

Assessing lepton-flavour non-universality from $B \rightarrow K^* \ell \ell$ angular analyses

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ABSTRACT: The $B \rightarrow K^* \mu \mu$ decay exhibits deviations with respect to Standard Model expectations and the measurement of the ratio R_K hints at a violation of lepton-flavour universality in $B \rightarrow K \ell \ell$ transitions. Both effects can be understood in model-independent fits as a short-distance contribution to the Wilson coefficient $C_{9\mu}$, with some room for similar contributions in other Wilson coefficients for $b \rightarrow s \mu \mu$ transitions. We discuss how a full angular analysis of $B \rightarrow K^* e e$ and its comparison with $B \rightarrow K^* \mu \mu$ could improve our understanding of these anomalies and help confirming their interpretation in terms of short-distance New Physics. We discuss several observables of interest in this context and provide predictions for them within the Standard Model as well as within several New Physics benchmark scenarios. We pay special attention to the sensitivity of these observables to hadronic uncertainties from SM contributions with charm loops.

KEYWORDS: Beyond Standard Model, Heavy Quark Physics

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1 Introduction

In recent years, several deviations from the Standard Model (SM) have arisen in B -physics observables, with the experimental confirmation of the anomaly [1] in the $B \rightarrow K^* \mu \mu$ observable P_5' [2–5], several tensions in branching ratios for $b \rightarrow s \mu \mu$ transitions [6–8] and evidence for the violation of lepton flavour universality (LFU) in different observables ($R_K, R(D), R(D^*)$) [9–13].¹ Global analyses of the deviations in $b \rightarrow s \ell \ell$ transitions point towards a large additional contribution to the Wilson coefficient $C_{9\mu}$ of the semileptonic

¹The observable P_2 [14, 15] exhibited also a coherent deviation in the bin [2,4,3] with the 1 fb^{-1} dataset [3]. Given the large experimental error in the 3 fb^{-1} dataset in the bin [2.5,4] due to $F_L^{\text{exp}} \simeq 1$ in that bin [4], it was not possible to confirm nor disprove this deviation. It would be very desirable to collect more data in that bin and in particular to measure F_L with a higher precision. In fact, a recent analysis by the Babar collaboration [16] hints at a deviation in the same region.

operator in the effective Hamiltonian [17] for $b \rightarrow s\mu\mu$, as initially discussed in ref. [1] and later confirmed by several works [18–24]. Even though $b \rightarrow s\mu\mu$ observables are affected by hadronic uncertainties which are difficult to estimate [1, 25–29], such a contribution to $C_{9\mu}$ in $b \rightarrow s\mu\mu$ appears as a very economical way of explaining a large set of deviations with respect to SM expectations, which could not be achieved by alternative explanations advocating different hadronic effects [26–29]. In addition, theory predictions for some $b \rightarrow s\mu\mu$ observables may also get a better agreement with data once additional contributions are allowed in other Wilson coefficients (such as $C_{9'\mu}$ or $C_{10\mu}$) [22]. On the other hand, $B \rightarrow K^*ee$ observables and the R_K ratio suggest that $b \rightarrow see$ transitions agree well with the SM [30], pointing to explanations with New Physics (NP) models with a maximal violation of LFU, affecting only muon and not electron modes. These hints of lepton flavour non-universality (LFNU) have triggered a lot of theoretical activity [31–51].

As discussed in several works [1, 25, 28, 29, 52–55], long-distance SM contributions from diagrams involving charm loops enter the computation of $b \rightarrow s\ell\ell$ processes, acting as additional contributions to the Wilson coefficient C_9 . These contributions are process-dependent and they must be estimated through different theoretical methods according to the dilepton invariant mass q^2 . The latest estimates of these contributions [25, 52] have been included in the global fits for $B \rightarrow K^*\mu\mu$ [1, 22–24], providing the consistent picture described above. In particular, bin-by-bin fits indicate that the data agrees well with a single, process-independent contribution to $C_{9\mu}$, independent of the dimuon invariant mass, and present only in muon modes, as expected from a short-distance (NP) flavour-non-universal contribution. In order to confirm this pattern, it would be very desirable to design observables probing:

- a) only the short-distance part of $C_{9\ell}$,
- b) other Wilson coefficients, such as $C_{10\ell}$, which do not receive long-distance contributions from the SM,
- c) the amount of lepton-flavour non-universality between electron and muon modes.

In all cases, hadronic uncertainties should remain controlled: while the *observation* of lepton non-universality is a clear signal of NP, the *interpretation* of the effect in terms of NP is affected by the same hadronic uncertainties as the individual $b \rightarrow s\ell\ell$ modes.

The purpose of this article is to investigate which observables can be built that match these criteria, once a full angular analysis of $B \rightarrow K^*ee$, with an accuracy comparable to that of $B \rightarrow K^*\mu\mu$, is available. If the most obvious quantity consists in comparing branching ratios through the ratio R_{K^*} (similar to R_K) (see ref. [22] for predictions for these ratios for different NP scenarios), it is also interesting to consider other ratios probing the violation of LFU using the angular coefficients J_i describing the whole angular kinematics of these decays.² In this note, we will discuss observables that can measure LFNU in $B \rightarrow K^*\ell\ell$. Some of them are variations around the basis of optimised observables introduced in

²Some of such ratios have been considered briefly in refs. [21, 24, 27], but a systematic study of hadronic uncertainties (in the presence of NP) and optimised ratios has not been made.

refs. [2, 15] and others can be built directly by combining angular coefficients from muon and electron modes. We will discuss the advantages of these observables in the context of hadronic uncertainties, and provide predictions in the SM and in several benchmark scenarios corresponding to the best-fit points obtained in our recent global analysis of $b \rightarrow s\ell\ell$ modes [22].

We begin with a presentation of the observables of interest in section 2. In addition to observables naturally derived from the angular coefficients J_i and the optimised observables $P_i^{(\prime)}$, we consider other observables, namely B_i and M (and \tilde{B}_i, \tilde{M}) which have a reduced sensitivity to charm contributions in some NP scenarios. In section 3 we present our predictions in the SM and in several NP benchmark points, illustrating how these observables can help in discerning among NP scenarios and how (in)sensitive they are with respect to hadronic uncertainties. We present our conclusions in section 4. In the appendices we discuss the dependence of M and \tilde{M} observables on charm contributions, we recall the definition of binned observables, and we provide further predictions for the various observables within the different benchmark scenarios.

2 $B \rightarrow K^*\ell\ell$ observables assessing lepton flavour universality

2.1 Observables derived from J_i, P_i and S_i

We want to exploit the angular analyses of both $B \rightarrow K^*\mu\mu$ and $B \rightarrow K^*ee$ decays in order to build observables that will probe the violation of LFU, the short-distance part of $C_{9\mu}$ and/or the other Wilson coefficients, with limited hadronic uncertainties. Natural combinations are³

$$Q_{FL} = F_L^\mu - F_L^e, \quad Q_i = P_i^\mu - P_i^e, \quad T_i = \frac{S_i^\mu - S_i^e}{S_i^\mu + S_i^e}, \quad B_i = \frac{J_i^\mu}{J_i^e} - 1, \quad \tilde{B}_i = \frac{\beta_e^2 J_i^\mu}{\beta_\mu^2 J_k^e} - 1, \quad (2.1)$$

where P_i should be replaced by P_i' for $Q_{i=4,5,6,8}$. B_i and \tilde{B}_i differ mostly at very low q^2 and become almost identical for large q^2 , where $\beta_\ell = \sqrt{1 - 4m_\ell^2/q^2} \simeq 1$ for both electrons and muons. The optimised observables $P_i^{(\prime)}$ have already a limited sensitivity to hadronic uncertainties [2, 15, 22, 56, 57], contrary to the angular averages S_i [2, 15, 26, 27, 57, 58]. We thus expect the Q_i observables to exhibit a correspondingly low sensitivity to hadronic uncertainties.⁴ Moreover, these observables are protected from long-distance charm-loop contributions in the SM.

A measurement of Q_i different from zero would point to NP in an unambiguous way, confirming the violation of LFU observed in R_K . A second step would then consist in identifying the pattern of NP, which requires to separate the residual hadronic uncertainties (in particular, charm-loop contributions) from the NP contributions. The set of observables Q_i, T_i and B_k (\tilde{B}_k) can be particularly instrumental at this second stage, with a sensitivity to the various Wilson coefficients depending on the particular angular coefficients considered.

³In the following, we always consider quantities obtained by combining CP-averaged angular coefficients.

⁴We also expect a reduced sensitivity to $K\pi$ S-wave contributions (see e.g. [59–61]).

We have already investigated this sensitivity [22, 56, 57], but we would like to highlight the difference of behaviour in the case of two of the relevant observables P'_4 and P'_5 , directly related to Q_4 and Q_5 respectively. Both LHCb and Belle collaborations [3, 5, 8] observed the same pattern, i.e., a significant deviation from the SM for P'_5 for q^2 between 4 and 8 GeV² and a result consistent with the SM within errors for P'_4 . This behaviour is expected in the presence of NP in the Wilson coefficient C_9 . From the large-recoil expressions of $A_{\perp,\parallel,0}^{L,R}$ (see eqs. (3.8)-(3.10) of ref. [62]) one finds that the right-handed amplitudes $|A_{0,\perp,\parallel}^R| \propto (C_9^{\text{eff}} + C_{10}) + \dots$ are suppressed compared to the left-handed ones in the SM, due to the approximated cancellation $C_9^{\text{eff}} + C_{10} \simeq 0$. This cancellation is not so effective in the presence of a negative NP contribution to C_9 , and $A_{0,\parallel}^R$, $|A_{\perp}^R|$ increase while $|A_{0,\parallel}^L|$, A_{\perp}^L decrease. Both effects add up coherently in the numerator of $P'_5 \propto \text{Re}(A_0^L A_{\perp}^{L*} - A_0^R A_{\perp}^{R*})$ due to the relative minus sign, and the effect is to reduce the value of $|P'_5|$ in the region far up from the photon pole, in agreement with the experimental observation. In $P'_4 \propto \text{Re}(A_0^L A_{\parallel}^{L*} + A_0^R A_{\parallel}^{R*})$, however, an increase in the right-handed amplitudes will compensate a decrease in the left-handed ones, due to the relative positive sign. For this reason, no deviation is expected in P'_4 in the presence of NP in C_9 (but in the absence of right-handed currents). The same mechanism is at work for Q_4 and Q_5 .

But the observables Q_i are more powerful than the P_i ones in diagnosing FLNU. If a deviation is observed in Q_i (signalling New Physics without ambiguity), their interpretation will still be affected by hadronic pollution, which will be smaller in Q_i than in the original optimised observable P_i . An illustration is provided by the case $C_9^{NP} = -1.1$: in appendix C of ref. [22], the error on $\langle P'_5 \rangle_{[4,6]}$ is ± 0.11 , whereas the error of Q_5 in the same bin is only ± 0.05 (see appendix C.2 below). We also stress that the observables Q_i and T_i have quite different New Physics sensitivities. First of all, the P_i^{ℓ} observables are soft-form factor independent at leading order, whereas the S_i observables are not. One thus needs to define the ratios T_i from S_i in order to tame the dependence on soft form factors, although the T_i observables maintain a residual (lepton-mass dependent) sensitivity at LO. This dependence is more pronounced if the difference $S_i^e - S_i^{\mu}$ is used instead of T_i (the size of the error can vary substantially according to the choice of form factors). We can illustrate again how the Q_i observables are sharper than T_i : in the case of $C_9^{NP} = -1.1$, one has $\langle T_5 \rangle_{[2.5,4]} = -0.61 \pm 0.32$ (50% error), while $\langle Q_5 \rangle_{[2.5,4]} = 0.37 \pm 0.02$ (6% error). This is not an isolated example, as can be seen from the tables in appendix C.2 below.

As discussed in section 2.3.1 of ref. [22], LHCb currently determines the polarisation fraction F_T and F_L using a simplified description of the angular kinematics. This means that these two quantities are actually measured from J_{1c} rather than J_{2s} and J_{2c} respectively. Both determinations are equivalent in the massless limit, and therefore this only has a limited impact, apart from the first bin [0.1,0.98]. In order to interpret the actual measurements more precisely, we define the \hat{P}_i observables involving \hat{F}_T and \hat{F}_L , as measured currently by LHC:

$$F_L = \frac{-J_{2c}}{d\Gamma/dq^2} \rightarrow \hat{F}_L = \frac{J_{1c}}{d\Gamma/dq^2} \quad F_T = \frac{4J_{2s}}{d\Gamma/dq^2} \rightarrow \hat{F}_T = 1 - \hat{F}_L \quad (2.2)$$

$$P_1 = \frac{J_3}{2J_{2s}} \rightarrow \hat{P}_1 = \frac{J_3}{2\hat{J}_{2s}} \quad P_2 = \frac{J_{6s}}{8J_{2s}} \rightarrow \hat{P}_2 = \frac{J_{6s}}{8\hat{J}_{2s}} \quad (2.3)$$

$$P_3 = -\frac{J_9}{4J_{2s}} \rightarrow \hat{P}_3 = -\frac{J_9}{4\hat{J}_{2s}} \quad P'_4 = \frac{J_4}{\sqrt{-J_{2s}J_{2c}}} \rightarrow \hat{P}'_4 = \frac{J_4}{\sqrt{\hat{J}_{2s}J_{1c}}} \quad (2.4)$$

$$P'_5 = \frac{J_5}{2\sqrt{-J_{2s}J_{2c}}} \rightarrow \hat{P}'_5 = \frac{J_5}{2\sqrt{\hat{J}_{2s}J_{1c}}} \quad P'_6 = -\frac{J_7}{2\sqrt{-J_{2s}J_{2c}}} \rightarrow \hat{P}'_6 = -\frac{J_7}{2\sqrt{\hat{J}_{2s}J_{1c}}} \quad (2.5)$$

$$P'_8 = -\frac{J_8}{\sqrt{-J_{2s}J_{2c}}} \rightarrow \hat{P}'_8 = -\frac{J_8}{\sqrt{\hat{J}_{2s}J_{1c}}} \quad \text{with } \hat{J}_{2s} = \frac{1}{16}(6J_{1s} - J_{1c} - 2J_{2s} - J_{2c}) \quad (2.6)$$

and we will provide predictions for both Q_i and \hat{Q}_i observables, in order to illustrate the differences in the first bin, as well as the insensitivity of the effect in higher bins.

In the case of the S_i , the consideration of the T_i ratio is also natural, but unfortunately these quantities are quite sensitive to hadronic uncertainties. They depend on soft form factors even in the large recoil limit due to lepton mass effects at very low q^2 , related to differences between muon and electron contributions in the normalization. Finally, the ratios B_i that are soft-form-factor independent at leading order in the large-recoil limit will be shown to complement the observables Q_i in an interesting way.

2.2 Observables with reduced sensitivity to charm effects

In the presence of NP, all observables Q_i, T_i and B_i are in principle affected by long-distance charm loop contributions in C_9 , both transversity-independent and transversity-dependent. We define these two terms in the following way: transversity-independent long-distance charm corresponds to an identical contribution to all $B \rightarrow K^* \ell \ell$ transversity amplitudes, whereas transversity-dependent contributions differ for each amplitude. Both of them are expected to exhibit a q^2 -dependence in general. The explicit computation of charm-loop contributions performed in ref. [25] using light-cone sum rules indicates that they are transversity-dependent, in agreement with general expectations that such hadronic contributions are different for different external hadronic states (including different K^* helicities). It is interesting to investigate these issues by considering specific observables with different sensitivity to transversity-dependent and independent long-distance charm contributions, as well as to LFNU New Physics.

One can think of exploiting the angular coefficients in electron and muon modes in order to build observables only sensitive to some of the Wilson coefficients, and in some cases, insensitive to transversity-independent long-distance charm contributions. It is easy to check that in the large-recoil limit and in the absence of right-handed or scalar operators, four angular coefficients exhibit a linear sensitivity to C_9 . Taking the results from refs. [15, 62] we have:

$$\beta_\ell J_{6s} - 2iJ_9 = 16\beta_\ell^2 N^2 m_B^2 (1 - \hat{s})^2 C_{10}^\ell \left[2C_7 \frac{\hat{m}_b}{\hat{s}} + C_9^\ell \right] \xi_\perp^2 + \dots \quad (2.7)$$

$$\beta_\ell J_5 - 2iJ_8 = 8\beta_\ell^2 N^2 m_B^2 (1 - \hat{s})^3 \frac{\hat{m}_{K^*}}{\sqrt{\hat{s}}} C_{10}^\ell \left[C_7 \hat{m}_b \left(\frac{1}{\hat{s}} + 1 \right) + C_9^\ell \right] \xi_\perp \xi_\parallel + \dots \quad (2.8)$$

where $\hat{s} = q^2/m_B^2$ and $\hat{m}_b = m_b/m_B$, ξ_\perp and ξ_\parallel correspond to the soft form factors [17], and the ellipses indicate terms suppressed in the large-recoil limit (including terms of order

m_ℓ^2/q^2). If we limit ourselves to real NP contributions,⁵ it is interesting to consider B_5 and B_{6s} (and \tilde{B}_5 and \tilde{B}_{6s}) in eq. (2.1), as well as a combination of them in the form⁶

$$M = \frac{(J_5^\mu - J_5^e)(J_{6s}^\mu - J_{6s}^e)}{J_{6s}^\mu J_5^e - J_{6s}^e J_5^\mu}, \quad \tilde{M} = \frac{(\beta_e^2 J_5^\mu - \beta_\mu^2 J_5^e)(\beta_e^2 J_{6s}^\mu - \beta_\mu^2 J_{6s}^e)}{\beta_e^2 \beta_\mu^2 (J_{6s}^\mu J_5^e - J_{6s}^e J_5^\mu)}. \quad (2.9)$$

By construction, B_5 and \tilde{B}_5 have a pole at the position of the zero of J_5^e in the SM (around $q^2 = 2 \text{ GeV}^2$) and B_{6s} , \tilde{B}_{6s} have a pole at the position of the zero of A_{FB} in the SM (around $q^2 = 4 \text{ GeV}^2$). We expect large uncertainties for these observables in the corresponding bins. On the contrary, M is well behaved in the same bins, but it will have large uncertainties when $B_5 \simeq B_{6s}$. In this sense, the observable M is well suited for NP scenarios and energy regions that yield very different contributions to B_5 and B_{6s} . While the B_i have a value in the SM slightly different from zero (specially the first bin) due to β_μ/β_e kinematic effects, the \tilde{B}_i observables vanish by construction in the SM.⁷

Even more interesting is the case of \tilde{M} , constructed in the same spirit as \tilde{B}_i , i.e. to cancel the dependence of the angular coefficients on β_ℓ . Its first bin can be accurately predicted even in the presence of NP, while its M counterpart suffers from large uncertainties in that bin. In the next section we will discuss some NP scenarios and show how these set of observables can become instrumental to disentangle them.

Let us write $C_{ie} = C_i$ and $C_{i\mu} = C_i + \delta C_i$ for $i \neq 9$, so that δC_i measure the LFU violation, whereas C_{ie} can include LFU NP effects. Furthermore, for $i = 9$ we take $C_{9e} = C_9 + \Delta C_9$ and $C_{9\mu} = C_9 + \delta C_9 + \Delta C_9$ where ΔC_9 is a long-distance charm contribution. In order to illustrate the relevant aspects of the various observables, within this section we will give analytic formulas assuming the contribution ΔC_9 is transversity independent and neglecting imaginary parts. But all our numerical evaluations will be based on complete expressions, as computed in ref. [22] where transversity-dependent charm contributions are included following ref. [25], and imaginary parts are properly accounted for. We see that $\delta C_{7,7'} = 0$,⁸ and δC_9 are directly related to short-distance physics, while ΔC_9 comes from long-distance contributions from $c\bar{c}$ loops where the lepton pair is created by an electromagnetic current, and thus identical for C_{9e} and $C_{9\mu}$. Any $\delta C_i \neq 0$ indicates the presence of LFNU New Physics.

In the large-recoil limit and in the absence of right-handed or scalar operators, we have:

$$B_5 = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2 \delta C_{10}}{\beta_e^2 C_{10}} + \frac{\beta_\mu^2}{\beta_e^2} \frac{(C_{10} + \delta C_{10})\delta C_9 \hat{s}}{C_{10}(C_7 \hat{m}_b(1 + \hat{s}) + (C_9 + \Delta C_9)\hat{s})} + \dots \quad (2.10)$$

$$B_{6s} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2 \delta C_{10}}{\beta_e^2 C_{10}} + \frac{\beta_\mu^2}{\beta_e^2} \frac{(C_{10} + \delta C_{10})\delta C_9 \hat{s}}{C_{10}(2C_7 \hat{m}_b + (C_9 + \Delta C_9)\hat{s})} + \dots \quad (2.11)$$

⁵A clear deviation from zero for the Q_i observables constructed out of P_3, P'_6 and P'_8 in the region $q^2 \geq 1 \text{ GeV}^2$ would signal the presence of large new weak phases contributions entering the semileptonic Wilson coefficients. There are no clear indications of such additional weak phases in the data already gathered by the LHCb collaboration.

⁶The definitions of $B_{5,6s}$ ($\tilde{B}_{5,6s}$) and M (\tilde{M}) could be adapted to the imaginary contributions $J_{8,9}$. However the latter vanish in the case of real NP contributions. Since current data does not indicate any need for complex NP contributions, we will not include these additional observables here.

⁷The measurement of \tilde{B}_i requires the measurement of the quantities $\langle J_i^e/\beta_\ell^2 \rangle$. Experimentally, this can be done by assigning a β_ℓ^2 factor to the data on an event-by-event basis [63].

⁸ C_7 includes both the SM C_7^{eff} plus possible LFU NP (the same applies to C_9).

$$M = \widetilde{M} + \Delta M + \mathcal{A}\Delta C_9 + \mathcal{B}\Delta C_9^2 + \dots \quad (2.12)$$

$$\widetilde{M} = \widetilde{M}_0 + \mathcal{A}'\delta C_{10}\Delta C_9 + \mathcal{B}'\delta C_{10}^2\Delta C_9^2 + \dots \quad (2.13)$$

where \widetilde{M}_0 , ΔM , $\mathcal{A}^{(\prime)}$ and $\mathcal{B}^{(\prime)}$ are defined in appendix A, and the ellipsis denote again terms neglected in eqs. (2.7) and (2.8) and suppressed in the large-recoil limit. The difference between the muon and electron masses relative to q^2 , induces a non-vanishing SM value for the B_i observables at low q^2 . \widetilde{B}_i are exactly zero in the SM, and can be obtained from eqs. (2.10), (2.11) in the limit $\beta_\ell \rightarrow 1$. Note that the B_i observables always have a residual charm dependence ΔC_9 in the denominator in the presence of NP.

From eq. (2.12), M appears sensitive to the muon-electron mass difference via ΔM , \mathcal{A} and \mathcal{B} , and the last two terms introduce a sensitivity to charm effects through ΔC_9 . Moreover, the first bin of M is very sensitive to this mass difference and will be affected by very large uncertainties in some NP scenarios. On the contrary, \widetilde{M} is blind to such mass effects. In addition, if there is no NP in δC_{10} then \widetilde{M} becomes also insensitive to transversity-independent charm effects at leading order and at large recoil. This means that \widetilde{M} is particularly clean at low q^2 (where large-recoil expressions are relevant), especially in the presence of NP in δC_9 . For larger values of q^2 and/or in the presence of NP in C_{10} , subleading charm effects are present and will enlarge the uncertainties, even though the impact of NP on this observable remains very large. \widetilde{M} at low q^2 will turn out to be very efficient to disentangle NP scenarios.

We have the following behaviour for $\delta C_9 = 0$:

$$B_5 = B_{6s} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_{10}}{C_{10}}. \quad (2.14)$$

For B_5 and B_{6s} , the limit of very small q^2 is equivalent to $\delta C_9 = 0$, and M is not well predicted in this limit (subleading effects dominate the computation). This is however not a problem in the current context where global analyses point towards a large NP contribution to C_9 . On the other hand, if $\delta C_{10} = 0$, we have⁹

$$B_5 = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_9 \hat{s}}{(C_7 \hat{m}_b (1 + \hat{s}) + (C_9 + \Delta C_9) \hat{s})} + \dots \quad (2.15)$$

$$B_{6s} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_9 \hat{s}}{(2C_7 \hat{m}_b + (C_9 + \Delta C_9) \hat{s})} + \dots \quad (2.16)$$

$$\widetilde{M} = -\frac{\delta C_9 \hat{s}}{C_7 \hat{m}_b (1 - \hat{s})} + \dots \quad (2.17)$$

B_5 and B_{6s} contain then a residual charm sensitivity through ΔC_9 , while \widetilde{M} is totally free from this transversity-independent long-distance charm at leading order. This is a very specific property of \widetilde{M} which is independent of transversity-independent charm contributions in the presence of New Physics in C_9 only. Transversity-dependent charm effects are kinematically suppressed at very low q^2 in these observables as it will be shown later on.

⁹The corresponding expressions for $\widetilde{B}_{5,6s}$ when $\delta C_{10} = 0$ can be easily obtained from eqs. (2.15)–(2.16) by taking $\beta \rightarrow 1$ and the one of M can be obtained from appendix A.

In the case where both δC_9 and δC_{10} are non-zero, a precise interpretation of these observables requires a more detailed study (including an assessment of all $c\bar{c}$ contributions to C_9). We see therefore that some of these observables will have a limited sensitivity to charm-loop contributions in some cases (SM, NP only in $C_{9\mu}$), but not in other cases (NP also in $C_{10,\mu}$ for instance).

As a conclusion, the behaviour of B_5 (\widetilde{B}_5), B_{6s} (\widetilde{B}_{6s}) and M (\widetilde{M}) in specific q^2 -regions should provide powerful tests of physics beyond the SM, with a limited sensitivity to hadronic uncertainties.

3 Predictions in the SM and in typical NP benchmark scenarios

3.1 Observables and scenarios

The above discussion assumed that one can determine exactly the value of the angular coefficients J_i differentially in q^2 . This is in principle possible using the method of amplitudes in ref. [64] even if for electrons it could be particularly difficult. The other methods (likelihood fit and method of moments) lead to binned observables, where the cancellations advocated above hold only in an approximate way, for bins small enough so that the angular coefficients do not exhibit steep variations. The modifications due to binning for the predictions of observables were described in detail in ref. [56], and are also recalled in appendix B for the observables described above. They will obviously have an impact on the previous discussion concerning the cancellation of hadronic uncertainties, which will then be only approximate.

In order to illustrate the interest of the various observables, in addition to the SM, we consider several NP benchmark scenarios corresponding to the best-fit points for hypotheses with a large pull in the global analysis of ref. [22] (with NP contributions in $b \rightarrow s\mu\mu$ but not in $b \rightarrow see$). For all theory predictions we follow the same approach as in ref. [22]. In particular, we use the form factors of ref. [25] together with the large-recoil symmetry relations [65] including symmetry-breaking perturbative [65] and power corrections [57]. While other form-factor determinations report smaller uncertainties (e.g. [66]) we prefer to use a more conservative input in order to illustrate in our predictions which observables are less dependent on form-factor uncertainties. We then compute the various observables following the definition of binned observables in appendix B. The results are shown in appendix C and in figures 1–8.

In the SM, Q_i , T_i and B_i are expected to be close to zero, as shown in appendix C. The binned observables B_5 and B_{6s} are actually different from zero due to the kinematic factors β_μ^2 and β_e^2 in the transversity amplitudes — one could imagine measuring the binned values of $J_{5,6s}^\ell/\beta_\ell^2$ and checking that the values for both lepton flavours are indeed identical. The difference between β_μ and β_e becomes less relevant for large q^2 (above 2.5 GeV^2), leading to B_5 and B_{6s} decreasing in magnitude and getting closer to each other. In the same region, M becomes larger as it involves the difference $B_5 - B_{6s}$ in the denominator. In the presence of NP affecting differently $C_{9\mu}$ and C_{9e} , B_5 and B_{6s} are different over the whole kinematic range. In the SM, the binned version of M is charm dependent due to β_μ/β_e terms. In the presence of LFNU in C_9 , it is interesting to focus instead on the observable \widetilde{M} , which is not

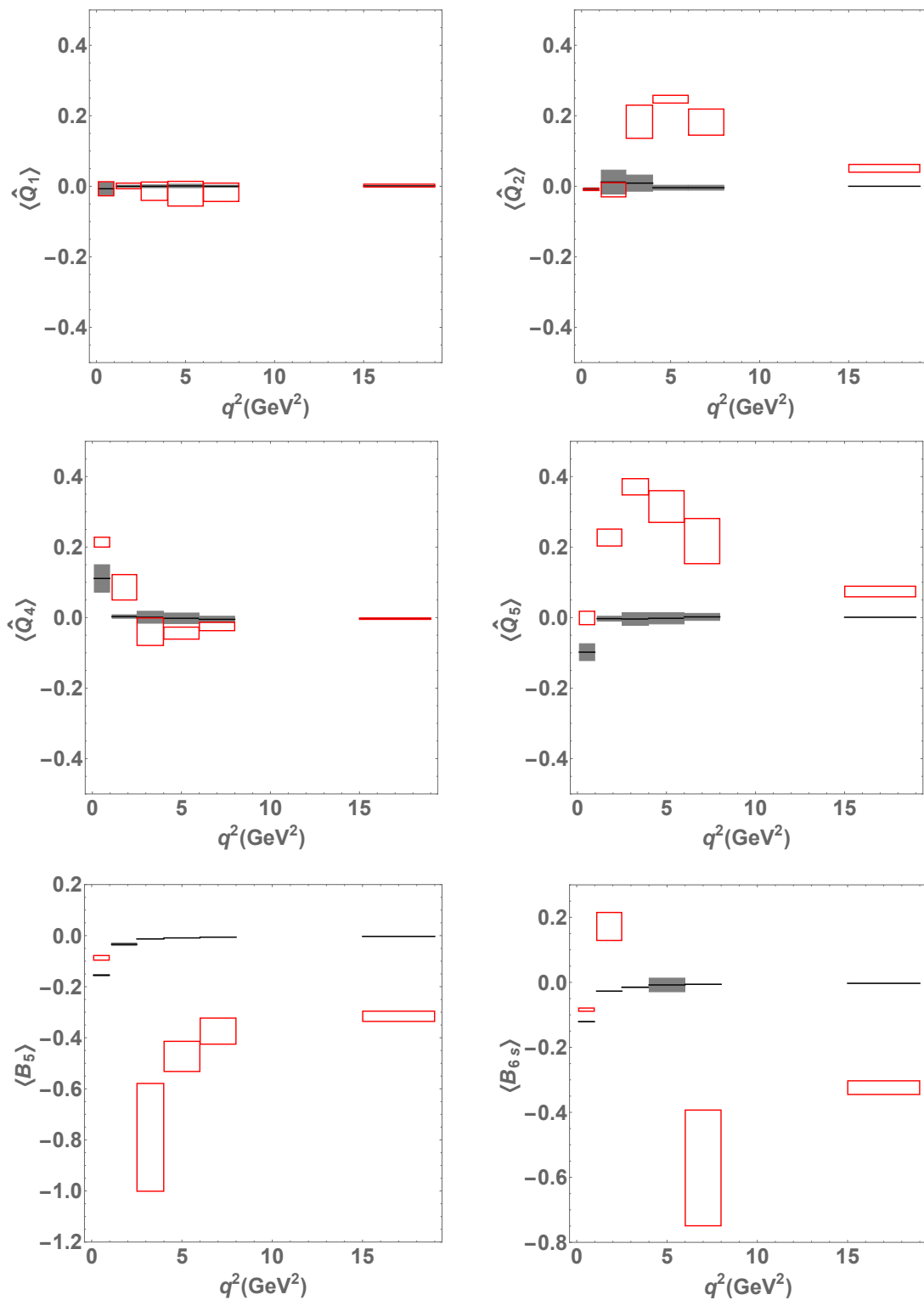


Figure 1. *Scenario 1.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -1.11$.

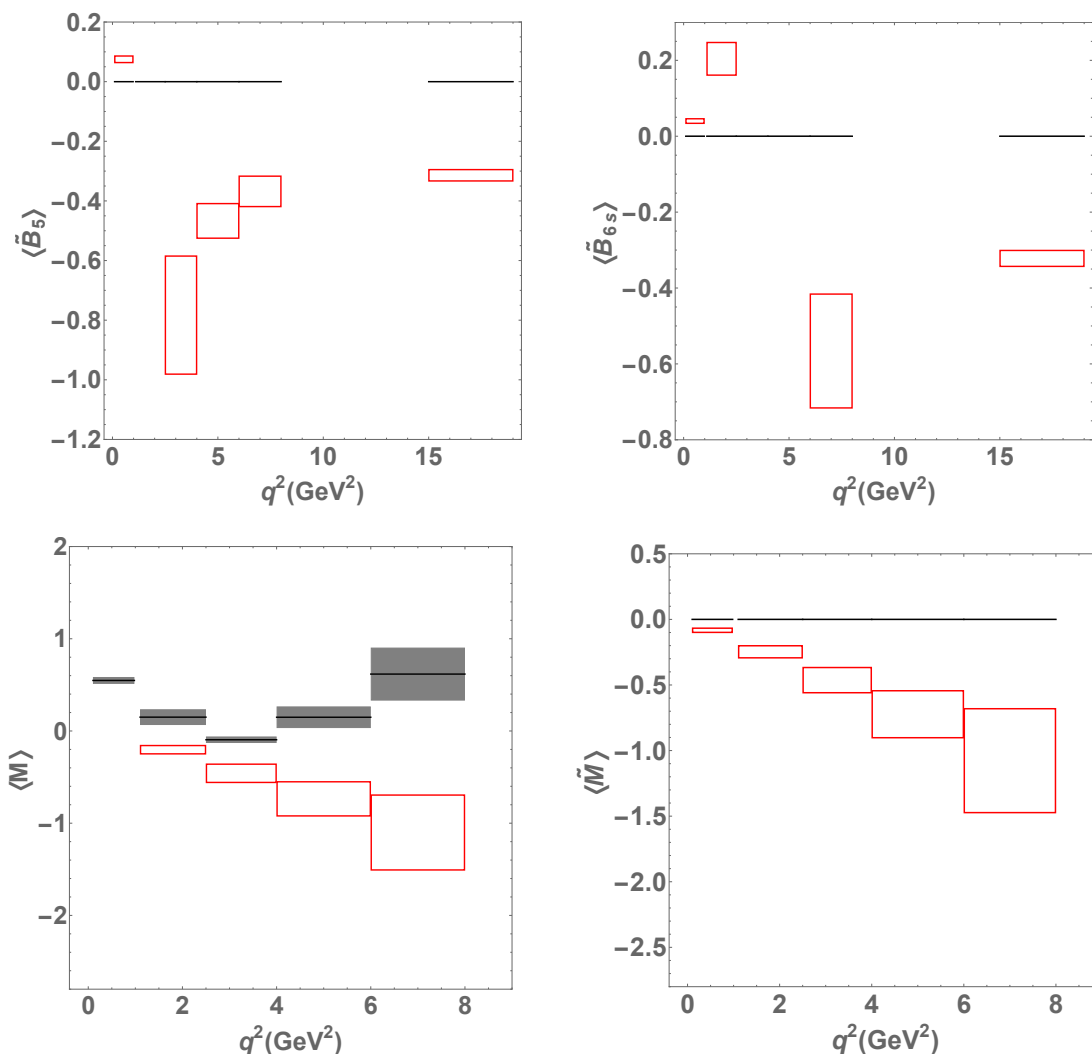


Figure 2. *Scenario 1.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -1.11$.

affected by lepton-mass effects and is essentially charm independent at very low- q^2 . If there are NP contributions in other Wilson coefficients, the situation becomes more complicated concerning the charm dependence of the observables. In the remainder of this section we will identify patterns based on the set of Q_i and \hat{Q}_i , and we will describe a very promising test based on B_5 , B_{6s} and M .

The observables \hat{Q}_i (see figures 1–8) show specific patterns for the different scenarios considered here:

- *Scenario 1:* $C_{9\mu}^{\text{NP}} = -1.1$. Both \hat{Q}_2 and \hat{Q}_5 are affected significantly, especially the latter. The most interesting region is $q^2 \gtrsim 6\text{GeV}^2$, taking into account that these observables receive essentially no charm contributions in the SM. No deviation should be observed in \hat{Q}_1 or \hat{Q}_4 in the same region within this scenario (see the discussion in section 2 concerning the sensitivity of P'_4 to C_9).

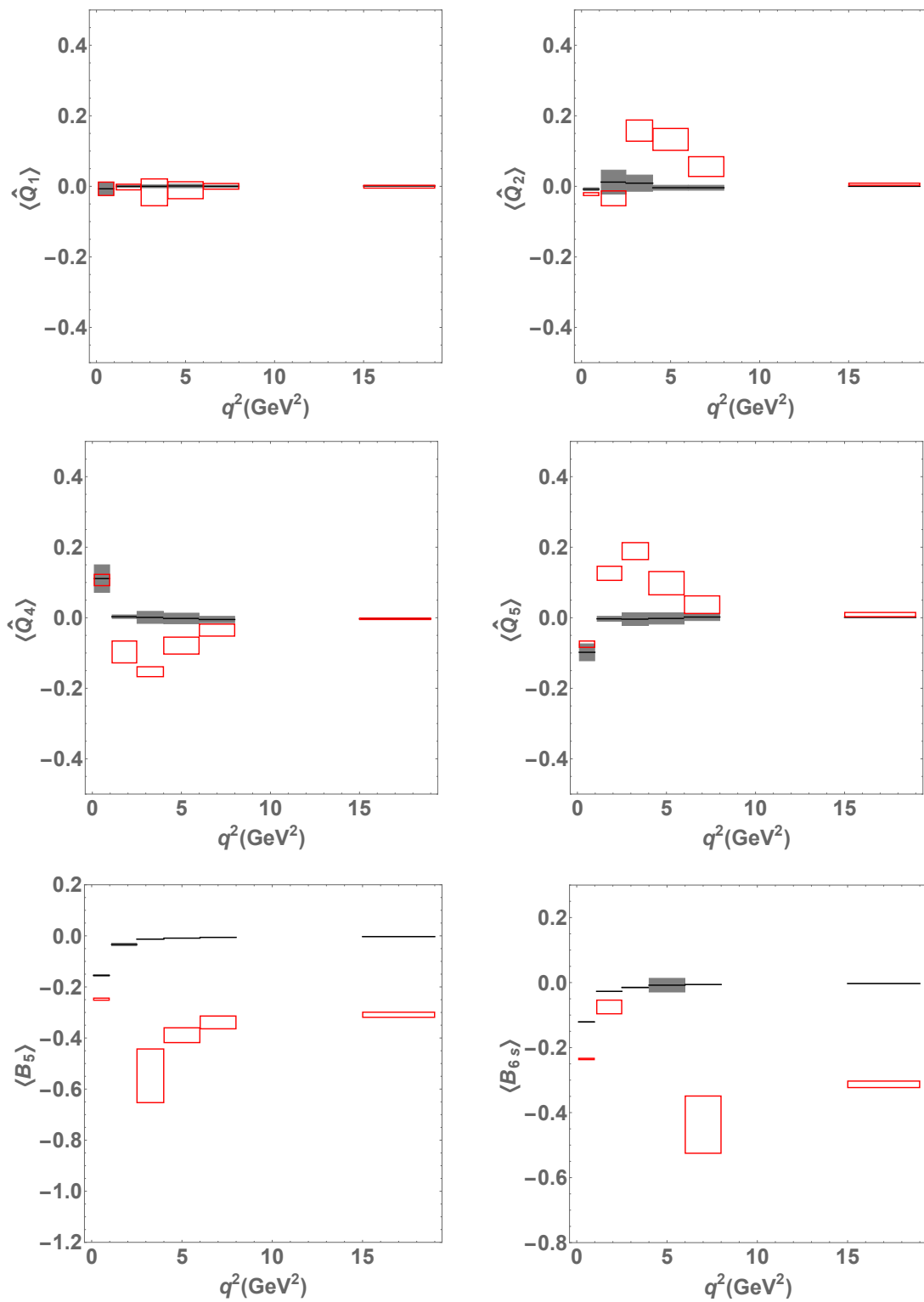


Figure 3. *Scenario 2.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.65$.

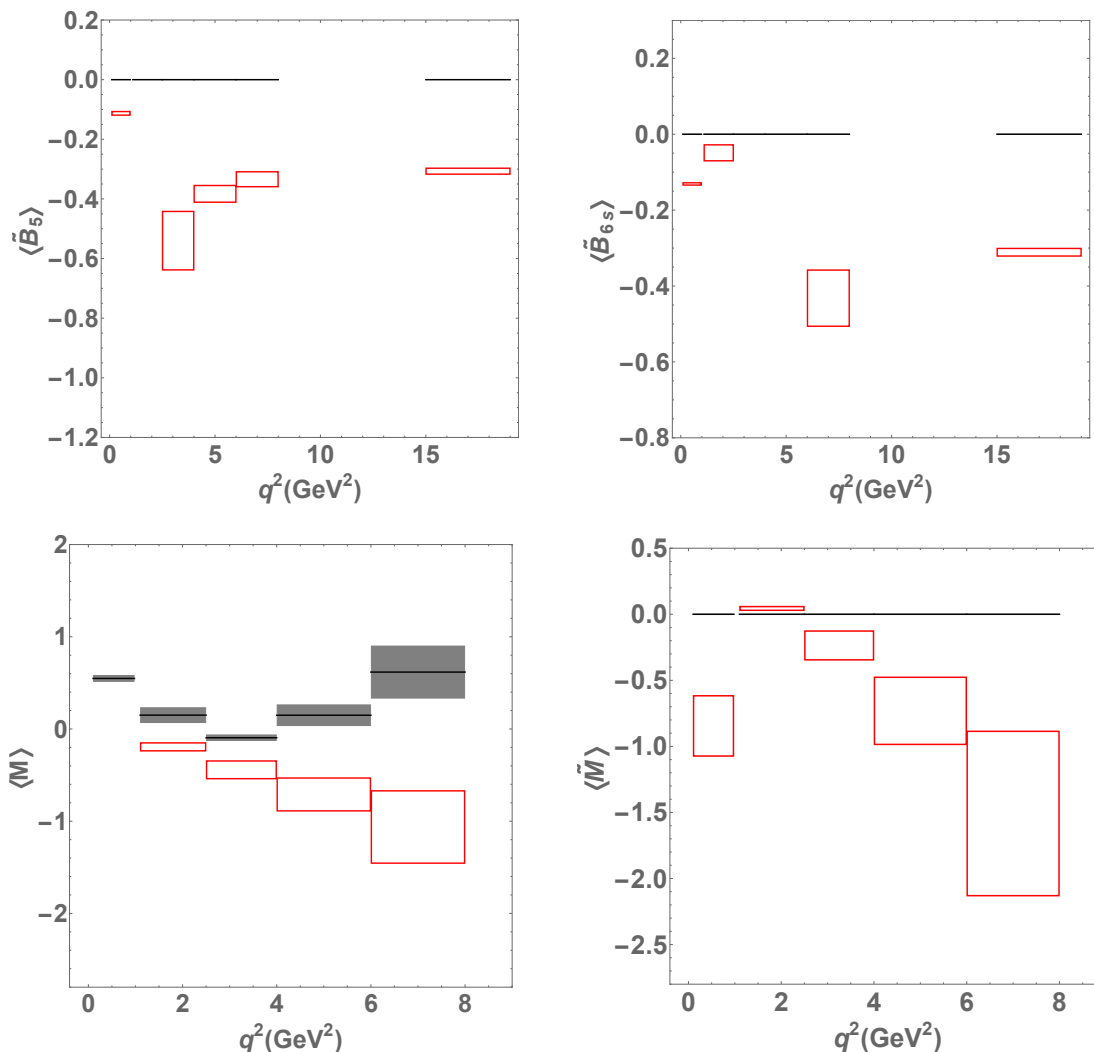


Figure 4. *Scenario 2.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.65$.

- *Scenario 2:* $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.65$. Within this scenario \hat{Q}_2 and \hat{Q}_5 show milder deviations, especially in the bin $6-8 \text{ GeV}^2$ where they are expected to be SM-like (contrary to Scenario 1). Indeed, the constraint from $B_s \rightarrow \mu\mu$ on $C_{10\mu}$ reduces the allowed size of the deviation in $C_{9\mu}$ in this particular scenario. On the contrary, \hat{Q}_4 could be particularly interesting in the region below 6 GeV^2 with a q^2 -dependence rather different from Scenario 1. No deviation is expected in \hat{Q}_1 .
- *Scenarios 3 and 4:* $C_{9\mu}^{\text{NP}} = -C'_{9\mu} = -1.07$ and $C_{9\mu}^{\text{NP}} = -C'_{9\mu} = -1.18$, $C_{10\mu}^{\text{NP}} = C'_{10\mu} = 0.38$ respectively. Both scenarios are quite difficult to distinguish using these observables. They have implications in all four relevant observables $\hat{Q}_{1,2,4,5}$. The behaviour of \hat{Q}_2 and \hat{Q}_5 is similar to Scenario 1, making the three scenarios difficult to disentangle when looking only to these observables. \hat{Q}_1 , which is designed to test the presence of right-handed currents, is affected significantly. Finally, \hat{Q}_4 both at

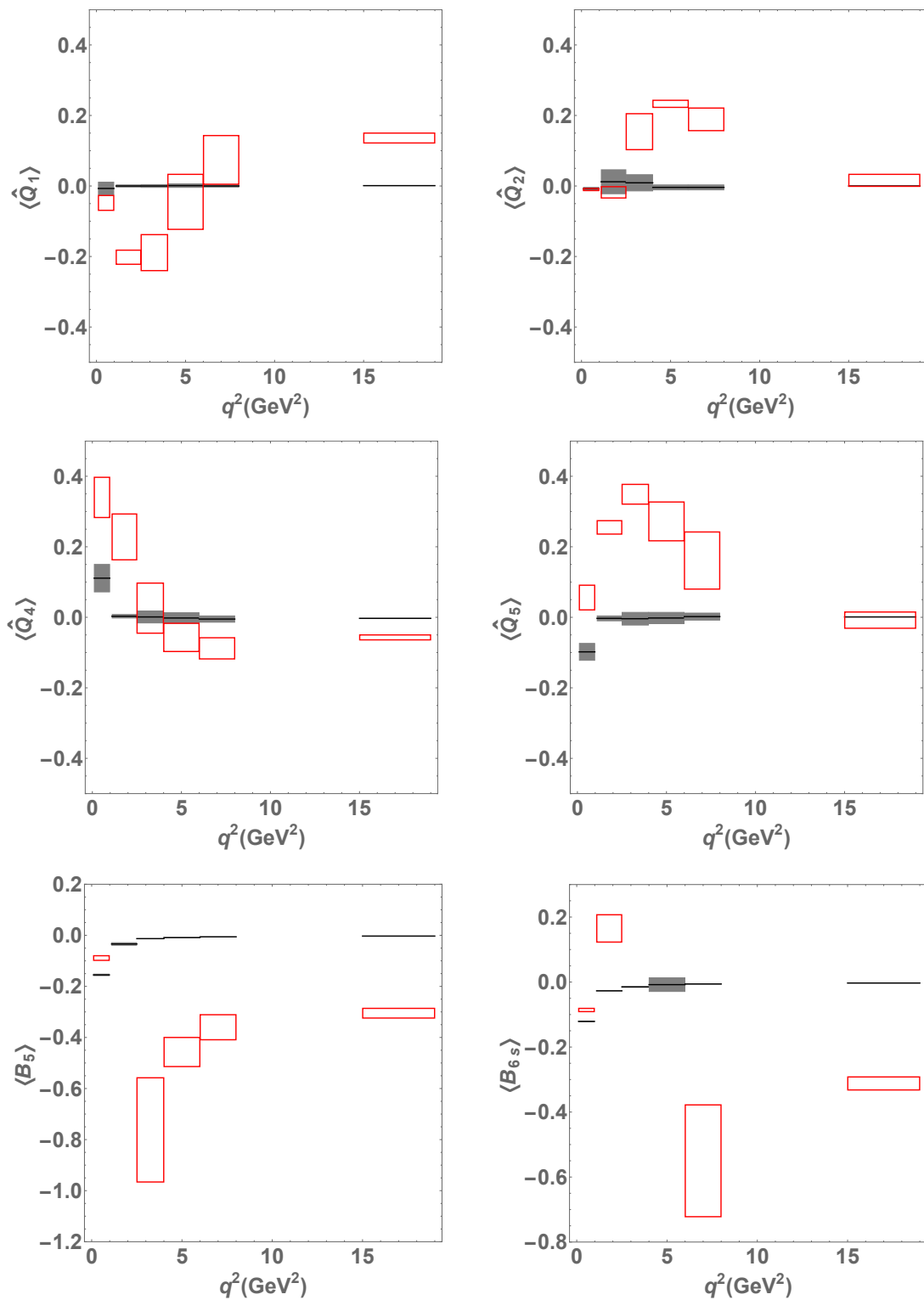


Figure 5. *Scenario 3.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -C_{9\mu}^{\text{NP}} = -1.07$.

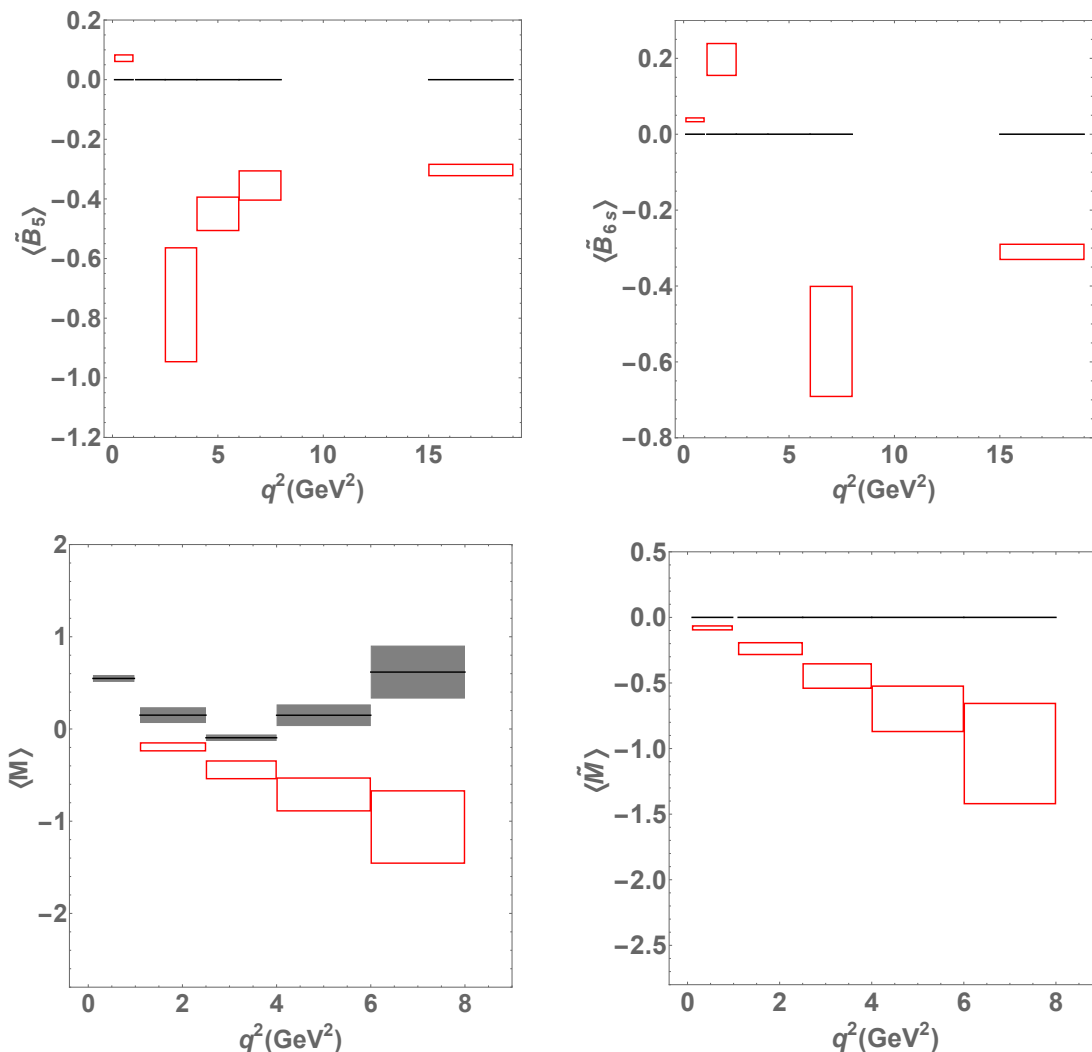


Figure 6. *Scenario 3.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -C_{9'\mu}^{\text{NP}} = -1.07$.

very low- and large- q^2 (but within the large recoil region) could be useful if accurate measurements are obtained. In particular, above 6 GeV^2 this observable is only sensitive to right-handed currents [67].

The same discussion applies to the observables Q_i . We note that \hat{Q}_i (Q_i) in the bin [6-8], which have no charm uncertainties in the SM, may play a central role in disentangling the first two scenarios.

These observables are quite complementary to R_{K^*} , for which we provide predictions in appendix C. Indeed, the value of R_{K^*} is very similar (within uncertainties) in the first two scenarios, whereas a larger suppression is expected for the other scenarios at moderately large q^2 , illustrating the complementarity with the \hat{Q}_i (Q_i) observables. For completeness we also present predictions for the observables T_i in the same appendix.

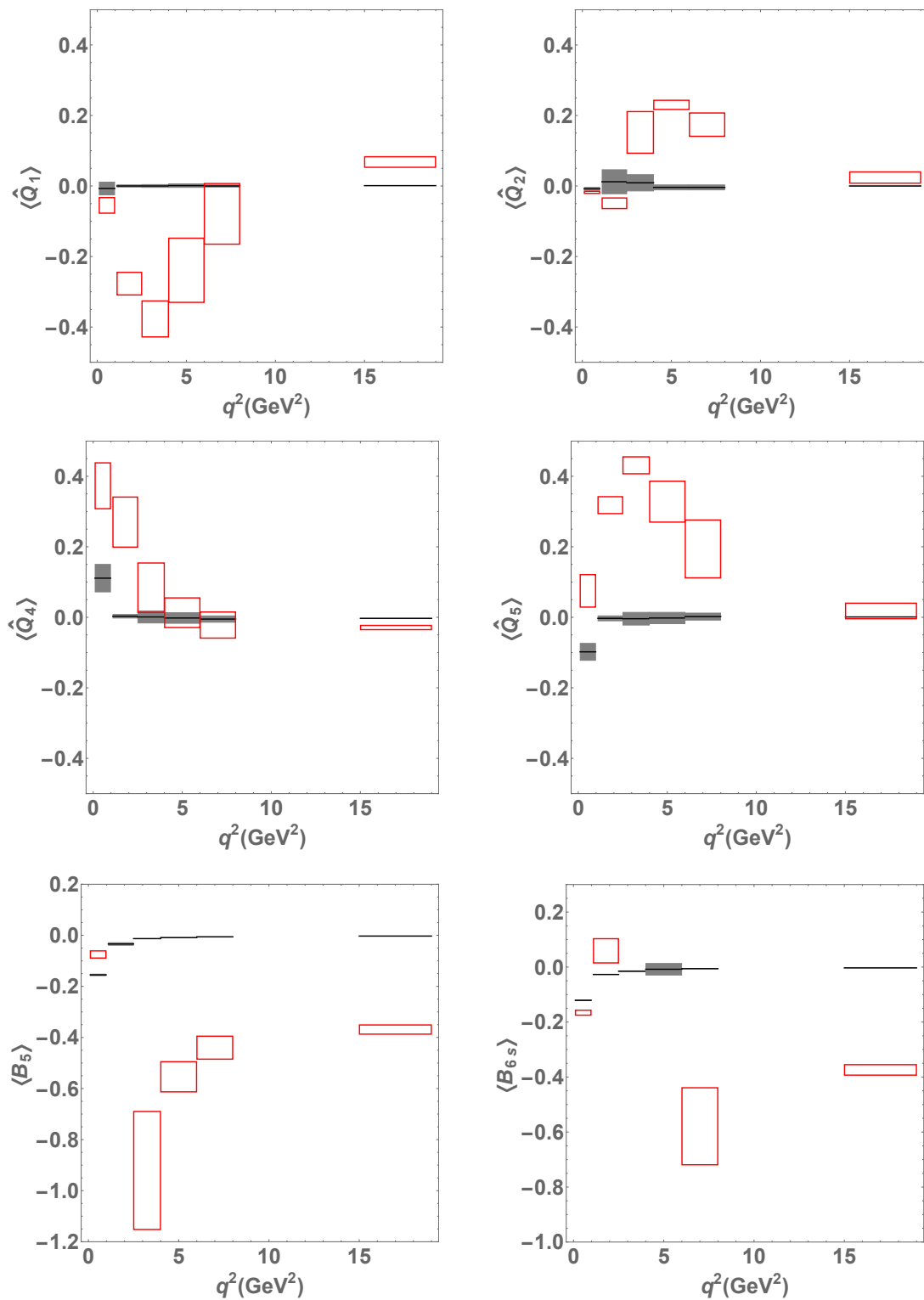


Figure 7. *Scenario 4.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -C_{9'\mu}^{\text{NP}} = -1.18$ and $C_{10\mu}^{\text{NP}} = C_{10'\mu}^{\text{NP}} = 0.38$.

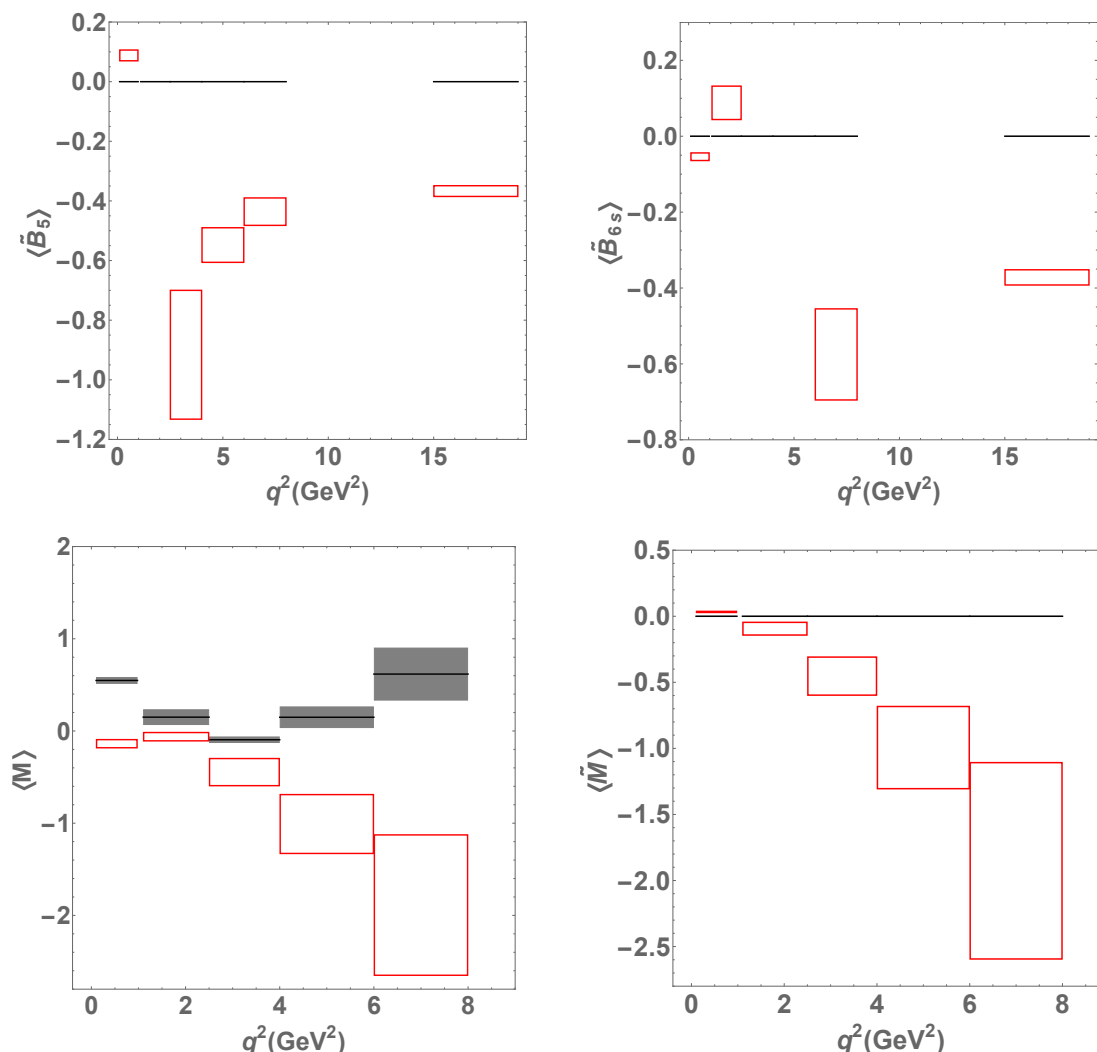


Figure 8. *Scenario 4.* SM predictions (grey boxes) and NP predictions (red boxes), assuming $C_{9\mu}^{\text{NP}} = -C_{9'\mu}^{\text{NP}} = -1.18$ and $C_{10\mu}^{\text{NP}} = C_{10'\mu}^{\text{NP}} = 0.38$.

3.2 B and \tilde{B} observables

We also give predictions for the B_i observables in appendix C and in figures 1–8 within each scenario. In the plots we have not shown the predictions in the bins where B_5 or B_{6s} have a pole ([1.1,2.5] for B_5 , [2.5,4] and [4,6] for B_{6s}) and cannot be predicted accurately. All scenarios give very similar predictions, apart from the first bin of B_5 and the two first bins of B_{6s} .

The first bin of these observables is predicted precisely both in the SM and in the presence of NP. Not only it is insensitive to form factors in the large-recoil limit at leading order, but it is also protected from long-distance charm contributions due to a kinematical suppression of the charm-dependent contribution at low q^2 (see also ref. [67]). The analysis of this bin in the SM and in the scenarios presented above is particularly interesting. As explained in the previous section, the SM predictions $B_5^{\text{SM}} = -0.155 \pm 0.003$ and

$B_{6s}^{\text{SM}} = -0.121 \pm 0.001$ are only different from zero due to β_μ/β_e effects integrated over the bin. This can be checked through the corresponding prediction for the \tilde{B}_i observables, which are free from these effects and equal to zero in the SM. In the case of a negative NP contribution to $C_{9\mu}$, both B_5 and B_{6s} receive a positive contribution that pushes them towards zero in the first bin. If there is a positive NP contribution in $C_{10\mu}$, the contribution to both observables is negative and large (of size $C_{10\mu}^{\text{NP}}/C_{10\mu}$). In summary, a contribution close to zero will favour a scenario with NP only in $C_{9\mu} < 0$, whereas values of B_5 and B_{6s} lower than the SM will signal NP in $C_{10\mu}$ (NP in $C_{9\mu}$ is better discriminated by other observables). In both cases B_5 and B_{6s} are almost equal, while a contribution to $C'_{10\mu}$ would break this degeneracy. The second bin of B_{6s} exhibits a similar pattern (above the SM in Scenario 1, below in Scenario 2).

The same discussion applies to \tilde{B}_i , which have a similar behaviour in those bins, the only difference being that they are centered around zero (SM prediction). For instance, the first bin of \tilde{B}_5 and \tilde{B}_{6s} in the Scenario 1 (Scenario 2) receives a positive (negative) contribution. The second bin of \tilde{B}_{6s} follows the same rules as B_{6s} .

The low-recoil behaviour of the B_i and \tilde{B}_i observables is particularly interesting because it points to large deviations that cannot be seen easily in the Q_i observables. Unfortunately, they are not useful in distinguishing Scenarios 1 and 2, except if compared together with the corresponding Q_i at low recoil, which show a slightly different behaviour in that region.

3.3 M and \tilde{M} observables

M is also an interesting observable to get information on the existence of NP contributions and identifying their nature. This can be seen from the results in appendix C and figures 1–8 by looking at the third bin, where it can be noted that this observable can help to disentangle Scenario 2 from Scenarios 1 and 3, thus testing for the presence of NP in $C_{10\mu}$.

However in the first bin, where $B_5 \simeq B_{6s}$, M is poorly predicted. In these region it proves instead very useful to exploit the alternative observable \tilde{M} , where effects related to β_ℓ are removed. This observable then gives additional information in discerning between Scenario 2 and Scenarios 1 and 3. The effects in this first bin can also be confirmed by looking at the second bin (notice that \tilde{M} is well defined in its second bin even if \tilde{B}_5 has a pole in its second bin).

3.4 Hadronic uncertainties

The observables presented here, specially Q_i , B_i and \tilde{B}_i , are built to be accurate in the SM, and almost insensitive to long-distance charm contributions. Moreover, whether NP is present or not, these observables are built to have no dependence on soft form factors at leading order in the large-recoil limit. In the presence of NP, these observables become again sensitive to charm-loop contributions, but in a very specific way that we discuss now.

Let us first recall that we introduced the observables \hat{Q}_i in order to provide predictions taking into account how LHC measures F_L currently. Here the cancellation of soft form factors between numerator and denominator is not fully operative and these observables are thus sensitive to soft form factors arising in J_{1c} but suppressed by powers of m_ℓ^2/q^2 .

This explains why the errors of \hat{Q}_i are larger (but still small in most of the bins) than for Q_i . The observables T_i exhibit a residual sensitivity to soft form factors in most of the bins. Finally, the observable M suffers from large uncertainties when $B_5 \simeq B_{6s}$, even though it is designed to have no dependence on soft form factors at leading order in the large-recoil limit.

Concerning long-distance charm-loop contributions, the most interesting observables are B_i (\tilde{B}_i) and M (\tilde{M}). In the analytic expressions provided in section 2.2, we have assumed that the charm contribution ΔC_9 entered all transversity amplitudes in the same way. One can generalize the expressions for $B_{5,6s}$ and \tilde{M} in eqs. (2.10), (2.11) and allow for transversity-dependent charm contributions $\Delta C_9^{\perp,\parallel,0}(q^2)$ associated to each amplitude:

$$B_5 = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_{10}}{C_{10}} + \frac{\beta_\mu^2}{\beta_e^2} \frac{2(C_{10} + \delta C_{10})\delta C_9 \hat{s}}{C_{10} (2C_7 \hat{m}_b (1 + \hat{s}) + (2C_9 + \Delta C_{9,0} + \Delta C_{9,\perp}) \hat{s})} \quad (3.1)$$

$$B_{6s} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_{10}}{C_{10}} + \frac{\beta_\mu^2}{\beta_e^2} \frac{2(C_{10} + \delta C_{10})\delta C_9 \hat{s}}{C_{10} (4C_7 \hat{m}_b + (2C_9 + \Delta C_{9,\perp} + \Delta C_{9,\parallel}) \hat{s})} \quad (3.2)$$

$$\begin{aligned} \tilde{M} &= \frac{(2C_{10}\delta C_9 \hat{s} + \delta C_{10} (2C_7 \hat{m}_b (1 + \hat{s}) + (2C_9 + 2\delta C_9 + \Delta C_{9,\perp} + \Delta C_{9,0}) \hat{s}))}{2C_{10}(C_{10} + \delta C_{10})\delta C_9 (2C_7 \hat{m}_b (\hat{s} - 1) + (\Delta C_{9,0} - \Delta C_{9,\parallel}) \hat{s}) \hat{s}} \\ &\times (2C_{10}\delta C_9 \hat{s} + \delta C_{10} (4C_7 \hat{m}_b + (2C_9 + 2\delta C_9 + \Delta C_{9,\perp} + \Delta C_{9,\parallel}) \hat{s})) \quad (3.3) \end{aligned}$$

The corresponding expressions for the $\tilde{B}_{5,6s}$ are obtained in the limit $\beta_\ell \rightarrow 1$. In the case of NP only in δC_9 they simplify to

$$B_5 = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_9 \hat{s}}{(C_7 \hat{m}_b (1 + \hat{s}) + (C_9 + (\Delta C_9^0 + \Delta C_9^\perp)/2) \hat{s})} + \dots \quad (3.4)$$

$$B_{6s} = \frac{\beta_\mu^2 - \beta_e^2}{\beta_e^2} + \frac{\beta_\mu^2}{\beta_e^2} \frac{\delta C_9 \hat{s}}{(2C_7 \hat{m}_b + (C_9 + (\Delta C_9^\parallel + \Delta C_9^\perp)/2) \hat{s})} + \dots \quad (3.5)$$

$$\tilde{M} = -\frac{\delta C_9 \hat{s}}{C_7 \hat{m}_b (1 - \hat{s}) - (\Delta C_9^0 - \Delta C_9^\parallel) \hat{s}/2} + \dots \quad (3.6)$$

The observable \tilde{M} was designed to cancel exactly a transversity-independent charm contribution ΔC_9 at leading order in the large recoil limit, which occurs in the denominator of the B_i observables. The above expressions indicate that for B_i , all the long-distance charm dependence is contained in the denominator, and its numerical impact is somehow reduced by a large C_9 , which explains their reduced sensitivity to ΔC_9 (this is even more efficient at very low q^2 due to the photon pole). In the case of \tilde{M} , C_9 cancels, leaving only the photon pole to tame the sensitivity to transversity-dependent charm-loop contributions. For this reason at higher q^2 values, where the photon pole contribution is smaller, the sensitivity to this transversity-dependent charm contribution is maximal in \tilde{M} as can be seen in appendix C and in figures 1–8. In addition, looking at eq. (3.3), it is interesting to note that \tilde{M} is sensitive to charm contributions only if *a*) there is LFNU New Physics in C_{10} or right-handed operators, or *b*) there are transversity-dependent charm-loop contributions (such that $\Delta C_9^0 \neq \Delta C_9^\parallel$).

We should finally comment on the fact that our predictions do not include any evaluation of Bremsstrahlung effects. Naively one expects these effects to be of order $\alpha \log(m_e^2/m_\mu^2) \sim 8\%$ [49]. Part of these effects are taken into account at the level of the experimental analysis by means of a Montecarlo simulation with PHOTOS [68], which accounts for soft-photon emission from the leptons. Other contributions (e.g., real emission from the mesons, virtual photons) should still be estimated by separating in the theoretical computations the radiative corrections already implemented experimentally and those to be estimated theoretically (see refs. [69, 70] for a discussion of this issue in the context of $K_{\ell 4}$ decays). Such a work goes far beyond the present note, but the impact of such effects should be expected to be of a few percent. A recent analysis [71] shows that indeed the experiment corrects for these effects already to very good accuracy.

4 Discussion and conclusion

The recent LHCb and Belle results on $b \rightarrow s\ell\ell$ transitions, with the anomalies observed in some angular observables such as $P_5'(B \rightarrow K^*\mu\mu)$, and the hints of LFNU in $B \rightarrow K\ell\ell$ have raised a considerable interest for these processes. In the present article we have discussed how angular analyses of $B \rightarrow K^*ee$ and $B \rightarrow K^*\mu\mu$ decay modes can be combined to understand better the pattern of anomalies observed and to get a solid handle on the size of some SM long-distance contributions.

We have proposed different sets of observables comparing $B \rightarrow K^*ee$ and $B \rightarrow K^*\mu\mu$, discussing their respective merits. A first set of observables is obtained directly from the observables that have been introduced for $B \rightarrow K^*\mu\mu$, namely Q_i (related to the optimised observables P_i), T_i (related to the angular averages S_i) and B_i (related to the angular coefficients J_i), measuring in each case the differences between muon and electron modes.

We have discussed further the merits of the observables B_5 and B_{6s} which are built from angular coefficients exhibiting only a linear dependence on $C_{9\ell}$ at large recoil. In principle, this allows us to disentangle the contributions coming from NP in C_9 and C_{10} , with a clean separation between lepton-flavour dependent (NP) and lepton-flavour universal (NP or SM long-distance) contributions to C_9 . We have also built an observable \widetilde{M} which exhibits very interesting features: in the presence of LFNU NP in $C_{9\ell}$ or $C_{10\ell}$ only, the large-recoil expression for \widetilde{M} is independent of long-distance LFU contributions (in particular transversity-independent charm contributions) and provides clean signals of NP. It proves also interesting to consider \widetilde{B}_5 and \widetilde{B}_{6s} , built from angular coefficients divided by appropriate powers of β_ℓ , thus removing some kinematic effects affecting B_5 and B_{6s} at very low q^2 .

We have then considered the situation for binned observables, and we have provided predictions for the SM and for several benchmark points inspired by our recent global analysis of $b \rightarrow s\ell\ell$ transitions. We can summarise our findings as follows. First, the Q_i observables are efficient to separate several NP scenarios where NP enter only $b \rightarrow s\mu\mu$ transitions due to very different q^2 dependences in the large-recoil region. Second, the observables $B_{5,6s}$ and $\widetilde{B}_{5,6s}$ at very large and low recoils provide further information, as NP in different muon Wilson coefficients will affect these observables significantly. Finally, the

\widetilde{M} observable at low q^2 proves particularly clean and efficient in identifying and interpreting NP in muon modes, with a limited sensitivity to charm contributions. These observables provide complementary information compared to the measurement of the ratio R_{K^*} that is expected very soon from the LHCb collaboration.

In view of these results, we are looking forward to the next measurements to be performed at LHCb and Belle-II. We expect their analysis to bring a significant improvement in our understanding of the exact origin of the anomalies currently observed in $b \rightarrow s\ell\ell$ modes.

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A Large-recoil expressions for M and \widetilde{M}

Under the notation and hypotheses in section 2.2, we can separate the charm contributions from the rest of the \widetilde{M} observable

$$\widetilde{M} = \widetilde{M}_0 + \mathcal{A}'\delta C_{10}\Delta C_9 + \mathcal{B}'\delta C_{10}^2\Delta C_9^2 \quad (\text{A.1})$$

with

$$\widetilde{M}_0 = \frac{(2C_7\delta C_{10}\hat{m}_b + \delta C_9 C_{10}\hat{s} + \delta C_{10}(C_9 + \delta C_9)\hat{s})}{C_7\delta C_9 C_{10}(C_{10} + \delta C_{10})\hat{m}_b(\hat{s} - 1)\hat{s}} \quad (\text{A.2})$$

$$\times (C_7\delta C_{10}\hat{m}_b + \delta C_9 C_{10}\hat{s} + \delta C_{10}(C_7\hat{m}_b + C_9 + \delta C_9)\hat{s})$$

$$\mathcal{A}' = \frac{2\delta C_9 C_{10}\hat{s} + \delta C_{10}(2(C_9 + \delta C_9)\hat{s} + C_7\hat{m}_b(3 + \hat{s}))}{C_7\delta C_9 C_{10}(C_{10} + \delta C_{10})\hat{m}_b(\hat{s} - 1)} \quad (\text{A.3})$$

$$\mathcal{B}' = \frac{\hat{s}}{C_7\delta C_9 C_{10}(C_{10} + \delta C_{10})\hat{m}_b(\hat{s} - 1)} \quad (\text{A.4})$$

M can be expressed in terms of \widetilde{M} and considering all the lepton mass effects coming from $\beta_\ell = \sqrt{1 - 4m_\ell^2/s}$ in the large recoil limit and up to leading order

$$M = \widetilde{M} + \Delta M + \mathcal{A}\Delta C_9 + \mathcal{B}\Delta C_9^2 \quad (\text{A.5})$$

$$\begin{aligned} \Delta M = & -\frac{\beta_e^2 - \beta_\mu^2}{\beta_e^2\beta_\mu^2} \frac{1}{C_7\delta C_9 C_{10}(C_{10} + \delta C_{10})\hat{m}_b(\hat{s} - 1)\hat{s}} \\ & \times \left[-C_{10}^2(2C_7\hat{m}_b + C_9\hat{s})(C_9\hat{s} + C_7\hat{m}_b(1 + \hat{s}))\beta_e^2 \right. \\ & \left. + (C_{10} + \delta C_{10})^2(2C_7\hat{m}_b + (C_9 + \delta C_9)\hat{s})((C_9 + \delta C_9)\hat{s} + C_7\hat{m}_b(1 + \hat{s}))\beta_\mu^2 \right] \quad (\text{A.6}) \end{aligned}$$

$$\mathcal{A} = \frac{\beta_e^2 - \beta_\mu^2}{\beta_e^2 \beta_\mu^2} \frac{1}{C_7 \delta C_9 C_{10} (C_{10} + \delta C_{10}) \hat{m}_b (\hat{s} - 1)} \quad (\text{A.7})$$

$$\times [C_{10}^2 (2C_9 \hat{s} + C_7 \hat{m}_b (3 + \hat{s})) \beta_e^2 - (C_{10} + \delta C_{10})^2 (2(C_9 + \delta C_9) \hat{s} + C_7 \hat{m}_b (3 + \hat{s})) \beta_\mu^2]$$

$$\mathcal{B} = \frac{\beta_e^2 - \beta_\mu^2}{\beta_e^2 \beta_\mu^2} \frac{\hat{s} (C_{10}^2 \beta_e^2 - (C_{10} + \delta C_{10})^2 \beta_\mu^2)}{C_7 \delta C_9 C_{10} (C_{10} + \delta C_{10}) \hat{m}_b (\hat{s} - 1)} \quad (\text{A.8})$$

B Definition of binned observables

The binned observables are defined following the same rules as in ref. [56]:

$$\langle Q_i \rangle = \langle P_i^\mu \rangle - \langle P_i^e \rangle \quad \langle \hat{Q}_i \rangle = \langle \hat{P}_i^\mu \rangle - \langle \hat{P}_i^e \rangle \quad \langle T_i \rangle = \frac{\langle S_i^\mu \rangle - \langle S_i^e \rangle}{\langle S_i^\mu \rangle + \langle S_i^e \rangle} \quad (\text{B.1})$$

$$\langle B_i \rangle = \frac{\langle J_i^\mu \rangle}{\langle J_i^e \rangle} - 1 \quad \langle \tilde{B}_i \rangle = \frac{\langle J_i^\mu / \beta_\mu^2 \rangle}{\langle J_i^e / \beta_e^2 \rangle} - 1 \quad (\text{B.2})$$

$$\langle M \rangle = \frac{(\langle J_5^\mu \rangle - \langle J_5^e \rangle) (\langle J_{6s}^\mu \rangle - \langle J_{6s}^e \rangle)}{\langle J_{6s}^\mu \rangle \langle J_5^e \rangle - \langle J_{6s}^e \rangle \langle J_5^\mu \rangle} \quad (\text{B.3})$$

$$\langle \tilde{M} \rangle = \frac{(\langle J_5^\mu / \beta_\mu^2 \rangle - \langle J_5^e / \beta_e^2 \rangle) (\langle J_{6s}^\mu / \beta_\mu^2 \rangle - \langle J_{6s}^e / \beta_e^2 \rangle)}{\langle J_{6s}^\mu / \beta_\mu^2 \rangle \langle J_5^e / \beta_e^2 \rangle - \langle J_{6s}^e / \beta_e^2 \rangle \langle J_5^\mu / \beta_\mu^2 \rangle} \quad (\text{B.4})$$

where $\langle P_i^\ell \rangle$ and $\langle S_i^\ell \rangle$ correspond to the observables defined in ref. [56] with $\ell = e$ or μ . Similarly, the $\langle \hat{P}_i^\ell \rangle$ are obtained from eqs. (2.2)–(2.6), substituting $J_i^\ell \rightarrow \langle J_i^\ell \rangle$.

C Predictions for the observables in various benchmark scenarios

Our predictions are obtained following ref. [22]. We quote two uncertainties, the second corresponding to the charm contributions, the first to all other sources of uncertainties. Bars denote predictions affected by a very large uncertainty (presence of a pole).

C.1 SM

Bin	Q_{F_L}	Q_1	Q_2	Q_3
[0.1, 0.98]	$-0.041 \pm 0.044 \pm 0.010$	$-0.001 \pm 0.001 \pm 0.001$	$0.019 \pm 0.003 \pm 0.001$	$0.000 \pm 0.000 \pm 0.000$
[1.1, 2.5]	$-0.027 \pm 0.014 \pm 0.001$	$-0.000 \pm 0.000 \pm 0.000$	$0.007 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[2.5, 4.]	$-0.016 \pm 0.009 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.001 \pm 0.001 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[4., 6.]	$-0.010 \pm 0.008 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-0.001 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[6., 8.]	$-0.006 \pm 0.006 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-0.001 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$-0.001 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
Bin	Q_4	Q_5	Q_6	Q_8
[0.1, 0.98]	$0.005 \pm 0.002 \pm 0.004$	$0.047 \pm 0.003 \pm 0.008$	$-0.005 \pm 0.002 \pm 0.001$	$0.001 \pm 0.000 \pm 0.000$
[1.1, 2.5]	$0.002 \pm 0.000 \pm 0.000$	$0.001 \pm 0.002 \pm 0.001$	$-0.001 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[2.5, 4.]	$0.000 \pm 0.000 \pm 0.000$	$-0.004 \pm 0.001 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$
[4., 6.]	$0.000 \pm 0.000 \pm 0.000$	$-0.004 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[6., 8.]	$0.000 \pm 0.000 \pm 0.000$	$-0.003 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$0.000 \pm 0.000 \pm 0.000$	$-0.001 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$

Bin	\hat{Q}_{FL}	\hat{Q}_1	\hat{Q}_2	\hat{Q}_3
[0.1, 0.98]	$0.018 \pm 0.017 \pm 0.004$	$-0.007 \pm 0.006 \pm 0.018$	$-0.008 \pm 0.004 \pm 0.001$	$0.000 \pm 0.001 \pm 0.001$
[1.1, 2.5]	$0.014 \pm 0.002 \pm 0.000$	$-0.000 \pm 0.003 \pm 0.000$	$0.013 \pm 0.032 \pm 0.002$	$0.000 \pm 0.000 \pm 0.000$
[2.5, 4.]	$0.010 \pm 0.002 \pm 0.000$	$0.000 \pm 0.003 \pm 0.000$	$0.010 \pm 0.025 \pm 0.001$	$0.000 \pm 0.001 \pm 0.000$
[4., 6.]	$0.008 \pm 0.001 \pm 0.000$	$0.001 \pm 0.006 \pm 0.000$	$-0.004 \pm 0.005 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[6., 8.]	$0.006 \pm 0.002 \pm 0.000$	$0.000 \pm 0.003 \pm 0.000$	$-0.004 \pm 0.007 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$0.001 \pm 0.000 \pm 0.000$	$0.001 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
Bin	\hat{Q}_4	\hat{Q}_5	\hat{Q}_6	\hat{Q}_8
[0.1, 0.98]	$0.111 \pm 0.007 \pm 0.037$	$-0.097 \pm 0.013 \pm 0.019$	$0.008 \pm 0.003 \pm 0.001$	$-0.004 \pm 0.004 \pm 0.003$
[1.1, 2.5]	$0.003 \pm 0.005 \pm 0.002$	$-0.003 \pm 0.007 \pm 0.001$	$0.001 \pm 0.003 \pm 0.000$	$-0.001 \pm 0.002 \pm 0.000$
[2.5, 4.]	$0.001 \pm 0.016 \pm 0.001$	$-0.005 \pm 0.017 \pm 0.001$	$-0.001 \pm 0.003 \pm 0.000$	$0.000 \pm 0.002 \pm 0.000$
[4., 6.]	$-0.002 \pm 0.015 \pm 0.000$	$-0.002 \pm 0.017 \pm 0.000$	$-0.000 \pm 0.001 \pm 0.000$	$-0.000 \pm 0.001 \pm 0.000$
[6., 8.]	$-0.005 \pm 0.009 \pm 0.001$	$0.002 \pm 0.010 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$-0.003 \pm 0.000 \pm 0.000$	$0.001 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$

Bin	T_3	T_4	T_5
[0.1, 0.98]	--	$-0.116 \pm 0.002 \pm 0.005$	$-0.075 \pm 0.003 \pm 0.001$
[1.1, 2.5]	--	--	$-0.017 \pm 0.004 \pm 0.001$
[2.5, 4.]	--	$-0.010 \pm 0.003 \pm 0.000$	$-0.006 \pm 0.003 \pm 0.000$
[4., 6.]	$-0.007 \pm 0.006 \pm 0.000$	$-0.007 \pm 0.003 \pm 0.000$	$-0.004 \pm 0.003 \pm 0.000$
[6., 8.]	$-0.005 \pm 0.004 \pm 0.060$	$-0.005 \pm 0.002 \pm 0.000$	$-0.003 \pm 0.002 \pm 0.000$
[15., 19.]	$-0.001 \pm 0.000 \pm 0.000$	$-0.001 \pm 0.000 \pm 0.000$	$-0.000 \pm 0.000 \pm 0.000$
Bin	T_7	T_8	T_9
[0.1, 0.98]	$-0.067 \pm 0.003 \pm 0.000$	$-0.081 \pm 0.025 \pm 0.051$	--
[1.1, 2.5]	$-0.013 \pm 0.003 \pm 0.000$	$-0.020 \pm 0.003 \pm 0.000$	--
[2.5, 4.]	$-0.007 \pm 0.003 \pm 0.000$	$-0.010 \pm 0.003 \pm 0.000$	$-0.010 \pm 0.027 \pm 0.000$
[4., 6.]	$-0.005 \pm 0.003 \pm 0.000$	$-0.007 \pm 0.003 \pm 0.000$	$-0.007 \pm 0.003 \pm 0.000$
[6., 8.]	$-0.003 \pm 0.002 \pm 0.000$	$-0.005 \pm 0.002 \pm 0.000$	$-0.005 \pm 0.004 \pm 0.000$
[15., 19.]	$-0.000 \pm 0.000 \pm 0.000$	$-0.001 \pm 0.001 \pm 0.004$	$-0.001 \pm 0.002 \pm 0.001$

Bin	B_5	B_{6s}	M
[0.1, 0.98]	$-0.155 \pm 0.002 \pm 0.002$	$-0.121 \pm 0.001 \pm 0.000$	$0.548 \pm 0.021 \pm 0.024$
[1.1, 2.5]	$-0.034 \pm 0.005 \pm 0.002$	$-0.027 \pm 0.000 \pm 0.000$	$0.150 \pm 0.071 \pm 0.037$
[2.5, 4.]	$-0.013 \pm 0.000 \pm 0.000$	$-0.015 \pm 0.001 \pm 0.000$	$-0.095 \pm 0.033 \pm 0.007$
[4., 6.]	$-0.009 \pm 0.000 \pm 0.000$	$-0.008 \pm 0.021 \pm 0.000$	$0.149 \pm 0.122 \pm 0.019$
[6., 8.]	$-0.006 \pm 0.000 \pm 0.000$	$-0.006 \pm 0.000 \pm 0.000$	$0.617 \pm 0.253 \pm 0.204$
[15., 19.]	$-0.003 \pm 0.000 \pm 0.000$	$-0.003 \pm 0.000 \pm 0.000$	--

Bin	\tilde{B}_5	\tilde{B}_{6s}	\tilde{M}
[0.1, 0.98]	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[1.1, 2.5]	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[2.5, 4.]	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[4., 6.]	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[6., 8.]	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$

C.2 Scenario 1: $C_{9\mu}^{\text{NP}} = -1.11$

Bin	Q_{FL}	Q_1	Q_2	Q_3
[0.1, 0.98]	$-0.085 \pm 0.073 \pm 0.021$	$-0.001 \pm 0.002 \pm 0.003$	$0.017 \pm 0.002 \pm 0.001$	$0.000 \pm 0.000 \pm 0.000$
[1.1, 2.5]	$-0.122 \pm 0.032 \pm 0.001$	$0.001 \pm 0.008 \pm 0.003$	$-0.008 \pm 0.010 \pm 0.001$	$-0.000 \pm 0.001 \pm 0.000$
[2.5, 4.]	$-0.086 \pm 0.037 \pm 0.002$	$-0.013 \pm 0.026 \pm 0.007$	$0.174 \pm 0.058 \pm 0.006$	$-0.001 \pm 0.002 \pm 0.000$
[4., 6.]	$-0.051 \pm 0.016 \pm 0.002$	$-0.022 \pm 0.038 \pm 0.010$	$0.246 \pm 0.009 \pm 0.002$	$-0.000 \pm 0.001 \pm 0.000$
[6., 8.]	$-0.027 \pm 0.008 \pm 0.003$	$-0.017 \pm 0.028 \pm 0.009$	$0.184 \pm 0.036 \pm 0.009$	$0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$-0.002 \pm 0.000 \pm 0.003$	$0.002 \pm 0.001 \pm 0.004$	$0.051 \pm 0.004 \pm 0.010$	$0.000 \pm 0.000 \pm 0.003$

Bin	Q_4	Q_5	Q_6	Q_8
[0.1, 0.98]	$0.136 \pm 0.011 \pm 0.049$	$0.172 \pm 0.004 \pm 0.016$	$-0.011 \pm 0.004 \pm 0.001$	$-0.012 \pm 0.004 \pm 0.003$
[1.1, 2.5]	$0.087 \pm 0.033 \pm 0.019$	$0.241 \pm 0.021 \pm 0.013$	$-0.002 \pm 0.001 \pm 0.000$	$-0.018 \pm 0.007 \pm 0.001$
[2.5, 4.]	$-0.037 \pm 0.035 \pm 0.010$	$0.370 \pm 0.017 \pm 0.014$	$-0.003 \pm 0.001 \pm 0.000$	$-0.014 \pm 0.007 \pm 0.001$
[4., 6.]	$-0.041 \pm 0.008 \pm 0.008$	$0.312 \pm 0.044 \pm 0.017$	$-0.006 \pm 0.002 \pm 0.000$	$-0.006 \pm 0.004 \pm 0.000$
[6., 8.]	$-0.020 \pm 0.005 \pm 0.010$	$0.212 \pm 0.056 \pm 0.029$	$-0.004 \pm 0.003 \pm 0.000$	$-0.002 \pm 0.002 \pm 0.001$
[15., 19.]	$-0.001 \pm 0.000 \pm 0.002$	$0.073 \pm 0.007 \pm 0.013$	$-0.001 \pm 0.000 \pm 0.020$	$-0.001 \pm 0.000 \pm 0.004$

Bin	\hat{Q}_{FL}	\hat{Q}_1	\hat{Q}_2	\hat{Q}_3
[0.1, 0.98]	$-0.037 \pm 0.022 \pm 0.011$	$-0.007 \pm 0.007 \pm 0.019$	$-0.009 \pm 0.003 \pm 0.000$	$0.000 \pm 0.001 \pm 0.001$
[1.1, 2.5]	$-0.086 \pm 0.049 \pm 0.001$	$0.001 \pm 0.008 \pm 0.003$	$-0.010 \pm 0.019 \pm 0.002$	$-0.000 \pm 0.001 \pm 0.000$
[2.5, 4.]	$-0.060 \pm 0.046 \pm 0.002$	$-0.014 \pm 0.026 \pm 0.007$	$0.183 \pm 0.048 \pm 0.006$	$-0.001 \pm 0.002 \pm 0.000$
[4., 6.]	$-0.033 \pm 0.021 \pm 0.002$	$-0.021 \pm 0.036 \pm 0.011$	$0.247 \pm 0.011 \pm 0.002$	$-0.000 \pm 0.001 \pm 0.000$
[6., 8.]	$-0.015 \pm 0.008 \pm 0.003$	$-0.017 \pm 0.026 \pm 0.009$	$0.182 \pm 0.035 \pm 0.009$	$0.000 \pm 0.000 \pm 0.000$
[15., 19.]	$-0.001 \pm 0.000 \pm 0.002$	$0.002 \pm 0.001 \pm 0.004$	$0.051 \pm 0.004 \pm 0.010$	$0.000 \pm 0.000 \pm 0.003$

Bin	\hat{Q}_4	\hat{Q}_5	\hat{Q}_6	\hat{Q}_8
[0.1, 0.98]	$0.214 \pm 0.008 \pm 0.010$	$-0.000 \pm 0.011 \pm 0.014$	$0.003 \pm 0.001 \pm 0.001$	$-0.014 \pm 0.007 \pm 0.001$
[1.1, 2.5]	$0.086 \pm 0.035 \pm 0.016$	$0.227 \pm 0.021 \pm 0.010$	$0.000 \pm 0.002 \pm 0.001$	$-0.019 \pm 0.007 \pm 0.001$
[2.5, 4.]	$-0.040 \pm 0.042 \pm 0.009$	$0.370 \pm 0.017 \pm 0.013$	$-0.003 \pm 0.002 \pm 0.000$	$-0.014 \pm 0.006 \pm 0.001$
[4., 6.]	$-0.045 \pm 0.016 \pm 0.008$	$0.314 \pm 0.043 \pm 0.017$	$-0.005 \pm 0.003 \pm 0.000$	$-0.006 \pm 0.004 \pm 0.000$
[6., 8.]	$-0.025 \pm 0.007 \pm 0.009$	$0.216 \pm 0.054 \pm 0.029$	$-0.004 \pm 0.003 \pm 0.000$	$-0.002 \pm 0.002 \pm 0.001$
[15., 19.]	$-0.003 \pm 0.000 \pm 0.002$	$0.074 \pm 0.007 \pm 0.013$	$-0.001 \pm 0.000 \pm 0.020$	$-0.001 \pm 0.000 \pm 0.004$

Bin	T_3	T_4	T_5
[0.1, 0.98]	--	--	$-0.026 \pm 0.038 \pm 0.011$
[1.1, 2.5]	--	--	$0.402 \pm 0.152 \pm 0.076$
[2.5, 4.]	--	$0.005 \pm 0.072 \pm 0.008$	$-0.608 \pm 0.295 \pm 0.121$
[4., 6.]	--	$-0.010 \pm 0.031 \pm 0.004$	$-0.224 \pm 0.061 \pm 0.026$
[6., 8.]	--	$-0.009 \pm 0.014 \pm 0.004$	$-0.126 \pm 0.042 \pm 0.025$
[15., 19.]	$-0.001 \pm 0.001 \pm 0.004$	$-0.002 \pm 0.000 \pm 0.001$	$-0.069 \pm 0.006 \pm 0.015$

Bin	T_7	T_8	T_9
[0.1, 0.98]	$-0.056 \pm 0.038 \pm 0.011$	--	--
[1.1, 2.5]	$0.029 \pm 0.071 \pm 0.010$	$-0.244 \pm 0.137 \pm 0.073$	--
[2.5, 4.]	$0.065 \pm 0.050 \pm 0.005$	$-0.143 \pm 0.075 \pm 0.023$	--
[4., 6.]	$0.087 \pm 0.028 \pm 0.003$	$-0.091 \pm 0.050 \pm 0.016$	--
[6., 8.]	$0.102 \pm 0.015 \pm 0.004$	$-0.067 \pm 0.083 \pm 0.025$	--
[15., 19.]	$0.118 \pm 0.001 \pm 0.003$	--	--

Bin	B_5	B_{6s}	M
[0.1, 0.98]	$-0.087 \pm 0.008 \pm 0.004$	$-0.084 \pm 0.005 \pm 0.001$	--
[1.1, 2.5]	--	$0.172 \pm 0.047 \pm 0.006$	$-0.203 \pm 0.049 \pm 0.012$
[2.5, 4.]	$-0.785 \pm 0.181 \pm 0.078$	--	$-0.459 \pm 0.106 \pm 0.026$
[4., 6.]	$-0.472 \pm 0.051 \pm 0.026$	--	$-0.736 \pm 0.188 \pm 0.062$
[6., 8.]	$-0.372 \pm 0.040 \pm 0.027$	$-0.569 \pm 0.150 \pm 0.032$	$-1.101 \pm 0.328 \pm 0.242$
[15., 19.]	$-0.316 \pm 0.007 \pm 0.018$	$-0.324 \pm 0.008 \pm 0.019$	--

Bin	\tilde{B}_5	\tilde{B}_{6s}	\tilde{M}
[0.1, 0.98]	$0.075 \pm 0.010 \pm 0.006$	$0.040 \pm 0.006 \pm 0.001$	$-0.083 \pm 0.017 \pm 0.006$
[1.1, 2.5]	--	$0.204 \pm 0.048 \pm 0.006$	$-0.247 \pm 0.049 \pm 0.015$
[2.5, 4.]	$-0.783 \pm 0.184 \pm 0.079$	--	$-0.463 \pm 0.102 \pm 0.027$
[4., 6.]	$-0.467 \pm 0.051 \pm 0.026$	--	$-0.723 \pm 0.182 \pm 0.061$
[6., 8.]	$-0.368 \pm 0.040 \pm 0.027$	$-0.566 \pm 0.151 \pm 0.032$	$-1.077 \pm 0.319 \pm 0.238$
[15., 19.]	$-0.314 \pm 0.007 \pm 0.018$	$-0.322 \pm 0.008 \pm 0.019$	--

C.3 Scenario 2: $C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.65$

Bin	Q_{FL}	Q_1	Q_2	Q_3
[0.1, 0.98]	$-0.096 \pm 0.081 \pm 0.013$	$-0.001 \pm 0.001 \pm 0.002$	$0.001 \pm 0.000 \pm 0.000$	$0.000 \pm 0.000 \pm 0.000$
[1.1, 2.5]	$-0.107 \pm 0.027 \pm 0.007$	$-0.002 \pm 0.008 \pm 0.002$	$-0.032 \pm 0.015 \pm 0.002$	$-0.000 \pm 0.001 \pm 0.000$
[2.5, 4.]	$-0.043 \pm 0.014 \pm 0.003$	$-0.017 \pm 0.039 \pm 0.008$	$0.148 \pm 0.037 \pm 0.003$	$0.000 \pm 0.001 \pm 0.000$
[4., 6.]	$-0.009 \pm 0.012 \pm 0.002$	$-0.011 \pm 0.027 \pm 0.005$	$0.134 \pm 0.029 \pm 0.006$	$0.001 \pm 0.001 \pm 0.000$
[6., 8.]	$0.003 \pm 0.011 \pm 0.003$	$-0.001 \pm 0.008 \pm 0.001$	$0.059 \pm 0.029 \pm 0.007$	$0.001 \pm 0.001 \pm 0.000$
[15., 19.]	$0.001 \pm 0.000 \pm 0.003$	$-0.002 \pm 0.001 \pm 0.005$	$0.005 \pm 0.001 \pm 0.003$	$0.000 \pm 0.000 \pm 0.003$

Bin	Q_4	Q_5	Q_6	Q_8
[0.1, 0.98]	$-0.003 \pm 0.007 \pm 0.027$	$0.078 \pm 0.007 \pm 0.029$	$-0.005 \pm 0.002 \pm 0.002$	$-0.005 \pm 0.001 \pm 0.003$
[1.1, 2.5]	$-0.102 \pm 0.028 \pm 0.014$	$0.136 \pm 0.017 \pm 0.012$	$-0.000 \pm 0.001 \pm 0.001$	$-0.005 \pm 0.002 \pm 0.001$
[2.5, 4.]	$-0.152 \pm 0.013 \pm 0.010$	$0.188 \pm 0.021 \pm 0.010$	$-0.007 \pm 0.002 \pm 0.001$	$0.002 \pm 0.003 \pm 0.001$
[4., 6.]	$-0.078 \pm 0.031 \pm 0.009$	$0.096 \pm 0.032 \pm 0.010$	$-0.008 \pm 0.004 \pm 0.000$	$0.005 \pm 0.004 \pm 0.000$
[6., 8.]	$-0.031 \pm 0.021 \pm 0.009$	$0.033 \pm 0.021 \pm 0.011$	$-0.004 \pm 0.003 \pm 0.000$	$0.004 \pm 0.003 \pm 0.001$
[15., 19.]	$0.000 \pm 0.000 \pm 0.002$	$0.007 \pm 0.001 \pm 0.006$	$-0.001 \pm 0.000 \pm 0.015$	$-0.001 \pm 0.001 \pm 0.005$

Bin	\hat{Q}_{FL}	\hat{Q}_1	\hat{Q}_2	\hat{Q}_3
[0.1, 0.98]	$-0.051 \pm 0.031 \pm 0.003$	$-0.007 \pm 0.006 \pm 0.019$	$-0.022 \pm 0.004 \pm 0.001$	$0.000 \pm 0.001 \pm 0.001$
[1.1, 2.5]	$-0.071 \pm 0.043 \pm 0.008$	$-0.002 \pm 0.008 \pm 0.002$	$-0.034 \pm 0.020 \pm 0.003$	$-0.000 \pm 0.001 \pm 0.000$
[2.5, 4.]	$-0.017 \pm 0.020 \pm 0.003$	$-0.017 \pm 0.040 \pm 0.008$	$0.159 \pm 0.028 \pm 0.003$	$0.000 \pm 0.001 \pm 0.000$
[4., 6.]	$0.009 \pm 0.007 \pm 0.002$	$-0.011 \pm 0.024 \pm 0.005$	$0.133 \pm 0.032 \pm 0.006$	$0.001 \pm 0.002 \pm 0.000$
[6., 8.]	$0.016 \pm 0.006 \pm 0.003$	$-0.000 \pm 0.005 \pm 0.001$	$0.056 \pm 0.027 \pm 0.007$	$0.001 \pm 0.002 \pm 0.000$
[15., 19.]	$0.002 \pm 0.001 \pm 0.004$	$-0.001 \pm 0.001 \pm 0.005$	$0.006 \pm 0.001 \pm 0.003$	$0.000 \pm 0.000 \pm 0.003$

Bin	\hat{Q}_4	\hat{Q}_5	\hat{Q}_6	\hat{Q}_8
[0.1, 0.98]	$0.107 \pm 0.007 \pm 0.015$	$-0.075 \pm 0.008 \pm 0.005$	$0.008 \pm 0.003 \pm 0.000$	$-0.008 \pm 0.004 \pm 0.001$
[1.1, 2.5]	$-0.097 \pm 0.030 \pm 0.012$	$0.126 \pm 0.017 \pm 0.009$	$0.002 \pm 0.002 \pm 0.000$	$-0.006 \pm 0.002 \pm 0.001$
[2.5, 4.]	$-0.154 \pm 0.009 \pm 0.010$	$0.189 \pm 0.022 \pm 0.010$	$-0.007 \pm 0.003 \pm 0.001$	$0.002 \pm 0.003 \pm 0.001$
[4., 6.]	$-0.079 \pm 0.023 \pm 0.008$	$0.098 \pm 0.030 \pm 0.010$	$-0.008 \pm 0.004 \pm 0.000$	$0.005 \pm 0.004 \pm 0.000$
[6., 8.]	$-0.035 \pm 0.015 \pm 0.008$	$0.037 \pm 0.021 \pm 0.011$	$-0.004 \pm 0.003 \pm 0.000$	$0.004 \pm 0.003 \pm 0.001$
[15., 19.]	$-0.003 \pm 0.000 \pm 0.002$	$0.009 \pm 0.001 \pm 0.006$	$-0.001 \pm 0.000 \pm 0.015$	$-0.001 \pm 0.001 \pm 0.005$

Bin	T_3	T_4	T_5
[0.1, 0.98]	--	$-0.158 \pm 0.050 \pm 0.043$	$-0.101 \pm 0.046 \pm 0.005$
[1.1, 2.5]	--	--	$0.276 \pm 0.131 \pm 0.056$
[2.5, 4.]	--	$-0.156 \pm 0.118 \pm 0.023$	$-0.234 \pm 0.100 \pm 0.039$
[4., 6.]	--	$-0.057 \pm 0.033 \pm 0.008$	$-0.070 \pm 0.022 \pm 0.008$
[6., 8.]	--	$-0.026 \pm 0.023 \pm 0.007$	$-0.026 \pm 0.015 \pm 0.005$
[15., 19.]	$-0.001 \pm 0.001 \pm 0.005$	$-0.000 \pm 0.000 \pm 0.001$	$-0.007 \pm 0.002 \pm 0.006$

Bin	T_7	T_8	T_9
[0.1, 0.98]	$-0.116 \pm 0.047 \pm 0.005$	--	--
[1.1, 2.5]	$0.015 \pm 0.056 \pm 0.002$	$-0.050 \pm 0.084 \pm 0.029$	--
[2.5, 4.]	$0.069 \pm 0.014 \pm 0.003$	$0.037 \pm 0.029 \pm 0.006$	--
[4., 6.]	$0.089 \pm 0.008 \pm 0.003$	$0.073 \pm 0.022 \pm 0.003$	--
[6., 8.]	$0.095 \pm 0.012 \pm 0.006$	$0.138 \pm 0.042 \pm 0.005$	--
[15., 19.]	$0.094 \pm 0.002 \pm 0.004$	--	--

Bin	B_5	B_{6s}	M
[0.1, 0.98]	$-0.248 \pm 0.003 \pm 0.002$	$-0.235 \pm 0.002 \pm 0.001$	--
[1.1, 2.5]	--	$-0.075 \pm 0.023 \pm 0.003$	$0.062 \pm 0.011 \pm 0.004$
[2.5, 4.]	$-0.546 \pm 0.090 \pm 0.039$	--	$-0.231 \pm 0.126 \pm 0.015$
[4., 6.]	$-0.389 \pm 0.025 \pm 0.013$	--	$-0.750 \pm 0.280 \pm 0.061$
[6., 8.]	$-0.338 \pm 0.020 \pm 0.013$	$-0.436 \pm 0.074 \pm 0.016$	$-1.550 \pm 0.570 \pm 0.305$
[15., 19.]	$-0.309 \pm 0.003 \pm 0.009$	$-0.313 \pm 0.004 \pm 0.009$	--

Bin	\tilde{B}_5	\tilde{B}_{6s}	\tilde{M}
[0.1, 0.98]	$-0.113 \pm 0.005 \pm 0.003$	$-0.131 \pm 0.003 \pm 0.001$	$-0.845 \pm 0.182 \pm 0.136$
[1.1, 2.5]	--	$-0.049 \pm 0.024 \pm 0.003$	$0.044 \pm 0.016 \pm 0.002$
[2.5, 4.]	$-0.540 \pm 0.091 \pm 0.039$	--	$-0.236 \pm 0.120 \pm 0.014$
[4., 6.]	$-0.383 \pm 0.025 \pm 0.013$	--	$-0.731 \pm 0.269 \pm 0.059$
[6., 8.]	$-0.334 \pm 0.020 \pm 0.013$	$-0.432 \pm 0.075 \pm 0.016$	$-1.508 \pm 0.551 \pm 0.297$
[15., 19.]	$-0.307 \pm 0.003 \pm 0.009$	$-0.311 \pm 0.004 \pm 0.009$	--

C.4 Scenario 3: $C_{9\mu}^{\text{NP}} = -C_{9'\mu}^{\text{NP}} = -1.07$

Bin	Q_{F_L}	Q_1	Q_2	Q_3
[0.1, 0.98]	$-0.109 \pm 0.094 \pm 0.034$	$-0.055 \pm 0.009 \pm 0.003$	$0.017 \pm 0.002 \pm 0.001$	$0.002 \pm 0.001 \pm 0.000$
[1.1, 2.5]	$-0.164 \pm 0.044 \pm 0.007$	$-0.204 \pm 0.024 \pm 0.005$	$-0.014 \pm 0.007 \pm 0.002$	$0.009 \pm 0.004 \pm 0.001$
[2.5, 4.]	$-0.133 \pm 0.060 \pm 0.003$	$-0.186 \pm 0.050 \pm 0.005$	$0.148 \pm 0.062 \pm 0.006$	$0.013 \pm 0.006 \pm 0.000$
[4., 6.]	$-0.106 \pm 0.037 \pm 0.004$	$-0.045 \pm 0.083 \pm 0.012$	$0.232 \pm 0.011 \pm 0.001$	$0.011 \pm 0.006 \pm 0.000$
[6., 8.]	$-0.089 \pm 0.021 \pm 0.007$	$0.074 \pm 0.072 \pm 0.015$	$0.190 \pm 0.032 \pm 0.008$	$0.007 \pm 0.005 \pm 0.000$
[15., 19.]	$-0.022 \pm 0.003 \pm 0.009$	$0.136 \pm 0.013 \pm 0.007$	$0.016 \pm 0.007 \pm 0.016$	$-0.017 \pm 0.007 \pm 0.007$

Bin	Q_4	Q_5	Q_6	Q_8
[0.1, 0.98]	$0.295 \pm 0.023 \pm 0.107$	$0.246 \pm 0.003 \pm 0.017$	$-0.017 \pm 0.007 \pm 0.002$	$-0.025 \pm 0.009 \pm 0.006$
[1.1, 2.5]	$0.233 \pm 0.050 \pm 0.045$	$0.271 \pm 0.016 \pm 0.013$	$-0.008 \pm 0.004 \pm 0.001$	$-0.030 \pm 0.012 \pm 0.003$
[2.5, 4.]	$0.031 \pm 0.068 \pm 0.021$	$0.347 \pm 0.021 \pm 0.017$	$-0.007 \pm 0.003 \pm 0.001$	$-0.025 \pm 0.013 \pm 0.001$
[4., 6.]	$-0.052 \pm 0.035 \pm 0.014$	$0.267 \pm 0.054 \pm 0.021$	$-0.008 \pm 0.003 \pm 0.000$	$-0.015 \pm 0.010 \pm 0.001$
[6., 8.]	$-0.082 \pm 0.022 \pm 0.017$	$0.153 \pm 0.071 \pm 0.038$	$-0.006 \pm 0.003 \pm 0.000$	$-0.008 \pm 0.008 \pm 0.001$
[15., 19.]	$-0.055 \pm 0.006 \pm 0.003$	$-0.009 \pm 0.008 \pm 0.021$	$-0.002 \pm 0.001 \pm 0.034$	$0.027 \pm 0.011 \pm 0.011$

Bin	\hat{Q}_{F_L}	\hat{Q}_1	\hat{Q}_2	\hat{Q}_3
[0.1, 0.98]	$-0.067 \pm 0.046 \pm 0.027$	$-0.048 \pm 0.011 \pm 0.019$	$-0.010 \pm 0.003 \pm 0.000$	$0.002 \pm 0.001 \pm 0.001$
[1.1, 2.5]	$-0.130 \pm 0.062 \pm 0.006$	$-0.202 \pm 0.021 \pm 0.005$	$-0.018 \pm 0.015 \pm 0.002$	$0.009 \pm 0.004 \pm 0.001$
[2.5, 4.]	$-0.108 \pm 0.070 \pm 0.003$	$-0.189 \pm 0.055 \pm 0.005$	$0.154 \pm 0.053 \pm 0.006$	$0.013 \pm 0.006 \pm 0.000$
[4., 6.]	$-0.089 \pm 0.045 \pm 0.004$	$-0.045 \pm 0.083 \pm 0.012$	$0.233 \pm 0.008 \pm 0.001$	$0.011 \pm 0.006 \pm 0.000$
[6., 8.]	$-0.076 \pm 0.025 \pm 0.007$	$0.075 \pm 0.071 \pm 0.015$	$0.189 \pm 0.031 \pm 0.008$	$0.007 \pm 0.006 \pm 0.000$
[15., 19.]	$-0.022 \pm 0.003 \pm 0.008$	$0.136 \pm 0.013 \pm 0.007$	$0.016 \pm 0.007 \pm 0.016$	$-0.017 \pm 0.007 \pm 0.007$
Bin	\hat{Q}_4	\hat{Q}_5	\hat{Q}_6	\hat{Q}_8
[0.1, 0.98]	$0.340 \pm 0.015 \pm 0.052$	$0.056 \pm 0.012 \pm 0.031$	$-0.002 \pm 0.002 \pm 0.003$	$-0.024 \pm 0.011 \pm 0.002$
[1.1, 2.5]	$0.227 \pm 0.053 \pm 0.041$	$0.255 \pm 0.015 \pm 0.010$	$-0.006 \pm 0.004 \pm 0.001$	$-0.031 \pm 0.013 \pm 0.003$
[2.5, 4.]	$0.025 \pm 0.074 \pm 0.020$	$0.348 \pm 0.021 \pm 0.017$	$-0.007 \pm 0.003 \pm 0.001$	$-0.025 \pm 0.013 \pm 0.001$
[4., 6.]	$-0.058 \pm 0.040 \pm 0.014$	$0.271 \pm 0.052 \pm 0.021$	$-0.008 \pm 0.003 \pm 0.000$	$-0.015 \pm 0.010 \pm 0.001$
[6., 8.]	$-0.089 \pm 0.024 \pm 0.016$	$0.159 \pm 0.069 \pm 0.038$	$-0.006 \pm 0.003 \pm 0.000$	$-0.009 \pm 0.008 \pm 0.001$
[15., 19.]	$-0.057 \pm 0.006 \pm 0.003$	$-0.008 \pm 0.008 \pm 0.021$	$-0.002 \pm 0.001 \pm 0.033$	$0.027 \pm 0.011 \pm 0.011$

Bin	T_3	T_4	T_5
[0.1, 0.98]	--	--	$-0.007 \pm 0.061 \pm 0.021$
[1.1, 2.5]	--	--	$0.436 \pm 0.158 \pm 0.080$
[2.5, 4.]	--	$0.091 \pm 0.149 \pm 0.012$	$-0.528 \pm 0.296 \pm 0.122$
[4., 6.]	--	$0.004 \pm 0.087 \pm 0.006$	$-0.161 \pm 0.090 \pm 0.029$
[6., 8.]	--	$-0.031 \pm 0.066 \pm 0.006$	$-0.074 \pm 0.075 \pm 0.032$
[15., 19.]	$-0.103 \pm 0.021 \pm 0.011$	$-0.031 \pm 0.004 \pm 0.003$	$-0.002 \pm 0.008 \pm 0.017$
Bin	T_7	T_8	T_9
[0.1, 0.98]	$-0.036 \pm 0.062 \pm 0.021$	--	--
[1.1, 2.5]	$0.081 \pm 0.105 \pm 0.021$	$-0.514 \pm 0.246 \pm 0.198$	--
[2.5, 4.]	$0.121 \pm 0.086 \pm 0.011$	$-0.322 \pm 0.117 \pm 0.059$	$0.830 \pm 0.290 \pm 0.082$
[4., 6.]	$0.136 \pm 0.069 \pm 0.008$	$-0.283 \pm 0.112 \pm 0.045$	$0.791 \pm 0.276 \pm 0.080$
[6., 8.]	$0.144 \pm 0.058 \pm 0.012$	$-0.304 \pm 0.312 \pm 0.100$	--
[15., 19.]	$0.177 \pm 0.005 \pm 0.009$	--	--

Bin	B_5	B_{6s}	M
[0.1, 0.98]	$-0.089 \pm 0.008 \pm 0.004$	$-0.086 \pm 0.005 \pm 0.001$	--
[1.1, 2.5]	--	$0.165 \pm 0.045 \pm 0.006$	$-0.194 \pm 0.047 \pm 0.011$
[2.5, 4.]	$-0.758 \pm 0.175 \pm 0.075$	--	$-0.443 \pm 0.102 \pm 0.026$
[4., 6.]	$-0.455 \pm 0.049 \pm 0.025$	--	$-0.710 \pm 0.182 \pm 0.060$
[6., 8.]	$-0.359 \pm 0.039 \pm 0.026$	$-0.549 \pm 0.145 \pm 0.031$	$-1.063 \pm 0.317 \pm 0.234$
[15., 19.]	$-0.305 \pm 0.007 \pm 0.017$	$-0.312 \pm 0.007 \pm 0.018$	--

Bin	\tilde{B}_5	\tilde{B}_{6s}	\tilde{M}
[0.1, 0.98]	$0.072 \pm 0.010 \pm 0.005$	$0.038 \pm 0.006 \pm 0.001$	$-0.080 \pm 0.016 \pm 0.006$
[1.1, 2.5]	--	$0.197 \pm 0.046 \pm 0.006$	$-0.238 \pm 0.047 \pm 0.015$
[2.5, 4.]	$-0.755 \pm 0.177 \pm 0.076$	--	$-0.447 \pm 0.098 \pm 0.026$
[4., 6.]	$-0.450 \pm 0.050 \pm 0.025$	--	$-0.697 \pm 0.176 \pm 0.059$
[6., 8.]	$-0.355 \pm 0.039 \pm 0.026$	$-0.546 \pm 0.146 \pm 0.031$	$-1.038 \pm 0.308 \pm 0.229$
[15., 19.]	$-0.303 \pm 0.007 \pm 0.018$	$-0.310 \pm 0.007 \pm 0.018$	--

C.5 Scenario 4: $C_{9\mu}^{\text{NP}} = -C_{9'\mu}^{\text{NP}} = -1.18$, $C_{10\mu}^{\text{NP}} = C_{10'\mu}^{\text{NP}} = 0.38$

Bin	Q_{F_L}	Q_1	Q_2	Q_3
[0.1, 0.98]	$-0.113 \pm 0.097 \pm 0.037$	$-0.063 \pm 0.010 \pm 0.004$	$0.006 \pm 0.001 \pm 0.001$	$0.002 \pm 0.001 \pm 0.000$
[1.1, 2.5]	$-0.167 \pm 0.044 \pm 0.009$	$-0.280 \pm 0.037 \pm 0.006$	$-0.044 \pm 0.009 \pm 0.003$	$0.010 \pm 0.004 \pm 0.001$
[2.5, 4.]	$-0.120 \pm 0.052 \pm 0.004$	$-0.371 \pm 0.045 \pm 0.005$	$0.146 \pm 0.071 \pm 0.007$	$0.016 \pm 0.007 \pm 0.000$
[4., 6.]	$-0.084 \pm 0.027 \pm 0.005$	$-0.236 \pm 0.092 \pm 0.013$	$0.230 \pm 0.014 \pm 0.004$	$0.014 \pm 0.008 \pm 0.000$
[6., 8.]	$-0.064 \pm 0.014 \pm 0.009$	$-0.078 \pm 0.087 \pm 0.018$	$0.175 \pm 0.033 \pm 0.008$	$0.009 \pm 0.007 \pm 0.000$
[15., 19.]	$-0.013 \pm 0.002 \pm 0.010$	$0.068 \pm 0.008 \pm 0.011$	$0.024 \pm 0.006 \pm 0.015$	$-0.020 \pm 0.009 \pm 0.008$

Bin	Q_4	Q_5	Q_6	Q_8
[0.1, 0.98]	$0.336 \pm 0.025 \pm 0.118$	$0.271 \pm 0.005 \pm 0.026$	$-0.018 \pm 0.007 \pm 0.003$	$-0.028 \pm 0.010 \pm 0.007$
[1.1, 2.5]	$0.276 \pm 0.052 \pm 0.052$	$0.337 \pm 0.022 \pm 0.006$	$-0.011 \pm 0.005 \pm 0.002$	$-0.034 \pm 0.014 \pm 0.003$
[2.5, 4.]	$0.089 \pm 0.066 \pm 0.025$	$0.430 \pm 0.021 \pm 0.013$	$-0.012 \pm 0.004 \pm 0.001$	$-0.026 \pm 0.014 \pm 0.002$
[4., 6.]	$0.018 \pm 0.035 \pm 0.017$	$0.324 \pm 0.059 \pm 0.019$	$-0.012 \pm 0.005 \pm 0.000$	$-0.016 \pm 0.011 \pm 0.001$
[6., 8.]	$-0.016 \pm 0.028 \pm 0.021$	$0.187 \pm 0.074 \pm 0.035$	$-0.008 \pm 0.005 \pm 0.000$	$-0.009 \pm 0.009 \pm 0.001$
[15., 19.]	$-0.027 \pm 0.004 \pm 0.004$	$0.017 \pm 0.008 \pm 0.020$	$-0.002 \pm 0.001 \pm 0.039$	$0.031 \pm 0.013 \pm 0.013$

Bin	\hat{Q}_{F_L}	\hat{Q}_1	\hat{Q}_2	\hat{Q}_3
[0.1, 0.98]	$-0.072 \pm 0.051 \pm 0.031$	$-0.055 \pm 0.012 \pm 0.020$	$-0.018 \pm 0.003 \pm 0.001$	$0.002 \pm 0.001 \pm 0.001$
[1.1, 2.5]	$-0.133 \pm 0.062 \pm 0.009$	$-0.277 \pm 0.034 \pm 0.006$	$-0.048 \pm 0.014 \pm 0.003$	$0.010 \pm 0.004 \pm 0.001$
[2.5, 4.]	$-0.094 \pm 0.062 \pm 0.004$	$-0.378 \pm 0.054 \pm 0.005$	$0.153 \pm 0.062 \pm 0.007$	$0.016 \pm 0.008 \pm 0.000$
[4., 6.]	$-0.065 \pm 0.034 \pm 0.005$	$-0.239 \pm 0.097 \pm 0.013$	$0.231 \pm 0.010 \pm 0.004$	$0.014 \pm 0.008 \pm 0.000$
[6., 8.]	$-0.051 \pm 0.017 \pm 0.009$	$-0.079 \pm 0.087 \pm 0.018$	$0.173 \pm 0.032 \pm 0.008$	$0.009 \pm 0.007 \pm 0.000$
[15., 19.]	$-0.013 \pm 0.002 \pm 0.010$	$0.068 \pm 0.009 \pm 0.011$	$0.024 \pm 0.006 \pm 0.015$	$-0.020 \pm 0.009 \pm 0.008$

Bin	\hat{Q}_4	\hat{Q}_5	\hat{Q}_6	\hat{Q}_8
[0.1, 0.98]	$0.372 \pm 0.016 \pm 0.060$	$0.076 \pm 0.014 \pm 0.041$	$-0.002 \pm 0.002 \pm 0.003$	$-0.027 \pm 0.012 \pm 0.003$
[1.1, 2.5]	$0.269 \pm 0.055 \pm 0.047$	$0.319 \pm 0.023 \pm 0.006$	$-0.008 \pm 0.005 \pm 0.002$	$-0.034 \pm 0.014 \pm 0.003$
[2.5, 4.]	$0.083 \pm 0.072 \pm 0.024$	$0.431 \pm 0.020 \pm 0.013$	$-0.011 \pm 0.005 \pm 0.001$	$-0.027 \pm 0.014 \pm 0.002$
[4., 6.]	$0.012 \pm 0.040 \pm 0.017$	$0.327 \pm 0.056 \pm 0.019$	$-0.012 \pm 0.005 \pm 0.000$	$-0.016 \pm 0.011 \pm 0.001$
[6., 8.]	$-0.023 \pm 0.030 \pm 0.021$	$0.192 \pm 0.072 \pm 0.034$	$-0.008 \pm 0.005 \pm 0.000$	$-0.009 \pm 0.009 \pm 0.001$
[15., 19.]	$-0.029 \pm 0.004 \pm 0.004$	$0.018 \pm 0.008 \pm 0.020$	$-0.002 \pm 0.001 \pm 0.039$	$0.031 \pm 0.013 \pm 0.013$

Bin	T_3	T_4	T_5
[0.1, 0.98]	--	--	$0.002 \pm 0.065 \pm 0.027$
[1.1, 2.5]	$0.991 \pm 0.188 \pm 0.182$	--	$0.488 \pm 0.162 \pm 0.090$
[2.5, 4.]	$1.010 \pm 0.231 \pm 0.028$	$0.133 \pm 0.149 \pm 0.012$	$-0.809 \pm 0.524 \pm 0.177$
[4., 6.]	--	$0.040 \pm 0.074 \pm 0.007$	$-0.222 \pm 0.085 \pm 0.032$
[6., 8.]	--	$0.002 \pm 0.050 \pm 0.008$	$-0.101 \pm 0.063 \pm 0.030$
[15., 19.]	$-0.047 \pm 0.010 \pm 0.014$	$-0.016 \pm 0.002 \pm 0.004$	$-0.021 \pm 0.007 \pm 0.017$

Bin	T_7	T_8	T_9
[0.1, 0.98]	$-0.034 \pm 0.066 \pm 0.023$	--	--
[1.1, 2.5]	$0.094 \pm 0.107 \pm 0.023$	$-0.614 \pm 0.296 \pm 0.250$	$0.974 \pm 0.486 \pm 0.234$
[2.5, 4.]	$0.146 \pm 0.076 \pm 0.012$	$-0.371 \pm 0.120 \pm 0.072$	$0.849 \pm 0.264 \pm 0.074$
[4., 6.]	$0.170 \pm 0.053 \pm 0.009$	$-0.319 \pm 0.112 \pm 0.055$	$0.817 \pm 0.252 \pm 0.071$
[6., 8.]	$0.183 \pm 0.040 \pm 0.013$	$-0.346 \pm 0.371 \pm 0.126$	--
[15., 19.]	$0.205 \pm 0.004 \pm 0.011$	--	--

Bin	B_5	B_{6s}	M
[0.1, 0.98]	$-0.075 \pm 0.010 \pm 0.009$	$-0.166 \pm 0.009 \pm 0.003$	$-0.138 \pm 0.031 \pm 0.031$
[1.1, 2.5]	--	$0.059 \pm 0.048 \pm 0.005$	$-0.062 \pm 0.051 \pm 0.006$
[2.5, 4.]	$-0.916 \pm 0.202 \pm 0.077$	--	$-0.446 \pm 0.163 \pm 0.022$
[4., 6.]	$-0.552 \pm 0.052 \pm 0.024$	--	$-1.009 \pm 0.337 \pm 0.079$
[6., 8.]	$-0.439 \pm 0.038 \pm 0.021$	$-0.577 \pm 0.119 \pm 0.028$	$-1.888 \pm 0.668 \pm 0.376$
[15., 19.]	$-0.369 \pm 0.007 \pm 0.016$	$-0.374 \pm 0.007 \pm 0.017$	--

Bin	\tilde{B}_5	\tilde{B}_{6s}	\tilde{M}
[0.1, 0.98]	$0.088 \pm 0.013 \pm 0.012$	$-0.054 \pm 0.011 \pm 0.003$	$0.033 \pm 0.003 \pm 0.002$
[1.1, 2.5]	--	$0.088 \pm 0.050 \pm 0.006$	$-0.094 \pm 0.054 \pm 0.007$
[2.5, 4.]	$-0.916 \pm 0.205 \pm 0.078$	--	$-0.453 \pm 0.159 \pm 0.023$
[4., 6.]	$-0.548 \pm 0.053 \pm 0.024$	--	$-0.994 \pm 0.328 \pm 0.078$
[6., 8.]	$-0.436 \pm 0.038 \pm 0.021$	$-0.575 \pm 0.119 \pm 0.028$	$-1.851 \pm 0.651 \pm 0.369$
[15., 19.]	$-0.367 \pm 0.007 \pm 0.016$	$-0.372 \pm 0.007 \pm 0.017$	--

C.6 R_{K^*}

R_{K^*}				
Bin	[0.1, 2]	[2, 4.3]	[4.3, 8.68]	[16., 19.]
SM	$0.988 \pm 0.007 \pm 0.001$	$1.000 \pm 0.006 \pm 0.000$	$1.000 \pm 0.005 \pm 0.000$	$0.998 \pm 0.000 \pm 0.000$
Scen.1	$0.951 \pm 0.096 \pm 0.021$	$0.871 \pm 0.093 \pm 0.009$	$0.813 \pm 0.026 \pm 0.029$	$0.786 \pm 0.001 \pm 0.004$
Scen.2	$0.889 \pm 0.102 \pm 0.008$	$0.737 \pm 0.028 \pm 0.005$	$0.701 \pm 0.016 \pm 0.045$	$0.701 \pm 0.003 \pm 0.006$
Scen.3	$0.898 \pm 0.142 \pm 0.039$	$0.780 \pm 0.142 \pm 0.018$	$0.747 \pm 0.090 \pm 0.045$	$0.692 \pm 0.006 \pm 0.013$
Scen.4	$0.890 \pm 0.149 \pm 0.043$	$0.742 \pm 0.123 \pm 0.019$	$0.690 \pm 0.059 \pm 0.052$	$0.655 \pm 0.005 \pm 0.015$

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