

# Highly symmetric D-brane-anti-D-brane effective actions

**Ehsan Hatefi**

*Institute for Theoretical Physics, TU Wien,  
Wiedner Hauptstrasse 8-10/136, A-1040 Vienna, Austria  
Mathematical Institute, Faculty of Mathematics,  
Charles University, P-18675, Czech Republic*

*E-mail:* [ehsan.hatefi@tuwien.ac.at](mailto:ehsan.hatefi@tuwien.ac.at)

**ABSTRACT:** The entire S-matrix elements of four, five and six point functions of D-brane-anti D-brane system are explored. To deal with symmetries of string amplitudes as well as their all order  $\alpha'$  corrections we first address a four point function of one closed string Ramond-Ramond (RR) and two real tachyons on the world volume of brane-anti brane system. We then focus on symmetries of string theory as well as universal tachyon expansion to achieve both string and effective field theory of an RR and three tachyons where the complete algebraic analysis for the whole S-matrix  $\langle V_{C-1} V_{T-1} V_{T^0} V_{T^0} \rangle$  was also revealed. Lastly, we employ all the conformal field theory techniques to  $\langle V_{C-1} V_{T-1} V_{T^0} V_{T^0} V_{T^0} \rangle$ , working out with symmetries of theory and find out the expansion for the amplitude to be able to precisely discover all order singularity structures of D-brane-anti-D-brane effective actions of string theory. Various remarks about the so called generalized Veneziano amplitude and new string couplings are elaborated as well.

**KEYWORDS:** p-branes, Superstrings and Heterotic Strings, Chern-Simons Theories, String Field Theory

**ARXIV EPRINT:** [1707.06609](https://arxiv.org/abs/1707.06609)

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**1 Introduction**

By dealing with unstable branes, one might have some motivations to gain not only more information about supersymmetry breaking but also explore new couplings on various time dependent backgrounds as well as working out with the properties of different string theories [1–9]. Insisting on D-brane anti-D-brane systems, one may reveal Sakai- Sugimoto model [10–12], the low energy physics of some of QCD models and spontaneous symmetry breaking in the appearance of holographic QCD models [13, 14] where flavour branes are inserted by different parallel branes and anti branes are taken into account within some backgrounds that are dual to colour confined phenomenon. A brane anti-brane configuration can be thought of a probe if  $N_f \ll N_c$ . As it is evident tachyonic strings play the fundamental role in instability of these systems, hence it is crucial to properly keep track of these tachyon modes in both String and Effective Field Theory (EFT) parts.

The effective actions that include tachyon modes was found by A.Sen and others in [15, 16] which could potentially explain a few remarks and phenomena like the decays of non-BPS branes [17, 18]. One could follow the established argument in [8, 9], clarifying how non-BPS EFT has inserted massless states and in particular tachyons. Within some reasonable field content we have also dealt with non-BPS actions in [19]. One may also talk about tachyon condensation for brane anti brane system [20] as well.

Whenever the distance between brane and anti-brane takes the value of smaller than string length scale two real tachyon modes would pop in this configuration, thus it made sense to replace them in an EFT and try to employ their dynamics. To do so, one needs to first learn how to embed them in an EFT so that the consistent results with string amplitudes come along. One can point out to a recent paper [21] on the dynamics of brane anti brane system which puts forward more evidences towards brane actions in the context of EFT and loop divergences. Other application for this brane-anti-brane system would be Brane production [22–30] as well as describing inflation in string theory in the context of KKLT [31–33].

We would like to make contact with scattering amplitudes to fix the ambiguities that appeared in each order of string theory and the reason for that is simply because not

only are we able to discover new couplings based on direct computations but also one fixes for once all the exact coefficients of string corrections in both IIB, IIA.<sup>1</sup> One even can go further and deal with the thermodynamical facets of brane anti brane system, where it is also recognized that at finite temperature this D-brane anti D-brane is being stabilised and can be entirely associated to black holes for which some implications have been devoted to either AdS/CFT or M-theory approach [49, 50]. D-brane-Anti-D-brane system has also been affecting on the stability of KKLT or in Large Volume Scenario and string compactifications [51, 52]. Given a tied relation between D-branes and RR fields [53] we just point out on the bound states of the branes [54] as well. Having set the fact that no duality transformation exists for non-BPS branes, one would have to emphasize that the only way of getting exact all order  $\alpha'$  corrections of effective couplings in string theory is just through CFT and scattering methods. To notify further comments we just demonstrate [44]. Last but not least one might head off reading all standard EFT methods for both Wess-Zumino (WZ) and (non)-BPS DBI effective actions that are verified in detail in [55, 56].

The paper is organised so that first we warm up with details on 3 and 4 point functions of D-brane anti-D-brane system, basically we fully address the S-matrix of a closed string Ramond-Ramond (RR) field and 2 tachyons [57] and explain how one does reconstruct all order exact  $\alpha'$  corrections in this context.

We then move on to observe more hidden symmetries in non-BPS context and as such we employ all the Conformal Field Theory (CFT) methods to a five point function of  $\langle V_{C-1}V_{T-1}V_{T^0}V_{T^0} \rangle$  in both type IIA and IIB. Given the selection rules [58], EFT and the entire algebraic solutions for integrals on 5-point functions we guess an expansion and test our guess with exact solutions of the integrals related to that amplitude and then produce all the infinite massless/ tachyon poles and come to an agreement with both string and field theory. Eventually we try to address for the first time a six point function of D-brane anti-D-brane system, deriving  $\langle V_{C-1}V_{T-1}V_{T^0}V_{T^0}V_{T^0} \rangle$  S-matrix and given all symmetries of amplitude we discover its expansion. We also show that in a particular limit that is being called soft limit  $4k_2.p \rightarrow 1$  the algebraic solutions for all 6-point functions can be found out. It is worth mentioning that within this limit all massless poles of the S-Matrix can be clearly observed and to be regenerated from an EFT argument as well. Lastly, we construct all order higher derivative corrections to four tachyon couplings in the context of brane anti-brane system.

## 2 Lower order D-brane-anti-D-brane effective actions

The so called effective actions of a  $D_p\bar{D}_p$ -brane of IIA(B) may be found out by inserting tachyonic modes in WZ and DBI effective actions. By taking into account just 2 unstable branes of IIB(A) and projecting them out through  $(-1)^{FL}$  operator one derives such an action. In order to simplify the field content, either 2 tachyons and a gauge (scalar) fields

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<sup>1</sup>For higher point functions we introduce [34–43] and to properly address string corrections we provide these references [44–46]. All order  $\alpha'$  BPS corrections as well as a conjecture [47, 48] on universal  $\alpha'$  corrections was illustrated that shockingly applied to both non-BPS and supersymmetric cases.

take part in the action whereas RR will contribute among Chern-Simons or WZ action and other fields act on DBI as follows [59]

$$S_{\text{DBI}} = - \int d^{p+1} \sigma \text{Tr} \left( V(\mathcal{T}) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a \mathcal{T} D_b \mathcal{T})} \right), \quad (2.1)$$

Trace in (2.1) has to be symmetric for  $F_{ab}, D_a \mathcal{T}, \mathcal{T}$  matrices. The definitions of the entire matrices are given by

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix}, \quad D_a \mathcal{T} = \begin{pmatrix} 0 & D_a T \\ (D_a T)^* & 0 \end{pmatrix}, \quad \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix} \quad (2.2)$$

with  $F_{ab}^{(i)} = \partial_a A_b^{(i)} - \partial_b A_a^{(i)}$  and  $D_a T = \partial_a T - i(A_a^{(1)} - A_a^{(2)})T$ .

Having made use of the ordinary trace (2.1) gets substituted to A. Sen's action [60] if one makes all kinetic terms symmetrized and evaluates the trace, however, it is shown by CFT and scattering amplitudes in [59, 61] that Sen's effective action does not produce consistent result with string amplitudes. Tachyon's potential in the context of type II scattering amplitude is shown by

$$V(|T|) = 1 + \pi\alpha' m^2 |T|^2 + \frac{1}{2} (\pi\alpha' m^2 |T|^2)^2 + \dots$$

where  $m_T^2 = -1/(2\alpha')$  and  $T_p$  is the tension of a p-brane. The expansion is also made consistent result with potential of  $V(|T|) = e^{\pi\alpha' m^2 |T|^2}$  that came from BSFT as well [62].

We argued in [59] that just above effective action can entirely and consistently generate all infinite singularities as well as contact terms of sting theory. Indeed we also dicussed that the presence of new mixing couplings like  $F^{(1)} \cdot F^{(2)}$  as well as  $D\phi^{(1)} \cdot D\phi^{(2)}$  is necessary to derive the actual and consistent results that are matched string computations with EFT. The Lagrangian of SYM (at quadratic level) in the presence of tachyons must be modified and has to have the following structures/interactions [59]:

$$\begin{aligned} \mathcal{L}_{\text{DBI}} = & -T_p(2\pi\alpha') \left( m^2 |T|^2 + DT \cdot (DT)^* - \frac{\pi\alpha'}{2} \left( F^{(1)} \cdot F^{(1)} + F^{(2)} \cdot F^{(2)} \right) \right) + T_p(\pi\alpha')^3 \\ & \times \left( \frac{2}{3} DT \cdot (DT)^* \left( F^{(1)} \cdot F^{(1)} + F^{(1)} \cdot F^{(2)} + F^{(2)} \cdot F^{(2)} \right) \right) \\ & + \frac{2m^2}{3} |\tau|^2 \left( F^{(1)} \cdot F^{(1)} + F^{(1)} \cdot F^{(2)} + F^{(2)} \cdot F^{(2)} \right) \\ & - \frac{4}{3} \left( (D^\mu T)^* D_\beta T + D^\mu T (D_\beta T)^* \right) \left( F^{(1)\mu\alpha} F_{\alpha\beta}^{(1)} + F^{(1)\mu\alpha} F_{\alpha\beta}^{(2)} + F^{(2)\mu\alpha} F_{\alpha\beta}^{(2)} \right) \end{aligned} \quad (2.3)$$

WZ action for brane anti brane system with  $C$  becomes a sum on RR potentials  $C = \sum_n (-i)^{\frac{p-m+1}{2}} C_m$  is [63–65]

$$S = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \left( e^{i2\pi\alpha' F^{(1)}} - e^{i2\pi\alpha' F^{(2)}} \right), \quad (2.4)$$

In [66, 67] it was justified how to consider the tachyons in the effective actions where another approach would be making contact with superconnection of the non-commutative

geometry [68–70] to be

$$S_{\text{WZ}} = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \text{STr} e^{i2\pi\alpha' \mathcal{F}} \quad (2.5)$$

where the curvature and super-connection are defined accordingly as

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$$

and

$$i\mathcal{A} = \begin{pmatrix} iA^{(1)} & \beta T^* \\ \beta T & iA^{(2)} \end{pmatrix},$$

In [59] it is also verified how to derive the curvature as follows

$$i\mathcal{F} = \begin{pmatrix} iF^{(1)} - \beta^2|T|^2 & \beta(DT)^* \\ \beta DT & iF^{(2)} - \beta^2|T|^2 \end{pmatrix},$$

with  $F^{(i)} = \frac{1}{2}F_{ab}^{(i)} dx^a \wedge dx^b$  and  $DT = [\partial_a T - i(A_a^{(1)} - A_a^{(2)})T] dx^a$ . Lastly one does extract various couplings of the WZ action (2.5) to be able to lead to diverse WZ couplings that are needed for consistent result of EFT with string side as follows

$$\begin{aligned} C \wedge \text{STr} i\mathcal{F} &= C_{p-1} \wedge (F^{(1)} - F^{(2)}) & (2.6) \\ C \wedge \text{STr} i\mathcal{F} \wedge i\mathcal{F} &= C_{p-3} \wedge \left\{ F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)} \right\} \\ &\quad + C_{p-1} \wedge \left\{ -2\beta^2|T|^2(F^{(1)} - F^{(2)}) + 2i\beta^2 DT \wedge (DT)^* \right\} \\ C \wedge \text{STr} i\mathcal{F} \wedge i\mathcal{F} \wedge i\mathcal{F} &= C_{p-5} \wedge \left\{ F^{(1)} \wedge F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)} \wedge F^{(2)} \right\} \\ &\quad + C_{p-3} \wedge \left\{ -3\beta^2|T|^2(F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)}) \right. \\ &\quad \quad \left. + 3i\beta^2(F^{(1)} + F^{(2)}) \wedge DT \wedge (DT)^* \right\} \\ &\quad + C_{p-1} \wedge \left\{ 3\beta^4|T|^4 \wedge (F^{(1)} - F^{(2)}) - 6i\beta^4|T|^2 DT \wedge (DT)^* \right\} \end{aligned}$$

The world volume of a non-BPS brane of both type IIA (IIB) string theory includes a real tachyon and the three point function of an RR and a tachyon  $\langle V_{C^{-1}} V_{T^{-1}} \rangle$  based on direct CFT methods was given in [71] to be

$$\mathcal{A}^{C^{-1}T^{-1}} \sim -2i \text{Tr} (P_- \mathbb{H}_{(n)} M_p) \quad (2.7)$$

Using momentum conservation one reveals that  $p^a p_a = k^2 = -\frac{1}{4}$ , the string amplitude (2.7) can be reconstructed out in an EFT by  $(2\pi\alpha'\beta'\mu'_p \int C_p \wedge DT)$  coupling where there is no singularity structure for this amplitude at all.

On the other hand, the world volume of a D-brane-anti D-brane system involves two real tachyons. Using direct CFT methods [72] the four point function of an RR and two

tachyons  $\langle V_{C-1}V_{T-1}V_{T^0} \rangle$  can be explored. To get familiar with notations we just warm up with this calculation where the vertices with their CP factors for brane- anti brane are

$$\begin{aligned} V_T^{(-1)}(x_1) &= e^{-\phi(x_1)} e^{\alpha' i k_1 \cdot X(x_1)} \lambda \otimes \sigma_2 \\ V_T^{(0)}(x_2) &= \alpha' i k_2 \cdot \psi(x_2) e^{\alpha' i k_2 \cdot X(x_2)} \lambda \otimes \sigma_1 \\ V_{RR}^{(-1)}(z, \bar{z}) &= (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{i \frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i \frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \otimes \sigma_3 \end{aligned} \quad (2.8)$$

world-sheet is taken as disk so that all open strings and RR are located on the boundary and the middle of disk accordingly. On-shell conditions are  $p^2 = 0, k_1^2 = k_2^2 = 1/4$ , notations are also followed by

$$P_- = \frac{1}{2}(1 - \gamma^{11}), \mathbb{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n}, (P_- \mathbb{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta} (P_- \mathbb{H}_{(n)})_{\delta\beta}. \quad (2.9)$$

where for type IIA (type IIB)  $n = 2, 4, a_n = i$  ( $n = 1, 3, 5, a_n = 1$ ) stands. To deal with standard holomorphic functions and based on various change of variables, the doubling trick has been set as follows

$$\tilde{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}), \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \rightarrow M_\alpha^\beta S_\beta(\bar{z}),$$

with further ingredients as

$$D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \frac{\pm i}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ even} \\ \frac{\pm 1}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \gamma_{11} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ odd} \end{cases}$$

Making use of the above doubling trick, we can now head off and start working with the following two point functions for all the fields of  $X^\mu, \psi^\mu, \phi$  as follows

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z-w), \\ \langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} (z-w)^{-1}, \\ \langle \phi(z) \phi(w) \rangle &= -\log(z-w). \end{aligned} \quad (2.10)$$

Using gauge fixing as  $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$  and taking  $u = -\frac{\alpha'}{2}(k_1 + k_2)^2$ , the ultimate form of the amplitude is<sup>2</sup>

$$\mathcal{A}^{C^{-1}T^{-1}T^0} \sim 4k_{2a} \int_{-\infty}^{\infty} dx (2x)^{-2u-1} (1+x^2)^{2u} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^a)$$

and the final result can be read [57]

$$\mathcal{A}^{C^{-1}T^{-1}T^0} = \frac{i\mu_p}{4} 2\pi \frac{\Gamma(-2u)}{\Gamma(1/2-u)^2} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \gamma^a) k_{2a} \quad (2.11)$$

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<sup>2</sup> $\alpha' = 2$  is set.

where  $\mu_p$  is RR charge and the trace for  $\gamma^{11}$  kept fixed for the following as well

$$p > 3, \quad H_n = *H_{10-n}, \quad n \geq 5.$$

We actually discussed all the proper expansion of S-matrices within detail in [73], so that the entire S-matrix elements are recovered in an EFT by sending either  $k_i \cdot k_j \rightarrow 0$  or  $(k_i + k_j)^2 \rightarrow 0$  which means that one indeed is able to regenerate massless /tachyonic singularities of different configurations. We knew that a non-zero coupling  $C_{p-1} \wedge F$  exists, and also there is non-vanishing two tachyons and a gauge field coupling, thus using momentum conservation along the world volume of brane the correct expansion for D-brane-anti D-brane was explored to be

$$u = -p^a p_a \rightarrow 0. \quad (2.12)$$

as also clarified in [61]. Note that as we have seen in an RR and a tachyon amplitude this constraint for non- BPS branes gets enhanced to

$$p^a p_a \rightarrow -m_T^2 = \frac{1}{4} \quad (2.13)$$

The expansion around  $u \rightarrow 0$  is

$$2\pi \frac{\Gamma(-2u)}{\Gamma(1/2-u)^2} = -\frac{1}{u} + \sum_{m=-1}^{\infty} c_m (u)^{m+1}, \quad c_{-1} = 4\ln(2), \quad c_0 = \left( \frac{\pi^2}{6} - 8\ln(2)^2 \right), \dots$$

The only u-channel massless gauge field pole of this particular *CTT* S-matrix can be regenerated by following sub-amplitude in an EFT

$$\mathcal{A} = V_a(C_{p-1}, A^{(1)}) G_{ab}(A) V_b(A, A^{(1)}, T_1, T_2) + V_a(C_{p-1}, A^{(2)}) G_{ab}(A) V_b(A, A^{(2)}, T_1, T_2)$$

where the presence of Chern-Simons coupling on the brane anti-brane system is needed as follows

$$i\mu_p(2\pi\alpha') \int_{\Sigma_{p+1}} \epsilon^{a_0 \dots a_p} \left( \text{Tr} (C_{a_0 \dots a_{p-2}} d_{a_{p-1}} (A_{1a_p} - A_{2a_p})) \right), \quad (2.14)$$

The off-shell propagator must be  $A^{(1)}$  and  $A^{(2)}$  to be able to have consistent result with both string and an EFT. The propagator and vertices are accordingly defined by

$$\begin{aligned} G_{ab}(A) &= \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p(k^2)} \\ V_b(A^{(1)}, T_1, T_2) &= iT_p(2\pi\alpha')(k_1 - k_2)_b \\ V_b(A^{(2)}, T_1, T_2) &= -iT_p(2\pi\alpha')(k_1 - k_2)_b \\ V_a(C_{p-1}, A^{(1)}) &= i\mu_p(2\pi\alpha') \frac{1}{(p-1)!} \epsilon_{a_0 \dots a_{p-1} a} C^{a_0 \dots a_{p-2}} k^{a_{p-1}} \\ V_a(C_{p-1}, A^{(2)}) &= -i\mu_p(2\pi\alpha') \frac{1}{(p-1)!} \epsilon_{a_0 \dots a_{p-1} a} C^{a_0 \dots a_{p-2}} k^{a_{p-1}} \end{aligned} \quad (2.15)$$

Taking into account  $k^a = (k_1 + k_2)^a = -p^a$  as well as replacing them in EFT amplitude, we gain the field theory amplitude as follows

$$\mathcal{A} = 4i\mu_p \frac{1}{p!u} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} k_{2a} \quad (2.16)$$

which is exactly as derived in string theory amplitude (2.11). In [74] we also constructed all order  $\alpha'$  corrections of one RR and two tachyons of D-brane anti D-brane system as

$$i\mu_p (2\pi\alpha')^2 C_{(p-1)} \wedge \text{Tr} \left( \sum_{m=-1}^{\infty} c_m (\alpha' (D^b D_b))^{m+1} DT \wedge DT^* \right) \quad (2.17)$$

### 3 All order $\langle V_{C^{-1}} V_{T^{-1}} V_{T^0} V_{T^0} \rangle$ S-matrix

All the correlators of this S-matrix can be investigated. We define

$$s = \frac{-\alpha'}{2} (k_1 + k_3)^2, \quad t = \frac{-\alpha'}{2} (k_1 + k_2)^2, \quad u = \frac{-\alpha'}{2} (k_2 + k_3)^2$$

One finds out the amplitude after the gauge fixing  $(x_1, x_2, x_3, z, \bar{z}) = (0, 1, \infty, z, \bar{z})$  to be

$$\begin{aligned} \mathcal{A}^{C^{-1}T^{-1}T^0T^0} &\sim \int \int dz d\bar{z} (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} (-4ik_{2a}k_{3b}) x_{45}^{-2(t+s+u+1)} |z|^{2t+2s} |1-z|^{2t+2u} \\ &\times \left[ (\Gamma^{ba} C^{-1})_{\alpha\beta} + 2\eta^{ab} (C^{-1})_{\alpha\beta} \left( \frac{1-x}{z-\bar{z}} \right) \right] \end{aligned}$$

where  $z = x_4 = x + iy, \bar{z} = x_5 = x - iy, x_{ij} = x_i - x_j$ . Notice that all integrations on upper half plane are carried out on RR position as follows

$$\int \int d^2z |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^d$$

and the final result on those moduli spaces has been verified in detail for  $d = 0, 1$  in [75] and for  $d = 2$  in [73]. Using some algebraic analysis the ultimate form of S-matrix is obtained to be

$$\begin{aligned} \mathcal{A}^{C^{-1}T^{-1}T^0T^0} &= \mathcal{A}_1 + \mathcal{A}_2 \\ \mathcal{A}_1 &\sim -4ik_{2a}k_{3b} \text{Tr} (P_- \mathbb{H}_{(n)} M_p \Gamma^{ba}) N_1 \\ \mathcal{A}_2 &\sim -4i \text{Tr} (P_- \mathbb{H}_{(n)} M_p) N_2 \end{aligned}$$

where the functions  $N_1, N_2$  are given as

$$\begin{aligned} N_1 &= (2)^{-2(t+s+u+1)} \pi \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u)}, \\ N_2 &= (2)^{-2(t+s+u)-3} \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u-1)}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u)} \end{aligned} \quad (3.1)$$

One normalizes the amplitude by  $3\pi^{-1/2}\beta'\mu'_p/8$  to be able to get consistent result with EFT. As can be explicitly seen, the amplitude is entirely symmetric with respect to interchanging



$s, t, u$  and for  $p - 1 = n$  case and it has an infinite massless gauge field poles in all  $s, t, u$ -channels and also for  $p + 1 = n$  case it does have an infinite tachyon poles in  $(s + t + u + 1)$ -channel poles ( $s' = s + \frac{1}{2}, t' = t + \frac{1}{2}, u' = u + \frac{1}{2}$ ). Given these arguments, the fact that the on-shell condition is

$$s + t + u = -p_a p^a - \frac{3}{4} \tag{3.2}$$

and  $p_a p^a \rightarrow 1/4$  for non-BPS D-branes, one obtains uniquely the expansion for  $CTTT$  as

$$\frac{1}{3} \left( (u \rightarrow 0, s, t \rightarrow -1/2), (s \rightarrow 0, u, t \rightarrow -1/2), (t \rightarrow 0, s, u \rightarrow -1/2) \right) \tag{3.3}$$

The expansion of  $N_1$  around (3.2) is

$$N_1 = \frac{2\pi\sqrt{\pi}}{3} \left( -\frac{1}{u} \sum_{n=-1}^{\infty} b_n (s' + t')^{n+1} + \sum_{p,n,m=0}^{\infty} c_{p,n,m} u^p (s't')^n (s' + t')^m + (u \leftrightarrow t) + (u \leftrightarrow s) \right)$$

where some coefficients are

$$b_{-1} = 1, \quad c_{p,0,0} = a_p, \quad a_0 = 4 \ln(2), \quad a_1 = \frac{\pi^2}{6} - 8 \ln(2)^2, \quad c_{0,1,0} = -14\zeta(3) \tag{3.4}$$

The string amplitude for this case is given by

$$\frac{24\beta'\mu'_p}{\sqrt{\pi}(p-1)!u} \sum_{n=-1}^{\infty} b_n (s' + t')^{n+1} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-2}} k_{2a_{p-1}} k_{3a_p} \tag{3.5}$$

These infinite u-channel gauge field poles can be produced in an EFT by the following sub-amplitude

$$\mathcal{A} = V_a^\alpha(C_{p-2}, T_1, A) G_{ab}^{\alpha\beta}(A) V_b^\beta(A, T_2, T_3) \tag{3.6}$$

where the vertices are

$$G_{ab}^{\alpha\beta}(A) = \frac{i\delta_{ab}\delta_{\alpha\beta}}{(2\pi\alpha')^2 T_p u}$$

$$V_b^\beta(A, T_2, T_3) = iT_p (2\pi\alpha') (k_2 - k_3)_b \text{Tr}(\lambda_2 \lambda_3 \Lambda^\beta)$$

$$V_a^\alpha(C_{p-2}, T_1, A) = 2\beta'\mu'_p (2\pi\alpha')^2 \frac{1}{(p-1)!} \epsilon_{a_1 \dots a_p} H^{a_1 \dots a_{p-1}} k_1^{a_p} \sum_{n=-1}^{\infty} b_n (\alpha' k_1 \cdot k)^{n+1} \text{Tr}(\lambda_1 \Lambda^\alpha)$$

$k$  is off-shell's gauge field momentum. Finally all contact interactions of this part of S-matrix or all order higher derivative corrections of  $C_{p-2} \wedge DT \wedge DT \wedge DT$  can be derived by the following coupling

$$8\beta'(\pi\alpha'^2)\mu'_p \sum_{p,n,m=0}^{\infty} c_{p,n,m} \left(\frac{\alpha'}{2}\right)^p (\alpha')^{2n+m} C_{p-2} \wedge \text{Tr} \left( D^{a_1} \dots D^{a_{2n}} D^{b_1} \dots D^{b_m} DT \wedge (D^a D_a)^p D_{b_1} \dots D_{b_m} (D_{a_1} \dots D_{a_n} DT \wedge D_{a_{n+1}} \dots D_{a_{2n}} DT) \right) \tag{3.7}$$

with  $\beta' = \frac{1}{\pi} \sqrt{\frac{6 \ln(2)}{\alpha'}}$  becomes normalization constant of WZ effective action of non-BPS branes. On the other hand all infinite tachyon channel poles in string side can be explored as

$$\left( -1 + \frac{1}{3} \sum_{n,m=0}^{\infty} e_{n,m} [(s' + t')^n (t' s')^{m+1} + (t', s' \rightarrow t', u') + (t', s' \rightarrow s', u')] \right) \quad (3.8)$$

$$\times \frac{24i\beta' \mu'_p}{(p+1)!(t+s+u+1)} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p}$$

with

$$e_{0,0} = -\pi^2/3, \quad e_{1,0} = 8\zeta(3), \quad e_{0,1} = \pi^4/45, \quad e_{1,1} = -32\zeta(5) + 8\zeta(3)\pi^2/3$$

All infinite tachyon poles are produced in an EFT by the following couplings and vertices

$$\mathcal{A} = V^\alpha(C_p, T) G^{\alpha\beta}(T) V^\beta(T, T, T, T) \quad (3.9)$$

The propagator and vertex  $V(C_p, T)$  are

$$G^{\alpha\beta}(T) = \frac{i\delta_{\alpha\beta}}{(2\pi\alpha') T_p (-k^2 - m^2)}$$

$$V^\alpha(C_p, T) = 2i\beta' \mu'_p (2\pi\alpha') \frac{1}{(p+1)!} \epsilon_{a_0 \dots a_p} H^{a_0 \dots a_p} \text{Tr}(\Lambda^\alpha)$$

Following Lagrangian is also needed

$$-T_p \text{Tr} \left( (\pi\alpha') m^2 T^2 + (\pi\alpha') D_a T D^a T - (\pi\alpha')^2 F_{ab} F^{ba} + T^4 \right) \quad (3.10)$$

Using (3.10) and the fact that off-shell tachyon in propagator is abelian, the vertex  $V^\beta(T, T, T, T)$  is constructed to be  $V^\beta(T, T, T, T) = -12iT_p \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \Lambda^\beta)$ . Considering it in (3.9), one reveals the tachyon pole of string amplitude in an EFT to be

$$24i\beta' \mu'_p \frac{1}{(p+1)!(s+t+u+1)} \epsilon_{a_0 \dots a_p} H^{a_0 \dots a_p}$$

In the next section we would like to perform an RR and four tachyon couplings on D-brane-anti-D-brane's world volume which is the generalization of Veneziano amplitude of four open string tachyons [76]. We observe that all infinite tachyon poles are reconstructed by  $(2\pi\alpha')^2 \mu_p \beta^2 \int C_{p-1} \wedge DT \wedge DT$  and all order extensions of four tachyon couplings.

#### 4 $\langle V_{C^{-1}(z, \bar{z})} V_{T^{-1}(x_1)} V_{T^0(x_2)} V_{T^0(x_3)} V_{T^0(x_4)} \rangle$ amplitude

In this section, given all symmetries that we discussed in the last sections and the fact that tachyonic expansion has been checked for all lower point functions of string amplitudes on the world volume of D-brane-Anti-D-brane systems such as  $CTT, CTT\phi, CTTA$  etc., we intend to show that all the infinite singularities of an RR and four tachyons on the world volume of D-brane-Anti-D-brane can be constructed out, although the precise and

algebraic forms of integrals are unknown. Various techniques have been demonstrated, fixing the position of open strings at  $x_1 = 0, 0 \leq x_2 \leq 1, x_3 = 1, x_4 = \infty$  and using 6 independent Mandelstam variables as  $s = -(\frac{1}{2} + 2k_1.k_3), t = -(\frac{1}{2} + 2k_1.k_2), v = -(\frac{1}{2} + 2k_1.k_4), u = -(\frac{1}{2} + 2k_2.k_3), r = -(\frac{1}{2} + 2k_2.k_4), w = -(\frac{1}{2} + 2k_3.k_4)$  the closed form of amplitude is

$$\begin{aligned}
 & 8k_{2a}k_{3b}k_{4c}2^{-1/2}(P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} \\
 & \times \int_0^1 dx_2 x_2^{-2t-1} (1-x_2)^{-2u-1} \iint d^2z |1-z|^{2s+2u+2w+1} |z|^{2t+2s+2v+1} \\
 & \times (z-\bar{z})^{-2(t+s+u+v+r+w+2)} |x_2-z|^{2t+2u+2r+1} \\
 & \times \left[ (\Gamma^{cba} C^{-1})_{\alpha\beta} + (z-\bar{z})^{-1} \left( 2\eta^{ab} (\gamma^c C^{-1})_{\alpha\beta} (1-x_2)^{-1} (x_2 - x x_2 - x + |z|^2) \right. \right. \\
 & \left. \left. - 2\eta^{ac} (\gamma^b C^{-1})_{\alpha\beta} (x_2 - x) + 2\eta^{bc} (\gamma^a C^{-1})_{\alpha\beta} (1-x) \right) \right] \quad (4.1)
 \end{aligned}$$

Hence  $CTTTT$  S-matrix makes sense for  $p-2 = n, p = n$  cases accordingly. If we use the particular limit  $4k_2.p \rightarrow 1$  that we called soft limit<sup>3</sup> then one reveals that the algebraic solutions for all the integrals on upper half plane can be entirely achieved, as explained in [73]. In this particular limit, the ultimate form of the amplitude for  $p-2 = n$  case will be given by

$$\begin{aligned}
 \mathcal{A}_1 & \sim 2^{-2(t+s+u+v+r+w)} \pi 2^{-1/2} k_{2a} k_{3b} k_{4c} \frac{8}{(p-2)!} \epsilon^{a_0 \dots a_{p-3} c b a} H_{a_0 \dots a_{p-3}} M_1, \quad (4.2) \\
 M_1 & = \frac{\Gamma(-2u)\Gamma(-2t)\Gamma(r-s)\Gamma(-t-v-r-\frac{1}{2})\Gamma(-u-r-w-\frac{1}{2})\Gamma(-t-s-u-v-r-w-\frac{3}{2})}{\Gamma(-2t-2u)\Gamma(-t-s-v-\frac{1}{2})\Gamma(-u-s-w-\frac{1}{2})\Gamma(-t-u-v-w-2r-1)}
 \end{aligned}$$

The second part of the amplitude makes sense for  $p = n$  case and after simplifications we obtain it as follows

$$\begin{aligned}
 \mathcal{A}_2 & \sim 2^{-2(t+s+u+v+r+w)} \pi 2^{-1/2} \frac{2}{p!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-1}} \left\{ -2k_{2a} \left( w + \frac{1}{2} \right) \left( r - s - \frac{1}{2} \right) M_3 \right. \\
 & \quad + 4k_{3a} \left( r + \frac{1}{2} \right) M_2 \left( t(1+t+r+v) - u(1+r+u+w) \right) \\
 & \quad \left. + k_{4a} M_3 \left( (1+2u)(1+r+u+w) + t + 2t(1+u+w+s) \right) \right\}, \\
 M_2 & = \frac{\Gamma(-2u)\Gamma(-2t)\Gamma(r-s+\frac{1}{2})\Gamma(-t-v-r-1)\Gamma(-u-r-w-1)\Gamma(-t-s-u-v-r-w-2)}{\Gamma(1-2t-2u)\Gamma(-t-s-v-\frac{1}{2})\Gamma(-u-s-w-\frac{1}{2})\Gamma(-t-u-v-w-2r-1)} \\
 M_3 & = \frac{\Gamma(-2u)\Gamma(-2t)\Gamma(r-s-\frac{1}{2})\Gamma(-t-v-r)\Gamma(-u-r-w-1)\Gamma(-t-s-u-v-r-w-2)}{\Gamma(-2t-2u)\Gamma(-t-s-v-\frac{1}{2})\Gamma(-u-s-w-\frac{1}{2})\Gamma(-t-u-v-w-2r-1)}
 \end{aligned}$$

Applying momentum conservation along the brane we derive

$$s + t + u + v + r + w = -p^a p_a - 2 \quad (4.3)$$

Given (4.3), the EFT methods that imposed to have an infinite massless either u or t-channel gauge field poles, also highly symmetries [77] of this amplitude ( it should be

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<sup>3</sup>As can be seen it is consistent with EFT couplings and produces all massless singularities of the S-matrix.

symmetric under exchanging Mandelstam variables  $u, t$  and other variables),  $k_i \cdot k_j \rightarrow 0$  and the fact that for tachyonic strings  $p^a p_a \rightarrow \frac{1}{4}$  we obtain the following expansion for this particular soft limit of the scattering amplitude as follows

$$\frac{1}{4} \left( v, w \rightarrow -\frac{1}{2} \right) \left\{ \begin{array}{l} u \rightarrow 0, \quad s \rightarrow -\frac{1}{4}, \quad (t, r, \rightarrow -\frac{1}{2}) \\ u \rightarrow 0, \quad r \rightarrow -\frac{1}{4}, \quad (t, s, \rightarrow -\frac{1}{2}) \\ t \rightarrow 0, \quad s \rightarrow -\frac{1}{4}, \quad (u, r, \rightarrow -\frac{1}{2}) \\ t \rightarrow 0, \quad r \rightarrow -\frac{1}{4}, \quad (u, s, \rightarrow -\frac{1}{2}) \end{array} \right.$$

Given the so called selection rules for non-BPS couplings of string theory [58] and also the fact that all kinetic terms have already been fixed in DBI action, one comes to know that for  $p-2 = n$  case we have just an infinite single massless gauge field poles in both u and t- channels. The expansion of  $M_1$  around the first above expansion is

$$2^{1/2} \Gamma(3/4)^2 \left( \frac{2}{u} - 8 \left( \frac{\ln 2}{2} + \frac{\pi}{4} - \frac{1}{2} \right) - \frac{8}{u} \left( \left( r + t + v + \frac{s+w}{2} \right) \ln 2 + \frac{s+w}{4} \pi - 2r + s - \frac{t+v+w}{2} \right) + \dots \right) \quad (4.4)$$

As it is clear from (4.2) and  $M_1$  for  $p-2 = n$  case the amplitude has an infinite u,t-channel massless gauge field poles and due to symmetries we just produce u-channel poles as follows.

$$\begin{aligned} & V^a(C_{p-3}, A^{(1)}, T_1, T_4) G^{ab}(A) V^b(A^{(1)}, T_2, T_3) \\ & + V^a(C_{p-3}, A^{(2)}, T_1, T_4) G^{ab}(A) V^b(A^{(2)}, T_2, T_3) \\ & G^{ab}(A) = \frac{i\delta^{ab}}{(2\pi\alpha')^2 T_p u} \\ & V^b(A^{(1)}, T_2, T_3) = iT_p(2\pi\alpha')(k_2 - k_3)^b \\ & V^b(A^{(2)}, T_2, T_3) = -iT_p(2\pi\alpha')(k_2 - k_3)^b \\ & V^a(C_{p-3}, A^{(1)}, T_1, T_4) = i\mu'_p \beta (2\pi\alpha')^3 \frac{1}{(p-2)!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-3}} k_{4a_{p-2}} k_{a_{p-1}} \\ & V^a(C_{p-3}, A^{(2)}, T_1, T_4) = -i\mu'_p \beta (2\pi\alpha')^3 \frac{1}{(p-2)!} \epsilon^{a_0 \dots a_{p-1} a} H_{a_0 \dots a_{p-3}} k_{4a_{p-2}} k_{a_{p-1}} \end{aligned} \quad (4.5)$$

where  $k_b = (k_2 + k_3)_b$  is the momentum of off-shell gauge field. Note that  $V^a(C_{p-3}, A, T_1, T_4)$  was derived from coupling

$$\beta \mu_p (2\pi\alpha')^3 \int_{\Sigma_{p+1}} \text{Tr} (C_{p-3} \wedge F \wedge DT_1 \wedge DT_4) \quad (4.6)$$

If we normalize the amplitude by  $\frac{2^{3/2} \beta \mu_p (2\pi)^2}{\Gamma(3/4)^2}$ , then one observes that the first u-channel gauge field pole is reconstructed in an EFT by (4.5) to be

$$i\mu_p \beta (2\pi\alpha')^2 \frac{4}{(p-2)! u} \epsilon^{a_0 \dots a_{p-3} cba} H_{a_0 \dots a_{p-3}} k_{2a} k_{3b} k_{4c}$$

The vertex of  $V^b(A^{(1)}, T_2, T_3)$  and single pole are fixed and have no corrections so to be able to generate all other massless u-channel poles one needs to apply higher derivative corrections to above WZ coupling (4.6), however, given the entire explanations of the previous sections we are no longer interested in doing so. We would rather carry out the rest of the analysis of the amplitude which has something to the extensions of four tachyon couplings.

On the other hand, having set the EFT arguments, one would become aware of the fact that for  $p = n$  case, the amplitude has an infinite  $(u + w + r + 1)$  channel (and due to symmetries  $(t + s + u + 1)$ ,  $(s + v + w + 1)$ ,  $(t + v + r + 1)$  channel as well) tachyon singularity structures and a double pole that can be shown later on.

As it is clear from  $\mathcal{A}_2$ , it has an infinite  $(u + w + r + 1)$  channel tachyon poles that can be explored in an EFT within the following sub-amplitude as follows

$$\begin{aligned} \mathcal{A} &= V^\alpha(C_{p-1}, T, T_1) G^{\alpha\beta}(T) V^\beta(T, T_2, T_3, T_4) \\ G^{\alpha\beta}(T) &= \frac{i\delta_{\alpha\beta}}{(2\pi\alpha') T_p(u + w + r + 1)} \\ V^\alpha(C_{p-1}, T, T_1) &= \mu_p \beta^2 (2\pi\alpha')^2 \frac{1}{(p-1)!} \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_{p-1}} k_{4a_p} \text{Tr}(\lambda_1 \Lambda^\alpha) \end{aligned} \quad (4.7)$$

To produce an infinite tachyon poles, one needs to employ the higher derivative corrections to four tachyon couplings as follows

$$T_p(\alpha')^{2+n+m} \sum_{m,n=0}^{\infty} (\mathcal{L}_1^{nm} + \mathcal{L}_2^{nm} + \mathcal{L}_3^{nm} + \mathcal{L}_4^{nm} + \mathcal{L}_5^{nm}) \quad (4.8)$$

where

$$\begin{aligned} \mathcal{L}_1^{nm} &= m^4 \text{Tr} \left( a_{n,m} D_{nm} [TTTT] - b_{n,m} D'_{nm} [TTTT] + \text{h.c.} \right) \\ \mathcal{L}_2^{nm} &= m^2 \text{Tr} \left( a_{n,m} [D_{nm} (TTD^\alpha T D_\alpha T) + D_{nm} (D^\alpha T D_\alpha TTT)] \right. \\ &\quad \left. - b_{n,m} [D'_{nm} (TD^\alpha T T D_\alpha T) + D'_{nm} (D^\alpha T T D_\alpha TT)] + \text{h.c.} \right) \\ \mathcal{L}_3^{nm} &= -\text{Tr} \left( a_{n,m} D_{nm} [D_\alpha T D_\beta T D^\beta T D^\alpha T] - b_{n,m} D'_{nm} [D_\alpha T D^\beta T D_\beta T D^\alpha T] + \text{h.c.} \right) \\ \mathcal{L}_4^{nm} &= -\text{Tr} \left( a_{n,m} D_{nm} [D_\alpha T D_\beta T D^\alpha T D^\beta T] - b_{n,m} D'_{nm} [D_\beta T D^\beta T D_\alpha T D^\alpha T] + \text{h.c.} \right) \\ \mathcal{L}_5^{nm} &= \text{Tr} \left( a_{n,m} D_{nm} [D_\alpha T D^\alpha T D_\beta T D^\beta T] - b_{n,m} D'_{nm} [D_\alpha T D_\beta T D^\alpha T D^\beta T] + \text{h.c.} \right) \end{aligned}$$

with following definition for  $D_{nm}$  and  $D'_{nm}$  derivative operators as

$$\begin{aligned} D_{nm}(EFGH) &\equiv D_{b_1} \dots D_{b_m} D_{a_1} \dots D_{a_n} E F D^{a_1} \dots D^{a_n} G D^{b_1} \dots D^{b_m} H \\ D'_{nm}(EFGH) &\equiv D_{b_1} \dots D_{b_m} D_{a_1} \dots D_{a_n} E D^{a_1} \dots D^{a_n} F G D^{b_1} \dots D^{b_m} H \end{aligned}$$

All order four tachyon extensions of  $V^\beta(T, T_2, T_3, T_4)$  might be explored from the higher derivative couplings (4.8) as below

$$\begin{aligned}
 & 2iT_p(\alpha')^{n+m}(a_{n,m} - b_{n,m})\text{Tr}(\lambda_2\lambda_3\lambda_4\Lambda^\beta) \left[ u'r' \left( (k_1 \cdot k)^m (k_2 \cdot k_4)^n + (k_1 \cdot k)^n (k_2 \cdot k_4)^m \right. \right. \\
 & \quad \left. \left. + (k_2 \cdot k)^m (k_2 \cdot k_3)^n + (k_2 \cdot k)^n (k_2 \cdot k_3)^m + (k_1 \cdot k)^m (k_2 \cdot k)^n + (k_1 \cdot k)^n (k_2 \cdot k)^m \right. \right. \\
 & \quad \left. \left. + (k_2 \cdot k_3)^m (k_2 \cdot k_4)^n + (k_2 \cdot k_3)^n (k_2 \cdot k_4)^m \right) + (u, r \rightarrow u, w) + (u, r \rightarrow r, w) \right]
 \end{aligned}$$

$k^a$  is off-shell tachyon's momentum. Taking into account this vertex in (4.7), one gains all tachyon poles in an EFT as follows

$$\begin{aligned}
 & 32i\beta^2\mu_p\epsilon^{a_0\cdots a_p}H_{a_0\cdots a_{p-1}}k_{4a_p}\frac{1}{(p-1)!(u+r+w+1)}\sum_{n,m=0}^{\infty}(a_{n,m} - b_{n,m}) \\
 & \quad \times \left[ u'r'(u'^m r'^n + u'^n r'^m) + u'w'(u'^m w'^n + u'^n w'^m) + r'w'(r'^m w'^n + r'^n w'^m) \right]
 \end{aligned}$$

One may re-write down the above amplitude as follows

$$\begin{aligned}
 & \sum_{n,m=0}^{\infty} e_{n,m} \left[ (u' + r')^n (u'r')^{m+1} + (u' + w')^n (u'w')^{m+1} + (r' + w')^n (r'w')^{m+1} \right] \\
 & \quad \times 16i\beta^2\mu_p\frac{\epsilon^{a_0\cdots a_p}H_{a_0\cdots a_{p-1}}k_{4a_p}}{(p-1)!(u+r+w+1)}
 \end{aligned}$$

These tachyon poles are exactly the same singularities that appeared in second part of string amplitude.

EFT imposes to us that the amplitude does have just a double pole for  $C_{p-1}$  case as

$$V(C_{p-1}, T_1, T_2)G(T_2)V_a(T_2, T_4, A)G_{ab}(A)V_b(A, T_2, T_3) \quad (4.9)$$

and the off-shell gauge field  $A$  needs to be both  $A^{(1)}$  and  $A^{(2)}$  with the following vertices derived

$$\begin{aligned}
 V(C_{p-1}, T_1, T_2) &= \beta^2\mu_p(2\pi\alpha')^2\frac{1}{p!}\epsilon^{a_0\cdots a_{p-1}c}H_{a_0\cdots a_{p-1}}k_{2c} \\
 V_a(T_2, T_4, A) &= T_p(2\pi\alpha')(k_{2a} + k_a) \\
 G_{ab}(A) &= \frac{i\delta^{ab}}{(2\pi\alpha')^2T_p u} \quad (4.10)
 \end{aligned}$$

$$\begin{aligned}
 V_b(A, T_2, T_3) &= T_p(2\pi\alpha')(k_2 - k_3)_b \\
 G(T_2) &= \frac{i}{(2\pi\alpha')T_p(u+r+w+1)} \quad (4.11)
 \end{aligned}$$

with  $k$  is off-shell gauge field's momentum. Inserting above vertices in (4.9) we are then able to reconstruct the double pole in an EFT as follows

$$\beta^2(2\pi\alpha')\mu_p\left(\frac{-1}{u(u+r+w+1)}\right)\frac{1}{2p!}\epsilon^{a_0\cdots a_{p-1}c}H_{a_0\cdots a_{p-1}}(p+k_1)_c(r-w) \quad (4.12)$$

It is worth pointing out that the appearance of  $F^{(1)} \cdot F^{(2)}$  and also  $D\phi^{i(1)} \cdot D\phi_{i(2)}$  couplings for D-brane-Anti D-brane DBI action have been confirmed by direct consistent string amplitudes of [59] and [74] accordingly. More crucially, we got to know that  $C_{p-1} \wedge F$  does not receive higher derivative correction of WZ couplings. The other important result that we found is that as the kinetic terms donot get any corrections hence all non-leading tachyonic singularities provide some information not only about the structure of higher derivative corrections to some couplings such as  $C_{p-1} \wedge DT \wedge DT$  but also could fix their coefficients on the world volume of brane-anti brane system as well. We could further move on and talk about all the contact terms of the string amplitude, for instance a new coupling such as  $C_{p-1} \wedge FTT^*$  can also be confirmed. We hope to further analyze the higher derivative corrections to those new couplings in near future and to be able to eventually fix all the higher derivative corrections to brane anti-brane DBI as well as WZ effective actions.

Remarks are in order. As we discussed these non-BPS couplings are worked out in the presence of the constraint  $p_a p^a \rightarrow 1/4$ , hence these couplings cannot be compared with BSFT couplings, although the tachyon potential is the same as the one that appeared in BSFT ( $V(T) = e^{\pi\alpha' m^2 T^2}$  [62]) i.e.,

$$V(T^i T^i) = 1 + \pi\alpha' m^2 T^i T^i + \frac{1}{2}(\pi\alpha' m^2 T^i T^i)^2 + \dots$$

where  $m_{\tilde{T}}^2 = -1/(2\alpha')$ . However, note that there is a symmetrized trace on the  $\sigma$  factors in DBI action, carrying out the symmetrized trace, one finds

$$\frac{1}{2} \text{STr} \left( V(T^i T^i) \sqrt{1 + [T^i, T^j][T^j, T^i]} \right) = \left( 1 - \frac{\pi}{2} T^2 + \frac{\pi^2}{24} T^4 + \dots \right) (1 + T^4 + \dots)$$

The tachyon does condensate at  $T \rightarrow \infty$  and hence, the tachyon potential goes to zero at that point as well.

## Acknowledgments

The author would like to thank L. Alvarez-Gaume, B. Jurco, O. Lechtenfeld, J.Kunz, P.H. Damgaard, N. E Bjerrum-Bohr, N. Arkani-Hamed, D. Jackson, C. Bachas, D. McGady, H. Steinacker, P. Schupp, J. Polchinski, J. Schwarz, A. Sen, W. Siegel, G. Veneziano and E. Witten for fruitful discussions. Some parts of this work are taken place at QMUL in London, CERN, ICTP, Charles University in math dep, IHES both physics and math deps, Leibniz University Hannover, Oldenburg University and Niels Bohr Institute in Copenhagen. He would like to thank those institutes for warm hospitality.

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