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# Complex linear Goldstino superfield and supergravity

**Sergei M. Kuzenko**

*School of Physics M013, The University of Western Australia,*
*35 Stirling Highway, Crawley W.A. 6009, Australia*

*E-mail:* sergei.kuzenko@uwa.edu.au

Abstract: The complex linear Goldstino superfield was proposed in arXiv:1102.3042 for the cases of global and local four-dimensional $\mathcal{N} = 1$ supersymmetry. Here we make use of this superfield to construct a supergravity action which is invariant under spontaneously broken local $\mathcal{N} = 1$ supersymmetry and has a positive cosmological constant for certain values of the parameters.

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## Contents



# 1 Introduction

Four years ago, we constructed the Goldstino model [1] described by a complex linear superfield $\Sigma$ constrained by

$$-\frac{1}{4}\bar{D}^2\Sigma = f\,, \qquad f = \text{const}\,. \tag{1.1}$$

Here $f$ is a parameter of mass dimension 2 which, without loss of generality, can be chosen to be real. To describe the Goldstino dynamics, $\Sigma$ was subject to the nonlinear constraints:

$$\Sigma^2 = 0\,, \tag{1.2}$$

$$-\frac{1}{4}\Sigma\bar{D}^2 D_\alpha\Sigma = f D_\alpha\Sigma\,. \tag{1.3}$$

The constraint (1.2) means that $\Sigma$ is nilpotent. The constraints (1.1), (1.2) and (1.3) imply that all component fields of $\Sigma$ are constructed in terms of a single spinor field $\bar{\rho}^{\dot\alpha}$. We recall that the general solution to the constraint (1.1) is

$$\Sigma(\theta,\bar\theta) = \mathrm{e}^{\mathrm{i}\theta\sigma^a\bar\theta\partial_a}\left(\phi + \theta\psi + \sqrt{2}\bar\theta\bar\rho + \theta^2 F + \bar\theta^2 f + \theta^\alpha\bar\theta^{\dot\alpha}U_{\alpha\dot\alpha} + \theta^2\bar\theta\bar\chi\right)\,. \tag{1.4}$$

The general solution to the constraint (1.2) fixes $\phi$ and two of the auxiliary fields

$$f\phi = \frac{1}{2}\bar\rho^2\,, \quad f\psi_\alpha = \frac{1}{\sqrt{2}}U_{\alpha\dot\alpha}\bar\rho^{\dot\alpha}\,, \quad fF = \frac{1}{\sqrt{2}}\bar\chi\bar\rho + \frac{1}{4}U^a U_a\,. \tag{1.5}$$

Finally, taking into account the constraint (1.3) fixes all of the components as functions of the Goldstino $\bar\rho$. The explicit expressions are:

$$f\phi = \frac{1}{2}\bar\rho^2\,, \qquad \sqrt{2}f^2\psi_\alpha = -\mathrm{i}\bar\rho^2(\partial\bar\rho)_\alpha\,, \qquad f^3 F = \bar\rho^2(\partial_a\bar\rho\tilde\sigma^{ab}\partial_b\bar\rho)\,,$$
$$fU_{\alpha\dot\alpha} = 2\mathrm{i}(\sigma^a\bar\rho)_\alpha\partial_a\bar\rho_{\dot\beta}\,, \qquad f^2\bar\chi_{\dot\alpha} = \sqrt{2}\Big((\bar\rho\tilde\sigma^a\sigma^b\partial_b\bar\rho)\partial_a\bar\rho_{\dot\beta} - \frac{1}{2}(\Box\bar\rho^2)\bar\rho_{\dot\beta}\Big)\,. \tag{1.6}$$



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The form of the Goldstino action coincides with the free action for the complex linear superfield,

$$S[\Sigma, \bar{\Sigma}] = -\int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, \Sigma\bar{\Sigma}\ . \tag{1.7}$$

At the component level, this action was shown [1] to be the same as the one described by Samuel and Wess [2]. A nonlinear field redefinition relating the component form of (1.7) to the Volkov-Akulov action [3, 4] follows from the results in [5].

It was also shown in [1] that all known Goldstino superfields [2, 6–8] can be obtained as composites constructed from spinor covariant derivatives of $\Sigma$ and its conjugate (see also [10, 11]).[1] This property and the universality [3, 4, 12, 13] of the Goldstino [3, 4] implies that any model for global supersymmetry breaking can be described in terms of $\Sigma$ and its conjugate.

Couplings of the complex linear Goldstino superfield to supersymmetric matter and $\mathcal{N} = 1$ supergravity were given in [1]. The results of [1] make it possible to derive a simple construction of models for spontaneously broken $\mathcal{N} = 1$ supergravity, similar to the old chiral construction of [8]. Recently, there have appeared models for spontaneously broken local supersymmetry [14, 15], which are based on the use of the chiral Goldstino superfield proposed in [16, 17]. Here we demonstrate how the complex linear Goldstino of [1] can be used to describe spontaneously broken supergravity.

## 2 Coupling to supergravity

The supergravity generalisation[2] of the constraints (1.1) and (1.3) given in [1] is as follows:

$$-\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\Sigma = Y\,, \qquad \bar{\mathcal{D}}_{\dot{\alpha}} Y = 0\,, \tag{2.1a}$$

$$-\frac{1}{4}\Sigma(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha\Sigma = Y\mathcal{D}_\alpha\Sigma\,, \tag{2.1b}$$

for some covariantly chiral scalar $Y$.[3] Here $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}})$ denote the superspace covariant derivative corresponding to the Wess-Zumino formulation for $\mathcal{N} = 1$ supergravity [19] (which at the component level is equivalent to approaches developed in [20, 21]), and $R$ is the covariantly chiral scalar component of the superspace torsion described in terms of $R$, $G_{\alpha\dot{\alpha}}$ and $W_{\alpha\beta\gamma}$ (see [18] for a review). Of course, the constraints (2.1a) and (2.1b) have to be accompanied by the nilpotent condition (1.2).

Here we consider the simplest case when $Y$ is a real non-zero constant,

$$Y = f = \text{const}\,, \tag{2.2}$$

[1]The Goldstino superfields constructed in [1, 2, 6, 8] can be derived using the general relationship between linear and nonlinear realisations of supersymmetry established in [7, 9]. In particular, the spinor Goldsino superfield advocated in [2] was first constructed in [7].

[2]Our conventions for $\mathcal{N} = 1$ supergravity mainly correspond to [33], with the only exception that the full superspace integration measure $E$ in (2.3) is denoted $E^{-1}$ in [18].

[3]In the super-Poincaré case, modified linear constraints of the form (2.1a) were first introduced in [22].



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as in the rigid supersymmetric case. To describe the dynamics of the Goldstino superfield coupled to supergravity, we propose the following action:

$$S = -\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\left(\frac{3}{\boldsymbol{\kappa}^2} + \bar{\Sigma}\Sigma\right) + \left\{\frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E} + \text{c.c.}\right\}, \tag{2.3}$$

where $\boldsymbol{\kappa}$ is the gravitational coupling constant and $\boldsymbol{\mu}$ a cosmological parameter. The integration measures $E$ and $\mathcal{E}$ correspond to the full superspace and its chiral subspace, respectively. The action (2.3) describes old minimal $\mathcal{N}=1$ supergravity if $\Sigma$ and $\bar{\Sigma}$ are switched off. With the Goldstino superfields $\Sigma$ and $\bar{\Sigma}$ included, the action proves to describe spontaneously broken $\mathcal{N}=1$ supergravity.

## 3 Super-Weyl invariant reformulation

To reduce the action (2.3) to components, one can follow, e.g., the component reduction procedure described in [18]. A less tedious calculation is required if one introduces a super-Weyl invariant extension of (2.3) by coupling this theory to a covariantly chiral conformal compensator $\Phi$, $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$, which is assumed to be nowhere vanishing. Such an extension can be obtained following the scheme described in [23] and based on the ideas due to Kugo and Uehara [24].

The super-Weyl transformation [25] in $\mathcal{N}=1$ old minimal supergravity is

$$\mathcal{D}'_\alpha = \mathrm{e}^{\frac{1}{2}\sigma-\bar{\sigma}}\Big(\mathcal{D}_\alpha - (\mathcal{D}^\beta\sigma)\,M_{\alpha\beta}\Big)\,, \qquad \bar{\mathcal{D}}'_{\dot{\alpha}} = \mathrm{e}^{\frac{1}{2}\bar{\sigma}-\sigma}\Big(\bar{\mathcal{D}}_{\dot{\alpha}} - (\bar{\mathcal{D}}^{\dot{\beta}}\bar{\sigma})\bar{M}_{\dot{\beta}\dot{\alpha}}\Big)\,, \tag{3.1}$$

where $\sigma$ is an arbitrary covariantly chiral scalar parameter, $\bar{\mathcal{D}}_{\dot{\alpha}}\sigma = 0$. The super-Weyl transformation of the chiral compensator $\Phi$ is

$$\Phi' = \mathrm{e}^{-\sigma}\Phi\,. \tag{3.2}$$

A super-Weyl invariant extension of the constraints (2.1) with $X$ given by (2.2) is

$$-\frac{1}{4}(\bar{\mathcal{D}}^2 - 4R)\Sigma = f\Phi^2\,, \tag{3.3a}$$

$$-\frac{1}{4}\Sigma(\bar{\mathcal{D}}^2 - 4R)\mathcal{D}_\alpha(\Sigma\bar{\Phi}^{-1}) = f\Phi^2\mathcal{D}_\alpha(\Sigma\bar{\Phi}^{-1})\,, \tag{3.3b}$$

provided the super-Weyl transformation of $\Sigma$ is chosen to be

$$\Sigma' = \mathrm{e}^{-\bar{\sigma}}\Sigma\,. \tag{3.4}$$

In the super-Weyl gauge $\Phi = 1$, the constraints (3.3) reduce to (2.1).[4]

The super-Weyl invariant extension of the action (2.3) is

$$S = -\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\left(\frac{3}{\boldsymbol{\kappa}^2}\bar{\Phi}\Phi + \bar{\Sigma}\Sigma\right) + \left\{\frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}^2}\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,\Phi^3 + \text{c.c.}\right\}. \tag{3.5}$$

[4] Applying a field redefinition $\Sigma \to= \Phi^n\Sigma$ leads to a different super-Weyl transformation law and modifies the explicit form of constraints (3.3).



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This action can be reduced to components using the reduction formula (5.8.50) in [18] and imposing suitable super-Weyl gauge conditions on the components of the chiral compensator $\Phi$, following the patterns described in [23] for four-dimensional $\mathcal{N}=1$ supergravity-matter systems and also in [26] for three-dimensional $\mathcal{N}=2$ supergravity-matter theories. Upon reducing the action to components and eliminating the supergravity auxiliary fields, for the cosmological constant one obtains

$$\Lambda = f^2 - 3\frac{|\boldsymbol{\mu}|^2}{\boldsymbol{\kappa}^2} \,. \tag{3.6}$$

This value agrees with the recent results in [14, 15, 27],[5] as well as with the ancient results [8, 30] (see also [31]). The cosmological constant is positive for $\boldsymbol{\kappa}^2 f^2 > 3|\boldsymbol{\mu}|^2$. Since the theory possesses local supersymmetry, the Goldstino can be eaten by the gravitino, in accordance with the Higgs mechanism for local supersymmetry [30, 32] (known as the super-Higgs effect). As a result, the gravitino becomes massive.

In conclusion, we recall that the constraint (1.1) is the only way to describe $\mathcal{N}=1$ anti-de Sitter supergravity using a non-minimal scalar multiplet as compensator [33]. We have shown that the same constraint can be used to describe spontaneously broken $\mathcal{N}=1$ supergravity with a positive cosmological constant.

## 4 Conclusion

Unlike the Goldstino superfields introduced in [2, 6–8], the complex linear Goldstino superfield allows nontrivial matter couplings [1]. This is achieved by choosing the chiral scalar $Y$ in (2.1) to be

$$Y = Y(\varphi^i)\,, \tag{4.1}$$

for some matter chiral superfields $\varphi^i$. In the presence of matter, the action (2.3) is replaced with

$$\begin{aligned} S = &-\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\left(\frac{3}{\boldsymbol{\kappa}^2}\mathrm{e}^{-\frac{1}{3}K(\varphi,\bar\varphi)} + \bar\Sigma\Sigma\right) \\ &+\left\{\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,\Phi^3\Big(\frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}^2} + W(\varphi)\Big) + \text{c.c.}\right\}\,, \end{aligned} \tag{4.2}$$

with $K(\varphi,\bar\varphi)$ being the Kähler potential of a Kähler manifold and $W(\varphi)$ a superpotential. Such matter couplings are analogous to the models considered in [14, 15, 27].

If $\Sigma$ obeys *only* the constraint (3.3a), the supergravity-matter system (3.5) possesses a dual formulation [1, 33]

$$S = -\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\left(\frac{3}{\boldsymbol{\kappa}^2}\bar\Phi\Phi - \bar X X\right) + \left\{\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,\Phi^3\Big(\frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}^2} + f\frac{X}{\Phi}\Big) + \text{c.c.}\right\}\,, \tag{4.3}$$

[5]Actually, the explicit expression (3.6) for the cosmological constant can be obtained without any calculation. The second term in (3.6) is the standard cosmological constant in pure $\mathcal{N}=1$ supergravity [28, 29], see, e.g., section 6.1.4 in [18] for a review. The first term in (3.6) follows from eq. (2.4) in [1].

where $X$ is a chiral scalar superfield, $\bar{\mathcal{D}}_{\dot\alpha} X = 0$, with the super-Weyl transformation

$$X' = \mathrm{e}^{-\sigma} X \ . \tag{4.4}$$

It is not clear to us how to modify the duality transformation in order to take account of the nilpotent constraint (1.2).

The action (4.3) with the chiral scalar $X$ constrained by

$$X^2 = 0 \tag{4.5}$$

describes coupling to supergravity of the Goldstino superfield introduced in [16, 17]. This model for spontaneously broken supergravity has been of much interest recently [14, 15, 27, 34, 35], in particular since it admits a nice geometric reformulation [27, 34, 35]. Here we would like to present a slightly different derivation of such a reformulation. In what follows, $X$ is assumed to obey the nilpotent constraint (4.5).

Varying (4.3) with respect to $\Phi$ gives the equation

$$\mathbb{R} - \boldsymbol{\mu} = \frac{2}{3} f \kappa^2 \frac{X}{\Phi} \,, \tag{4.6}$$

where we have the super-Weyl invariant chiral scalar

$$\mathbb{R} = -\frac{1}{4} \Phi^{-2} (\bar{\mathcal{D}}^2 - 4R) \bar\Phi \,, \tag{4.7}$$

which is related to the chiral scalar $\mathcal{R}$ of [34] by the rule $\mathcal{R} = \Phi \mathbb{R}$. Due to the nilpotent constraint (4.5), the equation of motion (4.6) implies that

$$(\mathbb{R} - \boldsymbol{\mu})^2 = 0 \,, \tag{4.8a}$$

which has the same form as the constraint put forward in [34, 35]. Making use of (4.6) once again, the action (4.3) takes the geometric form

$$S = \left( \frac{3}{2 f \kappa^2} \right)^2 \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \, E \, \bar\Phi \Phi |\mathbb{R} - \boldsymbol{\mu}|^2 - \left\{ \frac{1}{2} \frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}^2} \int \mathrm{d}^4x \mathrm{d}^2\theta \, \mathcal{E} \, \Phi^3 + \text{c.c.} \right\}, \tag{4.8b}$$

where $\mathbb{R}$ is subject to the constraint (4.8a). This action differs in its functional form from the one proposed in [34, 35]. The latter coincides with the minimal supergravity action

$$S = -\frac{3}{\boldsymbol{\kappa}^2} \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \, E \, \bar\Phi \Phi + \left\{ \frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}^2} \int \mathrm{d}^4x \mathrm{d}^2\theta \, \mathcal{E} \, \Phi^3 + \text{c.c.} \right\} , \tag{4.9a}$$

where $\mathbb{R}$ is subject to the nilpotent constraint

$$(\mathbb{R} - \boldsymbol{\lambda})^2 = 0 \,, \qquad \boldsymbol{\lambda} = \frac{\boldsymbol{\mu}}{\boldsymbol{\kappa}} + \frac{1}{\sqrt{3}} \boldsymbol{\kappa} f \ . \tag{4.9b}$$

The two descriptions should be equivalent.

An interesting problem is to understand whether our model (3.5) admits a geometric formulation similar to (4.8) or (4.9).

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