Chiral low-energy physics from squashed branes in deformed $\mathcal{N} = 4$ SYM

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ABSTRACT: We discuss the low-energy physics which arises on stacks of squashed brane solutions of $SU(N)$ $\mathcal{N} = 4$ SYM, deformed by a cubic soft SUSY breaking potential. A brane configuration is found which leads to a low-energy physics similar to the standard model in the broken phase, assuming suitable VEV's of the scalar zero modes. Due to the triple self-intersection of the branes, the matter content includes that of the MSSM with precisely 3 generations and right-handed neutrinos. No exotic quantum numbers arise, however there are extra chiral superfields with the quantum numbers of the Higgs doublets, the W , Z , e_R and u_R , whose fate depends on the details of the rich Higgs sector. The chiral low-energy sector is complemented by a heavy mirror sector with the opposite chiralities, as well as super-massive Kaluza-Klein towers completing the $\mathcal{N} = 4$ multiplets. The sectors are protected by two gauged global $U(1)$ symmetries.

KEYWORDS: Spontaneous Symmetry Breaking, Supersymmetric gauge theory, Extended Supersymmetry

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1 Introduction

$\mathcal{N} = 4$ Super-Yang-Mills (SYM) is the most (super)symmetric of all 4-dimensional field theories without gravity. As such it has played a prominent role ever since its discovery, even though it is usually considered as “too round” for real physics. More structure can be introduced by considering deformations of that model, notably by adding soft SUSY breaking terms to the potential. Then interesting patterns of spontaneous symmetry breaking can occur, inducing even more structure at low energy. A well-known example is the generation of fuzzy spheres, realized by the vacuum expectation values (VEV’s) of the matrix-valued scalar fields. Due to the Higgs effect, the model then behaves like a higher-dimensional model on $\mathbb{R}^4 \times S_N^2$ [1–9]. Recently, a much richer class of such solutions was found [10, 11] in the presence of a cubic SUSY-breaking potential corresponding to a holomorphic 3-form.

These solutions can be interpreted as projected or “squashed” fuzzy coadjoint orbits $\mathcal{C}[\mu]$ of $SU(3)$. Due to their self-intersecting geometry, they lead to 3 generations of massless fermions and scalar fields.

In this paper, we discuss these squashed brane solutions in more detail, and study some aspects of the resulting low-energy physics on stacks of such branes. Since there are massless scalar fields, it is natural (due to the presence of cubic interactions) to assume that some of them take non-trivial VEV’s. The main point to be emphasized here is that for suitable such VEV’s, the resulting low-energy sector behaves like a chiral gauge theory, in the sense that different chiralities of the fermionic (would-be) zero modes couple differently to the spontaneously broken massive gauge fields. Since this is a fundamental property of the standard model, that class of models becomes quite interesting for physics.

We first review and re-derive the fermionic and bosonic zero modes from a field-theory perspective, recovering results in [10]. The approach given here is based on two global symmetries $U(1)_{K_i}$ which are respected by the background up to gauge transformations; these allow a coherent treatment of the bosonic and fermionic modes, and are very useful in understanding the interactions. The “regular” zero modes on a stack of $\mathcal{C}[\mu_i]$ branes can be organized in terms of a quiver, with 3 families of chiral superfields transforming in the bi-fundamental of gauge group $U(n_i)$ arising on the coincident branes. They have specific $U(1)_{K_i}$ weights in the $\mathfrak{su}(3)$ weight lattice. Gauge fields and gauginos arise in vector supermultiplets. Nevertheless, the low-energy theory is not supersymmetric. These massless scalar modes will be dubbed “Higgs” modes henceforth.

Without attempting a full understanding of the rich Higgs sector in this paper, we consider some of the possible Higgs configurations, and elaborate the resulting physics in some detail using the new tools. In particular, we give a brane configuration which leads to the correct pattern of leptons and quarks coupling to the gauge fields of the standard model in its broken phase. This leads to an extension of the MSSM, where each chiral super-multiplet has an extra mirror copy with the opposite chirality, which acquires a higher (by assumption) mass from the mirror Higgs. The present scenario¹ improves upon the analogous background solutions in [12] and related proposals [13, 14] in several ways. First and foremost, there are necessarily 3 generations due to the triple self-intersecting geometries, resulting in a $\mathbb{Z}_3$ family symmetry (which may subsequently be broken). Moreover, the chiral low-energy sector is protected from recombining with the massive mirror fermions due to two exact $U(1)_{K_i}$ symmetries. These are combinations of the R -symmetry and the gauge symmetry, which are preserved by the background. In this way, a stable chiral low-energy physics can arise from the underlying non-chiral $\mathcal{N} = 4$ theory. Furthermore, the scale of the mirror fermions can in principle be much higher than the electroweak scale, for large branes.

The present scenario is somewhat reminiscent of higher-dimensional string-theoretical (and field-theoretical) models such as [15–19]. However it is much more radical and simple, since the chiral low-energy behavior is not put in by hand but arises from spontaneous symmetry breaking. Even if it may seem unlikely that such a scenario could be realistic,

¹This is basically a refinement of an outline given in [11], in a slightly different more conservative setting.

it is certainly worthwhile to explore the possible scope of these deformed $\mathcal{N} = 4$ models, given their special status in field theory.

Due to the complicated Higgs sector, no attempt is made in this paper to find the minima and to justify the assumed Higgs configuration. However, the basic result that certain Higgs configurations lead to chiral low-energy sector and a massive mirror sector is fully justified, and verified numerically. Also, the structure of leptons and quarks is very clear and convincing. However there is a rather complicated sector of modes with the quantum numbers of the two Higgs doublets and the electroweak gauge bosons, which is not worked out in detail. Some numerical computations are performed to gain some more insights, however this clearly requires more detailed investigations.

This paper is intended to be largely self-contained, and written from a field-theory perspective. Rather than just relying on the previous papers [10, 11], the necessary results are re-derived in a more transparent way, emphasizing the role of the two $U(1)_{K_i}$ symmetries. Although this increases the length, the paper should be more accessible in this way.

2 Squashed $SU(3)$ branes in deformed $\mathcal{N} = 4$ SYM

We start with the action of $\mathcal{N} = 4$ $SU(N)$ SYM, which is organized most transparently in terms of 10-dimensional SYM reduced to 4 dimensions:

$$S_{\text{YM}} = \int d^4x \frac{1}{4g_N^2} \text{tr} \left(-F^{\mu\nu} F_{\mu\nu} - 2D^\mu \Phi^a D_\mu \Phi_a + [\Phi^a, \Phi^b][\Phi_a, \Phi_b] \right) + \text{tr} \left(\bar{\Psi} \gamma^\mu i D_\mu \Psi + \bar{\Psi} \Gamma^a [\Phi_a, \Psi] \right). \quad (2.1)$$

Here $F_{\mu\nu}$ is the field strength, $D_\mu = \partial_\mu - i[A_\mu, \cdot]$ the covariant derivative, Φ^a , $a \in \{1, 2, 4, 5, 6, 7\}$ are 6 scalar fields, Ψ is a matrix-valued Majorana-Weyl (MW) spinor of $SO(9, 1)$ dimensionally reduced to 4-dimensions, and Γ^a arise from the 10-dimensional gamma matrices. All fields transform take values in $\mathfrak{u}(N)$ and transform in the adjoint of the $SU(N)$ gauge symmetry. The global $SO(6)_R$ symmetry is manifest. It will be useful to work with dimensionless scalar fields labeled by the six roots² $\pm\alpha_i$ of $\mathfrak{su}(3)$,

$$\Phi_\alpha = mX_\alpha, \quad \alpha \in \mathcal{I} = \{\pm\alpha_i, i = 1, 2, 3\} \subset \mathbb{R}^2 \quad (2.2)$$

with $\alpha_1 + \alpha_2 + \alpha_3 = 0$. These $\alpha \in \mathcal{I}$ are viewed as points in $\mathbb{R}^2$ forming a regular hexagon (see figure 3), with corresponding Weyl chambers defined by the Weyl group $\mathcal{W}$ of reflections along these roots. Here m has the dimension of a mass. Explicitly,

$$\begin{aligned} X_1^\pm &= \frac{1}{\sqrt{2}}(X_4 \pm iX_5) \equiv X_{\pm\alpha_1}, \\ X_2^\pm &= \frac{1}{\sqrt{2}}(X_6 \mp iX_7) \equiv X_{\pm\alpha_2}, \\ X_3^\pm &= \frac{1}{\sqrt{2}}(X_1 \mp iX_2) \equiv X_{\pm\alpha_3} \end{aligned} \quad (2.3)$$

²Here we use field theory conventions, while in [10] group-theory friendly conventions are used. In particular, the α_i are related to the standard basis $\tilde{\alpha}_i$ of positive roots of $\mathfrak{su}(3)$ used in group theory via $\alpha_1 = \tilde{\alpha}_1, \alpha_2 = \tilde{\alpha}_2, \alpha_3 = -\tilde{\alpha}_3$, such that $\alpha_1 + \alpha_2 + \alpha_3 = 0$; this is more natural here.

To introduce a scale and to allow non-trivial “brane” solutions, we add soft terms to the potential,

$$\mathcal{V}[\Phi] = \frac{m^4}{g_N^2} (V_4[X] + V_{\text{soft}}[X]) \quad (2.4)$$

where

$$\begin{aligned} V_4[X] &= -\frac{1}{4} \text{tr} \sum_{\alpha, \beta \in \mathcal{I}} [X_\alpha, X_\beta] [X^\alpha, X^\beta], \\ V_{\text{soft}}[X] &= 4 \text{tr} \left(-[X_1^+, X_2^+] X_3^+ - [X_2^-, X_1^-] X_3^- + M_i^2 X_i^- X_i^+ \right) \end{aligned} \quad (2.5)$$

thereby fixing the scale m . The cubic potential can be written as

$$V_3(X) = -\frac{4}{3} \text{tr} \left(\varepsilon_{ijk} X_i^+ X_j^+ X_k^+ + \text{h.c.} \right), \quad (2.6)$$

corresponding to a holomorphic 3-form on $\mathbb{R}^6$. Rewritten in a real basis, this is recognized as the structure constants of $\mathfrak{su}(3)$ projected to the root generators [10].

We will mostly set $M_i = 0$ in this paper. Then SUSY is explicitly broken, and the global $\text{SO}(6)_R$ symmetry is broken to $\text{SU}(3)_R$ by the cubic term. However as show in appendix A, some supersymmetry can be preserved for a suitable choice of M_i (and corresponding fermionic terms). More precisely, there is a specific $\mathcal{N} = 1^*$ deformation of $\mathcal{N} = 4$ SYM [3, 20, 21] with potential (2.5). However this requires M_i to be outside of the domain which admits the squashed brane solutions of interest here. Nevertheless, this observation should help to understand the quantum corrections of the model, which is left for future work. Here we focus on the classical aspects of the model.

Perturbation of the background. Let us add a perturbation ϕ^α to the background X^α ,

$$\Phi^\alpha = m (X^\alpha + \phi^\alpha). \quad (2.7)$$

This will lead to further symmetry breaking and interesting low-energy physics in the zero-mode sector of the background X . The complete potential is easily worked out,

$$\begin{aligned} V(X + \phi) &= V(X) + \text{tr} \left(\phi^\alpha \square_X X_\alpha + X^\alpha \square_\phi \phi_\alpha + \frac{1}{2} \phi^\alpha (\square_X + 2\mathcal{D}_{ad}) \phi_\alpha - \frac{1}{2} f^2 \right) \\ &\quad + 4 \text{tr} \left(-\varepsilon_{ijk} \phi_i^+ X_j^+ X_k^+ - \varepsilon_{ijk} \phi_i^+ \phi_j^+ X_k^+ + M_i^2 \phi_i^- X_i^+ + \frac{1}{2} M_i^2 \phi_i^- \phi_i^+ + \text{h.c.} \right). \end{aligned} \quad (2.8)$$

Here

$$f = i[\phi_\alpha, X^\alpha] \quad (2.9)$$

can be viewed as gauge-fixing function in extra dimensions, and we define

$$\square_X = \sum_{a \in \mathcal{I}} [X_a, [X^a, \cdot]] = [X_j^+, [X_j^-, \cdot]] + [X_j^-, [X_j^+, \cdot]], \quad (2.10)$$

$$\begin{aligned} (\mathcal{D}_{ad}\phi)_\alpha &= \sum_\beta [[X_\alpha, X^\beta], \phi_\beta] = ((\mathcal{D}_{\text{mix}} + \mathcal{D}_{\text{diag}})\phi)_\alpha \\ (\mathcal{D}_{\text{mix}}\phi)_\alpha &= \sum_{\beta \neq \alpha} [[X_\alpha, X^\beta], \phi_\beta] \\ (\mathcal{D}_{\text{diag}}\phi)_\alpha &= [[X_\alpha, X_{-\alpha}], \phi_\alpha] \quad (\text{no sum}) \end{aligned} \quad (2.11)$$

following [11], noting that

$$X^\alpha = X_{-\alpha}. \quad (2.12)$$

In particular, the equations of motion (eom) for the background X can be written as

$$0 = (\square_4 + m^2 (\square_X + 4M_i^2)) X_i^+ + 4m^2 \varepsilon_{ijk} X_j^- X_k^- \quad (2.13)$$

where $\square_4 = -D_\mu D^\mu$ is the 4-dimensional covariant d'Alembertian.

2.1 Squashed brane solutions

It is well-known that the above potential has fuzzy sphere solutions $X_i^\pm \sim c_i^\pm J_i$ where J_i are generators of $\mathfrak{su}(2)$ [1–8, 22]. However as shown in [10], there are also solutions with much richer structure corresponding to (stacks of) squashed fuzzy coadjoint $\text{SU}(3)$ orbits $\mathcal{C}_N[\mu]$, obtained by the following ansatz

$$X_i^\pm = r_i \pi(T_i^\pm). \quad (2.14)$$

Here

$$T_1^\pm \equiv T_{\pm\alpha_1}, \quad T_2^\pm \equiv T_{\pm\alpha_2}, \quad T_3^\pm \equiv T_{\pm\alpha_3} \quad (2.15)$$

are *root* generators of $\mathfrak{su}(3)_X$, π is any representation on $\mathcal{H} \cong \mathbb{C}^N$, and α_1, α_2 are the simple roots with $\alpha_3 = -(\alpha_1 + \alpha_2)$. In these conventions, the Lie algebra relations are

$$\begin{aligned} [T_\alpha, T_\beta] &= \pm T_{\alpha+\beta}, \quad 0 \neq \alpha + \beta \in \mathcal{I} \\ [T_{\alpha_i}, T_{-\alpha_i}] &= H_i \equiv H_{\alpha_i} \\ [H, T_\alpha] &= \alpha(H) T_\alpha, \end{aligned} \quad (2.16)$$

with $[T_1^+, T_2^+] = T_3^-$ and $\alpha_i(H_i) = (\alpha_i, \alpha_i) = 2$ where $(\cdot, \cdot)$ denotes the Killing form of $\mathfrak{su}(3)$. Using these Lie algebra relations, the equations of motion (2.13) become

$$\begin{aligned} 0 &= r_1 \left(2r_1^2 + r_2^2 + r_3^2 - 4 \frac{r_2 r_3}{r_1} + 4M_1^2 \right) T_1^+ \\ 0 &= r_2 \left(r_1^2 + 2r_2^2 + r_3^2 - 4 \frac{r_1 r_3}{r_2} + 4M_2^2 \right) T_2^+ \\ 0 &= r_3 \left(r_1^2 + r_2^2 + 2r_3^2 - 4 \frac{r_1 r_2}{r_3} + 4M_3^2 \right) T_3^+. \end{aligned} \quad (2.17)$$

Figure 1. 3-dimensional section of squashed $\mathbb{CP}^2$, taken from [10].

Assuming $M_i = 0$ for simplicity, these equations have the non-trivial solution

$$r_i = 1 \equiv r . \quad (2.18)$$

For $\pi = \pi_\mu$ an irreducible representation (irrep) with highest weight μ acting on $\mathcal{H}_\mu$, these solutions can be interpreted as quantized or fuzzy coadjoint orbits $\mathcal{C}[\mu] \subset \mathfrak{su}(3)_X \cong \mathbb{R}^8$ projected to $\mathbb{R}^6$ along two Cartan generators [10]. Generically these are 6-dimensional (fuzzy) varieties, while for $\mu = (n, 0)$ and $\mu = (0, n)$ they are 4-dimensional projections of (fuzzy) $\mathbb{CP}^2$. Here $\mu = (n_1, n_2)$ denotes the Dynkin labels of μ . Such a “squashed” $\mathbb{CP}^2$ has a triple self-intersection at the origin, as visualized in figure 1. We will see that pairs of fermionic zero modes arise at the intersections, connecting the different sheets.

To organize the degrees of freedom, we note that these solutions defines an embedding $\mathrm{SU}(3)_X \subset \mathrm{SU}(N)$, which acts via the adjoint on all the fields in the theory. Decomposing the $\mathfrak{su}(N)$ -valued fields into harmonics i.e. irreps of this $\mathrm{SU}(3)_X$

$$\mathfrak{su}(N) \cong \mathrm{End}(\mathcal{H}) = \oplus_\Lambda n_\Lambda \mathcal{H}_\Lambda \quad (2.19)$$

(here $\mathcal{H}_\Lambda$ denotes the highest weight irreps) allows to understand the physics of the fluctuations on such a background, even though the $\mathrm{SU}(N)$ gauge symmetry is broken completely for irreducible π_μ . In particular, the $\mathrm{SU}(3)_X$ gauge transformations act on the scalar fields as

$$X_\alpha \rightarrow \pi(g)\pi(T_\alpha)\pi(g^{-1}) = \Lambda(g)_\alpha^\beta \pi(T_\beta) + \Lambda(g)_\alpha^i \pi(H_i) \quad (2.20)$$

(here Λ_α^β is the 8-dimensional representation of $\mathrm{SU}(3)_X$). Now restrict to the Cartan subalgebra or the torus $\mathrm{U}(1) \times \mathrm{U}(1) \subset \mathrm{SU}(3)_X$, which is sufficient to specify the weights in the various $\mathfrak{su}(3)_X$ harmonics. Then the last term in (2.20) vanishes, and the six scalar fields X_α transform linearly, corresponding to the six non-zero weights in $(1, 1)$ of $\mathfrak{su}(3)_X$. This organization will be very useful.

The potential has a global $\mathrm{SU}(3)_R \subset \mathrm{SU}(4)_R$ symmetry, which is broken to $\mathrm{SU}(2) \times \mathrm{U}(1)$ or $\mathrm{U}(1)^2$ in the presence of masses $M_i \neq 0$. We denote with τ_i the traceless $\mathrm{U}(1)_i \subset \mathrm{U}(3)_R$ generator which has eigenvalue 1 on X_i^+ and $-\frac{1}{2}$ on the X_j^+ with $j \neq i$, or more formally

$$2\tau_i \phi_\alpha = (\alpha_i, \alpha) \phi_\alpha . \quad (2.21)$$

Then $\sum_i \tau_i = 0$, and the action of $2\tau_i$ on the scalar fields coincides with the adjoint action of the Cartan generators H_{α_i} of $\mathfrak{su}(3)_X$. In other words, the background X^α is annihilated by the following generators

$$K_i := 2\tau_i - [H_{\alpha_i}, \cdot], \quad i = 1, 2, 3 \quad (2.22)$$

which satisfy $K_1 + K_2 + K_3 = 0$, and generate a $U(1)_K \times U(1)_K$ symmetry of the background. Their charges are obtained by adding the (rescaled and rotated, cf. figure 4) $(1, 0) + (0, 1)$ weights of $\mathfrak{su}(3)_R$ to the non-zero weights of $(1, 1)$ of $\mathfrak{su}(3)_X$. In particular, the charges of $U(1)_i^K$ are points in the weight lattice of $\mathfrak{su}(3)_X$. This will be very important to characterize and protect the zero modes.

Now we can understand the Goldstone bosons arising from the broken global symmetries. The background breaks the global $SU(3)_R$ symmetry, but the traceless $U(1)_i$ with generators τ_i are equivalent to gauge transformations (i.e. “gauged”). Therefore there will be only $8 - 2$ physical Goldstone bosons, as the two $U(1)_{\tau_i}$ modes are eaten by the massive gauge bosons. These 6 physical Goldstone bosons are identified in appendix B with the 6 exceptional zero modes in the $(1, 1) \subset \text{End}(\mathcal{H})$ as discussed below.

Finally, the background admits a $\mathbb{Z}_3$ symmetry, which cyclically permutes the $X_i^\pm$. This is part of the $SU(3)_R$ symmetry, and also part of the Weyl group³ of $SU(3)_X$. It is also interesting to recall that the global $SU(4)_R$ is anomalous, and there is an associated Wess-Zumino term [23, 24]. This might be important for the effective description of the 6 physical Goldstone bosons.

2.2 Scalar zero modes on squashed branes

Let $M_i = 0$ from now on. Then the squashed brane backgrounds X_α admit a number of zero modes $\phi_\alpha^{(0)}$. To see this, we note that the bilinear form defined by $\mathcal{D}_{\text{mix}}$ on a background (2.14) can be simplified e.g. as follows

$$\text{tr} \left(\phi_i^- (\mathcal{D}_{\text{mix}} \phi)_i^+ \right) = \sum_{j \neq i} \text{tr} \left(\phi_i^- \left[[X_i^+, X_j^+], \phi_j^- \right] \right) = -\varepsilon_{ijk} \text{tr} \left(\phi_i^- [X_k^-, \phi_j^-] \right) \quad (2.23)$$

using $[T_i^+, T_j^+] = \varepsilon_{ijk} T_k^-$ and $r = 1$. This has precisely the form of the quadratic contribution from the cubic potential (2.8). Therefore the quadratic terms in the potential can be written as

$$V_2[\phi] = \text{tr} \phi^\alpha \mathcal{O}_V \phi_\alpha, \quad \mathcal{O}_V = \square_X + 2\mathcal{D}_{\text{diag}} - 2\mathcal{D}_{\text{mix}}. \quad (2.24)$$

It was shown in [10] that $\mathcal{O}_V$ is positive semi-definite for all representations π . The zero modes of $\mathcal{O}_V$ fall into two classes, denoted as regular and exceptional zero modes. Let us first focus on the regular zero modes, which are given by solutions of the decoupling condition [10, 11]

$$\mathcal{D}_{\text{mix}} \phi_\alpha^{(0)} = 0. \quad (2.25)$$

Here we shall provide a group-theoretic characterization of the regular zero modes, which implies (2.25); it is then straightforward to show that they are zero modes. Recall that

³The potential is in fact invariant under the full Weyl group S_3 of $SU(3)_X$.

the background respects the $U(1)$ generators⁴ $K_i = 2\tau_i - [H_{\alpha_i}, \cdot]$ (2.22), and consider the “ τ -parity” generator τ in $U(3)_R$ defined by

$$\tau\phi_i^\pm = \pm\phi_i^\pm, \quad (2.26)$$

which is broken by the cubic potential. Then

$$\begin{aligned} \tau\mathcal{D}_{\text{mix}} &= -\mathcal{D}_{\text{mix}}\tau \\ \mathcal{D}_{\text{mix}}K_i &= K_i\mathcal{D}_{\text{mix}}. \end{aligned} \quad (2.27)$$

Now fix some highest weight module $\mathcal{H}_\Lambda \subset \text{End}(\mathcal{H})$, and consider the set of $U(1)_{K_i}$ weights of $\phi_\alpha \in \mathcal{H}_\Lambda$, given by the 6 nonzero weights $\alpha \in (1, 1)$ of $\mathfrak{su}(3)_X$ minus the weights in $\mathcal{H}_\Lambda$. Among these, consider the 6 *extremal weights*⁵ Λ' , and denote the corresponding modes as

$$\phi_{\alpha, \Lambda'}^{(0)} = Y_\lambda, \quad \Lambda' = \alpha - \lambda, \quad Y_\lambda \in \mathcal{H}_\Lambda. \quad (2.28)$$

Here Y_λ is an extremal weight vector with weight λ in $\mathcal{H}_\Lambda$. These $\phi_{\alpha, \Lambda'}^{(0)}$ have charge $\Lambda' = \alpha - \lambda$ under the K_i , corresponding to a point of the $\mathfrak{su}(3)_X$ weight lattice in (the interior of) the Weyl chamber of α . These are the *regular zero modes*. They have eigenvalue $\tau = \pm 1$ determined by the parity of the Weyl chamber of $\alpha = \pm\alpha_i$. Since there is only one such state for any such (extremal!) Λ' and $\mathcal{D}_{\text{mix}}$ preserves Λ' , (2.27) implies that

$$\mathcal{D}_{\text{mix}}\phi_{\alpha, \Lambda'}^{(0)} = 0. \quad (2.29)$$

Using the extremal weight property, it is then easy to verify that these are zero modes

$$\mathcal{O}_V\phi_{\alpha, \Lambda'}^{(0)} = (\square_X + 2\mathcal{D}_{\text{diag}})\phi_{\alpha, \Lambda'}^{(0)} = 0; \quad (2.30)$$

e.g. for $\alpha = \alpha_3$, we have (cf. [11])

$$\begin{aligned} \square_X\phi_{\alpha_3, \Lambda'}^{(0)} &= r^2 ([T_1^+, [T_1^-, \cdot]] + [T_2^+, [T_2^-, \cdot]] + [T_3^-, [T_3^+, \cdot]])\phi_{\alpha_3, \Lambda'}^{(0)} \\ &= r^2 ([H_1, \cdot] + [H_2, \cdot] - [H_3, \cdot])\phi_{\alpha_3, \Lambda'}^{(0)}, \\ \mathcal{D}_{\text{diag}}\phi_{\alpha_3, \Lambda'}^{(0)} &= r^2 [T_3, T_{-3}]\phi_{\alpha_3}^{(0)} = r^2 [H_3, \phi_{\alpha_3}^{(0)}] \end{aligned} \quad (2.31)$$

hence (2.30) follows from $H_1 + H_2 + H_3 = 0$. We will find superpartners of these regular zero modes in section 4. Particular examples of such modes are given by

$$\phi_{\alpha, (n+1)\alpha}^{(0)} = (T_{-\alpha})^n. \quad (2.32)$$

Observe that they have eigenvalue $\tau = \pm 1$ determined by the Weyl chamber of $\alpha = \pm\alpha_i$. A possible background with such a “Higgs” switched on would then be

$$\Phi_\alpha = m(X_\alpha + \varepsilon_\alpha T_{-\alpha}^n). \quad (2.33)$$

⁴This symmetry was also used in [11] to classify excitations on spinning brane backgrounds.

⁵These are the corners of the convex set of weights in $\mathbb{R}^2$, or equivalently of the maximal irrep in $(1, 1) \otimes \mathcal{H}_{\Lambda^+}$. The conventions differ in an inessential way from the ones in [11].

On a single squashed $\mathbb{CP}_N^2$ brane, these exhausts all regular zero modes. Observe that there are 6 such zero modes even for degenerate Λ such as $\Lambda = (m, 0)$. Some intuition can be gained by noting that the regular zero modes with maximal Λ' on squashed $\mathbb{CP}_N^2$ link the 3 intersecting $\mathbb{R}^4$ sheets at the origin, with polarization along the common $\mathbb{R}^2$ [10]. More generally, the regular zero modes can be interpreted as strings linking these sheets, shifted along their intersection.⁶

For harmonics $\mathcal{H}_\Lambda \in \text{End}(\mathcal{H})$ with $\Lambda = (m-2, 1)$ and $\Lambda = (1, m-2)$, there are in addition 3 exceptional zero modes [10] with $\Lambda' = (m, 0)$ resp. $\Lambda' = (0, m)$, which have mixed polarizations. The most important among these arise for $\Lambda = (1, 1)$: these correspond to the 6 physical Goldstone bosons which arise from $\text{SU}(3)_R$ as discussed above, see appendix B. The full set of zero modes for squashed $\mathbb{CP}^2$ branes can then be obtained from the mode decomposition

$$\begin{aligned} (n, 0) \otimes (0, m) &= (n, m) + (n-1, m-1) + \dots + (n-m, 0) \\ (n, 0) \otimes (m, 0) &= (n+m, 0) + (n+m-2, 1) + (n+m-4, 2) + \dots + (\dots, m) \end{aligned} \quad (2.34)$$

assuming $n \geq m$. There is a set of 6 exceptional zero modes in $(n, 0) \otimes (0, n)$ given by the Goldstone bosons, and typically 3 exceptional zero modes for $(n, 0) \otimes (m, 0)$, or 6 for $n+m=3$.

From now on, we will collectively denote the set of these scalar (“would-be”) zero modes as Higgs sector, anticipating that they may acquire some VEV or some mass.

Higgs connecting different branes. Now consider backgrounds consisting of several branes, described by reducible representations in (2.14). For example, the matrix modes on a stack consisting of one $\mathcal{C}[\mu_L]$ and one $\mathcal{C}[\mu_R]$ brane decompose as

$$\text{End}(\mathcal{H}) = \text{End}(\mathcal{H}_{\mu_L}) + \text{End}(\mathcal{H}_{\mu_R}) + \mathcal{H}_{\mu_L} \otimes \mathcal{H}_{\mu_R}^* + \mathcal{H}_{\mu_L} \otimes \mathcal{H}_{\mu_R}^*. \quad (2.35)$$

To be specific, assume that $\mu_L = (1, 0)$ and $\mu_R = (0, 1)$. Then $\text{End}(\mathcal{H}_{\mu_L}) = (1, 1) + (0, 0) = \text{End}(\mathcal{H}_{\mu_R})$, each leading to 6 regular zero modes with $\Lambda' \in \mathcal{W}(2, 2)$ from $\mathcal{H}_{(1,1)}$, 6 regular zero modes with $\Lambda' \in \mathcal{W}(1, 1)$ from $\mathcal{H}_{(0,0)}$ corresponding to translations in the internal $\mathbb{R}^6$, and 6 exceptional zero modes with $\Lambda' \in \{\mathcal{W}(3, 0), \mathcal{W}(0, 3)\}$ from $\mathcal{H}_{(1,1)}$ corresponding to the $\text{SU}(3)_R$ Goldstone bosons. Here $\mathcal{W}(n_1, n_2)$ denotes the set of weights given by the action of the Weyl group $\mathcal{W}$ on $\Lambda' = (n_1, n_2)$. On the other hand, the regular modes connecting different branes can be written as

$$\phi_{\alpha, \Lambda'}^{(0)} = \varphi_{\alpha}^{ij} |\mu_L^i\rangle \langle \mu_R^j| \in \mathcal{H}_{\mu_L} \otimes \mathcal{H}_{\mu_R}^* \cong \mathcal{H}_{(2,0)} \oplus \mathcal{H}_{(0,1)}, \quad (2.36)$$

where $\mu_{L,R}^i$ are weights in $\mathcal{H}_{\mu_{L,R}}$, and $\Lambda' = \alpha - (\mu_L^i - \mu_R^j)$ is in the Weyl chamber opposite to α . This leads to 6 regular zero modes with $\Lambda' \in \mathcal{W}(3, 1)$ from $\mathcal{H}_{(2,0)}$, 6 regular zero modes with $\Lambda' \in \mathcal{W}(1, 2)$ from $\mathcal{H}_{(0,1)}$, and 3 exceptional zero modes with $\Lambda' \in \mathcal{W}(2, 0)$ from $\mathcal{H}_{(0,1)}$. Since the $|\mu_R^i\rangle$ can be viewed as coherent states on $\mathcal{C}[\mu_R]$ located at the origin [10], the $\mathcal{H}_{(0,1)}$ modes are strings linking the sheets of $\mathcal{C}[(1, 0)]$ and $\mathcal{C}[(0, 1)]$ with a 2-dimensional

⁶For examples of such string-like modes see e.g. [25].

intersection at the origin, while the $\mathcal{H}_{(2,0)}$ modes are strings linking coinciding sheets at the origin. Finally, the modes connecting $\mathcal{C}[(1,0)]$ with a point brane $\mathcal{C}[0]$ arise from

$$\phi_{\alpha,\Lambda'}^{(0)} = \varphi_{\alpha}^i |0\rangle \langle \mu_R^i| \in (0,0) \otimes (1,0) \cong \mathcal{H}_{(1,0)}, \quad (2.37)$$

leading to 6 regular zero modes with $\Lambda' \in \mathcal{W}(2,1)$, and 3 exceptional zero modes with $\Lambda' \in \mathcal{W}(0,2)$.

3 A standard-model-like brane configuration

Now consider a background consisting of two coincident (isomorphic) branes $\mathcal{C}[\mu_L]_u$, $\mathcal{C}[\mu_L]_d$, and two additional (typically different) branes $\mathcal{C}[\mu_{Ru}]$ and $\mathcal{C}[\mu_{Rd}]$. We assume that the scale m of these branes is very high. Furthermore we add a “leptonic” point brane $\mathcal{D}_l \cong \mathcal{C}[0]$, and 3 “baryonic” point branes $\mathcal{D}_{b_j} \cong \mathcal{C}[0]$, $j = 1, 2, 3$. Hence the matrices of $\mathcal{N} = 4$ SYM act on

$$\mathcal{H} = \mathcal{H}_L^2 \oplus \mathcal{H}_{Ru} \oplus \mathcal{H}_{Rd} \oplus \mathbb{C} \oplus \mathbb{C}^3. \quad (3.1)$$

If realized within $U(N)$ SYM (instead of $SU(N)$), this background admits a $U(2)_L \times U(1)_{Ru} \times U(1)_{Rd} \times U(1)_l \times U(3)_c$ gauge symmetry, or $U(2)_L \times U(2)_R \times U(1)_l \times U(3)_c$ if $\mu_{Ru} = \mu_{Rd}$. Such stacks of branes might be bound by quantum effects in $\mathcal{N} = 4$ SYM, which are related to supergravity and typically induce an attractive interaction between branes with different flux [26–29].

Now we switch on some “Higgs” links between these branes, realized by (would-be) zero modes ϕ_{α} linking some extremal states of the various $\mathcal{H}_i$ as in (2.36),

$$\phi_{\alpha} = \sum \varphi_{\alpha}^{ij} |\mu_L\rangle_i \langle \mu_R|_j + \dots \in \oplus \text{Hom}(\mathcal{H}_j, \mathcal{H}_i). \quad (3.2)$$

First, assume that the point-brane $\mathcal{D}_l$ is linked to the extremal weight states of $\mathcal{C}[\mu_{Ru}]$ as in (2.37),

$$\phi_S = \sum \varphi_S |0\rangle_l \langle \mu_{Ru}|_u \quad (3.3)$$

dropping the polarization indices. Assuming that the scale of φ_S is high,⁷ the unbroken gauge symmetry is reduced to $U(2)_L \times U(1)_{Ru,l} \times U(1)_{Rd} \times U(3)_c$, where $U(1)_{Ru,l}$ is generated by $\mathbb{1}_l + \mathbb{1}_{Ru}$. To write this in a more suggestive form, we introduce the hypercharge generator

$$Y := \mathbb{1}_{Ru} - \mathbb{1}_{Rd} + L - B, \quad (3.4)$$

and

$$T_5 = B - L + \Xi \quad (3.5)$$

(as in [12]), where

$$\begin{aligned} \Xi &= \mathbb{1}_{Lu} + \mathbb{1}_{Ld} - (\mathbb{1}_{Rd} + \mathbb{1}_{Ru}), \\ B &= \frac{1}{3} \mathbb{1}_b, \\ L &= \mathbb{1}_l. \end{aligned} \quad (3.6)$$

⁷We might as well consider $\mathcal{D}_l \cup_{\phi_S} \mathcal{C}[\mu_{Ru}]$ as a single brane.

Figure 2. Basic brane configuration for the standard model.

Ξ will act as chirality γ_5 in the light sector due to (4.31). Then the unbroken gauge symmetry can be written as $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_B \times U(1)_5 \times U(1)_{\text{tr}}$, where $U(1)_5$ and $U(1)_B$ will be anomalous in the light sector but not in the full model, and $U(1)_{\text{tr}}$ is the trace- $U(1)$. The latter can be dropped in $\mathcal{N} = 4$ SYM, but acquires an interesting role related to gravity in the IKKT matrix model [30]. This leaves exactly the gauge group of the standard model $SU(3)_c \times SU(2)_L \times U(1)_Y$, extended by $U(1)_B$, $U(1)_5$, and possibly $U(1)_{\text{tr}}$. The $U(1)_5$ will also be broken by the electroweak Higgs as elaborated below. All other gauge bosons are massive with mass set by the scale m or φ_S ; we will ignore these from now on.

Now assume that some “electroweak” Higgs arise such that the 4 squashed $\mathcal{C}[\mu_i]$ branes form two bound states

$$\begin{aligned}\mathcal{D}_u &= \mathcal{C}[\mu_L]_u \cup_{\phi_u} \mathcal{C}[\mu_Ru] \\ \mathcal{D}_d &= \mathcal{C}[\mu_L]_d \cup_{\phi_d} \mathcal{C}[\mu_Rd]\end{aligned}\tag{3.7}$$

as sketched in figure 2. For example, we could have $\mathcal{D}_u = \mathcal{C}[(1,0)] \cup \mathcal{C}[(0,1)]$ and $\mathcal{D}_d = \mathcal{C}[(1,0)] \cup \mathcal{C}[(0,2)]$. Dropping indices, we can write the corresponding Higgs suggestively as

$$\begin{aligned}\phi_d &\sim \sum (0|\mu_L\rangle_u + \varphi_d|\mu_L\rangle_d) \langle \mu_Rd|_d \cong \begin{pmatrix} 0 \\ \varphi_d \end{pmatrix} \langle \mu_Rd|_d \\ \phi_u &\sim \sum (\varphi_u|\mu_L\rangle_u + 0|\mu_L\rangle_d) \langle \mu_Ru|_u \cong \begin{pmatrix} \varphi_u \\ 0 \end{pmatrix} \langle \mu_Ru|_u\end{aligned}\tag{3.8}$$

connecting some of their extremal weight states of $\mathcal{H}_L$ and $\mathcal{H}_R$. Thus φ_d and φ_u can be viewed as non-vanishing entries of two $SU(2)_L$ doublets as in the MSSM (5.5), with

$$\tan \beta = \frac{\varphi_u}{\varphi_d} .\tag{3.9}$$

We assume that the scale of the $\mathcal{C}[\mu]$ branes is much larger than the (electroweak) scale of the Higgs, $\varphi \ll r = 1$ and $\varphi \ll \varphi_S$, so that we can neglect the back-reaction of the Higgs on the branes. This defines the background under consideration. The 3 coincident

“baryonic” point branes $\mathcal{D}_{b_j}$, $j = 1, 2, 3$ remain disconnected from the rest. As discussed above, such squashed branes are solutions of our model. The above Higgs are part of the zero mode sector, and we simply assume that they acquire some VEV.

Once these Higgs fields φ_d and φ_u are switched on, the gauge symmetry is broken⁸ to $SU(3)_c \times U(1)_Q \times U(1)_{tr} \times U(1)_B$, where B is the baryon number, and

$$Q := \frac{1}{2}(\mathbb{1}_{Ru} + \mathbb{1}_{Lu} - \mathbb{1}_{Rd} - \mathbb{1}_{Ld} + L - B) \quad (3.10)$$

is the electric charge generator. Here $\mathbb{1}_{Lu}, \mathbb{1}_{Ld}$ indicate the $\mathcal{H}_L$ which is part of $\mathcal{D}_u$ and $\mathcal{D}_d$, respectively. Note that Q is traceless provided $\dim \mathcal{H}_{Ru} = \dim \mathcal{H}_{Rd}$. We will see that Q and Y give the correct charge assignment of the standard model; in particular, we note the Gell-Mann-Nishijima formula

$$2Q - Y = \mathbb{1}_{Lu} - \mathbb{1}_{Ld} =: 2T_L^3. \quad (3.11)$$

Thus the low-energy broken gauge modes are given by three massive generators of $SU(2)_L \times U(1)_Y$ identified as $W^\pm$ and Z , and the $U(1)_5$ mode generated by T_5 . To elaborate the masses of these low-energy gauge bosons, we decompose the Hilbert space of scalar fields on the two $\mathcal{C}[\mu_L]$ as

$$\text{End}(\mathcal{H}_L^2) = \text{End}(\mathcal{H}_L) \otimes \mathfrak{u}(2)_L \quad (3.12)$$

where $\text{End}(\mathcal{H}_L)$ are the functions on $\mathcal{C}[\mu_L]$. Then the W bosons arise from the $\mathfrak{su}(2)_L \subset \mathfrak{u}(2)_L$ -valued gauge fields which are proportional to $\mathbb{1}$ on $\mathcal{H}_L$. The components of the $SU(2)_L \times U(1)_Y \subset U(N)$ gauge fields are accordingly given by

$$\begin{aligned} A_\mu(x) &= g_N (W_{\mu,i}(x) \tilde{t}_i + B_\mu(x) \tilde{t}_Y + B_{\mu 5} \tilde{t}_5) \\ &= g W_{\mu,i}(x) t_i + \frac{1}{2} g' B_\mu(x) t_Y + g_5 B_{\mu 5} t_5, \end{aligned} \quad (3.13)$$

where

$$\begin{aligned} t_i &= c_L \tilde{t}_i = \mathbb{1}_{\mathcal{H}_L} \otimes \frac{1}{2} \sigma_i, \quad i = 1, 2, 3 \\ t_Y &= c_Y \tilde{t}_Y = \mathbb{1}_{Ru} - \mathbb{1}_{Rd} + L - B, \\ t_5 &= c_5 \tilde{t}_5 = B - L + \Xi. \end{aligned} \quad (3.14)$$

They couple to the fermionic zero modes

$$D_\mu \psi = (\partial_\mu - i[\mathcal{A}_\mu, \cdot]) \psi = \left(\partial_\mu - ig W_i t_i - i \frac{g'}{2} B t_Y - ig_5 B_5 t_5 \right) \psi \quad (3.15)$$

and similarly to the Higgs fields ϕ_u, ϕ_d

$$D_\mu \phi = (\partial_\mu - i[\mathcal{A}_\mu, \cdot]) \phi = \left(\partial_\mu - ig W_i t_i - i \frac{g'}{2} B t_Y - ig_5 B_5 t_5 \right) \phi. \quad (3.16)$$

As explained in detail below, these reproduce precisely the couplings and charges of the standard model. We can therefore identify the gauge fields W_i, B , etc. with those of the

⁸Note that nonabelian VEVs in the scalar sector do reduce the rank of the gauge group.

the standard model, where g is the $SU(2)_L$ coupling constant, and g' is the $U(1)_Y$ coupling constant. The coupling constants of the $SU(2)_L \times U(1)_Y$ gauge bosons are therefore given by

$$g = \frac{g_N}{c_L}, \quad \frac{1}{2}g' = \frac{g_N}{c_Y}, \quad g_5 = \frac{g_{YM}}{c_5}. \quad (3.17)$$

The appropriate normalization is obtained such that the Lagrangian of the gauge fields is

$$\mathcal{L}_G = -\frac{1}{4}F_i^{\mu\nu}F_{\mu\nu}^i - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \dots \quad (3.18)$$

i.e. $\text{tr } \tilde{t}_i^2 = 1 = \text{tr } \tilde{t}_Y^2$, which gives

$$\begin{aligned} c_L &= \sqrt{\frac{1}{2} \dim \mathcal{H}_L}, \\ c_Y &= \sqrt{\dim \mathcal{H}_{Ru} + \dim \mathcal{H}_{Rd} + \frac{4}{3}}, \\ c_5 &= \sqrt{\dim \mathcal{H}_{Ru} + \dim \mathcal{H}_{Rd} + 2 \dim \mathcal{H}_L + \frac{4}{3}}. \end{aligned} \quad (3.19)$$

Then the masses of the gauge bosons are obtained from

$$\mathcal{L}_\phi[A] = -\frac{1}{2}\text{tr} D_\mu \phi^\dagger D^\mu \phi = -\frac{1}{2}\text{tr}[W_\mu, \phi]^\dagger [W^\mu, \phi] =: -\frac{1}{2}W_\mu W^\mu m_W^2 \quad (3.20)$$

where the covariant derivatives of scalar fields (3.8) are explicitly

$$\begin{aligned} iD\phi_d &= [A, \phi_d] = \left(gW_a t_a + \frac{1}{2}g'B + g_5 B_5 \right) \phi_d \\ &= \frac{\varphi_d}{2} \begin{pmatrix} g(W_1 + iW_2) \\ -gW_3 + g'B + 2g_5 B_5 \end{pmatrix} \langle \mu_R |_d = \frac{\varphi_d}{2} \begin{pmatrix} g(W_1 + iW_2) \\ -g_Z Z + 2g_5 B_5 \end{pmatrix} \langle \mu_R |_d, \\ iD\phi_u &= [A, \phi_u] = \left(gW_a t_a - \frac{1}{2}g'B + g_5 B_5 \right) \phi_u \\ &= \frac{\varphi_u}{2} \begin{pmatrix} g(W_1 - iW_2) \\ gW_3 - g'B + 2g_5 B_5 \end{pmatrix} \langle \mu_R |_u = \frac{\varphi_u}{2} \begin{pmatrix} g(W_1 - iW_2) \\ g_Z Z + 2g_5 B_5 \end{pmatrix} \langle \mu_R |_u. \end{aligned} \quad (3.21)$$

The Z boson is identified as the combination of W_3 and B which acquires a mass,

$$Z = gW_3 - g'B. \quad (3.22)$$

On the other hand, (3.10) guarantees that $U(1)_Q$ remains exactly massless, since $\mathcal{D}_{ul} = \mathcal{D}_u \cup_{\phi_S} \mathcal{D}_l$ and $\mathcal{D}_d$ are disconnected. The masses are obtained from

$$\mathcal{L}_\phi[A] = -\frac{\varphi^2}{8} \left(g^2 (W_1^\mu W_{1\mu} + W_2^\mu W_{2\mu}) + (g^2 + g'^2) Z^\mu Z_\mu + 4g_5^2 B_5^\mu B_{5\mu} \right) \quad (3.23)$$

for $\varphi_u = \varphi \sin \beta$, $\varphi_d = \varphi \cos \beta$. Here tr_N is evaluated using the explicit form (3.21) of ϕ connecting the extremal weight states of the squashed branes, and does *not* contribute any N -dependent factors.⁹ We can then read off the tree-level W and Z bosons masses,

$$m_W^2 = \frac{1}{4}g^2\varphi^2, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2)\varphi^2, \quad m_5^2 = g_5^2\varphi^2. \quad (3.24)$$

⁹This is an essential improvement compared with the background in [12], which lead to a factor N at this point, and to an equal scale of the mirror fermions and W bosons.

All scales are set by m . Note that as long as $\varphi \ll r$, m_W^2 is much lower than any of the higher KK gauge bosons which start at $12g_N^2 r^2$, where 12 is the lowest eigenvalue of $\square_X$ on $\mathcal{H}_{(1,1)}$. The photon and the Z -boson are now identified as usual

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W_3 \\ B \end{pmatrix}.$$

This gives the Weinberg angle

$$\sin^2 \theta_W = \frac{g'^2}{g'^2 + g^2} = \frac{2 \dim \mathcal{H}_L}{2 \dim \mathcal{H}_L + \dim \mathcal{H}_{Ru} + \dim \mathcal{H}_{Rd} + \frac{4}{3}}. \quad (3.25)$$

E.g. for $\dim \mathcal{H}_i = 3$ this gives $\sin^2 \theta_W = 0.45$, for $\dim \mathcal{H}_L = 3$, $\dim \mathcal{H}_R = 6$ this gives $\sin^2 \theta_W = 0.31$, and for $\dim \mathcal{H}_L = 3$, $\dim \mathcal{H}_R = 8$ this gives $\sin^2 \theta_W = 0.25$. These are of course tree-level formulae which should be viewed as GUT values at very high energies.

These formulae have to be generalized in the presence of several Higgs components, in particular $\tilde{\varphi}$ and φ which couple to the mirror fermions and standard-model fermions, respectively. All of them contribute to the W mass as above, and must be taken into account accordingly.

To put this into perspective, consider briefly the coupling of the Higgs to the fermionic zero modes (which are discussed in detail below). We will see that the off-diagonal fermionic zero modes connecting $\mathcal{D}_l$ with $\mathcal{D}_u$ or $\mathcal{D}_d$ have the structure

$$\Psi = |s_i\rangle \psi_{12}, \quad \psi_{12} \sim |\mu_{L,R}\rangle_{u,d} \langle 0|_l \quad (3.26)$$

and the Yukawa couplings among these arise from

$$\int d^4x g_N \text{tr}_N \bar{\Psi} \Gamma^a [\phi_a, \cdot] \Psi \sim 2 \int d^4x g_N \varphi \bar{\psi}_{12} \psi_{12} \quad (3.27)$$

The trace tr_N gives no extra factor since the fermions are made from coherent states, similar as the bosonic modes in (3.23). Therefore the fermion mass is given by

$$m_\psi \sim g_N \varphi = c_L g \varphi \sim \sqrt{\frac{1}{2} \dim \mathcal{H}_L} g \varphi, \quad (3.28)$$

which is much larger than the W scale (3.24) for large branes with $\dim \mathcal{H} \gg 1$. This implies that the mirror fermions can be much heavier than the W scale, which is essential, and resolves one of the main issues in [12]. On the other hand, this also entails that the $\text{SU}(N)$ coupling g_N is considerably larger than the electroweak coupling g . In the present paper, we focus on minimal or small branes.

In the next section we discuss the fermionic zero modes on such a background, and show how the fermions of the standard model can arise.

4 Fermionic zero modes

Now we turn to the fermionic zero modes, which provide the matter content of the low-energy field theory on the squashed $\mathcal{C}_N[\mu]$ backgrounds. The basic results are obtained

in [10, 11], however we emphasize again their group-theoretical organization which makes the relation with the scalar zero modes manifest.

The internal Dirac operator on a background X_α describes a stack of $\mathcal{C}_N[\mu]$ branes has the form

$$\mathcal{D}_{(\text{int})}^X \Psi = \sum_{\beta \in \mathcal{I}} \Delta^\beta [\Phi_\beta, \Psi] = \sqrt{2} \sum_{i=1}^3 (\Delta_i^- [X_i^+, \cdot] + \Delta_i^+ [X_i^-, \cdot]) \quad (4.1)$$

where the spinorial ladder operators

$$2\Delta_1^+ = \Delta_4 + i\Delta_5, \quad 2\Delta_2^+ = \Delta_6 - i\Delta_7, \quad 2\Delta_3^+ = \Delta_1 - i\Delta_2, \quad (4.2)$$

and $\Delta_i^- = (\Delta_i^+)^{\dagger}$ satisfy

$$\{\Delta_i^-, \Delta_j^+\} = \delta_{ij}. \quad (4.3)$$

We recall the traceless generators τ_i of the $U(1)_i \subset SU(3)_R$, and introduce their spinorial representation

$$\begin{aligned} \tilde{\tau}_i &= \Sigma_{(i)} - \frac{1}{2} \sum_{j \neq i} \Sigma_j, \\ \Sigma_{(i)} &= \frac{1}{2} [\Delta_i^-, \Delta_i^+] = \frac{1}{2} \chi_i, \quad i = 1, 2, 3 \end{aligned} \quad (4.4)$$

with $\tilde{\tau}_1 + \tilde{\tau}_2 + \tilde{\tau}_3 = 0$. The 8 states in a Dirac spinor of $SO(6)$ transform in the $(4)_L \oplus (\bar{4})_R$ of $SU(4)_R$, and decompose into $(3)_L \oplus (\bar{3})_R \oplus (1)_L \oplus (1)_R$ under $SU(3)_R$. The 6 non-vanishing charges $\alpha = \pm \alpha_i$ of $\tilde{\tau}_i$ thus form a regular hexagon $(3) \oplus (\bar{3})$ (just like the scalar fields ϕ_α)

$$|s_1, s_2, s_3\rangle \equiv |\alpha\rangle, \quad 2\tilde{\tau}_i |\alpha\rangle = (\alpha_i, \alpha) |\alpha\rangle, \quad (4.5)$$

while two singlet “gaugino” states

$$\chi_L = |\uparrow\uparrow\uparrow\rangle, \quad \chi_R = |\downarrow\downarrow\downarrow\rangle \quad (4.6)$$

have vanishing $\tilde{\tau}_i$ charge $\alpha = 0$. The spinors with $\alpha \neq 0$ have definite chirality determined by

$$\chi |\pm \alpha_i\rangle = \chi_1 \chi_2 \chi_3 |\pm \alpha_i\rangle = \pm |\pm \alpha_i\rangle = \tilde{\tau} |\pm \alpha_i\rangle \quad (4.7)$$

where $\tilde{\tau}$ is the trace- $U(1)_R$ generator acting on the spinors corresponding to τ (2.26).

Now we can exploit the fact that the background preserves the $U(1)_i^K$ symmetries (2.22). This implies that the Dirac operator $\mathcal{D}_{(\text{int})}^X$ commutes with

$$\tilde{K}_i := 2\tilde{\tau}_i - [H_{\alpha_i}, \cdot], \quad i = 1, 2, 3 \quad (4.8)$$

in analogy to (2.22). As in section 2.2, it follows that for each irreducible $\mathcal{H}_\Lambda \subset \text{End}(\mathcal{H})$, $\mathcal{D}_{(\text{int})}^X$ has 6 zero modes labeled by α

$$\begin{aligned} \mathcal{D}_{(\text{int})}^X \Psi_{\alpha, \Lambda'}^{(0)} &= 0, \\ \Psi_{\alpha, \Lambda'}^{(0)} &= |\alpha\rangle Y_\lambda, \quad \Lambda' = \alpha - \lambda, \quad Y_\lambda \in \mathcal{H}_\Lambda \end{aligned} \quad (4.9)$$

Figure 3. Fermionic zero modes in weight space with root basis α_i , and chirality sectors C_L, C_R indicated by color. The six Weyl chambers are separated by the dashed lines.

which are in one-to-one correspondence to the extremal weights Λ' of the $\tilde{K}_i$. Here Y_Λ is some extremal weight vector in $\mathcal{H}_\Lambda \subset \text{End}(\mathcal{H})$. This follows from 1) the multiplicity of the extremal weight states is one, 2) they are eigenvectors of χ , and 3)

$$\mathcal{D}_{(\text{int})}^X \chi = -\chi \mathcal{D}_{(\text{int})}^X. \quad (4.10)$$

In particular, these $\Psi_{\alpha, \Lambda'}^{(0)}$ can be viewed as superpartners the bosonic regular zero modes $\phi_{\alpha, \Lambda'}^{(0)}$ (2.30), with the same charge α under $U(1)_i \subset \text{SU}(3)_R$. For example,

$$\Psi_{\alpha_3, \alpha_3 - \Lambda}^{(0)} = |\downarrow\downarrow\uparrow\rangle Y_\Lambda \quad (4.11)$$

where Y_Λ is the highest weight vector of $\mathcal{H}_\Lambda \subset \text{End}(\mathcal{H})$. This can easily be verified directly using the form (4.1) of the Dirac operator, together with

$$\Delta_1^- |\downarrow\downarrow\uparrow\rangle = \Delta_2^- |\downarrow\downarrow\uparrow\rangle = \Delta_3^+ |\downarrow\downarrow\uparrow\rangle = 0. \quad (4.12)$$

These states are visualized¹⁰ in figure 3. They fall into chirality classes C_L and C_R with well-defined internal chirality

$$\chi \Psi_{\pm\alpha_i, \Lambda'}^{(0)} = \tilde{\tau} \Psi_{\pm\alpha_i, \Lambda'}^{(0)} = \pm \Psi_{\pm\alpha_i, \Lambda'}^{(0)}, \quad (4.13)$$

determined by the parity ± 1 of the Weyl chamber of $\alpha = \pm\alpha_i$; recall that the reflections along the τ_i divide weight space into the 6 Weyl chambers of $\mathfrak{su}(3)_X$. Now α is in the same Weyl chamber as $\Lambda' = \alpha - \Lambda$, and in the opposite Weyl chamber as the gauge charge Λ of the matrix wave-function Y_Λ . Thus the chirality of $\Psi_{\alpha, \Lambda'}^{(0)}$ is determined by the parity of α , hence¹¹ by Λ . This is an important statement, because it signals a chiral behavior of the low-energy gauge theory. All this is consistent with the vanishing index of $\mathcal{D}_{(\text{int})}^X$.

It turns out that there are no other fermionic zero modes besides these extremal zero modes, except for the trivial gaugino modes χ_L, χ_R with $\Lambda = 0$. The remaining fermionic

¹⁰This picture differs from figure 1 in [11] due to a different choice of roots.

¹¹This is strictly true only for Λ which are not on the border of two Weyl chambers. Otherwise, there are two zero modes with opposite chirality associated to Λ .

modes (including the gaugino modes for $\Lambda \neq 0$) acquire “Kaluza-Klein” masses with scale set by m . In particular, there are no fermionic zero modes corresponding to the exceptional scalar zero modes, hence supersymmetry is manifestly broken even in the low-energy spectrum.

So far, we only discussed the internal spinor structure of the zero modes. Taking into account the 10D Majorana-Weyl condition $\Psi^C = \Psi = \Gamma\Psi$, this translates directly to the space-time spinor structure. It is easy to see (cf. [10]) that the extremal modes $\Psi_{\alpha,\Lambda'}$ and $\Psi_{-\alpha,-\Lambda'}$ are related by the internal charge conjugation and have opposite chirality,

$$C^{(6)}\Psi_{\alpha,\Lambda'}^* = \Psi_{-\alpha,-\Lambda'}. \quad (4.14)$$

Let us use the short notation $\Psi_i^\pm = \Psi_{\pm\alpha_i,\Lambda'}^{(0)}$, where $\pm\alpha_i$ are the roots of $\mathfrak{su}(3)_X$. Taking into account the Majorana-Weyl condition, the corresponding solutions of the full Dirac operator have the form

$$\Psi_i(x) = \Psi_i^+ \otimes \psi_+^i(x) + \Psi_i^- \otimes \psi_-^i(x), \quad (4.15)$$

where the four-dimensional spinors $\psi_\pm^i$ satisfy

$$\not{D}_{(4)}\psi_\pm^i(x) = 0, \quad (\psi_\pm^i(x))^C = \psi_\mp^i(x). \quad (4.16)$$

and have specific chirality

$$\gamma_5\psi_\pm^i(x) = \pm\psi_\pm^i(x). \quad (4.17)$$

This means that the $\psi_\pm^i$ are not independent, as $\psi_+^i(x)$ determines $\psi_-^i(x)$. We can expand the general solution in terms of plane wave Weyl spinors $\psi_{i;k}^\pm(x)$ on $\mathbb{R}^4$ with momentum k ,

$$\Psi_i(x) = \int \frac{d^3k}{\omega_k} \left(\psi_{i;k}^+(x)\Psi_+^i + \psi_{i;k}^-(x)\Psi_-^i \right), \quad i = 1, 2, 3. \quad (4.18)$$

This can be viewed in terms of three 4-dimensional Weyl spinors ψ_i^+ , which naturally form 3 chiral supermultiplets with the corresponding bosonic zero modes.

Together with the relation between the internal chirality and the charges Λ' established above, it follows that the fermionic zero modes cannot acquire any mass terms even at the quantum level, as long as the $U(1)_i^K$ is unbroken. There are simply no other modes available with the opposite Λ' and the same 4D chirality to form a mass term in 4 dimensions. This holds even in the presence of mass terms such as in (2.5) or their fermionic analogs.

Since the above analysis is based entirely on group theory, the classification of zero modes carries over immediately to stacks of branes. The results can then be summarized by stating that a quiver gauge theory arises on stack of squashed branes $\oplus n_i \mathcal{C}[\mu_i]$, with gauge group $U(n_i)$ on each node μ_i and arrows corresponding to chiral superfields $\Phi_{\alpha,\Lambda'}$ labeled by the extremal weights Λ' obtained by adding the six non-vanishing weights $\alpha \in \mathcal{W}(1,1)$ to the (negative) weights of $\mathcal{H}_\Lambda \subset \text{Hom}(\mathcal{H}_{\mu_i}, \mathcal{H}_{\mu_j})$. Fields with opposite weights are conjugates of each other. The trivial modes $\Lambda = 0$ on each node lead to $\mathcal{N} = 4$ supermultiplets. However, this quiver does not give the full story, as there are exceptional scalar zero modes, heavy fields, and non-supersymmetric interactions which arise from the parent theory.

We will restrict ourselves to the fermionic zero modes $\Psi_{\alpha,\Lambda'}^{(0)}$ from now on. We emphasize again that all fermionic zero modes come in 3 generations, except for the two gaugino modes $\chi_{L,R}$ which arise for $\Lambda = 0$.

4.1 Higgs fields and Yukawa couplings on minimal branes

Adding a Higgs to the background $\Phi_\alpha = m(X_\alpha + \phi_\alpha)$, the fermionic zero modes may acquire masses through Yukawa couplings arising from $\mathcal{D}_{(\text{int})}^\phi$,

$$m \text{Tr} \bar{\Psi} \gamma_5 \mathcal{D}_{(\text{int})}^\phi \Psi = m \text{Tr} \bar{\Psi} \gamma_5 \Delta^\alpha [\phi_\alpha, \Psi] . \quad (4.19)$$

These Yukawas are non-vanishing only if the $U(1)_i$ charges of the three fields under K_i add up to zero, which provides a strong constraint for these couplings. Since the gaugino modes (4.6) arise only for the trivial $\Lambda = 0$ modes, they cannot contribute any non-vanishing Yukawas in the zero mode sector. Together with the $U(3)_R$ symmetry, this implies that the non-vanishing Yukawas in the zero-mode sector have the following form

$$\text{Tr} \bar{\Psi}_{-\alpha_i}^{(0)} \gamma_5 \Delta^{\alpha_j} [\phi_{\alpha_j}^{(0)}, \Psi_{\alpha_k}^{(0)}] \sim \varepsilon_{ijk} \quad (4.20)$$

or its conjugate. In particular, *the τ -parity of $\alpha_i, \alpha_j, \alpha_k$ are equal*. However, we do not know which Higgs assume a non-vanishing VEV. This should be determined largely by the cubic flux term (2.6), while the quartic potential will stabilize the Higgs, as discussed in section 5.2. Note that the structure of the Yukawa coupling (4.20) is very similar to the cubic flux term, which also couples only modes with the same τ -parity. Since the flux term is odd, it is plausible that non-trivial solutions with non-vanishing Yukawa couplings arise, with separate $\tau = \pm 1$ sectors. The latter will correspond to the light sector and the mirror sector below. However, a detailed analysis is beyond the scope of the present paper. We will thus make some simplifying assumptions in the following, in an attempt to identify physically interesting configurations for such Higgs and Yukawa couplings.

Our first assumption is that there are no Higgs modes *on* any given $\mathcal{C}[\mu]$ (linking a brane with itself). We restrict ourselves to Higgs fields ϕ_α arising as links between branes $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$ in (3.7). This suffices to exhibit the separation into light and mirror fermions.

Minimal branes and Higgs. We restrict ourselves to the minimal squashed CP^2 branes in this paper, with $\mu_L = (1, 0)$ and $\mu_R = (0, 1)$. Then among all possible Higgs modes linking $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$ in (2.36), we focus on the regular zero modes with $\Lambda' \in \mathcal{W}(1, 2)$

$$\phi_{\alpha, \Lambda'}^{(0)} = \varphi_\alpha^{ij} |\mu_L^i\rangle \langle \mu_R^j| \in (0, 1) \subset (1, 0) \otimes (1, 0), \quad \Lambda' \in \mathcal{W}(1, 2) \quad (4.21)$$

with antisymmetric¹² $\varphi_\alpha^{ij} = -\varphi_\alpha^{ji}$, and the $\Lambda' \in \mathcal{W}(2, 1)$ modes

$$\phi_{\alpha, \Lambda'}^{(0)} = \tilde{\varphi}_\alpha^{ij} |\mu_R^j\rangle \langle \mu_L^i| \in (1, 0) \subset (0, 1) \otimes (0, 1), \quad \Lambda' \in \mathcal{W}(2, 1) \quad (4.22)$$

which are determined by conjugation. They link adjacent weights μ_L^i and μ_R^j of $(1, 0)$ and $(0, 1)$, interpreted as strings linking the sheets of $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$ [10]. We will ignore the remaining regular zero modes with $\Lambda' \in \mathcal{W}(3, 1)$ and the 3 exceptional zero modes with $\Lambda' \in \mathcal{W}(2, 0)$ and $\Lambda \in \mathcal{W}(0, 1)$ here, since they would not lead to Yukawas between the

¹²the symmetric combination is part of $(2, 0)$.

Figure 4. $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$ with Higgs ϕ .

fermionic zero modes relevant to the SM. However they may give a mass to some of the extra (unwanted) fermions which arise besides the standard-model fermions, and should be taken into account eventually in a more complete analysis.

Consider these Higgs modes (4.21) in more detail. Since $(1,2)$ is the conjugate representation to $(2,1)$, the latter are determined by the 3+3 independent $(1,2)$ modes by conjugation. Equivalently, we can consider the three $\tau = +1$ modes with $\Lambda' \in \mathcal{W}(1,2)$ and the three $\tau = +1$ modes with $\Lambda' \in \mathcal{W}(2,1)$ as independent modes, which determine the remaining modes by conjugation. Explicitly, writing the $\mathcal{C}[\mu_L] + \mathcal{C}[\mu_R]$ background¹³ as

$$X_i^+ \equiv X_{\alpha_i} = r_i (|\mu_L^{i+1}\rangle\langle\mu_L^i| - |\mu_R^i\rangle\langle\mu_R^{i+1}|), \quad i = 1, 2, 3 \quad (4.23)$$

(cyclically) where $\mu_R^i = -\mu_L^i$ and $\alpha_1 = \mu_L^2 - \mu_L^1$ etc., these six independent Higgs fields are

$$\begin{aligned} \phi_i^+ &\equiv \phi_{+\alpha_i, \Lambda'}^{(0)} = \varphi_i (|\mu_R^{i-1}\rangle\langle\mu_L^{i+1}| - |\mu_R^{i+1}\rangle\langle\mu_L^{i-1}|), & \Lambda' \in \mathcal{W}(1,2) \\ \tilde{\phi}_i^+ &\equiv \tilde{\phi}_{+\alpha_i, \Lambda'}^{(0)} = \tilde{\varphi}_i (|\mu_L^{i-1}\rangle\langle\mu_R^i| - |\mu_L^i\rangle\langle\mu_R^{i-1}|), & \Lambda' \in \mathcal{W}(2,1) \end{aligned} \quad (4.24)$$

for $i = 1, 2, 3$, which determine their conjugates $\phi_i^-, \tilde{\phi}_i^-$ (see figure 4 and 5). The superscripts $\pm$ indicate the τ -parity $\tau = \pm 1$. Hence they are parametrized by 3 + 3 (complex) fields φ_i and $\tilde{\varphi}_i$, which will be referred to as “Higgs” and “mirror Higgs”.

Now we assume that only $\tilde{\varphi}_i \neq 0$, or more generally $|\tilde{\varphi}_i| \gg |\varphi_i|$. In other words, the τ -parity of the Higgs with $\Lambda' \in \mathcal{W}(2,1)$ is positive, and the τ -parity of the Higgs with $\Lambda' \in \mathcal{W}(1,2)$ is negative. This is the crucial assumption, which will lead to a chiral low-energy theory. It is reasonable, because the flux term only couples fields with the same τ -parity; however a detailed investigation is left for future work. We will see that under this assumption, the “mirror Higgs” $\tilde{\phi}_i$ gives a large mass to the “mirror” sector of the standard model, leaving the chiral standard model with massless chiral fermions (and some extra fields) at low energies. The (small) ϕ_i modes then play the role of the low-energy Higgs, giving mass to these standard-model fermions as usual.

¹³The minus in the second term reflects the fact that the generators of conjugate representations are related by minus transposition.

Figure 5. Weight lattice of $\mathfrak{su}(3)$, with $(1, 2)$ and $(2, 1)$ irreps, and Higgs modes $\phi_i^\pm$ and mirror Higgs $\tilde{\phi}_i^\pm$ in (4.24). Their τ -parity is indicated by $\pm$.

Finally, consider the ϕ_S Higgs (3.3) connecting a minimal $\mathcal{C}[\mu_R]$ brane with a “point” brane $\mathcal{D}_l = \mathcal{C}[0]$. Similar as the Higgs connecting minimal branes, we can organize them in terms of 3+3 modes with τ -parity $\tau = +1$

$$\begin{aligned}\phi_{iS}^+ &\equiv \phi_{\alpha i S}^{(0)} = \varphi_{iS} |\mu_R^{i+1}\rangle \langle 0|, & \Lambda' &\in \mathcal{W}(1, 2) \\ \tilde{\phi}_{iS}^+ &\equiv \tilde{\phi}_{\alpha i S}^{(0)} = \tilde{\varphi}_{iS} |0\rangle \langle \mu_R^i|, & \Lambda' &\in \mathcal{W}(2, 1)\end{aligned}\quad (4.25)$$

in the basis (4.24), and their conjugates. Those may be switched on independent of each other. There are also 3 exceptional scalar zero modes with $\Lambda' \in \mathcal{W}(0, 2)$, which we will ignore here.

Fermions between branes and points. Now consider the fermionic zero modes in more detail. The zero modes linking $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$ with a point brane $\mathcal{C}[0]$ are given by

$$\begin{aligned}\Psi_{\alpha, \Lambda'}^{(0)} &= |\alpha\rangle \psi_{\mu_L^\alpha 0}, & \psi_{\mu_L^\alpha 0} &= |\mu_L^\alpha\rangle \langle 0| \in (1, 0), & \Lambda' &= \alpha - \mu_L^\alpha \in \mathcal{W}(2, 1) \\ \tilde{\Psi}_{\alpha, \Lambda'}^{(0)} &= |\alpha\rangle \psi_{\mu_R^\alpha 0}, & \psi_{\mu_R^\alpha 0} &= |\mu_R^\alpha\rangle \langle 0| \in (0, 1), & \Lambda' &= \alpha - \mu_R^\alpha \in \mathcal{W}(1, 2).\end{aligned}\quad (4.26)$$

These are 6 zero modes with $\Lambda' \in \mathcal{W}(2, 1)$ and 6 zero modes with $\Lambda' \in \mathcal{W}(1, 2)$, with chirality determined by the τ -parity of α . The Yukawa coupling $\text{Tr } \bar{\Psi}_\beta^{(0)} \gamma_5 \Delta^\alpha [\phi_\alpha^{(0)}, \Psi_\gamma^{(0)}]$ of two such fermionic zero modes with the Higgs fields $\phi_\alpha^{(0)}$ is non-vanishing only if the $U(1)_i^K$ charges Λ' of ϕ_α and Ψ_γ add up to that of Ψ_β . A direct inspection of the $\mathfrak{su}(3)$ weight lattice (see figure 5) shows that $\mathcal{W}(2, 1) + \mathcal{W}(2, 1) = \mathcal{W}(1, 2)$ has indeed solutions provided the parities of γ and α are equal and opposite to that of β . There are no other couplings among these modes, consistent with the general discussion following (4.20). Together with the above assumption on the Higgs, this means that Yukawa couplings arise only between left-handed $\Psi_{+\alpha i, \Lambda'}^{(0)}$ with $\Lambda' \in \mathcal{W}(2, 1)$ and right-handed $\tilde{\Psi}_{-\alpha j, \Lambda'}^{(0)}$ with $\Lambda' \in \mathcal{W}(1, 2)$, mediated

by $\tilde{\phi}_{+\alpha_k}$ with $\Lambda' \in \mathcal{W}(2, 1)$. Hence the fermions (4.26) separate into “mirror” fermions

$$\{\psi^{\text{mirror}}\} = \left\{ \Psi_{+\alpha_i}^{(0)}; \chi = +1 \right\} \cup \left\{ \tilde{\Psi}_{-\alpha_i}^{(0)}; \chi = -1 \right\} \quad (4.27)$$

which acquire a large mass of order $\tilde{\varphi}$, while the remaining modes remain massless and constitute the “light” sector

$$\{\psi^{\text{light}}\} = \left\{ \Psi_{-\alpha_i}^{(0)}; \chi = -1 \right\} \cup \left\{ \tilde{\Psi}_{+\alpha_i}^{(0)}; \chi = +1 \right\}. \quad (4.28)$$

To summarize, the light fermions are left-handed links from $\mathcal{C}[0]$ to $\mathcal{C}[\mu_L]$, and right-handed links from $\mathcal{C}[0]$ to $\mathcal{C}[\mu_R]$. If $\varphi_i \neq 0$, then these light fermions couple among themselves and acquire a mass of order φ .

Now the crucial point is that the zero modes (4.26) with $\Lambda' \in \mathcal{W}(2, 1)$ and $\Lambda' \in \mathcal{W}(1, 2)$ are distinguished by their gauge charges:

$$\Xi \begin{pmatrix} \Psi_{\alpha}^{(0)} \\ \tilde{\Psi}_{\alpha}^{(0)} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \Psi_{\alpha}^{(0)} \\ \tilde{\Psi}_{\alpha}^{(0)} \end{pmatrix}, \quad \Xi = \mathbb{1}_L - \mathbb{1}_R \quad (4.29)$$

where $\Xi \in \mathfrak{su}(N)$ is the gauge generator (spontaneously broken by $\tilde{\phi}$) which assigns the charges ± 1 to the branes $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$. Combining these results, we conclude that

$$\begin{aligned} \chi \psi^{\text{mirror}} &= \Xi \psi^{\text{mirror}} \\ \chi \psi^{\text{light}} &= -\Xi \psi^{\text{light}} \end{aligned} \quad (4.30)$$

hence

$$\gamma_5 \psi^{\text{light}} = -\Xi \psi^{\text{light}} \quad (4.31)$$

using (4.17). This means that the low-energy fermions ψ^{light} are chiral as seen by the spontaneously broken gauge fields, just like the fermions in the standard model (in the broken phase). The basic result (4.30) will be verified numerically in section 6, and the relation with the standard model will be made more specific below.

Finally assume that in addition ϕ^S (4.25) is switched on, connecting $\mathcal{C}[\mu_R]$ with $\mathcal{C}[0]$. This will induce Yukawa couplings of $\psi_{\mu_R 0}$ with fermions on $\mathcal{C}[\mu_R]$ and on $\mathcal{C}[0]$, and possibly Yukawa couplings of $\psi_{\mu_L 0}$ with $\psi_{\mu_L \mu_R}$. Switching on $\phi^S \neq 0$ or $\tilde{\phi}^S = 0$ selectively, this should give a mass to $\psi_{\mu_R 0}$ while leaving the light fermions $\psi_{\mu_L 0}^{\text{light}}$ massless. This is desirable since it will give mass to ν_R , however a detailed investigation is left for future work.

Fermions on and between branes. Now consider the fermionic zero modes linking different $\mathcal{C}[\mu]$ branes. They are in one-to-one correspondence with the regular scalar zero modes discussed above. In particular, the zero modes connecting two minimal branes $\mathcal{C}[\mu_L]$ with $\mathcal{C}[\mu_R]$ have the form

$$\Psi_{\alpha, \Lambda'}^{(0)} = |\alpha\rangle \psi_{\alpha}, \quad \psi_{\alpha} = \psi_{\alpha}^{ij} |\mu_L^i\rangle \langle \mu_R^j| \in (1, 0) \otimes (1, 0) = (2, 0) + (0, 1) \quad (4.32)$$

corresponding to (2.36), where $|\alpha\rangle$ stands for the spinor (4.5) with weight α . This leads to 6 zero modes with $\Lambda' \in \mathcal{W}(3, 1)$ and 6 zero modes with $\Lambda' \in \mathcal{W}(1, 2)$. The latter are the superpartners of the Higgs fields (4.22).

There are also fermionic zero modes on some minimal branes $\mathcal{C}[\mu]$,

$$\Psi_{\alpha,\Lambda'}^{(0)} = |\alpha\rangle\psi_\alpha, \quad \psi_\alpha = \psi_\alpha^{ij}|\mu^i\rangle\langle\mu^j|. \quad (4.33)$$

Six of these have $\Lambda' \in \mathcal{W}(2,2)$, and 8 are trivial modes $\psi_\alpha = \mathbb{1}$ with $\Lambda' \in \mathcal{W}(1,1)$. If $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$ are connected with a Higgs $\phi_\alpha^{(0)}$ as above, then Yukawa couplings with structure $\text{tr}\bar{\psi}_{\mu_L\mu_R}\phi_\alpha^{(0)}\psi_{\mu_R\mu_R}$ and $\text{tr}\bar{\psi}_{\mu_R\mu_L}\phi_\alpha^{(0)}\psi_{\mu_L\mu_L}$ arise, giving mass to some of these fermions. Rather than attempting a detailed analytical explanation here, we will analyze this numerically in the next section.

5 Standard model fermions from branes

Now we apply these results to the brane configuration for the standard model (3.7). Consider the off-diagonal fermions linking the $2 \times \mathcal{C}[\mu_L] + \mathcal{C}[\mu_{Rd}] + \mathcal{C}[\mu_{Ru}] + \mathcal{D}_l + 3 \times \mathcal{D}_b$ branes. In the basis (Lu, Ld, Rd, Ru, l, b_i) , we denote these fermions as

$$\Psi = \begin{pmatrix} *2 & \tilde{H}_d & \tilde{H}_u & l_L & Q_L \\ & * & e' & e_R & d_R \\ & & * & \nu_R & u_R \\ & & & * & u' \\ & & & & *3 \end{pmatrix}. \quad (5.1)$$

The fermions of the SM arise as links between the point branes $\mathcal{D}_l$ and $\mathcal{D}_b$ and $\mathcal{D}_u = \mathcal{C}[\mu_L]_u \cup \mathcal{C}[\mu_{Ru}]$ resp. $\mathcal{D}_d = \mathcal{C}[\mu_L]_d \cup \mathcal{C}[\mu_{Rd}]$, i.e.

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad (5.2)$$

as well as the right-handed leptons and quarks. Furthermore there are slots for the Higgsinos $\tilde{H}_u, \tilde{H}_d$ as in the MSSM. The charge generators

$$\begin{aligned} Q &= \frac{1}{2} \text{diag} \left(1, -1, -1, 1, 1, -\frac{1}{3} \right), \\ Y &= \text{diag} \left(0, 0, -1, 1, 1, -\frac{1}{3} \right) \end{aligned} \quad (5.3)$$

assign the following quantum numbers (Q, Y) to these off-diagonal modes

$$(Q, Y)|_\Psi = \begin{pmatrix} * & \begin{pmatrix} (1,1) \\ (0,1) \end{pmatrix} & \begin{pmatrix} (0,-1) \\ (-1,-1) \end{pmatrix} & \begin{pmatrix} (0,-1) \\ (-1,-1) \end{pmatrix} & \begin{pmatrix} (\frac{2}{3}, \frac{1}{3}) \\ (-\frac{1}{3}, \frac{1}{3}) \end{pmatrix} \\ & * & (-1,-2) & (-1,-2) & (-\frac{1}{3}, -\frac{2}{3}) \\ & & * & (0,0) & (\frac{2}{3}, \frac{4}{3}) \\ & & & * & (\frac{2}{3}, \frac{4}{3}) \\ & & & & * \end{pmatrix} \quad (5.4)$$

(the SU(3) assignment is obvious, hence dropped). All quantum numbers of the standard model are correctly reproduced (cf. [26, 41]), and 3 families arise automatically due to the

$\mathbb{Z}_3$ symmetry. The Yukawa couplings may of course break the $\mathbb{Z}_3$, and will be discussed below. Thus the leptons arise as fermions linking $\mathcal{D}_u$ or $\mathcal{D}_d$ with $\mathcal{D}_l$, and the quarks arise as fermions linking $\mathcal{D}_u$ or $\mathcal{D}_d$ with $\mathcal{D}_l$.

All these modes have scalar superpartners given by the regular scalar zero modes. In particular, the two Higgs doublets¹⁴

$$H_d = \begin{pmatrix} 0 \\ \varphi_d \end{pmatrix}, \quad H_u = \begin{pmatrix} \varphi_u \\ 0 \end{pmatrix} \quad (5.5)$$

with $Y(H_d) = 1$ (as in the standard model) and $Y(H_u) = -1$ (as in the MSSM) fit into the above matrix structure as

$$\phi^a = \begin{pmatrix} 0_2 & H_d & H_u & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & & 0 & \varphi_S & 0 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \phi_u & 0 & 0 \\ 0 & 0 & \varphi_d & 0 & 0 & 0 \\ 0 & \varphi_d^\dagger & 0 & 0 & 0 & 0 \\ \varphi_u^\dagger & 0 & 0 & 0 & \varphi_S & 0 \\ 0 & 0 & 0 & \varphi_S^\dagger & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (5.6)$$

This indeed leads to the desired pattern of electroweak symmetry breaking, as shown in section 3. We also exhibit the “sterile” Higgs φ_S , which is a singlet under the standard model gauge group, occupying the same slot as ν_R . The chiralities and masses of the fermions depend on the Higgs expectation values. We will see in the next section that for $\tilde{\varphi} \gg \varphi$, the low-energy fermions linking point branes with $\mathcal{C}[\mu_L]$ are left-handed, and those linking with $\mathcal{C}[\mu_R]$ are right-handed. The fermions with the opposite chiralities — which necessarily exist due to the vanishing index in $\mathcal{N} = 4$ SYM — acquire a large mass terms of order $\tilde{\varphi}$, and are therefore invisible at low energies. Thus the fermions of the standard model have indeed the appropriate chirality at low energies, as suggested by their names l_L, e_R etc. Finally, recall that the modes in the lower-diagonal part of the matrices are identified by the MW condition with the upper-diagonal ones, and therefore do not constitute independent degrees of freedom.

It is remarkable that no exotic charges arise: all the charges in (5.4) correspond to the charges of the standard model, extended by the second Higgs doublet and the sterile ν_R . Thus we recover all fermions in the MSSM (including e.g. gluinos, winos and binos), extended by

$$u' \sim |0\rangle_l \langle 0|_b \quad (5.7)$$

which has the same quantum numbers as the u_R quarks (but it comes with both chiralities), and

$$e' \sim |\mu_{Rd}\rangle \langle \mu_{Ru}| \quad (5.8)$$

which has the same quantum numbers as e_R . This degeneracy can be understood by viewing $\mathcal{D}_{\mu_{Ru}} \cup \mathcal{D}_l$ as a single brane linked via ϕ_S . Thus u' may mix with u_R , and e'

¹⁴Unfortunately there is a conflict with the standard particle physics conventions, where the role of the $H_{u,d}$ is reversed, as is seen from their quantum numbers (5.4). The present notation is forced upon us by (3.7).

with e_R , and similarly ν_L may mix with neutral Higgsino $\tilde{\varphi}_u$ at low energies. The e' can be viewed as superpartner of the would-be $SU(2)_R$ gauge bosons connecting $\mathcal{C}[\mu_{Ru}]$ and $\mathcal{C}[\mu_{Rd}]$ if $\mu_{Ru} = \mu_{Rd}$. Finally there are a number of fermions which are neutral under the SM gauge groups. This includes the superpartner of the (broken) $U(1)_L$ gauge field

$$\lambda := |0\rangle_l \langle 0|_l, \quad (5.9)$$

some diagonal “neutralino” modes on $\mathcal{D}_u$ and $\mathcal{D}_d$, and of course ν_R . The multiplets come in several incarnations corresponding to different Λ' modes, which may acquire a mass from the Higgs(es). This is discussed next.

One might absorb the extra fields e' and u' by replacing $\mathcal{C}[\mu_{Ru}] \cup_{\phi_S} \mathcal{D}_l$ by a single brane $\mathcal{C}[\tilde{\mu}_{Ru}]$, as discussed in section 3. However, then there are typically extra modes connecting the extended branes. The main reason for keeping $\mathcal{D}_l$ separate is to keep things transparent by working with minimal branes $\mathcal{C}[\mu_i]$ and point branes $\mathcal{D}_{l,b}$.

5.1 Chiral fermions and Yukawas on the standard model branes

Now we apply our results on the Yukawa couplings to this brane configuration, with $\mu_L = (1, 0)$ and $\mu_R = (0, 1)$. In particular, we assume that $\mathcal{D}_u = \mathcal{C}[\mu_L]_u \cup \mathcal{C}[\mu_{Ru}]$ are linked by Higgs $\phi_\alpha, \tilde{\phi}_\alpha$ as above, and similarly for $\mathcal{D}_d = \mathcal{C}[\mu_L]_d \cup \mathcal{C}[\mu_{Rd}]$.

Consider first the fermions linking the point branes $\mathcal{D}_l, \mathcal{D}_b$ with $\mathcal{D}_u$ or $\mathcal{D}_d$. Assuming that $\tilde{\varphi} \gg \varphi$, the results of the previous section imply that these separate into light fermions with masses of order φ , and heavy mirror fermions with masses of order $\tilde{\varphi}$. This leaves only the light fermions at low energy, which comprise left-handed fermions linking $\mathcal{C}[0]$ to $\mathcal{C}[\mu_L]$, and right-handed fermions linking $\mathcal{C}[0]$ to $\mathcal{C}[\mu_R]$. They correspond to the standard-model-like chiral leptons and quarks. The mirror fermions have the same S.M. quantum numbers but the opposite chiralities, distinguished by the $U(1)_{K_i}$ quantum numbers. Due to the simple mode decomposition¹⁵ (4.26), we get precisely the same quark and lepton with their superpartners as in the MSSM, plus their mirror modes at higher energies (which also form supermultiplets).

Now consider the low-energy fermions which arise on the $\mathcal{D}_d$ and $\mathcal{D}_u$ branes (i.e. in the upper-left 4×4 block in (5.1)). This includes the superpartners of the electroweak sector, such as Higgsinos, Winos, Binos, charginos and neutralinos, as well as the e' . They come in different multiplets corresponding to the different Λ' modes in (4.32). Their precise Yukawa couplings and masses in this sector are rather complicated and will not be discussed in detail here; some illustrative numerical results are given in the next section. Since the $\Lambda = 0$ modes come as $\mathcal{N} = 4$ multiplets, there are also 3 generations of chiral supermultiplets corresponding to the W and Z bosons. The numerical results indicate that some but not all of these acquire a mass from the mirror Higgs $\tilde{\varphi}$, which suggests that some of the other Higgs discussed in section 4.1 should also acquire a VEV. We leave this for further investigations.

Finally some fermionic would-be zero modes arise within the 4 point branes $\mathcal{D}_l + 3\mathcal{D}_b$. This includes gluinos with $Y = Q = 0$, the color triplet u' (5.7) which is similar to u_R ,

¹⁵This holds also for genral non-minimal branes, as long as $\mathcal{D}_l, \mathcal{D}_b$ are point branes.

and the singlets λ on $\mathcal{D}_l$ (5.9). We only discuss some aspects here, postponing a detailed analysis to future work. First, the Higgs ϕ_S with $\Lambda' \in \mathcal{W}(2,1)$ should lead to a Yukawa coupling of the ν_R with the λ modes with $\Lambda' \in \mathcal{W}(1,1)$, and give a large mass to both ν_R and λ (except for the two gaugino polarizations (4.6) of λ). Similarly, the u' might couple to u_R via ϕ_S , (except for the two gaugino polarizations of u'), giving a mass to u' and u_R . It is tempting to speculate that the large Yukawa couplings of the top quark may be related to the presence of ϕ_S . The fate of the two gaugino polarizations of u' and λ is unclear. In any case, the sector containing $\mathcal{D}_l, \mathcal{D}_b$ and $\mathcal{C}[\mu_{uR}]$ is rather complex and should be studied elsewhere.

Due to the different parity modes of ϕ_S and $\tilde{\phi}_S$, it is possible that e.g. ν_R acquires a large mass but not its mirror $\tilde{\nu}_R$. Then the seesaw mechanism would apply to the physical neutrinos but not to the mirror neutrinos, and no new massless neutrinos would be introduced.

The main result here is the separation of leptons and quarks into light chiral and heavy mirror sectors, assuming a suitable Higgs configuration. The crucial decoupling of the light and mirror sector is guaranteed by the global $U(1)_i^K$ symmetry, and persists in the presence of explicit mass terms respecting that symmetry, such as in the $\mathcal{N} = 1^*$ model discussed in appendix A. This mechanism will be verified numerically below, along with some illustrative sample computations for the remaining sectors.

Extra $U(1)$'s and anomalies. In the presence of chiral fermions, the $U(1)_i$ gauge fields arising on backgrounds consisting of several $\mathcal{C}[\mu_i]$ branes deserve special attention. Consider first the field theory setting at hand. Assuming that the mirror Higgs $\tilde{\varphi}$ is much larger than the light Higgs φ as above, some of these $U(1)$'s acquire anomalous contributions from the low-energy sector (i.e. after integrating out the massive mirror fermions). In the present brane configuration this is the case for the $U(1)_B$ and the $U(1)_5$ fields. However, this anomaly from the light sector is canceled precisely by the anomaly from the mirror sector, so that there is no overall anomaly, in accordance with t'Hooft anomaly matching and the fact that $\mathcal{N} = 4$ SYM has no gauge anomalies. In particular, the $U(1)_B$ gauge field remains massless in this model.

To get a better perspective on these $U(1)$'s, it is useful to recall the analogous situation in string theory. The present softly broken $\mathcal{N} = 4$ SYM model would arise “locally” e.g. on stacks of N $D3$ -branes in a suitable flux background, cf. [15–18, 31, 32]. Then some of these $U(1)$'s are anomalous at low energy and acquire a mass through a Stückelberg mechanism [33–38], absorbing the corresponding Stückelberg field or axion which arises from the RR fields in string theory. This is a manifestation of the Green-Schwarz anomaly cancellation. In the present $\mathcal{N} = 4$ field theory setting, there is no manifest axion, hence the correspondence with the string theory case is not fully realized. However for analogous backgrounds in noncommutative $\mathcal{N} = 4$ in the IKKT matrix model, axion-like fields do arise [39, 40], as expected from the relation with string theory. One may then hope that an analogous Stückelberg mechanism applies and renders some of these $U(1)$'s massive, however the details remain to be understood.

5.2 Aspects of the Higgs potential

Now consider the interacting potential for the Higgs i.e. the scalar zero modes $\phi^{(0)}$ on a background solution X . The linear term in ϕ vanishes, so that the effective potential for ϕ obtained from (2.8) is

$$V(\phi) = \text{tr} \left(\frac{1}{2} \phi^\alpha (\square_X + 2\mathcal{D}_{\text{diag}}) \phi_\alpha + \left(X^\alpha + \frac{1}{4} \phi^\alpha \right) \square_\phi \phi_\alpha - \frac{1}{2} f^2 \right) + V_{\text{soft}}(\phi). \quad (5.10)$$

The cubic interaction arising from the quartic term can be written in different ways

$$\begin{aligned} \text{tr} X^\alpha \square_\phi \phi_\alpha &= \text{tr} [X_\alpha, \phi_\beta] [\phi^\alpha, \phi^\beta] \\ &= -\text{tr} \phi_\beta \left[[\phi^\alpha, \phi^\beta], X_\alpha \right] = \text{tr} \phi_\beta \left(\left[[\phi^\beta, X_\alpha], \phi^\alpha \right] + \left[[X_\alpha, \phi^\alpha], \phi^\beta \right] \right) \\ &= -\text{tr} \phi_\beta [[\phi_{-\beta}, X_\alpha], \phi^\alpha] \end{aligned} \quad (5.11)$$

using the Jacobi identity, $\phi^\beta = \phi_{-\beta}$, and the gauge condition $f = [X_\alpha, \phi^\alpha] = 0$. The latter is a special case of the following identities

$$[X_\alpha, \phi_\beta^{(0)}] = 0 \quad \text{if } \alpha + \beta \in \mathcal{I} \quad \text{or } \alpha + \beta = 0 \quad (5.12)$$

and

$$[X_\alpha, \phi_{-\beta}^{(0)}] = 0 \quad \text{if } \alpha - \beta \in \mathcal{I} \quad \text{or } \alpha - \beta = 0 \quad (5.13)$$

for the regular zero modes. These follow easily from their extremal weight property, see [11]. Since one of these two conditions is always satisfied for any pair of roots α, β of $\mathfrak{su}(3)$, this cubic term vanishes for the regular zero-modes, so that their interaction potential is

$$V(\phi) = \text{tr} \left(\frac{1}{2} \phi^\alpha (\square_X + 2\mathcal{D}_{\text{diag}}^X) \phi_\alpha + \frac{1}{4} [\phi_\alpha, \phi_\beta] [\phi^\alpha, \phi^\beta] \right) + V_{\text{soft}}(\phi). \quad (5.14)$$

where the quadratic term vanishes in the absence of mass terms. The argument applies also to Higgs modes connecting stacks of branes, as long as the X_α are proportional to $\mathfrak{su}(3)$ generators. Note that the Higgs potential has similar structure as our starting point (2.5). Although a full analysis of this potential is beyond the scope of this paper, it is plausible that the cubic flux term $V_3(\phi)$ again induces a non-trivial VEV to some of the Higgs modes, which are stabilized by the quartic term. A deformation of the branes by quantum corrections or mass terms¹⁶ might also play an important role here.

The above argument for (5.11) to vanish does not apply to the exceptional zero modes. Among those, the $\text{SU}(3)_R$ Goldstone bosons are exactly flat directions, but the $\Lambda \in \mathcal{W}(1,0)$, $\Lambda' \in \mathcal{W}(2,0)$ (or conjugate) modes connecting $\mathcal{C}[(0,1)]$ with $\mathcal{C}[(1,0)]$ might lead to non-trivial cubic terms. Again, this needs to be studied in more detail elsewhere.

Finally, we emphasize that even though the Higgs sector consists of many distinct fields $\tilde{\phi}_\alpha, \phi_\alpha$ etc., there should nevertheless be one lowest Higgs *fluctuation* mode around the common minimum, which is likely a combination of all the $\tilde{\phi}_\alpha$ and ϕ_α modes. Thus the assumption $\tilde{\varphi} \gg \varphi$ is not in obvious conflict with observation. At higher energies of course, several distinct Higgs modes will necessarily show up.

¹⁶Another conceivable mechanism is a rotation of the branes, see [11].

6 Numerical results and checks

Since the detailed structure of the various zero modes and their Yukawa couplings is quite complicated, a background consisting of two minimal branes with Higgs and an extra point brane was implemented in Mathematica. We consider two branes $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$ linked by a Higgs ϕ_α and $\tilde{\phi}_\alpha$ as in (4.24), and add a point brane $\mathcal{C}[0]$ to this configuration. We are interested in the Yukawa couplings and the masses of the fermions in this background, which is determined by the low-energy spectrum and the eigenmodes of the Dirac operator $\mathcal{D}_{(\text{int})}$ acting on spinors

$$\begin{aligned}\Psi &\in \mathbb{C}^8 \otimes \text{End}((1,0) + (0,1) + (0,0)) \\ &= \mathbb{C}^8 \otimes \text{End}((1,0) + (0,1)) + \mathbb{C}^8 \otimes (2(1,0) + 2(0,1)) + \mathbb{C}^8 \otimes (0,0).\end{aligned}\quad (6.1)$$

Here we note that $(0,0) \cong \mathbb{C}$ and

$$\text{Hom}((1,0) + (0,1), \mathbb{C}) \cong \text{Hom}(\mathbb{C}, (1,0) + (0,1)) \cong (1,0) + (0,1). \quad (6.2)$$

Due to the MW condition, these two contributions are identified, so that the last term in (6.1) reduces to $(1,0) + (0,1)$; a similar reduction should be applied to all modes. The lowest eigenvalues and multiplets of the Dirac operator $\mathcal{D}_{(\text{int})}$ on the background $X_\alpha + \phi_\alpha + \tilde{\phi}_\alpha$ were obtained as a function of the parameters $\varphi_i, \tilde{\varphi}_i$, for $r \gg \tilde{\varphi} \gg \varphi$. Their $U(1)_i^K$ eigenvalues Λ' have also been determined. The detailed results are as follows:

6.1 Fermions linking $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$ to a point brane

The most interesting sector are the fermionic links (6.2) of a point brane to $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$, which we discuss first. They are determined by the Dirac operator $\mathcal{D}_{(\text{int})}$ acting on spinors

$$\Psi \in \mathbb{C}^8 \otimes ((1,0) \oplus (0,1)) \quad (6.3)$$

as in (4.26). In the absence of any Higgs $\varphi_i = 0 = \tilde{\varphi}_i$, there are 6+6 exact zero modes as expected on $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$, and the non-zero eigenvalues of $\mathcal{D}_{(\text{int})}$ are of order r . Switching on $\tilde{\varphi}_i \equiv \tilde{\varphi}$ but leaving $\varphi_i = 0$, six of the would-be zero modes (“mirror fermions”) acquire non-vanishing eigenvalues¹⁷ $\tilde{\varphi}(4, -4, 2, 2, -2, -2)$, while 3+3 exact zero modes remain. The latter are the “light fermions” which constitute the fermions in the standard model, and one can verify that (4.30) holds,¹⁸ i.e. their chirality χ is measured by $\Xi = \mathbb{1}_3 - \mathbb{1}_{\bar{3}}$. These are indeed modes with $\Lambda' \in \mathcal{W}(1,2)$. Switching on also $\varphi_i \equiv \varphi \ll \tilde{\varphi}$, these light fermions acquire eigenvalues approximately given by $\varphi(4, -4, 2, 2, -2, -2)$. This precisely confirms the analysis in the previous sections, which means that the low-energy leptons and quarks on the SM brane configurations have indeed the appropriate chiral structure.

¹⁷these specific eigenvalues are not hard to understand.

¹⁸This holds to an excellent approximation as long as the background is undeformed, i.e. $r \gg \tilde{\varphi}$.

6.2 Fermions within $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$

Now consider the fermions on $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$, which live in

$$\begin{aligned}\Psi &\in \mathbb{C}^8 \otimes \text{End}((1,0) \oplus (0,1)) \\ &= \mathbb{C}^8 \otimes (2 \times (1,1) + (2,0) + (0,2) + (0,1) + (1,0) + 2 \times (0,0)) .\end{aligned}\quad (6.4)$$

In the absence of any Higgs $\varphi_i = 0 = \tilde{\varphi}_i$, we find indeed $52 = 6 * 6 + 2 * 8$ exact zero modes, which are the superpartners of the regular scalar zero modes in this sector. The $(1,1) + (0,0)$ preserve the branes, while the $(2,0) + (0,1)$ modes connect the two branes. The remaining non-zero eigenvalues of $\mathcal{D}_{(\text{int})}$ are of order r .

Switching on $\tilde{\varphi}_i \equiv \tilde{\varphi}$ but $\varphi_i = 0$ leaves $20 = 8 + 6 + 6$ exact zero modes, and 4 low-mass modes of order $\mathcal{O}(\tilde{\varphi}^2/r)$. Clearly 8 zero modes arise from the trivial matrix wavefunction $\psi \sim \mathbb{1}_{\mathcal{H}}$, which decompose into 6 zero modes with $\Lambda' \in \mathcal{W}(1,1)$, and two (gaugino) modes with $\Lambda' = 0$. Six further zero modes have $\Lambda' \in \mathcal{W}(3,1)$ or $\mathcal{W}(1,3)$ with $\Xi = \pm 2$, corresponding to “mirror” Higgsinos connecting the branes. The remaining 6 zero modes are a mixture of $\Lambda' \in \mathcal{W}(2,2)$ and $\Lambda' \in \mathcal{W}(1,1)$ modes on the branes, which are brane-preserving $\Xi = 0$.

Besides these 20 zero modes, the 4 lowest-mass modes have $\Xi = \pm 2$ and $\Lambda' \in \mathcal{W}(1,2)$ or $\Lambda' \in \mathcal{W}(2,1)$. Hence these are Higgsino modes connecting the branes.

Switching on also $\varphi_i = \varphi \ll \tilde{\varphi}$, only the 8 trivial zero modes $\sim id_{\mathcal{H}}$ modes remain, followed by a series of low-mass modes starting with 6 modes of order $\mathcal{O}(\varphi^2/r)$.

We note that (6.4) also describes the fermions connecting up and down branes, since the representations are the same. This therefore covers the entire upper-left 4×4 block in (5.1).

6.3 $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)] + \mathcal{C}[0]$ with ϕ_S

Now we take the full configuration $\mathcal{C}[\mu_L] + \mathcal{C}[\mu_R] + \mathcal{C}[0]$ with Higgs $\phi_\alpha, \tilde{\phi}_\alpha$ as above, organized as of 3+3 modes with τ -parity $\tau = +1$ as in (4.25)

$$\begin{aligned}\phi_{iS}^+ &\equiv \phi_{\alpha iS}^{(0)} = \varphi_{iS} |\mu_R^{i+1}\rangle \langle 0|, & \Lambda' \in \mathcal{W}(1,2) \\ \tilde{\phi}_{iS}^+ &\equiv \tilde{\phi}_{\alpha iS}^{(0)} = \tilde{\varphi}_{iS} |0\rangle \langle \mu_R^i|, & \Lambda' \in \mathcal{W}(2,1)\end{aligned}\quad (6.5)$$

in the basis (4.24). Just like the Higgs $\phi_\alpha, \tilde{\phi}_\alpha$, not all of them need to be switched on.

Consider first the case $\varphi_\alpha = 0, \tilde{\varphi}_\alpha \neq 0$. If both $\varphi_{iS} = 0 = \tilde{\varphi}_{iS}$, we have the situation discussed above, i.e. 20 zero modes on the $\mathcal{C}[(1,0)] + \mathcal{C}[(0,1)]$ branes, 2×6 massless fermions¹⁹ between $\mathcal{C}[0]$ and the others, and 8 trivial zero modes on $\mathcal{C}[0]$.

Switching on $\tilde{\varphi} \approx \varphi_{iS} \neq 0$ but keeping $\tilde{\varphi}_{iS} = 0$ gives 16 exact zero modes, while the lowest non-vanishing multiplet consists of 4 states with eigenvalue of order $\mathcal{O}(\frac{\varphi_S \tilde{\varphi}}{r})$. 8 of these zero modes are easily identified as trivial $\psi \sim \mathbb{1}_{\mathcal{H}}$ modes. The remaining 8 zero modes consist of six $\Lambda' \in \mathcal{W}(3,1)$ or $\mathcal{W}(1,3)$ with $\Xi = \pm 2$ corresponding to extra Higgsinos connecting the branes, and two $\Lambda' = 0$ modes which preserve the branes. Clearly $\varphi_S \neq 0$ gives mass to the 6 modes of $\Lambda' \in \mathcal{W}(2,2)$ and $\Lambda' \in \mathcal{W}(1,1)$ modes on the branes found

¹⁹The factor 2 comes from the doubling in (6.2), which is eliminated by the MW constraint.

in section 6.2. The 4 lowest non-zero modes are essentially $\Lambda' \in \mathcal{W}(2, 1)$ or $\Lambda' \in \mathcal{W}(1, 2)$ modes connecting $\mathcal{C}[\mu_L]$ and $\mathcal{C}[\mu_R]$ to $\mathcal{C}[0]$.

Exchanging the roles of $\tilde{\varphi}_{iS}$ and φ_{iS} gives a rather different picture. Switching on $\tilde{\varphi}_{iS} \neq 0$ but keeping $\varphi_{iS} = 0$ leaves only 8 exact zero modes, and a number of very low but nonzero modes. The 8 zero modes are again the trivial $\psi \sim \mathbf{1}_{\mathcal{H}}$ modes. The remaining 4 lowest non-trivial modes are found to be 4 brane-preserving $\Lambda' \in \mathcal{W}(1, 1)$ modes. Among the non-zero modes, there is clearly a seesaw-like mechanism at work, since the eigenvalues are much smaller than any of the φ, φ_S scales. For example setting $r = 10$ and $\tilde{\varphi} = \varphi_S = 1$ gives 10^{-4} as lowest non-trivial eigenvalue.

Finally switching on also $\varphi_i = \varphi \ll \tilde{\varphi}$ leaves only 8 exact zero modes $\sim id_{\mathcal{H}}$, and a number of low eigenvalues, again with a seesaw-like mechanism lowering some of the eigenvalues. For example setting $r = 10$ and $\tilde{\varphi} = \varphi_S = \tilde{\varphi}_S = 1$ gives 10^{-2} as lowest non-trivial eigenvalue. Again, half of these modes will be eliminated by the MW constraint.

It is interesting to observe that $\mathcal{C}[(0, 1)] + \mathcal{C}[(1, 0)] + \mathcal{C}[0]$ with both Higgs switched on corresponds to the decomposition of the $(7) = (3) + (\bar{3}) + (1)$ of G_2 under $\mathfrak{su}(3)_X$. There is in fact such a solution of our model, albeit an unstable one. The precise Higgs structure and its minima is clearly complicated and will be studied elsewhere.

6.4 Generic squashed $\mathcal{C}[\mu]$ branes

Finally, we briefly discuss the case of generic branes with non-minimal μ . If the Higgs modes are again realized as links between the extremal weight states of the $\mathcal{H}_{\mu_L}$ and $\mathcal{H}_{\mu_R}$, the story goes through with minor modifications. One important difference is that the masses of the (mirror) fermions will now be larger than the electroweak scale, due to the enhancement factor $\sqrt{\dim \mathcal{H}}$ in (3.28). This should help to make the present scenario more realistic. The quark and lepton sector which arises from $\text{Hom}(\mathbb{C}, \mathcal{H}_{\mu_{L,R}})$ is qualitatively the same as in the minimal case, since any $\mathcal{H}_{\mu_{L,R}}$ leads to precisely 3+3 chiral fermionic zero modes. Hence much of the discussion of this paper is in fact quite generic. Although the mode decomposition $\text{End}(\mathcal{H}_{\mu_L}, \mathcal{H}_{\mu_R})$ will be more complicated leading to more Higgs-like multiplets, the decomposition into chiral and mirror sectors should work as in the minimal case.

7 Summary and discussion

We have (re-)derived the fermionic and bosonic zero modes which arise on stacks of squashed $\mathcal{C}[\mu]$ brane solutions in $\mathcal{N} = 4$ SYM [10], deformed by a cubic SUSY-breaking potential corresponding to a holomorphic 3-form. These modes are organized in terms of two unbroken global gauged $U(1)_{K_i}$ symmetries, which provides a useful tool to understand their interactions. We use this to start exploring possible symmetry breaking patterns which arise from giving VEV's to these massless scalar fields (dubbed “Higgs” modes), and to study the resulting low-energy physics. One important result is that there are possible Higgs configurations which lead to a chiral low-energy theory, in the sense that different chiralities of the fermionic (would-be) zero modes couple differently to the spontaneously broken massive gauge fields.

To explore the possible implications, we discuss a brane configuration which leads to an extension of the standard model, correctly reproducing the leptons and quarks with the appropriate coupling to the low-energy gauge bosons, *assuming* an appropriate Higgs configuration. This can be viewed as an extension of the MSSM, where each chiral supermultiplet has an extra mirror copy with the opposite chirality, and acquires a higher (by assumption) mass from the mirror Higgs. This is reminiscent of mirror models [42, 43], with the particular feature that the Higgs multiplets also have mirror partners, which couple only to the mirror fermions. Thus the light and the mirror sectors communicate only via the common gauge fields, and through the lowest Higgs excitation modes which are expected to be a combination of the different multiplets. The mirror copies carry different quantum numbers under the $U(1)_{K_i}$ and the opposite τ -parity, and are thereby protected from recombining. Some fields come in different varieties, and might acquire masses from different Higgs modes. However due to the complicated Higgs sector, no attempt is made in this paper to find the minima and to justify the assumed Higgs configuration.

Even if it may seem unlikely that such a scenario could be realistic, it is certainly worthwhile to explore the possible scope of these deformed $\mathcal{N} = 4$ models, given their special status in field theory. The most obvious issue seems to be the requirement that the mirror Higgs $\tilde{\phi}_i$ should give a large mass to the mirror fermions, while it also couples to the W and Z bosons and thereby gives the dominant contribution to their masses. This means that $\langle \tilde{\phi} \rangle$ must be at the electroweak scale. On the other hand, the Yukawa couplings may be large for large branes (cf. (3.28)), so that the mirror fermions may indeed be much heavier than the electroweak scale. In any case, a more detailed knowledge of the Higgs sector and its lowest fluctuations is required before further conclusion can be drawn.

It is important to stress that although the low-energy spectrum of the squashed brane solutions is “mostly” supersymmetric, there are exceptional scalar zero modes which do not have any fermionic counterpart. Therefore SUSY is manifestly broken. One set of such exceptional zero modes are the $SU(3)_R$ Goldstone bosons. Two of them are equivalent to gauge transformations and hence unphysical, and the remaining would disappear in the presence of mass terms; these could also break the $\mathbb{Z}_3$ family symmetry. Since SUSY is broken, the low-energy action is extracted from the full underlying deformed $\mathcal{N} = 4$ theory.

There are many issues which should be addressed in further work. The most important problem is to elucidate the Higgs sector for stacks of branes, in particular to see if the configurations assumed in this paper can be justified dynamically. This could be addressed within the weak coupling regime. Another natural step is the generalization to non-minimal branes, which should allow to lift the mirror sector sufficiently high above the electroweak scale. Orbifold versions of the model might eliminate the mirror sector altogether (cf. [44, 45]). In the context of string theory, possible “global” realizations of analogous brane configurations in a suitable flux background should be sought, and the fate of the various $U(1)$ ’s should be clarified. Furthermore, a dual description in terms of supergravity might help to shed light on the strong coupling regime. Finally, the considerations in this paper can be carried over immediately to the IKKT matrix model [29] and suitable deformations, which reduces to $\mathcal{N} = 4$ SYM on $\mathbb{R}_\theta^4$ [46]. Then axion-like fields arise [39, 40], which might also give mass to the extra $U(1)_B$ gauge field. In any case, it is clear that the present

type of brane configurations in deformed $\mathcal{N} = 4$ SYM provides a remarkably rich basis for further investigations.

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A Relation with $\mathcal{N} = 1^*$

It is interesting to note that the present model can be viewed as a (mass deformation of a) supersymmetric $\mathcal{N} = 1^*$ deformation of $\mathcal{N} = 4$ SYM, with the superpotential [3, 20, 21]

$$W = \frac{\sqrt{2}}{g_N} \text{tr} \left([\Phi_1^+, \Phi_2^+] \Phi_3^- - m \Phi_3^- \Phi_3^- \right) \quad (\text{A.1})$$

choosing $\Phi_1^+, \Phi_2^+, \Phi_3^-$ as the three chiral superfields. This gives the following F-term contribution to the scalar potential

$$\begin{aligned} \mathcal{V}_F &= \frac{\partial W}{\partial \Phi_1^+} \left(\frac{\partial W}{\partial \Phi_1^+} \right)^* + \frac{\partial W}{\partial \Phi_2^+} \left(\frac{\partial W}{\partial \Phi_2^+} \right)^* + \frac{\partial W}{\partial \Phi_3^-} \left(\frac{\partial W}{\partial \Phi_3^-} \right)^* \\ &= \frac{2}{g_N^2} \text{tr} \left([\Phi_2^+, \Phi_3^-] [\Phi_3^+, \Phi_2^-] + [\Phi_1^+, \Phi_3^-] [\Phi_3^+, \Phi_1^-] \right. \\ &\quad \left. + ([\Phi_1^+, \Phi_2^+] - 2m\Phi_3^-) ([\Phi_2^-, \Phi_1^-] - 2m\Phi_3^+) \right). \end{aligned}$$

Writing $\Phi = mX$ and adding the D term, the full potential takes the form

$$\begin{aligned} \mathcal{V} &= \frac{m^4}{g_N^2} V(X), \\ V(X) &= -\text{tr} \left([X_i^+, X_j^+] [X_i^-, X_j^-] - \frac{1}{2} [X_i^+, X_i^-] [X_j^+, X_j^-] \right) \\ &\quad + 4\text{tr} \left(-[X_1^+, X_2^+] X_3^+ - [X_2^-, X_1^-] X_3^- + 2X_3^+ X_3^- \right) \\ &= -\frac{1}{4} \sum_{\alpha \neq \beta} \text{tr} [X^\alpha, X^\beta] [X_\alpha, X_\beta] + 4\text{tr} \left(-[X_1^+, X_2^+] X_3^+ - [X_2^-, X_1^-] X_3^- + 2X_3^+ X_3^- \right). \end{aligned} \quad (\text{A.2})$$

This is precisely the potential in (2.5), with

$$M_3^2 = 2, \quad M_1 = M_2 = 0. \quad (\text{A.3})$$

Then the global R-symmetry is reduced to $\text{SU}(2) \times \text{U}(1)$. However, this value of M_3^2 is too large for (2.17) to admit squashed brane solutions; these only exist for $M_3^2 < \frac{4}{3}$. Thus in

the range of M_i^2 of interest here, the model is not supersymmetric, but can be viewed as a mass deformation of the supersymmetric $\mathcal{N} = 1^*$ model, deformed by a negative mass term δM_3^2 . This might still be useful to obtain insights into the strong coupling regime.

Adding also mass terms M_1^2 and M_2^2 , the global symmetry is broken to $U(1) \times U(1)$. Then there are no physical Goldstone bosons on the squashed brane backgrounds, since both are equivalent to a gauge transformation.

B Exceptional modes as Goldstone bosons

Here we show that the 6 exceptional zero modes arising from $(1, 1) \subset \text{End}(\mathcal{H})$ are nothing but the 6 Goldstones arising from $SU(3)_R$ minus the two $U(1)$, which are gauged hence eaten by the massive gauge bosons. To start, recall from [10] that the exceptional zero modes from $(1, 1) \subset \text{End}(\mathcal{H})$ correspond to the extremal weight states with $\Lambda' \in \mathcal{W}(3, 0)$ and $\Lambda' \in \mathcal{W}(0, 3)$. Denoting the background as

$$X \sim \lambda^\alpha T_\alpha = \lambda_{-\alpha_1} T_{\alpha_1} + \lambda_{\alpha_3} T_{-\alpha_3} + \dots, \quad (\text{B.1})$$

these arise from the anti-symmetric tensor product $(3, 0) \subset (1, 1) \otimes (1, 1)$, e.g.

$$\phi = \lambda^\alpha \phi_\alpha = \lambda_{-\alpha_1} T_{\alpha_3} - \lambda_{\alpha_3} T_{-\alpha_1} \quad (\text{B.2})$$

etc. Thus

$$\begin{aligned} \delta T_{\alpha_1} &= \epsilon T_{\alpha_3} \\ \delta T_{-\alpha_3} &= -\epsilon T_{-\alpha_1} \end{aligned} \quad (\text{B.3})$$

for any $\epsilon \in \mathbb{C}$, with conjugate mode

$$\begin{aligned} \delta T_{-\alpha_1} &= \bar{\epsilon} T_{-\alpha_3} \\ \delta T_{\alpha_3} &= -\bar{\epsilon} T_{\alpha_1}. \end{aligned} \quad (\text{B.4})$$

Thus

$$\phi_\alpha = \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 0 & 0 \\ -\bar{\epsilon} & 0 & 0 \end{pmatrix}_{\alpha\beta} T_\beta \quad (\text{B.5})$$

etc., which generates precisely the $SU(3)/(U(1) \times U(1))$ R-symmetry.

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