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The $\mu \to e\gamma$ decay in a systematic effective field theory approach with dimension 6 operators

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Abstract: We implement a systematic effective field theory approach to the benchmark process $\mu \to e\gamma$, performing automated one-loop computations including dimension 6 operators and studying their anomalous dimensions. We obtain limits on Wilson coefficients of a relevant subset of lepton-flavour violating operators that contribute to the branching ratio $\mu \to e\gamma$ at one-loop. In addition, we illustrate a method to extract further constraints induced by the mixing of operators under renormalisation-group evolution. This results in limits on the corresponding Wilson coefficients directly at the high scale. The procedure can be applied to other processes as well and, as an example, we consider also lepton-flavour violating decays of the τ.

Keywords: Rare Decays, Effective field theories, Renormalization Group

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1 Introduction

The study of lepton-flavour violating (LFV) processes in the charged sector offers a possibility to probe the Standard Model (SM) to very high scales. Of particular importance is the LFV decay $\mu \to e\gamma$. First, there are very impressive experimental limits on this branching ratio. The current best limit $\mathrm{BR}(\mu^+ \to e^+\gamma) < 5.7 \times 10^{-13}$ [1] has been set by the MEG collaboration at PSI and an upgrade of the experiment is underway to improve the sensitivity further by an order of magnitude [2]. Second, in the SM with neutrino masses m_ν this branching ratio is suppressed by the tiny ratio $(m_\nu/m_W)^4$, where m_W is the mass of the W-boson. Thus, the SM branching ratio is well below any experimental limit that is achievable in the foreseeable future and any positive signal for $\mu \to e\gamma$ would be clear evidence for physics beyond the Standard Model (BSM). Conversely, improving limits on this branching ratio would put even more serious constraints on many BSM models. Given its importance the decay $\mu \to e\gamma$ has been studied in a large number of explicit BSM models. Here, a more model independent approach is taken.

The impact of a BSM model with new physics at a large energy scale $\Lambda \gg m_W$ to observables at much smaller scales can be described using an effective field theory (EFT) approach. The SM is considered to be an EFT valid up to a scale Λ and the BSM effects at lower energies are described by operators of dimension $n > 4$, suppressed by powers of Λ. These operators are generated from the BSM physics by integrating out the heavy non-SM degrees of freedom. In general, the dominant effects are expected to come from

dimension 5 and dimension 6 operators. A minimal list of all possible such operators formed from SM fields only and respecting the $SU(3) \times SU(2) \times U(1)$ gauge invariance consists of one dimension 5 operator [3] and 64 dimension 6 operators [4, 5], five of which are baryon number violating. As many of these operators actually represent matrices in generation space, the total number of coefficients needed to describe the most general case is rather large. Nevertheless, this is a systematic approach to study the impact of BSM physics to a large class of observables obtained from experiments at very different energy scales. It is used in Higgs physics, B-physics and the study of electric dipole moments to mention just a few of the applications.

Applying these ideas to the flavour changing decay $\mu \to e\gamma$ we note that there is a dimension 6 operator ($Q_{e\gamma}$ to be defined below) that induces such a decay directly at tree level. It is clear that the MEG limit provides an extremely strong constraint on the coefficient of this operator. However, such a decay can also be induced indirectly from other operators that are not immediately linked to $\mu \to e\gamma$. Thus, even if a particular BSM does not induce the operator $Q_{e\gamma}$ at the high scale Λ, it can lead to a non-vanishing contribution to $\mu \to e\gamma$. Broadly speaking, this can happen in two different ways.

First, some dimension 6 operators other than $Q_{e\gamma}$ can induce a decay $\mu \to e\gamma$ beyond tree level. The contribution to $\mu \to e\gamma$ from dimension 6 operators at one loop has partially been computed [6] and it has been found that several operators contribute. This can lead to very serious independent constraints on the coefficients of these operators.

The second possibility is through mixing in the renormalisation-group (RG) evolution of the Wilson coefficient $C_{e\gamma}$ of the operator $Q_{e\gamma}$. The Wilson coefficients $C_i(\Lambda)$ of the higher-dimensional operators are determined at the high scale Λ by integrating out the heavy fields. If these coefficients then are to be used to study the impact of the higher-dimensional operators to observables at a lower scale λ, say $\lambda \sim m_W$, the coefficients $C_i(m_W)$ have to be determined from $C_i(\Lambda)$ through RG evolution. The one-loop RG evolution of the dimension 6 operators has been studied [7–9] and, as expected, it has been found that other operators mix with $Q_{e\gamma}$ under the evolution.

The aim of this paper is to present a complete analysis of $\mu \to e\gamma$ in the context of an EFT approach including dimension 6 operators. To this end, we repeat and extend the one-loop calculation presented in [6] for this process with a RG analysis. The RG running is done in two steps. We first evolve from the large scale Λ to the electroweak scale $m_V \sim m_W \sim m_Z$ and then use a modified evolution suitable for the scales $m_\mu \lesssim \lambda \lesssim m_Z$, where the mass of the muon, m_μ, is the scale at which the coefficient $C_{e\gamma}$ has to be evaluated for the process $\mu \to e\gamma$. We consider the subset of all dimension 6 operators that are most directly linked to the LFV decay. The details of the Lagrangian and the setup for the calculations are given in section 2. In section 3 the relation between the Lagrangian and the branching ratio is discussed. Section 4 is the main part of the paper. Section 4.1 starts with the one-loop result of the branching ratio computed in the EFT. The experimental limit on the branching ratio can be translated directly into a limit for $C_{e\gamma}(m_\mu)$. From the explicit one-loop results, it is also possible to extract limits on other Wilson coefficients evaluated at the small scale. In a second step, in section 4.2, the anomalous dimensions of the operator $Q_{e\gamma}$ and those operators that mix with $Q_{e\gamma}$ are computed. These results

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are then used to obtain limits on the Wilson coefficients of these operators, evaluated directly at the large scale Λ. Our conclusions are presented in section 5. The details of the renormalisation needed for the one-loop result and the anomalous dimensions are given in appendix A. In appendix B the result for the (unrenormalised) one-loop branching ratio is listed. Finally, in appendix C we apply the same method to the LFV decays of the τ to obtain limits on the corresponding Wilson coefficients.

2 Effective D-6 extension of the SM: leptonic interactions

In this paper we take the point of view that the SM is an EFT valid up to some large scale Λ and BSM physics can be parametrised by operators of dimension 6 (D-6). Higher dimensional operators are not considered. A complete list of gauge invariant D-6 operators has been given, in [5]. In this section the subset of D-6 operators that are relevant for our analysis of $\mu \to e\gamma$ is presented and the implementation of these operators in automated computational tools is also briefly discussed.

The Lagrangian considered in this paper is the SM Lagrangian $\mathcal{L}_{\rm SM}$ extended by D-6 operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2}\sum_i C_i\, Q_i, \tag{2.1}$$

where the sum is over the D-6 operators listed in tables 1 and 2. These are the D-6 operators that can cause LFV interactions. The dimension 5 operator is not included in eq. (2.1): since the effect of this operator on $\mu \to e\gamma$ transitions has been studied before [10, 11] we do not consider it in our analysis. The notation and conventions are taken from [5]. In particular, $\{p, r, s, t\}$ denote generation indices. In the Lagrangian the operators appear multiplied by $C_i^{pr\cdots}/\Lambda^2$, where $C_i^{pr\cdots}$ are dimensionless coefficient matrices with two or four generation indices. With regard to the Hermitian conjugation, it is worth to remark that

- in the operator class $\psi^2\varphi^2 D$, it is self-realised by transposition of generation indices;
- in the operator classes $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$ and $(\bar{L}L)(\bar{R}R)$, it is self-realised by transposition of generation indices once the prescription $C^{prst} = C^{rpts}$ is assumed;
- for the other operator classes, adding the Hermitian conjugate (not listed explicitly in tables 1 and 2) is understood.

Working in the physical basis rather than in the gauge basis, the two operators of the $\psi^2 X\varphi$ set are rewritten using

$$Q_{eB} \to Q_{e\gamma} c_W - Q_{eZ} s_W, \tag{2.2}$$

$$Q_{eW} \to -Q_{e\gamma} s_W - Q_{eZ} c_W, \tag{2.3}$$

where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle. The term

$$\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi F_{\mu\nu} + \text{h.c.}, \tag{2.4}$$

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$\psi^2 X\varphi$		$\psi^2\varphi^2 D$		$\psi^2\varphi^3$	
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r\varphi)$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$		
		$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$		

Table 1. D-6 operators consisting of fermions and bosons, according to [5].

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{ledq}	$(\bar{l}^j_p e_r)(\bar{d}_s q^j_t)$
$Q^{(1)}_{lq}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$	$Q^{(1)}_{lequ}$	$(\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)$
$Q^{(3)}_{lq}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$	$Q^{(3)}_{lequ}$	$(\bar{l}^j_p\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}^k_s\sigma^{\mu\nu}u_t)$
				Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$		

Table 2. D-6 operators consisting of four fermions, according to [5].

where $F_{\mu\nu}$ is the electromagnetic field-strength tensor, is then the only term in the D-6 Lagrangian that induces a $\mu \to e\gamma$ transition at tree level. However, at one loop (and even higher order) the other operators listed in tables 1 and 2 also potentially contribute.

Finally, special attention is devoted to the operator $Q_{e\varphi}$: in Feynman gauge, the presence of such an operator produces Lagrangian terms of the form

$$\begin{aligned}\mathcal{L}_{e\varphi} =& \frac{v^3}{2\sqrt{2}\Lambda^2}C^{pr}_{e\varphi}\bar{e}_p e_r + \frac{3v^2}{2\sqrt{2}\Lambda^2}C^{pr}_{e\varphi}\bar{e}_p e_r h \\ &+ i\frac{v^2}{2\sqrt{2}\Lambda^2}C^{pr}_{e\varphi}\bar{e}_p e_r\widehat{Z} + i\frac{v^2}{2\Lambda^2}C^{pr}_{e\varphi}\bar{e}_p\nu_r\widehat{W}^+ + [\dots]\,.\end{aligned} \tag{2.5}$$

Apparently, this operator introduces Goldstone-boson ($\widehat{Z}$, $\widehat{W}^\pm$) interactions which are not compensated by any analogous vectorial term. However, the combination of eq. (2.5) with the D-4 SM Yukawa terms gives

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{e\varphi} =& \frac{v}{\sqrt{2}}\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C^{pr}_{e\varphi}\right)\bar{e}_p e_r \\ &+ \frac{1}{\sqrt{2}}\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C^{pr}_{e\varphi}\right)\bar{e}_p e_r h + \frac{v^2}{\sqrt{2}\Lambda^2}C^{pr}_{e\varphi}\bar{e}_p e_r h \\ &+ \frac{i}{\sqrt{2}}\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C^{pr}_{e\varphi}\right)\bar{e}_p e_r\widehat{Z} + i\left(-y_{pr} + \frac{v^2}{2\Lambda^2}C^{pr}_{e\varphi}\right)\bar{e}_p\nu_r\widehat{W}^+ + [\dots]\,.\end{aligned} \tag{2.6}$$

From eq. (2.6), it is understood that any 3-point off-diagonal interaction involving Goldstone bosons is not physical, i.e. it can be removed by an orthogonal transformation. However, this procedure results in

- a residual term with a physical Higgs supporting LFV currents;

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- a redefinition of the relation between leptonic Yukawa couplings and leptonic masses:

$$y_{pp} \to \frac{\sqrt{2}m_p}{v} + \frac{v^2}{2\Lambda^2} C^{pp}_{e\varphi}. \tag{2.7}$$

In the framework of LFV processes at tree level and one loop, the prescription of eq. (2.7) is never relevant. However, it is of fundamental importance in the case of flavour diagonal interactions and related analyses such as the study of the anomalous magnetic moment of the muon $(g-2)_\mu$.

In the following sections, one-loop calculations in the theory given by the Lagrangian eq. (2.1) will be presented. In order to perform such calculations in an automated way, several openly available tools were used:

- in order to obtain consistent Feynman rules, the described model was implemented both in `LanHEP v3.1.9` [12] and in `FeynRules v2.0` [13], and the agreement among the two packages was checked;
- in order to produce a model file for the `FeynArts v3.9` [14] and `FormCalc v8.3` [15, 16] packages, the FeynArts interface of FeynRules was exploited;
- the combined packages FeynArts/FormCalc were employed to generate non-integrated amplitudes to be elaborated afterwards with the symbolic manipulation system `Form v4.0` [17].

The list of resulting tree-level Feynman rules from the Lagrangian eq. (2.1) is too long to be given explicitly in this paper. It will be provided after the publication of this work: it will appear in the FeynRules model database[1] (in the format of a FeynRules model file). However, the Feynman rule for the $\mu - e - \gamma$ interaction (consisting of the effective tree-level interaction plus the one-loop wave-function renormalisation (WFR) of the relevant objects) is presented (see appendix A).

3 $\mu \to e\gamma$: branching ratio and constraints

It is well known that in the limit $m_\mu \gg m_e$ the partial width of the process $\mu \to e\gamma$ is given by

$$\Gamma_{\mu\to e\gamma} = \frac{1}{16\pi m_\mu} |\mathcal{M}|^2 , \tag{3.1}$$

where $\mathcal{M}$ is the transition amplitude, which contains the model-dependent information. Computing $\mathcal{M}$ in the theory given by eq. (2.1) and confronting the corresponding branching ratio $\mathrm{BR}(\mu^+ \to e^+\gamma)$ with the experimental limit [1] allows to put constraints on the Wilson coefficients C_i of some of the D-6 operators in eq. (2.1).

To make this connection more explicit we note that the Lagrangian eq. (2.1) induces flavour-violating interactions $\mu \to e\gamma$ that can be written as

$$V^\mu = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} \left(C_{TL}\, \omega_L + C_{TR}\, \omega_R\right) (p_2)_\nu , \tag{3.2}$$

[1] http://feynrules.irmp.ucl.ac.be/wiki/ModelDatabaseMainPage.

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where the conventions described in appendix A are used and $\omega_{L/R} = 1 \mp \gamma^5$. Note that no term $\sim \gamma^\mu$ appears in eq. (3.2) since such a term is forbidden by gauge invariance. C_{TL} and C_{TR} are coefficients of dimension one that depend on the Wilson coefficients of the D-6 operators and on the parameters of the SM. The unpolarised squared matrix element is expressed in terms of them as

$$|\mathcal{M}|^2 = \frac{4\left(|C_{TL}|^2 + |C_{TR}|^2\right) m_\mu^4}{\Lambda^4}, \tag{3.3}$$

and the branching ratio is

$$\mathrm{BR}(\mu \to e\gamma) = \frac{\Gamma_{\mu\to e\gamma}}{\Gamma_\mu} = \frac{m_\mu^3}{4\pi\Lambda^4\Gamma_\mu}\left(|C_{TL}|^2 + |C_{TR}|^2\right) = \frac{48\pi^2}{G_F^2 m_\mu^2}\frac{\left(|C_{TL}|^2 + |C_{TR}|^2\right)}{\Lambda^4}, \tag{3.4}$$

where $\Gamma_\mu = \left(G_F^2 m_\mu^5\right)/\left(192\pi^3\right)$ is the SM total decay width of the muon. The result eq. (3.4) is well known in the literature, see e.g. [18] and references therein. Confronting this result with the experimental upper limit [1] established by the MEG collaboration on the $\mu^+ \to e^+\gamma$ transition

$$\mathrm{BR}(\mu^+ \to e^+\gamma) \leq 5.7 \cdot 10^{-13}, \tag{3.5}$$

the limit

$$\frac{\sqrt{|C_{TL}|^2 + |C_{TR}|^2}}{\Lambda^2} \leq 4.3 \cdot 10^{-14}\,[\mathrm{GeV}]^{-1} \tag{3.6}$$

can be obtained.

At tree level, for the process $\mu^+ \to e^+\gamma$ the coefficients appearing in eq. (3.6) are given by $C_{TR}^{(0)} = -v\,C_{e\gamma}^{\mu e}/\sqrt{2}$ and $C_{TL}^{(0)} = -v\,(C_{e\gamma}^{e\mu})^*/\sqrt{2}$. In what follows, we will instead compute the coefficients for the process $\mu^- \to e^-\gamma$ where the tree-level results are given by $C_{TR}^{(0)} = -v\,C_{e\gamma}^{e\mu}/\sqrt{2}$ and $C_{TL}^{(0)} = -v\,(C_{e\gamma}^{\mu e})^*/\sqrt{2}$. From now on the generation indices will often be dropped and the simplified notation $C_{e\gamma}$ will be used for either $C_{e\gamma}^{\mu e}$ or $C_{e\gamma}^{e\mu}$. Similar remarks apply to C_{eZ} and $C_{e\varphi}$.[2] Applying the constraint eq. (3.6) then immediately results in a constraint on $C_{e\gamma}$.

It is clear that if the BSM physics is such that the matching at the scale Λ produces a sizable coefficient $C_{e\gamma}(\Lambda)$ this will be the dominant effect for $\mathrm{BR}(\mu \to e\gamma)$. On the other hand it is perfectly possible that the coefficient $C_{e\gamma}(\Lambda)$ is zero or strongly suppressed compared to Wilson coefficients of other D-6 operators. In this case effects of operators that enter C_{TL} and C_{TR} only at one loop can be important.

The result of C_{TL} (or C_{TR}) computed at one loop can schematically be written as

$$C_{TL}^{(1)} = -\sqrt{2}v\left(C_{e\gamma}\left(1 + e^2 c_{e\gamma}^{(1)}\right) + \sum_{i\neq e\gamma} e^2 c_i^{(1)}\,C_i\right), \tag{3.7}$$

[2]However, for the sake of completeness, generation indices are retained in the results provided in appendices A and B.

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where the electromagnetic coupling e stands for a generic coupling and the coefficients $c_{e\gamma}^{(1)}$ and $c_i^{(1)}$ depend on SM parameters such as m_Z, m_l etc. To compute the branching ratio at one loop, apart from wave-function renormalisation also the vacuum expectation value (VEV) v has to be renormalised. Even after this renormalisation, the coefficients $c_{e\gamma}^{(1)}$ and $c_i^{(1)}$ in general contain ultraviolet singularities. These singularities have to be absorbed by a renormalisation of the coefficient $C_{e\gamma}$. By choosing a particular scheme for this subtraction, a precise definition of the Wilson coefficient is given. In what follows, the $\overline{\text{MS}}$ scheme is used.

In passing, it should be mentioned that for the coefficient $c_{e\gamma}^{(1)}$ also infrared singularities have to be taken into consideration. However, the primary interest of considering one-loop corrections is in the contribution of operators other than $Q_{e\gamma}$ to C_{TL} and C_{TR}. The corrections $\sim e^2 c_{e\gamma}^{(1)} C_{e\gamma}$ only result in a small modification of the limit on $C_{e\gamma}$. Hence these corrections will not be considered in this paper.

The renormalised Wilson coefficients and, therefore, the coefficients C_{TL} and C_{TR} are scale dependent quantities. Hence, eq. (3.6) should be interpreted as a phenomenological constraint on the Wilson coefficients at the relevant energy scale. While $\lambda \sim m_\mu$ is the typical energy scale probed by the MEG experiment, the explicit results presented in the next section will show, that for some of the operators the relevant scale is the electroweak scale $\lambda \sim m_V$. In any case, these scales are much lower than Λ, the natural scale for the Wilson coefficients after integrating out the heavy non-SM fields. To stress this subtlety eq. (3.6) is rewritten as

$$\left.\frac{\sqrt{|C_{TL}(\lambda)|^2 + |C_{TR}(\lambda)|^2}}{\Lambda^2}\right|_{\lambda \ll \Lambda} \leq 4.3 \cdot 10^{-14}\,[\text{GeV}]^{-1}\,. \tag{3.8}$$

In the next section, the explicit result for the coefficients C_{TL} and C_{TR} of eq. (3.2) computed in the context of the Lagrangian eq. (2.1) at the tree level and one-loop level is given. Furthermore, various contributions coming from different operators are separately shown. Afterwards, the RG running of the Wilson coefficients is studied and eq. (3.8) is applied to obtain bounds on each relevant coefficient at the scale Λ. These limits provide the most direct link between the low-energy observable $\text{BR}(\mu \to e\gamma)$ and BSM scenarios within an EFT framework.

4 Results

In this section, analytical results and phenomenological studies concerning the impact of eq. (3.8) on the Wilson coefficients of D-6 operators in the Lagrangian eq. (2.1) are presented. The study is split into two parts:

1: The complete result for the decay $\mu \to e\gamma$ in the EFT up to the one-loop level is calculated. These results are then used to obtain bounds on the Wilson coefficients of D-6 operators at the fixed scale $\lambda = m_\mu$ or $\lambda = m_V$, applying the experimental constraint on the branching ratio $\text{BR}(\mu \to e\gamma)$.

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2: The mixing of a subset of D-6 operators with $Q_{e\gamma}$ under RG evolution is computed. Translating the experimental constraint on BR($\mu \to e\gamma$) to a limit on $C_{e\gamma}(m_Z)$, bounds on Wilson coefficients $C_i(\Lambda)$ of operators Q_i that mix with $Q_{e\gamma}$ are then obtained. The dependence on Λ of these bounds is discussed.

Due to the high level of automation, a certain number of cross checks was strongly required. Unless specified otherwise, every result of this paper was tested under the following aspects:

- with no exceptions, all the calculations were performed in a general R_ξ-gauge and it was verified that any physical result is independent of the gauge parameters ξ_γ, ξ_W, ξ_Z and ξ_G;
- intermediate expansions or truncations were never applied, i.e. only the complete and final result was expanded, to verify both the gauge invariance up to any order of $1/\Lambda^2$ and the numerical consistency of expansions with respect to the full result;
- if possible, some quantities were computed in different ways (e.g. the anomalous dimension of the operators $Q_{e\gamma}$ and Q_{eZ} were computed both with an Higgs boson in the final state and its VEV), further checking the complete agreement between(among) the two(many) results;
- if possible, any non-original outcome was compared with previous literature: in particular, SM results against [19, 20], fixed order calculations against [6], anomalous dimensions of the SM parameters against [21–23] and anomalous dimensions of D-6 operators against [7–9].[3]

In the following subsections, analytical results and phenomenological constraints are given.

4.1 Branching ratio: results and constraints

In this subsection, the explicit results of the one-loop calculations for the coefficients C_{TL} and C_{TR}, i.e. the coefficients $c_i^{(1)}$ as defined in eq. (3.7) are given. We use diagonal Yukawa matrices throughout.

First of all, it was verified that no term $\sim \gamma^\mu$ is generated by the Lagrangian eq. (2.1) for the LFV interaction V^μ, as dictated by gauge invariance. Then, the tree-level and one-loop results were calculated using standard techniques as described in section 3. Subsequently, the outcome was expanded around $m_l \ll m_V$, i.e. considering the leptonic masses to be much smaller than the bosonic ones. In this limit, the contribution from the operator $Q_{e\varphi}$ to C_{TL} reads

$$C_{TL} = C_{e\varphi}^{\mu e} \frac{m_W s_W}{48\sqrt{2} m_H^2 \pi^2} \left(4m_e^2 + 4m_\mu^2 + 3m_e^2 \log\left[\frac{m_e^2}{m_H^2}\right] + 3m_\mu^2 \log\left[\frac{m_\mu^2}{m_H^2}\right] \right) + C_{e\varphi}^{e\mu} \frac{m_W s_W}{48\sqrt{2} m_H^2 \pi^2} \left(-m_e m_\mu\right) + \dots , \tag{4.1}$$

[3] We thank the authors of [6–9] for help in clarifying any source of disagreement by private communication.

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Operator	C_{TL} or $C_{TR}(\mu \longleftrightarrow e)$
$Q_{e\gamma}$	$-C_{e\gamma}\dfrac{\sqrt{2}m_W s_W}{e}$
Q_{eZ}	$-C_{eZ}\dfrac{em_Z}{16\sqrt{2}\pi^2}\left(3-6c_W^2+4c_W^2\log\left[\dfrac{m_W^2}{m_Z^2}\right]+(12c_W^2-6)\log\left[\dfrac{m_Z^2}{\lambda^2}\right]\right)$
$Q_{\varphi l}^{(1)}$	$-C_{\varphi l}^{(1)}\dfrac{em_e\left(1+s_W^2\right)}{24\pi^2}$
$Q_{\varphi l}^{(3)}$	$C_{\varphi l}^{(3)}\dfrac{em_e\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{\varphi e}$	$C_{\varphi e}\dfrac{em_\mu\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{e\varphi}$	$C_{e\varphi}\dfrac{m_W s_W}{48\sqrt{2}m_H^2\pi^2}\left(4m_e^2+4m_\mu^2+3m_e^2\log\left[\dfrac{m_e^2}{m_H^2}\right]+3m_\mu^2\log\left[\dfrac{m_\mu^2}{m_H^2}\right]\right)$
$Q_{lequ}^{(3)}$	$-\dfrac{e}{2\pi^2}\sum_u m_u\left(C_{lequ}^{(3)}\right)^{\mu euu}\log\left[\dfrac{m_u^2}{\lambda^2}\right]$

Operator	C_{TL}	C_{TR}
Q_{le}	$\dfrac{e}{16\pi^2}\left(m_e C_{le}^{\mu eee}+m_\mu C_{le}^{\mu\mu\mu e}+m_\tau C_{le}^{\mu\tau\tau e}\right)$	$\dfrac{e}{16\pi^2}(m_e C_{le}^{eee\mu}+m_\mu C_{le}^{e\mu\mu\mu}+m_\tau C_{le}^{e\tau\tau\mu})$

Table 3. Complete set of results (up to one-loop) for the LO contributions of the various D-6 operators to the $\mu \to e\gamma$ decay. For C_{TL} (C_{TR}) the generation indices μe ($e\mu$) are understood.

where the ellipses stand for contributions from other operators. Since $m_e \ll m_\mu$ we can drop the term proportional to $C_{e\varphi}^{e\mu}$. Keeping the term $\sim m_e^2 C_{e\varphi}^{\mu e}$ in eq. (4.1) ensures that the result for C_{TR} can be obtained by $(\mu \longleftrightarrow e)$.

Finally, the complete set of LO contributions of D-6 operators in eq. (2.1) (up to one-loop in SM couplings) was obtained (see table 3). The full result without expansion around $m_l \ll m_V$ is lengthy and not suitable for a phenomenological analysis, but is given (truncated at the order $1/\Lambda^2$) in appendix B , including the complete information about the generation indices for the $Q_{e\gamma}$, Q_{eZ} and $Q_{e\varphi}$ operators.

The one-loop calculation leads to several UV-divergent terms in connection with three operators: $Q_{e\gamma}$, Q_{eZ} and $Q_{lequ}^{(3)}$. After $\overline{\text{MS}}$ renormalisation the remnants of these UV singularities are logarithms with an electroweak scale, $\log(m_V^2/\lambda^2)$, in the term proportional to C_{eZ} and logarithms with the various quark mass scales, $\log(m_u^2/\lambda^2)$ in the coefficient proportional to $C_{\mu euu}^{(3)} \equiv (C_{lequ}^{(3)})^{\mu euu}$. The one-loop corrections proportional to $C_{e\gamma}$ (not shown) also contain scale-dependent logarithms. Thus, as expected the coefficients C_{TL} and C_{TR} are scale dependent.

The impact on the phenomenology of the scale evolution from the large scale Λ to the electroweak scale is studied in section 4.2. Here the coefficients are evaluated at the small scale $\lambda \ll \Lambda$, in particular, $\lambda = m_Z$ for C_{eZ}. Thus, the result of table 3 can be combined

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3-P Coefficient	At fixed scale	4-P Coefficient	At fixed scale
$C_{e\gamma}^{\mu e}$	$2.5 \cdot 10^{-16} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\mu eee}$	$4.4 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{eZ}^{\mu e}(m_Z)$	$1.4 \cdot 10^{-13} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\mu\mu\mu e}$	$2.1 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$2.6 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\mu\tau\tau e}$	$1.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$2.5 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{\varphi e}$	$2.5 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{e\varphi}^{\mu e}$	$2.8 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		

Table 4. Limits on the Wilson coefficients contributing to the $\mu \to e\gamma$ transition up to the one-loop level.

directly with eq. (3.8) to put a limit on a set of coefficients coming from 7 operators (out of the ensemble of 19, see tables 1 and 2). The other operators do not contribute to the tree-level or one-loop fixed scale result.

Under the assumption that only one Wilson coefficient at a time is non-vanishing, the numerical limits of table 4 are obtained. They are given for the Wilson coefficients with generation indices μe. Since we consider the unpolarised decay, the corresponding limits with the generation indices $e\mu$ are of course the same. The numerical values of the input parameters have been taken from the Particle Data Group review [24]. Note that no limit on $C_{lequ}^{(3)}$ is given since its contribution vanishes if evaluated at the natural scale $\lambda = m_u$. It is of course possible that an interplay among the various coefficients leads to cancellations that invalidate the limits given in table 4. A possibility to pin down more specific limits concerns the study of the correlation among various experimental bounds (e.g., $\text{BR}(Z \to e\mu)$, $\text{BR}(\mu \to 3e)$, etc.), but this is outside the scope of this work. Similarly, the study of specific underlying theories that can lead to such cancellations is outside the strict EFT framework we are using.

The results of tables 3 and 4 were partially shown in the work of Crivellin, Najjari and Rosiek [6]; in addition to their results, here a complete treatment of the operators Q_{eZ} and $Q_{e\varphi}$ is shown. Regarding the latter, a comment is required: the coefficient $C_{e\varphi}$ is connected to a two-loop Barr-Zee effect [25], and it is well known [26–30] that such a two-loop contribution could be of the same order or even larger than the one-loop term of table 3. Even though such feature could surely be relevant, its analysis is not a purpose of this paper.

4.2 Anomalous dimensions: results and constraints

In the previous section, limits on the Wilson coefficients $C_i(m_V)$ or $C_i(m_l)$ have been obtained by a strict one-loop calculation. However, the most direct information on the underlying BSM theory can be obtained by information on the Wilson coefficients at the matching scale, $C_i(\Lambda)$. Thus, the anomalous dimensions of the D-6 operators that are relevant for the (tree-level) $\mu \to e\gamma$ transition have to be studied.

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The anomalous dimensions of D-6 operators have been calculated in [7–9]. We have repeated the computations of those that are relevant to our case and extended the treatment to include the running of the coefficient $C_{e\gamma}(\lambda)$ to scales $\lambda < m_V$.

By direct computation, one finds that the running of the $C^{\mu e}_{e\gamma}$ coefficient for $\lambda > m_V$ is governed by

$$\begin{aligned}
&16\pi^2 \frac{\partial C^{\mu e}_{e\gamma}}{\partial \log \lambda} \\
&= \left(e^2 \left(\frac{47}{3} + \frac{1}{4c_W^2} - \frac{9}{4s_W^2} \right) + 2\, Y_e^2 + \left(\frac{1}{2} + 2c_W^2 \right) Y_\mu^2 + \sum_l Y_l^2 + 3 \sum_q Y_q^2 \right) C^{\mu e}_{e\gamma} \\
&\quad + \left(6e^2 \left(\frac{c_W}{s_W} - \frac{s_W}{c_W} \right) - 2 c_W s_W Y_\mu^2 \right) C^{\mu e}_{eZ} + 16e \sum_u Y_u C^{(3)}_{\mu euu},
\end{aligned} \tag{4.2}$$

and the related quantity $C^{e\mu}_{e\gamma}$ can be obtained by interchanging the generation indices, i.e. $Y_\mu \longleftrightarrow Y_e$ and $C^{(3)}_{\mu euu} \longleftrightarrow C^{(3)}_{e\mu uu}$. Retaining only the dominant terms, eq. (4.2) becomes

$$\begin{aligned}
&16\pi^2 \frac{\partial C^{\mu e}_{e\gamma}}{\partial \log \lambda} \\
&\simeq \left(\frac{47e^2}{3} + \frac{e^2}{4c_W^2} - \frac{9e^2}{4s_W^2} + 3Y_t^2 \right) C^{\mu e}_{e\gamma} + 6e^2 \left(\frac{c_W}{s_W} - \frac{s_W}{c_W} \right) C^{\mu e}_{eZ} + 16e \sum_u Y_u C^{(3)}_{\mu euu}.
\end{aligned} \tag{4.3}$$

From eq. (4.3), it follows that direct contributions to the evolution of $C_{e\gamma}$ come from the operator $Q_{e\gamma}$ itself, plus the orthogonal operator Q_{eZ} and the four-fermion operator $Q^{(3)}_{lequ}$. Of course, the corresponding coefficients are precisely the UV singularities that appear in the renormalisation of $C_{e\gamma}$, discussed in section 4.1.

In the same way, a similar structure for the RG running of the $C^{\mu e}_{eZ}$ coefficient is found:

$$\begin{aligned}
&16\pi^2 \frac{\partial C^{\mu e}_{eZ}}{\partial \log \lambda} \\
&= \left(e^2 \left(-\frac{47}{3} + \frac{151}{12c_W^2} - \frac{11}{12s_W^2} \right) + 2\, Y_e^2 + \left(\frac{1}{2} + 2s_W^2 \right) Y_\mu^2 + \sum_l Y_l^2 + 3 \sum_q Y_q^2 \right) C^{\mu e}_{eZ} \\
&\quad - \left(\frac{2e^2}{3} \left(\frac{2c_W}{s_W} + \frac{31 s_W}{c_W} \right) + 2 c_W s_W Y_\mu^2 \right) C^{\mu e}_{e\gamma} + 2e \left(\frac{3c_W}{s_W} - \frac{5 s_W}{c_W} \right) \sum_u Y_u C^{(3)}_{\mu euu} \\
&\simeq -\frac{2e^2}{3} \left(\frac{2c_W}{s_W} + \frac{31 s_W}{c_W} \right) C^{\mu e}_{e\gamma} + \left(-\frac{47e^2}{3} + \frac{151e^2}{12c_W^2} - \frac{11e^2}{12s_W^2} + 3Y_t^2 \right) C^{\mu e}_{eZ} \\
&\quad + 2e \left(\frac{3c_W}{s_W} - \frac{5 s_W}{c_W} \right) \sum_u Y_u C^{(3)}_{\mu euu}.
\end{aligned} \tag{4.4}$$

From eqs. (4.3) and (4.4), it is understood that there is an interplay in the evolution of $C_{e\gamma}$ and C_{eZ}. Moreover their running is directly connected to $C^{(3)}_{\mu euu}$. Hence, if the underlying theory produces non-vanishing matching coefficients $C^{(3)}_{e\mu uu}(\Lambda)$ they will induce an non-vanishing $C_{e\gamma}(m_V)$, even if $C_{e\gamma}(\Lambda)$ happens to vanish. In fact, there are even further operators that contribute indirectly to $C_{e\gamma}(m_V)$, namely those operators that mix

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with $Q^{(3)}_{lequ}$ under RG evolution. To include these in the analysis, the contribution of operators listed in tables 1 and 2 to the anomalous dimension of $Q^{(3)}_{lequ}$ and $Q^{(1)}_{lequ}$ have been evaluated. The corresponding coefficients run according to

$$16\pi^2\frac{\partial C^{(3)}_{\mu ett}}{\partial\log\lambda} \simeq \frac{7eY_t}{3}C^{\mu e}_{e\gamma} + \frac{eY_t}{2}\left(\frac{3c_W}{s_W} - \frac{5s_W}{3c_W}\right)C^{\mu e}_{eZ} + \\ + \left(\frac{2e^2}{9c_W^2} - \frac{3e^2}{s_W^2} + \frac{3Y_t^2}{2} + \frac{8g_S^2}{3}\right)C^{(3)}_{\mu ett} + \frac{e^2}{8}\left(\frac{5}{c_W^2} + \frac{3}{s_W^2}\right)C^{(1)}_{\mu ett}, \tag{4.5}$$

$$16\pi^2\frac{\partial C^{(1)}_{\mu ett}}{\partial\log\lambda} \simeq \left(\frac{30e^2}{c_W^2} + \frac{18e^2}{s_W^2}\right)C^{(3)}_{\mu ett} + \left(-\frac{11e^2}{3c_W^2} + \frac{15Y_t^2}{2} - 8g_S^2\right)C^{(1)}_{\mu ett}. \tag{4.6}$$

Supposing that the coefficients $C^{(3)}_{\mu euu}$, $C^{(3)}_{\mu ecc}$ and $C^{(3)}_{\mu ett}$ are of the same order, any subleading term can be dropped by retaining only the top-Yukawa and gauge couplings in the above equations. Combining eqs. (4.3) and (4.4) with eqs. (4.5) and (4.6), a relatively simple system of ordinary differential equations (SoODE) can be built and used to study the impact of the operators in tables 1 and 2 to $\mu \to e\gamma$.

It should be noted that our analysis is restricted to the operators listed in tables 1 and 2 even though there are additional D-6 operators that also contribute directly or indirectly to the running of $C_{e\gamma}$ and C_{eZ} [7–9]. In principle, a complete analysis including all D-6 operators should be performed, extending the SoODE presented above. However, the case of the operator $Q^{(1)}_{lequ}$ presented in this analysis is the most relevant one and serves as an illustration on how to obtain limits on a large class of Wilson coefficients of operators that are not directly related to the process under consideration.

Now that the SoODE is established, we can obtain limits on the various Wilson coefficients. The main idea is as follows: an effective theory is defined through its Wilson coefficients at some large scale Λ. We will consider the relevant coefficients one-by-one, i.e. setting $C_i(\Lambda) \neq 0$ and all the other $C_j(\Lambda) = 0$; $j \neq i$. Then we let the system evolve from $\lambda = \Lambda$ to the electroweak scale $\lambda = m_V$. At this scale, we confront $C_{e\gamma}(\lambda = m_V)$ with the experimental limit according to table 4. This will result in a constraint on $C_i(\Lambda)$. The same procedure could of course also be carried out using $C_{eZ}(m_V)$ rather than $C_{e\gamma}(m_V)$. However, the corresponding limits on the various $C_i(\Lambda)$ would always be less stringent.

It should also be mentioned that a rigorous application of EFT ideas requires to properly evolve the fixed order coefficient $C_{e\gamma}$ from the scale $\lambda = m_\mu$ to $\lambda = m_V$. Obviously, the RG equations given above are only applicable for the scales $\lambda > m_V$. At the electroweak scale, another matching of the theory to a second EFT should be made by integrating out the heavy SM fields, i.e. the fields of mass $\sim m_V$, very similar to what is done in the context of B decays (see e.g. [31]). The new EFT, valid for scales $\lambda < m_V$ then consists of operators with only (light)quark- and lepton fields as well as gluons and the photon. The anomalous dimensions of these operators then have to be computed in order to determine the complete running of the Wilson coefficient $C_{e\gamma}$ for scales $m_\mu < \lambda < m_V$. As the numerical effects of this procedure are rather modest, a somewhat simplified analysis is performed. As previously investigated in [32], for the running of $C_{e\gamma}(\lambda)$ below the electroweak scale

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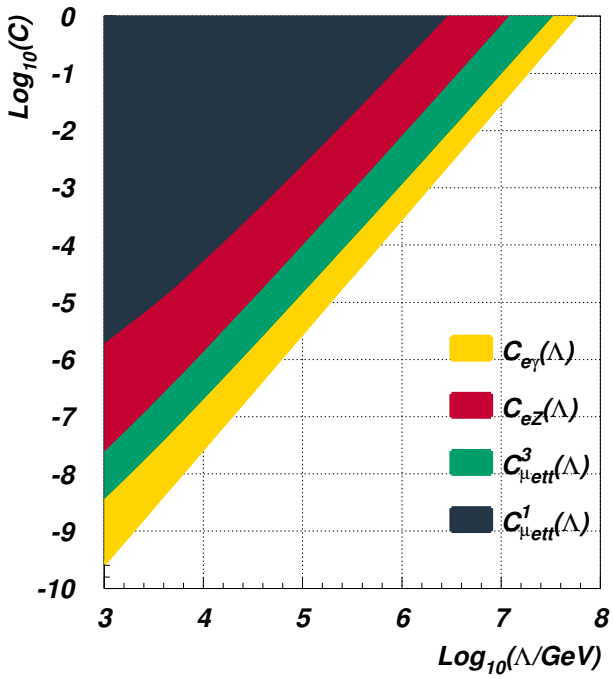

Figure 1. Constraints on $C^{\mu e}_{e\gamma}$ (yellow), $C^{\mu e}_{eZ}$ (green), $C^{(3)}_{\mu ett}$ (red) and $C^{(1)}_{\mu ett}$ (blue) plotted against the scale Λ at which they are defined. A $\log_{10}$-scale is adopted. The filled area represents the excluded regions.

only the QED contributions are taken into account. The corresponding RG equation reads

$$16\pi^2 \frac{\partial C_{e\gamma}}{\partial \log \lambda} \simeq e^2 \left(10 + \frac{4}{3}\sum_q e_q^2(\lambda)\right) C_{e\gamma}, \tag{4.7}$$

where the contribution of four-fermion operators has been omitted and $e_q(\lambda)$ denotes the electric charge of the fermion fields that are dynamical at the scale λ. Applying eq. (4.7) to the value of $C^{\mu e}_{e\gamma}(m_\mu)$ (and $C^{e\mu}_{e\gamma}(m_\mu)$) given in table 4 we obtain the limit

$$\sqrt{\frac{|C^{\mu e}_{e\gamma}(m_Z)|^2 + |C^{e\mu}_{e\gamma}(m_Z)|^2}{2}} < 1.8 \cdot 10^{-16} \frac{\Lambda^2}{[\mathrm{GeV}]^2}. \tag{4.8}$$

This is the limit that will be used to determine the constraints on the remaining Wilson coefficients at the scale Λ.

In the RG evolution only the Yukawa coupling of the top is kept and for all SM couplings one-loop running is implemented. Then the limits on the Wilson coefficients $C_{e\gamma}$, C_{eZ}, $C^{(3)}_{\mu ett}$ and $C^{(1)}_{\mu ett}$ are obtained as a function of the scale Λ. The results are displayed in figure 1. Not surprisingly, the most severe constraint is on $C_{e\gamma}$ itself. But also for C_{eZ} and $C^{(3)}_{\mu ett}$ which affect the running of $C_{e\gamma}$ directly, rather strong limits can be obtained. As expected, the limits on $C^{(1)}_{\mu ett}$ are weaker, as it affects $C_{e\gamma}$ only indirectly through $C^{(3)}_{\mu ett}$.

The dependence on Λ of the limits on $C_{e\gamma}$ is close to the canonical Λ^2 dependence, only slightly modified by the running of the Wilson coefficients. For the other Wilson

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3-P Coefficient	at $\Lambda = 10^3\,\text{GeV}$	at $\Lambda = 10^5\,\text{GeV}$	at $\Lambda = 10^7\,\text{GeV}$
$C^{\mu e}_{e\gamma}$	$2.7 \cdot 10^{-10}$	$2.9 \cdot 10^{-6}$	$3.1 \cdot 10^{-2}$
$C^{\mu e}_{eZ}$	$2.5 \cdot 10^{-8}$	$1.0 \cdot 10^{-4}$	$7.1 \cdot 10^{-1}$
$C^{(3)}_{\mu ett}$	$3.6 \cdot 10^{-9}$	$1.4 \cdot 10^{-5}$	$9.8 \cdot 10^{-2}$
$C^{(1)}_{\mu ett}$	$1.9 \cdot 10^{-6}$	$2.5 \cdot 10^{-3}$	n/a
$C^{(3)}_{\mu ecc}$	$4.8 \cdot 10^{-7}$	$1.9 \cdot 10^{-3}$	n/a
$C^{(1)}_{\mu ecc}$	$2.6 \cdot 10^{-4}$	$3.3 \cdot 10^{-1}$	n/a

Table 5. Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^5, 10^7\,\text{GeV}$.

coefficients, the effect of the running is somewhat larger. For illustrative purposes, in table 5, the numerical values for the Wilson coefficients for some choices of Λ are given. Relaxing the previous setup of only considering the top Yukawa coupling, the analysis can also be extended to include $C^{(3)}_{\mu ecc}$ and $C^{(1)}_{\mu ecc}$. Setting to zero all other Wilson coefficients at Λ, in particular, $C^{(3)}_{\mu ett}(\Lambda) = 0$ and $C^{(1)}_{\mu ett}(\Lambda) = 0$, it is then also possible to obtain limits on $C^{(3)}_{\mu ecc}(\Lambda)$ and $C^{(1)}_{\mu ecc}(\Lambda)$. It is clear that these limits get weaker with increasing Λ, ultimately reaching the limit of perturbativity $\sim 4\pi$.

Besides this, other assumptions can be made less strict: while eqs. (4.3) and (4.4) are complete, sub-leading terms can be gradually included in eqs. (4.5) and (4.6). As an example, reintroducing the bottom-Yukawa coupling and the CKM matrix off-diagonal terms, the following leading contributions arise:

$$16\pi^2 \frac{\partial C^{(3)}_{\mu ett}}{\partial \log \lambda} \simeq [\text{eq. (4.5)}] + Y_b^2 V_{33}^\dagger \left(C^{(3)}_{\mu eut} V_{13} + C^{(3)}_{\mu ect} V_{23} \right) + [\dots]\,, \tag{4.9}$$

$$16\pi^2 \frac{\partial C^{(1)}_{\mu ett}}{\partial \log \lambda} \simeq [\text{eq. (4.6)}] + 2 Y_b Y_t C_{\mu ebb} + [\dots]\,, \tag{4.10}$$

where $C_{\mu ebb}$ is a coefficient related to the Q_{ledq} operator, previously unconstrained. However, as soon as one includes other Yukawa couplings, the SoODE have to be enlarged to the point that many other computations are required. Nevertheless, in principle the method can be systematised and generalised to including each coefficient that could produce a (tree-level) $C_{e\gamma}$ transition at the muonic mass scale, even if the contribution to the evolution is not direct (as in the case of $C^{(1)}_{\mu ett}$).

To conclude this section, some limitations in our treatment are mentioned (again). First, this analysis has been done in a strict one-loop approximation, neglecting the possibility that for some operators two-loop contributions could be more important. This can happen in particular when through a two-loop effect a (small) Yukawa coupling is replaced by gauge couplings, as is the case in the Barr-Zee effect.

A second limitation regarding the limits presented in tables 4 and 5 is that they have been obtained assuming that only one coefficient at the time is non-zero. It is clear that such an assumption is rather unrealistic. A generic BSM model will usually introduce a

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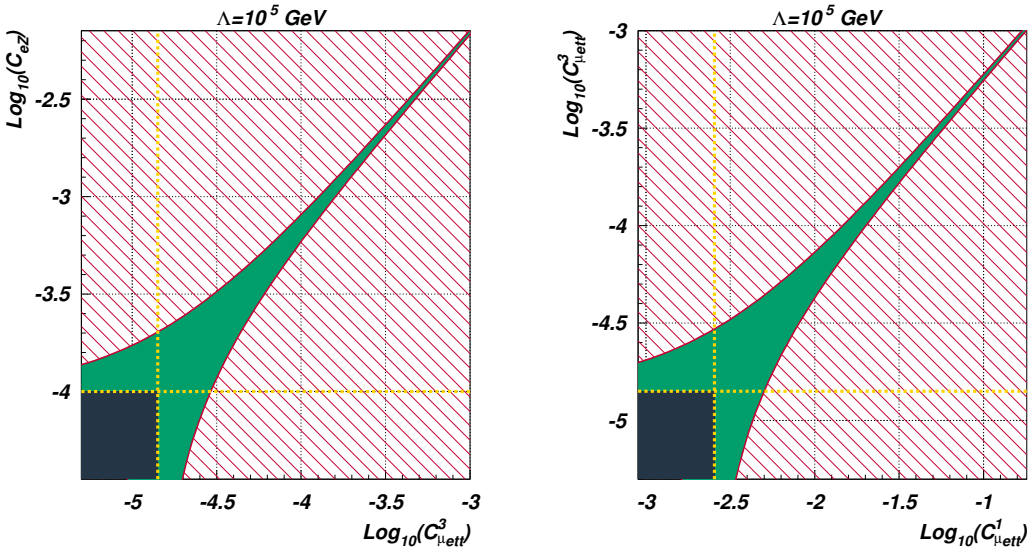

Figure 2. Correlations between $C_{eZ}^{\mu e}$ and $C_{\mu ett}^{(3)}$ (left) and $C_{\mu ett}^{(3)}$ and $C_{\mu ett}^{(1)}$ (right) at $\Lambda = 10^5$ GeV. The green area represents the allowed regions if both coefficients are allowed to deviate from zero.

large set of D-6 operators when heavy fields are integrated out. Allowing for more than one Wilson coefficient to be non zero, will introduce correlations that can lead to allowed regions that clearly violate the limits given in tables 4 and 5. As an example we consider the case when simultaneously $C_{eZ}(\Lambda)$ and $C_{\mu ett}^{(3)}(\Lambda)$ are non-vanishing (left panel of figure 2) as well as the case when simultaneously $C_{\mu ett}^{(3)}(\Lambda)$ and $C_{\mu ett}^{(1)}(\Lambda)$ are non-vanishing (right panel of figure 2). The allowed region (green) is clearly much larger than the allowed regions if only one non-vanishing coupling at the time is allowed (indicated by the yellow dotted lines). In principle, arbitrarily large values for $C_{\mu ett}^{(3)}(\Lambda)$ are allowed, as long as $C_{eZ}(\Lambda)$ or $C_{\mu ett}^{(1)}(\Lambda)$ are tuned to provide an almost perfect cancellation. Such a fine-tuned choice of couplings is of course very unnatural and at some point is in conflict with the fixed-order constraint of C_{eZ}. Nevertheless, it has to be mentioned that the limits presented in this analysis are to be taken more as guidelines rather than strict limits. A more complete analysis with several observables would be required to disentangle the correlations and get strict limits on the various Wilson coefficients.

Finally, we recall that for $\lambda < m_V$ we have considered only the running of $C_{e\gamma}$ induced by the pure QED contributions. The effect of the running of $C_{e\gamma}$ from $\lambda = m_\mu$ to $\lambda = m_V$ is below 10% and we have checked that the impact of the terms with Yukawa couplings is completely negligible. Hence, the use of this approximation will affect the limits presented here by a few percent at most. The only possible exception to this is the limit on $C_{\mu ecc}^{(3)}$. As can be seen from eq. (4.2), if $C_{\mu ecc}^{(3)}$ is much larger than $C_{e\gamma}$ the running of $C_{e\gamma}$ for $m_c < \lambda < m_V$ is modified noticeably. Such a situation can occur when considering the case $C_{\mu ecc}^{(3)}(\Lambda) \neq 0$ and all other $C_i(\Lambda) = 0$, as done in obtaining the limit on $C_{\mu ecc}^{(3)}$. In particular, if Λ is rather small, a very large $C_{\mu ecc}^{(3)}(\Lambda)$ is required to induce a sizable $C_{e\gamma}(m_V)$. We have checked that, depending on the choice of Λ, the naive limits obtained by having

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only $C^{(3)}_{\mu ecc}(\Lambda) \neq 0$ can be modified by up to a factor two when taking into account its contribution to the RG evolution for $\lambda < m_V$. The effect will be much smaller for a more realistic scenario with several non-vanishing coefficients at the large scale Λ.

5 Conclusions

In this paper a complete one-loop analysis of the LFV decay $\mu \to e\gamma$ in the context of an EFT with D-6 operators has been presented. The main results are the limits on the (scale-dependent) Wilson coefficients at the large matching scale. These limits provide the most direct information on possible BSM models that can be obtained from the $\mu \to e\gamma$ decay in an EFT framework.

It is not surprising that the limit on $\mathrm{BR}(\mu^+ \to e^+\gamma)$ results in a constraint on $C_{e\gamma}$, the Wilson coefficient of the operator $Q_{e\gamma}$ that induces a tree-level $\mu \to e\gamma$ transition. What is more remarkable is that constraints can be obtained also for a rather large number of further Wilson coefficients. These belong to operators that indirectly induce a LFV transition, either at one loop or through mixing under RG evolution. In this context it is important to note that the Wilson coefficients are scale dependent quantities and that in general operators mix under RG evolution. Thus, the presence at the large matching scale of any non-vanishing Wilson coefficient for an operator that mixes with $Q_{e\gamma}$ under RG evolution will induce a LFV transition $\mu \to e\gamma$ at the low scale.

It is clear that such an analysis can be applied to other processes as well. In particular, other LFV decays such as $\tau \to e\gamma$ or $\tau \to \mu\gamma$ lead immediately to similar constraints for the D-6 operators with other generation indices, as detailed in appendix C. But in principle, any observable for which there are strong experimental constraints can be used. A combined analysis with many observables will also potentially allow to disentangle correlations between Wilson coefficients. Such correlations in the RG running result in unnatural allowed regions which are governed by large cancellations.

Depending on the process under consideration the inclusion of all D-6 operators, not only those listed in tables 1 and 2 might be required. While this results in a more complicated system, such an analysis allows to combine consistently experimental results that have been obtained at completely different energy scales. In the absence of clear evidence for BSM physics at collider experiments, an extended EFT analysis providing constraints on many Wilson coefficients directly at the large scale can give useful clues in the search for a realistic BSM scenario and we consider this to be a very promising and useful strategy.

Acknowledgments

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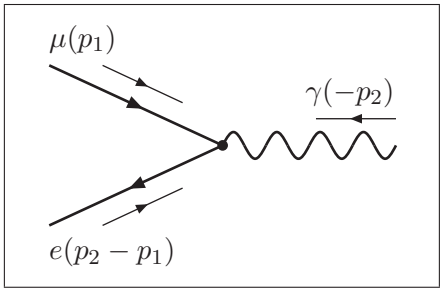
A D-6 effective $\mu - e - \gamma$ interaction at one-loop: the Feynman rule

In this appendix, the Feynman rule for the $\mu - e - \gamma$ interaction in the context of a D-6 ET is presented together with a complete treatment of the LFV wave-function renormalisation. Here and in appendix B we keep the generation indices of the Wilson coefficients $C_{e\gamma}$, C_{eZ} and $C_{e\varphi}$, but for notational simplicity drop the complex conjugate sign, i.e $(C^{\mu e}_{e\gamma})^* \to C^{\mu e}_{e\gamma}$.

In eq. (A.1), the structure of the interaction is introduced in terms of the new scale Λ and four effective coefficients related to the four possible contributions: vectorial left/right (K_{VL}/K_{VR}) and tensorial left/right (K_{TL}/K_{TR}). All momenta are considered to be incoming.



$$= \frac{1}{\Lambda^2}\left[\gamma^\mu \left(K_{VL}\,\omega_L + K_{VR}\,\omega_R\right) + i\sigma^{\mu\nu}\left(K_{TL}\,\omega_L + K_{TR}\,\omega_R\right)(p_2)_\nu\right]. \tag{A.1}$$

The coefficients of eq. (A.1) are connected to the one-loop wave-function renormalisation factors through

$$\frac{K_{VL}}{\Lambda^2} = -\frac{e}{2}\left(\frac{1}{2}\delta Z^L_{e\mu} + \frac{1}{2}\left(\delta Z^L_{e\mu}\right)^\dagger\right) - \frac{ev^2}{4c_W s_W \Lambda^2}\left(C^{(1)}_{\varphi l} + C^{(3)}_{\varphi l}\right)\frac{1}{2}\delta Z_{ZA}, \tag{A.2}$$

$$\frac{K_{VR}}{\Lambda^2} = -\frac{e}{2}\left(\frac{1}{2}\delta Z^R_{e\mu} + \frac{1}{2}\left(\delta Z^R_{e\mu}\right)^\dagger\right) - \frac{ev^2}{4c_W s_W \Lambda^2} C_{\varphi e}\frac{1}{2}\delta Z_{ZA}, \tag{A.3}$$

$$\frac{K_{TL}}{\Lambda^2} = -\frac{v}{\sqrt{2}\Lambda^2} C^{\mu e}_{e\gamma}\left(1 + \frac{1}{2}\delta Z^L_{\mu\mu} + \frac{1}{2}\left(\delta Z^R_{ee}\right)^\dagger + \frac{1}{2}\delta Z_{AA} + \frac{\delta v}{v}\right) - \frac{v}{\sqrt{2}\Lambda^2} C^{\mu e}_{eZ}\frac{1}{2}\delta Z_{ZA}, \tag{A.4}$$

$$\frac{K_{TR}}{\Lambda^2} = -\frac{v}{\sqrt{2}\Lambda^2} C^{e\mu}_{e\gamma}\left(1 + \frac{1}{2}\delta Z^R_{\mu\mu} + \frac{1}{2}\left(\delta Z^L_{ee}\right)^\dagger + \frac{1}{2}\delta Z_{AA} + \frac{\delta v}{v}\right) - \frac{v}{\sqrt{2}\Lambda^2} C^{e\mu}_{eZ}\frac{1}{2}\delta Z_{ZA}. \tag{A.5}$$

Several elements of eqs. (A.2)–(A.5) do not belong to the SM framework: the effective coefficients $C_{e\gamma}$, C_{eZ}, $C^{(1)}_{\varphi l}$, $C^{(3)}_{\varphi l}$ and $C_{\varphi e}$, plus the off-diagonal leptonic wave-function renormalisation. For further information, a complete treatment of LFV wave-function renormalisation in the on-shell scheme is given.

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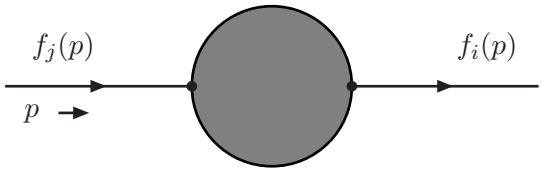

Figure 3. Conventions used for the one-particle irreducible two-point functions.

Making use of standard techniques (e.g., see [19]), the off-diagonal leptonic self-energy (for conventions used see figure 3) was calculated. Then, the renormalisation conditions in the on-shell scheme have been applied to obtain the various contributions to the off-diagonal wave-function renormalisation. The tensorial structure that corresponds to such transition consists of four possible coefficients:

$$\Gamma^f_{ij}(p) = i\delta_{ij}(\not{p} - m_i) + i\Big[\not{p}\,\omega_L\,\Sigma^{f,L}_{ij}(p^2) + \not{p}\,\omega_R\,\Sigma^{f,R}_{ij}(p^2) + \omega_L\,\Sigma^{f,l}_{ij}(p^2) + \omega_R\,\Sigma^{f,r}_{ij}(p^2)\Big]. \quad \text{(A.6)}$$

By applying the standard on-shell renormalisation conditions

$$\mathrm{Re}\Big[\Gamma^f_{ij}(p)\Big]\,u_j(p)\Big|_{p^2=m_j^2} = 0, \quad \text{(A.7)}$$

$$\bar{u}_i(p)\mathrm{Re}\Big[\Gamma^f_{ij}(p)\Big]_{p^2=m_i^2} = 0, \quad \text{(A.8)}$$

one finds the off-diagonal wave-function renormalisation that is required in eqs. (A.2) and (A.3) to determine the coefficients K_{VL} and K_{VR} of eq. (A.1):

$$\delta Z^L_{ij} = \frac{4}{m_i^2 - m_j^2}\left(m_j^2\Sigma^{f,L}_{ij}(m_j^2) + m_i m_j \Sigma^{f,R}_{ij}(m_j^2) + m_j\Sigma^{f,r}_{ij}(m_j^2) + m_i\Sigma^{f,l}_{ij}(m_j^2)\right), \quad \text{(A.9)}$$

$$\delta Z^R_{ij} = \frac{4}{m_i^2 - m_j^2}\left(m_j^2\Sigma^{f,R}_{ij}(m_j^2) + m_i m_j \Sigma^{f,L}_{ij}(m_j^2) + m_j\Sigma^{f,l}_{ij}(m_j^2) + m_i\Sigma^{f,r}_{ij}(m_j^2)\right). \quad \text{(A.10)}$$

The explicit result for the four coefficients of eq. (A.6) are as follows:

$$\begin{aligned}
&\Sigma^{f,L}_{e\mu}(p^2)\Lambda^2 \\
&= A_0\big[m_e^2\big]\left(-\frac{m_e}{64\pi^2}C^{(1)}_{\varphi l} - \frac{m_e}{64\pi^2}C^{(3)}_{\varphi l} - \frac{3m_Z s_W^2}{16\sqrt{2}\pi^2}C^{\mu e}_{eZ} + \frac{3m_Z(2s_W c_W)}{32\sqrt{2}\pi^2}C^{\mu e}_{e\gamma}\right) \\
&\quad + A_0\big[m_\mu^2\big]\left(\frac{m_\mu}{64\pi^2}C_{\varphi e} + \frac{3m_Z(c_W^2 - s_W^2)}{32\sqrt{2}\pi^2}C^{\mu e}_{eZ} + \frac{3m_Z(2s_W c_W)}{32\sqrt{2}\pi^2}C^{\mu e}_{e\gamma}\right) \\
&\quad + A_0\big[m_W^2\big]\left(\frac{3m_Z c_W^2}{16\sqrt{2}\pi^2}C^{\mu e}_{eZ} + \frac{3m_Z(2s_W c_W)}{32\sqrt{2}\pi^2}C^{\mu e}_{e\gamma}\right) \\
&\quad + A_0\big[\xi_W m_W^2\big]\left(-\frac{m_e}{32\pi^2}C^{(3)}_{\varphi l} - \frac{m_Z(2s_W c_W)}{32\sqrt{2}e\pi^2}C^{\mu e}_{e\varphi}\right) \\
&\quad + A_0\big[m_Z^2\big]\left(\frac{3m_Z(-1 + 2(c_W^2 - s_W^2))}{32\sqrt{2}\pi^2}C^{\mu e}_{eZ}\right)
\end{aligned}$$

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$$
\begin{aligned}
&+ A_0\big[\xi_Z m_Z^2\big]\left(\frac{m_\mu}{64\pi^2}C_{\varphi e} - \frac{m_e}{64\pi^2}C_{\varphi l}^{(1)} - \frac{m_e}{64\pi^2}C_{\varphi l}^{(3)} - \frac{m_Z c_W s_W}{32\sqrt{2}e\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ A_0\big[m_H^2\big]\left(-\frac{3m_Z c_W s_W}{32\sqrt{2}e\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ B_0\big[p^2, m_Z^2, m_e^2\big]\left(\frac{3m_e m_Z^2 s_W^2}{16\pi^2}C_{\varphi l}^{(1)} + \frac{3m_e m_Z^2 s_W^2}{16\pi^2}C_{\varphi l}^{(3)} - \frac{3m_Z\big(m_e^2 + m_Z^2 - p^2\big)s_W^2}{16\sqrt{2}\pi^2}C_{eZ}^{\mu e}\right)\\
&+ B_0\big[p^2, \xi_Z m_Z^2, m_e^2\big]\left(\frac{m_e\big(-m_e^2 + \xi_Z m_Z^2 + p^2 - 2\xi_Z m_Z^2(c_W^2 - s_W^2)\big)}{64\pi^2}(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)})\right)\\
&+ B_0\big[p^2, m_Z^2, m_\mu^2\big]\left(-\frac{3m_\mu m_Z^2(c_W^2 - s_W^2)}{32\pi^2}C_{\varphi e} + \frac{3m_Z\big(m_\mu^2 + m_Z^2 - p^2\big)(c_W^2 - s_W^2)}{32\sqrt{2}\pi^2}C_{eZ}^{\mu e}\right)\\
&+ B_0\big[p^2, \xi_Z m_Z^2, m_\mu^2\big]\left(\frac{m_\mu\big(m_\mu^2 + \xi_Z m_Z^2 - p^2 - 2\xi_Z m_Z^2(c_W^2 - s_W^2)\big)}{64\pi^2}C_{\varphi e}\right)\\
&+ B_0\big[p^2, m_W^2, 0\big]\left(\frac{3m_Z\big(m_W^2 - p^2\big)c_W^2}{16\sqrt{2}\pi^2}C_{eZ}^{\mu e} + \frac{3m_Z\big(m_W^2 - p^2\big)(2s_W c_W)}{32\sqrt{2}\pi^2}C_{e\gamma}^{\mu e}\right)\\
&+ B_0\big[p^2, \xi_W m_W^2, 0\big]\left(\frac{m_e\big(-\xi_W m_W^2 + p^2\big)}{32\pi^2}C_{\varphi l}^{(3)}\right)\\
&+ B_0\big[p^2, m_H^2, m_e^2\big]\left(-\frac{m_e^2 m_Z(2s_W c_W)}{32\sqrt{2}e\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ B_0\big[p^2, m_H^2, m_\mu^2\big]\left(-\frac{m_\mu^2 m_Z(2s_W c_W)}{32\sqrt{2}e\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ B_0\big[p^2, 0, m_e^2\big]\left(\frac{3m_Z\big(m_e^2 - p^2\big)(2s_W c_W)}{32\sqrt{2}\pi^2}C_{e\gamma}^{\mu e}\right)\\
&+ B_0\big[p^2, 0, m_\mu^2\big]\left(\frac{3m_Z\big(m_\mu^2 - p^2\big)(2s_W c_W)}{32\sqrt{2}\pi^2}C_{e\gamma}^{\mu e}\right)\\
&+ \frac{m_\mu m_Z^2(c_W^2 - s_W^2)}{16\pi^2}C_{\varphi e} - \frac{m_e m_Z^2 s_W^2}{8\pi^2}C_{\varphi l}^{(1)} - \frac{m_e m_Z^2 s_W^2}{8\pi^2}C_{\varphi l}^{(3)}\\
&- \frac{m_Z}{16\sqrt{2}\pi^2}\big(-2\big(m_e^2 - m_W^2 + m_Z^2\big) + \big(2m_e^2 + 2m_\mu^2 + 2m_W^2 + 4m_Z^2 - 3p^2\big)(c_W^2 - s_W^2)\big)C_{eZ}^{\mu e}\\
&- \frac{m_Z\big(2m_e^2 + 2m_\mu^2 + 2m_W^2 - 3p^2\big)c_W s_W}{8\sqrt{2}\pi^2}C_{e\gamma}^{\mu e}.
\end{aligned}
\tag{A.11}
$$

$$
\Sigma_{e\mu}^{f,R}(p^2) = \Sigma_{e\mu}^{f,L}(p^2)\big|_{\mu\leftrightarrow e}
\tag{A.12}
$$

$$
\begin{aligned}
&\Sigma_{e\mu}^{f,l}(p^2)\Lambda^2\\
&= A_0\big[m_e^2\big]\left(-\frac{\big(m_e^2 + 2m_Z^2(c_W^2 - s_W^2)\big)}{64p^2\pi^2}(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}) + \frac{m_e m_Z c_W s_W}{32\sqrt{2}ep^2\pi^2}C_{e\varphi}^{\mu e}\right.\\
&\quad\left. + \frac{3m_e m_Z(c_W^2 - s_W^2)}{32\sqrt{2}p^2\pi^2}C_{eZ}^{\mu e} + \frac{3m_e m_Z(2s_W c_W)}{32\sqrt{2}p^2\pi^2}C_{e\gamma}^{\mu e}\right)
\end{aligned}
$$

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\begin{aligned}
&+ A_0\big[m_\mu^2\big]\left(-\frac{\big(m_\mu^2+2m_Z^2(c_W^2-s_W^2)\big)}{64p^2\pi^2}(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})+\frac{m_\mu m_Z(2s_Wc_W)}{64\sqrt{2}ep^2\pi^2}C_{e\varphi}^{e\mu}\right.\\
&\left.+\frac{3m_\mu m_Z(c_W^2-s_W^2)}{32\sqrt{2}p^2\pi^2}C_{eZ}^{e\mu}+\frac{3m_\mu m_Z(2s_Wc_W)}{32\sqrt{2}p^2\pi^2}C_{e\gamma}^{e\mu}\right)\\
&+ A_0\big[m_W^2\big]\left(\frac{m_Z^2\big(2m_W^2-p^2\big)\,c_W^2}{16m_W^2p^2\pi^2}C_{\varphi l}^{(3)}\right)\\
&+ A_0\big[\xi_W m_W^2\big]\left(\frac{m_Z^2c_W^2}{16m_W^2\pi^2}C_{\varphi l}^{(3)}\right)\\
&+ A_0\big[m_Z^2\big]\left(\frac{\big(m_e^2+m_\mu^2+4m_Z^2-2p^2\big)\,(c_W^2-s_W^2)}{64p^2\pi^2}(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})\right.\\
&\left.-\frac{3m_\mu m_Z(c_W^2-s_W^2)}{32\sqrt{2}p^2\pi^2}C_{eZ}^{e\mu}-\frac{3m_e m_Z(c_W^2-s_W^2)}{32\sqrt{2}p^2\pi^2}C_{eZ}^{\mu e}\right)\\
&+ A_0\big[\xi_Z m_Z^2\big]\left(\frac{\big(m_e^2+m_\mu^2-\big(m_e^2+m_\mu^2-2p^2\big)\,(c_W^2-s_W^2)\big)}{64p^2\pi^2}(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})\right)\\
&+ A_0\big[m_H^2\big]\left(-\frac{m_\mu m_Z c_W s_W}{32\sqrt{2}ep^2\pi^2}C_{e\varphi}^{e\mu}-\frac{m_e m_Z c_W s_W}{32\sqrt{2}ep^2\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ B_0\big[p^2,m_Z^2,m_e^2\big]\left(\frac{\big(m_e^4-2m_Z^4+m_e^2\big(m_Z^2-2p^2\big)+m_Z^2p^2+p^4\big)\,(c_W^2-s_W^2)}{64p^2\pi^2}(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})\right.\\
&\left.+\frac{3m_e m_Z\big(-m_e^2+m_Z^2+p^2\big)\,(c_W^2-s_W^2)}{32\sqrt{2}p^2\pi^2}C_{eZ}^{\mu e}\right)\\
&+ B_0\big[p^2,\xi_Z m_Z^2,m_e^2\big]\left(\frac{1}{64p^2\pi^2}\big(m_e^2\big(m_e^2-\xi_Z m_Z^2-p^2\big)+\big(-m_e^4+\big(\xi_Z m_Z^2-p^2\big)\,p^2\right.\\
&\left.+m_e^2\big(\xi_Z m_Z^2+2p^2\big)\big)\,(c_W^2-s_W^2)\big)\,(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})\right)\\
&+ B_0\big[p^2,m_Z^2,m_\mu^2\big]\left(\frac{\big(m_\mu^4-2m_Z^4+m_\mu^2\big(m_Z^2-2p^2\big)+m_Z^2p^2+p^4\big)\,(c_W^2-s_W^2)}{64p^2\pi^2}(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})\right.\\
&\left.+\frac{3m_\mu m_Z\big(-m_\mu^2+m_Z^2+p^2\big)\,(c_W^2-s_W^2)}{32\sqrt{2}p^2\pi^2}C_{eZ}^{e\mu}\right)\\
&+ B_0\big[p^2,\xi_Z m_Z^2,m_\mu^2\big]\left(\frac{1}{64p^2\pi^2}\big(m_\mu^2\big(m_\mu^2-\xi_Z m_Z^2-p^2\big)+\big(-m_\mu^4+\big(\xi_Z m_Z^2-p^2\big)\,p^2\right.\\
&\left.+m_\mu^2\big(\xi_Z m_Z^2+2p^2\big)\big)\,(c_W^2-s_W^2)\big)\,(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})\right)\\
&+ B_0\big[p^2,m_W^2,0\big]\left(-\frac{m_Z^2\big(2m_W^4-m_W^2p^2-p^4\big)\,c_W^2}{16m_W^2p^2\pi^2}C_{\varphi l}^{(3)}\right)\\
&+ B_0\big[p^2,\xi_W m_W^2,0\big]\left(\frac{m_Z^2\big(\xi_W m_W^2-p^2\big)\,c_W^2}{16m_W^2\pi^2}C_{\varphi l}^{(3)}\right)\\
&+ B_0\big[p^2,m_H^2,m_e^2\big]\left(-\frac{m_e m_Z\big(m_e^2-m_H^2+p^2\big)\,(2s_Wc_W)}{64\sqrt{2}ep^2\pi^2}C_{e\varphi}^{\mu e}\right)
\end{aligned}
$$

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$$
\begin{aligned}
&+ B_0\left[p^2, m_H^2, m_\mu^2\right]\left(-\frac{m_\mu m_Z\left(-m_H^2+m_\mu^2+p^2\right)\left(2s_W c_W\right)}{64\sqrt{2}ep^2\pi^2}C_{e\varphi}^{e\mu}\right)\\
&+ B_0\left[p^2, 0, m_e^2\right]\left(-\frac{3m_e m_Z\left(m_e^2-p^2\right)\left(2s_W c_W\right)}{32\sqrt{2}p^2\pi^2}C_{e\gamma}^{\mu e}\right)\\
&+ B_0\left[p^2, 0, m_\mu^2\right]\left(-\frac{3m_\mu m_Z\left(m_\mu^2-p^2\right)\left(2s_W c_W\right)}{32\sqrt{2}p^2\pi^2}C_{e\gamma}^{e\mu}\right)\\
&- \frac{m_Z^2(c_W^2-s_W^2)}{16\pi^2}C_{\varphi l}^{(1)} - \frac{m_Z^2(1+2(c_W^2-s_W^2))}{16\pi^2}C_{\varphi l}^{(3)} - \frac{m_\mu m_Z(c_W^2-s_W^2)}{16\sqrt{2}\pi^2}C_{eZ}^{e\mu}\\
&- \frac{m_e m_Z(c_W^2-s_W^2)}{16\sqrt{2}\pi^2}C_{eZ}^{\mu e} - \frac{m_\mu m_Z c_W s_W}{8\sqrt{2}\pi^2}C_{e\gamma}^{e\mu} - \frac{m_e m_Z c_W s_W}{8\sqrt{2}\pi^2}C_{e\gamma}^{\mu e}. \qquad \text{(A.13)}
\end{aligned}
$$

$$
\begin{aligned}
&\Sigma_{e\mu}^{f,r}(p^2)\Lambda^2\\
&= A_0\left[m_e^2\right]\left(\frac{\left(m_e^2+2m_Z^2-2m_Z^2(c_W^2-s_W^2)\right)}{64p^2\pi^2}C_{\varphi e} + \frac{m_e m_Z c_W s_W}{32\sqrt{2}ep^2\pi^2}C_{e\varphi}^{e\mu}\right.\\
&\left.- \frac{3m_e m_Z s_W^2}{16\sqrt{2}p^2\pi^2}C_{eZ}^{e\mu} + \frac{3m_e m_Z(2s_W c_W)}{32\sqrt{2}p^2\pi^2}C_{e\gamma}^{e\mu}\right)\\
&+ A_0\left[m_\mu^2\right]\left(\frac{\left(m_\mu^2+2m_Z^2-2m_Z^2(c_W^2-s_W^2)\right)}{64p^2\pi^2}C_{\varphi e} + \frac{m_\mu m_Z(2s_W c_W)}{64\sqrt{2}ep^2\pi^2}C_{e\varphi}^{\mu e}\right.\\
&\left.- \frac{3m_\mu m_Z s_W^2}{16\sqrt{2}p^2\pi^2}C_{eZ}^{\mu e} + \frac{3m_\mu m_Z(2s_W c_W)}{32\sqrt{2}p^2\pi^2}C_{e\gamma}^{\mu e}\right)\\
&+ A_0\left[m_Z^2\right]\left(-\frac{\left(m_e^2+m_\mu^2+4m_Z^2-2p^2\right)s_W^2}{32p^2\pi^2}C_{\varphi e} + \frac{3m_e m_Z s_W^2}{16\sqrt{2}p^2\pi^2}C_{eZ}^{e\mu} + \frac{3m_\mu m_Z s_W^2}{16\sqrt{2}p^2\pi^2}C_{eZ}^{\mu e}\right)\\
&+ A_0\left[\xi_Z m_Z^2\right]\left(-\frac{\left(2p^2+\left(m_e^2+m_\mu^2-2p^2\right)(c_W^2-s_W^2)\right)}{64p^2\pi^2}C_{\varphi e}\right)\\
&+ A_0\left[m_H^2\right]\left(-\frac{m_e m_Z c_W s_W}{32\sqrt{2}ep^2\pi^2}C_{e\varphi}^{e\mu} - \frac{m_\mu m_Z c_W s_W}{32\sqrt{2}ep^2\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ B_0\left[p^2, m_Z^2, m_e^2\right]\left(-\frac{\left(m_e^4-2m_Z^4+m_e^2\left(m_Z^2-2p^2\right)+m_Z^2p^2+p^4\right)s_W^2}{32p^2\pi^2}C_{\varphi e}\right.\\
&\left.- \frac{3m_e m_Z\left(-m_e^2+m_Z^2+p^2\right)s_W^2}{16\sqrt{2}p^2\pi^2}C_{eZ}^{e\mu}\right)\\
&+ B_0\left[p^2, \xi_Z m_Z^2, m_e^2\right]\left(\frac{1}{64p^2\pi^2}\left(p^2\left(-m_e^2-\xi_Z m_Z^2+p^2\right)+\left(-m_e^4+\left(\xi_Z m_Z^2-p^2\right)p^2\right.\right.\right.\\
&\left.\left.\left.+m_e^2\left(\xi_Z m_Z^2+2p^2\right)\right)(c_W^2-s_W^2)\right)C_{\varphi e}\right)\\
&+ B_0\left[p^2, m_Z^2, m_\mu^2\right]\left(-\frac{\left(m_\mu^4-2m_Z^4+m_\mu^2\left(m_Z^2-2p^2\right)+m_Z^2p^2+p^4\right)s_W^2}{32p^2\pi^2}C_{\varphi e}\right.\\
&\left.- \frac{3m_\mu m_Z\left(-m_\mu^2+m_Z^2+p^2\right)s_W^2}{16\sqrt{2}p^2\pi^2}C_{eZ}^{\mu e}\right)
\end{aligned}
$$

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\begin{aligned}
&+ B_0\left[p^2, \xi_Z m_Z^2, m_\mu^2\right]\left(\frac{1}{64p^2\pi^2}\left(p^2\left(-m_\mu^2 - \xi_Z m_Z^2 + p^2\right) + \left(-m_\mu^4 + \left(\xi_Z m_Z^2 - p^2\right)p^2\right.\right.\right.\\
&\left.\left.\left.+ m_\mu^2\left(\xi_Z m_Z^2 + 2p^2\right)\right)\left(c_W^2 - s_W^2\right)\right) C_{\varphi e}\right)\\
&+ B_0\left[p^2, m_H^2, m_e^2\right]\left(-\frac{m_e m_Z\left(m_e^2 - m_H^2 + p^2\right)(2 s_W c_W)}{64\sqrt{2} e p^2 \pi^2} C_{e\varphi}^{e\mu}\right)\\
&+ B_0\left[p^2, m_H^2, m_\mu^2\right]\left(-\frac{m_\mu m_Z\left(-m_H^2 + m_\mu^2 + p^2\right)(2 s_W c_W)}{64\sqrt{2} e p^2 \pi^2} C_{e\varphi}^{\mu e}\right)\\
&+ B_0\left[p^2, 0, m_e^2\right]\left(-\frac{3 m_e m_Z\left(m_e^2 - p^2\right)(2 s_W c_W)}{32\sqrt{2} p^2 \pi^2} C_{e\gamma}^{e\mu}\right)\\
&+ B_0\left[p^2, 0, m_\mu^2\right]\left(-\frac{3 m_\mu m_Z\left(m_\mu^2 - p^2\right)(2 s_W c_W)}{32\sqrt{2} p^2 \pi^2} C_{e\gamma}^{\mu e}\right)\\
&+ \frac{m_Z^2 s_W^2}{8\pi^2} C_{\varphi e} - \frac{m_e m_Z c_W s_W}{8\sqrt{2}\pi^2} C_{e\gamma}^{e\mu} - \frac{m_\mu m_Z c_W s_W}{8\sqrt{2}\pi^2} C_{e\gamma}^{\mu e}\\
&+ \frac{m_e m_Z s_W^2}{8\sqrt{2}\pi^2} C_{eZ}^{e\mu} + \frac{m_\mu m_Z s_W^2}{8\sqrt{2}\pi^2} C_{eZ}^{\mu e}.
\end{aligned}
\tag{A.14}
$$

The explicit results for the four coefficients of the off-diagonal one-particle irreducible two-point function for leptons are sufficient to obtain the wave-function renormalisation factors eqs. (A.9) and (A.10).

Finally, for completeness we list the required SM expressions for the renormalisation. The expression

$$
\delta Z_{ZA} = \frac{\left(c_W e^2\left(2\left(-1 + \xi_W^2\right) m_W^2 - (-9 + \xi_W) A_0\left[m_W^2\right] - (5 + 3\xi_W) A_0\left[\xi_W m_W^2\right]\right)\right)}{\left(48(-1 + \xi_W) m_W^2 \pi^2 s_W\right)} \tag{A.15}
$$

is needed in eqs. (A.2)–(A.5) and the following expressions in the $\overline{\text{MS}}$ scheme are required for the computation of the anomalous dimensions analysed in section 4.2:

$$
\widehat{\Delta}^{-1}\delta Z_{AA} = -\frac{e^2(20 + 3\xi_W)}{48\pi^2}, \tag{A.16}
$$

$$
\widehat{\Delta}^{-1}\delta Z_{ZZ} = -\frac{e^2\left(-1 + 2s_W^2 + 40 s_W^4 + 6 c_W^4 \xi_W\right)}{96\pi^2 c_W^2 s_W^2}, \tag{A.17}
$$

$$
\widehat{\Delta}^{-1}\delta Z_e = -\frac{1}{2}\delta Z_{AA} + \frac{s_W}{c_W}\frac{1}{2}\delta Z_{ZA} = \frac{11 e^2}{96\pi^2}, \tag{A.18}
$$

$$
\begin{aligned}
\widehat{\Delta}^{-1}\delta m_W^2 = &+\frac{e^2 m_W^2}{32\pi^2 s_W^2}\xi_W + \frac{e^2 m_Z^2}{64\pi^2 s_W^2}\xi_Z\\
&- \frac{e^2\left(6\left(N_c \sum_q m_q^2 + \sum_l m_l^2\right) + 11 m_Z^2 - 20 m_Z^2 s_W^2\right)}{192\pi^2 s_W^2},
\end{aligned}
\tag{A.19}
$$

$$
\begin{aligned}
\widehat{\Delta}^{-1}\delta m_Z^2 = &+\frac{e^2 m_W^2}{32\pi^2 c_W^2 s_W^2}\xi_W + \frac{e^2 m_Z^2}{64\pi^2 c_W^2 s_W^2}\xi_Z\\
&- \frac{e^2\left(6\left(N_c \sum_q m_q^2 + \sum_l m_l^2\right) + 11 m_Z^2 - 58 m_Z^2 s_W^2 - 44 m_Z^2 s_W^4\right)}{192\pi^2 c_W^2 s_W^2},
\end{aligned}
\tag{A.20}
$$

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$$\widehat{\Delta}^{-1}\frac{\delta v}{v} = \left(1-\frac{1}{2s_W^2}\right)\frac{\delta m_W^2}{m_W^2} + \frac{c_W^2}{2s_W^2}\frac{\delta m_Z^2}{m_Z^2} - \delta Z_e = \frac{e^2}{64\pi^2 s_W^2}\xi_W + \frac{e^2}{128\pi^2 c_W^2 s_W^2}\xi_Z - \frac{e^2\left(2\left(N_c\sum_q m_q^2 + \sum_l m_l^2\right) - 9m_Z^2 + 6m_Z^2 s_W^2\right)}{128 m_Z^2\pi^2 c_W^2 s_W^2}, \quad \text{(A.21)}$$

$$\widehat{\Delta}^{-1}\delta Z_{ll}^R = -\frac{e^2\left(m_l^2 + 2m_Z^2 s_W^2\left(c_W^2\xi_\gamma + s_W^2\xi_Z\right)\right)}{32 m_Z^2\pi^2 c_W^2 s_W^2}, \quad \text{(A.22)}$$

$$\widehat{\Delta}^{-1}\delta Z_{ll}^L = -\frac{e^2\left(m_l^2 + m_Z^2\left(4c_W^2 s_W^2\xi_\gamma + 2c_W^2\xi_W + \left(1-2s_W^2\right)^2\xi_Z\right)\right)}{64 m_Z^2\pi^2 c_W^2 s_W^2}, \quad \text{(A.23)}$$

$$\widehat{\Delta}^{-1}\delta Z_{tt}^R = -\frac{e^2 m_t^2}{32 m_Z^2\pi^2 s_W^2 c_W^2} - \frac{e^2}{36\pi^2}\xi_\gamma - \frac{e^2 s_W^2}{36\pi^2 c_W^2}\xi_Z - \frac{g_S^2}{12\pi^2}\xi_G, \quad \text{(A.24)}$$

$$\widehat{\Delta}^{-1}\delta Z_{tt}^L = -\frac{e^2\left(m_t^2 + \sum_d |V_{d3}|^2 m_d^2\right)}{64 m_Z^2\pi^2 s_W^2 c_W^2} - \frac{e^2}{36\pi^2}\xi_\gamma - \frac{e^2}{32\pi^2 s_W^2}\xi_W - \frac{e^2\left(3-4s_W^2\right)^2}{576\pi^2 s_W^2 c_W^2}\xi_Z - \frac{g_S^2}{12\pi^2}\xi_G, \quad \text{(A.25)}$$

$$\widehat{\Delta}^{-1}\delta Z_{ct}^R = -\frac{e^2 m_c^2 m_t^2\sum_d V_{3d}V_{2d}^\dagger}{32 m_Z^2\pi^2 s_W^2 c_W^2} = 0, \quad \text{(A.26)}$$

$$\widehat{\Delta}^{-1}\delta Z_{ct}^L = -\frac{e^2\sum_d V_{3d}V_{2d}^\dagger m_d^2}{32 m_Z^2\pi^2 s_W^2 c_W^2} - \frac{e^2\sum_d V_{3d}V_{2d}^\dagger}{16\pi^2 s_W^2}\xi_W = -\frac{e^2\sum_d V_{3d}V_{2d}^\dagger m_d^2}{32 m_Z^2\pi^2 s_W^2 c_W^2}, \quad \text{(A.27)}$$

where

$$\widehat{\Delta} = \left[\frac{2}{4-D} - \gamma_E + \log 4\pi\right], \quad \text{(A.28)}$$

with D being the dimensional-regularisation parameter and γ_E the Euler-Mascheroni constant. All the above equations have been cross checked against [19][4] and [20].

B Explicit one-loop result for $\mu - e - \gamma$

In this appendix, the complete result for the unrenormalised coefficients $\bar{C}_{TL}$ and $\bar{C}_{TR}$ of the $\mu^- \to e^-\gamma$ decay in the EFT is given. After renormalisation, the formulae were further expanded around $m_l \ll m_V$ to obtain the results in table 3; then the public package `LoopTools 2.10` [15] was used to check the numerical stability of the aforementioned expansion. The result is presented in terms of Passarino-Veltman functions [33], following the convention described in [19]. Writing the coefficients as

$$\bar{C}_{TL} = C_{TL}^{(A_0)} + C_{TL}^{(B_0)} + C_{TL}^{(C_0)} + C_{TL}^{(c)}, \quad \text{(B.1)}$$

$$\bar{C}_{TR} = \bar{C}_{TL}\big|_{e\leftrightarrow\mu}, \quad \text{(B.2)}$$

[4] In the Feynman Gauge, i.e. $\xi \to 1$.

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the results read

$$
\begin{aligned}
&C_{TL}^{(A_0)}\\
&= A_0\big[m_e^2\big]\Bigg(\frac{e\big(m_e^2-m_\mu^2+4m_Z^2s_W^2\big)}{64m_\mu\big(m_e^2-m_\mu^2\big)\,\pi^2}C_{\varphi e}+\frac{em_Z^2\big(1-2s_W^2\big)}{32\big(m_e^3-m_em_\mu^2\big)\,\pi^2}\big(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}\big)\\
&\quad-\frac{m_em_Ws_W}{32\sqrt{2}\big(-m_e^2m_\mu+m_\mu^3\big)\,\pi^2}C_{e\varphi}^{e\mu}+\frac{m_\mu m_Ws_W}{32\sqrt{2}\big(-m_e^2m_\mu+m_\mu^3\big)\,\pi^2}C_{e\varphi}^{\mu e}\\
&\quad+\frac{em_\mu m_Z\big(3-4s_W^2\big)}{32\sqrt{2}\big(-m_e^2m_\mu+m_\mu^3\big)\,\pi^2}C_{eZ}^{\mu e}+\frac{em_Z\big(m_e+4m_es_W^2\big)}{32\sqrt{2}\big(-m_e^2m_\mu+m_\mu^3\big)\,\pi^2}C_{eZ}^{e\mu}\Bigg)\\
&\quad+A_0\big[m_\mu^2\big]\Bigg(\frac{em_Z^2s_W^2}{16m_e^2m_\mu\pi^2-16m_\mu^3\pi^2}C_{\varphi e}+\frac{e\big(-m_e^2+m_\mu^2+2m_Z^2\big(1-2s_W^2\big)\big)}{64\big(m_e^3-m_em_\mu^2\big)\,\pi^2}\big(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}\big)\\
&\quad+\frac{m_em_Ws_W}{32\sqrt{2}\big(m_e^3-m_em_\mu^2\big)\,\pi^2}C_{e\varphi}^{\mu e}-\frac{m_\mu m_Ws_W}{32\sqrt{2}\big(m_e^3-m_em_\mu^2\big)\,\pi^2}C_{e\varphi}^{e\mu}\\
&\quad-\frac{em_\mu m_Z\big(3-4s_W^2\big)}{32\sqrt{2}\big(m_e^3-m_em_\mu^2\big)\,\pi^2}C_{eZ}^{e\mu}-\frac{em_Z\big(m_e+4m_es_W^2\big)}{32\sqrt{2}\big(m_e^3-m_em_\mu^2\big)\,\pi^2}C_{eZ}^{\mu e}\Bigg)\\
&\quad+A_0\big[m_W^2\big]\Bigg(\frac{e\big(m_e^2+2m_Z^2\big(-1+s_W^2\big)\big)}{16\big(m_e^3-m_em_\mu^2\big)\,\pi^2}C_{\varphi l}^{(3)}\\
&\quad+\frac{e\big(-3m_e^2+3m_\mu^2-m_W^2\big)}{16\sqrt{2}\big(-m_e^2+m_\mu^2\big)\,m_Z\pi^2}C_{eZ}^{\mu e}-\frac{c_W^2em_em_Z}{16\sqrt{2}m_\mu\big(-m_e^2+m_\mu^2\big)\,\pi^2}C_{eZ}^{e\mu}\Bigg)\\
&\quad+A_0\big[m_Z^2\big]\Bigg(\frac{e\big(m_e^2-m_\mu^2+8m_Z^2s_W^2\big)}{64\big(-m_e^2m_\mu+m_\mu^3\big)\,\pi^2}C_{\varphi e}+\frac{e\big(m_e^2-m_\mu^2+4m_Z^2\big(-1+2s_W^2\big)\big)}{64\big(m_e^3-m_em_\mu^2\big)\,\pi^2}\big(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}\big)\\
&\quad+\frac{em_Z}{8\sqrt{2}\big(m_e^2-m_\mu^2\big)\,\pi^2}C_{eZ}^{\mu e}+\frac{em_Z\big(m_\mu^2\big(3-4s_W^2\big)+m_e^2\big(1+4s_W^2\big)\big)}{32\sqrt{2}m_em_\mu\big(m_e^2-m_\mu^2\big)\,\pi^2}C_{eZ}^{e\mu}\Bigg)\\
&\quad-A_0\big[m_H^2\big]\,\frac{m_Ws_W}{32\sqrt{2}m_em_\mu\pi^2}C_{e\varphi}^{e\mu},
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
&C_{TL}^{(B_0)}\\
&= B_0\big[m_e^2,0,m_W^2\big]\Bigg(-\frac{e\big(2m_\mu^2m_W^4+m_e^4\big(m_\mu^2+4m_W^2\big)-m_e^2m_W^2\big(5m_\mu^2+4m_W^2\big)\big)}{16m_e\big(m_e^2-m_\mu^2\big)^2\,\pi^2}C_{\varphi l}^{(3)}\\
&\quad-\frac{c_W^2em_em_\mu m_Z\big(m_e^2+m_Z^2-m_Z^2s_W^2\big)}{16\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\,\pi^2}C_{eZ}^{e\mu}\\
&\quad+\frac{c_W^2e\big(4m_e^4+3m_\mu^2m_W^2+m_e^2\big(-5m_\mu^2-4m_W^2\big)\big)\,m_Z}{16\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\,\pi^2}C_{eZ}^{\mu e}\Bigg)\\
&\quad+B_0\big[m_e^2,m_e^2,m_H^2\big]\Bigg(-\frac{m_e\big(2m_e^2-m_H^2\big)\,m_\mu m_Ws_W}{32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\,\pi^2}C_{e\varphi}^{e\mu}\\
&\quad+\frac{\big(2m_e^2-m_H^2\big)\big(2m_e^2-m_\mu^2\big)\,m_Ws_W}{32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\,\pi^2}C_{e\varphi}^{\mu e}\Bigg)
\end{aligned}
$$

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$$
\begin{aligned}
&+ B_0\big[m_e^2, m_e^2, m_Z^2\big]\left(\frac{em_\mu\big(-2m_e^4+m_e^2\big(2m_\mu^2-m_Z^2\big)+m_Z^2\big(4m_Z^2s_W^2+m_\mu^2\big(1-8s_W^2\big)\big)\big)}{64\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{\varphi e}\right.\\
&-\frac{em_Z^2\big(3m_e^4+m_\mu^2m_Z^2\big(1-2s_W^2\big)-m_e^2\big(2m_Z^2\big(1-2s_W^2\big)+m_\mu^2\big(1+4s_W^2\big)\big)\big)}{32m_e\big(m_e^2-m_\mu^2\big)^2\pi^2}\big(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}\big)\\
&+\frac{em_em_\mu m_Z\big(2m_e^2-m_Z^2\big)\big(1+4s_W^2\big)}{32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{eZ}^{e\mu}+\big(em_Z\big(m_e^4\big(8-16s_W^2\big)+3m_\mu^2m_Z^2\big(1-4s_W^2\big)\\
&\left.+2m_e^2\big(m_\mu^2\big(-1+4s_W^2\big)+m_Z^2\big(-3+8s_W^2\big)\big)\big)\big)\frac{C_{eZ}^{\mu e}}{\Big(32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\pi^2\Big)}\right)\\
&+ B_0\big[m_e^2, m_\mu^2, m_H^2\big]\left(\frac{m_\mu^2\big(m_e^2+m_H^2-3m_\mu^2\big)m_Ws_W}{32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{e\varphi}^{\mu e}\right.\\
&\left.+\frac{m_\mu\big(-2m_e^2m_H^2+\big(3m_e^2+m_H^2\big)m_\mu^2-m_\mu^4\big)m_Ws_W}{32\sqrt{2}m_e\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{e\varphi}^{e\mu}\right)\\
&+ B_0\big[m_e^2, m_\mu^2, m_Z^2\big]\left(\frac{em_\mu m_Z^2\big(3m_e^2-3m_\mu^2+2\big(-3m_e^2+m_\mu^2+m_Z^2\big)s_W^2\big)}{32\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{\varphi e}\right.\\
&+\big(e\big(m_\mu^2\big(m_\mu^2-m_Z^2\big)\big(m_\mu^2+2m_Z^2\big(1-2s_W^2\big)\big)+m_e^4\big(3m_\mu^2+4m_Z^2\big(-1+2s_W^2\big)\big)\\
&-m_e^2\big(4m_\mu^4+m_\mu^2m_Z^2\big(1-4s_W^2\big)+4m_Z^4\big(-1+2s_W^2\big)\big)\big)\frac{C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}}{64m_e\big(m_e^2-m_\mu^2\big)^2\pi^2}\\
&+\frac{em_\mu m_Z\big(m_\mu^4-m_\mu^2m_Z^2+m_e^2\big(-3m_\mu^2+2m_Z^2\big)\big)\big(-3+4s_W^2\big)}{32\sqrt{2}m_e\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{eZ}^{e\mu}\\
&+\big(em_Z\big(m_e^4\big(4-8s_W^2\big)+m_\mu^2\big(3m_Z^2\big(1-4s_W^2\big)+m_\mu^2\big(3+4s_W^2\big)\big)\\
&\left.+m_e^2\big(4m_Z^2\big(-1+2s_W^2\big)+m_\mu^2\big(-5+12s_W^2\big)\big)\big)\big)\frac{C_{eZ}^{\mu e}}{\Big(32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\pi^2\Big)}\right)\\
&+ B_0\big[m_\mu^2, 0, m_W^2\big]\left(\frac{em_e\big(2m_W^2\big(-2m_\mu^2-m_W^2\big)+m_e^2\big(m_\mu^2+3m_W^2\big)\big)}{16\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{\varphi l}^{(3)}\right.\\
&+\frac{c_W^2em_e\big(2m_\mu^2m_W^2+m_e^2\big(m_\mu^2-m_W^2\big)\big)m_Z}{16\sqrt{2}m_\mu\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{eZ}^{e\mu}-\big(e\big(m_e^4\big(m_\mu^2+3m_W^2\big)\\
&+m_\mu^2\big(m_\mu^4-m_\mu^2m_W^2+2c_W^4m_Z^4\big)\\
&\left.-m_e^2\big(2m_\mu^4+3m_\mu^2m_W^2+3c_W^4m_Z^4\big)\big)\frac{C_{eZ}^{\mu e}}{\Big(16\sqrt{2}\big(m_e^2-m_\mu^2\big)^2m_Z\pi^2\Big)}\right)\\
&+ B_0\big[m_\mu^2, m_e^2, m_H^2\big]\left(\frac{m_e^2\big(-3m_e^2+m_H^2+m_\mu^2\big)m_Ws_W}{32\sqrt{2}\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{e\varphi}^{\mu e}\right.\\
&\left.+\frac{m_e\big(-m_e^4-2m_H^2m_\mu^2+m_e^2\big(m_H^2+3m_\mu^2\big)\big)m_Ws_W}{32\sqrt{2}m_\mu\big(m_e^2-m_\mu^2\big)^2\pi^2}C_{e\varphi}^{e\mu}\right)
\end{aligned}
$$

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$$
\begin{aligned}
&+ B_0\big[m_\mu^2, m_e^2, m_Z^2\big]\left(\frac{em_Z^2\big(2m_e^3 - m_e m_Z^2 + 2m_e\big(m_e^2 - 3m_\mu^2 + m_Z^2\big)s_W^2\big)}{32\big(m_e^2 - m_\mu^2\big)^2\pi^2}(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)})\right.\\
&- \frac{em_e^2(m_e - m_\mu)(m_e + m_\mu)\big(m_e^2 - 3m_\mu^2 - m_Z^2\big) + 4e\big(m_e^2 - 2m_\mu^2\big)m_Z^2\big(m_e^2 + m_\mu^2 - m_Z^2\big)s_W^2}{64m_\mu\big(m_e^2 - m_\mu^2\big)^2\pi^2}C_{\varphi e}\\
&+ \frac{em_e m_Z\big(m_e^4 + 2m_\mu^2 m_Z^2 - m_e^2\big(3m_\mu^2 + m_Z^2\big)\big)\big(1 + 4s_W^2\big)}{32\sqrt{2}m_\mu\big(m_e^2 - m_\mu^2\big)^2\pi^2}C_{eZ}^{e\mu}\\
&+\big(em_Z\big(-m_e^2\big(5m_e^2 + m_\mu^2 - 3m_Z^2\big) + 4\big(m_e^4 + 3m_e^2(m_\mu - m_Z)(m_\mu + m_Z)\\
&\left.-2m_\mu^2(m_\mu - m_Z)(m_\mu + m_Z)\big)s_W^2\big)\big)\frac{C_{eZ}^{\mu e}}{\Big(32\sqrt{2}\big(m_e^2 - m_\mu^2\big)^2\pi^2\Big)}\right)\\
&+ B_0\big[m_\mu^2, m_\mu^2, m_H^2\big]\left(\frac{m_e m_\mu\big(m_H^2 - 2m_\mu^2\big)m_W s_W}{32\sqrt{2}\big(m_e^2 - m_\mu^2\big)^2\pi^2}C_{e\varphi}^{e\mu}\right.\\
&\left.+\frac{\big(m_e^2 - 2m_\mu^2\big)\big(m_H^2 - 2m_\mu^2\big)m_W s_W}{32\sqrt{2}\big(m_e^2 - m_\mu^2\big)^2\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ B_0\big[m_\mu^2, m_\mu^2, m_Z^2\big]\left(\frac{em_Z^2\big(3m_\mu^4 - 4m_\mu^2 m_Z^2 s_W^2 + m_e^2\big(2m_Z^2 s_W^2 + m_\mu^2\big(-3 + 4s_W^2\big)\big)\big)}{32m_\mu\big(m_e^2 - m_\mu^2\big)^2\pi^2}C_{\varphi e}\right.\\
&+ \frac{em_e\big(2m_\mu^4 + m_\mu^2 m_Z^2 + 2m_Z^4\big(-1 + 2s_W^2\big) + m_e^2\big(-2m_\mu^2 + m_Z^2\big(3 - 8s_W^2\big)\big)\big)}{64\big(m_e^2 - m_\mu^2\big)^2\pi^2}(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)})\\
&+ \frac{em_e m_\mu m_Z\big(2m_\mu^2 - m_Z^2\big)\big(-3 + 4s_W^2\big)}{32\sqrt{2}\big(m_e^2 - m_\mu^2\big)^2\pi^2}C_{eZ}^{e\mu} + \big(em_Z\big(m_e^2\big(2m_\mu^2 - 3m_Z^2\big)\big(-1 + 4s_W^2\big)\big)\\
&\left.-2m_\mu^2\big(m_Z^2 + 8(m_\mu - m_Z)(m_\mu + m_Z)s_W^2\big)\big)\frac{C_{eZ}^{\mu e}}{\Big(32\sqrt{2}\big(m_e^2 - m_\mu^2\big)^2\pi^2\Big)}\right), \qquad \text{(B.4)}
\end{aligned}
$$

$$
\begin{aligned}
&C_{TL}^{(C_0)}\\
&= C_0\big[m_\mu^2, m_e^2, 0, m_e^2, m_H^2, m_e^2\big]\left(-\frac{m_e^3 m_\mu m_W s_W}{16\sqrt{2}\big(m_e^2 - m_\mu^2\big)\pi^2}C_{e\varphi}^{e\mu} + \frac{m_e^2\big(2m_e^2 - m_\mu^2\big)m_W s_W}{16\sqrt{2}\big(m_e^2 - m_\mu^2\big)\pi^2}C_{e\varphi}^{\mu e}\right)\\
&+ C_0\big[m_\mu^2, m_e^2, 0, m_e^2, m_Z^2, m_e^2\big]\left(-\frac{em_e^2 m_\mu\big(m_e^2 - m_\mu^2 + 4m_Z^2 s_W^2\big)}{32\big(m_e^2 - m_\mu^2\big)\pi^2}C_{\varphi e}\right.\\
&+ \frac{em_e^3 m_Z^2\big(-1 + 2s_W^2\big)}{16\big(m_e^2 - m_\mu^2\big)\pi^2}(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)})\\
&\left.+ \frac{em_e^3 m_\mu m_Z\big(1 + 4s_W^2\big)}{16\sqrt{2}\big(m_e^2 - m_\mu^2\big)\pi^2}C_{eZ}^{e\mu} - \frac{em_e^2 m_Z\big(m_\mu^2\big(1 - 4s_W^2\big) + m_e^2\big(-4 + 8s_W^2\big)\big)}{16\sqrt{2}\big(m_e^2 - m_\mu^2\big)\pi^2}C_{eZ}^{\mu e}\right)\\
&+ C_0\big[m_\mu^2, m_e^2, 0, m_\mu^2, m_H^2, m_\mu^2\big]\left(\frac{m_e m_\mu^3 m_W s_W}{16\sqrt{2}\big(m_e^2 - m_\mu^2\big)\pi^2}C_{e\varphi}^{e\mu} + \frac{m_\mu^2\big(m_e^2 - 2m_\mu^2\big)m_W s_W}{16\sqrt{2}\big(m_e^2 - m_\mu^2\big)\pi^2}C_{e\varphi}^{\mu e}\right)
\end{aligned}
$$

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$$
\begin{aligned}
&+ C_0\left[m_\mu^2, m_e^2, 0, m_\mu^2, m_Z^2, m_\mu^2\right]\left(\frac{e m_\mu^3 m_Z^2 s_W^2}{8\left(-m_e^2+m_\mu^2\right)\pi^2} C_{\varphi e}\right.\\
&+ \frac{e m_e m_\mu^2\left(-m_e^2+m_\mu^2+2m_Z^2\left(1-2s_W^2\right)\right)}{32\left(-m_e^2+m_\mu^2\right)\pi^2}\left(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}\right)\\
&\left.- \frac{e m_\mu^2 m_Z\left(3 m_e m_\mu - 4 m_e m_\mu s_W^2\right)}{16\sqrt{2}\left(-m_e^2+m_\mu^2\right)\pi^2} C_{eZ}^{e\mu} - \frac{e m_\mu^2 m_Z\left(m_e^2 - 4\left(m_e^2-2m_\mu^2\right)s_W^2\right)}{16\sqrt{2}\left(-m_e^2+m_\mu^2\right)\pi^2} C_{eZ}^{\mu e}\right)\\
&+ C_0\left[m_\mu^2, m_e^2, 0, m_W^2, 0, m_W^2\right]\left(-\frac{e m_e m_W^2\left(2m_e^2-m_\mu^2-2m_W^2\right)}{8\left(m_e^2-m_\mu^2\right)\pi^2} C_{\varphi l}^{(3)}\right.\\
&\left.- \frac{c_W^4 e m_e m_\mu m_Z^3}{8\sqrt{2}\left(m_e^2-m_\mu^2\right)\pi^2} C_{eZ}^{e\mu} + \frac{c_W^2 e\left(2m_e^4+m_\mu^2 m_W^2-2m_e^2\left(m_\mu^2+m_W^2\right)\right)m_Z}{8\sqrt{2}\left(m_e^2-m_\mu^2\right)\pi^2} C_{eZ}^{\mu e}\right),
\end{aligned}
\tag{B.5}
$$

$$
\begin{aligned}
&C_{TL}^{(c)}\\
&= \frac{e m_\mu\left(-m_e^2+m_\mu^2+8m_Z^2 s_W^2\right)}{64\left(-m_e^2+m_\mu^2\right)\pi^2} C_{\varphi e} + \frac{e m_e\left(-m_e^2+m_\mu^2+4m_Z^2\left(-1+2s_W^2\right)\right)}{64\left(m_e^2-m_\mu^2\right)\pi^2} C_{\varphi l}^{(1)}\\
&- \frac{e m_e\left(m_e^2+3m_\mu^2-4m_Z^2\right)}{64\left(m_e^2-m_\mu^2\right)\pi^2} C_{\varphi l}^{(3)} + \frac{m_W s_W}{32\sqrt{2}\pi^2} C_{e\varphi}^{\mu e} - \frac{\sqrt{2} m_W s_W}{e} C_{e\gamma}^{\mu e}\\
&+ \frac{e m_e m_\mu m_Z\left(1+s_W^2\right)}{16\sqrt{2}\left(m_e^2-m_\mu^2\right)\pi^2} C_{eZ}^{e\mu} + \frac{e m_Z\left(m_\mu^2\left(7-2s_W^2\right)+m_e^2\left(-5+4s_W^2\right)\right)}{32\sqrt{2}\left(m_e^2-m_\mu^2\right)\pi^2} C_{eZ}^{\mu e}\\
&+ \frac{e}{16\pi^2}\left(m_e C_{le}^{e\mu ee} + m_\mu C_{le}^{\mu\mu e\mu} + m_\tau C_{le}^{\mu\tau\tau e}\right),
\end{aligned}
\tag{B.6}
$$

Note that eq. (B.2) applied to eq. (B.6) also implies that the generation indices in the operators C_{le} have to be swapped.

C Lepton-flavour violating τ decays and effective coefficient constraints

In this appendix, the strategy adopted in the main text is extended to the case of lepton-flavour violating tauonic transitions. By combining (see [24]) the experimental values obtained at the LEP collider (see [34–38]), the τ-lepton total width is inferred to be

$$
\Gamma_\tau = 2.3 \cdot 10^{-12}\ \text{GeV}. \tag{C.1}
$$

Recently, the BaBar Collaboration established [39] the following limits on the tauonic lepton-flavour violating decay rates:[5]

$$
\text{BR}(\tau^- \to e^-\gamma) \leq 3.3 \cdot 10^{-8}, \tag{C.2}
$$

$$
\text{BR}(\tau^- \to \mu^-\gamma) \leq 4.4 \cdot 10^{-8}. \tag{C.3}
$$

[5] Somewhat weaker limits have been obtained by the Belle collaboration [40].

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$\tau \to e\gamma$			
3-P Coefficient	At fixed scale	4-P Coefficient	At fixed scale
$C_{e\gamma}^{\tau e}$	$2.4 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\tau eee}$	$4.2 \cdot 10^{-4} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{eZ}^{\tau e}(m_Z)$	$1.3 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\tau\mu\mu e}$	$2.0 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$1.5 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\tau\tau\tau e}$	$1.2 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$1.4 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{\varphi e}$	$1.4 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{e\varphi}^{\tau e}$	$1.7 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$		

Table 6. Limits on the Wilson coefficients contributing to the $\tau \to e\gamma$ transition up to the one-loop level.

$\tau \to e\gamma$			
3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^4$ GeV	at $\Lambda = 10^5$ GeV
$C_{e\gamma}^{\tau e}$	$2.5 \cdot 10^{-6}$	$2.6 \cdot 10^{-4}$	$2.8 \cdot 10^{-2}$
$C_{eZ}^{\tau e}$	$2.3 \cdot 10^{-4}$	$1.3 \cdot 10^{-2}$	$9.5 \cdot 10^{-1}$
$C_{\tau ett}^{(3)}$	$3.4 \cdot 10^{-5}$	$1.9 \cdot 10^{-3}$	$1.4 \cdot 10^{-1}$
$C_{\tau ett}^{(1)}$	$1.8 \cdot 10^{-2}$	$5.0 \cdot 10^{-1}$	n/a
$C_{\tau ecc}^{(3)}$	$4.6 \cdot 10^{-3}$	$2.5 \cdot 10^{-1}$	n/a
$C_{\tau ecc}^{(1)}$	~ 2.4	n/a	n/a

Table 7. Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^4, 10^5$ GeV.

Putting together the information in eqs. (C.1) and (C.3) and adapting eq. (3.4) of section 3 to the tauonic case, the following limits are obtained:

$$\boxed{\tau \to e\gamma} \Longrightarrow \left.\frac{\sqrt{|C_{TL}(\lambda)|^2 + |C_{TR}(\lambda)|^2}}{\Lambda^2}\right|_{\lambda \ll \Lambda} \leq 4.1 \cdot 10^{-10} [\text{GeV}]^{-1}\,, \tag{C.4}$$

$$\boxed{\tau \to \mu\gamma} \Longrightarrow \left.\frac{\sqrt{|C_{TL}(\lambda)|^2 + |C_{TR}(\lambda)|^2}}{\Lambda^2}\right|_{\lambda \ll \Lambda} \leq 4.7 \cdot 10^{-10} [\text{GeV}]^{-1}\,. \tag{C.5}$$

The functional form of the coefficients C_{TL} and C_{TL} is not different from the result of table 3, apart from suitable changes of the mass parameters and generation indices (e.g. for the $\tau \to e\gamma$ case one should replace m_μ with m_τ except for the contribution from Q_{le}). Hence, exploiting the strategy that was presented in section 4, a set of both fixed-scale and Λ-dependent limits can be obtained for new coefficients involving a LFV connected to the third generation. Similarly to what has been done already, such results are summarised in tables 6–9. A final remark is required: as in eq. (4.8) the limits on $C_{e\gamma}$ at the m_Z scale

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$\tau \to \mu\gamma$			
3-P Coefficient	At fixed scale	4-P Coefficient	At fixed scale
$C_{e\gamma}^{\tau\mu}$	$2.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\tau ee\mu}$	$4.8 \cdot 10^{-4} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{eZ}^{\tau\mu}(m_Z)$	$1.5 \cdot 10^{-9} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\tau\mu\mu\mu}$	$2.3 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$1.7 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$	$C_{le}^{\tau\tau\tau\mu}$	$1.4 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{\varphi e}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{e\varphi}^{\tau\mu}$	$1.9 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$		

Table 8. Limits on the Wilson coefficients contributing to the $\tau \to \mu\gamma$ transition up to the one-loop level.

$\tau \to \mu\gamma$			
3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^4$ GeV	at $\Lambda = 10^5$ GeV
$C_{e\gamma}^{\tau\mu}$	$3.0 \cdot 10^{-6}$	$3.1 \cdot 10^{-4}$	$3.2 \cdot 10^{-2}$
$C_{eZ}^{\tau\mu}$	$2.8 \cdot 10^{-4}$	$1.5 \cdot 10^{-2}$	~ 1.1
$C_{\tau\mu tt}^{(3)}$	$4.0 \cdot 10^{-5}$	$2.2 \cdot 10^{-3}$	$1.6 \cdot 10^{-1}$
$C_{\tau\mu tt}^{(1)}$	$2.1 \cdot 10^{-2}$	$5.9 \cdot 10^{-1}$	n/a
$C_{\tau\mu cc}^{(3)}$	$5.4 \cdot 10^{-3}$	$3.0 \cdot 10^{-1}$	n/a
$C_{\tau\mu cc}^{(1)}$	~ 2.8	n/a	n/a

Table 9. Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^4, 10^5$ GeV.

are slightly different from the ones at the m_τ scale presented in tables 6 and 8. In fact, the limits evaluated at the electroweak scale read

$$\sqrt{\frac{|C_{e\gamma}^{\tau e}(m_Z)|^2 + |C_{e\gamma}^{e\tau}(m_Z)|^2}{2}} \leq 1.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}, \tag{C.6}$$

$$\sqrt{\frac{|C_{e\gamma}^{\tau\mu}(m_Z)|^2 + |C_{e\gamma}^{\mu\tau}(m_Z)|^2}{2}} \leq 2.0 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}. \tag{C.7}$$

Applying the RG evolution and using eqs. (C.6) and (C.7), one can extract the values of tables 7 and 9.

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