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Abelian tensor hierarchy in 4D $\mathcal{N} = 1$ conformal supergravity

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Abstract: We consider Abelian tensor hierarchy in four-dimensional $\mathcal{N} = 1$ supergravity in the conformal superspace formalism, where the so-called covariant approach is used to antisymmetric tensor fields. We introduce p-form gauge superfields as superforms in the conformal superspace. We solve the Bianchi identities under the constraints for the superforms. As a result, each of form fields is expressed by a single gauge invariant superfield. We also show the relation between the superspace formalism and the superconformal tensor calculus.

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1 Introduction

The superstring theory is regarded as a candidate for the unified theory of fundamental interactions including gravity. The theory contains strings/branes, which can describe gravity, gauge fields and matter ones in the low energy limit. For stable branes to describe our universe, anti-symmetric tensor fields are coupled to their preserved charges because they are extended objects. Further, unstable tachyons are avoided in the presence of the supersymmetry (SUSY). As a consequence, one may take supergravity (SUGRA), in which there exist such tensor fields on top of gravity, as a plausible low energy effective theory of the superstring theory.

Four-dimensional (4D) effective action is obtained through a compactification of extra dimensions. Hence, 4D $\mathcal{N} = 1$ SUGRA with tensor fields is a possible starting point to construct the effective description of the superstring theory, because $\mathcal{N} = 1$ SUGRA is a chiral theory, which has to contain the Standard Model of particle physics.

For these reasons, our interest is to construct the effective models of the superstring including p-form gauge fields within 4D $\mathcal{N} = 1$ SUGRA [1–5]. For the effective description of the superstring theory, we need to respect the structure of ten-dimensional antisymmetric tensors. In 4D effective theory, such antisymmetric tensors and their gauge transformations are described by the 4D form fields, whose transformations inevitably contain form fields with different ranks. Such a structure is called a tensor hierarchy [6–9], and is related to the anomaly cancellation conditions in the string theory. Therefore, the construction of a tensor hierarchy in 4D $\mathcal{N} = 1$ SUGRA is desirable in the context of string models.

In this paper, we consider the tensor hierarchy in 4D $\mathcal{N} = 1$ SUGRA. In particular, we focus on the construction which is inspired by structures of the geometries of extra dimensions. Becker et al. did such a construction in 4D $\mathcal{N} = 1$ global SUSY [8]. Our 4D SUGRA description can be applicable to discuss the roles of antisymmetric tensors, e.g. in cosmology [10–15] and SUSY breaking [16–18].

We will use the conformal superspace formalism [19], which is a superspace formalism of conformal supergravity. It has larger gauge symmetries than the superconformal tensor calculus [20–28] and Poincaré superspace [29, 30]. The symmetries will be useful to construct the SUGRA system coupled to the tensors and matters. We can straightforwardly reproduce the corresponding system in terms of the Poincaré superspace and also the superconformal tensor calculus due to their correspondences [19, 31].

We will adopt the so-called covariant approach [4, 29, 30, 32, 33]. In this approach, we regard bosonic tensors as components of differential superforms in superspace. This makes the local SUSY properties of the tensors manifest. In particular, it is straightforward to obtain gauge invariant superfields including bosonic field strengths.

This paper is organized as follows. In section 2, we review 4D $\mathcal{N} = 1$ conformal superspace briefly. Then, Abelian tensor hierarchy is introduced to the conformal superspace. Section 3 is devoted to impose constraints on field strength, and to show the solutions to the Bianchi identities. In section 4, we present the component formalism, which is written by superconformal tensor calculus. We give a summary of this paper and briefly discuss remaining issues in section 5. In appendix A, our notations are summarized. We present

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the explicit derivations of the solutions of Bianchi identities in appendix B. We show the explicit forms of bosonic field strengths in appendix C. Throughout this paper, the terms "form", "gauge field" and "field strength" are used to refer "superform", "gauge superfield" and "field strength superfield", respectively. We use the conventions of ref. [31] for conformal superspace except the notation of torsion. We use $T_{CB}{}^{A}$ to refer the torsion, which is equal to $R(P)_{CB}{}^{A}$ in ref. [31]. We also use the convention of ref. [30] for superforms, exterior derivatives and interior products.

2 Abelian tensor hierarchy in conformal superspace

In this section, we introduce the Abelian tensor hierarchy into conformal superspace. We begin with a brief review of conformal superspace. Abelian tensor hierarchy is then introduced into conformal superspace.

2.1 Conformal superspace

Conformal SUGRA is one of the most convenient formulation of SUGRA thanks to its larger gauge symmetries. Conformal superspace is a superspace approach to formulate conformal SUGRA. In the conformal superspace, we can formulate conformal SUGRA in a geometrical manner.

Conformal SUGRA is constructed as the gauge theory of superconformal group. We formulate conformal SUGRA in a superspace [19]. Superspace is a space where the coordinates are spanned by ordinary bosonic spacetime coordinates x^m and fermionic coordinates $(\theta^\mu, \bar{\theta}_{\dot{\mu}})$. Here, $m, n, \ldots$ are used for curved vector indices, $\mu, \nu, \ldots$ for undotted spinor indices, and $\dot{\mu}, \dot{\nu}, \ldots$ for dotted spinor indices. In the superspace, we can deal with bosonic translations and SUSY transformations at the same time, since SUSY transformations are understood as fermionic translations. Thus, we denote both bosonic and fermionic coordinates as $z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$, where capital Roman letters $M, N, \ldots$ express the sets of curved vector and spinor indices.

Conformal superspace is a superspace where gauge fields of the superconformal symmetry are introduced. The generators of the superconformal group are the following elements: spacetime translations P_a, SUSY transformations $Q_{\underline{\alpha}}$, Lorentz transformations M_{ab}, dilatation D, chiral rotation A, conformal boosts K_a and conformal SUSY transformations $S_{\underline{\alpha}}$. Here, Roman letters $a, b, \ldots$ denote flat vector indices, Greek letters $\alpha, \beta, \ldots$ and $\dot{\alpha}, \dot{\beta}, \ldots$ express undotted and dotted flat spinor indices, respectively. $\underline{\alpha}, \underline{\beta}, \ldots$ denote both spinor indices $\underline{\alpha} = (\alpha, \dot{\alpha})$. In the superspace, we can deal with bosonic translations and SUSY transformations at the same time. Therefore, we simply write P_A and K_A as $P_A = (P_a, Q_\alpha, \bar{Q}^{\dot{\alpha}})$ and $K_A = (K_a, S_\alpha, \bar{S}^{\dot{\alpha}})$, respectively. Here, we use Roman capital indices $A, B, \ldots$ for the sets of Lorentz vectors and spinors $A = (a, \alpha, \dot{\alpha})$. All the generators of superconformal group are denoted $X_{\mathcal{A}} = (P_A, M_{ab}, D, A, K_A)$, where the calligraphic letters $\mathcal{A}, \mathcal{B}, \ldots$ are used to refer the indices of the generators of the superconformal group.

The gauge fields of the superconformal symmetry are defined as

$$h_M{}^{\mathcal{A}} X_{\mathcal{A}} = E_M{}^{A} P_A + \frac{1}{2}\phi_M{}^{ab} M_{ba} + B_M D + A_M A + f_M{}^{A} K_A, \tag{2.1}$$

where $E_M{}^A$ is the vielbein, $\phi_M{}^{ab}$ is the spin connection, B_M, A_M and $f_M{}^A$ are the gauge fields corresponding to D, A and K_A, respectively. We assume that $E_M{}^A$ are invertible, and their inverses are denoted as $E_A{}^M$:

$$E_M{}^A E_A{}^N = \delta_M{}^N, \quad E_A{}^N E_N{}^B = \delta_A{}^B. \tag{2.2}$$

Using a differential form [30], the gauge fields are expressed as

$$h^{\mathcal{A}} = dz^M h_M{}^{\mathcal{A}}. \tag{2.3}$$

The gauged superconformal transformations are generated by $\delta_G(\xi^{\mathcal{A}} X_{\mathcal{A}})$. Here, $\xi^{\mathcal{A}}$ are real parameter superfields, and $\xi^{\mathcal{A}} X_{\mathcal{A}}$ denotes

$$\xi^{\mathcal{A}} X_{\mathcal{A}} = \xi(P)^A P_A + \frac{1}{2}\xi(M)^{ab} M_{ba} + \xi(D) D + \xi(A) A + \xi(K)^A K_A. \tag{2.4}$$

The gauge fields $h_M{}^{\mathcal{A}}$ receive the superconformal transformations $\delta_G(\xi^{\mathcal{A}'} X_{\mathcal{A}'})$ as

$$\delta_G(\xi^{\mathcal{B}'} X_{\mathcal{B}'}) h_M{}^{\mathcal{A}} = \partial_M \xi^{\mathcal{B}'} \delta_{\mathcal{B}'}{}^{\mathcal{A}} + h_M{}^{\mathcal{C}} \xi^{\mathcal{B}'} f_{\mathcal{B}'\mathcal{C}}{}^{\mathcal{A}}, \tag{2.5}$$

where primed calligraphic indices $\mathcal{A}', \mathcal{B}', \ldots$ mean all the superconformal generators except for P_A: $X_{\mathcal{A}'} = (M_{ab}, D, A, K_A)$.

We shall define SUSY transformations and spacetime translations. In the superspace approach, we can deal with SUSY transformations and spacetime translations at the same time, namely P_A-transformations. We relate P_A-transformations to the general coordinate transformation $\delta_{\rm GC}$ by using field-independent parameter superfields ξ^A as

$$\delta_G(\xi^A P_A) = \delta_{\rm GC}(\xi^M) - \delta_G(\xi^M h_M{}^{\mathcal{B}'} X_{\mathcal{B}'}), \tag{2.6}$$

where $\xi(P)^A$ are abbreviated to ξ^A, and ξ^M are defined by $\xi^M := \xi^A E_A{}^M$. The P_A-transformations acting on a superfield Φ with no curved index define the superconformally covariant derivatives ∇_A as

$$\delta_G(\xi^A P_A)\Phi = \xi^A P_A \Phi = \xi^A \nabla_A \Phi = \xi^M \nabla_M \Phi = \xi^M (\partial_M - h_M{}^{\mathcal{A}'} X_{\mathcal{A}'})\Phi. \tag{2.7}$$

Let us consider the curvatures associated with the superconformal symmetry, which appear in the Bianchi identities. They are defined by

$$R_{MN}{}^{\mathcal{A}} = \partial_M h_N{}^{\mathcal{A}} - \partial_N h_M{}^{\mathcal{A}} - (E_N{}^C h_M{}^{\mathcal{B}'} - E_M{}^C h_N{}^{\mathcal{B}'}) f_{\mathcal{B}'C}{}^{\mathcal{A}} - h_N{}^{\mathcal{C}'} h_M{}^{\mathcal{B}'} f_{\mathcal{B}'\mathcal{C}'}{}^{\mathcal{A}}. \tag{2.8}$$

Here, we use the convention of "implicit grading" [19]. Using differential forms, they are expressed as

$$R^{\mathcal{A}} = \frac{1}{2} dz^M \wedge dz^N R_{NM}{}^{\mathcal{A}} = dh^{\mathcal{A}} - E^B \wedge h^{\mathcal{C}'} f_{\mathcal{C}'B}{}^{\mathcal{A}} - \frac{1}{2} h^{\mathcal{B}'} \wedge h^{\mathcal{C}'} f_{\mathcal{C}'\mathcal{B}'}{}^{\mathcal{A}}, \tag{2.9}$$

where $E^A = dz^M E_M{}^A$. In particular, the curvatures associated with P_A are the torsion two-forms T^A. The torsions are given by explicitly

$$T^A = dE^A - E^C \wedge h^{\mathcal{B}'} f_{\mathcal{B}'C}{}^A = \frac{1}{2} E^B \wedge E^C T_{CB}{}^A, \tag{2.10}$$

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which appear in the Bianchi identities for p-form gauge fields discussed later. The curvatures are expressed also in terms of the (anti-)commutation relations of the superconformally covariant derivatives

$$[\nabla_A, \nabla_B] = -R_{AB}{}^{\mathcal{C}} X_{\mathcal{C}}. \tag{2.11}$$

The curvatures are constrained so that (anti-)commutation relations of the covariant derivatives are given by

$$\{\nabla_\alpha, \nabla_\beta\} = 0, \quad \{\bar\nabla_{\dot\alpha}, \bar\nabla_{\dot\beta}\} = 0, \quad \{\nabla_\alpha, \bar\nabla_{\dot\beta}\} = -2i\nabla_{\alpha\dot\beta}, \tag{2.12}$$

$$\begin{aligned} &[\nabla_\alpha, \nabla_{\beta\dot\gamma}] = -2i\epsilon_{\alpha\beta}\mathcal{W}_{\dot\gamma}, && [\bar\nabla_{\dot\alpha}, \nabla_{\dot\beta\gamma}] = -2i\epsilon_{\dot\alpha\dot\beta}\mathcal{W}_\gamma, \\ &[\nabla_{\alpha\dot\alpha}, \nabla_{\beta\dot\beta}] = \epsilon_{\dot\alpha\dot\beta}\{\nabla_{(\alpha}, \mathcal{W}_{\beta)}\} + \epsilon_{\alpha\beta}\{\bar\nabla_{(\dot\alpha}, \mathcal{W}_{\dot\beta)}\}, \end{aligned} \tag{2.13}$$

where

$$\begin{aligned} \mathcal{W}_\alpha &= (\epsilon\sigma^{bc})^{\beta\gamma} W_{\alpha\beta\gamma} M_{cb} + \frac{1}{2}\left(\nabla^\gamma W_{\gamma\alpha}{}^\beta\right) S_\beta - \frac{1}{2}\left(\nabla^{\gamma\dot\beta} W_{\gamma\alpha}{}^\beta\right) K_{\beta\dot\beta}, \\ \mathcal{W}^{\dot\alpha} &= (\bar\sigma^{bc}\epsilon)^{\dot\gamma\dot\beta} W^{\dot\alpha}{}_{\dot\beta\dot\gamma} M_{cb} - \frac{1}{2}\left(\bar\nabla_{\dot\gamma} W^{\dot\gamma\dot\alpha}{}_{\dot\beta}\right) \bar S^{\dot\beta} - \frac{1}{2}\left(\nabla^{\dot\gamma\beta} W_{\dot\gamma}{}^{\dot\alpha\dot\beta}\right) K_{\beta\dot\beta}, \end{aligned} \tag{2.14}$$

and the parentheses for indices mean symmetrizations of spinor indices: $\psi_{(\alpha}\chi_{\beta)} = \frac{1}{2}(\psi_\alpha\chi_\beta + \psi_\beta\chi_\alpha)$. $W_{\alpha\beta\gamma}$ are chiral primary superfields with Weyl weight 3/2 and chiral weight 1, and their indices are totally symmetric. Here, a primary superfield is a superfield that is invariant under the K_A-transformations: $K_A W_{\alpha\beta\gamma} = 0$. In particular, $T_{\alpha\dot\beta}{}^c$ are given by

$$T_{\alpha\dot\beta}{}^c = 2i(\sigma^c)_{\alpha\dot\beta}, \tag{2.15}$$

which is used to solve Bianchi identities in appendix B.

2.2 Abelian tensor hierarchy in conformal superspace

We introduce antisymmetric tensor gauge fields into conformal superspace. Antisymmetric tensor gauge fields are expressed in terms of p-form gauge fields. p-form gauge fields are transformed under Abelian internal gauge transformations using $(p-1)$-form parameter superfields. In addition, p-form gauge fields are shifted using p-form parameter superfields. This structure of gauge transformation of the tensors is called an Abelian tensor hierarchy.

We explain the hierarchy concretely. The p-form ($p \geq -1$) gauge fields $C_{[p]}^{I_p}$ are defined by[1]

$$C_{[p]}^{I_p} := \frac{1}{p!} dz^{M_1} \wedge \cdots \wedge dz^{M_p} C^{I_p}_{M_p \ldots M_1} = \frac{1}{p!} E^{A_1} \wedge \cdots \wedge E^{A_p} C^{I_p}_{A_p \ldots A_1}. \tag{2.16}$$

Here, the indices I_p denote the indices of internal space of p-form V_p, which are assumed to be real vector spaces. I_p run over $1, \ldots, \dim V_p$. We denote infinitesimal internal gauge transformations of p-forms as $\delta_T(\Lambda)$, where Λ is a set of real p-form parameter superfields $\Lambda_{[p]}^{I_{p+1}}$: $\Lambda = (\Lambda_{[0]}^{I_1}, \ldots, \Lambda_{[3]}^{I_4})$. The gauge transformation laws of $C_{[p]}^{I_p}$ are given by

$$\delta_T(\Lambda) C_{[p]}^{I_p} = d\Lambda_{[p-1]}^{I_p} + (q^{(p)} \cdot \Lambda_{[p]})^{I_p}, \tag{2.17}$$

[1] (-1)-forms are defined to be zero as in the ordinary differential geometry.

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where $q^{(p)}$ are matrices which map V_{p+1} to V_p. $(q^{(p)} \cdot \Lambda_{[p]})^{I_p}$ are given by explicitly

$$(q^{(p)} \cdot \Lambda_{[p]})^{I_p} = (q^{(p)})^{I_p}_{I_{p+1}} \Lambda_{[p]}{}^{I_{p+1}}. \tag{2.18}$$

We define the $X_{\mathcal{A}'}$ transformation laws of $C^{I_p}_{[p]}$ as

$$\delta_G(\xi^{\mathcal{C}'} X_{\mathcal{C}'}) C^{I_p}_{M_p \dots M_1} = 0. \tag{2.19}$$

Field strengths of p-form gauge fields are defined by

$$\begin{aligned} F^{I_p}_{[p+1]} &:= dC^{I_p}_{[p]} - (q^{(p)} \cdot C_{[p+1]})^{I_p} \\ &= \frac{1}{p!} dz^{M_1} \wedge \cdots \wedge dz^{M_p} \wedge dz^N \partial_N C^{I_p}_{M_p \dots M_1} \\ &\quad - \frac{1}{(p+1)!} dz^{M_1} \wedge \cdots \wedge dz^{M_{p+1}} (q^{(p)} \cdot C_{M_{p+1} \dots M_1})^{I_p}. \end{aligned} \tag{2.20}$$

They are transformed under the internal gauge transformations

$$\delta_T(\Lambda_{[p]}) F^{I_p}_{[p+1]} = -(q^{(p)} \cdot q^{(p+1)} \cdot \Lambda_{[p+1]})^{I_p}. \tag{2.21}$$

The invariances of the field strengths require that

$$q^{(p-1)} \cdot q^{(p)} = 0. \tag{2.22}$$

The SUSY transformations and spacetime translations are redefined with respect to δ_T transformations of $C^{I_p}_{[p]}$. The redefinitions are the same as the case that the tensor hierarchy does not exist [30]. P_A-transformations are redefined by

$$\delta_G(\xi^A P_A) = \delta_{\rm GC}(\xi^M) - \delta_G(\xi^M h_M{}^{\mathcal{A}'} X_{\mathcal{A}'}) - \delta_T(\Lambda(\xi)). \tag{2.23}$$

Here, $\Lambda(\xi)$ is defined by

$$\Lambda(\xi) = (\iota_\xi C^{I_1}_{[1]}, \dots, \iota_\xi C^{I_4}_{[4]}), \tag{2.24}$$

and ι_ξ is a interior product

$$\iota_\xi C^{I_p}_{[p]} = \frac{1}{(p-1)!} dz^{M_1} \wedge \cdots \wedge dz^{M_{p-1}} \xi^{M_p} C^{I_p}_{M_p \dots M_1}. \tag{2.25}$$

In particular, the P_A-transformations of $C^{I_p}_{[p]}$ are given by

$$\delta_G(\xi^A P_A) C^{I_p}_{[p]} = \delta_{\rm GC}(\xi^M) C^{I_p}_{[p]} - \delta_T(\Lambda(\xi)) C^{I_p}_{[p]} = \iota_\xi F^{I_p}_{[p+1]}. \tag{2.26}$$

The P_A-transformation laws of superfields which are invariant under δ_T transformations are not changed. Note that we obtain SUSY transformations of p-form gauge fields if we choose $\xi^A = \xi^{\underline{\alpha}}$.

Field strengths obey the following Bianchi identities:

$$dF^{I_p}_{[p+1]} = -(q^{(p)} \cdot F_{[p+2]})^{I_p}. \tag{2.27}$$

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form	gauge field	field strength	Bianchi identity
4-form	U^{I_4}	$G^{I_4} = dU^{I_4} = 0$	–
3-form	C^{I_3}	$\Sigma^{I_3} = dC^{I_3} - (q^{(3)} \cdot U)^{I_3}$	$d\Sigma^{I_3} = 0$
2-form	B^{I_2}	$H^{I_2} = dB^{I_2} - (q^{(2)} \cdot C)^{I_2}$	$dH = -(q^{(2)} \cdot \Sigma)^{I_2}$
1-form	A^{I_1}	$F^{I_1} = dA^{I_1} - (q^{(1)} \cdot B)^{I_1}$	$dF^{I_1} = -(q^{(1)} \cdot H)^{I_1}$
0-form	f^{I_0}	$g^{I_0} = df^{I_0} - (q^{(0)} \cdot A)^{I_0}$	$dg^{I_0} = -(q^{(0)} \cdot F)^{I_0}$
-1-form	0	$\omega^{I_{-1}} = -(q^{(-1)} \cdot f)^{I_{-1}}$	$d\omega^{I_{-1}} = -(q^{(-1)} \cdot g)^{I_{-1}}$

Table 1. The p-forms, their corresponding field strengths and Bianchi identities.

The existence of the tensor hierarchy deforms the Bianchi identites: the tensor hierarchy relates the exterior derivatives on the $(p+1)$-form field strengths to the $(p+2)$-form field strengths. The Bianchi identities play an important role in the next section.

Explicitly, we denote the p-form gauge fields, the field strengths and the Bianchi identities in table 1. In table 1, 4-form gauge fields appear. The bosonic component of gauge fields $U^{I_4}_{qpnm}$ exist in principle, but the bosonic components of the field strengths are zero: $G^{I_4}_{rqpnm} = 0$. This is because $G^{I_4}_{rqpnm}$ is the fifth rank antisymmetric tensor, which must be zero in 4D. Thus, we impose by hand that field strengths G^{I_4} are equal to zero as in ref. [32]. There are also 0-form "field strengths" in principle, but (-1)-form gauge field does not exist. Thus 0-form field strengths are defined by $d\omega^{I_{-1}} = -(q^{(-1)} \cdot g)^{I_{-1}}$.

From the higher-dimensional view point [8], V_p can be understood as spaces of differential forms on extra dimensions. The matrices $q^{(p)}$ are understood as the exterior derivative with respect to extra dimensions.

3 Constraints and Bianchi identities

In this section, constraints on the field strengths are imposed to construct irreducible superfields. We solve the Bianchi identities under these constraints. As a result, each of field strengths is expressed in terms of the corresponding gauge invariant superfields straightforwardly.

3.1 Constraints

In the previous section, we have defined the field strengths of the p-form gauge fields. The field strengths have redundant degrees of freedom, and we eliminate them by imposing constraints. We will take the constraints as the same ones without the tensor hierarchy [30, 33]. The constraints are explicitly given by table 2. Here, L^{I_2} and Ψ^{I_0} are real superfields. In addition, we have imposed that Ψ^{I_0} are primary superfields in table 2.

We solve the Bianchi identities under these constraints. On the one hand, the field strengths obey the Bianchi identities. On the other hand, we impose the constraints on the field strengths. The consistency between Bianchi identities and the constraints leads to new relations of the field strengths. Since the constraints are imposed Lorentz covariantly,

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form	constraints
4-form	$G^{I_4}_{EDCBA} = 0$
3-form	$\Sigma^{I_3}_{\underline{\delta}\,\underline{\gamma}\,\underline{\beta}A} = \Sigma^{I_3}_{\delta\dot{\gamma}ba} = 0$
2-form	$H^{I_2}_{\underline{\gamma}\,\underline{\beta}\,\underline{\alpha}} = H^{I_2}_{\gamma\beta a} = H^{I_2}_{\dot{\gamma}\dot{\beta}a} = 0, \quad H^{I_2}_{\gamma\dot{\beta}a} = +2i(\sigma_a)_{\gamma\dot{\beta}} L^{I_2}$
1-form	$F^{I_1}_{\underline{\alpha}\,\underline{\beta}} = 0$
0-form	$g^{I_0}_{\alpha} = i\nabla_\alpha \Psi^{I_0}, \quad g^{I_0}_{\dot{\beta}} = -i\bar{\nabla}_{\dot{\beta}} \Psi^{I_0}, \quad K_A \Psi^{I_0} = 0$

Table 2. The constraints on the field strengths.

it is convenient to express the Bianchi identities (2.27) by flat indices rather than curved indices:

$$\begin{aligned} &\frac{1}{(p+1)!} E^{A_1} \wedge \cdots \wedge E^{A_{p+1}} \wedge E^B \nabla_B F^{I_p}_{A_p \ldots A_{p+1}} \\ &+ \frac{1}{p!2!} E^{A_1} \wedge \cdots E^{A_p} \wedge E^B \wedge E^C T_{CB}{}^{A_{p+1}} F^{I_p}_{A_{p+1} \ldots A_1} \\ &= -\frac{1}{(p+2)!} E^{A_1} \wedge \cdots \wedge E^{A_{p+2}} (q^{(p)} \cdot F_{A_{p+2} \ldots A_1})^{I_p}. \end{aligned} \tag{3.1}$$

This equation follows from eq. (2.10) and (2.19). Equation (2.10) is used to express the exterior derivative on the vielbein 1-form dE^A in terms of the torsion 2-form. The exterior derivatives on gauge fields are written by covariant derivatives on the field strengths using eq. (2.19).

3.2 Solutions to the Bianchi identities

In this subsection we summarize the solutions to the Bianchi identities of table 1 under the constraints of table 2. The details of the derivations are discussed in appendix B. Each of the field strengths is expressed by a single gauge invariant superfield $(Y^{I_3}, L^{I_2}, W^{I_1}_\alpha, \Psi^{I_0})$. We find the Weyl weights and chiral ones (Δ, w) of the gauge invariant superfields. We also find that these gauge invariant field strengths are primary superfields (Ψ^{I_0} are imposed to be primary as in table 2). The weights and K_A-invariance play an important role in constructing superconformally invariant actions.

3.2.1 3-form gauge fields

For 3-form gauge fields, all the components of the field strengths are expressed in terms of chiral superfields Y^{I_3} and their complex conjugates $\bar{Y}^{I_3}$. They appear in the 2-spinor/2-vector components as

$$\Sigma^{I_3 \dot{\delta}\dot{\gamma}}{}_{ba} = \frac{1}{2}(\bar{\sigma}_{ba}\epsilon)^{\dot{\delta}\dot{\gamma}} Y^{I_3}, \quad \Sigma^{I_3}_{\delta\gamma ba} = \frac{1}{2}(\sigma_{ba}\epsilon)_{\delta\gamma} \bar{Y}^{I_3}. \tag{3.2}$$

From eqs. (B.58), (B.59), (B.60) and (B.61), Y^{I_3} obey

$$DY^{I_3} = 3Y^{I_3}, \quad AY^{I_3} = 2iY^{I_3}, \quad K_A Y^{I_3} = 0. \tag{3.3}$$

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They mean that Y^{I_3} are primary superfields and the weights are

$$(\Delta, w) = (3, 2). \tag{3.4}$$

Similarly, $\bar{Y}^{I_3}$ obey

$$D\bar{Y}^{I_3} = 3\bar{Y}^{I_3}, \quad A\bar{Y}^{I_3} = -2i\bar{Y}^{I_3}, \quad K_A\bar{Y}^{I_3} = 0. \tag{3.5}$$

Other Bianchi identities lead to

$$\nabla_\alpha \bar{Y}^{I_3} = 0, \quad \bar{\nabla}_{\dot{\alpha}} Y^{I_3} = 0, \tag{3.6}$$

which mean that Y^{I_3} and $\bar{Y}^{I_3}$ are chiral and anti-chiral superfields, respectively. Furthermore, 1-spinor/3-vector components are expressed in terms of spinor derivatives of Y^{I_3} and their conjugates:

$$\Sigma^{I_3\dot{\delta}}{}_{cba} = +\frac{1}{16}\bar{\sigma}^{d\dot{\delta}\delta}\epsilon_{dcba}\nabla_\delta Y^{I_3}, \quad \Sigma^{I_3}_{\delta cba} = -\frac{1}{16}(\sigma^d)_{\delta\dot{\delta}}\epsilon_{dcba}\bar{\nabla}^{\dot{\delta}}\bar{Y}^{I_3}. \tag{3.7}$$

$\Sigma^{I_3}_{dcba}$ are identified as the imaginary parts of $\nabla^2 Y^{I_3}$:

$$\Sigma^{I_3}_{dcba} = \frac{i}{64}\epsilon_{dcba}(\nabla^2 Y^{I_3} - \bar{\nabla}^2\bar{Y}^{I_3}). \tag{3.8}$$

We can understand the non-dynamical 4-form field strength in terms of the F-component of Y^{I_3} by the $\theta = \bar{\theta} = 0$ projection of both hand sides of this equation, where the $\theta = \bar{\theta} = 0$ projection is the projection from the superspace to the bosonic spacetime. The solutions to the Bianchi identities for 3-form gauge fields are the same as the case without tensor hierarchy [4, 30]. Note that our normalization of Y^{I_3} is equivalent to $8G^S$ in ref. [8].

3.2.2 2-form gauge fields

The field strengths of 2-form gauge fields are expressed by real superfields L^{I_2}. We list the solutions to the Bianchi identities.

- 1-spinor/2-vector components

$$H^{I_2}_{\delta ba} = 2(\sigma_{ba})_\delta{}^\beta\nabla_\beta L^{I_2}, \quad H^{I_2\dot{\delta}}{}_{ba} = 2(\bar{\sigma}_{ba})^{\dot{\delta}}{}_{\dot{\phi}}\bar{\nabla}^{\dot{\phi}}L^{I_2}, \tag{3.9}$$

- 3-vector components

$$H^{I_2}_{fba} = \frac{1}{4}\epsilon_{fbag}(\bar{\sigma}^g)^{\dot{\epsilon}\epsilon}[\nabla_\epsilon, \bar{\nabla}_{\dot{\epsilon}}]L^{I_2}. \tag{3.10}$$

- Deformed linearity conditions

$$\nabla^2 L^{I_2} = \frac{1}{4}(q^{(2)}\cdot\bar{Y})^{I_2}, \quad \bar{\nabla}^2 L^{I_2} = \frac{1}{4}(q^{(2)}\cdot Y)^{I_2}. \tag{3.11}$$

- D-, A-, K_A-transformation laws

$$DL^{I_2} = 2L^{I_2}, \quad AL^{I_2} = 0, \quad K_A L^{I_2} = 0. \tag{3.12}$$

  Hence, we find

$$(\Delta, w) = (2, 0). \tag{3.13}$$

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Note that the tensor hierarchy deforms ordinary linearity conditions of L^{I_2} by $q^{(2)}$.[2] If the tensor hierarchy does not exist, the deformed linearity conditions reduce to the ordinary linearity conditions $\nabla^2 L^{I_2} = \bar{\nabla}^2 L^{I_2} = 0$. Note that our normalization of L^{I_2} is equivalent to $\frac{1}{2}H^M$ in ref. [8].

3.2.3 1-form gauge fields

The solutions to 1-form gauge fields are mostly the same as an ordinary Abelian case. The field strengths are expressed in terms of the gaugino superfields $W_\alpha^{I_1}$ and their conjugates. The explicit solutions to the Bianchi identities are as follows.

- 1-spinor/2-vector components

$$F^{I_1}_{\dot\beta,\alpha\dot\alpha} = -2\epsilon_{\dot\beta\dot\alpha} W^{I_1}_\alpha, \quad F^{I_1}_{\beta,\alpha\dot\alpha} = -2\epsilon_{\beta\alpha}\bar{W}^{I_1}_{\dot\alpha}. \tag{3.14}$$

- Chirality conditions

$$\bar{\nabla}_{\dot\beta} W^{I_1}_\alpha = 0, \quad \nabla_\alpha \bar{W}^{I_1}_{\dot\beta} = 0. \tag{3.15}$$

- 2-vector components

$$F^{I_1}_{ba} = -\frac{i}{2}\left((\sigma_{ba})_\beta{}^\alpha \nabla^\beta W^{I_1}_\alpha - (\bar\sigma_{ba})^{\dot\beta}{}_{\dot\alpha}\bar{\nabla}_{\dot\beta}\bar{W}^{I_1\dot\alpha}\right). \tag{3.16}$$

- Deformed reality conditions

$$\nabla^\alpha W^{I_1}_\alpha - \bar{\nabla}_{\dot\alpha}\bar{W}^{I_1\dot\alpha} = -4i(q^{(1)}\cdot L)^{I_1}. \tag{3.17}$$

- D-, A-, K_A-transformation laws

$$\begin{aligned} DW^{I_1}_\alpha &= \frac{3}{2}W^{I_1}_\alpha, \qquad AW^{I_1}_\alpha = iW^{I_1}_\alpha, \qquad K_A W^{I_1}_\alpha = 0, \\ D\bar{W}^{I_1\dot\alpha} &= \frac{3}{2}\bar{W}^{I_1\dot\alpha}, \quad A\bar{W}^{I_1\dot\alpha} = -i\bar{W}^{I_1\dot\alpha}, \quad K_A\bar{W}^{I_1\dot\alpha} = 0. \end{aligned} \tag{3.18}$$

  Then, we find the weights of $W^{I_1}_\alpha$:

$$(\Delta, w) = (3/2, 1). \tag{3.19}$$

The reality conditions of $W^{I_1}_\alpha$ are deformed by the tensor hierarchy, i.e., $q^{(1)}$: L^{I_2} appear in the imaginary parts of $\nabla^\alpha W^{I_1}_\alpha$ in the presence of the tensor hierarchy. The deformed reality conditions reduce to the ordinary reality conditions $\nabla^\alpha W^{I_1}_\alpha = \bar{\nabla}_{\dot\alpha}\bar{W}^{I_1\dot\alpha}$ if the tensor hierarchy does not exist. Note that our normalization of $W^{I_1}_\alpha$ is equivalent to that of ref. [8].

[2] A deformed linear multiplet in 4D $\mathcal{N} = 1$ SUGRA is discussed in ref. [34].

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3.2.4 0-form gauge fields

The field strengths of 0-form gauge fields are expressed in terms of real primary superfields Ψ^{I_0}. The solutions to the Bianchi identities are as follows.

- Vector components

$$g_a^{I_0} = \frac{1}{4i}(\bar{\sigma}_a)^{\dot{\alpha}\beta}(\nabla_\beta g_{\dot{\alpha}}^{I_0} + \bar{\nabla}_{\dot{\alpha}} g_\beta^{I_0}) = -\frac{1}{4}(\bar{\sigma}_a)^{\dot{\alpha}\beta}[\nabla_\beta, \bar{\nabla}_{\dot{\alpha}}]\Psi^{I_0}. \tag{3.20}$$

- Modified higher component conditions

$$\frac{1}{4}\bar{\nabla}^2\nabla_\alpha\Psi^{I_0} = (q^{(0)} \cdot W_\alpha)^{I_0}, \quad \frac{1}{4}\nabla^2\bar{\nabla}_{\dot{\alpha}}\Psi^{I_0} = (q^{(0)} \cdot \bar{W}_{\dot{\alpha}})^{I_0}. \tag{3.21}$$

- D-, A-transformation laws

$$D\Psi^{I_0} = 0, \quad A\Psi^{I_0} = 0. \tag{3.22}$$

We find the weights of Ψ^{I_0}:

$$(\Delta, w) = (0, 0). \tag{3.23}$$

The conditions in eq. (3.21) are a bit peculiar in the presence of tensor hierarchy $q^{(0)}$: in the case of the absence of the tensor hierarchy, the Bianchi identities lead to the constraints $\bar{\nabla}^2\nabla_\alpha\Psi^{I_0} = 0$ and $\nabla^2\bar{\nabla}_{\dot{\alpha}}\Psi^{I_0} = 0$. We can find the expression of Ψ^{I_0} which can be consistent with the constraints. We can use chiral and anti-chiral primary superfields. Chiral primary superfields S^{I_0} as well as anti-chiral primary superfields $\bar{S}^{I_0}$ satisfy $\bar{\nabla}^2\nabla_\alpha S^{I_0} = \bar{\nabla}^2\nabla_\alpha\bar{S}^{I_0} = 0$. The field strengths of 0-form gauge fields Ψ^{I_0} can be defined as the imaginary part of the chiral superfields:

$$\Psi^{I_0} = \frac{1}{2i}(S^{I_0} - \bar{S}^{I_0}), \tag{3.24}$$

which are consistent with the constraints and the solutions to Bianchi identities. Note that S^{I_0} can be understood as the prepotentials for the 0-form gauge fields. Now we consider the case of the existence of the tensor hierarchy. Ψ^{I_0} are deformed to

$$\Psi^{I_0} = \frac{1}{2i}(S^{I_0} - \bar{S}^{I_0}) - (q^{(0)} \cdot V)^{I_0}, \tag{3.25}$$

where V^{I_1} are the prepotentials for 1-form gauge fields. Using $W_\alpha^{I_1} = -\frac{1}{4}\bar{\nabla}^2\nabla_\alpha V^{I_1}$, we obtain

$$\frac{1}{4}\bar{\nabla}^2\nabla_\alpha\Psi^{I_0} = (q^{(0)} \cdot W_\alpha)^{I_0}. \tag{3.26}$$

The results are consistent with eq. (3.21). Note that S^{I_0} are not gauge invariant in the presence of tensor hierarchy.

4 Component formalism

In this section we show the correspondence between the conformal superspace and superconformal tensor calculus [27] using the results in ref. [31]. Superconformal tensor calculus is presumably the most practically useful formalism. We focus on the correspondence of the superfields Y^{I_3}, L^{I_2}, $W_\alpha^{I_1}$ and Ψ^{I_0}, which characterize the corresponding field strengths.

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component	superspace
C_Γ	$\Phi_\Gamma\vert$
Z_Γ	$\begin{pmatrix}-i\nabla_\alpha\Phi_\Gamma\\ +i\bar\nabla^{\dot\alpha}\Phi_\Gamma\end{pmatrix}\Big\vert$
H_Γ	$+\frac{1}{4}(\nabla^2\Phi_\Gamma+\bar\nabla^2\Phi_\Gamma)\vert$
K_Γ	$-\frac{i}{4}(\nabla^2\Phi_\Gamma-\bar\nabla^2\Phi_\Gamma)\vert$
$B_{a\Gamma}$	$-\frac{1}{4}(\bar\sigma_a)^{\dot\beta\beta}[\nabla_\beta,\bar\nabla_{\dot\beta}]\Phi_\Gamma\vert$
Λ_Γ	$\frac{i}{4}\begin{pmatrix}-\bar\nabla^2\nabla_\alpha\Phi_\Gamma\\ +\nabla^2\bar\nabla^{\dot\alpha}\Phi_\Gamma\end{pmatrix}\Big\vert+2i\begin{pmatrix}\mathcal{W}_\alpha\\ \mathcal{W}^{\dot\alpha}\end{pmatrix}\Phi_\Gamma\vert$
D_Γ	$\frac{1}{8}\bar\nabla_{\dot\alpha}\nabla^2\bar\nabla^{\dot\alpha}\Phi_\Gamma\vert+\mathcal{W}_{\dot\alpha}\bar\nabla^{\dot\alpha}\Phi_\Gamma\vert$ $=\frac{1}{8}\nabla^\alpha\bar\nabla^2\nabla_\alpha\Phi_\Gamma\vert-\mathcal{W}^\alpha\nabla_\alpha\Phi_\Gamma\vert$

Table 3. The components of conformal multiplets. The components are expressed in terms of the $\theta=\bar\theta=0$ projections of the superfields Φ_Γ. In this table, $[C_\Gamma, Z_\Gamma, H_\Gamma, K_\Gamma, B_{a\Gamma}, \Lambda_\Gamma, D_\Gamma]$ correspond to those of $[\mathcal{C}_\Gamma, \mathcal{Z}_\Gamma, \mathcal{H}_\Gamma, \mathcal{K}_\Gamma, \mathcal{B}_{a\Gamma}, \Lambda_\Gamma, \mathcal{D}_\Gamma]$ in ref. [27].

4.1 Components of the superfields Y^{I_3}, L^{I_2}, $W_\alpha^{I_1}$ and Ψ^{I_0}

We express Y^{I_3}, L^{I_2}, $W_\alpha^{I_1}$ and Ψ^{I_0} within the superconformal tensor calculus. In the superconformal tensor calculus, we denote the components of a general complex multiplet V_Γ with arbitrary Lorentz indices Γ as

$$V_\Gamma=[C_\Gamma, Z_\Gamma, H_\Gamma, K_\Gamma, B_{a\Gamma}, \Lambda_\Gamma, D_\Gamma]. \tag{4.1}$$

The components of V_Γ are expressed by corresponding primary superfields Φ_Γ as in table 3. In this table, the symbol of "|" means the $\theta=\bar\theta=0$ projection. As already appeared in section 3, the $\theta=\bar\theta=0$ projection is the projection from the superspace to the bosonic spacetime. Component fields are obtained by $\theta=\bar\theta=0$ projections of superfields.

We also denote chiral conformal multiplets T_Γ as

$$T_\Gamma=[A_\Gamma, \mathcal{P}_{\rm R}\chi_\Gamma, F_\Gamma]. \tag{4.2}$$

T_Γ are embedded into the general complex multiplets as

$$V(T_\Gamma)=[A_\Gamma, -i\mathcal{P}_{\rm R}\chi_\Gamma, -F_\Gamma, iF_\Gamma, i\nabla_a A_\Gamma, 0, 0]. \tag{4.3}$$

The components of gauge invariant superfields Y^{I_3}, L^{I_2}, $W_\alpha^{I_1}$ and Ψ^{I_0} are given in tables 4, 5, 6 and 7, respectively. In these tables, note that the tensor hierarchy deforms the higher components of the L^{I_2}, $W_\alpha^{I_1}$ and Ψ^{I_0} in the presence of q's.

4.2 Bosonic field strengths

In the previous subsection, the lowest component of the field strengths $g_a^{I_0}\vert$, $F_{ab}^{I_1}\vert$, $H_{abc}^{I_2}\vert$ and $\Sigma_{abcd}^{I_3}\vert$ appear. They are covariantly transformed under SUSY transformations, because

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component	superfield
$A(Y^{I_3})$	$Y^{I_3}\vert$
$\mathcal{P}_{\mathrm{R}}\chi(Y^{I_3})$	$\nabla_\alpha Y^{I_3}\vert$
$F(Y^{I_3})$	$-\frac{1}{8}(\nabla^2 Y^{I_3}+\bar{\nabla}^2\bar{Y}^{I_3})\vert-\frac{i}{6}\epsilon^{dcba}\Sigma^{I_3}_{dcba}\vert$

Table 4. The components of the chiral primary superfields Y^{I_3}.

component	superfield
$C(L^{I_2})$	$L^{I_2}\vert$
$Z(L^{I_2})$	$\begin{pmatrix}-i\nabla_\alpha L^{I_2}\\ +i\bar{\nabla}^{\dot{\alpha}}L^{I_2}\end{pmatrix}\vert$
$H(L^{I_2})$	$-\frac{1}{16}(q^{(2)}\cdot(Y+\bar{Y}))^{I_2}\vert$
$K(L^{I_2})$	$\frac{i}{16}(q^{(2)}\cdot(Y-\bar{Y}))^{I_2}\vert$
$B_a(L^{I_2})$	$-\frac{1}{6}\epsilon_{adcb}H^{I_2dcb}\vert$
$\Lambda(L^{I_2})$	$-i\begin{pmatrix}0 & (\sigma^c)_{\alpha\dot{\beta}}\\ (\bar{\sigma}^c)^{\dot{\alpha}\beta} & 0\end{pmatrix}\nabla_c\begin{pmatrix}-i\nabla_\beta L^{I_2}\\ +i\bar{\nabla}^{\dot{\beta}}L^{I_2}\end{pmatrix}\vert+\frac{1}{16}\left(q^{(2)}\cdot\begin{pmatrix}-i\nabla_\alpha Y\\ +i\bar{\nabla}^{\dot{\alpha}}\bar{Y}\end{pmatrix}\right)^{I_2}\vert$
$D(L^{I_2})$	$-\nabla^a\nabla_a L^{I_2}\vert+\frac{1}{16}\left(q^{(2)}\cdot\frac{1}{4}(\nabla^2 Y+\bar{\nabla}^2\bar{Y})\right)^{I_2}\vert$

Table 5. The components of the real primary superfields L^{I_2}.

component	superfield
$A(W_\alpha^{I_1})$	$W_\alpha^{I_1}\vert$
$\mathcal{P}_{\mathrm{R}}\chi(W_\alpha^{I_1})$	$\frac{i}{2}(\sigma^{ba}\epsilon)_{\beta\alpha}F^{I_1}_{ba}\vert+\frac{1}{4}\epsilon_{\beta\alpha}(\nabla^\gamma W^{I_1}_\gamma+\bar{\nabla}_{\dot{\gamma}}\bar{W}^{I_1\dot{\gamma}})\vert-i\epsilon_{\beta\alpha}(q^{(1)}\cdot L)^{I_1}\vert$
$F(W_\alpha^{I_1})$	$-i\nabla_{\alpha\dot{\beta}}\bar{W}^{I_1\dot{\beta}}\vert-2i(q^{(1)}\cdot L)^{I_1}\vert$

Table 6. The components of the chiral primary superfields $W_\alpha^{I_1}$. The spinor index α is used for the external Lorentz index of $W_\alpha^{I_1}$.

they have only Lorentz indices. The lowest components of them are related to the lowest components of bosonic p-form gauge fields $C^{I_p}_{m_p\ldots m_1}\vert$. In SUGRA, they are also related to vierbein $e_m{}^a$ and gravitino $\psi_m{}^{\underline{\alpha}}$. Thus, we express the lowest components of field strengths in term of the lowest components of the bosonic p-form gauge fields, vierbein, and gravitino.

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component	superfield
$C(\Psi^{I_0})$	$\Psi^{I_0}\vert$
$Z(\Psi^{I_0})$	$\begin{pmatrix}-i\nabla_\alpha\Psi^{I_0}\\ +i\bar{\nabla}^{\dot\alpha}\Psi^{I_0}\end{pmatrix}\vert$
$H(\Psi^{I_0})$	$\frac{1}{4}(\nabla^2\Psi^{I_0}+\bar{\nabla}^2\Psi^{I_0})\vert$
$K(\Psi^{I_0})$	$-\frac{i}{4}(\nabla^2\Psi^{I_0}-\bar{\nabla}^2\Psi^{I_0})\vert$
$B_a(\Psi^{I_0})$	$g_a^{I_0}\vert=-\frac{1}{4}(\sigma_a)^{\dot\alpha\alpha}[\nabla_\alpha,\bar{\nabla}_{\dot\alpha}]\Psi^{I_0}\vert$
$\Lambda(\Psi^{I_0})$	$\left(q^{(0)}\cdot\begin{pmatrix}-iW_\alpha\\ +i\bar{W}^{\dot\alpha}\end{pmatrix}\right)^{I_0}\vert$
$D(\Psi^{I_0})$	$\frac{1}{4}\left(q^{(0)}\cdot(\nabla^\alpha W_\alpha+\bar{\nabla}_{\dot\alpha}\bar{W}^{\dot\alpha})\right)^{I_0}\vert$

Table 7. The components of real primary superfields Ψ^{I_0}.

The expressions are obtained by the so-called "double bar projection" [30, 35]. The double bar projections of the p-form gauge fields are defined as

$$\frac{1}{p!}dz^{M_1}\wedge\cdots\wedge dz^{M_p}C^{I_p}_{M_p\ldots M_1}\vert\vert:=\frac{1}{p!}dx^{m_1}\wedge\cdots\wedge dx^{m_p}C^{I_p}_{m_p\ldots m_1}\vert. \tag{4.4}$$

The symbol of "||" means the projection from superforms to forms on the bosonic spacetime: ction: $d\theta^{\underline{\mu}}=\theta^{\underline{\mu}}=0$. However, there still exist fermion parts through contractions between indices as shown below.

Explicitly, the double bar projections of the p-form gauge fields are as follows:

$$\begin{aligned}
f^{I_0}\vert\vert&=f^{I_0}\vert,\\
A^{I_1}\vert\vert&=dx^m A^{I_1}_m\vert,\\
B^{I_2}\vert\vert&=\frac{1}{2}dx^m\wedge dx^n B^{I_2}_{nm}\vert,\\
C^{I_3}\vert\vert&=\frac{1}{3!}dx^m\wedge dx^n\wedge dx^p C^{I_3}_{pnm}\vert,\\
U^{I_4}\vert\vert&=\frac{1}{4!}dx^m\wedge dx^n\wedge dx^p\wedge dx^q U^{I_4}_{qpnm}\vert.
\end{aligned} \tag{4.5}$$

The double bar projections of the field strengths systematically lead to the expressions of the bosonic field strengths. We consider the simplest case. For the 1-form field strength of 0-form gauge field, the double bar projections of g^{I_0} are

$$g^{I_0}\vert\vert=dx^m g^{I_0}_m\vert=dx^m E_m{}^A g^{I_0}_A\vert=dx^m E_m{}^a g^{I_0}_a\vert+dx^m E_m{}^{\underline{\alpha}} g^{I_0}_{\underline{\alpha}}\vert. \tag{4.6}$$

This relation gives rise to the component expression

$$\begin{aligned}
g^{I_0}_a\vert&=e_a{}^m g^{I_0}_m\vert-\frac{1}{2}e_a{}^m\psi_m{}^\alpha g^{I_0}_\alpha\vert-\frac{1}{2}e_a{}^m\bar{\psi}_{m\dot\alpha}g^{I_0\dot\alpha}\vert\\
&=e_a{}^m(\partial_m f^{I_0}-(q^{(0)}\cdot A_m)^{I_0})\vert-\frac{i}{2}e_a{}^m\psi_m{}^\alpha\nabla_\alpha\Psi^{I_0}\vert+\frac{i}{2}e_a{}^m\bar{\psi}_{m\dot\alpha}\bar{\nabla}^{\dot\alpha}\Psi^{I_0}\vert.
\end{aligned} \tag{4.7}$$

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Here, we used

$$dz^M g_M^{I_0}|| = df^{I_0}|| - (q^{(0)} \cdot A||)^{I_0} = dx^m(\partial_m f^{I_0}| - (q^{(0)} \cdot A_m|)^{I_0}). \tag{4.8}$$

The same procedure can be applied to higher forms. The results are summarized in appendix C.

5 Summary and discussion

In this paper, we have considered a way to couple the Abelian tensor hierarchy to 4D $\mathcal{N} = 1$ conformal supergravity. We have used the conformal superspace formalism and covariant approach. The constraints on the field strengths have been imposed. The constraints are the same as the case that Abelian tensor hierarchy does not exist. We have solved the Bianchi identities under the constraints. Each of the field strengths is expressed in terms of the single superfield and its conjugate. The linearity conditions which appear in 2-form gauge fields are deformed by the tensor hierarchy. The reality conditions which appear in 1-form gauge fields are also deformed. Furthermore, we have obtained nontrivial conditions of superconformal transformation laws.

We have obtained the irreducible superfields $Y^{I_3}, L^{I_2}, W_\alpha^{I_1}$ and Ψ^{I_0}, which contain p-forms in a covariant manner. We have also clarified the Weyl and chiral weights (Δ, w) of each superfield. Then, it is straightforward to construct the superconformally invariant kinetic terms of p-forms with action formulas shown in ref. [19], although the construction of Chern-Simons terms requires the so-called prepotential superfields. It is worth mentioning that the Weyl and chiral weights of irreducible superfields tell us possible couplings between conformal compensator and p-forms, which lead to possible interactions of p-forms due to SUGRA effects. Such effects are clarified within conformal SUGRA, and this is one of the utilities of conformal SUGRA. In particular, chiral superfields Y^{I_3} can appear also in superpotential terms, which would produce couplings of 4-form field-strengths. Such terms are expected to be important, e.g., for SUGRA realization of models in refs. [10, 11].

There are other remaining issues. As mentioned above, one can think the Chern-Simons couplings of tensor fields. To introduce these terms, we need to reconstruct the tensor hierarchy with the so-called prepotential approach. Further, when there exist (non-Abelian) gauge/matter fields, one has to take chiral anomalies into account. Then the prepotential S^{I_0} will appear also in the superpotential or gauge kinetic functions. It is also important to show the component expression of the kinetic and Chern-Simons terms not only for phenomenological applications but also for understanding the relation between 4D expressions and effective superstring models. We will address these issues elsewhere [36].

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A Notations

In this section, we summarize our notations. The notations are the same as those of Wess-Bagger [29]. The Minkowski metric and the totally antisymmetric tensor are is given by

$$\eta_{ab} = (-1, +1, +1, +1), \quad \epsilon^{0123} = -\epsilon_{0123} = +1. \tag{A.1}$$

The standard contractions of two-component spinors are given by $\xi^\alpha\psi_\alpha$ and $\bar{\xi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}$. The raising and lowering rules of indices are defined by

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}}, \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}, \tag{A.2}$$

where $\epsilon^{\alpha\beta}$, $\epsilon_{\alpha\beta}$, $\epsilon_{\dot{\alpha}\dot{\beta}}$ and $\epsilon^{\dot{\alpha}\dot{\beta}}$ are antisymmetric tensors that satisfy $\epsilon^{12} = \epsilon_{21} = +1$. The Hermitian conjugate of a spinors is defined as $(\psi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}}$. Hermitian conjugate reverses the order of the product of spinors:

$$(\psi_\alpha\xi_\beta)^\dagger = \bar{\xi}_{\dot{\beta}}\bar{\psi}_{\dot{\alpha}}. \tag{A.3}$$

Pauli matrices are defined as

$$(\sigma_0)_{\alpha\dot{\beta}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\sigma_1)_{\alpha\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (\sigma_2)_{\alpha\dot{\beta}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad (\sigma_3)_{\alpha\dot{\beta}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.4}$$

Their Hermitian conjugates are given by

$$(\bar{\sigma}_a)^{\dot{\alpha}\beta} = (\sigma_a)^{\beta\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\gamma}}\epsilon^{\beta\delta}(\sigma_a)_{\delta\dot{\gamma}}. \tag{A.5}$$

Pauli matrices satisfy

$$(\sigma_a)_{\alpha\dot{\beta}}(\bar{\sigma}_b)^{\dot{\beta}\gamma} + (\sigma_b)_{\alpha\dot{\beta}}(\bar{\sigma}_a)^{\dot{\beta}\gamma} = -2\eta_{ab}\delta_\alpha{}^\gamma, \quad (\bar{\sigma}_a)^{\dot{\alpha}\beta}(\sigma_b)_{\beta\dot{\gamma}} + (\bar{\sigma}_b)^{\dot{\alpha}\beta}(\sigma_a)_{\beta\dot{\gamma}} = -2\eta_{ab}\delta^{\dot{\alpha}}{}_{\dot{\gamma}}, \tag{A.6}$$

$$(\sigma^a)_{\alpha\dot{\beta}}(\bar{\sigma}_a)^{\dot{\gamma}\delta} = -2\delta_\alpha{}^\delta\delta^{\dot{\gamma}}{}_{\dot{\beta}}. \tag{A.7}$$

Lorentz vectors are expressed as mixed Lorentz spinors and vice versa:

$$v_{\alpha\dot{\beta}} = (\sigma^a)_{\alpha\dot{\beta}}v_a, \quad v_a = -\frac{1}{2}(\bar{\sigma}^a)^{\dot{\beta}\alpha}v_{\alpha\dot{\beta}}, \tag{A.8}$$

The matrices σ_{ab} and $\bar{\sigma}_{ab}$ are given by

$$(\sigma_{ab})_\alpha{}^\beta = \frac{1}{4}((\sigma_a)_{\alpha\dot{\gamma}}(\bar{\sigma}_b)^{\dot{\gamma}\beta} - (\sigma_b)_{\alpha\dot{\gamma}}(\bar{\sigma}_a)^{\dot{\gamma}\beta}), \quad (\bar{\sigma}_{ab})^{\dot{\alpha}}{}_{\dot{\beta}} = \frac{1}{4}((\bar{\sigma}_a)^{\dot{\alpha}\gamma}(\sigma_b)_{\gamma\dot{\beta}} - (\bar{\sigma}_b)^{\dot{\alpha}\gamma}(\sigma_a)_{\gamma\dot{\beta}}). \tag{A.9}$$

Any anti-symmetric tensor F_{ab} can be decomposed into chiral and anti-chiral parts:

$$F_{ab} = -(\epsilon\sigma_{ab})^{\alpha\beta}F^-_{\alpha\beta} + (\bar{\sigma}_{ab}\epsilon)^{\dot{\alpha}\dot{\beta}}F^+_{\dot{\alpha}\dot{\beta}}, \tag{A.10}$$

where

$$F^-_{\alpha\beta} := \frac{1}{2}(\sigma^{ab}\epsilon)_{\alpha\beta}F_{ab}, \qquad F^+_{\dot{\alpha}\dot{\beta}} := -\frac{1}{2}(\epsilon\bar{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}}F_{ab}. \tag{A.11}$$

In spinor notations, the equation is rewritten as

$$F_{\alpha\dot{\alpha},\beta\dot{\beta}} = (\sigma^a)_{\alpha\dot{\alpha}}(\sigma^b)_{\beta\dot{\beta}}F_{ab} = -2\epsilon_{\alpha\beta}F^+_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}F^-_{\alpha\beta}. \tag{A.12}$$

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B Solving the Bianchi identities

In this appendix, we show detailed derivations of solutions to the Bianchi identities. Subsections B.1, B.2, B.3 and B.4 are devoted to derive the results in 3.2.1, 3.2.2, 3.2.3 and 3.2.4, respectively. In the subsection B.5, we show derivations of the D-, A-, K_A-transformations of the gauge invariant superfields.

B.1 The Bianchi identities for 3-form gauge fields

Firstly, we solve the Bianchi identities for 3-form gauge fields. The Bianchi identities are given by

$$\frac{1}{4!}E^A\wedge E^B\wedge E^C\wedge E^D\wedge E^E\nabla_E\Sigma^{I_3}_{DCBA}+\frac{1}{3!2!}E^A\wedge E^B\wedge E^C\wedge E^D\wedge E^E T_{ED}{}^F\Sigma^{I_3}_{FCBA}=0. \tag{B.1}$$

Explicitly they are written by

$$\begin{aligned}0=&\nabla_E\Sigma^{I_3}_{DCBA}+\nabla_D\Sigma^{I_3}_{CBAE}+\nabla_C\Sigma^{I_3}_{BAED}+\nabla_B\Sigma^{I_3}_{AEDC}+\nabla_A\Sigma^{I_3}_{EDCB}\\&+T_{ED}{}^F\Sigma^{I_3}_{FCBA}-T_{EC}{}^F\Sigma^{I_3}_{FDBA}-T_{EB}{}^F\Sigma^{I_3}_{FCDA}-T_{EA}{}^F\Sigma^{I_3}_{FCBD}\\&+T_{DC}{}^F\Sigma^{I_3}_{FEBA}-T_{DB}{}^F\Sigma^{I_3}_{FECA}-T_{DA}{}^F\Sigma^{I_3}_{FEBC}\\&+T_{CB}{}^F\Sigma^{I_3}_{FEDA}-T_{CA}{}^F\Sigma^{I_3}_{FBED}\\&-T_{BA}{}^F\Sigma^{I_3}_{FECD}.\end{aligned} \tag{B.2}$$

The constraints for 3-form are given in table 2. Under these constraints, we solve the Bianchi identities.

For $E=\epsilon$, $D=\dot\delta$, $C=\gamma$, $B=\beta$, $A=a$, the Bianchi identities are

$$0=\quad T_{\epsilon\dot\delta}{}^f\Sigma^{I_3}_{f\gamma\beta a}+T_{\dot\delta\gamma}{}^f\Sigma^{I_3}_{f\epsilon\beta a}+T_{\dot\delta\beta}{}^f\Sigma^{I_3}_{f\epsilon\gamma a}. \tag{B.3}$$

This equation is equivalently written as

$$\Sigma^{I_3}_{\epsilon\dot\delta,\gamma\beta,\alpha\dot\alpha}+\Sigma^{I_3}_{\gamma\dot\delta,\epsilon\beta,\alpha\dot\alpha}=-\Sigma^{I_3}_{\beta\dot\delta,\epsilon\gamma,\alpha\dot\alpha}. \tag{B.4}$$

We decompose $\Sigma^{I_3}_{\epsilon\dot\delta,\gamma,\beta,\alpha\dot\alpha}$ into chiral part and anti-chiral part as

$$\Sigma^{I_3}_{\epsilon\dot\delta,\gamma,\beta,\alpha\dot\alpha}=-2\epsilon_{\epsilon\alpha}\Sigma^{+I_3}_{\dot\delta\dot\alpha,\gamma,\beta}+2\epsilon_{\dot\delta\dot\alpha}\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta}. \tag{B.5}$$

Substituting this decomposition into eq. (B.4), we obtain

$$0=-2\epsilon_{\epsilon\alpha}\Sigma^{+I_3}_{\dot\delta\dot\alpha,\gamma,\beta}+2\epsilon_{\dot\delta\dot\alpha}\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta}-2\epsilon_{\gamma\alpha}\Sigma^{+I_3}_{\dot\delta\dot\alpha,\epsilon,\beta}+2\epsilon_{\dot\delta\dot\alpha}\Sigma^{-I_3}_{\gamma\alpha,\epsilon,\beta}-2\epsilon_{\beta\alpha}\Sigma^{+I_3}_{\dot\delta\dot\alpha,\gamma,\epsilon}+2\epsilon_{\dot\delta\dot\alpha}\Sigma^{-I_3}_{\beta\alpha,\gamma,\epsilon}. \tag{B.6}$$

We find that anti-chiral parts vanish:

$$\Sigma^{+I_3}_{\dot\delta\dot\alpha,\gamma,\beta}=0. \tag{B.7}$$

Eq. (B.6) is then expressed as

$$\Sigma^{-I_3}_{\gamma\alpha,\epsilon,\beta}+\Sigma^{-I_3}_{\beta\alpha,\epsilon,\gamma}+\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta}=0, \tag{B.8}$$

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where we used $\Sigma^{-I_3}_{\gamma\alpha,\epsilon,\beta} = \Sigma^{-I_3}_{\gamma\alpha,\beta,\epsilon}$. Contracting γ and α by $\epsilon^{\gamma\alpha}$, we obtain

$$0 = \Sigma^{-I_3}{}_{\beta}{}^{\gamma}{}_{,\epsilon,\gamma} + \Sigma^{-I_3}{}_{\epsilon}{}^{\gamma}{}_{,\gamma,\beta}. \tag{B.9}$$

This equation means that

$$\Sigma^{-I_3}{}_{\beta}{}^{\gamma}{}_{,\epsilon,\gamma} = \frac{1}{2}\epsilon_{\beta\epsilon}\Sigma^{-I_3\delta\gamma}{}_{,\delta,\gamma}. \tag{B.10}$$

Then, $\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta}$ are calculated as

$$\begin{aligned}
\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta} &= \frac{1}{2}(\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta} + \Sigma^{-I_3}_{\beta\alpha,\gamma,\epsilon}) + \frac{1}{2}\epsilon_{\epsilon\beta}\Sigma^{-I_3\delta}{}_{\alpha,\gamma,\delta} \\
&= -\frac{1}{2}\Sigma^{-I_3}_{\alpha\gamma,\epsilon,\beta} + \frac{1}{2}\epsilon_{\epsilon\beta}\Sigma^{-I_3\delta}{}_{\alpha,\gamma,\delta} \\
&= -\frac{1}{2}\Sigma^{-I_3}_{\alpha\epsilon,\gamma,\beta} - \frac{1}{2}(\Sigma^{-I_3}_{\alpha\gamma,\epsilon,\beta} - \Sigma^{-I_3}_{\alpha\epsilon,\gamma,\beta}) + \frac{1}{2}\epsilon_{\epsilon\beta}\Sigma^{-I_3\delta}{}_{\alpha,\gamma,\delta} \\
&= -\frac{1}{2}\Sigma^{-I_3}_{\alpha\epsilon,\gamma,\beta} - \frac{1}{2}\epsilon_{\gamma\epsilon}\Sigma^{-I_3}{}_{\alpha}{}^{\delta}{}_{,\delta,\beta} + \frac{1}{2}\epsilon_{\epsilon\beta}\Sigma^{-I_3\delta}{}_{\alpha,\gamma,\delta} \\
&= -\frac{1}{2}\Sigma^{-I_3}_{\alpha\epsilon,\gamma,\beta} - \frac{1}{4}\epsilon_{\gamma\epsilon}\epsilon_{\alpha\beta}\Sigma^{-I_3\zeta\delta}{}_{,\delta,\zeta} + \frac{1}{4}\epsilon_{\epsilon\beta}\epsilon_{\alpha\gamma}\Sigma^{-I_3\delta\zeta}{}_{,\zeta,\delta}.
\end{aligned} \tag{B.11}$$

Therefore, $\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta}$ have only scalar components:

$$\Sigma^{-I_3}_{\epsilon\alpha,\gamma,\beta} = \frac{1}{6}(\epsilon_{\alpha\beta}\epsilon_{\epsilon\gamma} + \epsilon_{\alpha\gamma}\epsilon_{\epsilon\beta})\Sigma^{-I_3\eta\zeta}{}_{,\zeta,\eta}. \tag{B.12}$$

Thus, we obtain

$$\Sigma^{I_3}_{\epsilon\dot\delta,\gamma,\beta,\alpha\dot\alpha} = \frac{1}{3}\epsilon_{\dot\delta\dot\alpha}(\epsilon_{\alpha\beta}\epsilon_{\epsilon\gamma} + \epsilon_{\alpha\gamma}\epsilon_{\epsilon\beta})\Sigma^{-I_3\eta\zeta}{}_{,\zeta,\eta}. \tag{B.13}$$

We define $\bar{Y}^{I_3}$ as

$$\bar{Y}^{I_3} := \frac{2}{3}\Sigma^{-I_3\eta\zeta}{}_{,\zeta,\eta}. \tag{B.14}$$

This definitions of $\bar{Y}^{I_3}$ agree with those of eq. (VI-2.7) in ref. [30]. Equation (B.13) is equivalently expressed as in eq. (3.2). Similarly, dotted versions of the Bianchi identities (B.4) lead to eq. (3.2).

For $E = \epsilon$, $D = \delta$, $C = \gamma$, $B = \beta\dot\beta$, $A = \alpha\dot\alpha$, eq. (B.2) is written as

$$\begin{aligned}
0 &= \nabla_\epsilon\Sigma^{I_3}_{\delta,\gamma,\beta\dot\beta,\alpha\dot\alpha} + \nabla_\delta\Sigma^{I_3}_{\gamma,\beta\dot\beta,\alpha\dot\alpha,\epsilon} + \nabla_\gamma\Sigma^{I_3}_{\beta\dot\beta,\alpha\dot\alpha,\epsilon,\delta} \\
&= \epsilon_{\dot\beta\dot\alpha}(\epsilon_{\beta\delta}\epsilon_{\alpha\gamma} + \epsilon_{\alpha\delta}\epsilon_{\beta\gamma})\nabla_\epsilon\bar{Y}^{I_3} + \epsilon_{\beta\dot\alpha}(\epsilon_{\beta\gamma}\epsilon_{\alpha\epsilon} + \epsilon_{\alpha\gamma}\epsilon_{\beta\epsilon})\nabla_\delta\bar{Y}^{I_3} + \epsilon_{\dot\beta\dot\alpha}(\epsilon_{\beta\epsilon}\epsilon_{\alpha\delta} + \epsilon_{\alpha\epsilon}\epsilon_{\beta\delta})\nabla_\gamma\bar{Y}^{I_3}.
\end{aligned} \tag{B.15}$$

Contracting spinors by $\epsilon^{\dot\alpha\dot\beta}\epsilon^{\delta\beta}\epsilon^{\gamma\alpha}$, we obtain the anti-chirality conditions of $\bar{Y}^{I_3}$ in eq. (3.6). Similarly, we obtain chirality conditions of Y^{I_3} as in eq. (3.6).

For $E = \epsilon$, $D = \dot\delta$, $C = \dot\gamma$ $B = \beta\dot\beta$, $A = \alpha\dot\alpha$, eq. (B.2) is written as

$$0 = \nabla_\epsilon\Sigma^{I_3}_{\dot\delta,\dot\gamma,\beta\dot\beta,\alpha\dot\alpha} + T_{\epsilon\dot\delta}{}^{f}\Sigma^{I_3}_{f,\dot\gamma,\beta\dot\beta,\alpha\dot\alpha} + T_{\epsilon\dot\gamma}{}^{f}\Sigma^{I_3}_{f,\dot\delta,\beta\dot\beta,\alpha\dot\alpha}. \tag{B.16}$$

This is solved as

$$\nabla_\delta Y^{I_3} = +\frac{2}{3}\epsilon^{dcba}\sigma_{d\delta\dot\delta}\Sigma^{I_3\dot\delta}{}_{cba}, \tag{B.17}$$

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or equivalently expressed as in eq. (3.7). Similarly, $\Sigma^{I_3}_{\delta cba}$ are expressed in terms of $\bar{\nabla}^{\dot{\delta}}\bar{Y}^{I_3}$:

$$\bar{\nabla}^{\dot{\delta}}\bar{Y}^{I_3} = -\frac{2}{3}\epsilon^{dcba}\bar{\sigma}_d^{\dot{\delta}\delta}\Sigma^{I_3}_{\delta cba}, \tag{B.18}$$

or eq. (3.7).

For $E=\epsilon$, $D=\dot{\delta}$, $C=c$, $B=b$, $A=a$, eq. (B.2) is expressed as

$$0 = \nabla_\epsilon \Sigma^{I_3\dot{\delta}}{}_{cba} - \bar{\nabla}^{\dot{\delta}}\Sigma^{I_3}_{cba\epsilon} + T_\epsilon{}^{\dot{\delta}f}\Sigma^{I_3}_{fcba}. \tag{B.19}$$

Using (3.7), and contracting spinors, we obtain

$$\frac{8}{3}i\epsilon^{dcba}\Sigma^{I_3}_{dcba} = \nabla^2 Y^{I_3} - \bar{\nabla}^2\bar{Y}^{I_3}. \tag{B.20}$$

They are equivalently written as in eq. (3.8). There is no more non-trivial Bianchi identity from constraints.

B.2 The Bianchi identities for 2-form gauge fields

Next, we solve the Bianchi identities for 2-form gauge fields. The Bianchi identities are written as

$$\begin{aligned} 0 =& \nabla_D H^{I_2}_{CBA} - \nabla_C H^{I_2}_{BAD} + \nabla_B H^{I_2}_{ADC} - \nabla_A H^{I_2}_{DCB} \\ &+ T_{DC}{}^E H^{I_2}_{EBA} - T_{DB}{}^E H^{I_2}_{ECA} + T_{DA}{}^E H^{I_2}_{ECB} - T_{CB}{}^E H^{I_2}_{EAD} + T_{CA}{}^E H^{I_2}_{EBD} + T_{BA}{}^E H^{I_2}_{EDC} \\ &+ (q^{(2)}\cdot\Sigma_{DCBA})^{I_2}. \end{aligned} \tag{B.21}$$

The constraints on the field strengths of 2-form gauge fields are given in table 2.

For $D=\delta$, $C=\gamma$, $B=\dot{\beta}$, $a=\alpha\dot{\alpha}$, eq. (B.21) is

$$0 = \nabla_\delta H^{I_2}_{\gamma,\dot{\beta},\alpha\dot{\alpha}} - \nabla_\gamma H^{I_2}_{\dot{\beta},\alpha\dot{\alpha},\delta} + T_{\delta\dot{\beta}}{}^e H^{I_2}_{e,\gamma,\alpha\dot{\alpha}} - T_{\dot{\gamma}\beta}{}^e H^{I_2}_{e,\alpha\dot{\alpha},\delta}. \tag{B.22}$$

Using the constraints in table 2, we obtain

$$-4i\epsilon_{\gamma\alpha}\epsilon_{\dot{\beta}\dot{\alpha}}\nabla_\delta L^{I_2} - 4i\epsilon_{\delta\alpha}\epsilon_{\dot{\beta}\dot{\alpha}}\nabla_\gamma L^{I_2} + 2iH^{I_2}_{\delta\dot{\beta},\gamma,\alpha\dot{\alpha}} - 2iH^{I_2}_{\gamma\dot{\beta},\alpha\dot{\alpha},\delta} = 0. \tag{B.23}$$

We decompose $H^{I_2}_{\delta,\gamma\dot{\beta},\alpha\dot{\alpha}}$ as

$$H^{I_2}_{\delta,\gamma\dot{\beta},\alpha\dot{\alpha}} = -2\epsilon_{\gamma\alpha}H^{I_2+}{}_{\delta,\dot{\beta}\dot{\alpha}} + 2\epsilon_{\dot{\beta}\dot{\alpha}}H^{I_2-}{}_{\delta,\gamma\alpha}. \tag{B.24}$$

Substituting this into eq. (B.23) and contracting spinors by $\epsilon^{\dot{\beta}\dot{\alpha}}$, we obtain

$$\epsilon_{\alpha\gamma}\nabla_\delta L^{I_2} + \epsilon_{\alpha\delta}\nabla_\gamma L^{I_2} - H^{I_2-}{}_{\gamma,\delta\alpha} - H^{I_2-}{}_{\delta,\gamma\alpha} = 0. \tag{B.25}$$

Furthermore, contracting spinor indices by $\epsilon^{\alpha\gamma}$, we find that

$$H^{I_2-\alpha,}{}_{\delta\alpha} = -3\nabla_\delta L^{I_2}. \tag{B.26}$$

Substituting this equation to eq. (B.25), we obtain

$$H^{I_2-}{}_{\alpha,\gamma\delta} = -\epsilon_{\alpha\gamma}\nabla_\delta L^{I_2} - \epsilon_{\alpha\delta}\nabla_\gamma L^{I_2}, \tag{B.27}$$

where we used $H^{I_2-}{}_{\gamma,\delta\alpha} = \epsilon_{\gamma\alpha}H^{I_2-\phi}{}_{,\delta\phi} + H^{I_2-}{}_{\alpha,\delta\gamma}$ and $H^{I_2-}{}_{\delta,\gamma\alpha} = \epsilon_{\delta\alpha}H^{I_2-\phi}{}_{,\gamma\phi} + H^{I_2-}{}_{\alpha,\gamma\delta}$.

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The symmetrization of $\dot{\alpha} \leftrightarrow \dot{\beta}$ in eq. (B.23) with eq. (B.24) reads

$$H^{I_2+}{}_{\delta,\dot{\beta}\dot{\alpha}} = 0. \tag{B.28}$$

Therefore, we obtain

$$H^{I_2}_{\delta,\gamma\dot{\beta},\alpha\dot{\alpha}} = -2\epsilon_{\dot{\beta}\dot{\alpha}}(\epsilon_{\delta\gamma}\nabla_\alpha L^{I_2} + \epsilon_{\delta\alpha}\nabla_\gamma L^{I_2}), \tag{B.29}$$

which is equivalent to eq. (3.9). Similarly, for $D = \dot{\delta}$, $C = \dot{\gamma}$, $B = \beta$, $a = \alpha\dot{\alpha}$, we obtain

$$H^{I_2}_{\dot{\gamma},\beta\dot{\delta},\alpha\dot{\alpha}} = -2\epsilon_{\beta\alpha}(\epsilon_{\dot{\gamma}\dot{\delta}}\bar{\nabla}_{\dot{\alpha}} L^{I_2} + \epsilon_{\dot{\gamma}\dot{\alpha}}\bar{\nabla}_{\dot{\delta}} L^{I_2}), \tag{B.30}$$

which is equivalent to eq. (3.9).

For $D = \delta$, $C = \dot{\gamma}$, $B = b$, $A = a$, eq. (B.21) is written as

$$0 = \nabla_\delta H^{I_2}_{\dot{\gamma}ba} + \bar{\nabla}_{\dot{\gamma}} H^{I_2}_{ba\delta} + \nabla_b H^{I_2}_{a\delta\dot{\gamma}} - \nabla_a H^{I_2}_{\delta\dot{\gamma}b} + T_{\delta\dot{\gamma}}{}^e H^{I_2}_{eba}. \tag{B.31}$$

This equation implies

$$0 = 2(\bar{\sigma}_{ba})_{\dot{\gamma}\dot{\phi}}\nabla_\delta\bar{\nabla}^{\dot{\phi}} L^{I_2} + 2(\sigma_{ba})_\delta{}^\phi\bar{\nabla}_{\dot{\gamma}}\nabla_\phi L^{I_2} + 2i(\sigma_a)_{\delta\dot{\gamma}}\nabla_b L^{I_2} - 2i(\sigma_b)_{\delta\dot{\gamma}}\nabla_a L^{I_2} + 2i(\sigma_e)_{\delta\dot{\gamma}} H^{I_2}_{eba}. \tag{B.32}$$

From this identity, we obtain

$$\epsilon^{cfba}(\sigma_c)_{\gamma\dot{\gamma}} H^{I_2}_{fba} = -3[\nabla_\gamma, \bar{\nabla}_{\dot{\gamma}}] L^{I_2}, \tag{B.33}$$

which is equivalent to eq. (3.10).

For $D = \delta$, $C = \gamma$, $B = b$, $A = a$, eq. (B.21) is expressed as

$$0 = \nabla_\delta H^{I_2}_{\gamma ba} + \nabla_\gamma H^{I_2}_{\delta ba} + (q^{(2)} \cdot \Sigma_{\delta\gamma ba})^{I_2}. \tag{B.34}$$

Using eqs. (3.2) and (3.9), we obtain eq. (3.11). Similarly, for $D = \dot{\delta}$, $C = \dot{\gamma}$, $B = b$, $A = a$, we find eq. (3.11).

B.3 The Bianchi identities for 1-form gauge fields

Thirdly, we solve the Bianchi identities for the field strengths of 1-form gauge fields [29]:

$$0 = \nabla_C F^{I_1}_{BA} + \nabla_B F^{I_1}_{AC} + \nabla_A F^{I_1}_{CB} + T_{CB}{}^D F^{I_1}_{DA} + T_{BA}{}^D F^{I_1}_{DC} + T_{AC}{}^D F^{I_1}_{DB} + (q^{(1)} \cdot H_{CBA})^{I_1}. \tag{B.35}$$

For $C = \dot{\gamma}$, $B = \beta$, $A = \alpha$, Bianchi identities are

$$0 = T_{\dot{\gamma}\beta}{}^d F^{I_1}_{d\alpha} + T_{\alpha\dot{\gamma}}{}^d F^{I_1}_{d\beta}. \tag{B.36}$$

This means symmetric part of undotted spinors in $F^{I_1}_{\dot{\gamma}\beta,\alpha}$ is equal to zero. Then, we can write

$$F^{I_1}_{\alpha,\beta\dot{\gamma}} = -2\epsilon_{\alpha\beta} W^{I_1}_{\dot{\gamma}}. \tag{B.37}$$

Similarly, for $C = \gamma$, $B = \dot{\beta}$, $A = \dot{\alpha}$, we obtain

$$F^{I_1}_{\dot{\alpha},\gamma\dot{\beta}} = -2\epsilon_{\dot{\alpha}\dot{\beta}} W^{I_1}_{\gamma}. \tag{B.38}$$

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For $C=\dot{\gamma}$, $B=\dot{\beta}$, $A=\alpha\dot{\alpha}$, the Bianchi identities are

$$0=\bar{\nabla}_{\dot{\gamma}}F^{I_1}_{\dot{\beta},\alpha\dot{\alpha}}-\bar{\nabla}_{\dot{\beta}}F^{I_1}_{\alpha\dot{\alpha},\dot{\gamma}}. \tag{B.39}$$

Using (B.37) we obtain chirality condition for $W^{I_1}_{\alpha}$ as

$$\bar{\nabla}_{\dot{\gamma}}W^{I_1}_{\alpha}=0. \tag{B.40}$$

Similarly for $C=\gamma$, $B=\beta$, $A=\alpha\dot{\alpha}$, we find

$$\nabla_{\gamma}W^{I_1}_{\dot{\alpha}}=0. \tag{B.41}$$

For $C=\gamma$, $B=\dot{\beta}$, $A=\alpha\dot{\alpha}$, the Bianchi identities are

$$0=\nabla_{\gamma}F^{I_1}_{\dot{\beta},\alpha\dot{\alpha}}-\bar{\nabla}_{\dot{\beta}}F^{I_1}_{\alpha\dot{\alpha},\gamma}+T_{\gamma\dot{\beta}}{}^{d}F^{I_1}_{d,\alpha\dot{\alpha}}-4i\epsilon_{\gamma\alpha}\epsilon_{\dot{\beta}\dot{\alpha}}(q^{(1)}\cdot L)^{I_1}. \tag{B.42}$$

Contracting the spinor indices by $\epsilon^{\alpha\gamma}\epsilon^{\dot{\alpha}\dot{\beta}}$, we obtain eq. (3.17). Then, symmetrizing spinors in eq. (B.42), we also obtain eq. (3.16).

B.4 The Bianchi identities for 0-form gauge fields

Finally, we solve the Bianchi identities for 0-form gauge fields. The Bianchi identities are given by

$$0=\nabla_B g^{I_0}_A-\nabla_A g^{I_0}_B+T_{BA}{}^{C}g^{I_0}_C+(q^{(0)}\cdot F_{BA})^{I_0}. \tag{B.43}$$

For $B=\beta$, $A=\alpha$, eq. (B.43) is

$$0=\nabla_{\beta}g^{I_0}_{\alpha}+\nabla_{\alpha}g^{I_0}_{\beta}. \tag{B.44}$$

This means that

$$\nabla_{\beta}g^{I_0}_{\alpha}=\frac{1}{2}\epsilon_{\beta\alpha}\nabla^{\gamma}g^{I_0}_{\gamma}. \tag{B.45}$$

Furthermore, the actions of ∇^{β} on both hand sides lead to

$$\nabla^2 g^{I_0}_{\alpha}=0. \tag{B.46}$$

Similarly, for $B=\dot{\beta}$, $A=\dot{\alpha}$, we obtain

$$\bar{\nabla}_{\dot{\beta}}g^{I_0}_{\dot{\alpha}}=-\frac{1}{2}\epsilon_{\dot{\beta}\dot{\alpha}}\bar{\nabla}_{\dot{\gamma}}g^{I_0\dot{\gamma}}, \tag{B.47}$$

$$\bar{\nabla}^2 g^{I_0}_{\dot{\alpha}}=0. \tag{B.48}$$

These consequences suggest that we may impose the constraints

$$g^{I_0}_{\alpha}=\lambda\nabla_{\alpha}\Psi^{I_0},\quad g^{I_0}_{\dot{\alpha}}=\lambda^{*}\bar{\nabla}_{\dot{\alpha}}\Psi^{I_0}, \tag{B.49}$$

where we took λ as a complex constant, and Ψ^{I_0} are real primary superfields.

For $B=\beta$, $A=\dot{\alpha}$, eq. (B.43) is

$$0=\nabla_{\beta}g^{I_0}_{\dot{\alpha}}+\bar{\nabla}_{\dot{\alpha}}g^{I_0}_{\beta}+T_{\beta\dot{\alpha}}{}^{c}g^{I_0}_{c}. \tag{B.50}$$

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If we take $\lambda = i$, this equation reproduces the results in ref. [8]. In this choice, $g^{I_0}_{\alpha\dot{\alpha}}$ are written as

$$g^{I_0}_{\alpha\dot{\alpha}} = \frac{1}{2}[\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}]\Psi^{I_0}. \tag{B.51}$$

This equation is equivalently written as in eq. (3.20). Eq. (B.51) means that Ψ^{I_0} contain the field strengths of 0-form gauge fields in the vector components.

For $B = \dot{\beta}$, $A = \alpha\dot{\alpha}$, eq. (B.43) is

$$0 = \bar{\nabla}_{\dot{\beta}} g^{I_0}_{\alpha\dot{\alpha}} - \nabla_{\alpha\dot{\alpha}} g^{I_0}_{\dot{\beta}} + (q^{(0)} \cdot F_{\dot{\beta},\alpha\dot{\alpha}})^{I_0}. \tag{B.52}$$

Using eqs. (B.49), (3.20) and the identity $\nabla_\alpha \bar{\nabla}^2 + 4i\nabla_{\alpha\dot{\gamma}}\bar{\nabla}^{\dot{\gamma}} = \bar{\nabla}^2\nabla_\alpha - 8\mathcal{W}_\alpha$, we obtain the former equation in eq. (3.21). Similarly for $B = \beta$, $A = \alpha\dot{\alpha}$, the latter equation in eq. (3.21) is obtained.

Note that the degrees of freedom between bosons and fermions in Ψ^{I_0} are matched. If the tensor hierarchy does not exist, eq. (3.21) means that the higher fermion components of Ψ^{I_0} vanish:

$$\nabla^2\bar{\nabla}_{\dot{\alpha}}\Psi^{I_0} = 0, \quad \bar{\nabla}^2\nabla_\alpha\Psi^{I_0} = 0. \tag{B.53}$$

So it seems that degrees of freedom in Ψ^{I_0} are mismatched. In a general real superfield case, the degrees of freedom of the components $[C, Z, H, K, B_a, \lambda, D]$ are $[1, 4, 1, 1, 4, 4, 1]$. In this case, Bianchi identity (3.20) follows that vector components of Ψ^{I_0} are the bosonic field strengths: $[\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}]\Psi^{I_0}| \propto \partial_m f^{I_0}|$. Thus, vector components have only one freedom. Then, under the constraint (B.53), degrees of freedom are $[1, 4, 1, 1, 1, 0, 0]$. So the degrees of freedom between bosons and fermions in Ψ^{I_0} are matched. The same argument holds even if the tensor hierarchy exists.

B.5 D-, A-, K_A-transformation laws

We present the derivations of the D-, A-, K_A-transformation laws of $(Y^{I_3}, L^{I_2}, W^{I_1}_\alpha, \Psi^{I_0})$. The transformation laws of the superfields follow from those of $F^{I_p}_{A_{p+1}\ldots A_1}$. Since $F^{I_p}_{M_{p+1}\ldots M_1}$ are invariant under $X_{\mathcal{A}'}$ transformations, the properties are reduced to those of the vielbein:

$$\delta_G(\xi^{\mathcal{A}'}X_{\mathcal{A}'})E_B{}^M = -E_B{}^N\left(\delta_G(\xi^{\mathcal{A}'}X_{\mathcal{A}'})E_N{}^C\right)E_C{}^M. \tag{B.54}$$

The D-, A- and K_A-transformation laws of the vielbein are obtained as follows.

- D-transformations

$$\delta_G(\xi(D)D)E_b{}^M = +\xi(D)E_b{}^M, \quad \delta_G(\xi(D)D)E_{\underline{\beta}}{}^M = +\frac{1}{2}\xi(D)E_{\underline{\beta}}{}^M. \tag{B.55}$$

- A-transformations

$$\begin{aligned}\delta_G(\xi(A)A)E_b{}^M = 0, \quad & \delta_G(\xi(A)A)E_\beta{}^M = -i\xi(A)E_\beta{}^M, \\ & \delta_G(\xi(A)A)E^{\dot{\beta}M} = +i\xi(A)E^{\dot{\beta}M}.\end{aligned} \tag{B.56}$$

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- S-transformations

$$\begin{aligned}
&\delta_G(\xi(K)^\alpha S_\alpha)E_b{}^M = iE_b{}^N E_N{}^e \xi(K)^\delta (\sigma_e)_{\delta\dot\gamma} E^{\dot\gamma M} = i\xi(K)^\delta (\sigma_b)_{\delta\dot\gamma} E^{\dot\gamma M},\\
&\delta_G(\xi(K)_{\dot\alpha}\bar S^{\dot\alpha})E_b{}^M = iE_b{}^N E_N{}^e \xi(K)_{\dot\delta} (\bar\sigma_e)^{\dot\delta\gamma} E_\gamma{}^M = i\xi(K)_{\dot\delta} (\bar\sigma_b)^{\dot\delta\gamma} E_\gamma{}^M.\\
&\delta_G(\xi(K)^{\underline\alpha} S_{\underline\alpha})E_{\underline\beta}{}^M = 0.
\end{aligned} \tag{B.57}$$

- All the K_a-transformations of the vielbein are equal to zero.

Using these equations, the D-, A- and K_A-transformation laws of $(Y^{I_3}, L^{I_2}, W_\alpha^{I_1}, \Psi^{I_0})$ are determined. Note that the M-transformation laws of $(Y^{I_3}, L^{I_2}, W_\alpha^{I_1}, \Psi^{I_0})$ are obtained by their spinor indices.

B.5.1 3-form gauge fields

We show the D-, A-, K_A-transformation laws of Y^{I_3}. Y^{I_3} are given in terms of $\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba}$ as in eq. (3.2). The D-, A-, K_A-transformations of $\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba}$ are determined as follows.

- D-transformations

$$\delta_G(\xi(D)D)\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba} = \delta_G(\xi(D)D)E^{\dot\delta Q}E^{\dot\gamma P}E_b{}^N E_a{}^M \Sigma^{I_3}_{QPNM} = 3\xi(D)\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba} \tag{B.58}$$

- A-transformations

$$\delta_G(\xi(A)A)\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba} = \delta_G(\xi(A)A)E^{\dot\delta Q}E^{\dot\gamma P}E_b{}^N E_a{}^M \Sigma^{I_3}_{QPNM} = +2i\xi(A)\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba} \tag{B.59}$$

- S_α-transformations

$$\begin{aligned}
&\delta_G(\xi(K)^\alpha S_\alpha)\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba}\\
&= \delta_G(\xi(K)^\alpha S_\alpha)E^{\dot\delta Q}E^{\dot\gamma P}E_b{}^N E_a{}^M \Sigma^{I_3}_{QPNM}\\
&= E^{\dot\delta Q}E^{\dot\gamma P}(i\xi(K)^\alpha(\sigma_b)_{\alpha\dot\epsilon}E^{\dot\epsilon N})E_a{}^M\Sigma^{I_3}_{QPNM} + E^{\dot\delta Q}E^{\dot\gamma P}E_b{}^N(i\xi(K)^\alpha(\sigma_a)_{\alpha\dot\epsilon}E^{\dot\epsilon M})\Sigma^{I_3}_{QPNM}\\
&= i\xi(K)^\alpha(\sigma_b)_{\alpha\dot\epsilon}\Sigma^{I_3\dot\delta\dot\gamma\dot\epsilon}{}_a + i\xi(K)^\alpha(\sigma_a)_{\alpha\dot\epsilon}\Sigma^{I_3\dot\delta\dot\gamma}{}_b{}^{\dot\epsilon}\\
&= 0.
\end{aligned} \tag{B.60}$$

- $\bar S^{\dot\alpha}$-transformations

$$\begin{aligned}
&\delta_G(\xi(K)_{\dot\alpha}\bar S^{\dot\alpha})\Sigma^{I_3\dot\delta\dot\gamma}{}_{ba}\\
&= \delta_G(\xi(K)_{\dot\alpha}\bar S^{\dot\alpha})E^{\dot\delta Q}E^{\dot\gamma P}E_b{}^N E_a{}^M \Sigma^{I_3}_{QPNM}\\
&= E^{\dot\delta Q}E^{\dot\gamma P}(i\xi(K)_{\dot\alpha}(\bar\sigma_b)^{\dot\alpha\epsilon}E_\epsilon{}^N)E_a{}^M\Sigma^{I_3}_{QPNM} + E^{\dot\delta Q}E^{\dot\gamma P}E_b{}^N(i\xi(K)_{\dot\alpha}(\bar\sigma_a)^{\dot\alpha\epsilon}E_\epsilon{}^M)\Sigma^{I_3}_{QPNM}\\
&= i\xi(K)_{\dot\alpha}(\bar\sigma_b)^{\dot\alpha\epsilon}\Sigma^{I_3\dot\delta\dot\gamma}{}_{\epsilon a} + i\xi(K)_{\dot\alpha}(\bar\sigma_a)^{\dot\alpha\epsilon}\Sigma^{I_3\dot\delta\dot\gamma}{}_{b\epsilon}\\
&= 0.
\end{aligned} \tag{B.61}$$

Here, we used the constraints $\Sigma^{I_3\dot\delta\dot\gamma}{}_b{}^{\dot\epsilon} = \Sigma^{I_3\dot\delta\dot\gamma}{}_{b\epsilon} = 0$ in the last lines of S_α and $\bar S^{\dot\alpha}$ transformation laws. These equations lead to the superconformal transformation laws of Y in eq. (3.3). Those of $\bar Y$ are obtained similarly.

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B.5.2 2-form gauge fields

The D-, A-, K_A-transformation laws of L^{I_2} are obtained by the same procedure as the case of 3-form gauge fields. We summarize the results.

- D-transformations
$$\delta_G(\xi(D)D)H^{I_2\dot{\gamma}}{}_{\beta a} = 2\xi(D)H^{I_2\dot{\gamma}}{}_{\beta a} \tag{B.62}$$

- A-transformations
$$\delta_G(\xi(A)A)H^{I_2\dot{\gamma}}{}_{\beta a} = 0. \tag{B.63}$$

- S_α-transformations
$$\delta_G(\xi(K)^\alpha S_\alpha)H^{I_2\dot{\gamma}}{}_{\beta a} = i\xi(K)^\alpha(\sigma_a)_{\alpha\dot{\epsilon}}H_\beta^{I_2\dot{\gamma}\dot{\epsilon}} = 0. \tag{B.64}$$

- $\bar{S}^{\dot{\alpha}}$-transformations
$$\delta_G(\xi(K)_{\dot{\alpha}}\bar{S}^{\dot{\alpha}})H^{I_2\dot{\gamma}}{}_{\beta a} = i\xi(K)_{\dot{\alpha}}(\bar{\sigma}_a)^{\dot{\alpha}\epsilon}H^{I_2\dot{\gamma}}{}_{\beta\epsilon} = 0. \tag{B.65}$$

B.5.3 1-form gauge fields

The D-, A-, K_A-transformation laws of $W_\alpha^{I_2}$ are the same as the case that the tensor hierarchy does not exist. The results are as follows.

- D-transformations
$$\delta_G(\xi(D)D)F^{I_1\dot{\beta}}{}_a = \frac{3}{2}\xi(D)F^{I_1\dot{\beta}}{}_a \tag{B.66}$$

- A-transformations
$$\delta_G(\xi(A)A)F^{I_1\dot{\beta}}{}_a = +i\xi(A)F^{I_1\dot{\beta}}{}_a. \tag{B.67}$$

- S_α-transformations
$$\delta_G(\xi(K)^\alpha S_\alpha)F^{I_1\dot{\beta}}{}_a = -i\xi(K)^\alpha(\sigma_a)_{\alpha\dot{\epsilon}}F^{I_1\dot{\beta}\dot{\epsilon}} = 0. \tag{B.68}$$

- $\bar{S}^{\dot{\alpha}}$-transformations
$$\delta_G(\xi(K)_{\dot{\alpha}}\bar{S}^{\dot{\alpha}})F^{I_1\dot{\beta}}{}_a = -i\xi(K)_{\dot{\alpha}}(\bar{\sigma}_a)^{\dot{\alpha}\epsilon}F^{I_1\dot{\beta}}{}_\epsilon = 0. \tag{B.69}$$

B.5.4 0-form gauge fields

The D-, A-, K_A-transformation laws of Ψ^{I_0} are determined as follows.

- D-transformations
$$\delta_G(\xi(D)D)g_\alpha^{I_0} = \frac{1}{2}\xi(D)g_\alpha^{I_0} \tag{B.70}$$

  lead to
$$D\Psi^{I_0} = 0. \tag{B.71}$$

  This is because
$$[D, \nabla_\alpha] = \frac{1}{2}\nabla_\alpha. \tag{B.72}$$

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- A-transformations

$$\delta_G(\xi(A)A)g_\alpha^{I_0} = -i\xi(A)g_\alpha^{I_0} \tag{B.73}$$

lead to

$$A\Psi^{I_0} = 0. \tag{B.74}$$

This is because

$$[A, \nabla_\alpha] = -i\nabla_\alpha. \tag{B.75}$$

- S_α-transformations

$$\delta_G(\xi(K)^\beta S_\beta)g_\alpha^{I_0} = 0. \tag{B.76}$$

- $\bar{S}^{\dot{\alpha}}$-transformations

$$\delta_G(\xi(K)_{\dot{\beta}}\bar{S}^{\dot{\beta}})g_\alpha^{I_0} = 0. \tag{B.77}$$

We need to check the consistency between the weights of Ψ^{I_0} and the $S_{\underline{\alpha}}$-invariances of Ψ^{I_0}. The $S_{\underline{\alpha}}$-invariances of $g_{\underline{\alpha}}^{I_0}$ require that the weights of Ψ^{I_0} must be $(\Delta, w) = (0, 0)$ [27]. The requirements are understood as follows. The $S_{\underline{\alpha}}$-transformations of $\nabla_{\underline{\alpha}}\Psi^{I_0}$ are generally given by

$$S_\alpha\nabla_\beta\Psi^{I_0} = \epsilon_{\alpha\beta}(2D - 3iA)\Psi^{I_0}, \quad \bar{S}^{\dot{\alpha}}\bar{\nabla}^{\dot{\beta}}\Psi^{I_0} = \epsilon^{\dot{\alpha}\dot{\beta}}(2D + 3iA)\Psi^{I_0}. \tag{B.78}$$

These equations lead to the conditions for the $S_{\underline{\alpha}}$-invariances of $\nabla_{\underline{\alpha}}\Psi^{I_0}$:

$$D\Psi^{I_0} = 0, \quad A\Psi^{I_0} = 0. \tag{B.79}$$

Actually, Ψ^{I_0} satisfy the weight conditions. Thus, the weights are consistent with $S_{\underline{\alpha}}$-invariances of $g_{\underline{\alpha}}^{I_0}$.

C The explicit forms of bosonic field strengths

In this appendix, we summarize the explicit forms of the bosonic field strengths.

For 3-form gauge fields, the double bar projections of Σ^{I_3} lead to the following relations

$$\begin{aligned}&\frac{1}{4!}dx^m \wedge dx^n \wedge dx^p \wedge dx^q \Sigma^{I_3}_{qpnm}| \\ &= \frac{1}{4!}dx^m \wedge dx^n \wedge dx^p \wedge dx^q E_m{}^A E_n{}^B E_p{}^C E_q{}^D \Sigma^{I_3}_{DCBA}|.\end{aligned} \tag{C.1}$$

We expand this relation, and obtain

$$
\begin{aligned}
&e_m{}^a e_n{}^b e_p{}^c e_q{}^d \Sigma^{I_3}_{dcba}| \\
&= \partial_q C^{I_3}_{pnm}| + (-1)^1 \partial_p C^{I_3}_{qnm}| + (-1)^2 \partial_n C^{I_3}_{qpm}| + (-1)^3 \partial_m C^{I_3}_{qpn}| - (q^{(3)} \cdot U_{qpnm}|)^{I_3} \\
&\quad - \frac{1}{2}(e_m{}^a e_n{}^b e_p{}^c \psi_q{}^\delta + (-1)^1 e_m{}^a e_n{}^b e_q{}^c \psi_p{}^\delta \\
&\quad + (-1)^2 e_m{}^a e_p{}^b e_q{}^c \psi_n{}^\delta + (-1)^3 e_n{}^a e_p{}^b e_q{}^c \psi_m{}^\delta) \left(-\frac{1}{16}\right) (\sigma^e)_{\delta\dot\zeta} \epsilon_{ecba} \bar\nabla^{\dot\zeta} \bar Y^{I_3}| \\
&\quad - \frac{1}{2}(e_m{}^a e_n{}^b e_p{}^c \bar\psi_{q\dot\delta} + (-1)^1 e_m{}^a e_n{}^b e_q{}^c \bar\psi_{p\dot\delta} \\
&\quad + (-1)^2 e_m{}^a e_p{}^b e_q{}^c \bar\psi_{n\dot\delta} + (-1)^3 e_n{}^a e_p{}^b e_q{}^c \bar\psi_{m\dot\delta}) \left(+\frac{1}{16}\right) (\bar\sigma^e)^{\dot\delta\zeta} \epsilon_{ecba} \nabla_\zeta Y^{I_3}| \\
&\quad - \frac{1}{2}\cdot\frac{1}{2}(e_m{}^a e_n{}^b \psi_p{}^\gamma \psi_q{}^\delta + (-1)^1 e_m{}^a e_p{}^b \psi_n{}^\gamma \psi_q{}^\delta + (-1)^2 e_m{}^a e_q{}^b \psi_n{}^\gamma \psi_p{}^\delta \\
&\quad + (-1)^2 e_n{}^a e_p{}^b \psi_m{}^\gamma \psi_q{}^\delta + (-1)^{2+2} e_n{}^a e_q{}^b \psi_p{}^\gamma \psi_m{}^\delta + (-1)^{2+2} e_p{}^a e_q{}^b \psi_m{}^\gamma \psi_n{}^\delta) \frac{1}{2}(\sigma_{ba}\epsilon)_{\delta\gamma} \bar Y^{I_3}| \\
&\quad - \frac{1}{2}\cdot\frac{1}{2}(e_m{}^a e_n{}^b \bar\psi_{p\dot\gamma} \bar\psi_{q\dot\delta} + (-1)^1 e_m{}^a e_p{}^b \bar\psi_{n\dot\gamma} \bar\psi_{q\dot\delta} + (-1)^2 e_m{}^a e_q{}^b \bar\psi_{n\dot\gamma} \bar\psi_{p\dot\delta} \\
&\quad + (-1)^2 e_n{}^a e_p{}^b \bar\psi_{m\dot\gamma} \bar\psi_{q\dot\delta} + (-1)^{2+2} e_n{}^a e_q{}^b \bar\psi_{p\dot\gamma} \bar\psi_{m\dot\delta} + (-1)^{2+2} e_p{}^a e_q{}^b \bar\psi_{m\dot\gamma} \bar\psi_{n\dot\delta}) (\bar\sigma_{ba}\epsilon)^{\dot\delta\dot\gamma} Y^{I_3}|
\end{aligned}
\tag{C.2}
$$

For the 2-form gauge fields, the double bar projections are

$$
H^{I_2}|| = \frac{1}{3!} dx^m \wedge dx^n \wedge dx^p H^{I_2}_{pnm}| = \frac{1}{3!} dx^m \wedge dx^n \wedge dx^p E_m{}^A E_n{}^B E_p{}^C H^{I_2}_{CBA}|. \tag{C.3}
$$

We obtain the component expressions of bosonic field strengths

$$
\begin{aligned}
&e_m{}^a e_n{}^b e_p{}^c H^{I_2}_{cba}| \\
&= \partial_p B^{I_2}_{nm}| + \partial_n B^{I_2}_{mp}| + \partial_m B^{I_2}_{pn}| - (q^{(2)} \cdot C_{pnm}|)^{I_2} \\
&\quad - \frac{1}{2}(e_m{}^a e_n{}^b \psi_p{}^\gamma + (-1)^1 e_m{}^a \psi_n{}^\gamma e_p{}^b + (-1)^3 \psi_m{}^\gamma e_n{}^b e_p{}^a)(+2)(\sigma_{ba})_\gamma{}^\delta \nabla_\delta L^{I_2}| \\
&\quad - \frac{1}{2}(e_m{}^a e_n{}^b \bar\psi_{p\dot\gamma} + (-1)^1 e_m{}^a \bar\psi_{n\dot\gamma} e_p{}^b + (-1)^3 \bar\psi_{m\dot\gamma} e_n{}^b e_p{}^a)(+2)(\bar\sigma_{ba})^{\dot\gamma}{}_{\dot\delta} \bar\nabla^{\dot\delta} L^{I_2}| \\
&\quad - \frac{1}{2}\cdot\frac{1}{2}(e_m{}^a \psi_n{}^\beta \bar\psi_p{}^{\dot\gamma} + (-1)^1 \psi_m{}^\beta e_n{}^a \bar\psi_p{}^{\dot\gamma} + (-1)^3 \psi_n{}^\beta \bar\psi_m{}^{\dot\gamma} e_p{}^a)(-1)(+2i)(\sigma_a)_{\beta\dot\gamma} L^{I_2}| \\
&\quad - \frac{1}{2}\cdot\frac{1}{2}(e_m{}^a \bar\psi_{n\dot\beta} \psi_{p\gamma} + (-1)^1 \bar\psi_{m\dot\beta} e_n{}^a \psi_{p\gamma} + (-1)^3 \bar\psi_{n\dot\beta} \psi_{m\gamma} e_p{}^a)(-1)(+2i)(\bar\sigma_a)^{\dot\beta\gamma} L^{I_2}|.
\end{aligned}
\tag{C.4}
$$

For 1-form gauge fields, the double bar projections are

$$
F^{I_1}|| = \frac{1}{2!} dx^m \wedge dx^n F^{I_1}_{nm}| = \frac{1}{2!} dx^m \wedge dx^n E_m{}^A E_n{}^B F^{I_1}_{BA}|. \tag{C.5}
$$

We obtain the expressions of the bosonic field strengths

$$
\begin{aligned}
e_m{}^a e_n{}^b F^{I_1}_{ba}| &= \partial_n A^{I_1}_m| - \partial_m A^{I_1}_n| - (q^{(1)} \cdot B_{nm}|)^{I_1} \\
&\quad - \frac{1}{2}(e_m{}^c \psi_n{}^\beta - e_n{}^c \psi_m{}^\beta)(-1)(\sigma_c)_{\beta\dot\gamma} \bar W^{I_1\dot\gamma}| \\
&\quad - \frac{1}{2}(e_m{}^c \bar\psi_{n\dot\beta} - e_n{}^c \bar\psi_{m\dot\beta})(+1)(\bar\sigma_c)^{\dot\beta\gamma} W^{I_1}_\gamma|.
\end{aligned}
\tag{C.6}
$$

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The above expressions are basic building blocks in the constructions of component field actions.

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