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The ν_R -philic scalar: its loop-induced interactions and Yukawa forces in LIGO observations

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ABSTRACT: Right-handed neutrinos (ν_R) are often considered as a portal to new hidden physics. It is tempting to consider a gauge singlet scalar (ϕ) that exclusively couples to ν_R via a $\nu_R\nu_R\phi$ term. Such a ν_R -philic scalar does not interact with charged fermions at tree level but loop-induced effective interactions are inevitable, which are systematically investigated in this work. The magnitude of the loop-induced couplings coincidentally meets the current sensitivity of fifth-force searches. In particular, the loop-induced coupling to muons could be tested in the recent LIGO observations of neutron star mergers as there might be a sizable Yukawa force in the binary system mediated by the ν_R -philic scalar.

KEYWORDS: Beyond Standard Model, Neutrino Physics, Solar and Atmospheric Neutrinos

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1 Introduction

Right-handed neutrinos (ν_R) are one of the most intriguing pieces to be added to the Standard Model (SM). Not only can they resolve several problems of the SM including neutrinos masses, dark mater, and baryon asymmetry of the universe,¹ their singlet nature under the SM gauge symmetry also allows for couplings to hidden or dark sectors, a feature known as the neutrino portal to physics beyond the SM.

Among various new physics extensions built on ν_R , a gauge singlet scalar ϕ coupled exclusively to ν_R , referred to as the ν_R -philic scalar, is arguably the simplest.² At tree level, the ν_R -philic scalar does not interact directly with normal matter that consists of electrons and quarks, which implies that it might have been well hidden from low-energy laboratory

¹See, e.g., the so-called ν MSM [1, 2] which extends the SM by ν_R to incorporate neutrino masses, dark mater, and leptogenesis simultaneously.

²It has recently been shown that the ν_R -philic scalar could assist low-scale leptogenesis [3].

searches. At the one-loop level, there are loop-induced couplings of ϕ to charged fermions, which are suppressed by neutrino masses (m_ν) in the framework of Type I seesaw [4–8]. The suppression can be understood from that in the zero limit of neutrino masses, which corresponds to vanishing couplings of the SM Higgs to ν_R and left-handed neutrinos (ν_L), the ν_R sector would be entirely decoupled from the SM content. As we will show, for electrons, the loop-induced effective Yukawa coupling is of the order of

$$\frac{G_F m_e m_\nu}{16\pi^2} \sim \mathcal{O}(10^{-21}), \quad (1.1)$$

where G_F is the Fermi constant and m_e is the electron mass.

Despite the small value of the loop-induced coupling, the magnitude coincides with the sensitivity of current precision tests of gravity. For long-range forces mediated by ultra-light bosons coupled to electrons or quarks, experimental tests of the strong (based on the lunar laser-ranging technology [9]) and weak (e.g., torsion-balance experiments [10, 11]) equivalence principles are sensitive to Yukawa/gauge couplings spanning from 10^{-20} to 10^{-24} . Very recently, gravitational waves from black hole (BH) and neutron star (NS) binary mergers have been detected by the LIGO/VIRGO collaboration [12, 13], providing novel methods to test theories of gravity as well as other long-range forces [14–24]. For instance, the process of BH superradiance can be used to exclude a wide range of ultra-light boson masses [15]. The sizable abundance of muons in NS binary systems allows us to probe muonic forces as they could modify the orbital dynamics. It is expected that [24] current and future observations of NS binaries are sensitive to muonic Yukawa/gauge couplings ranging from 10^{-18} to 10^{-22} which, again, coincidentally covers the theoretical expectation of the loop-induced coupling for muons, $G_F m_\mu m_\nu / (16\pi^2) \sim 10^{-19}$.

In light of the frontiers of precision and novel tests of gravity and gravity-like forces, it is important to perform an in-depth study on the loop-induced interactions of the ν_R -philic scalar, which is the main goal of this work. We note here that in the seminal work on majorons [25], similar loop-induced interactions have been computed and confronted with experimental limits in the 1980s. More recently, ref. [26] studied majoron decay caused by the loop-induced couplings to charged fermions. In addition, majoron decay to photons is also possible at two-loop level [27]. While the majoron considered in refs. [25–27] is a pseudo-scalar boson, in this work we compute loop-induced interactions for a generic scalar and take three lepton flavors into account, with loop calculation details presented. The loop-induced interactions computed in this work could be of importance in phenomenological studies of long-range forces [28–46].

The paper is organized as follows. In section 2, we briefly review the Type I seesaw extended by a gauge singlet scalar, and derive the tree-level interactions for later use. In section 3, we compute the loop-induced interactions of ϕ with charged fermions. The calculation, for simplicity, is first performed assuming only one generation of leptons and then generalized to three flavors in section 4. In section 5, we confront the theoretical predictions to experimental limits including searches for long-range forces of normal matter and the LIGO observations of NS events which are sensitive to muonic couplings. We conclude in section 6 and delegate some details of our calculations to the appendix.

2 The model

2.1 Notations

Throughout this paper, Weyl spinors are frequently used in our discussions for simplicity. On the other hand, for Feynman diagram calculations, Dirac or Majorana spinors are more convenient due to a variety of techniques and especially many modern computation packages that have been developed. As both will be used in this paper, it is necessary to clarify our notations regarding Weyl spinors versus Dirac/Majorana spinors.

All four-component Dirac/Majorana spinors in this paper are denoted by ψ_X with some interpretative subscripts X . Otherwise, they are Weyl spinors. For instance, ν_L and ℓ_R are Weyl spinors of a left-handed neutrino and a right-handed charged lepton, respectively. In contrast to that, ψ_ℓ is a Dirac spinor of a charged lepton containing both left- and right-handed components.

For Weyl spinors, our notation follows the convention in ref. [47]. For example, the mass and kinetic terms of ν_R are

$$M_R \nu_R \nu_R \equiv M_R (\nu_R)^\alpha (\nu_R)_\alpha, \quad \nu_R^\dagger \bar{\sigma}^\mu i \partial_\mu \nu_R \equiv \left(\nu_R^\dagger \right)_{\dot{\alpha}} (\bar{\sigma}^\mu)^{\dot{\alpha}\beta} i \partial_\mu (\nu_R)_\beta. \quad (2.1)$$

Here and henceforth, the Weyl spinor indices $\alpha, \dot{\alpha}, \beta$ will be suppressed.

Dirac and Majorana spinors can be built from Weyl spinors. Hence the Dirac spinors of charged leptons and neutrinos can be written as

$$\psi_\ell = \begin{pmatrix} \ell_L \\ \ell_R^\dagger \end{pmatrix}, \quad \psi_\nu = \begin{pmatrix} \nu_L \\ \nu_R^\dagger \end{pmatrix}. \quad (2.2)$$

The Majorana spinor of a neutrino mass eigenstate ν_i (where $i = 1, 2, 3, \dots$) is defined as

$$\psi_i \equiv \begin{pmatrix} \nu_i \\ \nu_i^\dagger \end{pmatrix}. \quad (2.3)$$

Note that it is self-conjugate: $\psi_i^c = \psi_i$. For later convenience, some identities are listed below to convert Weyl spinors into Dirac/Majorana spinors:

$$\nu_i \nu_j = \nu_j \nu_i = \bar{\psi}_i P_L \psi_j, \quad \nu_i^\dagger \nu_j^\dagger = \nu_j^\dagger \nu_i^\dagger = \bar{\psi}_i P_R \psi_j, \quad \nu_i^\dagger \bar{\sigma}^\mu \nu_j = \bar{\psi}_i \gamma^\mu P_L \psi_j, \quad (2.4)$$

$$\ell_L \nu_i = \nu_i \ell_L = \bar{\psi}_i P_L \psi_\ell, \quad \ell_R \nu_i = \nu_i \ell_R = \bar{\psi}_i P_R \psi_\ell, \quad \ell_L^\dagger \bar{\sigma}^\mu \nu_i = \bar{\psi}_i \gamma^\mu P_L \psi_i, \quad (2.5)$$

where $P_{L/R} \equiv (1 \mp \gamma^5)/2$ and $\gamma_L^\mu \equiv \gamma^\mu P_L$.

2.2 Lagrangian

We consider the SM extended by several right-handed neutrinos ν_R and a singlet scalar ϕ . In Type I seesaw, the number of ν_R needs to be ≥ 2 in order to accommodate the observed neutrino oscillation data. Let us start with one generation of leptons and ignore the flavor structure (for the realistic case including three generations, see section 4). The Lagrangian of ν_R and ϕ reads:

$$\mathcal{L} \supset \nu_R^\dagger \bar{\sigma}^\mu i \partial_\mu \nu_R + \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} m_\phi^2 \phi^2 + \left[\frac{M_R}{2} \nu_R \nu_R + \frac{y_R}{2} \nu_R \nu_R \phi + \text{h.c.} \right]. \quad (2.6)$$

Here we assume ϕ is a real scalar or pseudo-scalar field. If it is a complex field, one can decompose it as $\phi = \phi_r + i\phi_i$ with ϕ_r and ϕ_i being real scalar and pseudo-scalar fields respectively. To make our calculation applicable to both scalar and pseudo-scalar cases, we allow y_R to be a complex coupling.

The Dirac masses of leptons are generated by

$$\mathcal{L} \supset y_\nu \tilde{H}^\dagger L \nu_R + y_\ell H^\dagger L \ell_R + \text{h.c.}, \quad (2.7)$$

where H is the SM Higgs doublet ($\tilde{H} \equiv i\sigma_2 H^*$), $L = (\nu_L, \ell_L)^T$ is a left-handed lepton doublet, and ℓ_R is a right-handed charged lepton. After electroweak symmetry breaking, $\langle H \rangle = (0, v)^T/\sqrt{2}$, eq. (2.7) leads to the following mass terms:

$$\mathcal{L} \supset m_D \nu_L \nu_R + m_\ell \ell_L \ell_R + \text{h.c.}, \quad (2.8)$$

where

$$m_D \equiv y_\nu \frac{v}{\sqrt{2}}, \quad m_\ell \equiv y_\ell \frac{v}{\sqrt{2}}. \quad (2.9)$$

The Dirac and Majorana mass terms of neutrinos can be formulated as

$$\mathcal{L}_{\nu \text{ mass}} = \frac{1}{2} (\nu_L, \nu_R) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \quad (2.10)$$

which then can be diagonalized by

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_4 \end{pmatrix}, \quad U^T \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} U = \begin{pmatrix} m_1 & \\ & m_4 \end{pmatrix}. \quad (2.11)$$

Here ν_1 and ν_4 are the light and heavy mass eigenstates with their masses determined by

$$m_1 = \frac{1}{2} \left(\sqrt{4m_D^2 + M_R^2} - M_R \right), \quad m_4 = \frac{1}{2} \left(\sqrt{4m_D^2 + M_R^2} + M_R \right). \quad (2.12)$$

The unitary matrix U is parametrized as

$$U = \begin{pmatrix} -i c_\theta & s_\theta \\ i s_\theta & c_\theta \end{pmatrix}, \quad (2.13)$$

where $c_\theta \equiv \cos \theta$, $s_\theta \equiv \sin \theta$, and

$$\theta = \arctan \sqrt{m_1/m_4}. \quad (2.14)$$

Eq. (2.13) has been parametrized in such a way that m_D , M_R , m_1 and m_4 are all positive numbers.

2.3 Interactions in the mass basis

Since ν_L and ν_R are not mass eigenstates, we need to reformulate neutrino interactions in the mass basis, i.e., the basis of ν_1 and ν_4 . The two bases are related by

$$\nu_L = -i c_\theta \nu_1 + s_\theta \nu_4, \quad (2.15)$$

$$\nu_R = i s_\theta \nu_1 + c_\theta \nu_4. \quad (2.16)$$

Neutrino interactions in the original basis (chiral basis) include gauge interactions and Yukawa interactions, summarized as follows:

$$\mathcal{L} \supset \frac{g}{2c_W} Z_\mu \nu_L^\dagger \bar{\sigma}^\mu \nu_L + \left[\frac{g}{\sqrt{2}} W_\mu^- \ell_L^\dagger \bar{\sigma}^\mu \nu_L - y_\nu H^+ \ell_L \nu_R + y_\ell H^- \nu_L \ell_R + \frac{y_R}{2} \nu_R \nu_R \phi + \text{h.c.} \right], \quad (2.17)$$

where g is the gauge coupling of $SU(2)_L$ in the SM, c_W is the cosine of the Weinberg angle, and $H^\pm$ is the charged component of H , i.e. the Goldstone boson associated to $W^\pm$.

Now applying the basis transformation in eqs. (2.15) and (2.16) to eq. (2.17), we get

$$\mathcal{L} \supset g_Z^{ij} Z_\mu \nu_i^\dagger \bar{\sigma}^\mu \nu_j + \left[g_W^i W_\mu^- \ell_L^\dagger \bar{\sigma}^\mu \nu_i - y_\nu^i H^+ \ell_L \nu_i + y_\ell^i H^- \nu_i \ell_R + \frac{y_R^{ij}}{2} \nu_i \nu_j \phi + \text{h.c.} \right]. \quad (2.18)$$

Here i and j take either 1 or 4. The couplings g_Z^{ij} , g_W^i , y_ν^i , y_ℓ^i , y_R^{ij} are given by the following matrices or vectors:

$$g_Z^{ij} = \frac{g}{2c_W} \begin{pmatrix} c_\theta^2 & i c_\theta s_\theta \\ -i c_\theta s_\theta & s_\theta^2 \end{pmatrix}, \quad y_R^{ij} = y_R \begin{pmatrix} -s_\theta^2 & i c_\theta s_\theta \\ i c_\theta s_\theta & c_\theta^2 \end{pmatrix}, \quad (2.19)$$

$$g_W^i = \frac{g}{\sqrt{2}} (-i c_\theta, s_\theta), \quad y_\nu^i = y_\nu (i s_\theta, c_\theta), \quad y_\ell^i = y_\ell (-i c_\theta, s_\theta). \quad (2.20)$$

Eq. (2.18) can be straightforwardly expressed in terms of Dirac and Majorana spinors according to eqs. (2.4) and (2.5):

$$\mathcal{L} \supset g_Z^{ij} Z_\mu \bar{\psi}_i \gamma_L^\mu \psi_j + \left[g_W^i W_\mu^- \bar{\psi}_i \gamma_L^\mu \psi_i + H^- \bar{\psi}_i (y_\ell^i P_L - y_\nu^{i*} P_R) \psi_i + \frac{y_R^{ij}}{2} \bar{\psi}_i P_L \psi_j \phi + \text{h.c.} \right]. \quad (2.21)$$

Note that in the mass basis, ϕ couples to both heavy and light neutrinos but the coupling of the latter is suppressed by s_θ .

3 Loop-induced interactions of ϕ with charged leptons

As shown in the previous section, at tree level the scalar singlet ϕ only couples to neutrinos, including light and heavy ones in the mass basis. It does not interact with other fermions directly. In this section, we show that one-loop corrections lead to effective interactions of ϕ with charged leptons.

From eq. (2.18), it is straightforward to check that at the one-loop level, in the unitarity gauge (which means Goldstone boson interactions can be ignored), there are only two

Figure 1. One-loop diagrams that give rise to effective couplings of ϕ with charged leptons (ℓ) or quarks (u, d). The left diagram is computed in eqs. (3.10)–(A.2), and the right diagram leads to a pseudo-scalar coupling (with γ^5), the effect of which however is suppressed in unpolarized matter. The diagrams are presented in the mass basis (ν_i and ν_j are mass eigenstates). For an equivalent description in the chiral basis, see figure 2.

Figure 2. The $W^\pm$ -mediated loop diagram in the chiral basis, which is equivalent to the left diagram in figure 1 in the mass basis. It shows explicitly how chirality changes in the process. Since in the chiral basis $W^\pm$ only couples to left-handed leptons and ϕ only to ν_R , we need two mass insertions of m_D to connect ν_L and ν_R . Other two mass insertions, M_R and m_ℓ , are also necessary due to additional requirements — see discussions in the text.

possible diagrams that can connect ϕ to charged leptons or quarks, as shown in figure 1. The second diagram involving the Z boson actually leads to a pseudo-scalar coupling (see calculations later on). In unpolarized matter, pseudo-scalar interactions cannot cause significant long-range forces [48, 49] because the Yukawa potential between two fermions are spin dependent. When taking an average over the spins, the effect of pseudo-scalar interactions vanishes. Therefore, we will focus our discussions on the first diagram where the external fermion lines have to be charged leptons.

The diagrams in figure 1 are in the mass basis which is technically convenient for evaluation. Nonetheless it is illuminating to show figure 2, another diagram in the chiral basis which explicitly shows how chirality changes in the process. The physical results should be basis independent.

Figure 2 follows directly from eq. (2.17), which suggests that ϕ only couples to ν_R while $W^\pm$ interacts with ν_L . Therefore, two Dirac mass insertions ($m_D \nu_L \nu_R$ and $m_D \nu_L^\dagger \nu_R^\dagger$) are necessarily introduced to connect ν_R and ν_L , or $\nu_R^\dagger$ and $\nu_L^\dagger$. Note that the two $W^\pm$ vertices have to be conjugate to each other, which implies that from the $W^\pm$ side, a pair of ν_L and $\nu_L^\dagger$ is provided. On the other hand, the Yukawa vertex couples ϕ to two ν_R 's rather than a pair of ν_R and $\nu_R^\dagger$. So a Majorana mass insertion is required to flip the lepton number and convert one of them to $\nu_R^\dagger$. The direction of lepton-number flow in this diagram are represented by the arrows. Note that according to the conventions in section 2.1, ν_L and ν_R have opposite lepton numbers. So for $\nu_R \nu_R \phi$, the arrow of ν_R should be outgoing. In contrast to that, the arrow of ν_L in the $W_\mu^- \ell_L^\dagger \bar{\sigma}^\mu \nu_L$ vertex goes inwardly. Finally, there should be a mass insertion of $m_\ell \ell_L \ell_R$ on one of the external fermion lines because it is impossible to write down an effective operator that consists of ϕ and two ℓ_L 's — the operator $\phi \ell_L \ell_L$ is not allowed due to electric charge conservation.

The chirality analysis in figure 2 indicates that the diagram would be proportional to $m_D^2 M_R m_\ell$ if all these masses are sufficiently small. If M_R is much larger than the typical scale of the loop momentum, then the propagators of ν_R also contribute an additional factor of M_R^{-2} . In this case, the diagram is expected to be proportional to $m_D^2 M_R^{-1} m_\ell \sim m_\nu m_\ell$ where m_ν is the light neutrino mass.

Now let us compute the loop diagrams explicitly. Using the Dirac/Majorana spinor representation in eq. (2.21), we can write down the amplitudes of the two diagrams in figure 1:

$$i\mathcal{M}_W = (i)^3 \int \frac{d^4 k}{(2\pi)^4} \overline{u(p_2)} g_W^j \gamma_L^\mu \Delta_j(p_j) \frac{y_R^{ji} P_L + y_R^{ji*} P_R}{2} \Delta_i(p_i) g_W^{i*} \gamma_L^\nu u(p_1) \Delta_{\mu\nu}^W(k), \quad (3.1)$$

$$i\mathcal{M}_Z = (i)^3 \int \frac{d^4 p_i}{(2\pi)^4} \overline{u(p_2)} g_Z^{(\ell)} \gamma_L^\mu u(p_1) \text{tr} \left[-g_Z^{ij} \gamma_L^\nu \Delta_j(p_j) \frac{y_R^{ji} P_L + y_R^{ji*} P_R}{2} \Delta_i(p_i) \right] \Delta_{\mu\nu}^Z(q), \quad (3.2)$$

where $(i)^3$ comes from three vertices; p_1 and p_2 are the momenta of the upper and lower external fermion lines; p_i and p_j are the momenta of ν_i and ν_j ; $q = p_2 - p_1 = p_j - p_i$; k is the momentum of W propagator; and $g_Z^{(\ell)}$ is the gauge coupling of Z to the charge fermion ℓ . The symbol Δ denotes propagators. For Majorana spinors in the mass basis, their propagators have the same form as Dirac propagators:

$$\Delta_i(p) = \frac{i}{\not{p} - m_i}. \quad (3.3)$$

The propagators of $W^\pm$ and Z are gauge dependent. Most generally, in R_ξ gauges, they are:

$$\Delta_{\mu\nu}^W(k) = \frac{-i}{k^2 - m_W^2} \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 - \xi m_W^2} (1 - \xi) \right], \quad (3.4)$$

$$\Delta_{\mu\nu}^Z(k) = \frac{-i}{k^2 - m_Z^2} \left[g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 - \xi m_Z^2} (1 - \xi) \right]. \quad (3.5)$$

The unitarity gauge corresponds to $\xi \rightarrow \infty$. Except for the unitarity gauge, other gauges with finite ξ , e.g., the Feynman-'t Hooft gauge ($\xi = 1$) and the Lorentz gauge ($\xi = 0$),

require the inclusion of Goldstone boson diagrams. The unitarity gauge, albeit involving fewer diagrams by virtue of infinitely large masses of Goldstone boson propagators, has a disadvantage in that the cancellation of UV divergences is less obvious — see discussions in section B.1. Nonetheless, it is straightforward to compute $i\mathcal{M}_W$ and $i\mathcal{M}_Z$ for general values of ξ .

First, let us inspect the $i\mathcal{M}_Z$ amplitude. The loop integral of the trace part gives rise to a quantity proportional to q^ν :

$$\int \frac{d^4 p_i}{(2\pi)^4} \text{tr} [\gamma_L^\nu \Delta_j(p_j) P_{L/R} \Delta_i(p_i)] \propto q^\nu, \quad (3.6)$$

which can be expected from Lorentz invariance, explained as follows. On the left-hand side of eq. (3.6) there are only two independent momenta $p_j = p_i + q$ and p_i . After p_i being integrated out, the only quantity that can carry a Lorentz index is q so the result is proportional to q^ν . Now plugging this in eq. (3.2), we can immediately get a γ^5 sandwiched between $\overline{u(p_2)}$ and $u(p_1)$:

$$\begin{aligned} \overline{u(p_2)} \not{q} P_L u(p_1) &= \overline{u(p_2)} (\not{p}_2 P_L - P_R \not{p}_1) u(p_1) \\ &= m_\ell \overline{u(p_2)} (P_L - P_R) u(p_1) \\ &= -m_\ell \overline{u(p_2)} \gamma^5 u(p_1). \end{aligned} \quad (3.7)$$

Therefore, the Z -mediated diagram induces a pseudo-scalar coupling, which is computed in appendix C.

The $i\mathcal{M}_W$ amplitude can be computed by splitting the $W^\pm$ propagator in eq. (3.4) to two parts:

$$\Delta_{\mu\nu}^W(k) = -i \frac{g_{\mu\nu} - k_\mu k_\nu / m_W^2}{k^2 - m_W^2} - i \frac{k_\mu k_\nu / m_W^2}{k^2 - \xi m_W^2}, \quad (3.8)$$

where the first part does not contain ξ and the second part is important for cancellation of UV divergences. Note that when computing eq. (3.1), because of the chiral projectors in $y_R^{ji} P_L + y_R^{ji*} P_R$, the product of Dirac matrices gives

$$\gamma_L^\mu \frac{\not{p}_j + m_j}{p_j^2 - m_j^2} [y_R^{ji} P_L + y_R^{ji*} P_R] \frac{\not{p}_i + m_i}{p_i^2 - m_i^2} \gamma_L^\nu = \gamma_L^\mu \frac{\not{p}_j m_i y_R^{ji*} + y_R^{ji} m_j \not{p}_i}{(p_j^2 - m_j^2)(p_i^2 - m_i^2)} \gamma_L^\nu. \quad (3.9)$$

It implies that if $m_i \rightarrow 0$ and $m_j \rightarrow 0$, the result would be zero, which agrees with our analysis in the chiral basis.

With the above details being noted, we compute³ eq. (3.1) in the soft scattering limit ($q \rightarrow 0$) with the approximation of $m_\ell \ll m_W$ and obtain:

$$i\mathcal{M}_W = i \frac{m_\ell G^{ij}}{256\pi^2 m_W^2} [F_1(m_i, m_j) + F_2(m_i, m_j)] \overline{u(p_2)} u(p_1) + i \lambda_{\phi\ell\ell}^{(W)} \overline{u(p_2)} i \gamma^5 u(p_1), \quad (3.10)$$

where

$$G^{ij} \equiv g_W^{i*} g_W^j (m_j y_R^{ij} + m_i y_R^{ij*}) = \frac{g^2 c_\theta^2 s_\theta^2}{2} \begin{bmatrix} -m_1(y_R + y_R^*) & m_1 y_R^* - m_4 y_R \\ m_1 y_R - m_4 y_R^* & m_4(y_R + y_R^*) \end{bmatrix}, \quad (3.11)$$

³We use `Package-X` [50] to compute loop integrals analytically and our code is available from [https://github.com/xunjiexu/vR.loop].

and F_1 and F_2 correspond to the contributions of the first and second parts of the $W^\pm$ propagator in eq. (3.8), respectively. Their explicit forms are given in appendix A. The second term of eq. (3.10) leads to pseudo-scalar couplings which cannot cause significant effect in unpolarized matter. Nevertheless, we compute the loop-induced pseudo-scalar couplings in appendix C.

We need to sum over i and j in eq. (3.10) to get a finite and gauge independent result. There are several cancellations involved in the summation, which are discussed in detail in appendix B. After a careful treatment of these cancellations, we obtain:

$$i\mathcal{M}_W \approx i\overline{u(p_2)}y_{\phi\ell\ell}u(p_1), \quad (3.12)$$

with

$$y_{\phi\ell\ell} = -\frac{3G_F m_1 m_\ell \text{Re}(y_R)}{16\sqrt{2}\pi^2}. \quad (3.13)$$

It implies that the loop diagram generates the effective interaction

$$\mathcal{L} \supset y_{\phi\ell\ell} \phi \overline{\psi_\ell} \psi_\ell, \quad (3.14)$$

where the effective coupling $y_{\phi\ell\ell}$, given in eq. (3.13), is suppressed by the neutrino mass m_ν and the charged lepton mass m_ℓ .

4 Generalization to three flavors

So far we have only considered leptons of a single flavor for which we have computed the loop-induced coupling $y_{\phi\ell\ell}$, as given in eq. (3.13). Now we would like to generalize it to the realistic scenario with three flavors.

Assuming there are three generations of ν_L and ν_R , we can express the neutrino mass terms in a similar way to eq. (2.10) except that now the mass matrix is interpreted as a 6×6 matrix:

$$M_{6\nu} = \begin{bmatrix} 0 & m_D \\ m_D^T & M_R \end{bmatrix}_{6 \times 6}, \quad (4.1)$$

where m_D and M_R are 3×3 Dirac and Majorana mass matrices respectively. In principle, the number of right-handed neutrinos does not have to be three. It can be two or more. But to make it concrete, let us concentrate on the case with three ν_L plus three ν_R .

The neutrino mass terms and Yukawa terms are formulated as:

$$\mathcal{L} \supset \frac{1}{2}(\nu_L^T, \nu_R^T) M_{6\nu} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} + \frac{1}{2} \phi \nu_R^T Y_R^0 \nu_R + \text{h.c.}, \quad (4.2)$$

where Y_R^0 is a 3×3 Yukawa coupling matrix.

A detailed analysis of this scenario is delegated to appendix D. Here we simply summarize the results. In general, without any requirements of m_D , M_R and Y_R^0 , the loop-induced coupling can be numerically obtained from

$$y_{\phi\ell\ell} = \frac{G_F m_\ell}{64\sqrt{2}\pi^2} \sum_{i,j} U_{\ell i}^* U_{\ell j} \left(Y_R M_d + M_d Y_R^\dagger \right)_{ij} F_{12}(m_i, m_j), \quad (4.3)$$

where F_{12} can be computed using eq. (B.11), U is the full 6×6 mixing matrix that can diagonalize $M_{6\nu}$, $M_d \equiv U^T M_{6\nu} U = \text{diag}(m_1, m_2, \dots, m_6)$ is the diagonalized form of $M_{6\nu}$, and Y_R is the mass-basis form of Y_R^0 :

$$Y_R \equiv U^T \text{diag}(0_{3 \times 3}, Y_R^0) U. \quad (4.4)$$

If M_R and Y_R^0 can be simultaneously diagonalized,⁴ then without loss of generality, we can assume M_R and Y_R^0 are diagonal. Under this assumption, the result can be further simplified to

$$y_{\phi\ell\ell} \approx -\frac{3G_F m_\ell}{32\sqrt{2}\pi^2} \left[m_D (Y_R^0 + Y_R^{0\dagger}) M_R^{-1} m_D^\dagger \right]_{\ell\ell}. \quad (4.5)$$

Eq. (4.5) can also be expressed in the Casas-Ibarra parametrization [54]:

$$y_{\phi\ell\ell} \approx -\frac{3G_F m_\ell}{32\sqrt{2}\pi^2} \left[U_L^* \sqrt{m_\nu^d} R^T (Y_R^0 + Y_R^{0\dagger}) R^* \sqrt{m_\nu^d} U_L^T \right]_{\ell\ell}, \quad (4.6)$$

where U_L is the PMNS matrix, $m_\nu^d = \text{diag}(m_1, m_2, m_3)$, and R is a complex orthogonal matrix ($RR^T = 1$), which is determined by $m_D = iU_L^* \sqrt{m_\nu^d} R^T \sqrt{M_R^{-1}}$ in the Casas-Ibarra parametrization. Note that our convention of U_L is chosen in the way that $U_L^T m_\nu U_L = m_\nu^d$ for $m_\nu \equiv -m_D M_R^{-1} m_D^T$.

5 Phenomenology

The loop-induced interaction of ϕ with electrons leads to a Yukawa potential between two objects containing N_1 and N_2 electrons,

$$V(r) = -\frac{y_{\phi ee}^2 N_1 N_2}{4\pi r} e^{-m_\phi r}. \quad (5.1)$$

The effective Yukawa coupling $y_{\phi ee}$ is of order $G_F m_e m_\nu / (16\pi^2) \sim \mathcal{O}(10^{-21})$, which reaches the current sensitivity of long-range force searches. If we replace electrons with muons, the effective coupling is generally two orders of magnitude larger because $m_\mu/m_e \approx 200$. The muonic long-range force can be tested in binary systems of neutron stars (NS) which contain $\mathcal{O}(0.1 \sim 1\%)$ muons of the total mass [55]. In particular, the recent gravitational wave observation of NS binary mergers by the LIGO collaboration [12, 13] are able to test the muonic force with unprecedented sensitivity.

As indicated by eq. (4.3), the value of $y_{\phi\ell\ell}$ depends on neutrino masses and the Yukawa couplings of ϕ to ν_R . Since there are many free parameters in Y_R and M_ν (where Majorana phases, the Dirac CP phase, the lightest neutrino mass are still unknown), we would like to simply parametrize $y_{\phi\ell\ell}$ as follows:

$$y_{\phi ee} = \frac{3G_F m_e Y_R^{(e)} m_\nu^{(e)}}{16\sqrt{2}\pi^2} \approx 8.0 \times 10^{-22} Y_R^{(e)} \left(\frac{m_\nu^{(e)}}{0.01 \text{ eV}} \right), \quad (5.2)$$

$$y_{\phi\mu\mu} = \frac{3G_F m_\mu Y_R^{(\mu)} m_\nu^{(\mu)}}{16\sqrt{2}\pi^2} \approx 5.0 \times 10^{-19} Y_R^{(\mu)} \left(\frac{m_\nu^{(\mu)}}{0.03 \text{ eV}} \right), \quad (5.3)$$

⁴Such a feature could arise from flavor symmetries, see models in refs. [51–53].

Figure 3. The effective Yukawa coupling of ϕ to e , compared with experimental limits. The predictions of our model (red) are evaluated according to eq. (5.2) with $m_{\nu}^{(e)} = 0.01$ eV. The experimental limits come from the Eöt-Wash torsion-balance tests of the equivalence principle (blue) [56], tests of gravitational inverse-square law (orange) [57], lunar laser-ranging (LLR, green) measurements [9, 56], and black hole superradiance (hatched bands) [15].

where $Y_R^{(e)}$ and $Y_R^{(\mu)}$ account for the suppression caused by the original Yukawa couplings if they are not of $\mathcal{O}(1)$, while $m_{\nu}^{(e)}$ and $m_{\nu}^{(\mu)}$ account for the suppression due to neutrino masses. In the limit of $Y_{R1} = Y_{R2} = Y_{R3}$ and $U_{\text{PMNS}}^* = U_{\text{PMNS}}$, $m_{\nu}^{(e)}$ would be identical to the neutrino mass matrix element responsible for neutrinoless double beta decay (often denoted as m_{ee} in the literature). But in general, they are different. Since $Y_R^{(e)} m_{\nu}^{(e)}$ and $Y_R^{(\mu)} m_{\nu}^{(\mu)}$ depend on a lot of unknown fundamental parameters, it is possible that the Majorana phases and other free parameters conspire in such a way that $Y_R^{(e)} m_{\nu}^{(e)} = 0$ while $Y_R^{(\mu)} m_{\nu}^{(\mu)}$ is not suppressed or vice versa, analogous to the well-known fact that m_{ee} for neutrinoless double beta decay can vanish in the normal mass ordering.

Next, we shall confront the theoretical predictions with experimental limits, as shown in figures 3 and 4 for $y_{\phi ee}$ and $y_{\phi \mu \mu}$ respectively.

For $y_{\phi ee}$, current limits come from long-range force searches of normal matter, which have long been investigated in precision tests of gravity, in particular, in tests of the equivalence principle. The Yukawa force mediated by ϕ can affect the former by contributing an exponential term to the total force and affect the latter due to its leptophilic coupling, which causes differential free-fall accelerations for different materials. So far, the Eöt-Wash torsion-balance experiment has performed tests of the weak equivalence principle with the highest precision [10, 11], leading to the most stringent constraint on $y_{\phi ee}$ in the regime of very small m_{ϕ} . In addition, the lunar laser-ranging (LLR) technology which is able to measure the varying distance between the moon and the earth to high precision using laser

Figure 4. The effective Yukawa coupling of ϕ to μ , compared with experimental limits. The predictions of our model (red) are evaluated according to eq. (5.3) with $m_{\nu}^{(\mu)} = 0.03$ eV. The muonic force could be probed in binary systems of neutron stars (NS) due to the considerable abundance of muons. The blue and green curves represent current sensitivity of the LIGO observations of GW170817 (NS-NS merger) and GW190814 (NS-BH merger) events, respectively. Solid (dashed) curves take conservative (optimistic) estimates of the muon abundance [24]. In addition, precision measurements of binary pulsar systems are also sensitive to the muonic force (orange curves) [24].

pulses is also sensitive to new long-range forces [9]. These two bounds, reviewed in ref. [56], are presented in figure 3 and overlap with the theoretically most favored region (red lines).

For larger masses, $y_{\phi ee}$ is constrained by tests of the inverse-square law of gravity [57, 58], the Casimir effect [59], stellar cooling processes [60, 61], N_{eff} in cosmology [62–66], supernovae [67–73], neutrino scattering [74–79], etc. But all these bounds are significantly higher than the largest expected values of $y_{\phi ee}$ — see ref. [37] for a recent compilation of these bounds.

In figure 3 (also figure 4), we add hatched bands to represent the constraint from black hole superradiance [15], which is independent of the Yukawa couplings because the effect is caused by ϕ coupling to the spacetime.

For $y_{\phi\mu\mu}$, the aforementioned laboratory constraints do not apply since normal matter does not contain muons. Neutron stars, however, can be a powerful probe of muonic forces due to a significant abundance muons in them, which is expected when the Fermi energy exceeds the muon mass. According to the calculations in refs. [24, 55], the number density of muons is typically of $\mathcal{O}(1 \sim 10)\%$ of the total number density, which is lower than but still comparable to the electron number density.⁵

⁵See figure 23 in ref. [55] and figure 3 in ref. [24]. In the former, the number densities of protons and electrons are presented. Assuming charge neutrality of the NS, the difference between proton and electron number densities is approximately the muon number density. The latter needs to be converted from mass ratios to number density ratios by multiplying a factor of m_{μ}/m_n where m_n is the neutron mass.

In fact, since the electron and muon number densities are of the same order of magnitude while $y_{\phi\ell\ell} \propto m_\ell$, for NS binaries we have

$$F^{(\mu)} \sim \left(\frac{m_\mu}{m_e}\right)^2 F^{(e)} \gg F^{(e)}, \quad (5.4)$$

where $F^{(\mu)}$ and $F^{(e)}$ are the forces caused by muons and electrons respectively.

The recent observations of NS-NS and NS-BH mergers by the LIGO collaboration provide very promising data to probe the muonic force in this model. For a NS-NS merger, the effect of ϕ is two-fold [19]. First, the attractive force affects the orbital dynamics in a classical way, i.e., modifying the Kepler's law when $r \sim m_\phi^{-1}$. Second, since ϕ is a ultra-light boson, there is radiation of ϕ due to the rotating dipole, which causes extra energy loss. For a NS-BH merger, only the effect of ϕ radiation is relevant. An in-depth analysis of the sensitivity to muonic forces based on the recent two events GW170817 (NS-NS merger) and GW190814 (NS-BH merger) has been performed in ref. [24]. Their results have been incorporated in figure 4, where solid (dashed) curves are derived using a conservative (optimistic) estimate of the muon abundance. For GW170817, the sensitivity curves of the two effects are evaluated and presented separately. The first effect (orbital dynamics) are more sensitive than the second to m_ϕ when it is in the large-mass ($10^{-12} \sim 10^{-10}$ eV) regime. In addition to binary mergers, precision measurements of binary pulsars can also be sensitive to muonic forces [24, 80].

As shown in figure 4, the LIGO curves cross the red lines of $Y_R^{(\mu)} = 10^{-3} \sim 1$, which implies that the loop-induced muonic force in this model could be probed in the theoretically most favored regime. Future experiments such as the Einstein Telescope⁶ and Cosmic Explorer [81] can substantially improve the sensitivity to muonic forces and thus have great potential of probing this scenario.

6 Conclusions and discussions

The ν_R -philic scalar model naturally gives rise to extremely small couplings of charged leptons to a long-range force mediator via loop-level processes. The small values of the loop-induced couplings coincidentally meet the current sensitivity of long-range force searches in laboratories and in astrophysical observations such as the recent detection of GW from NS mergers by LIGO, as we have shown in figures 3 and 4.

In this model, loop-induced couplings to quarks also exist, due to the Z -mediated diagram in figure 1. However, our calculation shows that only pseudo-scalar couplings are generated in this case, the effect of which is suppressed in unpolarized matter.

Our loop calculation result for the most general three-flavor case is given by eq. (4.3) which, though involving diagonalization of the full 6×6 mass matrix, can be numerically evaluated. For the special case where M_R and Y_R^0 can be simultaneously diagonalized, the result can be further simplified to eq. (4.6), where the dependence on the PMNS matrix is manifestly extracted.

⁶See the ET conceptual design document: https://tds.virgo-gw.eu/?call_file=ET-0106C-10.pdf.

Our results can also be used to obtain loop-induced interactions for other similar models that contain the diagrams in figure 1, via proper replacements of the couplings in vertices and masses in propagators. However, one caveat should be noted here that incomplete models where the tree-level couplings of ϕ to light neutrino mass eigenstates are not governed by the active-sterile neutrino mixing would lead to gauge dependent results.

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A Full expressions of F_1 and F_2

The explicit expressions of F_1 and F_2 read as follows:

$$\begin{aligned}
 F_1 = & 6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{m_W^2} \right) - 2 \frac{m_j^2 + 2m_W^2}{m_j^2 - m_W^2} \log \frac{m_j^2}{m_W^2} \\
 & + \frac{5m_i^2 m_j^2 - 5m_i^2 m_W^2 - 5m_j^2 m_W^2 + 11m_W^4}{(m_i^2 - m_W^2)(m_j^2 - m_W^2)} \\
 & - \frac{2(m_i^2 m_j^2 m_W^2 + m_i^2 m_j^4 - 2m_i^2 m_W^4 - 7m_j^4 m_W^2 + 2m_j^2 m_W^4 + 2m_j^6) \log \frac{m_i^2}{m_j^2}}{(m_i^2 - m_j^2)(m_j^2 - m_W^2)^2} \\
 & - \frac{2m_W^4 (17m_i^2 m_j^2 - 10m_i^2 m_W^2 + 5m_i^4 - 7m_j^2 m_W^2 + 2m_j^4 + 2m_W^4) \log \frac{m_i^2}{m_W^2}}{(m_i^2 - m_W^2)^2 (m_j^2 - m_W^2)^2} \\
 & - \frac{2m_i^2 m_j^2 (2m_i^2 m_j^2 - 7m_i^2 m_W^2 - 4m_j^2 m_W^2) \log \frac{m_i^2}{m_W^2}}{(m_i^2 - m_W^2)^2 (m_j^2 - m_W^2)^2}, \tag{A.1}
 \end{aligned}$$

$$\begin{aligned}
 F_2 = & -6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{\xi m_W^2} \right) + 2 \frac{m_j^2}{m_j^2 - \xi m_W^2} \log \frac{m_j^2}{\xi m_W^2} \\
 & - \frac{5m_i^2 (m_j^2 - \xi m_W^2) + \xi m_W^2 (7\xi m_W^2 - 5m_j^2)}{(m_i^2 - \xi m_W^2)(m_j^2 - \xi m_W^2)} \\
 & + \frac{2m_j^2 [m_i^2 (m_j^2 - \xi m_W^2) - 3\xi m_j^2 m_W^2 + 2m_j^4] \log \frac{m_i^2}{m_j^2}}{(m_i^2 - m_j^2)(m_j^2 - \xi m_W^2)^2} \\
 & - \frac{2\xi^2 m_j^2 m_W^4 (2m_j^2 - 3\xi m_W^2) \log \frac{m_i^2}{\xi m_W^2}}{(m_i^2 - \xi m_W^2)^2 (m_j^2 - \xi m_W^2)^2} \\
 & + \frac{2\xi m_i^2 m_W^2 (9\xi m_j^2 m_W^2 - 4m_j^4 - 4\xi^2 m_W^4) \log \frac{m_i^2}{\xi m_W^2}}{(m_i^2 - \xi m_W^2)^2 (m_j^2 - \xi m_W^2)^2} \\
 & + \frac{2m_i^4 (2m_j^4 - 5\xi m_j^2 m_W^2 + 3\xi^2 m_W^4) \log \frac{m_i^2}{\xi m_W^2}}{(m_i^2 - \xi m_W^2)^2 (m_j^2 - \xi m_W^2)^2}. \tag{A.2}
 \end{aligned}$$

Here we have used dimensional regularization which means the integrals are computed in a $d = 4 - 2\epsilon$ dimensional spacetime. And the generalization of integration measure $\int \frac{d^4 k}{(2\pi)^4} \rightarrow \mu^{2\epsilon} \int \frac{d^d k}{(2\pi)^d}$ introduces a dimensional constant μ which, together with $1/\epsilon$, should be canceled out in physical results.

We have verified that the above expressions are symmetric under $i \leftrightarrow j$:

$$F_1 = F_1|_{i \leftrightarrow j}, \quad F_2 = F_2|_{i \leftrightarrow j}. \quad (\text{A.3})$$

In addition, though $m_i^2 - m_j^2$ appears in some of the denominators, it does not cause additional divergences when $m_i \rightarrow m_j$:

$$\begin{aligned} \lim_{m_j \rightarrow m_i} F_1 = & 6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{m_W^2} \right) \\ & + \frac{3m_i^2 m_W^4 \left[3 - 8 \log \frac{m_i^2}{m_W^2} \right] + (3m_i^4 m_W^2 - m_i^6) \left[1 + 6 \log \frac{m_i^2}{m_W^2} \right] - 11m_W^6}{(m_i^2 - m_W^2)^3}, \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \lim_{m_j \rightarrow m_i} F_2 = & -6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{\xi m_W^2} \right) \\ & + \frac{2m_i^2 (-9\xi m_i^2 m_W^2 + 3m_i^4 + 8\xi^2 m_W^4)}{(m_i^2 - \xi m_W^2)^3} \log \frac{m_i^2}{\xi m_W^2} \\ & + \frac{-9\xi^2 m_i^2 m_W^4 + \xi m_i^4 m_W^2 + m_i^6 + 7\xi^3 m_W^6}{(m_i^2 - \xi m_W^2)^3}. \end{aligned} \quad (\text{A.5})$$

To obtain the final result of $i\mathcal{M}_W$, one needs both eqs. (A.1)–(A.2) and eqs. (A.4)–(A.5) to sum over i and j as it involves cases of $i \neq j$ and $i = j$.

B Some cancellations

In this appendix, we discuss several noteworthy cancellations in our calculation.

B.1 Cancellation of UV divergences

As can be seen from eqs. (A.1) and (A.2), both F_1 and F_2 contain UV divergences $1/\epsilon$ in their first terms. When combined together in eq. (3.10), there is obviously a cancellation between the two divergences:

$$6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{m_W^2} \right) - 6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{\xi m_W^2} \right) = 6 \log \xi. \quad (\text{B.1})$$

Further cancellations of $\log \xi$ will be discussed in the next subsection.

Here we would like to address a subtlety concerning UV divergences in the unitarity gauge. If we had naively taken the $\xi \rightarrow \infty$ limit of eq. (3.4) at the beginning of the above calculations, we would get a divergent result because

$$\lim_{\xi \rightarrow \infty} \Delta_{\mu\nu}^W(k) = -i \frac{g_{\mu\nu} - k_\mu k_\nu / m_W^2}{k^2 - m_W^2}, \quad (\text{B.2})$$

which corresponds to exactly the F_1 contribution according to eq. (3.8). And our calculation has shown that the F_1 contribution itself is UV divergent. We also know that the divergence is actually canceled out by the F_2 contribution, which however would vanish if $\xi \rightarrow \infty$ had been taken in the naive way. That implies that taking $\xi \rightarrow \infty$ should be after the loop integration. Actually from the second term of (3.8), one can see that when the loop integral contains $\frac{k_\mu k_\nu / m_W^2}{k^2 - \xi m_W^2}$, the $\xi \rightarrow \infty$ limit does not commute with $k \rightarrow \infty$ in the integral. Taking $\xi \rightarrow \infty$ after the integration can make the large momentum contribution with $k^2 > \xi m_W^2$ be included, which is crucial for the UV cancellation.

In other gauges, it is more straightforward to see that $i\mathcal{M}_W$ is finite. Taking the Feynman-'t Hooft gauge for example,

$$\lim_{\xi \rightarrow 1} \Delta_{\mu\nu}^W(k) = \frac{-ig_{\mu\nu}}{k^2 - m_W^2}, \quad (\text{B.3})$$

when it is applied to eq. (3.1), using eq. (3.9), the loop integral becomes

$$\int \frac{d^4k}{(2\pi)^4} \gamma_L^\mu \frac{\not{p}_j \lambda_j + \not{p}_i \lambda_i}{(p_j^2 - m_j^2)(p_i^2 - m_i^2)} \gamma_L^\nu \frac{g_{\mu\nu}}{k^2 - m_W^2} \xrightarrow{\text{large } k} (\lambda_i + \lambda_j) \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_L^\mu \not{k} \gamma_{L\mu}}{k^6}, \quad (\text{B.4})$$

where $\lambda_j \equiv m_j y_R^{ji*}$ and $\lambda_i \equiv y_R^{ji} m_j$. It is now obvious to see that the loop integral converges because the integrand is proportional to k^{-5} as $k \rightarrow \infty$.

B.2 Cancellation of ξ dependence in R_ξ gauges

The F_1 contribution is ξ independent. So we are only concerned with F_2 . Let us make a series expansion of F_2 with respect to ξ^{-1} :

$$F_2 = -6 \left(\frac{1}{\epsilon} + \log \frac{\mu^2}{m_W^2} \right) - 7 + 6 \log \xi + \mathcal{O}(\xi^{-1}). \quad (\text{B.5})$$

Usually in R_ξ gauges, it is expected that the ξ dependence of a $W^\pm$ diagram is canceled by a similar diagram with $W^\pm$ replaced by its Goldstone boson $H^\pm$. In our case, it would be the diagram in figure 5. However, a straightforward calculation shows that the amplitude of this diagram is

$$i\mathcal{M}_{H^\pm} \propto m_\ell \frac{m_j y_R^{ij} + m_i y_R^{ij*}}{m_W^2 \xi} + \mathcal{O}(\xi^{-2}), \quad (\text{B.6})$$

which is impossible to cancel the $\log \xi$ term in eq. (B.5) when ξ increases to sufficiently large values.

This problem is essentially related to the completeness of the model. For an arbitrary matrix of y_R^{ij} , indeed the result would be gauge dependent and the $\log \xi$ term remains for each case of $(i, j) = (1, 1), (1, 4), (4, 1)$, and $(4, 4)$. However, in section 2 we have shown that the elements in y_R^{ij} are correlated by active-sterile neutrino mixing — see eq. (2.19). Besides, g_W^i also depends on the mixing — see eq. (2.20). As a consequence, when summing up the contributions of both light and heavy neutrinos in eq. (3.10), the $\log \xi$ term cancels out because

$$\sum_{i,j} G^{ij} 6 \log \xi = 0. \quad (\text{B.7})$$

Figure 5. The Goldstone boson diagram that complements the $W^\pm$ diagram to cancel the ξ dependence in R_ξ gauges.

Therefore, the $\log \xi$ term can be safely ignored when computing the full amplitude. Actually if we inspect G^{ij} in the chiral basis, the cancellation is more manifest. From eq. (3.11), we can express $\sum_{i,j} G^{ij}$ in the matrix form:

$$\sum_{i,j} G^{ij} = w^\dagger Y_R \begin{bmatrix} m_1 & \\ & m_4 \end{bmatrix}^\dagger w + w^\dagger \begin{bmatrix} m_1 & \\ & m_4 \end{bmatrix} Y_R^* w, \quad (\text{B.8})$$

where w and Y_R are the vector and matrix of g_W^i and y_R^{ij} in eqs. (2.20) and (2.19) respectively. They are transformed from the chiral basis by:

$$w \equiv \frac{g}{\sqrt{2}} U^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad Y_R \equiv y_R U^T \begin{bmatrix} 0 & \\ & 1 \end{bmatrix} U. \quad (\text{B.9})$$

Therefore, in the chiral basis, we have

$$\begin{aligned} \sum_{i,j} G^{ij} &\propto (1, 0) U^* \left\{ U^T \begin{bmatrix} 0 & \\ & 1 \end{bmatrix} U U^\dagger M_{2\nu}^* U^* + U^T M_\nu U U^\dagger \begin{bmatrix} 0 & \\ & 1 \end{bmatrix} U^* \right\} U^T \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= (1, 0) \left\{ \begin{bmatrix} 0 & \\ & 1 \end{bmatrix} M_{2\nu}^* + M_{2\nu} \begin{bmatrix} 0 & \\ & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ &= 0. \end{aligned} \quad (\text{B.10})$$

Here $M_{2\nu}$ is the neutrino mass matrix in eq. (2.10). It shows that the vanishing product of w and Y_R (more specifically, $w^\dagger Y_R = 0$ and $Y_R^\dagger w = 0$), which is due to the absence of $W^\pm\text{-}\nu_R$ and $\phi\text{-}\nu_L$ couplings, leads to $\sum_{i,j} G^{ij} = 0$.

B.3 GIM-like cancellation

If all the neutrino masses (including heavy ones) are much smaller than m_W , when summing over i and j in eq. (3.10), the leading-order contribution vanishes in a way similar to the Glashow-Iliopoulos-Maiani (GIM) mechanism [82]. At the next-to-leading order (NLO), a nonzero result can be obtained. But in case of zero mass splitting of neutrinos, the NLO contribution would vanish again. This is also similar to the GIM cancellation, where if u and c quarks were of equal mass, the $K^0 \rightarrow \bar{K}^0$ amplitude would be zero.

Let us compute $i\mathcal{M}_W$ in the unitarity gauge. The preceding discussion in section B.2 concludes that the $\log \xi$ term can be safely ignored in this complete model. Hence we define

$$F_{12} \equiv \lim_{\xi \rightarrow \infty} (F_1 + F_2 - 6 \log \xi), \quad (\text{B.11})$$

which is finite. Then $i\mathcal{M}_W$ in the unitarity gauge can be computed by:

$$i\mathcal{M}_W = \overline{iu(p_2)u(p_1)} \sum_{i,j} \frac{m_\ell G^{ij} F_{12}(m_i, m_j)}{256\pi^2 m_W^2}. \quad (\text{B.12})$$

Now if we assume $m_W \gg m_i$ and m_j , F_{12} can be expanded as follows:

$$F_{12} = 4 + \frac{6m_i^4 \left(1 + 4 \log \frac{m_i}{m_W}\right) - 6m_j^4 \left(1 + 4 \log \frac{m_j}{m_W}\right)}{m_W^2 (m_i^2 - m_j^2)} + \mathcal{O}\left(\frac{m_{i,j}^4}{m_W^4}\right). \quad (\text{B.13})$$

The constant term in F_{12} does not contribute to eq. (B.12) due to $\sum_{i,j} G^{ij} = 0$. So the leading-order contribution to $i\mathcal{M}_W$ vanishes. Only the second or higher-order terms in eq. (B.13) further suppressed by $m_{i,j}^2/m_W^2$ contribute to nonzero $i\mathcal{M}_W$.

Plugging eq. (B.13) into eq. (B.12) and using the explicit form of G^{ij} in eq. (3.11), we obtain

$$\frac{i\mathcal{M}_W}{\overline{iu(p_2)u(p_1)}} \approx \frac{3g^2 m_\ell s_\theta^2 c_\theta^2 (y_R + y_R^*) \left[4(2m_1 + m_4) m_4^3 \log\left(\frac{m_4}{m_W}\right) + 3m_4^4 + 4m_1 m_4^3 - (1 \leftrightarrow 4) \right]}{256\pi^2 m_W^4 (m_1 + m_4)}. \quad (\text{B.14})$$

Note that the expression in the square bracket is antisymmetric under the interchange of m_1 and m_4 . Therefore if the mass splitting $m_1 - m_4$ is zero, the NLO contribution vanishes as well, similar to the GIM cancellation.

In Type I seesaw, the scale of heavy neutrino masses is often assumed to be much higher than the electroweak scale. Hence a more likely scenario is $m_4 \gg m_W \gg m_1$. For such a hierarchy, there is no GIM-like cancellation, as we shall show below.

First, we need to expand F_{12} in other regimes. If the diagram contains heavy neutrinos running in the loop, we expand it with respect to m_W :

$$F_{12} \approx \frac{2m_j^2 \left[1 - 6 \log\left(\frac{m_W}{m_j}\right)\right] - 2m_i^2 \left[1 - 6 \log\left(\frac{m_W}{m_i}\right)\right]}{m_i^2 - m_j^2}, \quad \text{for } m_W \ll m_i, m_j. \quad (\text{B.15})$$

If the diagram contains one light and one heavy neutrinos, we have

$$F_{12} \approx -2 + \left[12 \log\left(\frac{m_W}{m_j}\right) - \frac{6m_W^2}{m_j^2} \right] \left(1 + \frac{m_i^2}{m_j^2} \right) - \frac{6m_i^2}{m_j^2}, \quad \text{for } m_i \ll m_W \ll m_j. \quad (\text{B.16})$$

For $m_j \ll m_W \ll m_i$, the result can be obtained by an interchange of i and j in eq. (B.16).

Combining the results in eqs. (B.13), (B.15), and (B.16), we sum over i and j in eq. (B.12), which gives:

$$\frac{i\mathcal{M}_W}{\overline{iu(p_2)u(p_1)}} = -3g^2 m_\ell m^3 s_\theta^2 \text{Re}(y_R) \frac{m^4 - 4m^2 m_W^2 + m_W^4 \left(3 + 2 \log \frac{m^2}{m_W^2}\right)}{128\pi^2 m_W^2 (m^2 - m_W^2)^3} + \mathcal{O}(s_\theta^4),$$

where $m \equiv \sqrt{m_1^2 + m_4^2}$. Now taking $G_F = \sqrt{2}g^2/(8m_W^2)$, $ms_\theta^2 = m_1$, and $m \gg m_W$, we obtain the result in eqs. (3.12) and (3.13).

C Pseudo-scalar couplings

As mentioned in section 3, the W diagram in figure 1 leads to loop-induced couplings of both scalar ($\bar{\psi}\phi\psi$) and pseudo-scalar ($\bar{\psi}i\gamma^5\phi\psi$) forms. The Z diagram leads to only pseudo-scalar couplings. Although the pseudo-scalar couplings are not relevant to the phenomenology considered in this work, we would like to present our calculation of the pseudo-scalar couplings in this appendix.

Let us first compute the Z diagram. Starting from eq. (3.2), we have:

$$\text{tr} \left[-g_Z^{ij} \gamma_L^\nu \Delta_j(p_j) \frac{y_R^{ji} P_L + y_R^{ji*} P_R}{2} \Delta_i(p_i) \right] = g_Z^{ij} \frac{m_j y_R^{ji} p_i^\nu + m_i y_R^{ji*} p_j^\nu}{(p_j^2 - m_j^2)(p_i^2 - m_i^2)}. \quad (\text{C.1})$$

So eq. (3.2) can be written as

$$i\mathcal{M}_Z = (i)^3 \int \frac{d^4 p_i}{(2\pi)^4} \overline{u(p_2)} g_Z^{(\ell)} \gamma_L^\mu u(p_1) g_Z^{ij} \frac{m_j y_R^{ji} p_i^\nu + m_i y_R^{ji*} p_j^\nu}{(p_j^2 - m_j^2)(p_i^2 - m_i^2)} \Delta_{\mu\nu}^Z(q). \quad (\text{C.2})$$

In the limit of $q \rightarrow 0$,

$$\Delta_{\mu\nu}^Z(q) \rightarrow \frac{ig_{\mu\nu}}{m_Z^2},$$

we replace p_j with $p_i + q$ and extract the p_i -independent part out of the loop integral:

$$i\mathcal{M}_Z = \frac{i}{16\pi^2} \overline{u(p_2)} g_Z^{(\ell)} \gamma_{L\mu} u(p_1) \frac{g_Z^{ij}}{m_Z^2} I_{ij}^\mu(q), \quad (\text{C.3})$$

where the loop integral $I^\mu(q)$ reads:

$$I_{ij}^\mu(q) \equiv \left[\frac{i}{16\pi^2} \right]^{-1} \int \frac{d^4 p_i}{(2\pi)^4} \frac{m_j y_R^{ji} p_i^\mu + m_i y_R^{ji*} (p_i^\mu + q^\mu)}{[(p_i^\mu + q^\mu)^2 - m_j^2](p_i^2 - m_i^2)}. \quad (\text{C.4})$$

Performing the loop integration, we get

$$\begin{aligned} I_{ij}^\mu(q) = & q^\mu \frac{-m_j y_R^{ji} + m_i y_R^{ji*}}{2} \left(\frac{1}{\epsilon} + \log \mu^2 - \log m_j^2 \right) \\ & + q^\mu \frac{y_R^{ji} m_j (m_j^2 - 3m_i^2) + y_R^{ji*} m_i (m_i^2 - 3m_j^2)}{4(m_i^2 - m_j^2)} \\ & + q^\mu \frac{-y_R^{ji} m_j m_i + y_R^{ji*} (m_i^2 - 2m_j^2)}{2(m_i^2 - m_j^2)^2} m_i^3 \log \frac{m_i^2}{m_j^2} \\ & + q^\mu \mathcal{O}(q^2). \end{aligned} \quad (\text{C.5})$$

Next, we sum over i and j and expand the result in s_θ :

$$\sum_{i,j} g_Z^{ij} I_{ij}^\mu(q) \approx q^\mu \frac{y_R - y_R^*}{4} ms_\theta^2 + \mathcal{O}(s_\theta^3). \quad (\text{C.6})$$

The UV divergence cancels out in the summation because

$$\sum_{i,j} g_Z^{ij} m_j y_R^{ji} = \sum_{i,j} g_Z^{ij} m_i y_R^{ji*} = 0, \quad (\text{C.7})$$

which can be proven straightforwardly from eqs. (2.19).

Plugging eq. (C.6) into eq. (C.3), we obtain

$$i\mathcal{M}_Z = i \frac{g_Z^{(\ell)} m_1 (y_R - y_R^*)}{64\pi^2 m_Z^2} \overline{u(p_2)} \not{q} P_L u(p_1) \quad (\text{C.8})$$

$$= -i \frac{g_Z^{(\ell)} m_\ell m_1 (y_R - y_R^*)}{64\pi^2 m_Z^2} \overline{u(p_2)} \gamma^5 u(p_1), \quad (\text{C.9})$$

where in the second step we have used eq. (3.7).

Computing the pseudo-scalar coupling from the W diagram is similar, except that the bilinear $\overline{u(p_2)}(C_1 \not{p}_1 + C_2 \not{p}_2) P_L u(p_1)$ where C_1 and C_2 are different scalar quantities cannot be converted to $\overline{u(p_2)} \not{q} P_L u(p_1)$. It actually contributes to both scalar and pseudo-scalar couplings because

$$\begin{aligned} \overline{u(p_2)}(C_1 \not{p}_1 + C_2 \not{p}_2) P_L u(p_1) &= m_\ell \overline{u(p_2)}(C_1 P_R + C_2 P_L) u(p_1) \\ &= m_\ell \frac{C_1 + C_2}{2} \overline{u(p_2)} u(p_1) + m_\ell \frac{C_1 - C_2}{2} \overline{u(p_2)} \gamma^5 u(p_1). \end{aligned}$$

With this detail being noted, the calculation is straightforward and gives:

$$i\mathcal{M}_W \approx -i \frac{G_F m_1 m_\ell}{8\sqrt{2}\pi^2} \overline{u(p_2)} \left[\frac{3}{4}(y_R + y_R^*) + \frac{1}{2}(y_R - y_R^*) \gamma^5 \right] u(p_1). \quad (\text{C.10})$$

The first and second terms in the square bracket give rise to the loop-induced scalar and pseudo-scalar couplings respectively. The former has been considered in section 3. The latter and the $i\mathcal{M}_Z$ amplitude in eq. (C.9) lead to the following pseudo-scalar interaction:

$$\mathcal{L} \supset \lambda_{\phi\ell\ell}^{(W)} \phi \bar{\psi}_\ell i \gamma^5 \psi_\ell + \lambda_{\phi\ell\ell}^{(Z)} \phi \bar{\psi}_\ell i \gamma^5 \psi_\ell, \quad (\text{C.11})$$

where

$$\lambda_{\phi\ell\ell}^{(W)} = -\frac{G_F m_1 m_\ell \text{Im}(y_R)}{8\sqrt{2}\pi^2}, \quad \lambda_{\phi\ell\ell}^{(Z)} = -\frac{g_Z^{(\ell)} m_1 m_\ell \text{Im}(y_R)}{32\pi^2 m_Z^2}. \quad (\text{C.12})$$

D Generalization to three flavors

In this appendix, we present the detailed three-flavor analysis.

The 6×6 symmetric mass matrix can be diagonalized by a 6×6 unitary matrix U :

$$U^T M_{6\nu} U = \text{diag}(m_1, m_2, m_3, \dots, m_6) \equiv M_d. \quad (\text{D.1})$$

The neutrino flavor basis and the mass basis are connected by

$$\begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix}, \quad (\text{D.2})$$

where both ν_L and ν_R are 3×1 vectors. First, we convert the gauge interaction $\frac{g}{\sqrt{2}} W_\mu^- \ell_L^\dagger \bar{\sigma}^\mu \nu_L$ from the flavor basis to the mass basis:

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} W_\mu^- (e_L^\dagger, \mu_L^\dagger, \tau_L^\dagger) \bar{\sigma}^\mu \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} U \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix}. \quad (\text{D.3})$$

This generalizes g_W^i in eq. (2.20) from a 1×2 vector to a 3×6 matrix:

$$g_W^{\ell i} = \frac{g}{\sqrt{2}} U_{\ell i}, \quad (\ell = e, \mu, \tau, \text{ and } i = 1 \cdots 6). \quad (\text{D.4})$$

Next, we perform a similar transformation for the Yukawa interactions of ν_R :

$$\frac{1}{2} \phi \nu_R^T Y_R^0 \nu_R = \frac{\phi}{2} (\nu_1, \dots, \nu_6) U^T \begin{pmatrix} 0 & & \\ & 0 & \\ & & 0 \\ & & & Y_R^0 \end{pmatrix} U \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix} \equiv \frac{\phi}{2} (\nu_1 \cdots \nu_6) Y_R \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_6 \end{pmatrix}, \quad (\text{D.5})$$

where the 6×6 matrix Y_R is a generalization of the 2×2 y_R^{ij} matrix in eq. (2.19).

The generalization of G^{ij} is quite straightforward. By replacing g_W^i and y_R^{ij} in eq. (3.11) with $g_W^{\ell i}$ and Y_R^{ij} , we get:

$$G^{ij} = g_W^{\ell i*} g_W^{\ell j} (m_j Y_R^{ij} + m_i Y_R^{ij*}). \quad (\text{D.6})$$

As for F_1 and F_2 , the expressions in eqs. (A.1) and (A.2) can be used directly except that now i and j run from 1 to 6 instead of 1 and 4.

With the generalized G^{ij} , F_1 and F_2 , it is straightforward to get the loop-induced effective Yukawa coupling in eq. (4.3).

Note that any constant terms in F_{12} can be ignored because

$$\begin{aligned} & \sum_{i,j} U_{\ell i}^* U_{\ell j} \left(Y_R M_d + M_d Y_R^\dagger \right)_{ij} \\ &= \left[\begin{pmatrix} 1_{3 \times 3} & \\ & 0_{3 \times 3} \end{pmatrix} U^* U^T \begin{pmatrix} 0_{3 \times 3} & \\ & Y_R^0 \end{pmatrix} U M_d U^T \begin{pmatrix} 1_{3 \times 3} & \\ & 0_{3 \times 3} \end{pmatrix} + \text{h.c.} \right]_{\ell\ell} \\ &= \left[\begin{pmatrix} 1_{3 \times 3} & \\ & 0_{3 \times 3} \end{pmatrix} \begin{pmatrix} 0_{3 \times 3} & \\ & Y_R^0 \end{pmatrix} M_{6\nu}^* \begin{pmatrix} 1_{3 \times 3} & \\ & 0_{3 \times 3} \end{pmatrix} + \text{h.c.} \right]_{\ell\ell} \\ &= 0, \end{aligned} \quad (\text{D.7})$$

which is similar to eq. (B.10).

Eq. (4.3) applies for the most general $3\nu_L + 3\nu_R$ scenario. Although its dependence on the PMNS matrix and light neutrino masses is not manifest, each quantity in eq. (4.3) can be readily evaluated using numerical methods.

Below we would like to discuss a special case in which eq. (4.3) can be further simplified and expressed in the Casas-Ibarra parametrization [54].

If M_R and Y_R^0 can be simultaneously diagonalized and $m_{1,2,3} \ll m_{4,5,6}$, without loss of generality we can assume M_R and Y_R^0 are diagonal and m_D can be expressed, according to the Casas-Ibarra parametrization, as

$$m_D = iU_L^* \sqrt{m_\nu^d} R^T \sqrt{M_R}, \quad (\text{D.8})$$

where U_L , m_ν^d and R have been defined in section 4.

Then the full 6×6 mixing matrix U can be approximately decomposed as

$$U \approx \begin{pmatrix} U_L & \\ & 1_{3 \times 3} \end{pmatrix} \begin{pmatrix} 1_{3 \times 3} & -iT \\ -iT^\dagger & 1_{3 \times 3} \end{pmatrix}, \quad (\text{D.9})$$

where

$$T \equiv \sqrt{m_\nu^d} R^\dagger \sqrt{M_R^{-1}}. \quad (\text{D.10})$$

From eqs. (D.8) and (D.10), we have

$$m_D M_R^{-1} = iU_L^* T^*.$$

In the mass basis, the Yukawa coupling matrix Y_R , defined in eq. (D.5), now reads

$$Y_R \approx \begin{pmatrix} \mathcal{O}(T^2) & -iT^* Y_R^0 \\ -iY_R^0 T^\dagger & Y_R^0 \end{pmatrix}. \quad (\text{D.11})$$

Next, we need to compute $F_{12}(m_i, m_j)$ in eq. (4.3). Assuming $m_{4,5,6} \gg m_W \gg m_{1,2,3}$, the result is

$$F_{12} \approx \begin{pmatrix} 4 & 4 & 4 & f_4 - 2 & f_5 - 2 & f_6 - 2 \\ 4 & 4 & 4 & f_4 - 2 & f_5 - 2 & f_6 - 2 \\ 4 & 4 & 4 & f_4 - 2 & f_5 - 2 & f_6 - 2 \\ f_4 - 2 & f_4 - 2 & f_4 - 2 & f_4 - 8 & . & . \\ f_5 - 2 & f_5 - 2 & f_5 - 2 & . & f_5 - 8 & . \\ f_6 - 2 & f_6 - 2 & f_6 - 2 & . & . & f_6 - 8 \end{pmatrix}, \quad (\text{D.12})$$

where $f_i \equiv 12 \log \frac{m_i}{m_W}$ and “.” denotes more complicated expressions which are irrelevant to our calculation.

By introducing the following matrix:

$$H_{ij} \equiv \left(Y_R M_d + M_d Y_R^\dagger \right)_{ij} F_{12}(m_i, m_j), \quad (\text{D.13})$$

we can reformulate eq. (4.3) as

$$y_{\phi \ell \ell} = \frac{G_F m_\ell}{64 \sqrt{2} \pi^2} [U^* H U^T]_{\ell \ell}. \quad (\text{D.14})$$

Combining eqs. (D.11) and (D.12), we obtain the H matrix:

$$H = \begin{pmatrix} \mathcal{O}(T^4) & -iT^* Y_R^0 M_R D_f + \mathcal{O}(T^3) \\ (-iT^* Y_R^0 M_R D_f)^\dagger + \mathcal{O}(T^3) & Y_R^0 M_R (D_f - 6I_3) + \text{h.c.} \end{pmatrix}, \quad (\text{D.15})$$

where $D_f \equiv \text{diag}(f_4 - 2, f_5 - 2, f_6 - 2)$ and I_3 is a 3×3 identity matrix. Now supplying all the matrices required by eq. (D.14), we obtain the results in eqs. (4.5) and (4.6).

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