BPS black holes in a non-homogeneous deformation of the stu model of $N = 2$, $D = 4$ gauged supergravity

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ABSTRACT: We consider a deformation of the well-known stu model of $N = 2$, $D = 4$ supergravity, characterized by a non-homogeneous special Kähler manifold, and by the smallest electric-magnetic duality Lie algebra consistent with its upliftability to five dimensions. We explicitly solve the BPS attractor equations and construct static supersymmetric black holes with radial symmetry, in the context of U(1) dyonic Fayet-Iliopoulos gauging, focussing on axion-free solutions. Due to non-homogeneity of the scalar manifold, the model evades the analysis recently given in the literature. The relevant physical properties of the resulting black hole solution are discussed.

Contents

1 Introduction

Black holes in gauged supergravity theories provide an important testground to address fundamental questions of gravity, both at the classical and quantum level. Among the most prominent of these are perhaps the problems of black hole microstates, uniqueness theorems, or the attractor mechanism. In gauged supergravity, the solutions often (but not necessarily) have AdS asymptotics, and one can then try to study at least some of these issues guided by the AdS/CFT correspondence. On the other hand, black hole solutions to these theories are also relevant for a number of recent developments in fluid mechanics, high energy- and especially in condensed matter physics, since they provide the dual description of certain strongly coupled condensed matter systems at finite temperature, cf. [1] for a review. In particular, models similar to the one that we shall consider below, containing Einstein gravity coupled to U(1) gauge fields and neutral scalars, have been instrumental to study transitions from Fermi-liquid to non-Fermi-liquid behaviour, cf. [2, 3] and references therein.

    For these reasons, the construction of analytic black holes in gauged supergravity as well as the exploration of their physics has been an active field of research recently, especially in four-dimensional models with $N = 2$ supersymmetry and Fayet-Iliopoulos (FI) gaugings, cf. [4–30] for an (incomplete) list of references. Although we are still far from understanding the underlying general structure[1] of such solutions (if there is any),

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[1]By this we mean a possible gauged supergravity analogue of the well-known fact that asymptotically flat black holes are typically given (in the extremal limit) in terms of harmonic functions on a flat base space.

many important partial results have been obtained. These studies have also revealed some surprises, like for instance the existence of so-called superentropic black holes, which have noncompact event horizon but nevertheless finite area. These were first discovered in [24], and their physics was further discussed in [31–33].

Up to now, the construction and discussion of black holes in $N = 2$, $D = 4$ Fayet-Iliopoulos gauged supergravity theories has been mainly limited to models where the vector multiplet scalars parametrize a *symmetric* special Kähler manifold.[2] Here we shall go one step further w.r.t. the results that appeared in the literature so far, by considering a non-symmetric (and even non-homogeneous) deformation of the stu model, defined by the prepotential (3.1). We will deal with a particular FI gauging of this model, that leads to a scalar potential with two critical points corresponding to AdS vacua. One of these extremizes also the superpotential and is thus supersymmetric, while the other vacuum breaks supersymmetry.

The remainder of the paper is organized as follows: in section 2 we introduce some basics of the theoretical framework of our investigation, namely $N = 2$, $D = 4$ supergravity coupled to n_V vector multiplets, and its dyonic U(1) Fayet-Iliopoulos gauging. Then, in section 3 we focus on a specific model, whose three complex scalars parametrize a non-homogeneous special Kähler manifold. At the level of the prepotential, this is a one-parameter extension of the well-known stu model [35–37], and thus we call it a non-homogeneous deformation of the stu model (nh-stu). In particular, respectively in subsections 3.1 and 3.2, we compute the symplectic embedding of the electric-magnetic duality algebra, and we present some axion-free geometric data. In section 4 we perform a near-horizon analysis of the FI-gauged system, in particular axion-free charge configurations, and for specific choice of the dyonic FI gauging parameters. A new, explicit BPS black hole solution for the FI-gauged nh-stu model is presented in section 5, and its physical properties are then discussed in section 6. The concluding section 7 contains some outlook and considerations for future developments.

2 The setup

We consider $N = 2$, $D = 4$ gauged supergravity coupled to n_V Abelian vector multiplets (for notation and general treatment, cf. e.g. [38]). Besides the Vierbein e_μ^a, the bosonic field content includes the vectors A_μ^Λ enumerated by $\Lambda = 0, \ldots, n_V$ (with the naught index denoting the graviphoton), and the complex scalars z^i where $i = 1, \ldots, n_V$. These scalars parametrize a special Kähler manifold, i.e., an n_V-dimensional Hodge-Kähler manifold that is the base of a symplectic bundle, with covariantly holomorphic sections

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix}, \qquad D_{\bar{i}}\mathcal{V} = \partial_{\bar{i}}\mathcal{V} - \frac{1}{2}(\partial_{\bar{i}}\mathcal{K})\mathcal{V} = 0, \tag{2.1}$$

where $\mathcal{K}$ is the Kähler potential and D denotes the Kähler-covariant derivative. $\mathcal{V}$ obeys the symplectic constraint[3] $i\langle \mathcal{V}, \bar{\mathcal{V}} \rangle = 1$, and it is related to the holomorphic symplectic

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[2]For some notable exceptions cf. e.g. [34].
[3]The brackets represent the symplectic inner product $\langle A, B \rangle = A^T \Omega B = A_\Lambda B^\Lambda - A^\Lambda B_\Lambda$.

vector $(X^\Lambda, F_\Lambda)^T$ by

$$\mathcal{V} = e^{\mathcal{K}/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}. \tag{2.2}$$

The matrix $\mathcal{N}_{\Lambda\Sigma}$ determining the coupling between the scalars z^i and the vectors A_μ^Λ is defined by the relations[4]

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma, \qquad D_{\bar{i}} \bar{M}_\Lambda = \mathcal{N}_{\Lambda\Sigma} D_{\bar{i}} \bar{L}^\Sigma. \tag{2.3}$$

If a prepotential $F(X)$ exists, it is a homogeneous function of degree two which allows to determine the lower part of the symplectic sections (2.1) and the matrix $\mathcal{N}$ in terms of F itself, according to

$$\mathcal{V} = e^{\mathcal{K}/2} \begin{pmatrix} X^\Lambda \\ \partial_\Lambda F \end{pmatrix}, \qquad \mathcal{N}_{\Lambda\Sigma} = \bar{F}_{\Lambda\Sigma} + 2i \frac{\mathrm{Im}F_{\Lambda\Lambda'} X^{\Lambda'} \mathrm{Im}F_{\Sigma\Sigma'} X^{\Sigma'}}{X^\Omega \mathrm{Im}F_{\Omega\Omega'} X^{\Omega'}}, \tag{2.4}$$

where $F_{\Lambda\Sigma} = \partial_\Lambda \partial_\Sigma F$.

The bosonic Lagrangian reads

$$\mathscr{L} = \frac{R}{2} - g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + \frac{1}{4} \mathcal{I}_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{8\sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} \mathcal{R}_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma - V_g, \tag{2.5}$$

with the special Kähler metric $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}$. The scalar potential is

$$V_g = g^{i\bar{j}} D_i \mathcal{L} \bar{D}_{\bar{j}} \bar{\mathcal{L}} - 3|\mathcal{L}|^2, \tag{2.6}$$

where the superpotential $\mathcal{L}$ is determined by the dyonic Fayet-Iliopoulos (FI) gauging,

$$\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle = e^{\mathcal{K}/2} (X^\Lambda g_\Lambda - F_\Lambda g^\Lambda), \tag{2.7}$$

with FI parameters $\mathcal{G} = (g^\Lambda, g_\Lambda)$.

Since we are interested in static black holes with radial symmetry, we employ the Ansatz

$$ds^2 = -e^{2U(r)} dt^2 + e^{-2U(r)} (dr^2 + e^{2\psi(r)} d\Omega_\kappa^2), \tag{2.8}$$

where $d\Omega_\kappa^2 = d\theta^2 + f_\kappa^2(\theta) d\phi^2$ is the metric on the two-surfaces $\Sigma = \{\mathrm{S}^2, \mathbb{E}^2, \mathrm{H}^2\}$ of constant scalar curvature $R = 2\kappa$, with $\kappa = \{1, 0, -1\}$ respectively. Here the function $f_\kappa(\theta)$ is given by

$$f_\kappa(\theta) = \begin{cases} \sin\theta, & \kappa = 1, \\ \theta, & \kappa = 0, \\ \sinh\theta, & \kappa = -1. \end{cases} \tag{2.9}$$

The scalars are assumed to depend only on the radial coordinate r, $z^i = z^i(r)$, while the gauge fields should have an appropriate profile to satisfy

$$p^\Lambda = \frac{1}{\mathrm{vol}(\Sigma)} \int_\Sigma F^\Lambda, \qquad q_\Lambda = \frac{1}{\mathrm{vol}(\Sigma)} \int_\Sigma G_\Lambda, \tag{2.10}$$

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[4]In what follows we use the notation $\mathcal{I} = \mathrm{Im}\,\mathcal{N}$ and $\mathcal{R} = \mathrm{Re}\,\mathcal{N}$.

with p^Λ and q_Λ being the magnetic and electric charges associated to the black hole and G_Λ denoting the dual field strength,

$$G_\Lambda = \frac{\delta \mathscr{L}}{\delta \star F^\Lambda} \,. \tag{2.11}$$

The symplectic invariant central charge is given by

$$\mathcal{Z} = \langle \mathcal{Q}, \mathcal{V} \rangle \,, \tag{2.12}$$

where we introduced the vector of magnetic and electric charges, $\mathcal{Q} = (p^\Lambda, q_\Lambda)$.

Following the procedure outlined in [12], the previous Ansätze are plugged into the Lagrangian (2.5) and give rise to an effective one-dimensional action involving the scalar fields and the warp functions $U(r), \psi(r)$,

$$\begin{aligned}
S_{1d} = \int \mathrm{d}r \left\{ e^{2\psi} \left[U'^2 - \psi'^2 + g_{i\bar{j}} z^{i'} \bar{z}^{\bar{j}'} + e^{2U-4\psi} V_{\mathrm{BH}} + e^{-2U} V_g \right] - 1 \right\} \\
+ \int \mathrm{d}r \frac{\mathrm{d}}{\mathrm{d}r} \left[e^{2\psi} (2\psi' - U') \right] \,.
\end{aligned} \tag{2.13}$$

Here V_{BH} denotes the so-called black hole potential [39], defined by

$$V_{\mathrm{BH}} = -\frac{1}{2} \mathcal{Q}^T \mathcal{M} \mathcal{Q} \,, \tag{2.14}$$

where

$$\mathcal{M} = \begin{pmatrix} \mathcal{I} + \mathcal{R}(\mathcal{I})^{-1}\mathcal{R} & -\mathcal{R}(\mathcal{I})^{-1} \\ -(\mathcal{I})^{-1}\mathcal{R} & (\mathcal{I})^{-1} \end{pmatrix} \,. \tag{2.15}$$

If the charges satisfy the condition

$$\langle \mathcal{G}, \mathcal{Q} \rangle = -\kappa \,, \tag{2.16}$$

the effective action (2.13) can be rewritten as a sum of squares of first order differential conditions and a boundary term. As in [12], setting to zero each of these terms, a system of first order equations is obtained,

$$\begin{aligned}
& 2e^{2\psi} \left(e^{-U} \mathrm{Im}(e^{-i\alpha}\mathcal{V}) \right)' + e^{2(\psi-U)} \Omega \mathcal{M} \mathcal{G} + 4e^{-U}(\alpha' + \mathcal{A}_r)\mathrm{Re}(e^{-i\alpha}\mathcal{V}) + \mathcal{Q} = 0 \,, \\
& \psi' = 2e^{-U} \mathrm{Im}(e^{-i\alpha}\mathcal{L}) \,, \\
& \alpha' + \mathcal{A}_r = -2e^{-U} \mathrm{Re}(e^{-i\alpha}\mathcal{L}) \,.
\end{aligned} \tag{2.17}$$

Here $\mathcal{A}_\mu = \mathrm{Im}(\partial_\mu z^i (\partial_i \mathcal{K}))$ is the connection associated to the Kähler transformations and the phase α can be expressed in terms of the other fields as

$$e^{2i\alpha} = \frac{\mathcal{Z} - ie^{2(\psi-U)}\mathcal{L}}{\bar{\mathcal{Z}} + ie^{2(\psi-U)}\bar{\mathcal{L}}} \,. \tag{2.18}$$

It is possible to show [12] that the supersymmetry variations of $N = 2$ gauged supergravity reproduce the set of equations (2.17) by requiring the existence of a certain Killing spinor.

In this way, both the equations of motion and the supersymmetry conditions are satisfied by solutions of (2.17), and the resulting configuration will be 1/4 BPS.

As is the case for many other known solutions [10, 25, 40, 41], we shall assume vanishing axions. This is realized by purely imaginary scalars (with $\lambda^i > 0$),

$$z^i = x^i - i\lambda^i, \qquad x^i = 0. \tag{2.19}$$

The advantage of this choice will become evident in the next section: for some values of the FI parameters in $\mathcal{G}$, it indeed simplifies the equations of motion (2.17), setting α to a constant.

3 A non-homogeneous deformation of the stu model

In this paper, we will specialize our treatment on the special Kähler 3-moduli model based on the holomorphic prepotential[5]

$$F = \frac{X^1 X^2 X^3}{X^0} - \frac{A}{3} \frac{(X^3)^3}{X^0}, \tag{3.1}$$

where A is an arbitrary real constant. For $A = -1$, the prepotential reads

$$F = \frac{X^1 X^2 X^3}{X^0} + \frac{1}{3} \frac{(X^3)^3}{X^0}, \tag{3.2}$$

which has been constructed in the context of Type IIA string theory compactified on Calabi-Yau manifolds in [44]. In particular, analyzing string vacua with three complex moduli (section 3.2 therein), different bases for the toric construction of such a model have been considered; (3.2) corresponds to the basis $\mathbb{F}_0$ of [44], while other toric constructions determine the same model in different symplectic frames. The prepotential (3.2) can also be obtained as $c = 0$ limit of the heterotic prepotential appearing in [45] and the corresponding one-loop prepotential V_{GS} is given by considering its $c = 0$ limit.

In absence of gauging, the BPS attractor equations for this model have been discussed in [45]; a solution for a generic supporting black hole charge configuration was obtained in this context and, as a consequence, the BPS black hole entropy was determined as a function of the charges.

A full-fledged, explicit determination of the BPS black hole entropy of the model based on (3.2) was later given by Shmakova in the investigation of BPS attractor equations for black holes based on Calabi-Yau cubic prepotentials [46]. We report here the expression of the ungauged BPS black hole entropy, for later convenience:

$$\frac{S_{\mathrm{BH}}}{\pi} = \frac{\sqrt{f(\mathcal{Q})}}{3p^0}, \tag{3.3}$$

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[5]Black holes of type IIA Calabi-Yau compactifications in the presence of perturbative quantum corrections, leading to a prepotential of the form $F = d_{ijk} X^i X^j X^k / X^0 + ic(X^0)^2$ (for some constant c), were constructed and studied in [42, 43].

where

$$f\left(\mathcal{Q}\right) := 2\left\{\left(p^1p^2+\left(p^3\right)^2-p^0q_3\right)\left[\left(p^1p^2+\left(p^3\right)^2-p^0q_3\right)^2+12\left(p^2p^3-p^0q_1\right)\left(p^1p^3-p^0q_2\right)\right]\right.$$
$$\left.+\left[\left(p^1p^2+\left(p^3\right)^2-p^0q_3\right)^2-4\left(p^2p^3-p^0q_1\right)\left(p^1p^3-p^0q_2\right)\right]^{3/2}\right\}$$
$$-9\left[p^0\left(p^0q_0+p^1q_1+p^2q_2+p^3q_3\right)-2p^1p^2p^3-\frac{2}{3}\left(p^3\right)^3\right]^2,$$

$$(3.4)$$

and the conditions $f\left(\mathcal{Q}\right) > 0$ and $p^0 > 0$ define the BPS-supporting black hole charge vector $\mathcal{Q}$. It is immediate to check that (3.3) and (3.4) imply the entropy S_{BH} to be homogeneous of degree two in the charges, as it must be in four dimensions for 0-branes.

The model (3.1) under consideration, where A has to be considered a parameter, belongs to the broad class of the so-called very special Kähler manifolds, that can be obtained by dimensional reduction from the vector multiplets' scalar geometries coupled to minimal supergravity in $D = 5$, known as special real manifolds. All the models originating from this kind of geometry are described, in the so-called *4D/5D special coordinates'* symplectic frame (cf. e.g. [47, 48]), by a cubic prepotential of the form

$$F = d_{ijk}\frac{X^iX^jX^k}{X^0},$$

$$(3.5)$$

where d_{ijk} is a real and symmetric tensor and the corresponding special Kähler space is usually dubbed a *d-space* [48]. In particular, the model (3.1) is defined by $d_{123} = 1/6$ and $d_{333} = -A/3$.

It is worth pointing out that the *d*-space corresponding to (3.1) is neither symmetric nor homogeneous[6] [41, 49]. In particular, it does not fall within the class of symmetric models examined in [25], that are characterized by a constant tensor[7]

$$\hat{d}^{lmn} = \frac{g^{il}g^{jm}g^{kn}}{(d_{pqr}\lambda^p\lambda^q\lambda^r)^2}d_{ijk}.$$

$$(3.6)$$

In fact, it can be easily checked that the prepotential (3.1) implies a non-constant $\hat{d}^{lmn}$. For this reason, we will henceforth dub the cubic model (3.1) as a *non-homogeneous deformation* of the homogeneous and symmetric stu model (shortly, nh-stu), to which it reduces[8] when $A = 0$.

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[6]After [48] and [49], homogeneous special Kähler *d*-spaces, either symmetric or non-symmetric, have been classified in terms of the corresponding *d*-tensor, which uniquely determines their geometry. No homogeneous, non-symmetric, special Kähler (non-compact, Riemannian) spaces which are not based on cubic prepotentials (3.5) are known, although a proof of this fact does not exist, as far as we know.

[7]For some considerations on the completely contravariant *d*-tensor in generic *d*-spaces (and the corresponding definition of the so-called *E*-tensor for non-symmetric special Kähler spaces), cf. e.g. [50], and refs. therein.

[8]Consistently, for $A = 0$ the expression (3.22) below enhances to an 8-dimensional *U*-duality group, given by the $\mathrm{SL}(2,\mathbb{R})^{\otimes 3}$ group of the stu model [35–37] (cf. e.g. section 8 of [51]).

It is here worth pointing out that, however, at the level of the solution discussed in sections 4 and 5 (characterized by proportionality between λ^2 and λ^3), $A = 0$ yields the (axion-free) st^2 model, with some subtleties mentioned at the end of section 5.

3.1 Electric-magnetic duality algebra

The vector multiplets' scalar manifold of the nh-stu model is neither symmetric nor homogeneous; namely, the non-compact Riemannian space endowed with the special Kähler geometry specified by the cubic holomorphic prepotential (3.1) (with non-vanishing A) cannot be described as a coset[9] G/H, where H is a local, compact isotropy group (linearly realized on the scalar fields, which generally sit in its representations) and G is a global, non-compact symmetry group (non-linearly realized by the scalar fields, but linearly realized by the vectors). In theories of Abelian Maxwell fields, the group G describes the electric-magnetic duality symmetry, and its non-compactness in presence of scalar fields was firstly discussed by Gaillard and Zumino in [52].

Linearly realized electric-magnetic duality (U-duality[10]) plays a key role in Einstein-Maxwell theories coupled to scalar fields in presence of local supersymmetry, and consequently in their regular solutions, such as the dyonic black holes discussed in the present paper. Even if the scalar manifold is not a coset G/H, a global U-duality symmetry group G always exists, even if it may be non-reductive or also discrete in generic, (semi-)realistic models of string compactifications.

A general feature of Einstein-Maxwell theories coupled to non-linear sigma models of scalar fields in four dimensions is the symplectic structure of the field strength 2-forms and of their duals, which in turn allows to define the symplectic invariant scalar product specified in footnote 3. It results in the maximal, generally non-symmetric embedding (see theorem 1.5 in [52, 56])

$$G \subset \mathrm{Sp}(2n, \mathbb{R}) \tag{3.7}$$

$$\mathbf{R} = \mathbf{2n}, \tag{3.8}$$

where n is the number of vector fields, $\mathbf{2n}$ is the fundamental representation of $\mathrm{Sp}(2n, \mathbb{R})$ and $\mathbf{R}$ is the representation of G, not necessarily irreducible.

Thus, it is interesting to determine the (continuous, Lie component of the) U-duality algebra $\mathfrak{g}_{\text{nh-stu}}$ of the nh-stu model of $N = 2$, $D = 4$ supergravity. In this case we have $n = 4$, since one graviphoton and three vectors from the vector multiplets are present. We aim to explicitly find the realization of the maximal, non-symmetric embedding

$$\mathfrak{g}_{\text{nh-stu}} \subset \mathfrak{sp}(8, \mathbb{R}). \tag{3.9}$$

This is worth also in view of the fact that $G_{\text{nh-stu}}$, the Lie group generated by $\mathfrak{g}_{\text{nh-stu}}$, has not a transitive action on the non-linear sigma model described by the $N = 2$ holomorphic prepotential (3.1).

Since the semiclassical Bekenstein-Hawking entropy in the ungauged theory is generally invariant under linearly realized global symmetries, $\mathfrak{g}_{\text{nh-stu}}$ can be determined by finding all

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[9]In order for the coset G/H to be non-compact, H must *at least* be the maximal compact subgroup of G. When this is the case, and when both G and H are reductive Lie groups, the corresponding coset is symmetric.

[10]Here, U-duality is referred to as the 'continuous' symmetries of [53, 54]. Their discrete versions are the U-duality non-perturbative string theory symmetries introduced by Hull and Townsend [55].

infinitesimal symplectic transformations which leave the BPS black hole entropy S_{BH} (3.3)–(3.4) invariant.

Let us choose $A = -1$. From (3.3)–(3.4), the infinitesimal invariance condition reads

$$\delta S_{\text{BH}} = \frac{1}{2S_{\text{BH}}} \delta S_{\text{BH}}^2$$

$$= \frac{1}{2S_{\text{BH}}} \left[\left(-\frac{2f}{p^0} + \frac{\partial f}{\partial p^0} \right) \delta p^0 + \frac{\partial f}{\partial p^i} \delta p^i + \frac{\partial f}{\partial q_0} \delta q_0 + \frac{\partial f}{\partial q_i} \delta q_i \right] = 0, \qquad (3.10)$$

or equivalently

$$-6f\delta p^0 + p^0 \delta f = 0, \qquad (3.11)$$

where $\delta f = \frac{\partial f}{\partial \mathcal{Q}} \delta \mathcal{Q}$ and

$$\delta \mathcal{Q} = \left(\delta p^0, \delta p^i, \delta q_0, \delta q_i \right)^T = \mathcal{S}\mathcal{Q}, \qquad (3.12)$$

with $\mathcal{S}$ belonging to the symplectic Lie algebra. It is an 8×8 matrix which can be written in blocks as

$$\mathfrak{sp}\left(8, \mathbb{R}\right) \ni \mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad A^T = -D, \quad B^T = B, \quad C^T = C, \qquad (3.13)$$

where each block is a 4×4 matrix. Thus, $\mathcal{S}$ depends on ten real parameters.

By solving (3.11) for a BPS-supporting configuration with generic charges $\mathcal{Q}$ satisfying $f\left(\mathcal{Q}\right) > 0$ and $p^0 > 0$, the symplectic embedding of the U-duality Lie algebra $\mathfrak{g}_{\text{nh-stu}}$ of the nh-stu model into $\mathfrak{sp}(8, \mathbb{R})$ is realized by the following four-dimensional, lower triangular matrix subalgebra (cf. (3.9); $a, b, c \in \mathbb{R}$, $\phi \in \mathbb{R}_0^+$)

$$\mathcal{S}_{\text{nh-stu}}(a, b, c, \phi) = \begin{pmatrix} -3\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a & -\phi & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & -\phi & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & -\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\phi & -a & -b & -c \\ 0 & 0 & c & b & 0 & \phi & 0 & 0 \\ 0 & c & 0 & a & 0 & 0 & \phi & 0 \\ 0 & b & a & 2c & 0 & 0 & 0 & \phi \end{pmatrix} \in \mathfrak{g}_{\text{nh-stu}} \subset \mathfrak{sp}(8, \mathbb{R}). \qquad (3.14)$$

For a generic A, this can be generalized as follows:

$$\mathcal{S}_{\text{nh-stu}}(a, b, c, \phi; A) = \begin{pmatrix} -3\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a & -\phi & 0 & 0 & 0 & 0 & 0 & 0 \\ b & 0 & -\phi & 0 & 0 & 0 & 0 & 0 \\ c & 0 & 0 & -\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3\phi & -a & -b & -c \\ 0 & 0 & c & b & 0 & \phi & 0 & 0 \\ 0 & c & 0 & a & 0 & 0 & \phi & 0 \\ 0 & b & a & -2Ac & 0 & 0 & 0 & \phi \end{pmatrix} \in \mathfrak{g}_{\text{nh-stu}} \subset \mathfrak{sp}(8, \mathbb{R}). \qquad (3.15)$$

It can be noticed that (3.15) (which reduces to (3.14) for $A = -1$) determines a maximal Abelian subalgebra of $\mathfrak{sp}(8, \mathbb{R})$, whose four generators commute. Moreover, the generators corresponding to a, b, c in (3.14) span an axionic Peccei-Quinn translational three-dimensional algebra, nilpotent of degree four. Indeed, the part of (3.14) generated by a, b, c can be recast in the following generic, d-parametrized form [57]

$$\mathcal{S} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a^j & 0 & 0 & 0 \\ 0 & 0 & 0 & -a^i \\ 0 & d_{a,ij} & 0 & 0 \end{pmatrix} \subset \mathfrak{sp}(2n, \mathbb{R}), \tag{3.16}$$

where $(i = 1, \ldots, n-1)$

$$\begin{aligned} d_{a,ij} &:= d_{ijk}a^k, & d_{a,i} &:= d_{ijk}a^j a^k, & d_a &:= d_{ijk}a^i a^j a^k, \\ a^1 &:= 6a, & a^2 &:= 6b, & a^3 &:= 6c. \end{aligned} \tag{3.17}$$

$\mathcal{S}$ in (3.16) can be easily checked to be nilpotent of degree four,[11]

$$\mathcal{S}^4 = 0 \Rightarrow \exp(\mathcal{S}) = \mathbb{I}_{2n} + \mathcal{S} + \frac{1}{2}\mathcal{S}^2 + \frac{1}{3!}\mathcal{S}^3, \tag{3.18}$$

yielding, at group level [58, 59],

$$\exp(\mathcal{S}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ a^j & \mathbb{I}_{n-1} & 0 & 0 \\ -\frac{1}{6}d_a & -\frac{1}{2}d_{a,i} & 1 & -a^i \\ \frac{1}{2}d_{a,j} & d_{a,ij} & 0 & \mathbb{I}_{n-1} \end{pmatrix} \subset \mathrm{Sp}(2n, \mathbb{R}). \tag{3.19}$$

Such an Abelian $(n-1)$-dimensional global symmetry algebra/group, as discussed in [59] (see also refs. therein, in particular [60]), characterizes *every* model of $D = 4$ supergravity based on a cubic scalar geometry, even not of special Kähler type (i.e. the scalar geometries of $N = 4$, 6 and 8 supergravity theories, dubbed 'generalized d-geometries' in [59]): the representation of axions in $D = 4$ is *always* nilpotent of degree four.

Besides the $(n-1)$-dimensional axionic Peccei-Quinn translational algebra, the universal sector of the electric-magnetic duality algebra of every (generalized) d-geometry (also cf. [61]) is given by the $2n \times 2n$ generalization of the ϕ-parametrized part of (3.15), where ϕ can be thus regarded as the Kaluza-Klein radius/real dilaton of the Kaluza-Klein (KK) $\mathfrak{so}_{\mathrm{KK}}(1, 1)$,

$$K(\phi) = \begin{pmatrix} -3\phi & 0 & 0 & 0 \\ 0 & -\phi\mathbb{I}_{n-1} & 0 & 0 \\ 0 & 0 & 3\phi & 0 \\ 0 & 0 & 0 & \phi\mathbb{I}_{n-1} \end{pmatrix} \in \mathfrak{so}_{\mathrm{KK}}(1, 1) \subset \mathfrak{sp}(2n, \mathbb{R}). \tag{3.20}$$

---

[11]$\mathbb{I}_d$ denotes the $d \times d$ identity matrix throughout.

Therefore, the $2n \times 2n$ matrix realization of the universal sector of the global electric-magnetic duality symmetry of an Einstein-Maxwell theory whose scalar manifold is endowed with a 'generalized d-geometry' can be written at the Lie algebra level as [58, 59]

$$\mathcal{S}(a) + K(\phi) = \begin{pmatrix} -3\phi & 0 & 0 & 0 \\ a^j & -\phi\delta_i^j & 0 & 0 \\ 0 & 0 & 3\phi & -a^i \\ 0 & d_{a,ij} & 0 & \phi\delta_j^i \end{pmatrix} \subset \mathfrak{sp}(2n,\mathbb{R}),$$ (3.21)

and at the Lie group level as [58, 59]

$$\exp(\mathcal{S}(a))\exp(K(\phi)) = \begin{pmatrix} e^{-3\phi} & 0 & 0 & 0 \\ a^j & e^{-\phi}\delta_i^j & 0 & 0 \\ -\frac{1}{6}d_a & -\frac{1}{2}d_{a,i} & e^{3\phi} & -a^i \\ \frac{1}{2}d_{a,j} & d_{a,ij} & 0 & e^{\phi}\delta_j^i \end{pmatrix} \subset \mathrm{Sp}(2n,\mathbb{R}).$$ (3.22)

When considering $N = 2$, $D = 4$ theories, this result for special Kähler d-geometries was known after[12] [48].

Thus, in this sense, one can conclude that the nh-stu model has the *smallest possible* electric-magnetic duality algebra, consistent with its cubic nature (and thus with its upliftability to $N = 1$, $D = 5$ supergravity).

3.2 Axion-free geometry

As stated above, in the present investigation we consider only the axion-free case, thus parametrising the purely imaginary scalar fields as $z^i = -i\lambda^i$, with λ^i real and positive ($i = 1, 2, 3$); we are also choosing the projective coordinates as

$$\frac{X^1}{X^0} = -i\lambda^1, \qquad \frac{X^2}{X^0} = -i\lambda^2, \qquad \frac{X^3}{X^0} = -i\lambda^3.$$ (3.23)

Thus, the symplectic sections (2.1) become ($\Lambda = 0, 1, 2, 3$)

$$L^\Lambda = e^{\mathcal{K}/2}\left(1, -i\lambda^1, -i\lambda^2, -i\lambda^3\right)^T,$$
$$M_\Lambda = e^{\mathcal{K}/2}\left(-i\left(\lambda^1\lambda^2\lambda^3 - \frac{A}{3}(\lambda^3)^3\right), -\lambda^2\lambda^3, -\lambda^1\lambda^3, -\lambda^1\lambda^2 + A(\lambda^3)^2\right)^T,$$ (3.24)

while the Kähler potential reads

$$e^{-\mathcal{K}} = 8\left(\lambda^1\lambda^2\lambda^3 - \frac{A}{3}(\lambda^3)^3\right).$$ (3.25)

---

[12]In [59], (3.22) was shown also to pertain to the universal sector of axionic and KK coordinates in the scalar manifolds of $D = 4$ theories based on 'generalized d-geometries' (for non-homogeneous $N = 2$ very special Kähler geometries, the same parametrization provides a general description of the generic element of the *flat* symplectic bundle over the vector multiplets' scalar manifold [59, 60]).

For vanishing axions, the special Kähler metric takes the form

$$g_{i\bar{j}} = \frac{1}{4\left(\lambda^1\lambda^2\lambda^3 - \frac{A}{3}(\lambda^3)^3\right)^2} \begin{pmatrix} (\lambda^2)^2(\lambda^3)^2 & \frac{A}{3}(\lambda^3)^4 & -\frac{2}{3}A\lambda^2(\lambda^3)^3 \\ \frac{A}{3}(\lambda^3)^4 & (\lambda^1)^2(\lambda^3)^2 & -\frac{2}{3}A\lambda^1(\lambda^3)^3 \\ -\frac{2}{3}A\lambda^2(\lambda^3)^3 & -\frac{2}{3}A\lambda^1(\lambda^3)^3 & (\lambda^1)^2(\lambda^2)^2 + \frac{A^2}{3}(\lambda^3)^4 \end{pmatrix}.$$

(3.26)

The symplectic matrix $\mathcal{N}_{\Lambda\Sigma}$ has, in the axion-free case under consideration, vanishing real part $\mathcal{R}_{\Lambda\Sigma}$, while $\mathcal{I}_{\Lambda\Sigma}$ is given by

$$\mathcal{I}_{\Lambda\Sigma} = -\frac{1}{8}e^{-\mathcal{K}}\begin{pmatrix} 1 & 0 \\ 0 & 4g_{i\bar{j}} \end{pmatrix},$$

(3.27)

which is thus consistently negative definite.

4 Dyonic Fayet-Iliopoulos gaugings and near-horizon analysis

To proceed further, we shall assume a specific form for the FI parameters $\mathcal{G}$. The choice

$$\mathcal{G}^T = (0, g^1, g^2, g^3, g_0, 0, 0, 0)^T,$$

(4.1)

together with the vanishing axion condition (2.19), fixes the phase α in (2.18) to the constant value[13] $\alpha = \pm\pi/2$. This can be checked from the explicit expressions of the symplectic invariants $\mathcal{Z}$ and $\mathcal{L}$,

$$\mathcal{Z} = ie^{\mathcal{K}/2}\left(p^0\left(\lambda^1\lambda^2\lambda^3 - \frac{A}{3}\lambda^3\right) - q_1\lambda^1 - q_2\lambda^2 - q_3\lambda^3\right),$$

$$\mathcal{L} = e^{\mathcal{K}/2}\left(g_0 + g^1\lambda^2\lambda^3 + g^2\lambda^1\lambda^3 + g^3(\lambda^1\lambda^2 - A(\lambda^3)^2)\right).$$

(4.2)

As can be inferred from the BPS equations (2.17), the choice (4.1) requires some charges to vanish, so that the vector $\mathcal{Q}$ takes the form

$$\mathcal{Q}^T = (p^0, 0, 0, 0, 0, q_1, q_2, q_3)^T.$$

(4.3)

With the choice (4.1), the scalar potential (2.6) becomes

$$V_g = -g^2 g^3 \lambda^1 - g^1 g^3 \lambda^2 - \left(g^1 g^2 - A(g^3)^2\right)\lambda^3$$
$$- \frac{g_0}{\lambda^1\lambda^2\lambda^3 - \frac{A}{3}(\lambda^3)^3}\left(g^2\lambda^1\lambda^3 + g^1\lambda^2\lambda^3 + g^3\left(\lambda^1\lambda^2 - A(\lambda^3)^2\right)\right),$$

(4.4)

which matches the known expression for the stu model [10, 37, 47] for $A = 0$. In what follows we shall assume that all gauge coupling constants g_0, g^i are positive. Then the potential (4.4) has two critical points, namely one for

$$\lambda^1 = \frac{g^1}{g^3}\lambda^3, \qquad \lambda^2 = \frac{g^2}{g^3}\lambda^3, \qquad \lambda^3 = \sqrt{\frac{g_0 g^3}{g^1 g^2 - \frac{A}{3}(g^3)^2}},$$

(4.5)

---

[13]Another possible choice yielding the same constant value for α is $\mathcal{G}^T = (g^0, 0, 0, 0, 0, g_1, g_2, g_3)^T$, which would in turn require $\mathcal{Q}$ to assume the (magnetic) form $\mathcal{Q}^T = (0, p^1, p^2, p^3, q_0, 0, 0, 0)^T$.

and the other for

$$\lambda^1 = \frac{g^1}{g^3}\lambda^3, \qquad \lambda^2 = -\frac{1}{g^1 g^3}\left(g^1 g^2 - \frac{2}{3}A(g^3)^2\right)\lambda^3, \qquad \lambda^3 = \sqrt{\frac{g_0 g^3}{g^1 g^2 - \frac{A}{3}(g^3)^2}}. \quad (4.6)$$

The first has $V_g = -3\ell^{-2}$, and the second $V_g = -\ell^{-2}$, with ℓ defined in (6.2), so both correspond to AdS vacua. One easily shows that (4.5) is also a critical point of the superpotential (2.7), while (4.6) is not. The vacuum (4.5) is thus supersymmetric, whereas (4.6) breaks supersymmetry. Moreover, reality and positivity of the scalars λ^i implies that the second vacuum exists only in the range

$$\frac{3}{2}\frac{g^1 g^2}{(g^3)^2} < A < 3\frac{g^1 g^2}{(g^3)^2}, \quad (4.7)$$

in particular it is not present for zero deformation parameter A.

Owing to the constancy of α, the equations of motion (2.17) boil down to

$$2e^{2\psi}\left(e^{-U}\mathrm{Re}\mathcal{V}\right)' + e^{2(\psi-U)}\Omega\mathcal{M}\mathcal{G} + \mathcal{Q} = 0,$$
$$(e^\psi)' = 2e^{\psi-U}\mathrm{Re}\mathcal{L}. \quad (4.8)$$

The near-horizon geometry is required to be $\mathrm{AdS}_2 \times \Sigma$, i.e., the metric functions in (2.8) should take the form

$$e^U = \frac{r}{R_1}, \qquad e^\psi = r\frac{R_2}{R_1}, \quad (4.9)$$

while the scalar fields $z^i(r) = -i\lambda^i(r)$ assume a constant value on the horizon. Under this assumption, the BPS equations (4.8) simplify to

$$\mathcal{Q} + R_2^2\Omega\mathcal{M}\mathcal{G} = -4\mathrm{Im}\left(\overline{\mathcal{Z}}\mathcal{V}\right),$$
$$\mathcal{Z} = i\frac{R_2^2}{2R_1}. \quad (4.10)$$

In addition, one has to impose the constraint (2.16).

Following the procedure described in [22],[14] the BPS equations in the near-horizon limit (4.10) provide a set of equations for the variables $\{R_1, R_2, \lambda^i\}$ as functions of the gaugings g_0, g^i and the charges p^0, q_i.

In particular, since R_2 is directly related to the black hole entropy S, this yields an expression for S in terms of the gaugings and charges. In the model described above, the attractor equations (4.10) are implicitly solved by

$$R_2^4 d_{g,i} + \frac{1}{3}\left(\kappa + \frac{1}{2}\right)p^0 q_i = \frac{1}{36}\left(d_\lambda^{-1}\right)^{ij}q_j q_i - \frac{1}{4}\left(p^0\right)^2 d_{\lambda,i},$$
$$\lambda^i\left(1 - \frac{\kappa}{2}\right) = \frac{\kappa}{p^0}\left(-R_2^2 g^i + \frac{1}{6}\left(d_\lambda^{-1}\right)^{ij}q_j\right),$$
$$\frac{R_2^2}{R_1} = \left(p^0 e^{-\frac{\kappa}{2}}\left(\kappa - \frac{3}{4}\right) - 2e^{\frac{\kappa}{2}}\lambda^j q_j\right), \quad (4.11)$$
$$R_2^6 d_g + \frac{1}{2}R_2^2 p^0\left(p^0 g_0 + \kappa g^i q_i\right) = \frac{1}{216}\left(d_\lambda^{-1}\right)^k\left(d_\lambda^{-1}\right)^{ij}q_i q_j q_k$$
$$+ \frac{1}{64}p^0 q_i q_j\left(\left(d_\lambda^{-1}\right)^j\lambda^i + 2\left(d_\lambda^{-1}\right)^{ij}\right) + \frac{1}{8}\left(p^0\right)^2\left(\lambda^i q_i + p^0 d_\lambda\right),$$

---

[14]The equations (4.11) are based on [22], with some misprints corrected.

where the contractions of the tensor d_{ijk} are defined as in (3.17). Note that the non-homogeneity enters through $(d_\lambda^{-1})^{ij}$, that depends on the special Kähler metric, since

$$g^{ij} = -\frac{2}{3}d_\lambda(d_\lambda^{-1})^{ij} + 2\lambda^i\lambda^j \, ,$$

cf. eq. (A.6) of [22].

An explicit solution to (4.11) cannot be obtained by applying the analysis developed in [22] for the case of symmetric special Kähler manifolds, because the model under consideration is neither symmetric nor homogeneous.

5 The full black hole solution

The present section is devoted to the presentation of an exact black hole solution for the nh-stu model introduced in section 3. In order to simplify the BPS equations (4.8), we introduce the functions[15]

$$H^0 = \frac{e^{-U}}{\sqrt{2}}\left(\lambda^1\lambda^2\lambda^3 - \frac{A}{3}(\lambda^3)^3\right)^{-\frac{1}{2}} ,$$

$$H_1 = \lambda^2\lambda^3 H^0 \, , \qquad H_2 = \lambda^1\lambda^3 H^0 \, , \qquad H_3 = (\lambda^3)^2 H^0 \, . \tag{5.1}$$

In terms of the latter, the equations (4.8) become

$$(H^0)' + 2g_0(H^0)^2 = -e^{-2\psi}p^0 \, ,$$

$$H_1'^1 H_1^2 + \frac{2}{3}Ag^2H_3^2 - \frac{4}{3}Ag^3H_1H_3 = e^{-2\psi}q_1 \, ,$$

$$H_2'^2 H_2^2 + \frac{2}{3}Ag^1H_3^2 - \frac{4}{3}Ag^3H_2H_3 = e^{-2\psi}q_2 \, ,$$

$$H_3' + 2H_3(g^1H_1 + g^2H_2) - 2g^3\left(H_1H_2 + \frac{A}{3}H_3^2\right) \tag{5.2}$$

$$= e^{-2\psi}\frac{H_3}{H_1H_2 + AH_3^2}(q_1H_2 + q_2H_1 - q_3H_3) \, ,$$

$$\psi' = g_0H^0 + g^1H_1 + g^2H_2 + g^3\left(\frac{H_1H_2}{H_3} - AH_3\right) \, .$$

A remarkable feature of the nh-stu model is that, contrary to e.g. the case considered in [10], the equations (5.2) cannot be decoupled, due to the nondiagonal terms in the metric (3.26). Following the strategy of [10], we use the Ansatz

$$\psi = \log\left(a\,r^2 + c\right) \, ,$$

$$H^0 = e^{-\psi}\left(\alpha^0 r + \beta^0\right) \, , \tag{5.3}$$

$$H_i = e^{-\psi}\left(\alpha_i r + \beta_i\right) , \qquad i = 1, 2, 3 \, .$$

---

[15]A common choice for the functions H_i is to make them coincide with the components of the symplectic sections. For the present situation, we preferred to choose H_3 in a different way, in order to simplify the structure of the equations.

The solution for the fields is then expressed in terms of the functions H^0, H_i by inverting the relations (5.1). This yields

$$e^{2U} = \frac{1}{2} \left(\frac{H_3}{H^0} \right)^{\frac{1}{2}} \left(H_1 H_2 - \frac{A}{3} H_3^2 \right)^{-1} , \tag{5.4}$$

and

$$\lambda^1 = H_2 \left(H_3 H^0 \right)^{-\frac{1}{2}} , \qquad \lambda^2 = H_1 \left(H_3 H^0 \right)^{-\frac{1}{2}} , \qquad \lambda^3 = \left(\frac{H_3}{H^0} \right)^{\frac{1}{2}} , \tag{5.5}$$

for the warp factor and the scalars respectively. By means of the Ansatz (5.3), the differential equations (4.8) boil down to a system of algebraic conditions on the parameters and the charges characterizing the solution, i.e., $\{\alpha^0, \alpha_i, \beta^0, \beta_i, a, c, p^0, q_i\}$. The set of equations obtained in this way reduces, after some algebraic manipulations, to

$$\alpha^0 = \frac{a}{2g_0} , \qquad \alpha_1 = \frac{g^2}{g^3} \alpha_3 , \qquad \alpha_2 = \frac{g^1}{g^3} \alpha_3 , \qquad \alpha_3 = \frac{a\, g^3}{2 \left(g^1 g^2 - \frac{A}{3} (g^3)^2 \right)} ,$$

$$\beta_1 = \frac{g^2}{g^3} \beta_3 , \qquad \beta_2 = -\frac{1}{2} \beta_3 \left(\frac{g^1}{g^3} - A \frac{g^3}{g^2} \right) - \frac{1}{2} \beta^0 \frac{g_0}{g^2} ,$$

$$q_1 = 2\, \beta_3^2 \frac{g^2}{(g^3)^2} \left(g^1 g^2 - \frac{A}{3} (g^3)^2 \right) + g^2 \frac{ac}{2 \left(g^1 g^2 - \frac{A}{3} (g^3)^2 \right)} ,$$

$$q_2 = \frac{1}{2g^2} \left(\beta^0 g_0 + \beta_3 \frac{g^1 g^2}{g^3} \right)^2 + g^1 \frac{ac}{2 \left(g^1 g^2 - \frac{A}{3} (g^3)^2 \right)} \tag{5.6}$$

$$+ \frac{A}{3} \beta_3 \frac{g^3}{g^2} \left(\beta_3 \frac{g^1 g^2}{g^3} - \beta^0 g_0 - \frac{A}{2} \beta_3 g^3 \right) ,$$

$$q_3 = \frac{g^2}{g^3} q_2 - A \frac{g^3}{g^2} q_1 , \qquad\qquad p^0 = -\frac{ac}{2g_0} - 2g_0 \left(\beta^0 \right)^2 .$$

The solution for the scalars is obtained by plugging the parameters written in (5.6) into the expressions (5.5). In this way, the scalars assume the explicit form

$$\lambda^1 = \frac{a \frac{g^1}{g^3} \left(\lambda_\infty^3 \right)^2 r - g_0 \beta_3 \left(\frac{g^1}{g^3} - A \frac{g^3}{g^2} \right) - \beta^0 \frac{g_0^2}{g^2}}{\sqrt{\left(2g_0\, \beta^0 + a\, r \right) \left(2g_0\, \beta_3 + a\, r\, (\lambda_\infty^3)^2 \right)}} ,$$

$$\lambda^2 = \frac{g^2}{g^3} \lambda^3 , \qquad \lambda^3 = \lambda_\infty^3 \sqrt{\frac{ar + \frac{2\, g_0}{(\lambda_\infty^3)^2} \beta_3}{ar + 2g_0 \beta^0}} , \tag{5.7}$$

where λ_∞^3 is the asymptotic value of λ^3,

$$\lambda_\infty^3 = \sqrt{\frac{g_0 g^3}{g^1 g^2 - \frac{A}{3} (g^3)^2}} . \tag{5.8}$$

The warp factor in the metric reads

$$\mathrm{e}^{2U} = \frac{2g_0 g^3 (ar^2 + c)^2}{\lambda_\infty^3 \left(ar - g_0 \beta^0 - \frac{g_0}{(\lambda_\infty^3)^2}\beta_3 \right) \sqrt{(ar + 2g_0\beta^0)\left(ar + \frac{2g_0}{(\lambda_\infty^3)^2}\beta_3\right)}} \,. \tag{5.9}$$

This solution represents a black hole, with a horizon at the largest zero of e^{2U}, i.e., at $r_{\mathrm{h}} = \sqrt{-c/a}$, where we assumed $a > 0$ and $c < 0$. The curvature invariants diverge where the angular component of the metric $e^{2\psi - 2U}$ vanishes. Note that all the scalar fields λ_i should be well-defined and positive outside the horizon. Moreover, we still have to impose the condition (2.16), i.e.,

$$g_0 p^0 - g^i q_i = -\kappa \tag{5.10}$$

on the solution (5.6). We checked that these requirements are compatible with any of the three possible choices for $\kappa = 0, \pm 1$, i.e., the horizon topology can be either spherical, flat or hyperbolic.

The Dirac quantization condition (5.10) fixes one of the four parameters $\{a, c, \beta^0, \beta_3\}$ that determine the solution, for example c. Furthermore, one easily sees that the solution enjoys the scaling symmetry

$$(t, r, \theta, \phi, a, c, \beta^0, \beta_3, \kappa) \mapsto (t/s, sr, \theta, \phi, a/s, sc, \beta^0, \beta_3, \kappa), \qquad s \in \mathbb{R}, \tag{5.11}$$

that can be used to set $a = 1$ without loss of generality. Consequently, there are only two physical parameters left, on which the solution depends.

Notice that the solution (5.6) is characterized by the proportionality between the scalars λ_2 and λ_3, as is evident from (5.7). However, it is worth stressing that this fact does not trivialize our results, since the locus $\lambda^2 = \frac{g^2}{g^3}\lambda^3$ in the scalar manifold does not yield a consistent two-moduli truncation for the model (3.1). In other words, the Kähler geometry that can be derived from the truncated model $F\left(X^1, X^2, X^3\right)\big|_{\lambda^2 \propto \lambda^3}$ is not equivalent to the two-dimensional one characterized by the prepotential

$$F = \frac{\tilde{X}^1 \left(X^3\right)^2}{X^0}, \qquad \text{with} \qquad \tilde{X}^1 = X^1 - \frac{A}{3} X^3, \tag{5.12}$$

which is homogeneous and symmetric (the so-called st^2 model, cf. e.g. [62] and refs. therein). This difference is evident, for example, in terms of the Kähler metric. In fact one has

$$g_{ij}^{(3)} d\lambda^i d\lambda^j \big|_{\lambda_2 \propto \lambda_3} \neq g_{MN}^{(2)} d\lambda^M d\lambda^N, \qquad i, j = 1, 2, 3, \qquad M, N = 1, 2, \tag{5.13}$$

where the left-hand side is the line element obtained with the metric (3.26) when the condition $\lambda_2 \propto \lambda_3$ is imposed, while the right-hand side describes the geometry associated to the prepotential (5.12).

We conclude this section with a comment on the behaviour of the solution for $A = 0$. Due to the particular definition of H_3 we have chosen (with respect to the more common one used for example in [10, 12, 25]), setting $A = 0$ and $\lambda^2 = \frac{g^2}{g^3}\lambda^3$ is not sufficient to match

exactly the stu black hole solution with two independent parameters, known as st^2 solution, that can be derived from [10]. However, the parameters in (5.3) can be redefined as

$$\alpha_3' = \frac{\alpha_1 \alpha_2}{\alpha_3} - \frac{A}{3}\alpha_3 \,, \qquad \beta_3' = \frac{\beta_1 \beta_2}{\beta_3} - \frac{A}{3}\beta_3 \,, \tag{5.14}$$

in terms of which the solution (5.6) matches explicitly the known one when $A = 0$. This redefinition of the parameters is a way to recover the choice for the functions that is usually made when solving the BPS equations (2.17), whose analogue for the present case is

$$H_3' = \left(\lambda^1 \lambda^2 - A(\lambda^3)^2\right) H^0 \,, \quad \text{or} \quad H_3' = e^{-\psi}(\alpha_3' r + \beta_3') \,. \tag{5.15}$$

6 Physical discussion

In this section, we discuss some properties of our solution, like near-horizon limit, entropy or area-product formula.

In the asymptotic limit $r \to \infty$, the metric (5.9) becomes AdS$_4$, i.e., at leading order one has

$$ds^2 \to -\frac{r^2}{\ell^2}dt^2 + \ell^2 \frac{dr^2}{r^2} + r^2 d\Omega_\kappa^2 \,, \tag{6.1}$$

where we defined the asymptotic AdS$_4$ curvature radius ℓ by

$$\ell^2 = \frac{\lambda_\infty^3}{2g_0 g^3} \,, \tag{6.2}$$

and rescaled the coordinates according to $t \to \ell t$, $r \to r/\ell$. Notice that the asymptotic value of the cosmological constant is

$$\Lambda = -\frac{3}{\ell^2} = -\frac{6g_0 g^3}{\lambda_\infty^3} \,. \tag{6.3}$$

On the other hand, when r approaches the horizon $r_{\rm h}$, the functions U and ψ assume, after shifting $r \to r + r_{\rm h}$, the form (4.9), with R_1 and R_2 given by

$$R_1^2 = -\frac{\lambda_\infty^3 f(r_{\rm h})}{8g_0 g^3 c} \,, \qquad R_2^2 = \frac{\lambda_\infty^3 f(r_{\rm h})}{2g_0 g^3} \,, \tag{6.4}$$

where

$$f(r_{\rm h}) \equiv \left(r_{\rm h} - g_0\beta^0 - \frac{g_0}{(\lambda_\infty^3)^2}\beta_3\right) \sqrt{\left(r_{\rm h} + 2g_0\beta^0\right)\left(r_{\rm h} + \frac{2g_0}{(\lambda_\infty^3)^2}\beta_3\right)} \,.$$

In this limit, the spacetime becomes AdS$_2 \times \Sigma$, with metric

$$ds^2 = -\frac{r^2}{R_1^2}dt^2 + \frac{R_1^2}{r^2}dr^2 + R_2^2 d\Omega_\kappa^2 \,. \tag{6.5}$$

The Bekenstein-Hawking entropy is given by

$$S_{\rm BH} = \frac{A_{\rm h}}{4} = \frac{R_2^2 \,{\rm vol}(\Sigma)}{4} \,. \tag{6.6}$$

This expression can be written in terms of the charges p^0, q_i and the gaugings g_0, g^i only. To this aim, the eqs. (5.6) need to be inverted, in order to use the charges p^0, q_1, q_2 as parameters. This result sustains the presence of the *attractor mechanism* also in the case under consideration, which is a nontrivial statement, due to the non-homogeneity of the model we have been discussing.

Finally, the product of the areas of all the horizons $r = r_I$, $I = 1, \ldots, 4$ (i.e., all the roots of e^{2U}) assumes the remarkably simple form

$$\prod_{I=1}^{4} A(r_I) = -\frac{36}{\Lambda^2} \frac{\text{vol}(\Sigma)^4 g^2}{g^3} p^0 q_1 \tilde{q}_2^2 \,, \tag{6.7}$$

where we defined

$$\tilde{q}_2 \equiv q_2 - \frac{A}{3} \left(\frac{g^3}{g^2} \right)^2 q_1 \,. \tag{6.8}$$

Note that (6.7) depends only on the charges and the gauge parameters. Similar formulas have been proven to be true in a number of examples (see for instance [18, 19, 24, 50, 63–65]), a fact that calls for an underlying microscopic interpretation.

7 Conclusions

In this paper, we considered a non-homogeneous deformation of the stu model of $N = 2$, $D = 4$ supergravity, and computed the symplectic embedding of the electric-magnetic duality algebra. We then focused on a particular FI gauging of this model, that leads to a scalar potential with two AdS critical points, a supersymmetric one, and another that breaks supersymmetry and that exists only when the deformation parameter lies within a specific range.

Exploiting the construction of this non-homogeneous deformation in string theory (mentioned at the beginning of section 3), it would be interesting to investigate the origin of the FI gauging in this context, also in relation to the $A = 0$ limit [66].

Furthermore, we wrote down the attractor equations for this model, and constructed an explicit BPS black hole solution that interpolates between this attractor geometry and the supersymmetric AdS vacuum at infinity. Various physical properties of this solution were also discussed. In this context, it would be interesting to compute the mass of the black hole, and to see how it depends on the deformation parameter A.

A natural question is whether there exist also black holes in this theory that asymptotically yield the non-BPS vacuum. Since the first-order flow equations (2.17) that we used here are related to the existence of a Killing spinor [12], they cannot be used to obtain such non-BPS solutions. A possible way out, that in principle even allows to construct nonextremal black holes, would be to use the framework of the Hamilton-Jacobi formalism, leading to first-order equations similar in spirit to those in (2.17).

It would also be interesting to investigate solutions of AdS$_4$ BPS (and non-BPS) extremal black holes in $N = 2$, $D = 4$ FI-gauged supergravity coupled to hypermultiplets whose quaternionic scalars span non-symmetric (or non-homogeneous) manifolds, along

the lines of [67–69]. In presence of vector multiplets, a particularly interesting (self-mirror) case consists in the nh-stu model coupled to four hypermultiplets, whose scalar manifold is the non-homogeneous c-map image [70] of the non-homogeneous special Kähler manifold of the nh-stu model itself.

Finally, non-Abelian gaugings of the vector multiplets' sector (giving rise to the so-called Einstein-Yang-Mills $N = 2$, $D = 4$ supergravity theories) are very little known, especially in relation to the existence and properties of regular black hole solutions, of the related attractor mechanism, and of supersymmetry-preserving features. It would be very interesting to study such issues, e.g. along the lines of [71–74].

We hope to come back to these points in future publications.

Acknowledgments

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