Hilbert series and plethystics: paving the path towards 2HDM- and MLRSM-EFT

Anisha, Supratim Das Bakshi, Joydeep Chakraborty and Suraj Prakash

*Indian Institute of Technology Kanpur,
Kalyanpur, Kanpur 208016, India*

E-mail: anisha@iitk.ac.in, sdbakshi@iitk.ac.in, joydeep@iitk.ac.in,
surajprk@iitk.ac.in

ABSTRACT: Effective Field Theory (EFT) technique is one of the most elegant ways to capture the impact of high scale theory, if any, at some low energy by incorporating higher mass dimensional (≥ 5) effective operators ($\mathcal{O}_i$). The low energy EFT is described in terms of only light degrees of freedom (DOF) which can appear on-shell. An essential task while developing the EFT framework is to compute these $\mathcal{O}_i$'s. Hilbert Series (HS) is a novel and mathematically robust method to construct the complete set of gauge invariant independent, effective operators. The HS requires the knowledge of the transformation properties of the light DOF and the covariant derivatives under the internal gauge symmetries and conformal groups. The Hilbert Series method, by its virtue, automatically takes care of the redundancies in the operator set due to the Equations of Motion (EOMs) of fields and Integration by Parts (IBPs) with impeccable accuracy.

In this paper, we have adopted this methodology to construct the complete set of independent operators up to dimension-6 in the “Warsaw”-like basis for two different Beyond Standard Model scenarios — Two Higgs Doublet Model (2HDM) and Minimal Left-Right Symmetric Model (MLRSM). For both these cases, we have calculated the corrections to the scalar, gauge boson and fermion mass spectra due to the dimension-6 operators. The additional contributions to all the Feynman vertices are computed and their impact on different observables, namely Weak mixing angle, Fermi constant, ρ and oblique (S, T, U) parameters. We have further discussed how the magnetic moments of charged leptons and production and decay of the massive BSM particles, e.g., charged scalar and different rare processes are affected in the presence of effective operators. We have also constructed the effective scalar four-point interactions and commented on the possible reinvestigation of the theoretical constraints, e.g., unitarity and vacuum stability within these frameworks.

KEYWORDS: Effective Field Theories, Gauge Symmetry, Spontaneous Symmetry Breaking

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1 Introduction

The Standard Model (SM) of particle physics is the most successful theory to describe the dynamics of the sub-atomic particles. Most of the predictions of the SM are verified experimentally with impeccable accuracy. Nevertheless, the SM fails to explain many experimental observations, e.g., neutrino mass, the anomalous magnetic moment of the muon, dark matter, etc. Thus we are confident about the necessary existence of some theory Beyond Standard Model (BSM). The actual form of the BSM is yet to be unveiled, but it is expected that low energy physics, e.g., SM has emerged from some more fundamental theory lying at some higher scale. If we believe in such a proposal, then one needs to derive the low energy lagrangian from that relatively complete theory. To do so, the massive modes of that complete theory, i.e., the heavier particles having mass $>$ scale of low energy theory must be integrated out. In the process, higher mass dimensional operators, suppressed

by the mass of the integrated out particles, are generated alongside the renormalizable lagrangian describing relatively low energy physics. Then the *new* lagrangian at low energy will be written as $\mathcal{L} = \mathcal{L}^4 + \sum_{i \in \mathbb{N}} \mathcal{C}_{i+4} \frac{\mathcal{O}_{i+4}}{\Lambda^i}$, where $\mathcal{L}^4$ contains all renormalizable terms up to mass dimension-4, and $\mathcal{O}_{i+4}$'s are the effective operators of mass dimension $[M]^{i+4}$. The $\mathcal{C}_{i+4}$'s are the associated Wilson Coefficients (WCs). These effective operators and their respective WCs are expected to capture the footprints of the high scale theory. This idea is the basic anthem of the so-called “Effective Field Theory” (EFT) which provides a platform to realise the high scale physics in terms of low energy effective lagrangian. One can consult refs. [1–28] where different aspects of effective field theory have been discussed.

But can we construct a low energy effective field theory even if we do not know the exact nature of the high scale physics? Yes, that is indeed possible. One can always add gauge invariant operators of different mass dimensions to the renormalizable low energy lagrangian [5–14, 14, 15, 15, 16, 16, 18, 27, 29–44]. Based on this idea, over the years, EFT has become one of the most sophisticated tools to sense the presence of new physics, even without knowing it exactly. This version of EFT relies only on the light degrees of freedom (DOF). The new physics is captured in the WCs, which are free parameters and blind to the exact form of the BSM theory. Here, one must include the “complete set of gauge invariant independent operators of particular mass dimension” such that the operators can form a basis [45, 46]. This idea was first implemented in the context of the SM in [45, 46]. It has been noted in refs. [46–49] that based on the SM interactions and the particle content one can construct 59 gauge invariant independent dimension-6 operators in a specific basis, known as the “Warsaw” basis. Here, the lepton, as well as baryon number symmetries, are assumed to be respected. While constructing these operators, the most subtle aspect is the identification of independent operators. Since we are dealing with the dynamical field variables, the derivatives (more specifically covariant derivative) appear in the operator structures almost in the same footing as the quantum fields. Thus it is very much possible that two differently structured operators may be related to each other by Equations of Motion (EOMs) of the fields or they may differ by a total derivative. The appearance of the total derivative is related to one of the properties of the derivative operator — Integration by Parts (IBPs) [45, 46, 50, 51]. These redundancies can be avoided by incorporating the EOMs and IBPs explicitly. One can look into the refs. [45, 46] where the SM-Effective Field Theory (SMEFT) with dimension-6 operators was constructed based on this principle. In [52] the complete set of dimension-7 operators has been computed. The methodology discussed in these papers was adequate, but can be tedious to implement for BSM models with extended symmetries and(or) more particles.

Recently a new approach has been developed to construct the higher mass dimensional (≥ 5) operators based on the Hilbert Series (HS) method within the SM framework [9, 37, 38, 53–55, 55–59]. This provides a mathematically elegant, and rigorous platform to reveal the algebraic structures of the SMEFT. In this construction, information of the underlying global, gauge, and also space-time symmetries is utilised in the form of “characters” of representations of the field variables. We would like to specifically mention here that the derivative operators are treated in the same footing as the quantum fields in

this construction. The derivatives transform under the conformal group, which is $SO(4, 2)$ in our case. This method automatically takes care of the redundancy in the operator set and generates all independent operators, which is its best merit. Thus explicit implementations of the EOMs and IBPs can be avoided which may be cumbersome for general gauge structures involving multiple field representations. This algebraic method certainly widens the reach of the EFT program beyond the SM.

Now, coming back to the utility of the EFT framework, we have to remember that a finite number of WCs are capable of representing almost all possible BSM scenarios which can affect the low energy physics. In general, effective field theory provides a better handle to deal with the unknown BSM physics, if it is there and interferes with the SM interactions. Most of the BSM models contain too many free parameters and suffer from a lack of predictability. Therefore, from that perspective, the EFT approach is very economical and captures the new physics in a much more concise form. For example, if there is a deviation between the experimental data and the SM predictions, then it can be explained in terms of a few effective operators. The corresponding WCs can be constrained very acutely as the DOF of the analysis will be less as compared to a complete BSM theory. The fitted values of these WCs are obtained using experimental data and based on the low energy effective lagrangian.

An essential aspect of phenomenological analyses is identifying the parameter space for different BSM models allowed by the experimental data till date. In the SMEFT framework, the BSM effects appear in the form of effective operators along with the renormalizable SM lagrangian. For decades, the precision low energy observables, e.g., Weak mixing angle, Fermi constant, ρ and oblique (S, T, U) parameters have been used to determine the precise room left for the physics beyond the Standard Model [60–81]. The impact of these effective operators on the low energy precision observables, magnetic moments, and theoretical constraints in the scalar sector are discussed in [45, 63, 68, 81–83, 83–95]. Their role in the context of collider phenomenology, neutrino masses, and rare processes are addressed in [39, 56, 69, 89–91, 96–150].

In the process of adjudging the BSM parameter space under the light of present and future proposed colliders, and other low energy experiments we consider mostly the renormalizable BSM lagrangian. But these TeV scale BSM scenarios (NP1) could be effective theories which are generated from more complete theories (NP2) lying at an energy scale just beyond the access of present experiments. Then considering only the renormalizable BSM lagrangian may be misleading while drawing the exclusion limits in BSM parameter space. It has also been noted that the final remarks on the parameters of NP1 may alter significantly once we include the effective operators [56, 112, 116–119]. Also, in the light of future colliders, like FCC [151–153], the realisation of the effective TeV scale BSM scenarios could be significant. Recent works have already introduced the novelty of effective BSM (BSM-EFT) scenario in the context of 2HDM-EFT, see refs. [55, 112, 113, 154–156].

In this paper, we have borrowed the concept and methodology of Hilbert Series (HS) that has been used for SMEFT construction in [9, 38, 53, 54] and implemented to understand BSM-EFT. This paper, to our knowledge, is one of the earlier attempts to construct the complete BSM-EFT lagrangian up to dimension-6 effective operators using the HS

method. To start with, we have briefly discussed the underlying principle of effective operator construction using Hilbert Series. Around the TeV scale, many BSM theories have been developed. Among them, we have adopted the following scenarios: two Higgs doublet (2HDM) and Minimal Left-Right Symmetric (MLRSM) models. We have computed the complete set of independent dimension-6 operators in the “Warsaw”-like basis for these two BSM scenarios using the HS method. We have considered the most generic Two Higgs Doublet Model, i.e., no Z_2 symmetry has been imposed. In [113, 155] the dimension-6 operators are computed for Z_2 symmetric 2HDM based on the method discussed in [45, 46]. We have noted a few erroneous operators in [113] and corrected them. We have further revisited the spectrum computation performed in [113, 155] but with the most generic operator set. We have carried a similar task for the effective MLRSM case. For both the models, we have discussed the complete effective lagrangian exhaustively. We have further computed all the modified Feynman vertices after the inclusion of dimension-6 operators. We have rigorously estimated the impacts of those effective operators on different low energy observables, e.g., Weak mixing angle, Fermi constant, ρ and oblique (S, T, U) parameters. Then we have mentioned how the production, and decay of the massive BSM particles can be affected within this EFT framework in the presence of the modified interactions. In this connection, we have outlined the possible sources of additional contributions to the magnetic moment interactions and rare lepton flavor, and number violating processes, e.g., neutrinoless double beta decay, radiative and three-body decays of charged leptons. We have also derived the modified amplitude for the scalar four-point interactions and reasoned the necessity of further investigation of the theoretical constraints, e.g., unitarity, and vacuum stability. Then we have concluded with a note mentioning the possible future directions one can think of, based on this work.

2 Hilbert series and effective operators

In this section, we have briefly sketched the Hilbert Series (HS) method [9, 37, 38, 53–55, 57–59] based on which our BSM-EFT construction has been developed. There are two important ingredients to define the HS: (i) Plethystic Exponentials (PEs) and (ii) the Haar measure. The generic form of the Hilbert Series can be given as [9, 57–59]:

$$\mathcal{H}[\phi] = \prod_{j=1}^n \int_{\mathcal{G}_j} \underbrace{d\mu_j}_{\text{Haar Measure}} \underbrace{PE[\varphi, R]}_{\text{Plethystic Exponential}}, \quad (2.1)$$

where, φ is a spurion variable which can be either a scalar (ϕ), a fermion (ψ) or a gauge field (A_μ). Here, the PEs are integrated on the symmetry (group) space using the Haar measure $d\mu$. The Plethystic Exponentials for bosonic and fermionic fields are written as [9, 57–59]:

$$PE[\phi, R] = \exp \left[\sum_{r=1}^{\infty} \frac{\phi^r \chi_R(z_j^r)}{r} \right], \quad (2.2)$$

$$PE[\psi, R] = \exp \left[\sum_{r=1}^{\infty} (-1)^{r+1} \frac{\psi^r \chi_R(z_j^r)}{r} \right], \quad (2.3)$$

respectively. Here, $\chi_R(z_j^r)$ is the “Weyl” character corresponding to the representation R of the spurious fields (ϕ, ψ) under the symmetry group $\mathcal{G}_j$.

For a given group $\mathcal{G}$, the form of the Haar measure can be expressed as [157–159]:

$$\int_{\mathcal{G}} d\mu_{\mathcal{G}} = \frac{1}{(2\pi i)^k} \oint_{|z_1|=1} \cdots \oint_{|z_\sigma|=1} \frac{dz_1}{z_1} \cdots \frac{dz_\sigma}{z_\sigma} \prod_{\alpha^+} \left[1 - \prod_{m=1}^k (z_m)^{\alpha_m^+} \right], \quad (2.4)$$

where α_m^+ , and k are the positive roots and rank of the group $\mathcal{G}$ respectively.

Thus it is evident that the HS construction relies on two pieces of information: (a) the DOF, i.e., particle content and (b) the transformation properties of the particles under the given symmetries. If we restrict ourselves to the quantum fields, i.e., particles then the information we have gathered so far is sufficient to compute the HS. But we are interested in a complete set of operators, and that includes operators involving *derivatives*. Since the derivatives transform under space-time symmetry, we need knowledge of the conformal group along with the internal symmetries.¹ Due to the intrinsic properties of derivative, additional constraints might appear in the form of Equations of Motion (EOMs), and Integration by Parts (IBPs). It leads to the overcounting in the operators. Thus we should carefully identify the redundancy in the operator set to construct only independent operators so that they can form a basis. Now we will discuss how the HS construction can automatically take care of this issue, which is one of the most significant merits of this program.

To do so, we need to construct the characters of the derivative operator $\mathcal{D}$ under the conformal group $\text{SO}(4, 2)$ ² [9, 53, 57, 160–166]. The extra two dimensions (+2) increase the rank of the conformal group by one unit which appears as a scaling dimension (Δ) of the representation. In our analysis, Δ is the mass dimension of the operator/fields, e.g., 1 for boson, 3/2 for fermions, etc. The remaining part of the conformal group is the Lorentz group. But as the Lorentz group (LG) is not compact, its unitary representation is infinite dimensional [167]. Thus we will realize Lorentz group as $\text{SO}(4, \mathbb{C}) \simeq \text{SU}(2)_L \otimes \text{SU}(2)_R$. The Verma module characters [167] of $\text{SO}(4, \mathbb{C})$ with highest weight (l_1, l_2) $[\chi_{(l_1, l_2)}^{(4)}(x_1, x_2)]$ can be written as the product of the Verma module characters of $\text{SU}(2)_L$ with highest weight j_1 $[\chi_{j_1}(x)]$, and $\text{SU}(2)_R$ with highest weight j_2 $(\chi_{j_2}(y))$ as [9, 53, 57, 160–163, 168]:

$$\chi_{(l_1, l_2)}^{(4)}(x_1, x_2) = \chi_{j_1}(y_1) * \chi_{j_2}(y_2), \quad (2.5)$$

where $l_1 = j_1 + j_2$, $l_2 = j_1 - j_2$, $x_1 = y_1^{1/2} y_2^{1/2}$, and $x_2 = y_1^{1/2} y_2^{-1/2}$. Using this generic notion as discussed by F. A. Dolan in [160], the *characters of unitary irreducible representations* of $\text{SO}(4, 2)$ can be given as [9, 53, 57, 160, 169–171]:

$$\begin{aligned} \chi_{[j_1+j_2+2; j_1, j_2]}^{(4)}(s, y_1, y_2) &= s^{j_1+j_2+2} \left(\chi_{j_1}(y_1) \chi_{j_2}(y_2) - s \chi_{j_1-\frac{1}{2}}(y_1) \chi_{j_2-\frac{1}{2}}(y_2) \right) P^{(4)}(s, y_1, y_2), \\ \chi_{[j_1+1; j_1]}^{(4)}(s, y_1, y_2) &= s^{j_1+1} \left(\chi_{j_1}(y_1) - s \chi_{j_1-\frac{1}{2}}(y_1) \chi_{\frac{1}{2}}(y_2) + s^2 \chi_{j_1-1}(y_1) \right) P^{(4)}(s, y_1, y_2), \\ \chi_{[j_2+1; j_2]}^{(4)}(s, y_1, y_2) &= s^{j_2+1} \left(\chi_{j_2}(y_2) - s \chi_{j_2-\frac{1}{2}}(y_2) \chi_{\frac{1}{2}}(y_1) + s^2 \chi_{j_2-1}(y_2) \right) P^{(4)}(s, y_1, y_2), \end{aligned}$$

¹The transformation properties of the fields, having non-zero spin, under the space-time symmetry group must be taken into account. The forthcoming discussion addresses this as well.

²As we are working in 3+1 dimensional space-time.

where $P^{(4)}(s, y_1, y_2)$ is the momentum generating function for $SO(4, \mathbb{C})$. Now using this construction, we can write down the necessary and relevant characters for our analysis [9, 53, 57, 160]:

$$\chi_{[1;(0,0)]}^{(4)}(\mathcal{D}, \alpha, \beta) = \mathcal{D} P^{(4)}(\mathcal{D}, \alpha, \beta) \times [1 - \mathcal{D}^2], \quad (2.6)$$

$$\chi_{[\frac{3}{2},(\frac{1}{2},0)]}^{(4)}(\mathcal{D}, \alpha, \beta) = \mathcal{D}^{\frac{3}{2}} P^{(4)}(\mathcal{D}, \alpha, \beta) \times \left[\alpha + \frac{1}{\alpha} - \mathcal{D} \left(\beta + \frac{1}{\beta} \right) \right], \quad (2.7)$$

$$\chi_{[\frac{3}{2};(0,\frac{1}{2})]}^{(4)}(\mathcal{D}, \alpha, \beta) = \mathcal{D}^{\frac{3}{2}} P^{(4)}(\mathcal{D}, \alpha, \beta) \times \left[\beta + \frac{1}{\beta} - \mathcal{D} \left(\alpha + \frac{1}{\alpha} \right) \right], \quad (2.8)$$

$$\chi_{[2;(1,0)]}^{(4)}(\mathcal{D}, \alpha, \beta) = \mathcal{D}^2 P^{(4)}(\mathcal{D}, \alpha, \beta) \times \left[\alpha^2 + \frac{1}{\alpha^2} + 1 - \mathcal{D} \left(\alpha + \frac{1}{\alpha} \right) \left(\beta + \frac{1}{\beta} \right) + \mathcal{D}^2 \right], \quad (2.9)$$

$$\chi_{[2;(0,1)]}^{(4)}(\mathcal{D}, \alpha, \beta) = \mathcal{D}^2 P^{(4)}(\mathcal{D}, \alpha, \beta) \times \left[\beta^2 + \frac{1}{\beta^2} + 1 - \mathcal{D} \left(\alpha + \frac{1}{\alpha} \right) \left(\beta + \frac{1}{\beta} \right) + \mathcal{D}^2 \right], \quad (2.10)$$

where the momentum generating function $P^{(4)}(\mathcal{D}, \alpha, \beta)$ can be written as [9, 53, 57],

$$P^{(4)}(\mathcal{D}, \alpha, \beta) = \left[(1 - \mathcal{D}\alpha\beta) \left(1 - \frac{\mathcal{D}}{\alpha\beta} \right) \left(1 - \frac{\mathcal{D}\alpha}{\beta} \right) \left(1 - \frac{\mathcal{D}\beta}{\alpha} \right) \right]^{-1}. \quad (2.11)$$

In the presence of derivative operators which transform under the Lorentz group, the Hilbert Series in eq. (2.1) is redefined in the following form [9, 53, 57]:

$$\mathcal{H}[\phi] = \prod_{j=1}^n \int_{LG} d\mu_{LG} \int_{\mathcal{G}_j} d\mu_j PE[\phi, \mathcal{D}, R] \frac{1}{P^{(4)}(\mathcal{D}, \alpha, \beta)}, \quad (2.12)$$

where, $\mathcal{D}$ is the spurion variable corresponding to the covariant derivative operator. The Plethystics in eq. (2.2) also get modified as [9, 53, 57]:

$$PE[\phi, \mathcal{D}, R] = \exp \left[\sum_{r=1}^{\infty} \left(\frac{\phi}{\mathcal{D}^{\Delta_\phi}} \right)^r \frac{\chi_R(z_j^r, \alpha^r, \beta^r)}{r} \right], \quad (2.13)$$

$$PE[\psi, \mathcal{D}, R] = \exp \left[\sum_{r=1}^{\infty} (-1)^{r+1} \left(\frac{\psi}{\mathcal{D}^{\Delta_\psi}} \right)^r \frac{\chi_R(z_j^r, \alpha^r, \beta^r)}{r} \right]. \quad (2.14)$$

The Haar measure for Lorentz group is given as [9, 53, 57]:

$$d\mu_{LG} = \left[\frac{1}{2\alpha} (1 - \alpha^2) \left(1 - \frac{1}{\alpha^2} \right) \right] \left[\frac{1}{2\beta} (1 - \beta^2) \left(1 - \frac{1}{\beta^2} \right) \right]. \quad (2.15)$$

In this paper, we have worked with the following groups: $U(1), SU(2), SU(3)$. The Haar measure for these groups are given as [157–159]:

$$d\mu_{U(1)} = \frac{1}{x}, \quad (2.16)$$

$$d\mu_{SU(2)} = \frac{1}{2y} (1 - y^2) \left(1 - \frac{1}{y^2} \right),$$

$$d\mu_{SU(3)} = \frac{1}{6z_1 z_2} (1 - z_1 z_2) \left(1 - \frac{z_1^2}{z_2} \right) \left(1 - \frac{z_2^2}{z_1} \right) \left(1 - \frac{1}{z_1 z_2} \right) \left(1 - \frac{z_1}{z_2^2} \right) \left(1 - \frac{z_2}{z_1^2} \right).$$

We have worked with few specific representations under these above mentioned internal symmetry groups. We have enlisted the characters of those representations [172] below:

- U(1)

$$z^q, \quad q \text{ is the U(1) charge of the representation,} \quad (2.17)$$

- SU(2)

$$[1] \equiv 2 = \bar{2} = z_1 + \frac{1}{z_1}, \quad (2.18)$$

$$[2] \equiv 3 \equiv 2 \otimes 2 - 1 \equiv \left(z_1 + \frac{1}{z_1}\right)^2 - 1 = z_1^2 + \frac{1}{z_1^2} + 1, \quad (2.19)$$

- SU(3)

$$[1, 0] \equiv 3 = z_1 + \frac{1}{z_2} + \frac{z_2}{z_1}, \quad (2.20)$$

$$[0, 1] \equiv \bar{3} = \bar{2} = z_2 + \frac{1}{z_1} + \frac{z_1}{z_2}, \quad (2.21)$$

$$[1, 1] \equiv 8 \equiv 3 \otimes \bar{3} - 1 = \left(z_1 + \frac{1}{z_2} + \frac{z_2}{z_1}\right) \left(z_2 + \frac{1}{z_1} + \frac{z_1}{z_2}\right) - 1 \quad (2.22)$$

$$= \frac{z_1^2}{z_2} + \frac{z_1}{z_2^2} + z_1 z_2 + \frac{1}{z_1 z_2} + \frac{z_2}{z_1^2} + \frac{z_2^2}{z_1} + 2. \quad (2.23)$$

In the above equations, the Dynkin indices of the respective representations are depicted as [...].

Based on the HS method, using the information above the complete and independent set of effective operators up to dimension-6 has been constructed for 2HDM- and MLRSM-EFT frameworks mimicking the “Warsaw” basis. First we have constructed the raw HS³ [9, 53, 57], then we have refined those operators and presented them in a workable format. In figure 1, we have sketched the integrated steps that lead to the Hilbert Series.

3 Two Higgs Doublet Model (2HDM)

The Two Higgs Doublet Model (2HDM) extends the Standard Model (SM) in the most elementary way through the addition of an identical second Higgs doublet [113, 177–186]. In 2HDM the internal gauge symmetry is the same as the SM: $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. In table 1, we have listed the particle content of this model, and have depicted their representations and charges. In the following section, we have revisited the work of [113]

³In ref. [53] the derivative operator construction for specific cases are performed using “Macaulay2” [173], an “algorithm based on Commutative Algebra and Algebraic Geometry” [174–176].

Figure 1. Flow chart depicting the Hilbert Series construction.

using our set of operators (which are not the same as in ref. [113]). Before going into the EFT description of the 2HDM scenario, we have provided a brief discussion on the spectrum computation of this model following the path paved in [113].

The renormalizable Lagrangian. Based on the transformation properties of the particles, the renormalizable lagrangian for 2HDM can be written as [113, 177, 178]:

$$\begin{aligned}
 \mathcal{L}_{2HDM}^{(4)} = & -\frac{1}{4}\text{Tr}[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{4}\text{Tr}[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
 & + (D^\mu\phi_1)^\dagger(D_\mu\phi_1) + (D^\mu\phi_2)^\dagger(D_\mu\phi_2) - V(\phi_1, \phi_2) \\
 & + i(\bar{L}\gamma^\mu D_\mu L + \bar{Q}\gamma^\mu D_\mu Q + \bar{e}\gamma^\mu D_\mu e + \bar{u}\gamma^\mu D_\mu u + \bar{d}\gamma^\mu D_\mu d) + L_Y + h.c.,
 \end{aligned}
 \tag{3.1}$$

where the components of field strength tensors are given as [113, 177, 178]:

$$\begin{aligned}
 G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_3 f^{ABC} G_\mu^B G_\nu^C, \\
 W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g\epsilon^{IJK} W_\mu^J W_\nu^K, \\
 B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu.
 \end{aligned}
 \tag{3.2}$$

for $SU(3)_C, SU(2)_L, U(1)_Y$ respectively with $\{A, B, C\} \in (1, \dots, 8)$, $\{I, J, K\} \in (1, 2, 3)$. The general form of the covariant derivative can be given as [113, 177, 178]:

$$D_\mu \equiv \left(\partial_\mu + ig_3 \frac{T^A}{2} G_\mu^A + ig \frac{\tau^I}{2} W_\mu^I + ig' Y B_\mu \right).
 \tag{3.3}$$

Field	SU(3) _C	SU(2) _L	U(1) _Y	Spin	Lorentz Group
ϕ_1	1	2	1/2	0	Scalar
ϕ_2	1	2	1/2	0	Scalar
Q_L	3	2	1/6	1/2	Spinor
$(u^c)_L$	$\bar{3}$	1	2/3	1/2	Spinor
$(d^c)_L$	$\bar{3}$	1	-1/3	1/2	Spinor
L_L	1	2	-1/2	1/2	Spinor
$(e^c)_L$	1	1	-1	1/2	Spinor
G_μ^A	8	1	0	1	Vector
W_μ^I	1	3	0	1	Vector
B_μ	1	1	0	1	Vector

Table 1. 2HDM: quantum numbers of the fields.

Let us consider that D_μ acts on a quantum field (φ) which is charged under some gauge quantum number, e.g., SU(3)_C. Then the covariant derivative D_μ will contain a term proportional to the respective gauge field, e.g., $ig_3 \frac{T^A}{2} G_\mu^A$. Here, g_3, g, g' are the gauge couplings and G_μ^A, W_μ^I, B_μ are gauge fields corresponding to SU(3)_C, SU(2)_L, U(1)_Y respectively. Y is the hypercharge of the field on which this operator is acting. Here, $\frac{T^A}{2}$ and $\frac{\tau^I}{2}$ are the normalized generators of SU(3)_C and SU(2)_L groups respectively.

The Yukawa coupling is given as [113, 177, 178]:

$$\mathcal{L}_Y = -y_1^e \bar{L} \phi_1 e - y_2^e \bar{L} \phi_2 e - y_1^d \bar{Q} \phi_1 d - y_2^d \bar{Q} \phi_2 d - y_1^u \bar{Q} \tilde{\phi}_1 u - y_2^u \bar{Q} \tilde{\phi}_2 u + h.c., \quad (3.4)$$

where $\tilde{\phi} = i\tau_2 \phi^*$, and $y_{1,2}^{e,u,d}$ are the respective Yukawa coupling matrices.

The scalar potential for generic 2HDM is given as [177, 178]:

$$\begin{aligned} V(\phi_1, \phi_2) = & m_{11}^2 (\phi_1^\dagger \phi_1) + m_{22}^2 (\phi_2^\dagger \phi_2) - m_{12}^2 (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 \\ & + \lambda_3 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) + \frac{1}{2} \lambda_5 [(\phi_1^\dagger \phi_2)^2 + (\phi_2^\dagger \phi_1)^2] \\ & + (\lambda_6 (\phi_1^\dagger \phi_1) + \lambda_7 (\phi_2^\dagger \phi_2)) (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1). \end{aligned} \quad (3.5)$$

Here, the colored parts signify the Z_2 symmetry ($\phi_1 \rightarrow -\phi_1, \phi_2 \rightarrow \phi_2$) violation. These Z_2 violating terms in the above equation are not considered in [113].

In this model the spontaneous symmetry breaking (SSB) of SM gauge symmetry is induced by the vacuum expectation values (VEVs) of both the Higgs doublets. After SSB,

the Higgs fields are given as [113, 177, 178]:

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ (v_1 + h_1 + ia_1)/\sqrt{2} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ (v_2 + h_2 + ia_2)/\sqrt{2} \end{pmatrix}, \quad (3.6)$$

where v_1 and v_2 are the VEVs of ϕ_1 and ϕ_2 respectively. The electro-weak VEV is given as $v = \sqrt{v_1^2 + v_2^2}$. Among these unphysical scalar fields, we have two CP-odd scalars (a_1, a_2), two CP-even scalars (h_1, h_2) and two charged scalars (ϕ_1^+, ϕ_2^+).

After SSB, the scalar mass terms in the lagrangian is given as [113, 177, 178]:

$$\mathcal{L}_{MH}^{(4)} = \frac{1}{2} \begin{pmatrix} a_1 & a_2 \end{pmatrix} m_i^2 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \phi_1^- & \phi_2^- \end{pmatrix} m_+^2 \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} + \frac{1}{2} \begin{pmatrix} h_1 & h_2 \end{pmatrix} m_r^2 \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}. \quad (3.7)$$

The scalar mass matrices are explicitly written as [113]:

$$\begin{aligned} m_i^2 &= \begin{pmatrix} -m_{12}^2 + v_1 v_2 \lambda_5 + \frac{1}{2} v_1^2 \lambda_6 + \frac{1}{2} v_2^2 \lambda_7 & -\frac{v_2}{v_1} & 1 \\ 1 & -\frac{v_1}{v_2} & \end{pmatrix}, \\ m_r^2 &= \begin{pmatrix} -\frac{\lambda_7 v_2^3}{2v_1} + \frac{3}{2} v_1 \lambda_6 v_2 + \frac{m_{12}^2 v_2}{v_1} + 2v_1^2 \lambda_1 & -m_{12}^2 + v_1 v_2 (\lambda_3 + \lambda_4 + \lambda_5) + \frac{3}{2} v_1^2 \lambda_6 + \frac{3}{2} v_2^2 \lambda_7 \\ -m_{12}^2 + v_1 v_2 (\lambda_3 + \lambda_4 + \lambda_5) + \frac{3}{2} v_1^2 \lambda_6 + \frac{3}{2} v_2^2 \lambda_7 & -\frac{\lambda_6 v_1^3}{2v_2} + \frac{3}{2} v_2 \lambda_7 v_1 + \frac{m_{12}^2 v_1}{v_2} + 2v_2^2 \lambda_2 \end{pmatrix}, \\ m_+^2 &= \begin{pmatrix} -m_{12}^2 + \frac{1}{2} v_1 v_2 (\lambda_4 + \lambda_5) + \frac{1}{2} v_1^2 \lambda_6 + \frac{1}{2} v_2^2 \lambda_7 & -\frac{v_2}{v_1} & 1 \\ 1 & -\frac{v_1}{v_2} & \end{pmatrix}. \end{aligned} \quad (3.8)$$

Using the potential minimization conditions [113, 177, 178]

$$\begin{aligned} \frac{\partial V(\phi_1, \phi_2)}{\partial v_1} &= m_{11}^2 v_1 - m_{12}^2 v_2 + \lambda_1 v_1^3 + \frac{1}{2} \lambda_3 v_2^2 v_1 + \frac{1}{2} \lambda_4 v_2^2 v_1 + \frac{1}{2} \lambda_5 v_2^2 v_1 + \frac{3}{2} \lambda_6 v_2 v_1^2 + \frac{1}{2} \lambda_7 v_2^3 \\ &= 0, \\ \frac{\partial V(\phi_1, \phi_2)}{\partial v_2} &= -m_{12}^2 v_1 + m_{22}^2 v_2 + \frac{1}{2} \lambda_3 v_2 v_1^2 + \frac{1}{2} \lambda_4 v_2 v_1^2 + \frac{1}{2} \lambda_5 v_2 v_1^2 + \lambda_2 v_2^3 + \frac{1}{2} \lambda_6 v_1^3 + \frac{3}{2} \lambda_7 v_2^2 v_1 \\ &= 0, \end{aligned} \quad (3.9)$$

we can recast the mass-square parameters of the potential as [113, 177, 178]:

$$\begin{aligned} m_{11}^2 &= \frac{m_{12}^2 v_2}{v_1} - \frac{1}{2} \lambda_3 v_2^2 - \frac{1}{2} \lambda_4 v_2^2 - \frac{1}{2} \lambda_5 v_2^2 - \lambda_1 v_1^2 - \frac{3}{2} \lambda_6 v_1 v_2 - \frac{\lambda_7 v_2^3}{2v_1}, \\ m_{22}^2 &= \frac{m_{12}^2 v_1}{v_2} - \frac{1}{2} \lambda_3 v_1^2 - \frac{1}{2} \lambda_4 v_1^2 - \frac{1}{2} \lambda_5 v_1^2 - \lambda_2 v_2^2 - \frac{\lambda_6 v_1^3}{2v_2} - \frac{3}{2} \lambda_7 v_2 v_1. \end{aligned} \quad (3.10)$$

The physical scalar fields (Φ') are related to the unphysical ones (Φ) through the relations below, which further can be used to obtain physical scalar mass matrices [113, 177, 178]:

$$(\Phi') = \begin{pmatrix} \cos \beta_j & \sin \beta_j \\ -\sin \beta_j & \cos \beta_j \end{pmatrix} (\Phi), \quad \beta_j \equiv \{r, i, +\}. \quad (3.11)$$

Here,

$$\Phi \equiv \left\{ \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} \right\}, \quad \Phi' \equiv \left\{ \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}, \begin{pmatrix} A \\ \tilde{G} \end{pmatrix}, \begin{pmatrix} H^\pm \\ \tilde{G}^\pm \end{pmatrix} \right\}. \quad (3.12)$$

While computing the physical states, we find two Goldstone bosons (CP-odd: $\tilde{G}$, charged: $\tilde{G}^\pm$) which get eaten up through the Higgs mechanism and generate masses for Z and $W^\pm$ gauge bosons. The CP-odd, charged scalar and CP-even mass matrices are diagonalized by rotation angles [113, 177, 178]:

$$\tan 2\beta_+ = \tan 2\beta_i = \frac{2v_2v_1}{v_2^2 - v_1^2}, \quad (3.13)$$

$$\tan 2\beta_r = \frac{2v_1v_2(-2m_{12}^2 + 3\lambda_6v_1^2 + v_2(2\lambda_3v_1 + 2\lambda_4v_1 + 2\lambda_5v_1 + 3\lambda_7v_2))}{2m_{12}^2(v_1^2 - v_2^2) - \lambda_6(v_1^4 + 3v_2^2v_1^2) + v_2(-4\lambda_1v_1^3 + 3\lambda_7v_2v_1^2 + 4\lambda_2v_2^2v_1 + \lambda_7v_2^3)},$$

leading to physical masses for one CP-odd scalar (A), one charged scalar ($H^\pm$), and two CP-even scalars (H_1, H_2) respectively:

$$m_A^2 = -v^2 \left(\lambda_5 - \frac{m_{12}^2}{v_1v_2} + \frac{\lambda_6v_1}{2v_2} + \frac{\lambda_7v_2}{2v_1} \right), \quad (3.14)$$

$$m_{H^\pm}^2 = -v^2 \left(\frac{\lambda_4}{2} + \frac{\lambda_5}{2} - \frac{m_{12}^2}{v_1v_2} + \frac{\lambda_6v_1}{2v_2} + \frac{\lambda_7v_2}{2v_1} \right), \quad (3.15)$$

$$m_{H_1}^2 = \frac{1}{2v_1v_2} \left[v_1 \sin^2 \beta_h (2m_{12}^2v_1 - \lambda_6v_1^3 + v_2^2(4\lambda_2v_2 + 3\lambda_7v_1)) \right. \\ \left. - v_1v_2 \sin(2\beta_h) (-2m_{12}^2 + 3\lambda_6v_1^2 + v_2(2\lambda_3v_1 + 2\lambda_4v_1 + 2\lambda_5v_1 + 3\lambda_7v_2)) \right. \\ \left. + v_2 \cos^2 \beta_h (2m_{12}^2v_2 + 4\lambda_1v_1^3 + 3\lambda_6v_2v_1^2 - \lambda_7v_2^3) \right], \quad (3.16)$$

$$m_{H_2}^2 = \frac{1}{2v_1v_2} \left[v_2 \sin^2 \beta_h (2m_{12}^2v_2 + 4\lambda_1v_1^3 + 3\lambda_6v_2v_1^2 - \lambda_7v_2^3) \right. \\ \left. + v_1v_2 \sin(2\beta_h) (-2m_{12}^2 + 3\lambda_6v_1^2 + v_2(2\lambda_3v_1 + 2\lambda_4v_1 + 2\lambda_5v_1 + 3\lambda_7v_2)) \right. \\ \left. + v_1 \cos^2 \beta_h (2m_{12}^2v_1 - \lambda_6v_1^3 + v_2^2(4\lambda_2v_2 + 3\lambda_7v_1)) \right]. \quad (3.17)$$

The Higgs kinetic terms $(D^\mu \phi_1)^\dagger (D_\mu \phi_1)$, $(D^\mu \phi_2)^\dagger (D_\mu \phi_2)$ contain the informations regarding the gauge boson masses which acquire the following forms after SSB:

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2, \quad (3.18)$$

$$m_W^2 = \frac{1}{4}g^2v^2. \quad (3.19)$$

The physical gauge bosons ($W_\mu^\pm, Z_\mu, A_\mu$) are related to the unphysical ones ($W_\mu^1, W_\mu^2, W_\mu^3, B_\mu$) as:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad (3.20)$$

$$W_\mu^3 = Z_\mu \cos \theta_w + A_\mu \sin \theta_w, \quad (3.21)$$

$$B_\mu = -Z_\mu \sin \theta_w + A_\mu \cos \theta_w. \quad (3.22)$$

After SSB, the fermion masses are generated through the Yukawa terms as:

$$m_f = \frac{1}{\sqrt{2}} \left(v_1 y_1^f + v_2 y_2^f \right), \quad f = e, u, d. \quad (3.23)$$

where $y_{1,2}^f$ are the (3×3) Yukawa matrices for three generations of fermions.

Figure 2. Effective operators in “Warsaw”-like basis representing the following class of operators: (a) ϕ^6 $[6 + 7 \times 2 = 20]$, (b) $\phi^2 X^2$ $[32]$, (c) $\psi^2 \phi^2 D$ $[14 + 10 \times 2 = 34]$, (d) ψ^4 $[20 + 5 \times 2 = 30]$ (Baryon Number Conserving) + $[4 \times 2 = 8]$ (Baryon Number Violating), (e) $\phi^4 D^2$ $[8 + 6 \times 2 = 20]$, (f) $\psi^2 \phi^3$ $[24 \times 2 = 48]$, (g) $\psi^2 \phi X$ $[16 \times 2 = 32]$, (h) X^3 $[4]$.

4 Effective operators for 2HDM, their categorisation and impact on spectrum

We have constructed the Hilbert Series for 2HDM using the information from table 1 and following the thumb rules mentioned in the flowchart of figure 1. The complete set of dimension-6 operators are classified as: $\{\phi^6, \phi^2 X^2, \psi^2 \phi^2 D, \psi^4, \phi^4 D^2, \psi^2 \phi^3, \psi^2 \phi X, X^3\}$. These operators are depicted in tables 4–12, and they are schematically shown in figure 2. The number of independent operators are also mentioned in the caption of figure 2 as $\mathcal{O} [\dots]$. The effective operators in the context of 2HDM have been discussed in [112–116, 155, 156, 187]. As we are working with the most generic 2HDM framework, we have also included the Z_2 violating operators, which are written in colored texts. Our method predicts the existence of $\mathcal{O}_{\square}^{21(21)}$ operator in $\phi^4 D^2$ class instead of $Q_{\varphi D}^{12(12)}$ and $Q_{\varphi D}^{(1)21(2)}$ operators given in [113]. For 2HDM, the Hilbert Series yields 228 independent operators of mass dimension-6. This includes the distinct Hermitian Conjugates (h.c.) as well. In our analysis, we have assumed the Wilson Coefficients to be real. While writing the explicit structures of these operators apart from the field variables we have used few tensors, e.g., $\delta_{\mu\nu}$ (Kronecker delta), $\epsilon^{\mu\nu\rho}$ (Levi-Civita), etc. to form the invariants. As the operator set is enormous and their intricate structures are a bit involved, we have provided a suitable table where we have remarked on the nomenclature of the operators. We hope this would help the readers to follow the operators ($\mathcal{O}$), and their respective Wilson Coefficients ($\mathcal{C}$) which are also named following the same rules, see table 2.

We have started with mentioning the dimension-5 operators which are equivalent to the Weinberg operator. These operators lead to the Majorana mass terms for the SM light neutrinos. Here, we would like to note that the dimension-5, and -6 operators are suppressed by a mass scale Λ^{-1} and Λ^{-2} respectively, but they are not explicitly mentioned in the tables to keep them tidy. But in the later sections where the explicit computation with these operators has been performed, the scales are mentioned appropriately.

Operator Class	Symbol	Superscript	Subscript	Remarks
ϕ^6	$\mathcal{O}_6^{i_1 i_2 \dots i_6}$	$\{i_1, \dots, i_6\} \rightarrow \{1, 2\}$	$6 \rightarrow \text{mass dimension of } \phi^6$	$1 \rightarrow \{\phi_1, \phi_1^\dagger\}; 2 \rightarrow \{\phi_2, \phi_2^\dagger\}$ for $X \equiv WB, \tilde{X} \rightarrow \tilde{W}B$
$\phi^2 X^2$	$\mathcal{O}_{\phi X / \phi \tilde{X}}^{ij}$	$\{i, j\} \rightarrow \{1, 2\}$	$X \rightarrow \{W, B, G, WB\}$	
X^3	$\mathcal{O}_{X/\tilde{X}}$		$X \rightarrow \{W, G\}$	
$\psi^2 \phi X$	$\mathcal{O}_{fX}^i$	$i \rightarrow \{1, 2, \tilde{1}, \tilde{2}\}$	$f \rightarrow \{u, d, e\}$	For (u, d) , the operator contains Q and for (e) , one has L u couples with $\tilde{\phi}_1, \tilde{\phi}_2$ e, d couple with ϕ_1, ϕ_2
$\psi^2 \phi^3$	$\mathcal{O}_{f_1 f_2}^{i(jk)}$	$\{i, j, k\} \rightarrow \{1, 2, \tilde{1}, \tilde{2}\}$	$f_1 \rightarrow \{Q, L\}, f_2 \rightarrow \{u, d, e\}$	The field outside “()” is coupled directly to the fermions, while fields inside “()” are contracted with each other
$\psi^2 \phi^2 D$	$\mathcal{O}_{f\phi D}^{ij}$ $\mathcal{O}_{f\phi D}^{ij[1]/ij[3]}$	$\{i, j\} \rightarrow \{1, 2, \tilde{1}, \tilde{2}\}$	$f \rightarrow \{Q, L, u, d, e\}$	[1] $\Rightarrow$ SU(2) Singlet [3] $\Rightarrow$ SU(2) Triplet (involves τ^I)
$\phi^4 D^2$	$\mathcal{O}_{\square}^{ij(kl)}$ $\mathcal{O}_{\phi D}^{(i)jk(l)}$ $\mathcal{O}_{\phi D}^{ij(k)(l)}$	$\{i, j, k, l\} \rightarrow \{1, 2\}$	$\square$ for terms like $(\phi_i^\dagger \phi_j) \square (\phi_k^\dagger \phi_l)$, else ϕD	(i) $\Rightarrow D$ acting on ϕ_i
ψ^4	$\mathcal{O}_{f_1 f_2}^{[i]} / \mathcal{O}_{f_1 f_2 f_3 f_4}^{[i]}$	$i \rightarrow \{1, 3, 8\}$	$\{f_1, \dots, f_4\} \rightarrow \{Q, L, u, d, e\}$	[1] $\Rightarrow$ SU(2), SU(3) Singlet [3] $\Rightarrow$ SU(2) Triplet (involves τ^I) [8] $\Rightarrow$ SU(3) Octet (involves T^A)

Table 2. 2HDM: summary of the nomenclature of operators.

$\mathcal{O}_{\phi L}^{11}$	$(L^T C i \tau_2 \phi_1)(\tilde{\phi}_1^\dagger L) + h.c.$	$\mathcal{O}_{\phi L}^{22}$	$(L^T C i \tau_2 \phi_2)(\tilde{\phi}_2^\dagger L) + h.c.$	$\mathcal{O}_{\phi L}^{12}$	$(L^T C i \tau_2 \phi_1)(\tilde{\phi}_2^\dagger L) + h.c.$
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Table 3. 2HDM: $\phi^2 L^2$ [3 $\times$ 2 = 6] class of operators.

Dimension-5 and dimension-6 operators: spectrum of 2HDM-EFT.

Majorana mass of neutrinos from $\phi^2 L^2$ operators.

The neutrino mass term $[\overline{(\nu_L)^c} \nu_L]$ is generated from the dimension-5 operators, see table 3, once the Higgs fields acquire VEVs. The coefficient of this term which is essentially

$\mathcal{O}_6^{111111}$	$(\phi_1^\dagger \phi_1)^3$	$\mathcal{O}_6^{222222}$	$(\phi_2^\dagger \phi_2)^3$
$\mathcal{O}_6^{111122}$	$(\phi_1^\dagger \phi_1)^2 (\phi_2^\dagger \phi_2)$	$\mathcal{O}_6^{112222}$	$(\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2)^2$
$\mathcal{O}_6^{122111}$	$(\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) (\phi_1^\dagger \phi_1)$	$\mathcal{O}_6^{122122}$	$(\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) (\phi_2^\dagger \phi_2)$
$\mathcal{O}_6^{121211}$	$(\phi_1^\dagger \phi_2)^2 (\phi_1^\dagger \phi_1) + \text{h.c.}$	$\mathcal{O}_6^{121222}$	$(\phi_1^\dagger \phi_2)^2 (\phi_2^\dagger \phi_2) + \text{h.c.}$
$\mathcal{O}_6^{111112}$	$(\phi_1^\dagger \phi_1)^2 (\phi_1^\dagger \phi_2) + \text{h.c.}$	$\mathcal{O}_6^{122222}$	$(\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_2)^2 + \text{h.c.}$
$\mathcal{O}_6^{121221}$	$(\phi_1^\dagger \phi_2)^2 (\phi_2^\dagger \phi_1) + \text{h.c.}$	$\mathcal{O}_6^{121212}$	$(\phi_1^\dagger \phi_2)^3 + \text{h.c.}$
$\mathcal{O}_6^{112212}$	$(\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) (\phi_1^\dagger \phi_2) + \text{h.c.}$		

Table 4. 2HDM: ϕ^6 $[6 + 7 \times 2 = 20]$ class of operators.

$\mathcal{O}_{\phi G}^{11}$	$(\phi_1^\dagger \phi_1) G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{11}$	$(\phi_1^\dagger \phi_1) G_{\mu\nu}^A \tilde{G}^{A\mu\nu}$	$\mathcal{O}_{\phi G}^{22}$	$(\phi_2^\dagger \phi_2) G_{\mu\nu}^A G^{A\mu\nu}$
$\mathcal{O}_{\phi W}^{11}$	$(\phi_1^\dagger \phi_1) W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}}^{11}$	$(\phi_1^\dagger \phi_1) W_{\mu\nu}^I \tilde{W}^{I\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{22}$	$(\phi_2^\dagger \phi_2) G_{\mu\nu}^A \tilde{G}^{A\mu\nu}$
$\mathcal{O}_{\phi B}^{11}$	$(\phi_1^\dagger \phi_1) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}^{11}$	$(\phi_1^\dagger \phi_1) B_{\mu\nu} \tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi W}^{22}$	$(\phi_2^\dagger \phi_2) W_{\mu\nu}^I W^{I\mu\nu}$
$\mathcal{O}_{\phi B}^{22}$	$(\phi_2^\dagger \phi_2) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}^{22}$	$(\phi_2^\dagger \phi_2) B_{\mu\nu} \tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}}^{22}$	$(\phi_2^\dagger \phi_2) W_{\mu\nu}^I \tilde{W}^{I\mu\nu}$
$\mathcal{O}_{\phi WB}^{11}$	$(\phi_1^\dagger \tau^I \phi_1) W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W} B}^{11}$	$(\phi_1^\dagger \tau^I \phi_1) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi WB}^{22}$	$(\phi_2^\dagger \tau^I \phi_2) W_{\mu\nu}^I B^{\mu\nu}$
$\mathcal{O}_{\phi \tilde{W} B}^{22}$	$(\phi_2^\dagger \tau^I \phi_2) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi G}^{12}$	$(\phi_1^\dagger \phi_2) G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{12}$	$(\phi_1^\dagger \phi_2) G_{\mu\nu}^A \tilde{G}^{A\mu\nu}$
$\mathcal{O}_{\phi G}^{21}$	$(\phi_2^\dagger \phi_1) G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{21}$	$(\phi_2^\dagger \phi_1) G_{\mu\nu}^A \tilde{G}^{A\mu\nu}$	$\mathcal{O}_{\phi W}^{12}$	$(\phi_1^\dagger \phi_2) W_{\mu\nu}^I W^{I\mu\nu}$
$\mathcal{O}_{\phi \tilde{W}}^{12}$	$(\phi_1^\dagger \phi_2) W_{\mu\nu}^I \tilde{W}^{I\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}}^{21}$	$(\phi_2^\dagger \phi_1) W_{\mu\nu}^I \tilde{W}^{I\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}}^{21}$	$(\phi_2^\dagger \phi_1) W_{\mu\nu}^I \tilde{W}^{I\mu\nu}$
$\mathcal{O}_{\phi B}^{12}$	$(\phi_1^\dagger \phi_2) B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}^{12}$	$(\phi_1^\dagger \phi_2) B_{\mu\nu} \tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi B}^{21}$	$(\phi_2^\dagger \phi_1) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{\phi \tilde{B}}^{21}$	$(\phi_2^\dagger \phi_1) B_{\mu\nu} \tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi WB}^{12}$	$(\phi_1^\dagger \tau^I \phi_2) W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W} B}^{12}$	$(\phi_1^\dagger \tau^I \phi_2) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$
$\mathcal{O}_{\phi WB}^{21}$	$(\phi_2^\dagger \tau^I \phi_1) W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W} B}^{21}$	$(\phi_2^\dagger \tau^I \phi_1) \tilde{W}_{\mu\nu}^I B^{\mu\nu}$		

Table 5. 2HDM: $\phi^2 X^2$ [32] class of operators.

$\mathcal{O}_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$\mathcal{O}_W$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$\mathcal{O}_{\tilde{W}}$	$\epsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

Table 6. 2HDM: X^3 [4] class of operators.

$\mathcal{O}_{dG}^1$	$(\bar{Q}\sigma^{\mu\nu}T^A d)\phi_1 G_{\mu\nu}^A$	$\mathcal{O}_{dG}^2$	$(\bar{Q}\sigma^{\mu\nu}T^A d)\phi_2 G_{\mu\nu}^A$
$\mathcal{O}_{uG}^{\tilde{1}}$	$(\bar{Q}\sigma^{\mu\nu}T^A u)\tilde{\phi}_1 G_{\mu\nu}^A$	$\mathcal{O}_{uG}^{\tilde{2}}$	$(\bar{Q}\sigma^{\mu\nu}T^A u)\tilde{\phi}_2 G_{\mu\nu}^A$
$\mathcal{O}_{dW}^1$	$(\bar{Q}\sigma^{\mu\nu}d)\tau^I\phi_1 W_{\mu\nu}^I$	$\mathcal{O}_{dW}^2$	$(\bar{Q}\sigma^{\mu\nu}d)\tau^I\phi_2 W_{\mu\nu}^I$
$\mathcal{O}_{uW}^{\tilde{1}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tau^I\tilde{\phi}_1 W_{\mu\nu}^I$	$\mathcal{O}_{uW}^{\tilde{2}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tau^I\tilde{\phi}_2 W_{\mu\nu}^I$
$\mathcal{O}_{dB}^1$	$(\bar{Q}\sigma^{\mu\nu}d)\phi_1 B_{\mu\nu}$	$\mathcal{O}_{dB}^2$	$(\bar{Q}\sigma^{\mu\nu}d)\phi_2 B_{\mu\nu}$
$\mathcal{O}_{uB}^{\tilde{1}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tilde{\phi}_1 B_{\mu\nu}$	$\mathcal{O}_{uB}^{\tilde{2}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tilde{\phi}_2 B_{\mu\nu}$
$\mathcal{O}_{eW}^1$	$(\bar{L}\sigma^{\mu\nu}e)\tau^I\phi_1 W_{\mu\nu}^I$	$\mathcal{O}_{eW}^2$	$(\bar{L}\sigma^{\mu\nu}e)\tau^I\phi_2 W_{\mu\nu}^I$
$\mathcal{O}_{eB}^1$	$(\bar{L}\sigma^{\mu\nu}e)\phi_1 B_{\mu\nu}$	$\mathcal{O}_{eB}^2$	$(\bar{L}\sigma^{\mu\nu}e)\phi_2 B_{\mu\nu}$

Table 7. 2HDM: $\psi^2\phi X$ [$16\times 2 = 32$] class of operators. Each of these operators also has a distinct Hermitian Conjugate.

$\mathcal{O}_{Le}^{1(21)}$	$(\bar{L}e\phi_1)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Le}^{2(22)}$	$(\bar{L}e\phi_2)(\phi_2^\dagger\phi_2)$	$\mathcal{O}_{Le}^{2(11)}$	$(\bar{L}e\phi_2)(\phi_1^\dagger\phi_1)$
$\mathcal{O}_{Le}^{1(12)}$	$(\bar{L}e\phi_1)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Qd}^{1(21)}$	$(\bar{Q}d\phi_1)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{2(22)}$	$(\bar{Q}d\phi_2)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_{Qd}^{2(11)}$	$(\bar{Q}d\phi_2)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{1(12)}$	$(\bar{Q}d\phi_1)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Qu}^{\tilde{2}(22)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_{Qu}^{\tilde{1}(12)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Qu}^{\tilde{2}(11)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Qu}^{\tilde{1}(21)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_2^\dagger\phi_1)$
$\mathcal{O}_{Le}^{1(11)}$	$(\bar{L}e\phi_1)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Le}^{2(12)}$	$(\bar{L}e\phi_2)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Le}^{1(22)}$	$(\bar{L}e\phi_1)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_{Le}^{2(21)}$	$(\bar{L}e\phi_2)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{1(11)}$	$(\bar{Q}d\phi_1)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{2(12)}$	$(\bar{Q}d\phi_2)(\phi_1^\dagger\phi_2)$
$\mathcal{O}_{Qd}^{1(22)}$	$(\bar{Q}d\phi_1)(\phi_2^\dagger\phi_2)$	$\mathcal{O}_{Qd}^{2(21)}$	$(\bar{Q}d\phi_2)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qu}^{\tilde{1}(11)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_1^\dagger\phi_1)$
$\mathcal{O}_{Qu}^{\tilde{2}(21)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qu}^{\tilde{1}(22)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_2^\dagger\phi_2)$	$\mathcal{O}_{Qu}^{\tilde{2}(12)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_1^\dagger\phi_2)$

Table 8. 2HDM: $\psi^2\phi^3$ [$24\times 2 = 48$] class of operators. Each of these operators also has a distinct Hermitian Conjugate.

$\mathcal{O}_{L\phi D}^{11[1]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{L}\gamma^\mu L)$	$\mathcal{O}_{L\phi D}^{22[1]}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{L}\gamma^\mu L)$	$\mathcal{O}_{L\phi D}^{12[1]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{L}\gamma^\mu L) + h.c.$
$\mathcal{O}_{Q\phi D}^{11[1]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}_{Q\phi D}^{22[1]}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}_{Q\phi D}^{12[1]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{Q}\gamma^\mu Q) + h.c.$
$\mathcal{O}_{e\phi D}^{11}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{e}\gamma^\mu e)$	$\mathcal{O}_{e\phi D}^{22}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{e}\gamma^\mu e)$	$\mathcal{O}_{e\phi D}^{12}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{e}\gamma^\mu e) + h.c.$
$\mathcal{O}_{d\phi D}^{11}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{d}\gamma^\mu d)$	$\mathcal{O}_{d\phi D}^{22}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{d}\gamma^\mu d)$	$\mathcal{O}_{d\phi D}^{12}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{d}\gamma^\mu d) + h.c.$
$\mathcal{O}_{u\phi D}^{11}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{u\phi D}^{22}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{u\phi D}^{12}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{u}\gamma^\mu u) + h.c.$
$\mathcal{O}_{L\phi D}^{11[3]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{L}\tau^I\gamma^\mu L)$	$\mathcal{O}_{L\phi D}^{22[3]}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{L}\tau^I\gamma^\mu L)$	$\mathcal{O}_{L\phi D}^{12[3]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{L}\tau^I\gamma^\mu L) + h.c.$
$\mathcal{O}_{Q\phi D}^{11[3]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{Q}\tau^I\gamma^\mu Q)$	$\mathcal{O}_{Q\phi D}^{22[3]}$	$i(\phi_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{Q}\tau^I\gamma^\mu Q)$	$\mathcal{O}_{Q\phi D}^{12[3]}$	$i(\phi_1^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{Q}\tau^I\gamma^\mu Q) + h.c.$
$\mathcal{O}_{ud\phi D}^{\tilde{1}1}$	$i(\tilde{\phi}_1^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{u}\gamma^\mu d) + h.c.$	$\mathcal{O}_{ud\phi D}^{\tilde{2}2}$	$i(\tilde{\phi}_2^\dagger\overleftrightarrow{D}_\mu\phi_2)(\bar{u}\gamma^\mu d) + h.c.$	$\mathcal{O}_{ud\phi D}^{\tilde{2}1}$	$i(\tilde{\phi}_2^\dagger\overleftrightarrow{D}_\mu\phi_1)(\bar{u}\gamma^\mu d) + h.c.$

Table 9. 2HDM: $\psi^2\phi^2 D$ [$14 + 10\times 2 = 20$] class of operators.

$\mathcal{O}_{\square}^{11(11)}$	$(\phi_1^\dagger \phi_1) \square (\phi_1^\dagger \phi_1)$	$\mathcal{O}_{\square}^{22(22)}$	$(\phi_2^\dagger \phi_2) \square (\phi_2^\dagger \phi_2)$
$\mathcal{O}_{\square}^{11(22)}$	$(\phi_1^\dagger \phi_1) \square (\phi_2^\dagger \phi_2)$	$\mathcal{O}_{\square}^{22(11)}$	$(\phi_2^\dagger \phi_2) \square (\phi_1^\dagger \phi_1)$
$\mathcal{O}_{\square}^{21(21)}$	$(\phi_2^\dagger \phi_1) \square (\phi_2^\dagger \phi_1) + h.c.$	$\mathcal{O}_{\phi D}^{12(1)(2)}$	$(\phi_1^\dagger \phi_2) [(D^\mu \phi_1)^\dagger (D_\mu \phi_2)] + h.c.$
$\mathcal{O}_{\phi D}^{(1)11(1)}$	$[(D^\mu \phi_1)^\dagger \phi_1] [\phi_1^\dagger (D_\mu \phi_1)]$	$\mathcal{O}_{\phi D}^{(2)22(2)}$	$[(D^\mu \phi_2)^\dagger \phi_2] [\phi_2^\dagger (D_\mu \phi_2)]$
$\mathcal{O}_{\phi D}^{(1)22(1)}$	$[(D^\mu \phi_1)^\dagger \phi_2] [\phi_2^\dagger (D_\mu \phi_1)]$	$\mathcal{O}_{\phi D}^{(2)11(2)}$	$[(D^\mu \phi_2)^\dagger \phi_1] [\phi_1^\dagger (D_\mu \phi_2)]$
$\mathcal{O}_{\phi D}^{(2)22(1)}$	$[(D^\mu \phi_2)^\dagger \phi_2] [\phi_2^\dagger (D_\mu \phi_1)] + h.c.$	$\mathcal{O}_{\phi D}^{(1)21(1)}$	$[(D^\mu \phi_1)^\dagger \phi_2] [\phi_1^\dagger (D_\mu \phi_1)] + h.c.$
$\mathcal{O}_{\phi D}^{21(2)(2)}$	$(\phi_2^\dagger \phi_1) [(D^\mu \phi_2)^\dagger ((D_\mu \phi_2))] + h.c.$	$\mathcal{O}_{\phi D}^{12(1)(1)}$	$(\phi_1^\dagger \phi_2) [(D^\mu \phi_1)^\dagger ((D_\mu \phi_1))] + h.c.$

Table 10. 2HDM: $\phi^4 D^2$ $[8 + 6 \times 2 = 20]$ class of operators. Note the presence of $\mathcal{O}_{\square}^{21(21)}$ operator instead of operators ($Q_{\varphi D}^{12(12)}$ and $Q_{\varphi D}^{(1)21(2)}$) as given in [113].

$\mathcal{O}_{dd}$	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$	$\mathcal{O}_{uu}$	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$
$\mathcal{O}_{Le}$	$(\bar{L}\gamma_\mu L)(\bar{e}\gamma^\mu e)$	$\mathcal{O}_{Qe}$	$(\bar{Q}\gamma_\mu Q)(\bar{e}\gamma^\mu e)$
$\mathcal{O}_{ee}$	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$	$\mathcal{O}_{LL}$	$(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$
$\mathcal{O}_{eu}$	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{ed}$	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$
$\mathcal{O}_{Lu}$	$(\bar{L}\gamma_\mu L)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{Ld}$	$(\bar{L}\gamma_\mu L)(\bar{d}\gamma^\mu d)$
$\mathcal{O}_{LQ}^{[1]}$	$(\bar{L}\gamma_\mu L)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}_{LQ}^{[3]}$	$(\bar{L}\gamma_\mu \tau^I L)(\bar{Q}\gamma^\mu \tau^I Q)$
$\mathcal{O}_{QQ}^{[1]}$	$(\bar{Q}\gamma_\mu Q)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}_{QQ}^{[3]}$	$(\bar{Q}\gamma_\mu \tau^I Q)(\bar{Q}\gamma^\mu \tau^I Q)$
$\mathcal{O}_{ud}^{[1]}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$	$\mathcal{O}_{ud}^{[8]}$	$(\bar{u}\gamma_\mu T^A u)(\bar{u}\gamma^\mu T^A u)$
$\mathcal{O}_{Qu}^{[1]}$	$(\bar{Q}\gamma_\mu Q)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{Qu}^{[8]}$	$(\bar{Q}\gamma_\mu T^A Q)(\bar{u}\gamma^\mu T^A u)$
$\mathcal{O}_{Qd}^{[1]}$	$(\bar{Q}\gamma_\mu Q)(\bar{d}\gamma^\mu d)$	$\mathcal{O}_{Qd}^{[8]}$	$(\bar{Q}\gamma_\mu T^A Q)(\bar{d}\gamma^\mu T^A d)$
$\mathcal{O}_{QuQd}^{[1]}$	$\epsilon_{jk}(\bar{Q}^j u)(\bar{Q}^k d) + h.c.$		
$\mathcal{O}_{QuQd}^{[8]}$	$\epsilon_{jk}(\bar{Q}^j T^A u)(\bar{Q}^k T^A d) + h.c.$		
$\mathcal{O}_{LeQu}^{[1]}$	$\epsilon_{jk}(\bar{L}^j e)(\bar{Q}^k u) + h.c.$		
$\mathcal{O}_{LeQu}^{[3]}$	$\epsilon_{jk}(\bar{L}^j \sigma_{\mu\nu} e)(\bar{Q}^k \sigma^{\mu\nu} u) + h.c.$		
$\mathcal{O}_{LedQ}$	$(\bar{L}^j e)(\bar{d}Q^j) + h.c.$		

Table 11. 2HDM: Baryon Number Conserving ψ^4 $[20 + 5 \times 2 = 30]$ class of operators. Here j, k are SU(2) indices.

$\mathcal{O}_{QQQ}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}[(Q^{\alpha j})^T C Q^{\beta k}][(Q^{\gamma m})^T C L^n]$
$\mathcal{O}_{QQu}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}[(Q^{\alpha j})^T C Q^{\beta k}][(u^\gamma)^T C e]$
$\mathcal{O}_{duu}$	$\epsilon^{\alpha\beta\gamma}[(d^\alpha)^T C u^\beta][(u^\gamma)^T C e]$
$\mathcal{O}_{duQ}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}[(d^\alpha)^T C u^\beta][(Q^{\gamma j})^T C L^k]$

Table 12. 2HDM: Baryon Number Violating ψ^4 $[4 \times 2 = 8]$ class of operators. Each operator has a distinct Hermitian Conjugate. Here, $C = i\gamma^2\gamma^0$ in Dirac representation. Here α, β, γ are SU(3) indices and j, k are SU(2) indices.

the Majorana mass of the neutrino is given as:

$$\Delta m_\nu = \frac{v_1^2}{2\Lambda} \mathcal{C}_{\phi L}^{11} + \frac{v_2^2}{2\Lambda} \mathcal{C}_{\phi L}^{22} + \frac{v_1 v_2}{2\Lambda} \mathcal{C}_{\phi L}^{12}. \quad (4.1)$$

ϕ^6 operators: modification in the scalar potential.

The inclusion of ϕ^6 operators modifies the scalar potential as $V(\phi_1, \phi_2) + \mathcal{L}_{\phi^6}$ where

$$\mathcal{L}_{\phi^6} = \sum_{j=1}^{20} \mathcal{C}_6^j \mathcal{O}_\phi^j. \quad (4.2)$$

The additional minimization criteria along with eq. (3.9), are noted as:

$$\begin{aligned} \frac{\partial \mathcal{L}_{\phi^6}}{\partial v_1} &= 3\mathcal{C}_6^{111111} v_1^4 + 2\mathcal{C}_6^{111122} v_1^2 v_2^2 + 4\mathcal{C}_6^{121211} v_1^2 v_2^2 + 2\mathcal{C}_6^{121222} v_2^4 + \mathcal{C}_6^{112222} v_2^4 + 2\mathcal{C}_6^{122111} v_1^2 v_2^2 \\ &\quad + \mathcal{C}_6^{122122} v_2^4 + 3\mathcal{C}_6^{121221} v_1 v_2^3 + \frac{v_2^5}{v_1} \mathcal{C}_6^{122222} + 5\mathcal{C}_6^{111112} v_1^3 v_2 + 3\mathcal{C}_6^{112212} v_1 v_2^3 + 3\mathcal{C}_6^{121212} v_1 v_2^3 \\ &= 0, \end{aligned} \quad (4.3)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_{\phi^6}}{\partial v_2} &= \mathcal{C}_6^{111122} v_1^4 + 2\mathcal{C}_6^{121211} v_1^4 + 4\mathcal{C}_6^{121222} v_2^2 v_1^2 + 2\mathcal{C}_6^{112222} v_2^2 v_1^2 + \mathcal{C}_6^{122111} v_1^4 + 3\mathcal{C}_6^{222222} v_2^4 \\ &\quad + 2\mathcal{C}_6^{122122} v_2^2 v_1^2 + 3\mathcal{C}_6^{121221} v_2 v_1^3 + \frac{v_1^5}{v_2} \mathcal{C}_6^{111112} + 5\mathcal{C}_6^{122222} v_2^3 v_1 + 3\mathcal{C}_6^{112212} v_2 v_1^3 + 3\mathcal{C}_6^{121212} v_2 v_1^3 \\ &= 0. \end{aligned} \quad (4.4)$$

In passing, we would like to mention that our minimization criteria are in well agreement with the ref. [113] in the absence of the Z_2 violating operators.

$\phi^4 D^2$ and ϕ^6 operators: scalar field redefinitions.

The scalar-kinetic lagrangian, see eq. (3.1), gets modified in presence of additional

$\phi^4 D^2$ operators, table 10, as:

$$\begin{aligned}
 \mathcal{L}_{\text{kin}}^{(4)+(6)} = & \frac{1}{2} \begin{pmatrix} \partial^\mu h_1 & \partial^\mu h_2 \end{pmatrix} \begin{pmatrix} 1 + \frac{A_r^{11}}{\Lambda^2} & \frac{A_r^{12}}{\Lambda^2} \\ \frac{A_r^{12}}{\Lambda^2} & 1 + \frac{A_r^{11}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu h_1 \\ \partial_\mu h_2 \end{pmatrix} \\
 & + \frac{1}{2} \begin{pmatrix} \partial^\mu a_1 & \partial^\mu a_2 \end{pmatrix} \begin{pmatrix} 1 + \frac{A_i^{11}}{\Lambda^2} & \frac{A_i^{12}}{\Lambda^2} \\ \frac{A_i^{12}}{\Lambda^2} & 1 + \frac{A_i^{11}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu a_1 \\ \partial_\mu a_2 \end{pmatrix} \\
 & + \begin{pmatrix} \partial^\mu \phi_1^- & \partial^\mu \phi_2^- \end{pmatrix} \begin{pmatrix} 1 + \frac{A_+^{11}}{\Lambda^2} & \frac{A_+^{12}}{\Lambda^2} \\ \frac{A_+^{12}}{\Lambda^2} & 1 + \frac{A_+^{11}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \phi_1^+ \\ \partial_\mu \phi_2^+ \end{pmatrix},
 \end{aligned} \tag{4.5}$$

where (note the additional contributions due to the Z_2 violating operators in comparison with [113]),

$$\begin{aligned}
 A_r^{11} &= \frac{1}{2} \mathcal{C}_{\phi D}^{(1)11(1)} v_1^2 + \frac{1}{2} \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 - \mathcal{C}_{\square}^{21(21)} v_2^2 - 2 \mathcal{C}_{\square}^{11(11)} v_1^2 + \mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1 + \mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1, \\
 A_r^{12} &= \frac{1}{2} \mathcal{C}_{\phi D}^{12(1)(2)} v_1 v_2 - \mathcal{C}_{\square}^{11(22)} v_1 v_2 - \mathcal{C}_{\square}^{22(11)} v_1 v_2 - \mathcal{C}_{\square}^{21(21)} v_1 v_2 + \frac{1}{2} \mathcal{C}_{\phi D}^{(2)22(1)} v_2^2, \\
 A_r^{22} &= \frac{1}{2} \mathcal{C}_{\phi D}^{(2)11(2)} v_1^2 + \frac{1}{2} \mathcal{C}_{\phi D}^{(2)22(2)} v_2^2 - \mathcal{C}_{\square}^{21(21)} v_1^2 - 2 \mathcal{C}_{\square}^{22(22)} v_2^2 + \mathcal{C}_{\phi D}^{21(2)(2)} v_2 v_1, \\
 A_i^{11} &= \frac{1}{2} \mathcal{C}_{\phi D}^{(1)11(1)} v_1^2 + \frac{1}{2} \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 + \mathcal{C}_{\square}^{21(21)} v_2^2 + \mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1 + \mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1, \\
 A_i^{12} &= \frac{1}{2} \mathcal{C}_{\phi D}^{12(1)(2)} v_1 v_2 - \mathcal{C}_{\square}^{21(21)} v_1 v_2 + \frac{1}{2} \mathcal{C}_{\phi D}^{(2)22(1)} v_2^2, \\
 A_i^{22} &= \frac{1}{2} \mathcal{C}_{\phi D}^{(2)11(2)} v_1^2 + \frac{1}{2} \mathcal{C}_{\phi D}^{(2)22(2)} v_2^2 + \mathcal{C}_{\square}^{21(21)} v_1^2 + \mathcal{C}_{\phi D}^{21(2)(2)} v_2 v_1, \\
 A_+^{11} &= \mathcal{C}_{\phi D}^{12(1)(1)} v_1 v_2, \\
 A_+^{22} &= \mathcal{C}_{\phi D}^{21(2)(2)} v_1 v_2, \\
 A_+^{12} &= \frac{1}{2} \mathcal{C}_{\phi D}^{12(1)(2)} v_1 v_2.
 \end{aligned}$$

In order to reduce the scalar kinetic terms to their canonical forms, scalar fields are redefined as [113]:

$$\begin{aligned}
 h_1 &\rightarrow \left(1 - \frac{A_r^{11}}{2\Lambda^2}\right) h_1 - \left(\frac{A_r^{12}}{2\Lambda^2}\right) h_2, \\
 h_2 &\rightarrow \left(1 - \frac{A_r^{22}}{2\Lambda^2}\right) h_2 - \left(\frac{A_r^{12}}{2\Lambda^2}\right) h_1, \\
 a_1 &\rightarrow \left(1 - \frac{A_i^{11}}{2\Lambda^2}\right) a_1 - \left(\frac{A_i^{12}}{2\Lambda^2}\right) a_2, \\
 a_2 &\rightarrow \left(1 - \frac{A_i^{22}}{2\Lambda^2}\right) a_2 - \left(\frac{A_i^{12}}{2\Lambda^2}\right) a_1, \\
 \phi_1^+ &\rightarrow \left(1 - \frac{A_+^{11}}{2\Lambda^2}\right) \phi_1^+ - \left(\frac{A_+^{12}}{2\Lambda^2}\right) \phi_2^+, \\
 \phi_2^+ &\rightarrow \left(1 - \frac{A_+^{22}}{2\Lambda^2}\right) \phi_2^+ - \left(\frac{A_+^{12}}{2\Lambda^2}\right) \phi_1^+.
 \end{aligned} \tag{4.6}$$

These redefinitions induce modifications in the scalar mass matrices [113]

$$\mathcal{L}_{MH}^{(4)+(6)} = \frac{1}{2} \begin{bmatrix} h_1 & h_2 \end{bmatrix} \mathcal{M}_r^2 \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} a_1 & a_2 \end{bmatrix} \mathcal{M}_i^2 \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} \phi_1^- & \phi_2^- \end{bmatrix} \mathcal{M}_+^2 \begin{bmatrix} \phi_1^+ \\ \phi_2^+ \end{bmatrix}. \quad (4.7)$$

We have to remember that there will be further contributions to the scalar masses from the ϕ^6 operators, e.g., when four out of six scalar fields acquire VEVs. Encapsulating all these contributions, the final scalar mass matrix can be expressed as [113]:

$$\mathcal{M}_j^2 = \begin{pmatrix} (m_j^2 + \Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{11} & (m_j^2 + \Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{12} \\ (m_j^2 + \Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{12} & (m_j^2 + \Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{22} \end{pmatrix}, \quad \{j \equiv r, i, +\}, \quad (4.8)$$

where, $\Delta m_{j,\phi D}^2$ and $\Delta m_{j,\phi^6}^2$ are the corrections to the scalar spectrum due to $\phi^4 D^2$ and ϕ^6 operators respectively. First, we have discussed the modification in the scalar mass spectrum due to the field redefinitions ($\Delta m_{\phi D}^2$), and then we have computed the contributions from ϕ^6 operators ($\Delta m_{\phi^6}^2$).

CP-even scalar mass matrix.

$$(\Delta m_{r,\phi D}^2)_{11} = \frac{m_{12}^2 (v_1 A_r^{12} - v_2 A_r^{11})}{\Lambda^2 v_1} - \frac{(\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 A_r^{12}}{\Lambda^2} - \frac{2\lambda_1 v_1^2 A_r^{11}}{\Lambda^2} - \frac{3\lambda_6 v_1 (v_1 A_r^{12} + v_2 A_r^{11})}{2\Lambda^2} + \frac{\lambda_7 v_2^2 (v_2 A_r^{11} - 3v_1 A_r^{12})}{2\Lambda^2 v_1}, \quad (4.9)$$

$$(\Delta m_{r,\phi D}^2)_{12} = \frac{m_{12}^2 (v_1 v_2 (A_r^{22} + A_r^{11}) - v^2 A_r^{12})}{2\Lambda^2 v_1 v_2} - \frac{\lambda_1 v_1^2 A_r^{12}}{\Lambda^2} - \frac{\lambda_2 v_2^2 A_r^{12}}{\Lambda^2} - \frac{(\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 (A_r^{22} + A_r^{11})}{2\Lambda^2} + \frac{\lambda_6 v_1 ((v_1^2 - 3v_2^2) A_r^{12} - 3v_1 v_2 (A_r^{22} + A_r^{11}))}{4\Lambda^2 v_2} + \frac{\lambda_7 v_2 ((v_2^2 - 3v_1^2) A_r^{12} - 3v_1 v_2 (A_r^{22} + A_r^{11}))}{4\Lambda^2 v_1}, \quad (4.10)$$

$$(\Delta m_{r,\phi D}^2)_{22} = \frac{m_{12}^2 \left(A_r^{12} - \frac{v_1 A_r^{22}}{v_2} \right)}{\Lambda^2} - \frac{2\lambda_2 v_2^2 A_r^{22}}{\Lambda^2} - \frac{(\lambda_3 + \lambda_4 + \lambda_5) v_1 v_2 A_r^{12}}{\Lambda^2} + \frac{\lambda_6 v_1^2 (v_1 A_r^{22} - 3v_2 A_r^{12})}{2\Lambda^2 v_2} - \frac{3\lambda_7 v_2 (v_1 A_r^{22} + v_2 A_h^{12})}{2\Lambda^2}, \quad (4.11)$$

CP-odd scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{11} = & \frac{m_{12}^2 \left(A_i^{12} - \frac{v_2 A_i^{11}}{v_1} \right)}{\Lambda^2} + \frac{\lambda_5 v_2 (v_2 A_i^{11} - v_1 A_i^{12})}{\Lambda^2} + \\
 & + \frac{\lambda_6 v_1 (v_2 A_i^{11} - v_1 A_i^{12})}{2\Lambda^2} + \frac{\lambda_7 v_2^2 (v_2 A_i^{11} - v_1 A_i^{12})}{2\Lambda^2 v_1}, \quad (4.12)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{12} = & \frac{m_{12}^2 (v_1 v_2 (A_i^{22} + A_i^{11}) - v^2 A_i^{12})}{2\Lambda^2 v_1 v_2} + \frac{\lambda_5 (v^2 A_i^{12})}{2\Lambda^2} \\
 & - \frac{\lambda_5 (v_1 v_2 (A_i^{22} + A_i^{11}))}{2\Lambda^2} + \frac{\lambda_6 v_1 (v^2 A_i^{12} - v_1 v_2 (A_i^{22} + A_i^{11}))}{4\Lambda^2 v_2} \\
 & + \frac{\lambda_7 v_2 (v^2 A_i^{12} - v_1 v_2 (A_i^{22} + A_i^{11}))}{4\Lambda^2 v_1}, \quad (4.13)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{22} = & \frac{m_{12}^2 (v_2 A_i^{12} - v_1 A_i^{22})}{\Lambda^2 v_2} + \frac{\lambda_5 v_1 (v_1 A_i^{22} - v_2 A_i^{12})}{\Lambda^2} \\
 & + \frac{\lambda_6 v_1^2 (v_1 A_i^{22} - v_2 A_i^{12})}{2\Lambda^2 v_2} + \frac{\lambda_7 v_2 (v_1 A_i^{22} - v_2 A_i^{12})}{2\Lambda^2}, \quad (4.14)
 \end{aligned}$$

Singly charged scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{11} = & -\frac{m_{12}^2 v_2 A_+^{11}}{\Lambda^2 v_1} + \frac{(\lambda_4 + \lambda_5) v_2^2 A_+^{11}}{2\Lambda^2} + \frac{\lambda_6 v_1 v_2 A_+^{11}}{2\Lambda^2} + \frac{\lambda_7 v_2^3 A_+^{11}}{2\Lambda^2 v_1}, \\
 (\Delta m_{+,\phi D}^2)_{12} = & \frac{m_{12}^2 (A_+^{22} + A_+^{11})}{2\Lambda^2} - \frac{(\lambda_4 + \lambda_5) v_1 v_2 (A_+^{22} + A_+^{11})}{4\Lambda^2} \\
 & - \frac{\lambda_6 v_1^2 (A_+^{22} + A_+^{11})}{4\Lambda^2} - \frac{\lambda_7 v_2^2 (A_+^{22} + A_+^{11})}{4\Lambda^2}, \\
 (\Delta m_{+,\phi D}^2)_{22} = & -\frac{m_{12}^2 v_1 A_+^{22}}{\Lambda^2 v_2} + \frac{(\lambda_4 + \lambda_5) v_1^2 A_+^{22}}{2\Lambda^2} + \frac{\lambda_6 v_1^3 A_+^{22}}{2\Lambda^2 v_2} + \frac{\lambda_7 v_1 v_2 A_+^{22}}{2\Lambda^2}.
 \end{aligned}$$

Contributions from ϕ^6 operators. Here, we would like to emphasize that the additional contributions due to the Z_2 violating ϕ^6 -operators in comparison with [113] are given in color throughout the text.

CP-even scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{11} = & \frac{1}{\Lambda^2} \left[3C_6^{111111} v_1^4 + C_6^{111122} v_1^2 v_2^2 + 2C_6^{121211} v_1^2 v_2^2 + C_6^{122111} v_1^2 v_2^2 \right. \\
 & \left. + \frac{3}{4} C_6^{121221} v_1 v_2^3 - \frac{v_2^5}{4v_1} C_6^{122222} + \frac{15}{4} C_6^{111112} v_1^3 v_2 + \frac{3}{4} C_6^{112212} v_1 v_2^3 + \frac{3}{4} C_6^{121212} v_1 v_2^3 \right], \\
 (\Delta m_{r,\phi^6}^2)_{12} = & \frac{1}{\Lambda^2} \left[C_6^{111122} v_2 v_1^3 + 2C_6^{121211} v_2 v_1^3 + 2C_6^{121222} v_2^3 v_1 + C_6^{112222} v_2^3 v_1 + C_6^{122111} v_2 v_1^3 + C_6^{122122} v_2^3 v_1 \right. \\
 & \left. + \frac{9}{4} C_6^{121221} v_2^2 v_1^2 + \frac{5v_2^4}{4} C_6^{122222} + \frac{9}{4} C_6^{112212} v_2^2 v_1^2 + \frac{9}{4} C_6^{121212} v_2^2 v_1^2 + \frac{5v_1^4}{4} C_6^{111112} \right], \\
 (\Delta m_{r,\phi^6}^2)_{22} = & \frac{1}{\Lambda^2} \left[2C_6^{121222} v_2^2 v_1^2 + C_6^{112222} v_2^2 v_1^2 + C_6^{122122} v_2^2 v_1^2 + 3C_6^{222222} v_2^4 \right. \\
 & \left. + \frac{3}{4} C_6^{121221} v_2 v_1^3 + \frac{15}{4} C_6^{122222} v_2^3 v_1 + \frac{3}{4} C_6^{112212} v_2 v_1^3 + \frac{3}{4} C_6^{121212} v_2 v_1^3 - \frac{v_1^5}{4v_2} C_6^{111112} \right],
 \end{aligned}$$

CP-odd scalar mass matrix.

$$(\Delta m_{i,\phi^6}^2)_{11} = \frac{1}{\Lambda^2} \left[-\mathcal{C}_6^{121211} v_1^2 v_2^2 - \mathcal{C}_6^{121222} v_2^4 \right. \\ \left. - \frac{1}{4} \mathcal{C}_6^{121221} v_1 v_2^3 - \frac{v_2^5}{4v_1} \mathcal{C}_6^{122222} - \frac{1}{4} \mathcal{C}_6^{111112} v_1^3 v_2 - \frac{1}{4} \mathcal{C}_6^{112212} v_1 v_2^3 - \frac{9}{4} \mathcal{C}_6^{121212} v_1 v_2^3 \right] \quad (4.15)$$

$$(\Delta m_{i,\phi^6}^2)_{12} = \frac{1}{\Lambda^2} \left[\mathcal{C}_6^{121211} v_2 v_1^3 + \mathcal{C}_6^{121222} v_2^3 v_1 \right. \\ \left. + \frac{v_1^4}{4} \mathcal{C}_6^{111112} + \frac{1}{4} \mathcal{C}_6^{112212} v_2^2 v_1^2 + \frac{9}{4} \mathcal{C}_6^{121212} v_2^2 v_1^2 + \frac{1}{4} \mathcal{C}_6^{121221} v_2^2 v_1^2 + \frac{v_2^4}{4} \mathcal{C}_6^{122222} \right] \quad (4.16)$$

$$(\Delta m_{i,\phi^6}^2)_{22} = \frac{1}{\Lambda^2} \left[-\mathcal{C}_6^{121211} v_1^4 - \mathcal{C}_6^{121222} v_2^2 v_1^2 \right. \\ \left. - \frac{v_1^5}{4v_2} \mathcal{C}_6^{111112} - \frac{1}{4} \mathcal{C}_6^{112212} v_2 v_1^3 - \frac{9}{4} \mathcal{C}_6^{121212} v_2 v_1^3 - \frac{1}{4} \mathcal{C}_6^{121221} v_2 v_1^3 - \frac{1}{4} \mathcal{C}_6^{122222} v_2^3 v_1 \right]. \quad (4.17)$$

Charged scalar mass matrix.

$$(\Delta m_{+,\phi^6}^2)_{11} = \frac{1}{\Lambda^2} \left[-\frac{1}{2} \mathcal{C}_6^{121211} v_1^2 v_2^2 - \frac{v_2^4}{2} \mathcal{C}_6^{121222} - \frac{1}{4} \mathcal{C}_6^{122111} v_1^2 v_2^2 - \frac{v_2^4}{4} \mathcal{C}_6^{122122} \right. \\ \left. - \frac{3}{4} \mathcal{C}_6^{121221} v_1 v_2^3 - \frac{v_2^5}{4v_1} \mathcal{C}_6^{122222} - \frac{1}{4} \mathcal{C}_6^{111112} v_1^3 v_2 - \frac{1}{4} \mathcal{C}_6^{112212} v_1 v_2^3 - \frac{3}{4} \mathcal{C}_6^{121212} v_1 v_2^3 \right], \quad (4.18)$$

$$(\Delta m_{+,\phi^6}^2)_{12} = \frac{1}{\Lambda^2} \left[\frac{1}{4} \mathcal{C}_6^{122122} v_2^3 v_1 + \frac{1}{2} \mathcal{C}_6^{121211} v_2 v_1^3 + \frac{1}{2} \mathcal{C}_6^{121222} v_2^3 v_1 + \frac{1}{4} \mathcal{C}_6^{122111} v_2 v_1^3 \right. \\ \left. + \frac{3}{4} \mathcal{C}_6^{121212} v_2^2 v_1^2 + \frac{3}{4} \mathcal{C}_6^{121221} v_2^2 v_1^2 + \frac{v_2^4}{4} \mathcal{C}_6^{122222} + \frac{v_1^4}{4} \mathcal{C}_6^{111112} + \frac{1}{4} \mathcal{C}_6^{112212} v_2^2 v_1^2 \right], \quad (4.19)$$

$$(\Delta m_{+,\phi^6}^2)_{22} = \frac{1}{\Lambda^2} \left[-\frac{v_1^4}{2} \mathcal{C}_6^{121211} - \frac{1}{2} \mathcal{C}_6^{121222} v_2^2 v_1^2 - \frac{v_1^4}{4} \mathcal{C}_6^{122111} - \frac{1}{4} \mathcal{C}_6^{122122} v_2^2 v_1^2 \right. \\ \left. - \frac{3}{4} \mathcal{C}_6^{121212} v_2 v_1^3 - \frac{3}{4} \mathcal{C}_6^{121221} v_2 v_1^3 - \frac{1}{4} \mathcal{C}_6^{122222} v_2^3 v_1 - \frac{v_1^5}{4v_2} \mathcal{C}_6^{111112} - \frac{1}{4} \mathcal{C}_6^{112212} v_2 v_1^3 \right]. \quad (4.20)$$

These scalar mass matrices $\mathcal{M}_j^2$, eq. (4.8), can be diagonalized by the new rotation angles $\bar{\beta}_j$ which are related to the older ones β_j (for $\{j \equiv r, i, +\}$) as:

$$\tan 2\bar{\beta}_j = \tan 2\beta_j \left(1 - \frac{(\Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{12}}{(m_j^2)_{12}} - \frac{(\Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{22} - (\Delta m_{j,\phi^6}^2 + \Delta m_{j,\phi D}^2)_{11}}{(m_j^2)_{22} - (m_j^2)_{11}} \right). \quad (4.21)$$

We would like to mention that through out our calculation we have dropped terms which are beyond $\mathcal{O}(\frac{1}{\Lambda^2})$. After diagonalizing the scalar mass matrices, we can express the physical scalar spectrum as [113]:

CP-even scalar (H_1, H_2).

$$\begin{aligned}\mathcal{M}_{H_1}^2 &= (m_r^2 + \Delta m_{r,\phi^6}^2 + \Delta m_{r,\phi D}^2)_{11} \cos^2(\bar{\beta}_r) - (m_r^2 + \Delta m_{r,\phi^6}^2 + \Delta m_{r,\phi D}^2)_{12} \sin(2\bar{\beta}_r) \\ &\quad + (m_r^2 + \Delta m_{r,\phi^6}^2 + \Delta m_{r,\phi D}^2)_{22} \sin^2(\bar{\beta}_r),\end{aligned}\quad (4.22)$$

$$\begin{aligned}\mathcal{M}_{H_2}^2 &= (m_r^2 + \Delta m_{r,\phi^6}^2 + \Delta m_{r,\phi D}^2)_{11} \sin^2(\bar{\beta}_r) + (m_r^2 + \Delta m_{r,\phi^6}^2 + \Delta m_{r,\phi D}^2)_{12} \sin(2\bar{\beta}_r) \\ &\quad + (m_r^2 + \Delta m_{r,\phi^6}^2 + \Delta m_{r,\phi D}^2)_{22} \cos^2(\bar{\beta}_r),\end{aligned}\quad (4.23)$$

CP-odd scalar (A).

$$\begin{aligned}\mathcal{M}_A^2 &= (m_i^2 + \Delta m_{i,\phi^6}^2 + \Delta m_{i,\phi D}^2)_{11} \cos^2(\bar{\beta}_i) - (m_i^2 + \Delta m_{i,\phi^6}^2 + \Delta m_{i,\phi D}^2)_{12} \sin(2\bar{\beta}_i) \\ &\quad + (m_i^2 + \Delta m_{i,\phi^6}^2 + \Delta m_{i,\phi D}^2)_{22} \sin^2(\bar{\beta}_i),\end{aligned}\quad (4.24)$$

Charged scalar ($H^\pm$).

$$\begin{aligned}\mathcal{M}_{H^\pm}^2 &= (m_+^2 + \Delta m_{+,\phi^6}^2 + \Delta m_{+,\phi D}^2)_{11} \cos^2(\bar{\beta}_+) - (m_+^2 + \Delta m_{+,\phi^6}^2 + \Delta m_{+,\phi D}^2)_{12} \sin(2\bar{\beta}_+) \\ &\quad + (m_+^2 + \Delta m_{+,\phi^6}^2 + \Delta m_{+,\phi D}^2)_{22} \sin^2(\bar{\beta}_+).\end{aligned}\quad (4.25)$$

$\phi^2 X^2$ and $\phi^4 D^2$ operators: gauge field redefinition and spectrum.

The gauge-kinetic lagrangian, see eq. (3.1), gets modified in the presence of $\phi^2 X^2$ operators, table 5, as [113]:

$$\mathcal{L}_{\text{gauge,kin}}^{(4)+(6)} = -\left(1 - \frac{2\Theta_{WW}}{\Lambda^2}\right)(\partial_\mu W_\nu^+)(\partial^\mu W^{-\nu}) - \frac{1}{2}(\partial_\mu W_\nu^3 \quad \partial_\mu B_\nu) \begin{pmatrix} 1 - \frac{2\Theta_{WW}}{\Lambda^2} & \frac{\Theta_{W_3 B}}{\Lambda^2} \\ \frac{\Theta_{W_3 B}}{\Lambda^2} & 1 - \frac{2\Theta_{BB}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial^\mu W^{3\nu} \\ \partial^\mu B^\nu \end{pmatrix},$$

where,

$$\begin{aligned}\Theta_{WW} &= v_1^2 \mathcal{C}_{\phi W}^{11} + v_2^2 \mathcal{C}_{\phi W}^{22} + v_1 v_2 (\mathcal{C}_{\phi W}^{12} + \mathcal{C}_{\phi W}^{21}), \\ \Theta_{BB} &= v_1^2 \mathcal{C}_{\phi B}^{11} + v_2^2 \mathcal{C}_{\phi B}^{22} + v_1 v_2 (\mathcal{C}_{\phi B}^{12} + \mathcal{C}_{\phi B}^{21}), \\ \Theta_{W_3 B} &= v_1^2 \mathcal{C}_{\phi W B}^{11} + v_2^2 \mathcal{C}_{\phi W B}^{22} + v_1 v_2 (\mathcal{C}_{\phi W B}^{12} + \mathcal{C}_{\phi W B}^{21}).\end{aligned}\quad (4.26)$$

Similar to the scalar sector, we need to redefine the gauge fields such that the above gauge kinetic lagrangian can be brought into the canonical form. The redefined gauge fields can be written as [113]:

$$W_\mu^\pm \rightarrow W_\mu^\pm \left(1 + \frac{\Theta_{WW}}{\Lambda^2}\right), \quad (4.27)$$

$$W_\mu^3 \rightarrow W_\mu^3 \left(1 + \frac{\Theta_{WW}}{\Lambda^2}\right) - \frac{\Theta_{W_3 B}}{2\Lambda^2} B_\mu, \quad (4.28)$$

$$B_\mu \rightarrow B_\mu \left(1 + \frac{\Theta_{BB}}{\Lambda^2}\right) - \frac{\Theta_{W_3 B}}{2\Lambda^2} W_\mu^3. \quad (4.29)$$

The redefinitions of the gauge fields lead to corrections in the gauge boson mass matrix as:

Charged gauge boson mass.

$$m_W^2 + (\Delta m_W^2)_{\phi^2 X^2} = \frac{1}{4} g^2 v^2 \left(1 + \frac{2\Theta_{WW}}{\Lambda^2} \right). \quad (4.30)$$

Neutral gauge boson mass.

$$\begin{aligned} (m_0^2 + (\Delta m_0^2)_{\phi^2 X^2})_{11} &= \frac{1}{4} v^2 \left[g^2 \left(1 + \frac{2\Theta_{WW}}{\Lambda^2} \right) + gg' \frac{\Theta_{W_3 B}}{\Lambda^2} \right], \\ (m_0^2 + (\Delta m_0^2)_{\phi^2 X^2})_{12} &= \frac{1}{4} v^2 \left[-gg' \left(1 + \frac{\Theta_{BB} + \Theta_{WW}}{\Lambda^2} \right) - (g^2 + g'^2) \frac{\Theta_{W_3 B}}{2\Lambda^2} \right] \\ &= (m_0^2 + (\Delta m_0^2)_{\phi^2 X^2})_{21}, \\ (m_0^2 + (\Delta m_0^2)_{\phi^2 X^2})_{22} &= \frac{1}{4} v^2 \left[g'^2 \left(1 + \frac{2\Theta_{BB}}{\Lambda^2} \right) + gg' \frac{\Theta_{W_3 B}}{\Lambda^2} \right]. \end{aligned} \quad (4.31)$$

There will be additional contributions to these mass matrices from $\phi^4 D^2$ operators, table 10. These shifts can be given as (note the additional contributions due to the Z_2 violating operators in comparison with [113]):

$$(\Delta m_W^2)_{\phi^4 D^2} = \frac{1}{4\Lambda^2} g^2 v_1^2 v_2^2 \mathcal{C}_{\phi D}^{12(1)(2)} + \frac{1}{4\Lambda^2} g^2 v_1 v_2 \left(v_1^2 \mathcal{C}_{\phi D}^{12(1)(1)} + v_2^2 \mathcal{C}_{\phi D}^{21(2)(2)} \right), \quad (4.32)$$

$$(\Delta m_0^2)_{\phi^4 D^2} = \frac{\Theta_6^{WB}}{8\Lambda^2} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}, \quad (4.33)$$

where

$$\begin{aligned} \Theta_6^{WB} &= v_1^2 (v_1^2 \mathcal{C}_{\phi D}^{(1)11(1)} + v_2^2 \mathcal{C}_{\phi D}^{(1)22(1)}) + v_2^2 (v_1^2 \mathcal{C}_{\phi D}^{(2)11(2)} + v_2^2 \mathcal{C}_{\phi D}^{(2)22(2)}) + 2v_1^2 v_2^2 \mathcal{C}_{\phi D}^{12(1)(2)} \\ &\quad + 2v_1 v_2 \left(v_1^2 (\mathcal{C}_{\phi D}^{(1)21(1)} + \mathcal{C}_{\phi D}^{12(1)(1)}) + v_2^2 (\mathcal{C}_{\phi D}^{(2)22(1)} + \mathcal{C}_{\phi D}^{21(2)(2)}) \right). \end{aligned} \quad (4.34)$$

The neutral gauge boson mass matrix can be diagonalized using the following relation:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos \tilde{\theta}_w & \sin \tilde{\theta}_w \\ -\sin \tilde{\theta}_w & \cos \tilde{\theta}_w \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \quad (4.35)$$

where the modified rotation angle $\tilde{\theta}_w$ is related to θ_w by the following relation:

$$\tan 2\tilde{\theta}_w = \tan 2\theta_w \left[1 + \frac{1}{\Lambda^2} \left(\sec 2\theta_w (\Theta_{BB} - \Theta_{WW}) + \csc 2\theta_w \Theta_{W_3 B} \right) \right] \quad (4.36)$$

$$= \frac{2gg'}{g^2 - g'^2} \left[1 + \frac{1}{\Lambda^2} \left(\frac{g^2 + g'^2}{g^2 - g'^2} (\Theta_{BB} - \Theta_{WW}) + \frac{g^2 + g'^2}{2gg'} \Theta_{W_3 B} \right) \right]. \quad (4.37)$$

Here, θ_w is the Weak mixing angle and $\tan \theta_w = g'/g$.

After diagonalizing the gauge boson mass matrices, we have noted the physical gauge boson spectrum as [113]:

$$\begin{aligned}\mathcal{M}_W^2 &= \frac{1}{4}g^2 \left[v^2 + \frac{2v^2\Theta_{WW}}{\Lambda^2} + \frac{v_1^2v_2^2}{\Lambda^2}\mathcal{C}_{\phi D}^{12(1)(2)} + \frac{v_1v_2}{\Lambda^2}(v_1^2\mathcal{C}_{\phi D}^{12(1)(1)} + v_2^2\mathcal{C}_{\phi D}^{21(2)(2)}) \right], \\ \mathcal{M}_Z^2 &= \frac{1}{4}(g^2 + g'^2) \left[v^2 + \frac{2v^2}{\Lambda^2}(\cos^2\theta_w\Theta_{WW} + \sin^2\theta_w\Theta_{BB} + \cos\theta_w\sin\theta_w\Theta_{W3B}) + \frac{\Theta_6^{WB}}{2\Lambda^2} \right], \\ \mathcal{M}_A^2 &= 0.\end{aligned}\tag{4.38}$$

Here, we would like to draw the reader's attention towards the fact that even after incorporating all these corrections, photon is massless, i.e., $\mathcal{M}_A^2$ is zero, which is expected.

$\psi^2\phi^3$ operators: modification in spectrum.

The $\psi^2\phi^3$ operators lead to the additional contributions to the fermion masses as [113]:

$$\begin{aligned}\Delta m_e &= \frac{1}{2\sqrt{2}\Lambda^2} [\mathcal{C}_{Le}^{1(11)}v_1^3 + v_1^2v_2(\mathcal{C}_{Le}^{1(12)} + \mathcal{C}_{Le}^{1(21)} + \mathcal{C}_{Le}^{2(11)}) + v_1v_2^2(\mathcal{C}_{Le}^{1(22)} + \mathcal{C}_{Le}^{2(12)} + \mathcal{C}_{Le}^{2(21)}) + \mathcal{C}_{Le}^{2(22)}v_2^3], \\ \Delta m_u &= \frac{1}{2\sqrt{2}\Lambda^2} [\mathcal{C}_{Qu}^{\bar{1}(11)}v_1^3 + v_1^2v_2(\mathcal{C}_{Qu}^{\bar{1}(12)} + \mathcal{C}_{Qu}^{\bar{1}(21)} + \mathcal{C}_{Qu}^{\bar{2}(11)}) + v_1v_2^2(\mathcal{C}_{Qu}^{\bar{1}(22)} + \mathcal{C}_{Qu}^{\bar{2}(12)} + \mathcal{C}_{Qu}^{\bar{2}(21)}) + \mathcal{C}_{Qu}^{\bar{2}(22)}v_2^3], \\ \Delta m_d &= \frac{1}{2\sqrt{2}\Lambda^2} [\mathcal{C}_{Qd}^{1(11)}v_1^3 + v_1^2v_2(\mathcal{C}_{Qd}^{1(12)} + \mathcal{C}_{Qd}^{1(21)} + \mathcal{C}_{Qd}^{2(11)}) + v_1v_2^2(\mathcal{C}_{Qd}^{1(22)} + \mathcal{C}_{Qd}^{2(12)} + \mathcal{C}_{Qd}^{2(21)}) + \mathcal{C}_{Qd}^{2(22)}v_2^3].\end{aligned}$$

In the presence of these dimension-6 operators, the total fermion spectrum is given as:

$$\mathcal{M}_f = \frac{1}{\sqrt{2}} (v_1 y_1^f + v_2 y_2^f) - \Delta m_f, \quad \text{with } f \equiv \{e, u, d\}.\tag{4.39}$$

5 Minimal Left-Right Symmetric Model (MLRSM)

The Minimal Left-Right Symmetric Model (MLRSM) is a widely discussed beyond SM scenario where parity (P) and Charge conjugation (CP) symmetries are given the same status as the gauge ones. In this framework, P and CP symmetries are broken simultaneously with the right-handed gauge symmetry. Unlike the 2HDM case, here, the gauge sector is extended in the form of $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [188–191]. There are 16 fermions per generation, unlike the SM as the right-handed neutrino is a natural inclusion in this model. The particle content and their representations, and the charges under this gauge group are given in table 13.

The renormalizable Lagrangian. Based upon the quantum numbers and transformation properties of the particles, see table 13, the renormalizable lagrangian can be written as [191–193]:

$$\begin{aligned}\mathcal{L}^{(4)} &= -\frac{1}{4}Tr[G^{\mu\nu}G_{\mu\nu}] - \frac{1}{4}Tr[W_L^{\mu\nu}W_{\mu\nu,L}] - \frac{1}{4}Tr[W_R^{\mu\nu}W_{\mu\nu,R}] - \frac{1}{4}(B^{\mu\nu}B_{\mu\nu}) \\ &+ Tr[(D_\mu\Delta_L)^\dagger(D^\mu\Delta_L)] + Tr[(D_\mu\Delta_R)^\dagger(D^\mu\Delta_R)] + Tr[(D_\mu\Phi)^\dagger(D^\mu\Phi)] \\ &+ \bar{L}_L i \not{D} L_L + \bar{L}_R i \not{D} L_R + \bar{Q}_L i \not{D} Q_L + \bar{Q}_R i \not{D} Q_R + V(\Phi, \Delta_L, \Delta_R) + \mathcal{L}_Y + h.c.,\end{aligned}\tag{5.1}$$

Field	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{B-L}$	Spin	Lorentz Group
Φ	1	2	2	0	0	Scalar
Δ_L	1	3	1	2	0	Scalar
Δ_R	1	1	3	2	0	Scalar
Q_L	3	2	1	1/3	1/2	Spinor
Q_R	3	1	2	1/3	1/2	Spinor
L_L	1	2	1	-1	1/2	Spinor
L_R	1	1	2	-1	1/2	Spinor
G_μ^A	8	1	1	0	1	Vector
$W_{\mu,L}^I$	1	3	1	0	1	Vector
$W_{\mu,R}^I$	1	1	3	0	1	Vector
B_μ	1	1	1	0	1	Vector

Table 13. MLRSM: quantum number of the fields.

where the components of field strength tensors corresponding to $SU(3)_C, SU(2)_L, SU(2)_R, U(1)_{B-L}$ gauge groups are given as [191–193]:

$$\begin{aligned}
 G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_3 f^{ABC} G_\mu^B G_\nu^C, \\
 W_{L,\mu\nu}^I &= \partial_\mu W_{L,\nu}^I - \partial_\nu W_{L,\mu}^I + g_L \epsilon^{IJK} W_{L,\mu}^J W_{L,\nu}^K, \\
 W_{R,\mu\nu}^I &= \partial_\mu W_{R,\nu}^I - \partial_\nu W_{R,\mu}^I + g_R \epsilon^{IJK} W_{R,\mu}^J W_{R,\nu}^K, \\
 B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,
 \end{aligned}$$

with $\{A, B, C\} \in (1, \dots, 8)$, $\{I, J, K\} \in (1, 2, 3)$. In this scenario defining covariant derivative is not very straight forward unlike the 2HDM case. Thus we have explicitly mentioned them for fermions (L, Q) and scalars (Φ, Δ) as [191–193]:

$$D^\mu L_{L,R} = \left(\partial^\mu - ig_{L,R} \frac{\tau^I}{2} W_{L,R}^{I,\mu} - i\tilde{g} \frac{Y}{2} B^\mu \right) L_{L,R}, \quad (5.2)$$

$$D^\mu Q_{L,R} = \left(\partial^\mu - ig_3 \frac{T^A}{2} G_\mu^A - ig_{L,R} \frac{\tau^I}{2} W_{L,R}^{I,\mu} - i\tilde{g} \frac{Y}{2} B^\mu \right) Q_{L,R}, \quad (5.3)$$

$$\begin{aligned}
 D^\mu \Phi &= \partial^\mu \Phi - ig_L W_L^{I,\mu} \frac{\tau^I}{2} \Phi + ig_R \Phi \frac{\tau^I}{2} W_R^{I,\mu}, \\
 D^\mu \Delta_{L,R} &= \partial^\mu \Delta_{L,R} - ig_{L,R} \left[\frac{\tau^I}{2} W_{L,R}^{I,\mu}, \Delta_{L,R} \right] - i\tilde{g} B^\mu \Delta_{L,R},
 \end{aligned} \quad (5.4)$$

where g_3, g_L, g_R , and $\tilde{g}$ are the $SU(3)_C, SU(2)_L, SU(2)_R$, and $U(1)_{B-L}$ gauge couplings respectively. Here, we would like to note that the indices “ L, R ” appearing in the subscript

of the fields, e.g., $Q_{L,R}$, are to signify whether that field is transforming under $SU(2)_L$ or $SU(2)_R$ gauge groups respectively. We would like to further add that as we are working within MLRSM, we have considered $g_L = g_R = g$. In the rest of our analysis we will follow these conventions.

The scalar potential for MLRSM is given as [194–219]:

$$\begin{aligned}
 V(\Delta_L, \Delta_R, \Phi) = & -\mu_1^2 \left(\text{Tr} \left[\Phi^\dagger \Phi \right] \right) - \mu_2^2 \left(\text{Tr} \left[\tilde{\Phi} \Phi^\dagger \right] + \text{Tr} \left[\tilde{\Phi}^\dagger \Phi \right] \right) \\
 & - \mu_3^2 \left(\text{Tr} \left[\Delta_L (\Delta_L)^\dagger \right] + \text{Tr} \left[\Delta_R (\Delta_R)^\dagger \right] \right) \\
 & + \lambda_1 \text{Tr} \left[\Phi \Phi^\dagger \right]^2 + \lambda_2 \text{Tr} \left[\tilde{\Phi} \Phi^\dagger \right]^2 + \lambda_3 \text{Tr} \left[\tilde{\Phi} \Phi^\dagger \right] \text{Tr} \left[\tilde{\Phi}^\dagger \Phi \right] + \lambda_2 \text{Tr} \left[\tilde{\Phi}^\dagger \Phi \right]^2 \\
 & + \lambda_4 \text{Tr} \left[\Phi \Phi^\dagger \right] \left(\text{Tr} \left[\tilde{\Phi} \Phi^\dagger \right] + \text{Tr} \left[\tilde{\Phi}^\dagger \Phi \right] \right) \\
 & + \rho_1 \left(\text{Tr} \left[\Delta_L (\Delta_L)^\dagger \right]^2 + \text{Tr} \left[\Delta_R (\Delta_R)^\dagger \right]^2 \right) \\
 & + \rho_2 \left(\text{Tr} \left[\Delta_R \Delta_R \right] \text{Tr} \left[(\Delta_R)^\dagger (\Delta_R)^\dagger \right] + \text{Tr} \left[\Delta_L \Delta_L \right] \text{Tr} \left[(\Delta_L)^\dagger (\Delta_L)^\dagger \right] \right) \\
 & + \rho_3 \text{Tr} \left[\Delta_L (\Delta_L)^\dagger \right] \text{Tr} \left[\Delta_R (\Delta_R)^\dagger \right] \\
 & + \rho_4 \left(\text{Tr} \left[\Delta_R \Delta_R \right] \text{Tr} \left[(\Delta_L)^\dagger (\Delta_L)^\dagger \right] + \text{Tr} \left[\Delta_L \Delta_L \right] \text{Tr} \left[(\Delta_R)^\dagger (\Delta_R)^\dagger \right] \right) \\
 & + \alpha_1 \left(\text{Tr} \left[\Phi \Phi^\dagger \right] \text{Tr} \left[\Delta_L (\Delta_L)^\dagger \right] + \text{Tr} \left[\Phi \Phi^\dagger \right] \text{Tr} \left[\Delta_R (\Delta_R)^\dagger \right] \right) \\
 & + \alpha_2 \left(\text{Tr} \left[\Phi \tilde{\Phi}^\dagger \right] \text{Tr} \left[\Delta_R (\Delta_R)^\dagger \right] + \text{Tr} \left[\Phi^\dagger \tilde{\Phi} \right] \text{Tr} \left[\Delta_L (\Delta_L)^\dagger \right] \right) \\
 & + \text{Tr} \left[\Phi^\dagger \tilde{\Phi} \right] \text{Tr} \left[\Delta_R (\Delta_R)^\dagger \right] + \text{Tr} \left[\tilde{\Phi}^\dagger \Phi \right] \text{Tr} \left[\Delta_L (\Delta_L)^\dagger \right] \\
 & + \alpha_3 \left(\text{Tr} \left[\Phi \Phi^\dagger \Delta_L (\Delta_L)^\dagger \right] + \text{Tr} \left[\Phi^\dagger \Phi \Delta_R (\Delta_R)^\dagger \right] \right) \\
 & + \beta_1 \left(\text{Tr} \left[\Phi \Delta_R \Phi^\dagger (\Delta_L)^\dagger \right] + \text{Tr} \left[\Phi^\dagger \Delta_L \Phi (\Delta_R)^\dagger \right] \right) \\
 & + \beta_2 \left(\text{Tr} \left[\tilde{\Phi} \Delta_R \Phi^\dagger (\Delta_L)^\dagger \right] + \text{Tr} \left[\tilde{\Phi}^\dagger \Delta_L \Phi (\Delta_R)^\dagger \right] \right) \\
 & + \beta_3 \left(\text{Tr} \left[\Phi^\dagger \Delta_L \tilde{\Phi} (\Delta_R)^\dagger \right] + \text{Tr} \left[\Phi \Delta_R \tilde{\Phi}^\dagger (\Delta_L)^\dagger \right] \right), \tag{5.5}
 \end{aligned}$$

where $\tilde{\Phi} = \tau_2 \Phi^* \tau_2$. We have assumed the potential parameters μ_i , λ_i , ρ_i , β_i , and α_2 to be real to avoid any kind of explicit CP-violation in this scenario.

The Yukawa lagrangian for this model can be written as [193]:

$$\begin{aligned}
 \mathcal{L}_Y = & - \left\{ y_D \bar{L}_L \Phi L_R + \tilde{y}_D \bar{L}_L \tilde{\Phi} L_R + h.c. \right\} - \left\{ y_q \bar{Q}_L \Phi Q_R + \tilde{y}_q \bar{Q}_L \tilde{\Phi} Q_R + h.c. \right\} \\
 & - y_M \left\{ L_L^T C i \tau_2 \Delta_L L_L + L_R^T C i \tau_2 \Delta_R L_R + h.c. \right\}, \tag{5.6}
 \end{aligned}$$

where the fermion doublets are expressed as:

$$L_{L,R} = \begin{pmatrix} \nu_{L,R} \\ e_{L,R} \end{pmatrix}, \quad Q_{L,R} = \begin{pmatrix} u_{L,R} \\ d_{L,R} \end{pmatrix}. \tag{5.7}$$

Here, $y_D, \tilde{y}_D, y_q, \tilde{y}_q, y_M$ are the Yukawa couplings. The first two terms in eq. (5.6) are Chiral (Lepton Number Conserving) in nature, whereas the last term is a source of Lepton Number Violation (LNV).

In this model the spontaneous gauge symmetry breaking occurs at two levels: first $SU(2)_R \otimes U(1)_{B-L}$ is broken to $U(1)_Y$, and then $SU(2)_L \otimes U(1)_Y$ is broken to $U(1)_{em}$. The scalar fields after acquiring VEVs can be given as [191–193, 220]:

$$\begin{aligned}\Phi &= \begin{pmatrix} (\kappa_1 + h_1 + ia_1)/\sqrt{2} & \phi_1^+ \\ \phi_2^- & (\kappa_2 + h_2 + ia_2)/\sqrt{2} \end{pmatrix}, \\ \Delta_{L,R} &= \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ (v + \delta^{0r} + i\delta^{0i})/\sqrt{2} & -\delta^+/\sqrt{2} \end{pmatrix}_{L,R}.\end{aligned}\quad (5.8)$$

Following similar steps as in 2HDM, scalar potential minimization conditions are read as:

$$\frac{\partial V}{\partial \kappa_1} = \frac{\partial V}{\partial \kappa_2} = \frac{\partial V}{\partial v_R} = \frac{\partial V}{\partial v_L} = 0. \quad (5.9)$$

Simultaneously solving these equations, we obtain following expressions for μ_1^2 , μ_2^2 , μ_3^2 , and β_2 :

$$\begin{aligned}\mu_1^2 &= \frac{1}{2\kappa_-^2} \left(2\kappa_-^2 (\kappa_1^2 \lambda_1 + 2\kappa_2 \kappa_1 \lambda_4 + \kappa_2^2 \lambda_1) + v_L^2 (-\alpha_3 \kappa_-^2 + \alpha_1 \kappa_-^2 + (4\rho_1 - 2\rho_3) v_R^2) \right. \\ &\quad \left. + v_R^2 (\alpha_1 \kappa_-^2 - \alpha_3 \kappa_-^2) - 2\kappa_2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_L v_R \right),\end{aligned}\quad (5.10)$$

$$\begin{aligned}\mu_2^2 &= \frac{1}{4\kappa_1 \kappa_-^2} \left(v_L^2 (\kappa_1 (\alpha_3 \kappa_1 \kappa_2 + 2\alpha_2 \kappa_-^2) + 2\kappa_2 (\rho_3 - 2\rho_1) v_R^2) + \kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_L v_R \right. \\ &\quad \left. + \kappa_1 (2\kappa_-^2 (\kappa_1^2 \lambda_4 + 2\kappa_2 \kappa_1 (2\lambda_2 + \lambda_3) + \kappa_2^2 \lambda_4) + v_R^2 (\alpha_3 \kappa_1 \kappa_2 + 2\alpha_2 \kappa_-^2)) \right),\end{aligned}\quad (5.11)$$

$$\mu_3^2 = \frac{1}{2} (\alpha_1 \kappa_1^2 + \alpha_1 \kappa_2^2 + \alpha_3 \kappa_2^2 + 4\alpha_2 \kappa_1 \kappa_2 + 2\rho_1 v_L^2 + 2\rho_1 v_R^2), \quad (5.12)$$

$$\beta_2 = \frac{1}{\kappa_1^2} (2\rho_1 - \rho_3) v_L v_R - \kappa_2 (\beta_1 \kappa_1 + \beta_3 \kappa_2). \quad (5.13)$$

To keep our results in the same footing with [193, 220], we have performed a sub-rotation in the scalar sector as:

$$\begin{aligned}\phi_1^{0r} &= \frac{1}{\kappa_+} (\kappa_1 h_1 + \kappa_2 h_2), & \phi_2^{0r} &= \frac{1}{\kappa_+} (-\kappa_2 h_1 + \kappa_1 h_2), \\ \phi_1^{0i} &= \frac{1}{\kappa_+} (\kappa_1 a_1 - \kappa_2 a_2), & \phi_2^{0i} &= \frac{1}{\kappa_+} (-\kappa_2 a_1 - \kappa_1 a_2), \\ \phi_1'^+ &= \frac{1}{\kappa_+} (\kappa_1 \phi_1^+ + \kappa_2 \phi_2^+), & \phi_2'^+ &= \frac{1}{\kappa_+} (\kappa_1 \phi_2^+ - \kappa_2 \phi_1^+).\end{aligned}\quad (5.14)$$

This allows us to define the following basis to proceed for further computation:

$$\{\phi_1^{0r}, \phi_2^{0r}, \delta_R^{0r}, \delta_L^{0r}\} \Rightarrow \text{CP-even scalar}, \quad (5.15)$$

$$\{\phi_1^{0i}, \phi_2^{0i}, \delta_R^{0i}, \delta_L^{0i}\} \Rightarrow \text{CP-odd scalar}, \quad (5.16)$$

$$\{\phi_1'^+, \phi_2'^+, \delta_R^+, \delta_L^+\} \Rightarrow \text{Singly charged scalar}, \quad (5.17)$$

$$\{\delta_R^{++}, \delta_L^{++}\} \Rightarrow \text{Doubly charged scalar}. \quad (5.18)$$

After SSB, the part of the lagrangian containing the scalar mass terms is given as [191–193, 220]:

$$\mathcal{L}_{M_H}^{(4)} = \frac{1}{2} \begin{bmatrix} \phi_1^{0r} \\ \phi_2^{0r} \\ \delta_R^{0r} \\ \delta_L^{0r} \end{bmatrix}^T m_r^2 \begin{bmatrix} \phi_1^{0r} \\ \phi_2^{0r} \\ \delta_R^{0r} \\ \delta_L^{0r} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \phi_1^{0i} \\ \phi_2^{0i} \\ \delta_R^{0i} \\ \delta_L^{0i} \end{bmatrix}^T m_i^2 \begin{bmatrix} \phi_1^{0i} \\ \phi_2^{0i} \\ \delta_R^{0i} \\ \delta_L^{0i} \end{bmatrix} + \begin{bmatrix} \phi_1'^{++} \\ \phi_2'^{++} \\ \delta_R^+ \\ \delta_L^+ \end{bmatrix}^\dagger m_{++}^2 \begin{bmatrix} \phi_1'^{++} \\ \phi_2'^{++} \\ \delta_R^+ \\ \delta_L^+ \end{bmatrix} + \begin{bmatrix} \delta_R^{++} \\ \delta_L^{++} \end{bmatrix}^\dagger m_{++}^2 \begin{bmatrix} \delta_R^{++} \\ \delta_L^{++} \end{bmatrix}.$$

Here, m_r^2, m_i^2 , and m_{++}^2 are (4×4) matrices while m_{++}^2 is (2×2) matrix.

CP-even scalar mass matrix (m_r^2).

$$\begin{aligned} (m_r^2)_{11} &= \frac{2}{\kappa_+^2} (\kappa_1^4 \lambda_1 + 4\kappa_2 \kappa_1^3 \lambda_4 + 2\kappa_2^2 \kappa_1^2 (\lambda_1 + 4\lambda_2 + 2\lambda_3) + 4\kappa_2^3 \kappa_1 \lambda_4 + \kappa_2^4 \lambda_1), \\ (m_r^2)_{12} &= \frac{2\kappa_-^2}{\kappa_+^2} (\kappa_1^2 \lambda_4 + 2\kappa_2 \kappa_1 (2\lambda_2 + \lambda_3) + \kappa_2^2 \lambda_4), \\ (m_r^2)_{13} &= \frac{v_R}{\kappa_+} (\kappa_2 (4\alpha_2 \kappa_1 + \alpha_3 \kappa_2) + \alpha_1 \kappa_+^2 + (2\rho_1 - \rho_3) v_L^2), \\ (m_r^2)_{14} &= \frac{v_L}{\kappa_+} (\kappa_2 (4\alpha_2 \kappa_1 + \alpha_3 \kappa_2) + \alpha_1 \kappa_+^2 + (2\rho_1 - \rho_3) v_R^2), \\ (m_r^2)_{22} &= \frac{\kappa_+^4 v_L^2}{2\kappa_1^2 \kappa_+^2 \kappa_-^2} (\alpha_3 \kappa_1^2 + (2\rho_3 - 4\rho_1) v_R^2) + 2\kappa_+^4 v_L v_R (\beta_1 \kappa_1 \kappa_2 + \beta_3 \kappa_+^2) \\ &\quad + \kappa_1^2 (4(2\lambda_2 + \lambda_3) \kappa_-^6 + \alpha_3 \kappa_+^4 v_R^2), \\ (m_r^2)_{23} &= \frac{1}{2\kappa_1 \kappa_+} \left(2v_R (\kappa_1 (\alpha_3 \kappa_1 \kappa_2 + 2\alpha_2 \kappa_-^2) + \kappa_2 (\rho_3 - 2\rho_1) v_L^2) + \kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_L \right), \\ (m_r^2)_{24} &= \frac{1}{2\kappa_1 \kappa_+} \left(2v_L (\kappa_1 (\alpha_3 \kappa_1 \kappa_2 + 2\alpha_2 \kappa_-^2) + \kappa_2 (\rho_3 - 2\rho_1) v_R^2) + \kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_R \right), \\ (m_r^2)_{33} &= \frac{1}{2} (\rho_3 - 2\rho_1) v_L^2 + 2\rho_1 v_R^2, \\ (m_r^2)_{34} &= (2\rho_1 + \rho_3) v_L v_R, \\ (m_r^2)_{44} &= 2\rho_1 v_L^2 + \frac{1}{2} (\rho_3 - 2\rho_1) v_R^2. \end{aligned} \tag{5.19}$$

CP-odd scalar mass matrix (m_i^2).

$$\begin{aligned}
 (m_i^2)_{11} &= \frac{2v_L^2 v_R^2}{\kappa_+^2} (\rho_3 - 2\rho_1), \\
 (m_i^2)_{12} &= \frac{v_L v_R}{\kappa_1} \left(-\beta_1 \kappa_1 - 2\beta_3 \kappa_2 + \frac{2\kappa_2 (2\rho_1 - \rho_3) v_L v_R}{\kappa_+^2} \right), \\
 (m_i^2)_{13} &= \frac{1}{\kappa_+} (2\rho_1 - \rho_3) v_L^2 v_R, \\
 (m_i^2)_{14} &= \frac{v_L v_R^2}{\kappa_+} (\rho_3 - 2\rho_1), \\
 (m_i^2)_{22} &= \frac{1}{2\kappa_1^2 \kappa_+^2 \kappa_-^2} \left(v_L^2 (\alpha_3 \kappa_1^2 \kappa_+^4 - 2(\kappa_1^4 + 4\kappa_2^2 \kappa_1^2 - \kappa_2^4) (2\rho_1 - \rho_3) v_R^2) - 2\kappa_+^2 (\beta_1 \kappa_1 \kappa_2 (\kappa_2^2 - 3\kappa_1^2) \right. \\
 &\quad \left. + \beta_3 (\kappa_1^4 - 6\kappa_2^2 \kappa_1^2 + \kappa_2^4)) v_L v_R + \kappa_1^2 \kappa_+^4 (\alpha_3 v_R^2 - 4\kappa_-^2 (2\lambda_2 - \lambda_3)) \right), \\
 (m_i^2)_{23} &= \frac{v_L}{2\kappa_1 \kappa_+} (\kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) + 2\kappa_2 (\rho_3 - 2\rho_1) v_L v_R), \\
 (m_i^2)_{24} &= -\frac{v_R}{2\kappa_1 \kappa_+} (\kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) + 2\kappa_2 (\rho_3 - 2\rho_1) v_L v_R), \\
 (m_i^2)_{33} &= \frac{1}{2} (\rho_3 - 2\rho_1) v_L^2, \\
 (m_i^2)_{34} &= \frac{1}{2} (2\rho_1 - \rho_3) v_L v_R, \\
 (m_i^2)_{44} &= \frac{1}{2} (\rho_3 - 2\rho_1) v_R^2.
 \end{aligned} \tag{5.20}$$

Singly charged scalar mass matrix (m_+^2).

$$\begin{aligned}
 (m_+^2)_{11} &= \frac{1}{\kappa_+^2 \kappa_-^2} \left(v_R^2 (\alpha_3 \kappa_+^4 - 2(\kappa_1^2 + 3\kappa_2^2) (2\rho_1 - \rho_3) v_L^2) + 4\kappa_2 \kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_L v_R \right. \\
 &\quad \left. + 4\alpha_3 \kappa_1^2 \kappa_2^2 v_L^2 \right), \\
 (m_+^2)_{12} &= \frac{v_L}{\kappa_1 \kappa_+^2} (\kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_R + 2\kappa_2 (\rho_3 - 2\rho_1) v_L v_R^2 + 2\alpha_3 \kappa_1^2 \kappa_2 v_L), \\
 (m_+^2)_{13} &= \frac{1}{\sqrt{2} \kappa_+} v_R (\alpha_3 \kappa_+^2 + (2\rho_3 - 4\rho_1) v_L^2) + 2\kappa_2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_L, \\
 (m_+^2)_{14} &= \frac{1}{\sqrt{2} \kappa_1 \kappa_+} (\kappa_+^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) v_R + 2\kappa_2 (\rho_3 - 2\rho_1) v_L v_R^2 + 2\alpha_3 \kappa_1^2 \kappa_2 v_L), \\
 (m_+^2)_{22} &= \frac{v_L^2}{\kappa_+^2} (\alpha_3 \kappa_-^2 + (2\rho_3 - 4\rho_1) v_R^2), \\
 (m_+^2)_{23} &= \frac{v_L}{\sqrt{2} \kappa_1 \kappa_+} (\kappa_-^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) + 2\kappa_2 (2\rho_1 - \rho_3) v_L v_R), \\
 (m_+^2)_{24} &= \frac{v_L}{\sqrt{2} \kappa_+} (\alpha_3 \kappa_-^2 + (2\rho_3 - 4\rho_1) v_R^2), \\
 (m_+^2)_{33} &= \frac{1}{2} (\alpha_3 \kappa_-^2 + (2\rho_3 - 4\rho_1) v_L^2),
 \end{aligned}$$

$$\begin{aligned}
 (m_+^2)_{34} &= \frac{1}{2\kappa_1} \left(\kappa_-^2 (\beta_1 \kappa_1 + 2\beta_3 \kappa_2) + 2\kappa_2 (2\rho_1 - \rho_3) v_L v_R \right), \\
 (m_+^2)_{44} &= \frac{1}{2} \left(\alpha_3 \kappa_-^2 + (2\rho_3 - 4\rho_1) v_R^2 \right).
 \end{aligned} \tag{5.21}$$

Doubly charged scalar mass matrix (m_{++}^2).

$$(m_{++}^2)_{11} = \alpha_3 \kappa_-^2 + (\rho_3 - 2\rho_1) v_L^2 + 4\rho_2 v_R^2, \tag{5.22}$$

$$\begin{aligned}
 (m_{++}^2)_{12} &= \frac{1}{\kappa_1^2} \left(\kappa_-^2 (\beta_1 \kappa_1 \kappa_2 + \beta_3 \kappa_+^2) + (4\kappa_1^2 \rho_4 + \kappa_2^2 (2\rho_1 - \rho_3)) v_L v_R \right), \\
 (m_{++}^2)_{22} &= \alpha_3 \kappa_-^2 + 4\rho_2 v_L^2 + (\rho_3 - 2\rho_1) v_R^2.
 \end{aligned} \tag{5.23}$$

Once the cascade of spontaneous symmetry is completed, two charged and two neutral gauge bosons become massive, while one neutral gauge boson is massless identified as photon. As color symmetry remains intact, we will focus on the uncolored sector $(\text{SU}(2)_L \otimes \text{SU}(2)_R \otimes \text{U}(1)_{B-L})$ only for this purpose. The unphysical states $\{W_{1L}^\mu, W_{2L}^\mu, W_{3L}^\mu\} \in W_L^\mu$, $\{W_{1R}^\mu, W_{2R}^\mu, W_{3R}^\mu\} \in W_R^\mu$, and B^μ are the gauge bosons corresponding to the $\text{SU}(2)_L, \text{SU}(2)_R, \text{U}(1)_{B-L}$ groups respectively. After the SSB, the lagrangian containing gauge boson mass matrices are written as [193, 220]:

$$\mathcal{L}_M^{\text{gauge}} = \begin{pmatrix} W_{L\mu}^- & W_{R\mu}^- \end{pmatrix} \tilde{m}_W^2 \begin{pmatrix} W_L^{+\mu} \\ W_R^{+\mu} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} W_{3L\mu} & W_{3R\mu} & B_\mu \end{pmatrix} \tilde{m}_0^2 \begin{pmatrix} W_{3L}^\mu \\ W_{3R}^\mu \\ B^\mu \end{pmatrix}, \tag{5.24}$$

where the charged states are defined as

$$W_{L,R}^{\pm\mu} = \frac{1}{\sqrt{2}} (W_1^\mu \mp iW_2^\mu)_{L,R}. \tag{5.25}$$

The charged and neutral gauge boson mass matrices have the forms:

$$\tilde{m}_W^2 = \frac{g^2}{4} \begin{pmatrix} \kappa_+^2 + 2v_L^2 & -2\kappa_1 \kappa_2 \\ -2\kappa_1 \kappa_2 & \kappa_+^2 + 2v_R^2 \end{pmatrix}, \tag{5.26}$$

and

$$\tilde{m}_0^2 = \frac{1}{2} \begin{pmatrix} \frac{1}{2}g^2(\kappa_+^2 + 4v_L^2) & -\frac{1}{2}g^2\kappa_+^2 & -2g\tilde{g}v_L^2 \\ -\frac{1}{2}g^2\kappa_+^2 & \frac{1}{2}g^2(\kappa_+^2 + 4v_L^2) & -2g\tilde{g}v_R^2 \\ -2g\tilde{g}v_L^2 & -2g\tilde{g}v_R^2 & 2\tilde{g}^2(v_L^2 + v_R^2) \end{pmatrix}, \tag{5.27}$$

respectively. Here, we have defined $\kappa_\pm^2 = \kappa_1^2 \pm \kappa_2^2$, to express our result with the same convention of [193, 220].

Now to find the physical spectrum, we need to diagonalize the mass matrices. We have introduced the following rotation matrices to connect the physical (charged: $W_1^{\pm\mu}, W_2^{\pm\mu}$, neutral: Z_1^μ, Z_2^μ, A^μ), and unphysical basis of gauge bosons [193, 220]:

$$\begin{pmatrix} W_L^{\pm\mu} \\ W_R^{\pm\mu} \end{pmatrix} = \mathcal{R}_a \begin{pmatrix} W_1^{\pm\mu} \\ W_2^{\pm\mu} \end{pmatrix}, \quad \begin{pmatrix} W_{3L}^\mu \\ W_{3R}^\mu \\ B^\mu \end{pmatrix} = \mathcal{R}_b \begin{pmatrix} Z_1^\mu \\ Z_2^\mu \\ A^\mu \end{pmatrix}. \tag{5.28}$$

Here, the rotation matrices are given as:

$$\mathcal{R}_a = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix}, \quad (5.29)$$

$$\mathcal{R}_b = \begin{pmatrix} \cos \theta_w \cos \theta_2 & \cos \theta_w \sin \theta_2 & \sin \theta_w \\ -\sin \theta_w \sin \theta_1 \cos \theta_2 - \cos \theta_1 \sin \theta_2 & -\sin \theta_w \sin \theta_1 \sin \theta_2 + \cos \theta_1 \cos \theta_2 & \cos \theta_w \sin \theta_1 \\ -\sin \theta_w \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 & -\sin \theta_w \cos \theta_1 \sin \theta_2 - \sin \theta_1 \cos \theta_2 & \cos \theta_w \cos \theta_1 \end{pmatrix}.$$

The rotation “angles” are related to each other as [193, 220]:

$$\cos \theta_1 = \frac{\sqrt{\cos 2\theta_w}}{\cos \theta_w}, \quad \sin \theta_1 = \tan \theta_w, \quad g = \frac{e}{\sin \theta_w}, \quad \tilde{g} = \frac{e}{\sqrt{\cos 2\theta_w}}, \quad (5.30)$$

and

$$\tan 2\xi = \frac{-2k_1 k_2}{v_R^2 - v_L^2}, \quad \tan 2\theta_2 = \frac{a}{b}, \quad (5.31)$$

where

$$a = \left(\frac{1}{4} g^2 \kappa_+^2 - \tilde{g}^2 v_L^2 \right) \sqrt{\cos 2\theta_w}, \quad (5.32)$$

$$b = \left(\frac{1}{4} g^2 \kappa_+^2 \sin^2 \theta_w + \frac{1}{2} g^2 v_L^2 - \frac{1}{2} \tilde{g}^2 v_L^2 \sin^2 \theta_w - \frac{1}{2} (g^2 + \tilde{g}^2) v_R^2 \cos^2 \theta_w \right). \quad (5.33)$$

We would like to note, here, that θ_w is the Weak mixing angle.

The physical gauge boson spectrum can be written as:

$$m_{W_{1,2}}^2 = \frac{g^2}{4} \left[\kappa_+^2 + v_L^2 + v_R^2 \mp \sqrt{(v_R^2 - v_L^2)^2 + 4\kappa_1^2 \kappa_2^2} \right], \quad (5.34)$$

$$m_{Z_{1,2}}^2 = \frac{1}{4} g^2 \kappa_+^2 + \frac{1}{2} (g^2 + \tilde{g}^2) (v_L^2 + v_R^2) \mp \frac{1}{\cos^2 \theta_w} \sqrt{a^2 + b^2}. \quad (5.35)$$

After the SSB, the charged fermion masses are generated through the Yukawa couplings, eq. (5.6) in the following form [193, 220]:

$$m_e = \frac{1}{\sqrt{2}} (\kappa_1 \tilde{y}_D + \kappa_2 y_D) \Rightarrow \text{charged lepton mass}, \quad (5.36)$$

$$m_u = \frac{1}{\sqrt{2}} (\kappa_2 \tilde{y}_q + \kappa_1 y_q) \Rightarrow \text{up-type quark mass}, \quad (5.37)$$

$$m_d = \frac{1}{\sqrt{2}} (\kappa_1 \tilde{y}_q + \kappa_2 y_q) \Rightarrow \text{down-type quark mass}. \quad (5.38)$$

In this model, left- and right-handed neutrinos also acquire masses. We have defined a Majorana basis for light and heavy neutrinos as [193, 220]:

$$\nu = \frac{\nu_L + (\nu_L)^c}{\sqrt{2}}, \quad N = \frac{\nu_R + (\nu_R)^c}{\sqrt{2}}, \quad (5.39)$$

where the charge conjugated state is defined as $\psi^c = C\bar{\psi}^T$ using the charge conjugation operator $C = i\gamma_2\gamma_0$. Once the bi-doublet (Φ) gets VEV, see eq. (5.8), the Dirac-mass

terms for the neutrinos are generated as: $\frac{1}{\sqrt{2}}(\kappa_2 \tilde{y}_D + \kappa_1 y_D)$. The VEVs of Δ_L, Δ_R lead to the Majorana mass terms for ν and N respectively. Including all these, we can define the neutrino mass matrix in the basis $\{\nu, N\}$ as [193, 220]:

$$m_\nu = \begin{pmatrix} \sqrt{2}y_M v_L & \frac{1}{\sqrt{2}}(\kappa_2 \tilde{y}_D + \kappa_1 y_D) \\ \frac{1}{\sqrt{2}}(\kappa_2 \tilde{y}_D + \kappa_1 y_D) & \sqrt{2}y_M v_R \end{pmatrix}. \quad (5.40)$$

6 Effective operators for MLRSM, their categorization and their impact on spectrum

In this section, we have listed all the dimension-6 operators that are computed using the Hilbert Series method. These operators are computed using the information given in tables 13, and following the thumb rules mentioned in the flowchart of figure 1. We have classified all the dimension-6 operators in the following categories: $\{\phi^6, \phi^2 X^2, \psi^2 \phi^2 D, \phi^4 D^2, \psi^2 \phi^3, \psi^2 \phi X, \psi^4, X^3\}$. For MLRSM, the Hilbert Series yields 445 independent operators of mass dimension-6. This includes the distinct Hermitian Conjugates (h.c.) as well. Similar to the 2HDM case, the Wilson Coefficients are also taken to be real for this particular scenario. These operators are tabulated in tables 15–24, and also schematically depicted in figure 3 where the number of operators is also mentioned for each class as ($\mathcal{O}$ [...]). While writing the explicit structures of these operators along with the field variables we have also employed few tensors, e.g. $\delta_{\mu\nu}$ (Kronecker delta), $\epsilon^{\mu\nu\rho}$ (Levi-Civita) etc. to construct the invariants. We have provided a suitable table, similar to the 2HDM case, where we have outlined the nomenclature of the operators. This would be helpful for the readers to follow the operators ($\mathcal{O}$), and the respective Wilson Coefficients ($\mathcal{C}$) which are also defined following the same rule, see tables 14.

Operator Class	Symbol	Superscript	Subscript	Remarks
ϕ^6	$\mathcal{O}_6^{ijklmn}$ $\mathcal{O}_6^{\{ij\}\{klmn\}}$	$\{i, j, k, l, m, n\}$ $\downarrow$ $\{1, 2, 3, 4, L, l, R, r\}$ $\{\} \implies$ “Tr” no $\{\} \implies$ “Tr” over the full operator	$6 \rightarrow$ mass dimension of ϕ^6	$1 \rightarrow \phi; 2 \rightarrow \phi^\dagger;$ $3 \rightarrow \tilde{\phi}; 4 \rightarrow \tilde{\phi}^\dagger;$ $l \rightarrow \Delta_L; L \rightarrow \Delta_L^\dagger;$ $r \rightarrow \Delta_R; R \rightarrow \Delta_R^\dagger;$
X^3	$\mathcal{O}_{L,R}^{X^3/\tilde{X}^3}$	$X \rightarrow \{W, G\}$	L, R for W_L, W_R	$X \equiv WB, \tilde{X} \rightarrow \tilde{W}B$
$\phi^2 X^2$	$\mathcal{O}_{HX/H\tilde{X}}^{ij}$	$(i, j) \equiv (1, 2), (2, 3),$ $(4, 1), (L, l), (R, r)$	$H \equiv \phi, \Delta_L, \Delta_R$ $X \rightarrow \{B, G, W_L, W_R\}$	“Tr” is over the full operator
	$\mathcal{O}_{HX_1 X_2}^{ij}$		$\{X_1, X_2\} \rightarrow \{B, W_L, W_R\}$	
	$\mathcal{O}_{\Delta W}^{iX_1 j X_2}$	$(i, j) \equiv (L, l), (L, r), (R, l), (R, r)$ $\{X_1, X_2\} \rightarrow \{W_L, W_R\}$		
$\psi^2 \phi X$	$\mathcal{O}_{fL fR}^{iX}$	$i = 1 \implies \phi$ $i = 3 \implies \tilde{\phi}$ $X \rightarrow \{B, G, W_L, W_R\}$	$f \rightarrow q(\text{quark}),$ $f \rightarrow l(\text{lepton})$	**within the operator a quark is denoted by Q and a lepton by L
$\psi^2 \phi^3$	$\mathcal{O}_f^{i(jk)}/\mathcal{O}_f^{ijk}$	$\{i, j, k\}$ $\downarrow$ $\{1, 2, 3, 4, L, l, R, r\}$	$f \rightarrow \{l_L, l_R\}$	The field outside “()” are coupled directly to the fermions, “Tr” is taken over the fields inside “()”.
	$\mathcal{O}_{fL fR}^{i(jk)}/\mathcal{O}_{fL fR}^{ijk}$		$\tilde{f} \rightarrow \{q, l\}$	
$\psi^2 \phi^2 D$	$\mathcal{O}_{iDj}^f$ $\mathcal{O}_{iDj}^{f[1]/f[3]}$	$f \rightarrow \{l_L, l_R, q_L, q_R\}$	$\{i, j\}$ $\downarrow$ $\{1, 2, 3, 4, L, l, R, r\}$	[1] $\implies$ SU(2) Singlet [3] $\implies$ SU(2) Triplet (involves τ^I) “Tr” is to be taken over the “iDj” part
	$\mathcal{O}_{iDj}^{fL fR}$	$f \rightarrow \{l, q\}$		
$\phi^4 D^2$	$\mathcal{O}_{\square}^{ij(kl)}$	$\{i, j, k, l\}$ $\downarrow$ $\{1, 2, L, l, R, r\}$ $\{\} \implies$ “Tr”	$\square$ for terms like $(\phi_i^\dagger \phi_j) \square (\phi_k^\dagger \phi_l)$, else ϕD	(i) $\implies$ D acting on ϕ_i $\phi_i \rightarrow \{\phi, \phi^\dagger, \Delta_L, \Delta_L^\dagger, \Delta_R, \Delta_R^\dagger\}$
	$\mathcal{O}_{\phi D}^{(i)jk(l)}$			
	$\mathcal{O}_{\phi D}^{ij(k)(l)}$			
	$\mathcal{O}_{\phi D}^{i(j)k(l)}$			
	$\mathcal{O}_{\phi D}^{(ij)(kl)}$ $\mathcal{O}_{\phi D}^{\{ij\}\{(k)(l)\}}$	no $\{\} \implies$ “Tr” over the full operator		
ψ^4	$\mathcal{O}_{f_1 f_2}^{[i]}/\mathcal{O}_{f_1 f_2 f_3 f_4}^{[i]}$	$i \rightarrow \{1, 3, 8\}$	$\{f_1, \dots f_4\} \rightarrow \{q_L, q_R, l_L, l_R\}$	[1] $\implies$ SU(2), SU(3) Singlet [3] $\implies$ SU(2) Triplet (involves τ^I) [8] $\implies$ SU(3) Octet (involves T^A)

Table 14. MLRSM: summary of the nomenclature of operators. Note: ** — applies to all operator classes. “Tr” — trace must be taken over internal symmetry indices.

Figure 3. Effective operators in “Warsaw”-like basis representing the following class of operators: (a) ϕ^6 [42×2 + 28 = 112], (b) $\phi^2 X^2$ [62], (c) $\psi^2 \phi^2 D$ [20 + 8×2 = 36], (d) $\phi^4 D^2$ [20 + 11×2 = 42], (e) $\psi^2 \phi^3$ [22×2 = 44] (Majorana type) + [36 × 2 = 72] (Dirac type), (f) $\psi^2 \phi X$ [18×2 = 36], (g) ψ^4 [15 + 6×2 = 27] (Baryon Number Conserving) + [4×2 = 8] (Baryon Number Violating), (h) X^3 [6].

Scalar potential in the presence of ϕ^6 operators. In the presence of ϕ^6 operators, the scalar potential is modified as $V(\phi_1, \phi_2) + \mathcal{L}_{\phi^6}$ where

$$\mathcal{L}_{\phi^6} = \sum_{j=1}^{112} \mathcal{C}_6^j \mathcal{O}_6^j. \quad (6.1)$$

Apart from the earlier minimisation conditions eq. (5.9), we have further noted the following minimisation criteria:

$$\begin{aligned} \frac{\partial \mathcal{L}_{\phi^6}}{\partial v_R} &= \frac{1}{4\Lambda^2} \left[\kappa_1 v_L (3v_R^2 (\kappa_2 \mathcal{C}_6^{RrL2r1} + \kappa_1 (\mathcal{C}_6^{RrL4r1} + \mathcal{C}_6^{RrR4l1})) + \kappa_2 \kappa_1^2 \mathcal{C}_6^{21L2r1} + \kappa_2^3 \mathcal{C}_6^{41L2r3} + \mathcal{C}_6^{RrRrRr} v_R^3 \right. \\ &\quad + \kappa_1^3 (\mathcal{C}_6^{21L4r1} + \mathcal{C}_6^{21R4l1}) + \kappa_2^2 \kappa_1 (\mathcal{C}_6^{2L41r3} + \mathcal{C}_6^{41L41r} + \mathcal{C}_6^{41R2l1} + \mathcal{C}_6^{41R4l1})) + v_L^2 v_R (2\mathcal{C}_6^{RrRrLl} v_R^2 \\ &\quad + \kappa_1^2 \mathcal{C}_6^{21LlRr} + 2\kappa_1 \kappa_2 \mathcal{C}_6^{41LlRr} + \kappa_2^2 (\mathcal{C}_6^{LlR21r} + \mathcal{C}_6^{RrL21l})) + \mathcal{C}_6^{LlLlRr} v_L^4 v_R + \kappa_1 v_L^3 (\kappa_1 (\mathcal{C}_6^{LlL4r1} + \mathcal{C}_6^{LlR4l1}) \\ &\quad + \kappa_2 \mathcal{C}_6^{LlR2l1}) + v_R (v_R^2 (\kappa_1^2 (\mathcal{C}_6^{\{21\}\{RrRr\}} + 2\mathcal{C}_6^{21RrRr}) + \kappa_2^2 (\mathcal{C}_6^{\{21\}\{RrRr\}} + 2\mathcal{C}_6^{RrR21r}) + 4\kappa_2 \kappa_1 \mathcal{C}_6^{41RrRr}) \\ &\quad + \kappa_1^2 (\kappa_1^2 (\mathcal{C}_6^{2121Rr} + \mathcal{C}_6^{2211Rr}) + 2\kappa_2 \kappa_1 \mathcal{C}_6^{2141Rr} + \kappa_2^2 (\mathcal{C}_6^{23Rr41} + \mathcal{C}_6^{2R21r1} + 2(\mathcal{C}_6^{4141Rr} + \mathcal{C}_6^{41R2r1})))) \Big] \\ &= 0, \\ \frac{\partial \mathcal{L}_{\phi^6}}{\partial v_L} &= \frac{1}{4\Lambda^2} \left[v_L (v_R^2 (\kappa_1^2 \mathcal{C}_6^{21LlRr} + 2\kappa_2 \kappa_1 \mathcal{C}_6^{41LlRr} + \kappa_2^2 (\mathcal{C}_6^{LlR21r} + \mathcal{C}_6^{RrL21l})) + \mathcal{C}_6^{RrRrLl} v_R^4 \right. \\ &\quad + \kappa_1^2 (\kappa_1^2 (\mathcal{C}_6^{2121Ll} + \mathcal{C}_6^{2211Ll}) + 2\kappa_2 \kappa_1 \mathcal{C}_6^{2141Ll} + \kappa_2^2 (\mathcal{C}_6^{23Ll41} + \mathcal{C}_6^{2L21l1} + 2(\mathcal{C}_6^{4141Ll} + \mathcal{C}_6^{41L2l1})))) \Big] \end{aligned} \quad (6.2)$$

$\mathcal{O}_6^{414141}$	$Tr[\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_6^{412121}$	$Tr[\tilde{\Phi}^\dagger \Phi \Phi^\dagger \Phi \Phi^\dagger \Phi]$
$\mathcal{O}_6^{414123}$	$Tr[\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi \Phi^\dagger \tilde{\Phi}]$	$\mathcal{O}_6^{414121}$	$Tr[\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi \Phi^\dagger \Phi]$
$\mathcal{O}_6^{4141Rr}$	$Tr[\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Delta_R]$	$\mathcal{O}_6^{41R2l1}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Phi^\dagger \Delta_L \Phi]$
$\mathcal{O}_6^{4141Ll}$	$Tr[\tilde{\Phi}^\dagger \Phi \tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Delta_L]$	$\mathcal{O}_6^{41R2r1}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Phi^\dagger \Delta_R \Phi]$
$\mathcal{O}_6^{2L41r3}$	$Tr[\Phi^\dagger \Delta_L^\dagger \tilde{\Phi}^\dagger \Phi \Delta_R \tilde{\Phi}]$	$\mathcal{O}_6^{21L4r1}$	$Tr[\Phi^\dagger \Phi \Delta_L^\dagger \tilde{\Phi}^\dagger \Delta_R \Phi]$
$\mathcal{O}_6^{LR4rr1}$	$Tr[\Delta_L^\dagger \Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_R \Delta_R \Phi]$	$\mathcal{O}_6^{41L2l1}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Phi^\dagger \Delta_L \Phi]$
$\mathcal{O}_6^{LR4lr1}$	$Tr[\Delta_L^\dagger \Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_L \Delta_R \Phi]$	$\mathcal{O}_6^{41R4ll}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \tilde{\Phi}^\dagger \Phi \Delta_L]$
$\mathcal{O}_6^{R4Rlr1}$	$Tr[\Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_R^\dagger \Delta_L \Delta_R \Phi]$	$\mathcal{O}_6^{41L4lr}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \tilde{\Phi}^\dagger \Phi \Delta_R]$
$\mathcal{O}_6^{L4Lrl1}$	$Tr[\Delta_L^\dagger \tilde{\Phi}^\dagger \Delta_L^\dagger \Delta_R \Delta_L \Phi]$	$\mathcal{O}_6^{21L2r1}$	$Tr[\Phi^\dagger \Phi \Delta_L^\dagger \Phi^\dagger \Delta_R \Phi]$
$\mathcal{O}_6^{LR4ll1}$	$Tr[\Delta_L^\dagger \Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_L \Delta_L \Phi]$	$\mathcal{O}_6^{41L2r3}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Phi^\dagger \Delta_R \tilde{\Phi}]$
$\mathcal{O}_6^{LR2LL1}$	$Tr[\Delta_L^\dagger \Delta_R^\dagger \Phi^\dagger \Delta_L^\dagger \Delta_L^\dagger \Phi]$	$\mathcal{O}_6^{RrL4r1}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_L^\dagger \tilde{\Phi}^\dagger \Delta_R \Phi]$
$\mathcal{O}_6^{LR2rr1}$	$Tr[\Delta_L^\dagger \Delta_R^\dagger \Phi^\dagger \Delta_R \Delta_R \Phi]$	$\mathcal{O}_6^{RrR4l1}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_L \Phi]$
$\mathcal{O}_6^{21R4l1}$	$Tr[\Phi^\dagger \Phi \Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_L \Phi]$	$\mathcal{O}_6^{LlL4r1}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \tilde{\Phi}^\dagger \Delta_R \Phi]$
$\mathcal{O}_6^{41l2r1}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L \Phi^\dagger \Delta_R \Phi]$	$\mathcal{O}_6^{LlR4l1}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_R^\dagger \tilde{\Phi}^\dagger \Delta_L \Phi]$
$\mathcal{O}_6^{41RrRr}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R]$	$\mathcal{O}_6^{LlR2l1}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Phi^\dagger \Delta_L \Phi]$
$\mathcal{O}_6^{41LlLl}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L]$	$\mathcal{O}_6^{RrL2r1}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_L^\dagger \Phi^\dagger \Delta_R \Phi]$
$\mathcal{O}_6^{2141Rr}$	$Tr[\Phi^\dagger \Phi \tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Delta_R]$	$\mathcal{O}_6^{2141Ll}$	$Tr[\Phi^\dagger \Phi \tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Delta_L]$
$\mathcal{O}_6^{41RRrr}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Delta_R^\dagger \Delta_R \Delta_R]$	$\mathcal{O}_6^{41LlRr}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R]$
$\mathcal{O}_6^{41RRll}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_R^\dagger \Delta_R^\dagger \Delta_L \Delta_L]$	$\mathcal{O}_6^{41LLll}$	$Tr[\tilde{\Phi}^\dagger \Phi \Delta_L^\dagger \Delta_L^\dagger \Delta_L \Delta_L]$
$\mathcal{O}_6^{21LLrr}$	$Tr[\Phi^\dagger \Phi \Delta_L^\dagger \Delta_L^\dagger \Delta_R \Delta_R]$	$\mathcal{O}_6^{21LLrr}$	$Tr[\Phi^\dagger \Phi \Delta_L^\dagger \Delta_L^\dagger \Delta_R \Delta_R]$
$\mathcal{O}_6^{llRRRr}$	$Tr[\Delta_L \Delta_L \Delta_R^\dagger \Delta_R^\dagger \Delta_R^\dagger \Delta_R]$	$\mathcal{O}_6^{LlLLrr}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L^\dagger \Delta_R \Delta_R]$

Table 15. MLRSM: ϕ^6 $[42 \times 2 = 84]$ class of operators. Each of these operators also has a distinct Hermitian Conjugate.

$$\begin{aligned}
 & +2v_L^3 (\mathcal{C}_6^{LlLlRr} v_R^2 + \kappa_1^2 (\mathcal{C}_6^{\{21\}\{LlLl\}} + \mathcal{C}_6^{21LlLl}) + \kappa_2^2 (\mathcal{C}_6^{\{21\}\{LlLl\}} + \mathcal{C}_6^{LlL21l}) + 2\kappa_1 \kappa_2 \mathcal{C}_6^{41LlLl}) \\
 & + 3\kappa_1 v_L^2 v_R (\kappa_1 (\mathcal{C}_6^{LlL4r1} + \mathcal{C}_6^{LlR4l1}) + \kappa_2 \mathcal{C}_6^{LlR2l1}) + 3\mathcal{C}_6^{LlLlLl} v_L^5 + \kappa_1 v_R (v_R^2 (\kappa_2 \mathcal{C}_6^{RrL2r1} + \kappa_1 (\mathcal{C}_6^{RrL4r1} \\
 & + \mathcal{C}_6^{RrR4l1})) + \kappa_2 \kappa_1^2 \mathcal{C}_6^{21L2r1} + \kappa_1^3 (\mathcal{C}_6^{21L4r1} + \mathcal{C}_6^{21R4l1}) + \kappa_2^2 \kappa_1 (\mathcal{C}_6^{2L41r3} + \mathcal{C}_6^{41L41r} + \mathcal{C}_6^{41R2l1} + \mathcal{C}_6^{41R4l1}) \\
 & + \kappa_2^3 \mathcal{C}_6^{41L2r3})] \\
 & = 0, \tag{6.3} \\
 \frac{\partial \mathcal{L}_{\phi^6}}{\partial \kappa_1} &= \frac{1}{4\Lambda^2} [v_L^2 (v_R^2 (\kappa_1 \mathcal{C}_6^{21LlRr} + \kappa_2 \mathcal{C}_6^{41LlRr}) + \kappa_1 (2\kappa_1^2 (\mathcal{C}_6^{2121Ll} + \mathcal{C}_6^{2211Ll}) + 3\kappa_2 \kappa_1 \mathcal{C}_6^{2141Ll} + \kappa_2^2 (\mathcal{C}_6^{23Ll41} \\
 & + \mathcal{C}_6^{2L21l1} + 2(\mathcal{C}_6^{4141Ll} + \mathcal{C}_6^{41L2l1})))) + v_L v_R (v_R^2 (\kappa_2 \mathcal{C}_6^{RrL2r1} + 2\kappa_1 (\mathcal{C}_6^{RrL4r1} + \mathcal{C}_6^{RrR4l1})) + 3\kappa_2 \kappa_1^2 \mathcal{C}_6^{21L2r1}
 \end{aligned}$$

$\mathcal{O}_{l_L l_R}^{1B}$	$(\bar{L}_L \sigma_{\mu\nu} \Phi L_R) B^{\mu\nu}$	$\mathcal{O}_{l_L l_R}^{3B}$	$(\bar{L}_L \sigma_{\mu\nu} \tilde{\Phi} L_R) B^{\mu\nu}$	$\mathcal{O}_{l_L l_R}^{1W_L}$	$(\bar{L}_L \sigma_{\mu\nu} \tau^I \Phi L_R) W_L^{I\mu\nu}$
$\mathcal{O}_{l_L l_R}^{3W_L}$	$(\bar{L}_L \sigma_{\mu\nu} \tau^I \tilde{\Phi} L_R) W_L^{I\mu\nu}$	$\mathcal{O}_{l_L l_R}^{1W_R}$	$(\bar{L}_L \sigma_{\mu\nu} \tau^I \Phi L_R) W_R^{I\mu\nu}$	$\mathcal{O}_{l_L l_R}^{3W_R}$	$(\bar{L}_L \sigma_{\mu\nu} \tau^I \tilde{\Phi} L_R) W_R^{I\mu\nu}$
$\mathcal{O}_{q_L q_R}^{1B}$	$(\bar{Q}_L \sigma_{\mu\nu} \Phi Q_R) B^{\mu\nu}$	$\mathcal{O}_{q_L q_R}^{3B}$	$(\bar{Q}_L \sigma_{\mu\nu} \tilde{\Phi} Q_R) B^{\mu\nu}$	$\mathcal{O}_{q_L q_R}^{1W_L}$	$(\bar{Q}_L \sigma_{\mu\nu} \tau^I \Phi Q_R) W_L^{I\mu\nu}$
$\mathcal{O}_{q_L q_R}^{3W_L}$	$(\bar{Q}_L \sigma_{\mu\nu} \tau^I \tilde{\Phi} Q_R) W_L^{I\mu\nu}$	$\mathcal{O}_{q_L q_R}^{1W_R}$	$(\bar{Q}_L \sigma_{\mu\nu} \tau^I \Phi Q_R) W_R^{I\mu\nu}$	$\mathcal{O}_{q_L q_R}^{3W_R}$	$(\bar{Q}_L \sigma_{\mu\nu} \tau^I \tilde{\Phi} Q_R) W_R^{I\mu\nu}$
$\mathcal{O}_{q_L q_R}^{1G}$	$(\bar{Q}_L \sigma_{\mu\nu} T^A \Phi Q_R) G^{A\mu\nu}$	$\mathcal{O}_{q_L q_R}^{3G}$	$(\bar{Q}_L \sigma_{\mu\nu} T^A \tilde{\Phi} Q_R) G^{A\mu\nu}$		
$\mathcal{O}_{l_L l_L}^{\Delta_L W_L}$	$(L_L^T C \sigma_{\mu\nu} \tau^I \Sigma_L L_L) W_L^{I\mu\nu}$	$\mathcal{O}_{l_L l_L}^{\Delta_R W_R}$	$(L_L^T C \sigma_{\mu\nu} \tau^I \Sigma_R L_L) W_R^{I\mu\nu}$	$\mathcal{O}_{l_R l_R}^{\Delta_L W_L}$	$(L_R^T C \sigma_{\mu\nu} \tau^I \Sigma_L L_R) W_L^{I\mu\nu}$
$\mathcal{O}_{l_R l_R}^{\Delta_R W_R}$	$(L_R^T C \sigma_{\mu\nu} \tau^I \Sigma_R L_R) W_R^{I\mu\nu}$				

Table 16. MLRSM: $\psi^2 \phi X$ [18×2 = 36] class of operators. Each of these operators also has a distinct Hermitian Conjugate. Note $I \in \{1, 2, 3\}$ and $A \in \{1, 2, \dots, 8\}$ are the SU(2) and SU(3) indices respectively. Here, $\Sigma_{L,R} = i\tau_2 \Delta_{L,R}$.

$\mathcal{O}^{G^3}$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_L^{W^3}$	$\epsilon^{IJK} W_{L\mu}^{I\nu} W_{L\nu}^{J\rho} W_{L\rho}^{K\mu}$	$\mathcal{O}_R^{W^3}$	$\epsilon^{IJK} W_{R\mu}^{I\nu} W_{R\nu}^{J\rho} W_{R\rho}^{K\mu}$
$\mathcal{O}^{\tilde{G}^3}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$\mathcal{O}_L^{\tilde{W}^3}$	$\epsilon^{IJK} \tilde{W}_{L\mu}^{I\nu} W_{L\nu}^{J\rho} W_{L\rho}^{K\mu}$	$\mathcal{O}_R^{\tilde{W}^3}$	$\epsilon^{IJK} \tilde{W}_{R\mu}^{I\nu} W_{R\nu}^{J\rho} W_{R\rho}^{K\mu}$

Table 17. MLRSM: X^3 [6] class of operators.

$\mathcal{O}_6^{212121}$	$Tr[\Phi^\dagger \Phi \Phi^\dagger \Phi \Phi^\dagger \Phi]$	$\mathcal{O}_6^{\{21\}\{2121\}}$	$Tr[\Phi^\dagger \Phi] Tr[\Phi^\dagger \Phi \Phi^\dagger \Phi]$
$\mathcal{O}_6^{LlLLl}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L^\dagger \Delta_L \Delta_L]$	$\mathcal{O}_6^{LlLLl}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L]$
$\mathcal{O}_6^{RrRRrr}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R^\dagger \Delta_R \Delta_R]$	$\mathcal{O}_6^{RrRRrr}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R]$
$\mathcal{O}_6^{RrLLl}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_L^\dagger \Delta_L^\dagger \Delta_L \Delta_L]$	$\mathcal{O}_6^{LlLLr}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R]$
$\mathcal{O}_6^{LlRRrr}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R^\dagger \Delta_R \Delta_R]$	$\mathcal{O}_6^{RrRRl}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R \Delta_L^\dagger \Delta_L]$
$\mathcal{O}_6^{21LlLl}$	$Tr[\Phi^\dagger \Phi \Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L]$	$\mathcal{O}_6^{\{21\}\{LlLl\}}$	$Tr[\Phi^\dagger \Phi] Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L]$
$\mathcal{O}_6^{LlL21l}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Phi^\dagger \Phi \Delta_L]$	$\mathcal{O}_6^{21RrRr}$	$Tr[\Phi^\dagger \Phi \Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R]$
$\mathcal{O}_6^{RrR21r}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Phi^\dagger \Phi \Delta_R]$	$\mathcal{O}_6^{\{21\}\{RrRr\}}$	$Tr[\Phi^\dagger \Phi] Tr[\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R]$
$\mathcal{O}_6^{2121Ll}$	$Tr[\Phi^\dagger \Phi \Phi^\dagger \Phi \Delta_L^\dagger \Delta_L]$	$\mathcal{O}_6^{2211Ll}$	$Tr[\Phi^\dagger \Phi^\dagger \Phi \Phi \Delta_L^\dagger \Delta_L]$
$\mathcal{O}_6^{23Ll41}$	$Tr[\Phi^\dagger \tilde{\Phi} \Delta_L^\dagger \Delta_L \tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_6^{2L21l1}$	$Tr[\Phi^\dagger \Delta_L^\dagger \Phi^\dagger \Phi \Delta_L \Phi]$
$\mathcal{O}_6^{2121Rr}$	$Tr[\Phi^\dagger \Phi \Phi^\dagger \Phi \Delta_R^\dagger \Delta_R]$	$\mathcal{O}_6^{2211Rr}$	$Tr[\Phi^\dagger \Phi^\dagger \Phi \Phi \Delta_R^\dagger \Delta_R]$
$\mathcal{O}_6^{23Rr41}$	$Tr[\Phi^\dagger \tilde{\Phi} \Delta_R^\dagger \Delta_R \tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_6^{2R21r1}$	$Tr[\Phi^\dagger \Delta_R^\dagger \Phi^\dagger \Phi \Delta_R \Phi]$
$\mathcal{O}_6^{LlR21r}$	$Tr[\Delta_L^\dagger \Delta_L \Delta_R^\dagger \Phi^\dagger \Phi \Delta_R]$	$\mathcal{O}_6^{RrL21l}$	$Tr[\Delta_R^\dagger \Delta_R \Delta_L^\dagger \Phi^\dagger \Phi \Delta_L]$
$\mathcal{O}_6^{LR21rl}$	$Tr[\Delta_L^\dagger \Delta_R^\dagger \Phi^\dagger \Phi \Delta_R \Delta_L]$	$\mathcal{O}_6^{21LlRr}$	$Tr[\Phi^\dagger \Phi \Delta_L^\dagger \Delta_L \Delta_R^\dagger \Delta_R]$

Table 18. MLRSM: ϕ^6 [28] class of operators. These operators are Self-Hermitian.

$\mathcal{O}_{l_L l_L}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L)$	$\mathcal{O}_{l_R l_R}$	$(\bar{L}_R \gamma_\mu L_R)(\bar{L}_R \gamma^\mu L_R)$
$\mathcal{O}_{l_R q_L}$	$(\bar{L}_R \gamma_\mu L_R)(\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{l_L q_R}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{Q}_R \gamma^\mu Q_R)$
$\mathcal{O}_{l_L l_R}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_R \gamma^\mu L_R)$		
$\mathcal{O}_{q_L q_L}^{[1]}$	$(\bar{Q}_L \gamma_\mu Q_L)(\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{q_L q_L}^{[3]}$	$(\bar{Q}_L \gamma_\mu \tau^I Q_L)(\bar{Q}_L \gamma^\mu \tau^I Q_L)$
$\mathcal{O}_{l_L q_L}^{[1]}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{l_L q_L}^{[3]}$	$(\bar{L}_L \gamma_\mu \tau^I L_L)(\bar{Q}_L \gamma^\mu \tau^I Q_L)$
$\mathcal{O}_{q_R q_R}^{[1]}$	$(\bar{Q}_R \gamma_\mu Q_R)(\bar{Q}_R \gamma^\mu Q_R)$	$\mathcal{O}_{q_R q_R}^{[3]}$	$(\bar{Q}_R \gamma_\mu \tau^I Q_R)(\bar{Q}_R \gamma^\mu \tau^I Q_R)$
$\mathcal{O}_{l_R q_R}^{[1]}$	$(\bar{L}_R \gamma_\mu L_R)(\bar{Q}_R \gamma^\mu Q_R)$	$\mathcal{O}_{l_R q_R}^{[3]}$	$(\bar{L}_R \gamma_\mu \tau^I L_R)(\bar{Q}_R \gamma^\mu \tau^I Q_R)$
$\mathcal{O}_{q_L q_R}^{[1]}$	$(\bar{Q}_L \gamma_\mu Q_L)(\bar{Q}_R \gamma^\mu Q_R)$	$\mathcal{O}_{q_L q_R}^{[8]}$	$(\bar{Q}_L \gamma_\mu T^A Q_L)(\bar{Q}_R \gamma^\mu T^A Q_R)$
$\mathcal{O}_{l_L l_R q_L q_R}^{(1)}$	$\epsilon_{jk} \epsilon_{lm} (\bar{L}_L^j L_R^l)(\bar{Q}_L^k Q_R^m) + h.c.$		
$\mathcal{O}_{l_L l_R q_L q_R}^{(3)}$	$\epsilon_{jk} \epsilon_{lm} (\bar{L}_L^j \sigma_{\mu\nu} L_R^l)(\bar{Q}_L^k \sigma^{\mu\nu} Q_R^m) + h.c.$		
$\mathcal{O}_{l_L l_R q_R q_L}$	$\epsilon_{jk} \epsilon_{lm} (\bar{L}_L^j L_R^l)(\bar{Q}_R^m Q_L^k) + h.c.$		
$\mathcal{O}_{l_L l_R l_L l_R}$	$\epsilon_{jk} \epsilon_{lm} (\bar{L}_L^j L_R^l)(\bar{L}_L^k L_R^m) + h.c.$		
$\mathcal{O}_{q_L q_R q_L q_R}^{(1)}$	$\epsilon_{jk} \epsilon_{lm} (\bar{Q}_L^j Q_R^l)(\bar{Q}_L^k Q_R^m) + h.c.$		
$\mathcal{O}_{q_L q_R q_L q_R}^{(8)}$	$\epsilon_{jk} \epsilon_{lm} (\bar{Q}_L^j T^A Q_R^l)(\bar{Q}_L^k T^A Q_R^m) + h.c.$		

Table 19. MLRSM: Baryon Number Conserving ψ^4 [15 + 6×2 = 27] class of operators.

$\mathcal{O}_{q_L q_L q_L l_L}$	$\epsilon_{\alpha\beta\gamma} \epsilon_{jn} \epsilon_{km} [(Q_L^{\alpha j})^T C Q_L^{\beta k}] [(Q_L^{\gamma m})^T C L_L^n]$	$\mathcal{O}_{q_R q_R q_R l_R}$	$\epsilon_{\alpha\beta\gamma} \epsilon_{jn} \epsilon_{km} [(Q_R^{\alpha j})^T C Q_R^{\beta k}] [(Q_R^{\gamma m})^T C L_R^n]$
$\mathcal{O}_{q_L q_L q_R l_R}$	$\epsilon_{\alpha\beta\gamma} \epsilon_{jk} \epsilon_{lm} [(Q_L^{\alpha j})^T C Q_L^{\beta k}] [(Q_R^{\gamma l})^T C L_R^m]$	$\mathcal{O}_{q_R q_R q_L l_L}$	$\epsilon_{\alpha\beta\gamma} \epsilon_{jk} \epsilon_{lm} [(Q_R^{\alpha j})^T C Q_R^{\beta k}] [(Q_L^{\gamma l})^T C L_L^m]$

Table 20. MLRSM: Baryon Number Violating ψ^4 [4×2 = 8] class of operators. Each operator has a distinct Hermitian Conjugate.

$$\begin{aligned}
 & +4\kappa_1^3 (\mathcal{C}_6^{21L4r1} + \mathcal{C}_6^{21R4l1}) + 2\kappa_2^2 \kappa_1 (\mathcal{C}_6^{2L41r3} + \mathcal{C}_6^{41L41r} + \mathcal{C}_6^{41R2l1} + \mathcal{C}_6^{41R4l1}) + \kappa_2^3 \mathcal{C}_6^{41L2r3} \\
 & + v_L^3 v_R (2\kappa_1 (\mathcal{C}_6^{LlL4r1} + \mathcal{C}_6^{LlR4l1}) + \kappa_2 \mathcal{C}_6^{LlR2l1}) + v_L^4 (\kappa_1 (\mathcal{C}_6^{\{21\}\{LlLl\}} + \mathcal{C}_6^{21LlLl}) + \kappa_2 \mathcal{C}_6^{41LlLl}) \\
 & + \kappa_1 v_R^2 (2\kappa_1^2 (\mathcal{C}_6^{2121Rr} + \mathcal{C}_6^{2211Rr}) + 3\kappa_2 \kappa_1 \mathcal{C}_6^{2141Rr} + \kappa_2^2 (\mathcal{C}_6^{23Rr41} + \mathcal{C}_6^{2R21r1} + 2(\mathcal{C}_6^{4141Rr} + \mathcal{C}_6^{41R2r1}))) \\
 & + v_R^4 (\kappa_1 (\mathcal{C}_6^{\{21\}\{RrRr\}} + \mathcal{C}_6^{21RrRr}) + \kappa_2 \mathcal{C}_6^{41RrRr}) + 3\kappa_1^5 \mathcal{C}_6^{\{21\}\{2121\}} + 2\kappa_2^2 \kappa_1^3 \mathcal{C}_6^{\{21\}\{2121\}} + \kappa_2^4 \kappa_1 \mathcal{C}_6^{\{21\}\{2121\}} \\
 & + 3\kappa_1^5 \mathcal{C}_6^{212121} + 5\kappa_2 \kappa_1^4 \mathcal{C}_6^{412121} + \kappa_2^5 \mathcal{C}_6^{412121} + 4\kappa_2^2 \kappa_1^3 \mathcal{C}_6^{414121} + 2\kappa_2^4 \kappa_1 \mathcal{C}_6^{414121} + 6\kappa_2^3 \kappa_1^2 \mathcal{C}_6^{414123} + 6\kappa_2^3 \kappa_1^2 \mathcal{C}_6^{414141} \\
 & = 0, \\
 & \frac{\partial \mathcal{L}_{\phi^6}}{\partial \kappa_2} = \frac{1}{4\Lambda^2} [v_L^2 (v_R^2 (\kappa_1 \mathcal{C}_6^{41LlRr} + \kappa_2 (\mathcal{C}_6^{LlR21r} + \mathcal{C}_6^{RrL21l})) + \kappa_1^2 (\kappa_1 \mathcal{C}_6^{2141Ll} + \kappa_2 (\mathcal{C}_6^{23Ll41} + \mathcal{C}_6^{2L21l1} \\
 & + 2(\mathcal{C}_6^{4141Ll} + \mathcal{C}_6^{41L2l1})))) + \kappa_1 v_L v_R (\mathcal{C}_6^{RrL2r1} v_R^2 + \kappa_1^2 \mathcal{C}_6^{21L2r1} + 2\kappa_1 \kappa_2 (\mathcal{C}_6^{2L41r3} + \mathcal{C}_6^{41L41r} + \mathcal{C}_6^{41R2l1} \\
 & + \mathcal{C}_6^{41R4l1}) + 3\kappa_2^2 \mathcal{C}_6^{41L2r3}) + \kappa_1 \mathcal{C}_6^{LlR2l1} v_L^3 v_R + v_L^4 (\kappa_2 (\mathcal{C}_6^{\{21\}\{LlLl\}} + \mathcal{C}_6^{LlL21l}) + \kappa_1 \mathcal{C}_6^{41LlLl}) \\
 & + \kappa_1^2 v_R^2 (\kappa_1 \mathcal{C}_6^{2141Rr} + \kappa_2 (\mathcal{C}_6^{23Rr41} + \mathcal{C}_6^{2R21r1} + 2(\mathcal{C}_6^{4141Rr} + \mathcal{C}_6^{41R2r1}))) + v_R^4 (\kappa_2 (\mathcal{C}_6^{\{21\}\{RrRr\}} + \mathcal{C}_6^{RrR21r})
 \end{aligned}
 \tag{6.4}$$

$\mathcal{O}_{l_L}^{rLr}$	$L_L^T C \Sigma_R \Sigma_L^\dagger \Sigma_R L_L$	$\mathcal{O}_{l_L}^{r41}$	$L_L^T C \Sigma_R \tilde{\Phi}^\dagger \Phi L_L$	$\mathcal{O}_{l_L}^{l(23)}$	$[L_L^T C \Sigma_L L_L] Tr[\Phi^\dagger \tilde{\Phi}]$
$\mathcal{O}_{l_L}^{lLl}$	$L_L^T C \Sigma_L \Sigma_L^\dagger \Sigma_L L_L$	$\mathcal{O}_{l_L}^{r21}$	$L_L^T C \Sigma_R \Phi^\dagger \Phi L_L$	$\mathcal{O}_{l_L}^{l(41)}$	$[L_L^T C \Sigma_L L_L] Tr[\tilde{\Phi}^\dagger \Phi]$
$\mathcal{O}_{l_L}^{l(Rr)}$	$[L_L^T C \Sigma_L L_L] Tr[\Sigma_R^\dagger \Sigma_R]$	$\mathcal{O}_{l_L}^{l21}$	$L_L^T C \Sigma_L \Phi^\dagger \Phi L_L$	$\mathcal{O}_{l_L}^{l(21)}$	$[L_L^T C \Sigma_L L_L] Tr[\Phi^\dagger \Phi]$
$\mathcal{O}_{l_L}^{l(Ll)}$	$[L_L^T C \Sigma_L L_L] Tr[\Sigma_L^\dagger \Sigma_L]$	$\mathcal{O}_{l_L}^{r23}$	$L_L^T C \Sigma_R \Phi^\dagger \tilde{\Phi} L_L$	$\mathcal{O}_{l_R}^{r(41)}$	$[L_R^T C \Sigma_R L_R] Tr[\tilde{\Phi}^\dagger \Phi]$
$\mathcal{O}_{l_R}^{r(Ll)}$	$[L_R^T C \Sigma_R L_R] Tr[\Sigma_L^\dagger \Sigma_L]$	$\mathcal{O}_{l_R}^{l23}$	$L_R^T C \Sigma_L \Phi^\dagger \tilde{\Phi} L_R$	$\mathcal{O}_{l_R}^{r(23)}$	$[L_R^T C \Sigma_R L_R] Tr[\Phi^\dagger \tilde{\Phi}]$
$\mathcal{O}_{l_R}^{r(Rr)}$	$[L_R^T C \Sigma_R L_R] Tr[\Sigma_R^\dagger \Sigma_R]$	$\mathcal{O}_{l_R}^{r21}$	$L_R^T C \Sigma_R \Phi^\dagger \Phi L_R$	$\mathcal{O}_{l_R}^{r(21)}$	$[L_R^T C \Sigma_R L_R] Tr[\Phi^\dagger \Phi]$
$\mathcal{O}_{l_R}^{lRl}$	$L_R^T C \Sigma_L \Sigma_R^\dagger \Sigma_L L_R$	$\mathcal{O}_{l_R}^{l41}$	$L_R^T C \Sigma_L \tilde{\Phi}^\dagger \Phi L_R$		
$\mathcal{O}_{l_R}^{rRr}$	$L_R^T C \Sigma_R \Sigma_R^\dagger \Sigma_R L_R$	$\mathcal{O}_{l_R}^{l21}$	$L_R^T C \Sigma_L \Phi^\dagger \Phi L_R$		

Table 21. MLRSM: $\psi^2 \phi^3$ [22×2 = 44] (Majorana type) class of operators. Each of these operators also has a distinct Hermitian Conjugate. Note $\Sigma_{L,R} = i\tau_2 \Delta_{L,R}$.

$\mathcal{O}_{l_L l_R}^{1(41)}$	$(\bar{L}_L \Phi L_R) Tr[\tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_{l_L l_R}^{3(23)}$	$(\bar{L}_L \tilde{\Phi} L_R) Tr[\Phi^\dagger \tilde{\Phi}]$	$\mathcal{O}_{l_L l_R}^{1(21)}$	$(\bar{L}_L \Phi L_R) Tr[\Phi^\dagger \Phi]$
$\mathcal{O}_{l_L l_R}^{3(41)}$	$(\bar{L}_L \tilde{\Phi} L_R) Tr[\tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_{l_L l_R}^{3(21)}$	$(\bar{L}_L \tilde{\Phi} L_R) Tr[\Phi^\dagger \Phi]$	$\mathcal{O}_{l_L l_R}^{1(23)}$	$(\bar{L}_L \Phi L_R) Tr[\Phi^\dagger \tilde{\Phi}]$
$\mathcal{O}_{q_L q_R}^{1(41)}$	$(\bar{Q}_L \Phi Q_R) Tr[\tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_{q_L q_R}^{3(23)}$	$(\bar{Q}_L \tilde{\Phi} Q_R) Tr[\Phi^\dagger \tilde{\Phi}]$	$\mathcal{O}_{q_L q_R}^{1(21)}$	$(\bar{Q}_L \Phi Q_R) Tr[\Phi^\dagger \Phi]$
$\mathcal{O}_{q_L q_R}^{3(41)}$	$(\bar{Q}_L \tilde{\Phi} Q_R) Tr[\tilde{\Phi}^\dagger \Phi]$	$\mathcal{O}_{q_L q_R}^{3(21)}$	$(\bar{Q}_L \tilde{\Phi} Q_R) Tr[\Phi^\dagger \Phi]$	$\mathcal{O}_{l_L l_R}^{1(23)}$	$(\bar{Q}_L \Phi Q_R) Tr[\Phi^\dagger \tilde{\Phi}]$
$\mathcal{O}_{l_L l_R}^{L1l}$	$\bar{L}_L \Delta_L^\dagger \Phi \Delta_L L_R$	$\mathcal{O}_{l_L l_R}^{l1L}$	$\bar{L}_L \Delta_L \Phi \Delta_L^\dagger L_R$	$\mathcal{O}_{l_L l_R}^{L1r}$	$\bar{L}_L \Delta_L^\dagger \Phi \Delta_R L_R$
$\mathcal{O}_{l_L l_R}^{R1r}$	$\bar{L}_L \Delta_R^\dagger \Phi \Delta_R L_R$	$\mathcal{O}_{l_L l_R}^{r1R}$	$\bar{L}_L \Delta_R \Phi \Delta_R^\dagger L_R$	$\mathcal{O}_{l_L l_R}^{l1R}$	$\bar{L}_L \Delta_L \Phi \Delta_R^\dagger L_R$
$\mathcal{O}_{l_L l_R}^{L3l}$	$\bar{L}_L \Delta_L^\dagger \tilde{\Phi} \Delta_L L_R$	$\mathcal{O}_{l_L l_R}^{l3L}$	$\bar{L}_L \Delta_L \tilde{\Phi} \Delta_L^\dagger L_R$	$\mathcal{O}_{l_L l_R}^{L3r}$	$\bar{L}_L \Delta_L^\dagger \tilde{\Phi} \Delta_R L_R$
$\mathcal{O}_{l_L l_R}^{R3r}$	$\bar{L}_L \Delta_R^\dagger \tilde{\Phi} \Delta_R L_R$	$\mathcal{O}_{l_L l_R}^{r3R}$	$\bar{L}_L \Delta_R \tilde{\Phi} \Delta_R^\dagger L_R$	$\mathcal{O}_{l_L l_R}^{l3R}$	$\bar{L}_L \Delta_L \tilde{\Phi} \Delta_R^\dagger L_R$
$\mathcal{O}_{q_L q_R}^{L1l}$	$\bar{Q}_L \Delta_L^\dagger \Phi \Delta_L Q_R$	$\mathcal{O}_{q_L q_R}^{l1L}$	$\bar{Q}_L \Delta_L \Phi \Delta_L^\dagger Q_R$	$\mathcal{O}_{l_L l_R}^{L1r}$	$\bar{Q}_L \Delta_L^\dagger \Phi \Delta_R Q_R$
$\mathcal{O}_{q_L q_R}^{R1r}$	$\bar{Q}_L \Delta_R^\dagger \Phi \Delta_R Q_R$	$\mathcal{O}_{q_L q_R}^{r1R}$	$\bar{Q}_L \Delta_R \Phi \Delta_R^\dagger Q_R$	$\mathcal{O}_{q_L q_R}^{l1R}$	$\bar{Q}_L \Delta_L \Phi \Delta_R^\dagger Q_R$
$\mathcal{O}_{q_L q_R}^{L3l}$	$\bar{Q}_L \Delta_L^\dagger \tilde{\Phi} \Delta_L Q_R$	$\mathcal{O}_{q_L q_R}^{l3L}$	$\bar{Q}_L \Delta_L \tilde{\Phi} \Delta_L^\dagger Q_R$	$\mathcal{O}_{q_L q_R}^{L3r}$	$\bar{Q}_L \Delta_L^\dagger \tilde{\Phi} \Delta_R Q_R$
$\mathcal{O}_{q_L q_R}^{R3r}$	$\bar{Q}_L \Delta_R^\dagger \tilde{\Phi} \Delta_R Q_R$	$\mathcal{O}_{q_L q_R}^{r3R}$	$\bar{Q}_L \Delta_R \tilde{\Phi} \Delta_R^\dagger Q_R$	$\mathcal{O}_{q_L q_R}^{l3R}$	$\bar{Q}_L \Delta_L \tilde{\Phi} \Delta_R^\dagger Q_R$

Table 22. MLRSM: $\psi^2 \phi^3$ [36×2 = 72] (Dirac type) class of operators. Each of these operators also has a distinct Hermitian Conjugate.

$\mathcal{O}_{2D1}^{l_R[1]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{L}_R \gamma^\mu L_R)$	$\mathcal{O}_{2D1}^{l_R[3]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu^I \Phi](\bar{L}_R \gamma^\mu \tau^I L_R)$
$\mathcal{O}_{2D1}^{l_L[1]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{L}_L \gamma^\mu L_L)$	$\mathcal{O}_{2D1}^{l_L[3]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu^I \Phi](\bar{L}_L \gamma^\mu \tau^I L_L)$
$\mathcal{O}_{2D1}^{q_R[1]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{Q}_R \gamma^\mu Q_R)$	$\mathcal{O}_{2D1}^{q_R[3]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu^I \Phi](\bar{Q}_R \gamma^\mu \tau^I Q_R)$
$\mathcal{O}_{2D1}^{q_L[1]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{2D1}^{q_L[3]}$	$Tr[\Phi^\dagger i\overleftrightarrow{D}_\mu^I \Phi](\bar{Q}_L \gamma^\mu \tau^I Q_L)$
$\mathcal{O}_{LDl}^{l_L[1]}$	$Tr[\Delta_L^\dagger i\overleftrightarrow{D}_\mu \Delta_L](\bar{L}_L \gamma^\mu L_L)$	$\mathcal{O}_{LDl}^{l_L[3]}$	$Tr[\Delta_L^\dagger i\overleftrightarrow{D}_\mu^I \Delta_L](\bar{L}_L \gamma^\mu \tau^I L_L)$
$\mathcal{O}_{RDl}^{l_R[1]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu \Delta_R](\bar{L}_L \gamma^\mu L_L)$	$\mathcal{O}_{RDl}^{l_R[3]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu^I \Delta_R](\bar{L}_R \gamma^\mu L_R)$
$\mathcal{O}_{RDl}^{l_R[1]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu \Delta_R](\bar{L}_R \gamma^\mu L_R)$	$\mathcal{O}_{RDl}^{l_R[3]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu^I \Delta_R](\bar{L}_R \gamma^\mu \tau^I L_R)$
$\mathcal{O}_{LDl}^{q_L[1]}$	$Tr[\Delta_L^\dagger i\overleftrightarrow{D}_\mu \Delta_L](\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{LDl}^{q_L[3]}$	$Tr[\Delta_L^\dagger i\overleftrightarrow{D}_\mu^I \Delta_L](\bar{Q}_L \gamma^\mu \tau^I Q_L)$
$\mathcal{O}_{RDl}^{q_L[1]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu \Delta_R](\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{LDl}^{q_R[1]}$	$Tr[\Delta_L^\dagger i\overleftrightarrow{D}_\mu \Delta_L](\bar{Q}_R \gamma^\mu Q_R)$
$\mathcal{O}_{Rr}^{q_R[1]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu \Delta_R](\bar{Q}_R \gamma^\mu Q_R)$	$\mathcal{O}_{Rr}^{q_R[3]}$	$Tr[\Delta_R^\dagger i\overleftrightarrow{D}_\mu^I \Delta_R](\bar{Q}_R \gamma^\mu \tau^I Q_R)$
$\mathcal{O}_{4D1}^{l_L}$	$Tr[\tilde{\Phi}^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{L}_L \gamma^\mu L_L) + h.c.$	$\mathcal{O}_{4D1}^{l_R}$	$Tr[\tilde{\Phi}^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{L}_R \gamma^\mu L_R) + h.c.$
$\mathcal{O}_{4D1}^{q_L}$	$Tr[\tilde{\Phi}^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{Q}_L \gamma^\mu Q_L) + h.c.$	$\mathcal{O}_{4D1}^{q_R}$	$Tr[\tilde{\Phi}^\dagger i\overleftrightarrow{D}_\mu \Phi](\bar{Q}_R \gamma^\mu Q_R) + h.c.$
$\mathcal{O}_{lD1}^{l_L l_R}$	$L_L^T C i \tau_2 \Delta_L (\gamma_\mu D^\mu \Phi) L_R + h.c.$	$\mathcal{O}_{rD1}^{l_L l_R}$	$L_R^T C i \tau_2 \Delta_R (\gamma_\mu D^\mu \Phi) L_L + h.c.$
$\mathcal{O}_{lD3}^{l_L l_R}$	$L_L^T C i \tau_2 \Delta_L (\gamma_\mu D^\mu \tilde{\Phi}) L_R + h.c.$	$\mathcal{O}_{rD3}^{l_L l_R}$	$L_R^T C i \tau_2 \Delta_R (\gamma_\mu D^\mu \tilde{\Phi}) L_L + h.c.$

Table 23. MLRSM: $\psi^2 \phi^2 D$ [20 + 8×2 = 36] class of operators.

$$\begin{aligned}
 & + \kappa_1 C_6^{41RrRr} + \kappa_2 \kappa_1^4 C_6^{\{21\}\{2121\}} + 2\kappa_2^3 \kappa_1^2 C_6^{\{21\}\{2121\}} + 3\kappa_2^5 C_6^{\{21\}\{2121\}} + 3\kappa_2^5 C_6^{212121} + \kappa_1^5 C_6^{412121} \\
 & + 5\kappa_2^4 \kappa_1 C_6^{412121} + 2\kappa_2 \kappa_1^4 C_6^{414121} + 4\kappa_2^3 \kappa_1^2 C_6^{414121} + 6\kappa_2^2 \kappa_1^3 C_6^{414123} + 6\kappa_2^2 \kappa_1^3 C_6^{414141}] \\
 & = 0.
 \end{aligned} \tag{6.5}$$

$\phi^4 D^2$ and ϕ^6 operators: redefinitions of the scalar fields and modifications in spectrum.

The scalar kinetic lagrangian, eq. (5.1), gets modified in presence of the $\phi^4 D^2$ operators, table 24, as:

$$\begin{aligned}
 \mathcal{L}^{(4)+(6)} = & \frac{1}{2} \begin{pmatrix} \partial^\mu \phi_1^{0r} \\ \partial^\mu \phi_2^{0r} \\ \partial^\mu \delta_R^{0r} \\ \partial^\mu \delta_L^{0r} \end{pmatrix}^T \begin{pmatrix} 1 + \frac{A_r^{11}}{\Lambda^2} & \frac{A_r^{12}}{\Lambda^2} & \frac{A_r^{13}}{\Lambda^2} & \frac{A_r^{14}}{\Lambda^2} \\ \frac{A_r^{12}}{\Lambda^2} & 1 + \frac{A_r^{22}}{\Lambda^2} & \frac{A_r^{23}}{\Lambda^2} & \frac{A_r^{24}}{\Lambda^2} \\ \frac{A_r^{13}}{\Lambda^2} & \frac{A_r^{23}}{\Lambda^2} & 1 + \frac{A_r^{33}}{\Lambda^2} & \frac{A_r^{34}}{\Lambda^2} \\ \frac{A_r^{14}}{\Lambda^2} & \frac{A_r^{24}}{\Lambda^2} & \frac{A_r^{34}}{\Lambda^2} & 1 + \frac{A_r^{44}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \phi_1^{0r} \\ \partial_\mu \phi_2^{0r} \\ \partial_\mu \delta_R^{0r} \\ \partial_\mu \delta_L^{0r} \end{pmatrix} \\
 & + \frac{1}{2} \begin{pmatrix} \partial^\mu \phi_1^{0i} \\ \partial^\mu \phi_2^{0i} \\ \partial^\mu \delta_R^{0i} \\ \partial^\mu \delta_L^{0i} \end{pmatrix}^T \begin{pmatrix} 1 + \frac{A_i^{11}}{\Lambda^2} & \frac{A_i^{12}}{\Lambda^2} & \frac{A_i^{13}}{\Lambda^2} & \frac{A_i^{14}}{\Lambda^2} \\ \frac{A_i^{12}}{\Lambda^2} & 1 + \frac{A_i^{22}}{\Lambda^2} & \frac{A_i^{23}}{\Lambda^2} & \frac{A_i^{24}}{\Lambda^2} \\ \frac{A_i^{13}}{\Lambda^2} & \frac{A_i^{23}}{\Lambda^2} & 1 + \frac{A_i^{33}}{\Lambda^2} & \frac{A_i^{34}}{\Lambda^2} \\ \frac{A_i^{14}}{\Lambda^2} & \frac{A_i^{24}}{\Lambda^2} & \frac{A_i^{34}}{\Lambda^2} & 1 + \frac{A_i^{44}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \phi_1^{0i} \\ \partial_\mu \phi_2^{0i} \\ \partial_\mu \delta_R^{0i} \\ \partial_\mu \delta_L^{0i} \end{pmatrix} \tag{6.6}
 \end{aligned}$$

$\mathcal{O}_{\phi D}^{2(1)2(1)}$	$Tr[(\Phi^\dagger i\overleftrightarrow{D}_\mu \Phi)(\Phi^\dagger i\overleftrightarrow{D}^\mu \Phi)]$	$\mathcal{O}_{\phi D}^{(2)(1)21}$	$Tr[(D_\mu \Phi)^\dagger (D^\mu \Phi) \Phi^\dagger \Phi]$
$\mathcal{O}_{\phi D}^{(11)(22)}$	$Tr[D_\mu(\Phi \Phi)(D^\mu(\Phi \Phi))^\dagger]$	$\mathcal{O}_{\phi D}^{\{(2)(1)\}\{21\}}$	$Tr[(D_\mu \Phi)^\dagger (D^\mu \Phi)] Tr[\Phi^\dagger \Phi]$
$\mathcal{O}_{\phi D}^{2(1)1(1)}$	$Tr[\Phi^\dagger (D_\mu \Phi) \Phi (D^\mu \Phi)] + h.c.$	$\mathcal{O}_{\phi D}^{1(1)1(1)}$	$Tr[\Phi (D_\mu \Phi) \Phi (D^\mu \Phi)] + h.c.$
$\mathcal{O}_{\phi D}^{Ll(2)(1)}$	$Tr[(\Delta_L^\dagger \Delta_L)(D^\mu \Phi)^\dagger (D_\mu \Phi)]$	$\mathcal{O}_{\phi D}^{Rr(2)(1)}$	$Tr[(\Delta_R^\dagger \Delta_R)(D^\mu \Phi)^\dagger (D_\mu \Phi)]$
$\mathcal{O}_{\phi D}^{21(L)(l)}$	$Tr[(\Phi^\dagger \Phi)(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)]$	$\mathcal{O}_{\phi D}^{21(R)(r)}$	$Tr[(\Phi^\dagger \Phi)(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)]$
$\mathcal{O}_{\phi D}^{\{Ll\}\{(2)(1)\}}$	$Tr[\Delta_L^\dagger \Delta_L] Tr[(D_\mu \Phi)^\dagger (D^\mu \Phi)]$	$\mathcal{O}_{\phi D}^{\{Rr\}\{(2)(1)\}}$	$Tr[\Delta_R^\dagger \Delta_R] Tr[(D_\mu \Phi)^\dagger (D^\mu \Phi)]$
$\mathcal{O}_{\phi D}^{\{21\}\{(L)(l)\}}$	$Tr[\Phi^\dagger \Phi] Tr[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)]$	$\mathcal{O}_{\phi D}^{\{21\}\{(R)(r)\}}$	$Tr[\Phi^\dagger \Phi] Tr[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)]$
$\mathcal{O}_{\square}^{Rr(Rr)}$	$Tr[(\Delta_R^\dagger \Delta_R) \square (\Delta_R^\dagger \Delta_R)]$	$\mathcal{O}_{\square}^{Ll(Ll)}$	$Tr[(\Delta_L^\dagger \Delta_L) \square (\Delta_L^\dagger \Delta_L)]$
$\mathcal{O}_{\phi D}^{R(r)R(r)}$	$Tr[(\Delta_R^\dagger i\overleftrightarrow{D}_\mu \Delta_R)(\Delta_R^\dagger i\overleftrightarrow{D}^\mu \Delta_R)]$	$\mathcal{O}_{\phi D}^{L(l)L(l)}$	$Tr[(\Delta_L^\dagger i\overleftrightarrow{D}_\mu \Delta_L)(\Delta_L^\dagger i\overleftrightarrow{D}^\mu \Delta_L)]$
$\mathcal{O}_{\phi D}^{\{Rr\}\{(R)(r)\}}$	$Tr[\Delta_R^\dagger \Delta_R] Tr[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)]$	$\mathcal{O}_{\phi D}^{\{Ll\}\{(L)(l)\}}$	$Tr[\Delta_L^\dagger \Delta_L] Tr[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)]$
$\mathcal{O}_{\phi D}^{R(r)1(1)}$	$Tr[\Delta_R^\dagger (D_\mu \Delta_R) \Phi (D^\mu \Phi)] + h.c.$	$\mathcal{O}_{\phi D}^{L(l)1(1)}$	$Tr[\Delta_L^\dagger (D_\mu \Delta_L) \Phi (D^\mu \Phi)] + h.c.$
$\mathcal{O}_{\phi D}^{(R)11(r)}$	$Tr\{[(D_\mu \Delta_R)^\dagger \Phi] [\Phi (D^\mu \Delta_R)]\} + h.c.$	$\mathcal{O}_{\phi D}^{(L)11(l)}$	$Tr\{[(D_\mu \Delta_L)^\dagger \Phi] [\Phi (D^\mu \Delta_L)]\} + h.c.$
$\mathcal{O}_{\phi D}^{(R)11(l)}$	$Tr\{[(D_\mu \Delta_R)^\dagger \Phi] [\Phi (D^\mu \Delta_L)]\} + h.c.$	$\mathcal{O}_{\phi D}^{(L)11(r)}$	$Tr\{[(D_\mu \Delta_L)^\dagger \Phi] [\Phi (D^\mu \Delta_R)]\} + h.c.$
$\mathcal{O}_{\phi D}^{(2)rL(1)}$	$Tr\{[(D_\mu \Phi)^\dagger \Delta_R] [\Delta_L^\dagger (D^\mu \Phi)]\} + h.c.$	$\mathcal{O}_{\phi D}^{(L)12(r)}$	$Tr\{[(D_\mu \Delta_L)^\dagger \Phi] [\Phi^\dagger (D^\mu \Delta_R)]\} + h.c.$
$\mathcal{O}_{\phi D}^{(R)lL(r)}$	$Tr\{[(D_\mu \Delta_R)^\dagger \Delta_L] [\Delta_L^\dagger (D^\mu \Delta_R)]\}$	$\mathcal{O}_{\phi D}^{(L)rR(l)}$	$Tr\{[(D_\mu \Delta_L)^\dagger \Delta_R] [\Delta_R^\dagger (D^\mu \Delta_L)]\}$
$\mathcal{O}_{\phi D}^{(L)rL(r)}$	$Tr\{[(D_\mu \Delta_L)^\dagger \Delta_R] [\Delta_L^\dagger (D^\mu \Delta_R)]\} + h.c.$		

Table 24. MLRSM: $\phi^4 D^2$ [20 + 11×2 = 42] class of operators.

$$\begin{aligned}
 & + \begin{pmatrix} \partial^\mu \phi_1'^+ \\ \partial^\mu \phi_2'^+ \\ \partial^\mu \delta_R^+ \\ \partial^\mu \delta_L^+ \end{pmatrix}^\dagger \begin{pmatrix} 1 + \frac{A_{++}^{11}}{\Lambda^2} & \frac{A_{++}^{12}}{\Lambda^2} & \frac{A_{++}^{13}}{\Lambda^2} & \frac{A_{++}^{14}}{\Lambda^2} \\ \frac{A_{++}^{12}}{\Lambda^2} & 1 + \frac{A_{++}^{22}}{\Lambda^2} & \frac{A_{++}^{23}}{\Lambda^2} & \frac{A_{++}^{24}}{\Lambda^2} \\ \frac{A_{++}^{13}}{\Lambda^2} & \frac{A_{++}^{23}}{\Lambda^2} & 1 + \frac{A_{++}^{33}}{\Lambda^2} & \frac{A_{++}^{34}}{\Lambda^2} \\ \frac{A_{++}^{14}}{\Lambda^2} & \frac{A_{++}^{24}}{\Lambda^2} & \frac{A_{++}^{34}}{\Lambda^2} & 1 + \frac{A_{++}^{44}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \phi_1'^+ \\ \partial_\mu \phi_2'^+ \\ \partial_\mu \delta_R^+ \\ \partial_\mu \delta_L^+ \end{pmatrix} \\
 & + \begin{pmatrix} \partial^\mu \delta_R^{++} \\ \partial^\mu \delta_L^{++} \end{pmatrix}^\dagger \begin{pmatrix} 1 + \frac{A_{++}^{11}}{\Lambda^2} & \frac{A_{++}^{12}}{\Lambda^2} \\ \frac{A_{++}^{12}}{\Lambda^2} & 1 + \frac{A_{++}^{22}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial_\mu \delta_R^{++} \\ \partial_\mu \delta_L^{++} \end{pmatrix}.
 \end{aligned}$$

The components of the scalar mass matrices are given as:

$$\begin{aligned}
 A_r^{11} &= \frac{1}{2\kappa_+^2} \left[2\kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \right. \\
 &\quad + 2\kappa_1^4 \mathcal{C}_{\phi D}^{1(1)1(1)} + 2\kappa_2^4 \mathcal{C}_{\phi D}^{1(1)1(1)} + 4\kappa_1^4 \mathcal{C}_{\phi D}^{(11)(22)} + 4\kappa_2^4 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_1^4 \mathcal{C}_{\phi D}^{2(1)1(1)} + 2\kappa_2^4 \mathcal{C}_{\phi D}^{2(1)1(1)} \\
 &\quad \left. + \kappa_1^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_2^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_1^4 \mathcal{C}_{\phi D}^{(2)(1)21} + \kappa_2^4 \mathcal{C}_{\phi D}^{(2)(1)21} \right], \\
 A_r^{12} &= -\frac{\kappa_1 \kappa_2}{2\kappa_+^2} \left[-2\mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + \mathcal{C}_{\phi D}^{Ll(2)(1)} v_L^2 + \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 + \kappa_-^2 (2\mathcal{C}_{\phi D}^{1(1)1(1)} + 4\mathcal{C}_{\phi D}^{(11)(22)} + 2\mathcal{C}_{\phi D}^{2(1)1(1)} \right. \\
 &\quad \left. + \mathcal{C}_{\phi D}^{(2)(1)21} \right], \\
 A_r^{13} &= \frac{\kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R}{2\kappa_+},
 \end{aligned}$$

$\mathcal{O}_{\phi B}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi] B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi] B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\mathcal{O}_{\phi B}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi}] B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi}] B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\mathcal{O}_{\phi B}^{21}$	$Tr[\Phi^\dagger \Phi] B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\phi \tilde{B}}^{21}$	$Tr[\Phi^\dagger \Phi] B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\mathcal{O}_{\phi G}^{21}$	$Tr[\Phi^\dagger \Phi] G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{21}$	$Tr[\Phi^\dagger \Phi] G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$
$\mathcal{O}_{\phi G}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi}] G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi}] G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$
$\mathcal{O}_{\phi G}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi] G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\phi \tilde{G}}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi] G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$
$\mathcal{O}_{\phi W_L}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi} W_{L\mu\nu} W_L^{\mu\nu}]$	$\mathcal{O}_{\phi \tilde{W}_L}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi} W_{L\mu\nu} \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\phi W_L}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi W_{L\mu\nu} W_L^{\mu\nu}]$	$\mathcal{O}_{\phi \tilde{W}_L}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi W_{L\mu\nu} \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\phi W_L}^{21}$	$Tr[\Phi^\dagger \Phi W_{L\mu\nu} W_L^{\mu\nu}]$	$\mathcal{O}_{\phi \tilde{W}_L}^{21}$	$Tr[\Phi^\dagger \Phi W_{L\mu\nu} \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\phi W_R}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi} W_{R\mu\nu} W_R^{\mu\nu}]$	$\mathcal{O}_{\phi \tilde{W}_R}^{23}$	$Tr[\Phi^\dagger \tilde{\Phi} W_{R\mu\nu} \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\phi W_R}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi W_{R\mu\nu} W_R^{\mu\nu}]$	$\mathcal{O}_{\phi \tilde{W}_R}^{41}$	$Tr[\tilde{\Phi}^\dagger \Phi W_{R\mu\nu} \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\phi W_R}^{21}$	$Tr[\Phi^\dagger \Phi W_{R\mu\nu} W_R^{\mu\nu}]$	$\mathcal{O}_{\phi \tilde{W}_R}^{21}$	$Tr[\Phi^\dagger \Phi W_{R\mu\nu} \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\Delta B}^{Ll}$	$Tr[\Delta_L^\dagger \Delta_L] B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Delta \tilde{B}}^{Ll}$	$Tr[\Delta_L^\dagger \Delta_L] B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\mathcal{O}_{\Delta B}^{Rr}$	$Tr[\Delta_R^\dagger \Delta_R] B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\Delta \tilde{B}}^{Rr}$	$Tr[\Delta_R^\dagger \Delta_R] B_{\mu\nu} \tilde{B}^{\mu\nu}$
$\mathcal{O}_{\Delta G}^{Ll}$	$Tr[\Delta_L^\dagger \Delta_L] G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Delta \tilde{G}}^{Ll}$	$Tr[\Delta_L^\dagger \Delta_L] G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$
$\mathcal{O}_{\Delta G}^{Rr}$	$Tr[\Delta_R^\dagger \Delta_R] G_{\mu\nu}^a G^{a\mu\nu}$	$\mathcal{O}_{\Delta \tilde{G}}^{Rr}$	$Tr[\Delta_R^\dagger \Delta_R] G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$
$\mathcal{O}_{\phi W_L B}^{21}$	$Tr[\Phi^\dagger W_L^{\mu\nu} \Phi] B_{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}_L B}^{21}$	$Tr[\Phi^\dagger \tilde{W}_L^{\mu\nu} \Phi] B_{\mu\nu}$
$\mathcal{O}_{\phi W_R B}^{21}$	$Tr[\Phi^\dagger W_R^{\mu\nu} \Phi] B_{\mu\nu}$	$\mathcal{O}_{\phi \tilde{W}_R B}^{21}$	$Tr[\Phi^\dagger \tilde{W}_R^{\mu\nu} \Phi] B_{\mu\nu}$
$\mathcal{O}_{\phi W_L W_R}^{21}$	$Tr[\Phi^\dagger W_{L\mu\nu} \Phi W_R^{\mu\nu}]$	$\mathcal{O}_{\phi W_L \tilde{W}_R}^{21}$	$Tr[\Phi^\dagger W_{L\mu\nu} \Phi \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\phi W_L W_R}^{23}$	$Tr[\Phi^\dagger W_{L\mu\nu} \tilde{\Phi} W_R^{\mu\nu}]$	$\mathcal{O}_{\phi W_L \tilde{W}_R}^{23}$	$Tr[\Phi^\dagger W_{L\mu\nu} \tilde{\Phi} \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\phi W_L W_R}^{41}$	$Tr[\tilde{\Phi}^\dagger W_{L\mu\nu} \Phi W_R^{\mu\nu}]$	$\mathcal{O}_{\phi W_L \tilde{W}_R}^{41}$	$Tr[\tilde{\Phi}^\dagger W_{L\mu\nu} \Phi \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\Delta W_L B}^{Ll}$	$Tr[\Delta_L^\dagger W_L^{\mu\nu} \Delta_L] B_{\mu\nu}$	$\mathcal{O}_{\Delta \tilde{W}_L B}^{Ll}$	$Tr[\Delta_L^\dagger \tilde{W}_L^{\mu\nu} \Delta_L] B_{\mu\nu}$
$\mathcal{O}_{\Delta W_R B}^{Rr}$	$Tr[\Delta_R^\dagger W_R^{\mu\nu} \Delta_R] B_{\mu\nu}$	$\mathcal{O}_{\Delta \tilde{W}_R B}^{Rr}$	$Tr[\Delta_R^\dagger \tilde{W}_R^{\mu\nu} \Delta_R] B_{\mu\nu}$
$\mathcal{O}_{\Delta W}^{Ll W_L W_L}$	$Tr[\Delta_L^\dagger \Delta_L W_{L\mu\nu} W_L^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{Ll W_L W_L}$	$Tr[\Delta_L^\dagger \Delta_L W_{L\mu\nu} \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{Rr W_L W_L}$	$Tr[\Delta_R^\dagger \Delta_R W_{L\mu\nu} W_L^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{Rr W_L W_L}$	$Tr[\Delta_R^\dagger \Delta_R W_{L\mu\nu} \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{Ll W_R W_R}$	$Tr[\Delta_L^\dagger \Delta_L W_{R\mu\nu} W_R^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{Ll W_R W_R}$	$Tr[\Delta_L^\dagger \Delta_L W_{R\mu\nu} \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{Rr W_R W_R}$	$Tr[\Delta_R^\dagger \Delta_R W_{R\mu\nu} W_R^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{Rr W_R W_R}$	$Tr[\Delta_R^\dagger \Delta_R W_{R\mu\nu} \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{L W_L l W_L}$	$Tr[\Delta_L W_{L\mu\nu} \Delta_L^\dagger W_L^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{L W_L l W_L}$	$Tr[\Delta_L W_{L\mu\nu} \Delta_L^\dagger \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{L W_R r W_L}$	$Tr[\Delta_R W_{R\mu\nu} \Delta_L^\dagger W_L^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{L W_R r W_L}$	$Tr[\Delta_R W_{R\mu\nu} \Delta_L^\dagger \tilde{W}_L^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{R W_L l W_R}$	$Tr[\Delta_L W_{L\mu\nu} \Delta_R^\dagger W_R^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{R W_L l W_R}$	$Tr[\Delta_L W_{L\mu\nu} \Delta_R^\dagger \tilde{W}_R^{\mu\nu}]$
$\mathcal{O}_{\Delta W}^{R W_R r W_R}$	$Tr[\Delta_R W_{R\mu\nu} \Delta_R^\dagger W_R^{\mu\nu}]$	$\mathcal{O}_{\Delta \tilde{W}}^{R W_R r W_R}$	$Tr[\Delta_R W_{R\mu\nu} \Delta_R^\dagger \tilde{W}_R^{\mu\nu}]$

Table 25. MLRSM: $\phi^2 X^2$ [62] class of operators. Note $W_{L,R}^{\mu\nu} = W_{L,R}^{I\mu\nu} \cdot \tau^I$, $I = 1, 2, 3$.

$$\begin{aligned}
 A_r^{14} &= \frac{\kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L}{2\kappa_+}, \\
 A_r^{22} &= \frac{1}{2\kappa_+^2} \left[2\kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \right. \\
 &\quad + 4\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + 8\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(11)(22)} + 4\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{2(1)1(1)} + \kappa_1^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \\
 &\quad \left. + \kappa_2^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(2)(1)21} \right], \\
 A_r^{23} &= -\frac{\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R}{2\kappa_+}, \\
 A_r^{24} &= -\frac{\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L}{2\kappa_+}, \\
 A_r^{33} &= \frac{1}{2} \left[\mathcal{C}_{\phi D}^{(R)lL(r)} v_L^2 + v_R^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} - 4\mathcal{C}_{\square}^{Rr(Rr)}) + \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} \right. \\
 &\quad \left. + 2\kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} \right], \\
 A_r^{34} &= \frac{1}{2} \left[\mathcal{C}_{\phi D}^{(L)rL(r)} v_L v_R + \kappa_2^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) \right], \\
 A_r^{44} &= \frac{1}{2} \left[v_L^2 (\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} - 4\mathcal{C}_{\square}^{Ll(Ll)}) + \mathcal{C}_{\phi D}^{(L)rR(l)} v_R^2 + \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{21(L)(l)} \right. \\
 &\quad \left. + 2\kappa_2^2 \mathcal{C}_{\phi D}^{(L)11(l)} \right], \\
 A_i^{11} &= \frac{1}{2\kappa_+^2} \left[2\kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \right. \\
 &\quad + \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} - 2\kappa_1^4 \mathcal{C}_{\phi D}^{1(1)1(1)} - 2\kappa_2^4 \mathcal{C}_{\phi D}^{1(1)1(1)} + 4\kappa_1^4 \mathcal{C}_{\phi D}^{(11)(22)} + 4\kappa_2^4 \mathcal{C}_{\phi D}^{(11)(22)} - 2\kappa_1^4 \mathcal{C}_{\phi D}^{2(1)1(1)} - 2\kappa_2^4 \mathcal{C}_{\phi D}^{2(1)1(1)} \\
 &\quad + \kappa_1^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_2^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_1^4 \mathcal{C}_{\phi D}^{(2)(1)21} + \kappa_2^4 \mathcal{C}_{\phi D}^{(2)(1)21} + 4\kappa_1^4 \mathcal{C}_{\phi D}^{2(1)2(1)} \\
 &\quad \left. + 4\kappa_2^4 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \\
 A_i^{12} &= \frac{\kappa_1 \kappa_2}{2\kappa_+^2} \left[2\mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R - \mathcal{C}_{\phi D}^{Ll(2)(1)} v_L^2 - \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 \right. \\
 &\quad \left. + \kappa_-^2 (2\mathcal{C}_{\phi D}^{1(1)1(1)} - 4\mathcal{C}_{\phi D}^{(11)(22)} + 2\mathcal{C}_{\phi D}^{2(1)1(1)} - \mathcal{C}_{\phi D}^{(2)(1)21} - 4\mathcal{C}_{\phi D}^{2(1)2(1)}) \right], \\
 A_i^{13} &= -\frac{\kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R}{2\kappa_+}, \\
 A_i^{14} &= -\frac{\kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L}{2\kappa_+}, \\
 A_i^{22} &= \frac{1}{2\kappa_+^2} \left[2\kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \right. \\
 &\quad - 4\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + 8\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(11)(22)} - 4\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{2(1)1(1)} + \kappa_1^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \\
 &\quad \left. + \kappa_2^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(2)(1)21} + 8\kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \\
 A_i^{23} &= \frac{\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R}{2\kappa_+}, \\
 A_i^{24} &= \frac{\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L}{2\kappa_+}, \\
 A_i^{33} &= \frac{1}{2} \left[\mathcal{C}_{\phi D}^{(R)lL(r)} v_L^2 + v_R^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 4\mathcal{C}_{\phi D}^{R(r)R(r)}) + \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} \right. \\
 &\quad \left. + 2\kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} \right], \\
 A_i^{34} &= \frac{1}{2} \left[\mathcal{C}_{\phi D}^{(L)rL(r)} v_L v_R + \kappa_2^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) \right],
 \end{aligned}$$

$$\begin{aligned}
 A_i^{44} &= \frac{1}{2} \left[v_L^2 (\mathcal{C}_{\phi D}^{Ll\{(L)(l)\}} + 4\mathcal{C}_{\phi D}^{L(l)L(l)}) + \mathcal{C}_{\phi D}^{(L)rR(l)} v_R^2 + \kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \kappa_+^2 \mathcal{C}_{\phi D}^{21(L)(l)} \right. \\
 &\quad \left. + 2\kappa_-^2 \mathcal{C}_{\phi D}^{(L)11(l)} \right], \\
 A_+^{11} &= \frac{1}{2\kappa_+^2} \left[2\kappa_-^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \right. \\
 &\quad + 4\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + \kappa_+^4 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_-^2 \kappa_+^3 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_-^3 \kappa_+ \mathcal{C}_{\phi D}^{(11)(22)} + \kappa_-^4 \mathcal{C}_{\phi D}^{(11)(22)} \\
 &\quad + 4\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{2(1)1(1)} + \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_-^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{(2)(1)21} \\
 &\quad \left. + 2\kappa_+^4 \mathcal{C}_{\phi D}^{2(1)2(1)} - 4\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{2(1)2(1)} + 2\kappa_-^4 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \\
 A_+^{12} &= \frac{\kappa_+ \kappa_-}{2\kappa_+^2} \left[2\mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + \mathcal{C}_{\phi D}^{Ll(2)(1)} v_L^2 + \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 + \kappa_-^2 (2\mathcal{C}_{\phi D}^{1(1)1(1)} + 2\mathcal{C}_{\phi D}^{2(1)1(1)} + \mathcal{C}_{\phi D}^{(2)(1)21} \right. \\
 &\quad \left. - 4\mathcal{C}_{\phi D}^{2(1)2(1)}) \right], \\
 A_+^{13} &= -\frac{\kappa_-^2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R}{2\sqrt{2}\kappa_+}, \\
 A_+^{14} &= -\frac{\kappa_-^2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L}{2\sqrt{2}\kappa_+}, \\
 A_+^{22} &= \frac{1}{2\kappa_+^2} \left[2\kappa_-^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \right. \\
 &\quad - 4\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + \kappa_+^4 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_-^2 \kappa_+^3 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{(11)(22)} + 2\kappa_-^3 \kappa_+ \mathcal{C}_{\phi D}^{(11)(22)} + \kappa_-^4 \mathcal{C}_{\phi D}^{(11)(22)} \\
 &\quad - 4\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{2(1)1(1)} + \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_-^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + \kappa_+^4 \mathcal{C}_{\phi D}^{(2)(1)21} + \kappa_-^4 \mathcal{C}_{\phi D}^{(2)(1)21} \\
 &\quad \left. + 8\kappa_-^2 \kappa_+^2 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \\
 A_+^{23} &= -\frac{\kappa_+ \kappa_- \mathcal{C}_{\phi D}^{R(r)1(1)} v_R}{2\sqrt{2}\kappa_+}, \\
 A_+^{24} &= -\frac{\kappa_+ \kappa_- \mathcal{C}_{\phi D}^{L(l)1(1)} v_L}{2\sqrt{2}\kappa_+}, \\
 A_+^{33} &= \frac{1}{4} \left[\mathcal{C}_{\phi D}^{(R)lL(r)} v_L^2 + v_R^2 (2\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 2\mathcal{C}_{\phi D}^{R(r)R(r)} - 2\mathcal{C}_{\phi D}^{Rr(Rr)}) + \kappa_+^2 (2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \mathcal{C}_{\phi D}^{21(R)(r)} \right. \\
 &\quad \left. + 2\mathcal{C}_{\phi D}^{(R)11(r)}) \right], \\
 A_+^{34} &= \frac{1}{4} \left[\mathcal{C}_{\phi D}^{(L)rL(r)} v_L v_R + \kappa_+^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) \right], \\
 A_+^{44} &= \frac{1}{4} \left[v_L^2 (2\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + 2\mathcal{C}_{\phi D}^{L(l)L(l)} - 2\mathcal{C}_{\phi D}^{Ll(Ll)}) + \mathcal{C}_{\phi D}^{(L)rR(l)} v_R^2 + \kappa_+^2 (2\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \mathcal{C}_{\phi D}^{21(L)(l)} \right. \\
 &\quad \left. + 2\mathcal{C}_{\phi D}^{(L)11(l)}) \right], \\
 A_{++}^{11} &= \frac{1}{2} \left[\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{21(R)(r)} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{(R)11(r)} + v_R^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} \right], \\
 A_{++}^{12} &= \frac{1}{2} \kappa_+^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}), \\
 A_{++}^{22} &= \frac{1}{2} \left[\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \kappa_-^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{(L)11(l)} + v_L^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} \right].
 \end{aligned}$$

In order to reduce the scalar kinetic terms in their canonical forms, we have redefined the scalar fields as:

$$\begin{aligned}
 \phi_1^{0r} &\rightarrow \left(1 - \frac{A_r^{11}}{2\Lambda^2}\right) \phi_1^{0r} - \left(\frac{A_r^{12}}{2\Lambda^2}\right) \phi_2^{0r} - \left(\frac{A_r^{13}}{2\Lambda^2}\right) \delta_R^{0r} - \left(\frac{A_r^{14}}{2\Lambda^2}\right) \delta_L^{0r}, \\
 \phi_2^{0r} &\rightarrow \left(1 - \frac{A_r^{22}}{2\Lambda^2}\right) \phi_2^{0r} - \left(\frac{A_r^{12}}{2\Lambda^2}\right) \phi_1^{0r} - \left(\frac{A_r^{23}}{2\Lambda^2}\right) \delta_R^{0r} - \left(\frac{A_r^{24}}{2\Lambda^2}\right) \delta_L^{0r},
 \end{aligned} \tag{6.7}$$

$$\begin{aligned}
 \delta_R^{0r} &\rightarrow \left(1 - \frac{A_r^{33}}{2\Lambda^2}\right) \delta_R^{0r} - \left(\frac{A_r^{13}}{2\Lambda^2}\right) \phi_1^{0r} - \left(\frac{A_r^{23}}{2\Lambda^2}\right) \phi_2^{0r} - \left(\frac{A_r^{34}}{2\Lambda^2}\right) \delta_L^{0r}, \\
 \delta_L^{0r} &\rightarrow \left(1 - \frac{A_r^{44}}{2\Lambda^2}\right) \delta_L^{0r} - \left(\frac{A_r^{14}}{2\Lambda^2}\right) \phi_1^{0r} - \left(\frac{A_r^{24}}{2\Lambda^2}\right) \phi_2^{0r} - \left(\frac{A_r^{34}}{2\Lambda^2}\right) \delta_R^{0r}, \\
 \phi_1^{0i} &\rightarrow \left(1 - \frac{A_i^{11}}{2\Lambda^2}\right) \phi_1^{0i} - \left(\frac{A_i^{12}}{2\Lambda^2}\right) \phi_2^{0i} - \left(\frac{A_i^{13}}{2\Lambda^2}\right) \delta_R^{0i} - \left(\frac{A_i^{14}}{2\Lambda^2}\right) \delta_L^{0i}, \\
 \phi_2^{0i} &\rightarrow \left(1 - \frac{A_i^{22}}{2\Lambda^2}\right) \phi_2^{0i} - \left(\frac{A_i^{12}}{2\Lambda^2}\right) \phi_1^{0i} - \left(\frac{A_i^{23}}{2\Lambda^2}\right) \delta_R^{0i} - \left(\frac{A_i^{24}}{2\Lambda^2}\right) \delta_L^{0i}, \\
 \delta_R^{0i} &\rightarrow \left(1 - \frac{A_i^{33}}{2\Lambda^2}\right) \delta_R^{0i} - \left(\frac{A_i^{13}}{2\Lambda^2}\right) \phi_1^{0i} - \left(\frac{A_i^{23}}{2\Lambda^2}\right) \phi_2^{0i} - \left(\frac{A_i^{34}}{2\Lambda^2}\right) \delta_L^{0i}, \\
 \delta_L^{0i} &\rightarrow \left(1 - \frac{A_i^{44}}{2\Lambda^2}\right) \delta_L^{0i} - \left(\frac{A_i^{14}}{2\Lambda^2}\right) \phi_1^{0i} - \left(\frac{A_i^{24}}{2\Lambda^2}\right) \phi_2^{0i} - \left(\frac{A_i^{34}}{2\Lambda^2}\right) \delta_R^{0i},
 \end{aligned} \tag{6.8}$$

$$\begin{aligned}
 \phi_1'^{+} &\rightarrow \left(1 - \frac{A_+^{11}}{2\Lambda^2}\right) \phi_1'^{+} - \left(\frac{A_+^{12}}{2\Lambda^2}\right) \phi_2'^{+} - \left(\frac{A_+^{13}}{2\Lambda^2}\right) \delta_R^{+} - \left(\frac{A_+^{14}}{2\Lambda^2}\right) \delta_L^{+}, \\
 \phi_2'^{+} &\rightarrow \left(1 - \frac{A_+^{22}}{2\Lambda^2}\right) \phi_2'^{+} - \left(\frac{A_+^{12}}{2\Lambda^2}\right) \phi_1'^{+} - \left(\frac{A_+^{23}}{2\Lambda^2}\right) \delta_R^{+} - \left(\frac{A_+^{24}}{2\Lambda^2}\right) \delta_L^{+}, \\
 \delta_R^{+} &\rightarrow \left(1 - \frac{A_+^{33}}{2\Lambda^2}\right) \delta_R^{+} - \left(\frac{A_+^{13}}{2\Lambda^2}\right) \phi_1'^{+} - \left(\frac{A_+^{23}}{2\Lambda^2}\right) \phi_2'^{+} - \left(\frac{A_+^{34}}{2\Lambda^2}\right) \delta_L^{+}, \\
 \delta_L^{+} &\rightarrow \left(1 - \frac{A_+^{44}}{2\Lambda^2}\right) \delta_L^{+} - \left(\frac{A_+^{14}}{2\Lambda^2}\right) \phi_1'^{+} - \left(\frac{A_+^{24}}{2\Lambda^2}\right) \phi_2'^{+} - \left(\frac{A_+^{34}}{2\Lambda^2}\right) \delta_R^{+},
 \end{aligned} \tag{6.9}$$

$$\begin{aligned}
 \delta_R^{++} &\rightarrow \left(1 - \frac{A_{++}^{11}}{2\Lambda^2}\right) \delta_R^{++} - \left(\frac{A_{++}^{12}}{2\Lambda^2}\right) \delta_L^{++}, \\
 \delta_L^{++} &\rightarrow \left(1 - \frac{A_{++}^{22}}{2\Lambda^2}\right) \delta_L^{++} - \left(\frac{A_{++}^{12}}{2\Lambda^2}\right) \delta_R^{++}.
 \end{aligned} \tag{6.10}$$

These redefined scalar fields modify the scalar mass matrices. There will be further contributions to the scalar masses from the ϕ^6 operators. Encapsulating all these contributions, the total scalar mass matrices can be expressed as:

$$\mathcal{L}_{M_H}^{(4)+(6)} = \frac{1}{2} \begin{bmatrix} \phi_1^{0r} \\ \phi_2^{0r} \\ \delta_R^{0r} \\ \delta_L^{0r} \end{bmatrix}^T \mathcal{M}_r^2 \begin{bmatrix} \phi_1^{0r} \\ \phi_2^{0r} \\ \delta_R^{0r} \\ \delta_L^{0r} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \phi_1^{0i} \\ \phi_2^{0i} \\ \delta_R^{0i} \\ \delta_L^{0i} \end{bmatrix}^T \mathcal{M}_i^2 \begin{bmatrix} \phi_1^{0i} \\ \phi_2^{0i} \\ \delta_R^{0i} \\ \delta_L^{0i} \end{bmatrix} + \begin{bmatrix} \phi_1'^{+} \\ \phi_2'^{+} \\ \delta_R^{+} \\ \delta_L^{+} \end{bmatrix}^\dagger \mathcal{M}_+^2 \begin{bmatrix} \phi_1'^{+} \\ \phi_2'^{+} \\ \delta_R^{+} \\ \delta_L^{+} \end{bmatrix} + \begin{bmatrix} \delta_R^{++} \\ \delta_L^{++} \end{bmatrix}^\dagger \mathcal{M}_{++}^2 \begin{bmatrix} \delta_R^{++} \\ \delta_L^{++} \end{bmatrix}.$$

Here, $\mathcal{M}_r^2, \mathcal{M}_i^2$ and $\mathcal{M}_+^2$ are (4×4) matrices while $\mathcal{M}_{++}^2$ is (2×2) matrix. Elements of these mass matrices can be written as:

$$(\mathcal{M}_j)_{mn}^2 = \left(m_j^2 + (\Delta m_{j,\phi D}^2) + (\Delta m_{j,\phi^6}^2) \right)_{mn}; \quad j \equiv \{r, i, +, ++\}. \tag{6.11}$$

Here, we have provided the individual matrix elements:

CP-even scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{11} = & \frac{1}{\kappa_+^3 \Lambda^2} \left[-\kappa_1^2 (\kappa_2^2 (2\alpha_1 A_r^{14} v_L + \alpha_3 A_r^{14} v_L + 2\alpha_1 A_r^{13} v_R + \alpha_3 A_r^{13} v_R + 4\kappa_+ \lambda_1 A_r^{11} + 16\kappa_+ \lambda_2 A_r^{11}) \right. \\
 & + 8\kappa_+ \lambda_3 A_r^{11} + (2\rho_1 - \rho_3) v_L v_R (A_r^{13} v_L + A_r^{14} v_R)) - \kappa_2^2 (\kappa_2^2 ((\alpha_1 + \alpha_3) (A_r^{14} v_L + A_r^{13} v_R) \\
 & - 2\kappa_+ \lambda_4 A_r^{12} + 2\kappa_+ \lambda_1 A_r^{11}) + (2\rho_1 - \rho_3) v_L v_R (A_r^{13} v_L + A_r^{14} v_R)) + \kappa_1^4 (-(\alpha_1 (A_r^{14} v_L + A_r^{13} v_R) \\
 & + 2\kappa_+ \lambda_4 A_r^{12} + 2\kappa_+ \lambda_1 A_r^{11})) - 4\kappa_2 \kappa_1^3 (\alpha_2 A_r^{14} v_L + 2\kappa_+ \lambda_2 A_r^{12} + \kappa_+ \lambda_3 A_r^{12} + \alpha_2 A_r^{13} v_R + 2\kappa_+ \lambda_4 A_r^{11}) \\
 & \left. - 4\kappa_2^3 \kappa_1 (\alpha_2 A_r^{14} v_L - 2\kappa_+ \lambda_2 A_r^{12} - \kappa_+ \lambda_3 A_r^{12} + \alpha_2 A_r^{13} v_R + 2\kappa_+ \lambda_4 A_r^{11}) \right], \quad (6.12)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{12} = & \frac{1}{4\Lambda^2 \kappa_-^2 \kappa_+^3 \kappa_1^2} \left[-2\kappa_+ v_L v_R \kappa_2^4 (\beta_3 \kappa_2^2 + v_L v_R (\rho_3 - 2\rho_1)) A_r^{12} - 2\kappa_1^7 \kappa_2 (4v_L \alpha_2 A_r^{24} + 4v_R \alpha_2 A_r^{23} \right. \\
 & + v_L \alpha_3 A_r^{14} + v_R \beta_3 A_r^{14} + v_R \alpha_3 A_r^{13} + v_L \beta_3 A_r^{13} + 8\kappa_+ \lambda_4 A_r^{12} + 8\kappa_+ (A_r^{22} + A_r^{11}) \lambda_2 \\
 & + 4\kappa_+ (A_r^{22} + A_r^{11}) \lambda_3) - \kappa_1^8 (2v_L \alpha_1 A_r^{24} + 2v_R \alpha_1 A_r^{23} + 4v_L \alpha_2 A_r^{14} + v_R \beta_1 A_r^{14} + 4v_R \alpha_2 A_r^{13} \\
 & + v_L \beta_1 A_r^{13} + 4\kappa_+ \lambda_1 A_r^{12} + 8\kappa_+ \lambda_2 A_r^{12} + 4\kappa_+ \lambda_3 A_r^{12} + 4\kappa_+ (A_r^{22} + A_r^{11}) \lambda_4) + \kappa_1^3 (2\kappa_2^5 (4v_L \alpha_2 A_r^{24} \\
 & + 4v_R \alpha_2 A_r^{23} + v_L \alpha_3 A_r^{14} + v_R \beta_3 A_r^{14} + v_R \alpha_3 A_r^{13} + v_L \beta_3 A_r^{13} + 8\kappa_+ \lambda_4 A_r^{12} - 8\kappa_+ (A_r^{22} + A_r^{11}) \lambda_2 \\
 & - 4\kappa_+ (A_r^{22} + A_r^{11}) \lambda_3) - 4\kappa_+ A_r^{12} v_L v_R \beta_1 \kappa_2^3) + \kappa_1^6 (-\kappa_+ v_R^2 \alpha_3 A_r^{12} + \kappa_2^2 (-2v_L \alpha_1 A_r^{24} - 2v_L \alpha_3 A_r^{24} \\
 & - 2v_R \alpha_1 A_r^{23} - 2v_R \alpha_3 A_r^{23} + 4v_L \alpha_2 A_r^{14} - v_R \beta_1 A_r^{14} + 4v_R \alpha_2 A_r^{13} - v_L \beta_1 A_r^{13} - 4\kappa_+ \lambda_1 A_r^{12} - 8\kappa_+ \lambda_2 A_r^{12} \\
 & - 4\kappa_+ \lambda_3 A_r^{12} + 4\kappa_+ (A_r^{22} + A_r^{11}) \lambda_4) - v_L^2 (2v_R (2\rho_1 - \rho_3) A_r^{23} + \kappa_+ \alpha_3 A_r^{12}) - 2v_L v_R (v_R (2\rho_1 - \rho_3) A_r^{24} \\
 & + \kappa_+ \beta_3 A_r^{12})) + 2\kappa_1^5 ((- (v_R A_r^{14} + v_L A_r^{13}) \beta_3 + 16\kappa_+ (A_r^{22} + A_r^{11}) \lambda_2 + 8\kappa_+ (A_r^{22} + A_r^{11}) \lambda_3) \kappa_2^3 \\
 & + v_L v_R \kappa_2 ((v_R A_r^{14} + v_L A_r^{13}) (2\rho_1 - \rho_3) - \kappa_+ A_r^{12} \beta_1)) + \kappa_1^4 (-2\kappa_+ (\alpha_3 v_L^2 + 3v_R \beta_3 v_L + v_R^2 \alpha_3) \kappa_2^2 A_r^{12} \\
 & + 2\kappa_+ v_L^2 v_R^2 (2\rho_1 - \rho_3) A_r^{12} + \kappa_2^2 (2v_L \alpha_1 A_r^{24} + 2v_R \alpha_1 A_r^{23} + 4v_L \alpha_2 A_r^{14} + v_R \beta_1 A_r^{14} + 4v_R \alpha_2 A_r^{13} \\
 & + v_L \beta_1 A_r^{13} + 4\kappa_+ \lambda_1 A_r^{12} + 8\kappa_+ \lambda_2 A_r^{12} + 4\kappa_+ \lambda_3 A_r^{12} + 4\kappa_+ (A_r^{22} + A_r^{11}) \lambda_4) + 2\kappa_1 \kappa_2^5 (v_R \beta_3 \kappa_2^2 A_r^{14} \\
 & + v_L^2 v_R (\rho_3 - 2\rho_1) A_r^{13} + v_L (v_R^2 (\rho_3 - 2\rho_1) A_r^{14} + \beta_3 \kappa_2^2 A_r^{13} - \kappa_+ v_R \beta_1 A_r^{12})) + \kappa_1^2 (4\kappa_+ v_L^2 v_R^2 \kappa_2^2 (2\rho_1 - \rho_3) A_r^{12} \\
 & + \kappa_2^6 (2v_L \alpha_1 A_r^{24} + 2v_L \alpha_3 A_r^{24} + 2v_R \alpha_1 A_r^{23} + 2v_R \alpha_3 A_r^{23} - 4v_L \alpha_2 A_r^{14} + v_R \beta_1 A_r^{14} - 4v_R \alpha_2 A_r^{13} + v_L \beta_1 A_r^{13} \\
 & + 4\kappa_+ \lambda_1 A_r^{12} + 8\kappa_+ \lambda_2 A_r^{12} + 4\kappa_+ \lambda_3 A_r^{12} - 4\kappa_+ (A_r^{22} + A_r^{11}) \lambda_4) - \kappa_2^4 (\kappa_+ v_R^2 \alpha_3 A_r^{12} \\
 & + v_L^2 (2v_R (\rho_3 - 2\rho_1) A_r^{23} + \kappa_+ \alpha_3 A_r^{12}) + 2v_L v_R (v_R (\rho_3 - 2\rho_1) A_r^{24} + 3\kappa_+ \beta_3 A_r^{12}))) \Big], \quad (6.13)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{13} = & \frac{1}{4\kappa_+^3 \kappa_1 \Lambda^2} \\
 & \times \left[-\kappa_+^2 v_R (4\kappa_1 \rho_1 A_r^{33} v_L^2 - 2\kappa_1 \rho_3 A_r^{33} v_L^2 + 2\kappa_+ \kappa_1 \rho_1 A_r^{14} v_L + \kappa_+ \kappa_1 \rho_3 A_r^{14} v_L - 4\kappa_2 \rho_1 A_r^{12} v_L^2 \right. \\
 & + 2\kappa_2 \rho_3 A_r^{12} v_L^2 + 2\alpha_3 \kappa_1 \kappa_2^2 A_r^{33} + 2\alpha_3 \kappa_1^2 \kappa_2 A_r^{12} + 2\alpha_1 \kappa_+^2 \kappa_1 (A_r^{33} + A_r^{11}) \\
 & + 4\alpha_2 \kappa_1 (\kappa_1^2 A_r^{12} - \kappa_2^2 A_r^{12} + 2\kappa_1 \kappa_2 (A_r^{33} + A_r^{11})) + 4\kappa_1 \rho_1 A_r^{11} v_L^2 - 2\kappa_1 \rho_3 A_r^{11} v_L^2 + 2\alpha_3 \kappa_1 \kappa_2^2 A_r^{11}) \\
 & - 2\kappa_+^2 \kappa_1 v_R^2 (2\rho_1 (A_r^{34} v_L + \kappa_+ A_r^{13}) - \rho_3 A_r^{34} v_L) - \kappa_+^2 v_L (2\kappa_1 (\alpha_1 \kappa_+^2 + \kappa_2 (4\alpha_2 \kappa_1 + \alpha_3 \kappa_2)) A_r^{34} + \beta_1 \kappa_+^2 \kappa_1 A_r^{12} \\
 & + 2\beta_3 \kappa_+^2 \kappa_2 A_r^{12}) + \kappa_+^3 \kappa_1 (2\rho_1 - \rho_3) A_r^{13} v_L^2 - 4\kappa_+ \kappa_1 (2\kappa_1^2 \kappa_2^2 (\lambda_1 + 4\lambda_2 + 2\lambda_3) A_r^{13} \\
 & + 2\kappa_1 \kappa_2^3 (2\lambda_4 A_r^{13} - (2\lambda_2 + \lambda_3) A_r^{23}) + 2\kappa_1^3 \kappa_2 ((2\lambda_2 + \lambda_3) A_r^{23} + 2\lambda_4 A_r^{13}) + \kappa_2^4 (\lambda_1 A_r^{13} - \lambda_4 A_r^{23}) \\
 & \left. + \kappa_1^4 (\lambda_4 A_r^{23} + \lambda_1 A_r^{13})) \right], \quad (6.14)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{14} = & \frac{1}{4\kappa_+^3 \kappa_1 \Lambda^2} \\
 & \times \left[-\kappa_+^2 v_L (2\alpha_3 \kappa_1 \kappa_2^2 A_r^{44} + 2\alpha_3 \kappa_1^2 \kappa_2 A_r^{12} + 4\kappa_1 \rho_1 A_r^{44} v_R^2 - 2\kappa_1 \rho_3 A_r^{44} v_R^2 + 2\kappa_+ \kappa_1 \rho_1 A_r^{13} v_R \right. \\
 & + \kappa_+ \kappa_1 \rho_3 A_r^{13} v_R - 4\kappa_2 \rho_1 A_r^{12} v_R^2 + 2\kappa_2 \rho_3 A_r^{12} v_R^2 + 2\alpha_1 \kappa_+^2 \kappa_1 (A_r^{44} + A_r^{11}) + 4\alpha_2 \kappa_1 (\kappa_1^2 A_r^{12} - \kappa_2^2 A_r^{12} \\
 & + 2\kappa_1 \kappa_2 (A_r^{44} + A_r^{11})) + 4\kappa_1 \rho_1 A_r^{11} v_R^2 - 2\kappa_1 \rho_3 A_r^{11} v_R^2 + 2\alpha_3 \kappa_1 \kappa_2^2 A_r^{11}) - 2\kappa_+^2 \kappa_1 v_L^2 (2\rho_1 (\kappa_+ A_r^{14} + A_r^{34} v_R) \\
 & - \rho_3 A_r^{34} v_R) - 4\kappa_+ \kappa_1 (2\kappa_1^2 \kappa_2^2 (\lambda_1 + 4\lambda_2 + 2\lambda_3) A_r^{14} + 2\kappa_1 \kappa_2^3 (2\lambda_4 A_r^{14} - (2\lambda_2 + \lambda_3) A_r^{24}) \\
 & + 2\kappa_1^3 \kappa_2 ((2\lambda_2 + \lambda_3) A_r^{24} + 2\lambda_4 A_r^{14}) + \kappa_2^4 (\lambda_1 A_r^{14} - \lambda_4 A_r^{24}) + \kappa_1^4 (\lambda_4 A_r^{24} + \lambda_1 A_r^{14})) \\
 & \left. - \kappa_+^2 v_R (2\kappa_1 (\alpha_1 \kappa_+^2 + \kappa_2 (4\alpha_2 \kappa_1 + \alpha_3 \kappa_2)) A_r^{34} + \beta_1 \kappa_+^2 \kappa_1 A_r^{12} + 2\beta_3 \kappa_+^2 \kappa_2 A_r^{12}) + \kappa_+^3 \kappa_1 (2\rho_1 - \rho_3) A_r^{14} v_R^2 \right], \quad (6.15)
 \end{aligned}$$

$$(\Delta m_{r,\phi D}^2)_{22} = \frac{1}{2\Lambda^2 \kappa_-^2 \kappa_+^3 \kappa_1^2} \times \left[-2\kappa_+ v_L v_R \kappa_2^4 (\beta_3 \kappa_2^2 + v_L v_R (\rho_3 - 2\rho_1)) A_r^{22} - 2\kappa_1^7 \kappa_2 (v_L \alpha_3 A_r^{24} + v_R \beta_3 A_r^{24} + v_R \alpha_3 A_r^{23} + v_L \beta_3 A_r^{23} - 8\kappa_+ \lambda_2 A_r^{12} - 4\kappa_+ \lambda_3 A_r^{12}) - 4\kappa_+ A_r^{22} v_L v_R \beta_1 \kappa_2^3 - \kappa_1^8 (4v_L \alpha_2 A_r^{24} + v_R \beta_1 A_r^{24} + 4v_R \alpha_2 A_r^{23} + v_L \beta_1 A_r^{23} + 8\kappa_+ \lambda_2 A_r^{22} + 4\kappa_+ \lambda_3 A_r^{22} + 4\kappa_+ \lambda_4 A_r^{12}) + \kappa_1^6 (\kappa_2^2 (4v_L \alpha_2 A_r^{24} - v_R \beta_1 A_r^{24} + 4v_R \alpha_2 A_r^{23} - v_L \beta_1 A_r^{23} + 24\kappa_+ \lambda_2 A_r^{22} + 12\kappa_+ \lambda_3 A_r^{22} + 4\kappa_+ \lambda_4 A_r^{12}) - \kappa_+ A_r^{22} (\alpha_3 v_L^2 + 2v_R \beta_3 v_L + v_R^2 \alpha_3)) + 2\kappa_1^5 ((16\kappa_+ \lambda_2 A_r^{12} + 8\kappa_+ \lambda_3 A_r^{12} - (v_R A_r^{24} + v_L A_r^{23}) \beta_3) \kappa_2^3 + v_L v_R ((v_R A_r^{24} + v_L A_r^{23}) (2\rho_1 - \rho_3) - \kappa_+ A_r^{22} \beta_1) \kappa_2) + \kappa_1^4 (-2\kappa_+ (\alpha_3 v_L^2 + 3v_R \beta_3 v_L + v_R^2 \alpha_3) \kappa_2^2 A_r^{22} + 2\kappa_+ v_L^2 v_R^2 (2\rho_1 - \rho_3) A_r^{22} + \kappa_2^4 (4v_L \alpha_2 A_r^{24} + v_R \beta_1 A_r^{24} + 4v_R \alpha_2 A_r^{23} + v_L \beta_1 A_r^{23} - 24\kappa_+ \lambda_2 A_r^{22} - 12\kappa_+ \lambda_3 A_r^{22} + 4\kappa_+ \lambda_4 A_r^{12})) - \kappa_1^2 (\kappa_+ (\alpha_3 v_L^2 + 6v_R \beta_3 v_L + v_R^2 \alpha_3) \kappa_2^4 A_r^{22} + 4\kappa_+ v_L^2 v_R^2 \kappa_2^2 (\rho_3 - 2\rho_1) A_r^{22} + \kappa_2^6 (4v_L \alpha_2 A_r^{24} - v_R \beta_1 A_r^{24} + 4v_R \alpha_2 A_r^{23} - v_L \beta_1 A_r^{23} - 8\kappa_+ \lambda_2 A_r^{22} - 4\kappa_+ \lambda_3 A_r^{22} + 4\kappa_+ \lambda_4 A_r^{12})) + 2\kappa_1 \kappa_2^5 (v_R \beta_3 \kappa_2^2 A_r^{24} + v_L^2 v_R (\rho_3 - 2\rho_1) A_r^{23} + v_L (v_R^2 (\rho_3 - 2\rho_1) A_r^{24} + \beta_3 \kappa_2^2 A_r^{23} - \kappa_+ v_R \beta_1 A_r^{22})) \right], \quad (6.16)$$

$$(\Delta m_{r,\phi D}^2)_{23} = \frac{1}{4\Lambda^2 \kappa_-^2 \kappa_+^3 \kappa_1^2} \left[-v_R^2 (-2\kappa_-^2 v_L \kappa_1 \kappa_2 (2\rho_1 - \rho_3) A_r^{34} + \kappa_+^3 \alpha_3 \kappa_1^2 A_r^{23} + 4\kappa_-^2 \kappa_+ \kappa_1^2 \rho_1 A_r^{23} - 2\kappa_+^3 v_L^2 (2\rho_1 - \rho_3) A_r^{23}) \kappa_+^2 - v_R (\beta_1 \kappa_1^6 A_r^{34} - 2\beta_3 \kappa_1 \kappa_2^5 A_r^{34} - \beta_1 \kappa_1^2 \kappa_2^4 A_r^{34} + 2\beta_3 \kappa_1^5 \kappa_2 A_r^{34} - 2\alpha_3 \kappa_1^3 \kappa_2^3 A_r^{33} + 2\alpha_3 \kappa_1^5 \kappa_2 A_r^{33} + 4v_L^2 \kappa_1 \kappa_2^3 \rho_1 A_r^{33} - 4v_L^2 \kappa_1^3 \kappa_2 \rho_1 A_r^{33} - 2v_L^2 \kappa_1 \kappa_2^3 \rho_3 A_r^{33} + 2v_L^2 \kappa_1^3 \kappa_2 \rho_3 A_r^{33} + 2\kappa_+ v_L \kappa_1^4 \rho_1 A_r^{24} - 2\kappa_+ v_L \kappa_1^2 \kappa_2^2 \rho_1 A_r^{24} + \kappa_+ v_L \kappa_1^4 \rho_3 A_r^{24} - \kappa_+ v_L \kappa_1^2 \kappa_2^2 \rho_3 A_r^{24} + 2\kappa_+ v_L \beta_3 \kappa_1^4 A_r^{23} + 2\kappa_+ v_L \beta_3 \kappa_2^4 A_r^{23} + 2\kappa_+ v_L \beta_1 \kappa_1 \kappa_2^3 A_r^{23} + 4\kappa_+ v_L \beta_3 \kappa_1^2 \kappa_2^2 A_r^{23} + 2\kappa_+ v_L \beta_1 \kappa_1^3 \kappa_2 A_r^{23} - 2\alpha_3 \kappa_1^3 \kappa_2^3 A_r^{22} + 2\alpha_3 \kappa_1^5 \kappa_2 A_r^{22} + 4v_L^2 \kappa_1 \kappa_2^3 \rho_1 A_r^{22} - 4v_L^2 \kappa_1^3 \kappa_2 \rho_1 A_r^{22} - 2v_L^2 \kappa_1 \kappa_2^3 \rho_3 A_r^{22} + 2v_L^2 \kappa_1^3 \kappa_2 \rho_3 A_r^{22} - 2\alpha_3 \kappa_1^2 \kappa_2^4 A_r^{12} + 2\alpha_3 \kappa_1^4 \kappa_2^2 A_r^{12} + 2\alpha_1 \kappa_1^2 (\kappa_1^4 - \kappa_2^4) A_r^{12} + 4v_L^2 \kappa_1^4 \rho_1 A_r^{12} - 4v_L^2 \kappa_1^2 \kappa_2^2 \rho_1 A_r^{12} - 2v_L^2 \kappa_1^4 \rho_3 A_r^{12} + 2v_L^2 \kappa_1^2 \kappa_2^2 \rho_3 A_r^{12} + 4\kappa_-^2 \alpha_2 \kappa_1^2 (2\kappa_1 \kappa_2 A_r^{12} + (A_r^{33} + A_r^{22}) \kappa_1^2 - (A_r^{33} + A_r^{22}) \kappa_2^2)) \kappa_+^2 - \kappa_1 ((4\kappa_+ (2\lambda_2 + \lambda_3) A_r^{23} + 4\kappa_+ \lambda_4 A_r^{13} + v_L (4\alpha_2 A_r^{34} + (A_r^{33} + A_r^{22}) \beta_1)) \kappa_1^7 + 2\kappa_2 (8\kappa_+ \lambda_2 A_r^{13} + 4\kappa_+ \lambda_3 A_r^{13} + v_L (\alpha_3 A_r^{34} + (A_r^{33} + A_r^{22}) \beta_3)) \kappa_1^6 + \kappa_1^5 (\kappa_+ v_L^2 (\alpha_3 - 2\rho_1 + \rho_3) A_r^{23} + \kappa_2^2 (-12\kappa_+ (2\lambda_2 + \lambda_3) A_r^{23} - 4\kappa_+ \lambda_4 A_r^{13} + v_L ((A_r^{33} + A_r^{22}) \beta_1 - 4A_r^{34} \alpha_2))) + 2\kappa_2^3 (-16\kappa_+ \lambda_2 A_r^{13} - 8\kappa_+ \lambda_3 A_r^{13} + (A_r^{33} + A_r^{22}) v_L \beta_3) \kappa_1^4 - (\kappa_2^4 (-12\kappa_+ (2\lambda_2 + \lambda_3) A_r^{23} + 4\kappa_+ \lambda_4 A_r^{13} + v_L (4\alpha_2 A_r^{34} + (A_r^{33} + A_r^{22}) \beta_1)) - 2\kappa_+ A_r^{23} v_L^2 \alpha_3 \kappa_2^2) \kappa_1^3 + 2\kappa_2^5 (8\kappa_+ \lambda_2 A_r^{13} + 4\kappa_+ \lambda_3 A_r^{13} - v_L (\alpha_3 A_r^{34} + (A_r^{33} + A_r^{22}) \beta_3)) \kappa_1^2 + \kappa_2^4 (\kappa_+ v_L^2 (\alpha_3 + 2\rho_1 - \rho_3) A_r^{23} + \kappa_2^2 (-4\kappa_+ (2\lambda_2 + \lambda_3) A_r^{23} + 4\kappa_+ \lambda_4 A_r^{13} - v_L ((A_r^{33} + A_r^{22}) \beta_1 - 4A_r^{34} \alpha_2))) \kappa_1 - 2(A_r^{33} + A_r^{22}) v_L \beta_3 \kappa_2^7 \right], \quad (6.17)$$

$$(\Delta m_{r,\phi D}^2)_{24} = \frac{1}{4\Lambda^2 \kappa_-^2 \kappa_+^3 \kappa_1^2} \left[-v_L^2 (-2\kappa_-^2 v_R \kappa_1 \kappa_2 (2\rho_1 - \rho_3) A_r^{34} + \kappa_+^3 \alpha_3 \kappa_1^2 A_r^{24} + 4\kappa_-^2 \kappa_+ \kappa_1^2 \rho_1 A_r^{24} - 2\kappa_+^3 v_R^2 (2\rho_1 - \rho_3) A_r^{24}) \kappa_+^2 - v_L (-2\alpha_3 \kappa_1^3 \kappa_2^3 A_r^{44} + 2\alpha_3 \kappa_1^5 \kappa_2 A_r^{44} + 4v_R^2 \kappa_1 \kappa_2^3 \rho_1 A_r^{44} - 4v_R^2 \kappa_1^3 \kappa_2 \rho_1 A_r^{44} - 2v_R^2 \kappa_1 \kappa_2^3 \rho_3 A_r^{44} + 2v_R^2 \kappa_1^3 \kappa_2 \rho_3 A_r^{44} + \beta_1 \kappa_1^6 A_r^{34} - 2\beta_3 \kappa_1 \kappa_2^5 A_r^{34} - \beta_1 \kappa_1^2 \kappa_2^4 A_r^{34} + 2\beta_3 \kappa_1^5 \kappa_2 A_r^{34} + 2\kappa_+ v_R \beta_3 \kappa_1^4 A_r^{24} + 2\kappa_+ v_R \beta_3 \kappa_2^4 A_r^{24} + 2\kappa_+ v_R \beta_1 \kappa_1 \kappa_2^3 A_r^{24} + 4\kappa_+ v_R \beta_3 \kappa_1^2 \kappa_2^2 A_r^{24} + 2\kappa_+ v_R \beta_1 \kappa_1^3 \kappa_2 A_r^{24} + 2\kappa_+ v_R \kappa_1^4 \rho_1 A_r^{23} - 2\kappa_+ v_R \kappa_1^2 \kappa_2^2 \rho_1 A_r^{23} + \kappa_+ v_R \kappa_1^4 \rho_3 A_r^{23} - \kappa_+ v_R \kappa_1^2 \kappa_2^2 \rho_3 A_r^{23} - 2\alpha_3 \kappa_1^3 \kappa_2^3 A_r^{22} + 2\alpha_3 \kappa_1^5 \kappa_2 A_r^{22} + 4v_R^2 \kappa_1 \kappa_2^3 \rho_1 A_r^{22} - 4v_R^2 \kappa_1^3 \kappa_2 \rho_1 A_r^{22} - 2v_R^2 \kappa_1 \kappa_2^3 \rho_3 A_r^{22} + 2v_R^2 \kappa_1^3 \kappa_2 \rho_3 A_r^{22} - 2\alpha_3 \kappa_1^2 \kappa_2^4 A_r^{12} + 2\alpha_3 \kappa_1^4 \kappa_2^2 A_r^{12} + 2\alpha_1 \kappa_1^2 (\kappa_1^4 - \kappa_2^4) A_r^{12} + 4v_R^2 \kappa_1^4 \rho_1 A_r^{12} - 4v_R^2 \kappa_1^2 \kappa_2^2 \rho_1 A_r^{12} - 2v_R^2 \kappa_1^4 \rho_3 A_r^{12} + 2v_R^2 \kappa_1^2 \kappa_2^2 \rho_3 A_r^{12} + 4\kappa_-^2 \alpha_2 \kappa_1^2 (2\kappa_1 \kappa_2 A_r^{12} + (A_r^{44} + A_r^{22}) \kappa_1^2 - (A_r^{44} + A_r^{22}) \kappa_2^2)) \kappa_+^2 - \kappa_1 ((4\kappa_+ (2\lambda_2 + \lambda_3) A_r^{24} + 4\kappa_+ \lambda_4 A_r^{14} + v_R (4\alpha_2 A_r^{34} + (A_r^{44} + A_r^{22}) \beta_1)) \kappa_1^7 + 2\kappa_2 (8\kappa_+ \lambda_2 A_r^{14} + 4\kappa_+ \lambda_3 A_r^{14} + v_R (\alpha_3 A_r^{34} + (A_r^{44} + A_r^{22}) \beta_3)) \kappa_1^6 + (\kappa_+ v_R^2 (\alpha_3 - 2\rho_1 + \rho_3) A_r^{24} + \kappa_2^2 (-12\kappa_+ (2\lambda_2 + \lambda_3) A_r^{24} - 4\kappa_+ \lambda_4 A_r^{14} + v_R ((A_r^{44} + A_r^{22}) \beta_1 - 4A_r^{34} \alpha_2))) \kappa_1^5 + 2\kappa_2^3 \kappa_1^4 (-16\kappa_+ \lambda_2 A_r^{14} - 8\kappa_+ \lambda_3 A_r^{14} + (A_r^{44} + A_r^{22}) v_R \beta_3) - (\kappa_2^4 (-12\kappa_+ (2\lambda_2 + \lambda_3) A_r^{24} + 4\kappa_+ \lambda_4 A_r^{14} + v_R (4\alpha_2 A_r^{34} + (A_r^{44} + A_r^{22}) \beta_1)) - 2\kappa_+ A_r^{24} v_R^2 \alpha_3 \kappa_2^2) \kappa_1^3 + 2\kappa_2^5 (8\kappa_+ \lambda_2 A_r^{14} + 4\kappa_+ \lambda_3 A_r^{14} - v_R (\alpha_3 A_r^{34} + (A_r^{44} + A_r^{22}) \beta_3)) \kappa_1^2$$

$$\begin{aligned}
 & +\kappa_+^4(\kappa_+v_R^2(\alpha_3+2\rho_1-\rho_3)A_r^{24}+\kappa_+^2(-4\kappa_+(2\lambda_2+\lambda_3)A_r^{24}+4\kappa_+\lambda_4A_r^{14} \\
 & -v_R((A_r^{44}+A_r^{22})\beta_1-4A_r^{34}\alpha_2)))\kappa_1-2(A_r^{44}+A_r^{22})v_R\beta_3\kappa_2^7], \quad (6.18)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{33} = & \frac{1}{2\kappa_+\kappa_1\Lambda^2} \left[-v_R(2\kappa_+\kappa_1\rho_1A_r^{34}v_L+\kappa_+\kappa_1\rho_3A_r^{34}v_L-4\kappa_2\rho_1A_r^{23}v_L^2+2\kappa_2\rho_3A_r^{23}v_L^2+4\kappa_1\rho_1A_r^{13}v_L^2 \right. \\
 & -2\kappa_1\rho_3A_r^{13}v_L^2+2\alpha_3\kappa_1^2\kappa_2A_r^{23}+2\alpha_3\kappa_1\kappa_2^2A_r^{13}+2\alpha_1\kappa_+^2\kappa_1A_r^{13}+4\alpha_2\kappa_1(\kappa_1^2A_r^{23}-\kappa_2^2A_r^{23}+2\kappa_1\kappa_2A_r^{13})) \\
 & \left. +v_L(\kappa_+\kappa_1(2\rho_1-\rho_3)A_r^{33}v_L-\beta_1\kappa_+^2\kappa_1A_r^{23}-2\beta_3\kappa_+^2\kappa_2A_r^{23})-4\kappa_+\kappa_1\rho_1A_r^{33}v_R^2 \right], \quad (6.19)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{34} = & \frac{1}{4\kappa_+\kappa_1\Lambda^2} \\
 & \times \left[-v_L(2\alpha_3\kappa_1^2\kappa_2A_r^{23}+2\alpha_3\kappa_1\kappa_2^2A_r^{13}+2\alpha_1\kappa_+^2\kappa_1A_r^{13}+4\alpha_2\kappa_1(\kappa_1^2A_r^{23}-\kappa_2^2A_r^{23}+2\kappa_1\kappa_2A_r^{13})) \right. \\
 & +\beta_1\kappa_1^3A_r^{24}+2\beta_3\kappa_2^3A_r^{24}+\beta_1\kappa_1\kappa_2^2A_r^{24}+2\beta_3\kappa_1^2\kappa_2A_r^{24}+2\kappa_+\kappa_1\rho_1A_r^{44}v_R+\kappa_+\kappa_1\rho_3A_r^{44}v_R+2\kappa_+\kappa_1\rho_1A_r^{33}v_R \\
 & +\kappa_+\kappa_1\rho_3A_r^{33}v_R-4\kappa_2\rho_1A_r^{23}v_R^2+2\kappa_2\rho_3A_r^{23}v_R^2+4\kappa_1\rho_1A_r^{13}v_R^2-2\kappa_1\rho_3A_r^{13}v_R^2) \\
 & +v_L^2(-(\kappa_+\kappa_1(2\rho_1+\rho_3)A_r^{34}+2(2\rho_1-\rho_3)v_R(\kappa_1A_r^{14}-\kappa_2A_r^{24}))) \\
 & -v_R(2\alpha_3\kappa_1^2\kappa_2A_r^{24}+2\alpha_3\kappa_1\kappa_2^2A_r^{14}+2\alpha_1\kappa_+^2\kappa_1A_r^{14}+4\alpha_2\kappa_1(\kappa_1^2A_r^{24}-\kappa_2^2A_r^{24}+2\kappa_1\kappa_2A_r^{14}))+\beta_1\kappa_1^3A_r^{23} \\
 & \left. +2\beta_3\kappa_2^3A_r^{23}+\beta_1\kappa_1\kappa_2^2A_r^{23}+2\beta_3\kappa_1^2\kappa_2A_r^{23}+2\kappa_+\kappa_1\rho_1A_r^{34}v_R+\kappa_+\kappa_1\rho_3A_r^{34}v_R) \right], \quad (6.20)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi D}^2)_{44} = & \frac{1}{2\kappa_+\kappa_1\Lambda^2} \\
 & \times \left[-v_L(2\alpha_3\kappa_1^2\kappa_2A_r^{24}+2\alpha_3\kappa_1\kappa_2^2A_r^{14}+2\alpha_1\kappa_+^2\kappa_1A_r^{14}+4\alpha_2\kappa_1(\kappa_1^2A_r^{24}-\kappa_2^2A_r^{24}+2\kappa_1\kappa_2A_r^{14})) \right. \\
 & +2\kappa_+\kappa_1\rho_1A_r^{34}v_R+\kappa_+\kappa_1\rho_3A_r^{34}v_R-4\kappa_2\rho_1A_r^{24}v_R^2+2\kappa_2\rho_3A_r^{24}v_R^2+4\kappa_1\rho_1A_r^{14}v_R^2-2\kappa_1\rho_3A_r^{14}v_R^2) \\
 & \left. -4\kappa_+\kappa_1\rho_1A_r^{44}v_L^2+v_R(-\beta_1\kappa_+^2\kappa_1A_r^{24}-2\beta_3\kappa_+^2\kappa_2A_r^{24}+\kappa_+\kappa_1(2\rho_1-\rho_3)A_r^{44}v_R) \right]. \quad (6.21)
 \end{aligned}$$

CP-odd scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{11} = & \frac{v_L v_R}{\kappa_+^3\kappa_1\Lambda^2} \left[(2\rho_1-\rho_3)v_L(2\kappa_+v_R(\kappa_1A_i^{11}-\kappa_2A_i^{12})-\kappa_+^2\kappa_1A_i^{13})+\kappa_+^2(\beta_1\kappa_+\kappa_1A_i^{12}+2\beta_3\kappa_+\kappa_2A_i^{12} \right. \\
 & \left. +\kappa_1(2\rho_1-\rho_3)A_i^{14}v_R) \right], \quad (6.22)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{12} = & \frac{1}{4\kappa_-^2\kappa_+^3\kappa_1\Lambda^2} \left[-v_L^2(\alpha_3\kappa_+^5\kappa_1^2A_i^{12}+2\kappa_1\kappa_+^2\kappa_-^2(2\rho_1-\rho_3)v_R(\kappa_1A_i^{23}-\kappa_2A_i^{13}) \right. \\
 & -2\kappa_+(2\rho_1-\rho_3)v_R^2(3\kappa_1^4A_i^{12}-\kappa_2^4A_i^{12}+2\kappa_1^2\kappa_2^2A_i^{12}+2\kappa_1\kappa_2^3(A_i^{22}+A_i^{11}))-2\kappa_1^3\kappa_2(A_i^{22}+A_i^{11})) \\
 & +\kappa_+^2v_L(-\kappa_1(\kappa_1^4-\kappa_2^4)A_i^{13}(\beta_1\kappa_1+2\beta_3\kappa_2)+2\kappa_+v_R(\beta_1\kappa_1(\kappa_2^3A_i^{12}-3\kappa_1^2\kappa_2A_i^{12}+\kappa_1^3(A_i^{22}+A_i^{11})) \\
 & -\kappa_1\kappa_2^2(A_i^{22}+A_i^{11}))+\beta_3(\kappa_1^4A_i^{12}+\kappa_2^4A_i^{12}-6\kappa_1^2\kappa_2^2A_i^{12}-2\kappa_1\kappa_2^3(A_i^{22}+A_i^{11}))+2\kappa_1^3\kappa_2(A_i^{22}+A_i^{11}))) \\
 & +2\kappa_-^2\kappa_1(2\rho_1-\rho_3)v_R^2(\kappa_1A_i^{24}-\kappa_2A_i^{14}))+\kappa_1(4\kappa_-^2\kappa_+^5\kappa_1(2\lambda_2-\lambda_3)A_i^{12}-\alpha_3\kappa_+^5\kappa_1A_i^{12}v_R^2 \\
 & \left. +\kappa_-^2\kappa_+^4A_i^{14}(\beta_1\kappa_1+2\beta_3\kappa_2)v_R) \right], \quad (6.23)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{13} = & \frac{v_L}{4\kappa_+^3\kappa_1\Lambda^2} \\
 & \times \left[(2\rho_1-\rho_3)v_L(\kappa_+\kappa_1A_i^{13}-2\kappa_+^2v_R(\kappa_1(A_i^{33}+A_i^{11})-\kappa_2A_i^{12}))+4\kappa_+v_R^2(\kappa_1A_i^{13}-\kappa_2A_i^{23}) \right. \\
 & -\kappa_+^2(\beta_1\kappa_1(\kappa_1^2A_i^{12}+\kappa_2^2A_i^{12}-2\kappa_+A_i^{23}v_R)+2\beta_3\kappa_2(\kappa_1^2A_i^{12}+\kappa_2^2A_i^{12}-2\kappa_+A_i^{23}v_R) \\
 & \left. +\kappa_1(2\rho_1-\rho_3)v_R(\kappa_+A_i^{14}-2A_i^{34}v_R)) \right], \quad (6.24)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi D}^2)_{14} = & \frac{v_R}{4\kappa_+^3\kappa_1\Lambda^2} \\
 & \times \left[\kappa_+^2v_L(\kappa_+(\kappa_1(2\beta_1A_i^{24}-2\rho_1A_i^{13}+\rho_3A_i^{13}))+4\beta_3\kappa_2A_i^{24}))+2(2\rho_1-\rho_3)v_R(\kappa_1(A_i^{44}+A_i^{11}) \right. \\
 & \left. -\kappa_2A_i^{12}))+2(2\rho_1-\rho_3)v_L^2(2\kappa_+v_R(\kappa_1A_i^{14}-\kappa_2A_i^{24}))+\kappa_+^2\kappa_1A_i^{34} \right]
 \end{aligned}$$

$$+\kappa_+^2(\beta_1\kappa_+^2\kappa_1A_i^{12}+2\beta_3\kappa_+^2\kappa_2A_i^{12}+\kappa_+\kappa_1(2\rho_1-\rho_3)A_i^{14}v_R)], \quad (6.25)$$

$$\begin{aligned} (\Delta m_{i,\phi D}^2)_{22} = & \frac{1}{2\kappa_-^2\kappa_+^3\kappa_1^2\Lambda^2} \left[-v_L^2(\alpha_3\kappa_+^5\kappa_1^2A_i^{22}-2\kappa_1\kappa_2(\kappa_1^4-\kappa_2^4)(2\rho_1-\rho_3)A_i^{23}v_R \right. \\ & -2\kappa_+(2\rho_1-\rho_3)v_R^2(\kappa_1^4A_i^{22}-\kappa_2^4A_i^{22}+4\kappa_1^2\kappa_2^2A_i^{22}+2\kappa_1\kappa_2^3A_i^{12}-2\kappa_1^3\kappa_2A_i^{12})) \\ & +\kappa_+^2v_L(-\kappa_1(\kappa_1^4-\kappa_2^4)A_i^{23}(\beta_1\kappa_1+2\beta_3\kappa_2)+2\kappa_+v_R(\beta_1\kappa_1(\kappa_2^3A_i^{22}-3\kappa_1^2\kappa_2A_i^{22}+\kappa_1^3A_i^{12}-\kappa_1\kappa_2^2A_i^{12}) \\ & +\beta_3(\kappa_1^4A_i^{22}+\kappa_2^4A_i^{22}-6\kappa_1^2\kappa_2^2A_i^{22}-2\kappa_1\kappa_2^3A_i^{12}+2\kappa_1^3\kappa_2A_i^{12}))-2\kappa_-^2\kappa_1\kappa_2(2\rho_1-\rho_3)A_i^{24}v_R^2) \\ & \left. +\kappa_1(4\kappa_-^2\kappa_+^5\kappa_1(2\lambda_2-\lambda_3)A_i^{22}-\alpha_3\kappa_+^5\kappa_1A_i^{22}v_R^2+\kappa_-^2\kappa_+^4A_i^{24}(\beta_1\kappa_1+2\beta_3\kappa_2)v_R) \right], \quad (6.26) \end{aligned}$$

$$\begin{aligned} (\Delta m_{i,\phi D}^2)_{23} = & \frac{v_L^2}{4\kappa_-^2\kappa_+^3\kappa_1^2\Lambda^2} \left[(-\kappa_+^3\kappa_1^2A_i^{23}(\alpha_3\kappa_+^2+\kappa_-^2(2\rho_1-\rho_3))-2\kappa_1\kappa_+^2\kappa_-^2(2\rho_1-\rho_3)v_R(\kappa_1A_i^{12} \right. \\ & -\kappa_2(A_i^{33}+A_i^{22}))+2\kappa_+(2\rho_1-\rho_3)v_R^2(\kappa_1^4A_i^{23}-\kappa_2^4A_i^{23}+4\kappa_1^2\kappa_2^2A_i^{23}+2\kappa_1\kappa_2^3A_i^{13}-2\kappa_1^3\kappa_2A_i^{13})) \\ & +\kappa_+^2v_L(-\kappa_1\kappa_+^2\kappa_-^2(A_i^{33}+A_i^{22})(\beta_1\kappa_1+2\beta_3\kappa_2)+\kappa_+v_R(2\beta_1\kappa_1(\kappa_2^3A_i^{23}-3\kappa_1^2\kappa_2A_i^{23}+\kappa_1^3A_i^{13} \\ & -\kappa_1\kappa_2^2A_i^{13}))+2\beta_3(\kappa_1^4A_i^{23}+\kappa_2^4A_i^{23}-6\kappa_1^2\kappa_2^2A_i^{23}-2\kappa_1\kappa_2^3A_i^{13}+2\kappa_1^3\kappa_2A_i^{13}))- \kappa_-^2\kappa_1^2(2\rho_1-\rho_3)A_i^{24}) \\ & -2\kappa_-^2\kappa_1\kappa_2(2\rho_1-\rho_3)A_i^{34}v_R^2)+\kappa_1(4\kappa_-^2\kappa_+^5\kappa_1(2\lambda_2-\lambda_3)A_i^{23}-\alpha_3\kappa_+^5\kappa_1A_i^{23}v_R^2 \\ & \left. +\kappa_-^2\kappa_+^4A_i^{34}(\beta_1\kappa_1+2\beta_3\kappa_2)v_R) \right], \quad (6.27) \end{aligned}$$

$$\begin{aligned} (\Delta m_{i,\phi D}^2)_{24} = & \frac{1}{4\kappa_-^2\kappa_+^3\kappa_1^2\Lambda^2} \left[-v_L^2(\alpha_3\kappa_+^5\kappa_1^2A_i^{24}-2\kappa_1\kappa_2\kappa_+^2\kappa_-^2A_i^{34}v_R(2\rho_1-\rho_3) \right. \\ & -2\kappa_+(2\rho_1-\rho_3)v_R^2(\kappa_1^4A_i^{24}-\kappa_2^4A_i^{24}+4\kappa_1^2\kappa_2^2A_i^{24}+2\kappa_1\kappa_2^3A_i^{14}-2\kappa_1^3\kappa_2A_i^{14})) \\ & +\kappa_+^2v_L(-\kappa_1\kappa_+^2\kappa_-^2A_i^{34}(\beta_1\kappa_1+2\beta_3\kappa_2)+\kappa_+v_R(2\beta_1\kappa_1(\kappa_2^3A_i^{24}-3\kappa_1^2\kappa_2A_i^{24}+\kappa_1^3A_i^{14}-\kappa_1\kappa_2^2A_i^{14}) \\ & +2\beta_3(\kappa_1^4A_i^{24}+\kappa_2^4A_i^{24}-6\kappa_1^2\kappa_2^2A_i^{24}-2\kappa_1\kappa_2^3A_i^{14}+2\kappa_1^3\kappa_2A_i^{14}))- \kappa_-^2\kappa_1^2(2\rho_1-\rho_3)A_i^{23}) \\ & +2\kappa_-^2\kappa_1(2\rho_1-\rho_3)v_R^2(\kappa_1A_i^{12}-\kappa_2(A_i^{44}+A_i^{22}))- \kappa_1\kappa_+^2(-4\kappa_+\kappa_1\kappa_+^2\kappa_-^2s(2\lambda_2-\lambda_3)A_i^{24} \\ & \left. +\kappa_+\kappa_1A_i^{24}v_R(\alpha_3\kappa_+^2+\kappa_-^2(2\rho_1-\rho_3))- \kappa_+^2\kappa_-^2(A_i^{44}+A_i^{22})(\beta_1\kappa_1+2\beta_3\kappa_2)v_R) \right], \quad (6.28) \end{aligned}$$

$$\begin{aligned} (\Delta m_{i,\phi D}^2)_{33} = & -\frac{v_L}{2\kappa_+\kappa_1\Lambda^2} \left[2\beta_3\kappa_2^3A_i^{23}+\beta_1\kappa_+^2\kappa_1A_i^{23}+2\beta_3\kappa_1^2\kappa_2A_i^{23}+(2\rho_1-\rho_3)v_L(2v_R(\kappa_1A_i^{13}-\kappa_2A_i^{23}) \right. \\ & \left. -\kappa_+\kappa_1A_i^{33})+2\kappa_+\kappa_1\rho_1A_i^{34}v_R-\kappa_+\kappa_1\rho_3A_i^{34}v_R \right], \quad (6.29) \end{aligned}$$

$$\begin{aligned} (\Delta m_{i,\phi D}^2)_{34} = & \frac{1}{4\kappa_+\kappa_1\Lambda^2} \left[v_L(-\kappa_+^2A_i^{24}(\beta_1\kappa_1+2\beta_3\kappa_2)-\kappa_+\kappa_1(2\rho_1-\rho_3)(A_i^{44}+A_i^{33})v_R \right. \\ & +2(2\rho_1-\rho_3)v_R^2(\kappa_1A_i^{13}-\kappa_2A_i^{23}))+ (2\rho_1-\rho_3)v_L^2(-(2v_R(\kappa_1A_i^{14}-\kappa_2A_i^{24}))+\kappa_+\kappa_1A_i^{34})) \\ & \left. +v_R(\beta_1\kappa_+^2\kappa_1A_i^{23}+2\beta_3\kappa_+^2\kappa_2A_i^{23}+\kappa_+\kappa_1(2\rho_1-\rho_3)A_i^{34}v_R) \right], \quad (6.30) \end{aligned}$$

$$\begin{aligned} (\Delta m_{i,\phi D}^2)_{44} = & \frac{v_R}{2\kappa_+\kappa_1\Lambda^2} \left[2\beta_3\kappa_2^3A_i^{24}+\beta_1\kappa_+^2\kappa_1A_i^{24}+2\beta_3\kappa_1^2\kappa_2A_i^{24}+(2\rho_1-\rho_3)v_L(2v_R(\kappa_1A_i^{14}-\kappa_2A_i^{24}) \right. \\ & \left. -\kappa_+\kappa_1A_i^{34})+2\kappa_+\kappa_1\rho_1A_i^{44}v_R-\kappa_+\kappa_1\rho_3A_i^{44}v_R \right]. \quad (6.31) \end{aligned}$$

Singly charged scalar mass matrix.

$$\begin{aligned} (\Delta m_{+, \phi D}^2)_{11} = & \frac{1}{4\kappa_+^3\kappa_1\kappa_-^2\Lambda^2} \\ & \times \left[-\kappa_+^2v_R(\sqrt{2}\kappa_+^2\kappa_-^2(\alpha_3A_+^{13}\kappa_1+A_+^{14}(\beta_1\kappa_1+2\beta_3\kappa_2)))-2\sqrt{2}A_+^{13}\kappa_-^2\kappa_1(2\rho_1-\rho_3)v_L^2 \right. \\ & +2\kappa_+(A_+^{12}\kappa_1^2-A_+^{12}\kappa_2^2+4A_+^{11}\kappa_1\kappa_2)(\beta_1\kappa_1+2\beta_3\kappa_2)v_L)-2v_R^2(\alpha_3A_+^{11}\kappa_+^5\kappa_1 \\ & -(2\rho_1-\rho_3)v_L(\sqrt{2}A_+^{14}\kappa_2\kappa_+^2\kappa_-^2+2\kappa_+(-A_+^{12}\kappa_2^3+A_+^{12}\kappa_1^2\kappa_2+A_+^{11}\kappa_1^3+3A_+^{11}\kappa_1\kappa_2^2)v_L)) \\ & \left. -2\kappa_1\kappa_2v_L(\sqrt{2}(\kappa_1^4-\kappa_2^4)(\alpha_3A_+^{14}\kappa_1+A_+^{13}\beta_1\kappa_1+2A_+^{13}\beta_3\kappa_2)+2\alpha_3\kappa_+\kappa_1(A_+^{12}\kappa_1^2-A_+^{12}\kappa_2^2+2A_+^{11}\kappa_1\kappa_2)v_L) \right], \quad (6.32) \end{aligned}$$

$$\begin{aligned} (\Delta m_{+, \phi D}^2)_{12} = & \frac{1}{8\kappa_+^3\kappa_1\kappa_-^2\Lambda^2} \\ & \times \left[-\kappa_+^2v_L(\sqrt{2}\kappa_-^2(\alpha_3\kappa_1(2A_+^{24}\kappa_1\kappa_2+A_+^{14}\kappa_1^2-A_+^{14}\kappa_2^2)+\beta_1\kappa_1(2A_+^{23}\kappa_1\kappa_2+A_+^{13}\kappa_1^2-A_+^{13}\kappa_2^2) \right. \end{aligned}$$

$$\begin{aligned}
 & +2\beta_3\kappa_2(2A_+^{23}\kappa_1\kappa_2+A_+^{13}\kappa_1^2-A_+^{13}\kappa_2^2)+2\kappa_+v_R(\beta_1\kappa_1+2\beta_3\kappa_2)(4A_+^{12}\kappa_1\kappa_2+(A_+^{22}+A_+^{11})\kappa_-^2) \\
 & -2\sqrt{2}\kappa_-^2(2\rho_1-\rho_3)(A_+^{24}\kappa_2+A_+^{14}\kappa_1)v_R^2-2v_L^2(\alpha_3\kappa_+\kappa_1(A_+^{12}\kappa_1^4+A_+^{12}\kappa_2^4+2A_+^{12}\kappa_1^2\kappa_2^2 \\
 & -2(A_+^{22}+A_+^{11})\kappa_1\kappa_2^3+2(A_+^{22}+A_+^{11})\kappa_1^3\kappa_2)-(2\rho_1-\rho_3)v_R(\sqrt{2}\kappa_+^2\kappa_-^2(A_+^{23}\kappa_1-A_+^{13}\kappa_2) \\
 & +2\kappa_+(2A_+^{12}\kappa_1^3+2A_+^{12}\kappa_1\kappa_2^2-(A_+^{22}+A_+^{11})\kappa_2^3+(A_+^{22}+A_+^{11})\kappa_1^2\kappa_2)v_R)) \\
 & -v_R(\sqrt{2}\kappa_-^2\kappa_+^4(\alpha_3A_+^{23}\kappa_1+A_+^{24}(\beta_1\kappa_1+2\beta_3\kappa_2))+2\alpha_3A_+^{12}\kappa_+^5\kappa_1v_R)\Big], \tag{6.33}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{13} &= \frac{1}{8\kappa_+^3\kappa_1\kappa_-^2\Lambda^2} \\
 & \times \left[-\kappa_+^3\kappa_-^4(\alpha_3A_+^{13}\kappa_1+A_+^{14}(\beta_1\kappa_1+2\beta_3\kappa_2))-\kappa_+^2v_R(\sqrt{2}\alpha_3\kappa_1\kappa_+^2\kappa_-^2(A_+^{33}+A_+^{11}) \right. \\
 & +\sqrt{2}A_+^{34}\kappa_+^2\kappa_-^2(\beta_1\kappa_1+2\beta_3\kappa_2)+2\kappa_+v_L(\kappa_2(2\beta_3(A_+^{23}\kappa_1^2-A_+^{23}\kappa_2^2+4A_+^{13}\kappa_1\kappa_2)+A_+^{14}\kappa_-^2(2\rho_1-\rho_3)) \\
 & +\beta_1\kappa_1(A_+^{23}\kappa_1^2-A_+^{23}\kappa_2^2+4A_+^{13}\kappa_1\kappa_2))-2\sqrt{2}\kappa_-^2(2\rho_1-\rho_3)((A_+^{33}+A_+^{11})\kappa_1-A_+^{12}\kappa_2)v_L^2) \\
 & -2v_R^2(\alpha_3A_+^{13}\kappa_+^5\kappa_1-(2\rho_1-\rho_3)v_L(\sqrt{2}A_+^{34}\kappa_2\kappa_+^2\kappa_-^2+2\kappa_+(-A_+^{23}\kappa_2^3+A_+^{23}\kappa_1^2\kappa_2+A_+^{13}\kappa_1^3+3A_+^{13}\kappa_1\kappa_2^2)v_L)) \\
 & -\sqrt{2}\kappa_+^2\kappa_-^2v_L(2\kappa_2(\alpha_3A_+^{34}\kappa_1^2+\beta_3(A_+^{12}\kappa_1^2-A_+^{12}\kappa_2^2+2(A_+^{33}+A_+^{11})\kappa_1\kappa_2))+\beta_1\kappa_1(A_+^{12}\kappa_1^2-A_+^{12}\kappa_2^2 \\
 & \left. +2(A_+^{33}+A_+^{11})\kappa_1\kappa_2))+2\kappa_+\kappa_1v_L^2(A_+^{13}(\kappa_1^4-\kappa_2^4)(2\rho_1-\rho_3)-2\alpha_3\kappa_1\kappa_2(A_+^{23}\kappa_1^2-A_+^{23}\kappa_2^2+2A_+^{13}\kappa_1\kappa_2))\right], \tag{6.34}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{14} &= \frac{1}{8\kappa_+^3\kappa_1\kappa_-^2\Lambda^2} \left[-\kappa_+^3\kappa_-^4(\alpha_3A_+^{14}\kappa_1+A_+^{13}\beta_1\kappa_1+2A_+^{13}\beta_3\kappa_2) \right. \\
 & -\kappa_+^2v_R(\kappa_-^2(\sqrt{2}\alpha_3A_+^{34}\kappa_+^2\kappa_1+2\sqrt{2}A_+^{34}\kappa_1(\rho_3-2\rho_1)v_L^2+2A_+^{13}\kappa_+\kappa_2(2\rho_1-\rho_3)v_L) \\
 & +\beta_1\kappa_1(\sqrt{2}(A_+^{44}+A_+^{11})\kappa_1^4-\sqrt{2}(A_+^{44}+A_+^{11})\kappa_2^4+2A_+^{24}\kappa_+\kappa_1^2v_L-2A_+^{24}\kappa_+\kappa_2^2v_L+8A_+^{14}\kappa_+\kappa_1\kappa_2v_L) \\
 & +2\beta_3\kappa_2(\sqrt{2}(A_+^{44}+A_+^{11})\kappa_1^4-\sqrt{2}(A_+^{44}+A_+^{11})\kappa_2^4+2A_+^{24}\kappa_+\kappa_1^2v_L-2A_+^{24}\kappa_+\kappa_2^2v_L+8A_+^{14}\kappa_+\kappa_1\kappa_2v_L)) \\
 & +2v_R^2(-A_+^{14}\kappa_+^3\kappa_1(\alpha_3\kappa_+^2\kappa_-^2(2\rho_1-\rho_3))+2\kappa_+(2\rho_1-\rho_3)(-A_+^{24}\kappa_2^3+A_+^{24}\kappa_1^2\kappa_2+A_+^{14}\kappa_1^3+3A_+^{14}\kappa_1\kappa_2^2)v_L^2 \\
 & +\sqrt{2}\kappa_+^2\kappa_-^2(2\rho_1-\rho_3)(A_+^{12}\kappa_1+(A_+^{44}+A_+^{11})\kappa_2)v_L) \\
 & -\sqrt{2}\kappa_1\kappa_+^2\kappa_-^2v_L(\alpha_3(A_+^{12}\kappa_1^2-A_+^{12}\kappa_2^2+2(A_+^{44}+A_+^{11})\kappa_1\kappa_2)+2A_+^{34}\kappa_2(\beta_1\kappa_1+2\beta_3\kappa_2)) \\
 & \left. -4\alpha_3\kappa_+\kappa_1\kappa_2(A_+^{24}\kappa_1^2-A_+^{24}\kappa_2^2+2A_+^{14}\kappa_1\kappa_2)v_L^2\right], \tag{6.35}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{22} &= -\frac{v_L}{4\kappa_+^3\kappa_1\Lambda^2} \left[\sqrt{2}(\kappa_1^4-\kappa_2^4)(\alpha_3A_+^{24}\kappa_1+A_+^{23}\beta_1\kappa_1+2A_+^{23}\beta_3\kappa_2)+2\kappa_+^2v_R(A_+^{12}\beta_1\kappa_+\kappa_1 \right. \\
 & +\kappa_2(2A_+^{12}\beta_3\kappa_++\sqrt{2}A_+^{23}(2\rho_1-\rho_3)v_L))-2(2\rho_1-\rho_3)v_R^2(\sqrt{2}A_+^{24}\kappa_+^2\kappa_1+2\kappa_+(A_+^{22}\kappa_1+A_+^{12}\kappa_2)v_L) \\
 & \left. +2\alpha_3\kappa_1\kappa_+(A_+^{22}\kappa_1^2-A_+^{22}\kappa_2^2+2A_+^{12}\kappa_1\kappa_2)v_L\right], \tag{6.36}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{23} &= \frac{1}{8\kappa_+^3\kappa_1\Lambda^2} \left[-\kappa_+^3\kappa_-^2(\alpha_3A_+^{23}\kappa_1+A_+^{24}(\beta_1\kappa_1+2\beta_3\kappa_2))-\kappa_+^2v_R(\sqrt{2}\alpha_3A_+^{12}\kappa_+^2\kappa_1 \right. \\
 & +2v_L(\kappa_+(\kappa_2(2A_+^{13}\beta_3+A_+^{24}(2\rho_1-\rho_3))+A_+^{13}\beta_1\kappa_1)-\sqrt{2}(2\rho_1-\rho_3)(A_+^{12}\kappa_1-(A_+^{33}+A_+^{22})\kappa_2)v_L)) \\
 & +2(2\rho_1-\rho_3)v_Lv_R^2(\sqrt{2}A_+^{34}\kappa_+^2\kappa_1+2\kappa_+(A_+^{23}\kappa_1+A_+^{13}\kappa_2)v_L)-\sqrt{2}\kappa_+^2v_L(\alpha_3A_+^{34}\kappa_-^2\kappa_1 \\
 & +\beta_1\kappa_1(2A_+^{12}\kappa_1\kappa_2+(A_+^{33}+A_+^{22})\kappa_1^2-(A_+^{33}+A_+^{22})\kappa_2^2) \\
 & +2\beta_3\kappa_2(2A_+^{12}\kappa_1\kappa_2+(A_+^{33}+A_+^{22})\kappa_1^2-(A_+^{33}+A_+^{22})\kappa_2^2))-2\kappa_1\kappa_+v_L^2(\alpha_3(A_+^{23}\kappa_1^2-A_+^{23}\kappa_2^2+2A_+^{13}\kappa_1\kappa_2) \\
 & \left. -A_+^{23}\kappa_+^2(2\rho_1-\rho_3))\right], \tag{6.37}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{23} &= \frac{1}{8\kappa_+^3\kappa_1\Lambda^2} \left[-\kappa_+^3\kappa_-^2(\alpha_3A_+^{23}\kappa_1+A_+^{24}(\beta_1\kappa_1+2\beta_3\kappa_2))-\kappa_+^2v_R(\sqrt{2}\alpha_3A_+^{12}\kappa_+^2\kappa_1 \right. \\
 & +2v_L(\kappa_+(\kappa_2(2A_+^{13}\beta_3+A_+^{24}(2\rho_1-\rho_3))+A_+^{13}\beta_1\kappa_1)-\sqrt{2}(2\rho_1-\rho_3)(A_+^{12}\kappa_1-(A_+^{33}+A_+^{22})\kappa_2)v_L)) \\
 & +2(2\rho_1-\rho_3)v_Lv_R^2(\sqrt{2}A_+^{34}\kappa_+^2\kappa_1+2\kappa_+(A_+^{23}\kappa_1+A_+^{13}\kappa_2)v_L)-\sqrt{2}\kappa_+^2v_L(\alpha_3A_+^{34}\kappa_-^2\kappa_1 \\
 & +\beta_1\kappa_1(2A_+^{12}\kappa_1\kappa_2+(A_+^{33}+A_+^{22})\kappa_1^2-(A_+^{33}+A_+^{22})\kappa_2^2) \\
 & +2\beta_3\kappa_2(2A_+^{12}\kappa_1\kappa_2+(A_+^{33}+A_+^{22})\kappa_1^2-(A_+^{33}+A_+^{22})\kappa_2^2))-2\kappa_1\kappa_+v_L^2(\alpha_3(A_+^{23}\kappa_1^2-A_+^{23}\kappa_2^2+2A_+^{13}\kappa_1\kappa_2) \\
 & \left. -A_+^{23}\kappa_+^2(2\rho_1-\rho_3))\right], \tag{6.38}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{24} &= \frac{1}{8\kappa_+^3\kappa_1\Lambda^2} \left[-\kappa_+^3\kappa_-^2(\alpha_3A_+^{24}\kappa_1+A_+^{23}\beta_1\kappa_1+2A_+^{23}\beta_3\kappa_2) \right. \\
 & -\kappa_+^2v_R(2\kappa_2(\beta_3(\sqrt{2}A_+^{12}\kappa_1^2+\sqrt{2}A_+^{12}\kappa_2^2+2A_+^{14}\kappa_+v_L)+(2\rho_1-\rho_3)v_L(A_+^{23}\kappa_++\sqrt{2}A_+^{34}v_L)) \\
 & +\beta_1\kappa_1(\sqrt{2}A_+^{12}\kappa_1^2+\sqrt{2}A_+^{12}\kappa_2^2+2A_+^{14}\kappa_+v_L))+(4\rho_1-2\rho_3)v_R^2(A_+^{24}\kappa_+^3\kappa_1 \\
 & +\sqrt{2}\kappa_+^2(A_+^{12}\kappa_2+(A_+^{44}+A_+^{22})\kappa_1)v_L+2\kappa_+(A_+^{24}\kappa_1+A_+^{14}\kappa_2)v_L^2) \\
 & -\sqrt{2}\kappa_+^2v_L(\alpha_3\kappa_1(2A_+^{12}\kappa_1\kappa_2+(A_+^{44}+A_+^{22})\kappa_1^2-(A_+^{44}+A_+^{22})\kappa_2^2)+A_+^{34}\kappa_-^2(\beta_1\kappa_1+2\beta_3\kappa_2)) \\
 & \left. -2\alpha_3\kappa_1\kappa_+(A_+^{24}\kappa_1^2-A_+^{24}\kappa_2^2+2A_+^{14}\kappa_1\kappa_2)v_L^2\right], \tag{6.39}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{33} &= \frac{1}{4\kappa_+\kappa_1\Lambda^2} \left[-\kappa_-^2\kappa_+(\alpha_3A_+^{33}\kappa_1+A_+^{34}(\beta_1\kappa_1+2\beta_3\kappa_2)) \right. \\
 & \left. +\kappa_+^2\kappa_-(\alpha_3A_+^{33}\kappa_1+A_+^{34}(\beta_1\kappa_1+2\beta_3\kappa_2)) \right] \tag{6.40}
 \end{aligned}$$

$$\begin{aligned}
 & -v_R(\sqrt{2}\alpha_3 A_+^{13}\kappa_+^2\kappa_1 - 2(2\rho_1 - \rho_3)v_L(\sqrt{2}(A_+^{13}\kappa_1 - A_+^{23}\kappa_2)v_L - A_+^{34}\kappa_+\kappa_2)) \\
 & -\sqrt{2}(A_+^{23}\kappa_1^2 - A_+^{23}\kappa_2^2 + 2A_+^{13}\kappa_1\kappa_2)(\beta_1\kappa_1 + 2\beta_3\kappa_2)v_L + 2A_+^{33}\kappa_+\kappa_1(2\rho_1 - \rho_3)v_L^2], \\
 (\Delta m_{+,\phi D}^2)_{34} = & \frac{1}{8\kappa_+\kappa_1\Lambda^2} \left[-\kappa_-^2\kappa_+ (2\alpha_3 A_+^{34}\kappa_1 + (A_+^{44} + A_+^{33})\beta_1\kappa_1 + 2(A_+^{44} + A_+^{33})\beta_3\kappa_2) - v_R(\sqrt{2}\alpha_3 A_+^{14}\kappa_1^3 \right. \\
 & + \sqrt{2}\alpha_3 A_+^{14}\kappa_1\kappa_2^2 + \sqrt{2}A_+^{13}\beta_1\kappa_+^2\kappa_1 + 2\sqrt{2}A_+^{13}\beta_3\kappa_+^2\kappa_2 + 4A_+^{44}\kappa_+\kappa_2\rho_1 v_L - 2A_+^{44}\kappa_+\kappa_2\rho_3 v_L \\
 & + 4A_+^{33}\kappa_+\kappa_2\rho_1 v_L - 2A_+^{33}\kappa_+\kappa_2\rho_3 v_L + 4\sqrt{2}A_+^{24}\kappa_2\rho_1 v_L^2 - 2\sqrt{2}A_+^{24}\kappa_2\rho_3 v_L^2 - 4\sqrt{2}A_+^{14}\kappa_1\rho_1 v_L^2 \\
 & + 2\sqrt{2}A_+^{14}\kappa_1\rho_3 v_L^2) + 2(2\rho_1 - \rho_3)v_R^2(A_+^{34}\kappa_+\kappa_1 + \sqrt{2}(A_+^{23}\kappa_1 + A_+^{13}\kappa_2)v_L) \\
 & - \sqrt{2}v_L(\alpha_3\kappa_1(A_+^{23}\kappa_1^2 - A_+^{23}\kappa_2^2 + 2A_+^{13}\kappa_1\kappa_2) + (A_+^{24}\kappa_1^2 - A_+^{24}\kappa_2^2 + 2A_+^{14}\kappa_1\kappa_2)(\beta_1\kappa_1 + 2\beta_3\kappa_2)) \\
 & \left. + 2A_+^{34}\kappa_+\kappa_1(2\rho_1 - \rho_3)v_L^2 \right], \tag{6.41}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{+,\phi D}^2)_{44} = & \frac{1}{4\kappa_+\kappa_1\Lambda^2} \left[-\kappa_+\kappa_-^2 (\alpha_3 A_+^{44}\kappa_1 + A_+^{34}\beta_1\kappa_1 + 2A_+^{34}\beta_3\kappa_2) - v_R(\sqrt{2}A_+^{14}\beta_1\kappa_+^2\kappa_1 \right. \\
 & + 2\kappa_2(\sqrt{2}A_+^{14}\beta_3\kappa_+^2 + A_+^{34}\kappa_+(2\rho_1 - \rho_3)v_L)) + 2(2\rho_1 - \rho_3)v_R^2(A_+^{44}\kappa_+\kappa_1 + \sqrt{2}(A_+^{24}\kappa_1 + A_+^{14}\kappa_2)v_L) \\
 & \left. + \sqrt{2}\alpha_3\kappa_1(A_+^{24}(-\kappa_1^2) + A_+^{24}\kappa_2^2 - 2A_+^{14}\kappa_1\kappa_2)v_L \right]. \tag{6.42}
 \end{aligned}$$

Doubly charged scalar mass matrix.

$$(\Delta m_{++,\phi D}^2)_{11} = \frac{1}{2\kappa_1^2\Lambda^2} \left[A_{++}^{12}(\kappa_2^2(\rho_3 - 2\rho_1) - 4\kappa_1^2\rho_4)v_L v_R + \kappa_1^2(2\rho_1 - \rho_3)A_{++}^{11}v_L^2 + \alpha_3\kappa_1^2(\kappa_2^2 - \kappa_1^2)A_{++}^{11} \right. \\
 \left. - \beta_3\kappa_1^4 A_{++}^{12} + \beta_3\kappa_2^4 A_{++}^{12} + \beta_1\kappa_1\kappa_2^3 A_{++}^{12} - \beta_1\kappa_1^3\kappa_2 A_{++}^{12} - 4\kappa_1^2\rho_2 A_{++}^{11}v_R^2 \right], \tag{6.43}$$

$$\begin{aligned}
 (\Delta m_{++,\phi D}^2)_{12} = & -\frac{1}{4\kappa_1^2\Lambda^2} \\
 & \times \left[2\kappa_2^2\rho_1 A_{++}^{22}v_L v_R - \kappa_2^2\rho_3 A_{++}^{22}v_L v_R + 4\kappa_1^2\rho_4 A_{++}^{22}v_L v_R + 2\kappa_2^2\rho_1 A_{++}^{11}v_L v_R - \kappa_2^2\rho_3 A_{++}^{11}v_L v_R \right. \\
 & + 4\kappa_1^2\rho_4 A_{++}^{11}v_L v_R - 2\kappa_1^2\rho_1 A_{++}^{12}v_L^2 + 4\kappa_1^2\rho_2 A_{++}^{12}v_L^2 + \kappa_1^2\rho_3 A_{++}^{12}v_L^2 + 2\alpha_3\kappa_1^4 A_{++}^{12} - 2\alpha_3\kappa_1^2\kappa_2^2 A_{++}^{12} \\
 & + \beta_1\kappa_2^2\kappa_1\kappa_2(A_{++}^{22} + A_{++}^{11}) + \beta_3(\kappa_1^4 - \kappa_2^4)(A_{++}^{22} + A_{++}^{11}) - 2\kappa_1^2\rho_1 A_{++}^{12}v_R^2 + 4\kappa_1^2\rho_2 A_{++}^{12}v_R^2 \\
 & \left. + \kappa_1^2\rho_3 A_{++}^{12}v_R^2 \right], \tag{6.44}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{++,\phi D}^2)_{22} = & \frac{1}{2\kappa_1^2\Lambda^2} \\
 & \times \left[-2\kappa_2^2\rho_1 A_{++}^{12}v_L v_R + \kappa_2^2\rho_3 A_{++}^{12}v_L v_R - 4\kappa_1^2\rho_4 A_{++}^{12}v_L v_R - 4\kappa_1^2\rho_2 A_{++}^{22}v_L^2 - \alpha_3\kappa_1^4 A_{++}^{22} \right. \\
 & \left. + \alpha_3\kappa_1^2\kappa_2^2 A_{++}^{22} + \beta_1\kappa_1\kappa_2(\kappa_2^2 - \kappa_1^2)A_{++}^{12} + \beta_3(\kappa_2^4 - \kappa_1^4)A_{++}^{12} + 2\kappa_1^2\rho_1 A_{++}^{22}v_R^2 - \kappa_1^2\rho_3 A_{++}^{22}v_R^2 \right]. \tag{6.45}
 \end{aligned}$$

Contributions from ϕ^6 operators.

CP-even scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{11} = & \frac{1}{4\kappa_+^2\Lambda^2} \left[v_L^2(v_R^2(\kappa_1^2 C_6^{21LlRr} + 2\kappa_2\kappa_1 C_6^{41LlRr} + \kappa_2^2(C_6^{LlR21r} + C_6^{RrL21l})) + 6\kappa_1^2(\kappa_1^2(C_6^{2121Ll} \right. \\
 & + C_6^{2211Ll}) + 2\kappa_2\kappa_1 C_6^{2141Ll} + \kappa_2^2(C_6^{23Ll41} + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1}))) + 2\kappa_1 v_L v_R(v_R^2(\kappa_2 C_6^{RrL2r1} \\
 & + \kappa_1(C_6^{RrL4r1} + C_6^{RrR4l1})) + 6(\kappa_2\kappa_1^2 C_6^{21L2r1} + \kappa_1^3(C_6^{21L4r1} + C_6^{21R4l1}) + \kappa_2^2\kappa_1(C_6^{2L41r3} + C_6^{41L41r} \\
 & + C_6^{41R2l1} + C_6^{41R4l1}) + \kappa_2^3 C_6^{41L2r3})) + 2\kappa_1 v_L^3 v_R(\kappa_1(C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR2l1}) + v_L^4(\kappa_1^2(C_6^{\{21\}\{LlLl\}} \\
 & + C_6^{LlLlLl}) + \kappa_2^2(C_6^{\{21\}\{LlLl\}} + C_6^{LlL2l1}) + 2\kappa_2\kappa_1 C_6^{41LlLl}) + 6\kappa_1^2 v_R^2(\kappa_1^2(C_6^{2121Rr} + C_6^{2211Rr}) \\
 & + 2\kappa_2\kappa_1 C_6^{2141Rr} + \kappa_2^2(C_6^{23Rr41} + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))) + v_R^4(\kappa_1^2(C_6^{\{21\}\{RrRr\}} + C_6^{21RrRr}) \\
 & \left. + \kappa_2^2(C_6^{\{21\}\{RrRr\}} + C_6^{RrR21r}) + 2\kappa_2\kappa_1 C_6^{41RrRr}) + 15(\kappa_1^6(C_6^{\{21\}\{2121\}} + C_6^{212121}) + \kappa_2^6(C_6^{\{21\}\{2121\}} + C_6^{212121}) \right]
 \end{aligned}$$

$$\begin{aligned}
 & +\kappa_2^2\kappa_1^4(C_6^{\{21\}\{2121\}}+2C_6^{414121})+\kappa_2^4\kappa_1^2(C_6^{\{21\}\{2121\}}+2C_6^{414121})+2\kappa_2\kappa_1^5C_6^{412121}+2\kappa_2^5\kappa_1C_6^{412121} \\
 & +4\kappa_2^3\kappa_1^3(C_6^{414123}+C_6^{414141}))], \tag{6.46}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{13} = & \frac{1}{2\kappa_+\Lambda^2} [\kappa_1 v_L (3v_R^2 (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) + 2(\kappa_2 \kappa_1^2 C_6^{21L2r1} \\
 & + \kappa_1^3 (C_6^{21L4r1} + C_6^{21R4l1}) + \kappa_2^2 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) + \kappa_2^3 C_6^{41L2r3})) + v_L^2 v_R (\kappa_1^2 C_6^{21LlRr} \\
 & + 2\kappa_2 \kappa_1 C_6^{41LlRr} + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + \kappa_1 v_L^3 (\kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR21l}) \\
 & + v_R (v_R^2 (\kappa_1^2 (C_6^{\{21\}\{RrRr\}} + 2C_6^{21RrRr}) + \kappa_2^2 (C_6^{\{21\}\{RrRr\}} + 2C_6^{RrR21r}) + 4\kappa_2 \kappa_1 C_6^{41RrRr}) + 2\kappa_1^2 (\kappa_1^2 (C_6^{2121Rr} \\
 & + C_6^{2211Rr}) + 2\kappa_2 \kappa_1 C_6^{2141Rr} + \kappa_2^2 (C_6^{23Rr41} + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))))], \tag{6.47}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{12} = & \frac{1}{4\kappa_+^2\Lambda^2} \\
 & \times [v_L^2 (v_R^2 (\kappa_2 \kappa_1 (-C_6^{21LlRr} + C_6^{LlR21r} + C_6^{RrL21l}) + \kappa_1^2 C_6^{41LlRr} - \kappa_2^2 C_6^{41LlRr}) + 3\kappa_1 (\kappa_2 \kappa_1^2 (-2C_6^{2121Ll} \\
 & - 2C_6^{2211Ll} + C_6^{23Ll41} + C_6^{2L21l1} + 2C_6^{4141Ll} + 2C_6^{41L2l1})) + \kappa_1^3 C_6^{2141Ll} - 3\kappa_2^2 \kappa_1 C_6^{2141Ll} - \kappa_2^3 (C_6^{23Ll41} + C_6^{2L21l1} \\
 & + 2(C_6^{4141Ll} + C_6^{41L2l1}))) + v_L v_R (v_R^2 (\kappa_1^2 C_6^{RrL2r1} - \kappa_2^2 C_6^{RrL2r1} - 2\kappa_2 \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) \\
 & + 3(\kappa_1^4 C_6^{21L2r1} + 3\kappa_2^2 \kappa_1^2 (C_6^{41L2r3} - C_6^{21L2r1}) + 2\kappa_2 \kappa_1^3 (-2C_6^{21L4r1} - 2C_6^{21R4l1} + C_6^{2L41r3} \\
 & + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) - 2\kappa_2^3 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) - \kappa_2^4 C_6^{41L2r3})) \\
 & + v_L^3 v_R (-2\kappa_2 \kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_1^2 C_6^{LlR21l} - \kappa_2^2 C_6^{LlR21l}) + v_L^4 (\kappa_2 \kappa_1 (C_6^{LlL21l} - C_6^{21LlLl}) \\
 & + \kappa_1^2 C_6^{41LlLl} - \kappa_2^2 C_6^{41LlLl}) + 3\kappa_1 v_R^2 (\kappa_2 \kappa_1^2 (-2C_6^{2121Rr} - 2C_6^{2211Rr} + C_6^{23Rr41} + C_6^{2R21r1} + 2C_6^{4141Rr} + 2C_6^{41R2r1}) \\
 & + \kappa_1^3 C_6^{2141Rr} - 3\kappa_2^2 \kappa_1 C_6^{2141Rr} - \kappa_2^3 (C_6^{23Rr41} + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))) + v_R^4 (\kappa_2 \kappa_1 (C_6^{RrR21r} \\
 & - C_6^{21RrRr}) + \kappa_1^2 C_6^{41RrRr} - \kappa_2^2 C_6^{41RrRr}) + 5\kappa_1^2 (-\kappa_2 \kappa_1^3 (-2C_6^{\{21\}\{2121\}} - 3C_6^{212121} + 2C_6^{414121}) \\
 & + \kappa_2^3 \kappa_1 (-2C_6^{\{21\}\{2121\}} - 3C_6^{212121} + 2C_6^{414121}) + \kappa_1^4 C_6^{412121} + \kappa_2^4 C_6^{412121} + 2\kappa_2^2 \kappa_1^2 (3(C_6^{414123} + C_6^{414141}) \\
 & - 2C_6^{412121})))], \tag{6.48}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{14} = & \frac{1}{2\kappa_+\Lambda^2} [v_L (v_R^2 (\kappa_1^2 C_6^{21LlRr} + 2\kappa_2 \kappa_1 C_6^{41LlRr} + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + 2\kappa_1^2 (\kappa_1^2 (C_6^{2121Ll} \\
 & + C_6^{2211Ll}) + 2\kappa_2 \kappa_1 C_6^{2141Ll} + \kappa_2^2 (C_6^{23Ll41} + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1}))) + 3\kappa_1 v_L^2 v_R (\kappa_1 (C_6^{LlL4r1} \\
 & + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR21l}) + 2v_L^3 (\kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_2^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL21l}) + 2\kappa_2 \kappa_1 C_6^{41LlLl}) \\
 & + \kappa_1 v_R (v_R^2 (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) + 2(\kappa_2 \kappa_1^2 C_6^{21L2r1} + \kappa_1^3 (C_6^{21L4r1} + C_6^{21R4l1}) \\
 & + \kappa_2^2 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) + \kappa_2^3 C_6^{41L2r3})))],
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{22} = & \frac{1}{4\kappa_+^2\Lambda^2} \\
 & \times [v_L^2 (v_R^2 (\kappa_2^2 C_6^{21LlRr} - 2\kappa_2 \kappa_1 C_6^{41LlRr} + \kappa_1^2 (C_6^{LlR21r} + C_6^{RrL21l})) + 2\kappa_2^2 \kappa_1^2 (3C_6^{2121Ll} + 3C_6^{2211Ll} \\
 & - 2(C_6^{23Ll41} + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1}))) - 6\kappa_2 \kappa_1^3 C_6^{2141Ll} + 6\kappa_2^3 \kappa_1 C_6^{2141Ll} + \kappa_1^4 (C_6^{23Ll41} + C_6^{2L21l1} \\
 & + 2(C_6^{4141Ll} + C_6^{41L2l1})) + \kappa_2^2 (C_6^{23Ll41} + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1}))) + 2v_L v_R (-\kappa_1 (\kappa_2 C_6^{RrL2r1} v_R^2 \\
 & + 3\kappa_2^3 (C_6^{41L2r3} - C_6^{21L2r1})) + \kappa_2^2 (v_R^2 (C_6^{RrL4r1} + C_6^{RrR4l1}) + \kappa_2^2 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} \\
 & + C_6^{41R4l1})) + 3\kappa_2 \kappa_1^3 (C_6^{41L2r3} - C_6^{21L2r1}) + 2\kappa_2^2 \kappa_1^2 (3C_6^{21L4r1} + 3C_6^{21R4l1} - 2(C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} \\
 & + C_6^{41R4l1})) + \kappa_1^4 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1})) + 2\kappa_2 v_L^3 v_R (\kappa_2 (C_6^{LlL4r1} + C_6^{LlR4l1}) - \kappa_1 C_6^{LlR21l}) \\
 & + v_L^4 (\kappa_2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL21l}) - 2\kappa_2 \kappa_1 C_6^{41LlLl}) + v_R^2 (2\kappa_2 \kappa_1^2 (3C_6^{2121Rr} \\
 & + 3C_6^{2211Rr} - 2(C_6^{23Rr41} + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))) - 6\kappa_2 \kappa_1^3 C_6^{2141Rr} + 6\kappa_2^3 \kappa_1 C_6^{2141Rr} + \kappa_1^4 (C_6^{23Rr41} \\
 & + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1})) + \kappa_2^4 (C_6^{23Rr41} + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))) + v_R^4 (\kappa_2^2 (C_6^{\{21\}\{RrRr\}} \\
 & + C_6^{21RrRr}) + \kappa_1^2 (C_6^{\{21\}\{RrRr\}} + C_6^{RrR21r}) - 2\kappa_2 \kappa_1 C_6^{41RrRr}) + \kappa_1^6 C_6^{\{21\}\{2121\}} + 13\kappa_2^2 \kappa_1^4 C_6^{\{21\}\{2121\}} \\
 & + 13\kappa_2^4 \kappa_1^2 C_6^{\{21\}\{2121\}} + \kappa_2^2 C_6^{\{21\}\{2121\}} + 15\kappa_2^2 \kappa_1^4 C_6^{212121} + 15\kappa_2^4 \kappa_1^2 C_6^{212121} - 10\kappa_2 \kappa_1^5 C_6^{412121} + 40\kappa_2^3 \kappa_1^3 C_6^{412121} \\
 & - 10\kappa_2^5 \kappa_1 C_6^{412121} + 2\kappa_1^6 C_6^{414121} - 4\kappa_2^2 \kappa_1^4 C_6^{414121} - 4\kappa_2^4 \kappa_1^2 C_6^{414121} + 2\kappa_2^6 C_6^{414121} + 12\kappa_2 \kappa_1^5 C_6^{414123} \\
 & - 36\kappa_2^3 \kappa_1^3 C_6^{414123} + 12\kappa_2^5 \kappa_1 C_6^{414123} + 12\kappa_2 \kappa_1^5 C_6^{414141} - 36\kappa_2^3 \kappa_1^3 C_6^{414141} + 12\kappa_2^5 \kappa_1 C_6^{414141}]], \tag{6.49}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{23} = & \frac{1}{4\kappa_+ \Lambda^2} \left[v_L (3v_R^2 (\kappa_1^2 C_6^{RrL2r1} - \kappa_2^2 C_6^{RrL2r1} - 2\kappa_2 \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) + \kappa_1^4 C_6^{21L2r1} \right. \\
 & + 3\kappa_2^2 \kappa_1^2 (C_6^{41L2r3} - C_6^{21L2r1}) + 2\kappa_2 \kappa_1^3 (-2C_6^{21L4r1} - 2C_6^{21R4l1} + C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) \\
 & - 2\kappa_2^3 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) - \kappa_2^4 C_6^{41L2r3}) + 2v_L^2 v_R (\kappa_2 \kappa_1 (-C_6^{21LlRr} + C_6^{LlR21r} \\
 & + C_6^{RrL21l}) + \kappa_1^2 C_6^{41LlRr} - \kappa_2^2 C_6^{41LlRr}) + v_L^3 (-2\kappa_2 \kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_1^2 C_6^{LlR21l} - \kappa_2^2 C_6^{LlR21l}) \\
 & - 2v_R (\kappa_1 (\kappa_2 \kappa_1^2 (2C_6^{2121Rr} + 2C_6^{2211Rr} - C_6^{23Rr41} - C_6^{2R21r1} - 2C_6^{4141Rr} - 2C_6^{41R2r1}) + \kappa_1^3 (-C_6^{2141Rr} \\
 & + 3\kappa_2^2 \kappa_1 C_6^{2141Rr} + \kappa_2^3 (C_6^{23Rr41} + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))) - 2v_R^2 (\kappa_2 \kappa_1 (C_6^{RrR21r} \\
 & - C_6^{21RrRr}) + \kappa_1^2 C_6^{41RrRr} - \kappa_2^2 C_6^{41RrRr}))], \tag{6.50}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{24} = & \frac{1}{4\kappa_+ \Lambda^2} \\
 & \times [2v_L (v_R^2 (\kappa_2 \kappa_1 (-C_6^{21LlRr} + C_6^{LlR21r} + C_6^{RrL21l}) + \kappa_1^2 C_6^{41LlRr} - \kappa_2^2 C_6^{41LlRr}) + \kappa_1 (\kappa_2 \kappa_1^2 (-2C_6^{2121Ll} \\
 & - 2C_6^{2211Ll} + C_6^{23Ll41} + C_6^{2L21l1} + 2C_6^{4141Ll} + 2C_6^{41L2l1}) + \kappa_1^3 C_6^{2141Ll} - 3\kappa_2^2 \kappa_1 C_6^{2141Ll} - \kappa_2^3 (C_6^{23Ll41} \\
 & + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1}))) + 3v_L^2 v_R (-2\kappa_2 \kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_1^2 C_6^{LlR21l} - \kappa_2^2 C_6^{LlR21l}) \\
 & + 4v_L^3 (\kappa_2 \kappa_1 (C_6^{LlL21l} - C_6^{21LlLl}) + \kappa_1^2 C_6^{41LlLl} - \kappa_2^2 C_6^{41LlLl}) + v_R (v_R^2 (\kappa_1^2 C_6^{RrL2r1} - \kappa_2^2 C_6^{RrL2r1} \\
 & - 2\kappa_2 \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) + \kappa_1^4 C_6^{21L2r1} + 3\kappa_2^2 \kappa_1^2 (C_6^{41L2r3} - C_6^{21L2r1}) + 2\kappa_2 \kappa_1^3 (-2C_6^{21L4r1} \\
 & - 2C_6^{21R4l1} + C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) - 2\kappa_2^3 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) \\
 & - \kappa_2^4 C_6^{41L2r3})], \tag{6.51}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{33} = & \frac{1}{4\Lambda^2} \\
 & \times [v_L^2 (6C_6^{RrRrLl} v_R^2 + \kappa_1^2 C_6^{21LlRr} + 2\kappa_1 \kappa_2 C_6^{41LlRr} + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + 6\kappa_1 v_L v_R (\kappa_2 C_6^{RrL2r1} \\
 & + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) + C_6^{LlLlRr} v_L^4 + v_R^2 (\kappa_1^2 (C_6^{\{21\}\{RrRr\}} + 6C_6^{21RrRr}) + \kappa_2^2 (C_6^{\{21\}\{RrRr\}} + 6C_6^{RrR21r}) \\
 & + 12\kappa_2 \kappa_1 C_6^{41RrRr}) + 15C_6^{RrRrRr} v_R^4 + \kappa_1^2 (\kappa_1^2 (C_6^{2121Rr} + C_6^{2211Rr}) + 2\kappa_2 \kappa_1 C_6^{2141Rr} + \kappa_2^2 (C_6^{23Rr41} \\
 & + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1})))], \tag{6.52}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{34} = & \frac{1}{4\Lambda^2} [2v_L v_R (2C_6^{RrRrLl} v_R^2 + \kappa_1^2 C_6^{21LlRr} + 2\kappa_1 \kappa_2 C_6^{41LlRr} + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + 4C_6^{LlLlRr} v_L^3 v_R \\
 & + 3\kappa_1 v_L^2 (\kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR21l}) + \kappa_1 (3v_R^2 (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) \\
 & + \kappa_2^2 C_6^{21L2r1} + \kappa_1^3 (C_6^{21L4r1} + C_6^{21R4l1}) + \kappa_2^2 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) + \kappa_2^3 C_6^{41L2r3})], \tag{6.53}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{r,\phi^6}^2)_{44} = & \frac{1}{4\Lambda^2} [6v_L^2 (C_6^{LlLlRr} v_R^2 + \kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_2^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL21l}) + 2\kappa_1 \kappa_2 C_6^{41LlLl}) \\
 & + 6\kappa_1 v_L v_R (\kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR21l}) + 15C_6^{LlLlLl} v_L^4 + v_R^2 (\kappa_1^2 C_6^{21LlRr} + 2\kappa_2 \kappa_1 C_6^{41LlRr} \\
 & + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + C_6^{RrRrLl} v_R^4 + \kappa_1^2 (\kappa_1^2 (C_6^{2121Ll} + C_6^{2211Ll}) + 2\kappa_2 \kappa_1 C_6^{2141Ll} + \kappa_2^2 (C_6^{23Ll41} \\
 & + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1})))]. \tag{6.54}
 \end{aligned}$$

CP-odd scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{11} = & \frac{1}{4\kappa_+^2 \Lambda^2} [v_L^2 (v_R^2 (\kappa_1^2 C_6^{21LlRr} + 2\kappa_2 \kappa_1 C_6^{41LlRr} + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + 2\kappa_1^2 (\kappa_1^2 (C_6^{2121Ll} \\
 & + C_6^{2211Ll}) + 2\kappa_2 \kappa_1 C_6^{2141Ll} + \kappa_2^2 (C_6^{23Ll41} + C_6^{2L21l1} + 2C_6^{4141Ll}))) - 2\kappa_1 v_L v_R (v_R^2 (\kappa_2 C_6^{RrL2r1} \\
 & + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) - 2\kappa_1 \kappa_2^2 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R4l1})) - 2\kappa_1 v_L^3 v_R (\kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) \\
 & + \kappa_2 C_6^{LlR21l}) + v_L^4 (\kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_2^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL21l}) + 2\kappa_2 \kappa_1 C_6^{41LlLl}) \\
 & + 2\kappa_1^2 v_R^2 (\kappa_1^2 (C_6^{2121Rr} + C_6^{2211Rr}) + 2\kappa_2 \kappa_1 C_6^{2141Rr} + \kappa_2^2 (C_6^{23Rr41} + C_6^{2R21r1} + 2C_6^{4141Rr})) \\
 & + v_R^4 (\kappa_1^2 (C_6^{\{21\}\{RrRr\}} + C_6^{21RrRr}) + \kappa_2^2 (C_6^{\{21\}\{RrRr\}} + C_6^{RrR21r}) + 2\kappa_2 \kappa_1 C_6^{41RrRr}) + 3(2\kappa_2 \kappa_1^5 C_6^{412121} \\
 & + \kappa_1^6 (C_6^{\{21\}\{2121\}} + C_6^{212121}) + \kappa_2^6 (C_6^{\{21\}\{2121\}} + C_6^{212121}) + \kappa_2^4 \kappa_1^4 (C_6^{\{21\}\{2121\}} + 2C_6^{414121}) \\
 & + \kappa_2^4 \kappa_1^2 (C_6^{\{21\}\{2121\}} + 2C_6^{414121}) + 2\kappa_2^5 \kappa_1 C_6^{412121} + 4\kappa_2^3 \kappa_1^3 (C_6^{414123} + C_6^{414141}))), \tag{6.55}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{12} = & \frac{1}{4\kappa_+^2 \Lambda^2} \left[v_L^2 \left(v_R^2 (\kappa_2 \kappa_1 (-C_6^{21LlRr} + C_6^{LlR21r} + C_6^{RrL21l}) + \kappa_1^2 C_6^{41LlRr} - \kappa_2^2 C_6^{41LlRr}) \right. \right. \\
 & + \kappa_1 (\kappa_2 \kappa_1^2 (-2C_6^{2121Ll} - 2C_6^{2211Ll} + C_6^{23Ll4l} + C_6^{2L21l1} + 2C_6^{4141Ll} + 2C_6^{41L2l1}) + \kappa_1^3 C_6^{2141Ll} \\
 & - 3\kappa_2^2 \kappa_1 C_6^{2141Ll} - \kappa_2^3 (C_6^{23Ll4l} + C_6^{2L21l1} + 2C_6^{4141Ll} - 2C_6^{41L2l1})) + v_L v_R (v_R^2 (-\kappa_1^2 C_6^{RrL2r1} \\
 & + \kappa_2^2 C_6^{RrL2r1} + 2\kappa_2 \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) - \kappa_1^4 C_6^{21L2r1} + \kappa_2^2 \kappa_1^2 (C_6^{41L2r3} - C_6^{21L2r1}) + \kappa_2^4 C_6^{41L2r3} \\
 & + 2\kappa_2 \kappa_1^3 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1}) - 2\kappa_2^3 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} - C_6^{41R2l1} + C_6^{41R4l1})) \\
 & + v_L^3 v_R (2\kappa_2 \kappa_1 (C_6^{LlL4r1} + C_6^{LlR4l1}) - \kappa_1^2 C_6^{LlR2l1} + \kappa_2^2 C_6^{LlR2l1}) + v_L^4 (\kappa_2 \kappa_1 (C_6^{LlL2l1} - C_6^{21LlLl}) \\
 & + \kappa_1^2 C_6^{41LlLl} - \kappa_2^2 C_6^{41LlLl}) + \kappa_1 v_R^2 (\kappa_2 \kappa_1^2 (-2C_6^{2121Rr} - 2C_6^{2211Rr} + C_6^{23Rr4l} + C_6^{2R21r1} + 2C_6^{4141Rr} \\
 & - 2C_6^{41R2r1}) + \kappa_1^3 C_6^{2141Rr} - 3\kappa_2^2 \kappa_1 C_6^{2141Rr} - \kappa_2^3 (C_6^{23Rr4l} + C_6^{2R21r1} + 2C_6^{4141Rr} - 2C_6^{41R2r1})) \\
 & + v_R^4 (\kappa_2 \kappa_1 (C_6^{RrR21r} - C_6^{21RrRr}) + \kappa_1^2 C_6^{41RrRr} - \kappa_2^2 C_6^{41RrRr}) + \kappa_- (\kappa_2 \kappa_1^3 (-2C_6^{\{21\}\{2121\}} - 3C_6^{212121} \\
 & + 2C_6^{414121}) + \kappa_2^3 \kappa_1 (-2C_6^{\{21\}\{2121\}} - 3C_6^{212121} + 2C_6^{414121}) + \kappa_1^4 C_6^{412121} + \kappa_2^4 C_6^{412121} \\
 & \left. \left. + 2\kappa_2^2 \kappa_1^2 (3(C_6^{414123} + C_6^{414141}) - 2C_6^{412121}) \right) \right], \quad (6.56)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{13} = & -\frac{\kappa_1 v_L}{2\kappa_+ \Lambda^2} \left[v_L^2 (\kappa_1 (C_6^{LlL4r1} - C_6^{LlR4l1}) - \kappa_2 C_6^{LlR2l1}) + v_R^2 (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} - C_6^{RrR4l1})) \right. \\
 & \left. + \kappa_2 \kappa_1^2 C_6^{21L2r1} + \kappa_1^3 C_6^{21L4r1} - \kappa_1^3 C_6^{21R4l1} + \kappa_2^3 C_6^{41L2r3} - \kappa_2^2 \kappa_1 C_6^{41R2l1} \right], \quad (6.57)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{14} = & \frac{\kappa_1 v_R}{2\kappa_+ \Lambda^2} \left[v_L^2 (\kappa_1 (C_6^{LlL4r1} - C_6^{LlR4l1}) - \kappa_2 C_6^{LlR2l1}) + v_R^2 (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} - C_6^{RrR4l1})) \right. \\
 & \left. + \kappa_2 \kappa_1^2 C_6^{21L2r1} + \kappa_1^3 C_6^{21L4r1} - \kappa_1^3 C_6^{21R4l1} + \kappa_2^3 C_6^{41L2r3} - \kappa_2^2 \kappa_1 C_6^{41R2l1} \right], \quad (6.58)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{22} = & \frac{1}{4\kappa_+^2 \Lambda^2} \left[v_L^2 \left(v_R^2 (\kappa_2^2 C_6^{21LlRr} - 2\kappa_2 \kappa_1 C_6^{41LlRr} + \kappa_1^2 (C_6^{LlR21r} + C_6^{RrL21l})) + 2\kappa_2^2 \kappa_1^2 (C_6^{2121Ll} \right. \right. \\
 & + C_6^{2211Ll} - 4C_6^{4141Ll}) - 2\kappa_2 \kappa_1^3 C_6^{2141Ll} + 2\kappa_2^3 \kappa_1 C_6^{2141Ll} + \kappa_1^4 (C_6^{23Ll4l} + C_6^{2L21l1} - 2C_6^{4141Ll} + 2C_6^{41L2l1}) \\
 & + \kappa_2^4 (C_6^{23Ll4l} + C_6^{2L21l1} - 2(C_6^{4141Ll} + C_6^{41L2l1})) + 2v_L v_R (\kappa_1 (\kappa_2 C_6^{RrL2r1} v_R^2 + \kappa_2^3 (C_6^{21L2r1} + C_6^{41L2r3})) \\
 & + \kappa_2^2 (\kappa_2^2 (C_6^{2L41r3} - C_6^{41L41r} - C_6^{41R2l1} - C_6^{41R4l1}) - v_R^2 (C_6^{RrL4r1} + C_6^{RrR4l1})) + \kappa_2 \kappa_1^3 (C_6^{21L2r1} \\
 & + C_6^{41L2r3}) + \kappa_1^4 (C_6^{2L41r3} - C_6^{41L41r} + C_6^{41R2l1} - C_6^{41R4l1}) - 4\kappa_2^2 \kappa_1^2 (C_6^{41L41r} + C_6^{41R4l1})) \\
 & + C_6^{LlR4l1}) - \kappa_1 C_6^{LlR2l1}) + v_L^4 (\kappa_2^2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL2l1}) - 2\kappa_2 \kappa_1 C_6^{41LlLl}) \\
 & + v_R^2 (2\kappa_2^2 \kappa_1^2 (C_6^{2121Rr} + C_6^{2211Rr} - 4C_6^{4141Rr}) - 2\kappa_2 \kappa_1^3 C_6^{2141Rr} + 2\kappa_2^3 \kappa_1 C_6^{2141Rr} + \kappa_1^4 (C_6^{23Rr4l} + C_6^{2R21r1} \\
 & - 2C_6^{4141Rr} + 2C_6^{41R2r1}) + \kappa_2^4 (C_6^{23Rr4l} + C_6^{2R21r1} - 2(C_6^{4141Rr} + C_6^{41R2r1}))) + v_R^4 (\kappa_2^2 (C_6^{\{21\}\{RrRr\}} + C_6^{21RrRr}) \\
 & + \kappa_1^2 (C_6^{\{21\}\{RrRr\}} + C_6^{RrR21r}) - 2\kappa_2 \kappa_1 C_6^{41RrRr}) + \kappa_1^6 C_6^{\{21\}\{2121\}} + 5\kappa_2^4 \kappa_1^2 C_6^{\{21\}\{2121\}} + 5\kappa_2^4 \kappa_1^2 C_6^{\{21\}\{2121\}} \\
 & + \kappa_2^6 C_6^{\{21\}\{2121\}} + 3\kappa_2^4 \kappa_1^4 C_6^{212121} + 3\kappa_2^4 \kappa_1^2 C_6^{212121} - 2\kappa_2 \kappa_1^5 C_6^{412121} + 8\kappa_2^3 \kappa_1^3 C_6^{412121} - 2\kappa_2^5 \kappa_1 C_6^{412121} \\
 & - 2\kappa_1^6 C_6^{414121} - 8\kappa_2^4 \kappa_1^4 C_6^{414121} - 8\kappa_2^4 \kappa_1^2 C_6^{414121} - 2\kappa_2^6 C_6^{414121} + 4\kappa_2^5 \kappa_1 C_6^{414123} - 4\kappa_2^3 \kappa_1^3 C_6^{414123} \\
 & \left. \left. + 4\kappa_2^5 \kappa_1 C_6^{414123} - 12\kappa_2 \kappa_1^5 C_6^{414141} - 36\kappa_2^3 \kappa_1^3 C_6^{414141} - 12\kappa_2^5 \kappa_1 C_6^{414141} \right) \right], \quad (6.59)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{23} = & \frac{v_L}{4\kappa_+ \Lambda^2} \left[v_L^2 (2\kappa_2 \kappa_1 (C_6^{LlL4r1} - C_6^{LlR4l1}) + \kappa_1^2 C_6^{LlR2l1} - \kappa_2^2 C_6^{LlR2l1}) + v_R^2 (-\kappa_1^2 C_6^{RrL2r1} + \kappa_2^2 C_6^{RrL2r1} \right. \\
 & + 2\kappa_2 \kappa_1 (C_6^{RrL4r1} - C_6^{RrR4l1})) - \kappa_1^4 C_6^{21L2r1} + \kappa_2^2 \kappa_1^2 C_6^{21L2r1} + 2\kappa_2 \kappa_1^3 C_6^{21L4r1} - 2\kappa_2 \kappa_1^3 C_6^{21R4l1} - \kappa_2^2 \kappa_1^2 C_6^{41L2r3} \\
 & \left. + \kappa_2^4 C_6^{41L2r3} + 2\kappa_2 \kappa_1^3 C_6^{41L41r} + 2\kappa_2^3 \kappa_1 C_6^{41L41r} - 2\kappa_2^3 \kappa_1 C_6^{41R2l1} - 2\kappa_2 \kappa_1^3 C_6^{41R4l1} - 2\kappa_2^3 \kappa_1 C_6^{41R4l1} \right], \quad (6.60)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{24} = & \frac{v_R}{4\kappa_+ \Lambda^2} \left[v_L^2 (2\kappa_2 \kappa_1 (C_6^{LlR4l1} - C_6^{LlL4r1}) - \kappa_1^2 C_6^{LlR2l1} + \kappa_2^2 C_6^{LlR2l1}) + v_R^2 (\kappa_1^2 C_6^{RrL2r1} - \kappa_2^2 C_6^{RrL2r1} \right. \\
 & + 2\kappa_2 \kappa_1 (C_6^{RrR4l1} - C_6^{RrL4r1})) + \kappa_1^4 C_6^{21L2r1} - \kappa_2^2 \kappa_1^2 C_6^{21L2r1} - 2\kappa_2 \kappa_1^3 C_6^{21L4r1} + 2\kappa_2 \kappa_1^3 C_6^{21R4l1} + \kappa_2^2 \kappa_1^2 C_6^{41L2r3} \\
 & \left. - \kappa_2^4 C_6^{41L2r3} - 2\kappa_2 \kappa_1^3 C_6^{41L41r} - 2\kappa_2^3 \kappa_1 C_6^{41L41r} + 2\kappa_2^3 \kappa_1 C_6^{41R2l1} + 2\kappa_2 \kappa_1^3 C_6^{41R4l1} + 2\kappa_2^3 \kappa_1 C_6^{41R4l1} \right], \quad (6.61)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{i,\phi^6}^2)_{33} = & \frac{1}{4\Lambda^2} \left[v_L^2 (2C_6^{RrRrLl} v_R^2 + \kappa_1^2 C_6^{21LlRr} + 2\kappa_1 \kappa_2 C_6^{41LlRr} + \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) \right. \\
 & + 2\kappa_1 v_L v_R (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1})) + C_6^{LlLlRr} v_L^4 \\
 & + v_R^2 (\kappa_1^2 (-C_6^{\{21\}\{RrRr\}} - 2C_6^{21RrRr}) - \kappa_2^2 (C_6^{\{21\}\{RrRr\}} - 2C_6^{RrR21r})) \\
 & + 4\kappa_2 \kappa_1 C_6^{41RrRr} + 3C_6^{RrRrRr} v_R^4 + \kappa_1^2 (\kappa_1^2 (C_6^{2121Rr} + C_6^{2211Rr}) + 2\kappa_2 \kappa_1 C_6^{2141Rr} + \kappa_2^2 (C_6^{23Rr4l} \\
 & \left. + C_6^{2R21r1} + 2(C_6^{4141Rr} + C_6^{41R2r1}))) \right], \quad (6.62)
 \end{aligned}$$

$$(\Delta m_{i,\phi^6}^2)_{34} = \frac{\kappa_1}{4\Lambda^2} [v_L^2 (\kappa_1 (C_6^{LL4r1} + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR2l1}) + v_R^2 (\kappa_2 C_6^{RrL2r1} + \kappa_1 (C_6^{RrL4r1} + C_6^{RrR4l1}))] \quad (6.63)$$

$$+ \kappa_2 \kappa_1^2 C_6^{21L2r1} + \kappa_1^3 (C_6^{21L4r1} + C_6^{21R4l1}) + \kappa_2^3 C_6^{41L2r3} + \kappa_2^2 \kappa_1 (C_6^{2L41r3} + C_6^{41L41r} + C_6^{41R2l1} + C_6^{41R4l1})],$$

$$(\Delta m_{i,\phi^6}^2)_{44} = \frac{1}{4\Lambda^2} [2v_L^2 (C_6^{LlLlRr} v_R^2 + \kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_2^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL2l1}) + 2\kappa_1 \kappa_2 C_6^{41LlLl})$$

$$+ 2\kappa_1 v_L v_R (\kappa_1 (C_6^{LL4r1} + C_6^{LlR4l1}) + \kappa_2 C_6^{LlR2l1}) + 3C_6^{LlLlLl} v_L^4 + v_R^2 (\kappa_1^2 C_6^{21LlRr} + 2\kappa_2 \kappa_1 C_6^{41LlRr}$$

$$+ \kappa_2^2 (C_6^{LlR21r} + C_6^{RrL21l})) + C_6^{RrRrLl} v_R^4 + \kappa_1^2 (\kappa_1^2 (C_6^{2121Ll} + C_6^{2211Ll}) + 2\kappa_2 \kappa_1 C_6^{2141Ll}$$

$$+ \kappa_2^2 (C_6^{23Ll41} + C_6^{2L21l1} + 2(C_6^{4141Ll} + C_6^{41L2l1})))]. \quad (6.64)$$

Singly charged scalar mass matrix.

$$(\Delta m_{+,\phi^6}^2)_{11} = \frac{1}{4\kappa_+^2 \Lambda^2} [v_L^2 (v_R^2 (\kappa_2^2 C_6^{21LlRr} - 2\kappa_2 \kappa_1 C_6^{41LlRr} + \kappa_1^2 (C_6^{LlR21r} + C_6^{RrL21l})) + \kappa_2^4 (C_6^{2121Ll}$$

$$+ C_6^{2211Ll}) + \kappa_2^2 \kappa_1^2 (4C_6^{2121Ll} + C_6^{2211Ll} - 2C_6^{23Ll41} + C_6^{2L21l1} - 4C_6^{4141Ll} - 2C_6^{41L2l1}) + \kappa_1^4 (C_6^{2121Ll}$$

$$+ C_6^{2L21l1}) + 2\kappa_2 \kappa_1^3 (C_6^{2211Ll} - C_6^{2141Ll}) + 2\kappa_2^3 \kappa_1 (C_6^{2141Ll} + C_6^{2211Ll})) + 2\kappa_1 v_L v_R (\kappa_2 \kappa_1^2 C_6^{21L2r1}$$

$$+ \kappa_2^3 (2C_6^{21L2r1} - C_6^{41L2r3}) + \kappa_1^3 (C_6^{21L4r1} + C_6^{21R4l1}) + \kappa_2^2 \kappa_1 (2C_6^{21L4r1} + 2C_6^{21R4l1} - 2C_6^{2L41r3} + C_6^{41l2r1}$$

$$- 2C_6^{41L41r} - C_6^{41R2l1} - 2C_6^{41R4l1})) + v_L^4 (\kappa_2^2 (C_6^{\{21\}\{LlLl\}} + C_6^{21LlLl}) + \kappa_1^2 (C_6^{\{21\}\{LlLl\}} + C_6^{LlL2l1})$$

$$- 2\kappa_2 \kappa_1 C_6^{41LlLl}) + v_R^2 (\kappa_2^2 (C_6^{2121Rr} + C_6^{2211Rr}) + \kappa_2^2 \kappa_1^2 (4C_6^{2121Rr} + C_6^{2211Rr} - 2C_6^{23Rr41} + C_6^{2R21r1}$$

$$- 4C_6^{4141Rr} - 2C_6^{41R2r1}) + \kappa_1^4 (C_6^{2121Rr} + C_6^{2211Rr}) + 2\kappa_2 \kappa_1^3 (C_6^{2121Rr} - C_6^{2141Rr}) + 2\kappa_2^3 \kappa_1 (C_6^{2141Rr}$$

$$+ C_6^{2211Rr})) + v_R^4 (\kappa_2^2 (C_6^{\{21\}\{RrRr\}} + C_6^{21RrRr}) + \kappa_1^2 (C_6^{\{21\}\{RrRr\}} + C_6^{RrR21r}) - 2\kappa_2 \kappa_1 C_6^{41RrRr})$$

$$+ 3\kappa_1^6 C_6^{\{21\}\{2121\}} + 11\kappa_2^2 \kappa_1^4 C_6^{\{21\}\{2121\}} + 11\kappa_2^4 \kappa_1^2 C_6^{\{21\}\{2121\}} + 3\kappa_2^6 C_6^{\{21\}\{2121\}} + 3\kappa_1^6 C_6^{212121}$$

$$+ 12\kappa_2^2 \kappa_1^4 C_6^{212121} + 12\kappa_2^4 \kappa_1^2 C_6^{212121} + 3\kappa_2^6 C_6^{212121} + 2\kappa_2 \kappa_1^5 C_6^{412121} + 16\kappa_2^3 \kappa_1^3 C_6^{412121} + 2\kappa_2^5 \kappa_1 C_6^{412121}$$

$$- 2\kappa_2^2 \kappa_1^4 C_6^{414121} - 2\kappa_2^4 \kappa_1^2 C_6^{414121} - 12\kappa_2^3 \kappa_1^3 C_6^{414123} - 12\kappa_2^3 \kappa_1^3 C_6^{414141}]), \quad (6.65)$$

$$(\Delta m_{+,\phi^6}^2)_{12} = \frac{1}{4\kappa_+^2 \Lambda^2} [v_L^2 (\kappa_1 (\kappa_2 \kappa_1^2 (2C_6^{2121Ll} + C_6^{2211Ll} - C_6^{23Ll41} - C_6^{2L21l1} - 2C_6^{4141Ll} - C_6^{41L2l1}) + \kappa_1^3 C_6^{2211Ll}$$

$$- C_6^{2141Ll}) + \kappa_2^2 \kappa_1 (3C_6^{2141Ll} + C_6^{2211Ll}) + \kappa_2^3 (C_6^{2211Ll} + C_6^{23Ll41} - C_6^{2L21l1} + 2C_6^{4141Ll} + C_6^{41L2l1}))$$

$$- v_R^2 (\kappa_2 \kappa_1 (-C_6^{21LlRr} + C_6^{LlR21r} + C_6^{RrL21l}) + \kappa_1^2 C_6^{41LlRr} - \kappa_2^2 C_6^{41LlRr}))$$

$$+ \kappa_2^2 \kappa_1 v_L v_R (\kappa_2 (C_6^{21L2r1} - C_6^{41L2r3}) + \kappa_1 (C_6^{21L4r1} + C_6^{21R4l1} - 2C_6^{2L41r3} + C_6^{41l2r1}$$

$$- 2C_6^{41L41r} - C_6^{41R2l1} - 2C_6^{41R4l1})) + v_L^4 (\kappa_2 \kappa_1 (C_6^{21LlLl} - C_6^{LlL2l1}) + \kappa_1^2 (-C_6^{41LlLl}) + \kappa_2^2 C_6^{41LlLl})$$

$$+ \kappa_1 v_R^2 (\kappa_2 \kappa_1^2 (2C_6^{2121Rr} + C_6^{2211Rr} - C_6^{23Rr41} - C_6^{2R21r1} - 2C_6^{4141Rr} - C_6^{41R2r1}) + \kappa_1^3 (C_6^{2211Rr} - C_6^{2141Rr})$$

$$+ \kappa_2^2 \kappa_1 (3C_6^{2141Rr} + C_6^{2211Rr}) + \kappa_2^3 (C_6^{2211Rr} + C_6^{23Rr41} - C_6^{2R21r1} + 2C_6^{4141Rr} + C_6^{41R2r1}))$$

$$+ v_R^4 (\kappa_2 \kappa_1 (C_6^{21RrRr} - C_6^{RrR21r}) + \kappa_1^2 (-C_6^{41RrRr}) + \kappa_2^2 C_6^{41RrRr})$$

$$- \kappa_-^2 (\kappa_2 \kappa_1^3 (-2C_6^{\{21\}\{2121\}} - 3C_6^{212121} + 2C_6^{414121}) + \kappa_2^3 \kappa_1 (-2C_6^{\{21\}\{2121\}} - 3C_6^{212121} + 2C_6^{414121})$$

$$+ \kappa_1^4 C_6^{412121} + \kappa_2^4 C_6^{412121} + 2\kappa_2^2 \kappa_1^2 (3(C_6^{414123} + C_6^{414141}) - 2C_6^{412121}))), \quad (6.66)$$

$$(\Delta m_{+,\phi^6}^2)_{13} = -\frac{1}{8\sqrt{2}\kappa_+ \Lambda^2}$$

$$\times [2v_L (v_R^2 (\kappa_2 \kappa_1 (C_6^{R4Rl1r1} + C_6^{RrL2r1} + 2C_6^{RrL4r1}) + \kappa_2^2 C_6^{RrL2r1} + \kappa_1^2 (-C_6^{RrL2r1} + C_6^{RrL4r1}$$

$$+ 2C_6^{RrR4l1})) + \kappa_2^4 C_6^{21L2r1} + \kappa_1^4 (-C_6^{21L2r1} + 2C_6^{21R4l1}) + 2\kappa_2^2 \kappa_1^2 (C_6^{21L2r1} + C_6^{21R4l1} - C_6^{41L2r3})$$

$$+ \kappa_2 \kappa_1^3 (3C_6^{21L4r1} - C_6^{41l2r1}) + \kappa_2^3 \kappa_1 (C_6^{21L4r1} + C_6^{41l2r1})) + \kappa_+^2 v_L v_R (C_6^{21LlRr} - 2C_6^{LlR21r})$$

$$+ 2\kappa_1 v_L^3 (\kappa_1 C_6^{LlR4l1} - \kappa_2 (C_6^{L4Lr1l} - 2C_6^{LlL4r1} + C_6^{LlR2l1})) + 2v_R (\kappa_+^2 v_R^2 (C_6^{21RrRr} - C_6^{RrR21r})$$

$$+ \kappa_2^4 (C_6^{2121Rr} + C_6^{2211Rr}) + \kappa_1^4 (C_6^{2121Rr} + C_6^{2211Rr} - C_6^{2R21r1}) + \kappa_2^2 \kappa_1^2 (2C_6^{2121Rr} - C_6^{2R21r1})$$

$$+ \kappa_2^3 \kappa_1 (2C_6^{2141Rr} + C_6^{2211Rr} + 2C_6^{2R21r1} + C_6^{41R2r1}) + \kappa_2 \kappa_1^3 (2C_6^{2141Rr} + C_6^{2211Rr} - C_6^{41R2r1}))), \quad (6.67)$$

$$(\Delta m_{+,\phi^6}^2)_{14} = -\frac{1}{8\sqrt{2}\kappa_+ \Lambda^2} [v_L (\kappa_+^2 v_R^2 (C_6^{21LlRr} - 2C_6^{RrL21l}) + 2(\kappa_2^4 (C_6^{2121Ll} + C_6^{2211Ll}) + \kappa_1^4 (C_6^{2121Ll} + C_6^{2211Ll}$$

$$- C_6^{2L21l1}) + \kappa_2^2 \kappa_1^2 (2C_6^{2121Ll} - C_6^{2L21l1}) + \kappa_2^3 \kappa_1 (2C_6^{2141Ll} + C_6^{2211Ll} + 2C_6^{2L21l1} + C_6^{41L2l1})$$

$$\begin{aligned}
 & +\kappa_2\kappa_1^3(2C_6^{2141Ll}+C_6^{2211Ll}-C_6^{41L2l1})))+2v_L^2v_R(\kappa_2\kappa_1(C_6^{L4Lr1l}+C_6^{LlR2l1}+2C_6^{LlR4l1}) \\
 & +\kappa_1^2(2C_6^{LlL4r1}-C_6^{LlR2l1}+C_6^{LlR4l1})+\kappa_2^2C_6^{LlR2l1})+2\kappa_+^2v_L^3(C_6^{21LlLl}-C_6^{LlL2l1}) \\
 & +2\kappa_1v_R(v_R^2(\kappa_1C_6^{RrL4r1}-\kappa_2(C_6^{R4Rl1r}+C_6^{RrL2r1}-2C_6^{RrR4l1}))+\kappa_2^3(2C_6^{21L2r1}+C_6^{21R4l1} \\
 & -2C_6^{41L2r3}+C_6^{41R2l1}))+2\kappa_1^3C_6^{21L4r1}+2\kappa_2^2\kappa_1C_6^{21L4r1}+\kappa_2\kappa_1^2(3C_6^{21R4l1}-C_6^{41R2l1}))), \quad (6.68)
 \end{aligned}$$

$$(\Delta m_{+,\phi^6}^2)_{22} = \frac{1}{4\kappa_+^2\Lambda^2} [v_L^2(v_R^2(\kappa_1^2C_6^{21LlRr}+2\kappa_2\kappa_1C_6^{41LlRr}+\kappa_2^2(C_6^{LlR2l1r}+C_6^{RrL2l1}))+\kappa_1^4(2C_6^{2121Ll} \quad (6.69)$$

$$\begin{aligned}
 & +C_6^{2211Ll}))+4\kappa_2\kappa_1^3C_6^{2141Ll}+\kappa_2^2\kappa_1^2(C_6^{2211Ll}+2C_6^{23Ll41}+C_6^{2L2l1l}+4C_6^{4141Ll}+2C_6^{41L2l1}))+\kappa_2^4C_6^{2L2l1l}) \\
 & +2\kappa_1v_Lv_R(\kappa_2\kappa_1^2C_6^{21L2r1}+\kappa_1^3(C_6^{21L4r1}+C_6^{21R4l1}))+\kappa_2^2\kappa_1(2C_6^{2L41r3}-C_6^{41l2r1}+2C_6^{41L41r}+C_6^{41R2l1} \\
 & +2C_6^{41R4l1}))+\kappa_2^3C_6^{41L2r3}))+v_L^4(\kappa_1^2(C_6^{\{21\}\{LlLl\}}+C_6^{\{21\}\{LlLl\}})+\kappa_2^2(C_6^{\{21\}\{LlLl\}}+C_6^{LlL2l1}))+2\kappa_2\kappa_1C_6^{41LlLl}) \\
 & +v_R^2(\kappa_1^4(2C_6^{2121Rr}+C_6^{2211Rr}))+4\kappa_2\kappa_1^3C_6^{2141Rr}+\kappa_2^2\kappa_1^2(C_6^{2211Rr}+2C_6^{23Rr41}+C_6^{2R21r1}+4C_6^{4141Rr} \\
 & +2C_6^{41R2r1}))+\kappa_2^4C_6^{2R21r1}))+v_R^4(\kappa_1^2(C_6^{\{21\}\{RrRr\}}+C_6^{21RrRr}))+\kappa_2^2(C_6^{\{21\}\{RrRr\}}+C_6^{RrR21r}))+2\kappa_2\kappa_1C_6^{41RrRr}) \\
 & +3(\kappa_1^6(C_6^{\{21\}\{2121\}}+C_6^{212121}))+\kappa_2^6(C_6^{\{21\}\{2121\}}+C_6^{212121}))+\kappa_2^2\kappa_1^4(C_6^{\{21\}\{2121\}}+2C_6^{414121}) \\
 & +\kappa_2^4\kappa_1^2(C_6^{\{21\}\{2121\}}+2C_6^{414121}))+2\kappa_2\kappa_1^5C_6^{412121}+2\kappa_2^5\kappa_1C_6^{412121}+4\kappa_2^3\kappa_1^3(C_6^{414123}+C_6^{414141}))), \quad (6.70)
 \end{aligned}$$

$$(\Delta m_{+,\phi^6}^2)_{23} = \frac{1}{4\sqrt{2}\kappa_+\Lambda^2} \times [v_L(v_R^2(\kappa_1^2(-C_6^{R4Rl1r}+2C_6^{RrL4r1}))+\kappa_2^2C_6^{RrL2r1}+\kappa_2\kappa_1(-2C_6^{RrL2r1}+C_6^{RrL4r1}+2C_6^{RrR4l1}))) \quad (6.70)$$

$$\begin{aligned}
 & +\kappa_1(\kappa_2\kappa_1^2(-2C_6^{21L2r1}+C_6^{21R4l1}+C_6^{41R2l1}))-2\kappa_1^3C_6^{21L4r1}+\kappa_2^3(C_6^{21R4l1}-2C_6^{41L2r3}+C_6^{41R2l1}) \\
 & -2\kappa_2^2\kappa_1C_6^{41l2r1}))+\kappa_1v_L^3(\kappa_1(C_6^{L4Lr1l}-2C_6^{LlL4r1}+C_6^{LlR2l1}))+\kappa_2C_6^{LlR4l1}) \\
 & +(\kappa_1-\kappa_2)\kappa_2v_R(\kappa_2\kappa_1(C_6^{2211Rr}-C_6^{2R21r1}-C_6^{41R2r1}))+\kappa_1^2(C_6^{2211Rr}+C_6^{41R2r1}))- \kappa_2^2C_6^{2R21r1})), \quad (6.71)
 \end{aligned}$$

$$(\Delta m_{+,\phi^6}^2)_{33} = \frac{1}{8\Lambda^2} \times [v_L^2v_R(\kappa_1^2(-C_6^{L4Lr1l}+2C_6^{LlR4l1}))+\kappa_2\kappa_1(2C_6^{LlL4r1}-2C_6^{LlR2l1}+C_6^{LlR4l1}))+\kappa_2^2C_6^{LlR2l1}) \quad (6.72)$$

$$\begin{aligned}
 & +(\kappa_1-\kappa_2)\kappa_2v_L(\kappa_2\kappa_1(C_6^{2211Ll}-C_6^{2L2l1l}-C_6^{41L2l1}))+\kappa_1^2(C_6^{2211Ll}+C_6^{41L2l1}))- \kappa_2^2C_6^{2L2l1l}) \\
 & +v_R(\kappa_1v_R^2(\kappa_1(C_6^{R4Rl1r}+C_6^{RrL2r1}-2C_6^{RrR4l1}))+\kappa_2C_6^{RrL4r1}))+\kappa_2^4C_6^{21L2r1}+\kappa_1^4(C_6^{21L2r1}-2C_6^{21R4l1}) \\
 & +\kappa_2\kappa_1^3(C_6^{21L4r1}+C_6^{41l2r1}))+\kappa_2^3\kappa_1(C_6^{21L4r1}+C_6^{41l2r1}))+2\kappa_2^2\kappa_1^2(C_6^{41L2r3}-C_6^{41R2l1}))), \quad (6.73)
 \end{aligned}$$

$$(\Delta m_{+,\phi^6}^2)_{44} = \frac{1}{8\Lambda^2} \times [v_L^2(3C_6^{LlLlRr}v_R^2+\kappa_1^2(C_6^{21LlRr}+2(C_6^{LlR21r}-C_6^{Lr21rl}+C_6^{RrL2l1}))+\kappa_1^2(C_6^{21LlRr}+4C_6^{Lr4l1r}) \quad (6.73)$$

$$\begin{aligned}
 & +\kappa_+^2 \mathcal{C}_6^{LlR2l1}) + 6\mathcal{C}_6^{LlLlLl} v_L^4 + v_R^2 (\kappa_1^2 (\mathcal{C}_6^{21LlRr} - 2\mathcal{C}_6^{LR4lr1} + \mathcal{C}_6^{RrL2l1}) + 2\kappa_2 \kappa_1 \mathcal{C}_6^{41LlRr} + \kappa_2^2 (\mathcal{C}_6^{LlR21r} \\
 & + \mathcal{C}_6^{LR21rl}) + \mathcal{C}_6^{RrRrLl} v_R^4 + \kappa_1^4 \mathcal{C}_6^{2121Ll} + \kappa_2^4 \mathcal{C}_6^{2121Ll} + 2\kappa_1 \kappa_2^3 \mathcal{C}_6^{2141Ll} + 2\kappa_1^3 \kappa_2 \mathcal{C}_6^{2141Ll} + \kappa_1^4 \mathcal{C}_6^{2211Ll} \\
 & + \kappa_2^4 \mathcal{C}_6^{2211Ll} + 2\kappa_1^2 \kappa_2^2 \mathcal{C}_6^{23Ll41} + \kappa_1^4 \mathcal{C}_6^{2L21l1} + \kappa_2^4 \mathcal{C}_6^{2L21l1} + 4\kappa_1^2 \kappa_2^2 \mathcal{C}_6^{4141Ll} + 2\kappa_1 \kappa_2 \kappa_+^2 \mathcal{C}_6^{41L2l1}]. \quad (6.74)
 \end{aligned}$$

Doubly charged scalar mass matrix.

$$\begin{aligned}
 (\Delta m_{++,\phi^6}^2)_{11} = & \frac{1}{4\Lambda^2} [v_L^2 (\mathcal{C}_6^{RrLLll} v_R^2 + 4\kappa_1 \kappa_2 \mathcal{C}_6^{41LLll}) + 2\kappa_2 v_L v_R (\kappa_2 (\mathcal{C}_6^{L4Lrl1} + \mathcal{C}_6^{LR2LL1}) + \kappa_1 \mathcal{C}_6^{LR4ll1}) \\
 & + \mathcal{C}_6^{LlLlLl} v_L^4 + \kappa_2^2 (\mathcal{C}_6^{LR21rl} v_R^2 + \kappa_2^2 (\mathcal{C}_6^{2121Ll} + \mathcal{C}_6^{2211Ll}) + 2\kappa_1 \kappa_2 \mathcal{C}_6^{2141Ll} \\
 & + \kappa_1^2 (\mathcal{C}_6^{23Ll41} + \mathcal{C}_6^{2L21l1} + 2(\mathcal{C}_6^{4141Ll} + \mathcal{C}_6^{41L2l1})))], \quad (6.75)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{++,\phi^6}^2)_{12} = & \frac{1}{4\Lambda^2} [v_L v_R (\mathcal{C}_6^{lRRRrr} v_R^2 + 2(\kappa_1^2 \mathcal{C}_6^{21LLrr} + \kappa_2^2 \mathcal{C}_6^{21LLrr} + \kappa_2 \kappa_1 (\mathcal{C}_6^{41RRll} + \mathcal{C}_6^{LR4lr1}))) + \mathcal{C}_6^{LlLLrr} v_L^3 v_R \\
 & + \kappa_1 v_L^2 (\kappa_1 (\mathcal{C}_6^{L4Lrl1} + \mathcal{C}_6^{LR2LL1}) + \kappa_2 \mathcal{C}_6^{LR4ll1}) + v_R^2 (\kappa_2^2 \mathcal{C}_6^{LR2rr1} + \kappa_2 \kappa_1 \mathcal{C}_6^{LR4rr1} + \kappa_1^2 \mathcal{C}_6^{R4Rlrl1}) \\
 & + \kappa_2 (\kappa_2^2 \kappa_1 \mathcal{C}_6^{21L2r1} + \kappa_2^3 (\mathcal{C}_6^{21L4r1} + \mathcal{C}_6^{21R4l1}) + \kappa_2 \kappa_1^2 (\mathcal{C}_6^{2L41r3} + \mathcal{C}_6^{41L41r} + \mathcal{C}_6^{41R2l1} + \mathcal{C}_6^{41R4l1}) \\
 & + \kappa_1^3 \mathcal{C}_6^{41L2r3})], \quad (6.76)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{++,\phi^6}^2)_{22} = & \frac{1}{4\Lambda^2} [v_L^2 (\mathcal{C}_6^{LlRRrr} v_R^2 + \kappa_1^2 \mathcal{C}_6^{LR21rl}) + 2v_L v_R (\kappa_1^2 \mathcal{C}_6^{LR2rr1} + \kappa_2 \kappa_1 \mathcal{C}_6^{LR4rr1} + \kappa_2^2 \mathcal{C}_6^{R4Rlrl1}) \\
 & + 4\kappa_1 \kappa_2 \mathcal{C}_6^{41RRrr} v_R^2 + \mathcal{C}_6^{RrRRrr} v_R^4 + \kappa_2^2 (\kappa_2^2 (\mathcal{C}_6^{2121Rr} + \mathcal{C}_6^{2211Rr}) + 2\kappa_2 \kappa_1 \mathcal{C}_6^{2141Rr} \\
 & + \kappa_1^2 (\mathcal{C}_6^{23Rr41} + \mathcal{C}_6^{2R21r1} + 2(\mathcal{C}_6^{4141Rr} + \mathcal{C}_6^{41R2r1})))]. \quad (6.77)
 \end{aligned}$$

$\phi^2 X^2$ and $\phi^4 D^2$ operators: gauge field redefinitions and spectrum.

The gauge-kinetic terms, eq. (5.1), get modified in presence of $\phi^2 X^2$ operators as:

$$\begin{aligned}
 \mathcal{L}_{\text{gauge,kin}}^{(4)+(6)} = & - \left(\partial_\mu W_{L\nu}^- \partial_\mu W_{R\nu}^- \right) \begin{pmatrix} 1 - \frac{2\Theta_{W_{LL}}}{\Lambda^2} & -\frac{2\Theta_{W_{LR}}}{\Lambda^2} \\ -\frac{2\Theta_{W_{RL}}}{\Lambda^2} & 1 - \frac{2\Theta_{W_{RR}}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial^\mu W_L^{+\nu} \\ \partial^\mu W_R^{+\nu} \end{pmatrix} \\
 & - \frac{1}{2} \begin{pmatrix} \partial_\mu W_{3L\nu} \\ \partial_\mu W_{3R\nu} \\ \partial_\mu B_\nu \end{pmatrix}^T \begin{pmatrix} 1 - \frac{2\Theta_{3L3L}}{\Lambda^2} & -\frac{2\Theta_{3L3R}}{\Lambda^2} & -\frac{2\Theta_{3LB}}{\Lambda^2} \\ -\frac{2\Theta_{3L3R}}{\Lambda^2} & 1 - \frac{2\Theta_{3R3R}}{\Lambda^2} & -\frac{2\Theta_{3RB}}{\Lambda^2} \\ -\frac{2\Theta_{3LB}}{\Lambda^2} & -\frac{2\Theta_{3RB}}{\Lambda^2} & 1 - \frac{2\Theta_{BB}}{\Lambda^2} \end{pmatrix} \begin{pmatrix} \partial^\mu W_{3L}^\nu \\ \partial^\mu W_{3R}^\nu \\ \partial^\mu B^\nu \end{pmatrix}.
 \end{aligned}$$

Here, the parameters are given as:

$$\begin{aligned}
 \Theta_{W_{LL}} &= \kappa_+^2 \mathcal{C}_{\phi W_L}^{21} + 2\kappa_1 \kappa_2 (\mathcal{C}_{\phi W_L}^{23} + \mathcal{C}_{\phi W_L}^{41}) + v_L^2 \mathcal{C}_{\Delta W}^{LlW_L W_L} + v_R^2 \mathcal{C}_{\Delta W}^{RrW_L W_L}, \\
 \Theta_{W_{LR}} &= \kappa_+^2 \mathcal{C}_{\phi W_L W_R}^{23} + \kappa_1 \kappa_2 \mathcal{C}_{\phi W_L W_R}^{21} + \kappa_1^2 \mathcal{C}_{\phi W_L W_R}^{41}, \\
 \Theta_{W_{RL}} &= \kappa_+^2 \mathcal{C}_{\phi W_L W_R}^{41} + \kappa_1 \kappa_2 \mathcal{C}_{\phi W_L W_R}^{21} + \kappa_1^2 \mathcal{C}_{\phi W_L W_R}^{23}, \\
 \Theta_{W_{RR}} &= \kappa_+^2 \mathcal{C}_{\phi W_R}^{21} + 2\kappa_1 \kappa_2 (\mathcal{C}_{\phi W_R}^{23} + \mathcal{C}_{\phi W_R}^{41}) + v_L^2 \mathcal{C}_{\Delta W}^{LlW_R W_R} + v_R^2 \mathcal{C}_{\Delta W}^{RrW_R W_R}, \\
 \Theta_{3L3L} &= \kappa_+^2 \mathcal{C}_{\phi W_L}^{21} + 2\kappa_1 \kappa_2 (\mathcal{C}_{\phi W_L}^{23} + \mathcal{C}_{\phi W_L}^{41}) + v_L^2 (\mathcal{C}_{\Delta W}^{LlW_L W_L} - \mathcal{C}_{\Delta W}^{LW_L lW_L}) + v_R^2 \mathcal{C}_{\Delta W}^{RrW_L W_L}, \\
 \Theta_{3L3R} &= \frac{1}{2} \kappa_+^2 \mathcal{C}_{\phi W_L W_R}^{21} + \kappa_1 \kappa_2 (\mathcal{C}_{\phi W_L W_R}^{23} + \mathcal{C}_{\phi W_L W_R}^{41}) - \frac{1}{2} v_L v_R (\mathcal{C}_{\Delta W}^{LW_R rW_L} + \mathcal{C}_{\Delta W}^{RW_L lW_R}), \\
 \Theta_{3R3R} &= \kappa_+^2 \mathcal{C}_{\phi W_R}^{21} + 2\kappa_1 \kappa_2 (\mathcal{C}_{\phi W_R}^{23} + \mathcal{C}_{\phi W_R}^{41}) + v_R^2 (\mathcal{C}_{\Delta W}^{RrW_R W_R} - \mathcal{C}_{\Delta W}^{RW_R rW_R}) + v_L^2 \mathcal{C}_{\Delta W}^{LlW_R W_R}, \\
 \Theta_{3LB} &= \frac{1}{2} \kappa_+^2 \mathcal{C}_{\phi W_L B}^{21} - \frac{1}{2} v_L^2 \mathcal{C}_{\Delta W_L B}^{Ll}, \\
 \Theta_{3RB} &= \frac{1}{2} \kappa_+^2 \mathcal{C}_{\phi W_R B}^{21} - \frac{1}{2} v_R^2 \mathcal{C}_{\Delta W_R B}^{Rr}, \\
 \Theta_{BB} &= \kappa_+^2 \mathcal{C}_{\phi B}^{21} + 2\kappa_1 \kappa_2 (\mathcal{C}_{\phi B}^{23} + \mathcal{C}_{\phi B}^{41}) + v_L^2 \mathcal{C}_{\Delta B}^{Ll} + v_R^2 \mathcal{C}_{\Delta B}^{Rr}. \quad (6.78)
 \end{aligned}$$

Now similar to the scalar sector, we need to redefine the gauge fields such that the modified gauge kinetic lagrangian can be brought into the canonical form. The redefined gauge fields are written as:

$$\begin{aligned}
 W_L^{\pm\mu} &\rightarrow \left(1 + \frac{\Theta_{W_{LL}}}{\Lambda^2}\right) W_L^{\pm\mu} + \frac{\Theta_{W_{LR}}}{\Lambda^2} W_R^{\pm\mu}, \\
 W_R^{\pm\mu} &\rightarrow \left(1 + \frac{\Theta_{W_{RR}}}{\Lambda^2}\right) W_R^{\pm\mu} + \frac{\Theta_{W_{RL}}}{\Lambda^2} W_L^{\pm\mu}, \\
 W_{3L}^\mu &\rightarrow \left(1 + \frac{\Theta_{3L3L}}{\Lambda^2}\right) W_{3L}^\mu + \frac{\Theta_{3L3R}}{\Lambda^2} W_{3R}^\mu + \frac{\Theta_{3LB}}{\Lambda^2} B^\mu, \\
 W_{3R}^\mu &\rightarrow \left(1 + \frac{\Theta_{3R3R}}{\Lambda^2}\right) W_{3R}^\mu + \frac{\Theta_{3L3R}}{\Lambda^2} W_{3L}^\mu + \frac{\Theta_{3RB}}{\Lambda^2} B^\mu, \\
 B^\mu &\rightarrow \left(1 + \frac{\Theta_{BB}}{\Lambda^2}\right) B^\mu + \frac{\Theta_{3LB}}{\Lambda^2} W_{3L}^\mu + \frac{\Theta_{3RB}}{\Lambda^2} W_{3R}^\mu.
 \end{aligned} \tag{6.79}$$

After incorporating the redefined gauge fields in the renormalizable gauge kinetic terms, the gauge boson masses are shifted. The shifts in the charged and neutral gauge boson mass matrices can be recast in the following forms respectively:

$$\begin{aligned}
 (\Delta m_{W^\pm}^2)_{11}^{\phi^2 X^2} &= \frac{g^2}{\Lambda^2} \left(\frac{1}{2} (\kappa_+^2 + 2v_L^2) \Theta_{W_{LL}} - \kappa_1 \kappa_2 \Theta_{W_{RL}} \right), \\
 (\Delta m_{W^\pm}^2)_{12}^{\phi^2 X^2} &= \frac{g^2}{\Lambda^2} \left(\frac{1}{4} \kappa_+^2 (\Theta_{W_{LR}} + \Theta_{W_{RL}}) + \frac{1}{2} (v_L^2 \Theta_{W_{LR}} + v_R^2 \Theta_{W_{RL}}) - \frac{1}{2} \kappa_1 \kappa_2 (\Theta_{W_{LL}} + \Theta_{W_{RR}}) \right), \\
 (\Delta m_{W^\pm}^2)_{22}^{\phi^2 X^2} &= \frac{g^2}{\Lambda^2} \left(\frac{1}{2} (\kappa_+^2 + 2v_R^2) \Theta_{W_{RR}} - \kappa_1 \kappa_2 \Theta_{W_{LR}} \right),
 \end{aligned} \tag{6.80}$$

and

$$\begin{aligned}
 (\Delta m_{W_{3B}}^2)_{11}^{\phi^2 X^2} &= \frac{g}{\Lambda^2} \left(v_L^2 (-2\tilde{g}\Theta_{3LB} + 2g\Theta_{3L3L}) + \frac{g}{2} \kappa_+^2 (\Theta_{3L3L} - \Theta_{3L3R}) \right), \\
 (\Delta m_{W_{3B}}^2)_{12}^{\phi^2 X^2} &= \frac{g}{\Lambda^2} \left(v_L^2 (-\tilde{g}\Theta_{3RB} + g\Theta_{3L3R}) + v_R^2 (-\tilde{g}\Theta_{3LB} + g\Theta_{3L3R}) \right. \\
 &\quad \left. - \frac{g}{4} \kappa_+^2 (\Theta_{3L3L} + \Theta_{3R3R} - 2\Theta_{3L3R}) \right), \\
 (\Delta m_{W_{3B}}^2)_{13}^{\phi^2 X^2} &= \frac{1}{\Lambda^2} \left(\frac{g^2}{4} \kappa_+^2 (\Theta_{3LB} - \Theta_{3RB}) + (\tilde{g}^2 v_R^2 + (g^2 + \tilde{g}^2) v_L^2) \Theta_{3LB} \right. \\
 &\quad \left. - g\tilde{g} (v_L^2 (\Theta_{BB} + \Theta_{3L3L}) + v_R^2 \Theta_{3L3R}) \right), \\
 (\Delta m_{W_{3B}}^2)_{22}^{\phi^2 X^2} &= \frac{g}{\Lambda^2} \left(v_R^2 (-2\tilde{g}\Theta_{3RB} + 2g\Theta_{3R3R}) + \frac{g}{2} \kappa_+^2 (\Theta_{3R3R} - \Theta_{3L3R}) \right), \\
 (\Delta m_{W_{3B}}^2)_{23}^{\phi^2 X^2} &= \frac{1}{\Lambda^2} \left(\frac{g^2}{4} \kappa_+^2 (\Theta_{3RB} - \Theta_{3LB}) + (\tilde{g}^2 v_L^2 + (g^2 + \tilde{g}^2) v_R^2) \Theta_{3RB} \right. \\
 &\quad \left. - g\tilde{g} (v_R^2 (\Theta_{BB} + \Theta_{3R3R}) + v_L^2 \Theta_{3L3R}) \right), \\
 (\Delta m_{W_{3B}}^2)_{33}^{\phi^2 X^2} &= \frac{1}{\Lambda^2} (2\tilde{g}^2 (v_L^2 + v_R^2) \Theta_{BB} - 2g\tilde{g} (v_L^2 \Theta_{3LB} + v_R^2 \Theta_{3RB})).
 \end{aligned} \tag{6.81}$$

There will be an additional contribution to the gauge boson masses due to the $\phi^4 D^2$ operators, and their effects are captured as:

$$\begin{aligned}
 (\Delta m_{W^\pm}^2)_{11}^{\phi^4 D^2} = & \frac{g^2}{8\Lambda^2} \left[\mathcal{C}_{\phi D}^{(L)rR(l)} v_L^2 v_R^2 + 2v_L^2 \kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + v_L^2 \kappa_+^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 2v_L^2 \kappa_+^2 \mathcal{C}_{\phi D}^{(L)11(l)} \right. \\
 & + 2v_L^2 \kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} + v_L^2 \kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + v_L^2 \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} + 2\kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R \\
 & + 2v_L^4 (\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + \mathcal{C}_{\phi D}^{L(l)L(l)} + \mathcal{C}_{\phi D}^{Ll(Ll)}) + v_R^2 \kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + v_R^2 \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \\
 & - 4\kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + \kappa_+^2 (\kappa_1 + \kappa_2)^2 \mathcal{C}_{\phi D}^{(11)(22)} + \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + (\kappa_1^4 + \kappa_2^4) \mathcal{C}_{\phi D}^{(2)(1)21} \\
 & \left. - 4\kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)1(1)} + 8\kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \quad (6.82)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W^\pm}^2)_{12}^{\phi^4 D^2} = & \frac{g^2}{8\Lambda^2} \left[v_L v_R \left(-2\kappa_2 \kappa_1 \mathcal{C}_{\phi D}^{(2)rL(1)} + \kappa_+^2 \mathcal{C}_{\phi D}^{(L)11(r)} + \kappa_+^2 \mathcal{C}_{\phi D}^{(L)12(r)} + \kappa_+^2 \mathcal{C}_{\phi D}^{(R)11(l)} \right) \right. \\
 & - v_L^2 \left(\kappa_2 \left(\kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} + 2\kappa_1 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) - \mathcal{C}_{\phi D}^{L(r)L(r)} v_R^2 \right) \\
 & + \kappa_1 \kappa_2 \left(v_R^2 (\mathcal{C}_{\phi D}^{Rr(1)1(1)} - 2\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} - \mathcal{C}_{\phi D}^{Rr(2)(1)}) - 4\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{(11)(22)} \right) \\
 & \left. + \kappa_1 \kappa_2 \kappa_+^2 \left(2\mathcal{C}_{\phi D}^{2(1)1(1)} - 2\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} - \mathcal{C}_{\phi D}^{(2)(1)21} - 4\mathcal{C}_{\phi D}^{2(1)2(1)} + 2\mathcal{C}_{\phi D}^{1(1)1(1)} - 2\mathcal{C}_{\phi D}^{(11)(22)} \right) \right], \quad (6.83)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W^\pm}^2)_{22}^{\phi^4 D^2} = & \frac{g^2}{8\Lambda^2} \left[2\kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(\mathcal{C}_{\phi D}^{(R)lL(r)} v_R^2 + \kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) \right. \\
 & + v_R^2 \kappa_+^2 \left(2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{21(R)(r)} + 2\mathcal{C}_{\phi D}^{(R)11(r)} \right) + v_R^2 \left(-2\kappa_2^2 \mathcal{C}_{\phi D}^{R(r)1(1)} + \kappa_2^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \\
 & + 2v_R^4 \left(\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + \mathcal{C}_{\phi D}^{R(r)R(r)} + \mathcal{C}_{\phi D}^{Rr(Rr)} \right) + 2\kappa_1^2 \kappa_2^2 \left(-2\mathcal{C}_{\phi D}^{1(1)1(1)} - 2\mathcal{C}_{\phi D}^{2(1)1(1)} + \mathcal{C}_{\phi D}^{(2)(1)21} \right) \\
 & \left. + (\kappa_1 + \kappa_2)^2 \kappa_+^2 \mathcal{C}_{\phi D}^{(11)(22)} + \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2\kappa_+^4 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \quad (6.84)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{11}^{\phi^4 D^2} = & \frac{g^2}{8\Lambda^2} \left[v_L^2 \left(4\mathcal{C}_{\phi D}^{(L)rR(l)} v_R^2 + 4\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + 4\kappa_1^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 8\kappa_2^2 \mathcal{C}_{\phi D}^{(L)11(l)} + 4\kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} \right) \right. \\
 & + v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + 2\kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + 4v_L^4 \left(\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + 4\mathcal{C}_{\phi D}^{L(l)L(l)} \right) \\
 & + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) + \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \\
 & \left. + (\kappa_1^4 + \kappa_2^4) \left(\mathcal{C}_{\phi D}^{(2)(1)21} - 2\mathcal{C}_{\phi D}^{1(1)1(1)} - 2\mathcal{C}_{\phi D}^{2(1)1(1)} + 4\mathcal{C}_{\phi D}^{2(1)2(1)} + 4\mathcal{C}_{\phi D}^{(11)(22)} \right) \right], \quad (6.85)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{12}^{\phi^4 D^2} = & \frac{g^2}{8\Lambda^2} \left[2\kappa_2^2 v_L v_R \left(2(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - \mathcal{C}_{\phi D}^{(2)rL(1)} \right) - \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \right. \\
 & - v_L^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) + v_R^2 \left(2\kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} - \kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} - \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) \\
 & + (\kappa_1^4 + \kappa_2^4) \left(-\mathcal{C}_{\phi D}^{(2)(1)21} + 2\mathcal{C}_{\phi D}^{1(1)1(1)} + 2\mathcal{C}_{\phi D}^{2(1)1(1)} - 4\mathcal{C}_{\phi D}^{(11)(22)} - 4\mathcal{C}_{\phi D}^{2(1)2(1)} \right) \\
 & \left. - v_L^2 \left(-4\mathcal{C}_{\phi D}^{L(r)L(r)} v_R^2 + 2\kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} \right) \right], \quad (6.86)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{13}^{\phi^4 D^2} = & -\frac{g\tilde{g}}{4\Lambda^2} \left[v_L^2 (2v_R^2 (\mathcal{C}_{\phi D}^{L(r)L(r)} + \mathcal{C}_{\phi D}^{(L)rR(l)}) + 2\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + 2\kappa_1^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 4\kappa_2^2 \mathcal{C}_{\phi D}^{(L)11(l)}) \right. \\
 & + \kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} + 2\kappa_2^2 v_L v_R (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) + 2v_L^4 (\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + 4\mathcal{C}_{\phi D}^{L(l)L(l)}) \\
 & \left. + \kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R^2 \right], \quad (6.87)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{22}^{\phi^4 D^2} = & \frac{g^2}{8\Lambda^2} \left[2\kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + v_L^2 \left(4\mathcal{C}_{\phi D}^{(R)lL(r)} v_R^2 + \kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right) \right. \\
 & + \kappa_+^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + (\kappa_1^4 + \kappa_2^4) \left(\mathcal{C}_{\phi D}^{(2)(1)21} - 2\mathcal{C}_{\phi D}^{1(1)1(1)} - 2\mathcal{C}_{\phi D}^{2(1)1(1)} + 4\mathcal{C}_{\phi D}^{2(1)2(1)} + 4\mathcal{C}_{\phi D}^{(11)(22)} \right) \\
 & + v_R^2 \left(4\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 4\kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} + 8\kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} - 4\kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} \right) \\
 & \left. + v_R^2 \left(\kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right) + 4v_R^4 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 4\mathcal{C}_{\phi D}^{R(r)R(r)}) \right], \quad (6.88)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{23}^{\phi^4 D^2} = & -\frac{g\tilde{g}}{4\Lambda^2} \left[2\kappa_2^2 v_L v_R \left(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)} \right) + v_R^2 \left(4\kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} - \kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} \right) \right. \\
 & + v_R^2 \left(2v_R^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 4\mathcal{C}_{\phi D}^{R(r)R(r)}) + 2\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 2\kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} \right) \\
 & \left. + v_L^2 \left(2v_R^2 (\mathcal{C}_{\phi D}^{L(r)L(r)} + \mathcal{C}_{\phi D}^{(R)lL(r)}) - \kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} \right) \right], \quad (6.89)
 \end{aligned}$$

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{33}^{\phi^4 D^2} = & \frac{\tilde{g}^2}{2\Lambda^2} \left[2\kappa_2^2 v_L v_R \left(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)} \right) + v_L^4 \left(\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + 4\mathcal{C}_{\phi D}^{L(l)L(l)} \right) \right. \\
 & + v_L^2 \left(v_R^2 (2\mathcal{C}_{\phi D}^{L(r)L(r)} + \mathcal{C}_{\phi D}^{(L)rR(l)} + \mathcal{C}_{\phi D}^{(R)lL(r)}) + \kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 2\kappa_2^2 \mathcal{C}_{\phi D}^{L(l)11(l)} \right) \\
 & \left. + v_R^2 \left(v_R^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 4\mathcal{C}_{\phi D}^{R(r)R(r)}) + \kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} + 2\kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} \right) \right]. \quad (6.90)
 \end{aligned}$$

Once we incorporate the contributions from the dimension-6 operators, then the full gauge boson mass matrices can be written as:

$$\mathcal{M}_{W^\pm}^2 = \begin{pmatrix} (\tilde{m}_W^2)_{11} + (\Delta m_{W^\pm}^2)_{11} & (\tilde{m}_W^2)_{12} + (\Delta m_{W^\pm}^2)_{12} \\ (\tilde{m}_W^2)_{21} + (\Delta m_{W^\pm}^2)_{21} & (\tilde{m}_W^2)_{22} + (\Delta m_{W^\pm}^2)_{22} \end{pmatrix}, \quad (6.91)$$

and

$$\mathcal{M}_{W_3 B}^2 = \begin{pmatrix} (\tilde{m}_0^2)_{11} + (\Delta m_{W_3 B}^2)_{11} & (\tilde{m}_0^2)_{12} + (\Delta m_{W_3 B}^2)_{12} & (\tilde{m}_0^2)_{13} + (\Delta m_{W_3 B}^2)_{13} \\ (\tilde{m}_0^2)_{21} + (\Delta m_{W_3 B}^2)_{21} & (\tilde{m}_0^2)_{22} + (\Delta m_{W_3 B}^2)_{22} & (\tilde{m}_0^2)_{23} + (\Delta m_{W_3 B}^2)_{23} \\ (\tilde{m}_0^2)_{31} + (\Delta m_{W_3 B}^2)_{31} & (\tilde{m}_0^2)_{32} + (\Delta m_{W_3 B}^2)_{32} & (\tilde{m}_0^2)_{33} + (\Delta m_{W_3 B}^2)_{33} \end{pmatrix}, \quad (6.92)$$

where $(\tilde{m}_W^2)_{ij}$, and $(\tilde{m}_0^2)_{ij}$ are the elements of tree level charged and neutral gauge boson mass matrices respectively, see eq. (5.26) and eq. (5.27). Here, the shifted elements of the full mass matrices are:

Charged gauge boson mass matrix.

$$\begin{aligned}
 (\Delta m_{W^\pm}^2)_{11} &= (\Delta m_{W^\pm}^2)_{11}^{\phi^2 X^2} + (\Delta m_{W^\pm}^2)_{11}^{\phi^4 D^2}, \\
 (\Delta m_{W^\pm}^2)_{22} &= (\Delta m_{W^\pm}^2)_{22}^{\phi^2 X^2} + (\Delta m_{W^\pm}^2)_{22}^{\phi^4 D^2}, \\
 (\Delta m_{W^\pm}^2)_{12} &= (\Delta m_{W^\pm}^2)_{21} = (\Delta m_{W^\pm}^2)_{12}^{\phi^2 X^2} + (\Delta m_{W^\pm}^2)_{12}^{\phi^4 D^2}. \quad (6.93)
 \end{aligned}$$

Neutral gauge boson mass matrix.

$$\begin{aligned}
 (\Delta m_{W_3 B}^2)_{11} &= (\Delta m_{W_3 B}^2)_{11}^{\phi^2 X^2} + (\Delta m_{W_3 B}^2)_{11}^{\phi^4 D^2}, \\
 (\Delta m_{W_3 B}^2)_{22} &= (\Delta m_{W_3 B}^2)_{22}^{\phi^2 X^2} + (\Delta m_{W_3 B}^2)_{22}^{\phi^4 D^2}, \\
 (\Delta m_{W_3 B}^2)_{33} &= (\Delta m_{W_3 B}^2)_{33}^{\phi^2 X^2} + (\Delta m_{W_3 B}^2)_{33}^{\phi^4 D^2}, \\
 (\Delta m_{W_3 B}^2)_{12} &= (\Delta m_{W_3 B}^2)_{21} = (\Delta m_{W_3 B}^2)_{12}^{\phi^2 X^2} + (\Delta m_{W_3 B}^2)_{12}^{\phi^4 D^2}, \\
 (\Delta m_{W_3 B}^2)_{13} &= (\Delta m_{W_3 B}^2)_{31} = (\Delta m_{W_3 B}^2)_{13}^{\phi^2 X^2} + (\Delta m_{W_3 B}^2)_{13}^{\phi^4 D^2}, \\
 (\Delta m_{W_3 B}^2)_{23} &= (\Delta m_{W_3 B}^2)_{32} = (\Delta m_{W_3 B}^2)_{23}^{\phi^2 X^2} + (\Delta m_{W_3 B}^2)_{23}^{\phi^4 D^2}. \quad (6.94)
 \end{aligned}$$

The following matrix diagonalizes the modified charged gauge boson mass matrix:

$$\tilde{\mathcal{R}}_a = \begin{pmatrix} \cos \tilde{\xi} & \sin \tilde{\xi} \\ -\sin \tilde{\xi} & \cos \tilde{\xi} \end{pmatrix}, \quad (6.95)$$

where this new “angle” $\tilde{\xi}$ is related to the older one ξ as:

$$\tan 2\tilde{\xi} = \tan 2\xi \left[1 + \left(\frac{(\Delta \mathcal{M}_{W^\pm}^2)_{12}}{(\tilde{M}_W^2)_{12}} - \frac{(\Delta \mathcal{M}_{W^\pm}^2)_{22} - (\Delta \mathcal{M}_{W^\pm}^2)_{11}}{(\tilde{M}_W^2)_{22} - (\tilde{M}_W^2)_{11}} \right) + O\left(\frac{1}{\Lambda^4}\right) \right]. \quad (6.96)$$

After performing the diagonalization, the charged gauge boson masses are given as:

$$\mathcal{M}_{W_1}^2 = m_{W_1}^2 + ((\Delta m_{W^\pm}^2)_{11} \cos^2 \tilde{\xi} - (\Delta m_{W^\pm}^2)_{12} \sin 2\tilde{\xi} + (\Delta m_{W^\pm}^2)_{22} \sin^2 \tilde{\xi}), \quad (6.97)$$

$$\mathcal{M}_{W_2}^2 = m_{W_2}^2 + ((\Delta m_{W^\pm}^2)_{22} \cos^2 \tilde{\xi} + (\Delta m_{W^\pm}^2)_{12} \sin 2\tilde{\xi} + (\Delta m_{W^\pm}^2)_{11} \sin^2 \tilde{\xi}). \quad (6.98)$$

The modified neutral gauge boson mass matrix is diagonalized by a similar rotation matrix $\tilde{\mathcal{R}}_b$ which will be of the similar form as $\mathcal{R}_b$ see eq. (5.29). The only difference is that the earlier rotation angle $\theta_2 \in \mathcal{R}_b$ is replaced by $\tilde{\theta}_2 \in \tilde{\mathcal{R}}_b$ where as the other two angles remain unaltered. The neutral gauge boson masses are given as:

$$\begin{aligned} \mathcal{M}_{Z_1}^2 = & m_{Z_1}^2 + \sin^2 \tilde{\theta}_2 \left[(\Delta m_{W_3B}^2)_{22} \cos^2 \theta_1 + (\Delta m_{W_3B}^2)_{33} \sin^2 \theta_1 - (\Delta m_{W_3B}^2)_{23} \sin 2\theta_1 \right] \\ & + \sin 2\tilde{\theta}_2 \left[\sin \theta_w ((\Delta m_{W_3B}^2)_{23} \cos 2\theta_1 + \frac{1}{2}((\Delta m_{W_3B}^2)_{22} - (\Delta m_{W_3B}^2)_{33}) \sin 2\theta_1) \right. \\ & \quad \left. + (\Delta m_{W_3B}^2)_{13} \sin \theta_w - (\Delta m_{W_3B}^2)_{12} \sqrt{\cos 2\theta_w} \right] \\ & + \cos^2 \tilde{\theta}_2 \left[(\Delta m_{W_3B}^2)_{11} \cos^2 \theta_w - 2 \sin \theta_w ((\Delta m_{W_3B}^2)_{12} \sin \theta_w + (\Delta m_{W_3B}^2)_{13} \sqrt{\cos 2\theta_w}) \right. \\ & \quad \left. + \sin^2 \theta_w ((\Delta m_{W_3B}^2)_{33} \cos^2 \theta_1 + (\Delta m_{W_3B}^2)_{22} \sin^2 \theta_1 + (\Delta m_{W_3B}^2)_{23} \sin 2\theta_1) \right], \quad (6.99) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_{Z_2}^2 = & m_{Z_2}^2 + \cos^2 \tilde{\theta}_2 \left[(\Delta m_{W_3B}^2)_{22} \cos^2 \theta_1 + (\Delta m_{W_3B}^2)_{33} \sin^2 \theta_1 - (\Delta m_{W_3B}^2)_{23} \sin 2\theta_1 \right] \\ & - \sin 2\tilde{\theta}_2 \left[\sin \theta_w ((\Delta m_{W_3B}^2)_{23} \cos 2\theta_1 + \frac{1}{2}((\Delta m_{W_3B}^2)_{22} - (\Delta m_{W_3B}^2)_{33}) \sin 2\theta_1) \right. \\ & \quad \left. + (\Delta m_{W_3B}^2)_{13} \sin \theta_w - (\Delta m_{W_3B}^2)_{12} \sqrt{\cos 2\theta_w} \right] \\ & + \sin^2 \tilde{\theta}_2 \left[(\Delta m_{W_3B}^2)_{11} \cos^2 \theta_w - 2 \sin \theta_w ((\Delta m_{W_3B}^2)_{12} \sin \theta_w + (\Delta m_{W_3B}^2)_{13} \sqrt{\cos 2\theta_w}) \right. \\ & \quad \left. + \sin^2 \theta_w ((\Delta m_{W_3B}^2)_{33} \cos^2 \theta_1 + (\Delta m_{W_3B}^2)_{22} \sin^2 \theta_1 + (\Delta m_{W_3B}^2)_{23} \sin 2\theta_1) \right], \quad (6.100) \end{aligned}$$

$$\mathcal{M}_A^2 = 0. \quad (6.101)$$

Note that even after inclusion of all the corrections due to the effective operators the mass of photon turns out to be zero ($\mathcal{M}_A^2 = 0$) which is expected.

The modified rotation angle $\tilde{\theta}_2$ can be written in terms of θ_2 as

$$\tan 2\tilde{\theta}_2 = \tan 2\theta_2 \left[1 + \left(\frac{p}{q} - \frac{r}{s} \right) + O\left(\frac{1}{\Lambda^4}\right) \right], \quad (6.102)$$

where,

$$\begin{aligned}
 p &= 2\cos 2\theta_w((\Delta m_{W_3B}^2)_{13}\cos\theta_1 + (\Delta m_{W_3B}^2)_{12}\sin\theta_1) - \frac{1}{2}\sin 2\theta_w((\Delta m_{W_3B}^2)_{22} \\
 &\quad + (\Delta m_{W_3B}^2)_{33} - 2(\Delta m_{W_3B}^2)_{11} + ((\Delta m_{W_3B}^2)_{33} - (\Delta m_{W_3B}^2)_{22})\cos 2\theta_1 + 2(\Delta m_{W_3B}^2)_{23}\sin 2\theta_1), \\
 q &= 2\cos 2\theta_w((\tilde{m}_0^2)_{13}\cos\theta_1 + (\tilde{m}_0^2)_{12}\sin\theta_1) \\
 &\quad - \frac{1}{2}\sin 2\theta_w((\tilde{m}_0^2)_{22} + (\tilde{m}_0^2)_{33} - 2(\tilde{m}_0^2)_{11} + ((\tilde{m}_0^2)_{33} - (\tilde{m}_0^2)_{22})\cos 2\theta_1 + 2(\tilde{m}_0^2)_{23}\sin 2\theta_1), \\
 r &= \cos\theta_w(2(\Delta m_{W_3B}^2)_{23}\cos 2\theta_1 + ((\Delta m_{W_3B}^2)_{22} - (\Delta m_{W_3B}^2)_{33})\sin 2\theta_1 \\
 &\quad - 2\sin\theta_w((\Delta m_{W_3B}^2)_{13}\sin\theta_1 - (\Delta m_{W_3B}^2)_{12}\cos\theta_1)), \\
 s &= \cos\theta_w(2(\tilde{m}_0^2)_{23}\cos 2\theta_1 + ((\tilde{m}_0^2)_{22} - (\tilde{m}_0^2)_{33})\sin 2\theta_1 - 2\sin\theta_w((\tilde{m}_0^2)_{13}\sin\theta_1 - (\tilde{m}_0^2)_{12}\cos\theta_1)).
 \end{aligned}$$

Here, we would like to specify that the eigenvalues of the gauge boson masses remain unchanged upto $\mathcal{O}(1/\Lambda^2)$ even if we use the old rotation angles in the rotation matrices. But then the photon mass will have contributions $\mathcal{O}(1/\Lambda^4)$ which needs to be ignored.

$\psi^2\phi^3$ operators: modification in spectrum in the fermion sector.

The $\psi^2\phi^3$ operators generate the additional contributions to the charged fermion masses as:

$$\begin{aligned}
 \Delta m_e &= \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_2 \left(2\kappa_1\kappa_2(\mathcal{C}_{l_L l_R}^{1(41)} + \mathcal{C}_{l_L l_R}^{1(23)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{l_L l_R}^{1(21)} \right) \right. \\
 &\quad + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_1 \left(2\kappa_1\kappa_2(\mathcal{C}_{l_L l_R}^{3(23)} + \mathcal{C}_{l_L l_R}^{3(41)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{l_L l_R}^{3(21)} \right) \right. \\
 &\quad \left. \left. + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_2 (v_L^2 \mathcal{C}_{l_L l_R}^{l3L} + v_L v_R \mathcal{C}_{l_L l_R}^{l3R} + v_R^2 \mathcal{C}_{l_L l_R}^{r3R}) + \kappa_1 (v_L^2 \mathcal{C}_{l_L l_R}^{l1L} + v_L v_R \mathcal{C}_{l_L l_R}^{l1R} + v_R^2 \mathcal{C}_{l_L l_R}^{r1R}) \right] \right] \right] \quad (6.103)
 \end{aligned}$$

$$\begin{aligned}
 \Delta m_u &= \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_1 \left(2\kappa_1\kappa_2(\mathcal{C}_{q_L q_R}^{1(41)} + \mathcal{C}_{q_L q_R}^{1(23)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{q_L q_R}^{1(21)} \right) \right. \\
 &\quad + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_2 \left(2\kappa_1\kappa_2(\mathcal{C}_{q_L q_R}^{3(23)} + \mathcal{C}_{q_L q_R}^{3(41)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{q_L q_R}^{3(21)} \right) \right. \\
 &\quad \left. \left. + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_1 (v_L^2 \mathcal{C}_{q_L q_R}^{L3l} + v_L v_R \mathcal{C}_{q_L q_R}^{L3r} + v_R^2 \mathcal{C}_{q_L q_R}^{R3r}) + \kappa_2 (v_L^2 \mathcal{C}_{q_L q_R}^{L1l} + v_L v_R \mathcal{C}_{q_L q_R}^{L1r} + v_R^2 \mathcal{C}_{q_L q_R}^{R1r}) \right] \right] \right] \quad (6.104)
 \end{aligned}$$

$$\begin{aligned}
 \Delta m_d &= \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_2 \left(2\kappa_1\kappa_2(\mathcal{C}_{q_L q_R}^{1(41)} + \mathcal{C}_{q_L q_R}^{1(23)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{q_L q_R}^{1(21)} \right) \right. \\
 &\quad + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_1 \left(2\kappa_1\kappa_2(\mathcal{C}_{q_L q_R}^{3(23)} + \mathcal{C}_{q_L q_R}^{3(41)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{q_L q_R}^{3(21)} \right) \right. \\
 &\quad \left. \left. + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_2 (v_L^2 \mathcal{C}_{q_L q_R}^{l3L} + v_L v_R \mathcal{C}_{q_L q_R}^{l3R} + v_R^2 \mathcal{C}_{q_L q_R}^{r3R}) + \kappa_1 (v_L^2 \mathcal{C}_{q_L q_R}^{l1L} + v_L v_R \mathcal{C}_{q_L q_R}^{l1R} + v_R^2 \mathcal{C}_{q_L q_R}^{r1R}) \right] \right] \right] \quad (6.105)
 \end{aligned}$$

After incorporating these corrections, the resulting charged fermion mass matrices are written as:

$$\mathcal{M}_e = \frac{1}{\sqrt{2}} (\kappa_1 \tilde{y}_D + \kappa_2 y_D) - \Delta m_e, \quad (6.106)$$

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} (\kappa_2 \tilde{y}_q + \kappa_1 y_q) - \Delta m_u, \quad (6.107)$$

$$\mathcal{M}_d = \frac{1}{\sqrt{2}} (\kappa_1 \tilde{y}_q + \kappa_2 y_q) - \Delta m_d. \quad (6.108)$$

In the process, the Dirac-mass term for neutrinos also achieve correction (Δm_ν^D):

$$\begin{aligned}\Delta m_\nu^D &= \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_1 \left(2\kappa_1\kappa_2(\mathcal{C}_{l_L l_R}^{1(41)} + \mathcal{C}_{l_L l_R}^{1(23)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{l_L l_R}^{1(21)} \right) \right. \\ &\quad + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_2 \left(2\kappa_1\kappa_2(\mathcal{C}_{l_L l_R}^{3(23)} + \mathcal{C}_{l_L l_R}^{3(41)}) + (\kappa_1^2 + \kappa_2^2)\mathcal{C}_{l_L l_R}^{3(21)} \right) \right] \\ &\quad \left. + \frac{1}{2\sqrt{2}\Lambda^2} \left[\kappa_1 (v_L^2 \mathcal{C}_{l_L l_R}^{L3l} + v_L v_R \mathcal{C}_{l_L l_R}^{L3r} + v_R^2 \mathcal{C}_{l_L l_R}^{R3r}) + \kappa_2 (v_L^2 \mathcal{C}_{l_L l_R}^{L1l} + v_L v_R \mathcal{C}_{l_L l_R}^{L1r} + v_R^2 \mathcal{C}_{l_L l_R}^{R1r}) \right] \right],\end{aligned}\quad (6.109)$$

and the resulting Dirac-mass term for the neutrinos is read as:

$$\mathcal{M}_\nu^D = \frac{1}{\sqrt{2}} (\kappa_2 \tilde{y}_D + \kappa_1 y_D) - \Delta m_\nu^D. \quad (6.110)$$

The lepton number violating $\psi^2\phi^3$ operators will provide additional contributions to the Majorana masses for the light (ν) and heavy (N) neutrinos, eq. (5.39), respectively:

$$\begin{aligned}\Delta m_\nu^M &= \frac{1}{\sqrt{2}\Lambda^2} \left[v_L (v_R^2 (\mathcal{C}_{l_L}^{rLr} + \mathcal{C}_{l_L}^{l(Rr)}) + \kappa_1^2 (\mathcal{C}_{l_L}^{2l1} + \mathcal{C}_{l_L}^{l(21)}) + \kappa_2^2 \mathcal{C}_{l_L}^{l(21)} + 2\kappa_1\kappa_2 (\mathcal{C}_{l_L}^{l(23)} + \mathcal{C}_{l_L}^{l(41)})) \right. \\ &\quad \left. + v_L^3 (\mathcal{C}_{l_L}^{lLl} + \mathcal{C}_{l_L}^{l(Ll)}) + \kappa_1 v_R (\kappa_1 \mathcal{C}_{l_L}^{r21} + \kappa_2 (\mathcal{C}_{l_L}^{r23} + \mathcal{C}_{l_L}^{r41})) \right],\end{aligned}\quad (6.111)$$

$$\begin{aligned}\Delta m_N^M &= \frac{1}{\sqrt{2}\Lambda^2} \left[v_L^2 v_R (\mathcal{C}_{l_R}^{lRl} + \mathcal{C}_{l_R}^{r(Ll)}) + \kappa_1 v_L (\kappa_1 \mathcal{C}_{l_R}^{l21} + \kappa_2 (\mathcal{C}_{l_R}^{l23} + \mathcal{C}_{l_R}^{l41})) + v_R (v_R^2 (\mathcal{C}_{l_R}^{rRr} + \mathcal{C}_{l_R}^{r(Rr)}) \right. \\ &\quad \left. + \kappa_1^2 (\mathcal{C}_{l_R}^{r21} + \mathcal{C}_{l_R}^{r(21)}) + \kappa_2^2 \mathcal{C}_{l_R}^{r(21)} + 2\kappa_1\kappa_2 (\mathcal{C}_{l_R}^{r(23)} + \mathcal{C}_{l_R}^{r(41)})) \right].\end{aligned}\quad (6.112)$$

After incorporating the corrections due to the effective operators, the full neutrino mass matrix, eq. (5.40), can be written as:

$$m_\nu = \begin{pmatrix} \sqrt{2}y_M v_L - \Delta m_\nu^M & \mathcal{M}_\nu^D \\ \mathcal{M}_\nu^D & \sqrt{2}y_M v_R - \Delta m_N^M \end{pmatrix}. \quad (6.113)$$

7 Impact of effective operators on observables

7.1 Outline to grab the impact of effective operators

In this paper, the impact of the effective dimension-6 operators is catalogued in a two-fold way: (i) shift in the mass spectrum and (ii) modification in the Feynman vertices. In the earlier sections, we have discussed how the mass spectrum gets modified in the presence of higher dimensional operators and computed the complete spectra for two scenarios — 2HDM- and MLRSM-EFT in detail. We have prepared a schematic drawing, figure 4, to summarise the earlier sections.

In the presence of redefined fields (A'_μ , ϕ'), the effective lagrangian can be written as:

$$\begin{aligned}\mathcal{L}^{(4)+(6)} &= \mathcal{L}_A^{\text{kin}}(A'_\mu) + \mathcal{L}_\phi^{\text{kin}}(\phi', A'_\mu) + \mathcal{L}_\psi^{\text{kin}}(\psi, A'_\mu) + V(\phi') \\ &\quad + \mathcal{L}_{\text{Dirac}}^Y(\phi', \psi) + \mathcal{L}_{\text{Majorana}}^Y(\phi', \psi) + \frac{1}{\Lambda^2} \sum_j \mathcal{C}_j \mathcal{O}_j.\end{aligned}\quad (7.1)$$

Figure 4. Trajectories outlining the impact of effective operators on the spectrum.

But this does not tell the full story. In the following section, we have systematically discussed how the interaction vertices get modified in the presence of these effective operators, and how they affect the low energy observables. To get an idea of how different observables are connected to different parts of the lagrangian (which is a function of shifted fields), we have prepared table 26. We have organised the next part of the paper as follows: first, we have computed all the relevant modified Feynman vertices using the effective lagrangian in eq. (7.1). Then we have estimated the low energy observables, e.g., Weak mixing angle, Fermi constant, ρ and oblique parameters. We have also outlined the possible sources of anomalous magnetic moment and theoretical constraints on the scalar potential to capture the impact of the effective operators. We have further highlighted how different collider processes, e.g., production and decay of heavy BSM particles, get modified within this effective theory framework. Instead of going for a detailed exploration, we have chosen selective channels relevant to the considered models. We have concluded this section by paving the path towards the possible phenomenological explorations of these interesting BSM scenarios in colliders and other experiments. In passing, we would like to mention that Baryon Number Violating operators listed in tables 12 and 20, are not being used in the further analysis.

7.2 Weak mixing angle

The Weak mixing angle is measured using the low energy data of nucleon-neutrino scattering. This scattering is driven by the charged and neutral currents. One can define a ratio

Observables	Effective lagrangian terms
Weak mixing angle (θ_W)	$\mathcal{L}_\psi^{\text{kin}}(\psi, A'_\mu) + \frac{1}{\Lambda^2}[\psi^2 \phi^2 D + \psi^4]$
Fermi constant (G_F)	
ρ parameter	
Oblique parameters (S, T, U)	$\mathcal{L}_A^{\text{kin}}(A'_\mu) + \mathcal{L}_\phi^{\text{kin}}(\phi', A'_\mu) + \frac{1}{\Lambda^2}[\phi^4 D^2 + \phi^2 X^2]$
Magnetic moment	$\mathcal{L}_{\text{Majorana}}^Y(\phi', \psi) + \frac{1}{\Lambda^2}[\psi^2 \phi X]$
Lepton Flavor Violating (LFV) and Lepton Number Violating (LNV) processes	$\mathcal{L}_{\text{Majorana}}^Y(\phi', \psi) + \mathcal{L}_\phi^{\text{kin}}(\phi', A'_\mu) + \frac{1}{\Lambda^2}[\phi^4 D^2 + \psi^2 \phi^3]$
Scalar Quartic couplings	$V(\phi') + \frac{1}{\Lambda^2}[\phi^6]$

Table 26. Low energy observables and their origin in effective lagrangian.

($\hat{R}$) using this scattering information as [45]:

$$\hat{R} = \frac{\sigma^{\nu NC} - \sigma^{\bar{\nu} NC}}{\sigma^{\nu CC} - \sigma^{\bar{\nu} CC}}, \quad (7.2)$$

which can be recast in terms of the Weak mixing angle [45]:

$$\hat{R} = \frac{1}{2} - \sin^2 \bar{\theta}_w. \quad (7.3)$$

Here, $\sigma^{\nu(\bar{\nu})NC}$ and $\sigma^{\nu(\bar{\nu})CC}$ represent the cross section of neutrino (anti-neutrino)-nucleon scattering through neutral (NC) and charged (CC) currents respectively.

2HDM. For 2HDM scenario, the scattering cross-sections ($\sigma^{\nu(\bar{\nu})NC}$, and $\sigma^{\nu(\bar{\nu})CC}$) are derived using the following part of the lagrangian [45]:

$$\mathcal{L}_{\nu Q}^{CC} = \frac{g^2}{2\mathcal{M}_W^2} \bar{e}_L \gamma^\mu \nu_L \left(\epsilon_{\nu*(ud)_L} \bar{u}_L \gamma^\mu d_L + (\epsilon_{\nu*(ud)_R} \bar{u}_R \gamma^\mu d_R) \right) + h.c., \quad (7.4)$$

$$\begin{aligned} \mathcal{L}_{\nu Q}^{NC} = \frac{g^2}{\cos^2 \theta_w \mathcal{M}_Z^2} \bar{\nu}_L \gamma^\mu \nu_L \bigg(& \zeta_{\nu*u_L} \bar{u}_L \gamma^\mu u_L + \zeta_{\nu*u_R} \bar{u}_R \gamma^\mu u_R \\ & + \zeta_{\nu*d_L} \bar{d}_L \gamma^\mu d_L + \zeta_{\nu*d_R} \bar{d}_R \gamma^\mu d_R \bigg), \end{aligned} \quad (7.5)$$

Figure 5. The effective interactions among gauge bosons with the fermions. The $\bullet$ depicts the modified Feynman vertex (renormalizable + corrections due to dimension-6 operators) factors which are tabulated in the following sections for both 2HDM and MLRSM effective scenarios.

where $\epsilon_{\nu*(ud)_L}, \epsilon_{\nu*(ud)_R}, \zeta_{\nu*u_L}, \zeta_{\nu*d_L}, \zeta_{\nu*u_R}, \zeta_{\nu*d_R}$ are given as:

$$\begin{aligned} \epsilon_{\nu*(ud)_L} &= \epsilon_{(e\nu)_L} * \epsilon_{(ud)_L} - \frac{4\mathcal{M}_W^2}{g^2\Lambda^2} \mathcal{C}_{LQ}^{[3]}, & \epsilon_{\nu*(ud)_R} &= \epsilon_{(e\nu)_L} * \epsilon_{(ud)_R}, \\ \zeta_{\nu*u_L} &= \zeta_{\nu_L} \zeta_{u_L} - \frac{\cos^2 \theta_w \mathcal{M}_Z^2}{g^2\Lambda^2} [\mathcal{C}_{LQ}^{[1]} + \mathcal{C}_{LQ}^{[3]}], & \zeta_{\nu*d_L} &= \zeta_{d_L} * \zeta_{\nu_L} - \frac{\cos^2 \theta_w \mathcal{M}_Z^2}{g^2\Lambda^2} [\mathcal{C}_{LQ}^{[1]} - \mathcal{C}_{LQ}^{[3]}], \\ \zeta_{\nu*u_R} &= \zeta_{\nu_L} * \zeta_{u_R} - \frac{\cos^2 \theta_w \mathcal{M}_Z^2}{g^2\Lambda^2} \mathcal{C}_{Lu}, & \zeta_{\nu*d_R} &= \zeta_{\nu_L} * \zeta_{d_R} - \frac{\cos^2 \theta_w \mathcal{M}_Z^2}{g^2\Lambda^2} \mathcal{C}_{Ld}, \end{aligned}$$

and their associated parameters ϵ and ζ which encapsulate the features of the relevant modified interactions (schematically depicted in figure 5.) are given in table 27.

The parameters $a_{L/R}^i$ for $i = u, d, \nu, e$ in table 27, are given as:

$$\begin{aligned} a_L^\nu &= \frac{1}{2} + \frac{1}{2\Lambda^2} (\Theta_{BB} \sin^2 \theta_w + \Theta_{WW} \cos^2 \theta_w - 2\Theta_{W_3B} \sin \theta_w \cos \theta_w), \\ a_L^e &= -\frac{1}{2} + \sin^2 \theta_w + \frac{1}{2\Lambda^2} (\Theta_{BB} \sin^2 \theta_w - \Theta_{WW} \cos^2 \theta_w), \\ a_R^e &= \sin^2 \theta_w + \frac{1}{\Lambda^2} (\Theta_{BB} \sin^2 \theta_w - \Theta_{W_3B} \sin \theta_w \cos \theta_w), \\ a_L^u &= \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w + \frac{1}{\Lambda^2} \left(\frac{1}{2} \Theta_{WW} \cos^2 \theta_w - \frac{1}{3} \Theta_{W_3B} \sin \theta_w \cos \theta_w - \frac{1}{6} \Theta_{BB} \sin^2 \theta_w \right), \\ a_R^u &= -\frac{2}{3} \sin^2 \theta_w + \frac{2}{3\Lambda^2} (\Theta_{W_3B} \sin \theta_w \cos \theta_w - \Theta_{BB} \sin^2 \theta_w), \\ a_L^d &= -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_w + \frac{1}{\Lambda^2} \left(-\frac{1}{2} \Theta_{WW} \cos^2 \theta_w - \frac{1}{6} \Theta_{BB} \sin^2 \theta_w + \frac{2}{3} \Theta_{W_3B} \sin \theta_w \cos \theta_w \right), \\ a_R^d &= \frac{1}{3} \sin^2 \theta_w + \frac{1}{3\Lambda^2} (\Theta_{BB} \sin^2 \theta_w - \Theta_{W_3B} \sin \theta_w \cos \theta_w). \end{aligned}$$

$\bar{\psi}\gamma^\mu\psi X_\mu$	2HDM: charged current vertex factor
$\bar{\nu}_L\gamma^\mu e_L W_\mu^+$	$\frac{g}{\sqrt{2}}\epsilon_{(e\nu)_L} = \frac{g}{\sqrt{2}}\left[1 + \frac{\Theta_{WW}}{\Lambda^2} + \frac{\sqrt{2}}{g}A_{e_L\nu_L W}\right]$
$\bar{u}_L\gamma^\mu d_L W_\mu^+$	$\frac{g}{\sqrt{2}}\epsilon_{(ud)_L} = \frac{g}{\sqrt{2}}\left[1 + \frac{\Theta_{WW}}{\Lambda^2} + \frac{\sqrt{2}}{g}A_{u_L d_L W}\right]$
$\bar{u}_R\gamma^\mu d_R W_\mu^+$	$\frac{g}{\sqrt{2}}\epsilon_{(ud)_R} = A_{u_R d_R W}$
2HDM: neutral current vertex factor	
$\bar{\nu}_L\gamma^\mu\nu_L Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{\nu_L} = \frac{g}{\cos\theta_w}\left[a_L^\nu + \frac{\cos\theta_w}{g}A_{\nu_L\nu_L Z}\right]$
$\bar{e}_L\gamma^\mu e_L Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{e_L} = \frac{g}{\cos\theta_w}\left[a_L^e + \frac{\cos\theta_w}{g}A_{e_L e_L Z}\right]$
$\bar{e}_R\gamma^\mu e_R Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{e_R} = \frac{g}{\cos\theta_w}\left[a_R^e + \frac{\cos\theta_w}{g}A_{e_R e_R Z}\right]$
$\bar{u}_L\gamma^\mu u_L Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{u_L} = \frac{g}{\cos\theta_w}\left[a_L^u + \frac{\cos\theta_w}{g}A_{u_L u_L Z}\right]$
$\bar{u}_R\gamma^\mu u_R Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{u_R} = \frac{g}{\cos\theta_w}\left[a_R^u + \frac{\cos\theta_w}{g}A_{u_R u_R Z}\right]$
$\bar{d}_L\gamma^\mu d_L Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{d_L} = \frac{g}{\cos\theta_w}\left[a_L^d + \frac{\cos\theta_w}{g}A_{d_L d_L Z}\right]$
$\bar{d}_R\gamma^\mu d_R Z_\mu$	$\frac{g}{\cos\theta_w}\zeta_{d_R} = \frac{g}{\cos\theta_w}\left[a_R^d + \frac{\cos\theta_w}{g}A_{d_R d_R Z}\right]$

Table 27. 2HDM: coupling of gauge bosons and fermions. The detailed structures of the A 's are given in table 28. Hermitian conjugate vertices are not separately mentioned.

Implementing all these corrections due to the dimension-6 operators we can estimate the new Weak mixing angle as $\sin^2\bar{\theta}_w = \frac{1}{2} - \hat{R}$, where

$$\hat{R} = \frac{4\mathcal{M}_W^4}{\cos^4\theta_w\mathcal{M}_Z^4}\left(\frac{\zeta_{\nu^*u_L}^2 + \zeta_{\nu^*d_L}^2 - \zeta_{\nu^*u_R}^2 - \zeta_{\nu^*d_R}^2}{\epsilon_{\nu^*(ud)_L}^2 - \epsilon_{\nu^*(ud)_R}^2}\right). \quad (7.6)$$

MLRSM. Similar to the 2HDM case, we have constructed the lagrangian which contains the neutral and charged current interactions:

$$\begin{aligned} \mathcal{L}_{\nu Q}^{CC} &= \frac{g^2}{2\mathcal{M}_{W_1}^2}\bar{e}_L\gamma^\mu\nu_L\left(\epsilon_{(e\nu)_L^*(ud)_L}^1\bar{u}_L\gamma^\mu d_L + \epsilon_{(e\nu)_L^*(ud)_R}^1\bar{u}_R\gamma^\mu d_R\right) + h.c. \\ &\quad + \frac{g^2}{2\mathcal{M}_{W_2}^2}\bar{e}_L\gamma^\mu\nu_L\left(\epsilon_{(e\nu)_L^*(ud)_L}^2\bar{u}_L\gamma^\mu d_L + \epsilon_{(e\nu)_L^*(ud)_R}^2\bar{u}_R\gamma^\mu d_R\right) + h.c., \\ \mathcal{L}_{\nu Q}^{NC} &= \frac{g^2}{4\cos^2\theta_w\mathcal{M}_{Z_1}^2}\bar{\nu}_L\gamma^\mu\nu_L\left(\zeta_{\nu_L^*u_L}^1\bar{u}_L\gamma^\mu u_L + \zeta_{\nu_L^*u_R}^1\bar{u}_R\gamma^\mu u_R + \zeta_{\nu_L^*d_L}^1\bar{d}_L\gamma^\mu d_L + \zeta_{\nu_L^*d_R}^1\bar{d}_R\gamma^\mu d_R\right) \end{aligned}$$

$\overline{\nu}_L \gamma^\mu e_L W_\mu^+$	$A_{\nu_L e_L W} = \frac{gC_{L\phi D}^{11[3]} v_1^2}{\sqrt{2}\Lambda^2} + \frac{\sqrt{2}gC_{L\phi D}^{12[3]} v_2 v_1}{\Lambda^2} + \frac{gC_{L\phi D}^{22[3]} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{u}_L \gamma^\mu d_L W_\mu^+$	$A_{u_L d_L W} = \frac{gC_{Q\phi D}^{11[3]} v_1^2}{\sqrt{2}\Lambda^2} + \frac{\sqrt{2}gC_{Q\phi D}^{12[3]} v_2 v_1}{\Lambda^2} + \frac{gC_{Q\phi D}^{22[3]} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{u}_R \gamma^\mu d_R W_\mu^+$	$A_{u_R d_R W} = \frac{gC_{u\phi D}^{11} v_1^2}{\sqrt{2}\Lambda^2} + \frac{gC_{u\phi D}^{22} v_2^2}{\sqrt{2}\Lambda^2} + \frac{gC_{u\phi D}^{21} v_2 v_1}{\sqrt{2}\Lambda^2}$
$\overline{\nu}_L \gamma^\mu \nu_L Z_\mu$	$A_{\nu_L \nu_L Z} = -\frac{gC_{L\phi D}^{11[1]} v_1^2}{\sqrt{2}\Lambda^2} + \frac{gC_{L\phi D}^{11[3]} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{L\phi D}^{12[1]} v_2 v_1}{\Lambda^2}$ $+ \frac{\sqrt{2}gC_{L\phi D}^{12[3]} v_2 v_1}{\Lambda^2} - \frac{gC_{L\phi D}^{22[1]} v_2^2}{\sqrt{2}\Lambda^2} + \frac{gC_{L\phi D}^{22[3]} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{e}_L \gamma^\mu e_L Z_\mu$	$A_{e_L e_L Z} = -\frac{gC_{L\phi D}^{11[1]} v_1^2}{\sqrt{2}\Lambda^2} - \frac{gC_{L\phi D}^{11[3]} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{L\phi D}^{12[1]} v_2 v_1}{\Lambda^2}$ $- \frac{\sqrt{2}gC_{L\phi D}^{12[3]} v_2 v_1}{\Lambda^2} - \frac{gC_{L\phi D}^{22[1]} v_2^2}{\sqrt{2}\Lambda^2} - \frac{gC_{L\phi D}^{22[3]} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{e}_R \gamma^\mu e_R Z_\mu$	$A_{e_R e_R Z} = -\frac{gC_{e\phi D}^{11} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{e\phi D}^{12} v_2 v_1}{\Lambda^2} - \frac{gC_{e\phi D}^{22} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{u}_L \gamma^\mu u_L Z_\mu$	$A_{u_L u_L Z} = -\frac{gC_{Q\phi D}^{11[1]} v_1^2}{\sqrt{2}\Lambda^2} + \frac{gC_{Q\phi D}^{11[3]} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{Q\phi D}^{12[1]} v_2 v_1}{\Lambda^2}$ $+ \frac{\sqrt{2}gC_{Q\phi D}^{12[3]} v_2 v_1}{\Lambda^2} - \frac{gC_{Q\phi D}^{22[1]} v_2^2}{\sqrt{2}\Lambda^2} + \frac{gC_{Q\phi D}^{22[3]} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{d}_L \gamma^\mu d_L Z_\mu$	$A_{d_L d_L Z} = -\frac{gC_{Q\phi D}^{11[1]} v_1^2}{\sqrt{2}\Lambda^2} - \frac{gC_{Q\phi D}^{11[3]} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{Q\phi D}^{12[1]} v_2 v_1}{\Lambda^2}$ $- \frac{\sqrt{2}gC_{Q\phi D}^{12[3]} v_2 v_1}{\Lambda^2} - \frac{gC_{Q\phi D}^{22[1]} v_2^2}{\sqrt{2}\Lambda^2} - \frac{gC_{Q\phi D}^{22[3]} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{u}_R \gamma^\mu u_R Z_\mu$	$A_{u_R u_R Z} = -\frac{gC_{u\phi D}^{11} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{u\phi D}^{12} v_2 v_1}{\Lambda^2} - \frac{gC_{u\phi D}^{22} v_2^2}{\sqrt{2}\Lambda^2}$
$\overline{d}_R \gamma^\mu d_R Z_\mu$	$A_{d_R d_R Z} = -\frac{gC_{d\phi D}^{11} v_1^2}{\sqrt{2}\Lambda^2} - \frac{\sqrt{2}gC_{d\phi D}^{12} v_2 v_1}{\Lambda^2} - \frac{gC_{d\phi D}^{22} v_2^2}{\sqrt{2}\Lambda^2}$

Table 28. 2HDM: corrections to gauge bosons and fermion couplings from $\psi^2 \phi^2 D$ operators.

$\bar{\psi}\gamma^\mu\psi X_\mu$	MLRSM: charged current vertex factors
$\bar{\nu}_L\gamma^\mu e_L W_{1\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(e\nu)_L}^1 = \frac{g}{\sqrt{2}} \left[\cos\xi \left(1 + \frac{\Theta_{W_{LL}}}{\Lambda^2} \right) - \sin\xi \frac{\Theta_{W_{LR}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{\nu_L e_L W_2} \right]$
$\bar{\nu}_L\gamma^\mu e_L W_{2\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(e\nu)_L}^2 = \frac{g}{\sqrt{2}} \left[\sin\xi \left(1 + \frac{\Theta_{W_{LL}}}{\Lambda^2} \right) + \cos\xi \frac{\Theta_{W_{LR}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{\nu_L e_L W_2} \right]$
$\bar{\nu}_R\gamma^\mu e_R W_{1\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(e\nu)_R}^1 = \frac{g}{\sqrt{2}} \left[-\sin\xi \left(1 + \frac{\Theta_{W_{RR}}}{\Lambda^2} \right) + \cos\xi \frac{\Theta_{W_{RL}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{\nu_R e_R W_1} \right]$
$\bar{\nu}_R\gamma^\mu e_R W_{2\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(e\nu)_R}^2 = \frac{g}{\sqrt{2}} \left[\cos\xi \left(1 + \frac{\Theta_{W_{RR}}}{\Lambda^2} \right) + \sin\xi \frac{\Theta_{W_{RL}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{\nu_R e_R W_2} \right]$
$\bar{u}_L\gamma^\mu d_L W_{1\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(ud)_L}^1 = \frac{g}{\sqrt{2}} \left[\cos\xi \left(1 + \frac{\Theta_{W_{LL}}}{\Lambda^2} \right) - \sin\xi \frac{\Theta_{W_{LR}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{u_L d_L W_1} \right]$
$\bar{u}_L\gamma^\mu d_L W_{2\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(ud)_L}^2 = \frac{g}{\sqrt{2}} \left[\sin\xi \left(1 + \frac{\Theta_{W_{LL}}}{\Lambda^2} \right) + \cos\xi \frac{\Theta_{W_{LR}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{u_L d_L W_2} \right]$
$\bar{u}_R\gamma^\mu d_R W_{1\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(ud)_R}^1 = \frac{g}{\sqrt{2}} \left[-\sin\xi \left(1 + \frac{\Theta_{W_{RR}}}{\Lambda^2} \right) + \cos\xi \frac{\Theta_{W_{RL}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{u_R d_R W_1} \right]$
$\bar{u}_R\gamma^\mu d_R W_{2\mu}^+$	$\frac{g}{\sqrt{2}}\epsilon_{(ud)_R}^2 = \frac{g}{\sqrt{2}} \left[\cos\xi \left(1 + \frac{\Theta_{W_{RR}}}{\Lambda^2} \right) + \sin\xi \frac{\Theta_{W_{RL}}}{\Lambda^2} + \frac{\sqrt{2}}{g} A_{u_R d_R W_2} \right]$

Table 29. MLRSM: coupling of charged gauge boson and fermions. Here, $\tilde{e} = \frac{e}{2\cos\theta_w \sin\theta_w}$. Hermitian conjugate of these vertices are not separately mentioned. The detailed structures of A 's are depicted in table 30.

$$+ \frac{g^2}{4\cos^2\theta_w \mathcal{M}_{Z_2}^2} \bar{\nu}_L\gamma^\mu\nu_L \left(\zeta_{\nu_L^*u_L}^2 \bar{u}_L\gamma^\mu u_L + \zeta_{\nu_L^*u_R}^2 \bar{u}_R\gamma^\mu u_R + \zeta_{\nu_L^*d_L}^2 \bar{d}_L\gamma^\mu d_L + \zeta_{\nu_L^*d_R}^2 \bar{d}_R\gamma^\mu d_R \right).$$

The parameters $[\epsilon_{(e\nu)_L^*(ud)_L}^1, \epsilon_{(e\nu)_L^*(ud)_R}^1, \epsilon_{(e\nu)_L^*(ud)_L}^2, \epsilon_{(e\nu)_L^*(ud)_R}^2]$ contained in the charged current are expressed as:

$$\epsilon_{(e\nu)_L^*(ud)_L}^1 = \epsilon_{(e\nu)_L}^1 \epsilon_{(ud)_L}^1 - \frac{4\mathcal{M}_{W_1}^2}{g^2\Lambda^2} \mathcal{C}_{l_L q_L}^{[3]}, \quad \epsilon_{(e\nu)_L^*(ud)_R}^1 = \epsilon_{(e\nu)_L}^1 \epsilon_{(ud)_R}^1, \quad (7.7)$$

$$\epsilon_{(e\nu)_L^*(ud)_L}^2 = \epsilon_{(e\nu)_L}^2 \epsilon_{(ud)_L}^2 - \frac{4\mathcal{M}_{W_2}^2}{g^2\Lambda^2} \mathcal{C}_{l_L q_L}^{[3]}, \quad \epsilon_{(e\nu)_L^*(ud)_R}^2 = \epsilon_{(e\nu)_L}^2 \epsilon_{(ud)_R}^2. \quad (7.8)$$

The parameters $[\zeta_{\nu_L^*u_L}^1, \zeta_{\nu_L^*u_R}^1, \zeta_{\nu_L^*u_R}^2, \zeta_{\nu_L^*d_R}^1, \zeta_{\nu_L^*d_R}^2]$ contained in the neutral current are expressed as:

$$\begin{aligned} \zeta_{\nu_L^*u_L}^1 &= \zeta_{\nu_L}^1 \zeta_{u_L}^1 - \frac{4\mathcal{M}_{Z_1}^2 \cos^2\theta_w}{g^2\Lambda^2} \left(\mathcal{C}_{l_L q_L}^{[1]} + \mathcal{C}_{l_L q_L}^{[3]} \right), & \zeta_{\nu_L^*u_R}^1 &= \zeta_{\nu_L}^1 \zeta_{u_R}^1 - \frac{4\mathcal{M}_{Z_1}^2 \cos^2\theta_w}{g^2\Lambda^2} \mathcal{C}_{l_L q_R}, \\ \zeta_{\nu_L^*u_L}^2 &= \zeta_{\nu_L}^2 \zeta_{u_L}^2 - \frac{4\mathcal{M}_{Z_2}^2 \cos^2\theta_w}{g^2\Lambda^2} \left(\mathcal{C}_{l_L q_L}^{[1]} + \mathcal{C}_{l_L q_L}^{[3]} \right), & \zeta_{\nu_L^*u_R}^2 &= \zeta_{\nu_L}^2 \zeta_{u_R}^2 - \frac{4\mathcal{M}_{Z_2}^2 \cos^2\theta_w}{g^2\Lambda^2} \mathcal{C}_{l_L q_R}, \end{aligned}$$

$\overline{\nu_L} \gamma^\mu e_L W_{1\mu}^+$	$A_{\nu_L e_L W_1} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{11}^a \left(\mathcal{C}_{LDl}^{l_L[3]} v_L^2 + (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{l_L[3]} \right) - 2\mathcal{R}_{21}^a \kappa_1 \kappa_2 \mathcal{C}_{2D1}^{l_L[3]} \right)$
$\overline{\nu_L} \gamma^\mu e_L W_{2\mu}^+$	$A_{\nu_L e_L W_2} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{12}^a \left(\mathcal{C}_{LDl}^{l_L[3]} v_L^2 + (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{l_L[3]} \right) - 2\mathcal{R}_{22}^a \kappa_1 \kappa_2 \mathcal{C}_{2D1}^{l_L[3]} \right)$
$\overline{\nu_R} \gamma^\mu e_R W_{1\mu}^+$	$A_{\nu_R e_R W_1} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{11}^a (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{l_R[3]} + \mathcal{R}_{21}^a \left(\mathcal{C}_{RD r}^{l_R[3]} v_R^2 - 2\kappa_1 \kappa_2 \mathcal{C}_{2D1}^{l_R[3]} \right) \right)$
$\overline{\nu_R} \gamma^\mu e_R W_{2\mu}^+$	$A_{\nu_R e_R W_2} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{12}^a (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{l_R[3]} + \mathcal{R}_{22}^a \left(\mathcal{C}_{RD r}^{l_R[3]} v_R^2 - 2\kappa_1 \kappa_2 \mathcal{C}_{2D1}^{l_R[3]} \right) \right)$
$\overline{u_L} \gamma^\mu d_L W_{1\mu}^+$	$A_{u_L d_L W_1} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{11}^a \left(\mathcal{C}_{LDl}^{q_L[3]} v_L^2 + (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{q_L[3]} \right) - 2\mathcal{R}_{21}^a \kappa_1 \kappa_2 \mathcal{C}_{2D1}^{q_L[3]} \right)$
$\overline{u_L} \gamma^\mu d_L W_{2\mu}^+$	$A_{u_L d_L W_2} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{12}^a \left(\mathcal{C}_{LDl}^{q_L[3]} v_L^2 + (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{q_L[3]} \right) - 2\mathcal{R}_{22}^a \kappa_1 \kappa_2 \mathcal{C}_{2D1}^{q_L[3]} \right)$
$\overline{u_R} \gamma^\mu d_R W_{1\mu}^+$	$A_{u_R d_R W_1} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{11}^a (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{q_R[3]} + \mathcal{R}_{21}^a \left(\mathcal{C}_{RD r}^{q_R[3]} v_R^2 - 2\kappa_1 \kappa_2 \mathcal{C}_{2D1}^{q_R[3]} \right) \right)$
$\overline{u_R} \gamma^\mu d_R W_{2\mu}^+$	$A_{u_R d_R W_2} = \frac{g}{2\sqrt{2}\Lambda^2} \left(\mathcal{R}_{12}^a (\kappa_1^2 + \kappa_2^2) \mathcal{C}_{2D1}^{q_R[3]} + \mathcal{R}_{22}^a \left(\mathcal{C}_{RD r}^{q_R[3]} v_R^2 - 2\kappa_1 \kappa_2 \mathcal{C}_{2D1}^{q_R[3]} \right) \right)$

Table 30. MLRSM: corrections to the coupling of charged gauge bosons to fermions due to $\psi^2 \phi^2 D$ operators.

$$\begin{aligned}
 \zeta_{\nu_L * d_L}^1 &= \zeta_{\nu_L}^1 \zeta_{d_L}^1 - \frac{4\mathcal{M}_{Z_1}^2 \cos^2 \theta_w}{g^2 \Lambda^2} \left(\mathcal{C}_{l_L q_L}^{[1]} - \mathcal{C}_{l_L q_L}^{[3]} \right), & \zeta_{\nu_L * d_R}^1 &= \zeta_{\nu_L}^1 \zeta_{d_R}^1 - \frac{4\mathcal{M}_{Z_1}^2 \cos^2 \theta_w}{g^2 \Lambda^2} \mathcal{C}_{l_L q_R}, \\
 \zeta_{\nu_L * d_L}^2 &= \zeta_{\nu_L}^2 \zeta_{d_L}^2 - \frac{4\mathcal{M}_{Z_2}^2 \cos^2 \theta_w}{g^2 \Lambda^2} \left(\mathcal{C}_{l_L q_L}^{[1]} - \mathcal{C}_{l_L q_L}^{[3]} \right), & \zeta_{\nu_L * d_R}^2 &= \zeta_{\nu_L}^2 \zeta_{d_R}^2 - \frac{4\mathcal{M}_{Z_2}^2 \cos^2 \theta_w}{g^2 \Lambda^2} \mathcal{C}_{l_L q_R}.
 \end{aligned}$$

The “ a ” and “ b ” parameters in table 32 are given as:

$$\begin{aligned}
 a_L^i &= (A_1^{iL} \cos^2 \theta_w - A_2^{iL} \sin^2 \theta_w) + \frac{1}{\Lambda^2} \left[A_1^{iL} \left(\cos^2 \theta_w \Theta_{3L3L} - \sin^2 \theta_w \Theta_{3L3R} - \sin \theta_w \sqrt{\cos 2\theta_w} \Theta_{3LB} \right) \right. \\
 &\quad \left. + A_2^{iL} \left(\frac{\sin \theta_w \cos^2 \theta_w}{\sqrt{\cos 2\theta_w}} \Theta_{3LB} - \frac{\sin^3 \theta_w}{\sqrt{\cos 2\theta_w}} \Theta_{3RB} - \sin^2 \theta_w \Theta_{BB} \right) \right], \quad (7.9)
 \end{aligned}$$

$$\begin{aligned}
 b_L^i &= A_2^{iL} \frac{\sin^2 \theta_w}{\sqrt{\cos 2\theta_w}} + \frac{1}{\Lambda^2} \left[A_1^{iL} \left(-\sqrt{\cos 2\theta_w} \Theta_{3L3R} + \sin \theta_w \Theta_{3LB} \right) + A_2^{iL} \left(-\sin \theta_w \Theta_{3RB} \right. \right. \\
 &\quad \left. \left. + \frac{\sin^2 \theta_w}{\sqrt{\cos 2\theta_w}} \Theta_{BB} \right) \right], \quad (7.10)
 \end{aligned}$$

$$\begin{aligned}
 a_R^i &= -(A_1^{iR} + A_2^{iR}) \sin^2 \theta_w + \frac{1}{\Lambda^2} \left[A_1^{iR} \left(\cos^2 \theta_w \Theta_{3L3R} - \sin^2 \theta_w \Theta_{3R3R} - \sin \theta_w \sqrt{\cos 2\theta_w} \Theta_{3RB} \right) \right. \\
 &\quad \left. + A_2^{iR} \left(\frac{\sin \theta_w \cos^2 \theta_w}{\sqrt{\cos 2\theta_w}} \Theta_{3LB} - \frac{\sin^3 \theta_w}{\sqrt{\cos 2\theta_w}} \Theta_{3RB} - \sin^2 \theta_w \Theta_{BB} \right) \right], \quad (7.11)
 \end{aligned}$$

Quantum no.	ν_L	ν_R	e_L	e_R	u_L	u_R	d_L	d_R
T_{3L}	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0
T_{3R}	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
Q	0	0	-1	-1	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

Table 31. MLRSM: T_{3L} , T_{3R} , Q quantum number for fermions.

$$b_R^i = \left(-A_1^{iR} \sqrt{\cos 2\theta_w} + A_2^{iR} \frac{\sin^2 \theta_w}{\sqrt{\cos 2\theta_w}} \right) + \frac{1}{\Lambda^2} \left[A_1^{iR} \left(-\sqrt{\cos 2\theta_w} \Theta_{3R3R} + \sin \theta_w \Theta_{3RB} \right) + A_2^{iR} \left(-\sin \theta_w \Theta_{3RB} + \frac{\sin^2 \theta_w}{\sqrt{\cos 2\theta_w}} \Theta_{BB} \right) \right], \quad (7.12)$$

where

$$A_1^{iL,R} = 2T_{3L,R}^i, \quad A_2^{iL,R} = 2(Q^i - T_{3L,R}^i); \quad i = u, d, \nu, e. \quad (7.13)$$

The T_{3L} , T_{3R} , Q are the quantum numbers corresponding to $SU(2)_L$, $SU(2)_R$, and $U(1)_{em}$ gauge groups respectively. These are provided in table 31.

Now incorporating all the above mentioned corrections, we can express $\hat{R}$ as:

$$\hat{R} = \frac{4\mathcal{M}_{W_1}^4}{\cos^4 \theta_w \mathcal{M}_{Z_1}^4} \left[\frac{(\zeta_{\nu_L^* u_L}^1)^2 + (\zeta_{\nu_L^* d_L}^1)^2 - (\zeta_{\nu_L^* u_R}^1)^2 - (\zeta_{\nu_L^* d_R}^1)^2}{(\epsilon_{(\nu e)_L^* (ud)_L}^1)^2 - (\epsilon_{(\nu e)_L^* (ud)_R}^1)^2} \right]. \quad (7.14)$$

The modified Weak mixing angle can be derived using $\sin^2 \bar{\theta}_w = \frac{1}{2} - \hat{R}$.

7.3 Fermi constant ($\mathcal{G}_F$) and ρ parameter

The Fermi constant $(\mathcal{G}_F)_{SM}$ is defined using the information from muon decay $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ as [45]:

$$(\mathcal{G}_F)_{SM} = \frac{g^2}{4\sqrt{2}M_W^2}. \quad (7.15)$$

In the context of the Standard Model, the ρ parameter is defined as [45]:

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_w}. \quad (7.16)$$

In the presence of the effective operators, there can be two fold contributions to the Fermi constant: (i) through the modified $\mu - W - \nu_\mu$, $e - W - \bar{\nu}_e$ vertices and gauge boson mass $\mathcal{M}_W$, see figure 6(a), and (ii) direct contributions from the dimension-6 four-fermion operators, figure 6(b). We have encapsulated these contributions and computed the modified G_F and ρ parameters for both 2HDM and MLRSM effective theories.

$\bar{\psi}\gamma^\mu\psi X_\mu$	MLRSM: neutral current vertex factors
$\bar{\nu}_L\gamma^\mu\nu_L Z_{1\mu}$	$\tilde{e}\zeta_{\nu_L}^1 = \tilde{e} [a_L^\nu \cos\theta_2 + b_L^\nu \sin\theta_2 + \frac{1}{\tilde{e}}A_{\nu_L Z_1}]$
$\bar{\nu}_L\gamma^\mu\nu_L Z_{2\mu}$	$\tilde{e}\zeta_{\nu_L}^2 = \tilde{e} [a_L^\nu \sin\theta_2 - b_L^\nu \cos\theta_2 + \frac{1}{\tilde{e}}A_{\nu_L Z_2}]$
$\bar{e}_L\gamma^\mu e_L Z_{1\mu}$	$\tilde{e}\zeta_{e_L}^1 = \tilde{e} [a_L^e \cos\theta_2 + b_L^e \sin\theta_2 + \frac{1}{\tilde{e}}A_{e_L Z_1}]$
$\bar{e}_L\gamma^\mu e_L Z_{2\mu}$	$\tilde{e}\zeta_{e_L}^2 = \tilde{e} [a_L^e \sin\theta_2 - b_L^e \cos\theta_2 + \frac{1}{\tilde{e}}A_{e_L Z_2}]$
$\bar{e}_R\gamma^\mu e_R Z_{1\mu}$	$\tilde{e}\zeta_{e_R}^1 = \tilde{e} [a_R^e \cos\theta_2 + b_R^e \sin\theta_2 + \frac{1}{\tilde{e}}A_{e_R Z_1}]$
$\bar{e}_R\gamma^\mu e_R Z_{2\mu}$	$\tilde{e}\zeta_{e_R}^2 = \tilde{e} [a_R^e \sin\theta_2 - b_R^e \cos\theta_2 + \frac{1}{\tilde{e}}A_{e_R Z_2}]$
$\bar{u}_L\gamma^\mu u_L Z_{1\mu}$	$\tilde{e}\zeta_{u_L}^1 = \tilde{e} [a_L^u \cos\theta_2 + b_L^u \sin\theta_2 + \frac{1}{\tilde{e}}A_{u_L Z_1}]$
$\bar{u}_L\gamma^\mu u_L Z_{2\mu}$	$\tilde{e}\zeta_{u_L}^2 = \tilde{e} [a_L^u \sin\theta_2 - b_L^u \cos\theta_2 + \frac{1}{\tilde{e}}A_{u_L Z_2}]$
$\bar{u}_R\gamma^\mu u_R Z_{1\mu}$	$\tilde{e}\zeta_{u_R}^1 = \tilde{e} [a_R^u \cos\theta_2 + b_R^u \sin\theta_2 + \frac{1}{\tilde{e}}A_{u_R Z_1}]$
$\bar{u}_R\gamma^\mu u_R Z_{2\mu}$	$\tilde{e}\zeta_{u_R}^2 = \tilde{e} [a_R^u \sin\theta_2 - b_R^u \cos\theta_2 + \frac{1}{\tilde{e}}A_{u_R Z_2}]$
$\bar{d}_L\gamma^\mu d_L Z_{1\mu}$	$\tilde{e}\zeta_{d_L}^1 = \tilde{e} [a_L^d \cos\theta_2 + b_L^d \sin\theta_2 + \frac{1}{\tilde{e}}A_{d_L Z_1}]$
$\bar{d}_L\gamma^\mu d_L Z_{2\mu}$	$\tilde{e}\zeta_{d_L}^2 = \tilde{e} [a_L^d \sin\theta_2 - b_L^d \cos\theta_2 + \frac{1}{\tilde{e}}A_{d_L Z_2}]$
$\bar{d}_R\gamma^\mu d_R Z_{1\mu}$	$\tilde{e}\zeta_{d_R}^1 = \tilde{e} [a_R^d \cos\theta_2 + b_R^d \sin\theta_2 + \frac{1}{\tilde{e}}A_{d_R Z_1}]$
$\bar{d}_R\gamma^\mu d_R Z_{2\mu}$	$\tilde{e}\zeta_{d_R}^2 = \tilde{e} [a_R^d \sin\theta_2 - b_R^d \cos\theta_2 + \frac{1}{\tilde{e}}A_{d_R Z_2}]$

Table 32. MLRSM: coupling of neutral gauge bosons and fermions: here, $\tilde{e} = \frac{e}{2 \cos\theta_w \sin\theta_w}$. The detailed structures of A 's are mentioned in tables 33, 34.

[illegible]

Table 34. Table 33 continued.

Figure 6. Three-body decay of muon: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. This process gives an estimation for Fermi constant ($\mathcal{G}_F$). The (a) [●] depicts the modified Feynman vertex (renormalizable + corrections due to dimension-6 operators), and (b) [■] represents the effective four-Fermi vertex.

2HDM. The modified Fermi constant for 2HDM-EFT can be written as:

$$(\mathcal{G}_F)_{2HDM} = (\mathcal{G}_F)_{SM} \left[1 - \frac{\Delta M_W^2}{v^2} - \frac{1}{\Lambda^2} \left(\epsilon_{(e\nu)_L}^e + \epsilon_{(e\nu)_L}^\mu - \frac{2\mathcal{C}_{LL}M_W^2}{g^2} \right) \right]. \quad (7.17)$$

Here, $\mathcal{C}_{LL}$ contains information regarding the four-fermion interactions involving $\{e, \mu, \nu_e, \nu_\mu\}$, see table 11. The vertex corrections $\epsilon_{(e\nu)_L}^e, \epsilon_{(e\nu)_L}^\mu$ are depicted in tables 27, 28. The rho-parameter for this effective scenario can be defined as

$$\bar{\rho} = \frac{\mathcal{M}_W^2}{\mathcal{M}_Z^2(1 - \sin^2 \bar{\theta}_w)}. \quad (7.18)$$

MLRSM. The modified Fermi constant for MLRSM-EFT can be written as:

$$(\mathcal{G}_F)_{MLRSM} = \frac{g^2}{4\sqrt{2}\mathcal{M}_{W_1}^2} \left[\epsilon_{(e\nu)_L}^{1,e} + \epsilon_{(e\nu)_L}^{1,\mu} + \frac{1}{\Lambda^2} \frac{8\sqrt{2}\mathcal{C}_{l_L l_L} \mathcal{M}_{W_1}^2}{g^2} \right]. \quad (7.19)$$

Here, $\mathcal{C}_{l_L l_L}$ contains information regarding the four-fermion interactions involving $\{e, \mu, \nu_e, \nu_\mu\}$, see table 19. The vertex corrections $\epsilon_{(e\nu)_L}^{1,e}, \epsilon_{(e\nu)_L}^{1,\mu}$ are depicted tables 29, 30. The modified rho-parameter for this effective scenario can be expressed as

$$\bar{\rho} = \frac{\mathcal{M}_{W_1}^2}{\mathcal{M}_{Z_1}^2(1 - \sin^2 \bar{\theta}_w)}. \quad (7.20)$$

7.4 Oblique parameters

The two-point vector boson correlation functions after including the radiative corrections, i.e., the vacuum polarization can be written as [66, 70, 221]:

$$i\Pi_{V_i V_j}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi_{V_i V_j}(p^2) + \left(i \frac{p^\mu p^\nu}{p^2} \text{ terms} \right), \quad (7.21)$$

where V_i represents the vector boson in either unphysical $V_i \in \{W_1, W_2, W_3, B\}$ or physical $\{W, Z, \gamma\}$ basis and p^μ is the external momentum, see figure 7. Note that γ represents the photon field A_μ .

Figure 7. The effective two-point vector boson correlation function. The $[\bullet]$ represents the corrections due to the dimension-6 operators.

The vacuum polarization amplitudes $\Pi_{V_i V_j}(p^2)$ can be expanded as a polynomial in external momentum (p^2) as [66, 70, 221]:

$$\Pi_{V_i V_j}(p^2) = [\Pi_0 + \Pi_2 p^2 + \Pi_4 p^4 + \mathcal{O}(p^6)]_{V_i V_j}. \quad (7.22)$$

In [66], Peskin and Takeuchi had defined the oblique parameters S, T, U to grab the radiative corrections to the tree-level correlation functions. These variables also capture the new physics effects.

These S, T, U parameters can be defined in both unphysical and physical gauge boson basis as [66, 70, 221, 222]:

$$\begin{aligned} S &= -\frac{4 \cos \theta_w \sin \theta_w}{\alpha} \Pi'_{W_3 B}(0) \\ &= -\frac{4 \cos^2 \theta_w \sin^2 \theta_w}{m_Z^2 \alpha} \left[\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}(m_Z^2) \right. \\ &\quad \left. - \frac{\cos^2 \theta_w - \sin^2 \theta_w}{\cos \theta_w \sin \theta_w} (\Pi_{\gamma Z}(m_Z^2) - \Pi_{\gamma Z}(0)) \right], \end{aligned} \quad (7.23)$$

$$\begin{aligned} T &= \frac{1}{\alpha m_W^2} (\Pi_{W_1 W_1}(0) - \Pi_{W_3 W_3}(0)) \\ &= \frac{1}{\alpha} \left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{2 \sin \theta_w \Pi_{\gamma Z}(0)}{\cos \theta_w m_Z^2} \right], \end{aligned} \quad (7.24)$$

$$\begin{aligned} U &= \frac{4 \sin^2 \theta_w}{\alpha} (\Pi'_{W_1 W_1}(0) - \Pi'_{W_3 W_3}(0)) \\ &= -\frac{4 \sin^2 \theta_w}{\alpha} \left[\left(\frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} \right) - \cos^2 \theta_w \left(\frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} \right) \right. \\ &\quad \left. - 2 \cos \theta_w \sin \theta_w \left(\frac{\Pi_{\gamma Z}(m_Z^2) - \Pi_{\gamma Z}(0)}{m_Z^2} \right) - \sin^2 \theta_w \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2} \right], \end{aligned} \quad (7.25)$$

where $\alpha = e^2/4\pi$ is the Fine structure constant.

Here, for unphysical basis

$$\Pi'_{V_i V_j}(0) = \left. \frac{d\Pi_{V_i V_j}}{dp^2} \right|_{p^2=0}, \quad \Pi_{W_1 W_1}(p^2) = \Pi_{W_2 W_2}(p^2),$$

and for physical basis, we have

$$\Pi'_{V_i V_j}(0) = \left(\frac{\Pi_{V_i V_j}(p^2) - \Pi_{V_i V_j}(0)}{p^2} \right) \Big|_{p^2=0}, \quad \Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0.$$

$\phi^4 D^2$ and $\phi^2 X^2$ operators: modifications in vacuum polarisation functions.

In the earlier paragraph, we have discussed the general structure of the oblique parameters. Based on that, the S, T, U parameters are computed in the context of renormalizable 2HDM [223] and extended gauge scenarios [85, 224]. These oblique parameters get additional contributions in the presence of the effective operators [83, 87, 225]. Here, we will focus on the computation of S, T, U parameters specifically for 2HDM- and MLRSM-EFT scenarios.

We have noted how the $\phi^2 X^2$ operators redefine the gauge fields, and the $\phi^4 D^2$ operators affect the gauge boson mass terms which are bilinear in gauge fields. In the process, both classes of operators modify the two-point gauge field correlation functions, see figure 7.

2HDM. The part of the effective 2HDM lagrangian that is bilinear in gauge fields is given in unphysical basis as:

$$\begin{aligned} \mathcal{L}_{W_1 W_1} = & \frac{1}{2} W_{1\mu} (-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) W_{1\nu} \left(1 + \frac{2\Theta_{WW}}{\Lambda^2} \right) + \left(\frac{1}{2} W_{1\mu} W_1^\mu \right) \left[\frac{1}{4} g^2 (v_1^2 + v_2^2) \right. \\ & \left. + \frac{g^2}{4\Lambda^2} (\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + \mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2v_1^2 \Theta_{WW} + 2v_2^2 \Theta_{WW}) \right], \end{aligned} \quad (7.26)$$

$$\begin{aligned} \mathcal{L}_{W_3 W_3} = & \frac{1}{2} W_{3\mu} (-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) W_{3\nu} \left(1 + \frac{2\Theta_{WW}}{\Lambda^2} \right) + \left(\frac{1}{2} W_{3\mu} W_3^\mu \right) \left[\frac{1}{4} g^2 (v_1^2 + v_2^2) + \frac{gg' \Theta_{W_3 B}}{4\Lambda^2} (v_1^2 + v_2^2) \right. \\ & + \frac{g^2}{8\Lambda^2} (\mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 v_1^2 \\ & \left. + \mathcal{C}_{\phi D}^{(2)11(2)} v_2^2 v_1^2 + 2\mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2\mathcal{C}_{\phi D}^{(2)22(1)} v_2^3 v_1 + \mathcal{C}_{\phi D}^{2(22)2} v_2^4 + 4\Theta_{WW} (v_1^2 + v_2^2) \right)], \end{aligned} \quad (7.27)$$

$$\begin{aligned} \mathcal{L}_{W_3 B} = & W_{3\mu} (-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) B_\nu \left(-\frac{\Theta_{W_3 B}}{\Lambda^2} \right) \\ & + W_{3\mu} B^\mu \left[-\frac{1}{4} gg' (v_1^2 + v_2^2) - \frac{gg' (\Theta_{BB} + \Theta_{WW})}{4\Lambda^2} (v_1^2 + v_2^2) - \frac{\Theta_{W_3 B}}{8\Lambda^2} (g^2 + g'^2) (v_1^2 + v_2^2) \right. \\ & - \frac{gg'}{8\Lambda^2} (\mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 v_1^2 \\ & \left. + \mathcal{C}_{\phi D}^{(2)11(2)} v_2^2 v_1^2 + 2\mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2\mathcal{C}_{\phi D}^{(2)22(1)} v_2^3 v_1 + \mathcal{C}_{\phi D}^{2(22)2} v_2^4) \right], \end{aligned} \quad (7.28)$$

$$\begin{aligned} \mathcal{L}_{BB} = & \frac{1}{2} B_\mu (-\partial^2 g^{\mu\nu} + \partial^\mu \partial^\nu) B_\nu \left(1 + \frac{2\Theta_{BB}}{\Lambda^2} \right) \\ & + \left(\frac{1}{2} B_\mu B^\mu \right) \left[\frac{1}{4} g'^2 (v_1^2 + v_2^2) + \frac{\Theta_{BB} g'^2}{2\Lambda^2} (v_1^2 + v_2^2) + \frac{gg' \Theta_{W_3 B}}{4\Lambda^2} (v_1^2 + v_2^2) \right. \\ & + \frac{g'^2}{8\Lambda^2} (\mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 v_1^2 \\ & \left. + \mathcal{C}_{\phi D}^{(2)11(2)} v_2^2 v_1^2 + 2\mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2\mathcal{C}_{\phi D}^{(2)22(1)} v_2^3 v_1 + \mathcal{C}_{\phi D}^{2(22)2} v_2^4) \right]. \end{aligned} \quad (7.29)$$

We can relate the $\Pi_{VV}^{\mu\nu}(p^2)$ with the relevant part of the Fourier-transformed (FT) lagrangian as:

$$\Pi_{V_i V_j}^{\mu\nu}(p^2) V_{i\mu} V_{j\nu} = FT \{ \mathcal{L}_{V_i V_j} \},$$

and these can be written in terms of $\Pi_{V_i V_j}$:

$$i\Pi_{W_1 W_1}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi_{W_1 W_1}(p^2), \quad (7.30)$$

$$i\Pi_{W_3 W_3}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi_{W_3 W_3}(p^2), \quad (7.31)$$

$$i\Pi_{W_3 B}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi_{W_3 B}(p^2), \quad (7.32)$$

$$i\Pi_{BB}^{\mu\nu}(p^2) = i \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \Pi_{BB}(p^2). \quad (7.33)$$

Now the $\Pi_{V_i V_j}(p^2)$ can be expressed as:

$$\begin{aligned} \Pi_{W_1 W_1}(p^2) &= p^2 \left(1 + \frac{2\Theta_{WW}}{\Lambda^2} \right) + \frac{g^2}{4}(v_1^2 + v_2^2) \\ &\quad + \frac{g^2}{4\Lambda^2} \left[\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + \mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2v_1^2 \Theta_{WW} + 2v_2^2 \Theta_{WW} \right], \end{aligned} \quad (7.34)$$

$$\begin{aligned} \Pi_{W_3 W_3}(p^2) &= p^2 \left(1 + \frac{2\Theta_{WW}}{\Lambda^2} \right) + \frac{g^2}{4}(v_1^2 + v_2^2) + \frac{gg' \Theta_{W_3 B}}{4\Lambda^2} (v_1^2 + v_2^2) \\ &\quad + \frac{g^2}{8\Lambda^2} \left[\mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 v_1^2 \right. \\ &\quad \left. + \mathcal{C}_{\phi D}^{(2)11(2)} v_2^2 v_1^2 + 2\mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2\mathcal{C}_{\phi D}^{(2)22(1)} v_2^3 v_1 + \mathcal{C}_{\phi D}^{2(22)2} v_2^4 + 4\Theta_{WW}(v_1^2 + v_2^2) \right], \end{aligned} \quad (7.35)$$

$$\begin{aligned} \Pi_{W_3 B}(p^2) &= -\frac{p^2 \Theta_{W_3 B}}{\Lambda^2} - \frac{gg'}{4}(v_1^2 + v_2^2) - \frac{gg'(\Theta_{BB} + \Theta_{WW})}{4\Lambda^2} (v_1^2 + v_2^2) - \frac{\Theta_{W_3 B}}{8\Lambda^2} (g^2 + g'^2)(v_1^2 + v_2^2) \\ &\quad - \frac{gg'}{8\Lambda^2} \left[\mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 v_1^2 \right. \\ &\quad \left. + \mathcal{C}_{\phi D}^{(2)11(2)} v_2^2 v_1^2 + 2\mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2\mathcal{C}_{\phi D}^{(2)22(1)} v_2^3 v_1 + \mathcal{C}_{\phi D}^{2(22)2} v_2^4 \right], \end{aligned} \quad (7.36)$$

$$\begin{aligned} \Pi_{BB}(p^2) &= p^2 \left(1 + \frac{2\Theta_{BB}}{\Lambda^2} \right) + \frac{g'^2}{4}(v_1^2 + v_2^2) + \frac{\Theta_{BB} g'^2}{2\Lambda^2} (v_1^2 + v_2^2) + \frac{gg' \Theta_{W_3 B}}{4\Lambda^2} (v_1^2 + v_2^2) \\ &\quad + \frac{g'^2}{8\Lambda^2} \left[\mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi D}^{(1)21(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(1)} v_2 v_1^3 + 2\mathcal{C}_{\phi D}^{12(1)(2)} v_2^2 v_1^2 + \mathcal{C}_{\phi D}^{(1)22(1)} v_2^2 v_1^2 \right. \\ &\quad \left. + \mathcal{C}_{\phi D}^{(2)11(2)} v_2^2 v_1^2 + 2\mathcal{C}_{\phi D}^{21(2)(2)} v_2^3 v_1 + 2\mathcal{C}_{\phi D}^{(2)22(1)} v_2^3 v_1 + \mathcal{C}_{\phi D}^{2(22)2} v_2^4 \right]. \end{aligned} \quad (7.37)$$

Then recasting these $\Pi_{V_i V_j}(p^2)$ in eqs. (7.23), (7.24), (7.25), we can construct the oblique parameters for 2HDM-EFT scenario as:

$$S = \frac{4 \sin \theta_w \cos \theta_w}{\alpha \Lambda^2} \left[\mathcal{C}_{\phi W B}^{11} v_1^2 + v_2 v_1 (\mathcal{C}_{\phi W B}^{12} + \mathcal{C}_{\phi W B}^{21}) + \mathcal{C}_{\phi W B}^{22} v_2^2 \right], \quad (7.38)$$

$$\begin{aligned} T &= -\frac{g}{8\alpha \Lambda^2 M_W^2} \left[g \mathcal{C}_{\phi D}^{(1)11(1)} v_1^4 + 2\mathcal{C}_{\phi W B}^{11} (v_1^2 + v_2^2) v_1^2 g' \right. \\ &\quad \left. + v_2 \left(2\mathcal{C}_{\phi W B}^{22} v_2 v_1^2 g' + 2\mathcal{C}_{\phi W B}^{22} v_2^3 g' + g \mathcal{C}_{\phi D}^{(1)22(1)} v_2 v_1^2 + g \mathcal{C}_{\phi D}^{(2)11(2)} v_2 v_1^2 + g \mathcal{C}_{\phi D}^{2(22)2} v_2^3 \right. \right. \\ &\quad \left. \left. + 2g \mathcal{C}_{\phi D}^{(1)21(1)} v_1^3 + 2\mathcal{C}_{\phi W B}^{12} v_1 g' (v_1^2 + v_2^2) + 2\mathcal{C}_{\phi W B}^{21} v_1^3 g' + 2\mathcal{C}_{\phi W B}^{21} v_2^2 v_1 g' + 2g \mathcal{C}_{\phi D}^{(2)22(1)} v_2^2 v_1 \right) \right], \end{aligned}$$

$U = 0$.

MLRSM. Using the same principle followed in 2HDM-EFT, vacuum polarization amplitudes $\Pi_{V_i V_j}(p^2)$ in case of MLRSM-EFT can be given as:

$$\begin{aligned}
 \Pi_{W_L W_L}(p^2) = & p^2 \left(1 + 2 \frac{\Theta_{W_{LL}}}{\Lambda^2} \right) + \frac{1}{8} (2g^2 \kappa_1^2 + 2g^2 \kappa_2^2 + 4g^2 v_L^2) \\
 & + \frac{1}{8\Lambda^2} \left[2g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + g^2 \mathcal{C}_{\phi D}^{(L)rR(l)} v_L^2 v_R^2 + 8g^2 v_L^2 \Theta_{W_{LL}} + 2g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} v_L^2 \right. \\
 & + 2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} v_L^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{21(L)(l)} v_L^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{21(L)(l)} v_L^2 + 2g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(L)11(l)} v_L^2 + 2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(L)11(l)} v_L^2 \\
 & + 2g^2 \kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} v_L^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} v_L^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} v_L^2 \\
 & + 2g^2 v_L^4 (\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + \mathcal{C}_{\phi D}^{L(l)L(l)} + \mathcal{C}_{\square}^{Ll(Ll)}) + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 \\
 & + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 + 4g^2 \kappa_1^2 \Theta_{W_{LL}} + 4g^2 \kappa_2^2 \Theta_{W_{LL}} - 8g^2 \kappa_1 \kappa_2 \Theta_{W_{RL}} - 4g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{(11)(22)} \\
 & + g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{(11)(22)} + 2g^2 \kappa_1 \kappa_2^3 \mathcal{C}_{\phi D}^{(11)(22)} + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(11)(22)} + 2g^2 \kappa_1^3 \kappa_2 \mathcal{C}_{\phi D}^{(11)(22)} - 4g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)1(1)} \\
 & + g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{2(1)(1)\{21\}} + g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{2(1)(1)\{21\}} + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{(2)(1)21} + g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{(2)(1)21} \\
 & \left. + 8g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \tag{7.39}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{W_R W_R}(p^2) = & p^2 \left(1 + 2 \frac{\Theta_{W_{RR}}}{\Lambda^2} \right) + \frac{1}{8} (2g^2 \kappa_1^2 + 2g^2 \kappa_2^2 + 4g^2 v_R^2) \\
 & + \frac{1}{8\Lambda^2} \left[2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + g^2 \mathcal{C}_{\phi D}^{(R)lL(r)} v_L^2 v_R^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} v_L^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} v_L^2 \right. \\
 & + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} v_L^2 + 8g^2 v_R^2 \Theta_{W_{RR}} + 2g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} v_R^2 + 2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} v_R^2 \\
 & + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} v_R^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{21(R)(r)} v_R^2 + 2g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(R)11(r)} v_R^2 + 2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} v_R^2 \\
 & - 2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 \\
 & + 2g^2 v_R^4 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + \mathcal{C}_{\phi D}^{R(r)R(r)} + \mathcal{C}_{\square}^{Rr(Rr)}) - 8g^2 \kappa_1 \kappa_2 \Theta_{W_{LR}} + 4g^2 \kappa_1^2 \Theta_{W_{RR}} + 4g^2 \kappa_2^2 \Theta_{W_{RR}} \\
 & - 4g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{1(1)1(1)} + g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{(11)(22)} + g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{(11)(22)} + 2g^2 \kappa_1 \kappa_2^3 \mathcal{C}_{\phi D}^{(11)(22)} + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(11)(22)} \\
 & + 2g^2 \kappa_1^3 \kappa_2 \mathcal{C}_{\phi D}^{(11)(22)} - 4g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)1(1)} + g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \\
 & \left. + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(2)(1)21} + 2g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{2(1)2(1)} + 2g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{2(1)2(1)} + 4g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)2(1)} \right],
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{W_L W_R}(p^2) = & -\frac{1}{8\Lambda^2} p^2 (-8\Theta_{W_{LR}} - 8\Theta_{W_{RL}}) - \frac{1}{2} g^2 \kappa_1 \kappa_2 \\
 & + \frac{1}{8\Lambda^2} \left[g^2 v_L^2 \left(\mathcal{C}_{\phi D}^{(L)rL(r)} v_R^2 + 4\Theta_{W_{LR}} - \kappa_2 \left(\kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} + \kappa_1 (2\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Ll(2)(1)}) \right) \right) \right. \\
 & + g^2 v_L v_R (-2\kappa_2 \kappa_1 \mathcal{C}_{\phi D}^{(2)rL(1)} + \kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) + \kappa_2^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} \\
 & + \mathcal{C}_{\phi D}^{(R)11(l)})) + 4g^2 v_R^2 \Theta_{W_{RL}} + g^2 \kappa_2 \kappa_1 \mathcal{C}_{\phi D}^{R(r)1(1)} v_R^2 - 2g^2 \kappa_2 \kappa_1 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 - g^2 \kappa_2 \kappa_1 \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 \\
 & - 4g^2 \kappa_2 \kappa_1 \Theta_{W_{LL}} + 2g^2 \kappa_1^2 \Theta_{W_{LR}} + 2g^2 \kappa_2^2 \Theta_{W_{LR}} + 2g^2 \kappa_1^2 \Theta_{W_{RL}} + 2g^2 \kappa_2^2 \Theta_{W_{RL}} - 4g^2 \kappa_2 \kappa_1 \Theta_{W_{RR}} \\
 & + 2g^2 \kappa_2 \kappa_1^3 \mathcal{C}_{\phi D}^{1(1)1(1)} + 2g^2 \kappa_2^3 \kappa_1 \mathcal{C}_{\phi D}^{1(1)1(1)} - 2g^2 \kappa_2 \kappa_1^3 \mathcal{C}_{\phi D}^{(11)(22)} - 4g^2 \kappa_2^2 \kappa_1^2 \mathcal{C}_{\phi D}^{(11)(22)} - 2g^2 \kappa_2^3 \kappa_1 \mathcal{C}_{\phi D}^{(11)(22)} \\
 & + 2g^2 \kappa_2 \kappa_1^3 \mathcal{C}_{\phi D}^{2(1)1(1)} + 2g^2 \kappa_2^3 \kappa_1 \mathcal{C}_{\phi D}^{2(1)1(1)} - 2g^2 \kappa_2 \kappa_1^3 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} - 2g^2 \kappa_2^3 \kappa_1 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \\
 & \left. - g^2 \kappa_2 \kappa_1^3 \mathcal{C}_{\phi D}^{(2)(1)21} - g^2 \kappa_2^3 \kappa_1 \mathcal{C}_{\phi D}^{(2)(1)21} - 4g^2 \kappa_2 \kappa_1^3 \mathcal{C}_{\phi D}^{2(1)2(1)} - 4g^2 \kappa_2^3 \kappa_1 \mathcal{C}_{\phi D}^{2(1)2(1)} \right], \tag{7.41}
 \end{aligned}$$

$$\begin{aligned}
 \Pi_{W_{3L} W_{3L}}(p^2) = & p^2 \left(1 + 2 \frac{\Theta_{3L3L}}{\Lambda^2} \right) + \frac{1}{8} (2g^2 \kappa_1^2 + 2g^2 \kappa_2^2 + 8g^2 v_L^2) \\
 & + \frac{1}{8\Lambda^2} \left[4g^2 \kappa_1^2 \Theta_{3L3L} + 4g^2 \kappa_2^2 \Theta_{3L3L} + 16g^2 \Theta_{3L3L} v_L^2 - 4g^2 \kappa_1^2 \Theta_{3L3R} - 4g^2 \kappa_2^2 \Theta_{3L3R} - 16g \Theta_{3LB} \tilde{g} v_L^2 \right. \\
 & + 2g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} v_L v_R + 4g^2 \mathcal{C}_{\phi D}^{(L)rR(l)} v_L^2 v_R^2 + 4g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} v_L^2 + 4g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} v_L^2 \\
 & + 4g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{21(L)(l)} v_L^2 + 8g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(L)11(l)} v_L^2 + 4g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{L(l)1(1)} v_L^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} v_L^2 \\
 & + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} v_L^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} v_L^2 + 4g^2 v_L^4 (\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + 4\mathcal{C}_{\phi D}^{L(l)L(l)}) + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 \\
 & + g^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} v_R^2 + g^2 \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} v_R^2 - 2g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{1(1)1(1)} - 2g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{1(1)1(1)} + 4g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{(11)(22)} \\
 & + 4g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{(11)(22)} - 2g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{2(1)1(1)} - 2g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{2(1)1(1)} + g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} \\
 & \left. + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}} + 2g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{(2)(1)21} + 2g^2 \kappa_1^4 \mathcal{C}_{\phi D}^{2(1)2(1)} + 2g^2 \kappa_2^4 \mathcal{C}_{\phi D}^{2(1)2(1)} + 4g^2 \kappa_1^2 \kappa_2^2 \mathcal{C}_{\phi D}^{2(1)2(1)} \right]
 \end{aligned}$$

$$+2g^2\kappa_1^2\kappa_2^2\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}+g^2\kappa_1^4\mathcal{C}_{\phi D}^{(2)(1)21}+g^2\kappa_2^4\mathcal{C}_{\phi D}^{(2)(1)21}+4g^2\kappa_1^4\mathcal{C}_{\phi D}^{2(1)2(1)}+4g^2\kappa_2^4\mathcal{C}_{\phi D}^{2(1)2(1)}\Big], \quad (7.42)$$

$$\begin{aligned} \Pi_{W_{3L}W_{3R}}(p^2) &= 4p^2\frac{\Theta_{3L3R}}{\Lambda^2} + \frac{1}{4}(-2g^2\kappa_1^2-2g^2\kappa_2^2) \\ &+ \frac{1}{4\Lambda^2}\Big[-2g^2\kappa_1^2\Theta_{3L3L}-2g^2\kappa_2^2\Theta_{3L3L}+gv_L^2(8g\Theta_{3L3R}-8\Theta_{3RB}\tilde{g}+4g\mathcal{C}_{\phi D}^{(L)rL(r)}v_R^2-2g\kappa_1^2\mathcal{C}_{\phi D}^{L(l)1(1)}) \\ &-g\kappa_1^2\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}}-g\kappa_2^2\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}}-g\kappa_1^2\mathcal{C}_{\phi D}^{Ll(2)(1)})+4g^2\kappa_1^2\Theta_{3L3R}+4g^2\kappa_2^2\Theta_{3L3R}+8g^2\Theta_{3L3R}v_R^2 \\ &-8g\Theta_{3LB}\tilde{g}v_R^2-2g^2\kappa_1^2\Theta_{3R3R}-2g^2\kappa_2^2\Theta_{3R3R}+2g^2\kappa_2^2v_Lv_R(2(\mathcal{C}_{\phi D}^{(L)11(r)}+\mathcal{C}_{\phi D}^{(L)12(r)}+\mathcal{C}_{\phi D}^{(R)11(l)}) \\ &-\mathcal{C}_{\phi D}^{(2)rL(1)})+g^2\kappa_1^2v_R^2(2\mathcal{C}_{\phi D}^{R(r)1(1)}-\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}}-\mathcal{C}_{\phi D}^{Rr(2)(1)})-g^2\kappa_2^2\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}}v_R^2 \\ &+g^2\kappa_1^4(2\mathcal{C}_{\phi D}^{1(1)1(1)}-4\mathcal{C}_{\phi D}^{(11)(22)}+2\mathcal{C}_{\phi D}^{2(1)1(1)}-\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}-\mathcal{C}_{\phi D}^{(2)(1)21}-4\mathcal{C}_{\phi D}^{2(1)2(1)}) \\ &+g^2\kappa_2^4(2\mathcal{C}_{\phi D}^{1(1)1(1)}-4\mathcal{C}_{\phi D}^{(11)(22)}+2\mathcal{C}_{\phi D}^{2(1)1(1)}-\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}-\mathcal{C}_{\phi D}^{(2)(1)21}-4\mathcal{C}_{\phi D}^{2(1)2(1)}) \\ &-2g^2\kappa_2^2\kappa_1^2\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}\Big], \end{aligned} \quad (7.43)$$

$$\begin{aligned} \Pi_{W_{3R}W_{3R}}(p^2) &= p^2\left(1+2\frac{\Theta_{3R3R}}{\Lambda^2}\right) + \frac{1}{8}(2g^2\kappa_1^2+2g^2\kappa_2^2+8g^2v_R^2) \\ &+ \frac{1}{8\Lambda^2}\Big[-4g^2\kappa_1^2\Theta_{3L3R}-4g^2\kappa_2^2\Theta_{3L3R}+4g^2\kappa_1^2\Theta_{3R3R}+4g^2\kappa_2^2\Theta_{3R3R}+16g^2\Theta_{3R3R}v_R^2 \\ &-16g\Theta_{3RB}\tilde{g}v_R^2+2g^2\kappa_2^2\mathcal{C}_{\phi D}^{(2)rL(1)}v_Lv_R+4g^2\mathcal{C}_{\phi D}^{(R)lL(r)}v_L^2v_R^2+g^2\kappa_1^2\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}}v_L^2 \\ &+g^2\kappa_2^2\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}}v_L^2+g^2\kappa_1^2\mathcal{C}_{\phi D}^{Ll(2)(1)}v_L^2+4g^2\kappa_1^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}v_R^2+4g^2\kappa_2^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}v_R^2 \\ &+4g^2\kappa_1^2\mathcal{C}_{\phi D}^{21(R)(r)}v_R^2+8g^2\kappa_2^2\mathcal{C}_{\phi D}^{(R)11(r)}v_R^2-4g^2\kappa_1^2\mathcal{C}_{\phi D}^{R(r)1(1)}v_R^2+g^2\kappa_1^2\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}}v_R^2 \\ &+g^2\kappa_2^2\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}}v_R^2+g^2\kappa_1^2\mathcal{C}_{\phi D}^{Rr(2)(1)}v_R^2+4g^2v_R^4(\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}}+4\mathcal{C}_{\phi D}^{R(r)R(r)})-2g^2\kappa_1^4\mathcal{C}_{\phi D}^{1(1)1(1)} \\ &-2g^2\kappa_2^4\mathcal{C}_{\phi D}^{1(1)1(1)}+4g^2\kappa_1^4\mathcal{C}_{\phi D}^{(11)(22)}+4g^2\kappa_2^4\mathcal{C}_{\phi D}^{(11)(22)}-2g^2\kappa_1^4\mathcal{C}_{\phi D}^{2(1)1(1)}-2g^2\kappa_2^4\mathcal{C}_{\phi D}^{2(1)1(1)} \\ &+g^2\kappa_1^4\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}+g^2\kappa_2^4\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}+2g^2\kappa_1^2\kappa_2^2\mathcal{C}_{\phi D}^{\{(2)(1)\}\{21\}}+g^2\kappa_1^4\mathcal{C}_{\phi D}^{(2)(1)21}+g^2\kappa_2^4\mathcal{C}_{\phi D}^{(2)(1)21} \\ &+4g^2\kappa_1^4\mathcal{C}_{\phi D}^{2(1)2(1)}+4g^2\kappa_2^4\mathcal{C}_{\phi D}^{2(1)2(1)}\Big], \end{aligned} \quad (7.44)$$

$$\begin{aligned} \Pi_{W_{3LB}}(p^2) &= p^2\frac{\Theta_{3LB}}{\Lambda^2} - g\tilde{g}v_L^2 \\ &+ \frac{1}{4\Lambda^2}\Big[-4g\Theta_{3L3L}\tilde{g}v_L^2 - g\tilde{g}v_R^2(4\Theta_{3L3R}+\kappa_1^2\mathcal{C}_{\phi D}^{R(r)1(1)})+g^2\kappa_1^2\Theta_{3LB}+g^2\kappa_2^2\Theta_{3LB} \\ &+4\Theta_{3LB}(\tilde{g})^2v_R^2+4\Theta_{3LB}(g^2+(\tilde{g})^2)v_L^2-g^2(\kappa_1^2+\kappa_2^2)\Theta_{3RB}-4g\Theta_{BB}\tilde{g}v_L^2 \\ &-2g\kappa_2^2\tilde{g}v_Lv_R(\mathcal{C}_{\phi D}^{(L)11(r)}+\mathcal{C}_{\phi D}^{(L)12(r)}+\mathcal{C}_{\phi D}^{(R)11(l)})-2g\tilde{g}v_L^2v_R^2(\mathcal{C}_{\phi D}^{(L)rL(r)}+\mathcal{C}_{\phi D}^{(L)rR(l)}) \\ &-2g\kappa_1^2\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}}\tilde{g}v_L^2-2g\kappa_2^2\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}}\tilde{g}v_L^2-2g\kappa_1^2\mathcal{C}_{\phi D}^{21(L)(l)}\tilde{g}v_L^2-4g\kappa_2^2\mathcal{C}_{\phi D}^{(L)11(l)}\tilde{g}v_L^2 \\ &-g\kappa_1^2\mathcal{C}_{\phi D}^{L(l)1(1)}\tilde{g}v_L^2-2g\tilde{g}v_L^4(\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}}+4\mathcal{C}_{\phi D}^{L(l)L(l)})\Big], \end{aligned} \quad (7.45)$$

$$\begin{aligned} \Pi_{W_{3RB}}(p^2) &= p^2\frac{\Theta_{3RB}}{\Lambda^2} - g\tilde{g}v_R^2 \\ &+ \frac{1}{4\Lambda^2}\Big[-g\tilde{g}v_L^2(4\Theta_{3L3R}-\kappa_1^2\mathcal{C}_{\phi D}^{L(l)1(1)})-g^2(\kappa_1^2+\kappa_2^2)\Theta_{3LB}-4g\Theta_{3R3R}\tilde{g}v_R^2+g^2\kappa_1^2\Theta_{3RB} \\ &+g^2\kappa_2^2\Theta_{3RB}+4\Theta_{3RB}(\tilde{g})^2v_L^2+4\Theta_{3RB}(g^2+(\tilde{g})^2)v_R^2-4g\Theta_{BB}\tilde{g}v_R^2 \\ &-2g\kappa_2^2\tilde{g}v_Lv_R(\mathcal{C}_{\phi D}^{(L)11(r)}+\mathcal{C}_{\phi D}^{(L)12(r)}+\mathcal{C}_{\phi D}^{(R)11(l)})-2g\tilde{g}v_L^2v_R^2(\mathcal{C}_{\phi D}^{(L)rL(r)}+\mathcal{C}_{\phi D}^{(R)lL(r)}) \\ &-2g\kappa_1^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}v_R^2-2g\kappa_2^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}v_R^2-2g\kappa_1^2\mathcal{C}_{\phi D}^{21(R)(r)}\tilde{g}v_R^2-4g\kappa_2^2\mathcal{C}_{\phi D}^{(R)11(r)}\tilde{g}v_R^2 \\ &+g\kappa_1^2\mathcal{C}_{\phi D}^{R(r)1(1)}\tilde{g}v_R^2-2g\tilde{g}v_R^4(\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}}+4\mathcal{C}_{\phi D}^{R(r)R(r)})\Big], \end{aligned} \quad (7.46)$$

$$\begin{aligned} \Pi_{BB}(p^2) &= \frac{1}{2}\tilde{g}(2\tilde{g}v_L^2+2\tilde{g}v_R^2) + \frac{\tilde{g}}{2\Lambda^2}\Big[-4g\Theta_{3LB}v_L^2-4g\Theta_{3RB}v_R^2+4\Theta_{BB}\tilde{g}v_L^2+4\Theta_{BB}\tilde{g}v_R^2 \\ &+2\kappa_2^2\tilde{g}v_Lv_R(\mathcal{C}_{\phi D}^{(L)11(r)}+\mathcal{C}_{\phi D}^{(L)12(r)}+\mathcal{C}_{\phi D}^{(R)11(l)})+\tilde{g}v_L^2v_R^2(2\mathcal{C}_{\phi D}^{(L)rL(r)}+\mathcal{C}_{\phi D}^{(L)rR(l)}+\mathcal{C}_{\phi D}^{(R)lL(r)}) \\ &+\kappa_1^2\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}}\tilde{g}v_L^2+\kappa_2^2\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}}\tilde{g}v_L^2+\kappa_1^2\mathcal{C}_{\phi D}^{21(L)(l)}\tilde{g}v_L^2+2\kappa_2^2\mathcal{C}_{\phi D}^{(L)11(l)}\tilde{g}v_L^2+\tilde{g}v_L^4(\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}}) \end{aligned}$$

$$\begin{aligned}
 & +4\mathcal{C}_{\phi D}^{L(l)L(l)} + \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} v_R^2 + \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} v_R^2 + \kappa_1^2 \mathcal{C}_{\phi D}^{21(R)(r)} \tilde{g} v_R^2 + 2\kappa_2^2 \mathcal{C}_{\phi D}^{(R)11(r)} \tilde{g} v_R^2 \\
 & + \tilde{g} v_R^4 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 4\mathcal{C}_{\phi D}^{R(r)R(r)}) \Big]. \tag{7.47}
 \end{aligned}$$

Using the rotation matrices $\tilde{\mathcal{R}}_{a,b}$, see eq. (5.29), we can define these $\Pi_{V_i V_j}(p^2)$ in the physical basis as follows:

$$\begin{aligned}
 \Pi_{W_1 W_1} &= \Pi_{W_L W_L} (\tilde{\mathcal{R}}_a^{11})^2 + 2\Pi_{W_L W_R} \tilde{\mathcal{R}}_a^{11} \tilde{\mathcal{R}}_a^{21} + \Pi_{W_R W_R} (\tilde{\mathcal{R}}_a^{21})^2, \\
 \Pi_{W_1 W_2} &= \Pi_{W_L W_L} \tilde{\mathcal{R}}_a^{11} \tilde{\mathcal{R}}_{12}^a + \Pi_{W_L W_R} (\tilde{\mathcal{R}}_{11}^a \tilde{\mathcal{R}}_{22}^a + \tilde{\mathcal{R}}_{12}^a \tilde{\mathcal{R}}_{21}^a) + \Pi_{W_R W_R} \tilde{\mathcal{R}}_{21}^a \tilde{\mathcal{R}}_{22}^a, \\
 \Pi_{W_2 W_2} &= \Pi_{W_L W_L} (\tilde{\mathcal{R}}_{12}^a)^2 + 2\Pi_{W_L W_R} \tilde{\mathcal{R}}_{11}^a \tilde{\mathcal{R}}_{22}^a + \Pi_{W_R W_R} (\tilde{\mathcal{R}}_{22}^a)^2, \\
 \Pi_{Z_1 Z_1} &= \Pi_{BB} (\tilde{\mathcal{R}}_{31}^b)^2 + 2\Pi_{W_{3L} B} \tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{31}^b + \Pi_{W_{3L} W_{3L}} (\tilde{\mathcal{R}}_{11}^b)^2 + 2\Pi_{W_{3L} W_{3R}} \tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{21}^b + 2\Pi_{W_{3R} B} \tilde{\mathcal{R}}_{21}^b \tilde{\mathcal{R}}_{31}^b \\
 & \quad + \Pi_{W_{3R} W_{3R}} (\tilde{\mathcal{R}}_{21}^b)^2, \\
 \Pi_{Z_2 Z_2} &= \Pi_{BB} (\tilde{\mathcal{R}}_{32}^b)^2 + 2\Pi_{W_{3L} B} \tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{32}^b + \Pi_{W_{3L} W_{3L}} (\tilde{\mathcal{R}}_{12}^b)^2 + 2\Pi_{W_{3L} W_{3R}} \tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{22}^b + 2\Pi_{W_{3R} B} \tilde{\mathcal{R}}_{22}^b \tilde{\mathcal{R}}_{32}^b \\
 & \quad + \Pi_{W_{3R} W_{3R}} (\tilde{\mathcal{R}}_{22}^b)^2, \\
 \Pi_{Z_1 Z_2} &= \Pi_{BB} \tilde{\mathcal{R}}_{31}^b \tilde{\mathcal{R}}_{32}^b + \Pi_{W_{3L} B} (\tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{32}^b + \tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{31}^b) + \Pi_{W_{3L} W_{3L}} \tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{12}^b + \Pi_{W_{3L} W_{3R}} (\tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{22}^b + \tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{21}^b) \\
 & \quad + \Pi_{W_{3R} B} (\tilde{\mathcal{R}}_{21}^b \tilde{\mathcal{R}}_{32}^b + \tilde{\mathcal{R}}_{22}^b \tilde{\mathcal{R}}_{31}^b) + \Pi_{W_{3R} W_{3R}} \tilde{\mathcal{R}}_{21}^b \tilde{\mathcal{R}}_{22}^b, \\
 \Pi_{\gamma\gamma} &= \Pi_{BB} (\tilde{\mathcal{R}}_{33}^b)^2 + 2\Pi_{W_{3L} B} \tilde{\mathcal{R}}_{13}^b \tilde{\mathcal{R}}_{33}^b + \Pi_{W_{3L} W_{3L}} (\tilde{\mathcal{R}}_{13}^b)^2 + 2\Pi_{W_{3L} W_{3R}} \tilde{\mathcal{R}}_{13}^b \tilde{\mathcal{R}}_{23}^b + 2\Pi_{W_{3R} B} \tilde{\mathcal{R}}_{23}^b \tilde{\mathcal{R}}_{33}^b \\
 & \quad + \Pi_{W_{3R} W_{3R}} (\tilde{\mathcal{R}}_{23}^b)^2, \\
 \Pi_{Z_1 \gamma} &= \Pi_{BB} \tilde{\mathcal{R}}_{31}^b \tilde{\mathcal{R}}_{33}^b + \Pi_{W_{3L} B} (\tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{33}^b + \tilde{\mathcal{R}}_{13}^b \tilde{\mathcal{R}}_{31}^b) + \Pi_{W_{3L} W_{3L}} \tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{13}^b + \Pi_{W_{3L} W_{3R}} (\tilde{\mathcal{R}}_{11}^b \tilde{\mathcal{R}}_{23}^b + \tilde{\mathcal{R}}_{13}^b \tilde{\mathcal{R}}_{21}^b) \\
 & \quad + \Pi_{W_{3R} B} (\tilde{\mathcal{R}}_{21}^b \tilde{\mathcal{R}}_{33}^b + \tilde{\mathcal{R}}_{23}^b \tilde{\mathcal{R}}_{31}^b) + \Pi_{W_{3R} W_{3R}} \tilde{\mathcal{R}}_{21}^b \tilde{\mathcal{R}}_{23}^b, \\
 \Pi_{Z_2 \gamma} &= \Pi_{BB} \tilde{\mathcal{R}}_{32}^b \tilde{\mathcal{R}}_{33}^b + \Pi_{W_{3L} B} (\tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{33}^b + \tilde{\mathcal{R}}_{13}^b \tilde{\mathcal{R}}_{32}^b) + \Pi_{W_{3L} W_{3L}} \tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{13}^b + \Pi_{W_{3L} W_{3R}} (\tilde{\mathcal{R}}_{12}^b \tilde{\mathcal{R}}_{23}^b + \tilde{\mathcal{R}}_{13}^b \tilde{\mathcal{R}}_{22}^b) \\
 & \quad + \Pi_{W_{3R} B} (\tilde{\mathcal{R}}_{22}^b \tilde{\mathcal{R}}_{33}^b + \tilde{\mathcal{R}}_{23}^b \tilde{\mathcal{R}}_{32}^b) + \Pi_{W_{3R} W_{3R}} \tilde{\mathcal{R}}_{22}^b \tilde{\mathcal{R}}_{23}^b.
 \end{aligned}$$

Now the oblique parameters (S, T, U) for MLRSM-EFT scenario can be constructed using eqs. (7.23), (7.24), (7.25). We need to keep in mind that in order to define the S, T, U parameters, we should use only SM like gauge bosons $W_1^\pm, Z_1, \gamma$.

7.5 Magnetic moments of charged fermions

The charged fermions can couple to the gauge field strength tensors through effective $\psi^2 \phi X$ class of dimension-6 operators, see figure 8(a). Here, we are mostly interested in the coupling between the charged leptons and quarks with the electromagnetic field strength tensor ($\bar{f}_L \sigma_{\mu\nu} f_R A^{\mu\nu}$) which lead to their respective magnetic moments (f). There are contributions to the anomalous magnetic moments of the charged leptons (τ, μ, e) due to the modified interactions in the presence of charged scalars, see figures 8(b), (c).

2HDM. The magnetic moment interactions between the charged leptons, and quarks with the electromagnetic field tensor ($A_{\mu\nu}$) in presence of $\psi^2 \phi X$ operators (see table 7) within 2HDM-EFT framework are given as:

$$\begin{aligned}
 \bar{l}_L \sigma^{\mu\nu} l_R A_{\mu\nu} &: \frac{1}{\sqrt{2}\Lambda^2} \left[(\mathcal{C}_{eB}^1 v_1 + \mathcal{C}_{eB}^2 v_2) \cos \theta_w - (\mathcal{C}_{eW}^1 v_1 + \mathcal{C}_{eW}^2 v_2) \sin \theta_w \right], \\
 \bar{u}_L \sigma^{\mu\nu} u_R A_{\mu\nu} &: \frac{1}{\sqrt{2}\Lambda^2} \left[(\mathcal{C}_{uB}^1 v_1 + \mathcal{C}_{uB}^2 v_2) \cos \theta_w + (\mathcal{C}_{uW}^1 v_1 + \mathcal{C}_{uW}^2 v_2) \sin \theta_w \right], \\
 \bar{d}_L \sigma^{\mu\nu} d_R A_{\mu\nu} &: \frac{1}{\sqrt{2}\Lambda^2} \left[(\mathcal{C}_{dB}^1 v_1 + \mathcal{C}_{dB}^2 v_2) \cos \theta_w - (\mathcal{C}_{dW}^1 v_1 + \mathcal{C}_{dW}^2 v_2) \sin \theta_w \right].
 \end{aligned}$$

Figure 8. Feynman diagram representing the magnetic moment interactions of charged fermions: (a) $\psi^2\phi X \Rightarrow$ class of effective operators [■] leading to magnetic moment interactions $\bar{f}_L \sigma_{\mu\nu} f_R A^{\mu\nu}$ with f representing charged fermions. (b) and (c) modified vertices [●] lead to radiative decay of charged leptons ($l_i \rightarrow l_j \gamma$). For $i = j = \tau, \mu, e$, both diagrams contribute to the magnetic moment of charged leptons.

Vertex	2HDM: effective vertex ($\bar{\nu} e^- H^+$) factor
$[\bar{\nu} e^- H^+]_{lm}$	$\left[y_2^e s_{\beta_+} - y_1^e c_{\beta_+} + \frac{1}{2\Lambda^2} \left(-A_+^{11} y_1^e s_{\beta_+} - A_+^{22} y_2^e s_{\beta_+} + A_+^{11} y_1^e c_{\beta_+} + A_+^{12} y_2^e c_{\beta_+} \right. \right.$ $+ v_1^2 \left(\mathcal{C}_{Le}^{1(11)} c_{\beta_+} - \mathcal{C}_{Le}^{2(11)} s_{\beta_+} \right) + v_1 v_2 \left((\mathcal{C}_{Le}^{1(12)} + \mathcal{C}_{Le}^{1(21)}) c_{\beta_+} - (\mathcal{C}_{Le}^{2(12)} + \mathcal{C}_{Le}^{2(21)}) s_{\beta_+} \right)$ $\left. \left. + v_2^2 \left(\mathcal{C}_{Le}^{1(22)} c_{\beta_+} - \mathcal{C}_{Le}^{2(22)} s_{\beta_+} \right) \right) \right]_{lm}$

Table 35. 2HDM: coupling of Singly charged scalar with a charged lepton and a neutrino. Here, $s_{\theta_i} = \sin \theta_i$, $c_{\theta_i} = \cos \theta_i$ and l, m are the fermion family indices.

The magnetic moments of the charged leptons get additional contributions due to their modified couplings with the singly charged scalars ($H^\pm$), see table 35 and figure 8(b). There will be similar contributions to the quark magnetic moments which are not explicitly mentioned here.

Vertex	MLRSM: effective vertex ($\bar{\nu} e^- H^+$) factor
	Majorana type interactions
$\left[(\overline{e_L})^c \nu_L \delta_L^+ \right]_{lm}$	$\left[\frac{y_M}{\sqrt{2}} + \frac{1}{2\sqrt{2}\Lambda^2} \left(-y_M A_+^{44} + v_L^2 (-\mathcal{C}_{l_L}^{l(Ll)} - \mathcal{C}_{l_L}^{lLl}) - \mathcal{C}_{l_L}^{l(Rr)} v_R^2 - \kappa_+^2 \mathcal{C}_{l_L}^{l(21)} - \kappa_1^2 \mathcal{C}_{l_L}^{l21} - 2\kappa_1 \kappa_2 \mathcal{C}_{l_L}^{l(23)} - 2\kappa_1 \kappa_2 \mathcal{C}_{l_L}^{l(41)} \right) \right]_{lm}$
$\left[(\overline{e_R})^c \nu_R \delta_R^+ \right]_{lm}$	$\left[\frac{y_M}{\sqrt{2}} + \frac{1}{2\sqrt{2}\Lambda^2} \left(-y_M A_+^{33} - \mathcal{C}_{l_R}^{r(Ll)} v_L^2 + v_R^2 (-\mathcal{C}_{l_R}^{r(Rr)} - \mathcal{C}_{l_R}^{rRr}) - \kappa_+^2 \mathcal{C}_{l_R}^{r(21)} - \kappa_1^2 \mathcal{C}_{l_R}^{r21} - 2\kappa_1 \kappa_2 \mathcal{C}_{l_R}^{r(23)} - 2\kappa_1 \kappa_2 \mathcal{C}_{l_R}^{r(41)} \right) \right]_{lm}$
$\vdots$	$\vdots$
	Dirac type interactions
$\vdots$	$\vdots$
$\left[\bar{\nu}_L e_R \phi_1'^+ \right]_{lm}$	$\left[\frac{1}{\kappa_+} (\kappa_1 \tilde{y}_D - \kappa_1 y_D) + \frac{1}{2\kappa_+ \Lambda^2} \left(\kappa_1 (y_D - \tilde{y}_D) A_+^{11} + \kappa_2 (\tilde{y}_D - y_D) A_+^{12} + \kappa_1^3 \mathcal{C}_{l_L l_R}^{1(21)} + \kappa_1 \kappa_2^2 \mathcal{C}_{l_L l_R}^{1(21)} + 2\kappa_1^2 \kappa_2 \mathcal{C}_{l_L l_R}^{1(23)} + 2\kappa_1^2 \kappa_2 \mathcal{C}_{l_L l_R}^{1(41)} - \kappa_2^3 \mathcal{C}_{l_L l_R}^{3(21)} - \kappa_1^2 \kappa_2 \mathcal{C}_{l_L l_R}^{3(21)} - 2\kappa_1 \kappa_2^2 \mathcal{C}_{l_L l_R}^{3(23)} - 2\kappa_1 \kappa_2^2 \mathcal{C}_{l_L l_R}^{3(41)} \right) \right]_{lm}$
$\left[\bar{\nu}_R e_L \phi_2'^+ \right]_{lm}$	$\left[\frac{1}{\kappa_+} (\kappa_1 \tilde{y}_D - \kappa_1 y_D) + \frac{1}{2\kappa_+ \Lambda^2} \left(\kappa_2 (y_D - \tilde{y}_D) A_+^{12} + \kappa_1 (y_D - \tilde{y}_D) A_+^{22} + \kappa_1^3 \mathcal{C}_{l_L l_R}^{1(21)} + \kappa_1 \kappa_2^2 \mathcal{C}_{l_L l_R}^{1(21)} + 2\kappa_1^2 \kappa_2 \mathcal{C}_{l_L l_R}^{1(23)} + 2\kappa_1^2 \kappa_2 \mathcal{C}_{l_L l_R}^{1(41)} + \kappa_2^3 \mathcal{C}_{l_L l_R}^{3(21)} + \kappa_1^2 \kappa_2 \mathcal{C}_{l_L l_R}^{3(21)} + 2\kappa_1 \kappa_2^2 \mathcal{C}_{l_L l_R}^{3(23)} + 2\kappa_1 \kappa_2^2 \mathcal{C}_{l_L l_R}^{3(41)} \right) \right]_{lm}$

Table 36. MLRSM: couplings of Singly charged scalars with a charged lepton and a neutrino in unphysical basis. A few Majorana type and Dirac type interactions are listed. Here, L, R represent the fermions belonging to $SU(2)_L, SU(2)_R$ gauge groups respectively.

MLRSM. The magnetic moment interactions between the charged leptons and quarks with the electromagnetic field strength tensor ($A_{\mu\nu}$) in the presence of $\psi^2 \phi X$ operators (see table 16) within MLRSM-EFT framework are given as:

$$\begin{aligned}
 \bar{l}_L \sigma^{\mu\nu} l_R A_{\mu\nu} &: -\frac{1}{\sqrt{2}\Lambda^2} \left[\left(\kappa_2 \mathcal{C}_{l_L l_R}^{1W_L} + \kappa_1 \mathcal{C}_{l_L l_R}^{3W_L} \right) \sin \theta_w + \left(\kappa_2 \mathcal{C}_{l_L l_R}^{1W_R} + \kappa_1 \mathcal{C}_{l_L l_R}^{3W_R} \right) \sin \theta_1 \cos \theta_w \right], \\
 \bar{u}_L \sigma^{\mu\nu} u_R A_{\mu\nu} &: \frac{1}{\sqrt{2}\Lambda^2} \left[\left(\kappa_1 \mathcal{C}_{q_L q_R}^{1W_L} + \kappa_2 \mathcal{C}_{q_L q_R}^{3W_L} \right) \sin \theta_w + \left(\kappa_1 \mathcal{C}_{q_L q_R}^{1W_R} + \kappa_2 \mathcal{C}_{q_L q_R}^{3W_R} \right) \sin \theta_1 \cos \theta_w \right], \\
 \bar{d}_L \sigma^{\mu\nu} d_R A_{\mu\nu} &: -\frac{1}{\sqrt{2}\Lambda^2} \left[\left(\kappa_2 \mathcal{C}_{q_L q_R}^{1W_L} + \kappa_1 \mathcal{C}_{q_L q_R}^{3W_L} \right) \sin \theta_w + \left(\kappa_2 \mathcal{C}_{q_L q_R}^{1W_R} + \kappa_1 \mathcal{C}_{q_L q_R}^{3W_R} \right) \sin \theta_1 \cos \theta_w \right].
 \end{aligned}$$

The modified interactions involving singly ($H^\pm$) and doubly ($\delta^{\pm\pm}$) charged scalars provide additional contributions to the magnetic moments of the charged leptons, see tables 36, 37 and figures 8(b), (c). There will be similar contributions to the quark magnetic moments through their couplings with singly charged scalars. The vertices are not explicitly depicted here.

7.6 Phenomenology involving charged scalars

The SM scalar sector contains only one Higgs doublet which takes part in the spontaneous electro-weak symmetry breaking, and the Higgs mechanism predicts only one real scalar.

Vertex	MLRSM: effective vertex ($e^\mp e^\mp \delta^{\pm\pm}$) factor
$[e_L^\mp e_L^\mp \delta_L^{\pm\pm}]_{lm}$	$\left[y_M - \frac{1}{2\Lambda^2} \left(y_M A_{++}^{22} + C_{l_L}^{l(Ll)} v_L^2 + C_{l_L}^{l(Rr)} v_R^2 + \kappa_1^2 C_{l_L}^{l(21)} + \kappa_2^2 C_{l_L}^{l(21)} + \kappa_2^2 C_{l_L}^{l(21)} + 2\kappa_1 \kappa_2 C_{l_L}^{l(23)} + 2\kappa_1 \kappa_2 C_{l_L}^{l(41)} \right) \right]_{lm}$
$[e_R^\mp e_R^\mp \delta_R^{\pm\pm}]_{lm}$	$\left[y_M - \frac{1}{2\Lambda^2} \left(y_M A_{++}^{11} + C_{l_R}^{r(Ll)} v_L^2 + C_{l_R}^{r(Rr)} v_R^2 + \kappa_1^2 C_{l_R}^{r(21)} + \kappa_2^2 C_{l_R}^{r(21)} + \kappa_2^2 C_{l_R}^{r(21)} + 2\kappa_1 \kappa_2 C_{l_R}^{r(23)} + 2\kappa_1 \kappa_2 C_{l_R}^{r(41)} \right) \right]_{lm}$
$[e_R^\mp e_L^\mp \delta_L^{\pm\pm}]_{lm}$	$\frac{1}{\Lambda^2} \left[-\frac{y_M}{2} A_{++}^{12} - \frac{1}{2} \kappa_2^2 C_{l_R}^{l(21)} - \frac{1}{2} \kappa_1 \kappa_2 C_{l_R}^{l(23)} - \frac{1}{2} \kappa_1 \kappa_2 C_{l_R}^{l(41)} \right]_{lm}$
$[e_L^\mp e_R^\mp \delta_R^{\pm\pm}]_{lm}$	$\frac{1}{\Lambda^2} \left[-\frac{y_M}{2} A_{++}^{12} - \frac{1}{2} \kappa_2^2 C_{l_L}^{r(21)} - \frac{1}{2} \kappa_1 \kappa_2 C_{l_L}^{r(23)} - \frac{1}{2} \kappa_1 \kappa_2 C_{l_L}^{r(41)} \right]_{lm}$

Table 37. MLRSM: coupling between the doubly charged scalar ($\delta^{\pm\pm}$) and a pair of same signed charged lepton ($e^\pm$). Here, the subscript L, R in the charged lepton (e) correspond to the $SU(2)_{L,R}$.

Figure 9. Pair production of charged scalar (singly and doubly) through neutral gauge boson propagator in s -channel. The $[\bullet]$ depicts the modified Feynman vertex (renormalizable + corrections due to dimension-6 operators).

Figure 10. (a) Production of single doubly (singly) charged scalar and (b) Pair production of doubly (singly) charged scalars through Vector Boson Fusion (VBF). The $[\bullet]$ depicts the modified Feynman vertex (renormalizable + corrections due to dimension-6 operators).

Thus any signature supporting new scalars will signify the presence of BSM physics. In this section, we have briefly outlined the interactions involving the charged scalars within the EFT framework. Both the models, 2HDM and MLRSM, predict the existence of charged scalars. These charged scalars can be produced through s -channel and Vector Boson Fusion (VBF) processes, see figures 9, and 10.

2HDM: Feynman vertices involving singly charged scalars. In the 2HDM scenario, due to the presence of an extra Higgs doublet, we have a physical singly charged scalar particle ($H^\pm$) which signifies a distinct departure from the Standard Model. Thus analysis related to this charged scalar $H^\pm$ could pave a path to explore this model in the light of present and future colliders. In this context, their pair production $pp \rightarrow H^\pm H^\pm$, and lep-

tonic decay modes, see table 35, need to be analyzed. As the interaction vertices are modified in the presence of the dimension-6 operators, the predictions for cross sections and exclusion limits may alter significantly. Thus it will be worthwhile to revisit the phenomenology involving the charged scalars in [177–186, 226, 227] within the 2HDM-EFT framework.

MLRSM: Feynman vertices involving doubly charged scalars. The MLRSM scenario has been explored extensively in the context of the LHC and FCC, considering only renormalizable interactions. In this paper, we have shown how those interactions get modified once we include the dimension-6 effective operators. Here, instead of discussing all possible phenomenology, we have highlighted a few channels which are considered to be the smoking gun features of this model. One of the most striking aspects of this scenario is the presence of left (Δ_L) and right (Δ_R) triplet Higgs multiplets that contain neutral, singly and doubly charged scalars.

The presence of doubly charged scalars makes this model phenomenologically attractive and distinct. Our focus will be on the doubly charged scalars: their production ($pp \rightarrow \delta_i^{++}\delta_j^{--}$) and their decay modes to a pair of (i) charged leptons $l_a^\pm l_b^\pm$ and (ii) charged gauge bosons $W^\pm W^\pm$. One of the most significant final states in this model is hadronically quiet four leptons ($l_a^\pm l_b^\pm l_c^\mp l_d^\mp$) through neutral current s -channel processes [228–230], see figure 9. Apart from the s -channel process, through the Vector Boson Fusion (VBF) [231–233], the following final states can be achieved: (i) same-signed di-leptons ($l_a^\pm l_b^\pm$) with 2 forward jets, see figure 10(a) and (ii) four leptons with 2 forwards jets, see figure 10(b). In this paper, we have depicted the relevant Feynman vertices, see figure 11, in tables 37–46. In passing we would like to note that the phenomenology involving the singly charged scalars will be very similar to that for 2HDM case which is already discussed in the previous segment. The “Golden channel” [228, 234] involving heavy right-handed (N) leading to same signed charged leptons and two jets also gets modified in presence of the effective interactions, see tables 29, 30. The phenomenology involving the charged scalars and right-handed neutrino within the renormlizable MLRSM framework [214, 228–230, 235–244] needs to be revisited in the presence of the effective operators.

7.7 Rare processes

So far, we have discussed the impact of the modified Feynman vertices on different low energy observables and also on some significant and promising collider phenomenological processes. Now we would like to focus on possible rare processes which may get additional contributions from the higher dimensional operators. As these processes are yet to be observed, the rate of their occurrence must be very small, which in turn puts a stringent upper limit on the corresponding Wilson Coefficients. Thus not only for the sake of completeness but also to estimate the allowed maximum values of the WCs these channels must be taken into account. As we have a complete set of operators, there will be many rare processes, but we can not include all of them. We will provide an outline to show how these effective operators can affect different rare events.

Looking into the VBF channel, figure 10, it is expected that the amplitude for neutrinoless double beta decay ($0\nu 2\beta$), $2n \rightarrow 2p + 2e^-$, figure 12(b), will be modified as compared to the earlier analysis based on the renormalizable interactions [245, 246]. The

Figure 11. Effective interaction vertices between charged scalars and gauge bosons in MLRSM model. Decay of doubly charged scalar is driven by: (a) $\delta^{\pm\pm}-W^{\mp}-W^{\mp}$ and (b) $\delta^{\pm\pm}-l_a^{\mp}-l_b^{\mp}$ vertices. The vertex (c) $\delta^{++}(H^+)-\delta^{--}(H^-)-W^+-W^-$ is relevant for VBF process. The vertex (d) $\delta^{++}(H^+)-\delta^{--}(H^-)-Z(\gamma)$ participates in the s -channel pair production of doubly (singly) charged scalar. The [●] depicts the modified Feynman vertex (renormalizable + corrections due to dimension-6 operators).

Vertex	2HDM: effective vertex ($W^{\pm} Z H^{\mp}$) factor
$W^{\pm\mu} Z_{\mu} H^{\mp}$	$ \begin{aligned} & -\frac{g}{8\sqrt{2}}(4\tilde{g}\sin\tilde{\theta}_w)(v_1\cos\tilde{\beta}_+ - v_2\sin\tilde{\beta}_+) - \frac{g}{8\sqrt{2}\Lambda^2}(4\tilde{g}\sin\theta_w\Theta_{BB}(-v_2\sin\beta_+ + 4v_1\cos\beta_+) \\ & + \cos\tilde{\theta}_w(v_1(2\tilde{g}\cos\beta_+\Theta_{W_3B} + gv_2^2((C_{\phi D}^{(1)22(1)} + C_{\phi D}^{(2)11(2)})\cos\beta_+ - 3C_{\phi D}^{(2)22(1)}\sin\beta_+)) - 2v_2\tilde{g}\sin\beta_+\Theta_{W_3B} \\ & + gv_1^3(2C_{\phi D}^{(1)11(1)}\cos\beta_+ - C_{\phi D}^{(1)21(1)}\sin\beta_+) + gv_2v_1^2(3C_{\phi D}^{(1)21(1)}\cos\beta_+ - (C_{\phi D}^{(1)22(1)} + C_{\phi D}^{(2)11(2)})\sin\beta_+) \\ & + gv_2^3(C_{\phi D}^{(2)22(1)}\cos\beta_+ - 2C_{\phi D}^{(22)2}\sin\beta_+)) + v_1^3\tilde{g}\sin\theta_w(2C_{\phi D}^{(1)11(1)}\cos\beta_+ - C_{\phi D}^{(1)21(1)}\sin\beta_+) \\ & + v_1^2v_2\tilde{g}\sin\theta_w((3C_{\phi D}^{(1)21(1)} + 4C_{\phi D}^{(12)1(1)})\cos\beta_+ - (2C_{\phi D}^{(12)1(2)} + C_{\phi D}^{(1)22(1)} + C_{\phi D}^{(2)11(2)})\sin\beta_+) \\ & + v_1v_2^2\tilde{g}\sin\theta_w((2C_{\phi D}^{(12)1(2)} + C_{\phi D}^{(1)22(1)} + C_{\phi D}^{(2)11(2)})\cos\beta_+ - (4C_{\phi D}^{(21)2(2)} + 3C_{\phi D}^{(2)22(1)})\sin\beta_+) \\ & + v_2^3\tilde{g}\sin\theta_w(C_{\phi D}^{(2)22(1)}\cos\beta_+ - 2C_{\phi D}^{(22)2}\sin\beta_+) + 2v_2\tilde{g}A_+^{22}\sin\theta_w\sin\beta_+ + 2v_1\tilde{g}A_+^{11}\sin\theta_w\sin\beta_+ \\ & - 2v_2\tilde{g}A_+^{12}\sin\theta_w\cos\beta_+ - 2v_1\tilde{g}A_+^{11}\sin\theta_w\cos\beta_+ - 4\tilde{g}\sin\theta_w\Theta_{WW}(v_2\sin\beta_+ - v_1\cos\beta_+)) \end{aligned} $

Table 38. 2HDM: coupling of charged scalar ($H^{\mp}$) with a charged gauge boson ($W^{\pm}$) and a neutral gauge boson (Z).

Vertex	MLRSM: effective vertex ($W^\pm W^\pm \delta^{\mp\mp}$) factor
$W_L^{\pm\mu} W_{L\mu}^\pm \delta_L^{\mp\mp}$	$-\frac{g^2 v_L}{4\sqrt{2}} \left[4 + \frac{1}{\Lambda^2} \left(-2A_{++}^{22} + 8\Theta_{W_{LL}} + 2v_L^2 (\mathcal{C}_{\phi D}^{\{L1\}\{(L)(l)\}} + 2\mathcal{C}_{\phi D}^{L(l)L(l)}) + 2\mathcal{C}_{\phi D}^{(L)rR(l)} v_R^2 + 2\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \right. \right. \\ \left. \left. + \kappa_+^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{(L)11(l)} + 2\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} \right) \right]$
$W_L^{\pm\mu} W_{R\mu}^\pm \delta_L^{\mp\mp}$	$-\frac{g^2}{4\sqrt{2}\Lambda^2} \left[\left(v_L (\mathcal{C}_{\phi D}^{(L)rL(r)} v_R^2 + 8\Theta_{W_{LR}} - \kappa_+^2 \mathcal{C}_{\phi D}^{L(l)1(1)}) + \kappa_+^2 v_R (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)}) + \mathcal{C}_{\phi D}^{(R)11(l)} \right) \right]$
$W_R^{\pm\mu} W_{R\mu}^\pm \delta_L^{\mp\mp}$	$\frac{g^2 v_R A_{++}^{12}}{2\sqrt{2}\Lambda^2}$
$W_R^{\pm\mu} W_{R\mu}^\pm \delta_R^{\mp\mp}$	$-\frac{g^2 v_R}{4\sqrt{2}} \left[4 + \frac{1}{\Lambda^2} \left(-2A_{++}^{11} + 8\Theta_{W_{RR}} + \mathcal{C}_{\phi D}^{(R)lL(r)} v_L^2 + 2\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} v_R^2 + 4\mathcal{C}_{\phi D}^{R(r)R(r)} v_R^2 + 2\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \right. \right. \\ \left. \left. + \kappa_+^2 \mathcal{C}_{\phi D}^{21(R)(r)} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{(R)11(r)} - \kappa_+^2 \mathcal{C}_{\phi D}^{R(r)1(1)} \right) \right]$
$W_R^{\pm\mu} W_{L\mu}^\pm \delta_R^{\mp\mp}$	$-\frac{g^2}{4\sqrt{2}\Lambda^2} \left[\left(\mathcal{C}_{\phi D}^{(L)rL(r)} v_L^2 v_R + \kappa_+^2 v_L (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) + 2v_R (4\Theta_{W_{RL}} + \kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{R(r)1(1)}) \right) \right]$
$W_L^{\pm\mu} W_{L\mu}^\pm \delta_R^{\mp\mp}$	$\frac{g^2 v_L A_{++}^{12}}{2\sqrt{2}\Lambda^2}$

Table 39. MLRSM: coupling of doubly charged scalar ($\delta^{\pm\pm}$) with a pair of same signed charged gauge bosons ($W^\pm$).

Figure 12. Feynman diagrams representing the neutrinoless double beta decay: $2n \rightarrow 2p + 2e^-$ ($0\nu 2\beta$). The $[\bullet]$ represents effective vertex. $0\nu 2\beta$ can acquire additional contributions from (a) light and heavy Majorana neutrino exchange, and (b) from doubly charged scalars too.

Figure 13. Feynman diagram representing the lepton number and flavour violating three body decays of charged lepton: $l_i^\pm \rightarrow l_j^\pm l_k^\pm l_m^\pm$. The $[\bullet]$ represents effective vertex.

Vertex	MLRSM: effective vertex ($\delta^{\mp\mp} \delta^{\pm\pm} Z_1$) factor
$\delta_L^- \delta_L^{++} Z_1^\mu$	$ \begin{aligned} & (k_-^\mu + k_{++}^\mu) \left(-\tilde{g} c_{\theta_1} c_{\tilde{\theta}_2} s_{\theta_w} + \tilde{g} s_{\theta_1} s_{\tilde{\theta}_2} + g c_{\tilde{\theta}_2} c_{\theta_w} \right) + \frac{1}{4\Lambda^2} (k_-^\mu + k_{++}^\mu) \left(4\tilde{g} c_{\theta_1} c_{\theta_2} A_{++}^{22} s_{\theta_w} - 4\tilde{g} s_{\theta_1} s_{\theta_2} A_{++}^{22} \right. \\ & - c_{\theta_2} s_{\theta_1} s_{\theta_w} \left(4\tilde{g} \Theta_{3RB} + 4g \Theta_{3LR} + g \kappa_2^2 (-C_{\phi D}^{L(1)(1)}) \right) + 4\tilde{g} \Theta_{3LB} c_{\theta_2} c_{\theta_w} - 4\tilde{g} \Theta_{3RB} c_{\theta_1} s_{\theta_2} - 4\tilde{g} \Theta_{BB} c_{\theta_1} c_{\theta_2} s_{\theta_w} \\ & + 4\tilde{g} \Theta_{BB} s_{\theta_1} s_{\theta_2} - 2C_{\phi D}^{\{Ll\}\{(L)(l)\}} \tilde{g} c_{\theta_1} c_{\theta_2} v_L^2 s_{\theta_w} - 2\kappa_1^2 \tilde{g} c_{\theta_1} c_{\theta_2} (C_{\phi D}^{\{21\}\{(L)(l)\}} + 2C_{\phi D}^{(L)11(l)}) s_{\theta_w} \\ & - 2\kappa_2^2 C_{\phi D}^{21(l)(l)} \tilde{g} c_{\theta_1} c_{\theta_2} s_{\theta_w} + 2C_{\phi D}^{\{Ll\}\{(L)(l)\}} \tilde{g} v_L^2 s_{\theta_1} s_{\theta_2} + 2\kappa_2^2 C_{\phi D}^{\{21\}\{(L)(l)\}} \tilde{g} s_{\theta_1} s_{\theta_2} + 2\kappa_1^2 \tilde{g} s_{\theta_1} s_{\theta_2} (C_{\phi D}^{\{21\}\{(L)(l)\}} + 2C_{\phi D}^{(L)11(l)}) \\ & + 2\kappa_2^2 C_{\phi D}^{21(l)(l)} \tilde{g} s_{\theta_1} s_{\theta_2} - 4g c_{\theta_2} A_{++}^{22} c_{\theta_w} + 4g \Theta_{3LR} c_{\theta_2} c_{\theta_w} + g s_{\theta_2} \left(c_{\theta_1} \left(\kappa_2^2 C_{\phi D}^{L(1)(1)} - 4\Theta_{3LR} \right) + 4\Theta_{3LB} s_{\theta_1} \right) \\ & - 4g \Theta_{3LB} c_{\theta_1} c_{\theta_2} s_{\theta_w} + 2g C_{\phi D}^{\{Ll\}\{(L)(l)\}} c_{\theta_2} v_L^2 c_{\theta_w} + g \kappa_2^2 c_{\theta_2} c_{\theta_w} (2C_{\phi D}^{\{21\}\{(L)(l)\}} + 2C_{\phi D}^{21(L)(l)} + C_{\phi D}^{L(1)(1)}) \\ & \left. + 2g \kappa_1^2 c_{\theta_2} (C_{\phi D}^{\{21\}\{(L)(l)\}} + 2C_{\phi D}^{(L)11(l)}) c_{\theta_w} - 2\kappa_2^2 C_{\phi D}^{\{21\}\{(L)(l)\}} \tilde{g} c_{\theta_1} c_{\theta_2} s_{\theta_w} \right) \end{aligned} $
$\delta_R^- \delta_R^{++} Z_1^\mu$	$ \begin{aligned} & - (p_-^\mu + p_{++}^\mu) \left(\tilde{g} c_{\theta_1} c_{\tilde{\theta}_2} s_{\theta_w} - \tilde{g} s_{\theta_1} s_{\tilde{\theta}_2} + g c_{\theta_1} s_{\tilde{\theta}_2} + g c_{\tilde{\theta}_2} s_{\theta_1} s_{\theta_w} \right) - \frac{1}{4\Lambda^2} (p_-^\mu + p_{++}^\mu) \left(-4\tilde{g} c_{\theta_1} c_{\theta_2} A_{++}^{11} s_{\theta_w} \right. \\ & - 4\tilde{g} \Theta_{3LB} c_{\theta_2} c_{\theta_w} + 4\tilde{g} \Theta_{3RB} c_{\theta_1} s_{\theta_2} + 4\tilde{g} \Theta_{3RB} c_{\theta_2} s_{\theta_1} s_{\theta_w} + 4\tilde{g} \Theta_{BB} c_{\theta_1} c_{\theta_2} s_{\theta_w} - 4\tilde{g} \Theta_{BB} s_{\theta_1} s_{\theta_2} \\ & + 2\kappa_1^2 \tilde{g} c_{\theta_1} c_{\theta_2} (C_{\phi D}^{\{21\}\{(R)(r)\}} + 2C_{\phi D}^{(R)11(r)}) s_{\theta_w} + 2\kappa_2^2 C_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} c_{\theta_1} c_{\theta_2} s_{\theta_w} + 2\kappa_2^2 C_{\phi D}^{21(R)(r)} \tilde{g} c_{\theta_1} c_{\theta_2} s_{\theta_w} \\ & - 2C_{\phi D}^{\{Rr\}\{(R)(r)\}} \tilde{g} v_R^2 s_{\theta_1} s_{\theta_2} - 2\kappa_2^2 C_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} s_{\theta_1} s_{\theta_2} - 2\kappa_1^2 \tilde{g} s_{\theta_1} s_{\theta_2} (C_{\phi D}^{\{21\}\{(R)(r)\}} + 2C_{\phi D}^{(R)11(r)}) - 2\kappa_2^2 C_{\phi D}^{21(R)(r)} \tilde{g} s_{\theta_1} s_{\theta_2} \\ & - 4g c_{\theta_1} s_{\theta_2} A_{++}^{11} - 4g c_{\theta_2} s_{\theta_1} A_{++}^{11} s_{\theta_w} - g c_{\theta_2} c_{\theta_w} \left(4\Theta_{3LR} + \kappa_2^2 C_{\phi D}^{R(r)1(1)} \right) + 4g \Theta_{3R3R} c_{\theta_1} s_{\theta_2} + 4g \Theta_{3R3R} c_{\theta_2} s_{\theta_1} s_{\theta_w} \\ & + 4g \Theta_{3RB} c_{\theta_1} c_{\theta_2} s_{\theta_w} - 4g \Theta_{3RB} s_{\theta_1} s_{\theta_2} + 2g C_{\phi D}^{\{Rr\}\{(R)(r)\}} c_{\theta_1} v_R^2 s_{\theta_2} + 2g C_{\phi D}^{\{Rr\}\{(R)(r)\}} c_{\theta_2} v_R^2 s_{\theta_1} s_{\theta_w} \\ & + g \kappa_2^2 c_{\theta_1} s_{\theta_2} (2C_{\phi D}^{\{21\}\{(R)(r)\}} + 2C_{\phi D}^{21(R)(r)} - C_{\phi D}^{R(r)1(1)}) + 2g \kappa_1^2 c_{\theta_1} s_{\theta_2} (C_{\phi D}^{\{21\}\{(R)(r)\}} + 2C_{\phi D}^{(R)11(r)}) \\ & + 2g \kappa_1^2 c_{\theta_2} s_{\theta_1} (C_{\phi D}^{\{21\}\{(R)(r)\}} + 2C_{\phi D}^{(R)11(r)}) s_{\theta_w} + 2g \kappa_2^2 C_{\phi D}^{\{21\}\{(R)(r)\}} c_{\theta_2} s_{\theta_1} s_{\theta_w} + 2g \kappa_2^2 C_{\phi D}^{21(R)(r)} c_{\theta_2} s_{\theta_1} s_{\theta_w} \\ & \left. + 4\tilde{g} s_{\theta_1} s_{\theta_2} A_{++}^{11} + 2C_{\phi D}^{\{Rr\}\{(R)(r)\}} \tilde{g} c_{\theta_1} c_{\theta_2} v_R^2 s_{\theta_w} - g \kappa_2^2 C_{\phi D}^{R(r)1(1)} c_{\theta_2} s_{\theta_1} s_{\theta_w} \right) \end{aligned} $
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} Z_1^\mu$	$ \begin{aligned} & \frac{1}{2\Lambda^2} \left(-c_{\theta_1} \left(\tilde{g} c_{\theta_2} s_{\theta_w} (k_{\mp\mp}^\mu + p_{\pm\pm}^\mu) \left(\kappa_1^2 (C_{\phi D}^{(L)11(r)} + C_{\phi D}^{(L)12(r)} + C_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right) + g s_{\theta_2} \left(k_{\mp\mp}^\mu \left(\kappa_1^2 (C_{\phi D}^{(L)11(r)} \right. \right. \right. \right. \\ & \left. \left. \left. + C_{\phi D}^{(L)12(r)} + C_{\phi D}^{(R)11(l)}) - A_{++}^{12} \right) - A_{++}^{12} p_{\pm\pm}^\mu \right) \right) + \tilde{g} s_{\theta_1} s_{\theta_2} (k_{\mp\mp}^\mu + p_{\pm\pm}^\mu) \left(\kappa_1^2 (C_{\phi D}^{(L)11(r)} + C_{\phi D}^{(L)12(r)} + C_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right) \\ & + g c_{\theta_2} \left(c_{\theta_w} \left(p_{\pm\pm}^\mu \left(\kappa_1^2 (C_{\phi D}^{(L)11(r)} + C_{\phi D}^{(L)12(r)} + C_{\phi D}^{(R)11(l)}) - A_{++}^{12} \right) - k_{\mp\mp}^\mu A_{++}^{12} \right) - s_{\theta_1} s_{\theta_w} \left(k_{\mp\mp}^\mu \left(\kappa_1^2 (C_{\phi D}^{(L)11(r)} + C_{\phi D}^{(L)12(r)} \right. \right. \right. \\ & \left. \left. \left. + C_{\phi D}^{(R)11(l)}) - A_{++}^{12} \right) - A_{++}^{12} p_{\pm\pm}^\mu \right) \right) \right) \end{aligned} $

Table 40. MLRSM: coupling of a pair of doubly charged scalars ($\delta^{\pm\pm}$) with a neutral gauge boson (Z_1). Here, $s_{\theta_i} = \sin \theta_i$ and $c_{\theta_i} = \cos \theta_i$. $k_{\pm\pm}^\mu$ and $p_{\pm\pm}^\mu$ are the four momenta of $\delta_L^{\pm\pm}$ and $\delta_R^{\pm\pm}$ respectively. $\tilde{\theta}_2$ is the redefined angle defined in eq. (6.102).

other diagram for $0\nu 2\beta$ mediated by the Majorana neutrinos, figure 12(a), in MLRSM will be also be affected due to the effective gauge boson-fermion vertex, see tables 29, 30. So MLRSM-EFT scenario offers very convoluted contributions to $0\nu 2\beta$ process, and the usual constraints need to be revisited. The leptonic current for this process, and the meson decays ($M^+ \rightarrow M'^- l^+ l^+$) are the same [247–250]. Thus the prediction for meson decays through Majorana neutrinos and doubly charged scalars will also be modified. The lepton flavour as well as number violating three body decays of charged leptons, e.g., $l_i^\mp \rightarrow l_j^\pm l_k^\mp l_m^\mp$ [251], can get additional contributions from the effective operators, see figure 13. The possibility of having charged lepton flavour violating radiative decays, $l_i \rightarrow l_j + \gamma$, (with $i \neq j$) can

Vertex	MLRSM: effective vertex ($\delta^{\pm\pm} \delta^{\mp\mp} Z_2$) factor
$\delta_L^- \delta_L^{++} Z_2^\mu$	$ \begin{aligned} & - \left(k_{--}^\mu + k_{++}^\mu \right) \left(\tilde{g} c_{\tilde{\theta}_2} s_{\theta_1} + \tilde{g} c_{\theta_1} s_{\tilde{\theta}_2} s_{\theta_w} - g s_{\tilde{\theta}_2} c_{\theta_w} \right) - \frac{1}{4\Lambda^2} \left(k_{--}^\mu + k_{++}^\mu \right) \left(-4\tilde{g} c_{\theta_2} s_{\theta_1} A_{++}^{22} - 4\tilde{g} c_{\theta_1} s_{\theta_2} A_{++}^{22} s_{\theta_w} \right. \\ & + s_{\theta_1} s_{\theta_2} s_{\theta_w} \left(4\tilde{g} \Theta_{3RB} + 4g \Theta_{3LR} + g \kappa_2^2 (-\mathcal{C}_{\phi D}^{L(l)1(1)}) \right) - 4\tilde{g} \Theta_{3LB} s_{\theta_2} c_{\theta_w} - 4\tilde{g} \Theta_{3RC} c_{\theta_1} c_{\theta_2} + 4\tilde{g} \Theta_{BBC} c_{\theta_2} s_{\theta_1} \\ & + 4\tilde{g} \Theta_{BBC} c_{\theta_1} s_{\theta_2} s_{\theta_w} + 2\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} \tilde{g} c_{\theta_2} v_L^2 s_{\theta_1} + 2\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} \tilde{g} c_{\theta_1} v_L^2 s_{\theta_2} s_{\theta_w} + 2\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \tilde{g} c_{\theta_2} s_{\theta_1} \\ & + 2\kappa_1^2 \tilde{g} c_{\theta_2} s_{\theta_1} (\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + 2\mathcal{C}_{\phi D}^{(L)11(l)}) + 2\kappa_1^2 \tilde{g} c_{\theta_1} s_{\theta_2} (\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + 2\mathcal{C}_{\phi D}^{(L)11(l)}) s_{\theta_w} + 2\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \tilde{g} c_{\theta_1} s_{\theta_2} s_{\theta_w} \\ & + 2\kappa_2^2 \mathcal{C}_{\phi D}^{21(L)(l)} \tilde{g} c_{\theta_2} s_{\theta_1} + 2\kappa_2^2 \mathcal{C}_{\phi D}^{21(L)(l)} \tilde{g} c_{\theta_1} s_{\theta_2} s_{\theta_w} + 4g s_{\theta_2} A_{++}^{22} c_{\theta_w} - 4g \Theta_{3LR} s_{\theta_2} c_{\theta_w} + g c_{\theta_2} \left(\kappa_2^2 \mathcal{C}_{\phi D}^{L(l)1(1)} \right. \\ & \left. - 4\Theta_{3LR} + 4\Theta_{3LB} s_{\theta_1} \right) + 4g \Theta_{3LB} c_{\theta_1} s_{\theta_2} s_{\theta_w} - 2g \mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} v_L^2 s_{\theta_2} c_{\theta_w} + g \kappa_2^2 s_{\theta_2} c_{\theta_w} (-2\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} - 2\mathcal{C}_{\phi D}^{21(L)(l)}) \\ & \left. - \mathcal{C}_{\phi D}^{L(l)1(1)} - 2g \kappa_1^2 s_{\theta_2} (\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + 2\mathcal{C}_{\phi D}^{(L)11(l)}) c_{\theta_w} \right) \end{aligned} $
$\delta_R^- \delta_R^{++} Z_2^\mu$	$ \begin{aligned} & (p_{--}^\mu + p_{++}^\mu) (-\tilde{g} c_{\theta_2} s_{\theta_1} - \tilde{g} c_{\theta_1} s_{\theta_2} s_{\theta_w} + g c_{\theta_1} c_{\theta_2} - g s_{\theta_1} s_{\theta_2} s_{\theta_w}) + \frac{1}{4\Lambda^2} (p_{--}^\mu + p_{++}^\mu) (4\tilde{g} c_{\theta_2} s_{\theta_1} A_{++}^{11} + 4\tilde{g} c_{\theta_1} s_{\theta_2} A_{++}^{11} s_{\theta_w} \\ & + s_{\theta_2} c_{\theta_w} (4\tilde{g} \Theta_{3LB} + 4g \Theta_{3LR} + g \kappa_2^2 \mathcal{C}_{\phi D}^{R(r)1(1)}) + 4\tilde{g} \Theta_{3RC} c_{\theta_1} c_{\theta_2} - 4\tilde{g} \Theta_{3RB} s_{\theta_1} s_{\theta_2} s_{\theta_w} - 4\tilde{g} \Theta_{BBC} c_{\theta_2} s_{\theta_1} - 4\tilde{g} \Theta_{BBC} c_{\theta_1} s_{\theta_2} s_{\theta_w} \\ & - 2\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} \tilde{g} c_{\theta_2} v_R^2 s_{\theta_1} - 2\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} \tilde{g} c_{\theta_1} v_R^2 s_{\theta_2} s_{\theta_w} - 2\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} c_{\theta_2} s_{\theta_1} - 2\kappa_1^2 \tilde{g} c_{\theta_2} s_{\theta_1} (\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \\ & + 2\mathcal{C}_{\phi D}^{(R)11(r)}) - 2\kappa_1^2 \tilde{g} c_{\theta_1} s_{\theta_2} (\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 2\mathcal{C}_{\phi D}^{(R)11(r)}) s_{\theta_w} - 2\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} c_{\theta_1} s_{\theta_2} s_{\theta_w} - 2\kappa_2^2 \mathcal{C}_{\phi D}^{21(R)(r)} \tilde{g} c_{\theta_2} s_{\theta_1} \\ & - 2\kappa_2^2 \mathcal{C}_{\phi D}^{21(R)(r)} \tilde{g} c_{\theta_1} s_{\theta_2} s_{\theta_w} - 4g c_{\theta_1} c_{\theta_2} A_{++}^{11} + 4g s_{\theta_1} s_{\theta_2} A_{++}^{11} s_{\theta_w} + 4g \Theta_{3RC} c_{\theta_1} c_{\theta_2} - 4g \Theta_{3RB} s_{\theta_1} s_{\theta_2} s_{\theta_w} \\ & - 4g \Theta_{3LB} c_{\theta_2} s_{\theta_1} - 4g \Theta_{3RB} c_{\theta_1} s_{\theta_2} s_{\theta_w} + 2g \mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} c_{\theta_1} c_{\theta_2} v_R^2 + g \kappa_2^2 c_{\theta_1} c_{\theta_2} (2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 2\mathcal{C}_{\phi D}^{21(R)(r)} - \mathcal{C}_{\phi D}^{R(r)1(1)}) \\ & + 2g \kappa_1^2 c_{\theta_1} c_{\theta_2} (\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 2\mathcal{C}_{\phi D}^{(R)11(r)}) - 2g \mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} v_R^2 s_{\theta_1} s_{\theta_2} s_{\theta_w} + g \kappa_2^2 s_{\theta_1} s_{\theta_2} s_{\theta_w} (-2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} - 2\mathcal{C}_{\phi D}^{21(R)(r)}) \\ & + \mathcal{C}_{\phi D}^{R(r)1(1)} - 2g \kappa_1^2 s_{\theta_1} s_{\theta_2} (\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 2\mathcal{C}_{\phi D}^{(R)11(r)}) s_{\theta_w} \Big) \end{aligned} $
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} Z_2^\mu$	$ \begin{aligned} & \frac{1}{2\Lambda^2} \left(-\tilde{g} c_{\theta_2} s_{\theta_1} (k_{\mp\mp}^\mu + p_{\pm\pm}^\mu) \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right) + c_{\theta_1} \left(g c_{\theta_2} \left(\kappa_{\mp\mp}^2 (\mathcal{C}_{\phi D}^{(L)11(r)} \right. \right. \right. \\ & \left. \left. \left. + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - A_{++}^{12} \right) - A_{++}^{12} p_{\pm\pm}^\mu \right) - \tilde{g} s_{\theta_2} s_{\theta_w} (k_{\mp\mp}^\mu + p_{\pm\pm}^\mu) \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right) \right) \\ & + g s_{\theta_2} \left(c_{\theta_w} \left(p_{\pm\pm}^\mu \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - A_{++}^{12} \right) - k_{\mp\mp}^\mu A_{++}^{12} \right) - s_{\theta_1} s_{\theta_w} \left(k_{\mp\mp}^\mu \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} \right. \right. \right. \\ & \left. \left. \left. + \mathcal{C}_{\phi D}^{(R)11(l)}) - A_{++}^{12} \right) - A_{++}^{12} p_{\pm\pm}^\mu \right) \right) \end{aligned} $

Table 41. MLRSM: coupling of a pair of doubly charged scalars ($\delta^{\pm\pm}$) with a neutral gauge boson (Z_2). Here, $s_{\theta_i} = \sin \theta_i$ and $c_{\theta_i} = \cos \theta_i$. $k_{\pm\pm}^\mu$ and $p_{\pm\pm}^\mu$ are the four momenta of $\delta_L^{\pm\pm}$ and $\delta_R^{\pm\pm}$ respectively. $\tilde{\theta}_2$ is the redefined angle defined in eq. (6.102).

not be ignored, see figures 8(a), (b). The sensitivity of these rare events are expected to increase significantly with future experiments. Thus the respective WCs will be severely constrained and may play a crucial role to decide the minimum energy scale to unveil these rare events at the colliders.

7.8 Theoretical constraints in the scalar sector

The shape of the scalar potential is determined by the quartic couplings in the asymptotic limit. The physical masses of the scalar fields are linear functions of these quartic couplings. Thus it is evident that any constraint on these couplings can be translated appropriately to put some bounds on the scalar spectrum. Here, we have outlined two such theoretical constraints: (i) Tree Unitarity (TU) [252, 253] and (ii) Vacuum Stability (VS) [254]. Both the bounds are computed at the large value of the scalar field. Thus the relevant part of

Vertex	MLRSM: effective vertex ($\delta^{\pm\pm}\delta^{\mp\mp}\gamma$) and ($\delta^{\pm}\delta^{\mp}\gamma$) factor
$\delta_L^--\delta_L^{++}A^\mu$	$\frac{1}{4}(k_{\mp\mp}^\mu + k_{\pm\pm}^\mu)(4\tilde{g}c_{\theta_1}c_{\theta_w} + 4gs_{\theta_w}) + \frac{1}{4\Lambda^2}(k_{\mp\mp}^\mu + k_{\pm\pm}^\mu)(-4\tilde{g}c_{\theta_1}A_{++}^{22}c_{\theta_w} + 4\tilde{g}(\Theta_{3LB}s_{\theta_w} + \Theta_{3RB}s_{\theta_1}c_{\theta_w})$ $+ 4\tilde{g}\Theta_{BB}c_{\theta_1}c_{\theta_w} + 2\mathcal{C}_{\phi D}^{\{L\}\{(L)\}}\tilde{g}c_{\theta_1}v_L^2c_{\theta_w} + 2\kappa_2^2\mathcal{C}_{\phi D}^{\{21\}\{(L)\}}\tilde{g}c_{\theta_1}c_{\theta_w} + 2\kappa_2^2\mathcal{C}_{\phi D}^{21(L)(l)}\tilde{g}c_{\theta_1}c_{\theta_w}$ $+ 4\kappa_1^2\mathcal{C}_{\phi D}^{(L)(11)}\tilde{g}c_{\theta_1}c_{\theta_w} - 4gA_{++}^{22}s_{\theta_w} + 4g\Theta_{3L3L}s_{\theta_w} + gc_{\theta_w}(s_{\theta_1}(4\Theta_{3L3R} - \kappa_2^2\mathcal{C}_{\phi D}^{L(l)1(1)}) + 4\Theta_{3LB}c_{\theta_1})$ $+ 2g\mathcal{C}_{\phi D}^{\{L\}\{(L)\}}v_L^2s_{\theta_w} + 2g\kappa_+^2\mathcal{C}_{\phi D}^{\{21\}\{(L)\}}s_{\theta_w} + 2g\kappa_2^2\mathcal{C}_{\phi D}^{21(L)(l)}s_{\theta_w} + 4g\kappa_1^2\mathcal{C}_{\phi D}^{(L)11(l)}s_{\theta_w} + g\kappa_2^2\mathcal{C}_{\phi D}^{L(l)1(1)}s_{\theta_w})$
$\delta_R^--\delta_R^{++}A^\mu$	$\frac{1}{4}(p_{\mp\mp}^\mu + p_{\pm\pm}^\mu)(4\tilde{g}c_{\theta_1}c_{\theta_w} + 4gs_{\theta_1}c_{\theta_w}) + \frac{1}{4\Lambda^2}(p_{\mp\mp}^\mu + p_{\pm\pm}^\mu)(-4\tilde{g}c_{\theta_1}A_{++}^{11}c_{\theta_w} + 4\tilde{g}(\Theta_{3LB}s_{\theta_w} + \Theta_{3RB}s_{\theta_1}c_{\theta_w})$ $+ 4\tilde{g}\Theta_{BB}c_{\theta_1}c_{\theta_w} + 2\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}}\tilde{g}c_{\theta_1}v_R^2c_{\theta_w} + 2\kappa_+^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}c_{\theta_1}c_{\theta_w} + 2\kappa_2^2\mathcal{C}_{\phi D}^{21(R)(r)}\tilde{g}c_{\theta_1}c_{\theta_w} + 4\kappa_1^2\mathcal{C}_{\phi D}^{(R)11(r)}\tilde{g}c_{\theta_1}c_{\theta_w}$ $- 4gs_{\theta_1}A_{++}^{11}c_{\theta_w} + gs_{\theta_w}(4\Theta_{3L3R} + \kappa_2^2\mathcal{C}_{\phi D}^{R(r)1(1)}) + 4g\Theta_{3R3R}s_{\theta_1}c_{\theta_w} + 4g\Theta_{3RB}c_{\theta_1}c_{\theta_w} + 2g\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}}v_R^2s_{\theta_1}c_{\theta_w}$ $+ 2g\kappa_+^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}s_{\theta_1}c_{\theta_w} + 2g\kappa_2^2\mathcal{C}_{\phi D}^{21(R)(r)}s_{\theta_1}c_{\theta_w} + 4g\kappa_1^2\mathcal{C}_{\phi D}^{(R)11(r)}s_{\theta_1}c_{\theta_w} - g\kappa_2^2\mathcal{C}_{\phi D}^{R(r)1(1)}s_{\theta_1}c_{\theta_w})$
$\delta_L^{\mp\mp}\delta_R^{\pm\pm}A^\mu$	$-\frac{1}{2\Lambda^2}\tilde{g}c_{\theta_1}c_{\theta_w}(k_{\mp\mp}^\mu + p_{\pm\pm}^\mu)(2A_{++}^{12} - \kappa_1^2(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)})) + gA_{++}^{12}(k_{\mp\mp}^\mu + p_{\pm\pm}^\mu)(s_{\theta_1}c_{\theta_w} + s_{\theta_w})$ $- g\kappa_1^2(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)})(k_{\mp\mp}^\mu s_{\theta_1}c_{\theta_w} + p_{\pm\pm}^\mu s_{\theta_w})$
$\delta_L^{\mp\mp}\delta_L^{\pm\pm}A^\mu$	$\tilde{g}c_{\theta_1}(k_{\mp\mp}^\mu + k_{\pm\pm}^\mu)c_{\theta_w} - \frac{1}{8\Lambda^2}(k_{\mp\mp}^\mu + k_{\pm\pm}^\mu)(8\tilde{g}c_{\theta_1}A_{++}^{44}c_{\theta_w} - 8\tilde{g}(\Theta_{3LB}s_{\theta_w} + \Theta_{3RB}s_{\theta_1}c_{\theta_w}) - 8\tilde{g}\Theta_{BB}c_{\theta_1}c_{\theta_w})$ $- 4\mathcal{C}_{\phi D}^{\{L\}\{(L)\}}\tilde{g}c_{\theta_1}v_L^2c_{\theta_w} - 2\mathcal{C}_{\phi D}^{(L)rR(l)}\tilde{g}c_{\theta_1}v_R^2c_{\theta_w} - 4\kappa_+^2\mathcal{C}_{\phi D}^{\{21\}\{(L)\}}\tilde{g}c_{\theta_1}c_{\theta_w} - 2\kappa_+^2\mathcal{C}_{\phi D}^{21(L)(l)}\tilde{g}c_{\theta_1}c_{\theta_w}$ $- 4\kappa_+^2\mathcal{C}_{\phi D}^{(L)(11)}\tilde{g}c_{\theta_1}c_{\theta_w} + g(\kappa_2^2 - \kappa_1^2)\mathcal{C}_{\phi D}^{L(l)1(1)}s_{\theta_1}c_{\theta_w} + gs_{\theta_w}(8\mathcal{C}_{\phi D}^{L(l)1(1)}v_L^2 + \kappa_-^2\mathcal{C}_{\phi D}^{L(l)1(1)})$
$\delta_R^{\mp\mp}\delta_R^{\pm\pm}A^\mu$	$\tilde{g}c_{\theta_1}(p_{\mp\mp}^\mu + p_{\pm\pm}^\mu)c_{\theta_w} - \frac{1}{8\Lambda^2}(p_{\mp\mp}^\mu + p_{\pm\pm}^\mu)(8\tilde{g}c_{\theta_1}A_{++}^{33}c_{\theta_w} - 8\tilde{g}(\Theta_{3LB}s_{\theta_w} + \Theta_{3RB}s_{\theta_1}c_{\theta_w}) - 8\tilde{g}\Theta_{BB}c_{\theta_1}c_{\theta_w})$ $- 2\mathcal{C}_{\phi D}^{(R)lR(r)}\tilde{g}c_{\theta_1}v_L^2c_{\theta_w} - 4\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}}\tilde{g}c_{\theta_1}v_R^2c_{\theta_w} - 16\mathcal{C}_{\phi D}^{R(r)R(r)}\tilde{g}c_{\theta_1}v_R^2c_{\theta_w} - 4\kappa_+^2\mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}c_{\theta_1}c_{\theta_w}$ $- 2\kappa_+^2\mathcal{C}_{\phi D}^{21(R)(r)}\tilde{g}c_{\theta_1}c_{\theta_w} - 4\kappa_+^2\mathcal{C}_{\phi D}^{(R)11(r)}\tilde{g}c_{\theta_1}c_{\theta_w} + gs_{\theta_1}c_{\theta_w}(4v_R^2(3\mathcal{C}_{\phi D}^{R(r)R(r)} + \mathcal{C}_{\phi D}^{Rr(Rr)}) - \kappa_-^2\mathcal{C}_{\phi D}^{R(r)1(1)}) + g\kappa_-^2\mathcal{C}_{\phi D}^{R(r)1(1)}s_{\theta_w})$
$\delta_L^{\mp\mp}\delta_R^{\pm\pm}A^\mu$	$-\frac{1}{4\Lambda^2}\tilde{g}c_{\theta_1}c_{\theta_w}(4A_{++}^{34}(k_{\mp\mp}^\mu + p_{\pm\pm}^\mu) - k_{\mp\mp}^\mu(v_L v_R(2\mathcal{C}_{\phi D}^{(L)rL(r)} + \mathcal{C}_{\phi D}^{(L)rR(l)}) + \kappa_+^2(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}))$ $- p_{\pm\pm}^\mu(v_L v_R(2\mathcal{C}_{\phi D}^{(L)rL(r)} + \mathcal{C}_{\phi D}^{(R)lR(r)}) + \kappa_+^2(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)})))$ $+ g v_L v_R(s_{\theta_1}c_{\theta_w}(\mathcal{C}_{\phi D}^{(L)rL(r)}k_{\mp\mp}^\mu + \mathcal{C}_{\phi D}^{(R)lR(r)}p_{\pm\pm}^\mu) + s_{\theta_w}(\mathcal{C}_{\phi D}^{(L)rR(l)}k_{\mp\mp}^\mu + \mathcal{C}_{\phi D}^{(L)rL(r)}p_{\pm\pm}^\mu))$

Table 42. MLRSM: coupling of a pair of doubly charged scalars ($\delta^{\pm\pm}$) and singly charged scalars ($\delta^{\pm}$) with photon (A^μ). Here, $s_{\theta_i} = \sin \theta_i$ and $c_{\theta_i} = \cos \theta_i$. $k_{\pm\pm}^\mu$, $p_{\pm\pm}^\mu$, $k_{\pm}^\mu$ and $p_{\pm}^\mu$ are the four momenta of $\delta_L^{\pm\pm}$, $\delta_R^{\pm\pm}$, $\delta_L^{\pm}$ and $\delta_R^{\pm}$ respectively. $\tilde{\theta}_2$ is the redefined angle defined in eq. (6.102).

the scalar potential for us is the scalar four-point interactions. The TU sets an upper limit on the amplitudes of the scalar four-point interactions, and that is $\leq 8\pi$ [253]. Now we know that the amplitude for such processes is the quartic couplings. This, in turn, sets a maximum allowed limit on the quartic couplings. On the other side the VS criteria ensure that potential is bounded from below such that the vacuum is stable.⁴ These TU and VS criteria are modified in the presence of effective operators, and thus the limits on the scalar spectrum may be altered [122, 123].

In this paper we have computed the scalar four-point interactions, see figure 14, which also represent the amplitude for the process $\phi_i\phi_j \rightarrow \phi_k\phi_l$ including the modifications due

⁴Here, we have kept vacuum stability and boundedness criteria in the same footing. The meta-stability is overlooked for this discussion.

Vertex	MLRSM: effective vertex ($\delta^{\mp\mp} \delta^{\pm\pm} W_3[B] W_3[B]$) factor
$\delta_L^- \delta_L^{++} W_{3L}^\mu W_{3L\mu}$	$g^2 + \frac{g}{8\Lambda^2} \left(16\tilde{g}\Theta_{3LB} - 8gA_{++}^{22} + 16g\Theta_{3L3L} + 8g\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} v_L^2 + 4g\kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + 4g\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \right. \\ \left. + 4g\kappa_2^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 8g\kappa_1^2 \mathcal{C}_{\phi D}^{(L)11(l)} + 4g\kappa_2^2 \mathcal{C}_{\phi D}^{L(l)1(1)} + g\kappa_1^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + g\kappa_2^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + g\kappa_2^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_{3L}^\mu W_{3L\mu}$	$\frac{g^2}{8\Lambda^2} \left(\kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} - 4A_{++}^{12} \right)$
$\delta_R^- \delta_R^{++} W_{3R}^\mu W_{3R\mu}$	$\frac{g^2}{8\Lambda^2} \left(\kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_2^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Rr(2)(1)}) \right)$
$\delta_L^- \delta_L^{++} W_{3R}^\mu W_{3R\mu}$	$\frac{g^2}{8\Lambda^2} \left(\kappa_1^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_2^2 (\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Ll(2)(1)}) \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_{3R}^\mu W_{3R\mu}$	$\frac{g^2}{8\Lambda^2} \left(\kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} - 4A_{++}^{12} \right)$
$\delta_R^- \delta_R^{++} W_{3R}^\mu W_{3R\mu}$	$g^2 + \frac{g}{8\Lambda^2} \left(16\tilde{g}\Theta_{3RB} - 8gA_{++}^{11} + 16g\Theta_{3R3R} + 8g\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} v_R^2 + 4g\kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} + 4g\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \right. \\ \left. + 4g\kappa_2^2 \mathcal{C}_{\phi D}^{21(R)(r)} + 8g\kappa_1^2 \mathcal{C}_{\phi D}^{(R)11(r)} - 4g\kappa_2^2 \mathcal{C}_{\phi D}^{R(r)1(1)} + g\kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + g\kappa_2^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + g\kappa_2^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right)$
$\delta_L^- \delta_L^{++} B^\mu B_\mu$	$\tilde{g}^2 + \frac{\tilde{g}}{2\Lambda^2} \left(-2\tilde{g}A_{++}^{22} + 4\tilde{g}\Theta_{BB} + 2\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} \tilde{g} v_L^2 + \kappa_2^2 \tilde{g} (\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} + \mathcal{C}_{\phi D}^{21(L)(l)}) + \kappa_1^2 \tilde{g} (\mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \right. \\ \left. + 2\mathcal{C}_{\phi D}^{(L)11(l)}) + 4g\Theta_{3LB} \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} B^\mu B_\mu$	$\frac{\tilde{g}^2}{2\Lambda^2} \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right)$
$\delta_R^- \delta_R^{++} B^\mu B_\mu$	$\tilde{g}^2 + \frac{\tilde{g}}{2\Lambda^2} \left(-2\tilde{g}A_{++}^{11} + 4\tilde{g}\Theta_{BB} + 2\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} \tilde{g} v_R^2 + \kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} + \kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} + \kappa_2^2 \mathcal{C}_{\phi D}^{21(R)(r)} \tilde{g} \right. \\ \left. + 2\kappa_1^2 \mathcal{C}_{\phi D}^{(R)11(r)} \tilde{g} + 4g\Theta_{3RB} \right)$
$\delta_L^- \delta_L^{++} W_{3L}^\mu W_{3R\mu}$	$\frac{g}{4\Lambda^2} \left(8\tilde{g}\Theta_{3RB} + 8g\Theta_{3L3R} - g \left(\kappa_2^2 (2\mathcal{C}_{\phi D}^{L(l)1(1)} + \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Ll(2)(1)}) + \kappa_1^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} \right) \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_{3L}^\mu W_{3R\mu}$	$-\frac{g^2 \kappa_1^2}{4\Lambda^2} (\mathcal{C}_{\phi D}^{(2)rL(1)} - 2(\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}))$
$\delta_R^- \delta_R^{++} W_{3R}^\mu W_{3R\mu}$	$\frac{g}{4\Lambda^2} \left(8\tilde{g}\Theta_{3LB} + 8g\Theta_{3L3R} - g \left(\kappa_2^2 (-2\mathcal{C}_{\phi D}^{R(r)1(1)} + \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Rr(2)(1)}) + \kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} \right) \right)$
$\delta_L^- \delta_L^{++} W_{3L}^\mu B_\mu$	$2g\tilde{g} + \frac{1}{2\Lambda^2} \left(-4g\tilde{g}A_{++}^{22} + 4g\tilde{g}\Theta_{3L3L} + 4\tilde{g}^2\Theta_{3LB} + 4g\tilde{g}\Theta_{BB} + 2g\kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \tilde{g} \right. \\ \left. + 2g\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \tilde{g} + 2g\kappa_2^2 \mathcal{C}_{\phi D}^{21(L)(l)} \tilde{g} + 4g\kappa_1^2 \mathcal{C}_{\phi D}^{(L)11(l)} \tilde{g} + g\kappa_2^2 \mathcal{C}_{\phi D}^{L(l)1(1)} \tilde{g} + 4g^2\Theta_{3LB} \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_{3L}^\mu B_\mu$	$\frac{g\tilde{g}}{2\Lambda^2} \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right)$
$\delta_R^- \delta_R^{++} W_{3L}^\mu B_\mu$	$\frac{\tilde{g}}{2\Lambda^2} \left(4\tilde{g}\Theta_{3LB} + 4g\Theta_{3L3R} + g\kappa_2^2 \mathcal{C}_{\phi D}^{R(r)1(1)} \right)$
$\delta_L^- \delta_L^{++} W_{3R}^\mu B_\mu$	$\frac{\tilde{g}}{2\Lambda^2} \left(4\tilde{g}\Theta_{3RB} + 4g\Theta_{3L3R} + g\kappa_2^2 (-\mathcal{C}_{\phi D}^{L(l)1(1)}) \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_{3R}^\mu B_\mu$	$\frac{g\tilde{g}}{2\Lambda^2} \left(\kappa_1^2 (\mathcal{C}_{\phi D}^{(L)11(r)} + \mathcal{C}_{\phi D}^{(L)12(r)} + \mathcal{C}_{\phi D}^{(R)11(l)}) - 2A_{++}^{12} \right)$
$\delta_R^- \delta_R^{++} W_{3R}^\mu B_\mu$	$2g\tilde{g} + \frac{1}{2\Lambda^2} \left(-4g\tilde{g}A_{++}^{11} + 4g\tilde{g}\Theta_{3R3R} + 4\tilde{g}^2\Theta_{3RB} + 4g\tilde{g}\Theta_{BB} + 2g\kappa_1^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} + 2g\kappa_2^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \tilde{g} \right. \\ \left. + 2g\kappa_2^2 \mathcal{C}_{\phi D}^{21(R)(r)} \tilde{g} + 4g\kappa_1^2 \mathcal{C}_{\phi D}^{(R)11(r)} \tilde{g} - g\kappa_2^2 \mathcal{C}_{\phi D}^{R(r)1(1)} \tilde{g} + 4g^2\Theta_{3RB} \right)$

Table 43. MLRSM: coupling of a pair of doubly charged scalars ($\delta^{\pm\pm}$) with a pair two of neutral gauge bosons ($B, W_{3L,3R}$) in unphysical basis. The vertices can be written in terms of ($A, Z_{1,2}$) using rotation matrix $\mathcal{R}_b$.

Vertex	MLRSM: effective vertex ($\delta^{\mp\mp} \delta^{\pm\pm} W^\mp W^\pm$) factor
$\delta_L^{\mp\mp} \delta_L^{\pm\pm} W_L^{\pm\mu} W_{L\mu}^\mp$	$g^2 + \frac{g^2}{4\Lambda^2} \left(-4A_{++}^{22} + 4v_L^2 (\mathcal{C}_{\phi D}^{\{Ll\}\{(L)(l)\}} + 3\mathcal{C}_{\phi D}^{L(l)L(l)} + \mathcal{C}_{\square}^{Ll(Ll)}) + \mathcal{C}_{\phi D}^{(L)rR(l)} v_R^2 + 8\Theta_{W_{LL}} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(L)(l)\}} \right. \\ \left. + \kappa_+^2 \mathcal{C}_{\phi D}^{21(L)(l)} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{(L)11(l)} + 2\kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{L(l)1(1)} + \kappa_+^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_2^2 \mathcal{C}_{\phi D}^{Ll(2)(1)} \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_L^{\pm\mu} W_{L\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(\kappa_2^2 \mathcal{C}_{\phi D}^{(2)rL(1)} - 2A_{++}^{12} \right)$
$\delta_R^{\mp\mp} \delta_R^{\pm\pm} W_L^{\pm\mu} W_{L\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(\mathcal{C}_{\phi D}^{(L)rR(l)} v_L^2 + \kappa_1^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_2^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Rr(2)(1)}) \right)$
$\delta_L^{\mp\mp} \delta_L^{\pm\pm} W_R^{\pm\mu} W_{R\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(\mathcal{C}_{\phi D}^{(R)lL(r)} v_R^2 + \kappa_2^2 \mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \kappa_1^2 (\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Ll(2)(1)}) \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_R^{\pm\mu} W_{R\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(\kappa_1^2 \mathcal{C}_{\phi D}^{(2)rL(1)} - 2A_{++}^{12} \right)$
$\delta_R^{\mp\mp} \delta_R^{\pm\pm} W_R^{\pm\mu} W_{R\mu}^\mp$	$g^2 + \frac{g^2}{4\Lambda^2} \left(-4A_{++}^{11} + \mathcal{C}_{\phi D}^{(R)lL(r)} v_L^2 + 4v_R^2 (\mathcal{C}_{\phi D}^{\{Rr\}\{(R)(r)\}} + 3\mathcal{C}_{\phi D}^{R(r)R(r)} + \mathcal{C}_{\square}^{Rr(Rr)}) + 8\Theta_{W_{RR}} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{\{21\}\{(R)(r)\}} \right. \\ \left. + \kappa_+^2 \mathcal{C}_{\phi D}^{21(R)(r)} + 2\kappa_+^2 \mathcal{C}_{\phi D}^{(R)11(r)} - 2\kappa_1^2 \mathcal{C}_{\phi D}^{R(r)1(1)} + \kappa_+^2 \mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} + \kappa_1^2 \mathcal{C}_{\phi D}^{Rr(2)(1)} \right)$
$\delta_L^{\mp\mp} \delta_L^{\pm\pm} W_L^{\pm\mu} W_{R\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(4\Theta_{W_{LR}} - \kappa_1 \left(\kappa_1 \mathcal{C}_{\phi D}^{L(l)1(1)} + \kappa_2 (2\mathcal{C}_{\phi D}^{\{Ll\}\{(2)(1)\}} + \mathcal{C}_{\phi D}^{Ll(2)(1)}) \right) \right)$
$\delta_L^{\mp\mp} \delta_R^{\pm\pm} W_L^{\pm\mu} W_{R\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(\mathcal{C}_{\phi D}^{(L)rL(r)} v_L v_R - \kappa_1 \kappa_2 \mathcal{C}_{\phi D}^{(2)rL(1)} \right)$
$\delta_R^{\mp\mp} \delta_R^{\pm\pm} W_L^{\pm\mu} W_{R\mu}^\mp$	$\frac{g^2}{4\Lambda^2} \left(4\Theta_{W_{RL}} + \kappa_1 \kappa_2 (\mathcal{C}_{\phi D}^{R(r)1(1)} - 2\mathcal{C}_{\phi D}^{\{Rr\}\{(2)(1)\}} - \mathcal{C}_{\phi D}^{Rr(2)(1)}) \right)$

Table 44. MLRSM: coupling of a pair of doubly charged scalars ($\delta^{\pm\pm}$) with a pair of two charged gauge bosons ($W_{L,R}^\pm$) in unphysical basis. These can be easily translated in physical gauge boson basis using rotation matrix $\tilde{\mathcal{R}}_a$.

Figure 14. Scalar four-point interactions: amplitude is the effective scalar quartic coupling λ_{ijkl} . The $\bullet$ represents the effective four-point scalar vertex.

the dimension-6 operators. The added contributions are directly from ϕ^6 operators, and also from the $\phi^4 D^2$ operators through the redefinition of the scalar fields.

Vertex	MLRSM: effective vertex ($\delta^\mp \delta^\pm Z_{1,2}$) factor
$\delta_L^- \delta_L^+ Z_1^\mu$	$ \begin{aligned} & -\frac{1}{8} (k_-^\mu + k_+^\mu) (8\tilde{g}c_{\theta_1}c_{\tilde{\theta}_2}s_{\theta_w} - 8\tilde{g}s_{\theta_1}s_{\tilde{\theta}_2}) - \frac{1}{8\Lambda^2} (k_-^\mu + k_+^\mu) [-8\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w}A_+^{44} + 8\tilde{g}s_{\theta_1}s_{\theta_2}A_+^{44} - 8\tilde{g}c_{\theta_2}c_{\theta_w}\Theta_{3LB} \\ & + 8\tilde{g}c_{\theta_1}s_{\theta_2}\Theta_{3RB} + 8\tilde{g}c_{\theta_2}s_{\theta_1}s_{\theta_w}\Theta_{3RB} + 8\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w}\Theta_{BB} - 8\tilde{g}s_{\theta_1}s_{\theta_2}\Theta_{BB} + 4C_{\phi D}^{\{L\}\{(L)\}(l)}\tilde{g}c_{\theta_1}c_{\theta_2}v_L^2s_{\theta_w} \\ & + 2C_{\phi D}^{(L)rR(l)}\tilde{g}c_{\theta_1}c_{\theta_2}v_R^2s_{\theta_w} + 4\kappa_1^2C_{\phi D}^{\{21\}\{(L)\}(l)}\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w} + 4\kappa_2^2C_{\phi D}^{\{21\}\{(L)\}(l)}\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w} + 2\kappa_1^2C_{\phi D}^{21(L)(l)}\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w} \\ & + 2\kappa_2^2C_{\phi D}^{21(L)(l)}\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w} + 4\kappa_1^2C_{\phi D}^{(L)11(l)}\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w} + 4\kappa_2^2C_{\phi D}^{(L)11(l)}\tilde{g}c_{\theta_1}c_{\theta_2}s_{\theta_w} - 4C_{\phi D}^{\{L\}\{(L)\}(l)}\tilde{g}v_L^2s_{\theta_1}s_{\theta_2} \\ & - 2C_{\phi D}^{(L)rR(l)}\tilde{g}v_R^2s_{\theta_1}s_{\theta_2} - 4\kappa_1^2C_{\phi D}^{\{21\}\{(L)\}(l)}\tilde{g}s_{\theta_1}s_{\theta_2} - 4\kappa_2^2C_{\phi D}^{\{21\}\{(L)\}(l)}\tilde{g}s_{\theta_1}s_{\theta_2} - 2\kappa_1^2C_{\phi D}^{21(L)(l)}\tilde{g}s_{\theta_1}s_{\theta_2} - 2\kappa_2^2C_{\phi D}^{21(L)(l)}\tilde{g}s_{\theta_1}s_{\theta_2} \\ & - 4\kappa_1^2C_{\phi D}^{(L)11(l)}\tilde{g}s_{\theta_1}s_{\theta_2} - 4\kappa_2^2C_{\phi D}^{(L)11(l)}\tilde{g}s_{\theta_1}s_{\theta_2} + g \left(c_{\theta_2} \left(c_{\theta_w} \left(v_L^2 8C_{\square}^{Li(Li)} + \kappa_1^2 C_{\phi D}^{L(l)1(1)} - \kappa_2^2 C_{\phi D}^{L(l)1(1)} \right) \right. \right. \\ & \left. \left. + \kappa_-^2 C_{\phi D}^{L(l)1(1)} s_{\theta_1}s_{\theta_w} \right) + \kappa_-^2 C_{\phi D}^{L(l)1(1)} c_{\theta_1}s_{\theta_2} \right) \Big] \end{aligned} $
$\delta_L^\mp \delta_R^\pm Z_1^\mu$	$ \begin{aligned} & \frac{1}{4\Lambda^2} \left[g v_L v_R \left(c_{\theta_2} \left(s_{\theta_1}s_{\theta_w} \left(C_{\phi D}^{(L)rL(r)} k_\mp^\mu + C_{\phi D}^{(R)lL(r)} p_\pm^\mu \right) - c_{\theta_w} \left(C_{\phi D}^{(L)rR(l)} k_\mp^\mu + C_{\phi D}^{(L)rL(r)} p_\pm^\mu \right) \right) \right. \\ & \left. + c_{\theta_1}s_{\theta_2} \left(C_{\phi D}^{(L)rL(r)} k_\mp^\mu + C_{\phi D}^{(R)lL(r)} p_\pm^\mu \right) - \tilde{g} \left(s_{\theta_1}s_{\theta_2} - c_{\theta_1}c_{\theta_2}s_{\theta_w} \right) \left((k_\mp^\mu + p_\pm^\mu) \left(4A_+^{34} - \left(2v_L v_R C_{\phi D}^{(L)rL(r)} \right. \right. \right. \right. \\ & \left. \left. \left. + \kappa_+^2 \left(C_{\phi D}^{(L)11(r)} + C_{\phi D}^{(L)12(r)} + C_{\phi D}^{(R)11(l)} \right) \right) \right) - v_L v_R \left(k_\mp^\mu C_{\phi D}^{(L)rR(l)} + p_\pm^\mu C_{\phi D}^{(R)lL(r)} \right) \right] \end{aligned} $
$\vdots$	$\vdots$
$\delta_R^- \delta_R^+ Z_2^\mu$	$ \begin{aligned} & +\frac{1}{8} (p_-^\mu + p_+^\mu) (-8\tilde{g}c_{\theta_2}s_{\theta_1} - 8\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w}) + \frac{1}{8\Lambda^2} (p_-^\mu + p_+^\mu) [8\tilde{g}c_{\theta_2}s_{\theta_1}A_+^{33} + 8\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w}A_+^{33} + 8\tilde{g}s_{\theta_2}c_{\theta_w}\Theta_{3LB} \\ & + 8\tilde{g}c_{\theta_1}c_{\theta_2}\Theta_{3RB} - 8\tilde{g}s_{\theta_1}s_{\theta_2}s_{\theta_w}\Theta_{3RB} - 8\tilde{g}c_{\theta_2}s_{\theta_1}\Theta_{BB} - 8\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w}\Theta_{BB} - 2C_{\phi D}^{(R)lL(r)}\tilde{g}c_{\theta_2}v_L^2s_{\theta_1} - 2C_{\phi D}^{(R)lL(r)}\tilde{g}c_{\theta_1}v_L^2s_{\theta_2}s_{\theta_w} \\ & - 4C_{\phi D}^{\{Rr\}\{(R)(r)\}}\tilde{g}c_{\theta_2}v_R^2s_{\theta_1} - 4C_{\phi D}^{\{Rr\}\{(R)(r)\}}\tilde{g}c_{\theta_1}v_R^2s_{\theta_2}s_{\theta_w} - 16C_{\phi D}^{R(r)R(r)}\tilde{g}c_{\theta_2}v_R^2s_{\theta_1} - 16C_{\phi D}^{R(r)R(r)}\tilde{g}c_{\theta_1}v_R^2s_{\theta_2}s_{\theta_w} \\ & - 4\kappa_1^2C_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}c_{\theta_2}s_{\theta_1} - 4\kappa_2^2C_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}c_{\theta_2}s_{\theta_1} - 4\kappa_1^2C_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w} - 4\kappa_2^2C_{\phi D}^{\{21\}\{(R)(r)\}}\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w} \\ & - 2\kappa_1^2C_{\phi D}^{21(R)(r)}\tilde{g}c_{\theta_2}s_{\theta_1} - 2\kappa_2^2C_{\phi D}^{21(R)(r)}\tilde{g}c_{\theta_2}s_{\theta_1} - 2\kappa_1^2C_{\phi D}^{21(R)(r)}\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w} - 2\kappa_2^2C_{\phi D}^{21(R)(r)}\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w} - 4\kappa_1^2C_{\phi D}^{(R)11(r)}\tilde{g}c_{\theta_2}s_{\theta_1} \\ & - 4\kappa_2^2C_{\phi D}^{(R)11(r)}\tilde{g}c_{\theta_2}s_{\theta_1} - 4\kappa_1^2C_{\phi D}^{(R)11(r)}\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w} - 4\kappa_2^2C_{\phi D}^{(R)11(r)}\tilde{g}c_{\theta_1}s_{\theta_2}s_{\theta_w} + g \left(s_{\theta_2} \left(-\kappa_-^2 C_{\phi D}^{R(r)1(1)} c_{\theta_w} \right. \right. \\ & \left. \left. + s_{\theta_1}s_{\theta_w} \left(4v_R^2 (3C_{\phi D}^{R(r)R(r)} + C_{\square}^{Rr(Rr)}) - \kappa_-^2 C_{\phi D}^{R(r)1(1)} \right) \right) - c_{\theta_1}c_{\theta_2} \left(4v_R^2 (3C_{\phi D}^{R(r)R(r)} + C_{\square}^{Rr(Rr)}) - \kappa_-^2 C_{\phi D}^{R(r)1(1)} \right) \right) \Big] \end{aligned} $

Table 45. MLRSM: coupling of a pair of singly charged scalars ($\delta^\pm$) with a neutral gauge boson ($Z_{1,2}$). Here $s_{\theta_i} = \sin \theta_i$ and $c_{\theta_i} = \cos \theta_i$. $k_\pm^\mu$ and $p_\pm^\mu$ are the four momenta of $\delta_L^\pm$ and $\delta_R^\pm$. $\tilde{\theta}_2$ is the redefined angle defined in eq. (6.102).

2HDM: scalar-four point interactions. The TU bounds have been computed for 2HDM in [252, 255–257, 257–263] but with the only renormalizable scalar potential. We expect these bounds to be altered in the presence of these extra contributions from the effective operators. Thus the conclusion embracing the validity of 2HDM scenario including the renormalization group evolutions (RGEs) of the couplings is destined to change. As, after the inclusion of these effective operators, the anomalous dimension matrices need to be included to perform the RGEs of the Wilson coefficients. This will lead to an involved computation and it will be difficult to pass any conclusive remarks on the scale up to which

Vertex	MLRSM: effective vertex ($\delta^\mp \delta^\pm W_3[B] W_3[B]$) factor
$\delta_L^- \delta_L^+ W_L^{+\mu} W_{L\mu}^-$	$2g^2 + \frac{g^2}{8\Lambda^2} \left(-16A_+^{44} + 4v_L^2 (3C_{\phi D}^{\{Ll\}\{(L)l\}} + 9C_{\phi D}^{L(l)L(l)} - C_{\square}^{Ll(Ll)}) + 4C_{\phi D}^{(L)rR(l)} v_R^2 + 32\Theta_{W_{LL}} + 8\kappa_1^2 C_{\phi D}^{\{21\}\{(L)l\}} \right.$ $+ 8\kappa_2^2 C_{\phi D}^{\{21\}\{(L)l\}} + 4\kappa_1^2 C_{\phi D}^{21(L)l} + 4\kappa_2^2 C_{\phi D}^{21(L)l} + 8\kappa_1^2 C_{\phi D}^{(L)11(l)} + 8\kappa_2^2 C_{\phi D}^{(L)11(l)} + 8\kappa_1 \kappa_2 C_{\phi D}^{L(l)1(1)}$ $\left. + 2\kappa_1^2 C_{\phi D}^{\{Ll\}\{(2)1\}} + 2\kappa_2^2 C_{\phi D}^{\{Ll\}\{(2)1\}} + \kappa_1^2 C_{\phi D}^{Ll(2)(1)} + \kappa_2^2 C_{\phi D}^{Ll(2)(1)} \right)$
$\delta_L^- \delta_L^+ W_{3L}^\mu W_{3L\mu}$	$\frac{g^2}{16\Lambda^2} \left(8v_L^2 (C_{\phi D}^{\{Ll\}\{(L)l\}} + C_{\phi D}^{L(l)L(l)} - C_{\square}^{Ll(Ll)}) + (\kappa_1^2 + \kappa_2^2) (2C_{\phi D}^{\{Ll\}\{(2)1\}} + C_{\phi D}^{Ll(2)(1)}) \right)$
$\delta_L^- \delta_L^+ W_{3L}^\mu B_\mu$	$\frac{\tilde{g}}{4\Lambda^2} \left(8\tilde{g}\Theta_{3LB} - 4gv_L^2 (C_{\phi D}^{\{Ll\}\{(L)l\}} + 4C_{\phi D}^{L(l)L(l)}) + g(\kappa_2^2 - \kappa_1^2) C_{\phi D}^{L(l)1(1)} \right)$
$\delta_L^- \delta_L^+ B^\mu B_\mu$	$\tilde{g}^2 + \frac{\tilde{g}^2}{4\Lambda^2} \left(-4A_+^{44} + 8\Theta_{BB} + 4v_L^2 (C_{\phi D}^{\{Ll\}\{(L)l\}} + 4C_{\phi D}^{L(l)L(l)}) + C_{\phi D}^{(L)rR(l)} v_R^2 + C_{\phi D}^{(R)lL(r)} v_R^2 + 2\kappa_+^2 C_{\phi D}^{\{21\}\{(L)l\}} \right.$ $\left. + \kappa_+^2 C_{\phi D}^{21(L)l} + 2\kappa_+^2 C_{\phi D}^{(L)11(l)} \right)$
$\vdots$	$\vdots$
$\delta_R^- \delta_R^+ W_R^{+\mu} W_{R\mu}^-$	$2g^2 + \frac{g^2}{8\Lambda^2} \left(-16A_+^{33} + 4C_{\phi D}^{(R)lL(r)} v_L^2 + 12C_{\phi D}^{\{Rr\}\{(R)r\}} v_R^2 + 36C_{\phi D}^{R(r)R(r)} v_R^2 - 4C_{\square}^{Rr(Rr)} v_R^2 + 32\Theta_{W_{RR}} \right.$ $+ 8\kappa_1^2 C_{\phi D}^{\{21\}\{(R)r\}} + 8\kappa_2^2 C_{\phi D}^{\{21\}\{(R)r\}} + 4\kappa_1^2 C_{\phi D}^{21(R)r} + 4\kappa_2^2 C_{\phi D}^{21(R)r} + 8\kappa_1^2 C_{\phi D}^{(R)11(r)} + 8\kappa_2^2 C_{\phi D}^{(R)11(r)}$ $\left. - 4\kappa_1^2 C_{\phi D}^{R(r)1(1)} - 4\kappa_2^2 C_{\phi D}^{R(r)1(1)} + 2\kappa_1^2 C_{\phi D}^{\{Rr\}\{(2)1\}} + 2\kappa_2^2 C_{\phi D}^{\{Rr\}\{(2)1\}} + \kappa_1^2 C_{\phi D}^{Rr(2)(1)} + \kappa_2^2 C_{\phi D}^{Rr(2)(1)} \right)$
$\delta_R^- \delta_R^+ W_{3R}^\mu W_{3R\mu}$	$\frac{g^2}{16\Lambda^2} \left(8v_R^2 (C_{\phi D}^{\{Rr\}\{(R)r\}} + C_{\phi D}^{R(r)R(r)} - C_{\square}^{Rr(Rr)}) + (\kappa_1^2 + \kappa_2^2) (2C_{\phi D}^{\{Rr\}\{(2)1\}} + C_{\phi D}^{Rr(2)(1)}) \right)$
$\delta_R^- \delta_R^+ W_{3R}^\mu B_\mu$	$\frac{\tilde{g}}{4\Lambda^2} \left(8\tilde{g}\Theta_{3RB} - 4gv_R^2 (C_{\phi D}^{\{Rr\}\{(R)r\}} + 4C_{\phi D}^{R(r)R(r)}) + g(\kappa_1^2 - \kappa_2^2) C_{\phi D}^{R(r)1(1)} \right)$
$\delta_R^- \delta_R^+ B^\mu B_\mu$	$\tilde{g}^2 + \frac{\tilde{g}^2}{4\Lambda^2} \left(-4A_+^{33} + 8\Theta_{BB} + v_L^2 (C_{\phi D}^{(L)rR(l)} + C_{\phi D}^{(R)lL(r)}) + 4C_{\phi D}^{\{Rr\}\{(R)r\}} v_R^2 + 16C_{\phi D}^{R(r)R(r)} v_R^2 + 2\kappa_+^2 C_{\phi D}^{\{21\}\{(R)r\}} \right.$ $\left. + \kappa_+^2 C_{\phi D}^{21(R)r} + 2\kappa_+^2 C_{\phi D}^{(R)11(r)} \right)$

Table 46. MLRSM: coupling of a pair of singly charged scalars ($\delta^\pm$) with either a pair of charged ($W_{L,R}^\pm$) or neutral ($B, W_{3L,3R}$) gauge bosons in unphysical basis. These can be translated in physical gauge boson basis using rotation matrix $\tilde{\mathcal{R}}_{a,b}$.

this theory will be valid based on these criteria. In this section, we have summarised the complete set of four-point interactions in the unphysical basis of scalar fields in tables 47–49.

MLRSM: scalar four-point interactions Similar to the 2HDM case, the scalar quartic couplings in this model are also constrained by the TU criteria. For the renormalizable potential these criteria are derived and discussed in [192, 264, 265]. These scalar four-point vertex factors, i.e., quartic couplings must satisfy the vacuum stability criteria such that the effective potential is bounded from below. This should be further analyzed using the same method mentioned in [254, 266, 267]. As the scalar sector of the MLRSM scenario is quite extended, the number of four-point interaction vertices are very large. Thus instead of providing all of them, unlike the 2HDM case, here, we have listed a few selective vertices, see table 50. Interested readers can be provided with the Mathematica file containing all such vertices for MLRSM scenario.

Vertex	2HDM: effective vertex $(\phi_i \phi_j \phi_k \phi_l)$ factor
$\phi_1^- \phi_1^- \phi_1^+ \phi_1^+$	$\lambda_1 - \frac{\lambda_6 A_+^{12}}{\Lambda^2} - \frac{2\lambda_1 A_+^{11}}{\Lambda^2} + \frac{3C_6^{111111} v_1^2}{2\Lambda^2} + \frac{C_6^{111112} v_1 v_2}{\Lambda^2} + \frac{C_6^{111122} v_2^2}{2\Lambda^2}$
$\phi_1^- \phi_2^- \phi_1^+ \phi_2^+$	$\lambda_3 + \lambda_4 - \frac{\lambda_3 A_+^{22}}{\Lambda^2} - \frac{\lambda_4 A_+^{22}}{\Lambda^2} - \frac{2\lambda_7 A_+^{12}}{\Lambda^2} - \frac{\lambda_3 A_+^{11}}{\Lambda^2} - \frac{\lambda_4 A_+^{11}}{\Lambda^2} - \frac{2\lambda_6 A_+^{11}}{\Lambda^2} + \frac{C_6^{111122} v_1^2}{\Lambda^2}$ $+ \frac{C_6^{112212} v_2 v_1}{\Lambda^2} + \frac{C_6^{112222} v_2^2}{\Lambda^2} + \frac{C_6^{122111} v_1^2}{2\Lambda^2} + \frac{2C_6^{121221} v_2 v_1}{\Lambda^2} + \frac{C_6^{122122} v_2^2}{2\Lambda^2}$
$\rho_1^2 \phi_1^- \phi_1^+$	$\lambda_1 - \frac{\lambda_6 A_r^{12}}{2\Lambda^2} - \frac{\lambda_1 A_r^{11}}{\Lambda^2} - \frac{\lambda_1 A_+^{11}}{\Lambda^2} - \frac{\lambda_6 A_+^{12}}{2\Lambda^2} + \frac{9C_6^{111111} v_1^2}{2\Lambda^2} + \frac{3C_6^{111112} v_1 v_2}{\Lambda^2} + \frac{C_6^{111122} v_2^2}{2\Lambda^2} + \frac{C_6^{121211} v_2^2}{2\Lambda^2}$ $+ \frac{C_6^{122111} v_2^2}{4\Lambda^2}$
$\rho_1 \rho_2 \phi_1^- \phi_1^+$	$\lambda_6 - \frac{\lambda_6 A_r^{22}}{2\Lambda^2} - \frac{\lambda_1 A_r^{12}}{\Lambda^2} - \frac{\lambda_3 A_r^{12}}{2\Lambda^2} - \frac{\lambda_6 A_r^{11}}{2\Lambda^2} - \frac{\lambda_4 A_+^{12}}{2\Lambda^2} - \frac{\lambda_5 A_+^{12}}{2\Lambda^2} - \frac{\lambda_6 A_+^{11}}{\Lambda^2} + \frac{3C_6^{111112} v_1^2}{\Lambda^2}$ $+ \frac{2C_6^{111122} v_1 v_2}{\Lambda^2} + \frac{3C_6^{112212} v_2^2}{2\Lambda^2} + \frac{2C_6^{121211} v_1 v_2}{\Lambda^2} + \frac{C_6^{122111} v_1 v_2}{\Lambda^2}$
$\rho_2^2 \phi_1^- \phi_1^+$	$\frac{\lambda_3}{2} - \frac{\lambda_3 A_r^{22}}{2\Lambda^2} - \frac{\lambda_6 A_r^{12}}{2\Lambda^2} - \frac{\lambda_3 A_+^{11}}{2\Lambda^2} - \frac{\lambda_7 A_+^{12}}{2\Lambda^2} + \frac{C_6^{111122} v_1^2}{2\Lambda^2} + \frac{3C_6^{112212} v_1 v_2}{2\Lambda^2} + \frac{C_6^{121211} v_1^2}{2\Lambda^2} + \frac{C_6^{122111} v_1^2}{4\Lambda^2}$ $+ \frac{3C_6^{112222} v_2^2}{2\Lambda^2}$
$\eta_1^2 \phi_1^- \phi_1^+$	$\lambda_1 - \frac{\lambda_6 A_i^{12}}{2\Lambda^2} - \frac{\lambda_1 A_i^{11}}{\Lambda^2} - \frac{\lambda_1 A_+^{11}}{\Lambda^2} - \frac{\lambda_6 A_+^{12}}{2\Lambda^2} + \frac{3C_6^{111111} v_1^2}{2\Lambda^2} + \frac{C_6^{111112} v_1 v_2}{\Lambda^2} + \frac{C_6^{111122} v_2^2}{2\Lambda^2} - \frac{C_6^{121211} v_2^2}{2\Lambda^2}$ $+ \frac{C_6^{122111} v_2^2}{4\Lambda^2}$
$\eta_1 \eta_2 \phi_1^- \phi_1^+$	$\lambda_6 - \frac{\lambda_6 A_i^{22}}{2\Lambda^2} - \frac{\lambda_1 A_i^{12}}{\Lambda^2} - \frac{\lambda_3 A_i^{12}}{2\Lambda^2} - \frac{\lambda_6 A_i^{11}}{2\Lambda^2} - \frac{\lambda_4 A_+^{12}}{2\Lambda^2} - \frac{\lambda_5 A_+^{12}}{2\Lambda^2} - \frac{\lambda_6 A_+^{11}}{\Lambda^2} + \frac{C_6^{111112} v_1^2}{\Lambda^2}$ $+ \frac{C_6^{112212} v_2^2}{2\Lambda^2} + \frac{2C_6^{121211} v_1 v_2}{\Lambda^2}$
$\phi_2^- \phi_2^- \phi_2^+ \phi_2^+$	$\lambda_2 - \frac{2\lambda_2 A_+^{22}}{\Lambda^2} - \frac{\lambda_7 A_+^{11}}{\Lambda^2} + \frac{C_6^{112222} v_1^2}{2\Lambda^2} + \frac{C_6^{122222} v_1 v_2}{\Lambda^2} + \frac{3C_6^{222222} v_2^2}{2\Lambda^2}$
$\rho_1^2 \phi_2^- \phi_2^+$	$\frac{\lambda_3}{2} - \frac{\lambda_3 A_r^{11}}{2\Lambda^2} - \frac{\lambda_7 A_r^{12}}{2\Lambda^2} - \frac{\lambda_3 A_+^{22}}{2\Lambda^2} - \frac{\lambda_6 A_+^{11}}{2\Lambda^2} + \frac{3C_6^{111122} v_1^2}{2\Lambda^2} + \frac{3C_6^{112212} v_1 v_2}{2\Lambda^2} + \frac{C_6^{121222} v_2^2}{2\Lambda^2} + \frac{C_6^{112222} v_2^2}{2\Lambda^2}$ $+ \frac{C_6^{122122} v_2^2}{4\Lambda^2}$
$\rho_1 \rho_2 \phi_2^- \phi_2^+$	$\lambda_7 - \frac{\lambda_7 A_r^{22}}{2\Lambda^2} - \frac{\lambda_2 A_r^{12}}{\Lambda^2} - \frac{\lambda_3 A_r^{12}}{2\Lambda^2} - \frac{\lambda_7 A_r^{11}}{2\Lambda^2} - \frac{\lambda_4 A_+^{11}}{2\Lambda^2} - \frac{\lambda_5 A_+^{11}}{2\Lambda^2} - \frac{\lambda_7 A_+^{22}}{\Lambda^2} + \frac{3C_6^{112212} v_1^2}{2\Lambda^2}$ $+ \frac{2C_6^{121222} v_1 v_2}{\Lambda^2} + \frac{2C_6^{112222} v_1 v_2}{\Lambda^2} + \frac{C_6^{122122} v_1 v_2}{\Lambda^2} + \frac{3C_6^{122222} v_2^2}{\Lambda^2}$
$\rho_2^2 \phi_2^- \phi_2^+$	$\lambda_2 - \frac{\lambda_2 A_r^{22}}{\Lambda^2} - \frac{\lambda_7 A_r^{12}}{2\Lambda^2} - \frac{\lambda_2 A_+^{22}}{\Lambda^2} - \frac{\lambda_7 A_+^{11}}{2\Lambda^2} + \frac{C_6^{121222} v_1^2}{2\Lambda^2} + \frac{C_6^{112222} v_1^2}{2\Lambda^2} + \frac{C_6^{122122} v_1^2}{4\Lambda^2} + \frac{3C_6^{122222} v_1 v_2}{\Lambda^2}$ $+ \frac{9C_6^{222222} v_2^2}{2\Lambda^2}$
$\eta_1^2 \phi_2^- \phi_2^+$	$\frac{\lambda_3}{2} - \frac{\lambda_3 A_i^{11}}{2\Lambda^2} - \frac{\lambda_7 A_i^{12}}{2\Lambda^2} - \frac{\lambda_3 A_+^{22}}{2\Lambda^2} - \frac{\lambda_6 A_+^{11}}{2\Lambda^2} + \frac{C_6^{111122} v_1^2}{2\Lambda^2} + \frac{C_6^{112212} v_1 v_2}{2\Lambda^2} - \frac{C_6^{121222} v_2^2}{2\Lambda^2} + \frac{C_6^{112222} v_2^2}{2\Lambda^2}$ $+ \frac{C_6^{122122} v_2^2}{4\Lambda^2}$
$\eta_1 \eta_2 \phi_2^- \phi_2^+$	$\lambda_7 - \frac{\lambda_7 A_i^{22}}{2\Lambda^2} - \frac{\lambda_2 A_i^{12}}{\Lambda^2} - \frac{\lambda_3 A_i^{12}}{2\Lambda^2} - \frac{\lambda_7 A_i^{11}}{2\Lambda^2} - \frac{\lambda_4 A_+^{11}}{2\Lambda^2} - \frac{\lambda_5 A_+^{11}}{2\Lambda^2} - \frac{\lambda_7 A_+^{22}}{\Lambda^2} + \frac{C_6^{112212} v_1^2}{\Lambda^2}$ $+ \frac{2C_6^{121222} v_1 v_2}{\Lambda^2} + \frac{C_6^{122222} v_2^2}{\Lambda^2}$

Table 47. 2HDM: scalar four-point vertices in the unphysical basis.

Vertex	2HDM: effective vertex $(\phi_i \phi_j \phi_k \phi_l)$ factor
ρ_1^4	$\frac{\lambda_1}{4} - \frac{\lambda_6 A_r^{12}}{4\Lambda^2} - \frac{\lambda_1 A_r^{11}}{2\Lambda^2} + \frac{15C_6^{111111} v_1^2}{8\Lambda^2} + \frac{5C_6^{111112} v_1 v_2}{4\Lambda^2} + \frac{C_6^{111122} v_2^2}{8\Lambda^2} + \frac{C_6^{121211} v_2^2}{4\Lambda^2} + \frac{C_6^{122111} v_2^2}{8\Lambda^2}$
ρ_2^4	$\frac{\lambda_2}{4} - \frac{\lambda_2 A_r^{22}}{2\Lambda^2} - \frac{\lambda_7 A_r^{12}}{4\Lambda^2} + \frac{C_6^{121222} v_1^2}{4\Lambda^2} + \frac{C_6^{112222} v_1^2}{8\Lambda^2} + \frac{C_6^{122122} v_1^2}{8\Lambda^2} + \frac{5C_6^{122222} v_1 v_2}{4\Lambda^2} + \frac{15C_6^{222222} v_2^2}{8\Lambda^2}$
$\rho_1^3 \rho_2$	$\frac{\lambda_6}{2} - \frac{\lambda_6 A_r^{22}}{4\Lambda^2} - \frac{\lambda_1 A_r^{12}}{2\Lambda^2} - \frac{\lambda_3 A_r^{12}}{4\Lambda^2} - \frac{\lambda_4 A_r^{12}}{4\Lambda^2} - \frac{\lambda_5 A_r^{12}}{4\Lambda^2} - \frac{3\lambda_6 A_r^{11}}{4\Lambda^2} + \frac{5C_6^{111112} v_1^2}{2\Lambda^2} + \frac{C_6^{111122} v_1 v_2}{\Lambda^2}$ $+ \frac{3C_6^{112212} v_2^2}{4\Lambda^2} + \frac{2C_6^{121211} v_1 v_2}{\Lambda^2} + \frac{3C_6^{121212} v_2^2}{4\Lambda^2} + \frac{C_6^{122111} v_1 v_2}{\Lambda^2} + \frac{3C_6^{121221} v_2^2}{4\Lambda^2}$
$\rho_1^2 \rho_2^2$	$\frac{\lambda_3}{4} + \frac{\lambda_4}{4} + \frac{\lambda_5}{4} - \frac{\lambda_3 A_r^{22}}{4\Lambda^2} - \frac{3\lambda_6 A_r^{12}}{4\Lambda^2} - \frac{3\lambda_7 A_r^{12}}{4\Lambda^2} - \frac{\lambda_3 A_r^{11}}{4\Lambda^2} - \frac{\lambda_4 A_r^{11}}{4\Lambda^2} - \frac{\lambda_5 A_r^{11}}{4\Lambda^2}$ $- \frac{\lambda_4 A_r^{22}}{4\Lambda^2} - \frac{\lambda_5 A_r^{22}}{4\Lambda^2} + \frac{3C_6^{111122} v_1^2}{4\Lambda^2} + \frac{9C_6^{112212} v_1 v_2}{4\Lambda^2} + \frac{3C_6^{121211} v_1^2}{2\Lambda^2} + \frac{9C_6^{121212} v_1 v_2}{4\Lambda^2} + \frac{3C_6^{121222} v_2^2}{2\Lambda^2} + \frac{3C_6^{122111} v_1^2}{4\Lambda^2}$ $+ \frac{9C_6^{121221} v_1 v_2}{4\Lambda^2} + \frac{3C_6^{122222} v_2^2}{4\Lambda^2} + \frac{3C_6^{122122} v_2^2}{4\Lambda^2}$
$\eta_1^2 \rho_1^2$	$\frac{\lambda_1}{2} - \frac{\lambda_6 A_i^{12}}{4\Lambda^2} - \frac{\lambda_1 A_i^{11}}{2\Lambda^2} - \frac{\lambda_1 A_r^{11}}{2\Lambda^2} - \frac{\lambda_6 A_r^{12}}{4\Lambda^2} + \frac{9C_6^{111111} v_1^2}{4\Lambda^2} + \frac{3C_6^{111112} v_1 v_2}{2\Lambda^2} + \frac{C_6^{111122} v_2^2}{4\Lambda^2} + \frac{C_6^{122111} v_2^2}{4\Lambda^2}$
$\eta_1 \eta_2 \rho_1^2$	$\frac{\lambda_6}{2} - \frac{\lambda_6 A_i^{22}}{4\Lambda^2} - \frac{\lambda_1 A_i^{12}}{2\Lambda^2} - \frac{\lambda_3 A_i^{12}}{4\Lambda^2} - \frac{\lambda_4 A_i^{12}}{4\Lambda^2} + \frac{\lambda_5 A_i^{12}}{4\Lambda^2} - \frac{\lambda_6 A_r^{11}}{4\Lambda^2} - \frac{\lambda_5 A_r^{12}}{2\Lambda^2} - \frac{\lambda_6 A_r^{11}}{2\Lambda^2}$ $+ \frac{3C_6^{111112} v_1^2}{2\Lambda^2} + \frac{C_6^{112212} v_2^2}{4\Lambda^2} + \frac{3C_6^{121211} v_2 v_1}{\Lambda^2} + \frac{9C_6^{121212} v_2^2}{4\Lambda^2} + \frac{C_6^{121221} v_2^2}{4\Lambda^2}$
$\eta_1^2 \rho_1 \rho_2$	$\frac{\lambda_6}{2} - \frac{\lambda_5 A_i^{12}}{2\Lambda^2} - \frac{\lambda_6 A_i^{11}}{2\Lambda^2} - \frac{\lambda_6 A_r^{22}}{4\Lambda^2} - \frac{\lambda_1 A_r^{12}}{2\Lambda^2} - \frac{\lambda_3 A_r^{12}}{4\Lambda^2} - \frac{\lambda_4 A_r^{12}}{4\Lambda^2} + \frac{\lambda_5 A_r^{12}}{4\Lambda^2} - \frac{\lambda_6 A_r^{11}}{4\Lambda^2}$ $+ \frac{3C_6^{111112} v_1^2}{2\Lambda^2} + \frac{C_6^{111122} v_2 v_1}{\Lambda^2} + \frac{3C_6^{112212} v_2^2}{4\Lambda^2} - \frac{9C_6^{121212} v_2^2}{4\Lambda^2} + \frac{C_6^{122111} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121221} v_2^2}{4\Lambda^2}$
$\eta_1 \eta_2 \rho_1 \rho_2$	$\lambda_5 - \frac{\lambda_5 A_i^{22}}{2\Lambda^2} - \frac{\lambda_6 A_i^{12}}{2\Lambda^2} - \frac{\lambda_7 A_i^{12}}{2\Lambda^2} - \frac{\lambda_5 A_r^{11}}{2\Lambda^2} - \frac{\lambda_5 A_r^{22}}{2\Lambda^2} - \frac{\lambda_5 A_r^{11}}{2\Lambda^2} - \frac{\lambda_6 A_r^{12}}{2\Lambda^2} - \frac{\lambda_7 A_r^{12}}{2\Lambda^2}$ $+ \frac{C_6^{112212} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121211} v_1^2}{\Lambda^2} + \frac{9C_6^{121212} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121222} v_2^2}{\Lambda^2} + \frac{C_6^{121221} v_2 v_1}{\Lambda^2}$
$\rho_1^2 \rho_2^2$	$\frac{\lambda_3}{4} + \frac{\lambda_4}{4} + \frac{\lambda_5}{4} - \frac{\lambda_3 A_r^{22}}{4\Lambda^2} - \frac{3\lambda_6 A_r^{12}}{4\Lambda^2} - \frac{3\lambda_7 A_r^{12}}{4\Lambda^2} - \frac{\lambda_3 A_r^{11}}{4\Lambda^2} - \frac{\lambda_4 A_r^{11}}{4\Lambda^2} - \frac{\lambda_5 A_r^{11}}{4\Lambda^2}$ $- \frac{\lambda_4 A_r^{22}}{4\Lambda^2} - \frac{\lambda_5 A_r^{22}}{4\Lambda^2} + \frac{3C_6^{111122} v_1^2}{4\Lambda^2} + \frac{9C_6^{112212} v_1 v_2}{4\Lambda^2} + \frac{3C_6^{121211} v_1^2}{2\Lambda^2} + \frac{9C_6^{121212} v_1 v_2}{4\Lambda^2} + \frac{3C_6^{121222} v_2^2}{2\Lambda^2} + \frac{3C_6^{122111} v_1^2}{4\Lambda^2}$ $+ \frac{9C_6^{121221} v_1 v_2}{4\Lambda^2} + \frac{3C_6^{122222} v_2^2}{4\Lambda^2} + \frac{3C_6^{122122} v_2^2}{4\Lambda^2}$
$\rho_1 \rho_2^3$	$\frac{\lambda_7}{2} - \frac{3\lambda_7 A_r^{22}}{4\Lambda^2} - \frac{\lambda_2 A_r^{12}}{2\Lambda^2} - \frac{\lambda_3 A_r^{12}}{4\Lambda^2} - \frac{\lambda_4 A_r^{12}}{4\Lambda^2} - \frac{\lambda_5 A_r^{12}}{4\Lambda^2} - \frac{\lambda_7 A_r^{11}}{4\Lambda^2} + \frac{3C_6^{112212} v_1^2}{4\Lambda^2} + \frac{3C_6^{121212} v_1^2}{4\Lambda^2}$ $+ \frac{2C_6^{121222} v_2 v_1}{\Lambda^2} + \frac{C_6^{112222} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121221} v_2^2}{4\Lambda^2} + \frac{C_6^{122122} v_2 v_1}{\Lambda^2} + \frac{5C_6^{122222} v_2^2}{2\Lambda^2}$
$\eta_1^2 \rho_1 \rho_2$	$\frac{\lambda_6}{2} - \frac{\lambda_5 A_i^{12}}{2\Lambda^2} - \frac{\lambda_6 A_i^{11}}{2\Lambda^2} - \frac{\lambda_6 A_r^{22}}{4\Lambda^2} - \frac{\lambda_1 A_r^{12}}{2\Lambda^2} - \frac{\lambda_3 A_r^{12}}{4\Lambda^2} - \frac{\lambda_4 A_r^{12}}{4\Lambda^2} + \frac{\lambda_5 A_r^{12}}{4\Lambda^2} - \frac{\lambda_6 A_r^{11}}{4\Lambda^2}$ $+ \frac{3C_6^{111112} v_1^2}{2\Lambda^2} + \frac{C_6^{111122} v_2 v_1}{\Lambda^2} + \frac{3C_6^{112212} v_2^2}{4\Lambda^2} - \frac{9C_6^{121212} v_2^2}{4\Lambda^2} + \frac{C_6^{122111} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121221} v_2^2}{4\Lambda^2}$
$\eta_1 \eta_2 \rho_1 \rho_2$	$\lambda_5 - \frac{\lambda_5 A_i^{22}}{2\Lambda^2} - \frac{\lambda_6 A_i^{12}}{2\Lambda^2} - \frac{\lambda_7 A_i^{12}}{2\Lambda^2} - \frac{\lambda_5 A_r^{11}}{2\Lambda^2} - \frac{\lambda_5 A_r^{22}}{2\Lambda^2} - \frac{\lambda_5 A_r^{11}}{2\Lambda^2} - \frac{\lambda_6 A_r^{12}}{2\Lambda^2} - \frac{\lambda_7 A_r^{12}}{2\Lambda^2}$ $+ \frac{C_6^{112212} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121211} v_1^2}{\Lambda^2} + \frac{9C_6^{121212} v_2 v_1}{\Lambda^2} + \frac{3C_6^{121222} v_2^2}{\Lambda^2} + \frac{C_6^{121221} v_2 v_1}{\Lambda^2}$
$\eta_1^2 \rho_1^2$	$\frac{\lambda_1}{2} - \frac{\lambda_6 A_i^{12}}{4\Lambda^2} - \frac{\lambda_1 A_i^{11}}{2\Lambda^2} - \frac{\lambda_1 A_r^{11}}{2\Lambda^2} - \frac{\lambda_6 A_r^{12}}{4\Lambda^2} + \frac{9C_6^{111111} v_1^2}{4\Lambda^2} + \frac{3C_6^{111112} v_1 v_2}{2\Lambda^2} + \frac{C_6^{111122} v_2^2}{4\Lambda^2} + \frac{C_6^{122111} v_2^2}{4\Lambda^2}$ $+ \frac{C_6^{112212} v_1^2}{4\Lambda^2} + \frac{9C_6^{121212} v_1^2}{4\Lambda^2} + \frac{3C_6^{121222} v_2 v_1}{\Lambda^2} + \frac{C_6^{121221} v_2^2}{4\Lambda^2} + \frac{3C_6^{122222} v_2^2}{2\Lambda^2}$

Table 48. 2HDM: scalar four-point vertices in the unphysical basis.

Vertex	MLRSM: effective vertex $(\phi_i \phi_j \phi_k \phi_l)$ factor
$(\phi_1^{0r})^4$	$\frac{\lambda_1}{4} + \frac{\lambda_2}{2} + \frac{\lambda_3}{4} + \frac{\lambda_4}{2} + \frac{C_6^{2121Rr} v_R^2}{32\Lambda^2} + \frac{15\kappa^2 C_6^{\{21\}\{2121\}}}{8\Lambda^2} + \frac{15\kappa^2 C_6^{212121}}{16\Lambda^2} + \frac{15\kappa^2 C_6^{412121}}{8\Lambda^2} + \frac{15\kappa^2 C_6^{414121}}{8\Lambda^2}$ $+ \frac{15\kappa^2 C_6^{414123}}{8\Lambda^2} + \frac{15\kappa^2 C_6^{414141}}{8\Lambda^2} + \frac{C_6^{2141Rr} v_R^2}{16\Lambda^2} + \frac{C_6^{2211Rr} v_R^2}{32\Lambda^2} + \frac{C_6^{23Rr41} v_R^2}{32\Lambda^2} + \frac{C_6^{2R21r1} v_R^2}{32\Lambda^2} + \frac{C_6^{4141Rr} v_R^2}{16\Lambda^2}$ $+ \frac{C_6^{41R2r1} v_R^2}{16\Lambda^2} - \frac{\lambda_1 \tilde{A}_r^{11}}{2\Lambda^2} - \frac{\lambda_2 \tilde{A}_r^{11}}{\Lambda^2} - \frac{\lambda_3 \tilde{A}_r^{11}}{2\Lambda^2} - \frac{\lambda_4 \tilde{A}_r^{11}}{\Lambda^2}$
$(\phi_1^{0r})^3 \phi_1^{0i}$	$- \frac{C_6^{41R2r1} v_R^2}{8\Lambda^2}$
$(\phi_1^{0r})^3 \phi_2^{0r}$	$- \frac{\lambda_1 \tilde{A}_r^{12}}{\Lambda^2} - \frac{\lambda_4 \tilde{A}_r^{12}}{\Lambda^2} - \frac{C_6^{2121Rr} v_R^2}{8\Lambda^2} - \frac{C_6^{2141Rr} v_R^2}{8\Lambda^2} - \frac{C_6^{2211Rr} v_R^2}{8\Lambda^2}$
$(\phi_1^{0r})^3 \phi_2^{0i}$	$\frac{C_6^{2141Rr} v_R^2}{8\Lambda^2} + \frac{C_6^{4141Rr} v_R^2}{4\Lambda^2} + \frac{C_6^{41R2r1} v_R^2}{8\Lambda^2} + \frac{5\kappa^2 C_6^{412121}}{2\Lambda^2} + \frac{5\kappa^2 C_6^{414121}}{\Lambda^2} + \frac{5\kappa^2 C_6^{414123}}{2\Lambda^2} + \frac{15\kappa^2 C_6^{414141}}{2\Lambda^2}$
$(\phi_1^{0r})^3 \delta_L^{0r}$	$\frac{\kappa C_6^{21L2r1} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{21L4r1} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{21R4l1} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{2L41r3} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{41L2r3} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{41L41r} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{41R2l1} v_R}{2\sqrt{2}\Lambda^2}$ $- \frac{\alpha_1 \tilde{A}_r^{14}}{4\Lambda^2} - \frac{\alpha_2 \tilde{A}_r^{14}}{2\Lambda^2} - \frac{\alpha_3 \tilde{A}_r^{14}}{8\Lambda^2} - \frac{\lambda_1 \tilde{A}_r^{14}}{2\Lambda^2} - \frac{\lambda_2 \tilde{A}_r^{14}}{\Lambda^2} - \frac{\lambda_3 \tilde{A}_r^{14}}{2\Lambda^2} + \frac{\kappa C_6^{41R4l1} v_R}{2\sqrt{2}\Lambda^2} - \frac{\lambda_4 \tilde{A}_r^{14}}{\Lambda^2}$
$(\phi_1^{0r})^3 \delta_L^{0i}$	$\frac{\kappa C_6^{21L2r1} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{21L4r1} v_R}{2\sqrt{2}\Lambda^2} - \frac{\kappa C_6^{21R4l1} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{2L41r3} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{41L2r3} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{41L41r} v_R}{2\sqrt{2}\Lambda^2} - \frac{\kappa C_6^{41R2l1} v_R}{2\sqrt{2}\Lambda^2}$ $- \frac{\kappa C_6^{41R4l1} v_R}{2\sqrt{2}\Lambda^2}$
$\vdots$	$\vdots$
$(\phi_1^{0r})^3 \delta_R^{0r}$	$\frac{\kappa C_6^{2121Rr} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{2141Rr} v_R}{\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{2211Rr} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{23Rr41} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{2R21r1} v_R}{2\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{4141Rr} v_R}{\sqrt{2}\Lambda^2} + \frac{\kappa C_6^{41R2r1} v_R}{\sqrt{2}\Lambda^2}$ $- \frac{\alpha_1 \tilde{A}_r^{13}}{4\Lambda^2} - \frac{\alpha_2 \tilde{A}_r^{13}}{2\Lambda^2} - \frac{\alpha_3 \tilde{A}_r^{13}}{8\Lambda^2} - \frac{\lambda_1 \tilde{A}_r^{13}}{2\Lambda^2} - \frac{\lambda_2 \tilde{A}_r^{13}}{\Lambda^2} - \frac{\lambda_3 \tilde{A}_r^{13}}{2\Lambda^2} - \frac{\lambda_4 \tilde{A}_r^{13}}{\Lambda^2}$
$(\phi_1^{0r})^3 \delta_R^{0i}$	0

Table 50. MLRSM: scalar four-point vertices in the unphysical basis. As the number of such kind of vertices are very large, we have not mentioned all of them. These vertex factors are computed with the following assumptions: $\kappa_1 = \kappa_2 = \kappa, v_L = 0$. In these limits we have redefined A_r^{ij}, A_i^{ij} as $\tilde{A}_r^{ij}, \tilde{A}_i^{ij}$ respectively.

8 Conclusions and remarks

In this paper, we have implemented the Hilbert Series (HS) method to construct the complete set of independent effective operators up to dimension-6 for two BSM scenarios: 2HDM and MLRSM. To start with, we have demonstrated the building blocks for the HS: Plethysthic Exponentials (PEs) and Haar measure. The PEs are constructed for bosonic and fermionic fields using the transformation properties of the fields under the internal gauge symmetry and Lorentz group. In this connection, we have also mentioned how this information is encrypted in the characters of those representations. Once the PEs are constructed, they are integrated on the group space using the Haar measure. These allow us to obtain the full Hilbert Series in a polynomial form. In our case, the index parameter for this polynomial expansion is the mass dimension. Thus we have been able to truncate the series based on the mass dimensionality of the operators. We have restricted ourselves up to dimension-6 operators. This method also takes care of two significant constraints related to the derivative operators: Equations of Motion (EOMs) and Integration by Parts (IBPs).

Thus all the redundancies are removed and a complete set of independent operators for a given mass dimension is computed.

After constructing the operators, we have analysed how they can affect the predictions of the renormalizable version of the adopted scenarios: 2HDM and MLRSM. First, we have calculated the scalar and gauge field redefinitions due to $\phi^4 D^2$ and $\phi^2 X^2$ class of operators respectively and have estimated the corresponding modifications in the spectra. We have also focused on other sets of operators, which directly affect the spectrum, and included their contributions. Noting down all such contributions, we have provided the full mass spectra for scalar and gauge fields within the effective theory frameworks of 2HDM and MLRSM. We have further estimated the additional contributions to the charged and neutral fermion masses from the relevant operators. After completing the spectrum analysis, we have looked into the low energy observables which are precisely measured in the experiments, namely Weak mixing angle, Fermi constant, ρ and oblique (S, T, U) parameters. For both the scenarios, we have analytically computed the contributions to these observables. We have also listed all the relevant modified Feynman vertices which affect the above mentioned low energy processes, and are also involved in the production and decay modes of the BSM particles leading to interesting phenomenological signatures at the LHC, FCC. We have concluded with a discussion on the impacts of these effective operators on the magnetic moments of charged fermions, LNV and LFV processes, and the theoretical constraints, like tree unitarity using four-point scalar interactions and vacuum stability, i.e., boundedness criteria of the scalar potential.

Our future plan is to embed the 2HDM- and MLRSM-EFT frameworks in FeynRules using refs. [268–272] to adjudge them in the light of collider and low energy experiments and precision observable tests.

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