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Lagrangian formulation of the massive higher spin supermultiplets in three dimensional space-time

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Abstract: We give explicit construction for massive higher spin supermultiplets for the case of minimal supersymmetry in $d = 3$ and find the corresponding Lagrangian formulations. We show that all such massive supermultiplets can be straightforwardly constructed out of the appropriately chosen set of massless ones exactly in the same way as the gauge invariant description for the massive bosonic (fermionic) field with spin s can be obtained using a set of massless fields with spins $s, s-1, \ldots, 0(1/2)$. Moreover, such construction for the massive supermultiplets turns out to be perfectly consistent with our previous results on the gauge invariant Lagrangian formulation for massive higher spin bosons and fermions in $d = 3$.

Keywords: Supersymmetric gauge theory, Field Theories in Lower Dimensions, Higher Spin Symmetry, Gauge Symmetry

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Contents

1 Introduction

Last time the higher spin field theory is becoming one the the central directions in modern theoretical and mathematical physics (see e.g. the reviews [1–8]). In this paper we are going to study some aspects of the higher spin field theory related to constructing the massive supersymmetric higher spin models in three space-time dimensions.[1]

In higher spin field theories, in spite of their infinite dimensional gauge algebras, the supersymmetry still plays a distinguished role in many aspects. It is enough to remind that, for example, in the superstring theory all these massive higher spin states are perfectly combined into the massive supermultiplets and it is the supersymmetry that stays behind many nice feature of these theories.

In general, the classification of massless and massive supermultiplets is a rather straightforward algebraic task depending mainly on the space-time dimension and on the specific properties of the fermions in this space-time. But as far as explicit construction (in terms of fields and Lagrangians) is concerned, the situation with massless and massive

[1]Strictly speaking, the notion of spin does not exist in three dimensions in literal sense. For the massless fields the only characteristic is statistics (i.e. boson or fermion, see e.g. [9]), while massive fields are characterized by their helicities in the same way as massless ones in four dimensions. But the term spin is widely used in the literature on three dimensional theories and so we will also use it here.

higher spin supermultiplets drastically differ. For the massless supermultiplets it is not hard to find such realization following to the simple general pattern:

$$\delta B \sim F\zeta, \qquad \delta F \sim \partial B\zeta.$$

This is an essential reason why the supersymmetric massless higher spin theory is developed good enough (see [10–15] for massless higher spin models in four dimensions).

But the analogous construction for the massive supermultiplets appears to be very complicated even if one uses a powerful superfield technique (see e.g. [16–21] for some examples of higher superspin superfields in four dimensional theory and [22–24] in three dimensional ones). The reason is that shifting from massless to the massive case one has to introduce very complicated higher derivative corrections to the supertransformations. Moreover the higher the spins of the fields entering supermultiplet the higher the number of derivatives one has to consider.

In four dimensions the solution for this problem was proposed [25–27] in component approach[2] based on the gauge invariant formalism for the massive higher spin bosonic [28] and fermionic [29] fields. Recall that to obtain gauge invariant description for massive bosonic (fermionic) field with spin s, one introduces a set of massless fields with spins $s, s-1, \ldots, 0(\frac{1}{2})$ with their usual kinetic terms and local gauge transformations. Then one adds all possible low derivative terms into the Lagrangian mixing all these massless fields together as well as non-derivative corrections to the gauge transformations to restore the gauge symmetries broken by the mass terms. Now if one takes some massive supermultiplet and decomposes each massive field into the appropriate set of massless ones, one immediately sees that all these massless fields are perfectly combined into the set of massless supermultiplets. Thus the main idea of [25–27] was to generalize the gauge invariant description of massive particles to the case of massive supermultiplets. Namely, one introduces appropriate set of massless supermultiplets with all their fields, kinetic terms and initial supertransformations and than adds lower derivative terms to the Lagrangian as well *non-derivative corrections to the supertransformations for the fermions only*. It is the absence of any higher derivative terms that makes such construction to be pretty straightforward in spite of the large number of fields involved. At the same time if one tries to fix all these local symmetries then all these complicated higher derivatives corrections reappeared just as the transformations restoring the gauge.

In this paper we give an explicit construction for the massive supermultiplets with arbitrary spins in three dimensions for the case of minimal supersymmetry. Naturally, this construction is based on the gauge invariant description for the massive $d = 3$ bosonic [30] and fermionic [31] higher spin fields developed in our previous works.[3] Recall that in $d = 4$ it was crucial for the whole construction that there exists the possibility to consider a kind

[2] The Lagrangian formulation given in these papers is on-shell and does not include the auxiliary fields which are needed for closing the superalgebra. Off-shell supersymmetric formulation for these models is still unknown.

[3] Non gauge invariant formalism has been developed in [32].

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of dual mixing for the massless supermultiplets:

$$\begin{pmatrix} \Psi_{s+\frac{1}{2}} \\ A_s \end{pmatrix} \oplus \begin{pmatrix} B_s \\ \Phi_{s-\frac{1}{2}} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} & \Psi_{s+\frac{1}{2}} & \\ A_s & & B_s \\ & \Phi_{s-\frac{1}{2}} & \end{pmatrix},$$

where A_s and B_s are two bosonic fields with equal spins but opposite parities. Similarly, in $d = 3$ case we found that it is crucial that there exist the possibility to consider massless supermultiplets containing one bosonic and two fermionic fields:

$$\begin{pmatrix} \Psi_{s+\frac{1}{2}} \\ f_s \\ \Phi_{s-\frac{1}{2}} \end{pmatrix}.$$

Recall that in three dimensions massless higher spin bosonic and fermionic fields do not have any physical degrees of freedom. Thus such a structure of the supermultiplet does not contradict to the fact that in any supermultiplet the numbers of bosonic and fermionic physical degrees of freedom must be equal. Note here that massive higher spin supermultiplets do have physical degrees of freedom that originate form the massless supermultiplets containing spins 1, 1/2 and 0 (that inevitably appear in the decomposition of massive supermultiplet into the massless ones) exactly in the same way as in the gauge invariant description of massive higher spin fields physical degrees of freedom come from the components with spins 1, 1/2 and 0.

The paper is organized as follows. In sections 2 and 3 we provide all necessary formulas for massless high spin bosonic and fermionic fields and massless higher spin supermultiplets in the frame-like multispinor formalism we use in this work (see below). Section 4 contains two relatively simple examples, namely massive supermultiplets $(\frac{3}{2}, 1, \frac{1}{2})$ and $(2, \frac{3}{2}, \frac{3}{2})$,[4] illustrating our general technique. Sections 5 and 6 contains our main results: massive supermultiplets $(s + \frac{1}{2}, s, s - \frac{1}{2})$ and $(s, s - \frac{1}{2}, s - \frac{1}{2})$, correspondingly. To make our paper self-contained as much as possible, we include three appendices giving gauge invariant description of massive higher spin bosons, fermions with Majorana mass terms and fermions with Dirac mass terms, adopted to the formalism used in this work.

Notations and conventions. We will work in the frame-like multispinor formalism where all objects are one-forms or zero-forms completely symmetric on their local spinor indices. Spinor indices $\alpha, \beta, \dots = 1, 2$ are raised and lowered with the help of antisymmetric bi-spinor $\varepsilon^{\alpha\beta}$:

$$\varepsilon^{\alpha\gamma}\varepsilon_{\gamma\beta} = -\delta^\alpha{}_\beta, \qquad \varepsilon^{\alpha\beta}A_\beta = A^\alpha, \qquad \varepsilon_{\alpha\beta}A^\beta = -A_\alpha.$$

To simplify formulas we will often use a shorthand notations for spinor indices:

$$\Psi^{\alpha_1\alpha_2\dots\alpha_n} = \Psi^{\alpha(n)}.$$

We will also assume that spinor indices denoted by the same letter and placed on the same level are symmetrized:

$$\Psi^{\alpha(n)}\zeta^\alpha = \Psi^{(\alpha_1\dots\alpha_n}\zeta^{\alpha_{n+1})},$$

[4] Such supermultiplet has been constructed previously by dimensional reduction from $d = 4$ [33].

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where symmetrization contains the minimum number of terms necessary without any normalization. Basis elements of $1, 2, 3$-form spaces are respectively $e^{\alpha(2)}$, $E_2{}^{\alpha(2)}$, E_3 where the last two are defined as double and triple wedge product of $e^{\alpha(2)}$:

$$e^{\alpha\alpha} \wedge e^{\beta\beta} = \varepsilon^{\alpha\beta} E_2{}^{\alpha\beta},$$
$$E_2{}^{\alpha\alpha} \wedge e^{\beta\beta} = \varepsilon^{\alpha\beta}\varepsilon^{\alpha\beta} E_3.$$

Let us write some useful relations for these basis elements

$$E_2{}^{\alpha}{}_{\gamma} \wedge e^{\gamma\beta} = 3\varepsilon^{\alpha\beta} E_3, \qquad e^{\alpha}{}_{\gamma} \wedge e^{\gamma\beta} = 4E_2{}^{\alpha\beta}.$$

In what follows we will systematically omit the $\wedge$ symbol.

2 Massless fields

In this section we give all necessary formulas for massless bosonic and fermionic higher spin fields in the flat three-dimensional space-time.

Boson with spin $s = l + 1$, $l \geq 1$ requires two one-forms $\Omega^{\alpha(2l)}$ and $f^{\alpha(2l)}$ completely symmetric on their local spinor indices. The free Lagrangian (which is a three-form in our formalism) looks like:

$$(-1)^{l+1}\mathcal{L}_0 = l\Omega_{\alpha(2l-1)\beta} e^{\beta}{}_{\gamma}\Omega^{\alpha(2l-1)\gamma} + \Omega_{\alpha(2l)} d\Phi^{\alpha(2l)}, \tag{2.1}$$

where d is an external derivative. This Lagrangian is invariant under the following local gauge transformations:

$$\delta_0\Omega^{\alpha(2l)} = d\eta^{\alpha(2l)}, \qquad \delta_0\Phi^{\alpha(2l)} = d\xi^{\alpha(2l)} + e^{\alpha}{}_{\beta}\eta^{\alpha(2l-1)\beta}, \tag{2.2}$$

where $\eta^{\alpha(2l)}$ and $\xi^{\alpha(2l)}$ are zero forms also completely symmetric in their indices.

Boson with spin 1 requires zero-form $B^{\alpha\beta}$ and one-form A. The free Lagrangian and gauge transformations are:

$$\mathcal{L}_0 = E_3 B^{\alpha\beta} B_{\alpha\beta} - B_{\alpha\beta} e^{\alpha\beta} dA, \qquad \delta_0 A = d\xi. \tag{2.3}$$

Equation for the auxiliary field $B^{\alpha\beta}$ has the form:

$$2E_3 B^{\alpha\beta} = e^{\alpha\beta} dA \quad \Rightarrow \quad E_2{}^{\alpha\beta} B_{\alpha\beta} = dA \tag{2.4}$$

and as a result we have the following on-shell identity:

$$E^{\alpha\beta} dB_{\alpha\beta} = 0 \quad \Rightarrow \quad E^{\alpha}{}_{\gamma} dB^{\beta\gamma} = E^{\beta}{}_{\gamma} dB^{\alpha\gamma}.$$

Boson with spin 0 requires two zero-forms $\pi^{\alpha\beta}$ and φ. The free Lagrangian:

$$\mathcal{L}_0 = E_3 \pi^{\alpha\beta}\pi_{\alpha\beta} - E_2{}^{\alpha\beta}\pi_{\alpha\beta} d\varphi. \tag{2.5}$$

Equation for the auxiliary field $\pi^{\alpha\beta}$:

$$2E_3\pi^{\alpha\beta} = E_2{}^{\alpha\beta} d\varphi \quad \Rightarrow \quad e^{\alpha\beta}\pi_{\alpha\beta} = d\varphi \tag{2.6}$$

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leads to the following on-shell identity:

$$e^{\alpha\beta} d\pi_{\alpha\beta} = 0 \quad \Rightarrow \quad e^\alpha{}_\gamma d\pi^{\beta\gamma} = e^\beta{}_\gamma d\pi^{\alpha\gamma}.$$

Fermion with spin $s = l + \frac{1}{2}$, $l \geq 1$ is described by one-form $\Psi^{\alpha(2l-1)}$ (also completely symmetric on spinor indices) with the free Lagrangian and gauge transformations having the form:

$$(-1)^{l+1} \mathcal{L}_0 = \frac{i}{2} \Psi_{\alpha(2l-1)} d\Psi^{\alpha(2l-1)}, \qquad \delta_0 \Psi^{\alpha(2l-1)} = d\zeta^{\alpha(2l-1)}. \tag{2.7}$$

Fermion with spin $\frac{1}{2}$ is described by zero-form ϕ^α with the free Lagrangian:

$$\mathcal{L}_0 = \frac{i}{2} \phi_\alpha E^\alpha{}_\beta d\phi^\beta. \tag{2.8}$$

3 Massless supermultiplets

As it has been already noted in the Introduction, the main idea of this work is that massive supermultiplet can be straightforwardly constructed out of appropriate set of massless supermultiplets exactly in the same way as gauge invariant description of massive bosonic or fermionic higher spin field can be constructed out of appropriate set of massless ones. As it will be seen further on in the analysis of massive supermultiplets, the main role as a building block is played by the massless supermultiplet containing one bosonic and two fermionic fields, namely (here and in what follows s is always an integer) $(s + \frac{1}{2}, s, s - \frac{1}{2})$.

Supermultiplet $(l + \frac{3}{2}, l + 1, l + \frac{1}{2})$, $l \geq 1$ contains two fermionic $\Phi^{\alpha(2l+1)}$ and $\Psi^{\alpha(2l-1)}$ and two bosonic $\Omega^{\alpha(2l)}$, $f^{\alpha(2l)}$ one-forms. The sum of their kinetic terms:

$$\begin{aligned}\mathcal{L}_0 = (-1)^{l+1} \Big[& l\Omega_{\alpha(2l-1)\beta} e^\beta{}_\gamma \Omega^{\alpha(2l-1)\gamma} + \Omega_{\alpha(2l)} df^{\alpha(2l)} \\ & + \frac{i}{2} \Phi_{\alpha(2l+1)} d\Phi^{\alpha(2l+1)} - \frac{i}{2} \Psi_{\alpha(2l-1)} d\Psi^{\alpha(2l-1)} \Big]\end{aligned} \tag{3.1}$$

is invariant under the following global supertransformations:

$$\begin{aligned}\delta f^{\alpha(2l)} &= i\alpha_l \Psi^{\alpha(2l-1)} \zeta^\alpha + i(2l+1)\beta_l \Phi^{\alpha(2l)\beta} \zeta_\beta, \\ \delta\Phi^{\alpha(2l+1)} &= \beta_l \Omega^{\alpha(2l)} \zeta^\alpha, \\ \delta\Psi^{\alpha(2l-1)} &= 2l\alpha_l \Omega^{\alpha(2l-1)\beta} \zeta_\beta.\end{aligned} \tag{3.2}$$

In what follows we will fix the normalization of supertransformations so that

$$2l{\alpha_l}^2 + (2l+1){\beta_l}^2 = 2,$$

while the relative values of α_l and β_l will depend on the massive supermultiplet which this massless one enters in. Thus in general such supermultiplet, its Lagrangian and the supertransformations contain two fermionic and one bosonic fields. But as the particular cases we can obtain supermultiplets with one fermionic and one bosonic fields. Namely, putting $\alpha_l = 0$ we get supermultiplet $(l + 3/2, l + 1)$, while the case $\beta_l = 0$ corresponds to $(l + 1, l + 1/2)$.

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Supermultiplet $(\frac{3}{2},1,\frac{1}{2})$ contains fermionic one-form Φ^α and zero-form ψ^α as well as bosonic zero-form $B^{\alpha\beta}$ and one-form A. The sum of their kinetic terms

$$\mathcal{L}_0 = -\frac{i}{2}\Phi_\alpha d\Phi^\alpha + EB^{\alpha\beta}B_{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}dA + \frac{i}{2}\psi_\alpha E^\alpha{}_\beta d\psi^\alpha \tag{3.3}$$

is invariant under the following supertransformations:[5]

$$\begin{aligned}
\delta A &= i\beta_0\Phi_\alpha\zeta^\alpha + i\alpha_0\psi_\alpha e^{\alpha\beta}\zeta_\beta, \\
\delta\Phi^\alpha &= -\beta_0 e^{\beta\gamma}B_{\beta\gamma}\zeta^\alpha, \\
\delta\psi^\alpha &= 4\alpha_0 B^{\alpha\beta}\zeta_\beta.
\end{aligned} \tag{3.4}$$

Supermultiplet $(\frac{1}{2},0)$ contains fermionic zero-form ϕ_α and two bosonic zero-forms $\pi^{\alpha\beta}$ and φ. The sum of kinetic terms

$$\mathcal{L}_0 = \frac{i}{2}\phi_\alpha E^\alpha{}_\beta d\phi^\alpha - E\pi^{\alpha\beta}\pi_{\alpha\beta} + E^{\alpha\beta}\pi_{\alpha\beta}d\varphi \tag{3.5}$$

is invariant (provided one takes into account the equation for the auxiliary field $\pi^{\alpha\beta}$) under the following supertransformations:

$$\delta\varphi = i\tilde{\beta}_0\phi_\alpha\zeta^\alpha, \qquad \delta\phi^\alpha = 2\tilde{\beta}_0\pi^{\alpha\beta}\zeta_\beta. \tag{3.6}$$

4 Simple examples

4.1 Massive supermultiplet $(\frac{3}{2},1,\frac{1}{2})$

Gauge invariant description of massive spin-$\frac{3}{2}$ requires massless spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$ ones, while massive spin-1 requires massless spin-1 and spin-0. Thus to construct massive supermultiplet $(\frac{3}{2},1,\frac{1}{2})$ we need two massless supermultiplets $(\frac{3}{2},1,\frac{1}{2})$ and $(\frac{1}{2},0)$:

$$\begin{pmatrix} \frac{3}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \frac{3}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

We begin with the sum of kinetic terms for all necessary fields:

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{i}{2}\Phi_\alpha d\Phi^\alpha + EB^{\alpha\beta}B_{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}dA + \frac{i}{2}\psi_\alpha E^\alpha{}_\beta d\psi^\beta \\
&\quad + \frac{i}{2}\phi_\alpha E^\alpha{}_\beta d\phi^\beta - E\pi^{\alpha\beta}\pi_{\alpha\beta} + E^{\alpha\beta}\pi_{\alpha\beta}d\varphi
\end{aligned} \tag{4.1}$$

as well as their initial supertransformations:

$$\begin{aligned}
&\delta_0 A = i\beta_0\Phi_\alpha\zeta^\alpha + i\alpha_0\psi_\alpha e^{\alpha\beta}\zeta_\beta, \\
&\delta_0\Phi^\alpha = -\beta_0 e^{\beta\gamma}B_{\beta\gamma}\zeta^\alpha, \qquad\qquad \delta_0\psi^\alpha = 4\alpha_0 B^{\alpha\beta}\zeta_\beta, \\
&\delta_0\varphi = i\tilde{\beta}_0\phi_\alpha\zeta^\alpha, \qquad\qquad \delta_0\phi^\alpha = 2\tilde{\beta}_0\pi^{\alpha\beta}\zeta_\beta.
\end{aligned} \tag{4.2}$$

[5] Strictly speaking this Lagrangian is invariant up to the terms proportional to the auxiliary field $B^{\alpha\beta}$ equation only. Thus there are two possible approach here. From one hand one can introduce non-trivial corrections to the supertransformations for this auxiliary field. Another possibility, that we will systematically follow here and further on, is to use equations for the auxiliary fields in calculating all variations.

Now we add the most general low derivative terms:

$$\mathcal{L}_1 = 2mE_{\alpha\beta}\pi^{\alpha\beta}A + ia_1\Phi_\alpha e^\alpha{}_\beta\Phi^\beta + ia_2\Phi_\alpha E^\alpha{}_\beta\psi^\beta + ia_3\Phi_\alpha E^\alpha{}_\beta\phi^\beta$$
$$+ia_4E\psi_\alpha\psi^\alpha + ia_5E\psi_\alpha\phi^\alpha + ia_6E\phi_\alpha\phi^\alpha. \tag{4.3}$$

This breaks the invariance under the supertransformations producing

$$\delta_0\mathcal{L}_1 = -2i(2a_1\beta_0 - a_2\alpha_0)\Psi_\alpha E^{(\alpha}{}_\beta B^{\gamma)\beta}\zeta_\gamma + 2ia_2\alpha_0\Psi_\alpha(EB)\zeta^\alpha$$
$$+ia_3\tilde{\beta}_0\Psi_\alpha E^{(\alpha}{}_\beta\pi^{\gamma)\beta}\zeta_\gamma + i(2m\beta_0 + a_3\tilde{\beta}_0)\Psi_\alpha(E\pi)\zeta^\alpha$$
$$-2i(a_2\beta_0 - 4a_4\alpha_0)E\psi_\alpha B^{\alpha\beta}\zeta_\beta + 2i(2m\alpha_0 + a_5\tilde{\beta}_0)E\psi_\alpha\pi^{\alpha\beta}\zeta_\beta$$
$$-2i(a_3\beta_0 - 2a_5\alpha_0)E\phi_\alpha B^{\alpha\beta}\zeta_\beta + im\tilde{\beta}_0\phi_\alpha e^{\alpha\beta}dA\zeta_\beta + 4ia_6\tilde{\beta}_0E\phi_\alpha\pi^{\alpha\beta}\zeta_\beta.$$

Recall that, as we have already mentioned above, all calculations are performed up to the terms proportional to the auxiliary fields equations and in this case one has take into account that equation for the $\pi^{\alpha\beta}$ field was modified and looks like:

$$E^\alpha{}_\beta\pi^{\alpha\beta} = -\frac{1}{2}e^{\alpha(2)}[d\varphi + 2mA]. \tag{4.4}$$

To cancel these new variations we put

$$2a_1\beta_0 = a_2\alpha_0, \qquad 2m\beta_0 = -a_3\tilde{\beta}_0, \qquad a_2\beta_0 = 4a_4\alpha_0, \qquad 2a_3\beta_0 - 4a_5\alpha_0 = 2m\tilde{\beta}_0$$

and introduce the following corrections to the supertransformations:[6]

$$\delta_1\Psi^\alpha = \gamma_1A\zeta^\alpha + \gamma_2\varphi e^{\alpha\beta}\zeta_\beta, \qquad \delta_1\psi_\alpha = \gamma_3\varphi\zeta_\alpha, \qquad \delta_1\phi_\alpha = \gamma_4\varphi\zeta_\alpha. \tag{4.5}$$

Then all variations with one derivative vanish provided

$$\gamma_1 = 2a_2\alpha_0, \qquad \gamma_2 = \frac{a_3\tilde{\beta}_0}{2}, \qquad \gamma_3 = 2m\alpha_0 + a_5\tilde{\beta}_0, \qquad \gamma_4 = 2a_6\tilde{\beta}_0$$

leaving us with variations without derivatives:

$$\delta_1\mathcal{L}_1 = i(2a_1\gamma_1 + ma_3\alpha_1)\Psi_\alpha e^\alpha{}_\beta A\zeta^\beta + i(8a_1\gamma_2 - a_2\gamma_3 - a_3\gamma_4)\Psi_\alpha E^{\alpha\beta}\varphi\zeta_\beta$$
$$+i(a_2\gamma_1 + 2m\gamma_3)\psi_\alpha E^{\alpha\beta}A\zeta_\beta - i(3a_2\gamma_2 - 2a_4\gamma_3 - a_5\gamma_4)E\psi_\alpha\varphi\zeta^\alpha$$
$$+i(a_3\gamma_1 + 2m\gamma_4)\phi_\alpha E^{\alpha\beta}A\zeta_\beta - i(3a_3\gamma_2 - a_5\gamma_3 - 2a_6\gamma_4)E\phi_\alpha\varphi\zeta^\alpha.$$

Thus we obtain:

$$a_1 = a_4 = a_6 = \frac{m}{2}, \qquad a_2 = -a_3 = \sqrt{2}m, \qquad a_5 = -2m,$$
$$\tilde{\beta}_0{}^2 = 2\beta_0{}^2 = 4\alpha_0{}^2.$$

Now if we introduce new variables

$$\tilde{\psi}^\alpha = \frac{1}{\sqrt{2}}(\psi^\alpha - \phi^\alpha), \qquad \tilde{\phi}^\alpha = \frac{1}{\sqrt{2}}(\psi^\alpha + \phi^\alpha),$$

[6] Let us stress that here and in what follows the only corrections we have to introduce in our approach are the non-derivative ones to the fermionic fields.

then the fermionic mass terms take the form

$$\mathcal{L}_m = \frac{im}{2}[\Phi_\alpha e^\alpha{}_\beta \Phi^\alpha + 4\Phi_\alpha E^\alpha{}_\beta \tilde{\psi}^\beta + 3E\tilde{\psi}_\alpha \tilde{\psi}^\alpha] - \frac{im}{2} E\tilde{\phi}_\alpha \tilde{\phi}^\alpha, \tag{4.6}$$

which corresponds to massive spin-$\frac{3}{2}$ (in the gauge invariant formalism with the field $\tilde{\psi}^\alpha$ playing the role of Stueckelberg one) and massive spin-$\frac{1}{2}$ with equal masses. Note that though we begin with the most general form of the fermionic mass terms, the supersymmetry leads us to their form corresponding to gauge invariant formulation of massive spin-$\frac{3}{2}$. In what follows from the very beginning we will use gauge invariant description of massive bosonic and fermionic higher spin fields entering supermultiplets. It will greatly simplify all calculations and always happens to be compatible with the supersymmetry.

4.2 Massive supermultiplet $(2, \frac{3}{2}, \frac{3}{2})$

This supermultiplet has been constructed previously by dimensional reduction from four dimensions [33]. Here we will show how its construction can be worked out in our approach. In the massless limit massive spin-2 decomposes into massless spin-2, spin-1 and spin-0 ones, while massive spin-$\frac{3}{2}$ into massless spin-$\frac{3}{2}$ and spin-$\frac{1}{2}$. Thus in this case the decomposition of massive supermultiplet into the massless ones has the form:

$$\begin{pmatrix} & 2 & \\ \frac{3}{2} & & \frac{3}{2} \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{3}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

Again we begin with the sum of kinetic terms for all necessary fields:

$$\begin{aligned} \mathcal{L}_0 = {} & \Omega_{\alpha\beta} e^\beta{}_\gamma \Omega^{\alpha\gamma} + \Omega_{\alpha\beta} df^{\alpha\beta} - \frac{i}{2}\Psi_\alpha d\Psi^\alpha \\ & - \frac{i}{2}\Phi_\alpha d\Phi^\alpha + EB^{\alpha\beta}B_{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}dA + \frac{i}{2}\psi_\alpha E^\alpha{}_\beta d\psi^\beta \\ & + \frac{i}{2}\phi_\alpha E^\alpha{}_\beta d\phi^\beta - E\pi^{\alpha\beta}\pi_{\alpha\beta} + E^{\alpha\beta}\pi_{\alpha\beta} d\varphi \end{aligned} \tag{4.7}$$

as well as their initial supertransformations:

$$\begin{aligned} \delta f^{\alpha\beta} &= i\alpha_1 \Psi^{(\alpha}\zeta^{\beta)}, & \delta\Psi^\alpha &= 2\alpha_1 \Omega^{\alpha\beta}\zeta_\beta, \\ \delta A &= i\beta_0 \Phi_\alpha \zeta^\alpha + i\alpha_0 \psi_\alpha e^{\alpha\beta}\zeta_\beta, & & \\ \delta\Phi^\alpha &= -\beta_0 e^{\beta\gamma} B_{\beta\gamma}\zeta^\alpha, & \delta\psi^\alpha &= 4\alpha_0 B^{\alpha\beta}\zeta_\beta, \\ \delta\varphi &= i\tilde{\beta}_0 \phi_\alpha \zeta^\alpha, & \delta\phi^\alpha &= 2\tilde{\beta}_0 \pi^{\alpha\beta}\zeta_\beta. \end{aligned} \tag{4.8}$$

Now we have to add low derivative terms. As we have already mentioned, we will use gauge invariant description both for the massive spin-2 as well as massive spin-$\frac{3}{2}$ fields. As for the massive spin-2 here the choice is unambiguous — we have just one spin-1 and spin-0 zero fields to the roles of Stueckelberg ones. And for the two massive spin-$\frac{3}{2}$ fields by analogy with four-dimensional case [25] we will assume that two Majorana fields will combine into a Dirac one. Thus we introduce the following terms (see appendices A and C):

$$\begin{aligned}
\mathcal{L}_1 &= -2me_{\alpha\beta}\Omega^{\alpha\beta}A - mf_{\alpha\beta}E^\beta{}_\gamma B^{\alpha\gamma} - 4mE_{\alpha\beta}\pi^{\alpha\beta}A \\
&\quad + im[\Psi_\alpha e^\alpha{}_\beta\Phi^\beta + 2\Psi_\alpha E^\alpha{}_\beta\psi^\beta + 2\Phi_\alpha E^\alpha{}_\beta\phi^\beta + 3E\psi_\alpha\phi^\alpha], \qquad (4.9) \\
\mathcal{L}_2 &= \frac{m^2}{4}f_{\alpha\beta}e^\beta{}_\gamma f^{\alpha\gamma} - m^2E^{\alpha\beta}f_{\alpha\beta}\varphi + \frac{3m^2}{2}E\varphi^2. \qquad (4.10)
\end{aligned}$$

As in the previous case calculating all variations one has to use equations for the auxiliary fields which in this case have the form:

$$\begin{aligned}
0 &= e^\alpha{}_\beta\Omega^{\alpha\beta} + df^{\alpha(2)} + 2me^{\alpha(2)}A, \\
0 &= 2EB^{\alpha(2)} - e^{\alpha(2)}dA + \frac{m}{2}E^\alpha{}_\beta f^{\alpha\beta}, \qquad (4.11) \\
0 &= 2E\pi^{\alpha(2)} - E^{\alpha(2)}[d\varphi - 4mA].
\end{aligned}$$

To compensate for variations with one derivative $\delta_0\mathcal{L}_1$ we introduce the following corrections to the supertransformations:

$$\delta_1\Psi^\alpha = \gamma_1 A\zeta^\alpha, \qquad \delta_1\Phi^\alpha = \gamma_2 f^{\alpha\beta}\zeta_\beta + \gamma_3\varphi e^{\alpha\beta}\gamma_\beta, \qquad \delta_1\psi^\alpha = \gamma_4\varphi\zeta^\alpha, \qquad (4.12)$$

where

$$\gamma_1 = 4m\alpha_0, \qquad \gamma_2 = -m\alpha_1, \qquad \gamma_3 = m\tilde{\beta}_0, \qquad \gamma_4 = 3m\tilde{\beta}_0 + 4m\alpha_0.$$

Then all variations without derivatives $\delta_1\mathcal{L}_1 + \delta_0\mathcal{L}_2$ vanish provided

$$\alpha_1{}^2 = \tilde{\beta}_0{}^2 = 4\alpha_0{}^2, \qquad \beta_0{}^2 = \alpha_0{}^2.$$

5 Massive supermultiplet $\left(s+\frac{1}{2}, s, s-\frac{1}{2}\right)$

In this case the same line of reasoning leads us to the decomposition:

$$\begin{pmatrix} s+\frac{1}{2} \\ s \\ s-\frac{1}{2} \end{pmatrix} \Rightarrow \sum_{l=1}^{s}\begin{pmatrix} l+\frac{1}{2} \\ l \\ l-\frac{1}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

Correspondingly we begin with appropriate sum of kinetic terms for all fields

$$\begin{aligned}
\mathcal{L}_0 &= \sum_{l=1}^{s-1}(-1)^{l+1}[l\Omega_{\alpha(2l-1)\beta}e^\beta{}_\gamma\Omega^{\alpha(2l-1)\gamma} + \Omega_{\alpha(2l)}df^{\alpha(2l)}] \\
&\quad + EB_{\alpha(2)}B^{\alpha(2)} - B_{\alpha(2)}e^{\alpha(2)}dA - E\pi_{\alpha(2)}\pi^{\alpha(2)} + \pi_{\alpha(2)}E^{\alpha(2)}d\varphi \\
&\quad + \frac{i}{2}\left[\sum_{l=0}^{s-2}(-1)^{l+1}\Psi_{\alpha(2l+1)}d\Psi^{\alpha(2l+1)} + \sum_{l=0}^{s-1}(-1)^{l+1}\Phi_{\alpha(2l+1)}d\Phi^{\alpha(2l+1)}\right] \\
&\quad + \frac{i}{2}\phi_\alpha E^\alpha{}_\beta d\phi^\beta + \frac{i}{2}\chi_\alpha E^\alpha{}_\beta d\chi^\beta \qquad (5.1)
\end{aligned}$$

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and their initial supertransformations:

$$
\begin{aligned}
\delta f^{\alpha(2l)} &= i\alpha_l \Psi^{\alpha(2l-1)}\zeta^\alpha + i(2l+1)\beta_l \Phi^{\alpha(2l)\beta}\zeta_\beta, & l \geq 1 \\
\delta_0 \Phi^{\alpha(2l+1)} &= \beta_l \Omega^{\alpha(2l)}\zeta^\alpha, & \delta_0 \Psi^{\alpha(2l-1)} = 2l\alpha_l \Omega^{\alpha(2l-1)\beta}\zeta_\beta, \\
\delta A &= i\beta_0 \Phi_\alpha \zeta^\alpha + i\alpha_0 \psi_\alpha e^{\alpha\beta}\zeta_\beta, \\
\delta \Phi^\alpha &= -\beta_0 e^{\beta(2)} B_{\beta(2)}\zeta^\alpha, & \delta\psi^\alpha = 4\alpha_0 B^{\alpha\beta}\zeta_\beta, \\
\delta\varphi &= i\tilde{\beta}_0 \phi_\alpha \zeta^\alpha, & \delta\phi^\alpha = 2\tilde{\beta}_0 \pi^{\alpha\beta}\zeta_\beta.
\end{aligned}
\tag{5.2}
$$

Now we have to add the lower derivative terms. For the bosonic terms we take the ones corresponding to gauge invariant description of massive spin-s boson (see appendix A), while for the fermionic terms we introduce the most general ones compatible with the fact that they have to correspond to gauge invariant description of two massive fermions with spin-$(s+\frac{1}{2})$ and spin-$(s-\frac{1}{2})$ with equal masses (see appendix B):

$$
\begin{aligned}
\mathcal{L}_{1b} &= \sum_{l=1}^{s-2}(-1)^{l+1}a_l \left[-\frac{(l+2)}{l}\Omega_{\alpha(2l)\beta(2)}e^{\beta(2)} f^{\alpha(2l)} + \Omega_{\alpha(2l)}e_{\beta(2)} f^{\alpha(2l)\beta(2)} \right] \\
&\quad + a_0[2\Omega_{\alpha(2)}e^{\alpha(2)}A - f_{\alpha\beta}E^\beta{}_\gamma B^{\alpha\gamma}] + \tilde{a}_0 \pi_{\alpha(2)}E^{\alpha(2)}A,
\end{aligned}
\tag{5.3}
$$

$$
\mathcal{L}_2 = \sum_{l=1}^{s-1}(-1)^{l+1}c_l f_{\alpha(2l-1)\beta}e^\beta{}_\gamma f^{\alpha(2l-1)\gamma} + \tilde{c}_1 h_{\alpha(2)}E^{\alpha(2)}\varphi + c_0 E\varphi^2,
\tag{5.4}
$$

$$
\begin{aligned}
\mathcal{L}_{1f} &= i\sum_{l=0}^{s-1}(-1)^{l+1}\tilde{b}_l \Phi_{\alpha(2l)\beta}e^\beta{}_\gamma \Phi^{\alpha(2l)\gamma} + i\sum_{l=0}^{s-2}(-1)^{l+1}[b_l \Phi_{\alpha(2l)\beta}e^\beta{}_\gamma \Psi^{\alpha(2l)\gamma} + \hat{b}_l \Psi_{\alpha(2l)\beta}e^\beta{}_\gamma \Psi^{\alpha(2l)\gamma}] \\
&\quad + iE[\tilde{b}_{-1}\phi_\alpha \phi^\alpha + b_{-1}\phi_\alpha \psi^\alpha + \hat{b}_{-1}\psi_\alpha \psi^\alpha] \\
&\quad + i\sum_{l=1}^{s-1}(-1)^{l+1}\Phi_{\alpha(2l-1)\beta(2)}e^{\beta(2)}[\tilde{d}_l \Phi^{\alpha(2l-1)} + e_l \Psi^{\alpha(2l-1)}] \\
&\quad + i\sum_{l=1}^{s-2}(-1)^{l+1}\Psi_{\alpha(2l-1)\beta(2)}e^{\beta(2)}[d_l \Psi^{\alpha(2l-1)} + \tilde{e}_l \Phi^{\alpha(2l-1)}] \\
&\quad + i\Phi_\alpha E^\alpha{}_\beta[\tilde{d}_0 \phi^\beta + e_0 \psi^\beta] + i\Psi_\alpha E^\alpha{}_\beta[d_0 \psi^\beta + \tilde{e}_0 \phi^\beta].
\end{aligned}
\tag{5.5}
$$

As in the previous cases, calculating the variations we use auxiliary field equations:

$$
0 = e^\alpha{}_\beta \Omega^{\alpha(2l-1)\beta} + df^{\alpha(2l)} + a_l e_{\beta(2)} f^{\alpha(2l)\beta(2)} + \frac{(l+1)a_{l-1}}{l(l-1)(2l-1)}e^{\alpha(2)} f^{\alpha(2l-2)}.
$$

From the variations with one derivative we found that we must introduce the full set of corrections to the supertransformations:

$$
\begin{aligned}
\delta_1 \Phi^{\alpha(2l+1)} &= \tilde{\gamma}_l f^{\alpha(2l+1)\beta}\zeta_\beta + \tilde{\delta}_l f^{\alpha(2l)}\zeta^\alpha, \\
\delta_1 \Psi^{\alpha(2l+1)} &= \gamma_l f^{\alpha(2l+1)\beta}\zeta_\beta + \delta_l f^{\alpha(2l)}\zeta^\alpha, \\
\delta_1 \Phi^\alpha &= \tilde{\gamma}_0 f^{\alpha\beta}\zeta_\beta + \tilde{\delta}_0 A\zeta^\alpha + \rho_0 e^{\alpha\beta}\varphi\zeta_\beta, \\
\delta_1 \Psi^\alpha &= \delta_0 f^{\alpha\beta}\zeta_\beta + \gamma_0 A\zeta^\alpha + \tilde{\tilde{\rho}}_0 e^{\alpha\beta}\varphi\zeta_\beta, \\
\delta_1 \psi^\alpha &= \tilde{\rho}_0 \varphi\zeta^\alpha, & \delta_1 \phi^\alpha = \hat{\rho}_0 \varphi\zeta^\alpha.
\end{aligned}
\tag{5.6}
$$

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Moreover, supersymmetry requires that we put $\tilde{e}_l = 0$ and it is this constraint that allowed us to find the solution for the fermionic mass terms in appendix B. Now when all the coefficients in the Lagrangian are fixed, it is straightforward to find solution for the parameters of the supertransformations:

$$\begin{aligned}
&{\alpha_l}^2 = \frac{(l+1)}{l(2l+1)}, && {\beta_l}^2 = \frac{2l}{(2l+1)^2}, && \\
&{\alpha_0}^2 = \frac{1}{4}, && {\beta_0}^2 = \frac{1}{2}, && {\tilde{\beta}_0}{}^2 = 1,
\end{aligned} \tag{5.7}$$

where we set the normalization so that

$$2l{\alpha_l}^2 + (2l+1){\beta_l}^2 = 2.$$

Then the parameters determining corrections to the supertransformations are also fixed:

$$\begin{aligned}
&{\gamma_l}^2 = \frac{(s+l+1)(s-l-1)}{2l(l+1)^2(2l+1)^2}m^2, \\
&{\tilde{\gamma}_l}{}^2 = \frac{(s+l+1)(s-l-1)}{(l+1)(l+2)(2l+3)}m^2, \\
&{\delta_l}^2 = \frac{(l+1)}{(l+2)(2l+3)}m^2, \\
&{\tilde{\delta}_l}{}^2 = \frac{m^2}{2l(2l+1)^2}, \\
&{\gamma_0}^2 = 2(s-1)(s+1)m^2, \qquad {\tilde{\delta}_0}{}^2 = 2m^2, \\
&{\rho_0}^2 = {\tilde{\rho}_0}{}^2 = s^2m^2, \qquad {\hat{\rho}_0}{}^2 = m^2, \qquad \tilde{\hat{\rho}}_0 = 0.
\end{aligned} \tag{5.8}$$

6 Massive supermultiplet $\left(s, s-\frac{1}{2}, s-\frac{1}{2}\right)$

In this case the decomposition looks like:

$$\begin{pmatrix} s \\ s-\frac{1}{2}, s-\frac{1}{2} \end{pmatrix} \Rightarrow \begin{pmatrix} s \\ s-\frac{1}{2} \end{pmatrix} \oplus \sum_{l=1}^{s-1} \begin{pmatrix} l+\frac{1}{2} \\ l \\ l-\frac{1}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}.$$

Thus we need the same set of fields as in the previous case except the field $\Phi^{\alpha(2s-1)}$ (recall that the supermultiplet $(s, s-1/2)$ is just a particular case of $(s+1/2, s, s-1/2)$ one). So we take the same massless Lagrangian (5.1) with this field omitted and the same set of initial supertransformations (5.2) where now $\beta_{s-1} = 0$. As far as the low derivative terms, the bosonic terms will again have the same form (5.3) and (5.4), while by analogy with four dimensional case [27] we will assume that fermions have Dirac mass terms compatible with gauge invariant description (see appendix C):

$$
\begin{aligned}
\mathcal{L}_{1f} &= i\sum_{l=0}^{s-2}(-1)^{l+1}b_l\Psi_{\alpha(2l)\beta}e^{\beta}{}_{\gamma}\Phi^{\alpha(2l)\gamma} + i\tilde{b}_0E\phi_\alpha\chi^\alpha \\
&\quad +i\sum_{l=1}^{s-2}(-1)^{l+1}[d_l\Psi_{\alpha(2l-1)\beta(2)}e^{\beta(2)}\Psi^{\alpha(2l-1)} + \tilde{d}_l\Phi_{\alpha(2l-1)\beta(2)}e^{\beta(2)}\Phi^{\alpha(2l-1)}] \\
&\quad +id_0\Psi_\alpha E^\alpha{}_\beta\psi^\beta + i\tilde{d}_0\Phi_\alpha E^\alpha{}_\beta\phi^\beta.
\end{aligned}
\tag{6.1}
$$

Calculating all variations with one derivative $\delta_0\mathcal{L}_1$ we find that we have to introduce the following corrections to supertransformations:

$$
\begin{aligned}
\delta_1\Psi^{\alpha(2l+1)} &= \gamma_l f^{\alpha(2l)}\zeta^\alpha, & \delta\Phi^{\alpha(2l+1)} &= \tilde{\gamma}_l f^{\alpha(2l+1)\beta}\zeta_\beta, \\
\delta_1\Psi^\alpha &= \gamma_0 A\zeta^\alpha, & \delta_1\Phi^\alpha &= \tilde{\gamma}_0 f^{\alpha\beta}\zeta_\beta + \rho_0 e^{\alpha\beta}\varphi\zeta_\beta, \\
\delta_1\psi^\alpha &= \tilde{\rho}_0\varphi\zeta^\alpha
\end{aligned}
\tag{6.2}
$$

and obtain the following expressions for the parameters determining supertransformations:

$$
\begin{aligned}
{\alpha_l}^2 &= \frac{(l+1)(s+l)}{2sl(2l+1)}, & {\beta_l}^2 &= \frac{l(s-l-1)}{s(2l+1)^2}, & & \\
{\alpha_0}^2 &= \frac{1}{8}, & {\beta_0}^2 &= \frac{(s-1)}{4s}, & {\tilde{\beta}_0}^2 &= \frac{1}{2}, \\
{\gamma_l}^2 &= \frac{s(s-l-1)m^2}{4l(l+1)^2(2l+1)^2}, & & & & \\
{\tilde{\gamma}_l}^2 &= \frac{s(s+l+1)m^2}{2(l+1)(l+2)(2l+3)}, & & & &
\end{aligned}
\tag{6.3}
$$

$$
\begin{aligned}
{\gamma_0}^2 &= s(s-1)m^2, & {\tilde{\gamma}_0}^2 &= \frac{s(s+1)m^2}{12}, \\
{\rho_0}^2 &= \frac{s(s-1)m^2}{4}, & {\tilde{\rho}_0}^2 &= 4(s-1)^2m^2.
\end{aligned}
\tag{6.4}
$$

Conclusion. In this paper we have constructed the minimal supersymmetric Lagrangian formulation for all massive supermultiplets with arbitrary spins in $d=3$. We have shown that as in the $d=4$ case such massive supermultiplets can be straightforwardly built out of the appropriately chosen set of massless ones. Such procedure can be considered as a supersymmetric generalization for the gauge invariant formalism for massive higher spin bosonic and fermionic fields where the description for the massive field is obtained through the set of the massless ones. In most cases constructing the Lagrangians we from the very beginning choose mass terms compatible with such gauge invariant description for massive fields. But as we have shown in one case and checked in others even if one starts with the most general form of the mass terms without any preliminary assumptions the supersymmetry alone will unavoidably lead to such form. Thus the very idea of gauge invariant description for the massive higher spin fields is in the perfect agreement with the supersymmetry.

As the directions of the further development one can point out: **1.** The approach can be applied to the extended supersymmetries as well. **2.** It would be interesting to consider interaction of such massive higher spin supermultiplets with supergravity. **3.** The approach considered in this paper is on-shell. It would be interesting to develop a completely off-shell superfield Lagrangian formulation for the three-dimensional supersymmetric massive higher spin theories. Some preliminary results have already been obtained in [34].[7] **4.** We have constructed the supersymmetric massive higher spin models in flat 3d space It would be interesting to generalize the models under consideration to AdS_3 space and apply it to problem of AdS_3/CFT_2 duality.

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A Massive boson with spin $s \geq 2$

Gauge invariant description of massive boson with spin s [30] requires massless fields with spins $s \geq l \geq 0$. Thus we introduce a collection of one-forms $f^{\alpha(2l)}$, $\Omega^{\alpha(2l)}$, $s-1 \geq l \geq 1$ as well as one-form A and zero-forms $B^{\alpha\beta}$, $\pi^{\alpha\beta}$ and φ. As usual, we begin with the sum of kinetic terms for all fields:

$$\begin{aligned} \mathcal{L}_0 = & \sum_{l=1}^{s-1}(-1)^{l+1}[l\Omega_{\alpha(2l-1)\beta}e^\beta{}_\gamma\Omega^{\alpha(2l-1)\gamma} + \Omega_{\alpha(2l)}df^{\alpha(2l)}] \\ & + EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}dA - E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}d\varphi \end{aligned} \tag{A.1}$$

and their initial gauge transformations:

$$\delta_0 f^{\alpha(2l)} = d\xi^{\alpha(2l)} + e^\alpha{}_\beta\eta^{\alpha(2l-1)\beta}, \qquad \delta_0\Omega^{\alpha(2l)} = d\eta^{\alpha(2l)}, \qquad \delta_0 A = d\xi. \tag{A.2}$$

To proceed we add the most general terms with one derivative:

$$\begin{aligned} \mathcal{L}_1 = & \sum_{l=1}^{s-2}(-1)^{l+1}[b(l)\Omega_{\alpha(2)\beta(2l)}e^{\alpha(2)}f^{\beta(2l)} + a(l)\Omega_{\alpha(2l)}e_{\beta(2)}f^{\alpha(2l)\beta(2)}] \\ & - a_0\Omega_{\alpha\beta}e^{\alpha\beta}A - b_0 f_{\alpha\beta}E^\beta{}_\gamma B^{\alpha\gamma} + \tilde{a}_0\pi_{\alpha\beta}E^{\alpha\beta}A \end{aligned} \tag{A.3}$$

[7] Recently we have been informed by S.M. Kuzenko that he has unpublished yet results on superfield formulation for massless three-dimensional $N = 2$ supersymmetric models with arbitrary superspins.

as well as the most general corrections to the gauge transformations:

$$\begin{aligned}
\delta_1 \Omega^{\alpha(2l)} &= \kappa_1(l) e_{\beta(2)} \eta^{\alpha(2l)\beta(2)} + \kappa_2(l) e^{\alpha(2)} \eta^{\alpha(2l-2)} + \kappa_3(l) e^{\alpha}{}_{\beta} \xi^{\alpha(2l-1)\beta}, \\
\delta_1 \Phi^{\alpha(2l)} &= \kappa_4(l) e_{\beta(2)} \xi^{\alpha(2l)\beta(2)} + \kappa_5(l) e^{\alpha(2)} \xi^{\alpha(2l-2)}, \\
\delta_1 B^{\alpha\beta} &= \kappa_1(0) \eta^{\alpha\beta}, \qquad \delta_1 A = D\xi + \kappa_4(0) e_{\alpha\beta} \xi^{\alpha\beta}, \\
\delta_1 \pi^{\alpha\beta} &= \kappa_6 \xi^{\alpha\beta}, \qquad \delta_1 \varphi = \kappa_7 \xi.
\end{aligned} \tag{A.4}$$

All variations coming from $\delta_0 \mathcal{L}_1 + \delta_1 \mathcal{L}_0$ vanish provided

$$\begin{aligned}
&\kappa_1(l) = -b(l), && \kappa_2(l) = \frac{a(l-1)}{l(2l-1)}, && \kappa_4(l) = a(l), \\
&\kappa_5(l) = -\frac{b(l-1)}{l(2l-1)}, && b(l) = -\frac{(l+2)a(l)}{l}, \\
&\kappa_1(0) = -a_0, && \kappa_4(0) = \frac{b_0}{4}, && \kappa_7 = -\tilde{a}_0, \qquad a_0 + 2b_0 = 0.
\end{aligned}$$

To proceed we introduce the most general terms without derivatives:

$$\mathcal{L}_2 = \sum_{l=1}^{s-1} (-1)^{l+1} c(l) f_{\alpha(2l-1)\beta} e^{\beta}{}_{\gamma} f^{\alpha(2l-1)\gamma} + \tilde{c}_1 f_{\alpha\beta} E^{\alpha\beta} \varphi + c_0 E \varphi^2. \tag{A.5}$$

Then all remaining variations can be canceled if we put

$$\begin{aligned}
\frac{2(k+2)(2k+3)}{(k+1)(2k+1)} a(k)^2 - \frac{2(k+1)}{(k-1)} a(k-1)^2 + 4c(k) = 0, \\
\kappa_3(l) = \frac{c(l)}{l}, \\
(l+2)^2 c(l+1) = l(l+1)c(l), \\
\kappa_6 = \tilde{c}_1 = -\frac{\tilde{a}_0 b_0}{4}, \qquad {\tilde{a}_0}^2 = 64 c_1, \\
{b_0}^2 = \frac{(s+1)(s-1)}{3} m^2,
\end{aligned}$$

where we choose normalization

$$m^2 = \frac{2s(s-1)}{(s-2)} a(s-2)^2.$$

The last relation in the second line is just a recurrent relation on $c(l)$ and it gives

$$c(l) = \frac{s^2(s-1)}{l(l+1)^2} c(s-1).$$

Than the first line can be considered as a recurrent relation on $a(l)$. For $l = s-1$ we obtain

$$c(s-1) = \frac{m^2}{4(s-1)},$$

and then we get general solution

$$a(l)^2 = \frac{l(s+l+1)(s-l-1)}{2(l+1)(l+2)(2l+3)} m^2, \qquad {a_0}^2 = \frac{(s-1)(s+1)}{3} m^2.$$

B Massive fermions with Majorana mass terms

To construct gauge invariant description of massive fermion with spin-$(s+\frac{1}{2})$ [31] we introduce a set of one-forms $\Psi^{\alpha(2l+1)}$, $s-1 \geq l \geq 0$ and zero-form χ^α. As for the Lagrangian we take the sum of kinetic terms for all fields as well as the most general form for the mass-like terms:

$$\begin{aligned}\mathcal{L}_0 = & \frac{i}{2}\sum_{l=0}^{s-1}(-1)^{l+1}\Psi_{\alpha(2l+1)}d\Psi^{\alpha(2l+1)} + \frac{i}{2}\chi_\alpha E^\alpha{}_\beta d\chi^\beta \\ & +i\sum_{l=0}^{s-1}(-1)^{l+1}b_l\Psi_{\alpha(2l)\beta}e^\beta{}_\gamma\Psi^{\alpha(2l)\gamma} + i\tilde{b}_0 E\chi_\alpha\chi^\alpha \\ & +i\sum_{l=1}^{s-1}(-1)^{l+1}d_l\Psi_{\alpha(2l-1)\beta(2)}e^{\beta(2)}\Psi^{\alpha(2l-1)} + id_0\Psi_\alpha E^\alpha{}_\beta\chi^\beta. \end{aligned} \tag{B.1}$$

At the same time we introduce the most general ansatz for the local gauge transformations:

$$\begin{aligned}\delta\Psi^{\alpha(2l+1)} &= d\xi^{\alpha(2l+1)} + \alpha_l e^\alpha{}_\beta\xi^{\alpha(2l)\beta} + \beta_l e^{\alpha(2)}\xi^{\alpha(2l-1)} + \gamma_l e_{\beta(2)}\xi^{\alpha(2l+1)\beta(2)}, \\ \delta\chi^\alpha &= \tilde{\alpha}_0\xi^\alpha. \end{aligned} \tag{B.2}$$

For variations with one derivative to vanish we have to put

$$\alpha_l = \frac{2b_l}{(2l+1)}, \qquad \beta_l = \frac{d_l}{l(2l+1)}, \qquad \gamma_l = d_{l+1}, \qquad \tilde{\alpha}_0 = d_0.$$

Than all variations without derivatives vanish provided

$$\frac{4{b_l}^2}{(2l+1)} - {d_l}^2 + \frac{(l+2)(2l+1)}{(l+1)(2l+3)}{d_{l+1}}^2 = 0, \qquad b_{l-1} = \frac{(2l+3)}{(2l+1)}b_l.$$

These relations can be easily solved and give

$$\begin{aligned} b_l &= \frac{(2s+1)}{2(2l+3)}m, & \tilde{b}_0 &= -3b_0, \\ {d_l}^2 &= \frac{(s-l)(s+l+1)}{2(l+1)(2l+1)}m^2, & {d_0}^2 &= 2s(s+1)m^2. \end{aligned}$$

where we choose normalization by setting $b_{s-1} = \frac{m}{2}$. In what follows we will need analogous solution for the fermion with spin-$(s-\frac{1}{2})$ which has the form

$$\begin{aligned} b_l &= \frac{(2s-1)}{2(2l+3)}m, & \tilde{b}_0 &= -3b_0, \\ {d_l}^2 &= \frac{(s-l-1)(s+l)}{2(l+1)(2l+1)}m^2, & {d_0}^2 &= 2s(s-1)m^2. \end{aligned}$$

For the massive supermultiplet $(s+\frac{1}{2}, s, s-\frac{1}{2})$ we need two massive fermions with spin-$(s+\frac{1}{2})$ and $(s-\frac{1}{2})$. The simplest solution is to take just the sum of corresponding mass terms:

$$
\begin{aligned}
\mathcal{L}_1 = {} & i \sum_{l=0}^{s-1} \frac{(2s+1)m}{2(2l+3)} \tilde{\Phi}_{\alpha(2l)\beta} e^\beta{}_\gamma \tilde{\Phi}^{\alpha(2l)\gamma} - i \sum_{l=0}^{s-2} \frac{(2s-1)m}{2(2l+3)} \tilde{\Psi}_{\alpha(2l)\beta} e^\beta{}_\gamma \tilde{\Psi}^{\alpha(2l)\gamma} \\
& + i \sum_{l=1}^{s-1} A(s,l) \tilde{\Phi}_{\alpha(2l-1)\beta(2)} e^{\beta(2)} \tilde{\Phi}^{\alpha(2l-1)} - i \sum_{l=1}^{s-2} A(s-1,l) \tilde{\Psi}_{\alpha(2l-1)\beta(2)} e^{\beta(2)} \tilde{\Psi}^{\alpha(2l-1)} \\
& + im\sqrt{2s(s+1)} \tilde{\Phi}_\alpha E^\alpha{}_\beta \tilde{\phi}^\beta - im\sqrt{2s(s-1)} \tilde{\Psi}_\alpha E^\alpha{}_\beta \tilde{\psi}^\beta \\
& - \frac{i(2s+1)m}{2} E \tilde{\phi}_\alpha \tilde{\phi}^\alpha + \frac{i(2s-1)m}{2} E \tilde{\psi}_\alpha \tilde{\psi}^\alpha, \qquad \text{(B.3)}
\end{aligned}
$$

where we denote

$$
A(s,l) = \sqrt{\frac{(s-l)(s+l+1)}{2(l+1)(2l+1)}} m.
$$

But in general the variables in terms of which the mass terms turn out to be diagonal do not coincide with the ones entering massless supermultiplets. The most general situation corresponds to the possible mixings for the pairs of fermions with equal spins. Thus we introduce:

$$
\begin{aligned}
\tilde{\Phi}_{\alpha(2s-1)} &= \Phi_{\alpha(2s-1)}, \\
\tilde{\Phi}_{\alpha(2l+1)} &= \cos\theta_l \Phi_{\alpha(2l+1)} + \sin\theta_l \Psi_{\alpha(2l+1)}, \\
\tilde{\Psi}_{\alpha(2l+1)} &= -\sin\theta_l \Phi_{\alpha(2l+1)} + \cos\theta_l \Psi_{\alpha(2l+1)}, \qquad \text{(B.4)} \\
\tilde{\phi}^\alpha &= \cos\theta \phi^\alpha + \sin\theta \psi^\alpha, \\
\tilde{\psi}^\alpha &= -\sin\theta \phi^\alpha + \cos\theta \psi^\alpha.
\end{aligned}
$$

Than for the mass terms we obtain the Lagrangian (5.5) used in our construction of corresponding massive supermultiplet, where

$$
\begin{aligned}
\tilde{b}_{s-1} &= \frac{m}{2}, \\
\tilde{b}_l &= \frac{(2s+1)\cos^2\theta_l - (2s-1)\sin^2\theta_l}{2(2l+3)} m, \\
b_l &= \frac{4s\sin\theta_l\cos\theta_l}{(2l+3)} m, \\
\hat{b}_l &= \frac{(2s+1)\sin^2\theta_l - (2s-1)\cos^2\theta_l}{2(2l+3)} m, \\
\tilde{b}_{-1} &= -\frac{(2s+1)\cos^2\theta - (2s-1)\sin^2\theta}{2} m, \\
b_{-1} &= -4s\sin\theta\cos\theta m, \\
\hat{b}_{-1} &= -\frac{(2s+1)\sin^2\theta - (2s-1)\cos^2\theta}{2} m, \\
\tilde{d}_{s-1} &= \frac{m}{\sqrt{(2s-1)}} \cos\theta_{s-2},
\end{aligned}
$$

$$
\begin{aligned}
\tilde{d}_l &= A(s,l)\cos\theta_l\cos\theta_{l-1} - A(s-1,l)\sin\theta_l\sin\theta_{l-1}, \\
\tilde{d}_0 &= \sqrt{2s(s+1)}m\cos\theta_0\cos\theta - \sqrt{2s(s-1)}m\sin\theta_0\sin\theta, \\
d_l &= A(s,l)\sin\theta_l\sin\theta_{l-1} - A(s-1,l)\cos\theta_l\cos\theta_{l-1}, \\
d_0 &= \sqrt{2s(s+1)}m\sin\theta_0\sin\theta - \sqrt{2s(s-1)}m\cos\theta_0\cos\theta, \\
e_{s-1} &= \frac{m}{\sqrt{(2s-1)}}\sin\theta_{s-2}, \\
e_l &= A(s,l)\cos\theta_l\sin\theta_{l-1} + A(s-1,l)\sin\theta_l\cos\theta_{l-1}, \\
e_0 &= \sqrt{2s(s+1)}m\cos\theta_0\sin\theta + \sqrt{2s(s-1)}m\sin\theta_0\cos\theta, \\
\tilde{e}_l &= A(s,l)\sin\theta_l\cos\theta_{l-1} + A(s-1,l)\cos\theta_l\sin\theta_{l-1}, \\
\tilde{e}_0 &= \sqrt{2s(s+1)}m\sin\theta_0\cos\theta + \sqrt{2s(s-1)}m\cos\theta_0\sin\theta.
\end{aligned}
$$

But supersymmetry requires that $\tilde{e}_l = 0$ and this gives a recurrent relation on the mixing angles:

$$
\tan\theta_{l-1} = -\sqrt{\frac{(s-l)(s+l+1)}{(s-l-1)(s+l)}}\tan\theta_l.
$$

For the massive supermultiplet $(s+\frac{1}{2}, s, s-\frac{1}{2})$ we will need the following simple solution for this relation:

$$
\sin\theta_l = (-1)^l\sqrt{\frac{s-l-1}{2s}}, \qquad \cos\theta_l = \sqrt{\frac{s+l+1}{2s}}.
$$

Than for the coefficients in the fermionic mass terms (5.5) we obtain:

$$
\begin{aligned}
\tilde{b}_l &= -\tilde{b}_{-1} = -\hat{b}_{-1} = \frac{m}{2}, & \hat{b}_l &= -\frac{2l+1}{2(2l+3)}m, \\
b_l &= (-1)^l\frac{2\sqrt{(s-l-1)(s+l+1)}}{(2l+3)}m, & b_{-1} &= 2sm, \\
\tilde{d}_l &= \sqrt{\frac{(s-l)(s+l)}{2(l+1)(2l+1)}}m, & \tilde{d}_0 &= \sqrt{2}sm, \\
d_l &= -\sqrt{\frac{(s-l-1)(s+l+1)}{2(l+1)(2l+1)}}m, & d_0 &= -\sqrt{2(s+1)(s-1)}m, \\
e_l &= -(-1)^l\sqrt{\frac{1}{2(l+1)(2l+1)}}m, & e_0 &= -\sqrt{2}m.
\end{aligned}
$$

C Massive fermions with Dirac mass terms

For the massive supermultiplet $(s, s-\frac{1}{2}, s-\frac{1}{2})$ we need a pair of massive fermions with spin-$(s-\frac{1}{2})$ with equal masses and with mass-like terms having a Dirac form. The kinetic terms has the usual form:

$$\begin{aligned}\mathcal{L}_0 &= \frac{i}{2}\sum_{l=0}^{s-2}(-1)^{l+1}[\Psi_{\alpha(2l+1)}d\Psi^{\alpha(2l+1)} + \Phi_{\alpha(2l+1)}d\Phi^{\alpha(2l+1)}] \\ &\quad + \frac{i}{2}\psi_\alpha E^\alpha{}_\beta d\psi^\beta + \frac{i}{2}\phi_\alpha E^\alpha{}_\beta d\phi^\beta, \end{aligned} \tag{C.1}$$

while for the mass-like terms we choose the Lagrangian (6.1). For the field's $\Psi^{\alpha(2l+1)}$ local gauge transformations we consider the following ansatz:

$$\begin{aligned}\delta\Psi^{\alpha(2l+1)} &= d\xi^{\alpha(2l+1)} + \beta_l e^{\alpha(2)}\xi^{\alpha(2l-1)} + \gamma_l e_{\beta(2)}\xi^{\alpha(2l+1)\beta(2)}, \\ \delta\Phi^{\alpha(2l+1)} &= \alpha_l e^\alpha{}_\beta \xi^{\alpha(2l)\beta}, \qquad \delta\psi^\alpha = \tilde{\alpha}_0\xi^\alpha.\end{aligned}$$

For variations with one derivative to cancel we have to put

$$\alpha_l = \frac{b_l}{(2l+1)}, \qquad \beta_l = \frac{d_l}{l(2l+1)}, \qquad \gamma_l = d_{l+1}, \qquad \tilde{\alpha}_0 = d_0.$$

Then all variations without derivatives vanish provided

$$\begin{aligned}\frac{{b_l}^2}{(2l+1)} - {d_l}^2 + \frac{(l+2)(2l+1)}{(l+1)(2l+3)}{d_{l+1}}^2 &= 0, \\ \frac{(2l+3)}{(2l+1)}b_l\tilde{d}_l - b_{l-1}d_l &= 0.\end{aligned}$$

Analogously, the invariance under the local gauge transformations for the $\Phi^{\alpha(2l+1)}$ field gives:

$$\begin{aligned}\frac{{b_l}^2}{(2l+1)} - {\tilde{d}_l}^2 + \frac{(l+2)(2l+1)}{(l+1)(2l+3)}{\tilde{d}_{l+1}}^2 &= 0, \\ \frac{(2l+3)}{(2l+1)}b_l d_l - b_{l-1}\tilde{d}_l &= 0.\end{aligned}$$

From these equations we obtain the following solution:

$$\begin{aligned} b_l &= \frac{(2s-1)}{(2l+3)}m, \qquad & {d_l}^2 = {\tilde{d}_l}^2 &= \frac{(s-l-1)(s+l)}{2(l+1)(2l+1)}m^2, \\ \tilde{b}_0 &= 3b_0, & {d_0}^2 = {\tilde{d}_0}^2 &= 2s(s-1)m^2, \end{aligned}$$

where we set $b_{s-2} = m$.

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