{\mathcal{M}}_N, \quad \widetilde{\mathcal{M}}_N = \begin{pmatrix} \mathbf{0} & \epsilon_1 & \mathbf{0} \\ \epsilon_1^T & \mathbf{0} & \mathbf{1} + \delta \\ \mathbf{0} & \mathbf{1} + \delta & \epsilon_2 \end{pmatrix} \quad (4.5)$$

where ϵ_2 is a symmetric 3×3 matrix with elements $\{\epsilon_2\}_{ij} = \{\epsilon_2\}_{ji} \sim \mathcal{O}(y_2 v_t / v_L)$, ϵ_1 is a general 3×3 matrix with elements $\{\epsilon_1\}_{ij} \sim \mathcal{O}(y_1 v_t / v_L)$, and now δ is a 3×3 diagonal matrix $\delta = \text{diag}(0, \delta_1, \delta_2)$, $\delta_{1,2} \ll 1$, to accommodate the small non-degeneracy among the three heavy N s. First, the leading mass diagonalization can be made by the same rotation $\mathbf{V}_B$, similar to eq. (3.4), as in the charged lepton case. This results in a symmetric 3×3 matrix, $\delta_3 \equiv -\delta + \epsilon_2/2$, in the diagonal blocks as the perturbation. Assume there exists an orthogonal 3×3 transformation V , such that $V^T \cdot \delta_3 \cdot V = \text{diag}\{a_1, a_2, a_3\} \equiv \delta_4$, and $|a_{1,2,3}| \sim \mathcal{O}(\sqrt{\delta^2 + \epsilon_2^2}) \ll 1$. Then by using $U = \mathbf{V}_B \cdot \text{diag}\{\mathbf{1}, V, V\}$, the re-scaled neutrino mass matrix can be brought into the following form

$$(U)^T \cdot \widetilde{\mathcal{M}}_N \cdot U = \mathcal{M}^{(0)} + \Delta \mathcal{M}$$

$$\mathcal{M}^{(0)} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} + \delta_4 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} + \delta_4 \end{pmatrix}, \quad \Delta \mathcal{M} = \begin{pmatrix} \mathbf{0} & y & y \\ y^T & \mathbf{0} & -z \\ y^T & -z & w \end{pmatrix} \quad (4.6)$$

where $y = \frac{\epsilon_1 \cdot V}{\sqrt{2}} \sim \mathcal{O}(\epsilon_1)$, $z = \frac{V^T \cdot \epsilon_2 \cdot V}{2} \sim \mathcal{O}(\epsilon_2)$, and $w = 2V^T \cdot \delta \cdot V \sim \mathcal{O}(\delta)$. One can see that after this rotation, the leading mass eigenstates are nothing but the Cartesian basis. By the standard perturbation techniques, it is easy to see that the SM neutrinos will acquire nonzero masses at the second order perturbation. For example, at this order,

$$m_1 \left(\frac{fv_L}{\sqrt{2}} \right)^{-1} = \sum_{i=2}^9 \frac{(\Delta \mathcal{M}_{1i})^2}{0 - \mathcal{M}_{ii}^{(0)}} = - \sum_{j=1}^3 \left[\frac{(y_{1j})^2}{-1 + a_j} + \frac{(y_{1j})^2}{1 + a_j} \right] \simeq 2 \sum_{i=1}^3 (y_{1i})^2 a_i, \quad (4.7)$$

and it is indeed of the order of $\mathcal{O}(\epsilon_1^2 \epsilon_2)$ as in the 1-generation case.

The active neutrino masses can also be understood diagrammatically. By integrating out the heavy N , the corresponding Feynman diagram in the weak basis, displayed in figure 1, and can be seen to give the same conclusion. It also reveals that the low energy effective operator for active neutrino mass is not given by the Weinberg operator. If we assume a hierarchy that $v_L \gg v \gg v_t$, and T is the only beyond SM degree of freedom left below v_L , the active neutrino masses are generated by a dimension six operator $\mathcal{O}_6 = \frac{c}{(\Lambda_L)^2} (\bar{l}_L^c T^\dagger l_L) Tr(T^\dagger T)$ where c is a constant and Λ_L is the lepton number breaking scale

Figure 1. Diagrammatic representation of the ϵ^3 -suppression for the active neutrino masses. Superscripts denote family indices. Upper(green) arrows denote flow of lepton charge.

related to v_L . After T picks up a VEV, v_t , the neutrino mass is given by $m_\nu \simeq \frac{cv_t^3}{\Lambda_L^2}$. It is also clear that O_6 has a higher dimension than the Weinberg operator. Together with the fact that $v_t \ll v_*$, they allow the lepton breaking scale to be much lower than the usual type I seesaw mechanism.

Now, the upper-left (3×3) sub-matrix, denoted as $\mathbf{N}_\nu$, of U for active neutrinos is in general non-unitary, $\mathbf{N}_\nu \mathbf{N}_\nu^\dagger \neq 1$. This non-unitarity will result in various observable effects. However, one expects that the off-diagonal elements of $|\mathbf{N}_\nu \mathbf{N}_\nu^\dagger|$ are of the order of $\mathcal{O}(\epsilon_1^2) \sim 10^{-6} \times (v_t/\text{GeV})^2 \times (\text{TeV}/v_L)^2$, which is roughly below the current experimental limits, $\lesssim 10^{-5}$ [36, 37]. Therefore, we will leave the comprehensive study of these precision tests to future work.

5 Neutrino oscillations and data fitting

First, we provide a simple, realistic solution which can accommodate the neutrino data. Then we move on to the more general numerical survey where the solutions will be fed into the later study of lepton flavor changing processes.

To simplify the discussion, we assume that the heavy N 's are degenerate ($\delta = 0$), $y_2 \propto \mathbf{1}$, and all the Yukawa couplings in y_1 are of the same order and there is no hierarchy among them. The (9×9) mass matrix looks like

$$\mathcal{M}_N = v_L \begin{pmatrix} \mathbf{0} & \epsilon y_1 & \mathbf{0} \\ \epsilon y_1^T & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \epsilon y_2 \end{pmatrix}, \quad (5.1)$$

where $\epsilon \sim \mathcal{O}(v_t/v_L)$ is an unknown overall constant which controls the amplitude of perturbation and the elements of y_1 are of $\sim \mathcal{O}(1)$. As discussed previously, in the leading approximation, the (3×3) active neutrino mass matrix reads

$$\mathcal{M}_{ij}^\nu \sim \epsilon^3 v_L \{y_1\}_{i\alpha} \{y_2\}_{\alpha\beta} \{y_1\}_{j\beta} \sim y_2 \epsilon^3 v_L \{y_1\}_{i\alpha} \{y_1\}_{j\alpha}. \quad (5.2)$$

If y_1 is highly democratic, namely,

$$y_1 \propto \mathbf{I}_c \equiv \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (5.3)$$

the resulting active neutrino mass matrix also has the pattern $\mathcal{M}^\nu \propto \mathbf{I}_c$ which is of rank one and it has two zero eigenvalues. It naturally leads to the normal hierarchical neutrino masses. Taking into account the data, the realistic mass matrix for normal hierarchy(NH) instead takes the form

$$\mathcal{M}^\nu \sim \begin{pmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 1 & 1 \\ 0.1 & 1 & 1 \end{pmatrix} \times (0.03) \text{ eV} \quad (5.4)$$

if $m_1 \simeq 0$, and, to simplify the discussion we set $\delta_{CP} = 0$. A simple solution to arrive such pattern is

$$y_1 \sim \frac{y}{\sqrt{3}} \begin{pmatrix} -0.3 & 0.3 & 0.3 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad y_2 \sim y \mathbf{1} \quad (5.5)$$

which has the apparent $\mu - \tau$ symmetry. This can be realized in the extra-dimensional models by arranging the amount of overlap in higher dimensional fermion wavefunctions, see for example [33–35].

On the other hand, a more subtle construction of y_1 is required to accommodate the inverted hierarchy(IH) case. For example, if $m_3 \simeq 0, \delta_{CP} = 0$, the following realistic neutrino masses matrix

$$\mathcal{M}^\nu \sim \begin{pmatrix} 1.6 & -0.2 & -0.2 \\ -0.2 & 0.9 & -0.8 \\ -0.2 & -0.8 & 0.8 \end{pmatrix} \times (0.03) \text{ eV} \quad (5.6)$$

can be generated by

$$y_1 \sim y \begin{pmatrix} -0.7 & -0.7 & -0.7 \\ 0.3 & 0.6 & -0.7 \\ -0.1 & -0.4 & 0.8 \end{pmatrix}, \quad y_2 \sim y \mathbf{1}. \quad (5.7)$$

For both NH and IH cases, $y^3 v_t^3 / v_L^2 \sim 0.03 \text{ eV}$. Taking $v_t = 1 \text{ GeV}$ and $v_L = 1(5) \text{ TeV}$, we have $y \sim 0.03(0.09)$. This simple solution with $y_2 \sim y \cdot \mathbf{1}$ gives us a rough idea of the Yukawa coupling strengths.

For the realistic data fitting, we perform a comprehensive numerical scan with the working assumption that $|(y_2)_{ij}| \simeq y_2$ and that the heavy N 's are nearly degenerate. These assumption can be relaxed giving rise to more free parameters to fit the data. Moreover, the Yukawa couplings are taken to be complex in the numerical study to accommodate the nonzero CP phase, δ_{CP} which current data give a hint of. However, it is clear that the resulting neutrino mass is about $m_\nu \sim (y_1^2 y_2 v_t^3 / M_N^2)$. We adopt the following 3σ ranges from [38] for the neutrino oscillation parameters. For NH,

$$31.42^\circ < \theta_{12} < 36.05^\circ, \quad 40.3^\circ < \theta_{23} < 51.5^\circ, \quad 8.09^\circ < \theta_{13} < 8.98^\circ, \quad 144^\circ < \delta_{CP} < 374^\circ, \\ \Delta m_{21}^2 = (6.8 - 8.02) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = (2.399 - 2.593) \times 10^{-3} \text{ eV}^2. \quad (5.8)$$

Figure 2. Feynman diagrams for $\mu \rightarrow e\gamma$ transition, the photon can attach to any charged line.

As for the IH case, the corresponding 3σ ranges are:

$$31.43^\circ < \theta_{12} < 36.06^\circ, \quad 41.3^\circ < \theta_{23} < 51.7^\circ, \quad 8.14^\circ < \theta_{13} < 9.01^\circ, \quad 192^\circ < \delta_{CP} < 354^\circ, \\ \Delta m_{21}^2 = (6.8 - 8.02) \times 10^{-5} \text{ eV}^2, \quad \Delta m_{31}^2 = -(2.369 - 2.562) \times 10^{-3} \text{ eV}^2. \quad (5.9)$$

The lightest neutrino masses, m_{lightest} , for both NH and IH are allowed to vary in the range between 10^{-4}eV and 0.2eV so that the cosmological bound, $\sum_j m_j < 0.57\text{eV}$ at 95% C.L.(from CMB spectrum and gravitational lensing data only) [39],⁴ can be met. Once m_{lightest} is fixed, $m_{1,2,3}$ can be determined from the measured mass squared differences. Then the effective active neutrino mass matrix can be obtained by

$$\mathcal{M}_\nu = U_{PMNS}^* \cdot \text{diag}(m_1, m_2, m_3) \cdot U_{PMNS}^\dagger. \quad (5.10)$$

In the standard parametrization, the rotation matrix is given by⁵

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix} \quad (5.11)$$

where the shorthand $s_{12} \equiv \sin\theta_{12}$ and the like are used. Each element of the y_2 Yukawa matrix is a random number in between 0.7 and 1.3 times an overall unknown factor y_2 with either sign. And we require that the ratio of the largest to the smallest absolute values in y_1 to be smaller than 10. About 10^5 such solutions are prepared for both NH and IH cases. The realistic Yukawa coupling configurations can be used for predicting the lepton flavor violating processes. The results will be displayed in the next section.

6 $\mu \rightarrow e\gamma$, and a_μ

With this rich exotic leptons introduced for anomaly cancelations it is important to examine the constraints from charged lepton flavor violation (CLFV) searches. The $\mu \rightarrow e\gamma$ process can be mediated by triplet running in the loop, see figure 2. In the lepton mass basis, with

⁴This upper limit has been improved to $\sum_j m_j < 0.44\text{eV}$ (or $m_{\text{lightest}} \lesssim 0.15\text{eV}$) at 95% C.L. [40] recently.

⁵In general $U_{PMNS} = U_{c.l.}^\dagger U$ where $U_{c.l.}$ is the unitary matrix that diagonalizes the SM charged leptons mass matrix and U is the neutrino rotation matrix. In the limit that the $\mathbf{E}$ decouples from the $\mathbf{e}$ we can take $U_{c.l.} = \mathbf{1}$.

Figure 3. The loop function $I_1(x)$.

the $\mathbf{V}_B$ rotation, the triplet coupling can be approximated as

$$\begin{aligned} \frac{(y_1)_{ij}}{\sqrt{2}} \left\{ \bar{T}_0 \bar{\nu}^c_i (N_{j+} + N_{j-}) + T_{++} \bar{e}_i^c (E_{j+} + E_{j-}) \right. \\ \left. + \frac{1}{\sqrt{2}} T_+ [\bar{\nu}^c_i (E_{j+} + E_{j-}) + \bar{e}_i^c (N_{j+} + N_{j-})] \right\} + h.c. \end{aligned} \quad (6.1)$$

where $i, j = 1, 2, 3$ are the generation indices, and $\pm$ denote the different mass eigenstates within each generation.

The 1-loop contributions can be calculated to be

$$\Delta a_\mu = - \sum_{i=1}^6 \frac{|(y_1)_{\mu i}|^2 m_\mu^2}{32\pi^2} \left[\frac{I_1((\tau_i^{--})^{-1})}{M_{E_i}^2} + \frac{2I_1(\tau_i^{--})}{M_{T_{--}}^2} + \frac{1}{2} \frac{I_1(\tau_i^N)}{M_{T_-}^2} \right] \quad (6.2)$$

where $\tau_i^{--} \equiv \frac{M_{E_i}^2}{M_{T_{--}}^2}$, $\tau_i^N \equiv \frac{M_{N_i}^2}{M_{T_-}^2}$, and

$$I_1(x) = \int_0^1 dz \frac{z(1-z)^2}{1-z+xz} = \frac{1}{6(1-x)^4} [1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x] . \quad (6.3)$$

See figure 3 for the plot of this function. When $x \ll 1$ and $x \sim 1$, the loop function can be expanded as

$$I_1(x) \simeq \frac{1}{6} - \frac{x}{3} - \left(\frac{11}{6} + \ln x \right) x^2 + \mathcal{O}(x^3), \quad (6.4)$$

$$\simeq \frac{1}{12} - \frac{x-1}{20} + \frac{(x-1)^2}{30} + \mathcal{O}((x-1)^3). \quad (6.5)$$

The first term in the square bracket of eq. (6.2) is the contribution where the photon attaches to the heavy charged lepton. The second and third terms are the contributions where the photon attaches to the T_{--} and T_- , respectively. Because of the electric charge, the T_{--} contribution has an extra factor 2. Note also the one half factor associated with the T_- contribution which is due to the extra $1/\sqrt{2}$ factor in the singly charged triplet-fermion vertex coupling. Moreover, assuming that $M_E \sim M_N \sim M$ (so that all $I_1 \sim 1/12$)

the $(g-2)_\mu$ can be related to the neutrino mass and eliminating the y_1 dependence,

$$\Delta a_\mu \sim -\frac{3m_\mu^2 m_\nu}{16\pi^2 y_2 v_t^3} \left[I_1 ((\tau_i^{--})^{-1}) + 2I_1 (\tau_i^{--}) \frac{M^2}{M_T^2} + \frac{1}{2} I_1 (\tau_i^N) \frac{M^2}{M_T^2} \right] \sim -10^{-15} \frac{(\text{GeV})^3}{y_2 v_t^3} \quad (6.6)$$

which is negligibly small.

Similar calculation can be carried out for the $\mu \rightarrow e\gamma$ dipole transition amplitude.

$$\begin{aligned} \mathcal{M}_{\mu \rightarrow e\gamma}^{\mu} &= A_{\mu e} \bar{e}(p_2) (-i\sigma^{\mu q}) \hat{R} \mu(p_1), \\ A_{\mu e} &= \sum_{i=1}^6 \frac{e(y_1)_{\mu i} (y_1)_{ei}^* m_\mu}{64\pi^2} \left[\frac{I_1 ((\tau_i^{--})^{-1})}{M_{E_i}^2} + \frac{2I_1 (\tau_i^{--})}{M_{T-}^2} + \frac{1}{2} \frac{I_1 (\tau_i^N)}{M_{T-}^2} \right], \end{aligned} \quad (6.7)$$

where $q^\mu \equiv (p_2 - p_1)^\mu$ is the photon 4-momentum, and $\hat{R} = (1 + \gamma^5)/2$. Then, the branching ratio is [41]

$$Br(\mu \rightarrow e\gamma) = \frac{12\pi^2 A_{\mu e}^2}{G_F^2 m_\mu^2} \sim \frac{27\alpha (y_1)^4 (3.5I_1)^2}{64\pi G_F^2 M^4} \sim 10^{-13} \left(\frac{\text{TeV}}{M} \right)^4 \left(\frac{y_1}{0.01} \right)^4. \quad (6.8)$$

Or, assuming that $M_E \sim M_N \sim M$, the LFV process can be related to the neutrino mass,

$$A_{\mu e} \sim \frac{3em_\mu m_\nu}{32\pi^2 y_2 v_t^3} \left[I_1 ((\tau_i^{--})^{-1}) + 2I_1 (\tau_i^{--}) \frac{M^2}{M_T^2} + \frac{1}{2} I_1 (\tau_i^N) \frac{M^2}{M_T^2} \right]. \quad (6.9)$$

Note that the mass squared of heavy leptons in numerator and denominator cancel out, and the branching ratios is not very sensitive to the masses of heavy degree of freedom.

Comparing to the most recent bound, $Br(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$ at 90% C.L. [42], our numerical results are shown in figure 4. As can be seen from the plot, it is easier to find the solutions for larger m_{lightest} in the IH case. For $y_2 v_t^3 = 1(\text{GeV})^3$, the $\mu \rightarrow e\gamma$ branching ratio is right below the current experimental limit for $m_{\text{lightest}} \lesssim 10^{-2} \text{eV}$. Note that the branching ratios have lower bounds, around $\sim 10^{-16}/10^{-15}$ for NH/IH case with $y_2 v_t^3 = 1(\text{GeV})^3$. Therefore, for this model to admit a realistic solution which accommodates simultaneously the neutrino oscillation data and the current $\mu \rightarrow e\gamma$ bound, the predicted lower bounds must stay below the experimental limit. It is required that

$$\frac{1(\text{GeV})^3}{y_2 v_t^3} < 64.807(20.493) \quad (6.10)$$

for NH(IH). Thus, we arrive an interesting lower bound on the triplet VEV that

$$v_t > \frac{0.249(0.365)}{(y_2)^{1/3}} \text{ GeV} > 0.107(0.157) \text{ GeV} \quad (6.11)$$

for the NH(IH) case, where the ultima bound is obtained by taking the strong coupling limit $y_2 = 4\pi$.

From eq. (2.11), this lower bound implies that the triplet mass is roughly below $\lesssim 8 \text{TeV}$ if $\kappa \sim v$.

Figure 4. The log-log plot of $Br(\mu \rightarrow e\gamma)$ vs. $m_{\text{lightest}}(\text{eV})$ in our model. NH/IH is on the left/right panel. The red dash line indicates the current experimental bound [42]. The radius of each dot is proportional to the number of found solutions in the corresponding $Br - m_{\text{lightest}}$ cell. We take $y_2 v_t^3 = 1(\text{GeV})^3$ and $M_N = M_E = M_T$.

Since in our model the triplet does not carry lepton number, there is no tree-level contribution to $\mu \rightarrow 3e$ and the similar τ decays. The dipole induced $Br(\mu \rightarrow 3e)$ will be small comparing to $\mu \rightarrow e\gamma$. The ratio [41] is given by

$$\frac{Br(\mu \rightarrow 3e)}{Br(\mu \rightarrow e\gamma)} = \frac{2\alpha}{3\pi} \left(\ln \frac{m_\mu}{m_e} - \frac{11}{8} \right) \simeq 0.7 \times 10^{-2} \quad (6.12)$$

which makes $Br(\mu \rightarrow 3e) < 3 \times 10^{-15}$ in this model. Similarly, the branching ratios of $\tau \rightarrow l\gamma$ ($l = e, \mu$) are

$$Br(\tau \rightarrow l\gamma) \simeq \frac{12\pi^2 A_{\tau l}^2}{G_F^2 m_\tau^2} \times Br(\tau \rightarrow e\bar{\nu}_e \nu_\tau) \quad (6.13)$$

and we adopt the measured $Br(\tau \rightarrow e\bar{\nu}_e \nu_\tau) = 17.82\%$ [43]. The predicted branching ratios of $\tau \rightarrow l\gamma$ in our model are displayed in figure 5 which are much smaller than the current experimental bound; $Br(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$ and $Br(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8}$ at 90%C.L. [43]. Note that in the IH cases the two have same statistics which is due to the complex conjugated pair solutions to the y_1 Yukawa for a given m_{lightest} and U_{PMNS} . As pointed out in [44], the double ratios, for example, $Br(\mu \rightarrow e\gamma)/Br(\tau \rightarrow e\gamma)$, are independent of the unknown parameters y_2, v_t and the masses of the heavy degrees of freedom. They are complementary handles to the long baseline experiments for determining the type of neutrino mass hierarchy. Unfortunately, we have not found any notable statistical difference between the double ratios of NH and IH in this model.

7 Triplets at colliders

The phenomenology of the Z_ℓ and the charged heavy leptons are the same as in (I), and we shall not repeat them here. The triplets are the new players and we will discuss their signatures at the LHC below. We start with a list of their dominant decay modes.

Figure 5. $Br(\tau \rightarrow e\gamma)$ and $Br(\tau \rightarrow \mu\gamma)$ vs. m_{lightest} in our model. We take $y_2 v_t^3 = 1(\text{GeV})^3$ and $M_N = M_E = M_T$.

7.1 Decays of the triplet

Due to the gauge couplings and SSB, the triplet scalar can decay into (a) two SM gauge bosons collectively called V (b) a lighter triplet partner plus a V , e.g. $T_{--} \rightarrow T_- W^+$, and (c) two light triplets, e.g. $T_{--} \rightarrow 2T_-$. The later two require huge mass splitting or the rates are suppressed by v_t , thus can be ignored here.⁶ Therefore, $T \rightarrow V_1 V_2$ ($V_{1,2} = W^\pm, Z$) are the dominant decays since T does not couple to two SM fermions simultaneously in the weak basis. This is very different from the cases of triplet with $l = 2$ as discussed, for example, in [45]. Parameterizing the vertex $TV_1^\mu V_2^\nu$ Feynman rule as $i\kappa_{V_1, V_2} g^{\mu\nu}$, it's

⁶For example, $\Gamma(T \rightarrow T_1 W) = G_F M^3 \lambda_{cm}^3(x_1, x_W)/(2\sqrt{2}\pi)$, where $x_1 = (M_1/M)^2$ and $x_W = (M_W/M)^2$. However, there is no allowed phase space if the mass squared difference between T and T_1 is at most v^2 .

Vertex	$\Re T_0 W^+ W^-$	$\Re T_0 Z Z$	$T_{--} W^+ W^+$	$T_- W^+ Z$	$T_- W^+ P$
κ_{VV}	$i g_2^2 v_t$	$2i \frac{g_2^2}{c_W^2} v_t$	$i \sqrt{2} g_2^2 v_t$	$i \frac{g_2^2}{\sqrt{2} c_W} (1 + s_W^2) v_t$	$-i \frac{g_2^2 c}{\sqrt{2}} v_t$

Table 4. Feynman rules for TVV vertices. The $g^{\mu\nu}$ factors are omitted.

straightforward to calculate the following decay widths:

$$\Gamma(T \rightarrow V_1 V_2) = \frac{|\kappa_{V_1, V_2}|^2}{16\pi M_T} \lambda_{cm}(x_1^2, x_2^2) \left[2 + \left(\frac{1 - x_1^2 - x_2^2}{2x_1 x_2} \right)^2 \right], \quad (7.1)$$

$$\Gamma(T \rightarrow V_1 V_1) = \frac{|\kappa_{V_1, V_1}|^2}{32\pi M_T} \sqrt{1 - 4x_1^2} \left[2 + \left(\frac{1 - 2x_1^2}{2x_1^2} \right)^2 \right], \quad (7.2)$$

$$\Gamma(T \rightarrow V_1 \gamma) = \frac{3|\kappa_{V_1, \gamma}|^2}{16\pi M_T} (1 - x_1^2), \quad (7.3)$$

where $x_i \equiv M_{V_i}/M_T$ and $\lambda_{cm}(y, z) \equiv \sqrt{1 + y^2 + z^2 - 2y - 2z - 2yz}$. The couplings are listed in table 4. The typical decay widths for charged triplets are narrow, around $\mathcal{O}(10^{-2})$ MeV, for $v_t \sim 1$ GeV and $M_T \sim 1$ TeV. However, the charged triplet still decays promptly once produced. Moreover, the signal of triplet will be 4 fermion final state from the decay of two gauge bosons or 2 fermion plus a high energy photon.

On the other hand, if there is mixing between $\Re T_0$ and the Higgs boson, t_0 can decay into fermion pairs. The two body decay width of t_0 is given by

$$\Gamma(t_0 \rightarrow f \bar{f}) = \frac{|U_h^{12}|^2 G_F M_T}{4\pi\sqrt{2}} \sum_f N_c m_f^2 \left(1 - \frac{4m_f^2}{M_T^2} \right)^{3/2} \quad (7.4)$$

where U_h is given in eq. (2.17). This will be dominated by the $t\bar{t}$ final state if $M_T \gg M_t = 174$ GeV. LHC-1 gave a bound on the SM signal strength that $\mu = 1.09 \pm 0.11$ [46], which implies that $|U_h^{12}|^2 < 0.13$ at 2σ level. For $M_T = 0.5(1.0)$ TeV, the 2-body decay width has an upper bound $\Gamma(t_0 \rightarrow t\bar{t}) < 8(36)$ MeV, and $\Gamma(t_0 \rightarrow b\bar{b}) < 0.57(1.1)$ MeV. The mixing with the SM Higgs will also provide additional 2 gauge bosons decay widths,

$$\begin{aligned} \Gamma(t_0 \rightarrow W^+ W^-) &= \frac{|U_h^{12}|^2 G_F M_T^3}{32\pi\sqrt{2}} \sqrt{1 - x_W} (4 - 4x_W + 3x_W^2), \\ \Gamma(t_0 \rightarrow Z Z) &= \frac{|U_h^{12}|^2 G_F M_T^3}{64\pi\sqrt{2}} \sqrt{1 - x_Z} (4 - 4x_Z + 3x_Z^2), \end{aligned} \quad (7.5)$$

where $x_V \equiv 4M_V^2/M_T^2$. For $M_T = 0.5(1.0)$ TeV, $\Gamma(t_0 \rightarrow W^+ W^-) \simeq 2\Gamma(t_0 \rightarrow Z Z) < 5.3(42.6)$ GeV.

Finally, we discuss the $t_0 \rightarrow 2h_{SM}$ decay. Since $|U_h^{12}| \ll 1$, the relevant Lagrangian is roughly

$$\simeq \left[3\lambda_H v U_h^{12} + \frac{1}{2}(\lambda_1 v_t + \kappa) \right] h_{SM}^2 t_0 \quad (7.6)$$

and the κ term dominates. We have

$$\Gamma(t_0 \rightarrow 2h_{SM}) \simeq \frac{\kappa^2}{32\pi M_T} \sqrt{1 - x_H} = 0.172(0.096) \times \left(\frac{\kappa}{100 \text{ GeV}} \right)^2 \text{ GeV} \quad (7.7)$$

M_T (TeV)	$T_{++}T_{--}$	T_+T_-	T_+T_{--}	$T_{++}T_-$
0.5	1.77	0.179	0.872	2.40
0.7	0.345	3.50×10^{-2}	0.157	0.496
1.0	4.62×10^{-2}	0.47×10^{-2}	1.93×10^{-2}	7.01×10^{-2}

Table 5. Charged Triplet bosons pair production cross sections(in fb) at the LHC14. Here we have neglected effects from $\Re H_0$ and $\Re T_0$ since it is small and give subleading contributions.

for $M_T = 0.5(1.0)\text{TeV}$. With non-zero mixing with Higgs, the two gauge bosons are still the dominant decay channel of t_0 . Moreover, for $M_{t_0} \gg M_Z$, the decay branching ratios of $Br(t_0 \rightarrow ZZ) \simeq 1/3$ and $Br(t_0 \rightarrow W^+W^-) \simeq 2/3$ are quite robust.

7.2 Triplet production at hadron colliders

As seen in the previous section, the production and decay of t_0 is very sensitive to its mixing with the SM Higgs. We will start with the case that the mixing between $h, \Re T_0$ is negligible and focus on the production of the charged triplet at the collider. The pair production at the LHC is mainly by the Drell-Yan processes through the TTV vertices. The gauge boson associated production cross section, $\sigma(pp \rightarrow VT)$, is proportional to v_t^2 and negligible. If ignoring the mixing and mass differences, $\sigma(pp \rightarrow T_+T_{--}) = \sigma(pp \rightarrow T_0^*T_-)$ and $\sigma(pp \rightarrow T_-T_{++}) = \sigma(pp \rightarrow T_0T_+)$ for they have the same couplings and mediated by the s-channel W -exchange diagrams. The cross sections at LHC14 for some typical triplet masses, listed in table 5, are evaluated by the program CalcHep [47] with the CTEQ611 [48] PDF.

Note that $pp \rightarrow t\bar{t}W$ will be dominant SM background for $T_{--}T_{++}$. After applying proper cuts, a doubly charged of mass up to about 0.7 TeV, and it decays mainly into di-boson, can be probed at LHC14 with an integrated luminosity of $300 fb^{-1}$ [45]. However, we defer a full study of the signal and proper treatment of the background to a future study.

In contrast, the real part of the neutral triplet⁷ can be singly produced via gluon fusion through the mixing (U_h^{12}). Our estimates of the production cross sections at the LHC and future hadron colliders are given in table 6. The SM backgrounds are estimated by evaluating the production cross section with the di-boson invariant mass in the $M_T \pm 50\text{GeV}$ range. Derived from the numbers listed in table 6, the 5σ limits on the 2-dimensional $|U_h^{12}|^2$ and effective luminosity plane is shown in figure 6. The limit is determined by

$$\frac{\text{Signal}}{\sqrt{\text{Background}}} = \frac{\sqrt{\mathcal{L}_0 \times \xi_{VV}} \times \sigma_S \times Br(t_0 \rightarrow VV)}{\sqrt{\sigma_{BG}}} = 5 \quad (7.8)$$

where ξ_{VV} is the efficiency of detection of VV final states, and $\mathcal{L}_0$ is the integrated luminosity. It can be seen that t_0 with a mass of 1TeV and $|U_h^{12}|^2 = 0.05$ could be directly studied at the LHC14 with $\sim 1ab^{-1}$ effective luminosity.

⁷The single production of A_0 can be ignored for it has a v_t/v suppressing mixing with the Goldstone G_0 , otherwise it couples only to one SM lepton doublet and one L_1 , eq. (2.2).

$\sqrt{s}(\text{TeV})$	$M_{t_0}(\text{TeV})$	$\sigma(pp \rightarrow t_0)$	$\sigma(pp \rightarrow W^+W^-)_{SM}$	$\sigma(pp \rightarrow ZZ)_{SM}$
14	0.5	2.88×10^2	2.56×10^3	4.0×10^2
14	1.0	7.42	1.49×10^2	2.35×10^1
14	2.0	6.02×10^{-2}	4.93	0.879
30	0.5	1.68×10^3	7.27×10^3	1.17×10^3
30	1.0	6.72×10^1	5.49×10^2	9.45×10^1
30	2.0	1.16	3.56×10^1	8.30
100	0.5	1.63×10^4	3.14×10^4	5.27×10^3
100	1.0	9.73×10^2	3.13×10^3	4.26×10^2
100	2.0	3.03×10^1	4.09×10^2	1.40×10^2

Table 6. Gluon fusion neutral Triplet boson production cross sections(in fb) at the LHC and beyond. Here we assume that $|U_h^{12}|^2 = 0.1$.

Figure 6. The 5σ limits of detecting a Triplet t_0 at the LHC. The curves are for t_0 decays into W^+W^- , and dashed ones are for $t_0 \rightarrow ZZ$. The color codes, (blue, black, orange), are for $M_T = (0.5, 1.0, 2.0)$ TeV, respectively. The horizontal dashed red line is the LHC-1 upper limit on $|U_h^{12}|^2$.

7.3 Triplet pair productions at the e^+e^- machine

The triplet pair productions at the e^+e^- machine are mediated by the s-channel photon and Z diagrams. Ignoring the mixing between $\Re T_0$ and h , the cross sections can be easily

Figure 7. The pair production cross sections (in fb) for four different masses, $M_T = 0.5, 0.7, 1.0, 1.4$ TeV, vs. $\sqrt{s}$ at the e^+e^- collider.

calculated to be:

$$\sigma(e^+e^- \rightarrow t_0 A) = \frac{\pi\alpha^2}{3s} \frac{1}{s_W^2 c_W^2} \left(\frac{1}{1 - \frac{M_Z^2}{s}} \right)^2 \left(1 - \frac{4M_T^2}{s} \right)^{3/2}, \quad (7.9)$$

$$\sigma(e^+e^- \rightarrow T_+ T_-) = \frac{\pi\alpha^2}{3s} \left(1 - \frac{\tan \theta_W}{1 - \frac{M_Z^2}{s}} \right)^2 \left(1 - \frac{4M_T^2}{s} \right)^{3/2}, \quad (7.10)$$

$$\sigma(e^+e^- \rightarrow T_{++} T_{--}) = \frac{4\pi\alpha^2}{3s} \left(1 + \frac{\cot 2\theta_W}{1 - \frac{M_Z^2}{s}} \right)^2 \left(1 - \frac{4M_T^2}{s} \right)^{3/2}. \quad (7.11)$$

The cross sections are displayed in figure 7. Note that the interference between photon and Z contributions is destructive/constructive for $T_+ T_- / T_{--} T_{++}$ production cross section. Because the electric charge squared, $T_{\pm\pm}$ has the largest production cross section. We use CalcHEP to estimate the SM backgrounds and find that they are about three orders of magnitude smaller than the signals, and thus negligible.

8 EW precision, ΔS and ΔT from the exotic fermions and scalars

8.1 Tree-level ρ -parameter

Since T gets a VEV, v_t , the tree-level ρ -parameter is less than unit:

$$1 + \alpha \Delta T_{\text{tree}} = \rho_{\text{tree}} = \frac{v^2 + 2v_t^2}{v^2 + 4v_t^2} = 1 - \frac{2v_t^2}{v^2 + 4v_t^2}. \quad (8.1)$$

Therefore, the loop-induced $\Delta T_{\text{loop}} (> 0)$ can be compensated by $\Delta T_{\text{tree}} (< 0)$. For $\Delta T = 0.08 \pm 0.12$ [43], the 2σ range is

$$-0.16 < \Delta T = \Delta T_{\text{tree}} + \Delta T_{\text{loop}} < 0.32. \quad (8.2)$$

The above only uses tree level contributions from the SM triplet implies that $v_t < 5.94$ GeV. Combining with neutrino mass generation and the $\mu \rightarrow e\gamma$ limit, we obtain the following interesting limit

$$0.107 < v_t < 5.94 \text{ GeV}. \quad (8.3)$$

8.2 Loop corrections

Since anomaly cancelation mandates the addition of extra leptons, it is important to know how quantum corrections to ΔT and ΔS from these new states will alter the above bound on v_T .

For each generation, the contributions from exotic leptons are [1]

$$\Delta T_{F_i} = \frac{1}{16\pi s_W^2 M_W^2} \left(M_{N_i}^2 + M_{E_i}^2 - 2 \frac{M_{N_i}^2 M_{E_i}^2}{M_{N_i}^2 - M_{E_i}^2} \ln \frac{M_{N_i}^2}{M_{E_i}^2} \right), \quad (8.4)$$

$$\Delta S_{F_i} = \frac{1}{6\pi} \left(1 + \ln \frac{M_{N_i}^2}{M_{E_i}^2} \right), \quad (8.5)$$

where $i = 1, 2$. From the triplet $T = (T_0, T_-, T_{--})^T$, they are

$$\Delta T_T = \frac{1}{8\pi s_W^2 M_W^2} \left(M_{T_0}^2 + M_{T_-}^2 - 2 \frac{M_{T_0}^2 M_{T_-}^2}{M_{T_0}^2 - M_{T_-}^2} \ln \frac{M_{T_0}^2}{M_{T_-}^2} \right. \\ \left. M_{T_-}^2 + M_{T_{--}}^2 - 2 \frac{M_{T_-}^2 M_{T_{--}}^2}{M_{T_-}^2 - M_{T_{--}}^2} \ln \frac{M_{T_-}^2}{M_{T_{--}}^2} \right), \quad (8.6)$$

$$\Delta S_T = \frac{1}{3\pi} \ln \frac{M_{T_0}^2}{M_{T_{--}}^2}. \quad (8.7)$$

To simplify the discussion, we assume that all the exotic charged(neutral) leptons have the same mass $M_E(M_N)$, T_- and T_{--} are degenerate, and implement the current limit $\Delta S = 0.05 \pm 0.10$ [43]. To proceed, we assume that the mass squared differences, $|M_E^2 - M_N^2|, |M_{T_0}^2 - M_{T_-}^2|$, are at most v^2 (see section 2). It is easy to generalize to other values. We define two variables, $x_E \equiv (M_N/M_E)^2$ and $x_T = (M_{T_0}/M_{T_-})^2$, for the discussion. In terms of these variables

$$\Delta T = \frac{1}{8\pi s_W^2 M_W^2} \left[3M_E^2 I_2(x_E) + M_{T_-}^2 I_2(x_T) - 16\pi^2 v_t^2 \right], \quad (8.8)$$

$$\Delta S = \frac{1}{\pi} \left(1 + \ln x_E + \frac{1}{3} \ln x_T \right), \quad (8.9)$$

where

$$I_2(x) = 1 + x - \frac{2x \ln x}{x - 1}. \quad (8.10)$$

The function $I_2(x = 1) = 0$ and it is monotonically increasing when x goes to zero, $I_2(0) = 1$. The 2σ range of ΔS , $-0.15 < \Delta S < 0.25$, amounts to

$$0.3317 < (x_E^3 x_T)^{1/4} < 0.8513. \quad (8.11)$$

Apparently, $x < 1$ is preferred, and one needs either $M_N < M_E$, $M_{T_0} < M_{T_-}$ or both to satisfy the requirement from ΔS . Since we assume the mass squared difference is at most v^2 . In the cases of largest mass squared splitting, M_E, M_{T_-} can be related to x 's as

$$M_E^2 = \frac{v^2}{1 - x_E}, \quad M_{T_-}^2 = \frac{v^2}{1 - x_T}. \quad (8.12)$$

We consider two simple cases: $x_E = 1$, and $x_T = 1$. For $x_T = 1$, the direct search of charged Higgs sets a bound $M_{T_-} (= M_{T_0}) > 80\text{GeV}$ [43]. ΔS requires that $0.2296 < x_E < 0.8069$. From ΔT , one has

$$-0.16 < \frac{1}{8\pi s_W^2 M_W^2} [3M_E^2 I_2(x_E) - 16\pi^2 v_t^2] < 0.32. \quad (8.13)$$

The largest ΔT_F comes from the smallest x_E , namely, the largest mass squared difference. By using eq. (8.12), one obtains $v_t < 23.75\text{GeV}$, with $M_E = 280.3\text{GeV}$, and $M_N = 134.2\text{GeV}$.

By the same token, when $x_E = 1$, one has $0.0121 < x_T < 0.5253$, $v_t < 19.72\text{GeV}$, with $M_{T_-} = 247.5\text{GeV}$, and $M_{T_0} = 27.2\text{GeV}$. Since T does not carry lepton number, it interacts with SM fermions through the mixing U_h^{12} and $b\bar{b}$ will be the dominant decay channel. However t_0 has the SM gauge interaction, see table 4, and the process $e^+e^- \rightarrow Z^* \rightarrow Zb\bar{b}$ can go with an effective mixing squared $\simeq (4v_t/v)^2 |U_h^{12}|^2 < 0.013$. The effective mixing squared agrees with the bound, $\lesssim 0.02$, from the direct search of neutral scalar at LEP2 for this mass [49].

A full analysis yields an upper bound

$$v_t < 24.08\text{ GeV}, \quad (8.14)$$

which corresponds to $x_E = x_T = 0.3317$, $M_E = M_{T_-} = 300.9\text{GeV}$, $M_N = M_{T_0} = 173.3\text{GeV}$. The solution agrees with the current direct search bounds on the masses of charged heavy lepton, $\gtrsim 100\text{GeV}$ [50], and Higgs, $\gtrsim 80\text{GeV}$ [43]. Moreover, M_N and M_{T_0} are larger than the LEP2 bound from the Z decay and the direct search for the neutral Higgs. Comparing to the previous bound, eq. (8.3), where loop contributions are not included, the upper bound for v_t is pushed up by around factor of five.

This much has been said about the upper bound on v_t . We should remark that the oblique parameters do not impose any lower bound on v_t . For example, even $v_t = 0$, all requirements from ΔT and ΔS also that the mass squared differences are less than v^2 can be met when $x_E = x_T = 0.8513$, $M_E = M_{T_-} = 613.7\text{GeV}$, and $M_N = M_{T_0} = 566.2\text{GeV}$.

9 Higgs to 2γ

New electrically charged degrees of freedom which couple to h_{SM} modify the SM Higgs di-photon decay width. In addition to the new charged leptons introduced for anomaly cancelation, which have been studied in [1], the charged components of the triplet also contribute. For h_{SM} di-photon decay, the relevant Lagrangian are the $\lambda_{1,6}$ terms,

$$\simeq v(\lambda_1 + \lambda_6/2)h_{SM}T_+T_- + v(\lambda_1 + \lambda_6)h_{SM}T_{++}T_{--}, \quad (9.1)$$

assuming that $|U_h^{11}| \sim 1.0$. Although the $\lambda_{4,t}$ terms also contribute to $h_{SM}T_+T_-$ and $h_{SM}T_{++}T_{--}$ vertices, their strengthes are doubly suppressed by v_t and U_h^{12} , and thus can

be ignored. The di-photon decay width is thus

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \left| F_1(\tau_W) + \frac{4}{3}F_{1/2}(\tau_t) + \sum_{i=1}^6 y_{E_i} \frac{2M_W}{g_2 M_{E_i}} F_{1/2}(\tau_{E_i}) + (\lambda_1 + \lambda_6/2) \frac{v^2}{2M_{T_+}^2} F_0(\tau_{T_+}) + 4(\lambda_1 + \lambda_6) \frac{v^2}{2M_{T_{++}}^2} F_0(\tau_{T_{++}}) \right|^2, \quad (9.2)$$

where $\tau_i \equiv (m_H/2m_i)^2$, and all the loop functions can be found in [51]. For the exotic leptons, the Yukawa couplings are parameterized as $\mathcal{L} \supset -y_{E_i} \bar{E}_i E_i h_{SM}$ in the mass basis. Assuming that T_- and T_{--} are degenerate, the width reads

$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \times \left| -8.324 + 1.834 + \sum_{i=1}^6 0.32(3.64) \times y_{E_i} + 0.051(0.203) \times (\lambda_1 + 0.9\lambda_6) \right|^2 \quad (9.3)$$

for $M_{E_i} = 1000(100)\text{GeV}$, and $M_{T_-} = 1.0(0.5)\text{TeV}$. The first two numbers are the dominate SM contributions from $W^\pm$ and top quark, respectively. The dominant SM Higgs production channel at the LHC is through gluon fusion which is intact in this model. Therefore, the signal strength of $pp \rightarrow h \rightarrow \gamma\gamma$ is

$$\begin{aligned} \mu_{\gamma\gamma} &\simeq \Gamma(H \rightarrow \gamma\gamma)/\Gamma(H \rightarrow \gamma\gamma)_{SM} \\ &\sim 1 - \sum_i (0.049 - 0.561) \times y_{E_i} - (0.0157 - 0.063) \times (\lambda_1 + 0.9\lambda_6). \end{aligned} \quad (9.4)$$

It is expected that $|y_{E_i}| \sim m_l/v_h \ll 1$ [1], and the charged leptons contribution can be ignored. Comparing to the data $\mu_{\gamma\gamma} = 1.18(+0.17 - 0.14)$ [52], it is safe even $|\lambda_{1,6}| \sim \mathcal{O}(1)$. This agrees with the general analysis given in [53].

10 Conclusions

We have studied a novel neutrino mass generation mechanism in the recently proposed gauged lepton number model by us [1]. The model is free of anomalies by the addition of two sets of exotic chiral leptons for each generation. The $U(1)_l$ gauge symmetry is spontaneously broken when a $l = 1$ SM singlet, ϕ_1 , gets a VEV, v_L . In addition, one $l = 0$ SM triplet, T , is introduced for neutrino mass generation. The triplet in this model differs from the well-studied $l = 2$ triplet in the type-II see-saw model. Since it carries no lepton number, the triplet does not couple to the SM leptons. An immediate consequence is that there is no doubly charged triplet contribution to the neutrino-less double decays of nuclei which in our model is given mainly by the exchange of light neutrinos. The VEV of the charge-neutral parts of T , v_t , and the SM Higgs H , $v \simeq 246\text{GeV}$, breaks the SM electroweak gauge symmetry and the custodial symmetry. With only two exotic scalars, ϕ_1 and T , and no RH SM singlet neutrino, the resulting neutrino mass is of the inverse see-saw type. Since the phenomenology of the obligatory new gauge boson Z_ℓ and the

exotic leptons have been studied in [1], we have focused on the physics of neutrino mass and the new $l = 0$ triplet in this work.

We begin the discussion of the one-generation case since the physics is clear in this simple setting. Since the exotic leptons required for anomaly cancelations will in general mix with the SM leptons we require that the Yukawa couplings $f_{1,2}$ to be very small. This discussion is later extended to the realistic three-generation case, and we have carefully investigated the physics of active neutrino masses in this model. The active neutrino masses are of the order of v_t^3/v_L^2 given by the dimension-six operator O_6 . Since the electroweak precision requires a relatively small v_t , no further parameter fine-tuning is required other than taking $f_1 \simeq 0$ mentioned before. Both realistic NH and IH neutrino masses can be accommodated in this model. If assuming a democratic structure of the Yukawa couplings, it is more natural to get an NH pattern. For IH, it requires a more subtle Yukawa pattern and prefers to have the lightest neutrino mass $\gtrsim 10^{-2}$ eV, which is promising for the neutrinoless double beta decays searches.

It is worth noting that O_6 produces elements of the active neutrino mass matrix that is Majorana-like, i.e. of the form $\overline{\nu_L^{ic}}\nu_L^j$ where i, j are family indices. This is the same as O_5 would. Thus, low energy neutrino measurements such as neutrinoless double beta decays of nuclei, tritium β decays spectrum endpoint, and cosmological neutrino mass bounds cannot distinguish between O_6 or the Weinberg operator as the origin of neutrino masses. In order to do that one needs to explore the TeV scale to discover whether there are new degrees of freedom. O_5 assumes that there are none whereas O_6 requires new leptons below 10 TeV.⁸ In addition, a detailed program searching for CLFV decays of muon and τ will also be useful since O_5 and O_6 have very different UV completions and thus will yield different results for these processes.

We have calculated the 1-loop triplet contributions to a_μ and the LFV processes $l_1 \rightarrow l_2 \gamma$ ($l_{1,2} = e, \mu, \tau$). Δa_μ is negative but negligible in this model. Thus, it cannot resolve the discrepancy between the data and the SM expectation [54]. On the other hand, we have found an interesting connection between the neutrino masses and the LFV branching ratios. Taking into account the current limit on $Br(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13}$, we have obtained an interesting lower bound on $v_t \gtrsim 0.1$ GeV. Since T does not couple to SM leptons, the LFV process $\mu \rightarrow 3e$ and the τ counterparts are mediated by the photon dipole transition and thus predicted to be very small, $Br(\mu \rightarrow 3e) \lesssim 10^{-15}$.

The triplet gets a VEV so that the constraint from ΔT can be relaxed. We have carefully analyzed the limits from both ΔS and ΔT and arrived an upper bound for $v_t \lesssim 24.1$ GeV if assuming the mass squared differences among the isospin components of the triplet and the heavy leptons to be at most electroweak, $\lesssim v^2$. Combining with the neutrino masses and LFV bounds, we have $0.1 \lesssim v_t \lesssim 24.1$ GeV in this model. The lower bound of v_t also implies that $M_T \lesssim 8$ TeV provided that $\kappa \simeq v$.

We have studied the decays of the triplet. For $T_\pm$ and $T_{\pm\pm}$, the dominant decay channel is into di-boson. Depending on the scalar potential, the T_0 component of the triplet in

⁸We have seen previously that the mass splitting $|M_E^2 - M_N^2| \lesssim v^2$. Leptons with the mass around 10 TeV will give a splitting of < 1 GeV. This is much smaller than what we have encountered and will require very delicate tuning of parameters.

general mixes with the SM Higgs doublet, although the mixing squared is limited to be smaller than 0.13 at 2σ level [46]. However, even allowing for this mixing the dominant decay channel of T_0 is still the SM di-boson modes. Due to their SM gauge interactions, the charged triplets can be pair produced via Drell-Yan processes at the LHC. In addition to the SM gauge couplings, due to its mixing with the SM Higgs, the neutral triplet can be singly produced via the gluon fusion. At LHC14, it is possible to probe t_0 of mass up to 1TeV and $|U_h^{12}|^2 \sim 0.1$ with an integrated luminosity of 300 fb^{-1} . At the linear colliders, the signal of triplet pair production will be very clean once the center-of-mass energy is higher than the mass threshold. For the mass range of triplet we are interested in, we have found that the bound from the current $h_{SM} \rightarrow 2\gamma$ measurement is weak.

Acknowledgments

WFC is supported by the Taiwan Minister of Science and Technology under Grant No.106-2112-M-007-009-MY3 and No.105-2112-M-007-029. TRIUMF receives federal funding via a contribution through the National Research Council of Canada.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] W.-F. Chang and J.N. Ng, *Study of gauged lepton symmetry signatures at colliders*, *Phys. Rev. D* **98** (2018) 035015 [[arXiv:1805.10382](https://arxiv.org/abs/1805.10382)] [[INSPIRE](#)].
- [2] S. Weinberg, *Baryon and lepton nonconserving processes*, *Phys. Rev. Lett.* **43** (1979) 1566 [[INSPIRE](#)].
- [3] S.L. Glashow, *The future of elementary particle physics*, *NATO Sci. Ser. B* **61** (1980) 687 [[INSPIRE](#)].
- [4] M. Gell-Mann, P. Ramond and R. Slansky, *Complex spinors and unified theories*, *Conf. Proc. C* **790927** (1979) 315 [[arXiv:1306.4669](https://arxiv.org/abs/1306.4669)] [[INSPIRE](#)].
- [5] T. Yanagida, *Horizontal symmetry and masses of neutrinos*, *Conf. Proc. C* **7902131** (1979) 95 [[INSPIRE](#)].
- [6] R.N. Mohapatra and G. Senjanović, *Neutrino mass and spontaneous parity violation*, *Phys. Rev. Lett.* **44** (1980) 912 [[INSPIRE](#)].
- [7] J. Schechter and J.W.F. Valle, *Neutrino decay and spontaneous violation of lepton number*, *Phys. Rev. D* **25** (1982) 774 [[INSPIRE](#)].
- [8] W.-F. Chang, J.N. Ng and J.M.S. Wu, *Some consequences of the Majoron being the dark radiation*, *Phys. Lett. B* **730** (2014) 347 [[arXiv:1310.6513](https://arxiv.org/abs/1310.6513)] [[INSPIRE](#)].
- [9] W.-F. Chang and J.N. Ng, *Minimal model of Majoronic dark radiation and dark matter*, *Phys. Rev. D* **90** (2014) 065034 [[arXiv:1406.4601](https://arxiv.org/abs/1406.4601)] [[INSPIRE](#)].
- [10] P. Schwaller, T.M.P. Tait and R. Vega-Morales, *Dark matter and vectorlike leptons from gauged lepton number*, *Phys. Rev. D* **88** (2013) 035001 [[arXiv:1305.1108](https://arxiv.org/abs/1305.1108)] [[INSPIRE](#)].

- [11] P. Fileviez Perez and M.B. Wise, *Baryon and lepton number as local gauge symmetries*, *Phys. Rev. D* **82** (2010) 011901 [*Erratum ibid.* **D 82** (2010) 079901] [[arXiv:1002.1754](#)] [[INSPIRE](#)].
- [12] P. Fileviez Perez and M.B. Wise, *Breaking local baryon and lepton number at the TeV scale*, *JHEP* **08** (2011) 068 [[arXiv:1106.0343](#)] [[INSPIRE](#)].
- [13] M. Duerr, P. Fileviez Perez and M.B. Wise, *Gauge theory for baryon and lepton numbers with leptiquarks*, *Phys. Rev. Lett.* **110** (2013) 231801 [[arXiv:1304.0576](#)] [[INSPIRE](#)].
- [14] P. Fileviez Perez, S. Ohmer and H.H. Patel, *Minimal theory for lepto-baryons*, *Phys. Lett. B* **735** (2014) 283 [[arXiv:1403.8029](#)] [[INSPIRE](#)].
- [15] S. Ohmer and H.H. Patel, *Leptobaryons as Majorana dark matter*, *Phys. Rev. D* **92** (2015) 055020 [[arXiv:1506.00954](#)] [[INSPIRE](#)].
- [16] X.G. He, G.C. Joshi, H. Lew and R.R. Volkas, *New Z' phenomenology*, *Phys. Rev. D* **43** (1991) 22 [[INSPIRE](#)].
- [17] X.-G. He, G.C. Joshi, H. Lew and R.R. Volkas, *Simplest Z' model*, *Phys. Rev. D* **44** (1991) 2118 [[INSPIRE](#)].
- [18] D. Wyler and L. Wolfenstein, *Massless neutrinos in left-right symmetric models*, *Nucl. Phys. B* **218** (1983) 205 [[INSPIRE](#)].
- [19] R.N. Mohapatra and J.W.F. Valle, *Neutrino mass and baryon number nonconservation in superstring models*, *Phys. Rev. D* **34** (1986) 1642 [[INSPIRE](#)].
- [20] A. Abada and M. Lucente, *Looking for the minimal inverse seesaw realisation*, *Nucl. Phys. B* **885** (2014) 651 [[arXiv:1401.1507](#)] [[INSPIRE](#)].
- [21] T.G. Rizzo, *Doubly charged Higgs bosons and lepton number violating processes*, *Phys. Rev. D* **25** (1982) 1355 [[INSPIRE](#)].
- [22] A.G. Akeroyd and M. Aoki, *Single and pair production of doubly charged Higgs bosons at hadron colliders*, *Phys. Rev. D* **72** (2005) 035011 [[hep-ph/0506176](#)] [[INSPIRE](#)].
- [23] P. Fileviez Perez, T. Han, G.-Y. Huang, T. Li and K. Wang, *Neutrino masses and the CERN LHC: testing type II seesaw*, *Phys. Rev. D* **78** (2008) 015018 [[arXiv:0805.3536](#)] [[INSPIRE](#)].
- [24] G. Bambhaniya, J. Chakraborty, J. Gluza, M. Kordiaczyńska and R. Szafron, *Left-right symmetry and the charged Higgs bosons at the LHC*, *JHEP* **05** (2014) 033 [[arXiv:1311.4144](#)] [[INSPIRE](#)].
- [25] A. Das, P.S.B. Dev and R.N. Mohapatra, *Same sign versus opposite sign dileptons as a probe of low scale seesaw mechanisms*, *Phys. Rev. D* **97** (2018) 015018 [[arXiv:1709.06553](#)] [[INSPIRE](#)].
- [26] J. Schechter and J.W.F. Valle, *Neutrino masses in $SU(2) \times U(1)$ theories*, *Phys. Rev. D* **22** (1980) 2227 [[INSPIRE](#)].
- [27] W. Konetschny and W. Kummer, *Nonconservation of total lepton number with scalar bosons*, *Phys. Lett. B* **70** (1977) 433 [[INSPIRE](#)].
- [28] T.P. Cheng and L.-F. Li, *Neutrino masses, mixings and oscillations in $SU(2) \times U(1)$ models of electroweak interactions*, *Phys. Rev. D* **22** (1980) 2860 [[INSPIRE](#)].
- [29] G. Lazarides, Q. Shafi and C. Wetterich, *Proton lifetime and fermion masses in an $SO(10)$ model*, *Nucl. Phys. B* **181** (1981) 287 [[INSPIRE](#)].

- [30] R.N. Mohapatra and G. Senjanović, *Neutrino masses and mixings in gauge models with spontaneous parity violation*, *Phys. Rev. D* **23** (1981) 165 [[INSPIRE](#)].
- [31] E. Ma and U. Sarkar, *Neutrino masses and leptogenesis with heavy Higgs triplets*, *Phys. Rev. Lett.* **80** (1998) 5716 [[hep-ph/9802445](#)] [[INSPIRE](#)].
- [32] Y. Cai, T. Han, T. Li and R. Ruiz, *Lepton number violation: seesaw models and their collider tests*, *Front. in Phys.* **6** (2018) 40 [[arXiv:1711.02180](#)] [[INSPIRE](#)].
- [33] W.-F. Chang, I.-T. Chen and S.-C. Liou, *Neutrino masses via the Zee mechanism in 5D split fermions model*, *Phys. Rev. D* **83** (2011) 025017 [[arXiv:1008.5095](#)] [[INSPIRE](#)].
- [34] W.-F. Chang, J.N. Ng and J.M.S. Wu, *Testing realistic quark mass matrices in the custodial Randall-Sundrum model with flavor changing top decays*, *Phys. Rev. D* **78** (2008) 096003 [[arXiv:0806.0667](#)] [[INSPIRE](#)].
- [35] W.-F. Chang, J.N. Ng and J.M.S. Wu, *Flavour changing neutral current constraints from Kaluza-Klein gluons and quark mass matrices in RS1*, *Phys. Rev. D* **79** (2009) 056007 [[arXiv:0809.1390](#)] [[INSPIRE](#)].
- [36] G.C. Branco, R.G. Felipe and F.R. Joaquim, *Leptonic CP-violation*, *Rev. Mod. Phys.* **84** (2012) 515 [[arXiv:1111.5332](#)] [[INSPIRE](#)].
- [37] F.J. Escrihuela, D.V. Forero, O.G. Miranda, M. Tórtola and J.W.F. Valle, *Probing CP-violation with non-unitary mixing in long-baseline neutrino oscillation experiments: DUNE as a case study*, *New J. Phys.* **19** (2017) 093005 [[arXiv:1612.07377](#)] [[INSPIRE](#)].
- [38] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler and T. Schwetz, *Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity*, *JHEP* **01** (2017) 087 [[arXiv:1611.01514](#)] [[INSPIRE](#)].
- [39] PLANCK collaboration, P.A.R. Ade et al., *Planck 2015 results. XIII. Cosmological parameters*, *Astron. Astrophys.* **594** (2016) A13 [[arXiv:1502.01589](#)] [[INSPIRE](#)].
- [40] PLANCK collaboration, N. Aghanim et al., *Planck 2018 results. VI. Cosmological parameters*, [arXiv:1807.06209](#) [[INSPIRE](#)].
- [41] W.-F. Chang and J.N. Ng, *Lepton flavor violation in extra dimension models*, *Phys. Rev. D* **71** (2005) 053003 [[hep-ph/0501161](#)] [[INSPIRE](#)].
- [42] MEG collaboration, A.M. Baldini et al., *Search for the lepton flavour violating decay $\mu^+ \rightarrow e^+ \gamma$ with the full dataset of the MEG experiment*, *Eur. Phys. J. C* **76** (2016) 434 [[arXiv:1605.05081](#)] [[INSPIRE](#)].
- [43] PARTICLE DATA GROUP collaboration, C. Patrignani et al., *Review of particle physics*, *Chin. Phys. C* **40** (2016) 100001 [[INSPIRE](#)].
- [44] W.-F. Chang, S.-C. Liou, C.-F. Wong and F. Xu, *Charged lepton flavor violating processes and scalar leptoquark decay branching ratios in the colored Zee-Babu model*, *JHEP* **10** (2016) 106 [[arXiv:1608.05511](#)] [[INSPIRE](#)].
- [45] T. Han, B. Mukhopadhyaya, Z. Si and K. Wang, *Pair production of doubly-charged scalars: neutrino mass constraints and signals at the LHC*, *Phys. Rev. D* **76** (2007) 075013 [[arXiv:0706.0441](#)] [[INSPIRE](#)].
- [46] ATLAS and CMS collaborations, *Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV*, *JHEP* **08** (2016) 045 [[arXiv:1606.02266](#)] [[INSPIRE](#)].

- [47] A. Belyaev, N.D. Christensen and A. Pukhov, *CalcHEP 3.4 for collider physics within and beyond the Standard Model*, *Comput. Phys. Commun.* **184** (2013) 1729 [[arXiv:1207.6082](#)] [[INSPIRE](#)].
- [48] J. Pumplin, D.R. Stump, J. Huston, H.L. Lai, P.M. Nadolsky and W.K. Tung, *New generation of parton distributions with uncertainties from global QCD analysis*, *JHEP* **07** (2002) 012 [[hep-ph/0201195](#)] [[INSPIRE](#)].
- [49] OPAL, DELPHI, LEP WORKING GROUP FOR HIGGS BOSON SEARCHES, ALEPH and L3 collaborations, R. Barate et al., *Search for the Standard Model Higgs boson at LEP*, *Phys. Lett. B* **565** (2003) 61 [[hep-ex/0306033](#)] [[INSPIRE](#)].
- [50] L3 collaboration, P. Achard et al., *Search for heavy neutral and charged leptons in e^+e^- annihilation at LEP*, *Phys. Lett. B* **517** (2001) 75 [[hep-ex/0107015](#)] [[INSPIRE](#)].
- [51] A. Djouadi, *The anatomy of electro-weak symmetry breaking. I: the Higgs boson in the Standard Model*, *Phys. Rept.* **457** (2008) 1 [[hep-ph/0503172](#)] [[INSPIRE](#)].
- [52] CMS collaboration, *Measurements of Higgs boson properties in the diphoton decay channel in proton-proton collisions at $\sqrt{s} = 13$ TeV*, [arXiv:1804.02716](#) [[INSPIRE](#)].
- [53] W.-F. Chang, J.N. Ng and J.M.S. Wu, *Constraints on new scalars from the LHC 125 GeV Higgs signal*, *Phys. Rev. D* **86** (2012) 033003 [[arXiv:1206.5047](#)] [[INSPIRE](#)].
- [54] PARTICLE DATA GROUP collaboration, C. Patrignani et al., *Review of particle physics*, *Chin. Phys. C* **40** (2016) 100001 [[INSPIRE](#)].