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Equivariant A-twisted GLSM and Gromov-Witten invariants of CY 3-folds in Grassmannians

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Abstract: We compute genus-zero Gromov-Witten invariants of Calabi-Yau complete intersection 3-folds in Grassmannians using supersymmetric localization in A-twisted non-Abelian gauged linear sigma models. We also discuss a Seiberg-like duality interchanging $\mathrm{Gr}(n, m)$ and $\mathrm{Gr}(m - n, m)$.

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Contents

1 Introduction

In a Calabi-Yau (CY) compactifications of $E_8 \times E_8$ heterotic string theory with the standard embedding, there are four types of the Yukawa couplings $(\mathbf{1}^3, \mathbf{1}\cdot\mathbf{27}\cdot\mathbf{27}^*, \mathbf{27}^{*3}, \mathbf{27}^3)$ in the 4-dimensional low energy effective theory. It is known that $\mathbf{27}^3$-type Yukawa couplings do not receive either loop or world-sheet instanton corrections. On the other hand, the $\mathbf{27}^{*3}$-type Yukawa couplings receive corrections coming from world-sheet instantons. It was conjectured in [1] that the world-sheet instanton corrected $\mathbf{27}^{*3}$-type Yukawa couplings can be explicitly computed from the $\mathbf{27}^3$-type Yukawa couplings of the mirror manifold.

A generalization of A-twisted gauged linear sigma models (GLSMs) with one omega-background parameter on a 2-sphere S^2 has been constructed in [2]. Recently, the supersymmetric localization computations on this geometry have been performed in [3] (See also [4]). It gives an explicit formula for cubic correlation functions of scalars in the vector multiplet, which conjecturally give the Yukawa couplings of the mirror when the omega-background parameter is set to zero. An interesting point here is that one can compute the $\mathbf{27}^3$-type Yukawa couplings without knowing the mirror manifold. This formula goes back to [5] when the gauge group is Abelian. A mathematical conjecture in the Abelian case, called *toric residue mirror conjecture*, is formulated in [6, 7] and proved in [8–10]. The formula obtained in [3] works also for non-Abelian gauge theories, and can be regarded as a generalization of toric residue mirror conjecture to CY manifolds in non-toric manifolds. We also give an explicit computation of the mirror map in a framework of A-twisted GLSMs with omega-background parameter. This allows us to give a conjectural computation of the genus-zero Gromov-Witten invariants of the CY manifolds.

CY 3-folds with one-dimensional Kähler moduli defined as complete intersections of zero loci of sections of equivariant vector bundles on Grassmannians are classified in [11]. In this paper, we realize some of them as phases of GLSMs, and compute the Yukawa couplings in terms of A-twisted GLSM. This allows us to give a conjectural computation of genus-zero Gromov-Witten invariants of such CY 3-folds. The result agrees with a mathematically rigorous treatment obtained earlier in [12] based on Abelian/non-Abelian correspondence [13–15].

This paper is organized as follows: in section 2, we briefly review GLSMs with unitary gauge groups and their relation to CY 3-folds in Grassmannians. In section 3, we discuss the mirror map for CY 3-folds in Grassmannians in terms of A-twisted GLSMs with omega background. In section 4, we use the method in section 3 to compute the genus-zero Gromov-Witten invariants of some CY 3-folds in Grassmannians. In section 5, we discuss a Seiberg-like dual description of the Yukawa coupling. The last section is devoted to a summary.

2 GLSMs and CY 3-folds in Grassmannians

In this section, we consider 2d $\mathcal{N}=(2,2)$ GLSMs [16] with gauge group $G=\mathrm{U}(n)$ which flow to infrared non-linear sigma models (NLSMs) with large positive Fayet-Iliopoulos parameter (FI-parameter) $\xi \gg 0$. The target spaces are given by the Higgs branch moduli of GLSMs. In this paper, we study the case where the target spaces are CY 3-folds defined as complete intersections of zeros of sections of vector bundles constructed from the dual of the universal subbundle $\mathcal{S}$ on Grassmannians.

The matter multiplets consist of m fundamental chiral multiplets Φ^i for $i=1,\cdots,m$ and chiral multiplets P_l for $l=1,\cdots s$ in the gauge representation R_l. When all the P_l are absent, the D-term vacuum condition for Φ^i in the Higgs branch defines the Grassmannian $\mathrm{Gr}(n,m)$ in the positive FI-parameter region. The introduction of P_l modifies the D-term vacuum condition to the total space of the vector bundle on the Grassmannian $\oplus_{l=1}^{s} E_{R_l} \to \mathrm{Gr}(n,m)$ with appropriate choices of gauge representations R_l. The vector bundle E_{R_l} is determined by the gauge representation R_l. For examples, relations between R_l and E_l are

$$\begin{aligned}
R_l &= \mathbf{n}^* \longleftrightarrow E_l = \mathcal{S}, && (2.1)\\
R_l &= \det{}^{-q} \longleftrightarrow E_l = \mathcal{O}(-q) = \mathcal{O}(-1)^{\otimes q}, && (2.2)\\
R_l &= \mathrm{Sym}^q \mathbf{n}^* \longleftrightarrow E_l = \mathrm{Sym}^q \mathcal{S}, && (2.3)\\
R_l &= \Lambda^q \mathbf{n}^* \longleftrightarrow E_l = \Lambda^q \mathcal{S}, && (2.4)\\
&\cdots
\end{aligned}$$

Here $\mathbf{n}^*$, $\det^{-1}$, $\mathrm{Sym}^q\mathbf{n}^*$, and $\Lambda^q\mathbf{n}^*$ represent the anti-fundamental representation, the inverse of the determinant representation, the q-th symmetric products of the anti-fundamental representation, and the q-th anti-symmetric products of the anti-fundamental representation of the gauge group $\mathrm{U}(n)$, respectively. The bundle $\mathcal{S}$ is the universal subbundle and $\mathcal{O}(-1)=\Lambda^n\mathcal{S}$ is the inverse of the determinant line bundle.

We introduce the following superpotential term

$$W(P,\Phi) = \sum_{l=1}^{s} P_l G_l(\Phi). \tag{2.5}$$

Here $G_l(\Phi)$ is a homogeneous polynomial in Φ^i which belongs to the complex conjugate representation of R_l. The polynomial $G_l(\phi)$ defines a section of the bundle E_l^* on $\mathrm{Gr}(n,m)$, where E_l^* is the dual bundle of E_l. The F-term equations of (2.5) are given by

$$G_l(\phi) = 0, \tag{2.6}$$

$$\sum_{l=1}^{s} p_l \frac{\partial G_l}{\partial \phi_i} = 0. \tag{2.7}$$

If the equation (2.6) defines a smooth complete intersection in the Grassmannian, then it follows from the Jacobian criterion for smoothness that the rank of the matrix $\left(\frac{\partial G_l}{\partial \phi_i}\right)$ appearing in (2.7) is equal to the sum $\sum_{l=1}^{s} \dim R_l$ of the dimensions of p_l, so that the only solution to (2.7) is $p_l = 0$ for all l. The F-term and D-term equations reduce to

$$\sum_{i=1}^{m} \phi^i (\phi^i)^\dagger = \xi \mathbf{1}_n, \quad G_l(\phi) = 0. \tag{2.8}$$

Then the GLSM flows to non-linear sigma model whose target space is given by the complete intersection of zero section of E_l^* in the Grassmannian $\mathrm{Gr}(n,m)$;

$$X^{n,m}_{\oplus_{l=1}^{s} E_l^*} := \{[\phi^i] \in \mathrm{Gr}(n,m) | G_l(\phi) = 0, \ l = 1, 2, \cdots, s\}. \tag{2.9}$$

The complex dimension of $X^{n,m}_{\oplus_{l=1}^{s} E_l}$ is given by

$$\dim_{\mathbb{C}} X^{n,m}_{\oplus_{l=1}^{s} E_l} = mn - n^2 - \sum_{l=1}^{s} \dim R_l. \tag{2.10}$$

Recall that the dimensions of the symmetric and anti-symmetric representations are given by

$$\dim R_l = \begin{cases} \dfrac{(n+q+1)!}{q!(n-1)!} & R_l = \mathrm{Sym}^q \mathbf{n}^*, \\ \dfrac{n!}{(n-q)!q!} & R_l = \Lambda^q \mathbf{n}^*. \end{cases} \tag{2.11}$$

Since we are interested in the cases where the target spaces are CY manifolds, the axial anomaly has to be canceled:

$$2\pi i \partial_\mu J_A^\mu = m \mathrm{Tr}_{\mathbf{n}} F_{12} + \sum_{l=1}^{s} \mathrm{Tr}_{R_l} F_{12} = 0. \tag{2.12}$$

We will give a computation which conjecturally gives genus-zero Gromov-Witten invariants of CY 3-folds realized as axial anomaly free non-Abelian GLSMs.

3 Equivariant A-twisted GLSM on two sphere and mirror symmetry

Supersymmetric backgrounds in two dimensions have been studied from a rigid limit of linearized new minimal supergravity [2]. There exists a new supersymmetric background

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on S^2 which is an extension of the topological A-twist by one omega-background parameter $\hbar$ (equivariant A-twisted GLSM). If $\hbar$ is set to zero, the theory reduces to the ordinary A-twisted GLSM on S^2.

In [3, 4], the correlation functions of gauge invariant operators coming from the vector multiplet scalar σ inserted at the north and the south poles of S^2 have been evaluated by supersymmetric localization. For $G = \mathrm{U}(n)$, the saddle point value of the a-th diagonal component σ at the north and the south pole are given by

$$\sigma_a(x)|_{\mathrm{N}} = \sigma_a - \frac{k_a}{2}\hbar \quad \text{(north pole)}, \tag{3.1}$$

$$\sigma_a(x)|_{\mathrm{S}} = \sigma_a + \frac{k_a}{2}\hbar \quad \text{(south pole)}. \tag{3.2}$$

Here k_a for $a = 1, \cdots, n$ are the magnetic charges for the diagonal elements of the gauge fields. The correlation function of $\langle f(\sigma)|_{\mathrm{N}} g(\sigma)|_{\mathrm{S}}\rangle$ for $G = \mathrm{U}(n)$ is given by

$$\begin{aligned}\langle f(\sigma)|_{\mathrm{N}} g(\sigma)|_{\mathrm{S}}\rangle = &\frac{1}{n!}\sum_{\mathbf{k}\in\mathbb{Z}^n} z^{\sum_{a=1}^n k_a} \sum_{\sigma_*} \\ &\times \text{JK-Res}(\mathbf{Q}(\sigma_*), \eta) Z_{\mathbf{k}}^{\text{vec}} \left(\prod Z_{\mathbf{k}}^{\text{chiral}}\right) f\left(\sigma - k\frac{\hbar}{2}\right) g\left(\sigma + k\frac{\hbar}{2}\right).\end{aligned} \tag{3.3}$$

Here $f(\sigma)|_{\mathrm{N}}$ and $g(\sigma)|_{\mathrm{S}}$ are gauge invariant operators constructed from σ and inserted on the north and south pole respectively. The variable z is the exponential of the complexified FI-parameter defined as $z := e^{2\pi\sqrt{-1}(\theta + \frac{\sqrt{-1}}{2\pi}\xi)}$ with theta angle θ. The factors $Z_{\mathbf{k}}^{\text{vec}}$ and $\prod Z_{\mathbf{k}}^{\text{chiral}}$ are the contributions from the one-loop determinants of the $\mathrm{U}(n)$ vector multiplet and chiral multiplets with magnetic charge $\mathbf{k} = \mathrm{diag}(k_1, \cdots, k_n)$ respectively, and have the following forms:

$$Z_{\mathbf{k}}^{\text{vec}}(\sigma, \hbar) = (-1)^{\sum_{a<b}(k_a - k_b)} \prod_{1\le a\neq b\le n} \left(\sigma_a - \sigma_b + |k_a - k_b|\frac{\hbar}{2}\right), \tag{3.4}$$

$$Z_{\mathbf{k}}^{\text{chiral}}(\sigma, \lambda, \hbar) = \prod_{\rho\in\Delta(R)} \prod_{j=-\frac{|\rho(\mathbf{k})-r+1|-1}{2}}^{\frac{|\rho(\mathbf{k})-r+1|-1}{2}} (\rho(\sigma) + \lambda + j\hbar)^{-\mathrm{sign}(\rho(k)-r+1)}. \tag{3.5}$$

Here $\Delta(R)$ is the set of weights of R. λ is the twisted mass. JK-Res$(\mathbf{Q}(\sigma_*), \eta)$ is the Jeffrey-Kirwan residue operation determined by a charge vector η at a singular locus σ_*. The variable r is an integer R-charge of the lowest component scalar in the chiral multiplet. We assign $r = 0$ for the fundamental chiral multiplets which parametrize the coordinates of the target space in the low energy NLSM. This assignment is compatible with the R-charge assignment in an A-twisted NLSM which is relevant to the Yukawa coupling computation [19]. Then, according to (2.5), the R-charge of the lowest component scalar p^l in the chiral multiplet P^l is determined to be $r = 2$.

For all the GLSMs that we consider in the next section, the weights ρ of the chiral multiplet P^l satisfy the condition $\rho(\mathbf{k}) \le 0$ for $\mathbf{k} \in \mathbb{Z}^n_{\ge 0}$. Then with the choice of $\eta = (1, \cdots, 1)$, the Jeffrey-Kirwan residue is the sum of residues at the poles coming from the fundamental chiral multiplets, and has following simple form:

$$\langle f(\sigma)|_{\mathrm{N}} g(\sigma)|_{\mathrm{S}}\rangle = \frac{1}{n!}\sum_{\mathbf{k}\in\mathbb{Z}^n_{\geq 0}} ((-1)^{n-1}z)^{\sum_{a=1}^n k_a} \oint \prod_{a=1}^n \frac{d\sigma_a}{2\pi\sqrt{-1}} f\left(\sigma - k\frac{\hbar}{2}\right) g\left(\sigma + k\frac{\hbar}{2}\right)$$
$$\times \prod_{1\leq a\neq b\leq n}\left(\sigma_a-\sigma_b+|k_a-k_b|\frac{\hbar}{2}\right) \frac{\prod_{l=1}^s \prod_{\rho\in\Delta(R_l)} \prod_{j=\frac{\rho(\mathbf{k})}{2}}^{\frac{-\rho(\mathbf{k})}{2}} (\rho(\sigma)+\lambda'_l+j\hbar)}{\prod_{i=1}^m \prod_{a=1}^n \prod_{j=-\frac{k_a}{2}}^{\frac{k_a}{2}} (\sigma_a+\lambda_i+j\hbar)}. \tag{3.6}$$

Here λ_i is the twisted mass of Φ^i and λ'_l is the twisted mass of P_l. The contour integrals enclose all the poles $\sigma_a = -j\hbar - \lambda_i$ with $i = 1, \cdots, m$, $j = -\frac{k_a}{2}, \cdots, \frac{k_a}{2}$ and $k_a = 0, 1, \cdots$.

If we set $\hbar = 0$, equivariant A-twist reduces to ordinary A-twist on S^2. In this case, $\sigma(x)$ is invariant under the supersymmetric transformation at any point on S^2, and the saddle point value is simply given by a constant configuration σ_a. Then the correlation function of $(\mathrm{Tr}\sigma)^M$ with $\lambda_i = \lambda'_l = 0$ is given by

$$\langle(\mathrm{Tr}\sigma)^M\rangle_{\hbar=0} = \frac{(-1)^{\frac{n(n-1)}{2}}}{n!} \sum_{\mathbf{k}\in\mathbb{Z}^n_{\geq 0}} ((-1)^{n-1}z)^{\sum_{a=1}^n k_a} \oint_{\sigma=0} \prod_{a=1}^n \frac{d\sigma_a}{2\pi\sqrt{-1}} \left(\sum_{a=1}^n \sigma_a\right)^M$$
$$\times \prod_{1\leq a<b\leq n} (\sigma_a-\sigma_b)^2 \frac{\prod_{l=1}^s \prod_{\rho\in\Delta(R_l)} \rho(\sigma)^{-\rho(\mathbf{k})+1}}{\prod_{a=1}^n \sigma_a^{m(k_a+1)}}. \tag{3.7}$$

If the target space of the low energy NLSM is a CY 3-fold, the expectation values $\langle(\mathrm{Tr}\sigma)^M\rangle_{\hbar=0}$ except for $M = 3$ are zero, and it was conjectured that the expectation value $\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0}$ gives the $\mathbf{27}^3$-type Yukawa coupling

$$K_{zzz} := \int_{\check{X}} \Omega(z) \wedge \left(z\frac{\partial}{\partial z}\right)^3 \Omega(z) \tag{3.8}$$

in four dimensions compactified by the mirror manifold $\check{X}$. Here $\Omega(z)$ is the holomorphic $(3,0)$-form on $\check{X}$ and z is the complex structure moduli of $\check{X}$. We comment on the over all sign ambiguity and sign difference between exponentiated FI-parameter and the complex structure moduli in (3.7) and (3.8). Since the first coefficient of (3.8) in series expansion of z agrees with the triple intersection number of X which is positive, the overall sign of (3.7) is fixed by requiring the positivity of the triple intersection number. In general, the exponentiated FI-parameter z has a different sign from the complex structure. In the computation of Gromov-Witten invariants, this sign ambiguity is fixed by requiring the Gromov-Witten invariant to be positive. In the models treated in the next section, the sign shift $z \to (-1)^m z$ in the GLSM gives the correct sign of the mirror Yukawa coupling.

When the gauge group is U(1), the ambient space of the target space is the complex projective space $\mathbb{P}^{m-1}$, and (3.8) was first proposed by [5]. A mathematical interpretation of this formula was proposed by [6]. It has been shown in [3] that (3.7) for GLSMs studied in [18] gives correct mirror Yukawa coupling given in [17]. So we expect that (3.8) also works for the non-Abelian GLSMs considered in the next section.

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In order to extract genus-zero Gromov-Witten invariants from the mirror Yukawa coupling, we have to rewrite the mirror Yukawa coupling in terms of the flat coordinate t. The relation between z and t is given by the *mirror map*, defined as the ratio of two period integrals $I_0(z)$ and $I_1(z)$ of $\check{X}$;

$$t = \frac{I_1(z)}{I_0(z)}. \tag{3.9}$$

Here $I_0(z)$ is the fundamental period normalized as $I_0(0) = 1$, and $I_1(z)$ is the period with a logarithmic monodromy $I_1(z) = I_0(z)\log z + \tilde{I}_1(z)$ with $\tilde{I}_1(0) = 0$. By solving (3.9) recursively and expressing z as a function of $q := e^t$, the Yukawa coupling in the flat coordinate is written as

$$K_{ttt} = \frac{K_{zzz}(z(q))}{I_0^2(z(q))}\left(\frac{q}{z(q)}\frac{dz(q)}{dq}\right)^3 = n_0 + \sum_{d=1}^{\infty} n_d d^3 \frac{q^d}{1-q^d}, \tag{3.10}$$

$$N_d = \sum_{k|d} \frac{n_{\frac{d}{k}}}{k^3}. \tag{3.11}$$

Here n_d for $d = 0, 1, 2, \ldots$ are the instanton numbers of the target space X of genus 0 and degree d. N_d for $d = 0, 1, 2, \ldots$ are genus zero Gromov-Witten invariants. In particular, $n_0 = \int_X H^3$ is the triple intersection number of the hyperplane class H in X.

The period integrals and the mirror map can be extracted from the equivariant A-twisted GLSM as follows. Motivated by the factorization property of the physical S^2 partition function [20–22], we rewrite correlation functions as follows. After redefinitions of integration variables, the expectation values of $(\mathrm{Tr}\sigma)^M$ inserted at north and south can are rewritten as

$$\begin{aligned}
\langle(\mathrm{Tr}\sigma)^M|_{\mathrm{N}}\rangle_\hbar &= \sum_{\{j\}} \oint_{x_a=-\lambda_{j_a}} \prod_{a=1}^{n} \frac{dx_a}{2\pi\sqrt{-1}} \tilde{Z}(x,\lambda,\lambda') Z(z,x,\lambda,\lambda',\hbar) \\
&\quad \times \left(\sum_{\mathbf{k}\in\mathbb{Z}_{\geq 0}^n} ((-1)^{n-1}z)^{\sum_{a=1}^n k_a} \prod_{a=1}^{n} (x_a - k_a\hbar)^M Z_\Phi(k,x,\lambda,-\hbar) \prod_{l=1}^{s} Z_{P_l}(k,x,\lambda',-\hbar) \right), \\
\langle(\mathrm{Tr}\sigma)^M|_{\mathrm{S}}\rangle_\hbar &= \sum_{\{j\}} \oint_{x_a=-\lambda_{j_a}} \prod_{a=1}^{n} \frac{dx_a}{2\pi\sqrt{-1}} \tilde{Z}(x,\lambda,\lambda') Z(z,x,\lambda,\lambda',-\hbar) \\
&\quad \times \left(\sum_{\mathbf{k}\in\mathbb{Z}_{\geq 0}^n} ((-1)^{n-1}z)^{\sum_{a=1}^n k_a} \prod_{a=1}^{n} (x_a + k_a\hbar)^M Z_\Phi(k,x,\lambda,\hbar) \prod_{l=1}^{s} Z_{P_l}(k,x,\lambda',\hbar) \right),
\end{aligned} \tag{3.12}$$

with

$$\tilde{Z}(x,\lambda,\lambda') = \frac{\prod_{n\geq a>b\geq 1}(x_a - x_b)^2 \prod_{l=1}^{s}\prod_{\rho\in\Delta(R_l)}(\rho(x)+\lambda'_l)}{\prod_{i=1}^{m}\prod_{a=1}^{n}(x_a+\lambda_i)}, \tag{3.13}$$

$$Z(z,x,\lambda,\lambda',\hbar) = \sum_{\mathbf{k}\in\mathbb{Z}_{\geq 0}^n} ((-1)^{n-1}z)^{\sum_{a=1}^n k_a} Z_\Phi(k,x,\lambda,\hbar) \prod_{l=1}^{s} Z_{P_l}(k,x,\lambda',\hbar), \tag{3.14}$$

$$Z_\Phi(k,x,\lambda,\hbar) = \frac{\prod_{n\geq a>b\geq 1}(x_a - x_b + (k_a-k_b)\hbar)}{\prod_{n\geq a>b\geq 1}(x_a-x_b)\prod_{i=1}^m\prod_{a=1}^n\prod_{l=1}^{k_a}(x_a+\lambda_i+l\hbar)}, \tag{3.15}$$

$$Z_{P_l}(k,x,\lambda',\hbar) = \prod_{\rho\in\Delta(R_l)}\prod_{j=1}^{-\rho(\mathbf{k})}(\rho(x)+\lambda'_l - j\hbar). \tag{3.16}$$

Here $\{j\}=\{j_1,j_1,\cdots,j_n\}$ and $\sum_{\{j\}}=\sum_{1\leq j_1<j_2<\cdots<j_n\leq m}$. Then the generating function of the correlation functions $\langle(\mathrm{Tr}\sigma)^M|_{\mathrm{N}}(\mathrm{Tr}\sigma)^N|_{\mathrm{S}}\rangle_\hbar$ can be written as

$$\begin{aligned}\langle e^{\alpha\mathrm{Tr}\sigma}|_{\mathrm{N}}e^{\beta\mathrm{Tr}\sigma}|_{\mathrm{S}}\rangle_\hbar = \sum_{\{j\}}\oint_{x_a=-\lambda_{j_a}}\prod_{a=1}^n\frac{dx_a}{2\pi\sqrt{-1}}\tilde{Z}(x,\lambda,\lambda') \\ \times e^{\alpha\sum_{a=1}^n x_a}Z(e^{-\alpha\hbar}z,x,\lambda,\lambda',-\hbar)e^{\beta\sum_{a=1}^n x_a}Z(e^{\beta\hbar}z,x,\lambda,\lambda',\hbar).\end{aligned} \tag{3.17}$$

When the twisted masses are distinct, we can explicitly perform the contour integrals in (3.17) and obtain the following vortex factorization form of the generating function.

$$\begin{aligned}\langle e^{\alpha\mathrm{Tr}\sigma}|_{\mathrm{N}}e^{\beta\mathrm{Tr}\sigma}|_{\mathrm{S}}\rangle_\hbar = \sum_{\{j\}}\tilde{Z}_{\{j\}}(\lambda,\lambda')e^{\alpha\sum_{a=1}^n\lambda_{j_a}} \\ \times Z_{\mathrm{v},\{j\}}(e^{-\alpha\hbar}z,\lambda,\lambda',-\hbar)e^{\beta\sum_{a=1}^n\lambda_{j_a}}Z_{\mathrm{v},\{j\}}(e^{\beta\hbar}z,\lambda,\lambda',\hbar).\end{aligned} \tag{3.18}$$

with

$$\tilde{Z}_{\{j\}}(\lambda,\lambda') = \frac{\prod_{n\geq a>b\geq 1}(\lambda_{j_a}-\lambda_{j_b})^2\prod_{l=1}^s\prod_{\rho\in\Delta(R_l)}(\rho(-\lambda_{\{j\}})+\lambda'_l)}{\prod_{a=1}^n\prod_{i=1,i\neq j_a}^m(\lambda_i-\lambda_{j_a})}, \tag{3.19}$$

$$Z_{\mathrm{v},\{j\}}(z,\lambda,\lambda',\hbar) = Z(z,x,\lambda,\lambda',\hbar)\Big|_{x_a=-\lambda_{j_a}}. \tag{3.20}$$

Here $\lambda_{\{j\}}=\mathrm{diag}(\lambda_{j_1},\lambda_{j_2},\cdots,\lambda_{j_n})$. From the view point of Higgs branch localization, $\tilde{Z}_{\{j\}}(\lambda,\lambda')$ is interpreted as the 1-loop determinant and the vortex partition function $Z_{\mathrm{v},\{j\}}(z,\lambda,\lambda',-\hbar)$ ($Z_{\mathrm{v},\{j\}}(z,\lambda,\lambda',\hbar)$) is the point like vortex contribution on north (south) pole of S^2 at a root of Higgs branch specified by twisted masses $\lambda_{\{j\}}$.

Now we discuss the relation between (3.6), (3.18) and Givental's work [23, 24]. For clarity of exposition, we restrict ourselves to the case when the gauge group is $G=\mathrm{U}(1)$, the gauge charge of Φ^i is 1 for $i=1,\ldots,m$, and the gauge charge of P_l is $-q_l$ with $m=\sum_{l=1}^s q_l$. The target space of the low-energy NLSM is a CY complete intersection in $\mathbb{P}^{m-1}$. Then (3.6) with $f(\sigma)=e^{\alpha\sigma}$ and $g(\sigma)=1$ is expressed as

$$\langle e^{\alpha\sigma}|_{\mathrm{N}}\rangle_\hbar = (-1)^s\sum_{k=0}^\infty((-1)^m z)^k\oint\frac{dx}{2\pi\sqrt{-1}}e^{\alpha x}\frac{\prod_{l=1}^s\prod_{j=0}^{q_l k}(q_l x-\lambda'_l+j\hbar)}{\prod_{i=1}^m\prod_{j=0}^k(x+\lambda_i+j\hbar)}, \tag{3.21}$$

which agrees with the function

$$\Phi^* = \frac{1}{2\pi\sqrt{-1}}\oint e^{p(t-\tau)/\hbar}\sum_{d=0}^\infty e^{d\tau}\frac{\prod_{a=1}^r\prod_{m=0}^{l_a d}(l_a p-\lambda'_a-m\hbar)}{\prod_{i=0}^n\prod_{m=0}^d(p-\lambda_i-m\hbar)}dp \tag{3.22}$$

appearing in [24, page 650] by setting $\alpha\hbar = -(\tau - t)$, and $\log((-1)^m z) = \tau$ up to change of signs of λ_i, $\hbar$ and overall sign. The function Φ^* is the generating function of intersection numbers on the quasimap space, and goes back to the generating function

$$\sum_{d=0}^{\infty} e^{dt} \int_{M_d} E_{d,l} e^{(t-\tau)(A+\omega/\hbar)} \tag{3.23}$$

in [23, page 338], which is a regularized version of a '$\frac{\infty}{2}$-dimensional integral' on the loop space. The factorization

$$\Phi^* = \sum_i \frac{\prod_a (l_a \lambda_i - \lambda'_a)}{\prod_{j\neq i}(\lambda_i - \lambda_j)} e^{\lambda_i (t-\tau)/\hbar} Z_i^*(e^t, \hbar) Z_i^*(e^\tau, -\hbar), \tag{3.24}$$

$$Z_i^* = \sum_{d=0}^{\infty} q^d \frac{\prod_{a=1}^r \prod_{m=1}^{l_a d}(l_a\lambda_i - \lambda'_a + m\hbar)}{\prod_{\alpha=0}^n \prod_{m=1}^d (\lambda_i - \lambda_\alpha + m\hbar)}, \tag{3.25}$$

which agrees with the Abelian case

$$\langle e^{\alpha\sigma}|_{\mathrm{N}}\rangle_\hbar = \sum_{j=1}^m \tilde{Z}_j(\lambda,\lambda') e^{\alpha\lambda_j} Z_{\mathrm{v},j}(e^{-\alpha\hbar}z, \lambda, \lambda', -\hbar) Z_{\mathrm{v},j}(z,\lambda,\lambda',\hbar), \tag{3.26}$$

$$\tilde{Z}_j(\lambda,\lambda') = \frac{\prod_{l=1}^s (q_l\lambda_j + \lambda'_l)}{\prod_{i=1, i\neq j}^m (\lambda_i - \lambda_j)}, \tag{3.27}$$

$$Z_{\mathrm{v},j}(z,\lambda,\lambda',-\hbar) = \sum_{k=0}^{\infty} ((-1)^m z)^k \frac{\prod_{l=1}^s \prod_{p=1}^{q_l k}(-q_l\lambda_j - \lambda'_l - p\hbar)}{\prod_{i=1}^m \prod_{l=1}^k (\lambda_i - \lambda_j - l\hbar)} \tag{3.28}$$

of (3.18), also goes back to [23, page 338], and is an important ingredient in the proof of Givental's mirror theorem. The factorization for toric complete intersections is given in [26, Proposition 6.2].

Givental's I-function $I(z,x,\hbar)$ for the complete intersection in $\mathbb{P}^{m-1}$ is given by

$$I(z,x,\hbar) = z^{\frac{x}{\hbar}} Z((-1)^m z, x, \lambda, \lambda', \hbar)\Big|_{\lambda_i = \lambda'_l = 0} = z^{\frac{x}{\hbar}} \sum_{k=0}^{\infty} z^k \frac{\prod_{l=1}^s \prod_{j=1}^{q_l k}(q_l x + l\hbar)}{\prod_{l=1}^k (x + l\hbar)^m}. \tag{3.29}$$

The period integrals $I_i(z)$ appear in the expansion of the I-function as

$$I(z,x,\hbar) = \sum_{i=0}^3 I_i(z) \left(\frac{x}{\hbar}\right)^i \mod \left(\frac{x}{\hbar}\right)^4. \tag{3.30}$$

When the gauge group is non-Abelian, we expect that $Z(z,x,\lambda,\lambda',\hbar)$ is again related to Givental's I-function $I(z,x,\hbar)$ of $X^{n,m}_{\oplus_{l=1}^s E_l^*}$ by

$$I(z,x,\hbar) = z^{\sum_{a=1}^n \frac{x_a}{\hbar}} Z((-1)^m z, x, \lambda, \lambda', \hbar)\Big|_{\lambda_i = \lambda'_l = 0}. \tag{3.31}$$

Here the sign of z is fixed by requiring the first instanton number to be positive. If the I-function for a Calabi-Yau 3-fold is expanded as

$$I(z,x,\hbar) = I_0(z) + I_1(z)\frac{\sum_{a=1}^n x_a}{\hbar} + O\left(\frac{1}{\hbar^2}\right), \tag{3.32}$$

	$\phi_i,(i=1,\cdots,7)$	p_1	$p_l,(l=2,\cdots,5)$
$\mathrm{U}(2)_G$	$\mathbf{2}$	$\mathrm{Sym}^2\mathbf{2}^*$	$\det^{-1}$
$\mathrm{U}(1)_R$	0	2	2

Table 1. The charge assignment for lowest component scalars in the chiral multiplets of equivariant A-twisted GLSM for $X^{2,7}_{\mathrm{Sym}^2\mathcal{S}^*\oplus\mathcal{O}(1)^{\oplus 4}}$

then the mirror map is again given by the ratio of two periods as (3.9). If we define $Z_0(z)$ and $Z_1(z)$ as the first two coefficients of expansion

$$Z((-1)^m z,x,\hbar)=Z((-1)^m z,x,\lambda,\lambda',\hbar)\Big|_{\lambda_i=\lambda'_l=0}=Z_0(z)+Z_1(z)\frac{\sum_{a=1}^n x_a}{\hbar}+O\left(\frac{1}{\hbar^2}\right). \tag{3.33}$$

Then I_0 and I_1 are related to Z_0 and Z_1 by

$$I_0(z)=Z_0(z),\quad I_1(z)=Z_0(z)\log(z)+Z_1(z). \tag{3.34}$$

4 Computation of Gromov-Witten invariants

In this section, we compute the Yukawa couplings and genus-zero Gromov-Witten invariants of some examples of compact CY 3-folds in Grassmannians which are obtained as complete intersections of equivariant vector bundles.

Our first example is $X^{2,7}_{\mathrm{Sym}^2\mathcal{S}^*\oplus\mathcal{O}(1)^{\oplus 4}}$, which is a complete intersection CY 3-fold of $\mathrm{Sym}^2\mathcal{S}^*$ and four copies of $\mathcal{O}(1)$ in $\mathrm{Gr}(2,7)$. The gauge group is $\mathrm{U}(2)$, and there are seven fundamental chiral multiplets $\Phi_i,(i=1,\cdots,7)$, one chiral multiplet P_1 in the gauge representation $\mathrm{Sym}^2\mathbf{2}^*$, and four chiral multiplet $P_i(i=2,\cdots,5)$ in the gauge representation $\det^{-1}$. The charge assignment for the lowest component scalars in the chiral multiplets is listed in table 1. The superpotential is given by

$$W=\sum_{l=1}^{5}P_lG_l(\Phi)=P_1^{(ab)}S_{(ij)}\Phi^{(i}_{(a}\Phi^{j)}_{b)}+\sum_{l=2}^{5}P_lA^l_{[ij]}\epsilon^{ab}\Phi^{[i}_a\Phi^{j]}_b. \tag{4.1}$$

Here a,b denote color indices, and the notations (ij) and $[i,j]$ show that the indices i and j are symmetric and anti-symmetric under the permutation respectively. The F-term equation for P_l gives

$$A^l_{[ij]}\epsilon^{ab}\phi^{[i}_a\phi^{j]}_b=0,\quad S_{(ij)}\phi^{(i}_{(a}\phi^{j)}_{b)}=0. \tag{4.2}$$

Let us compute the mirror Yukawa coupling and instanton numbers. The set of weights of the chiral multiplet P_l evaluated at $\mathrm{diag}(\sigma_1,\sigma_2)$ is given by

$$\begin{aligned}&\{-2\sigma_1,-\sigma_1-\sigma_2,-2\sigma_2\}&&\text{for } P_1,\\&\{-\sigma_1-\sigma_2\}&&\text{for } P_2,\dots,P_5.\end{aligned} \tag{4.3}$$

Hence the contributions from the one-loop determinants of the chiral multiplets are

$$Z^{\text{chiral}}_{\mathbf{k}}(\sigma,\hbar=0)=\begin{cases}\prod_{a=1}^{2}\sigma_a^{-k_a-1}, & \text{for } \Phi_i,\\ (-\sigma_1-\sigma_2)^{k_1+k_2+1}\prod_{a=1}^{2}(-2\sigma_a)^{2k_a+1} & \text{for } P_1\\ (-\sigma_1-\sigma_2)^{k_1+k_2+1} & \text{for } P_2,\dots,P_5.\end{cases} \tag{4.4}$$

From (3.7), the Yukawa coupling of this model with the sign change $z\to -z$ is given by

$$\begin{aligned}\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0} &= \frac{1}{2}\sum_{\mathbf{k}\in\mathbb{Z}^2_{\geq 0}}\oint_{\sigma=0}\prod_{a=1}^{2}\frac{d\sigma_a}{2\pi\sqrt{-1}}z^{k_1+k_2}(\sigma_1+\sigma_2)^3(\sigma_1-\sigma_2)^2\\ &\qquad\times\frac{(-\sigma_1-\sigma_2)^{5k_1+5k_2+5}(-2\sigma_1)^{2k_1+1}(-2\sigma_2)^{2k_2+1}}{\prod_{a=1}^{2}\sigma_a^{7k_a+7}}\\ &=\sum_{k=0}^{\infty}\sum_{m=0}^{k}(-1)^{k+1}2^{2k+2}z^k\left(\binom{5k+6}{5m+1}-\binom{5k+6}{5m+3}\right)\\ &=\frac{8(7+6z)}{(1+4z)(1-44z-16z^2)}.\end{aligned} \tag{4.5}$$

Next we compute the Yukawa coupling in the flat coordinate. (3.33) is written in this model as

$$Z(-z,x,\hbar)=\sum_{\mathbf{k}\in\mathbb{Z}^2_{\geq 0}}(-z)^{k_1+k_2}\frac{(x_2-x_1+(k_2-k_1)\hbar)\prod_{l=1}^{k_1+k_2}(x_1+x_2+l\hbar)^5\prod_{a=1}^{2}\prod_{l=1}^{2k_a}(2x_a+l\hbar)}{(x_2-x_1)\prod_{a=1}^{2}\prod_{l=1}^{k_a}(x_a+l\hbar)^7}. \tag{4.6}$$

From this equation, the series $Z_0(z)$ and $Z_1(z)$ can be read off as

$$Z_0(z)=1+4z+64z^2+1408z^3+37216z^4+1093504z^5+\cdots \tag{4.7}$$

$$Z_1(z)=10z+189z^2+\frac{13528}{3}z^3+\frac{744743}{6}z^4+\frac{11218906}{3}z^5+\cdots \tag{4.8}$$

Eq. (3.9) is solved recursively as

$$z(q)=q-10q^2+q^3+20q^4-2412q^5+\cdots. \tag{4.9}$$

Then, we obtain the Yukawa coupling in the flat coordinate:

$$K_{ttt}=n_0+\sum_{d=1}^{\infty}n_d d^3\frac{q^d}{1-q^d}, \tag{4.10}$$

with

$$n_0=56,\ n_1=160,\ n_2=758,\ n_3=5824,\ n_4=65540,\ n_5=884064,\cdots. \tag{4.11}$$

	$X^{2,5}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(2)}$	$X^{2,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)^{\oplus 3}}$	$X^{2,6}_{\mathrm{Sym}^2\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(2)}$	$X^{2,6}_{\mathrm{Sym}^2\mathcal{S}^*\oplus\mathcal{S}^*\otimes\mathcal{O}(1)}$
n_0	24	33	40	48
n_1	336	252	160	112
n_2	3636	1854	1560	1102
n_3	83392	27156	14560	7104
n_4	2727936	567063	272000	98892
n_5	109897632	14514039	5299328	1389664

Table 2. The triple intersection number and genus zero instanton numbers for CY 3-folds in $\mathrm{Gr}(2,m)$.

	$\phi_i, (i=1,\cdots,5)$	p
$\mathrm{U}(3)_G$	$\mathbf{3}$	$(\Lambda^2\mathbf{3}^*)\otimes\det^{-1}$
$\mathrm{U}(1)_R$	0	2

Table 3. The matter contents of equivariant A-twisted GLSM for $X^{3,5}_{(\Lambda^2\mathcal{S}^*)\otimes\mathcal{O}(1)}$.

(4.11) reproduce the correct triple intersection number and the genus-zero instanton numbers for No. 212 in the Calabi-Yau date base [27]. Genus-zero Gromov-Witten invariants for other U(2) GLSMs are listed in the table 2. The Calabi-Yau 3-fold $X^{2,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)^{\oplus 3}}$ is known by [11] to be deformation-equivalent to a complete intersection Calabi-Yau 3-fold in a minuscule Schubert variety introduced by Miura [28]. Its genus-zero Gromov-Witten invariants are also computed in [28]. Gromov-Witten invariants for Miura's Calabi-Yau 3-fold was also computed in [29] by physical S^2 partition function method [30].

Our second example is $X^{3,5}_{(\Lambda^2\mathcal{S}^*)\otimes\mathcal{O}(1)}$, which is known by [11] to be deformation-equivalent to the complete intersection of two copies of $\mathrm{Gr}(2,5)$ in $\mathbb{P}^9$. This is a U(3) GLSM with five fundamental chiral multiplets and one chiral multiplet P in the gauge representation $(\Lambda^2\mathbf{3}^*)\otimes\det^{-1}$ as shown in table 3. The set of weights $\rho(\sigma)$ for the chiral multiplet P is given by

$$\{-(\sigma_1+2\sigma_2+2\sigma_3),\ -(2\sigma_1+\sigma_2+2\sigma_3),\ -(2\sigma_1+2\sigma_2+\sigma_3)\}. \tag{4.12}$$

The superpotential is

$$W=A_{[i_1i_2,i_3][i_4i_5]}\epsilon^{a_1a_2a_3}P^{[bc]}\Phi^{[i_1}_{a_1}\Phi^{i_2}_{a_2}\Phi^{i_3]}_{a_3}\Phi^{[i_4}_{[b}\Phi^{i_5]}_{c]}. \tag{4.13}$$

From (3.7), the Yukawa coupling of this model with the sign change $z\to -z$ is given by

$$\begin{aligned}\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0}&=\frac{1}{3!}\sum_{\mathbf{k}\in\mathbb{Z}^3_{\geq 0}}(-z)^{\sum_{a=1}^3k_a}\oint_{\sigma=0}\prod_{a=1}^3\frac{d\sigma_a}{2\pi\sqrt{-1}}\left(\sum_{a=1}^3\sigma_a\right)^3 \\ &\quad\times\prod_{a<b}(\sigma_a-\sigma_b)^2\prod_{a=1}^3\frac{(\sigma_a-2\sum_{b=1}^3\sigma_b)^{2\sum_{b=1}^3k_b-k_a+1}}{\sigma_a^{5k_a+5}} \\ &=25(1+121z+14884z^2+1830609z^3+225150025z^4+27691622464z^5)+\cdots.\end{aligned} \tag{4.14}$$

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The function (3.33) for this model is given by

$$Z(-z,x,\hbar)=\sum_{\mathbf{k}\in\mathbb{Z}^3_{\geq 0}} z^{\sum_{a=1}^3 k_a}\frac{\prod_{a>b}(x_a-x_b+(k_a-k_b)\hbar)\prod_{a=1}^{3}\prod_{j=1}^{2\sum_{b=1}^3 k_b-k_a}\left(2\sum_{b=1}^{3}x_b-x_a+j\hbar\right)}{\prod_{a>b}(x_a-x_b)\prod_{a=1}^{3}\prod_{j=1}^{k_a}(x_a+j\hbar)^5}. \tag{4.15}$$

From this equation, we obtain the first two coefficients as

$$Z_0(z)=1+9z+361z^2+21609z^3+1565001z^4+126630009z^5+\cdots, \tag{4.16}$$

$$Z_1(z)=30z+1425z^2+90895z^3+\frac{13604625}{2}z^4+\frac{1123000637}{2}z^5+\cdots. \tag{4.17}$$

The complex structure moduli is expressed as function of q as

$$z(q)=q-30q^2+195q^3-3070q^4-99495q^5+\cdots. \tag{4.18}$$

Then the expected genus zero instanton numbers are

$$n_0=25,\ n_1=325,\ n_2=3200,\ n_3=66250,\ n_4=1985000,\ n_5=73034875,\cdots. \tag{4.19}$$

These values reproduce the Gromov-Witten invariants of the complete intersection of two copies of $\mathrm{Gr}(2,5)$ in $\mathbb{P}^9$ calculated in [31]. Gromov-Witten invariants for some other CY 3-folds in Grassmannian $\mathrm{Gr}(n,m)$ is listed in tables 4, 5 and 6. The manifold $X^{3,8}_{(\mathrm{Sym}^2\mathcal{S}^*)^{\oplus 2}}$ in table 4 is an Abelian 3-fold, so that its Gromov-Witten invariants are zero except at degree 0.

5 Seiberg like description of the mirror Yukawa coupling

In this section, we study dual $\mathrm{U}(m-n)$ GLSM description of Yukawa couplings of $\mathrm{U}(n)$ GLSM with m fundamental chiral and chiral multiplet $P_l,(l=1,\cdots,s)$ in several examples. We start with the case where all P_l belong to the gauge representation $\det^{-q_l}$ and the target space of low energy NLSM is $X^{n,m}_{\oplus_{l=1}^s\mathcal{O}(q_l)}$ in $\mathrm{Gr}(n,m)$. The Seiberg-like duality of this model is studied in [18]. The CY 3-fold $X^{n,m}_{\oplus_{l=1}^s\mathcal{O}(q_l)}$ is isomorphic to a CY 3-fold $X^{m-n,m}_{\oplus_{l=1}^s\mathcal{O}(q_l)}$ in $\mathrm{Gr}(m-n,m)$. Then, in the dual side, the GLSM is $\mathrm{U}(m-n)$ gauge group with m fundamental chiral multiplets and chiral multiplet P_l in the $\det^{-q_l}$ representation for $l=1,\cdots,s$. For example, one has $X^{2,6}_{\mathcal{O}(1)^{\oplus 4}\oplus\mathcal{O}(2)}\simeq X^{4,6}_{\mathcal{O}(1)^{\oplus 4}\oplus\mathcal{O}(2)}$. In U(2) GLSM description, the Yukawa coupling of $X^{2,6}_{\mathcal{O}(1)^{\oplus 4}\oplus\mathcal{O}(2)}$ is given by

$$\begin{aligned}\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0}&=\frac{1}{2}\sum_{\mathbf{k}\in\mathbb{Z}^2_{\geq 0}}(-z)^{\sum_{a=1}^2 k_a}\oint_{\sigma=0}\prod_{a=1}^{2}\frac{d\sigma_a}{2\pi\sqrt{-1}}\left(\sum_{a=1}^{2}\sigma_a\right)^3\\&\quad\times(\sigma_1-\sigma_2)^2\frac{(-\sigma_1-\sigma_2)^{4(k_1+k_2)+4}(-2\sigma_1-2\sigma_2)^{2(k_1+k_2)+1}}{\prod_{a=1}^2\sigma_a^{6(k_a+1)}}\\&=28(1+104z+11248z^2+1214720z^3)+\cdots.\end{aligned} \tag{5.1}$$

	$X^{3,6}_{\Lambda^2\mathcal{S}^*\oplus\mathcal{O}(1)^{\oplus 2}\oplus\mathcal{O}(2)}$	$X^{3,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\Lambda^2\mathcal{S}^*}$	$X^{3,7}_{\mathrm{Sym}^2\mathcal{S}^*\oplus\mathcal{O}(1)^{\oplus 3}}$
n_0	32	42	128
n_1	256	210	0
n_2	2016	1176	4096
n_3	32000	13104	0
n_4	709904	201936	9280
n_5	19397376	3824016	0
	$X^{3,7}_{(\Lambda^2\mathcal{S}^*)^{\oplus 2}\oplus\mathcal{O}(1)^{\oplus 3}}$	$X^{3,8}_{(\Lambda^2\mathcal{S}^*)^{\oplus 4}}$	$X^{3,8}_{(\mathrm{Sym}^2\mathcal{S}^*)^{\oplus 2}}$
n_0	61	92	384
n_1	163	140	0
n_2	630	328	0
n_3	4795	1872	0
n_4	48422	12280	0
n_5	599809	100728	0

Table 4. The triple intersection number and instanton numbers for CY 3-folds in $\mathrm{Gr}(3, m)$.

	$X^{4,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)}$	$X^{4,7}_{\Lambda^2\mathcal{S}^*\oplus\mathcal{O}(1)^{\oplus 2}\oplus\mathcal{O}(2)}$	$X^{4,7}_{(\Lambda^3\mathcal{S}^*)^{\oplus 2}\oplus\mathcal{O}(1)}$
n_0	42	32	72
n_1	196	256	136
n_2	1225	2016	508
n_3	12740	32000	3088
n_4	198058	709904	25342

Table 5. The triple intersection number and instanton numbers for CY 3-folds in $\mathrm{Gr}(4, m)$.

	$X^{5,7}_{\Lambda^4\mathcal{S}^*\oplus\mathcal{O}(1)\oplus\mathcal{O}(2)}$	$X^{6,8}_{\Lambda^5\mathcal{S}^*\oplus\mathcal{O}(1)^{\oplus 3}}$
n_0	36	57
n_1	216	147
n_2	1674	756
n_3	21888	5283

Table 6. The triple intersection number and instanton numbers for CY 3-folds in $\mathrm{Gr}(5, 7)$ and $\mathrm{Gr}(6, 8)$.

In the dual U(4) GLSM description, the Yukawa coupling of $X^{4,6}_{\mathcal{O}(1)^{\oplus 4}\oplus\mathcal{O}(2)}$ is given by

$$\begin{aligned}\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0} &= -\frac{1}{24}\sum_{\mathbf{k}\in\mathbb{Z}^4_{\geq 0}}(-z)^{\sum_{a=1}^4 k_a}\oint_{\sigma=0}\prod_{a=1}^{4}\frac{d\sigma_a}{2\pi\sqrt{-1}}\left(\sum_{a=1}^{4}\sigma_a\right)^3 \\ &\quad\times\prod_{1\leq a<b\leq 4}(\sigma_a-\sigma_b)^2\,\frac{\left(-\sum_{a=1}^4\sigma_a\right)^{4(\sum_{a=1}^4 k_a+1)}\left(-2\sum_{a=1}^4\sigma_a\right)^{(\sum_{a=1}^4 2k_a+1)}}{\prod_{a=1}^4\sigma_a^{6(k_a+1)}} \\ &= 28(1+104z+11248z^2+1214720z^3)+\cdots\,. \end{aligned} \tag{5.2}$$

From (5.1) and (5.2), we find that two A-twisted GLSMs give the same Yukawa coupling.

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	$\phi_i, (i=1,\cdots,6)$	p_1	p_2
$\mathrm{U}(4)_G$	$\mathbf{4}$	$\mathbf{4}^* \otimes \det^{-1}$	$\det^{-1}$
$\mathrm{U}(1)_R$	0	2	2

Table 7. Field contents of U(4) GLSM for $X^{4,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)}$

Next, we study the dual $\mathrm{U}(m-n)$ GLSM description of the Yukawa coupling for a $\mathrm{U}(n)$ GLSM with m fundamental chiral multiplets, one chiral multiplet P_1 in the gauge representation $\mathbf{n}^*\otimes\det^{-1}$ and $P_2,\ldots,P_s$ in the gauge representation $\det^{-q_l}$, which flow to NLSM with target space $X^{n,m}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus_{l=2}^s\mathcal{O}(q_l)}$. This target space is a complete intersection of $\mathcal{S}^*\otimes\mathcal{O}(1)$ and $\mathcal{O}(q_l)$ for $l=2,\cdots,s$ in $\mathrm{Gr}(n,m)$.

To find the dual description, let us first consider the dual $\mathrm{U}(m-n)$ gauge theory description of $\mathrm{U}(n)$ with m fundamental chiral multiplet Φ_i and an anti-fundamental chiral multiplet P_1 [32]. In the $\mathrm{U}(n)$ GLSM, an anti-fundamental chiral multiplet define the fiber of the universal subbundle $\mathcal{S}$ on $\mathrm{Gr}(n,m)$. The universal subbundle $\mathcal{S}\to\mathrm{Gr}(n,m)$ is mapped to the dual of the universal quotient bundle $\mathcal{Q}^*\to\mathrm{Gr}(m-n,m)$. Then $\mathcal{Q}^*\to\mathrm{Gr}(m-n,m)$ can be realized by a $\mathrm{U}(m-n)$ gauge theory with m fundamental chiral multiplets, m mesonic chiral fields $M_1,\ldots,M_m$ and a chiral multiplet $\tilde{\Phi}$ in the representation $(\mathbf{m}-\mathbf{n})^*$ with the superpotential $W=\sum_{i=1}^m M_i\tilde{\Phi}\Phi_i$. The F-term equation gives $\mathcal{Q}^*$ on $\mathrm{Gr}(m-n,m)$.

In our case, the anti-fundamental chiral multiplet P_1 is modified to chiral multiplet in the gauge representation $\mathbf{n}^*\otimes\det^{-1}$. With the D-term equation, an anti-fundamental chiral multiplet defines the fiber of the vector bundle $\mathcal{S}\otimes\mathcal{O}(-1)$. The bundle $\mathcal{S}\otimes\mathcal{O}(-1)\to\mathrm{Gr}(n,m)$ is mapped to the tensor product $\mathcal{Q}^*\otimes\mathcal{O}(-1)$ of the dual of the universal quotient bundle and the inverse of the determinant line bundle on $\mathrm{Gr}(m-n,m)$. Similarly, the bundle $\mathcal{Q}^*\otimes\mathcal{O}(-1)$ on $\mathrm{Gr}(m-n,m)$ can be realized by a $\mathrm{U}(m-n)$ gauge theory with m fundamental chiral multiplet, m meson like chiral fields $M_1,\ldots,M_m$ in the representation $\det^{-1}$ of $\mathrm{U}(m-n)$ and a chiral multiplet $\tilde{\Phi}$ in the $(\mathbf{m}-\mathbf{n})^*\otimes\det$ representation with the superpotential $W=\sum_{i=1}^m M_i\tilde{\Phi}\Phi_i$. The F-term equation gives $\mathcal{Q}^*\otimes\mathcal{O}(-1)$ on $\mathrm{Gr}(m-n,m)$. We expect that the matter context of $\mathrm{U}(m-n)$ GLSM is m fundamental chiral multiplets Φ, m chiral multiplets M_i in the representation $\det^{-1}$, a chiral multiplet $\tilde{\Phi}$ in the representation $(\mathbf{m}-\mathbf{n})^*\otimes\det$ and chiral multiplets $P_2,\ldots,P_s$ in the representation $\det^{-q_l}$ with the superpotential $W=\sum_{i=1}^m M_i\tilde{\Phi}\Phi_i+\sum_{l=2}^s P_lG_l(\Phi)$.

We compute the Yukawa coupling in both sides and see the agreement. We first consider the U(4) GLSM description of $X^{4,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)}$. The field content is listed in table 7. The Yukawa coupling is given by

$$\begin{aligned}
\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0} &= -\frac{1}{24}\sum_{\mathbf{k}\in\mathbb{Z}^4_{\geq 0}}(-z)^{\sum_{a=1}^4 k_a}\oint_{\sigma=0}\prod_{a=1}^4\frac{d\sigma_a}{2\pi\sqrt{-1}}\left(\sum_{a=1}^4\sigma_a\right)^3\\
&\quad\times\prod_{1\leq a<b\leq 4}(\sigma_a-\sigma_b)^2\,\frac{\prod_{a=1}^4(-\sigma_a-\sum_{b=1}^4\sigma_b)^{k_a+\sum_{b=1}^4 k_a+1}(-\sum_{b=1}^4\sigma_b)^{\sum_{b=1}^4 k_a+1}}{\prod_{a=1}^4\sigma_a^{6(k_a+1)}}\\
&= 14(3+170z+10557z^2+650876z^3+40150735z^4)+\cdots\,. \qquad (5.3)
\end{aligned}$$

	ϕ_i	$M_i, (i = 1, \cdots, 5)$	p_1	$\tilde{\phi}$
$\mathrm{U}(2)_G$	$\mathbf{2}$	$\det^{-1}$	$\det^{-1}$	$\mathbf{2}^* \otimes \det$
$\mathrm{U}(1)_R$	0	2	2	0

Table 8. Field contents of dual U(2) GLSM for $X^{4,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)}$.

	$\phi_i, (i = 1, \cdots, 5)$	p_1	p_2
$\mathrm{U}(2)_G$	$\mathbf{2}$	$\mathbf{2}^* \otimes \det^{-1}$	$\det^{-2}$
$\mathrm{U}(1)_R$	0	2	2

Table 9. Field content of GLSM for $X^{2,5}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(2)}$.

The matter content of the dual U(2) description of $X^{4,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)}$ is given in table 8. The Yukawa coupling is given by

$$\begin{aligned}\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0} &= \frac{1}{2}\sum_{\mathbf{k}\in\mathbb{Z}^2_{\geq 0}}(-z)^{\sum_{a=1}^2 k_a}\oint_{\sigma=0}\prod_{a=1}^{2}\frac{d\sigma_a}{2\pi\sqrt{-1}}\left(\sum_{a=1}^{2}\sigma_a\right)^3 \\ &\quad\times(\sigma_1-\sigma_2)^2\,\frac{(-\sigma_1-\sigma_2)^{7(k_1+k_2)+7}}{\prod_{a=1}^2\sigma_a^{7(k_a+1)}} \\ &= 14(3-170z+10557z^2-650876z^3+40150735z^4)+\cdots. \end{aligned} \tag{5.4}$$

We find (5.4) agrees with (5.3) up to the change $z \to -z$ of signs. Note that (5.4) has the same Yukawa coupling as $X^{2,7}_{\mathcal{O}(1)^{\oplus 7}}$, which is known by [11] to be deformation-equivalent to $X^{4,6}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(1)}$.

Next we study the dual U(3) description of $X^{2,5}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(2)}$. The field content of U(2) GLSM is given in table 9. In the original U(2) GLSM, the Yukawa coupling of $X^{2,5}_{\mathcal{S}^*\otimes\mathcal{O}(1)\oplus\mathcal{O}(2)}$ is given by

$$\begin{aligned}\langle(\mathrm{Tr}\sigma)^3\rangle_{\hbar=0} &= \frac{1}{2}\sum_{\mathbf{k}\in\mathbb{Z}^2_{\geq 0}}(-z)^{\sum_{a=1}^2 k_a}\oint_{\sigma=0}\prod_{a=1}^{2}\frac{d\sigma_a}{2\pi\sqrt{-1}}\left(\sum_{a=1}^{2}\sigma_a\right)^3(\sigma_1-\sigma_2)^2 \\ &\quad\times\frac{\prod_{a=1}^2(-\sigma_a-\sigma_1-\sigma_2)^{k_a+k_1+k_2+1}(-2\sigma_1-2\sigma_2)^{2(k_1+k_2)+1}}{\prod_{a=1}^2\sigma_a^{5(k_a+1)}} \\ &= 24(1-136z+18480z^2-2511104z^3+341214464z^4)+\cdots \end{aligned} \tag{5.5}$$

From our observation, we expect that the matter content of U(3) GLSM is given by table 10 with the superpotential $W = \sum_{i=1}^5 M_i\tilde{\Phi}\Phi_i + P_1 G(\Phi)$. Here $G(\Phi)$ is a homogeneous polynomial of degree 4.

	ϕ_i	$M_i, (i = 1, \cdots, 5)$	p_1	$\tilde{\phi}$
$\mathrm{U}(3)_G$	$\mathbf{3}$	$\det^{-1}$	$\det^{-2}$	$\mathbf{3}^* \otimes \det$
$\mathrm{U}(1)_R$	0	2	2	0

Table 10. Field contents of dual U(3) GLSM for $X^{2,5}_{\mathcal{S}^* \otimes \mathcal{O}(1) \oplus \mathcal{O}(2)}$

The Yukawa coupling on the dual side is

$$\begin{aligned}
\langle (\mathrm{Tr}\sigma)^3 \rangle_{\hbar=0} =& \frac{1}{6} \sum_{\mathbf{k} \in \mathbb{Z}^n} z^{\sum_{a=1}^n k_a} \sum_{\sigma_*} \text{JK-Res}(\mathbf{Q}(\sigma_*), \eta) \\
&\times \left(\sum_{a=1}^3 \sigma_a \right)^3 \prod_{a<b} (\sigma_a - \sigma_b)^2 \frac{(-\sum_{a=1}^3 \sigma_a)^{5\sum_{a=1}^3 k_a + 5} (-2\sum_{a=1}^3 \sigma_a)^{2\sum_{a=1}^3 k_a + 1}}{\prod_{a<b} (\sigma_a + \sigma_b)^{k_a + k_b + 1} \prod_{a=1}^3 \sigma_a^{5k_a + 5}} d^3\sigma \\
=:& \sum_{\sigma_*} \text{JK-Res}(\mathbf{Q}(\sigma_*), \eta) \omega.
\end{aligned} \tag{5.6}$$

Eq. (5.6) is a degenerate case and we use a constructive definition of Jeffrey-Kirwan residue operation [33]. The singular hyperplanes $H_{ab} = \{\sigma_a + \sigma_b = 0\}$ and $H_a = \{\sigma_a = 0\}$ meet at the origin $\sigma_* = 0$. The Jeffrey-Kirwan operation is not defined in the physical choice of vector $\eta = (1, 1, 1)$, and we slightly shift η inside the geometric phase. For example, we can take $\eta = (1, 1 + \varepsilon, 1 - \varepsilon), \varepsilon < 1$. Then eight flags will contribute to the residue operation. But the iterated residue for six of them gives zero in an order-by-order computation. The following iterated residues give non-zero contributions;

$$\operatorname*{Res}_{\sigma_3=0} \operatorname*{Res}_{\sigma_1=0} \operatorname*{Res}_{\sigma_2=0} \omega = \frac{8}{3}(9 - 1216z + 171232z^2 - 19353984z^3 + 5393384448z^4) + \cdots, \tag{5.7}$$

and

$$\operatorname*{Res}_{\tilde{\sigma}_3=0} \operatorname*{Res}_{\tilde{\sigma}_2=0} \operatorname*{Res}_{\tilde{\sigma}_1=0} \omega = -\frac{64}{3}(z + 614z^2 + 405744z^3 + 290306784z^4) + \cdots, \tag{5.8}$$

with $\tilde{\sigma}_1 := \sigma_1 + \sigma_2, \tilde{\sigma}_2 := \sigma_2 + \sigma_3, \tilde{\sigma}_3 := \sigma_1 + \sigma_3$. Then the Yukawa coupling is

$$\begin{aligned}
\langle (\mathrm{Tr}\sigma)^3 \rangle_{\hbar=0} &= \operatorname*{Res}_{\sigma_3=0} \operatorname*{Res}_{\sigma_1=0} \operatorname*{Res}_{\sigma_2=0} \omega + \operatorname*{Res}_{\tilde{\sigma}_3=0} \operatorname*{Res}_{\tilde{\sigma}_2=0} \operatorname*{Res}_{\tilde{\sigma}_1=0} \omega \\
&= 24(1 - 136z + 18480z^2 - 2511104z^3 + 341214464z^4) + \cdots,
\end{aligned} \tag{5.9}$$

in complete agreement with (5.5).

6 Summary

We studied genus-zero Gromov-Witten invariants of CY 3-folds defined as complete intersections in Grassmannians by using equivariant A-twisted GLSM on S^2. The Yukawa coupling can be calculated from the cubic correlation function of the scalar in the vector multiplet. In order to obtain the Yukawa coupling in the flat coordinate, we have to compute the mirror map, which gives the complex structure moduli as a function of the flat coordinate. The mirror map can be computed from the Z-function appearing in the factorization of correlation functions. We have also studied Seiberg-like duality between GLSMs with different ranks. We studied only the cases when the gauge group is $\mathrm{U}(n)$, and it would be interesting to extend our analysis to other gauge groups and quiver gauge theories.

Cohomological Yang-Mills theories on curved backgrounds have recently been studied by coupling to background topological gravity in [34], which includes supersymmetric background studied in [2]. It is also interesting to perform the supersymmetric localization computation for GLSMs on these backgrounds, and figure out their interpretation as low energy target space geometry.

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