Model-independent bounds on $R(J/\psi)$

Thomas D. Cohen,[a] **Henry Lamm**[a] **and Richard F. Lebed**[b]

[a]*Department of Physics, University of Maryland,*
4150 Campus Dr, College Park, MD 20740, U.S.A.

[b]*Department of Physics, Arizona State University,*
550 E Tyler Drive, Tempe, AZ 85287, U.S.A.

E-mail: cohen@umd.edu, hlamm@umd.edu, Richard.Lebed@asu.edu

ABSTRACT: We present a model-independent bound on $R(J/\psi) \equiv \mathcal{BR}(B_c^+ \to J/\psi\,\tau^+\nu_\tau)/\mathcal{BR}(B_c^+ \to J/\psi\,\mu^+\nu_\mu)$. This bound is constructed by constraining the form factors through a combination of dispersive relations, heavy-quark relations at zero-recoil, and the limited existing determinations from lattice QCD. The resulting 95% confidence-level bound, $0.20 \le R(J/\psi) \le 0.39$, agrees with the recent LHCb result at $1.3\,\sigma$, and rules out some previously suggested model form factors.

Contents

1 Introduction

Within the Standard Model, *lepton universality* is broken only by the Higgs interaction, but the discovery of neutrino masses implies that at least one, and potentially several, relevant forms of beyond-Standard Model modification exist. The ratios of semileptonic heavy-meson decay branching fractions to distinct lepton flavors represent a group of observables particularly sensitive to new physics, because the QCD dynamics of the heavy-meson decays decouples from the electroweak interaction at leading order:

$$\left|\mathcal{M}_{\bar{b}\to\bar{c}\,\ell^+\nu_\ell}\right|^2 = \frac{L_{\mu\nu}H^{\mu\nu}}{q^2 - M_W^2} + \mathcal{O}(\alpha, G_F)\,. \tag{1.1}$$

This expression implies that the ratios of semileptonic heavy-meson decay branching fractions can differ from unity at this level of precision only due to kinematic factors. Measurements from BaBar, Belle, and LHCb of the ratios $R(D^{(*)})$ for heavy-light meson decays $B \to D^{(*)}\ell\bar{\nu}$ with $\ell = \tau$ to those with $\ell = \mu$ or e (or their average) exhibit tension with theoretical predictions. The HFLAV averages [1] of the experimental results $R(D^*) = 0.306(13)(7)$ [2–10] and $R(D) = 0.407(39)(24)$ [2–4] represent a combined 3.8σ discrepancy [1] from the HFLAV-suggested Standard-Model value of $R(D^*) = 0.258(5)$ [1] obtained by an averaging [11–13] that utilizes experimental data, lattice QCD results, and heavy-quark effective theory, and from $R(D) = 0.300(8)$ [14], which is an average of lattice QCD results [15, 16], as well as a value $R(D) = 0.299(3)$ obtained by also including experimental data supplemented by heavy-quark effective theory [17]. In light of this tension, the LHCb Collaboration has measured the rates for the heavy-heavy semileptonic meson decays $B_c^+ \to J/\psi\,\ell^+\nu_\ell$ (figure 1) in the $\ell = \tau, \mu$ channels, finding $R(J/\psi) = 0.71(17)(18)$ [18].
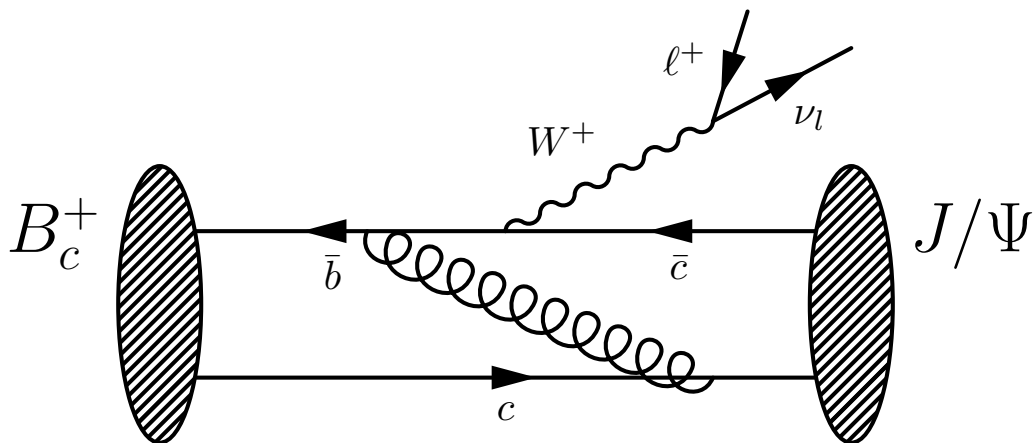
Figure 1. Schematic picture of the $B_c^+ \to J/\psi\,\ell^+\nu_\ell$ process.

At present, only model-dependent calculations of $R(J/\psi)$ exist (collected in table 1) [19–32]. Although most models' central values cluster in LHCb's quoted theory range of 0.25–0.28, one notes a wide spread in their estimated uncertainty. We take as a reasonable estimate of the model range $0 < R(J/\psi) < 0.48$, the union of the 95% confidence levels (CL) of the reported theoretical uncertainties, which in turn typically account only for parameter fitting. These results rely upon approximations such as nonrelativistic reduction, constituent quarks, or perturbative QCD to obtain transition form factors between the heavy-heavy B_c^+ and J/ψ mesons. Without a clear understanding of the systematic uncertainties these assumptions introduce, the reliability of these predictions is suspect.

In this paper, we present the first model-independent constraint, a 95% CL bound of $0.20 \le R(J/\psi) \le 0.39$ within the Standard Model, in which uncertainties are all quantifiable. In order to obtain this result, we begin in section 2 with a discussion of the $V-A$ structure of the Standard Model and the form factors. In section 3 we explain how heavy-quark spin symmetry can be applied at the zero-recoil point to relate the form factors, using the method of [20]. The initial lattice-QCD results of the HPQCD Collaboration [33, 34] for two of the transition form factors are discussed in section 4. The dispersive analysis framework utilized to constrain the form factors as functions of momentum transfer is presented in section 5. The results of our analysis, as well as future projections for the bound, appear in section 6, and we conclude in section 7.

2 Structure of $\langle J/\psi\,|(V-A)^\mu|B_c^+\rangle$

In the Standard Model, the factorization of eq. (1.1) into a leptonic and a hadronic tensor reduces the problem of calculating $R(J/\psi)$ to the computation of the hadronic matrix element $\langle J/\psi\,|(V-A)^\mu|B_c^+\rangle$. Using this factorization, the hadronic matrix element can be written in terms of four transition form factors. These form factors enter the matrix element in combination with the meson masses, $M \equiv M_{B_c^+}$ and $m \equiv M_{J/\psi}$, the corresponding meson momenta P^μ and p^μ, and the polarization ϵ^μ of the J/ψ. The form factors themselves depend only upon $q^2 = (P - p)^2$, the squared momentum transfer to the leptons. A

number of form-factor decompositions exist in the literature; one common set [35] used in lattice-QCD [33] and model calculations is given by $V(q^2)$, $A_i(q^2)$, $i = 0, 1, 2, 3$:

$$
\begin{aligned}
\langle J/\psi(p,\epsilon)|(V-A)^\mu|B_c^+(P)\rangle = {} & \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m}\epsilon_\nu^* p_\rho P_\sigma V\left(q^2\right) - (M+m)\epsilon^{*\mu}A_1\left(q^2\right) \\
& + \frac{\epsilon^* \cdot q}{M+m}(P+p)^\mu A_2\left(q^2\right) + 2m\frac{\epsilon^* \cdot q}{q^2}q^\mu A_3\left(q^2\right) \\
& - 2m\frac{\epsilon^* \cdot q}{q^2}q^\mu A_0\left(q^2\right) ,
\end{aligned}
\tag{2.1}
$$

where $q^\mu \equiv (P-p)^\mu$. While we have exhibited five form factors, only four are independent. In the physical set, $A_0(q^2)$ is defined as the unique form factor that couples to timelike virtual W polarizations ($\propto q^\mu$), while $A_3(q^2)$ is simply a convenient shorthand for a combination appearing in intermediate stages of calculations, and in fact satisfies

$$
A_3\left(q^2\right) = \frac{M+m}{2m}A_1\left(q^2\right) - \frac{M-m}{2m}A_2\left(q^2\right) .
\tag{2.2}
$$

Furthermore, the finiteness of eq. (2.1) as $q^2 \to 0$ requires $A_3(0) = A_0(0)$, which proves useful in constructing our bounds. In what follows, we also use the notation $t \equiv q^2$, and define two important kinematic points, $t_\pm = (M \pm m)^2$.

Using eq. (2.1) or an equivalent basis, model predictions include uncontrolled approximations for the form factors. Some models construct wave functions for the two mesons, while others attempt to compute a perturbative distribution amplitude at $q^2 \to 0$ and then extrapolate to larger values with some functional form. In addition, some models do not respect form-factor relations, such as the heavy-quark spin-symmetry relations discussed below. Due to these issues, the good agreement seen between the model predictions may more reflect the theoretical prejudice in modeling than a genuine estimate of the true Standard-Model value.

While this decomposition is useful for lattice QCD, it is not the best decomposition for the dispersive analysis. The second convention we use is the helicity basis, which exchanges the form factors V, A_i for g, f, $\mathcal{F}_1$, and $\mathcal{F}_2$.[1] They are related by

$$
\begin{aligned}
g &= \frac{2}{M+m}V , \\
f &= (M+m)A_1 , \\
\mathcal{F}_1 &= \frac{1}{m}\left[-\frac{2k^2t}{M+m}A_2 - \frac{1}{2}\left(t - M^2 + m^2\right)(M+m)A_1\right] , \\
\mathcal{F}_2 &= 2A_0 ,
\end{aligned}
\tag{2.3}
$$

where, in terms of the spatial momentum $\boldsymbol{p}$ of the J/ψ in the B_c^+ rest frame,

$$
k \equiv M\sqrt{\frac{\boldsymbol{p}^2}{t}} = \sqrt{\frac{(t_+ - t)(t_- - t)}{4t}} .
\tag{2.4}
$$

---

[1]Strictly speaking, $\mathcal{F}_{1,2}$ are helicity amplitudes (in conventional notation [36], proportional to $H_{0,t}$, respectively), while f, g are two linear combinations of them: $H_\pm(t) = f(t) \mp k\sqrt{t}g(t)$, where k is defined in eq. (2.4).

Model	R_{theory}	Year
CQM [19]	0.28	1998
QCDSR [20]	$0.25^{+0.09}_{-0.09}$	1999
RCQM [21]	0.26	2000
QCDSR [22]	0.25	2003
RCQM [23]	0.24	2006
NRQM [24]	$0.27^{+0.02}_{-0}$	2006
NRQCD [25]	$0.07^{+0.06}_{-0.04}$	2013
pQCD [26]	$0.29^{+0.09}_{-0.09}$	2013
pQCD [27]	$0.30^{+0.11}_{-0.08}$	2016
pQCD [28]	$0.29^{+0.07}_{-0.07}$	2017
CQM [29]	0.24	2017
pQCD [30]	$0.283^{+0.048}_{-0.048}$	2017
CQM [31]	$0.24^{+0.07}_{-0.07}$	2018
RCQM [32]	0.24	2018
Range	0–0.48	–

Table 1. Model predictions of $R(J/\psi)$ classified by method, which are abbreviated as: constituent quark model (CQM), relativistic quark model (RCQM), QCD sum rules (QCDSR), nonrelativistic quark model (NRQM), nonrelativistic QCD (NRQCD), and perturbative QCD calculations (pQCD).

Setting $t = t_-$, we see that $\mathcal{F}_1(t_-) = (M - m)f(t_-)$. In this decomposition, the constraint $A_3(0) = A_0(0)$ reads $\mathcal{F}_1(0) = \frac{1}{2}(M^2 - m^2)\mathcal{F}_2(0)$. The differential cross section for the semileptonic decay then reads

$$\frac{d\Gamma}{dt} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M^3} \frac{k}{t^{5/2}} \left(t - m_\ell^2\right)^2 \times \left\{ \left(2t + m_\ell^2\right)\left[2t|f|^2 + |\mathcal{F}_1|^2 + 2k^2 t^2|g|^2\right] + 3m_\ell^2 k^2 t|\mathcal{F}_2|^2 \right\}. \tag{2.5}$$

Inspecting eq. (2.5), one can see that in the light leptonic channels ($\ell = e, \mu$), the contribution to the $\mathcal{F}_2(t)$ can be neglected, while in the τ channel it cannot. As seen below, the uncertainty in our bound on $R(J/\psi)$ is dominated by the unknown form factor $\mathcal{F}_2(t)$.

3 Heavy-quark spin symmetry

Decays of heavy-light $Q\bar{q}$ systems possess enhanced symmetries in the heavy-quark limit because operators that distinguish between heavy quarks of different spin and flavor are suppressed by $1/m_Q$, and their matrix elements vanish when $m_Q \to \infty$. Consequently, all transition form factors $\langle Q'\bar{q}|\mathcal{O}|Q\bar{q}\rangle$ in this limit are proportional to a single, universal Isgur-Wise function $\xi(w)$ [37, 38], whose momentum-transfer argument is w, the dot product of the initial and final heavy-light hadron 4-velocities, $v^\mu \equiv p_M^\mu/M$ and $v'^\mu \equiv p_m^\mu/m$, respectively:

$$w \equiv v \cdot v' = \gamma_m = \frac{E_m}{m} = \frac{M^2 + m^2 - t}{2Mm} . \tag{3.1}$$

At the zero-recoil point $t = (M-m)^2$ or $w = 1$, the daughter hadron m is at rest with respect to the parent M. Indeed, one notes that w equals the Lorentz factor γ_m of m in the M rest frame. The maximum value of w corresponds to the minimum momentum transfer t through the virtual W to the lepton pair, which occurs when the leptons are created with minimal energy, $t = m_\ell^2$.

In heavy-light systems, the heavy-quark approximation corresponds to a light quark bound in a nearly static spin-independent color field. In the weak decay $Q \to Q'$ between two very heavy quark flavors, as the momentum transfer to the light quark $t \to 0$, q no longer changes states, and therefore the wave function of this light spectator quark remains unaffected. One thus concludes that $\xi(1) = 1$ at the zero-recoil (Isgur-Wise) point, yielding a absolute normalization for the form factors. These results are accurate up to corrections of $\mathcal{O}(\Lambda_{\rm QCD}/m_{Q'})$.

In the decay $B_c^+ \to J/\psi$, the spectator light quark is replaced by another heavy quark, c. This substitution results in a pair of related effects on the enhanced symmetries of the heavy-quark limit [39]. First, the difference between the heavy-quark kinetic energy operators produces energies no longer negligible compared to those of the spectator c, and this effect spoils the flavor symmetry in heavy-heavy systems. Furthermore, the spectator c receives a momentum transfer from the decay of $\bar{b} \to \bar{c}$ of the same order as the momentum imparted to the $\bar{c}$, so one cannot justify a normalization of the form factors at the zero-recoil point based purely upon symmetry.

While the heavy-flavor symmetry is lost, the separate spin symmetries of $\bar{b}$ and $\bar{c}$ quarks remain, with an additional spin symmetry from the heavy spectator c. Furthermore, the presence of the heavy c suggests a system that is closer to a nonrelativistic limit than heavy-light systems. In the $B_c^+ \to J/\psi$ semileptonic decays, one further finds that

$$
\begin{aligned}
w_{\max} &= w\left(t = m_\ell^2\right) = \frac{M^2 + m^2 - m_\ell^2}{2Mm} \approx 1.28\ (\mu),\ 1.20\ (\tau)\,, \\
w_{\min} &= w\left(t = (M-m)^2\right) = 1\,,
\end{aligned}
\tag{3.2}
$$

suggesting that an expansion about the zero-recoil point may still be reasonable. Together, the spin symmetries imply that the four form factors are related to a single, universal function h (Δ in ref. [39]), but only at the zero-recoil point, and no symmetry-based normalization for h can be derived [39].

Using the trace formalism of [40], in ref. [39] it was shown how to compute the relative normalization between the four $\bar{Q}q \to \bar{Q}'q$ form factors near the zero-recoil point [i.e., where the spatial momentum transfer to the spectator q is $\mathcal{O}(m_q)$]. Using these relations, h was derived for a color-Coulomb potential in ref. [39]. This approximation was improved in ref. [41], where a constituent quark-model calculation of $\mathcal{BR}(B_c^+ \to J/\psi\, \ell^+ \nu_\ell)$ for $\ell = e, \mu$ but not τ, was performed. The heavy-quark spin-symmetry relations of ref. [39] were generalized in [20] to account for a momentum transfer to the spectator quark occurring at leading order in NRQCD, specifically, to the case $v \neq v'$ but $w \to 1$. We reproduce here the relations of [20], where the form factors $g(w=1), \mathcal{F}_1(w=1), \mathcal{F}_2(w=1)$ are related to

$f(w = 1)$ by

$$
\begin{aligned}
g(w = 1) &= \frac{2\rho + (1 + \rho)\sigma}{4M^2 r\rho} f(w = 1) \,, \\
\mathcal{F}_1(w = 1) &= M(1 - r)f(w = 1) \,, \\
\mathcal{F}_2(w = 1) &= \frac{2(1 + r)\rho + (1 - r)(1 - \rho)\sigma}{4Mr\rho} f(w = 1),
\end{aligned}
\tag{3.3}
$$

where $r \equiv m/M$, $\rho \equiv m_{Q'}/m_Q$, and $\sigma \equiv m_q/m_Q$. These relations reproduce the standard Isgur-Wise result [37, 38, 42] when $\sigma = 0$. The relation between $\mathcal{F}_1(w = 1)$ and $f(w = 1)$ follows directly from the definition of eq. (2.3), independent of heavy-quark symmetries. Terms that break these relations should be $\mathcal{O}(m_c/m_b, \Lambda_{\text{QCD}}/m_c) \approx 30\%$, and we allow conservatively for up to 50% violations. The heavy-quark spin symmetry further relates the zero-recoil form factors of $B_c^+ \to J/\psi$ to those of $B_c^+ \to \eta_c$, which will be useful in the future to obtain further constraints.

In analogy with the heavy-light systems, we can enforce a further constraint from heavy-quark symmetries. The universal form factor h represents the overlap element of the initial and final states, and therefore should be maximized at $w = 1$. This statement is an assumption, but a very mild one: in the heavy-light system, the slope of the Isgur-Wise function is rigorously negative at $w = 1$ [43–45], and it would indeed be very surprising if the same did not hold for the form factors of heavy-heavy mesons, which are even more similar to idealized quark-model states.

4 Lattice QCD results

The state-of-the-art lattice QCD calculations for $B_c^+ \to J/\psi$ are limited to preliminary results from the HPQCD Collaboration for $V(q^2)$ at two q^2 values and $A_1(q^2)$ at three q^2 values [33, 34]. These results were obtained using 2+1+1 HISQ ensembles, in which the smallest lattice spacing is $a \approx 0.09$ fm, and the b quark is treated via NRQCD, and are reproduced in figure 2. For $q^2 = t_-, 0$ $A_1(q^2)$ has also been computed on coarser lattices and for lighter dynamical b-quark ensembles, in order to check the accuracy and assess the uncertainty of the $a \approx 0.09$ fm NRQCD results. At present, there are no lattice results for $A_0(q^2) = \frac{1}{2}\mathcal{F}_2(q^2)$ or $A_2(q^2)$. Below, we show that the most desirable piece of new information from the lattice is a computation of $A_0(0)$, which could cut our uncertainties in half.

5 Dispersive relations

In this work we derive constraints on the form factors of $B_c^+ \to J/\psi$ using analyticity and unitarity constraints on a particular two-point Green's function and a conformal parameterization in the manner implemented by Boyd, Grinstein, and Lebed (BGL) [46] for the decays of heavy-light hadrons to heavy-light or light-light hadrons. We utilize a slightly different set of free parameters to simplify the computation for our particular case of a heavy-heavy meson decaying to another heavy-heavy meson. Here we briefly sketch the necessary components, emphasizing where we differ from the literature.

To derive our constraints, one considers the two-point momentum-space Green's function $\Pi_J^{\mu\nu}$ of a vectorlike quark current, $J^\mu \equiv \bar{Q}\Gamma^\mu Q'$. $\Pi_J^{\mu\nu}$ can be decomposed in different ways [42, 47–50]; in this work we choose to separate it into spin-1 (Π_J^T) and spin-0 (Π_J^L) pieces *à la* [42] via

$$
\begin{aligned}
\Pi_J^{\mu\nu}(q) &\equiv i \int d^4x\, e^{iqx} \left\langle 0 \left| T J^\mu(x) J^{\dagger\nu}(0) \right| 0 \right\rangle \\
&= \frac{1}{q^2}\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi_J^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_J^L\left(q^2\right) .
\end{aligned}
\tag{5.1}
$$

From perturbative QCD (pQCD), the functions $\Pi_J^{L,T}$ are known to contain first- and second-order divergences, respectively, and must undergo subtractions in order to be rendered finite. The finite dispersion relations are:

$$
\begin{aligned}
\chi_J^L\left(q^2\right) &\equiv \frac{\partial \Pi_J^L}{\partial q^2} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^L(t)}{(t-q^2)^2}\,, \\
\chi_J^T\left(q^2\right) &\equiv \frac{1}{2}\frac{\partial^2 \Pi_J^T}{\partial\left(q^2\right)^2} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^T(t)}{(t-q^2)^3}\,.
\end{aligned}
\tag{5.2}
$$

The freedom to chose a value of q^2 can be leveraged to compute $\chi(q^2)$ reliably in pQCD, far in q^2 from where the two-point function receives nonperturbative contributions from effects such as bound states and resonances. The formal condition on q^2 to be in the perturbative regime is

$$
(m_Q + m_{Q'})\Lambda_{\mathrm{QCD}} \ll (m_Q + m_{Q'})^2 - q^2\,,
\tag{5.3}
$$

which, for $Q, Q' = c, b$, $q^2 = 0$ is clearly sufficient. Existing calculations of two-loop pQCD $\chi(q^2 = 0)$ modified by non-perturbative vacuum contributions [51–55] used in ref. [42] can be applied here. An example of the state of the art in this regard (although slightly different from the approach used here) appears in ref. [17].

The spectral functions $\mathrm{Im}\,\Pi_J$ can be decomposed into a sum over the complete set of states X that can couple the current J^μ to the vacuum:

$$
\mathrm{Im}\,\Pi_J^{T,L}\left(q^2\right) = \frac{1}{2}\sum_X (2\pi)^4 \delta^4(q - p_X)\left|\langle 0\left|J\right|X\rangle\right|^2\,.
\tag{5.4}
$$

Each term in the sum is semipositive definite, thereby producing a strict inequality for each X in eqs. (5.2). These inequalities can be made stronger by including multiple X at once, as discussed in refs. [12, 13, 42]. For X we include only below-threshold B_c^+ poles and a single two-body channel, $B_c^+ + J/\psi$, implying that our results provide very conservative bounds.

In contrast to many prior dispersive analyses, $B_c^+ \to J/\psi$, like the $\Lambda_b \to \Lambda_c$ process studied in ref. [50], does not give the lightest two-body threshold with the correct quantum numbers; these lighter thresholds must be taken into consideration. Depending upon the quantum numbers indicated by J, the first physically prominent two-body production threshold in t occurs at $B^{(*)} + D$ (see table 2). In early literature such as [50], the branch cut starting at the threshold for the process of interest was the one used in the dispersive analysis, while the effect of the cut from the lower threshold up to this threshold was

modeled and argued to amount to a slight loosening of the unitarity bound given below by eq. (5.11). Here, however, we represent the analytic features more faithfully by using the lower threshold directly. With this fact in mind, we define a new variable $t_{\rm bd} \equiv (M_{B^{(*)}} + M_D)^2$ that corresponds to the first branch point in a given two-point function, while the $B_c^+ + J/\psi$ branch point occurs at $t_+ > t_{\rm bd}$.

With these variables, one maps the complex t plane to the unit disk in a variable z (with the two sides of the branch cut forming the unit circle C) using the conformal variable transformation

$$z(t; t_0) \equiv \frac{\sqrt{t_* - t} - \sqrt{t_* - t_0}}{\sqrt{t_* - t} + \sqrt{t_* - t_0}}, \tag{5.5}$$

where t_* is the branch point around which one deforms the contour, and t_0 is a free parameter used to improve the convergence of functions at small z. In this mapping, z is real for $t \leq t_*$ and a pure phase for $t \geq t_*$.

Prior work that computed the form factors between baryons whose threshold was above that of the lightest pair in that channel (*i.e.*, $\Lambda_b \to \Lambda_c$, $\Lambda_b \to p$) took $t_* = t_+$ [42, 49], which introduces into the region $|z| < 1$ a subthreshold branch cut, meaning that the form factors have complex nonanalyticities that cannot trivially be removed. To avoid this issue, we instead set $t_* = t_{bd}$, which is possible because we are only interested in the semileptonic decay region, $m_\ell^2 \leq t \leq t_-$, which is always smaller than t_{bd}. This choice ensures that the only nonanalytic features within the unit circle $|z| = 1$ are simple poles corresponding to single particles $B_c^{(*)+}$, which can be removed by *Blaschke factors* described below. The need to avoid branch cuts but not poles from $|z| < 1$ derives from the unique feature of the Blaschke factors, which can remove each pole given only its location (i.e., mass), independent of its residue.[2] In contrast, correctly accounting for a branch cut requires knowledge of both the location of the branch point and the function along the cut.

To remove these subthreshold poles, one multiplies by $z(t; t_s)$ [using the definition of eq. (5.5)], a Blaschke factor, which eliminates a simple pole $t = t_s$. Using this formalism, the bound on each form factor $F_i(t)$ can be written as

$$\frac{1}{\pi} \sum_i \int_{t_{\rm bd}}^\infty dt \left| \frac{dz(t; t_0)}{dt} \right| |P_i(t)\phi_i(t; t_0)F_i(t)|^2 \leq 1. \tag{5.6}$$

The function $P_i(t)$ in eq. (5.6) is a product of Blaschke factors $z(t; t_p)$ that remove *dynamical* singularities due to the presence of subthreshold resonant poles. Masses corresponding to the poles that must be removed in $B_c^+ \to J/\psi$ are found in table 2, organized by the channel to which each one contributes. These masses have either been measured by LHCb [58, 59] or derived from model calculations [60], with uncertainties that are negligible for our purposes.

The weight function $\phi_i(t; t_0)$ is called an *outer function* in complex analysis, and is given by

$$\phi_i(t; t_0) = \tilde{P}_i(t) \left[\frac{W_i(t)}{|dz(t; t_0)/dt| \, \chi^j(q^2)(t - q^2)^{n_j}} \right]^{1/2}, \tag{5.7}$$

where $j = T, L$ (for which $n_j = 3, 2$, respectively), the function $\tilde{P}_i(t)$ is a product of factors $z(t; t_s)$ or $\sqrt{z(t; t_s)}$ designed to remove *kinematical* singularities at points $t = t_s < t_{\rm bc}$ from

---

[2]The analytic significance of Blaschke factors for heavy-hadron form factors was first noted in refs. [56, 57].

Type	J^P	Lowest Pair	M [GeV]
Vector	1^-	BD	6.337, 6.899, 7.012
Axial	1^+	B^*D	6.730, 6.736, 7.135, 7.142
Scalar	0^+	BD	6.700, 7.108
Pseudoscalar	0^-	B^*D	**6.2749(8)**, **6.842(9)**

Table 2. Lowest B_c^+ states needed for Blaschke factors with $t < t_{\mathrm{bc}}$ (whose relevant two-body threshold is indicated by "Lowest Pair") for the J^P channels of interest. Bold values indicate masses measured by LHCb.

F_i	K	χ	a	b	c
f	24	$\chi^T(-u)$	1	1	1
$\mathcal{F}_1$	48	$\chi^T(-u)$	1	1	2
g	96	$\chi^T(+u)$	3	3	1
$\mathcal{F}_2$	64	$\chi^L(-u)$	3	3	1

Table 3. Inputs entering $\phi_i(t; t_0)$ in eq. (5.8) for the meson form factors F_i.

the other factors in eq. (5.6), and $W_i(t)$ is computable weight function depending upon the particular form factor F_i. The outer function can be reexpressed in a general form for any particular F_i as

$$\phi_i(t; t_0) = \sqrt{\frac{n_I}{K\pi\chi}} \left(\frac{t_{\mathrm{bd}} - t}{t_{\mathrm{bd}} - t_0} \right)^{\frac{1}{4}} \left(\sqrt{t_{\mathrm{bd}} - t} + \sqrt{t_{\mathrm{bd}} - t_0} \right) (t_{\mathrm{bc}} - t)^{\frac{a}{4}}$$
$$\times \left(\sqrt{t_{\mathrm{bd}} - t} + \sqrt{t_{\mathrm{bd}} - t_-} \right)^{\frac{b}{2}} \left(\sqrt{t_{\mathrm{bd}} - t} + \sqrt{t_{\mathrm{bd}}} \right)^{-(c+3)}, \tag{5.8}$$

where n_I is an isospin Clebsch-Gordan factor, which is 1 for $B_c^+ \to J/\psi$. The remaining factors are found in table 3. Transforming the dispersion-relation inequality based upon eq. (5.4) into z-space, eq. (5.6) becomes

$$\frac{1}{2\pi i} \sum_i \oint_C \frac{dz}{z} |\phi_i(z) P_i(z) F_i(z)|^2 \le 1, \tag{5.9}$$

which, upon dividing out the non-analytic terms, allows the expansion in z corresponding to an analytic function:

$$F_i(t) = \frac{1}{|P_i(t)| \phi_i(t; t_0)} \sum_{n=0}^{\infty} a_{in} z(t; t_0)^n. \tag{5.10}$$

Inserting this form into eq. (5.9), one finds that the bound can be compactly written as a constraint on the Taylor series coefficients:

$$\sum_{i; n=0}^{\infty} a_{in}^2 \le 1. \tag{5.11}$$

All possible functional dependences of the form factor $F_i(t)$ consistent with eqs. (5.2) are now incorporated into the coefficients $a_{\rm in}$.

It is useful to introduce a number of dimensionless parameters that are functions of the meson masses:

$$r \equiv \frac{m}{M}, \qquad\qquad\qquad \delta \equiv \frac{m_\ell}{M},$$
$$\beta \equiv \frac{M_{B^{(*)}}}{M}, \qquad\qquad\qquad \Delta \equiv \frac{M_D}{M},$$
$$\kappa \equiv (\beta + \Delta)^2 - (1 - r)^2,$$
$$\lambda \equiv (\beta + \Delta)^2 - \delta^2, \tag{5.12}$$

and a parameter N related to t_0 in eq. (5.5) by

$$N \equiv \frac{t_{\rm bd} - t_0}{t_{\rm bd} - t_-} \,. \tag{5.13}$$

It is straightforward to compute the kinematical range for the semileptonic process given in terms of z:

$$z_{\rm max} = \frac{\sqrt{\lambda} - \sqrt{N\kappa}}{\sqrt{\lambda} + \sqrt{N\kappa}} \,,$$
$$z_{\rm min} = -\left(\frac{\sqrt{N} - 1}{\sqrt{N} + 1} \right) , \tag{5.14}$$

The minimal (optimized) truncation error is achieved when $z_{\rm min} = -z_{\rm max}$, which occurs when

$$N_{\rm opt} = \sqrt{\frac{\lambda}{\kappa}} \,. \tag{5.15}$$

Evaluating at $N = N_{\rm opt}$, one finds

$$z_{\rm max} = -z_{\rm min} = \frac{\lambda^{1/4} - \kappa^{1/4}}{\lambda^{1/4} + \kappa^{1/4}} \,, \tag{5.16}$$

From these expressions, we find that the semileptonic decays have $z_{{\rm max},\tau} \approx 0.019$ and $z_{{\rm max},\mu} \approx 0.027$, where each has a 1.5% variation, depending upon whether the BD or B^*D threshold is the lowest branch point, t_{bd}.

In the limit $t_{\rm bd} \to t_+$, one obtains $\Delta \to r$, $\beta \to 1$, $\kappa \to 4r$, and recovers the expressions in ref. [46].

6 Results

Before presenting our bound on $R(J/\psi)$, we summarize the constraints the form factors $g, f, \mathcal{F}_1, \mathcal{F}_2$ are required to satisfy:

- The coefficients a_n of each form factor are constrained by $\sum_n a_n^2 \leq 1$ [eq. (5.11)], in particular, for the cases $n = 1, 2$ investigated here.

- Using eq. (3.3), the values $g(t_-)$ and $\mathcal{F}_2(t_-)$ are related to the value of $f(t_-)$, which in turn is computed from lattice QCD, to within 50%.

- All form factors (which are defined to have the same sign convention as the Isgur-Wise function) are assumed maximal at the zero-recoil point $t = t_-$ since the universal form factor h represents an overlap matrix element between initial and final states. Although this condition is not required by the model-independent parametrization eq. (5.10), it appears to be supported by all the models cited in table 1 for which functional expressions of form factors are provided. We find this condition to be suitably implemented via the constraints $F_i(t_-) \geq F_i(0)$ and $\frac{dF_i}{dt}\big|_{t_-} \geq 0$, where F_i represents any of the form factors.

- The relation $\mathcal{F}_1(t_-) = M(1 - r)f(t_-)$ [above eq. (2.5)] is exact.

- $\mathcal{F}_1(0) = \frac{1}{2}M^2(1 - r^2)\mathcal{F}_2(0)$ [above eq. (2.5)] follows from the condition $A_3(0) = A_0(0)$.

Imposing these constraints, we perform our fit in two steps, reflecting the difference in information between the two form factors (V, A_1) for which lattice values have been computed, and the two (A_0, A_2) without.

In the first step, random Gaussian-distributed points are sampled for the form factors g and f [equivalently, by eq. (2.3), V and A_1] whose mean gives the HPQCD results. The combined uncertainties are given by the quadrature sum of the reported uncertainty δ_{lat} of the form-factor points and an additional systematic uncertainty, f_{lat} (expressed as a percentage of the form-factor point value) that we use to estimate the uncomputed lattice uncertainties (i.e., finite-volume corrections, quark-mass dependence, discretization errors). f_{lat} is taken to be 1, 5, or 20% of the value of the form factor from the lattice. For our final result, we suggest using $f_{\text{lat}} = 20\%$, while the other two values are helpful for understanding future prospects with improved lattice data. Using these sample points, we compute lines of best fit, from which we produce the coefficients a_n. The resulting bands of allowed form factors are shown for $f_{\text{lat}} = 20\%$ in figure 2, alongside the HPQCD results.

In the second step, we compute $\mathcal{F}_1$ and $\mathcal{F}_2$ (which include $A_{0,2}$), for which no lattice information exists. One could adopt the tactic of randomly generating points with some prior distribution, which, once accounting for the constraints, could be used to suggest a mean value of $R(J/\psi)$ with some prior-dependent uncertainty. We instead opt to remove this possible dependence by obtaining the numerical maximum and minimum $R(J/\psi)$ values, subject to the computed f, g values and the constraints listed above. In this way, the only uncertainties included are those from the lattice-QCD results and the violations of the heavy-quark spin-symmetry relations. The resulting bands of form factors for $\mathcal{F}_1$ and $\mathcal{F}_2$ that produce the minimum and maximum values of $R(J/\psi)$ subject to the constraints are plotted in figure 3.

Having computed all the form factors, we present the 95% CL ranges for $R(J/\psi)$ as a function of the truncation power $n = 1, 2$ in the dispersive analysis coefficients of eq. (5.10) and the $1, 5, 20\%$ systematic uncertainty f_{lat} associated with the lattice data. The full results are presented in table 4. The bound on $R(J/\psi)$ appears relatively insensitive to

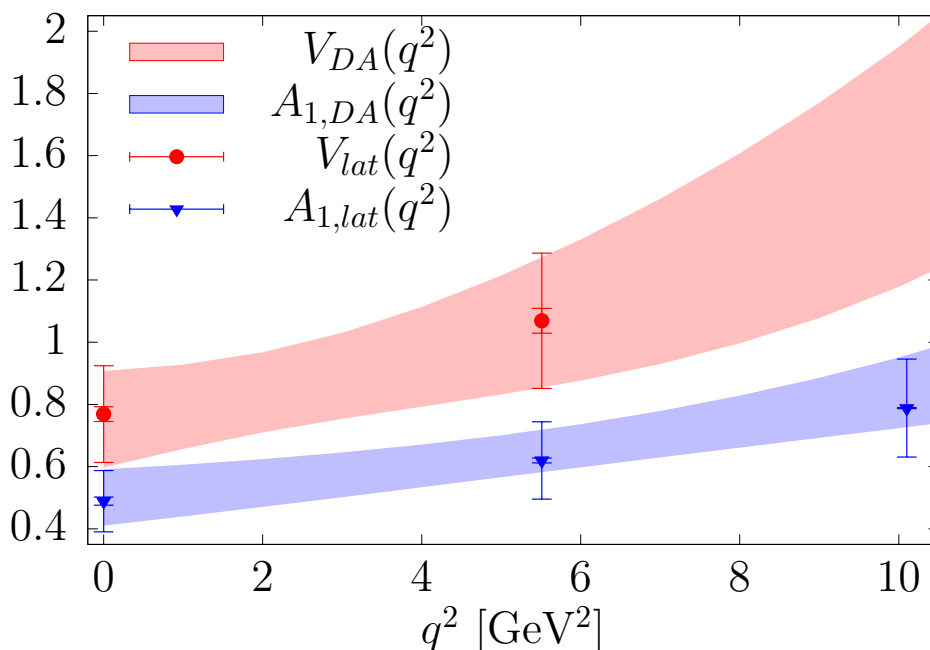**Figure 2**. $B_c^+ \to J/\psi$ form factors $V(q^2)$ (red circles) and $A_1(q^2)$ (blue triangles) from the HPQCD Collaboration [33, 34]. The interior bars represent the statistical uncertainty quoted by HPQCD. The exterior bars represent the result of including our $f_{\text{lat}} = 20\%$ systematic uncertainty. The colored bands DA (dispersive analysis) represent our one-standard-deviation (1σ) best-fit region.

f_{lat}	$n = 1$	$n = 2$
1	[0.21, 0.33]	[0.20, 0.35]
5	[0.20, 0.33]	[0.20, 0.35]
20	[0.20, 0.36]	[0.20, 0.39]

Table 4. 95% CL upper and lower bounds on $R_{J/\psi}$ as a function of the truncation power n of coefficients included from eq. (5.10) and the systematic lattice uncertainty f_{lat}.

increasing the number of free parameters a_n, and only mildly dependent upon f_{lat}. One might be concerned that increasing n could dramatically change these results, but we note that the typical value of $\sum_n a_n^2$ for $n = 1$ is $\mathcal{O}(10^{-2})$, while for $n = 2$ we find $a_2 \approx 1$. While the dispersive constraint is saturated in the $n = 2$ case, the bound on $R(J/\psi)$ is only enlarged by 5%. However, the saturation of any particular a_n is not necessary to find the effect of higher a_n to be negligible. Since higher-order terms are suppressed by $z_{\text{max}} \approx 0.03$, in order for these terms to contribute strongly, one must have $a_{n+1} z_{\text{max}} \gtrsim a_n$. Such an a_{n+1} value would either have to violate saturation $\sum_n a_n^2 \leq 1$ once the lower-order terms a_n are fixed, or else it would change the numerical results very little.

In figure 4 we plot the previous model-dependent values of $R(J/\psi)$ alongside the LHCb result and our 95% CL bound of $0.20 \leq R(J/\psi) \leq 0.39$, as a function of publication date. One can see that, while many of the previous model results lie within our 95% CL band,
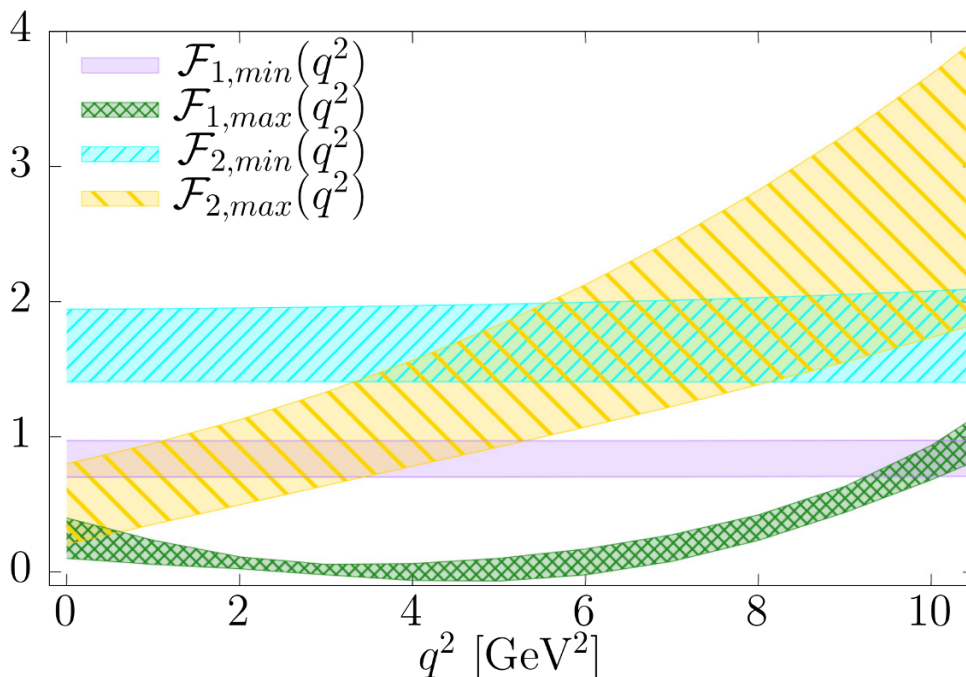
Figure 3. Dimensionless form factors $\mathcal{F}_1/[\frac{1}{2}M^2(1-r^2)]$ and $\mathcal{F}_2$ that provide the maximum and minimum $R(J/\psi)$ values consistent with lattice and heavy-quark spin-symmetry constraints. The colored bands represent the 1σ range due to the uncertainty associated with the HPQCD results, combined with an $f_{\text{lat}} = 20\%$ systematic uncertainty.

some are either partially or entirely excluded. The anomalously low NRQCD result of ref. [25] is in severe disagreement with our bound (the small $R(J/\psi)$ of ref. [25] can be attributed to a larger-than-typical muonic branching ratio), while all the other models that have included 1σ theory uncertainty estimates are seen to remain compatible with the dispersive bounds obtained from a fairly sparse set of lattice results. Our 95% CL band should be viewed as a model-independent upper limit (subject to the assumptions listed above) for the largest theory uncertainty any model can allow and remain consistent with analyticity, unitarity, and existing lattice "data." With better lattice results — and/or actual experimental measurements of the form factors at any values of q^2 — the allowed parameter space for any given model will become severely curtailed.

Considering that the fits to the lattice data already constrain the form factors g and f well, we can ask what new piece of information would most improve our bounds. Since $\mathcal{F}_2$ essentially affects only the τ channel, it is obvious that this form factor is the one most important to reducing the range in $R(J/\psi)$. By inspection of both $\mathcal{F}_1$ and $\mathcal{F}_2$ in figure 3, we find that the q^2 dependence of the form factors generating the maximum and minimum $R(J/\psi)$ values are quite different in shape. The minimum-$R(J/\psi)$ ones clearly prefer flat form factors, while the maximum ones are nearly zero at $q^2 = 0$ and rise to their maximum allowed values at $q^2 = t_-$. This dependence suggests that obtaining a value of $\mathcal{F}_2(q^2=0)$ would greatly improve the lower bound, while $\mathcal{F}_2(t_-)$ would restrict the upper bound. With the LHCb result already lying slightly above our bound, it would be most incisive to reduce
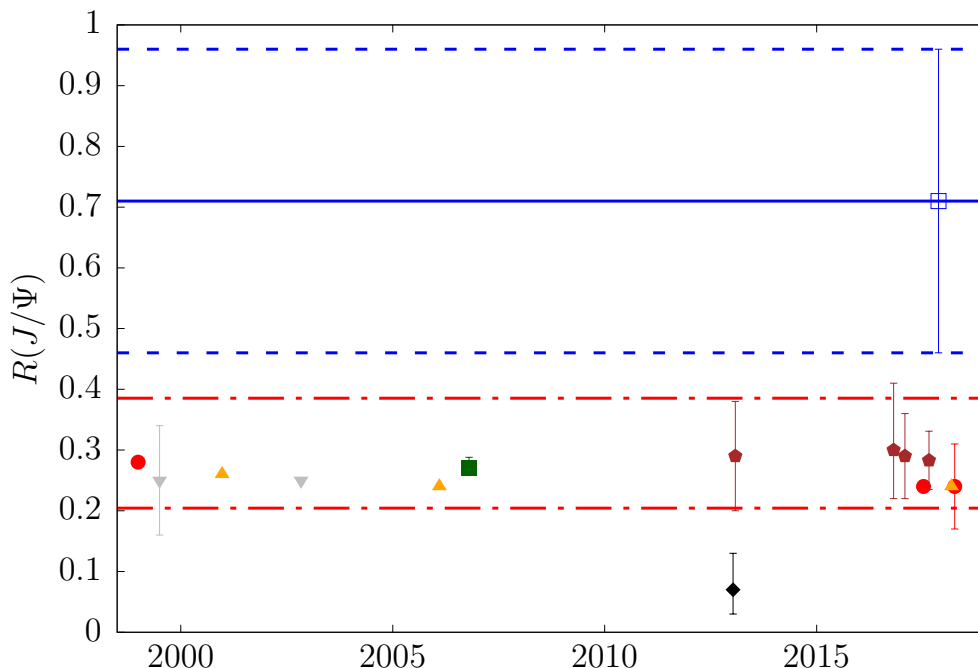
Figure 4. $R(J/\psi)$ from the LHCb experiment (blue open square, 1σ uncertainty denoted by blue dashed lines), our bound (red dash-dotted lines), and models (points colored by model type), as listed in table 1.

our upper bound. This zero-recoil form factor is directly related by $\mathcal{F}_2(t_-) = 2A_0(t_-)$ to a traditional lattice form factor, and therefore should be possible to compute.

To investigate the possible effect of this new information, we consider a synthetic point $\mathcal{F}_2(t_-) = 2(1 \pm f_{\text{lat}})$. This particular value is chosen because it lies near the average of the minimum and maximum preferred values, and is similar to the values suggested in models. Taking $f_{\text{lat}} = 20\%$, we find that the bound could tightened to $[0.20, 0.35]$. If this point and the existing 5 lattice points reached $f_{\text{lat}} = 1\%$, one could anticipate a range of $[0.21, 0.32]$. So, an additional lattice point at $\mathcal{F}_2(t_-)$ could improve the bound by the same amount as reducing the uncertainty f_{lat} from 20% to 1% (as seen in table 4), but with substantially less computing resources.

7 Discussion and conclusion

In contrast to the model-dependent previous works, we have presented a model-independent bound on $R(J/\psi)$, finding it constrained to lie in the range $0.20 \leq R(J/\psi) \leq 0.39$ at the 95% CL. At this level, we find that the LHCb result is consistent with the Standard Model at $1.3\,\sigma$. The near-term outlook for a higher-statistics LHCb measurement, coupled with new lattice results, promises to reduce the uncertainty on the experimental and theoretical values dramatically.

Even without a lattice QCD calculation of the $\mathcal{F}_2$ form factor, additional potential areas of improvement can be investigated. Experience in the heavy-light sector and the

fact that the $R(J/\psi)$ bounds require saturating $\sum a_n^2 = 1$ suggest that including multiple states appearing in the dispersion relation can provide complementary information to help constrain the form factors further, and in this case one can additionally include the lattice results for $B \to D^{(*)}$ [15, 16, 61–64] and $\Lambda_b \to \Lambda_c$ [65].

Acknowledgments

References

[1] HFLAV collaboration, Y. Amhis et al., *Averages of b-hadron, c-hadron and τ-lepton properties as of summer 2016*, *Eur. Phys. J.* **C 77** (2017) 895 [arXiv:1612.07233] [INSPIRE], updated results and plots available at https://hflav.web.cern.ch.

[2] BaBar collaboration, J.P. Lees et al., *Evidence for an excess of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ decays*, *Phys. Rev. Lett.* **109** (2012) 101802 [arXiv:1205.5442] [INSPIRE].

[3] BaBar collaboration, J.P. Lees et al., *Measurement of an Excess of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ Decays and Implications for Charged Higgs Bosons*, *Phys. Rev.* **D 88** (2013) 072012 [arXiv:1303.0571] [INSPIRE].

[4] Belle collaboration, M. Huschle et al., *Measurement of the branching ratio of $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ relative to $\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell$ decays with hadronic tagging at Belle*, *Phys. Rev.* **D 92** (2015) 072014 [arXiv:1507.03233] [INSPIRE].

[5] Belle collaboration, Y. Sato et al., *Measurement of the branching ratio of $\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}_\tau$ relative to $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ decays with a semileptonic tagging method*, *Phys. Rev.* **D 94** (2016) 072007 [arXiv:1607.07923] [INSPIRE].

[6] LHCb collaboration, *Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \to D^{*+}\mu^-\bar{\nu}_\mu)$*, *Phys. Rev. Lett.* **115** (2015) 111803 [arXiv:1506.08614] [INSPIRE] [*Erratum ibid.* **115** (2015) 159901].

[7] Belle collaboration, S. Hirose et al., *Measurement of the τ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$*, *Phys. Rev. Lett.* **118** (2017) 211801 [arXiv:1612.00529] [INSPIRE].

[8] LHCb collaboration, *Measurement of the ratio of the $B^0 \to D^{*-}\tau^+\nu_\tau$ and $B^0 \to D^{*-}\mu^+\nu_\mu$ branching fractions using three-prong τ-lepton decays*, *Phys. Rev. Lett.* **120** (2018) 171802 [arXiv:1708.08856] [INSPIRE].

[9] LHCb collaboration, *Test of Lepton Flavor Universality by the measurement of the $B^0 \to D^{*-}\tau^+\nu_\tau$ branching fraction using three-prong τ decays*, *Phys. Rev.* **D 97** (2018) 072013 [arXiv:1711.02505] [INSPIRE].

[10] Belle collaboration, S. Hirose et al., *Measurement of the τ lepton polarization and $R(D^*)$ in the decay $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$ with one-prong hadronic τ decays at Belle*, *Phys. Rev.* **D 97** (2018) 012004 [arXiv:1709.00129] [INSPIRE].

[11] F.U. Bernlochner, Z. Ligeti, M. Papucci and D.J. Robinson, *Combined analysis of semileptonic B decays to D and D^*: $R(D^{(*)})$, $|V_{cb}|$ and new physics*, *Phys. Rev.* **D 95** (2017) 115008 [arXiv:1703.05330] [INSPIRE] [*Erratum ibid.* **D 97** (2018) 059902].

[12] D. Bigi, P. Gambino and S. Schacht, *$R(D^*)$, $|V_{cb}|$ and the Heavy Quark Symmetry relations between form factors*, *JHEP* **11** (2017) 061 [arXiv:1707.09509] [INSPIRE].

[13] S. Jaiswal, S. Nandi and S.K. Patra, *Extraction of $|V_{cb}|$ from $B \to D^{(*)}\ell\nu_\ell$ and the Standard Model predictions of $R(D^{(*)})$*, *JHEP* **12** (2017) 060 [arXiv:1707.09977] [INSPIRE].

[14] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, *Eur. Phys. J.* **C 77** (2017) 112 [arXiv:1607.00299] [INSPIRE].

[15] MILC collaboration, J.A. Bailey et al., *$B \to D\ell\nu$ form factors at nonzero recoil and $-V_{cb}-$ from 2+1-flavor lattice QCD*, *Phys. Rev.* **D 92** (2015) 034506 [arXiv:1503.07237] [INSPIRE].

[16] HPQCD collaboration, H. Na, C.M. Bouchard, G.P. Lepage, C. Monahan and J. Shigemitsu, *$B \to Dl\nu$ form factors at nonzero recoil and extraction of $|V_{cb}|$*, *Phys. Rev.* **D 92** (2015) 054510 [arXiv:1505.03925] [INSPIRE] [*Erratum ibid.* **D 93** (2016) 119906].

[17] D. Bigi and P. Gambino, *Revisiting $B \to D\ell\nu$*, *Phys. Rev.* **D 94** (2016) 094008 [arXiv:1606.08030] [INSPIRE].

[18] LHCb collaboration, *Measurement of the ratio of branching fractions $\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)/\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)$*, *Phys. Rev. Lett.* **120** (2018) 121801 [arXiv:1711.05623] [INSPIRE].

[19] A. Yu. Anisimov, I.M. Narodetsky, C. Semay and B. Silvestre-Brac, *The B_c meson lifetime in the light front constituent quark model*, *Phys. Lett.* **B 452** (1999) 129 [hep-ph/9812514] [INSPIRE].

[20] V.V. Kiselev, A.K. Likhoded and A.I. Onishchenko, *Semileptonic B_c meson decays in sum rules of QCD and NRQCD*, *Nucl. Phys.* **B 569** (2000) 473 [hep-ph/9905359] [INSPIRE].

[21] M.A. Ivanov, J.G. Korner and P. Santorelli, *The Semileptonic decays of the B_c meson*, *Phys. Rev.* **D 63** (2001) 074010 [hep-ph/0007169] [INSPIRE].

[22] V.V. Kiselev, *Exclusive decays and lifetime of B_c meson in QCD sum rules*, hep-ph/0211021 [INSPIRE].

[23] M.A. Ivanov, J.G. Korner and P. Santorelli, *Exclusive semileptonic and nonleptonic decays of the B_c meson*, *Phys. Rev.* **D 73** (2006) 054024 [hep-ph/0602050] [INSPIRE].

[24] E. Hernandez, J. Nieves and J.M. Verde-Velasco, *Study of exclusive semileptonic and non-leptonic decays of B_c - in a nonrelativistic quark model*, *Phys. Rev.* **D 74** (2006) 074008 [hep-ph/0607150] [INSPIRE].

[25] C.-F. Qiao and R.-L. Zhu, *Estimation of semileptonic decays of B_c meson to S-wave charmonia with nonrelativistic QCD*, *Phys. Rev.* **D 87** (2013) 014009 [arXiv:1208.5916] [INSPIRE].

[26] W.-F. Wang, Y.-Y. Fan and Z.-J. Xiao, *Semileptonic decays $B_c \to (\eta_c, J/\Psi)l\nu$ in the perturbative QCD approach*, *Chin. Phys.* **C 37** (2013) 093102 [arXiv:1212.5903] [INSPIRE].

[27] Z. Rui, H. Li, G.-x. Wang and Y. Xiao, *Semileptonic decays of B_c meson to S-wave charmonium states in the perturbative QCD approach*, *Eur. Phys. J.* **C 76** (2016) 564 [arXiv:1602.08918] [INSPIRE].

[28] R. Dutta and A. Bhol, *$B_c \to (J/\psi, \eta_c)\tau\nu$ semileptonic decays within the standard model and beyond*, *Phys. Rev.* **D 96** (2017) 076001 [arXiv:1701.08598] [INSPIRE].

[29] A. Liptaj, S. Dubnicka, A.Z. Dubnickova, M.A. Ivanov and A. Issadykov, *Selected Decays of Heavy Mesons in Covariant Confined Quark Model: Semileptonic and Nonleptonic Decays of B_c Meson*, `PoS(EPS-HEP2017)667`.

[30] R. Watanabe, *New Physics effect on $B_c \to J/\psi \tau \bar{\nu}$ in relation to the $R_{D^{(*)}}$ anomaly*, *Phys. Lett.* **B 776** (2018) 5 [`arXiv:1709.08644`] [INSPIRE].

[31] A. Issadykov and M.A. Ivanov, *The decays $B_c \to J/\psi + \bar{\ell}\nu_\ell$ and $B_c \to J/\psi + \pi(K)$ in covariant confined quark model*, *Phys. Lett.* **B 783** (2018) 178 [`arXiv:1804.00472`] [INSPIRE].

[32] C.-T. Tran, M.A. Ivanov, J.G. Körner and P. Santorelli, *Implications of new physics in the decays $B_c \to (J/\psi, \eta_c)\tau\nu$*, *Phys. Rev.* **D 97** (2018) 054014 [`arXiv:1801.06927`] [INSPIRE].

[33] HPQCD collaboration, B. Colquhoun, C. Davies, J. Koponen, A. Lytle and C. McNeile, *B_c decays from highly improved staggered quarks and NRQCD*, `PoS(LATTICE2016)281` [`arXiv:1611.01987`] [INSPIRE].

[34] A. Lytle, private communication.

[35] M. Wirbel, B. Stech and M. Bauer, *Exclusive Semileptonic Decays of Heavy Mesons*, *Z. Phys.* **C 29** (1985) 637 [INSPIRE].

[36] J.D. Richman and P.R. Burchat, *Leptonic and semileptonic decays of charm and bottom hadrons*, *Rev. Mod. Phys.* **67** (1995) 893 [`hep-ph/9508250`] [INSPIRE].

[37] N. Isgur and M.B. Wise, *Weak Decays of Heavy Mesons in the Static Quark Approximation*, *Phys. Lett.* **B 232** (1989) 113 [INSPIRE].

[38] N. Isgur and M.B. Wise, *Weak transition form-factors between heavy mesons*, *Phys. Lett.* **B 237** (1990) 527 [INSPIRE].

[39] E.E. Jenkins, M.E. Luke, A.V. Manohar and M.J. Savage, *Semileptonic B(c) decay and heavy quark spin symmetry*, *Nucl. Phys.* **B 390** (1993) 463 [`hep-ph/9204238`] [INSPIRE].

[40] A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, *Heavy Meson Form-factors From QCD*, *Nucl. Phys.* **B 343** (1990) 1 [INSPIRE].

[41] P. Colangelo and F. De Fazio, *Using heavy quark spin symmetry in semileptonic B_c decays*, *Phys. Rev.* **D 61** (2000) 034012 [`hep-ph/9909423`] [INSPIRE].

[42] C.G. Boyd, B. Grinstein and R.F. Lebed, *Precision corrections to dispersive bounds on form-factors*, *Phys. Rev.* **D 56** (1997) 6895 [`hep-ph/9705252`] [INSPIRE].

[43] J.D. Bjorken, *New Symmetries in Heavy Flavor Physics*, proceedings of the *4th Les Rencontres de Physique de la Vallee d'Aoste: Results and Perspectives in Particle Physics*, La Thuile, France, 18–24 March 1990, **C90-03-18** (1990) 583.

[44] N. Isgur and M.B. Wise, *Excited charm mesons in semileptonic $\bar{B}$ decay and their contributions to a Bjorken sum rule*, *Phys. Rev.* **D 43** (1991) 819 [INSPIRE].

[45] N. Uraltsev, *New exact heavy quark sum rules*, *Phys. Lett.* **B 501** (2001) 86 [`hep-ph/0011124`] [INSPIRE].

[46] B. Grinstein and R.F. Lebed, *Above-Threshold Poles in Model-Independent Form Factor Parametrizations*, *Phys. Rev.* **D 92** (2015) 116001 [`arXiv:1509.04847`] [INSPIRE].

[47] C.G. Boyd, B. Grinstein and R.F. Lebed, *Constraints on form-factors for exclusive semileptonic heavy to light meson decays*, *Phys. Rev. Lett.* **74** (1995) 4603 [`hep-ph/9412324`] [INSPIRE].

[48] C.G. Boyd, B. Grinstein and R.F. Lebed, *Model independent extraction of $|V_{cb}|$ using dispersion relations*, *Phys. Lett.* **B 353** (1995) 306 [hep-ph/9504235] [INSPIRE].

[49] C.G. Boyd and R.F. Lebed, *Improved QCD form-factor constraints and $\Lambda_b \to \Lambda_c \ell \bar{\nu}$*, *Nucl. Phys.* **B 485** (1997) 275 [hep-ph/9512363] [INSPIRE].

[50] C.G. Boyd, B. Grinstein and R.F. Lebed, *Model independent determinations of $\bar{B} \to D\ell\nu$, $D^*\ell\bar{\nu}$ form-factors*, *Nucl. Phys.* **B 461** (1996) 493 [hep-ph/9508211] [INSPIRE].

[51] S.C. Generalis, *QCD sum rules. 1: Perturbative results for current correlators*, *J. Phys.* **G 16** (1990) 785 [INSPIRE].

[52] L.J. Reinders, H.R. Rubinstein and S. Yazaki, *QCD Contribution to Vacuum Polarization 2. The Pseudoscalar Unequal Mass Case*, *Phys. Lett.* **B 97** (1980) 257 [*Erratum ibid.* **B 100** (1981) 519].

[53] L.J. Reinders, S. Yazaki and H.R. Rubinstein, *Two Point Functions for Flavor Changing Currents in QCD*, *Phys. Lett.* **B 103** (1981) 63.

[54] L.J. Reinders, H. Rubinstein and S. Yazaki, *Hadron Properties from QCD Sum Rules*, *Phys. Rept.* **127** (1985) 1 [INSPIRE].

[55] A. Djouadi and P. Gambino, *Electroweak gauge bosons selfenergies: Complete QCD corrections*, *Phys. Rev.* **D 49** (1994) 3499 [*Erratum ibid.* **D 53** (1996) 4111] [hep-ph/9309298] [INSPIRE].

[56] I. Caprini, *Effect of upsilon poles on the analyticity constraints for heavy meson form-factors*, *Z. Phys.* **C 61** (1994) 651 [INSPIRE].

[57] I. Caprini, *Slope of the Isgur-Wise function from a QSSR constraint on the $\Upsilon B \bar{B}$ couplings*, *Phys. Lett.* **B 339** (1994) 187 [hep-ph/9408238] [INSPIRE].

[58] ATLAS collaboration, *Observation of an Excited $B_c^{\pm}$ Meson State with the ATLAS Detector*, *Phys. Rev. Lett.* **113** (2014) 212004 [arXiv:1407.1032] [INSPIRE].

[59] LHCb collaboration, *Observation of $B_c^+ \to J/\psi D^{(*)} K^{(*)}$ decays*, *Phys. Rev.* **D 95** (2017) 032005 [arXiv:1612.07421] [INSPIRE].

[60] E.J. Eichten and C. Quigg, *Mesons with beauty and charm: Spectroscopy*, *Phys. Rev.* **D 49** (1994) 5845 [hep-ph/9402210] [INSPIRE].

[61] FERMILAB LATTICE, MILC collaboration, J.A. Bailey et al., *Update of $|V_{cb}|$ from the $\bar{B} \to D^*\ell\bar{\nu}$ form factor at zero recoil with three-flavor lattice QCD*, *Phys. Rev.* **D 89** (2014) 114504 [arXiv:1403.0635] [INSPIRE].

[62] J. Harrison, C. Davies and M. Wingate, *$|V_{cb}|$ from the $\bar{B}^0 \to D^{*+}\ell^-\bar{\nu}$ zero-recoil form factor using $2+1+1$ flavour HISQ and NRQCD*, PoS(LATTICE2016)287 [arXiv:1612.06716] [INSPIRE].

[63] HPQCD collaboration, J. Harrison, C. Davies and M. Wingate, *Lattice QCD calculation of the $B_{(s)} \to D^*_{(s)}\ell\nu$ form factors at zero recoil and implications for $|V_{cb}|$*, *Phys. Rev.* **D 97** (2018) 054502 [arXiv:1711.11013] [INSPIRE].

[64] LANL-SWME collaboration, J.A. Bailey, T. Bhattacharya, R. Gupta, Y.-C. Jang, W. Lee, J. Leem et al., *Calculation of $\bar{B} \to D^*\ell\bar{\nu}$ form factor at zero recoil using the Oktay-Kronfeld action*, *EPJ Web Conf.* **175** (2018) 13012 [arXiv:1711.01786] [INSPIRE].

[65] W. Detmold, C. Lehner and S. Meinel, *$\Lambda_b \to p\ell^-\bar{\nu}_\ell$ and $\Lambda_b \to \Lambda_c \ell^-\bar{\nu}_\ell$ form factors from lattice QCD with relativistic heavy quarks*, *Phys. Rev.* **D 92** (2015) 034503 [arXiv:1503.01421] [INSPIRE].