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Current interactions and holography from the 0-form sector of nonlinear higher-spin equations

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ABSTRACT: The form of higher-spin current interactions in AdS_4 is derived from the full nonlinear higher-spin equations in the sector of Weyl 0-forms. The coupling constant in front of spin-one currents built from scalars and spinors as well as Yukawa coupling are determined explicitly. Couplings of all other higher-spin current interactions are determined implicitly. All couplings are shown to be independent of the phase parameter of the nonlinear higher-spin theory. The proper holographic dependence of the vertex on the higher-spin phase parameter is shown to result from the boundary conditions on the bulk fields.

KEYWORDS: Higher Spin Gravity, AdS-CFT Correspondence, Higher Spin Symmetry

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1 Introduction

Higher-spin (HS) gauge theory is a theory of interacting massless fields of all spins. Massless fields of spins $s \geq 1$ are gauge fields supporting various types of gauge symmetries [1–4]. Gauge fields of spins $s = 1$ and $s = 2$ are familiar from the conventional models of fundamental interactions, underlying the Standard Model and GR, respectively. Symmetries of gauge fields of spins $s > 2$ are HS symmetries. The latter are anticipated to play a role at ultra high energies possibly beyond Planck energy. Since such energies are unreachable by modern accelerators the conjecture that a fundamental theory exhibits HS symmetries at ultra high energies provides a unique chance to explore properties of this regime. HS symmetries severely restrict the structure of HS theory, underlying many of its unusual properties.

One is that, as shown by A. Bengtsson, I. Bengtsson, Brink [5, 6] and Berends, Burgers, van Dam [7, 8], HS interactions consistent with HS gauge symmetries contain higher derivatives (note however that no higher derivatives appear at the quadratic level in a maximally symmetric background geometry [1–4]). Another one, originally indicated by the analysis of HS symmetries in [7, 8] and later shown to follow from the structure of HS symmetry algebras [9–11], is that a HS theory containing a propagating field of any

spin $s > 2$ should necessarily contain an infinite tower of HS fields of infinitely increasing spins. Since higher spins induce higher derivatives in the interactions any HS theory with an infinite tower of HS fields is somewhat nonlocal. Of course some kind of nonlocality beyond Planck scale should be expected of a theory anticipated to capture the quantum gravity regime.

Appearance of higher derivatives in interactions demands a dimensionful constant ρ which was identified in [12, 13] with the radius of the background $(A)dS$ space. In this setup, the higher-derivative vertices admit no meaningful flat limit in agreement with numerous no-go statements ruling out nontrivial interactions of massless HS fields in Minkowski space [14, 15] (see [16] for more detail and references).

The feature that consistent HS interactions demand non-zero cosmological constant acquired a deep interpretation with the discovery of the AdS/CFT correspondence [17–19]. After general holographic aspects of HS theory were pointed out in [20–22], the precise proposal on the AdS₄/CFT₃ correspondence was put forward by Klebanov and Polyakov [23]. Its first explicit check was done by Giombi and Yin in [24] triggering sharp increase of interest in HS theories and HS holographic duality [25]–[44].

Analysis of holographic duality in the HS theory is hoped to shed light on the very origin of AdS/CFT correspondence. Since HS gauge symmetry principle is the defining property of HS theories, it is crucially important to elaborate a manifestly gauge invariant definition for the boundary generating functional. A new proposal in this direction was recently put forward in [45] which, along with the idea of derivation of holographic duality from the unfolded equations [29] respecting all necessary symmetries, may provide a useful setup for better understanding of holographic duality.

Geometric origin of the dimensionful parameter ρ has an important consequence that any HS gauge theory with unbroken HS symmetries does not allow a low-energy analysis because a dimensionless derivative $\rho \frac{\partial}{\partial x}$ that appears in the expansion in powers of derivatives cannot be treated as small. This is most obvious from the fact that the rescaled covariant derivatives $\mathcal{D} = \rho D$, which are non-commutative in the background AdS space-time of curvature ρ^{-2} , have commutator of order one, $[\mathcal{D}, \mathcal{D}] \sim 1$.

As a result, in a field redefinition

$$\phi \rightarrow \phi' = \phi + \sum_{n,m=0}^{\infty} a_{nm} \mathcal{D}^n \phi \mathcal{D}^m \phi, \quad (1.1)$$

all terms with higher derivatives may give comparable contributions. So, whether expansion (1.1) is local or not should depend solely on the behavior of the coefficients a_{nm} at $n, m \rightarrow \infty$. If at most a finite number of coefficients $a_{n,m}$ is nonzero, field redefinition (1.1) is genuinely local. Based on the results of this paper, in [46] we conjecture a restriction on the coefficients analogous to a_{nm} appropriate for the definition of the local frame in the twistor-like formalism of the standard formulation of nonlinear HS equations of motion in AdS₄ [47].

Importance of the proper definition of locality was originally raised in [48] where it was shown that by a field redefinition of the form (1.1) it is possible to get rid of the currents

from the r.h.s. of HS field equations. Recently this issue was reconsidered in [49–52] (see also a subsequent paper [53]). In particular in [49] a proposal was put forward on the part of the problem associated with the exponential factors resulting from so-called inner Klein operators while the structure of the preexponential factors was only partially determined. In [46], the results of [49] are extended to the preexponential factors at the quadratic order.

Analysis of this paper was motivated by the work [54] where current HS interactions were rewritten in the unfolded form. A natural question on the agenda was to derive these results from the nonlinear HS equations of [47]. This study was boosted by the recent papers [50, 52] where such an attempt was undertaken. Unfortunately, leaving the problem unsolved, the authors of these papers arrived at some counterintuitive conclusions raising the issue of physical interpretation of the HS equations of [47], thus urging us to reconsider the problem.

In this paper, confining ourselves to the 0-form sector of the HS equations, which is simpler than that of 1-forms, we present a simple field redefinition that reduces the first nonlinear corrections resulting from the nonlinear HS equations to the usual current interactions in the form obtained in [54]. The same time, it elucidates deep geometric structures underlying the perturbative analysis of HS theory, relating homotopy integrals over different types of simplexes. This field redefinition not only determines relative coefficients in front of different current interactions but also suggests a proper criterium of (non)locality in nonlinear AdS_4 HS theory, further elaborated in [46] where it is shown in particular that the results obtained in this paper are unambiguously selected by the proper locality criterion of higher-order corrections. It should be stressed that the analysis of the 0-form sector of this paper is fully informative implying locality in the 1-form sector up to possible gauge artifacts. Details for the 1-form sector are presented in [55].

Note that the analysis of [46] refers mainly to the locality of the final result that should not be confused with the issue of (non)locality of the field redefinition from the originally nonlocal setup to the local one. The same time the results of [46] showing that the field redefinition found in this paper is essentially nonlocal are in agreement with the analysis of [53] where it was argued that a field redefinition bringing the nonlocal setup resulting from the standard homotopy analysis of nonlinear equations to any local form is essentially nonlocal. The interpretation of the results of this paper given in [46] provides however a starting point for elaboration a perturbative scheme that operates entirely with local or minimally nonlocal results at higher orders with no reference to nonlocal field redefinitions at all.

One of the conclusions of this paper is that the current interactions in HS theory are independent of the phase of the free parameter η of the nonlinear HS equations, depending solely on $\eta\bar{\eta}$. Remarkably, this agrees with the conjectures on holographic duality of the HS theory with different η [25, 26] as well as with the structure of the boundary correlator found in [28]. Namely, following [29] we demonstrate how different boundary results originate from the η -dependent boundary conditions compatible with the boundary CFT.

The rest of the paper is organized as follows. In section 2 we recall unfolded formulation of free massless fields and conserved currents in four dimensions. The structure of the unfolded equations describing current interactions of massless fields of all spins is recalled

in section 3. The form of nonlinear HS equations is sketched in section 4. In section 5 quadratic corrections to massless equations resulting from the nonlinear HS equations are obtained and their relation to usual local current interactions is established. Holographic interpretation of the obtained results is discussed in section 6 with the emphasize on the dependence on the phase parameter of the HS theory. Main results are summarized and discussed in section 7.

2 Free massless equations

2.1 Central on-shell theorem

The infinite set of $4d$ massless fields of all spins $s = 0, 1/2, 1, 3/2, 2, \dots$ is conveniently described by a 1-form $\omega(y, \bar{y}|x) = dx^n \omega_n(y, \bar{y}|x)$ and 0-form $C(y, \bar{y}|x)$ [56]

$$f(y, \bar{y}|x) = \frac{1}{2i} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} y_{\alpha_1} \dots y_{\alpha_n} \bar{y}_{\dot{\beta}_1} \dots \bar{y}_{\dot{\beta}_m} f^{\alpha_1 \dots \alpha_n, \dot{\beta}_1 \dots \dot{\beta}_m}(x), \quad (2.1)$$

where x^n are $4d$ space-time coordinates and $Y^A = (y^\alpha, \bar{y}^{\dot{\alpha}})$ are auxiliary commuting spinor variables ($A = 1, \dots, 4$ is a Majorana spinor index while $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$ are two-component spinor indices). The key fact of the analysis of $4d$ massless fields referred to as Central on-shell theorem is that unfolded system of field equations for free massless fields of all spins has the form [56]

$$R_1(y, \bar{y}|x) = L(w, C) := \frac{i}{4} \left(\eta \bar{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \bar{y}^{\dot{\alpha}} \partial \bar{y}^{\dot{\beta}}} \bar{C}(0, \bar{y}|x) + \bar{\eta} H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0|x) \right), \quad (2.2)$$

$$\mathcal{D}_{\text{tw}} C(y, \bar{y}|x) = 0, \quad (2.3)$$

where

$$R_1(y, \bar{y}|x) := \mathcal{D}_{ad} \omega(y, \bar{y}|x) := D^L \omega(y, \bar{y}|x) + h^{\alpha\dot{\beta}} \left(y_\alpha \frac{\partial}{\partial \bar{y}^{\dot{\beta}}} + \frac{\partial}{\partial y^\alpha} \bar{y}_{\dot{\beta}} \right) \omega(y, \bar{y}|x), \quad (2.4)$$

$$\mathcal{D}_{\text{tw}} C(y, \bar{y}|x) := D^L C(y, \bar{y}|x) - i h^{\alpha\dot{\beta}} \left(y_\alpha \bar{y}_{\dot{\beta}} - \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\dot{\beta}}} \right) C(y, \bar{y}|x), \quad (2.5)$$

$$D^L f(y, \bar{y}|x) := df(y, \bar{y}|x) + \left(\omega^{\alpha\beta} y_\alpha \frac{\partial}{\partial y^\beta} + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \frac{\partial}{\partial \bar{y}^{\dot{\beta}}} \right) f(y, \bar{y}|x). \quad (2.6)$$

Here $d = dx^n \frac{\partial}{\partial x^n}$. Background AdS_4 space is described by a flat $sp(4)$ connection $w = (\omega_{\alpha\beta}, \bar{\omega}_{\dot{\alpha}\dot{\beta}}, h_{\alpha\dot{\beta}})$ containing Lorentz connection $\omega_{\alpha\beta}$, $\bar{\omega}_{\dot{\alpha}\dot{\beta}}$ and vierbein $h_{\alpha\dot{\beta}}$ that obey the equations

$$R_{\alpha\beta} = 0, \quad \bar{R}_{\dot{\alpha}\dot{\beta}} = 0, \quad R_{\alpha\dot{\alpha}} = 0, \quad (2.7)$$

where, here and after discarding the wedge product symbols,

$$R_{\alpha\beta} := d\omega_{\alpha\beta} + \omega_{\alpha\gamma} \omega_{\beta}{}^{\gamma} - H_{\alpha\beta}, \quad \bar{R}_{\dot{\alpha}\dot{\beta}} := d\bar{\omega}_{\dot{\alpha}\dot{\beta}} + \bar{\omega}_{\dot{\alpha}\dot{\gamma}} \bar{\omega}_{\dot{\beta}}{}^{\dot{\gamma}} - \bar{H}_{\dot{\alpha}\dot{\beta}}, \quad (2.8)$$

$$R_{\alpha\dot{\beta}} := d + \omega_{\alpha\gamma} h^{\gamma}{}_{\dot{\beta}} + \bar{\omega}_{\dot{\beta}\dot{\delta}} h_{\alpha}{}^{\dot{\delta}}. \quad (2.9)$$

Two-component indices are raised and lowered by $\varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}$, $\varepsilon_{12} = 1$: $A^\alpha = \varepsilon^{\alpha\beta} A_\beta$, $A_\alpha = A^\beta \varepsilon_{\beta\alpha}$ and analogously for dotted indices. $H^{\alpha\beta} = H^{\beta\alpha}$ and $\overline{H}^{\dot{\alpha}\dot{\beta}} = \overline{H}^{\dot{\beta}\dot{\alpha}}$ are the frame 2-forms

$$H^{\alpha\beta} := h^{\alpha\dot{\alpha}} h^{\beta\dot{\beta}}_{\dot{\alpha}}, \quad \overline{H}^{\dot{\alpha}\dot{\beta}} := h^{\alpha\dot{\alpha}} h_{\alpha\dot{\beta}}^{\dot{\beta}}. \quad (2.10)$$

η and $\bar{\eta}$ are complex conjugated free parameters introduced for future convenience. Their module will be identified with the coupling constant of the theory

$$\eta\bar{\eta} = f. \quad (2.11)$$

The phase of η matters at the full nonlinear level as well as in the holographic interpretation. The AdS radius $\rho = \lambda^{-1}$ which is set equal to 1 in this paper can be reintroduced via rescaling $h_{\alpha\dot{\beta}} \rightarrow \lambda h_{\alpha\dot{\beta}}$.

1-form HS connection $\omega(y, \bar{y}|x)$ contains HS gauge fields. For spins $s \geq 1$, equation (2.2) expresses the Weyl 0-forms $C(Y|x)$ via gauge invariant combinations of derivatives of the HS gauge connections. From this perspective the Weyl 0-forms $C(Y|x)$ generalize the spin-two Weyl tensor along with all its derivatives to any spin.

System (2.2), (2.3) decomposes into subsystems of different spins. In these terms, a massless spin s is described by the 1-forms $\omega(y, \bar{y}|x)$ and 0-form $C(y, \bar{y}|x)$ obeying

$$\omega(\mu y, \mu \bar{y} | x) = \mu^{2(s-1)} \omega(y, \bar{y} | x), \quad C(\mu y, \mu^{-1} \bar{y} | x) = \mu^{\pm 2s} C(y, \bar{y} | x), \quad (2.12)$$

where the signs $+$ and $-$ correspond to self-dual and anti-self-dual parts of the generalized Weyl tensors $C(y, \bar{y}|x)$ with helicities $h = \pm s$. More precisely, $C(y, 0|x)$ and $C(0, \bar{y}|x)$ describe the minimal gauge invariant combinations of derivatives of the gauge fields of spins $s \geq 1$ as well as the matter fields of spins $s = 0$ or $1/2$. For $s = 1, 2$, $C(y, 0|x)$ and $C(0, \bar{y}|x)$ parameterize Maxwell and Weyl tensors respectively. Those associated with higher powers of auxiliary variables y and $\bar{y}$ describe on-shell nontrivial combinations of derivatives of the generalized Weyl tensors as is obvious from equations (2.3), (2.5) relating second derivatives in $y, \bar{y}$ to the x derivatives of $C(Y|x)$ of lower degrees in Y . Higher space-time derivatives in the nonlinear system result from the components of $C(Y|x)$ of higher degrees in Y .

Since twisted covariant derivative (2.5) contains both the term with two derivatives in $y, \bar{y}$ and that with the product of y and $\bar{y}$ the components of $C(y, \bar{y}|x)$ of the degree

$$2N_x(C(Y|x)) := N_Y(C(Y|x)) - 2s, \quad N_Y(C(Y|x)) := \left(y^\alpha \frac{\partial}{\partial y^\alpha} + \bar{y}^{\dot{\alpha}} \frac{\partial}{\partial \bar{y}^{\dot{\alpha}}} \right) C(Y|x) \quad (2.13)$$

contain space-time derivatives of order $N_x(C(Y|x))$ and lower. In other words, AdS geometry induces filtration with respect to space-time derivatives rather than gradation as would be the case for massless fields in Minkowski space free of a dimensional parameter. Because of the lack of indices, at any given order in Y , $C(Y|x)$ only mixes leading and first subleading derivative of massless fields.

2.2 Currents

Conserved currents $J(Y_1, Y_2|x)$ are associated with bilinears of the 0-forms $C(Y|x)$ [57, 58]

$$J(Y_1, Y_2|x) := C(Y_1|x)\tilde{C}(Y_2|x), \quad \tilde{C}(y, \bar{y}|x) = C(-y, \bar{y}|x). \quad (2.14)$$

The two types of fields $C(y, \bar{y}|x)$ and $\tilde{C}(y, \bar{y}|x)$ are analogues of the fields $C_{\pm}(y, \bar{y}|x)$ of [54] characterized by opposite signs in the h -dependent term of twisted covariant derivative (2.5).

As a consequence of rank-one equation (2.5) $J(Y_1, Y_2|x)$ obeys the rank-two equation

$$\mathcal{D}_{\text{tw}}^2 J(Y_1, Y_2|x) = 0, \quad \mathcal{D}_{\text{tw}}^2 := D^L - ih^{\alpha\dot{\beta}} \left(y_{1\alpha} \bar{y}_{1\dot{\beta}} - y_{2\alpha} \bar{y}_{2\dot{\beta}} - \frac{\partial^2}{\partial y_1^\alpha \partial \bar{y}_{1\dot{\beta}}} + \frac{\partial^2}{\partial y_2^\alpha \partial \bar{y}_{2\dot{\beta}}} \right). \quad (2.15)$$

As shown in [54, 58], unfolded equation (2.15) describes conserved conformal currents along with some off-shell conformal currents in four dimensions.

The current J contains primary components J_0 , that cannot be expressed via derivatives of others, and descendants J_+ expressible via x -derivatives of the primaries. From (2.15) it is obvious that

$$J_0 \in \text{Ker } \sigma_-, \quad \sigma_- = \frac{\partial^2}{\partial y_1^\alpha \partial \bar{y}_{1\dot{\beta}}} - \frac{\partial^2}{\partial y_2^\alpha \partial \bar{y}_{2\dot{\beta}}}. \quad (2.16)$$

Descendants J_+ belong to a supplement of $\text{Ker } \sigma_-$. Being expressed via derivatives of J_0 , descendant currents, most of which are also conserved, correspond to so-called improvements which do not generate nontrivial charges. The space of J_0 was analyzed in detail in [54, 59].

At the free field level, HS equations (2.2), (2.3) can be supplemented with current conservation equation (2.15). For a flat background connection obeying (2.7) this unfolded system is consistent independently of whether J has bilinear form (2.14) or not. (For instance, J can be realized by a sum over the color index i of $C^i(Y|x)$.) This leads to the remarkable fact emphasized in [54] that the nonlinear equations describing current interactions in the lowest order amount to a linear system for ω , C and J .

3 Current deformation

Schematically, for a flat connection $D = d + w$, $w = (\omega, \bar{\omega}, h)$ obeying $(\mathcal{D}_{ad})^2 = 0$ (equivalently, $(\tilde{\mathcal{D}}_{\text{tw}})^2 = 0$) which is a concise form of (2.7), the current deformation of free equations (2.2), (2.3) has the form

$$\mathcal{D}_{\text{tw}}\omega - L(w, C) + \Gamma_{\text{cur}}(w, J) = 0, \quad (3.1)$$

$$\mathcal{D}_{ad}C + \mathcal{H}_{\text{cur}}(w, J) = 0, \quad (3.2)$$

where $L(w, C)$ is defined in (2.2) while the 2-form $\Gamma_{\text{cur}}(w, J)$ and 1-form $\mathcal{H}_{\text{cur}}(w, J)$ are some functionals of the background fields w and the current J . Current interactions are associated with $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ linear in J . Their form should respect the consistency of (3.1) and (3.2) with $d^2 = 0$ by virtue of current equation (2.15) that remains unchanged.

In this setup formal consistency of the system is insensitive to whether J is bilinear in the rank-one fields like in (2.14) or not. It is only important that J obeys (2.15). As a result, system of equations (3.1), (3.2) and (2.15) is linear, admitting an interesting interpretation [54] as mixing massless fields in four dimensions associated with ω and C and those in six dimensions associated with J . The same property implies a clear group-theoretical interpretation of equations (3.1), (3.2) and (2.15) as covariant constancy conditions in the appropriate deformation of the $o(3,2) \sim sp(4)$ -modules associated with ω , C and J .

The functionals $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ are determined by the compatibility conditions not uniquely. The freedom in $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ results from field redefinitions linear in J

$$\omega \rightarrow \omega' = \omega + \Omega(w, J), \quad C \rightarrow C' = C + \Phi(J). \quad (3.3)$$

Deformations $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ are nontrivial when they cannot be removed by a field redefinition and trivial otherwise. Usual current interactions are nontrivial. Trivial terms are associated with improvements. Schematically, we set

$$J = J_0 + \Delta J, \quad (3.4)$$

where ΔJ denotes an improvement that can be removed by a field redefinition (3.3).

The problem admits a cohomological interpretation with nontrivial current interactions identified with the quotient of all $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ respecting compatibility over the trivial ones. The resulting cohomology can be identified with certain Shevalley-Eilenberg cohomology of $sp(4)$ with coefficients in $sp(4)$ -modules associated with ω , C and J . Algebraically this is the semidirect sum of a rank-one and rank-two systems. (For more detail on these issues we refer the reader to [59] and references therein.) The cohomological interpretation of the current interactions raises a problem of a representative choice equivalent to the choice of variables modulo field redefinitions (3.3).

It should be stressed that the concept of (non)triviality of the currents depends crucially on the choice of a proper class of functions. Indeed, since the system in question is linear, it admits a solution in the form

$$\omega(J) = \omega_0 + G_\omega(J), \quad C(J) = C_0 + G_C(J), \quad (3.5)$$

where G_ω and G_C are appropriate Green functions while ω_0 and C_0 solve the J -independent part of the equations. The shift by a Green function is a nonlocal operation. So, currents can be nontrivial only with respect to local field redefinitions. In the HS theory in AdS space involving infinitely many spins and derivatives, the question what is a proper substitute of the concept of a local field redefinition among various field redefinitions (1.1), that preserves the concept of nontrivial current interactions, is not quite trivial.

The current deformation of $4d$ HS field equations was analyzed in [54] where the local functionals $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ were found from the consistency conditions in the basis where nontrivial currents correspond to the primaries of the current module J . This corresponds to $\Gamma_{\text{cur}}(w, J)$ and $\mathcal{H}_{\text{cur}}(w, J)$ in (3.1) and (3.2) with the minimal number of derivatives which is finite for any fixed spins s_1, s_2 of the constituent fields of the current

J (2.14) and the spin s_J of the current J equal to the spin of the fields ω and C in eqs. (3.1), (3.2).

The corresponding interactions classified by Metsaev [60] exist provided that for a conserved currents with $s_J \geq 1$

$$s_J \geq s_1 + s_2 \quad (3.6)$$

(Yukawa interaction with $s_J, s_1, s_2 = 0$ or $1/2$ not involving conserved currents does not obey this restriction). Later on HS cubic interactions were considered by many authors in various setups (see, e.g., [61]–[65] and also [66, 67] for the nonstandard full action proposal).

In [54] it was shown that deformation (3.1), (3.2) properly reproduces current interactions of HS fields. In the lowest nontrivial order in interactions it is impossible to determine the coefficients in front of individual currents solely from the consistency of the deformation. As such they remain arbitrary functions of s_J, s_1 and s_2 .

For simplicity, in this paper we focus on the deformation $\mathcal{H}_{\text{cur}}(w, J)$ in the 0-form sector, leaving consideration of $\Gamma_{\text{cur}}(w, J)$ to [55]. Since $\mathcal{H}_{\text{cur}}(w, J)$ and $\Gamma_{\text{cur}}(w, J)$ are tightly related by the compatibility conditions, analysis of the 0-form sector answers all qualitative questions on HS current interactions. Upon the change of variables from $y^\pm$ used in [54] to $y_{1,2}$ of this paper the final result of [54] in the 0-form sector can be represented in the form

$$\begin{aligned} \mathcal{H}_{\text{cur}}(w, J) = & \frac{1}{4} \int_0^1 d\tau \sum_{h_1, h_2, h_J} \\ & \left(a(h_1, h_2, h_J) \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_\beta \bar{t}^{\dot{\beta}}] h(y, \tau \bar{s} + (1-\tau)\bar{t}) J_{h_1, h_2, h_J}(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}) \right. \\ & \left. + \bar{a}(h_1, h_2, h_J) \int \frac{dsdt}{(2\pi)^2} \exp i[s_\beta t^{\dot{\beta}}] h(\tau s - (1-\tau)t, \bar{y}) J_{-h_1, -h_2, -h_J}(y + s; y + t, \tau \bar{y}; -(1-\tau)\bar{y}) \right), \end{aligned} \quad (3.7)$$

where we use notation

$$h(u, \bar{u}) = h^{\alpha\dot{\alpha}} u_\alpha \bar{u}_{\dot{\alpha}} \quad (3.8)$$

and J_{h_1, h_2, h_J} is the projection of J to the helicities h_1, h_2, h_J . Also we find it convenient to slightly re-order the arguments putting first left and then right spinors

$$J(Y_1; Y_2) = J(y_1; y_2, \bar{y}_1; \bar{y}_2). \quad (3.9)$$

The coefficients $a(h_1, h_2, h_J)$ and $\bar{a}(h_1, h_2, h_J)$ can be non-zero provided that condition (3.6) is true.

Using that, for a polynomial $P(y_1; y_2)$ of degrees n_1 and n_2 in y_1 and y_2 , respectively,

$$\int_0^1 d\tau P(\tau y_1; (1-\tau)y_2) = \frac{n_1! n_2!}{(n_1 + n_2 + 1)!} P(y_1; y_2), \quad (3.10)$$

eq. (3.7) just reproduces the coefficients of the deformation of [54]. In fact, it is easier to check compatibility of the deformation directly in the form (3.7) than in the component formalism of [54].

Indeed, an elementary computation shows that, skipping explicit reference to helicities,

$$\begin{aligned}
 \mathcal{D}_{\text{tw}} \mathcal{H}_{\text{cur}}(w, J) &= -\frac{1}{8} \int_0^1 d\tau \frac{\partial}{\partial \tau} \\
 &\left(a \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}} \bar{t}^{\dot{\beta}}] \bar{H}^{\dot{\alpha}\dot{\beta}} (\tau \bar{s}_{\dot{\alpha}} + (1-\tau) \bar{t}_{\dot{\alpha}}) (\tau \bar{s}_{\dot{\beta}} + (1-\tau) \bar{t}_{\dot{\beta}}) J(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}) \right. \\
 &\quad \left. + \bar{a} \int \frac{dsdt}{(2\pi)^2} \exp i[s_{\beta} t^{\beta}] H^{\alpha\beta} (\tau s_{\alpha} + (1-\tau) t_{\alpha}) (\tau s_{\beta} + (1-\tau) t_{\beta}) J(y+s; y+t, \tau \bar{y}; -(1-\tau)\bar{y}) \right) \\
 &= -\frac{1}{8} \left(a \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}} \bar{t}^{\dot{\beta}}] (\bar{H}^{\dot{\alpha}\dot{\beta}} \bar{s}_{\dot{\alpha}} \bar{s}_{\dot{\beta}} J(y, 0, \bar{y} + \bar{s}, \bar{y} + \bar{t}) - \bar{H}^{\dot{\alpha}\dot{\beta}} \bar{t}_{\dot{\alpha}} \bar{t}_{\dot{\beta}} J(0; -y, \bar{y} + \bar{s}; \bar{y} + \bar{t})) \right. \\
 &\quad \left. + \bar{a} \int \frac{dsdt}{(2\pi)^2} \exp i[s_{\beta} t^{\beta}] (H^{\alpha\beta} s_{\alpha} s_{\beta} J(y+s, y+t, \bar{y}, 0) - H^{\alpha\beta} t_{\alpha} t_{\beta} J(y+s; y+t, 0; -\bar{y})) \right). \tag{3.11}
 \end{aligned}$$

For appropriately adjusted coefficients $a(h_1, h_2, h_J)$ and $\bar{a}(h_1, h_2, h_J)$ the remaining nonzero terms on the r.h.s. of (3.11) precisely cancel those produced by differentiation of ω by virtue of First on-shell theorem (2.2) in the ω -dependent part of the nonlinear deformation (5.1). However, as shown in [59], such terms are nonzero only beyond the region (3.6). In other words, $\mathcal{H}_{\text{cur}}$ (3.7) is $\mathcal{D}_{\text{tw}}$ -closed for any coefficients provided that (3.6) is true.

The deformation obtained in [54] was checked to give rise to proper current interactions of different spins. This is also easy to see from (3.7).

Consider for instance the spin-one current. Let $h_J = 1$. In this case, the primary sector has degree 2 in y and 0 in $\bar{y}$. We observe that since $C(y, 0|x)$ enters (2.2) with the factor of $\bar{\eta}$ the contribution of the current to the Maxwell equations on the spin-one potential $\omega(0, 0|x)$ is

$$\begin{aligned}
 i\bar{\eta} \mathcal{H}_{\text{cur}}(w, J) \Big|_{h_J=1} &= \frac{i}{4} \bar{\eta} \int_0^1 d\tau \sum_{h_1, h_2} a(h_1, h_2, 1) \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}} \bar{t}^{\dot{\beta}}] \\
 &\quad h(y, \tau \bar{s} + (1-\tau) \bar{t}) \left(J_{h_1, h_2, 1}(\tau y; 0, \bar{s}; \bar{t}) + J_{h_1, h_2, 1}(0; (1-\tau)y, \bar{s}; \bar{t}) \right). \tag{3.12}
 \end{aligned}$$

For $h_1 = h_2 = 0$ this gives

$$\begin{aligned}
 i\bar{\eta} \mathcal{H}_{\text{cur}}(w, J) \Big|_{h_1=h_2=0, h_J=1} &= \frac{i}{4} \bar{\eta} a(0, 0, 1) \int_0^1 d\tau \tau (1-\tau) \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}} \bar{t}^{\dot{\beta}}] \\
 &\quad \left(h(y, \bar{t}) J_{0,0,1}(y; 0, \bar{s}; 0) + h(y, \bar{s}) J_{0,0,1}(0; y, 0; \bar{t}) \right) \tag{3.13}
 \end{aligned}$$

and, completing the integrations,

$$i\bar{\eta} \mathcal{H}_{\text{cur}}(w, J) \Big|_{h_1=h_2=0, h_J=1} = \frac{\bar{\eta}}{24} a(0, 0, 1) h \left(y, \frac{\partial}{\partial \bar{t}} \right) \left(J_{0,0,1}(y; 0, \bar{t}; 0) - J_{0,0,1}(0; y, 0; \bar{t}) \right)_{\bar{t}=0}. \tag{3.14}$$

Plugging in the bilinear expression (2.14) for J and using rank-one equation (2.3) gives the usual spin-one current built from scalars, $J_n \sim \bar{\eta} C_1(0|x) \frac{\partial}{\partial x^n} C_2(0|x) - (1 \leftrightarrow 2)$. (For antisymmetrization over color indices one can start with matrix-valued fields C .)

Analogously, eq. (3.12) at $h_1 = \frac{1}{2}$ and $h_2 = -\frac{1}{2}$ reproduces the canonical spin-one current built from spin-1/2 fields

$$\begin{aligned} i\bar{\eta}\mathcal{H}_{\text{cur}}(w, J) \Big|_{h_1=\frac{1}{2}, h_2=-\frac{1}{2}, h_J=1} \\ = \frac{\bar{\eta}}{12} a \left(\frac{1}{2}, -\frac{1}{2}, 1 \right) h \left(y, \frac{\partial}{\partial \bar{t}} \right) \left(J_{-\frac{1}{2}, \frac{1}{2}, 1}(0; y, \bar{t}; 0) - J_{\frac{1}{2}, -\frac{1}{2}, 1}(y; 0, 0; \bar{t}) \right)_{\bar{t}=0}. \end{aligned} \quad (3.15)$$

Also it is easy to see that formula (3.7) properly reproduces Yukawa interactions in the sector of spins 0 and 1/2 while the selfinteraction of scalars is absent in agreement with the conclusion of [59] that HS interactions in the 0-form sector are conformally invariant, as well as with the observation that cubic coupling of scalars in the nonlinear HS theory is zero [68].

Let us stress that no nontrivial deformations beyond usual current interactions can be expected in the lowest order because conserved currents fill in all σ_- cohomologies in free unfolded HS equations (2.2), (2.3), i.e. there is no room for further nontrivial interactions. (For more detail on the σ_- cohomology techniques see e.g. [69] and references therein.)

As mentioned above a trivial part of the current J can be removed by a (generalized) local field redefinition while the nontrivial part survives. However, there is a subtlety. Since HS theories are formulated in AdS space of non-zero radius $\rho = \lambda^{-1}$, nonlinear HS equations can give rise to infinite tails of higher-derivative terms. It is neither guaranteed nor needed that general conserved currents admit an expansion in power series of individual improvements $J = \sum_{n=0}^{\infty} a_n \rho^n J_n$ where J_0 is a spin- s primary current built from some constituent fields C , while J_n are descendants containing up to n derivatives of J_0 . What is really necessary is that the current J , that appears on the r.h.s. of the field equations, should allow a representation (3.4) with ΔJ removable by a generalized local field redefinition.

A nonlinear deformation following from nonlinear HS equations reproduces current interactions (3.7) with certain coefficients $a(h_1, h_1, h_J)$. Before going into detail of their derivation in section 5, we briefly sketch the form of nonlinear HS equations.

4 Nonlinear higher-spin equations in AdS₄

The key element of the construction of [47] is the dependence of the HS 1-forms and 0-forms on an additional Majorana spinor variable Z^A and Klein operators $K = (k, \bar{k})$

$$\omega(Y; K|x) \longrightarrow W(Z; Y; K|x), \quad C(Y; K|x) \longrightarrow B(Z; Y; K|x). \quad (4.1)$$

An additional spinor field $S_A(Z; Y; K|x)$, that carries only pure gauge degrees of freedom, plays a role of connection in Z^A directions. It is convenient to introduce anticommuting Z -differentials θ^A , $\theta^A \theta^B = -\theta^B \theta^A$, to interpret $S_A(Z; Y; K|x)$ as a 1-form in Z direction,

$$S = \theta^A S_A(Z; Y; K|x). \quad (4.2)$$

HS equations determining dependence on the variables Z_A in terms of “initial data”

$$\omega(Y; K|x) = W(0; Y; K|x), \quad C(Y; K|x) = B(0; Y; K|x), \quad (4.3)$$

are formulated in terms of the associative star product $*$ acting on functions of two spinor variables

$$(f * g)(Z; Y) = \int \frac{d^4 U d^4 V}{(2\pi)^4} \exp[iU^A V^B C_{AB}] f(Z + U; Y + U) g(Z - V; Y + V), \quad (4.4)$$

where $C_{AB} = (\epsilon_{\alpha\beta}, \bar{\epsilon}_{\dot{\alpha}\dot{\beta}})$ is the $4d$ charge conjugation matrix and U^A, V^B are real integration variables. 1 is a unit element of the star-product algebra, i.e. $f * 1 = 1 * f = f$. Star product (4.4) provides a particular realization of the Weyl algebra

$$[Y_A, Y_B]_* = -[Z_A, Z_B]_* = 2iC_{AB}, \quad [Y_A, Z_B]_* = 0, \quad [a, b]_* := a * b - b * a. \quad (4.5)$$

The Klein operators $K = (k, \bar{k})$ satisfy

$$k * w^\alpha = -w^\alpha * k, \quad k * \bar{w}^{\dot{\alpha}} = \bar{w}^{\dot{\alpha}} * k, \quad \bar{k} * w^\alpha = w^\alpha * \bar{k}, \quad \bar{k} * \bar{w}^{\dot{\alpha}} = -\bar{w}^{\dot{\alpha}} * \bar{k}, \quad (4.6)$$

$$k * k = \bar{k} * \bar{k} = 1, \quad k * \bar{k} = \bar{k} * k \quad (4.7)$$

with $w^\alpha = (y^\alpha, z^\alpha, \theta^\alpha)$, $\bar{w}^{\dot{\alpha}} = (\bar{y}^{\dot{\alpha}}, \bar{z}^{\dot{\alpha}}, \bar{\theta}^{\dot{\alpha}})$. These relations extend the action of the star product to the Klein operators.

The nonlinear HS equations are [47]

$$dW + W * W = 0, \quad (4.8)$$

$$dB + W * B - B * W = 0, \quad (4.9)$$

$$dS + W * S + S * W = 0, \quad (4.10)$$

$$S * B = B * S, \quad (4.11)$$

$$S * S = i(\theta^A \theta_A + \theta^\alpha \theta_\alpha F_*(B) * k * \kappa + \bar{\theta}^{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} \bar{F}_*(B) * \bar{k} * \bar{\kappa}), \quad (4.12)$$

where $F_*(B)$ is some star-product function of the field B . The simplest choice of linear functions

$$F_*(B) = \eta B, \quad \bar{F}_*(B) = \bar{\eta} B, \quad (4.13)$$

where η is a complex parameter

$$\eta = |\eta| \exp i\varphi, \quad \varphi \in [0, \pi), \quad (4.14)$$

leads to a class of pairwise nonequivalent nonlinear HS theories. The cases of $\varphi = 0$ and $\varphi = \frac{\pi}{2}$ correspond to so called A and B HS models distinguished by the property that they respect parity [68].

The left and right inner Klein operators

$$\kappa := \exp i z_\alpha y^\alpha, \quad \bar{\kappa} := \exp i \bar{z}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}, \quad (4.15)$$

which enter eq. (4.12), change a sign of undotted and dotted spinors, respectively,

$$(\kappa * f)(z, \bar{z}; y, \bar{y}) = \exp i z_\alpha y^\alpha f(y, \bar{z}; z, \bar{y}), \quad (\bar{\kappa} * f)(z, \bar{z}; y, \bar{y}) = \exp i \bar{z}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}} f(z, \bar{y}; y, \bar{z}), \quad (4.16)$$

$$\kappa * f(z, \bar{z}; y, \bar{y}) = f(-z, \bar{z}; -y, \bar{y}) * \kappa, \quad \bar{\kappa} * f(z, \bar{z}; y, \bar{y}) = f(z, -\bar{z}; y, -\bar{y}) * \bar{\kappa}, \quad (4.17)$$

$$\kappa * \kappa = \bar{\kappa} * \bar{\kappa} = 1, \quad \kappa * \bar{\kappa} = \bar{\kappa} * \kappa. \quad (4.18)$$

Perturbative analysis of eqs. (4.8)–(4.12) assumes their linearization around some vacuum solution. The simplest choice is

$$W_0(Z; Y; K|x) = w(Y|x), \quad S_0(Z; Y; K|x) = \theta^A Z_A, \quad B_0(Z; Y; K|x) = 0, \quad (4.19)$$

where $w(Y|x)$ is some solution to the flatness condition $dw + w * w = 0$. A flat connection $w(Y|x)$ bilinear in Y^A describes AdS_4

$$w(Y|x) = -\frac{i}{4} w^{AB} Y_A Y_B = -\frac{i}{4} (\omega^{AB} + h^{AB}) Y_A Y_B, \quad (4.20)$$

$$\omega^{AB} Y_A Y_B := \omega^{\alpha\beta} y_\alpha y_\beta + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}, \quad h^{AB} Y_A Y_B := 2h^{\alpha\dot{\beta}} y_\alpha \bar{y}_{\dot{\beta}}. \quad (4.21)$$

Propagating massless fields are described by K -even $W(Z; Y; K|x)$ and K -odd $B(Z; Y; K|x)$

$$W(Z; Y; -K|x) = W(Z; Y; K|x), \quad B(Z; Y; -K|x) = -B(Z; Y; K|x). \quad (4.22)$$

The fields of opposite parity in the Klein operators

$$W(Z; Y; -K|x) = -\mathcal{W}(Z; Y; K|x), \quad B(Z; Y; -K|x) = B(Z; Y; K|x) \quad (4.23)$$

are topological in the sense that every irreducible field describes at most a finite number of degrees of freedom. (For more detail see [47, 70, 71]). They can be treated as describing infinite sets of coupling constants in HS theory. In this paper all these fields are set to zero.

It is easy to see that in the first nontrivial order

$$B_1(Z; Y; K|x) = C(Y; K|x) \quad (4.24)$$

(for more detail see [47, 70, 71]). Thus, nonlinear HS equations (4.8)–(4.12) give rise to the doubled set of massless fields

$$C(Y; K|x) = C^{1,0}(Y|x)k + C^{0,1}(Y|x)\bar{k}. \quad (4.25)$$

In the sector of massless fields, the linearization of eqs. (4.8)–(4.12) just reproduces free equations (2.2), (2.3) [47, 70, 71] (see [72] for a simple derivation)

$$D_0 \omega(y, \bar{y}; K|x) = \frac{i}{4} \left(\eta \bar{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \bar{y}^{\dot{\alpha}} \partial \bar{y}^{\dot{\beta}}} C(0, \bar{y}; K|x)k + \bar{\eta} H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0; K|x)\bar{k} \right), \quad (4.26)$$

$$D_0 C(y, \bar{y}; K|x) = 0, \quad (4.27)$$

where

$$D_0 \omega := d\omega + w * \omega + \omega * w, \quad D_0 C := dC + w * C - C * w. \quad (4.28)$$

The twisted adjoint covariant derivative acting on $C^{i,1-i}(Y|x)$ in (2.3) results from the application of the adjoint covariant derivative to $C(Y; K|x)$ in (4.27) by virtue of (4.6).

For the analysis of HS field equations it is useful to unify the 1-forms W and S into a single field

$$\mathcal{W}(Z; Y; K; \theta|x) = d_x + W(Z; Y; K|x) + S(Z; Y; K; \theta|x), \quad (4.29)$$

where the space-time differentials dx^n are implicit. In these terms, system (4.8)–(4.12) acquires the concise form

$$\mathcal{W} * \mathcal{W} = i(\theta^A \theta_A + \theta^\alpha \theta_\alpha F_*(B) * k * \kappa + \bar{\theta}^{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} \bar{F}_*(B) * \bar{k} * \bar{\kappa}), \quad (4.30)$$

$$[\mathcal{W}, B]_* = 0. \quad (4.31)$$

5 Quadratic corrections in the 0-form sector

5.1 Quadratic deformation from nonlinear higher-spin equations

First nontrivial correction to equations on the 0-forms C has the form

$$D_0 C + [\omega, C]_* + \mathcal{H}(w, J) = 0, \quad (5.1)$$

where ω stands for the first-order (i.e. not containing the vacuum part w) part of the Z -independent part of HS connection (4.3) and J denotes bilinears of the 0-forms C

$$J(y_1; y_2, \bar{y}_1; \bar{y}_2; K|x) := C(y_1, \bar{y}_1; K|x) C(y_2, \bar{y}_2; K|x). \quad (5.2)$$

The presence of either k or $\bar{k}$ in the first factor of C leads to the change of a relative sign between the sector of Y_1 and Y_2 as in covariant derivative (2.15) upon the Klein operator in the first factor of $C(y_1, \bar{y}_1; K|x)$ is moved to the right through the second factor.

In these terms,

$$\mathcal{H}(w, J) = \mathcal{H}_\eta(w, J) + \mathcal{H}_{\bar{\eta}}(w, J), \quad (5.3)$$

where

$$\begin{aligned} \mathcal{H}_\eta(w, J) = & -\frac{i}{2} \eta \int \frac{dS dT}{(2\pi)^4} \exp i S_A T^A \int_0^1 d\tau \\ & [h(s, \tau \bar{y} - (1-\tau) \bar{t}) J(\tau s; -(1-\tau)y + t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) \\ & - h(t, \tau \bar{y} - (1-\tau) \bar{s}) J((1-\tau)y + s; \tau t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K)] * k, \end{aligned} \quad (5.4)$$

$$\begin{aligned} \mathcal{H}_{\bar{\eta}}(w, J) = & -\frac{i}{2} \bar{\eta} \int \frac{dS dT}{(2\pi)^4} \exp i S_A T^A \int_0^1 d\tau \\ & [h(\tau y - (1-\tau)t, \bar{s}) J(y + s; y + t, \tau \bar{s}; -(1-\tau)\bar{y} + \bar{t}; K) \\ & - h(\tau y - (1-\tau)s, \bar{t}) J(y + s; y + t, (1-\tau)\bar{y} + \bar{s}; \tau \bar{t}; K)] * \bar{k}. \end{aligned} \quad (5.5)$$

Formulae (5.4), (5.5) are simple consequences of the homotopy formulae obtained in [72]

$$D_0 C + \mathcal{H}_{\text{tw}}([W_1(Z; Y; K; \theta|x), C(Y; K|x)]_*) = 0, \quad (5.6)$$

where

$$\begin{aligned} \mathcal{H}_{\text{tw}} f(Z; Y; K|x) := & \exp \left\{ -\frac{i}{8} \omega^{AB} h_A{}^C \frac{\partial^2}{\partial \theta^B \partial \theta^C} + \frac{i}{2} h^{AB} Y_A \frac{\partial}{\partial \theta^B} \right\} \\ & \exp \left\{ -\frac{1}{2} \omega^{AB} \frac{\partial^2}{\partial Y^A \partial \theta^B} + \frac{1}{4} h^{AB} \frac{\partial^2}{\partial Z^A \partial \theta^B} \right\} f(Z; Y; K; \theta|x) \Big|_{Z=\theta=0} \end{aligned} \quad (5.7)$$

and $\mathcal{W}_1(Z; Y; K; \theta|x)$ is the first-order correction to $\mathcal{W}$ which, using (4.24), has the form

$$\mathcal{W}_1(Z; Y; K; \theta|x) = \omega(Y; K|x) + i\Delta_{ad}^*(\theta^\alpha \theta_\alpha \eta C * \kappa k + \bar{\theta}^{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} \bar{\eta} C * \bar{\kappa} \bar{k}), \quad (5.8)$$

where, as shown in [72],

$$\Delta_{ad}^* f(Z; Y; \theta) = \frac{i}{2} Z^A \frac{\partial}{\partial \theta^A} \int_0^1 \frac{d\tau}{\tau} \exp\left(-\frac{1-\tau}{2\tau} w^{BC} \frac{\partial^2}{\partial Y^B \partial \theta^C}\right) f(\tau Z; Y; \tau \theta). \quad (5.9)$$

In a slightly different form $\mathcal{H}(w, J)$ was also presented in [50].

Being derived from the consistent nonlinear equations, deformation terms (5.4) and (5.5) obey the compatibility conditions. Analogously to the analysis of the deformation in section 3 one can check that

$$D_0 \mathcal{H}_\eta(w, J) = \int_0^1 d\tau \frac{\partial}{\partial \tau} G(\tau) = G(1) - G(0), \quad (5.10)$$

where $G(1) = 0$ because of the cancellation between the two terms in (5.4) while $G(0)$ has the structure analogous to the r.h.s. of (3.11) canceling the terms resulting from the differentiation of ω in $[\omega, C]$ by virtue of (2.2).

The integration over S and T in (5.4), (5.5) brings infinite tails of contracted indices. As explained in section 2.1 this induces an infinite expansion in higher space-time derivatives of the constituent fields. Hence, formula (5.1) with $\mathcal{H}$ (5.4), (5.5) differs from the conventional current interactions (3.7) which, being free of the integration over s_α and t_α , contains a finite number of derivatives for any given spins s_1, s_2 of the constituent fields and s_J of the current.

To reproduce standard current interactions from nonlinear HS equations we have to find a field redefinition

$$C \rightarrow C'(Y; K|x) = C(Y; K|x) + \Phi(Y; K|x) \quad (5.11)$$

with Φ linear in J , bringing equation (5.1) to the form (3.2), (3.7) with some coefficients $a(s_1, s_2, s_J)$. Before going into details of its derivation note that, being somewhat analogous, formulae (3.7) and (5.4), (5.5) are essentially different. Namely, comparing the terms that contain usual star product with respect to the barred variables we observe that the arguments of the vierbein h depend on $\bar{y}$ in (5.4) but on y in (3.7). Naively, this makes it difficult to relate the two formulae by a field redefinition. Nevertheless, as shown below this can be achieved in two steps. Firstly we find in section 5.3 a field redefinition which, bringing the current terms to a local form, completely removes the dependence on y and $\bar{y}$ from the arguments of h . Secondly, in section 5.4, the deformation is brought by a local field redefinition to the canonical form (3.7) where h depends on y and $\bar{y}$ of opposite chiralities compared to (5.4), (5.5).

This chirality flip has a dramatic effect implying that the effective coupling constant in front of currents depends on $\eta\bar{\eta}$, hence resolving some paradoxical claims in the literature on the meaningless dependence on the parameter η (see [50, 52] and references therein). More precisely the authors of [50, 52] were assuming that the current deformation results

from the sectors η^2 and $\bar{\eta}^2$. If true, this would imply that the current contribution to the r.h.s. of the Fronsdal equations is not sign definite for an arbitrary phase of η , changing its sign under the phase rotation $\eta \rightarrow i\eta$, $\bar{\eta} \rightarrow -i\bar{\eta}$. In particular, this would imply that the coefficient in front of the stress tensor, i.e. the gravitational constant, is not positive definite that would indicate a serious inconsistency of either the applied approach or the field equations in question. Our results show that the approach proposed in this paper is free of this problem.

5.2 Ansatz

An appropriate Ansatz for the first field redefinition is

$$\Phi_{1\eta}(Y; K|x) = \int \frac{dS dT}{(2\pi)^4} \exp i S_A T^A \int \prod_{i=1}^3 d\tau_i \phi_{1\eta}(\tau_i) \frac{\partial}{\partial \tau_3} J(\tau_3 s + \tau_1 y; t - \tau_2 y, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k, \quad (5.12)$$

where $\phi_{1\eta}(\tau_i)$ is some function of three integration variables τ_i . The τ -integration is over $\mathbb{R}^3$ while $\phi_{1\eta}(\tau)$ is demanded to have a compact support allowing to freely integrate by parts.

An important feature of the Ansatz (5.12) is that the right sector in (5.12) is of the form of the star product with respect to dotted spinors, involving no homotopy parameters. The rationale behind this Ansatz is the usual separation variables with respect to z_α and $\bar{z}_{\dot{\alpha}}$ sectors expressed by the two facts. The first is that the action of the inner Klein operators κ and $\bar{\kappa}$ in the F and $\bar{F}$ -dependent terms on the r.h.s. of (4.12) leaves, respectively, the right and left sectors untouched. The second is that the vacuum part S_0 (4.19) is a sum of mutually commuting left and right components

$$S_0 = s_0 + \bar{s}_0, \quad s_0 := \theta^\alpha z_\alpha, \quad \bar{s}_0 := \bar{\theta}^{\dot{\alpha}} \bar{z}_{\dot{\alpha}}. \quad (5.13)$$

Reconstruction of nonlinear corrections to HS equations is based on the perturbative resolution for the dependence on Z variables from the equations (4.10), (4.11) and (4.12) using the fact that the star-commutators with s_0 and $\bar{s}_0$ are proportional to de Rham derivatives in the left and right sectors of z_α and $\bar{z}_{\dot{\alpha}}$, respectively. Following the standard separation of variables approach, it is natural to look for a proper resolution operator d_Z^* for S_0 in the factorized form

$$d_Z^* = d_z^* + d_{\bar{z}}^* \quad (5.14)$$

with independent left and right resolutions d_z^* and $d_{\bar{z}}^*$. This leads to the factorized Ansatz (5.12) for any d_z^* and $d_{\bar{z}}^*$. Under this factorization condition eq. (5.12) can be checked to describe the only field redefinition that gives rise to local quadratic corrections to the HS equations. The details of this analysis are not presented in this paper since it is elementary to prove the uniqueness using the Green function obtained in [46] where it is also shown that the solution presented in this paper plays a distinguished role from the locality perspective, providing a proper local frame in the HS theory.

5.3 Elimination of $\bar{y}$

An elementary computation using the identity

$$\int \frac{dsdt}{(2\pi)^2} \exp i[s_\alpha t^\alpha] \left(y_\alpha \frac{\partial}{\partial \tau_3} - i \left(\partial_{2\alpha} \frac{\partial}{\partial \tau_1} + \partial_{1\alpha} \frac{\partial}{\partial \tau_2} \right) \right) J(\tau_3 s + \tau_1 y; -\tau_2 y + t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) = 0, \quad (5.15)$$

expressing the fact that antisymmetrization over any three two-component indices is zero, yields

$$\begin{aligned} D_0 \Phi_{1\eta}(Y; K|x) &= i h^{\alpha\dot{\alpha}} \int \frac{dSdT}{(2\pi)^4} \exp i[S_A T^A] \int d^3 \tau_i \phi_{1\eta}(\tau) \\ &\quad \left[(\tau_2 \bar{\partial}_{1\dot{\beta}} - \tau_1 \bar{\partial}_{2\dot{\beta}}) \left(\partial_{2\alpha} \frac{\partial}{\partial \tau_1} + \partial_{1\alpha} \frac{\partial}{\partial \tau_2} \right) - i(1 - \tau_1 - \tau_2) \left(\partial_{2\alpha} \frac{\partial}{\partial \tau_1} + \partial_{1\alpha} \frac{\partial}{\partial \tau_2} \right) \bar{y}_{\dot{\beta}} \right. \\ &\quad + \frac{\partial}{\partial \tau_3} \left(i\tau_3 (\partial_{2\alpha} + \partial_{1\alpha}) \bar{y}_{\dot{\beta}} - (1 - \tau_1 - \tau_3) \partial_{1\alpha} \bar{\partial}_{1\dot{\beta}} + (1 - \tau_2 - \tau_3) \partial_{2\alpha} \bar{\partial}_{2\dot{\beta}} \right. \\ &\quad \left. \left. + \tau_1 \partial_{1\alpha} \bar{\partial}_{2\dot{\beta}} - \tau_2 \partial_{2\alpha} \bar{\partial}_{1\dot{\beta}} \right) \right] J(\tau_3 s + \tau_1 y; -\tau_2 y + t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k. \quad (5.16) \end{aligned}$$

Here $\partial_{1\alpha}(\bar{\partial}_{1\dot{\alpha}})$ and $\partial_{2\alpha}(\bar{\partial}_{2\dot{\alpha}})$ denote derivatives over the first and second undotted(dotted) spinorial arguments of J defined to anticommute with the respective Klein operators $k(\bar{k})$ so that

$$\partial_2 A(w_1) k B(w_2) = -A(w_1) k \partial_2 B(w_2). \quad (5.17)$$

This sign change matters in relations (5.15), (5.16) with the current J (5.2) built from two 0-forms C containing k or $\bar{k}$.

Setting

$$\phi_{1\eta}(\tau) = \phi'_{1\eta}(\tau) \delta \left(1 - \sum_{i=1}^3 \tau_i \right) \quad (5.18)$$

and integrating by parts we obtain

$$\begin{aligned} D_0 \Phi_{1\eta}(Y; K|x) &= i \int \frac{dSdT}{(2\pi)^4} \exp i[S_A T^A] \int d^3 \tau \times \phi'_{1\eta}(\tau) \delta \left(1 - \sum_{i=1}^3 \tau_i \right) \\ &\quad \left\{ i h(\partial_2, \bar{y}) \left(\frac{\overleftarrow{\partial}}{\partial \tau_1} - \frac{\overleftarrow{\partial}}{\partial \tau_3} \right) \tau_3 + i h(\partial_1, \bar{y}) \left(\frac{\overleftarrow{\partial}}{\partial \tau_2} - \frac{\overleftarrow{\partial}}{\partial \tau_3} \right) \tau_3 - h(\partial_1, \bar{\partial}_1) \left(\frac{\overleftarrow{\partial}}{\partial \tau_2} - \frac{\overleftarrow{\partial}}{\partial \tau_3} \right) \tau_2 \right. \\ &\quad \left. + h(\partial_2, \bar{\partial}_2) \left(\frac{\overleftarrow{\partial}}{\partial \tau_1} - \frac{\overleftarrow{\partial}}{\partial \tau_3} \right) \tau_1 - h(\partial_1, \bar{\partial}_2) \left(\frac{\overleftarrow{\partial}}{\partial \tau_3} - \frac{\overleftarrow{\partial}}{\partial \tau_2} \right) \tau_1 + h(\partial_2, \bar{\partial}_1) \left(\frac{\overleftarrow{\partial}}{\partial \tau_3} - \frac{\overleftarrow{\partial}}{\partial \tau_1} \right) \tau_2 \right\} \\ &\quad J(\tau_3 s + \tau_1 y; -\tau_2 y + t, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k. \quad (5.19) \end{aligned}$$

Finally, for

$$\phi'_{1\eta} = \frac{1}{2} \eta \theta(\tau_1) \theta(\tau_2) \theta(\tau_3), \quad (5.20)$$

this gives

$$\begin{aligned}
 D_0 \Phi_{1\eta}(Y; K|x) = & -\frac{i}{2} \eta \int \frac{dS dT}{(2\pi)^4} \exp i[S_A T^A] \int_0^1 d\tau \\
 & \left[h(s, \tau \bar{y} - (1-\tau) \bar{t}) J(\tau s; -(1-\tau)y + t, \bar{y} + \bar{s}; \bar{y} + \bar{t}) \right. \\
 & - h(t, \tau \bar{y} - (1-\tau) \bar{s}) J(s + (1-\tau)y; \tau t, \bar{y} + \bar{s}; \bar{y} + \bar{t}) \\
 & \left. - i h(\partial_1 + \partial_2, (1-\tau) \bar{t} + \tau \bar{s}) J(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) \right] * k. \quad (5.21)
 \end{aligned}$$

Comparing this with (5.4) we find that

$$\mathcal{H}_\eta(w, J) = D_0 \Phi_{1\eta}(J) + \mathcal{H}'_\eta(w, J), \quad (5.22)$$

where, completing integration over s and t ,

$$\begin{aligned}
 \mathcal{H}'_\eta(w, J) = & \frac{\eta}{2} \int \frac{d\bar{s} d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_\alpha \bar{t}^{\dot{\alpha}}] \\
 & \times \int_0^1 d\tau h(\partial_1 + \partial_2, (1-\tau) \bar{t} + \tau \bar{s}) J(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k. \quad (5.23)
 \end{aligned}$$

We observe that, being free of the integration over s and t , $\mathcal{H}'_\eta(w, J)$ is a local functional of J . However, containing a finite number of derivatives of physical (i.e. primary) fields for any three spins s_1, s_2 and s_J , $\mathcal{H}'_\eta(w, J)$ differs from $\mathcal{H}_{\text{cur}}$ (3.7). Depending on $\partial_1 + \partial_2$ instead of y , $\mathcal{H}'_\eta(w, J)$ contains one extra space-time derivative compared to (3.7). In Minkowski space this would imply that $\mathcal{H}'_\eta(w, J)$ was an improvement. However this is not the case in AdS_4 where derivatives of different orders talk to each other via nonzero commutators of covariant derivatives.

5.4 Reincarnation of y

Let

$$\Phi_{2\eta} = -\frac{1}{2} \eta \int \frac{d\bar{s} d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_\beta \bar{t}^{\dot{\beta}}] \int_0^1 d\tau J(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k. \quad (5.24)$$

An elementary computation yields

$$\begin{aligned}
 D_0 \Phi_{2\eta} = & -\frac{i}{2} \eta \int \frac{d\bar{s} d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_\beta \bar{t}^{\dot{\beta}}] \\
 & \times \int_0^1 d\tau h(y - i(\partial_2 + \partial_1), \tau \bar{s} + (1-\tau) \bar{t}) J(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k. \quad (5.25)
 \end{aligned}$$

This gives

$$\mathcal{H}_\eta(w, J) = \mathcal{H}_{\eta \text{ impr}}(w, J) + \mathcal{H}_{\eta \text{ cur}}(w, J), \quad (5.26)$$

where

$$\mathcal{H}_{\eta \text{ impr}}(w, J) := D_0(\Phi_{1\eta} + \Phi_{2\eta}) \quad (5.27)$$

and

$$\begin{aligned} \mathcal{H}_{\eta \text{ cur}}(w, J) = & -\frac{i}{2}\eta \int \frac{d\bar{s}d\bar{t}}{(2\pi)^2} \exp i[\bar{s}_{\dot{\beta}}\bar{t}^{\dot{\beta}}] \\ & \times \int_0^1 d\tau h(y, \tau\bar{s} + (1-\tau)\bar{t}) J(\tau y; -(1-\tau)y, \bar{y} + \bar{s}; \bar{y} + \bar{t}; K) * k. \end{aligned} \quad (5.28)$$

Analogously, in the $\bar{\eta}$ sector

$$\begin{aligned} \mathcal{H}_{\bar{\eta} \text{ cur}}(w, J) = & -\frac{i}{2}\bar{\eta} \int \frac{dsdt}{(2\pi)^2} \exp i[s_{\beta}t^{\beta}] \\ & \times \int_0^1 d\tau h(\tau s + (1-\tau)t, \bar{y}) J(y + s; y + t, \tau\bar{y}; -(1-\tau)\bar{y}; K) * \bar{k}. \end{aligned} \quad (5.29)$$

This expressions just reproduce (3.7) with

$$a(h_1, h_2, h_J) = -\frac{i}{2}\eta, \quad \bar{a}(h_1, h_2, h_J) = -\frac{i}{2}\bar{\eta}. \quad (5.30)$$

Thus, the current deformation of massless equations resulting from the nonlinear HS equations is shown to reproduce the standard local current interactions up to improvement $J_{\text{impr}}(w, J)$ which can be compensated by the field redefinition

$$C \rightarrow C' = C + \Phi_{1\eta} + \Phi_{2\eta} + \Phi_{1\bar{\eta}} + \Phi_{2\bar{\eta}}. \quad (5.31)$$

The flip of the $\bar{y}$ dependence of h in (3.7) to the y dependence in (5.28) has an important consequence. Since contribution (3.12) to the spin-one current contains a factor of $\bar{\eta}$ while (5.28) contains a factor of η they combine into the coupling constant f (2.11) while the phase of η cancels out. Analogously to the spin-one example of section 3, taking into account that left and right components of 0-forms contribute to the gauge field sector with the additional factors ηk and $\bar{\eta} \bar{k}$ as in (4.26) we conclude that the physical current contributing to the gauge field sector is proportional to $\eta\bar{\eta}$. Its explicit form is presented in [55].

The conclusion that current interactions resulting from the nonlinear HS equations are independent of the phase of η differs drastically from that of [50, 52] where it was claimed that the result (though divergent) depends on the phase of η and even vanishes or changes a sign for certain phases.

An important question is which field redefinitions (5.11) can be treated as local and which cannot. The transformation induced by Φ_2 (5.12) is genuinely local containing a finite number of derivatives. That induced by Φ_1 is not local in the standard sense, containing an infinite tale of derivatives. We postpone the discussion of this issue till the comprehensive paper [46] showing in particular that the nonlinear field redefinition described by Φ_1 is fixed uniquely by the locality condition at the higher orders.

6 Holographic interpretation

Naively, holographic interpretation of the obtained results may look problematic since the current interactions resulting from the nonlinear HS equations turn out to be independent

of the phase of η . One may wonder how the variety of models conjectured to be dual to HS theory with different η [25, 26] can be generated by the same lowest-order vertex. This seeming paradox gets a natural resolution by noticing that different boundary models result from different boundary conditions in the same bulk model. Within the approach of [29, 45] this works as follows.

6.1 Boundary conditions

Due to dependence on the Klein operators k and $\bar{k}$ the field pattern in the AdS₄ HS theory is doubled. Referring for more detail to sections 8–10 of [29] the asymptotic behaviour of the Weyl forms is

$$C^{j1-j}(y, \bar{y}|\mathbf{x}, \mathbf{z}) = \mathbf{z} \exp(y_\alpha \bar{y}^\alpha) T^{j1-j}(w, \bar{w}|\mathbf{x}, \mathbf{z}), \quad (6.1)$$

where C^{j1-j} is defined in (4.25), $\mathbf{z}$ is the Poincaré coordinate,

$$w^\alpha = \mathbf{z}^{1/2} y^\alpha, \quad \bar{w}^\alpha = \mathbf{z}^{1/2} \bar{y}^\alpha \quad (6.2)$$

and the 0-forms T^{j1-j} are associated with the boundary currents.

AdS/CFT correspondence assumes certain boundary conditions at infinity $\mathbf{z} \rightarrow 0$. In terms of AdS₄ HS Weyl forms they relate $T^{j1-j}(w, 0|\mathbf{x}, 0)$ to $T^{1-jj}(0, iw|\mathbf{x}, 0)$. Indeed, in eqs. (8.28), (8.29) of [29], the combinations of currents

$$\bar{\eta} T^{j1-j}(w, 0|\mathbf{x}, 0) - \eta T^{1-jj}(0, iw|\mathbf{x}, 0) \quad (6.3)$$

were shown to contribute the *r.h.s.s* of the equations for HS connections in the boundary limit $\mathbf{z} \rightarrow 0$. As a result, the contribution of HS connections at the boundary cannot be neglected except for the boundary conditions

$$\bar{\eta} T_+^{j1-j}(w, \bar{w}|\mathbf{x}, 0) - \eta T_-^{1-jj}(-i\bar{w}, iw|\mathbf{x}, 0) = 0, \quad (6.4)$$

where T_+ and T_- are the positive and negative helicity parts of $T(y, \bar{y}|x)$, respectively.¹ Note that the relative sign of the factors of i in the arguments of the second term of (6.4) is determined by the condition that the fields $T_+^{j1-j}(w, \bar{w}|\mathbf{x}, 0)$ obey the unfolded equations (8.18) of [29] in the coordinates $\mathbf{x}$

$$\left[d_{\mathbf{x}} - i dx^{\alpha\beta} \frac{\partial^2}{\partial w^{+\alpha} \partial w^{-\beta}} \right] T^{i1-j}(w^+, w^-|\mathbf{x}, \mathbf{z}) = 0. \quad (6.5)$$

As stressed in [29], the decoupling of boundary HS connections is necessary for the proper interpretation of the boundary model in terms of CFT. If boundary HS connections do couple to the HS currents, being conformally invariant, the boundary model is not a CFT in the standard sense lacking the gauge invariant stress tensor. Generally, one can use arbitrary boundary conditions reproducing different conformally invariant boundary field theories. However most of them, except for those associated with boundary conditions (6.4),

¹I acknowledge with gratitude a useful discussion of this option with Slava Didenko and Zgenya Skvortsov during the KITP program New methods in nonperturbative quantum field theory (Jan 6–Apr 11 2014).

are not usual CFT. Such models should correspond to conformal models considered in [73] where the effects of boundary HS gauge fields in the $\text{AdS}_4/\text{CFT}_3$ HS duality were explored.

For the A -model ($\eta = 1$) and B -model ($\eta = i$) that respect parity, conditions (6.4) are equivalent to

$$\mathcal{A}^{jj}(w^+, w^-|\mathbf{x}) := T^{j1-j}(w^+, w^-|\mathbf{x}, 0) - T^{1-jj}(-iw^-, iw^+|\mathbf{x}, 0) = 0, \quad (6.6)$$

$$\mathcal{B}^{jj}(w^+, w^-|\mathbf{x}) := T^{j1-j}(w^+, w^-|\mathbf{x}, 0) + T^{1-jj}(-iw^-, iw^+|\mathbf{x}, 0) = 0 \quad (6.7)$$

called in [29] $\mathcal{A}$ and $\mathcal{B}$ conditions, respectively. Being based on the parity automorphism of the nonlinear HS theory, the $\mathcal{A}$ condition at $\eta = \bar{\eta}$ and $\mathcal{B}$ condition at $\eta = -\bar{\eta}$ can be imposed in all orders of the perturbative expansion. This conclusion matches the well-known fact [23, 68, 74] that the A -model with $\mathcal{A}$ boundary conditions and B -model with $\mathcal{B}$ conditions correspond to free conformal bosons and fermions on the boundary (see also [30] and references therein). For any other boundary conditions and/or phases of η the boundary dual theory is not free. Hence the full HS theory containing all boundary models as specific reductions is not equivalent to a free boundary theory (cf [27]).

For general η , the HS theory is not P -symmetric. As a result, conditions (6.4), which distinguish between positive and negative helicities, break P -symmetry. It is convenient to use following [29] the mirror extension of the Poincaré coordinate $\mathbf{z}$ to negative and, more generally, complex values allowing to extend conditions (6.4) to

$$\bar{\eta}T_+^{j1-j}(w, \bar{w}|\mathbf{x}, \mathbf{z}) - \eta T_-^{1-jj}(-i\bar{w}, iw|\mathbf{x}, -\mathbf{z}) = 0. \quad (6.8)$$

These conditions relate self-dual fields in the original space with anti-self-dual fields in the mirror space and vice versa in a way manifestly dependent on the phase of η . Note that the form of the extension of conditions (6.4) with arbitrary phase of η to higher orders is not obvious and should be derived perturbatively.

Reduction of the universal HS vertex found in this paper to different subsectors associated with boundary conditions (6.4) reproduces the conformal structures identified by Maldacena and Zhiboedov in [27]. Indeed, after imposing the boundary conditions (6.4) the remaining real boundary fields are

$$j^j(w^+, w^-|\mathbf{x}) := \frac{1}{2} \left(\bar{\eta}T_+^{j1-j}(w^+, w^-|\mathbf{x}, 0) + \eta T_-^{1-jj}(-iw^-, iw^+|\mathbf{x}, 0) \right) = \bar{\eta}T_+^{j1-j}(w^+, w^-|\mathbf{x}, 0). \quad (6.9)$$

The fact that the bulk HS vertex is independent of the phase φ of η implies that the boundary vertex (to be represented by the Lagrangian 4-form in terms of [45]) has the structure

$$V = \sum_{i,j=1,2} (a_{ij}T_+^{i1-i}T_+^{j1-j} + b_{ij}T_-^{i1-i}T_-^{j1-j} + e_{ij}T_-^{i1-i}T_+^{j1-j}), \quad (6.10)$$

where a_{ij} , b_{ij} and e_{ij} are some φ -independent coefficients built from $\mathbf{z}$ -odd components of derivatives of the boundary HS connections and background fields (indices are implicit). In terms of real φ -independent currents (6.9) V reads

$$V = \frac{1}{\eta\bar{\eta}} \sum_{i,j=1,2} \left(\exp 2i\varphi a_{ij}j_+^{i1-i}j_+^{j1-j} + \exp -2i\varphi b_{ij}j_-^{i1-i}j_-^{j1-j} + e_{ij}j_-^{i1-i}j_+^{j1-j} \right). \quad (6.11)$$

Since the dependence on φ in (6.11) is manifest we can identify the parity-even boson and fermion vertices as those associated with $\varphi = 0$ and $\varphi = \pi/2$, respectively. This gives

$$V_b = \frac{1}{\eta\bar{\eta}} \sum_{i,j=1,2} (a_{ij} j_+^{i1-i} j_+^{j1-j} + b_{ij} j_-^{i1-i} j_-^{j1-j} + e_{ij} j_-^{i1-i} j_+^{j1-j}), \quad (6.12)$$

$$V_f = \frac{1}{\eta\bar{\eta}} \sum_{i,j=1,2} (-a_{ij} j_+^{i1-i} j_+^{j1-j} - b_{ij} j_-^{i1-i} j_-^{j1-j} + e_{ij} j_-^{i1-i} j_+^{j1-j}). \quad (6.13)$$

Since parity transformation exchanges the left and right sectors, i.e. positive and negative helicities, the remaining parity-odd vertex is

$$V_o = \frac{i}{\eta\bar{\eta}} \sum_{i,j=1,2} (a_{ij} j_+^{i1-i} j_+^{j1-j} - b_{ij} j_-^{i1-i} j_-^{j1-j}). \quad (6.14)$$

This results in the following formula

$$V = \cos^2(\varphi) V_b + \sin^2(\varphi) V_f + \frac{1}{2} \sin(2\varphi) V_o, \quad (6.15)$$

which precisely matches the form of the deformation of the HS current algebra found in [28] with $\sin^2(\varphi) = (1 + \tilde{\lambda}^2)^{-1}$ being in agreement with the AdS/CFT expectation and the conjecture of Giombi and Yin [75].

To summarize, the proper dependence on the phase parameter in the holographic duals of the AdS₄ HS theory is reproduced by the phase-independent vertex of the bulk HS theory derived in this paper via imposing appropriate phase-dependent boundary conditions.

6.2 Boundary limit

The important question is what is the role of the nonlinear field redefinition (5.31) from the boundary point of view? In particular, is it local or nonlocal in the boundary limit? Naively, formula (6.2) suggests that higher-derivative contributions resulting from infinite differentiations of the spinorial variables y and $\bar{y}$ in the arguments of $T^{j1-j}(w, \bar{w}|\mathbf{x}, \mathbf{z})$ are suppressed by additional powers of $\mathbf{z}$ which disappear at $\mathbf{z} \rightarrow 0$ so that only some local parts can survive. However, the story is less trivial.

The important issue here is what happens with the $\mathbf{z}$ -independent factor in (6.1)

$$F := \exp(y_\alpha \bar{y}^\alpha). \quad (6.16)$$

As shown in [29], F is a Fock projector

$$F * F = F. \quad (6.17)$$

A potential difficulty is that

$$\tilde{F} := k * F * k = \exp(-y_\alpha \bar{y}^\alpha), \quad (6.18)$$

which is also a Fock projector

$$\tilde{F} * \tilde{F} = \tilde{F}, \quad (6.19)$$

has divergent product with F

$$F * \tilde{F} = \tilde{F} * F = \infty. \quad (6.20)$$

This has a consequence that the homotopy integral

$$\int d\tau_1 d\tau_2 \rho(\tau) \exp \tau_1 y_\alpha \bar{y}^\alpha * \exp \tau_2 y_\alpha \bar{y}^\alpha \quad (6.21)$$

may diverge at $\tau_1, \tau_2 \rightarrow -1$ that can lead to divergencies in the boundary limit. (For more detail see [29, 45].) However, as shown recently in [76], for the solution found in this paper the divergency of this type is absent.

7 Discussion

Confining ourselves to the 0-form sector, we have shown how nonlinear HS equations in AdS_4 reproduce standard local current interactions upon a proper field redefinition. This prescription has to be compared with other suggestions proposed in recent papers [50–52].

Conceptually, our conclusions are just opposite to those of the authors of [50, 52] who arrived at the conclusion that there are difficulties with the nonlinear HS equations exhibiting a “naked singularity” in the analysis of locality, leading the authors of [50] to counterintuitive conclusions like, e.g., the cancellation of the current contributions at $\eta = \exp \frac{i\pi}{4}$. On the other hand, our conclusion is that nonlinear HS equations properly reproduce usual current interactions of massless fields with the coupling constant independent of the phase of η . (Strictly speaking we have shown this directly only for the spin-one currents. However, because interactions of various spins are controlled by the HS symmetry, the same is true for all other spins as shown explicitly in [55].)

The mechanism underlying cancelation of the phase of η is tricky. Naively, that the arguments of the vierbein h in (5.4) depend on $\bar{y}$ suggests that this part of the deformation should contribute to the right sector of $\bar{y}$. However this is not the case since the first field redefinition (5.12) leads to an expression with h independent of both $\bar{y}$ and y while the second one (5.24) results in deformation (5.28) with h depending on y . Hence the nontrivial part of deformation (5.4) lives in the left sector. This flip of chirality upon the field redefinition effectively replaces one of the factors of η by $\bar{\eta}$ and vice versa.

Nevertheless, as shown in section 6 along the lines of [29], the conclusion that the HS current interactions depend on $\eta\bar{\eta}$ is in agreement with various conjectures on holographic duality of the HS theory with different η [25, 26] as well as the Maldacena-Zhiboedov cubic correlator [28] since the proper dependence on η of the boundary models results from η -dependent boundary conditions in the same bulk model.

Since HS theory is based on HS symmetries it is crucially important to use a gauge invariant setup for the analysis of holography. This concerns not only the usual gauge symmetries associated with Fronsdal fields but also the Stueckelberg-like symmetries that control the dependence on the spinorial variables Z^A (see section 4). To the best of our knowledge, the only available proposal of this type to HS holography is that of [45] which is currently under study [77]. *A priori*, it could happen that the field redefinition bringing the current interactions to the canonical form can result from an allowed gauge transformation

in the full space of Z, Y spinorial variables. In that case it simply will not contribute to the final result.

For simplicity, in this paper we focus on the sector of 0-forms which directly reproduces currents of lower spins $s_J \leq 1$ and implicitly (i.e. via descendants) currents of all higher spins. The fact that it turns out to be possible to derive local current interactions in this sector suggests unambiguously that the same should be true in the 1-form sector to which primary HS currents contribute. Details of the analysis of this problem are given in [55]. In particular, results of [55] simplify direct comparison of the relative coefficients in front of contributions of currents of different spins with those derived recently in [78] for the case of $\eta = 1$ from the holographic principle. The two are anticipated to coincide because at least for the sectors associated with free boundary theories the relative coefficients are determined by the HS symmetry. Recently this coincidence was explicitly checked in [79].

The simple and elegant representation of the field redefinition bringing HS interactions to the canonical current form in terms of an integral over certain simplex suggests that there should exist a deeper way to understand HS perturbations in terms of certain geometry in the space of the homotopy parameters. Further understanding of this issue may be very interesting and suggestive from both HS and geometric perspectives.

We believe that the results of this paper properly illustrate efficiency of the methods of [29, 45] in application to HS holographic duality. It would be extremely interesting to reproduce the generating functional for boundary correlators directly from the invariant functional of [45] which problem is currently under study [77].

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